Update on Bodek-Yang Model

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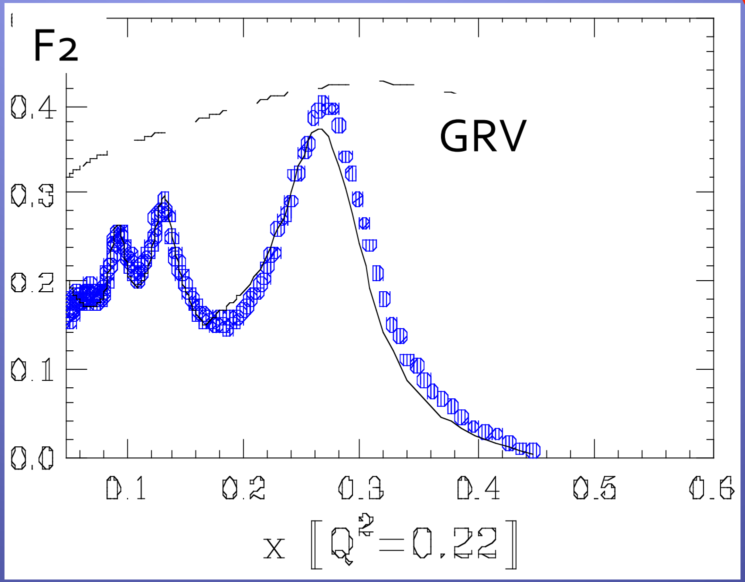
NuINT 2022 Workshop, October 24-29, 2022

Bodek-Yang Model

- Bodek-Yang model: describe DIS cross section in all Q^2 regions
- Challenges in e/μ-N DIS
  - High x PDFs at low Q^2
  - Resonance region overlapped with a DIS contribution
  - Hard to extrapolate DIS contribution to low Q^2 region from high Q^2 data due to non-perturbative QCD effects



- A model in terms of quark-parton model (easy to convert e/μ scattering to ν scattering)

✓ Understanding of high x PDFs at low Q^2? wealthy SLAC, JLAB data.

✓ Understanding of resonance scattering in terms of quark-parton model?

(duality works, many studies by JLAB)

Lessons from previous QCD studies

- NLO & NNLO analyses with DIS data: PRL 82, 2467 (1999), Eur. Phys. J. **C13**, 241 (2000) by Bodek and Yang
  - Kinematic higher twist (target mass) effects are large and must be included in the form of Georgi & Politzer x scaling.
  - Resonance region is also well described (duality works).
  - Most of dynamic higher twist corrections (in NLO analysis) are similar to missing NNLO higher order terms.

- NNLO pQCD+TM with NNLO PDFs can describe the non-perturbative QCD effects at low Q^2
- Thus, we reverse the approach to build the model:
  - Use LO PDFs and "effective target mass and final state masses" to account for initial target mass, final target mass, and even missing higher orders

$$\xi = \frac{2xQ'^2}{Q^2(1+\sqrt{1+4M^2x^2/Q^2})},$$

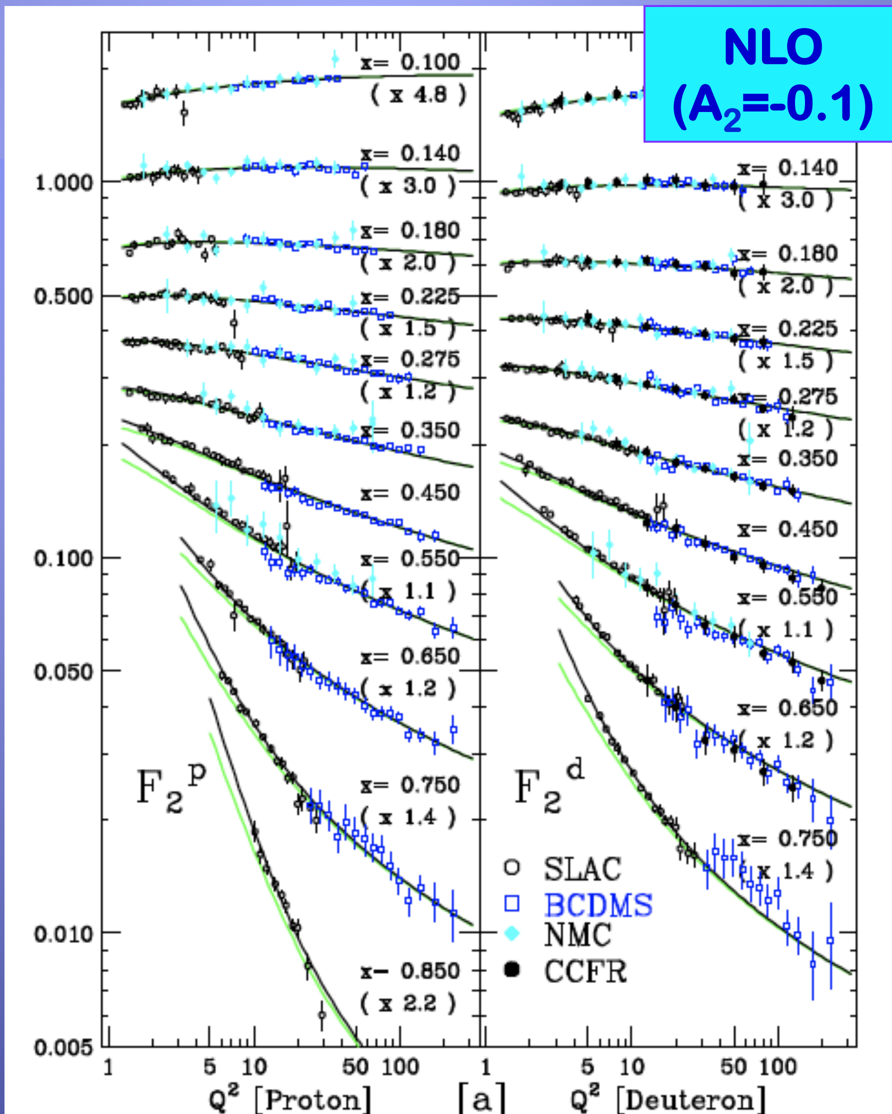
$$2Q'^2 = [Q^2 + M_f{}^2 - M_i{}^2] + \sqrt{(Q^2 + M_f{}^2 - M_i{}^2)^2 + 4Q^2(M_i{}^2 + P_T^2)}.$$

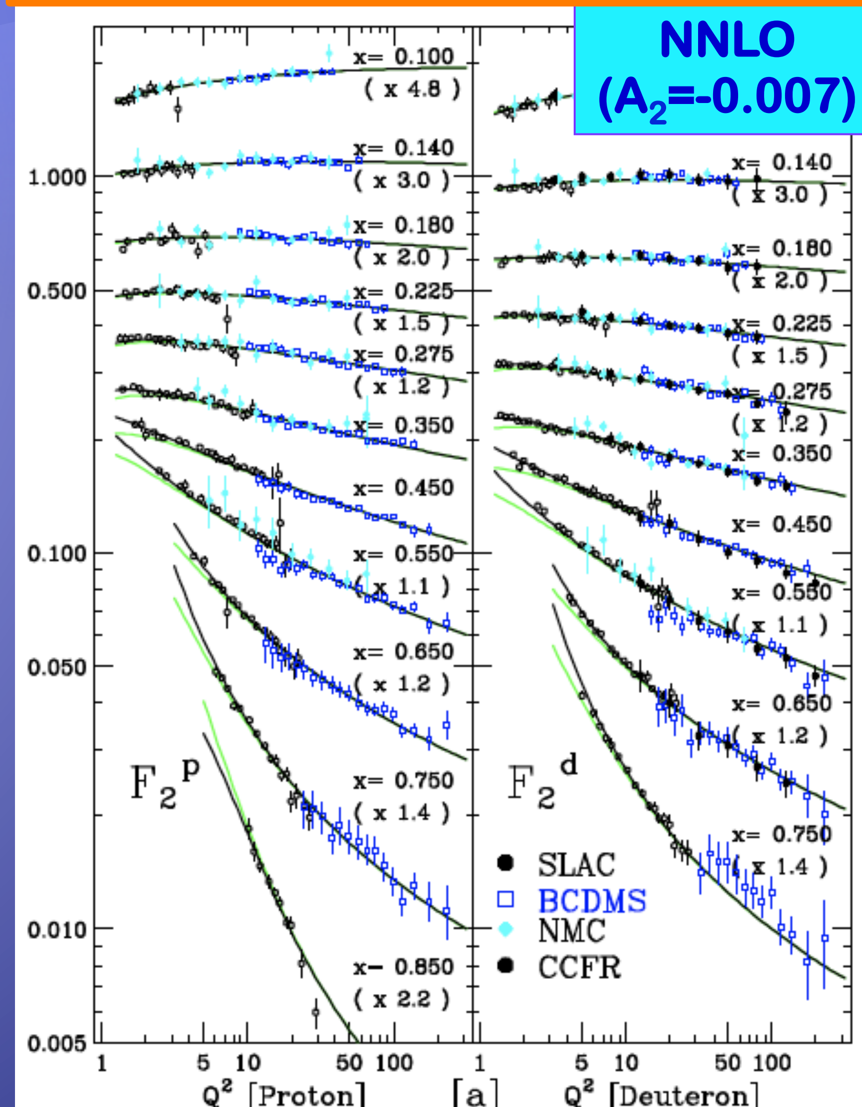
NLO vs NNLO

Studies of higher twist and higher order effects in NLO and NNLO QCD analysis of lepton-nucleon scattering data on F_2 and $R = \sigma_L/\sigma_T$

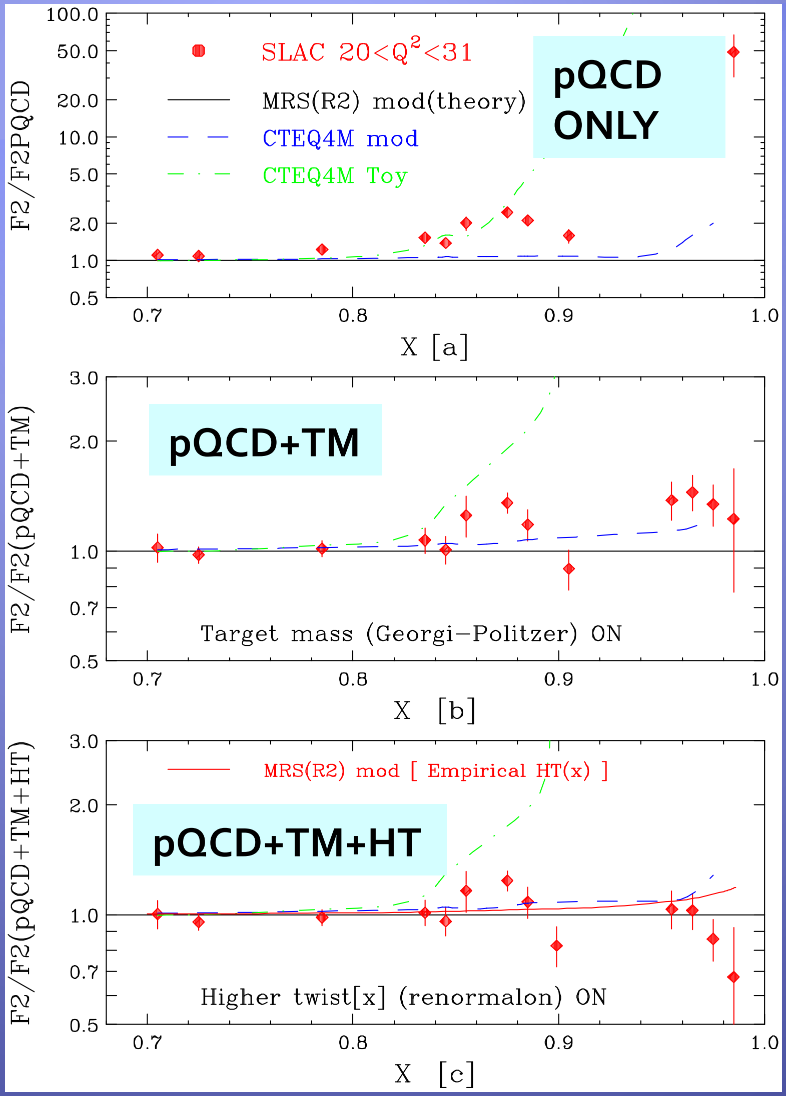
U.K. Yang & A. Bodek

The European Physical Journal C - Particles and Fields **13**, 241–245 (2000) | Cite this article





Very high x and low Q² data

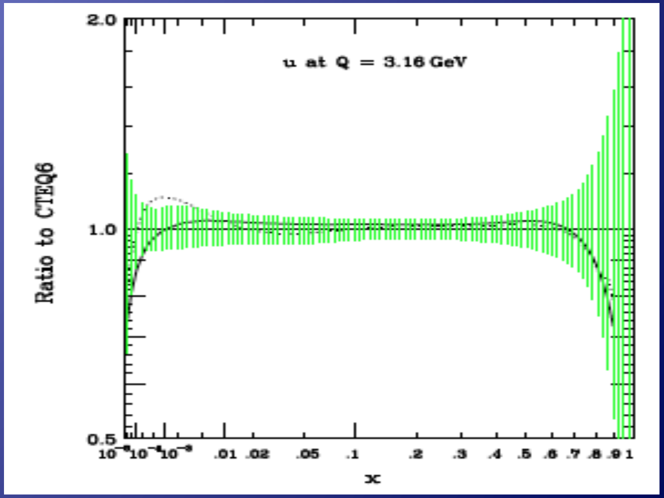


Parton Distributions, d/u, and Higher Twist Effects at High x

U. K. Yang and A. Bodek
Phys. Rev. Lett. **82**, 2467 – Published 22 March 1999

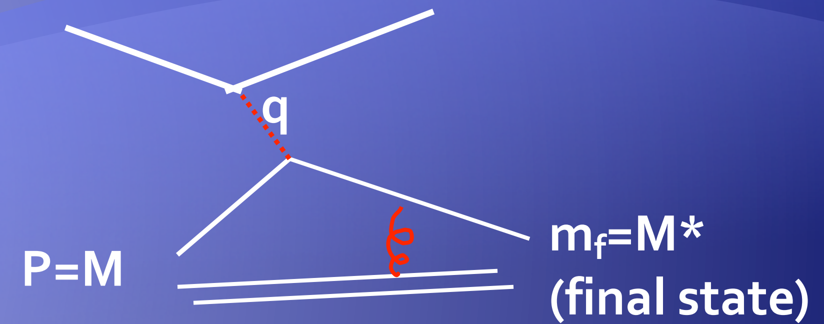
- Very high x and low Q^2 data is well described by the pQCD+TM+HT
- Extraction of the high x PDF is promising (1999)

  - still a large uncertainty (2022)



Modeling neutrino cross sections

- NNLO pQCD +TM approach: describes the DIS region and resonance data very well



- Bodek-Yang LO approach: (pseudo NNLO)
  - Use effective LO PDFs with a new scaling variable, ξ_w to absorb target mass, higher twist, missing QCD higher orders

$$x_{Bj} = \frac{Q^2}{2M\nu} \Rightarrow \xi_W = \frac{Q^2 + B}{\{M\nu[1+\sqrt{(1+Q^2/\nu^2)}] + A\}}$$

  - Multiply all PDFs by K factors for photo production limit and higher twist

$$F_2(x,Q^2) \rightarrow \frac{Q^2}{Q^2+C} F_2(\xi_w, Q^2)$$

Bodek-Yang Effective LO PDFs Model

1. Start with GRV98 LO (Q^2_{min}=0.80)
2. Replace x_{bj} with a new scaling, ξ_w
3. Multiply all PDFs by K factors for photo prod. limit and higher twist

   [$\sigma(\gamma) = 4\pi\alpha/Q^2 * F_2(x, Q^2)$]

   Ksea = $Q^2/[Q^2+Csea]$

   Kval = $[1- G_D{}^2 (Q^2)] * [Q^2+C_{2V}] / [Q^2+C_{1V}]$

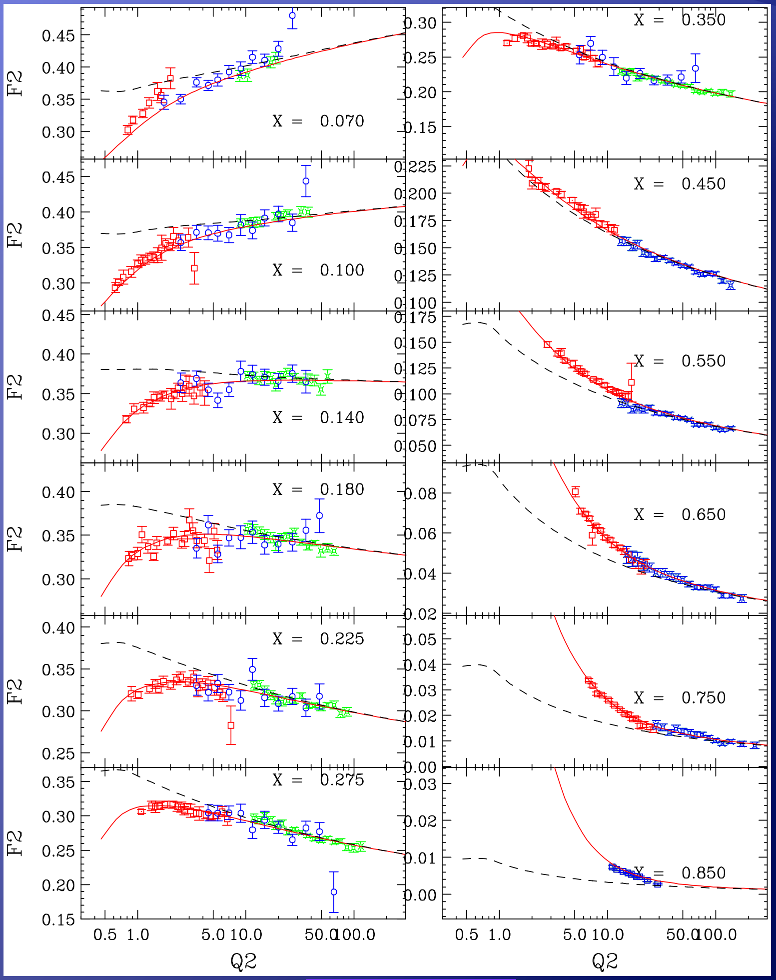
   motivated by Adler Sum rule

   where $G_D{}^2 (Q^2) = 1/ [1+Q^2 / 0.71]^4$
4. Freeze the evolution at $Q^2 = Q^2_{min}$

   - $F_2(x, Q^2 < 0.8) = K(Q^2) * F_2(\xi w, Q^2=0.8)$
5. Fit all DIS F_2(p/D) data: with *W>2 GeV*

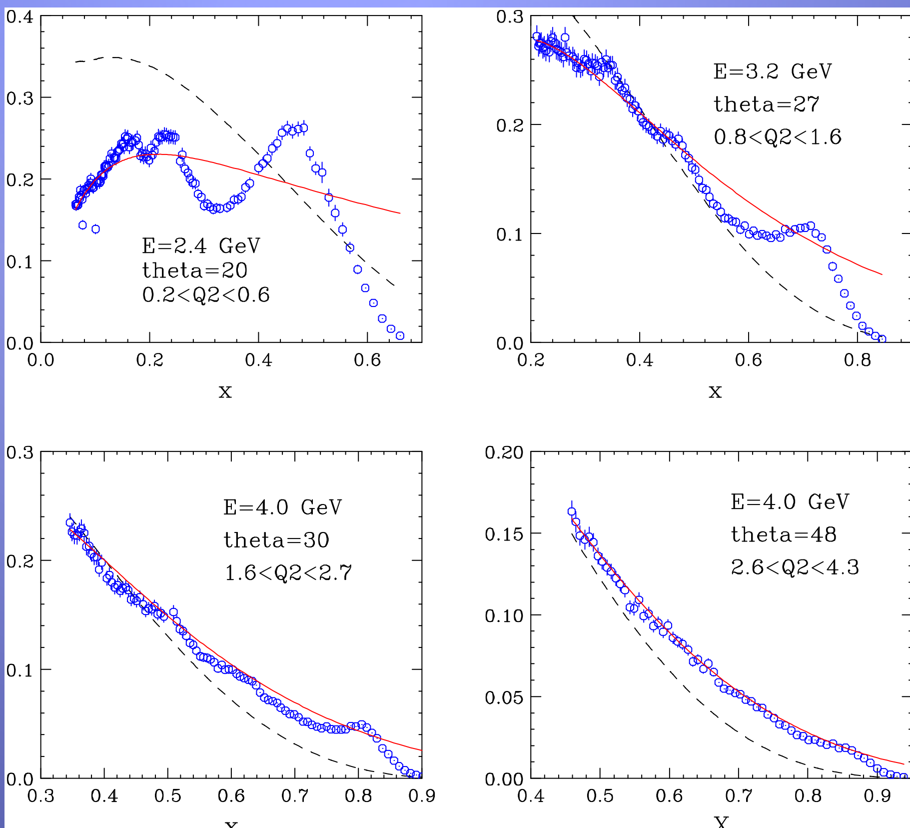
   SLAC/BCDMS/NMC/HERA data

$$\chi^2/DOF = 1235/1200$$



F_2(p)

Predictions for Resonance, Photo-production data



F$_2$(d) resonance

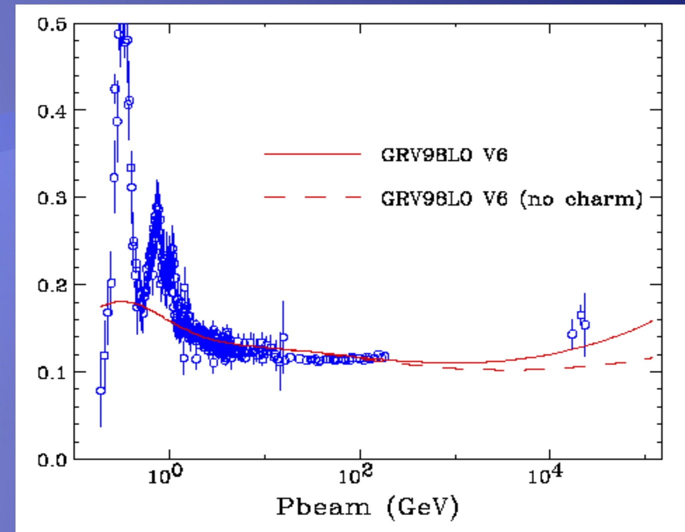


Photo-production (P)

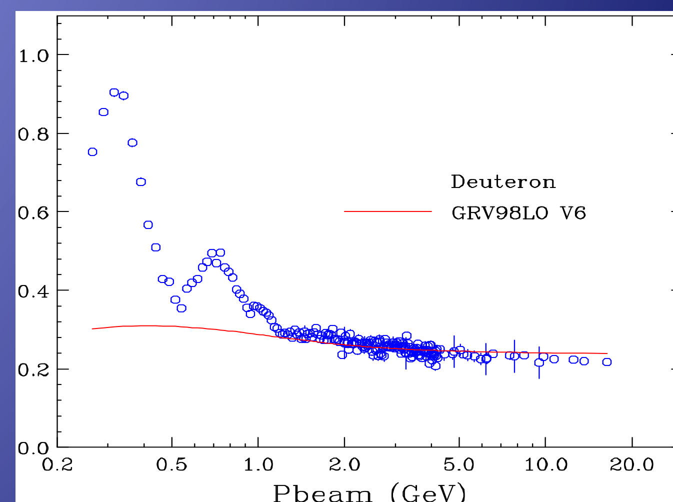


Photo-production (d)

Bodek-Yang Effective LO PDFs Model

- Include the photo-production data
- Use different K factors for up and down quark type separately

$$\text{Kval (u,d)} = [1- G_D^{\,2} (Q^2)] * [Q^2+C_{2V}] / [Q^2+C_{1V}]$$

$$\text{Ksea (u,d,s)} = Q^2/[Q^2+\text{Csea}]$$

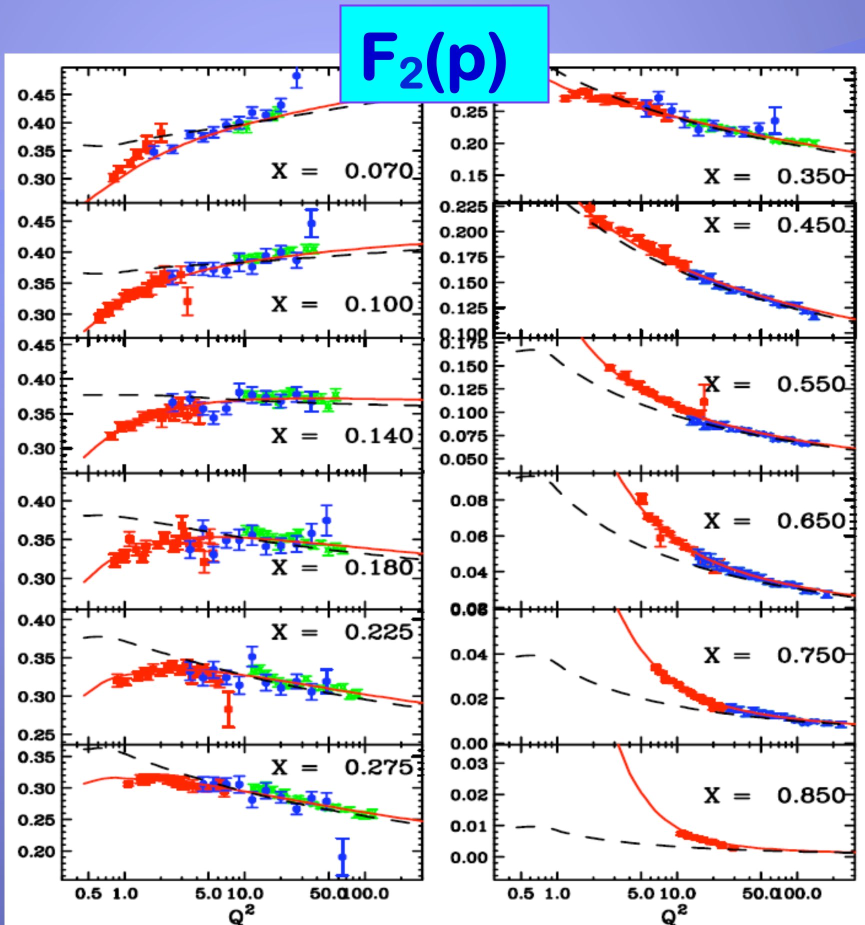
- Additional K^{LW} factor for valence quarks:

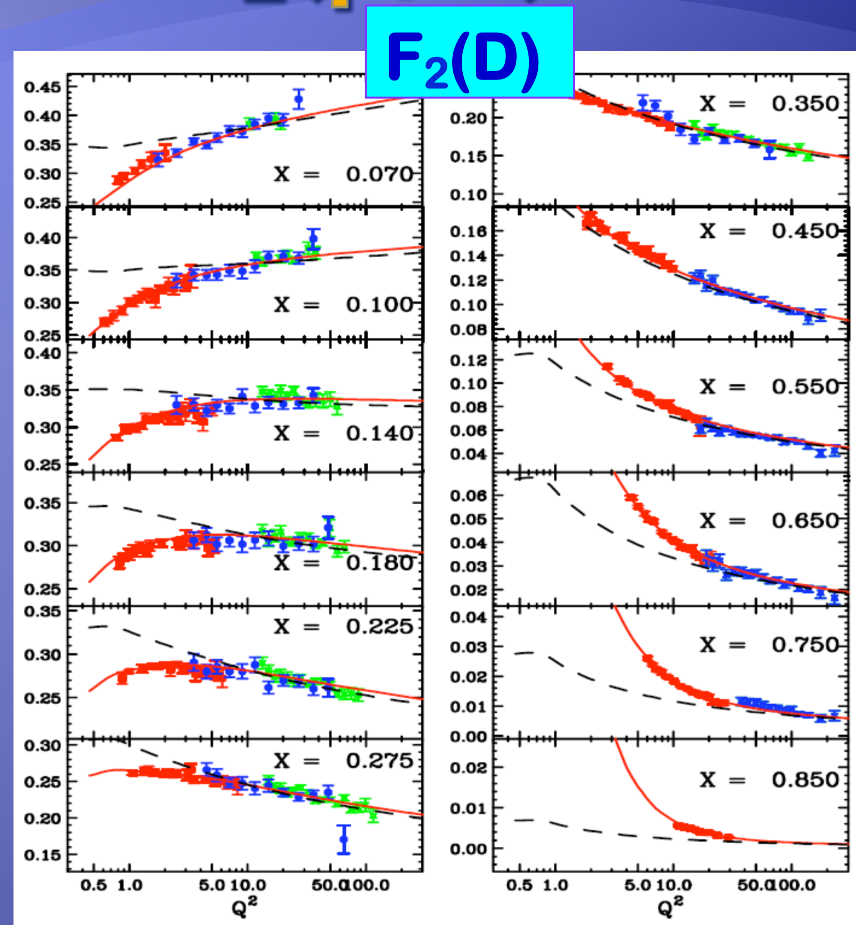
$$\text{Kval} = K^{LW}*[1- G_D^{\,2} (Q^2)] * [Q^2+C_{2V}] / [Q^2+C_{1V}]$$

where $K^{LW} = (\nu^2+C^{\nu})/\nu^2$

A 0.621	B 0.380	C_{v2d} 0.323	C_{v2u} 0.264
C_{sea}^{down} 0.561	C_{sea}^{up} 0.369	C_{v1d} 0.341	C_{v1u} 0.417
$C_{sea}^{strange}$ 0.561	$C^{low-\nu}$ 0.218	$\mathcal{F}_{valence}$ $[1 - G_D^2(Q^2)]$	N 1.026

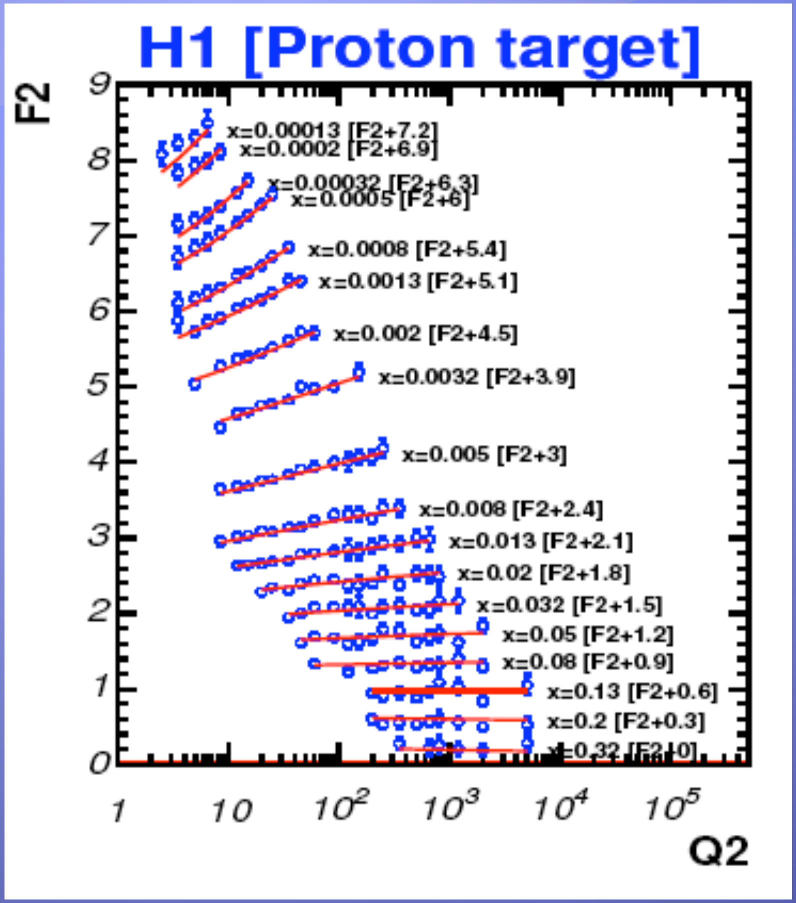
Fit Results on DIS F_2(p/D) data

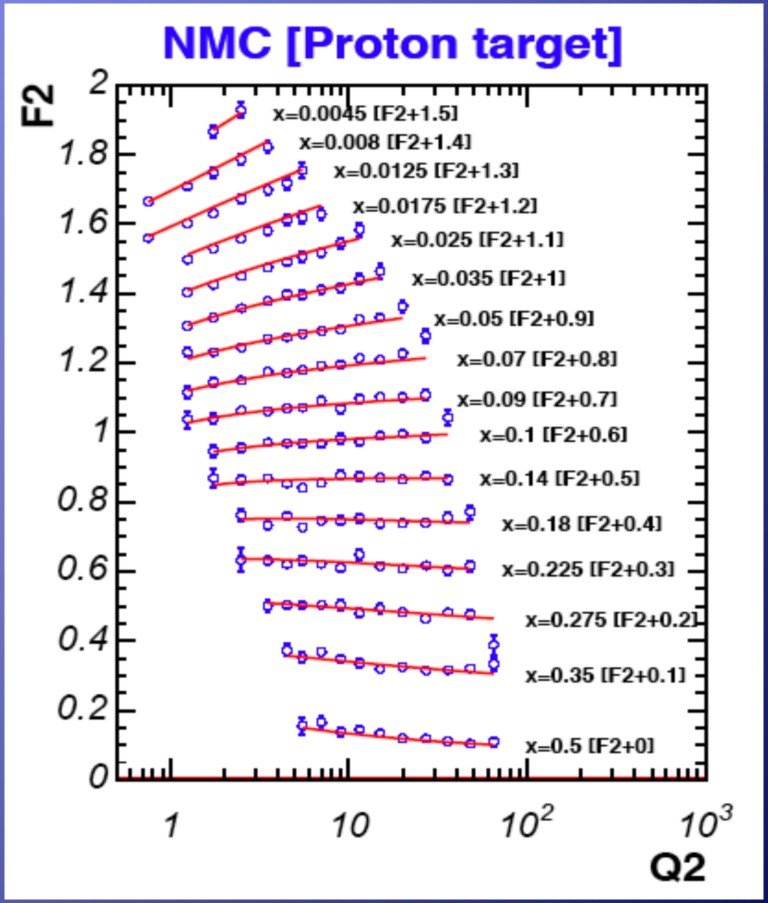




- Excellent Fitting:
  - red solid line: effective LO using ξw
  - black dashed line: x_{bj}

Low x HERA and NMC data





- Fit works at low x

Photo-production data

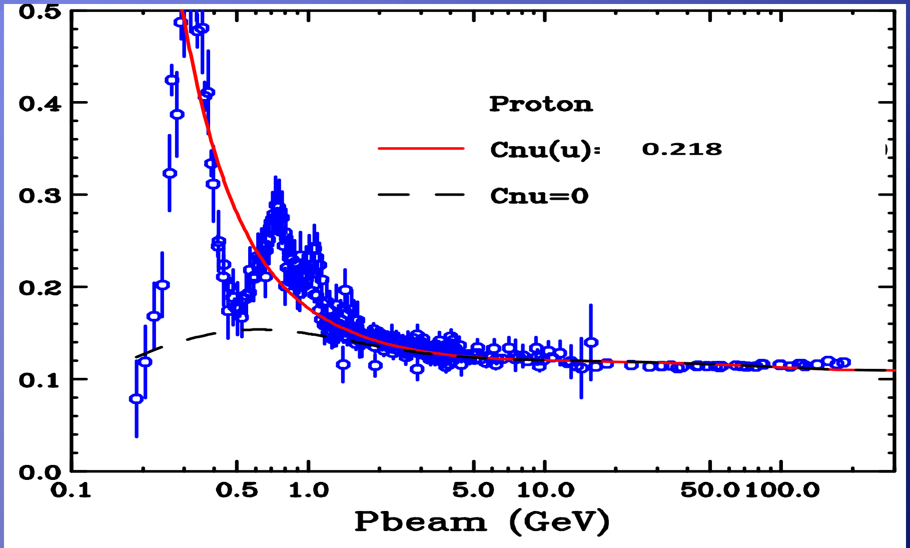
- Additional K^{LW} factor for valence quarks:

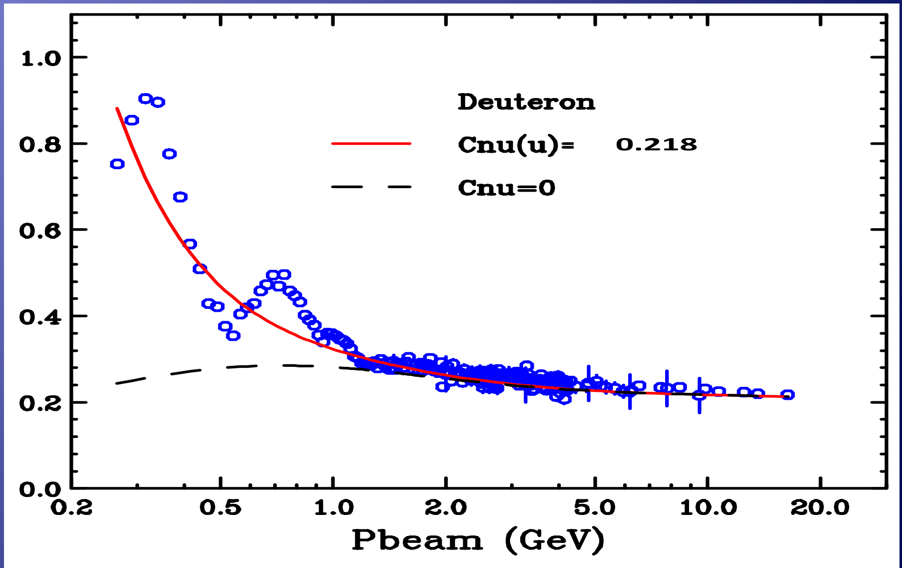
$$Kval = K^{LW} * [1 - G_D^{\,2}(Q^2)] * [Q^2 + C_{2V}] / [Q^2 + C_{1V}]$$

$$K^{LW} = (\nu^2 + C^{\nu}) / \nu^2$$

This makes a duality work all the way down to Q2=0 (for charged leptons)

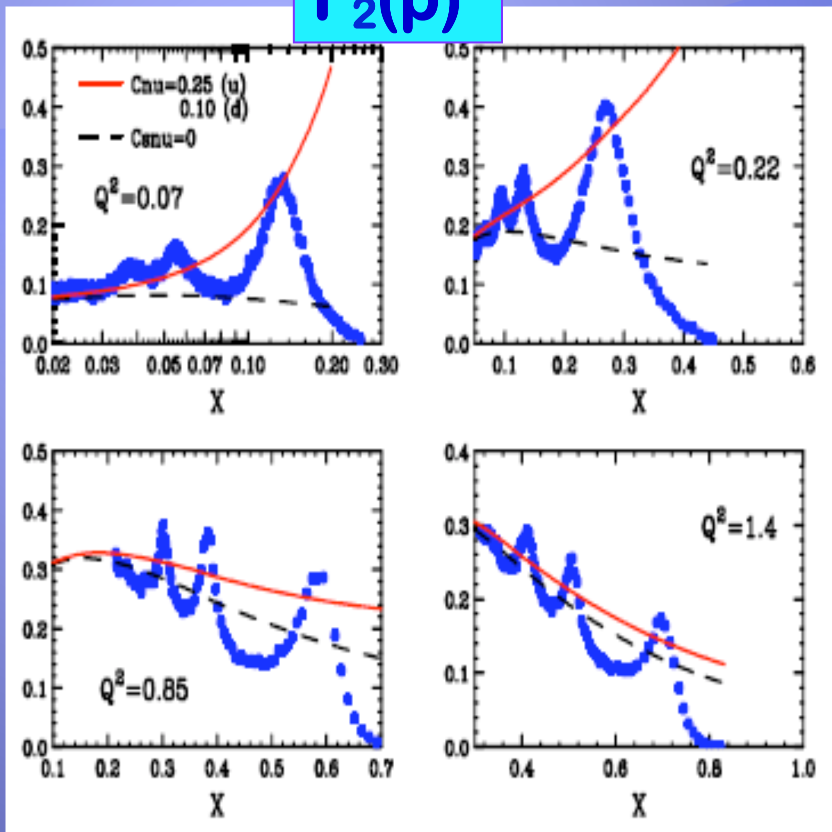
- Photo-production data with ν(Pbeam)>1 GeV included in the fitting



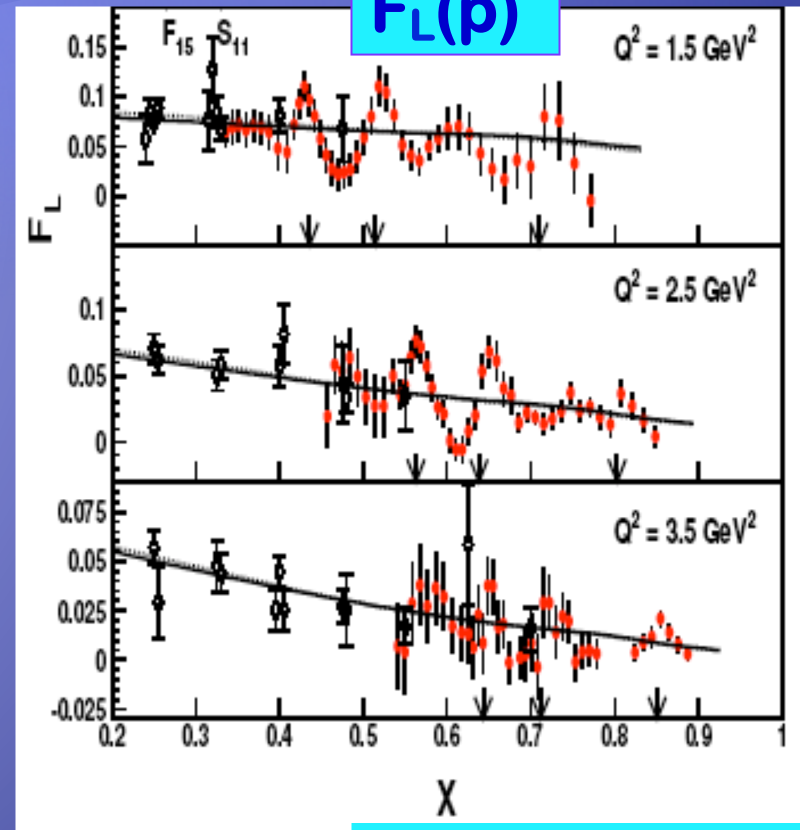


F2 & F_L Resonance data

$F_2(p)$



$F_L(p)$



$$F_L = F_2(1 + 4M^2x^2 / Q^2)\frac{R}{(1+R)}$$

- Predictions are in good agreement (not included in the fit) duality works
- F_L was calculated using F2 and R_{1998}

Neutrino cross sections

- Effective LO model with ξw describe all DIS and resonance F_2 data as well as photo-production data (Q^2=0 limit): vector contribution works well
- Neutrino Scattering:
  - Effective LO model works for xF_3?
  - Nuclear correction using e/μ scattering data
  - Axial vector contribution at low Q^2?
  - Use $R=R_{1998}$ to get $2xF_1$
  - Implement charm mass effect through ξw slow rescaling algorithm for F_2, $2xF_1$, and xF_3

Submitted to EPJC (2022), arXiv:2108.09240

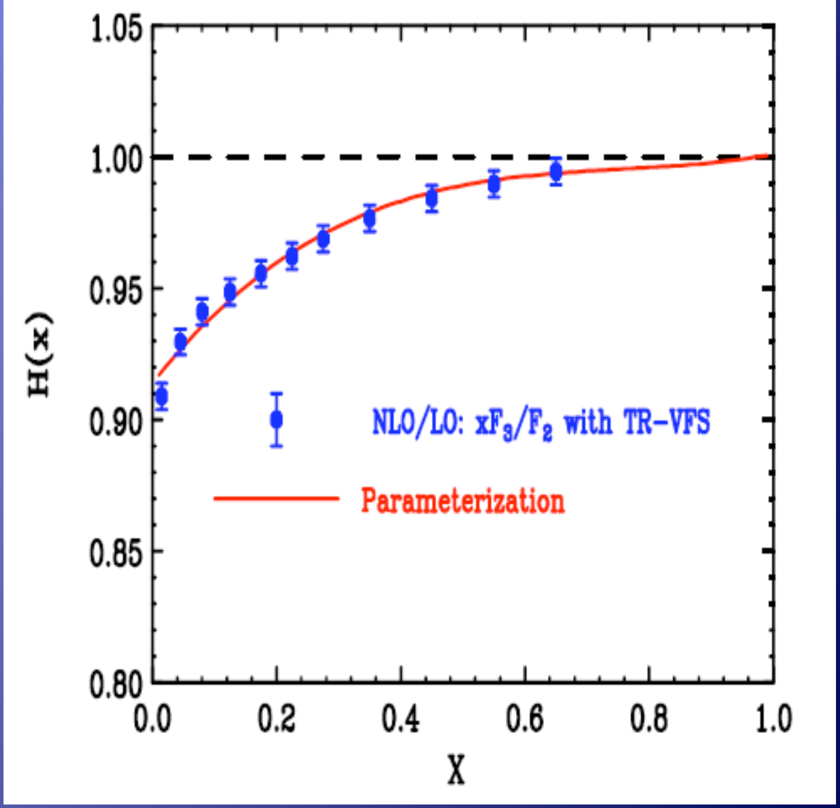
Effective LO model for xF_3?

- Scaling variable, ξw absorbs higher order effect for F_2, but the higher order effects for F_2 and xF_3 are not the same
- Use NLO QCD to get double ratio

$$H(x) = \frac{xF_3(\text{NLO})}{xF_3(\text{LO})} / \frac{F_2(\text{NLO})}{F_2(\text{LO})}$$

not 1 but almost indep. of Q^2

- Enhance anti-neutrino cross section by 3%



Axial Vector Structure Functions

- At high Q^2 , vector and axial vector contribution are same, but not at low Q2
- K factors for axial contributions: type II

$$K_{sea}^{vector} = \frac{Q^2}{Q^2 + C} \Rightarrow K_{sea}^{axial} = \frac{Q^2 + 0.55C_{sea}^{axial}}{Q^2 + C_{sea}^{axial}}$$

$$K_{val}^{axial} = \frac{Q^2 + 0.1C_{val}^{axial}}{Q^2 + C_{val}^{axial}}$$

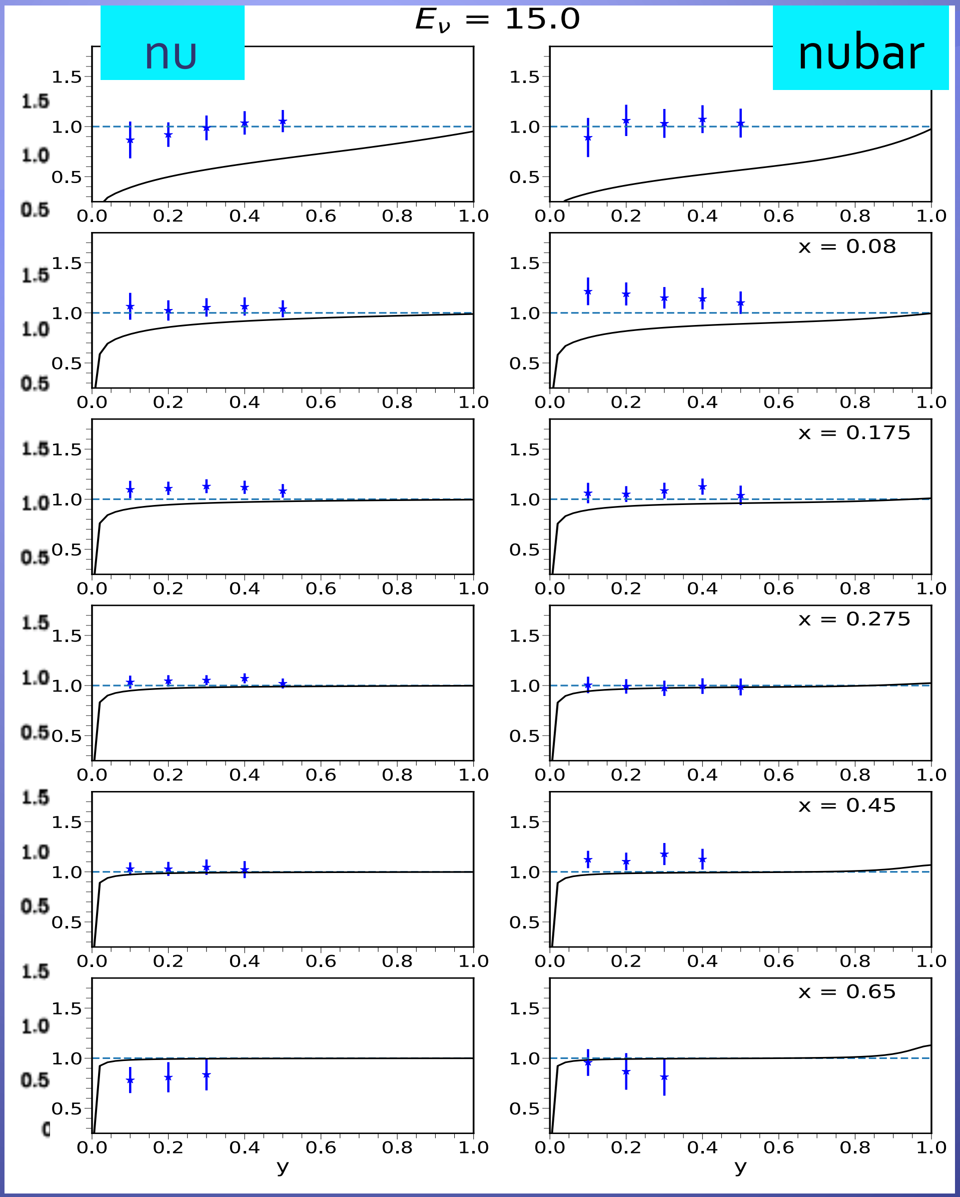
$$\text{where } C_{sea}^{axial} = 0.75, \ \ C_{val}^{axial} = 0.18$$

  - 0.55 was chosen to satisfy the prediction from PCAC by Kulagin, agrees with CCFR/CHROUS data for F_2 extrapolation to (Q^2=0)

- But, the non-zero PCAC component of F_2^{axial} at low Q^2: mostly longitudinal

$$2xF_1^{axial} = 2xF_1^{vector}$$

Comparison with CCFR (Fe), CHORUS (Pb) data



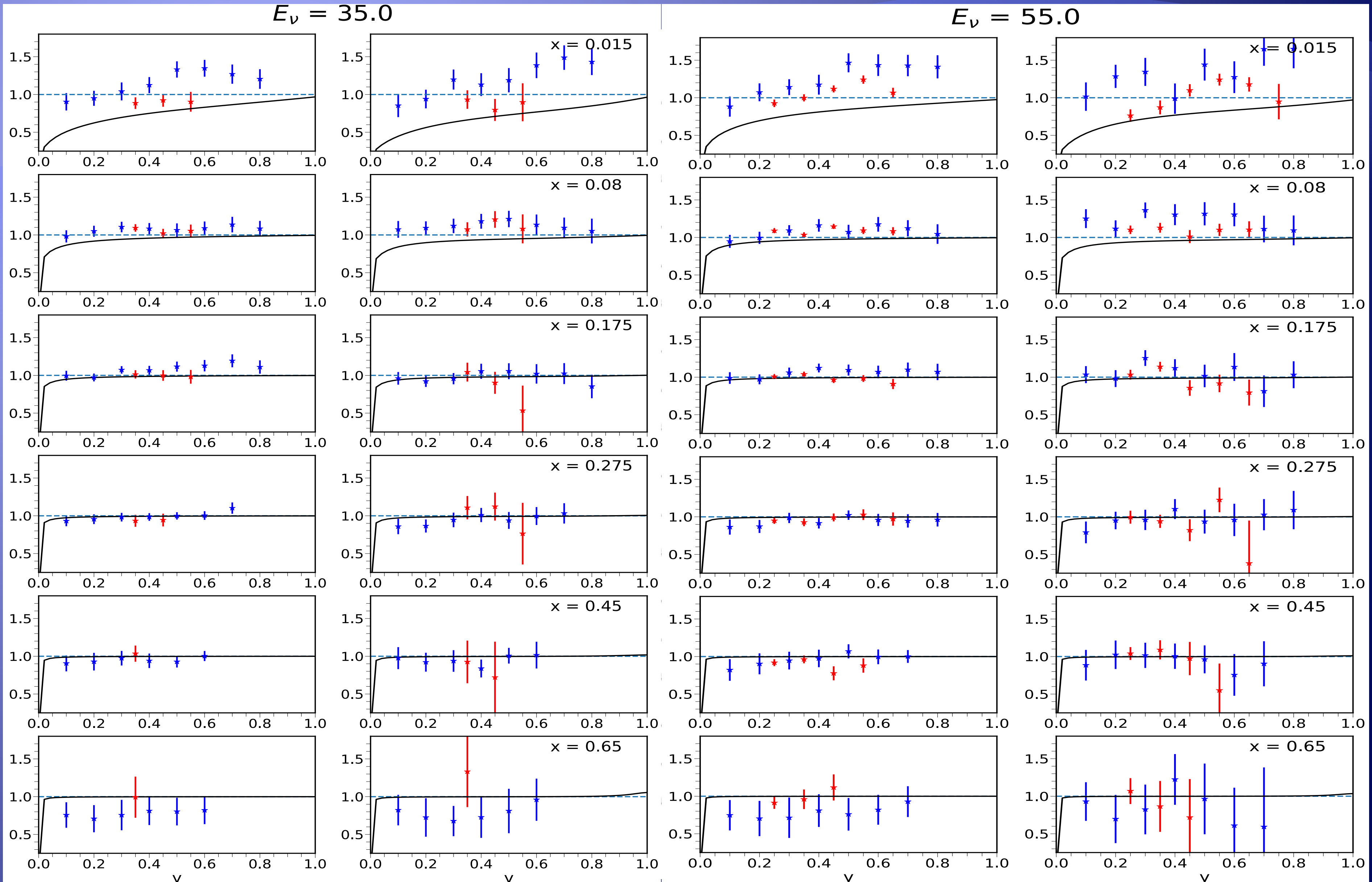
- Blue point: CHORUS/theory (type II)
- Solid line:theory (type I)/(type II)

- Type I (Vector = Axial at low Q^2)
- Type II (Vector < Axial at low Q^2)

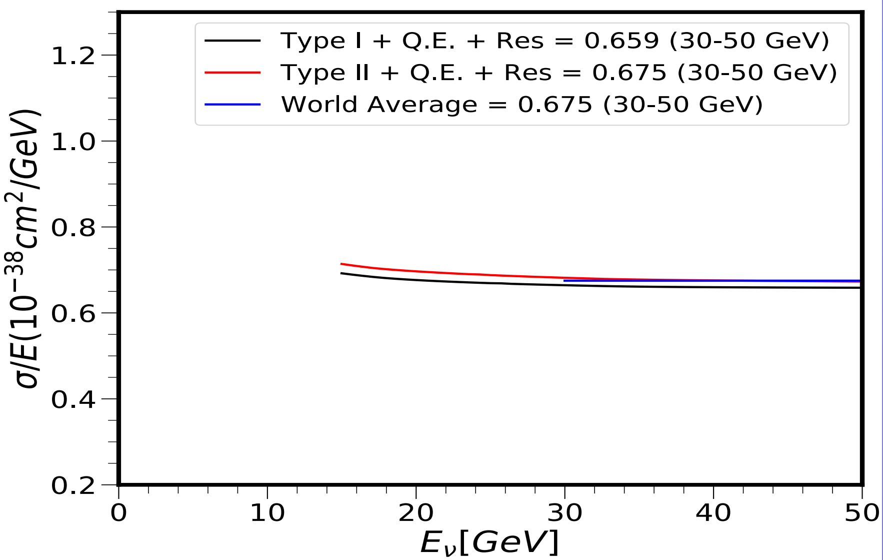
Red point: CCFR/type II

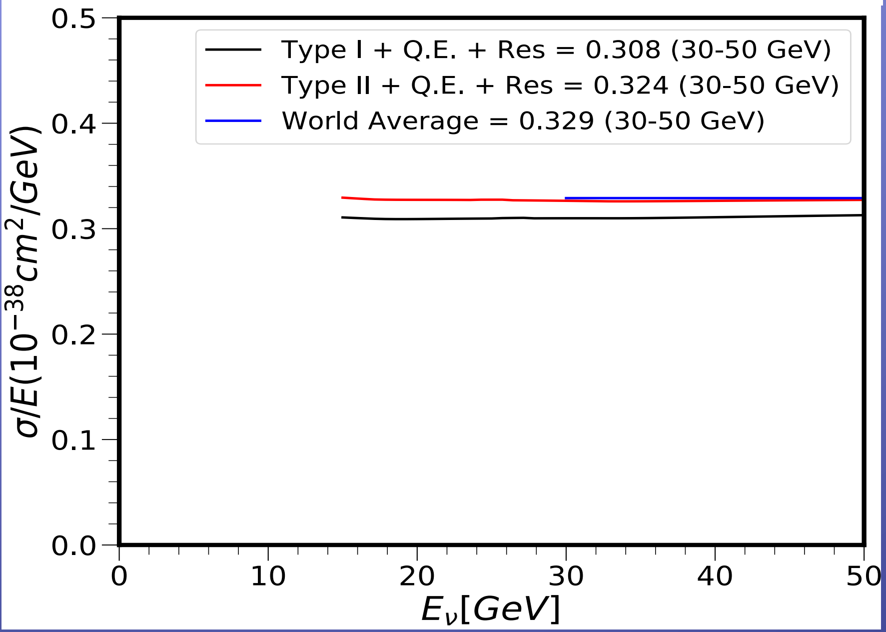
Blue point: CHORUS/type II

Comparison with CCFR(Fe) , CHORUS (Pb) data

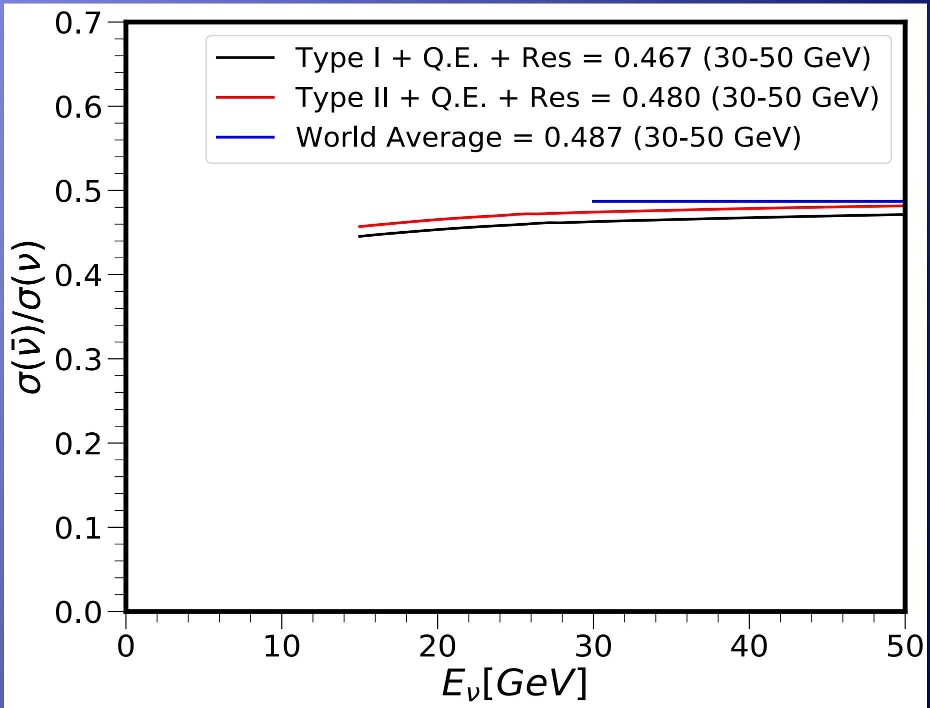


Total cross sections





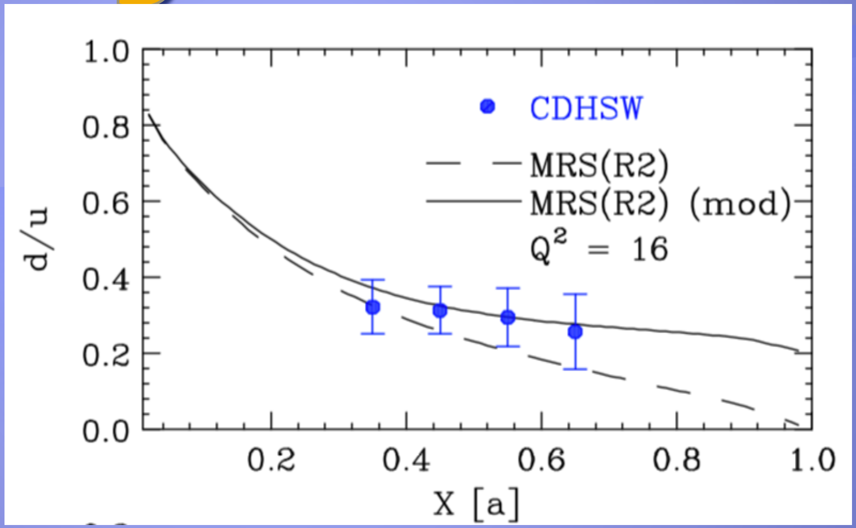
- BY(DIS, W>1.4) + Q.E. + Resonance

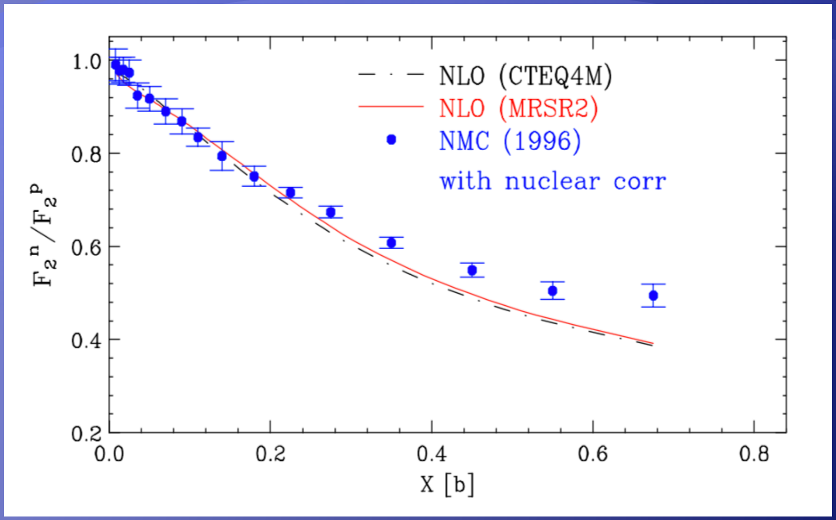


Summary & Discussions

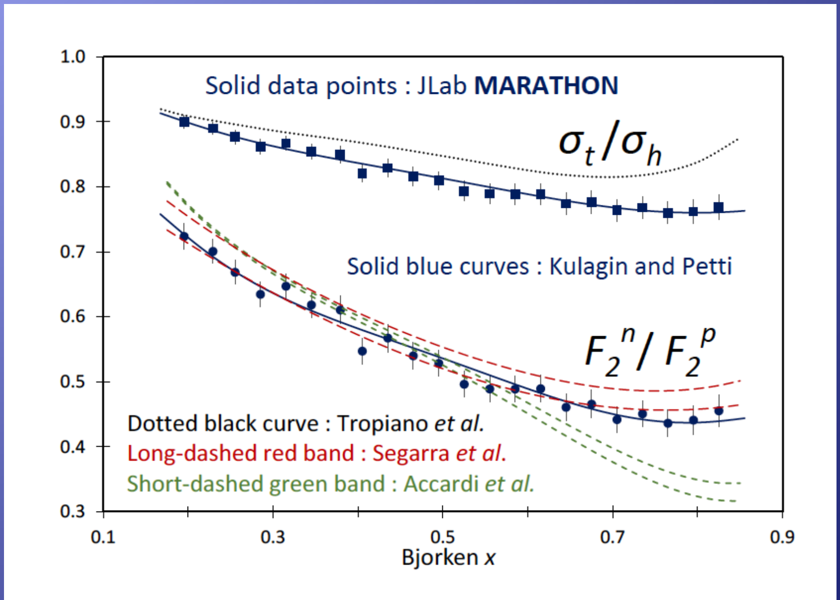
- BY Effective LO model with ξw describe all e/μ DIS and resonance data as well as photo-production data (down to Q^2=0): provide a good reference for vector SF for neutrino cross section
- $d\sigma$/dxdy data favor updated BY(DIS) type II model
- BY(DIS) type II model (low Q^2: axial>vector) provide a good reference for neutrino cross sections. Low energy neutrino experiments can normalize their data to our model to extract their flux
- Model also works well down to W=1.4 GeV, thus providing overlap with resonance models
- Future improvement: use very high-x data (nCTEQ effort)

High-x PDF





PRL 82, 2467 (1999)



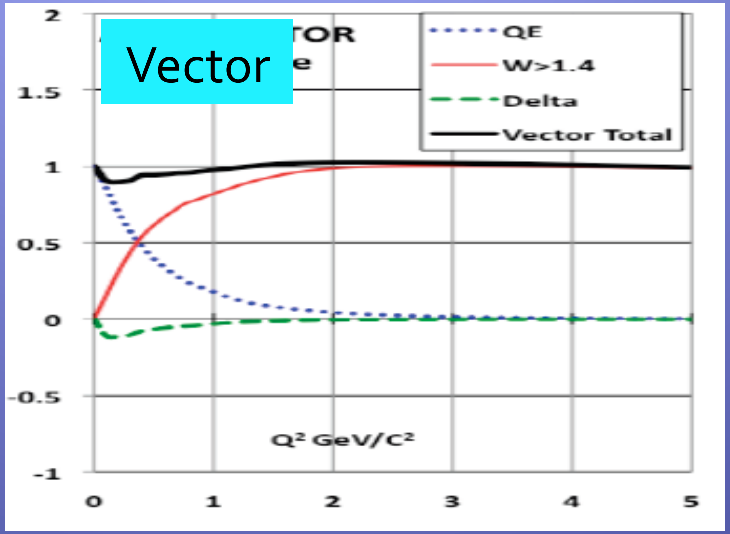
PRL 128, 132003 (2021)

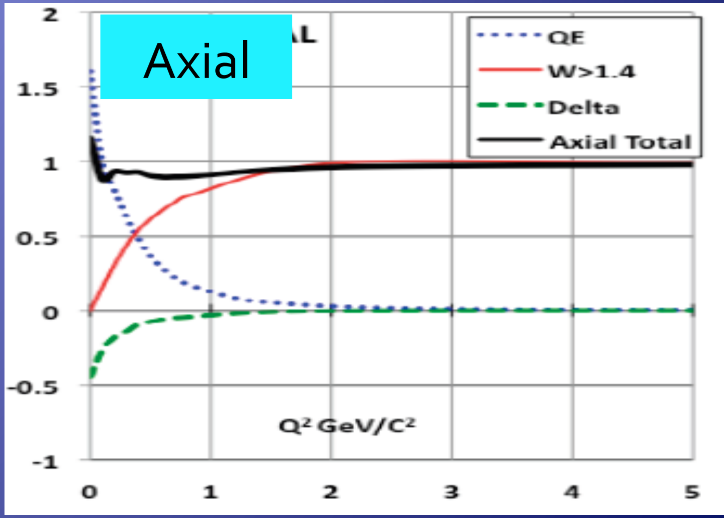
Test of the Adler Sum Rule

- This sum rule should be valid at all values of Q^2

$$|F_V(Q^2)|^2 + \int_{\nu_0}^{\infty} W_{2n-sc}^{\nu-vector}(\nu,Q^2)\,d\nu - \int_{\nu_0}^{\infty} W_{2p-sc}^{\nu-vector}(\nu,Q^2)\,d\nu = 1$$

$$|\mathcal{F}_A(Q^2)|^2 + \int_{\nu_0}^{\infty} W_{2n-sc}^{\nu-axial}(\nu,Q^2)\,d\nu - \int_{\nu_0}^{\infty} W_{2p-sc}^{\nu-axial}(\nu,Q^2)\,d\nu = 1$$





Nuclear Effects: use e/μ data