Checking backward evolution?

Pythia meeting 2020, April 22, 2020 (NB: Reused from PB meeting 2020-03-26)
Stefan Prestel (Lund)
LH 2019 study with L. Gellersen and D. Napoletano (arXiv:2003.01700)

Any self-respecting shower will “amend” DGLAP evolution to avoid soft double-counting.

But how bad are the consequences?

[Note: Derivation following arXiv:1506.05057]
DGLAP evolution equation:

$$\frac{df_a(x,t)}{d\ln t} = \sum_{b=q,g} \int_x^1 \frac{dz}{z} \frac{\alpha_s}{2\pi} \left[P_{ba}(z)\right]_+ f_b(x/z,t) \,,$$

where P_{ab} = regularized kernels. Assume: Write using unregularized kernels $\hat{P}_{ab}$, restricted to all but an ε-environment around the pole, plus an endpoint:

$$P_{ba}(z,\varepsilon) = \hat{P}_{ba}(z)\,\Theta(1-z-\varepsilon) - \delta_{ab}\,\frac{\Theta(z-1+\varepsilon)}{\varepsilon} \sum_{c=q,g} \int_0^{1-\varepsilon} d\zeta\,\zeta\,\hat{P}_{ac}(\zeta)$$

w/o momentum conservation, $\varepsilon \to 0$ is possible, allowing identification of $\left[P_{ba}(z)\right]_+$ as the $\varepsilon \to 0$ limit of $P_{ba}(z,\varepsilon)$.

With this, the DGLAP evolution equation becomes:

$$\frac{1}{f_a(x,t)}\frac{df_a(x,t)}{d\ln t} = - \sum_{c=q,g}\int_0^{1-\varepsilon} d\zeta\,\zeta\,\frac{\alpha_s}{2\pi}\hat{P}_{ac}(\zeta) + \sum_{b=q,g}\int_x^{1-\varepsilon}\frac{dz}{z}\,\frac{\alpha_s}{2\pi}\,\hat{P}_{ba}(z)\,\frac{f_b(x/z,t)}{f_a(x,t)}\,.$$

The first term is the “virtual” part, the second a “spectrum” contribution.

(Note: We can argue about the precise form of the “virtual” part, but that’s not really important here. The relevant point is $\varepsilon \ll 1$.)

Defing the Sudakov form factor Δ and the no-emission probability Π,

$$\begin{aligned}\Delta_a(t_1,t_0) &= \exp\left\{-\int_{t_1}^{t_0}\frac{dt}{t}\sum_{c=q,g}\int_0^{1-\varepsilon} d\zeta\,\zeta\,\frac{\alpha_s}{2\pi}\hat{P}_{ac}(\zeta)\right\}\\ \Pi_a(t_1,t_0;x) &= \exp\left\{-\int_{t_1}^{t_0}\frac{dt}{t}\sum_{b=q,g}\int_x^{1-\varepsilon}\frac{dz}{z}\,\frac{\alpha_s}{2\pi}\,\hat{P}_{ba}(z)\,\frac{f_b(x/z,t)}{f_a(x,t)}\right\}\end{aligned}$$

the DGLAP equation can be rewritten as

$$f_a(x,t)\Delta_a(t,\mu^2) = f_a(x,\mu^2)\,\Pi_a(t,\mu^2;x)\ .$$

The assumptions going into this are

1. P are the DGLAP kernels
2. $\varepsilon \ll 1$

DGLAP IV

The DGLAP equation can be rewritten as

$$f_a(x,t)\Delta_a(t,\mu^2) = f_a(x,\mu^2)\,\Pi_a(t,\mu^2;x)\ .$$

The assumptions going into this are

1. We use DGLAP kernels
2. $\varepsilon \ll 1$

Parton showers rely on sampling the spectrum (i.e. Π) to produce evolution.

We do not use vanilla DGLAP kernels (due to having to avoid soft double-counting), nor have $\varepsilon \ll 1$ in all regions of phase space, due to momentum conservation constraints.

How much is the relation violated in practise?

Note that the function

$$D_a(t, \mu^2; x) = \frac{f_a(x, \mu^2)}{f_a(x, t)} \, \Pi_a(t, \mu^2; x) \ .$$

should be x-independent for all a, and every combination t, μ^2.

Note that we don't really need to agree that $D_a = \Delta_a$, nor do we need to worry about the precise value of D_a yet.
Since the value of D_a depends heavily on t, μ^2 and the guts of the PS (ordering, etc.), it's best to normalize

$$d(t, \mu^2; x_i) = \frac{D(t, \mu^2; x_i)}{\sum_j D(t, \mu^2; x_j)} = \frac{\frac{f_a(x_i, \mu^2)}{f_a(x_i, t)} \, \Pi_a(t, \mu^2; x_i)}{\sum_j D(t, \mu^2; x_j)} \ ,$$

to compare the x-independence for various t on equal footing

We want to check the x-independence of

$$d(t,\mu^2;x_i) = \frac{D(t,\mu^2;x_i)}{\sum_j D(t,\mu^2;x_j)} = \frac{\frac{f_a(x_i,\mu^2)}{f_a(x_i,t)}\,\Pi_a(t,\mu^2;x_i)}{\sum_j D(t,\mu^2;x_j)}\ ,$$

Expectations:

1. Large separation between t,μ^2 gives long evolution, which could lead to small inconsistencies accumulating.
2. Usually $\varepsilon = \varepsilon(\bar{t}, m_{\mathrm{D}})$, where m_{D} is the dipole mass. For Pythia

$$\varepsilon_{\mathrm{Pythia}} = \frac{\sqrt{t_{\mathrm{Pythia}}}}{m_{\mathrm{D}}}\left(\sqrt{1+\frac{t_{\mathrm{Pythia}}}{4\,m_{\mathrm{D}}^2}} - \frac{\sqrt{t_{\mathrm{Pythia}}}}{2\,m_{\mathrm{D}}}\right)$$

Smaller m_{D} thus allows ε to be closer to 1, and violate the assumptions more severely.

We want to check the x-independence of

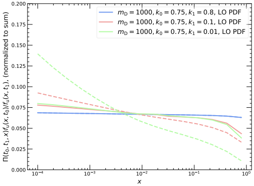
$$d(t,\mu^2;x_i) = \frac{D(t,\mu^2;x_i)}{\sum_j D(t,\mu^2;x_j)} = \frac{\frac{f_a(x_i,\mu^2)}{f_a(x_i,t)}\,\Pi_a(t,\mu^2;x_i)}{\sum_j D(t,\mu^2;x_j)}\ ,$$

Settings:

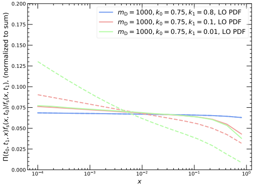
- For simplicity, stick to initial-initial color connection (=Drell-Yan-like)
- Pick a value for m_D, and then define $t_0 = k_0 m_\mathrm{D}$. We fixed $k_0 = 0.75$.
- Define $t_1 = k_1 t_0$. Use $k_1 \in [0.8, 0.1, 0.01]$ as proxy for short, moderate and very long evolution.
- Check both NNPDF23_lo_as_0119_qed and NNPDF23_nlo_as_0119_qed

...then calculate $D_a(t,\mu^2;x)$ for several values of x (by employing trial showers for "events" with two partons a, b with $(p_a+p_b)^2 = m_\mathrm{D}^2$, $x_a = x$, and only allowing radiation from the a-leg)

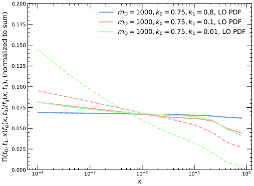
High mass plots (note: focus on the solid lines)



(a) d-quark evolution



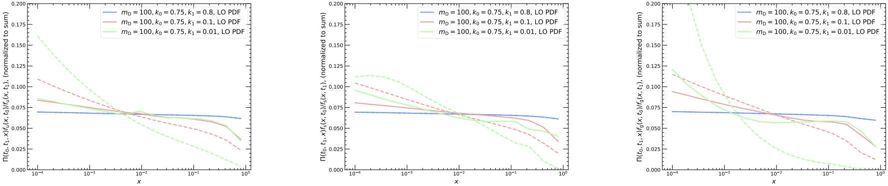
(b) s-quark evolution



(c) gluon evolution

Figure: x-distribution for different length of parton-shower evolution, for $m_{\mathrm{D}} = 1000$ GeV, leading-order PDF set NNPDF23_lo_as_0119_qed, and for both Pythia (solid curves) and Sherpa (dashed curves)

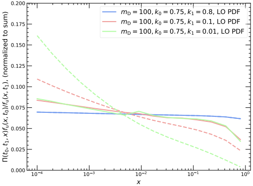
Low mass plots (note: focus on the solid lines)



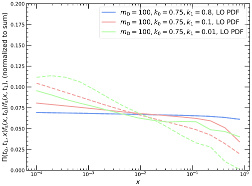
(a) d-quark evolution (b) s-quark evolution (c) gluon evolution

Figure: x-distribution for different length of parton-shower evolution, for $m_{\rm D} = 100$ GeV, leading-order PDF set NNPDF23_lo_as_0119_qed, and for both Pythia (solid curves) and Sherpa (dashed curves)

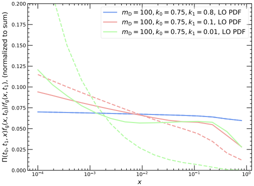
LO vs. NLO PDFs (note: focus on the solid lines)



(a) d-quark evolution

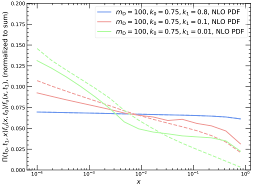


(b) s-quark evolution

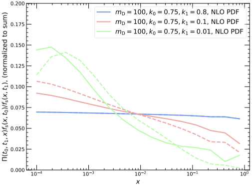


(c) gluon evolution

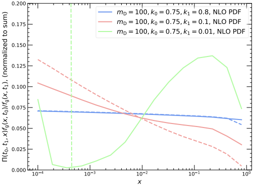
Figure: $m_D = 100$ GeV, leading-order PDF set NNPDF23_lo_as_0119_qed



(a) d-quark evolution



(b) s-quark evolution



(c) gluon evolution

Figure: $m_D = 100$ GeV, NLO PDF set NNPDF23_lo_as_0119_qed

Fun results, but probably needs further studies :)

- Lower mass $\longrightarrow$ stronger x-dependence: Makes sense from form of ε.
- Long evolution $\longrightarrow$ stronger x-dependence: Problems accumulate.
- NLO PDFs $\longrightarrow$ stronger x-dependence: No NLO kernels means NLO DGLAP not recovered.

Lots of further studies could be done...

Deductor has published a similar study, Cascade is working on it.