Discussion of Issues Related to MicroBooNE Cross-Section vs E_ν

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26 October 2022

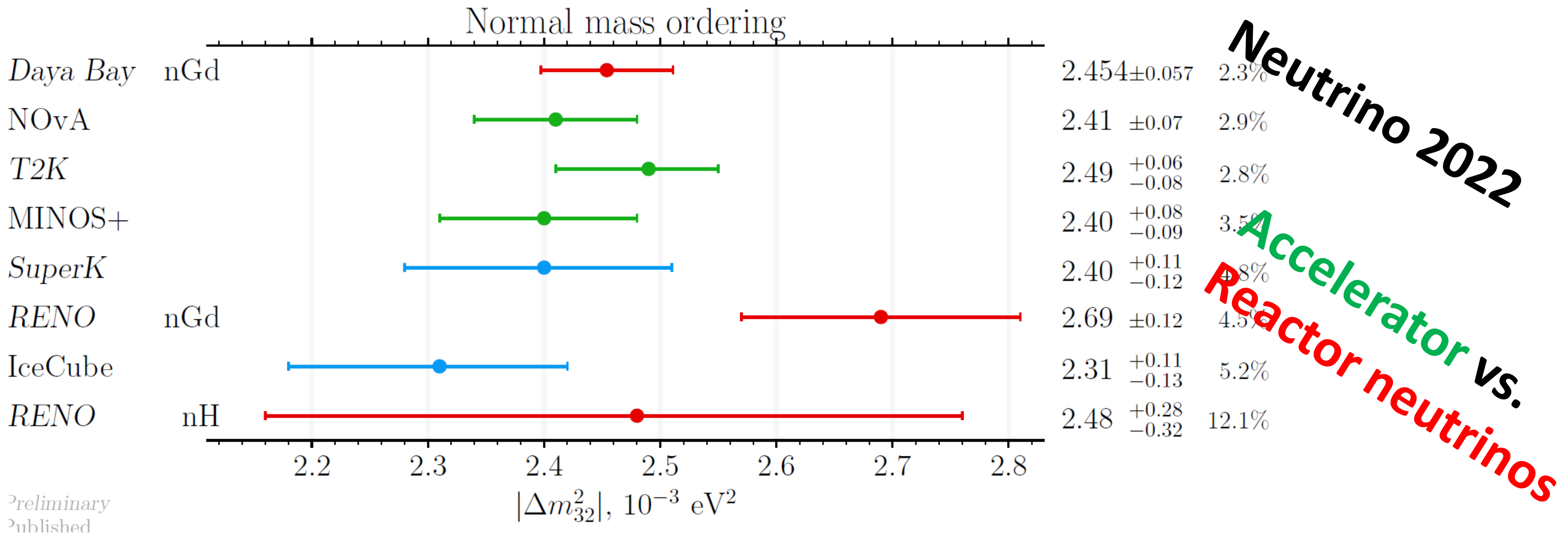
Purpose of this Discussion

- We’ve had some back and forth at the meeting already, but mostly in rushed Q&A sessions. Some session chairs had to cut off discussion.
- The idea is to give voice to a few of the main points so that others at the meeting can absorb them and ask questions.
- As chair for the discussion, I’m going to discourage a lot of back and forth among the presenters and hope for more questions and comments from others who have not expressed an opinion so far.

Xin

Point I

- There is **no principal difference** between extracting neutrino energy-dependent Xs and the oscillation parameters
  - If any, the requirement of extracting oscillation parameters is more stringent than that of extracting Xs
    - Only Xs uncertainties are suppressed in extracting Xs, while all uncertainties need to be suppressed in an oscillation analysis



Point II

- The claim of “model dependence” is **analysis dependent**

$$\frac{d^2\sigma}{dT_l cos\theta} = \frac{\sum_j U_{ij}(d_j - b_j)}{\Phi \cdot T \cdot \varepsilon_i \cdot (\Delta T_l, \Delta cos\theta)_i}$$

$$N_i = \sum_j R_{ij} \cdot S_j + B_i$$

  - The model dependence (flux, spectrum, Xs, missing hadronic energy …) enters the results through background **B** and efficiency **ε**
  - In some realistic situations, **B** can be high and **ε** can be low, which leads to sizable model dependence → **validations are needed to demonstrate that the model can describe data within the model uncertainties**

Point III

- In the recent MicroBooNE energy-dependent Xs analysis, we performed dedicated validations on the model of missing hadronic energy: M($\mathbf{E}_{\text{had}}^{\text{rec}}$) vs. $\mu(\mathbf{E}_{\text{had}}^{\text{rec}} \mid \mathbf{E}_{\nu}, \mathbf{E}_{\mu}^{\text{rec}})$
- There are principal understandings of the validation procedure
  - Energy conservation $\mathbf{E}_{\nu} = \mathbf{E}_{\mu} + \mathbf{E}_{\text{had,vis}} + \mathbf{E}_{\text{had,missing}}$
  - Reweighting events within the model uncertainties given different reaction mechanism
- These principals are confirmed through a set of fake data studies
  - We confirmed that the Model Validation is **much more stringent** than the Xs extraction (when comparing with truth)
- **MicroBooNE model passed validation with real data**

Callum

Fake Data Studies

- Take an alternative model → full detector simulation → treat as data → use analysis machinery with nominal model assumptions → extract XSEC → compare with known value
- An essential part of a robust analysis, but no solid prescription (that I know of) for design, or interpretation of results
- **Difficult (physics) question:** do the FDS span a plausible range of model variations?
- **Easy(?) question:** how much bias is too much?

Types of Fake Data Studies

- “Asimov style” – no statistical throw
  - Simple to interpret, probably acceptable limitations
  - I’ll focus on these
- “Other” – with expected data equivalent statistical throws
  - Can only be interpreted with an ensemble
  - Compare nominal MC distribution and shifted data distributions to look for a bias

How much bias is too much?

- **χ^2 (syst + stat) → 0 for a completely unbiased FDS**
  - The distribution is **not** χ^2 distributed
  - A naive “p-value” makes an incorrect assumption
- Possibly uncontroversial(?): χ^2/DOF << 1 when χ^2 calculated with all systs + stat uncertainties
- Useful(?) heuristic: the bias should be small relative to the size of differences analysis is trying to probe
- Also of concern: when individual bins have large biases → data consumers use data in weird and wonderful ways

How much bias is too much?

- But hang on, is χ^2 (XS systs only) $\rightarrow 0$???
- Probably not, but alternative XS models should be covered by XS systematics in the analysis
- Possibly useful criteria(?): χ^2 (XS systs only)/DOF < 1 for all models tested
- E.g., T2K oscillation analyses require that FDS show biases smaller than 50% of the systematic error for a bin

Stephen

Unfolding into true E_ν

- For future neutrino oscillation experiments to be successful it is clear we need a good modelling of both:
    1. The evolution of the cross-section as a function of neutrino energy
    2. The mapping from true to reconstructed neutrino energy

- We want cross-section measurements made today to allow us to tune/constrain state-of-the-art models in X years time

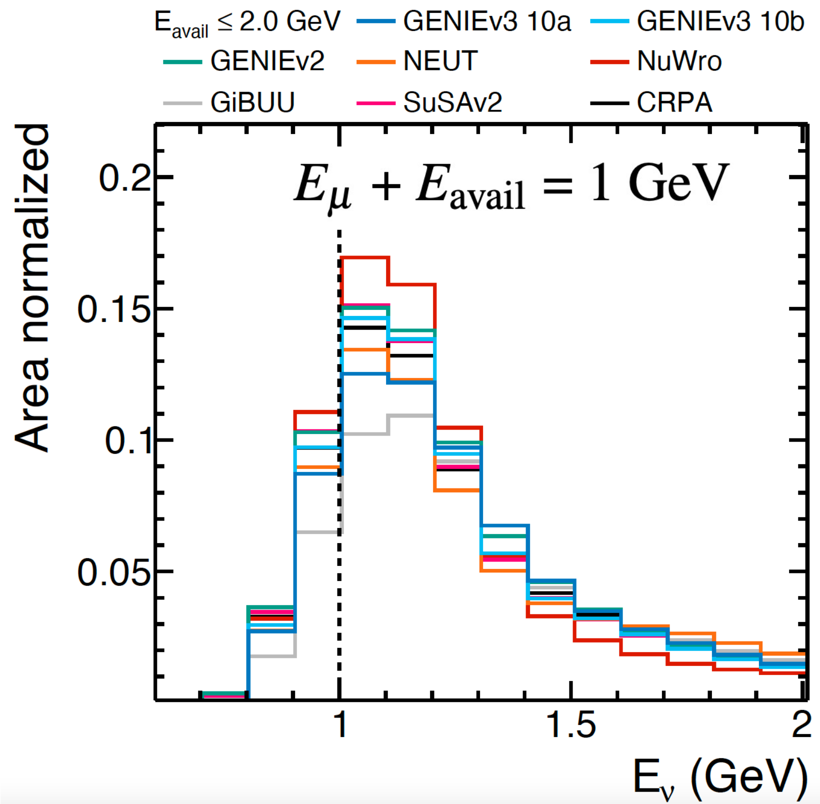
- When we unfold into true E_ν we assume we know 2. in order to determine 1.
    - At odds with our usual minimal-model-dependence approach to measurements

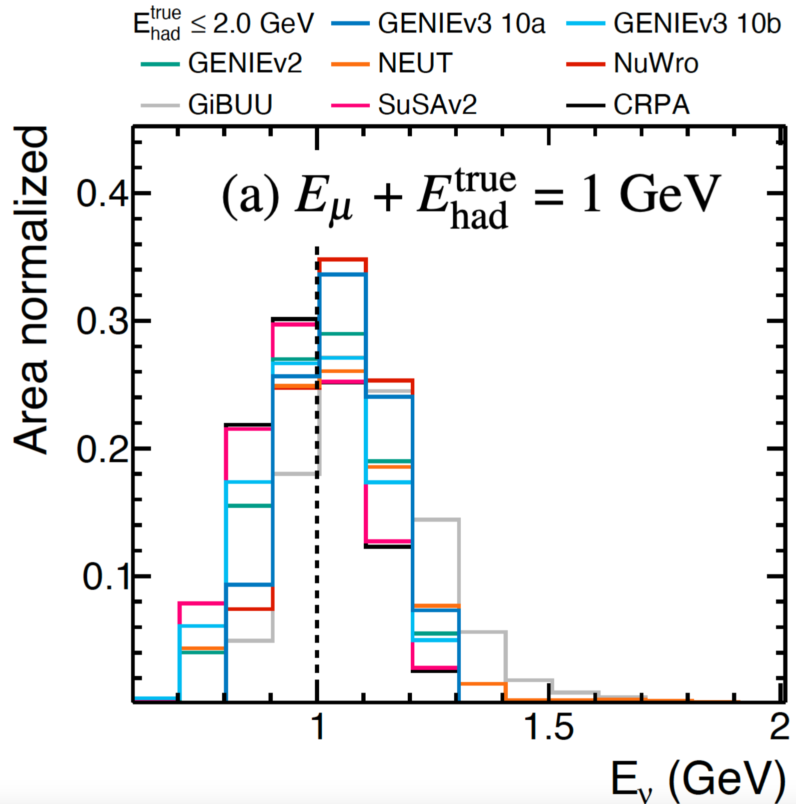
- Such measurements are only quantitatively useful whilst the uncertainties applied to 2. remain valid

How well might we know the E_ν^{rec} to E_ν^{true} map

- The mapping from E_ν^{rec} to E_ν^{true} depends on (among other things):
  1. The modelling of the nuclear initial state
  2. Final state interactions

- The modelling of these is not under well-control.





Plots adapted from those in:
[EPJC, **82**, 808 (2022)]

Xin’s Backup Slides

Backup [1]: Model dependence in ε

- Efficiency is a function of P and theta
- Model dependence ε (flux, spectrum, Xs, missing hadronic energy …) can change the distributions w.r.t. P and theta
  - Thus entering into the efficiency ε
- In another word, if one reweight the distribution according to the model, **the efficiency ε will change**

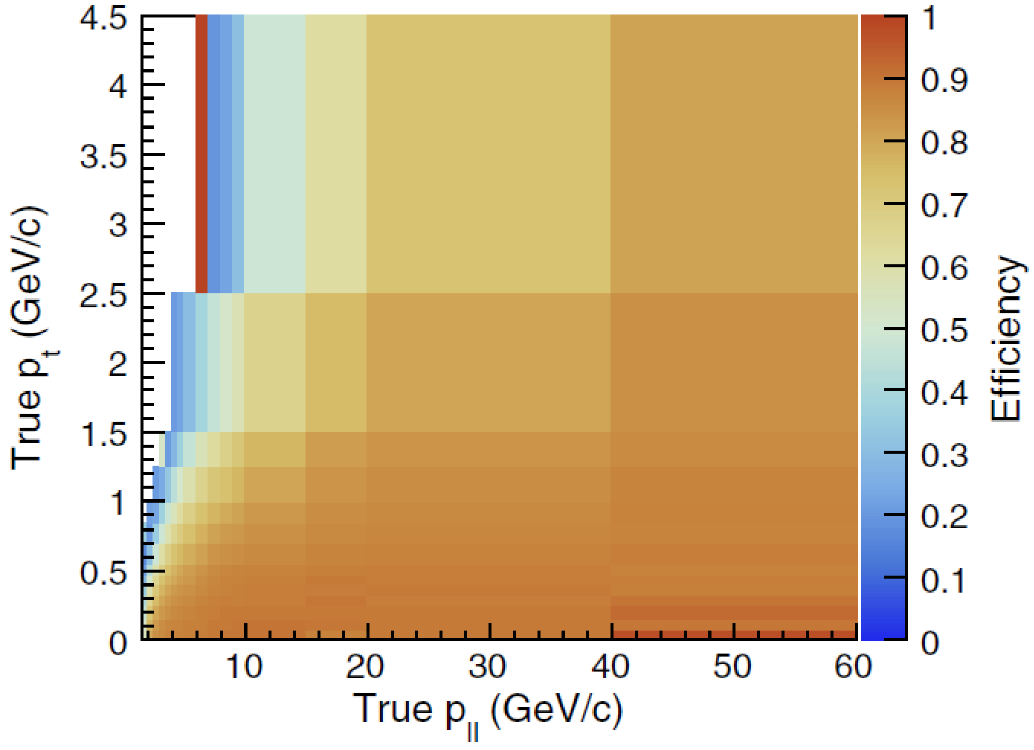


FIG. 5. Selection efficiency as a function of p_t and $p_{\parallel}$.

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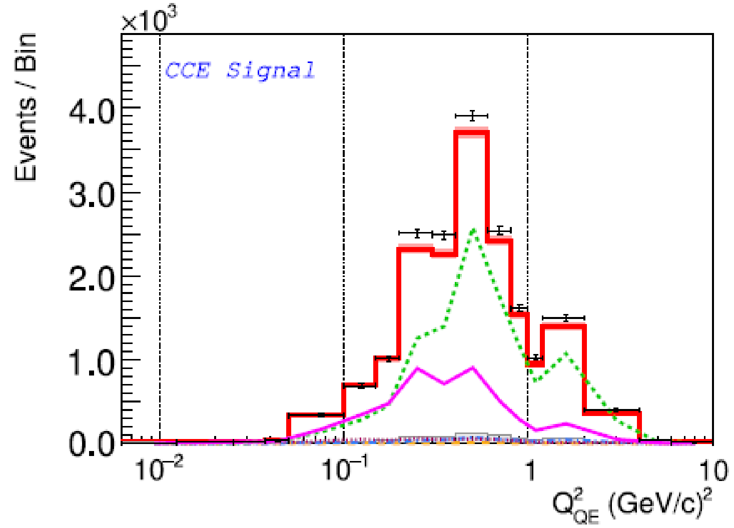
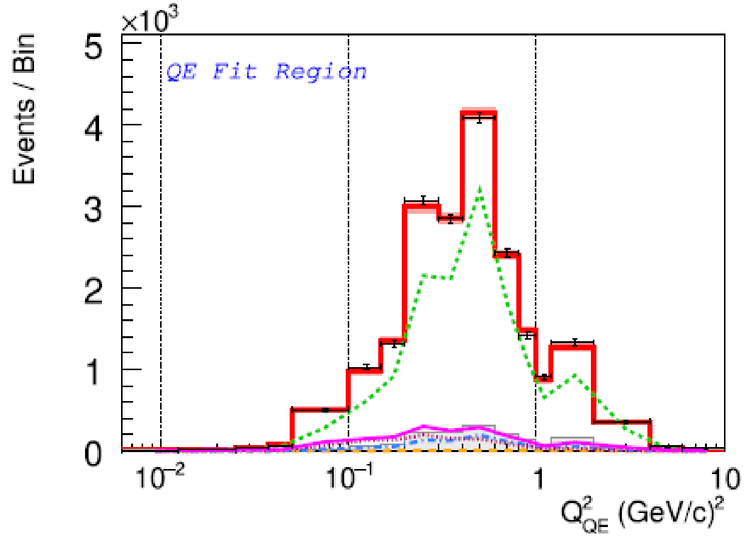
Backup [2]: Model Dependence in analysis

- While elastic scattering has a definite mapping between neutrino energy and the Q^2_{QE},
- The CCQE background from Carbon has a model dependence in mapping between the neutrino energy and the Q^2_{QE}
  - **Given the purity is not high, model validation is needed for the above mapping!**
- Simultaneous fit of signal and background, extrapolation of background from sideband to signal region is model-dependent

CeJin's talk





Backup [3]: Missing hadronic energy model always needed for comparison between theory and extracted Xs

- For example, dXs w.r.t. muon momentum
  - **It is obvious that 1 GeV neutrino cannot produce a muon with 2 GeV energy**

$$\frac{d^2\sigma}{dT_l cos\theta} = \frac{\sum_j U_{ij}(d_j - b_j)}{\Phi \cdot T \cdot \varepsilon_i \cdot (\Delta T_l, \Delta cos\theta)_i}$$

- When theorists compare their favorite model to the results, they need the model of missing hadronic energy to link the neutrino spectrum to the dXs
  - To do this, theorists need to consider the uncertainties associated with the neutrino spectrum and its uncertainties (e.g. shape) → a band of prediction

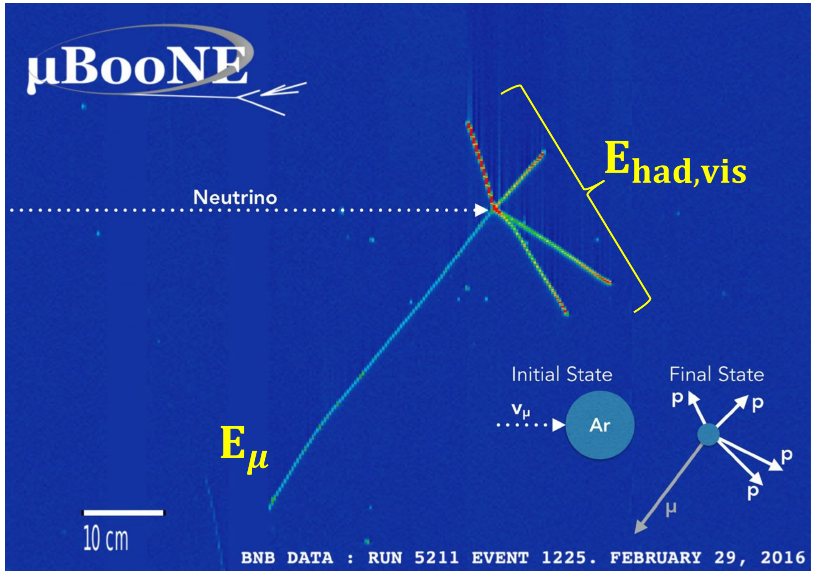
- Furthermore, since the extracted dXs has neutrino flux and spectrum uncertainties, **one needs to consider the correlation (covariance matrix) between the neutrino flux and spectrum and the extracted dXs**
  - Otherwise, they cannot do a proper comparison between measurement and theory

Backup [4]: PRISM Fake Data Example

- When the fake data is exactly same as the CV MC, the model validation method cannot detect a problem
  - This requires (at least) two things to be wrong (out of the model uncertainties): e.g. missing hadronic energy and neutrino flux
  - MicroBooNE model validation has demonstrated that incorrect model of missing hadronic energy (by its own) can be detected
- Of course, if the model of nu flux is wrong, we cannot extract a correct Xs independent on whether it is nu or mu energy
  - Even in the case that the other wrong thing is in Xs, the extracted Xs will be biased (out of the allowed Xs uncertainties)

Challenge in validating energy model $\mathbf{D}(E_\nu \rightarrow E_{reco})$

- How to verify the modeling of the undetected **missing hadronic energy**?
  - ➥ Mapping of $E_\nu \rightarrow \mathrm{E}_\nu^{\mathrm{rec}}$



True energy components:

$$\mathrm{E}_\nu = \mathrm{E}_\mu + \mathrm{E}_{\mathrm{had,vis}} + \mathrm{E}_{\mathrm{had,missing}}$$

Calorimetric energy reconstruction:

$$\mathrm{E}_\nu^{\mathrm{rec}} = \mathrm{E}_\mu^{\mathrm{rec}} + \mathrm{E}_{\mathrm{had,vis}}^{\mathrm{rec}}$$

Conditional constraining procedure

- Overcome the challenge by leveraging LArTPC's simultaneous measurements of **lepton energy** and **visible hadronic energy**

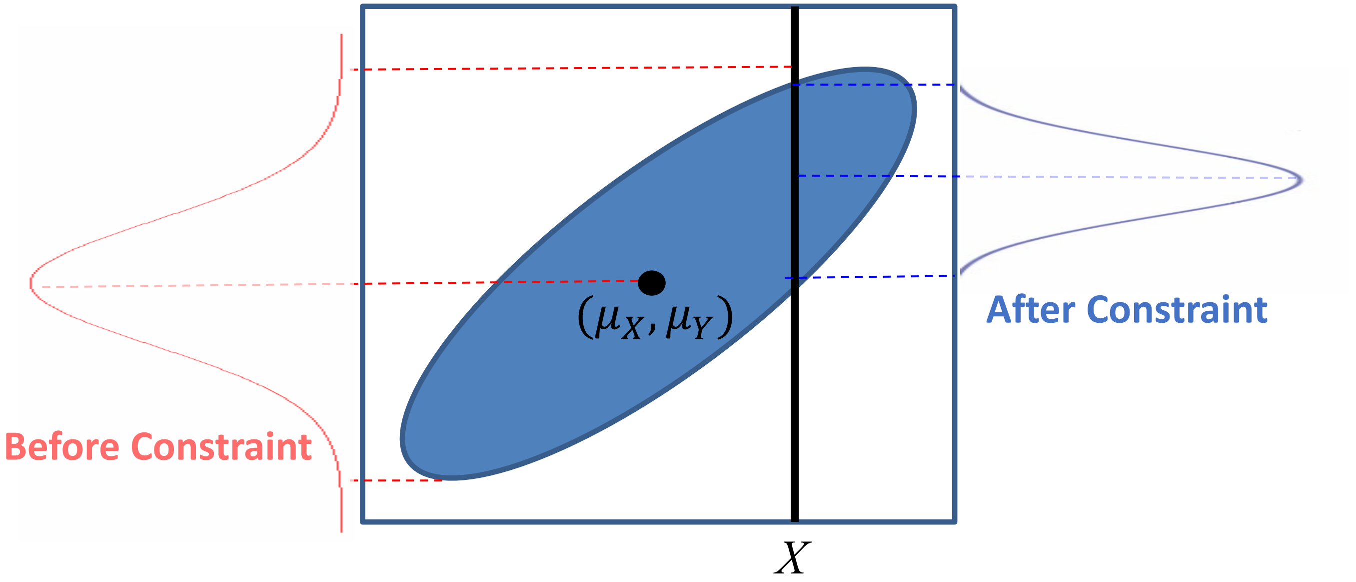
Conditional expectation & covariance

$$\boldsymbol{\mu}_{X,Y} = \begin{pmatrix} \boldsymbol{\mu}_X \\ \boldsymbol{\mu}_Y \end{pmatrix}, \qquad \boldsymbol{\Sigma}_{X,Y} = \begin{pmatrix} \boldsymbol{\Sigma}_{XX} & \boldsymbol{\Sigma}_{XY} \\ \boldsymbol{\Sigma}_{YX} & \boldsymbol{\Sigma}_{YY} \end{pmatrix}$$

$$\mu_{Y|X} = \mu_Y + \Sigma_{YX}\Sigma_{XX}^{-1}(X - \mu_X)$$

$$\boldsymbol{\Sigma}_{Y|X} = \boldsymbol{\Sigma}_{YY} - \boldsymbol{\Sigma}_{YX}\boldsymbol{\Sigma}_{XX}^{-1}\boldsymbol{\Sigma}_{XY}$$

* A variant of Gaussian Process regression



* Estimate correlated statistical uncertainty with bootstrapping (sampling w/ replacement)

$$\begin{pmatrix} \mu(E_{had}^{rec}) \\ \Sigma(E_{had}^{rec}) \end{pmatrix} + M(\mathrm{E}_\mu^{rec}) = \begin{pmatrix} \mu(E_{had}^{rec} \mid \mathrm{E}_\mu^{rec}, E_\nu) \\ \Sigma(E_{had}^{rec} \mid \mathrm{E}_\mu^{rec}, E_\nu) \end{pmatrix}$$

Prior model — Sideband — Posterior model

$$\mathbf{E}_\nu = \mathbf{E}_\mu + \mathbf{E}_{had,vis} + \mathbf{E}_{had,missing}$$

Model Validation: $\mathrm{M}(\mathbf{E}_{\mathbf{had}}^{\mathbf{rec}})$ vs. $\mu(\mathbf{E}_{\mathbf{had}}^{\mathbf{rec}} \mid \mathbf{E}_{\nu}, \mathbf{E}_{\mu}^{\mathbf{rec}})$

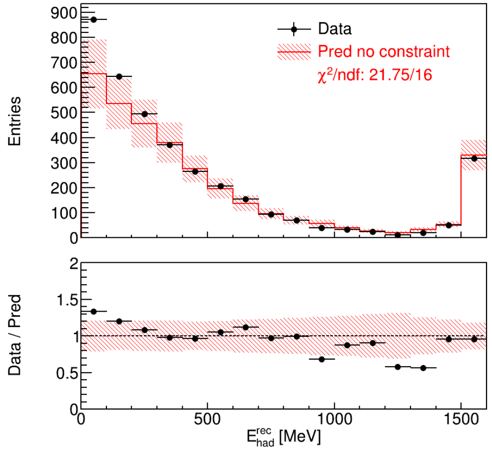
Neutrino flux modeling

Measurement of muon kinematics

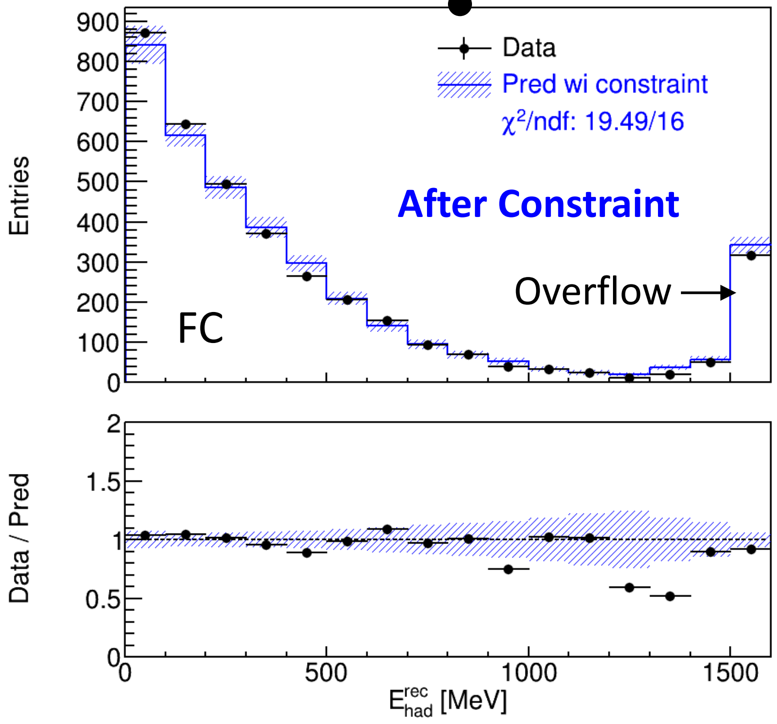
- New method to <u>validate modeling of neutrino energy</u> reconstruction given separated lepton and hadronic energy measurements in LArTPC

$$\mathbf{E}_{\nu} = \mathbf{E}_{\mu} + \mathbf{E}_{\mathbf{had,vis}} + \mathbf{E}_{\mathbf{had,missing}}$$

Before Constraint



Excess at low hadronic energy indicates mis-modeling of missing energy?

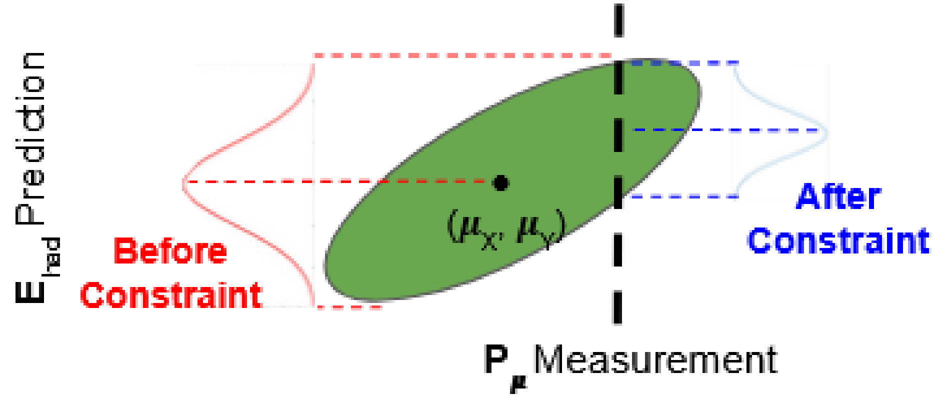


Measured muon kinematics are used to constrain the overall model (flux, cross section, etc.) for hadronic energy

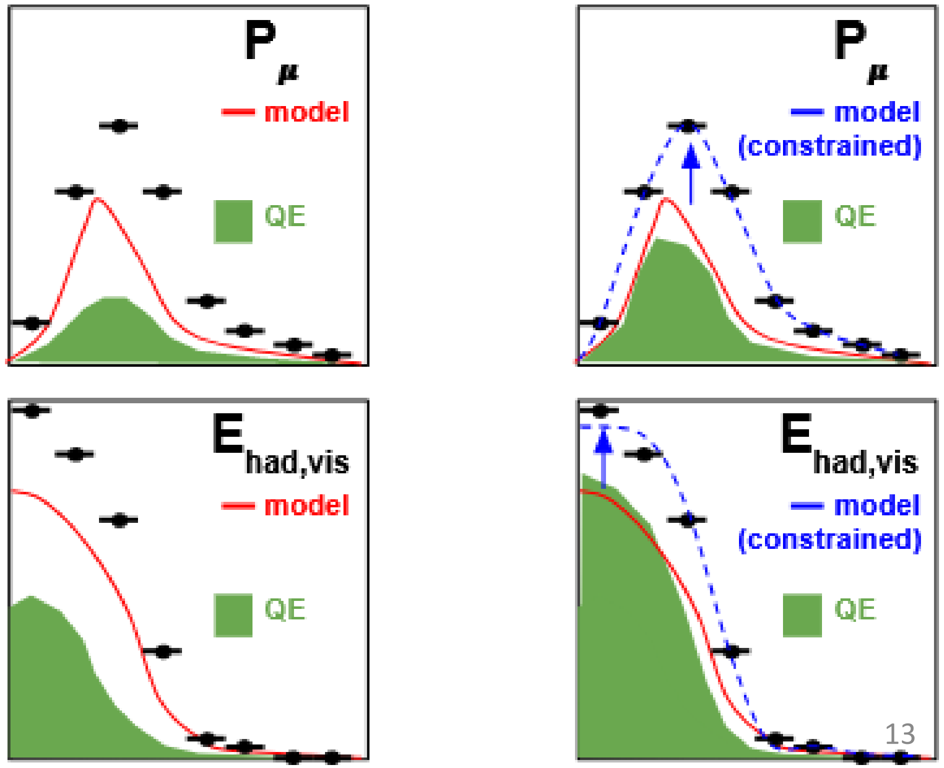
- Systematic uncertainties 20% → 5% in performing model validation
- No sign of mis-modeling of the **missing hadronic energy**
  - $\mathbf{D}(\boldsymbol{E_{\nu}} \rightarrow \boldsymbol{E_{reco}})$ is good!

Model Validation of Missing Hadronic Energy

- Conditional constraint procedure akin to reweighting based on P_μ measurement
- QE, RES, DIS predict different P_μ, $E_{had}^{missing}$, and E_{had}^{vis} distributions
  - The constrained prediction of E_{had}^{vis} is sensitive to the modeling of $E_{had}^{missing}$ in each process
- Measurement of constrained E_{had}^{vis} is thus sensitive to the model processes used in $E_{had}^{missing} \rightarrow$ validation of **the mapping between true and reconstructed E_ν**

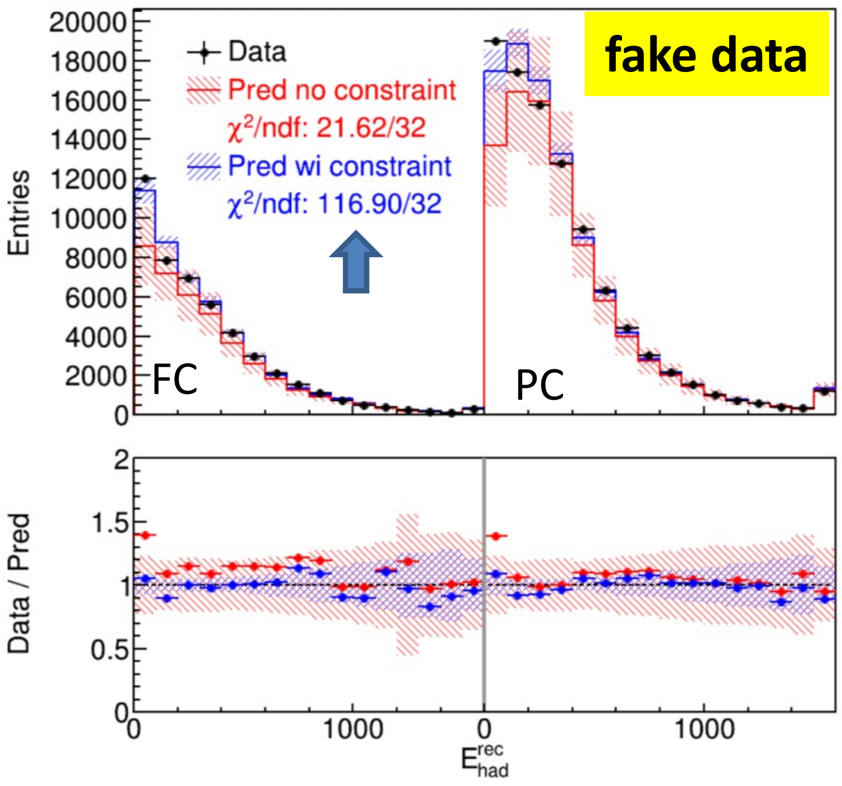


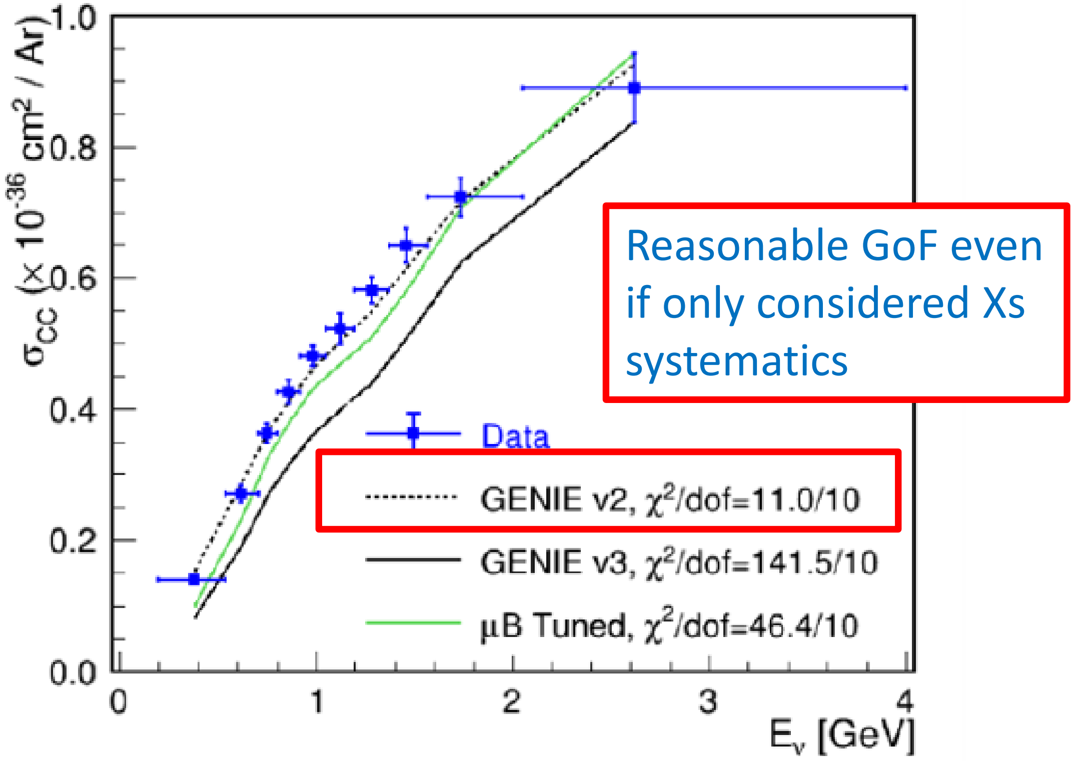
For Illustrative Purposes Only:



Constraint only used for validation, not unfolding

Fake Data: GENIE v2





- Fake data (GENIE v2) shows a very poor χ^2/ndf for $E^{\mathrm{rec}}_{\mathrm{had}}$ after constraint to muon kinematics

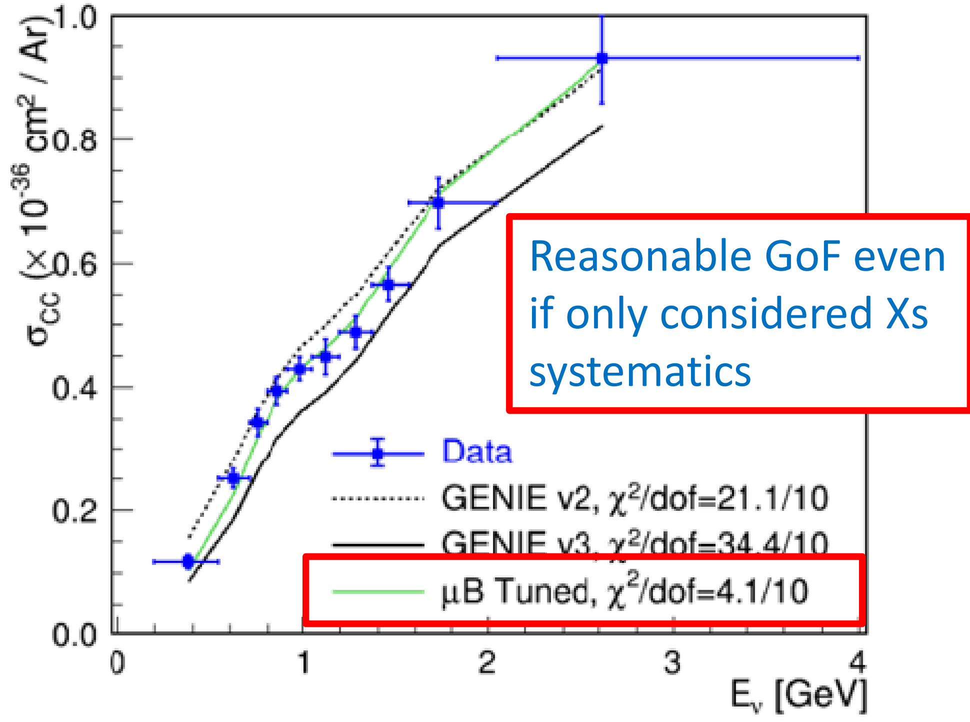
- Model validation procedure is much more sensitive (stringent) to the model defects than the extraction of energy-dependent Xs

Fake Data: Enhance Missing Hadronic Energy

E_p^{rec} scaling factor	FC events (ndf=16)	PC events (ndf=16)	FC+PC (ndf=32)
0.95	2.55 (1.00)	4.08 (1.00)	5.34 (1.00)
0.90	8.90 (0.92)	17.13 (0.38)	21.05 (0.93)
0.85	18.66 (0.29)	39.45 (0.00)	47.01 (0.04)
0.80	32.95 (0.01)	67.88 (0.00)	80.60 (0.00)

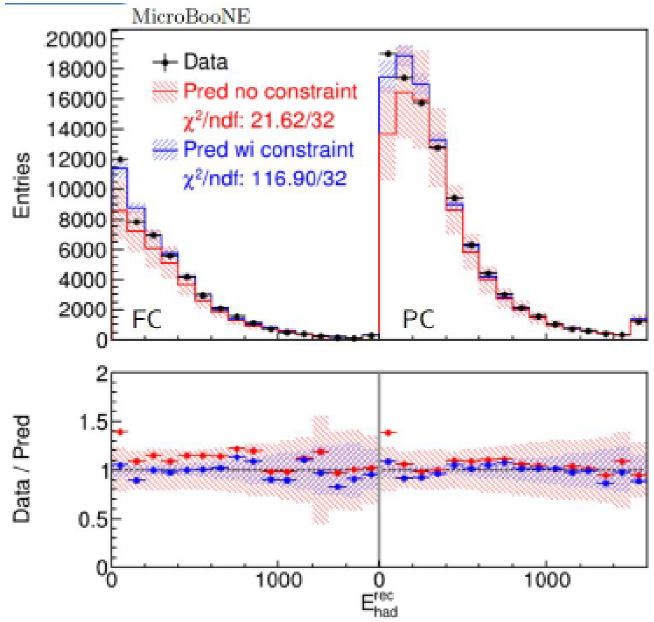
χ^2 P-value

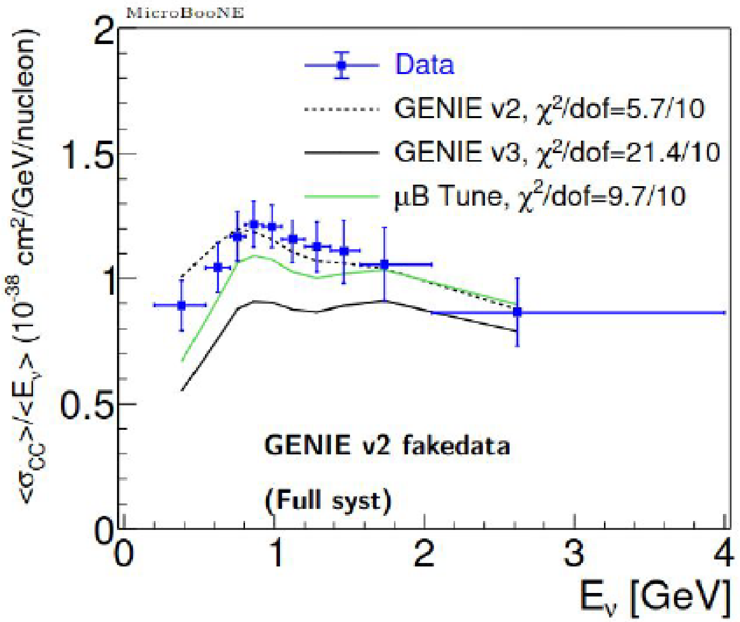
- χ^2/ndf has a significant increase with a shift of ~15% in the hadronic energy fraction allocated to protons (mimicking a variation of the proton-inelastic cross section)



- Model validation procedure is much more sensitive (stringent) to the model defects than the extraction of energy-dependent Xs

Testing Model Validation Procedure with Fake Data





Fake Data	GoF χ^2/ndf	Unfolded XS w.r.t truth χ^2/ndf	Type of Uncertainties Stat. + Syst.
Genie v2	**116.9/32**	**5.7/10**	**Fluctuations + Full**
-15% E_p	**39.5/16**	**4.1/10**	**Asimov + Xs only**
-30% E_p	**47.1/16**	**5.2/10**	**Asimov + Full**

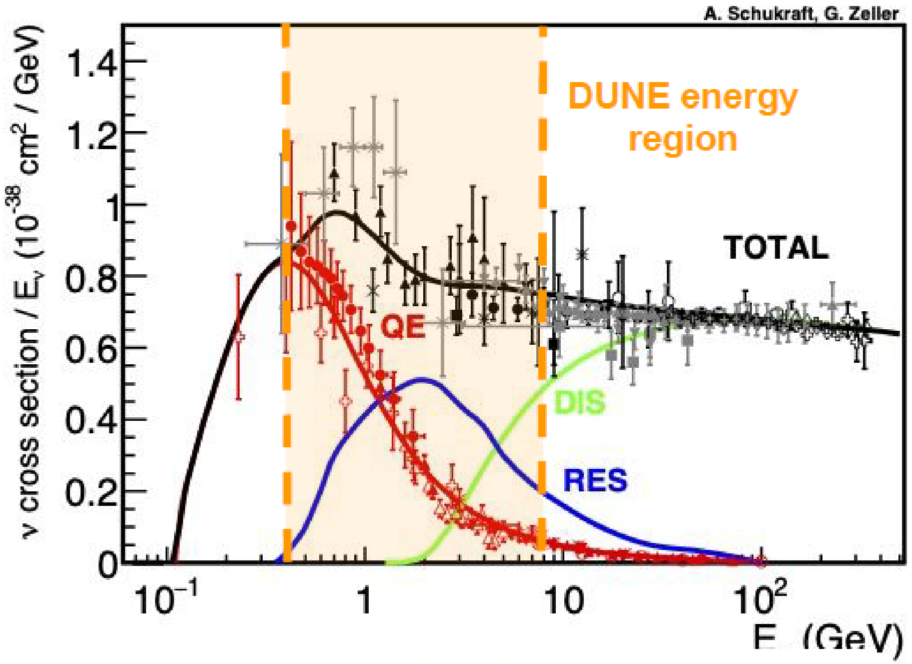
- Fake data generated from scratch with Genie v2 prediction
  - 7.2×10^{20} POT exposure used
- Constrained model prediction fails validation test (**χ^2/ndf = 116.9/32, p-value = 1.3×10^{-11}**) → $E_{had}^{missing}$ modeling disagreement
- Unfolded XS consistent with truth (**χ^2/ndf = 5.7/10, p-value = 0.84** → Xs extraction is less sensitive to data/model discrepancy than the model validation)
  - Consistent with expectation
  - Similar observation in other fake data sets

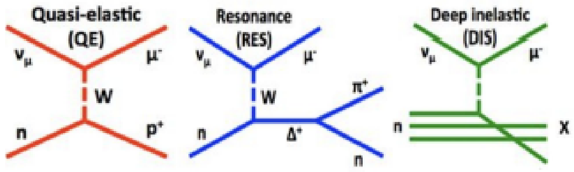
Why we are interested in E_ν-dependent Cross Sections

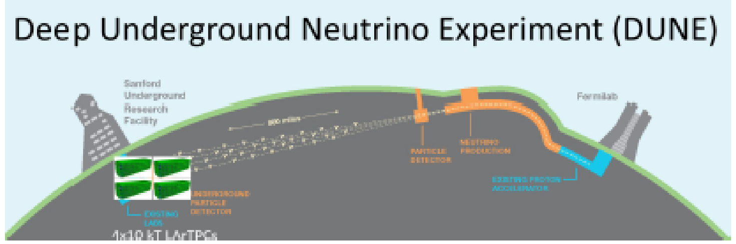
- Inclusive ν_μ CC channel, able to tag neutrino flavor, is an important channel for DUNE oscillation measurement

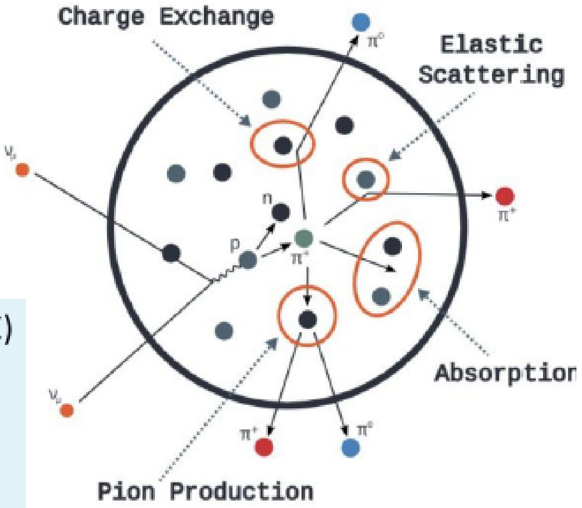
- Kinematics of inclusive ν_μ CC defined by 3 degrees of freedom: $\{E_\nu, P_\mu, \theta_\mu\}$
    - E_ν can be reconstructed with additional E_{had} measurement

- Inclusive ν_μ CC in the DUNE energy range consists of several major interaction modes (QE, RES, DIS,...)
    - While final-state particles can be used to separate these modes up to nuclear effects (2p2h, FSI,...), E_ν-dependent cross sections give additional discrimination capabilities









Credit: T. Golan