Persistent-currents magnetization in STEAM-LEDET

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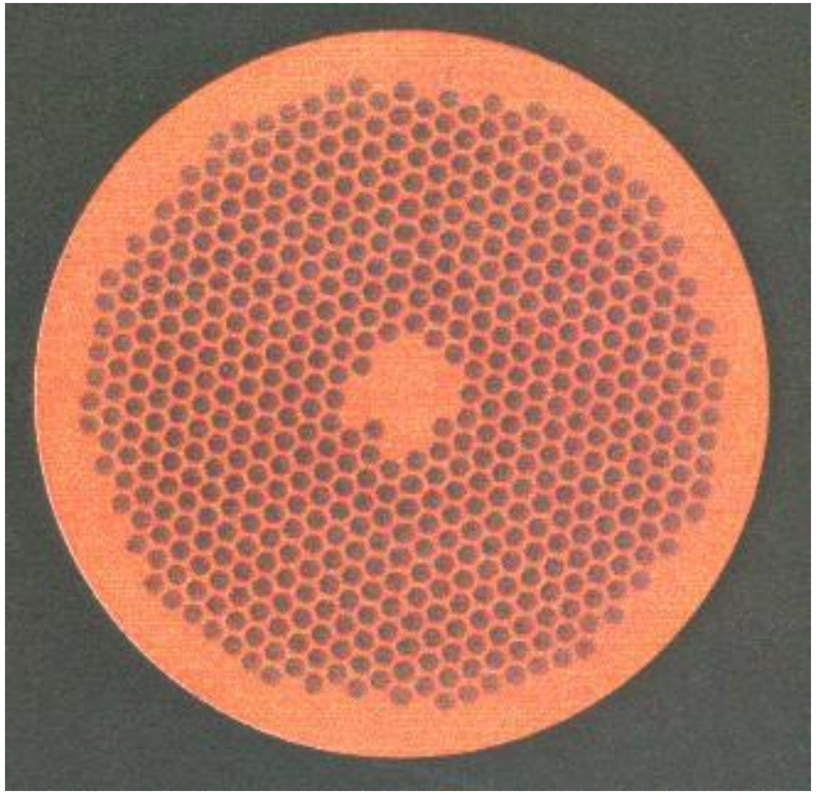
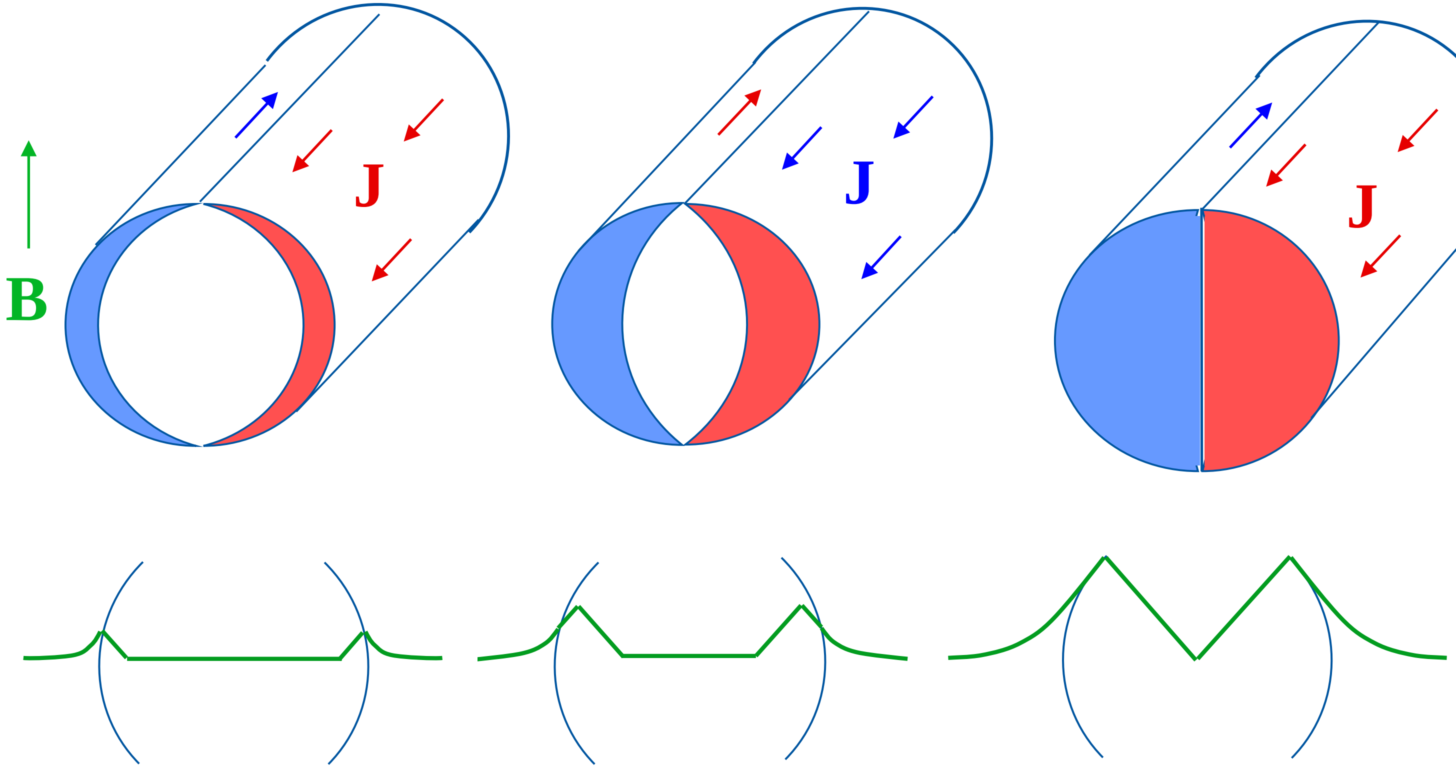
Thanks to A. Verweij (CERN)

13 August 2020

Persistent-current magnetization in the superconductor





Superconducting wire/strand
- Superconducting filaments
- Copper matrix

Persistent currents arise to screen the interior of the superconducting filaments from the applied field H and generate **magnetization** in the wire volume opposing to H [diamagnetism]

All figures taken from M. Wilson's lectures

References

[1] C.P. Bean, "Magnetization of Hard Superconductors", 1962, https://link.aps.org/doi/10.1103/PhysRevLett.8.250

[2] C.P. Bean, "Magnetization of High-Field Superconductors", 1964, https://link.aps.org/doi/10.1103/RevModPhys.36.31

[3] Y.B. Kim, C.F. Hempstead, A.R. Strnad, "Magnetization and Critical Supercurrents", 1963, https://link.aps.org/doi/10.1103/PhysRev.129.528

[4] M. Sorbi and V. Marinozzi, "Magnetization Heat in Superconductors and in Eddy Current Problems: A Classical Thermodynamic Approach", 2016

[5] N. Schwerg, "Estimation of the Instantaneously Dissipated Hysteresis Losses in Superconductors", 2012

[6] P. Campbell, "Comments on "Energy stored in permanent magnets"", 2000

[7] R.B. Goldfarb, M. Lelental, and C.A. Thompson, "Alternating-Field Susceptometry and Magnetic Susceptibility of Superconductors", 1991, https://doi.org/10.1007/978-1-4899-2379-0_3

[8] G. Goev, V. Masheva, J. Geshev, and M. Mikhov, "Irreversible Susceptibility of Initial Magnetization Curve", 2007, https://aip.scitation.org/doi/abs/10.1063/1.2733131

[9] S. Le Naour et al., "Magnetization measurements on LHC superconducting strands", 1999

[10] E. Ravaioli et al., "Lumped-Element Dynamic Electro-Thermal model of a superconducting magnet“, 2016, http://dx.doi.org/10.1016/j.cryogenics.2016.04.004

[11] E. Ravaioli et al., “Fast Method to Quantify the Collective Magnetization in Superconducting Magnets”, 2013

Persistent-current magnetization in STEAM-LEDET

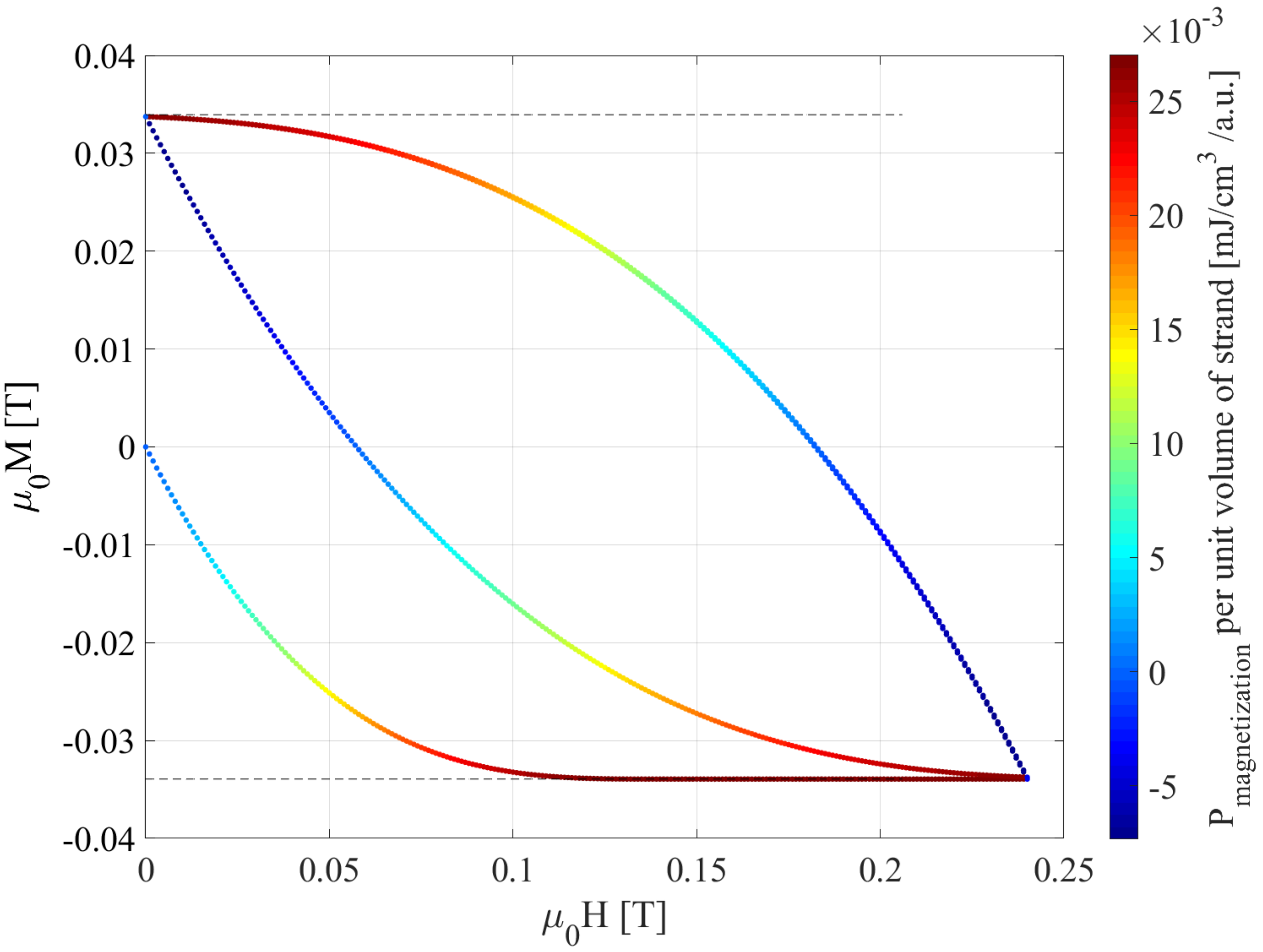
1. Calculation of **magnetization in the strand**
   - Bean's model
   - Jc(T,B) fits
   - "Virgin" magnetization curve and slope
   - Saturation magnetization
   - Magnetization for an arbitrary magnetic cycle

2. Calculation of **hysteresis loss**
   - Review from the literature
   - Energy stored and dissipated in magnetization loops
   - Comparison with ROXIE

3. Calculation of the **effect of magnetization on the magnet differential inductance**
   - Equivalent electrical circuit
   - First attempt at validation

This presentation focusses on results.
For the derivation of the model, see the Annex and/or let's have a coffee

From Bean's model [1-2]



Penetration field
Hp(B) = 1/π*Jc(T,B)*df

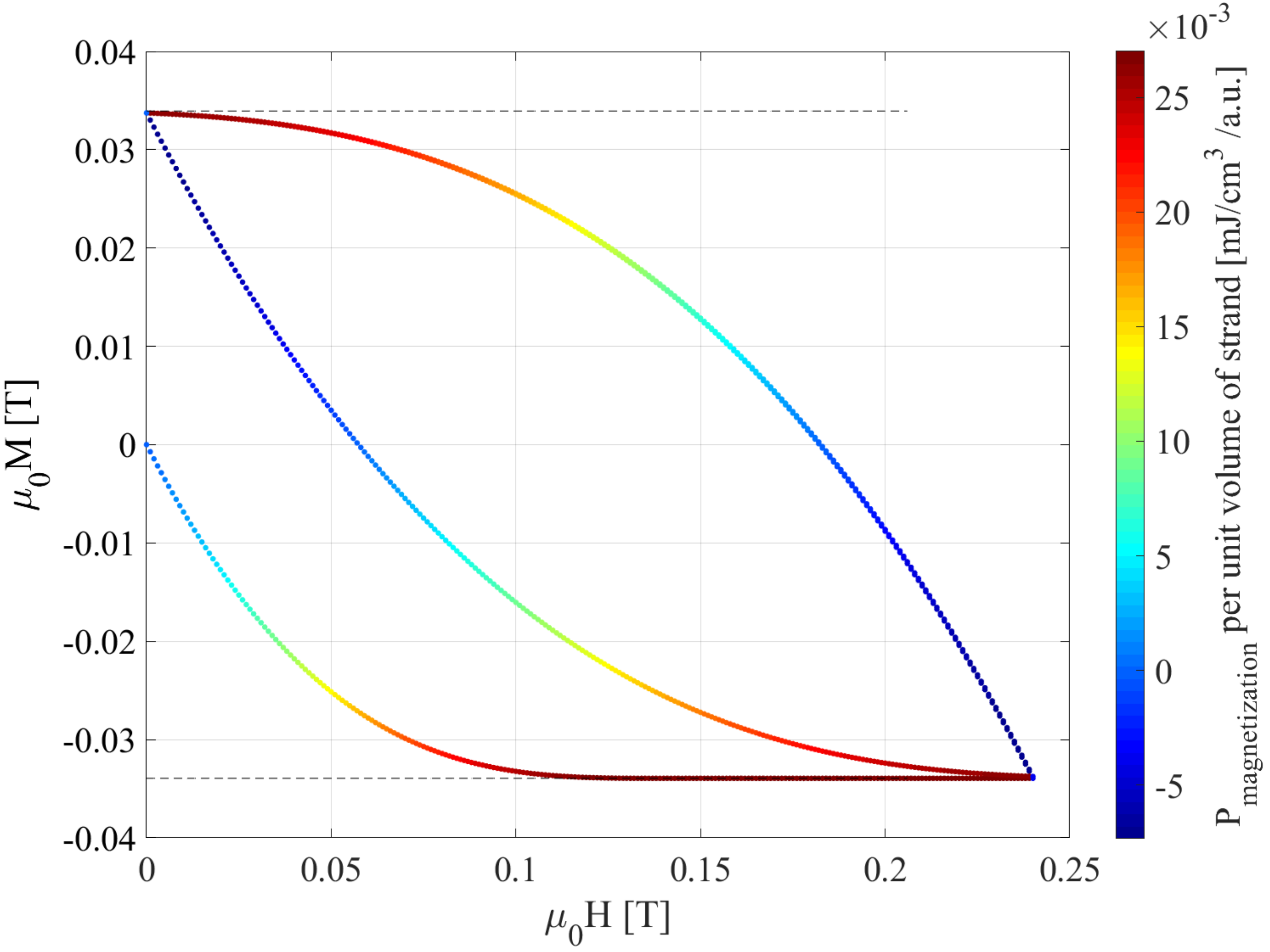
Saturation magnetization
M_sat_fil = ±2/3*Hp

Magnetization homogenized in the strand cross-section
M_sat = M_sat_fil*f_{SC}

Magnetization is calculated analytically

For this presentation, MQY outer wire parameters are used:
ds=0.48 mm
df=7 μm
f_{SC}~0.35
Jc(T,B): Bottura fit [fit 1 in ROXIE]

Magnetization model description



Magnetization M is calculated analytically

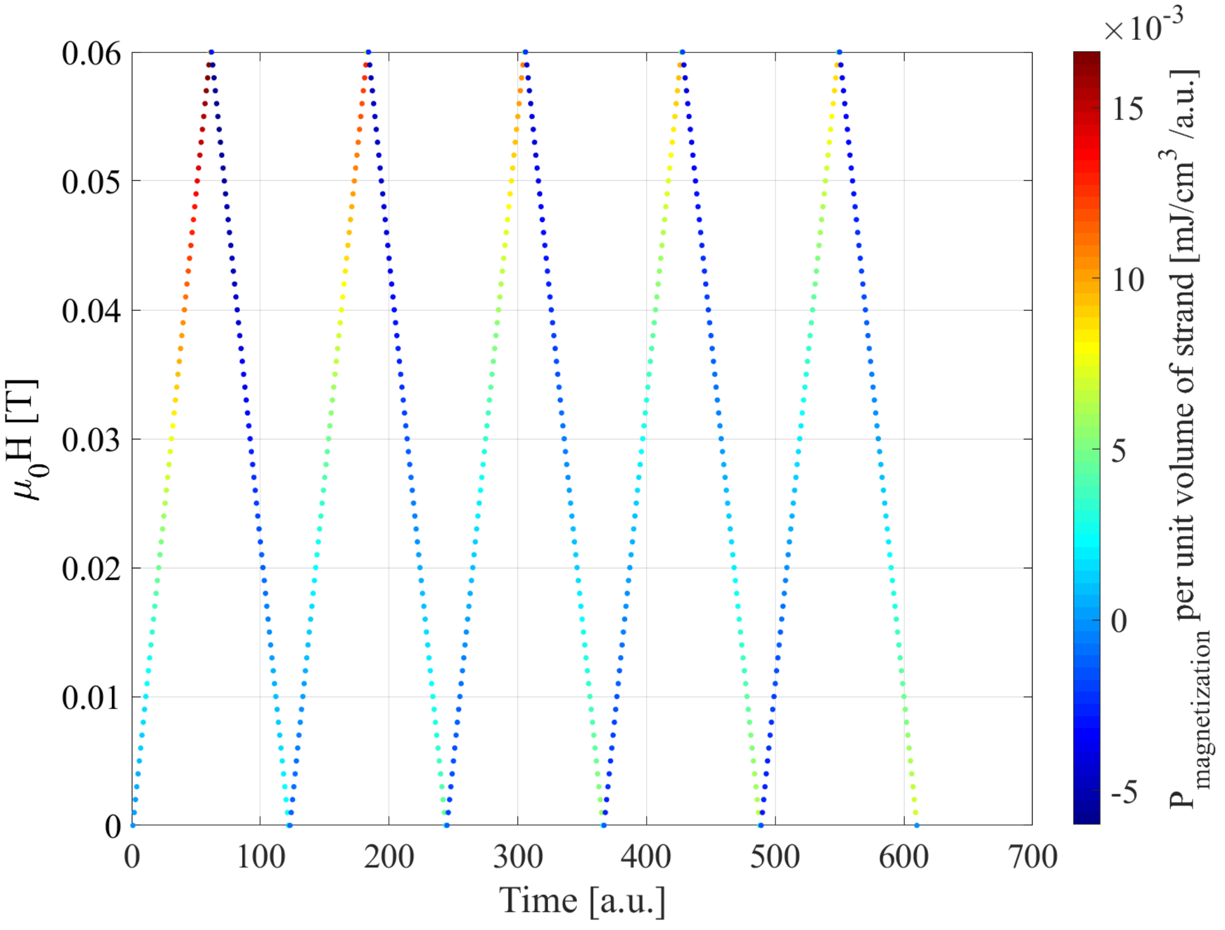
- Analytical formula from Bean's model for round wire
- Jc dependence on B introduced
- Formula adapted for calculating M also for incomplete magnetic loops

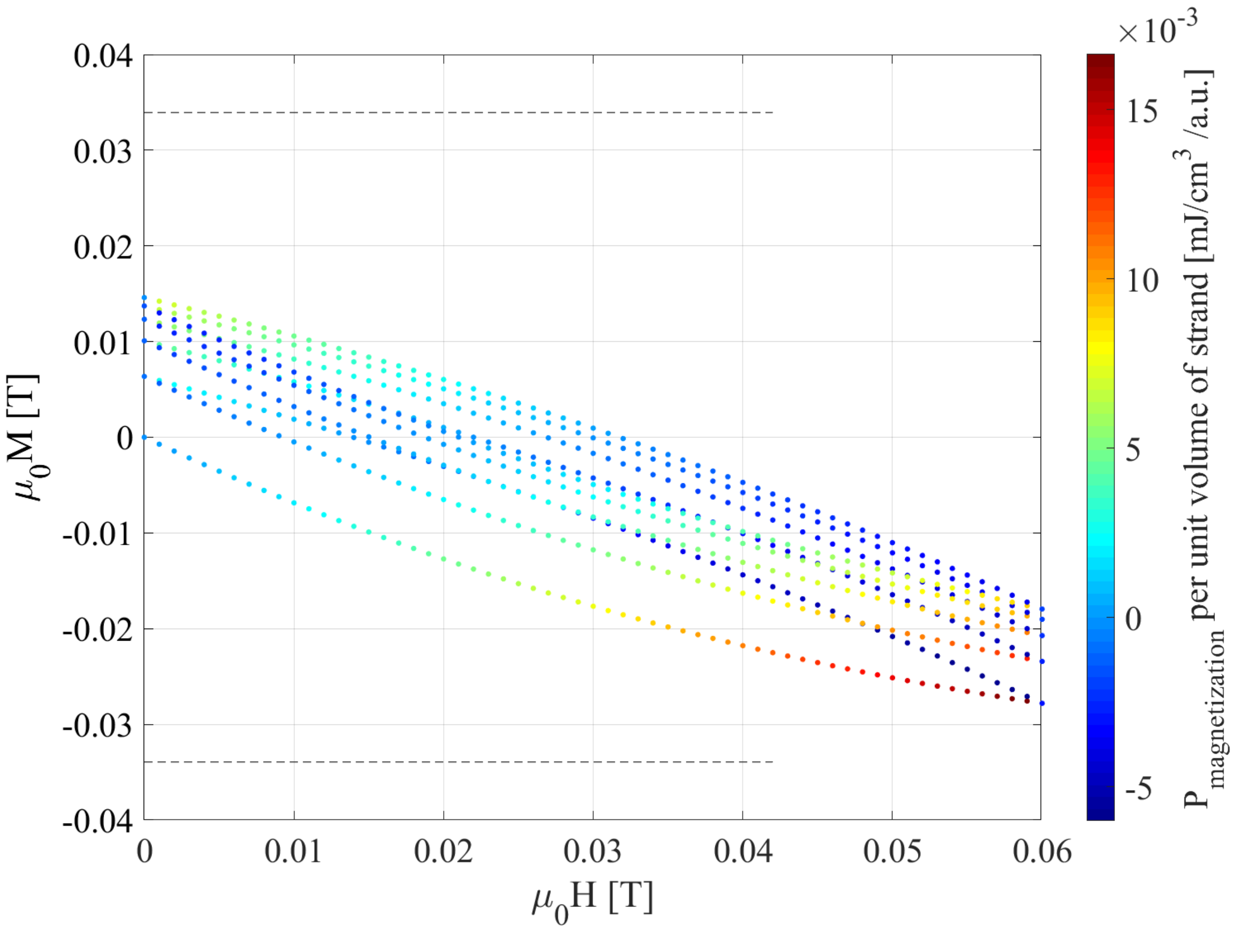
Assumptions/Simplifications

- All assumptions of Bean's model
- Magnetization homogenized within wire volume
- Interaction between magnetization and coupling currents neglected

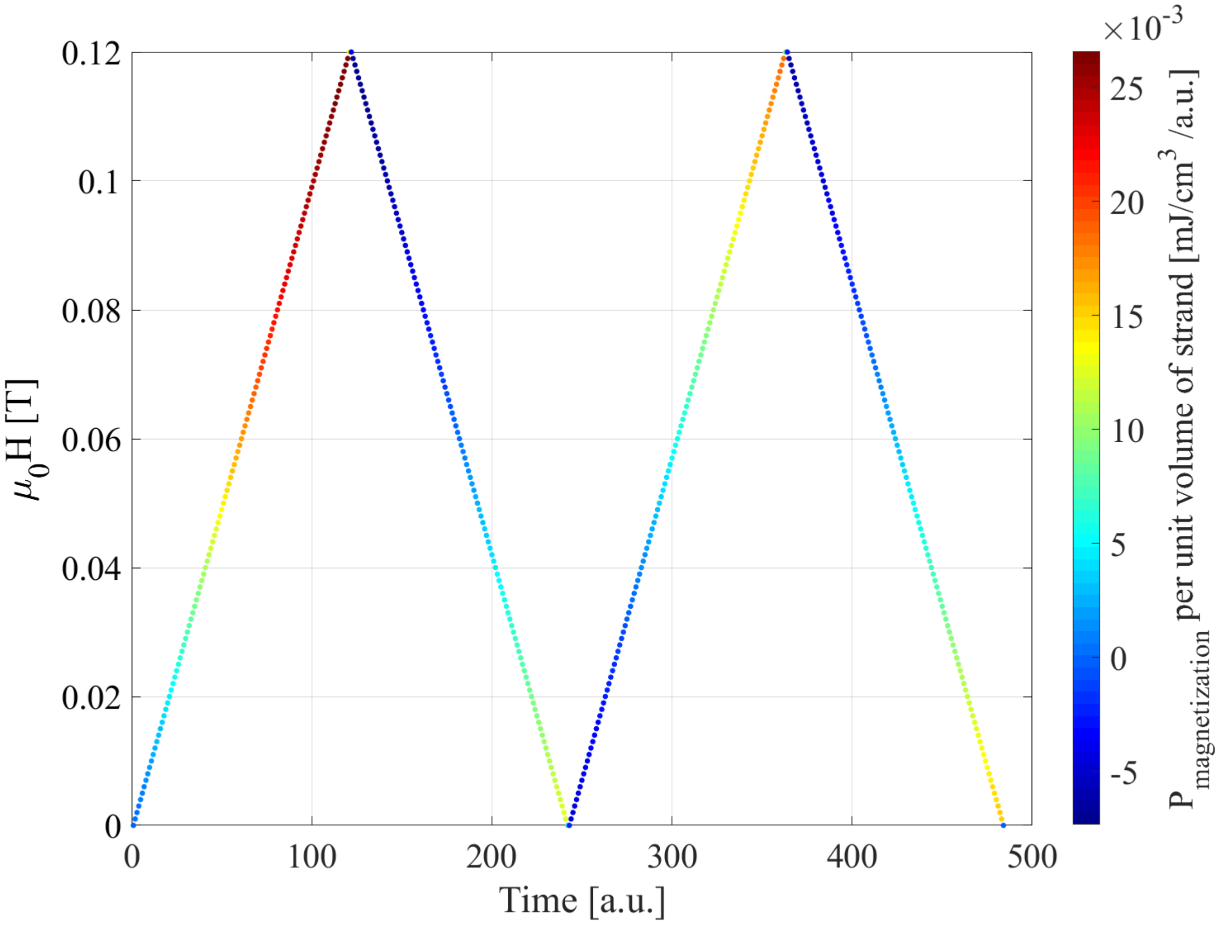
Some examples of simulated magnetization transients

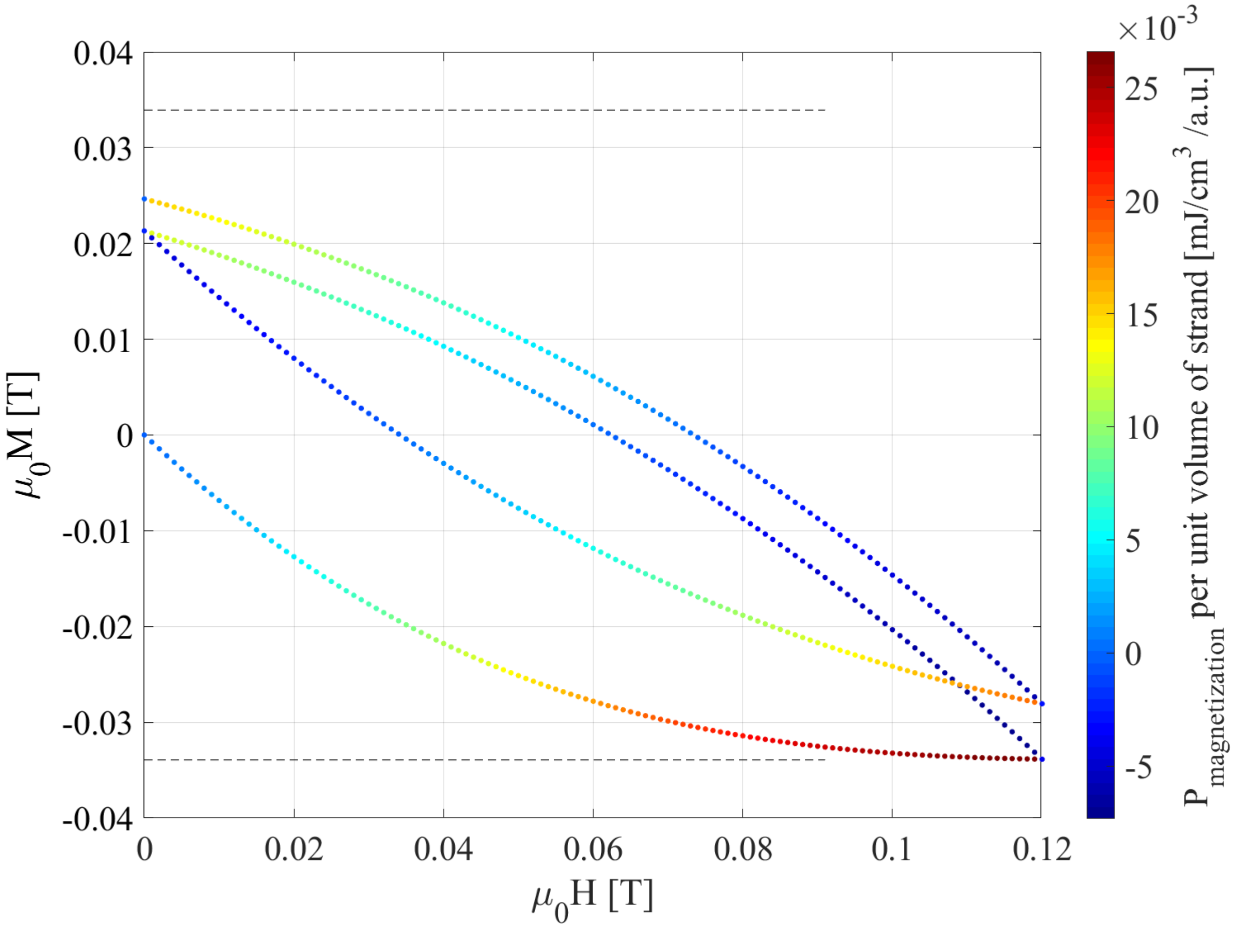
Cycle A : 0→+0.5*Hp→0



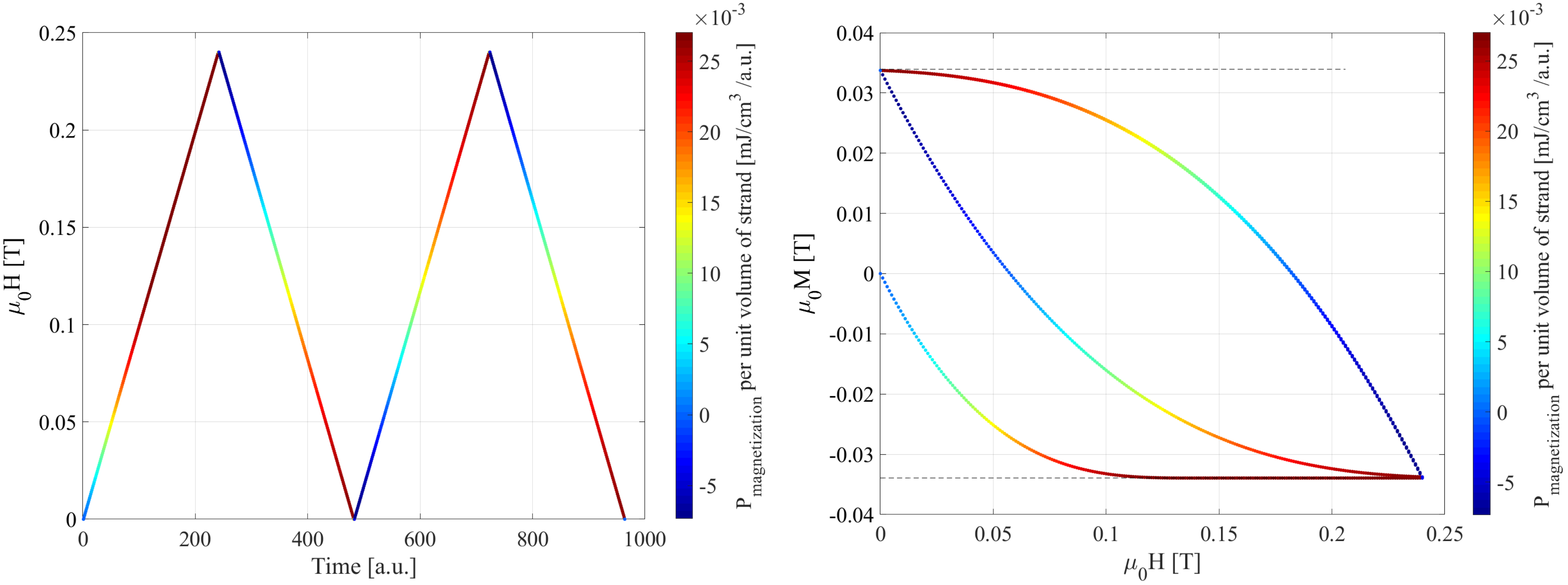


Cycle B : 0→+Hp→0→+Hp→0

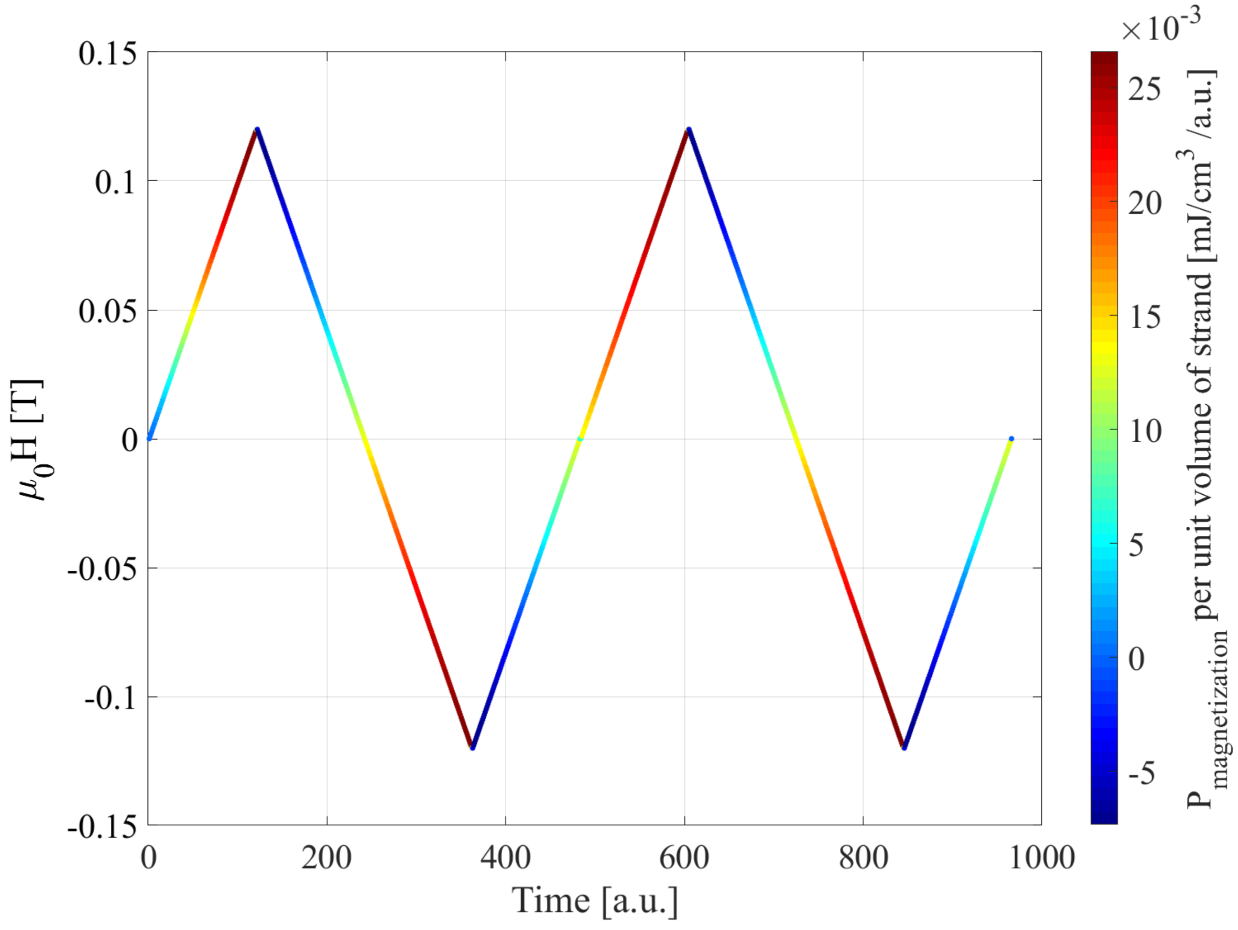


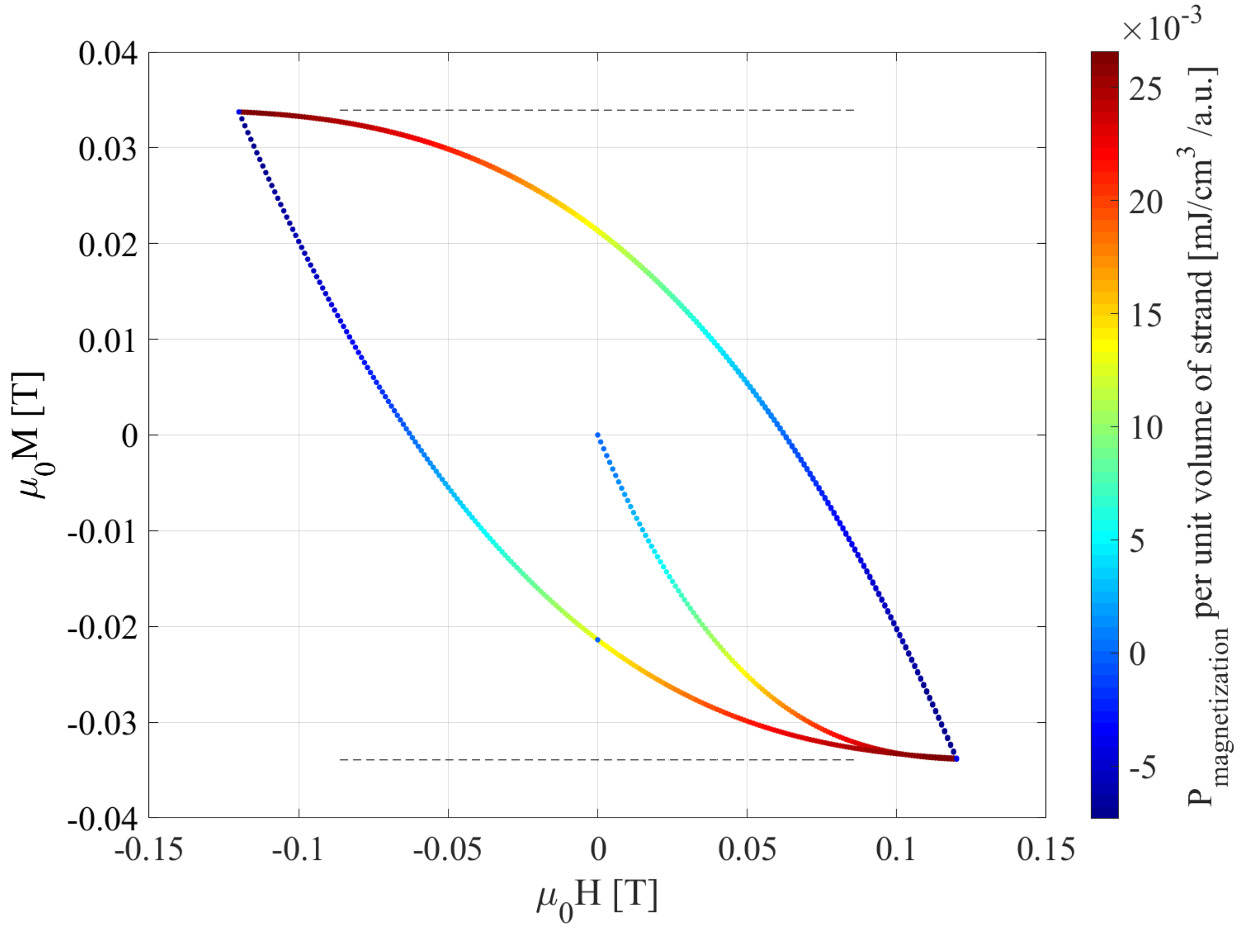


Cycle C : 0→+2*Hp→0→+2*Hp→0

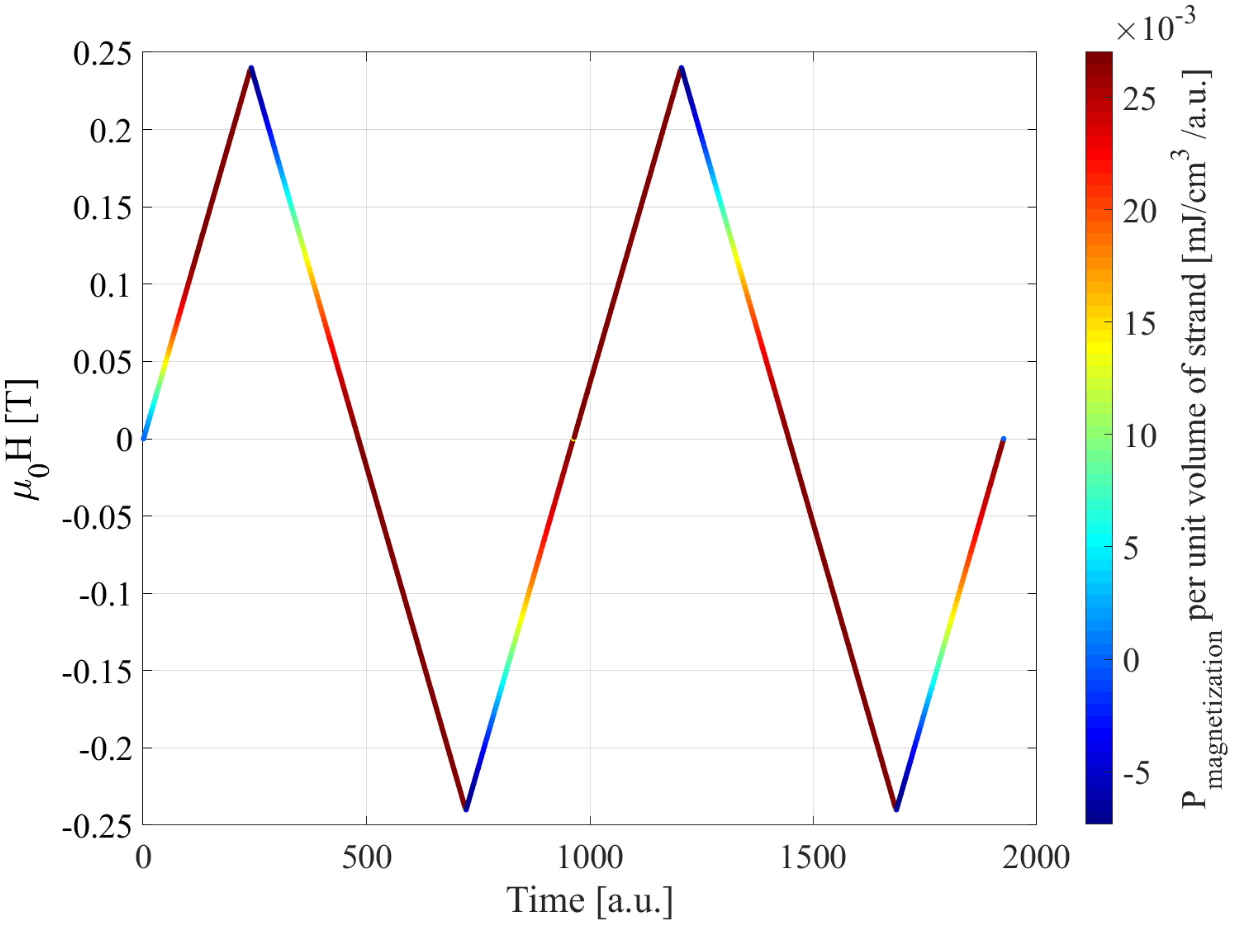


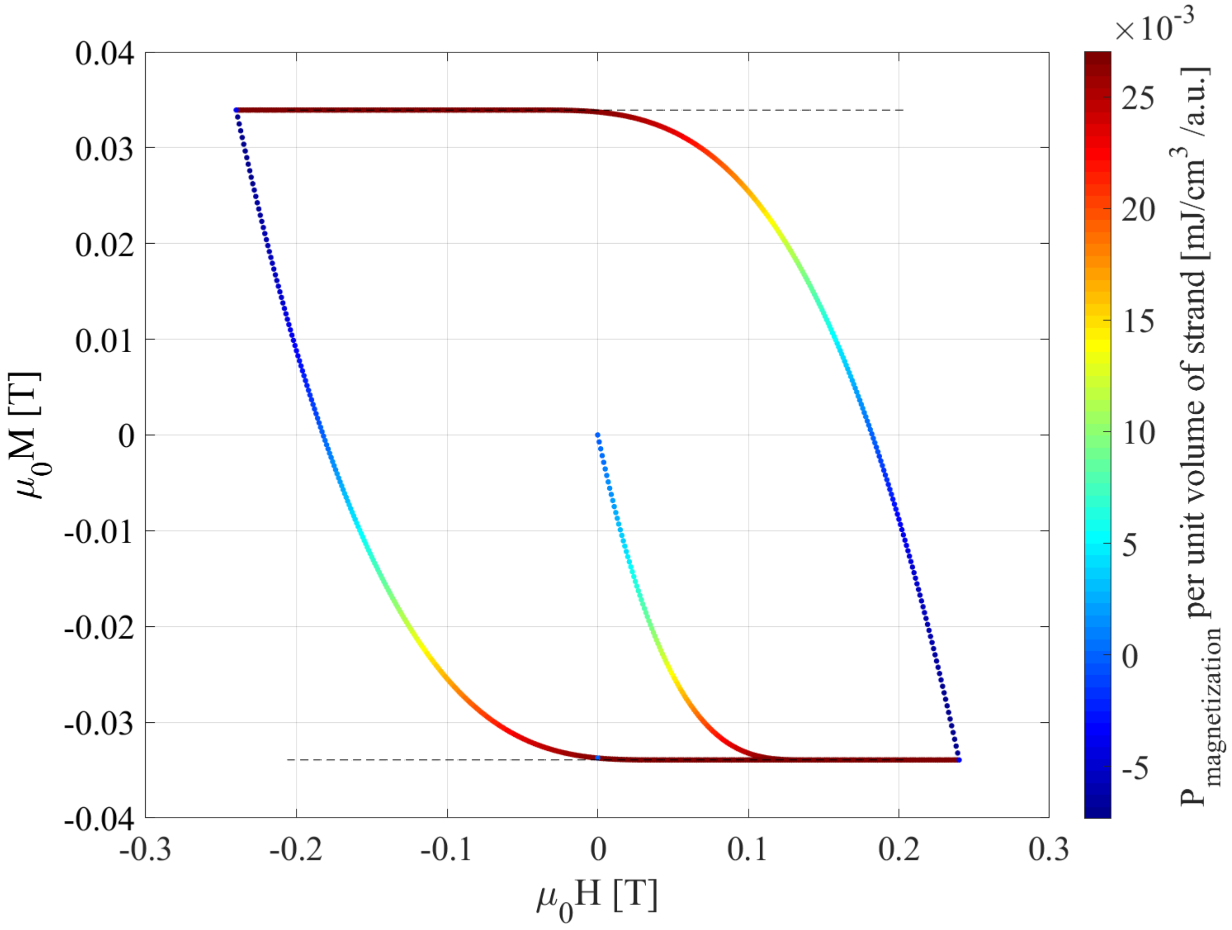
Cycle D : 0→+Hp→-Hp→+Hp→-Hp→0



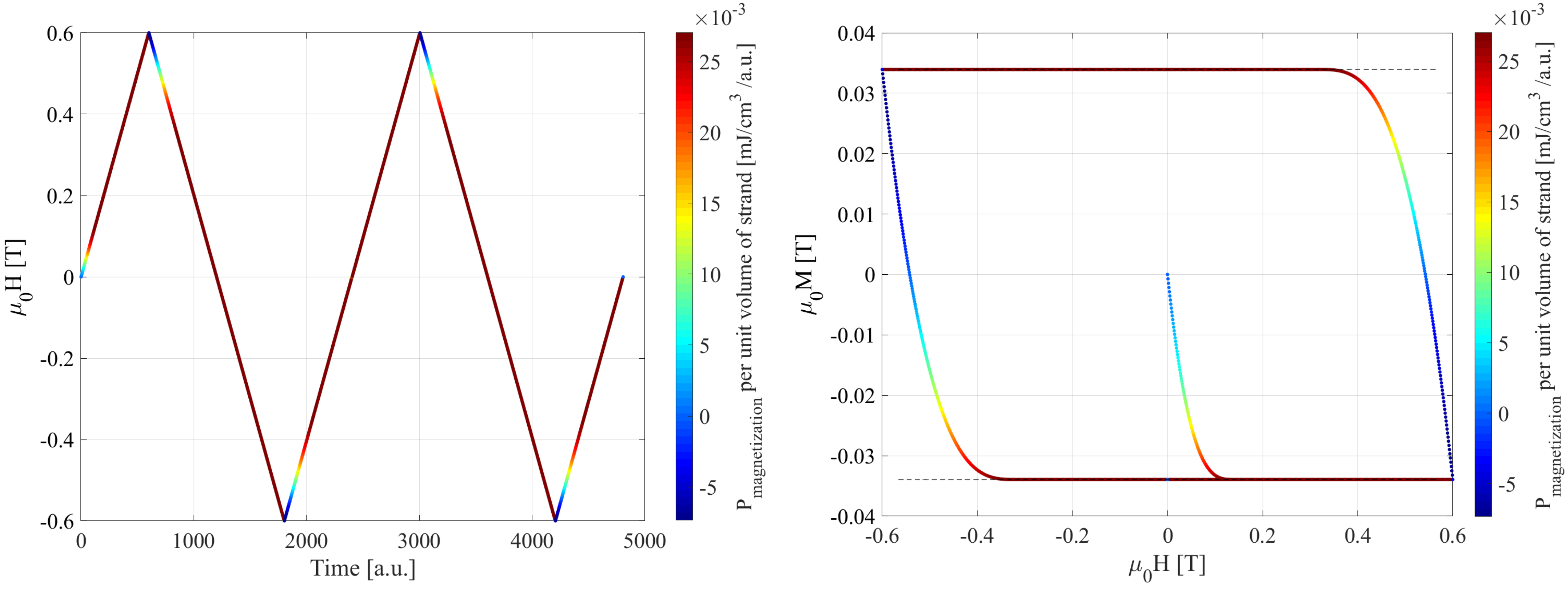


Cycle E : 0→+2*Hp→-2*Hp→+2*Hp→-2*Hp→0

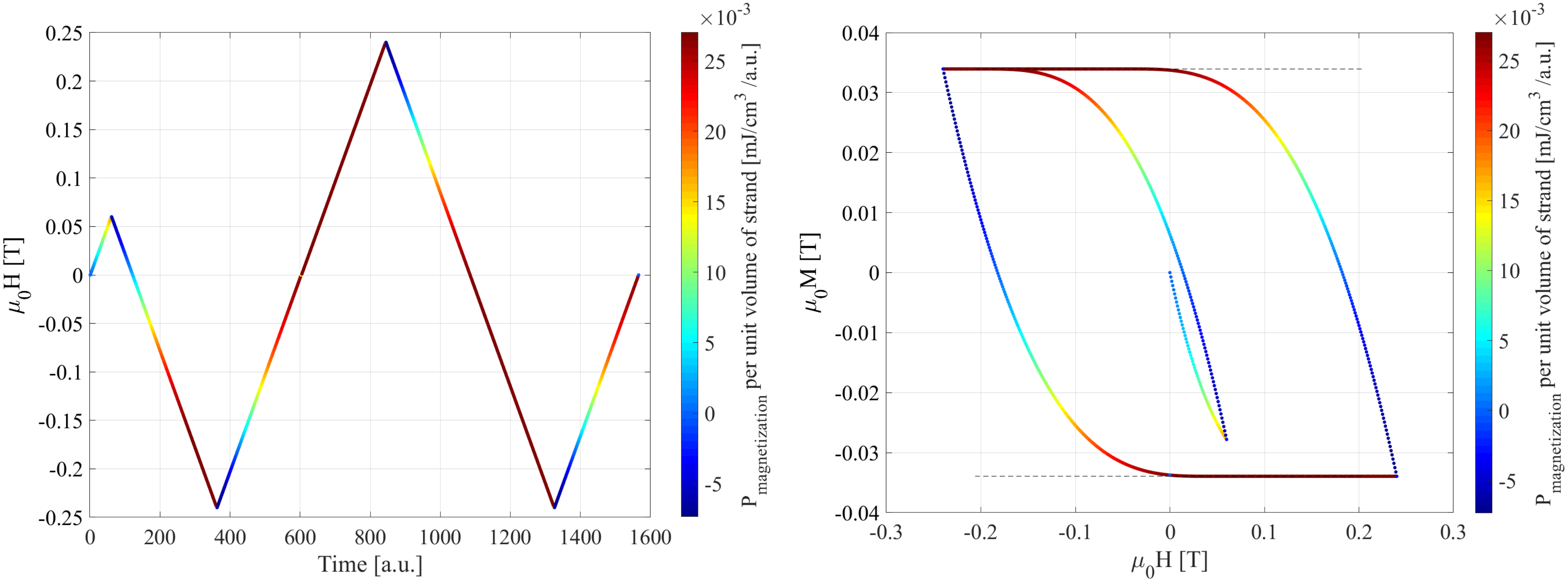




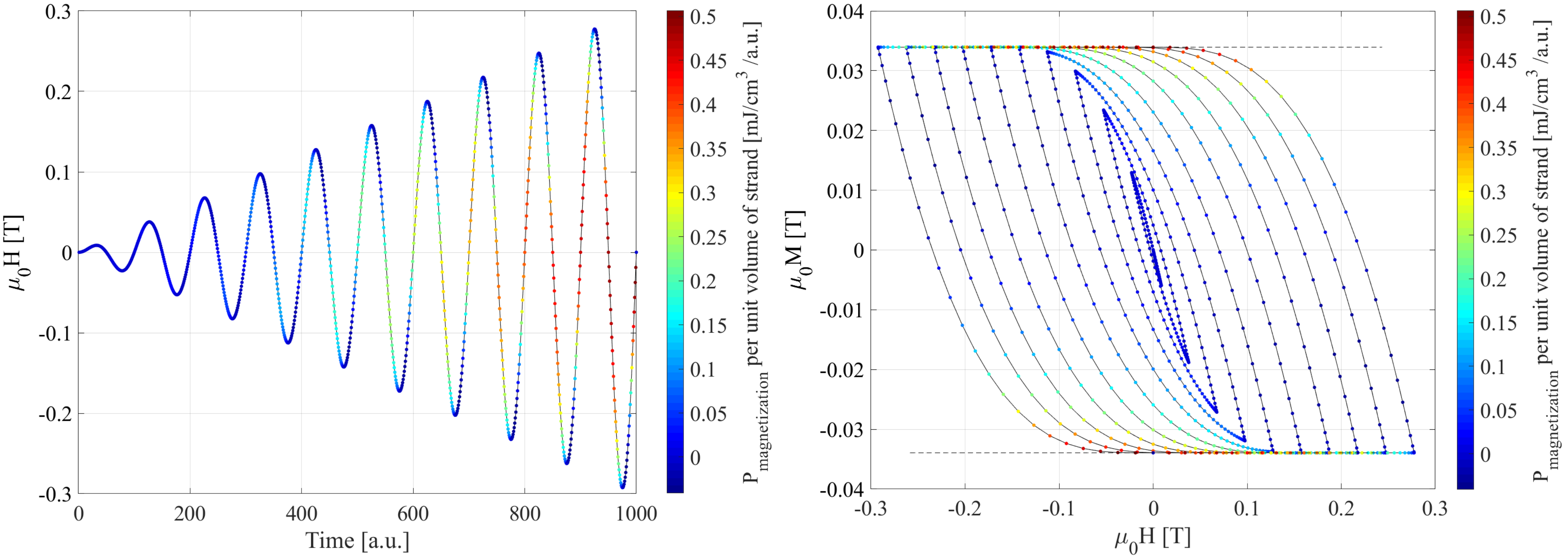
Cycle F : 0→+5*Hp→-5*Hp→+5*Hp→-5*Hp→0



Cycle G : 0→+0.5*Hp→-2*Hp→+2*Hp→-2*Hp→0

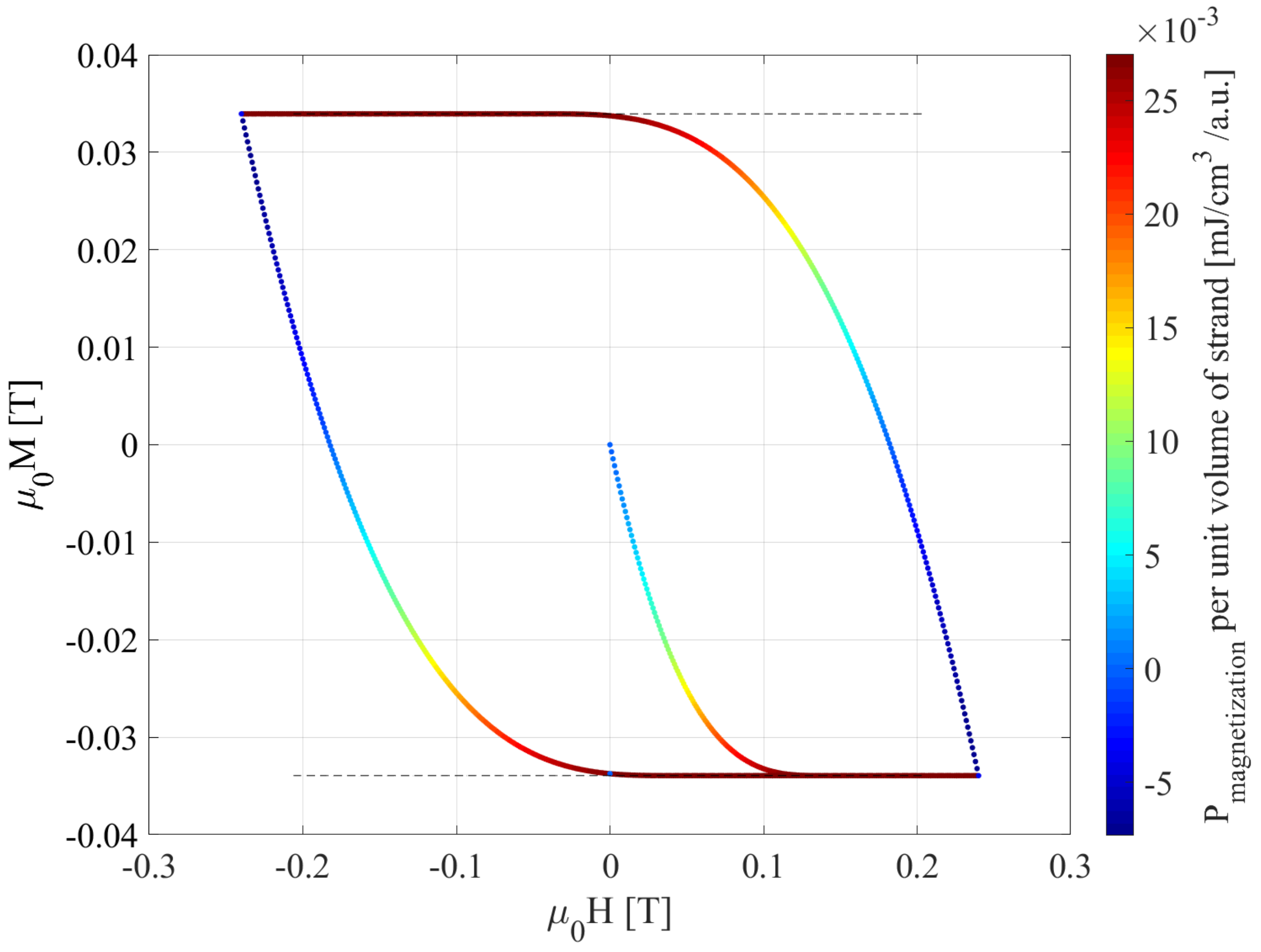


Cycle H : Sinusoid with linearly increasing amplitude



Hysteresis loss in the superconductor

Hysteresis loss per cycle and instantaneous hysteresis loss



For a closed magnetic cycle (starting and ending with the same H and M)
The loss per cycle per volume of wire is proportional to the area of the magnetization loop. This is shown in [1-2] and many others.

$E_{cycle}''' = int_{loop}(H*dM)$

Note that for a closed loop all these hold:

$int_{loop}(H*dH) = 0$

$int_{loop}(M*dM) = 0$

$E_{cycle}''' = int_{loop}(H*dB)$

$E_{cycle}''' = int_{loop}(H*dM)$

$E_{cycle}''' = -int_{loop}(M*dH)$

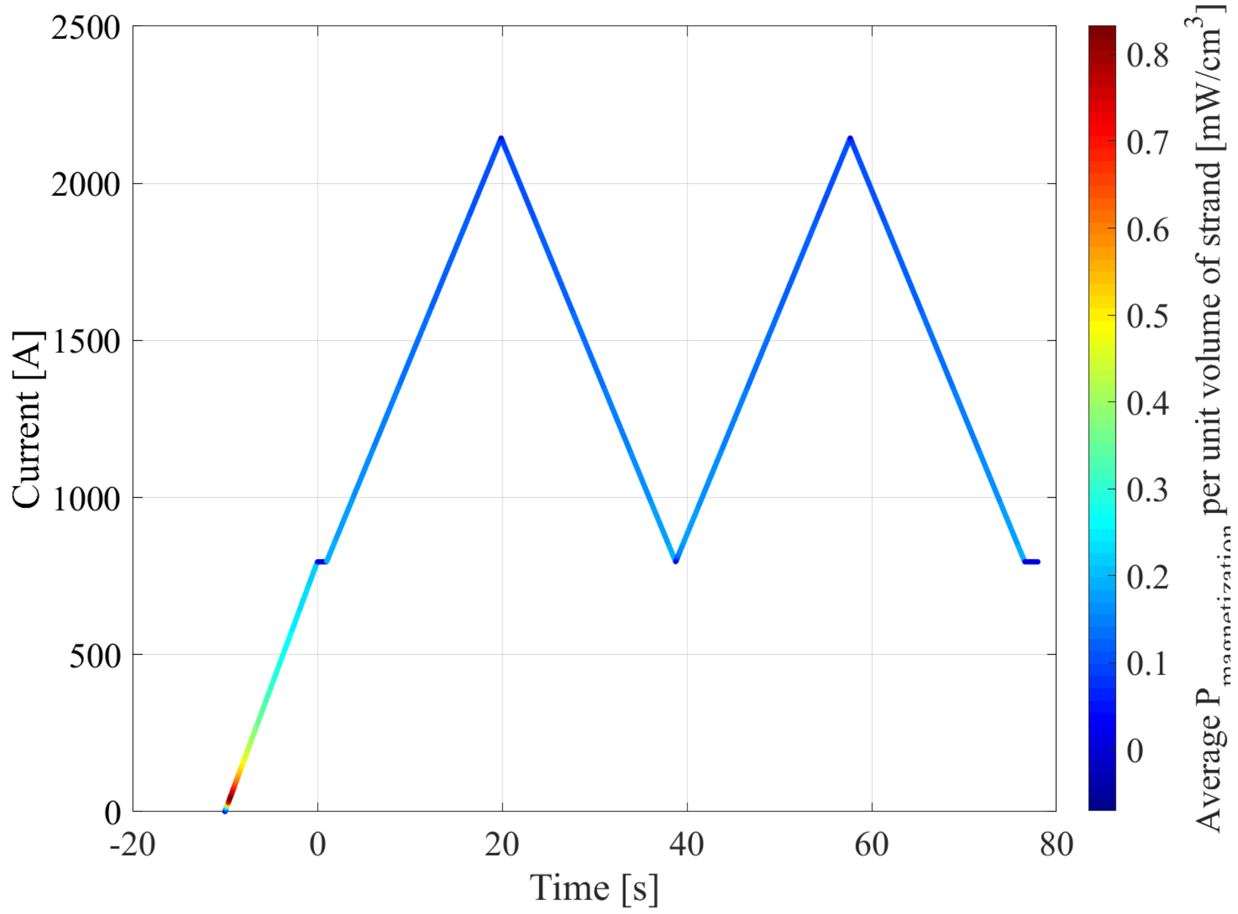
$E_{cycle}''' = -int_{loop}(M*dB)$

Instantaneous loss is P''' = -M*dB/dt [4]

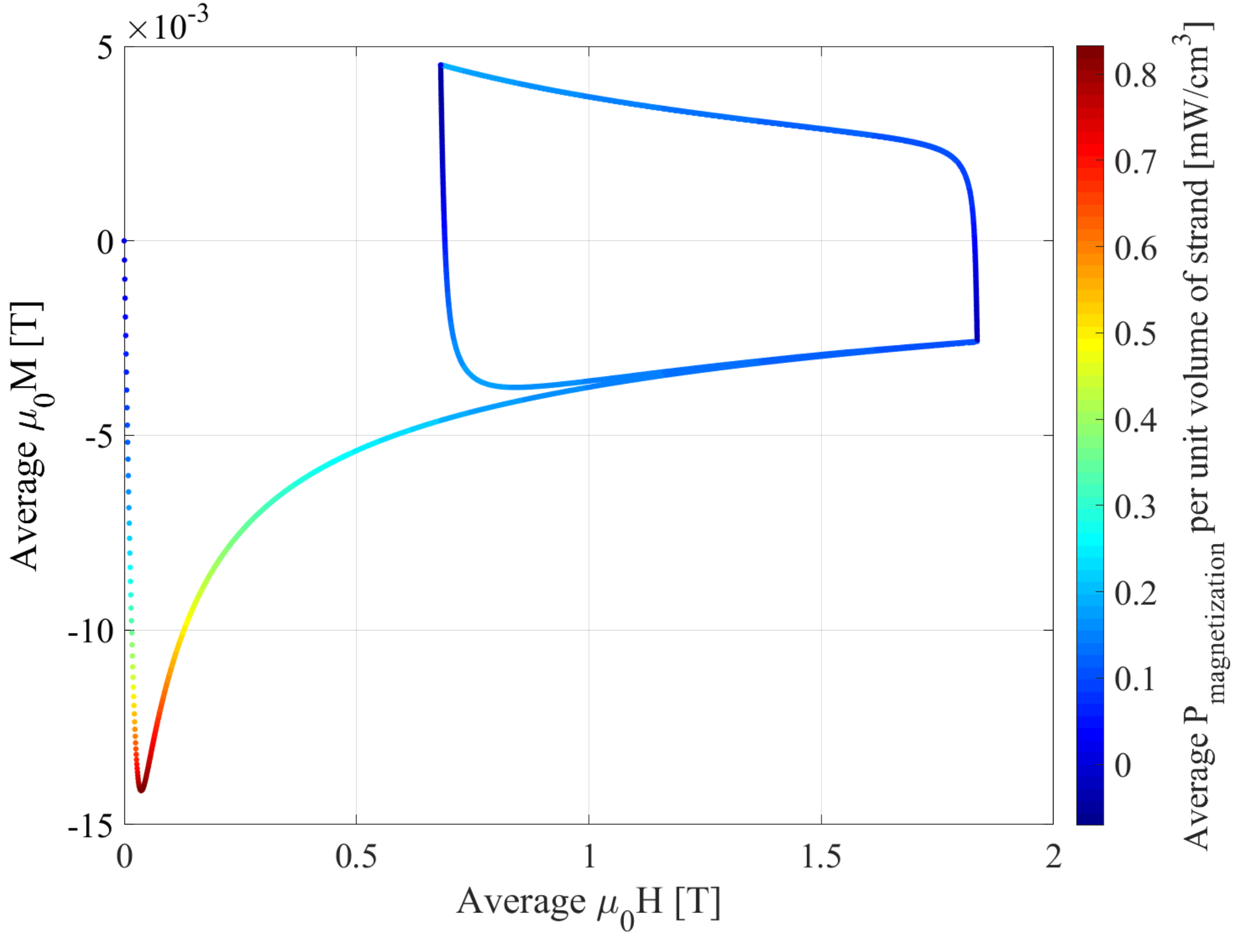
Cross-check of persistent-currents magnetization and hysteresis loss by comparing to ROXIE

Cycle #3

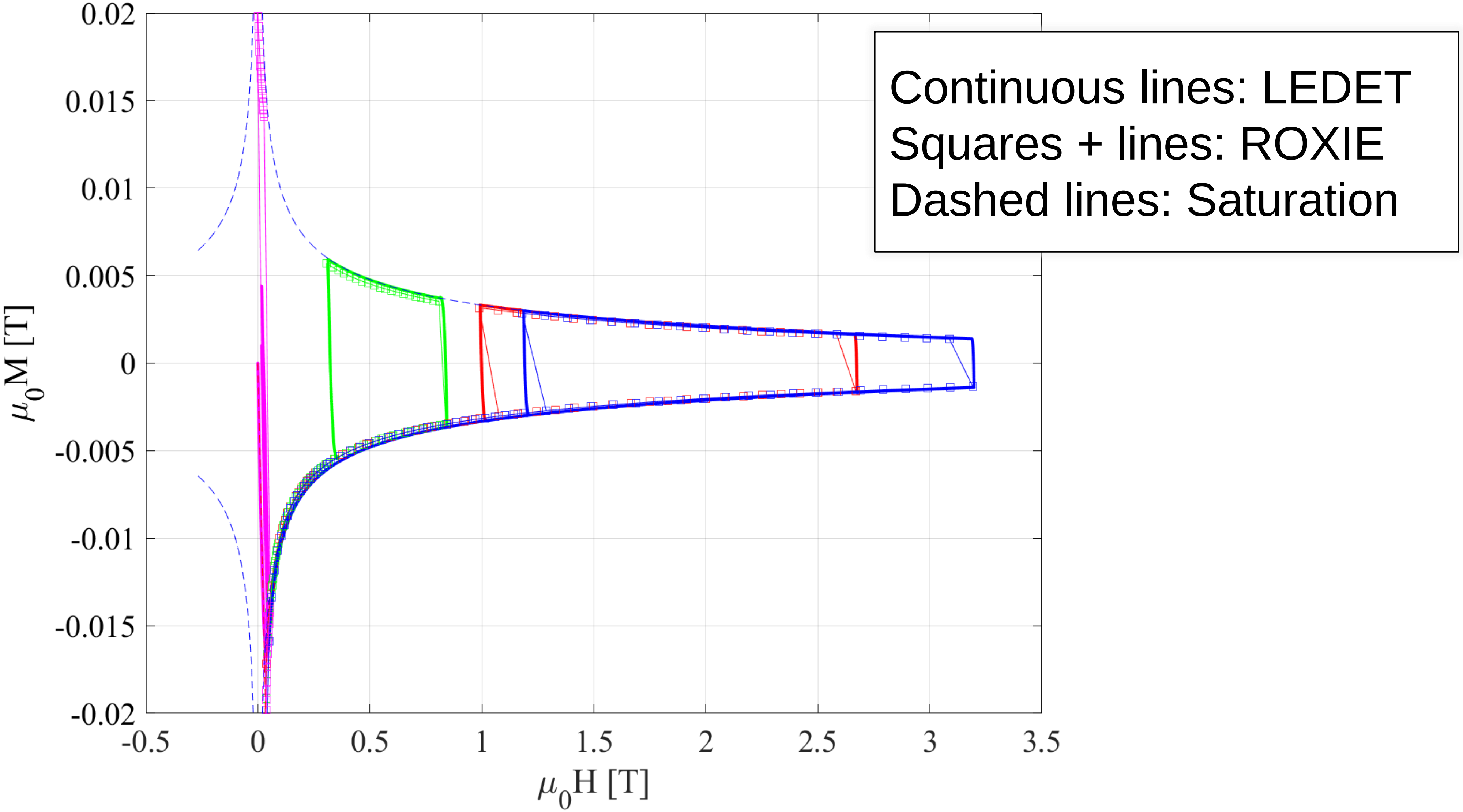
I vs time



Average μ_0*M vs Average μ_0*H

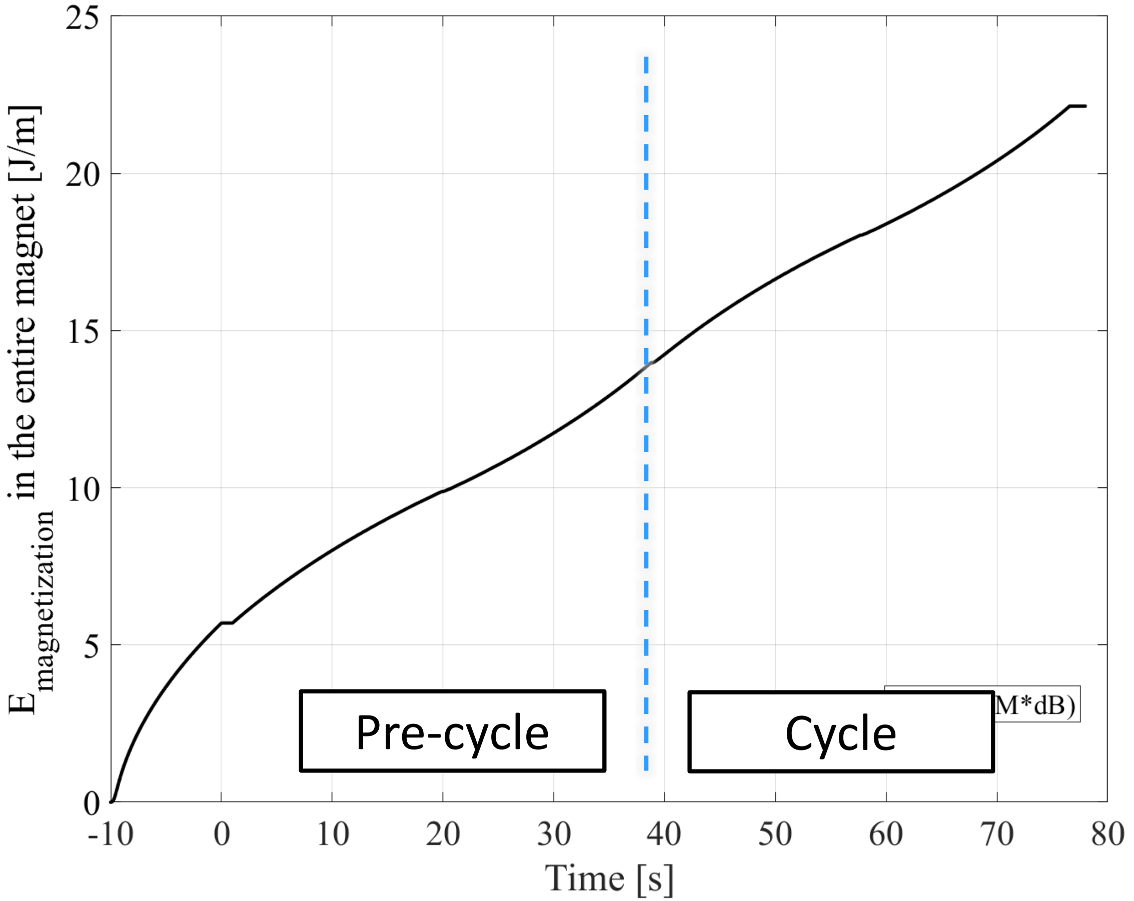


Cycle #3 – Four selected strands – Comparison with ROXIE

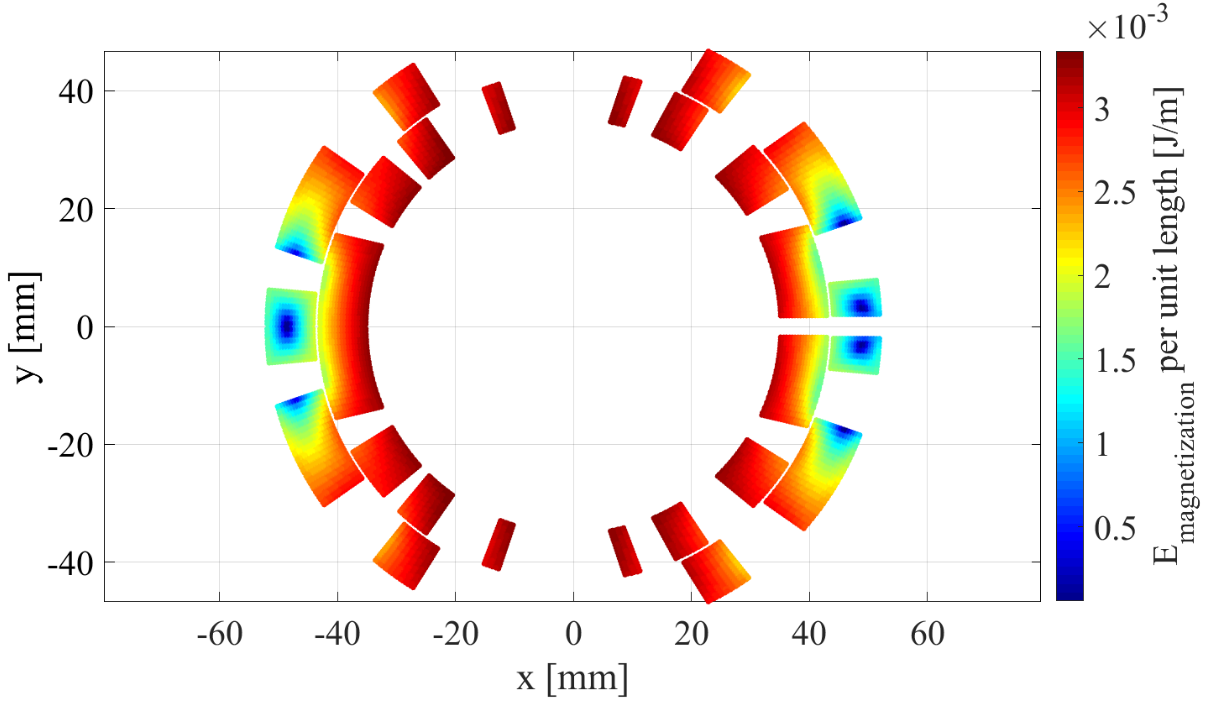


Cycle #3

Integrated loss per unit length



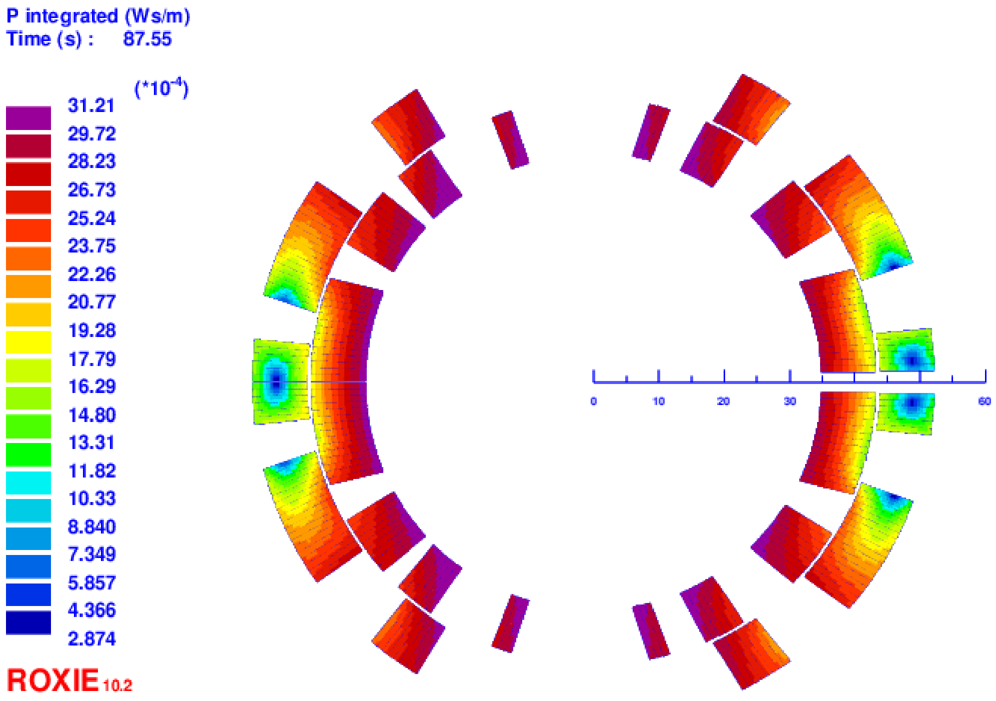
Integrated loss per unit length

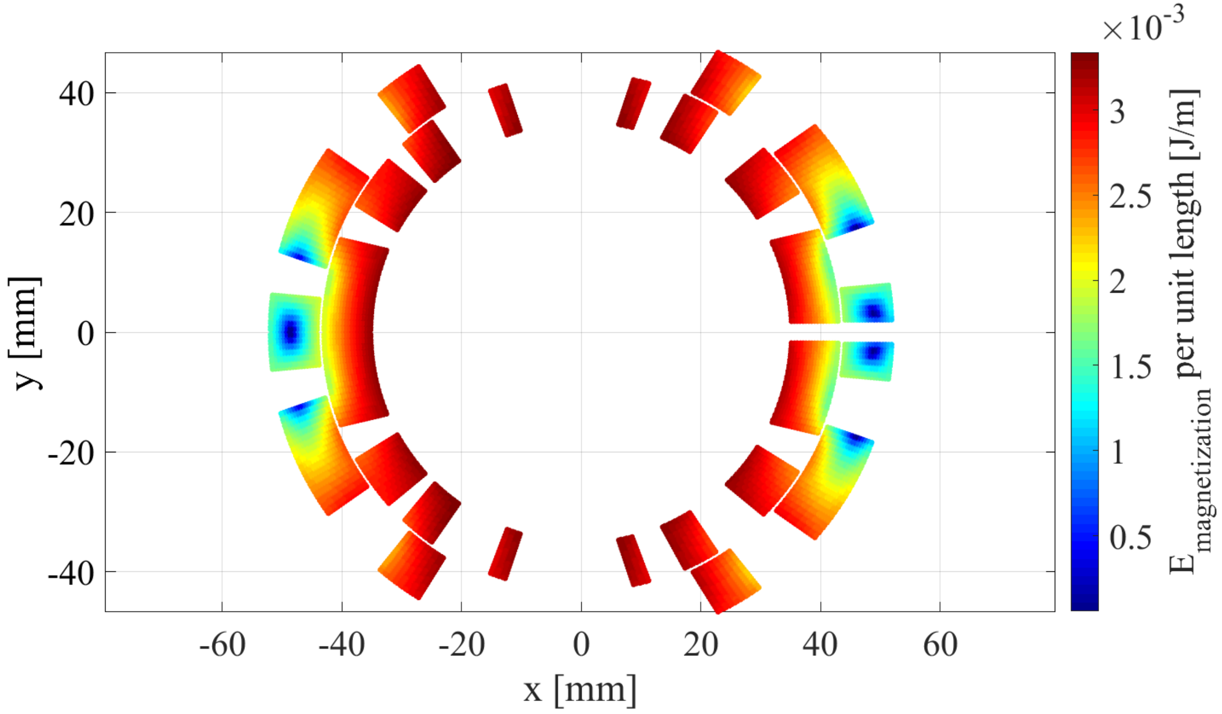


Pre-cycle + Cycle #3 – comparison with ROXIE

ROXIE – Integrated loss per unit length

LEDET – Integrated loss per unit length





Comparison with ROXIE – T=6 K

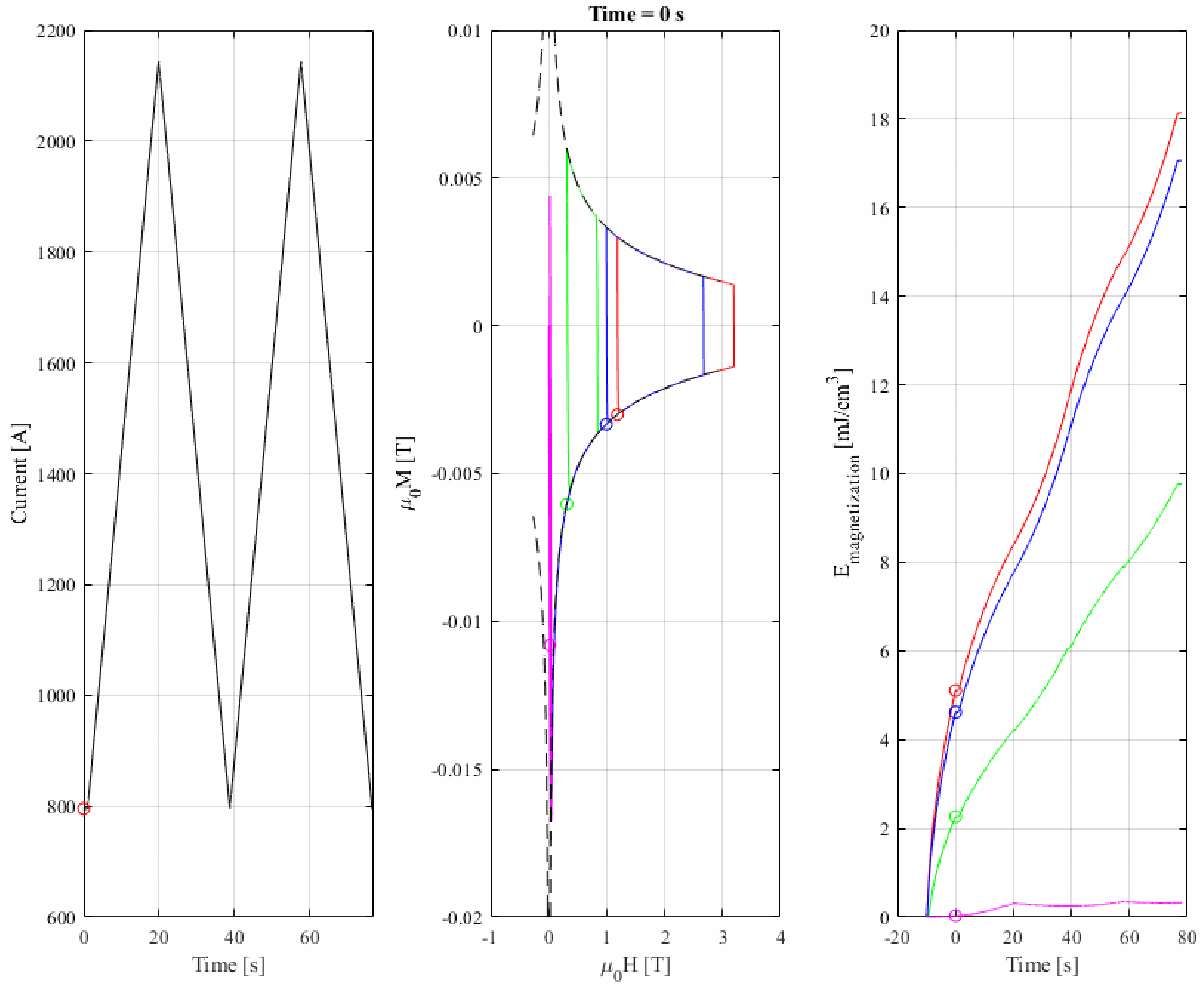
PC loss per cycle [J/m]	ROXIE Bottura's fit From Mikko	ROXIE Bottura's fit Critical state LITERNL on	ROXIE Bottura's fit Scalar model LITERNL on	LEDET Bottura's fit Df=7 um	Error LEDET Cpr ROXIE
Pre-cycle #2				19.22	
Cycle #2	17.71			17.64	-0.5%
Pre-C+C #2		37.40	35.56	36.86	-1%
Pre-cycle #3				14.00	
Cycle #3	7.84			8.14	+5%
Pre-C+C #3		20.99		22.14	+5%

Comparison with ROXIE – T=1.9 K

PC loss per cycle [J/m]	ROXIE Bottura's fit Critical state LITERNL on	LEDET Bottura's fit Df=7 um	Error LEDET Cpr ROXIE
Pre-cycle #2		53.84	
Cycle #2		45.55	
Pre-C+C #2	109.79	99.39	-9%
Pre-cycle #3		40.28	
Cycle #3		24.09	
Pre-C+C #3	62.96	64.37	+2%

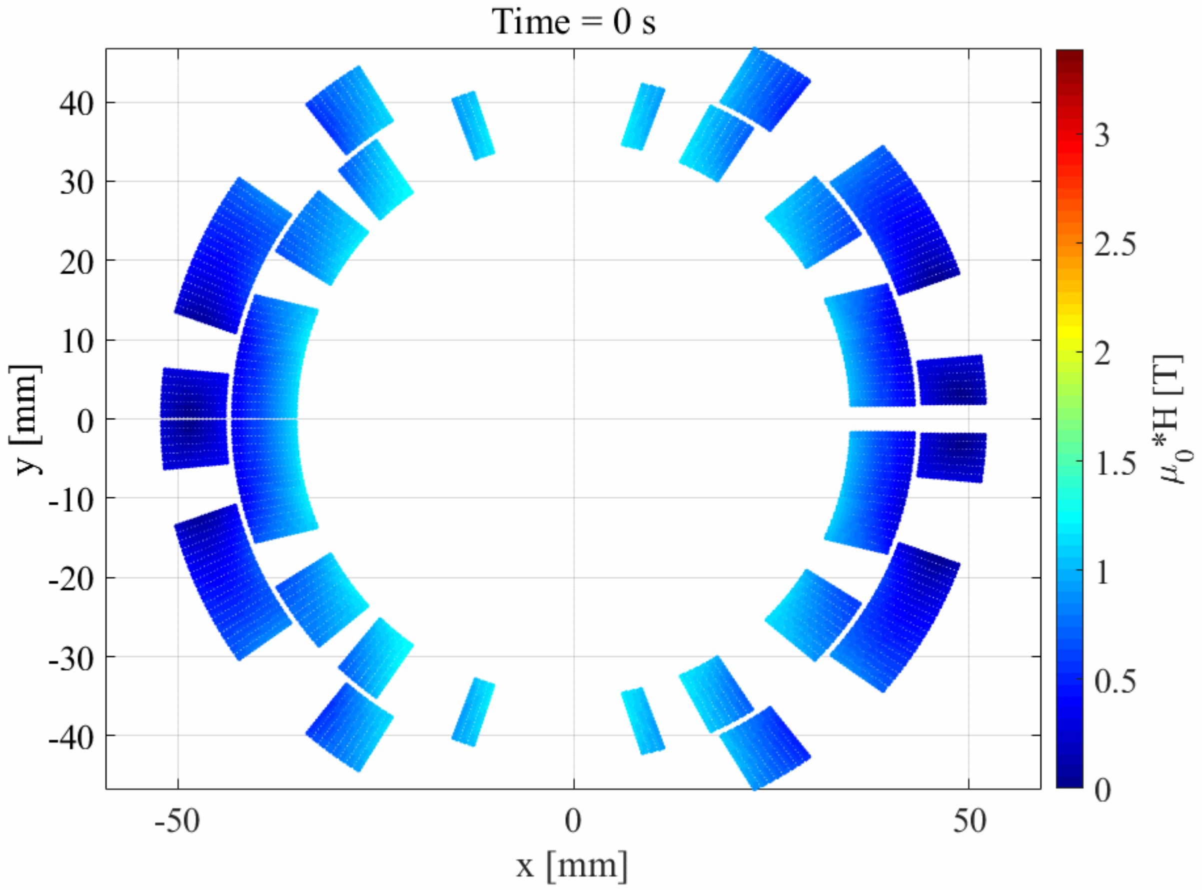
Detailed results of Cycle #3

Cycle #3 – Four selected strands

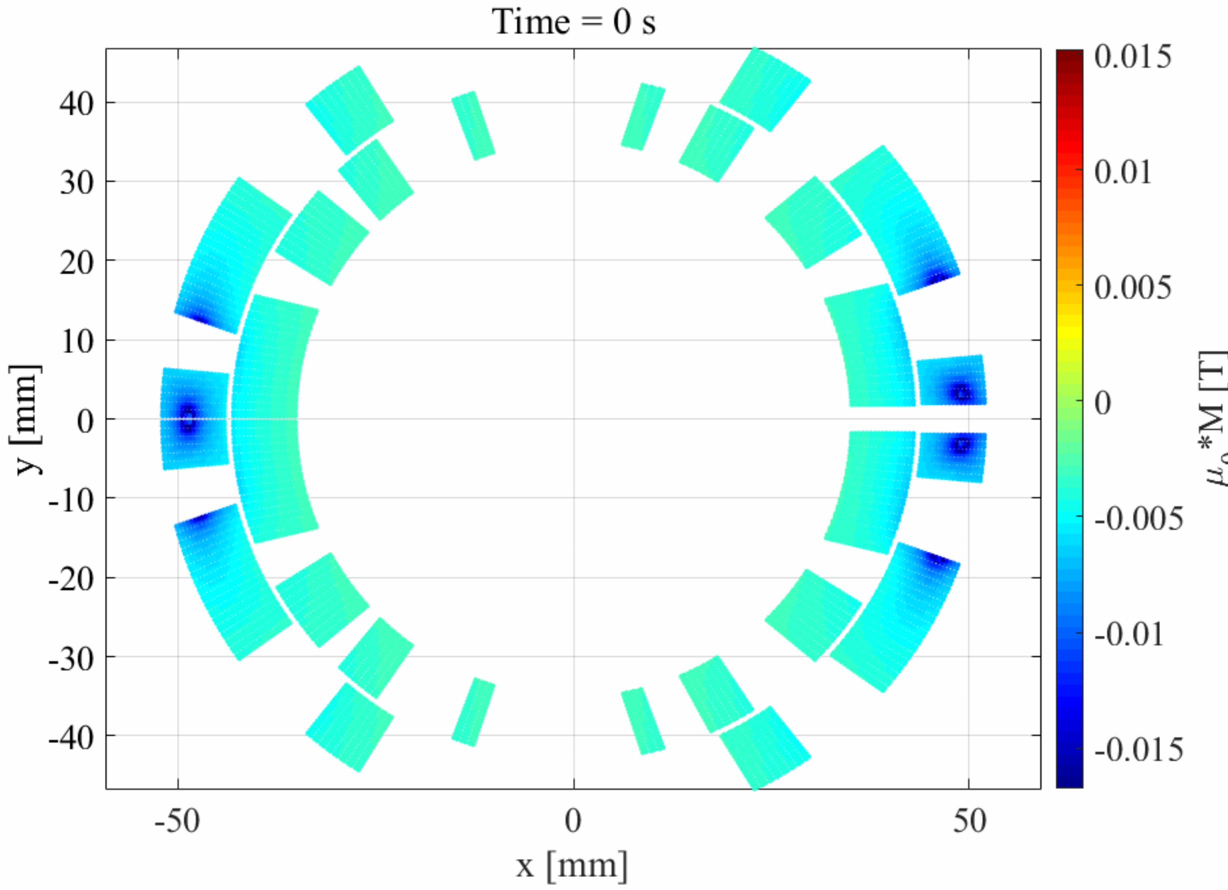


Cycle #3

μ_0*H

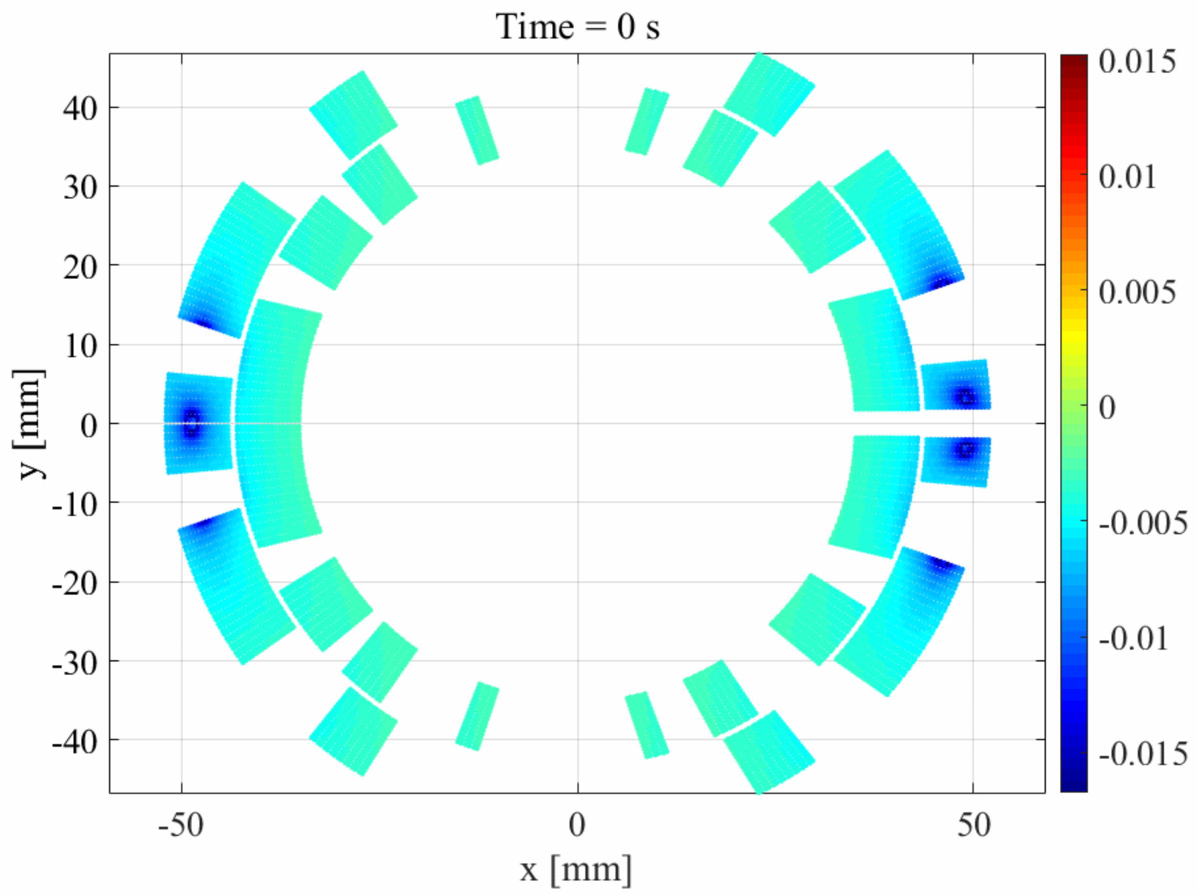


μ_0*M in the strand volume

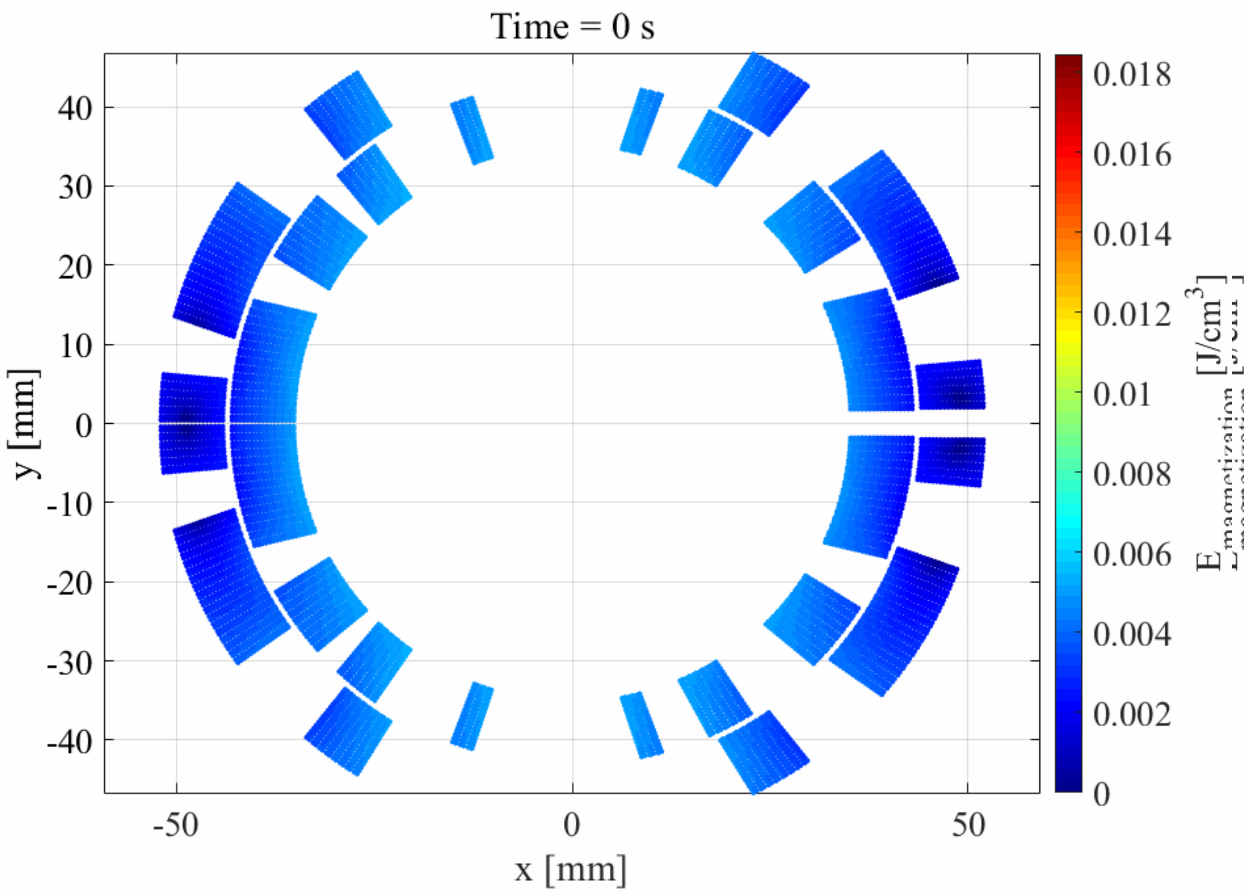


Cycle #3

μ_0*M in the strand volume

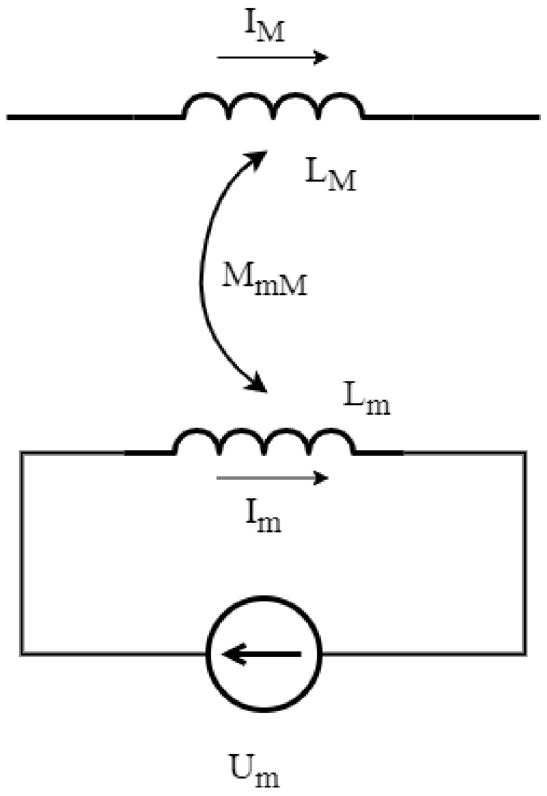


Integrated loss per unit volume



Effect of magnetization on magnet differential inductance

Equivalent electrical model for magnetization effects



This equivalent electrical model is proposed to better understand the energy exchanges involved in the magnetization process, and to evaluate the effect of magnetization on magnet differential inductance.

To model the stored/lost energy contributions in the volume of the superconductor:

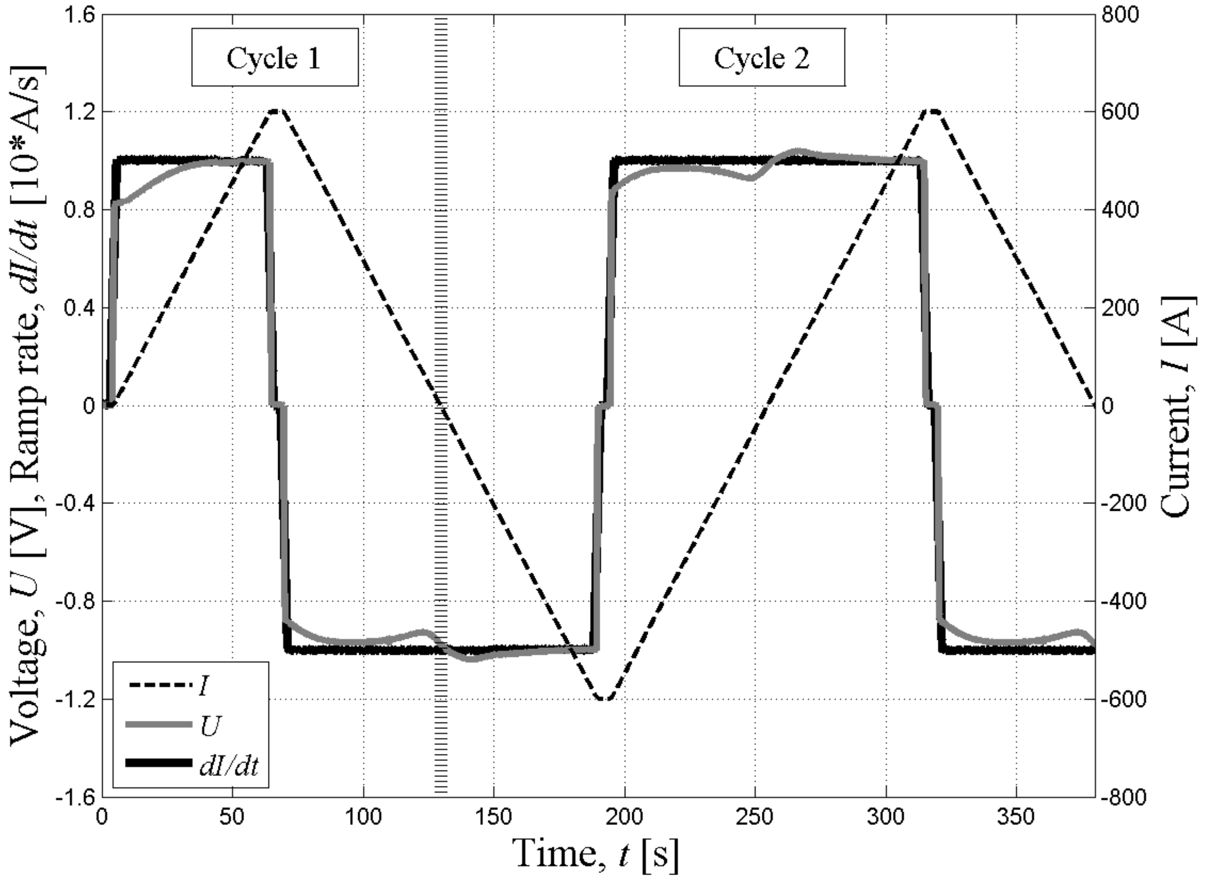
- H field is proportional to the magnet current I_M → $H = f_{mag} * I_M$ (transfer function f_{mag} defined in [1/m])
- M is proportional to magnetizing current I_m → $M = I_m/d_s$ (as proposed in [10])
- M is a function of H and its history → $M = f(H)$ ↔ $I_m = f(I_M)$
- Magnetization loss per unit volume is M*dB/dt → Total loss = $U_m * I_m$ [demonstrated in the next Annex]

Note the absence of resistors in the circuit: the loss comes from the current source.

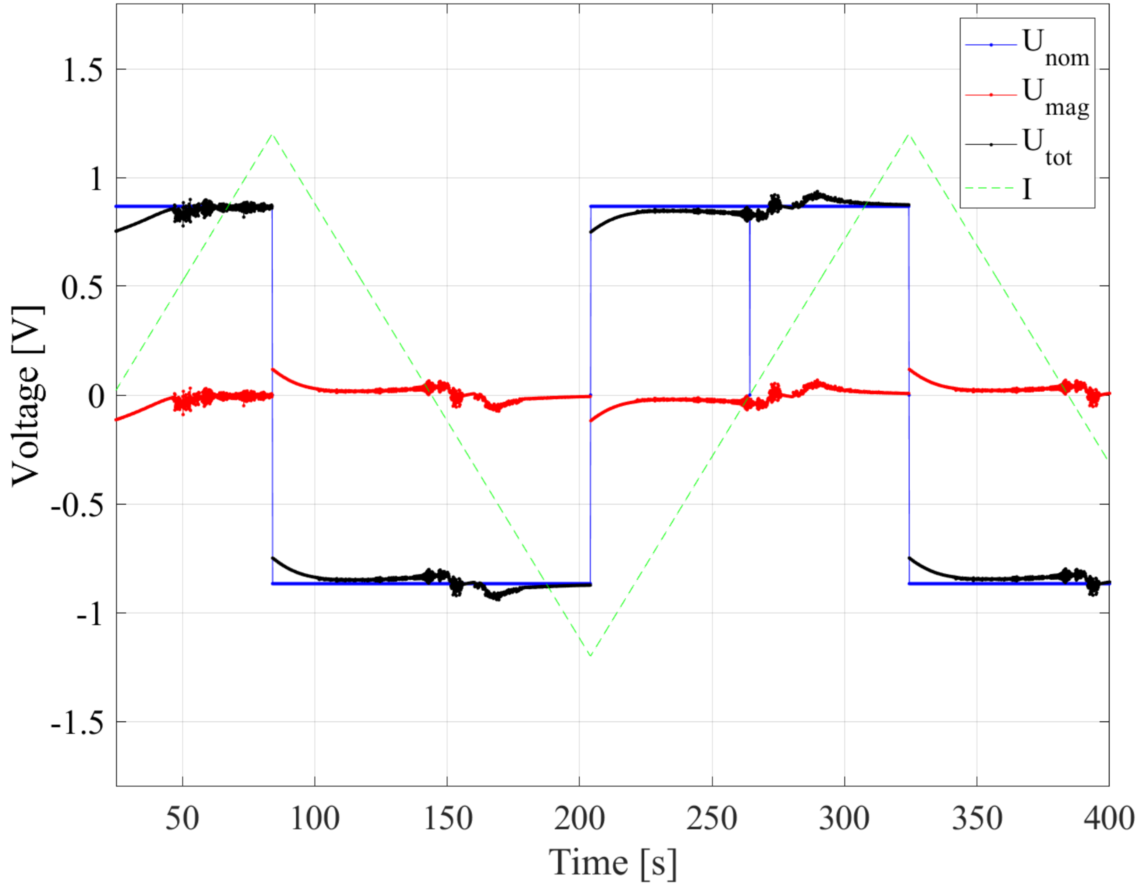
First attempt at validation using MB experimental data

Effect of magnetization on magnet differential inductance - PRELIMINARY

MB – Experimental [11]
Inductive voltage vs Time

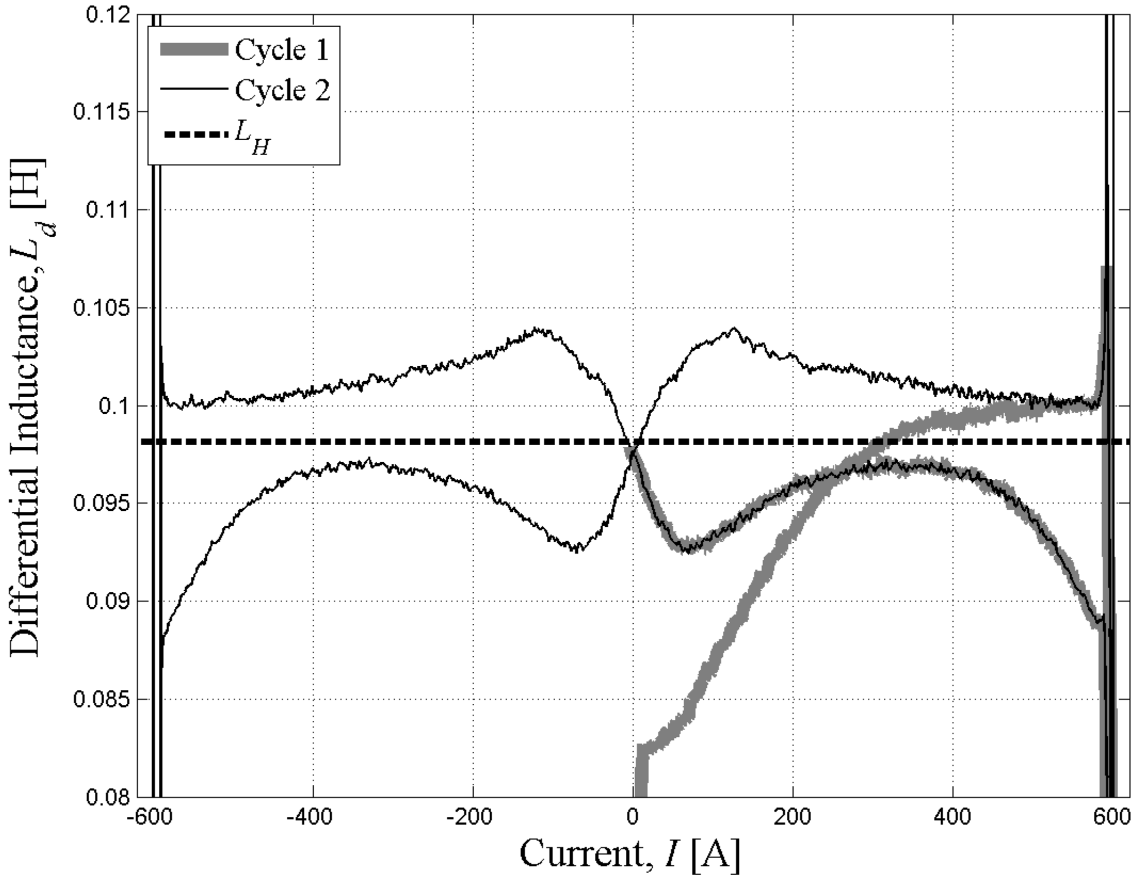


MB – Simulated
Inductive voltage vs Time

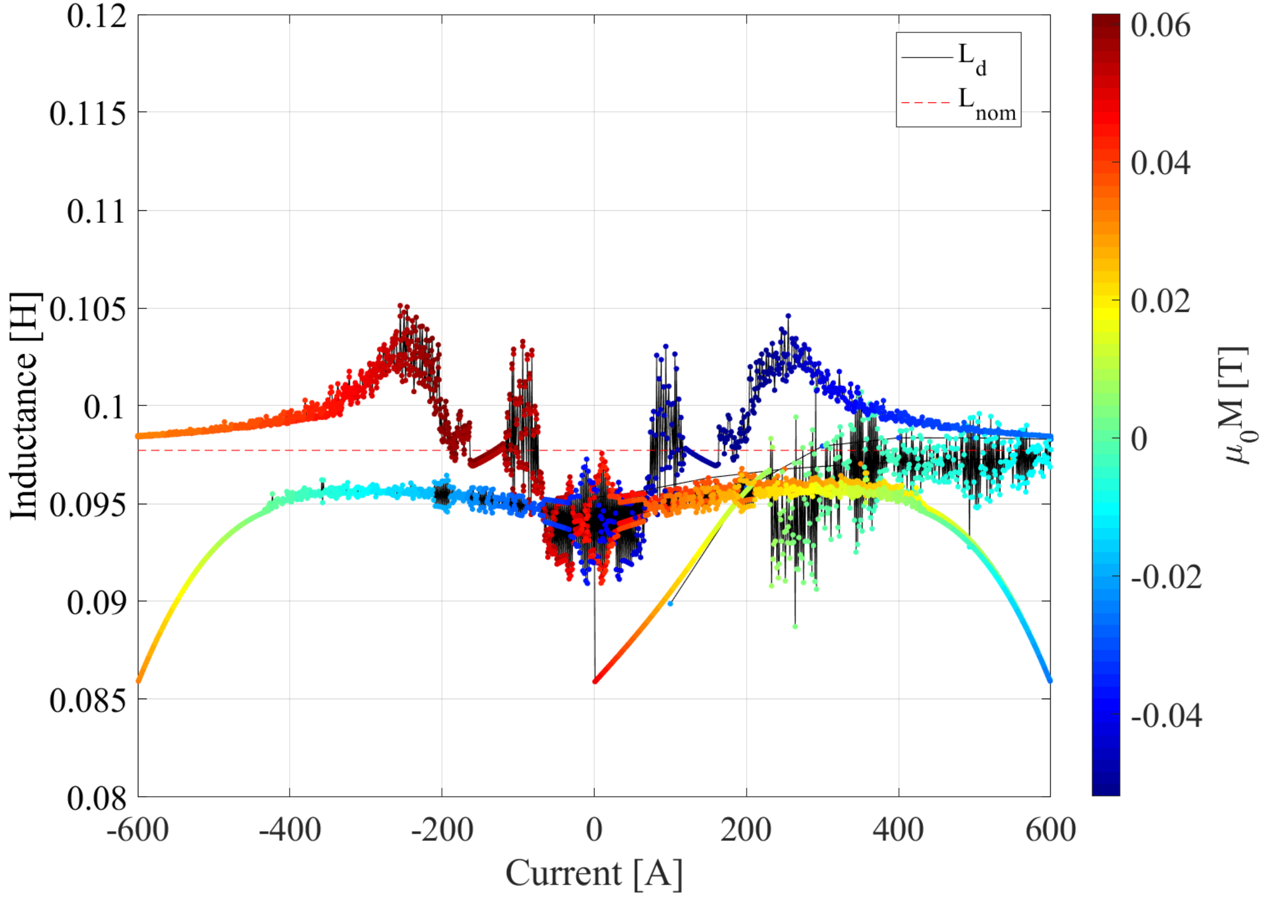


Effect of magnetization on magnet differential inductance - PRELIMINARY

MB – Experimental [11]
Differential inductance vs Current



MB – Simulated +11 mH [iron-yoke contribution]
Differential inductance vs Current



Magnetization in STEAM-LEDET – Status [Yellow=Next steps]

Feature	Studied	Implemented	Cross-checked	Validated
Jc(T,B) fits for Nb-Ti	X	X	X	
Jc(T,B) fits for Nb_3Sn	X	X		
Magnetization in a cycle	X	X	X	
"Virgin" curve slope	X	X		
Magnetization for any transient	X	X		
Loss in a cycle	X	X	X	
Instantaneous loss	X	X	X	
Effect on differential inductance	X	X		
Effect of field changing direction	X			
Interaction with IFCC	X			
Magnetization in CLIQ transient	X			
Implemented in LEDET exe	X			
Effect of iron-yoke				

Achievements and remaining challenges

Achievements

- Persistent currents and magnetization loss implemented in LEDET
- Four different Jc(T,B) fits implemented: Jc=constant [Bean's model], Bottura's fit, CUDI fit, Summer's fit
- Formulation is analytical and follows the Bean's model [1-2] adjusted for varying Jc(T,B), polarities, partial magnetic cycles
    - Calculation checked for different transients with different polarities, amplitudes, magnetic histories
    - Instantaneous loss per unit volume is calculated as M*dB/dt [4]
- Cross-checked with ROXIE for two different cycles
    - Good agreement with the magnetization amplitude vs B and distribution in the cross-section
    - Good agreement with the integrated PC loss [1-5% difference] and distribution in the cross-section
- Effect of magnetization on the magnet differential inductance [first attempt at validation]
- Simulation time: <5 mins for ~1e4 time steps, ~1e4 strands [ROXIE: ~2 hours for ~120 time steps]
- *Very interesting musing about the nature of magnetization and losses*

Remaining challenges

- Implement this feature in the main LEDET application
- Erratic magnetization at low field when Hp(B) changes quickly with H
- Double-check which part of the magnetization is reversible [stored energy] and which part is irreversible [heat]
- Validate, Validate, Validate, for different magnets and types of transients

Annex

More detailed description
of the analytical magnetization calculation

Description of magnetization transients

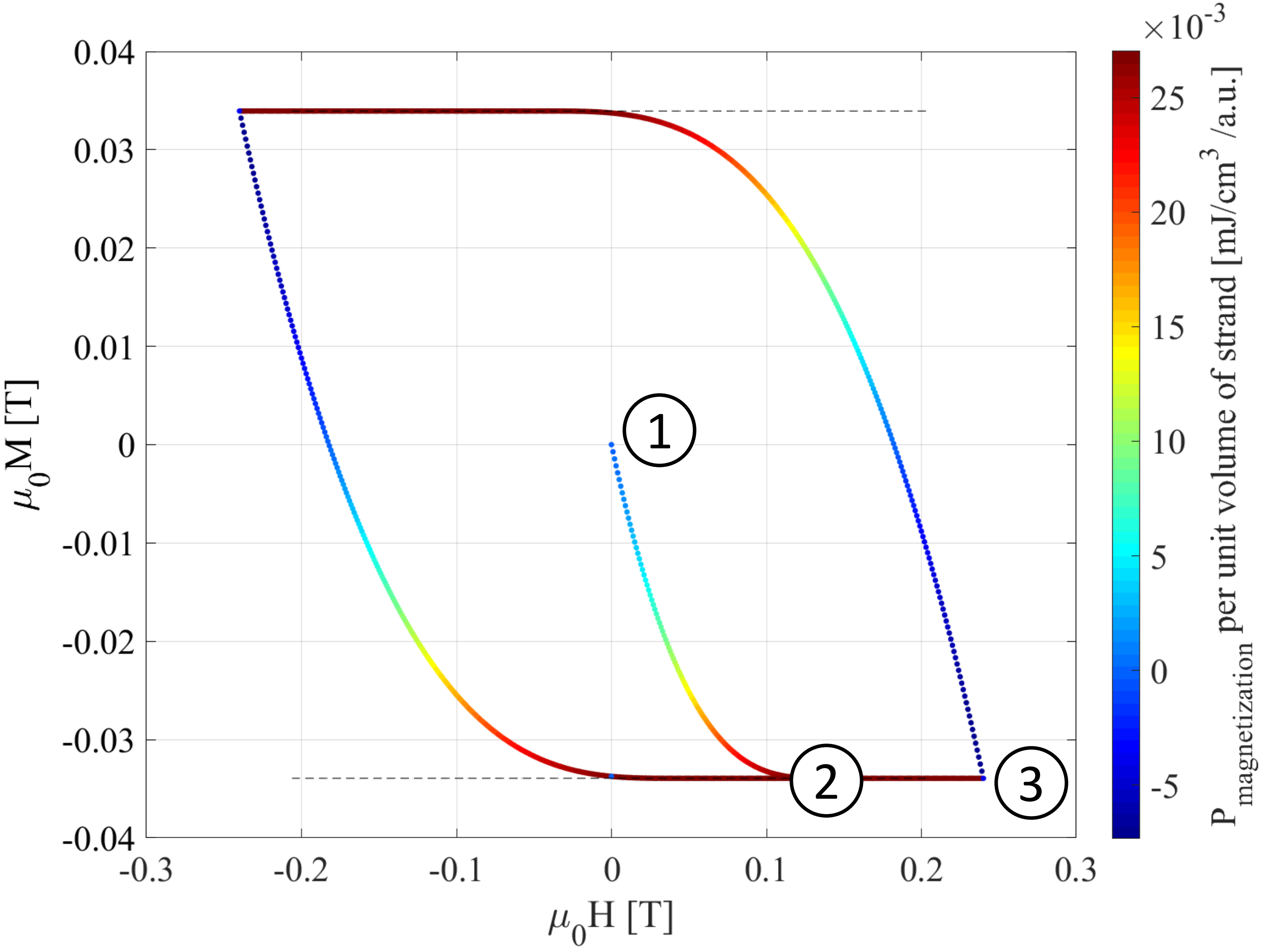
In the next slides, a magnetization cycle is described in detail.
The equations used to analytically calculate the magnetization as a function of H and its history are shown.
First, different equations for each part of the magnetization transient are shown.
Second, the equations for all parts of the magnetization transient are unified into one general equation.

Complications:

- “Virgin” curve is different from following transients (or is it?...)
- Polarity of the applied H field
- Case of ΔH inverted before the wire is fully magnetized

Note1: For the sake of simplicity, the transients plotted are calculated for constant Jc. However, in the actual implementation the Jc dependence on B is included.
Note2: The homogenized magnetization in the strand is presented, not the magnetization in the filaments

Magnetization in the “virgin” curve (1→3)



<u>Magnetization in the “virgin” curve</u>

Hp(B) = 1/π*Jc(T,B)*df

Before reaching saturation (1→2)

M = (3*H^2*Hp - H^3 - 3*H*Hp^2) / (3*Hp^2) *2 *f_{SC}

(from Bean’s model [1-2])

Saturation if:

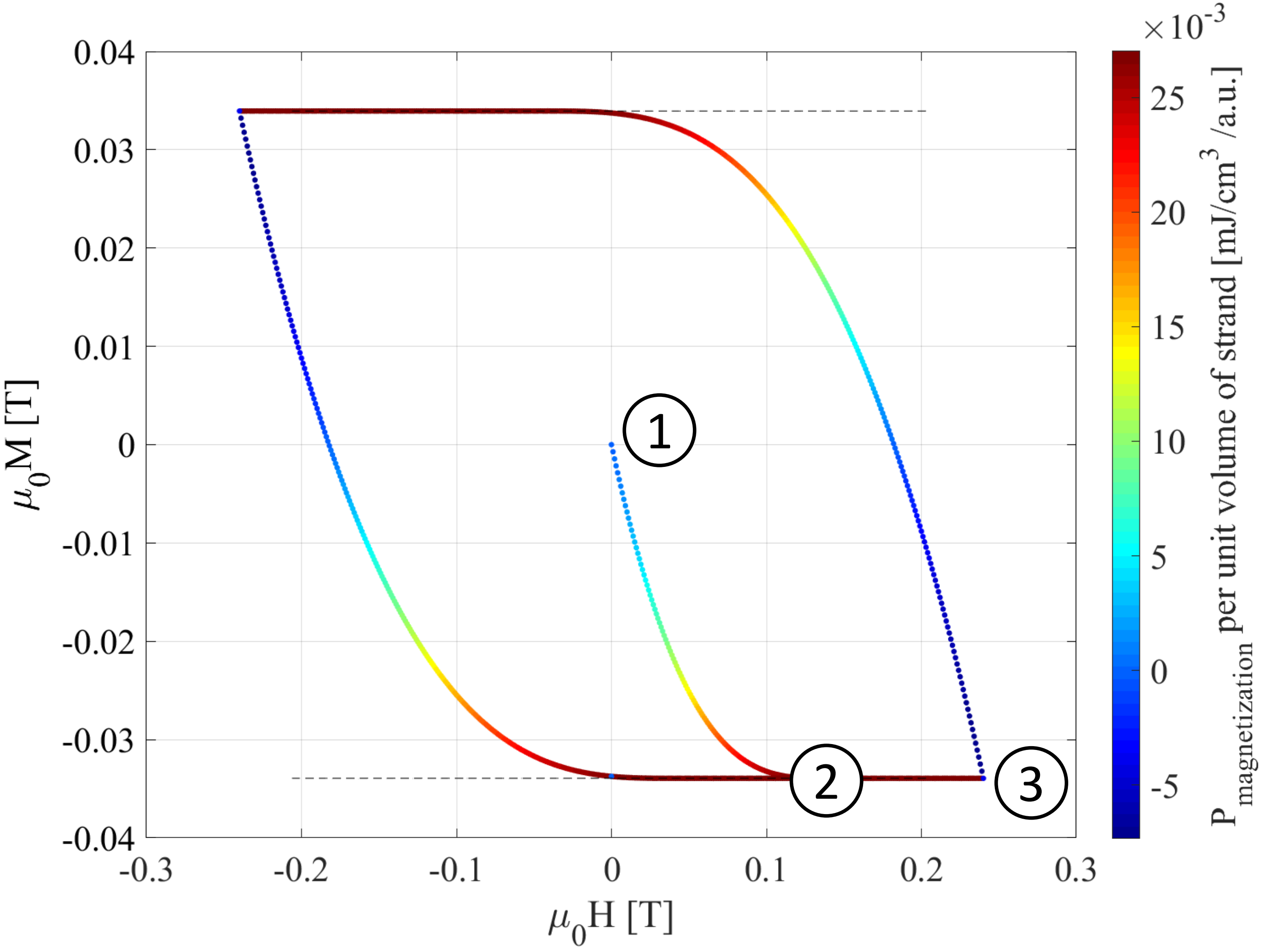
H > Hp

After reaching saturation (2→3)

M = -2/3/π*Jc(T,B)*df* f_{SC}

= -2/3*Hp* f_{SC}

Correction for the sign of the applied H field



<u>Magnetization in the "virgin" curve</u>

Hp(B) = 1/π*Jc(T,B)*df

S=sign(deltaH)

Before reaching saturation (1→2)

M = (+S*3*H^2*Hp - H^3 - 3*H*Hp^2) / (3*Hp^2) *2 *f_{SC}

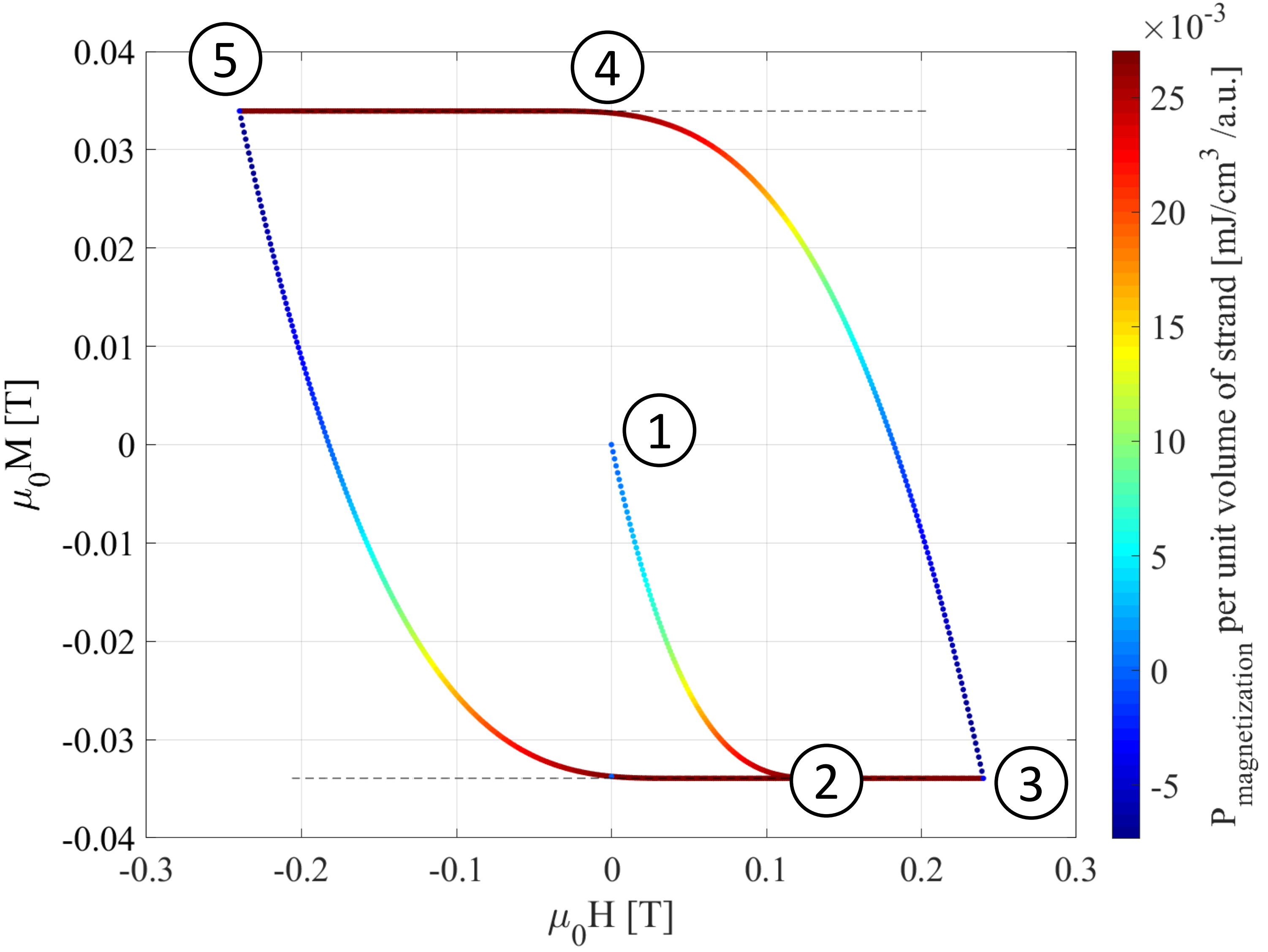
Saturation if:

H*S > Hp

After reaching saturation (2→3)

M = -S*2/3*Hp* f_{SC}

Magnetization after inverting the field change (3→5)



<u>Magnetization after inverting ($H-H_{last}$)</u>

Hp(B) = 1/π*Jc(T,B)*df

S=sign(deltaH)

M_3 H→H-H_3 Hp→2*Hp

Before reaching saturation (3→4)

M = M_3 + (+S*3*(H-H_3)^2*(2*Hp) - (H-H_3)^3 - 3*(H-H_3)*(2*Hp)^2) / (3*(2*Hp)^2) *2 * f_{SC}

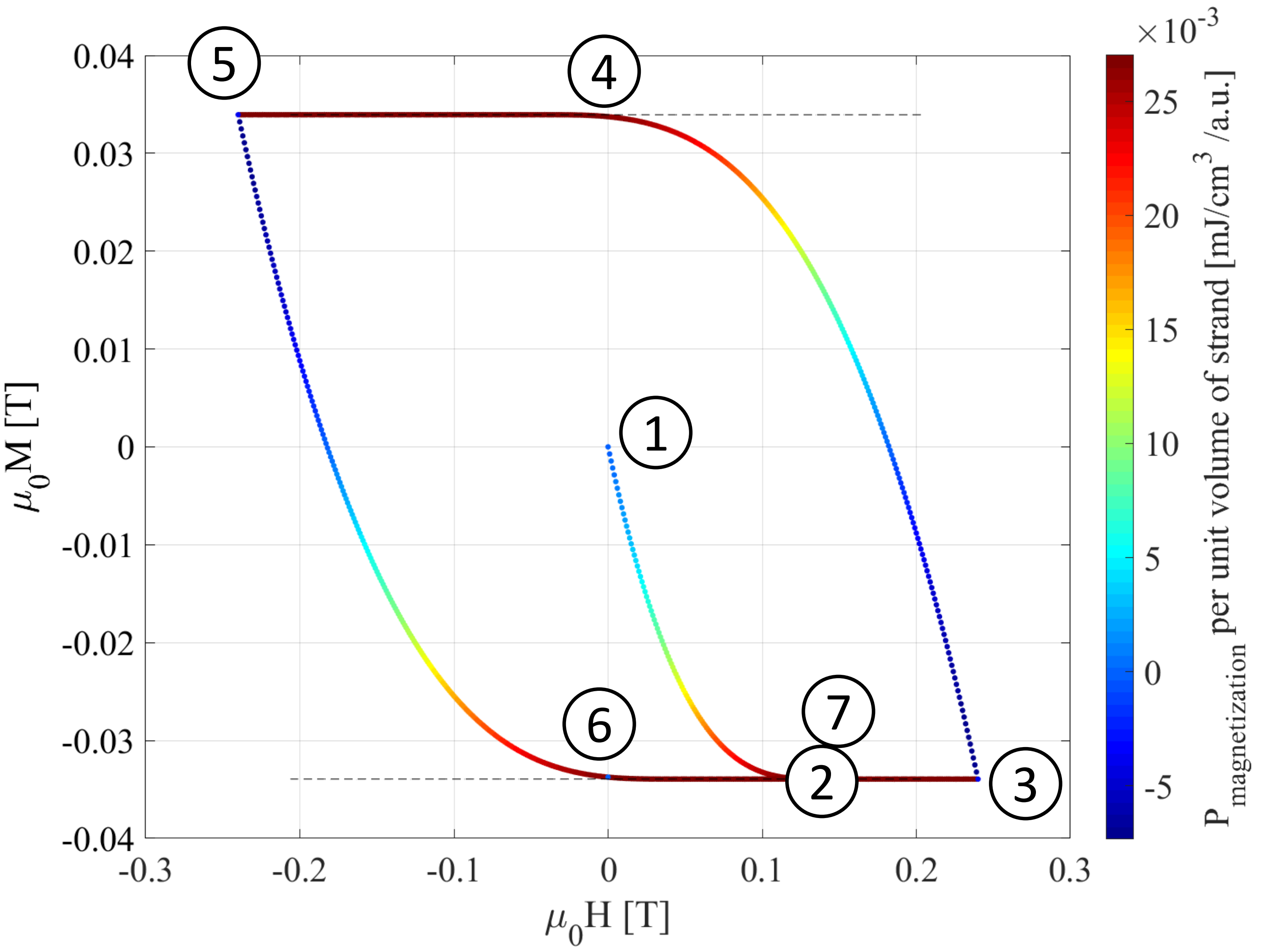
Saturation if:

(H-H_3)*S > 2*Hp

After reaching saturation (4→5)

M = -S*2/3*Hp* f_{SC}

Magnetization after inverting the field change (5→7≡2)



Same formula as (3→5) is correct with H_3→H_5 and M_3→M_5

Magnetization after inverting (H-H_{last})

Hp(B) = 1/π*Jc(T,B)*df

S=sign(deltaH)

M_5 H→H-H_5 Hp→2*Hp

Before reaching saturation (5→6)

M = M_5 + (+S*3*(H-H_5)^2*(2*Hp) - (H-H_5)^3 - 3*(H-H_5)*(2*Hp)^2) / (3*(2*Hp)^2) *2 * f_{SC}

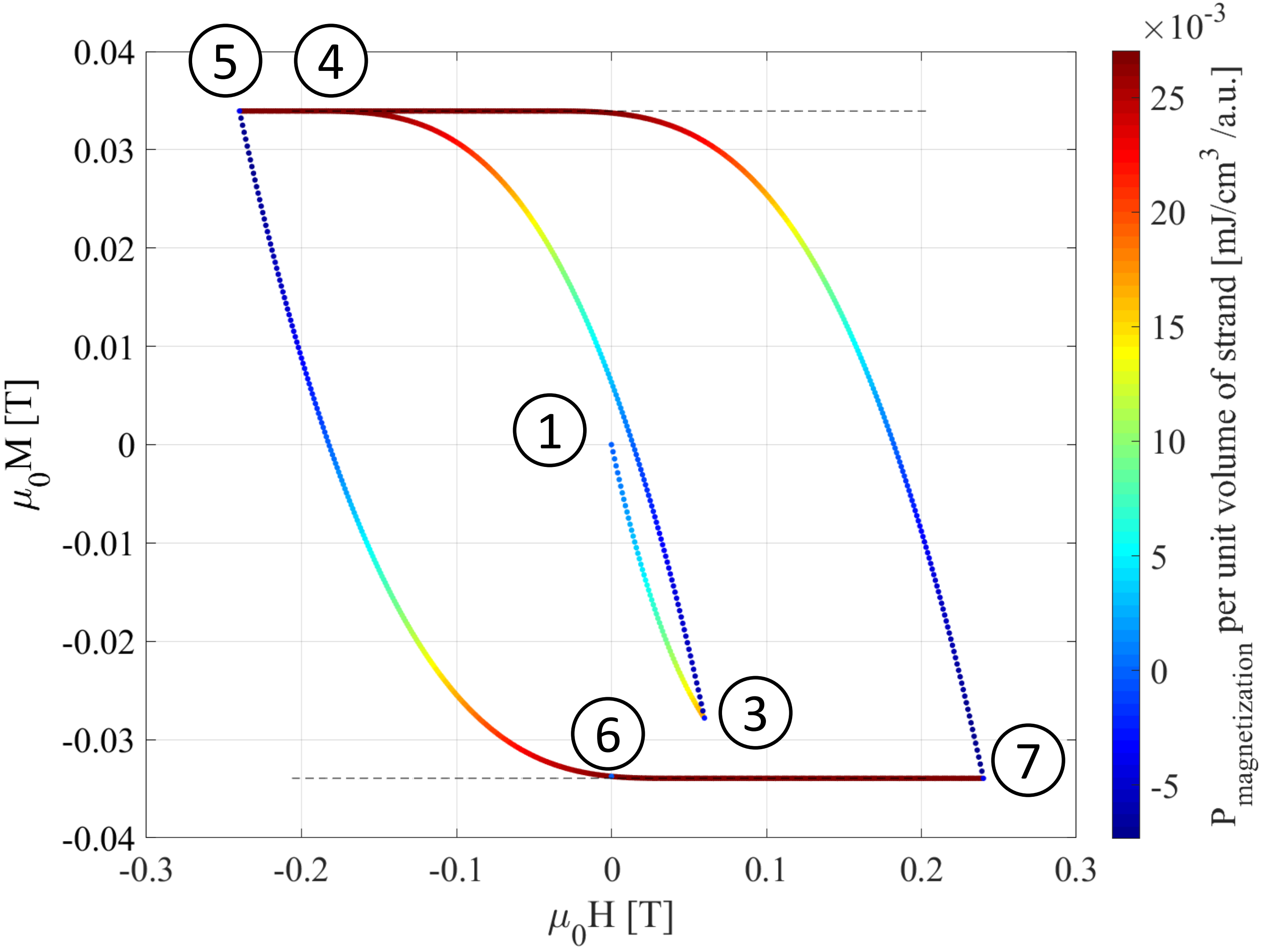
Saturation if:

(H-H_5)*S > 2*Hp

After reaching saturation (6→7)

M = -S*2/3*Hp* f_{SC}

Correction needed if the wire was not fully magnetized (3→5)



Magnetization after inverting ($H-H_{last}$)

$Hp(B) = 1/\pi * Jc(T,B) * df$

$S=sign(deltaH)$

M_3 $H \rightarrow H-H_3$ $Hp \rightarrow 2*Hp$

$f_{sat3} = max(-1, min(1, M_3/(+2/3*Hp*f_{SC})))$

Before reaching saturation (3→4)

$M = M_3 + (+S*3*(H-H_3)^2*((1+S*f_{sat3})*Hp) - (H-H_3)^3 - 3*(H-H_3)*((1+S*f_{sat3})*Hp)^2) / (3*((1+S*f_{sat3})*Hp)^2) *2 * f_{SC}$

Saturation if:

$(H-H_3)*S > (1+S*f_{sat3})*Hp$

After reaching saturation (4→5)

$M = -S*2/3*Hp* f_{SC}$

One formula that works for all the transients, including “virgin” curve

Analytical formula for the magnetization in any transient

$$Hp(B) = 1/\pi * Jc(T,B) * df$$

$$S = sign(deltaH)$$

$$f_{sat,last} = max(-1,\ min(1,\ M_{last}/(+2/3*Hp*f_{SC})\ ))$$

Saturation if:

$$(H-H_{last})*S > (1+S*f_{sat,last})*Hp$$

If wire is not saturated

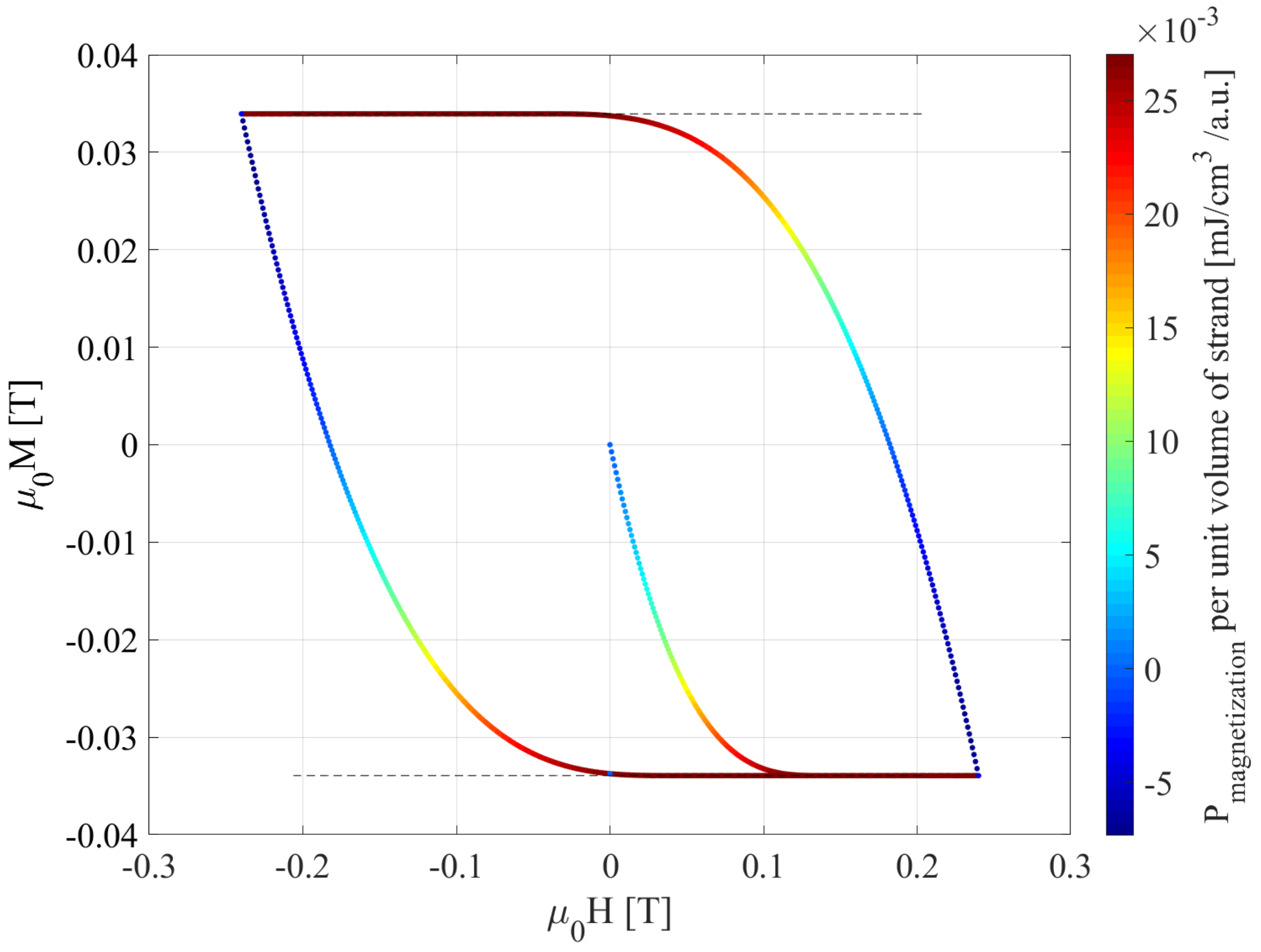
$$M = M_{last} + (\ +S*3*(H-H_{last})^\wedge 2*((1+S*f_{sat,last})*Hp) - (H-H_{last})^\wedge 3 - 3*(H-H_{last})*((1+S*f_{sat,last})*Hp)^\wedge 2)\ /\ (3*((1+S*f_{sat,last})*Hp)^\wedge 2)\ \ *2\ *lambda$$

If wire is saturated

$$M = -S*2/3*Hp*\ f_{SC}$$

The formula requires keeping track of values of H_{last}, M_{last}, $f_{sat,last}$ at the moment at which $(H-H_{last})$ is inverted

Instantaneous hysteresis loss



Modeling the instantaneous magnetization loss is important for simulating transients that are not closed magnetic loops.
The problem was analyzed in [4-5] and others.
In [4], it is shown that the instantaneous magnetization loss per unit volume deposited as heat in the superconductor is
P''' = -M*dB/dt

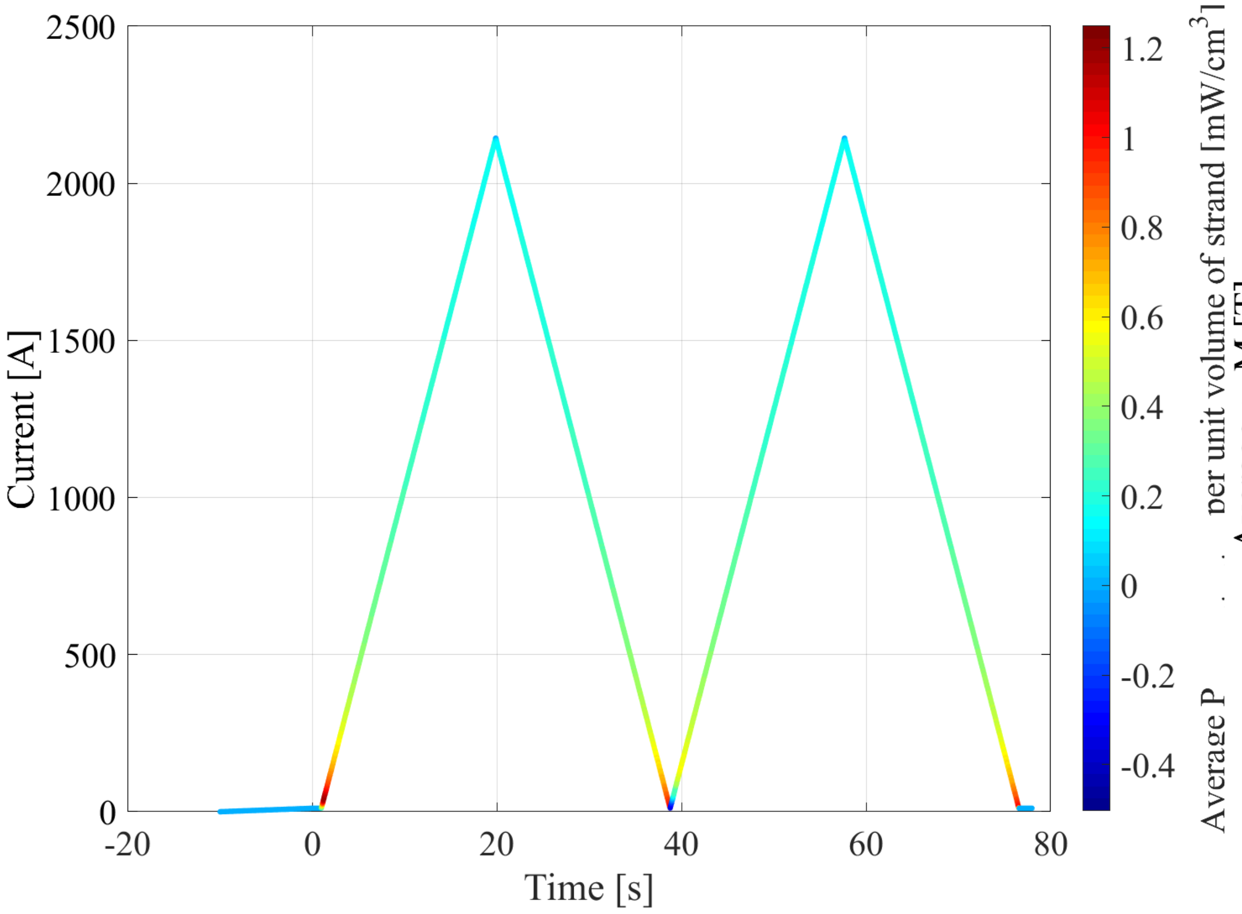
Note that integrating P''' over a closed magnetic cycle gives
$E_{cycle}''' = int_{loop}(P''') = -int_{loop}(M*dH)$

One doubt remains because the quantity P''' can have negative value. In this context, this is equivalent to subtracting heat, i.e. cooling.
To be followed up

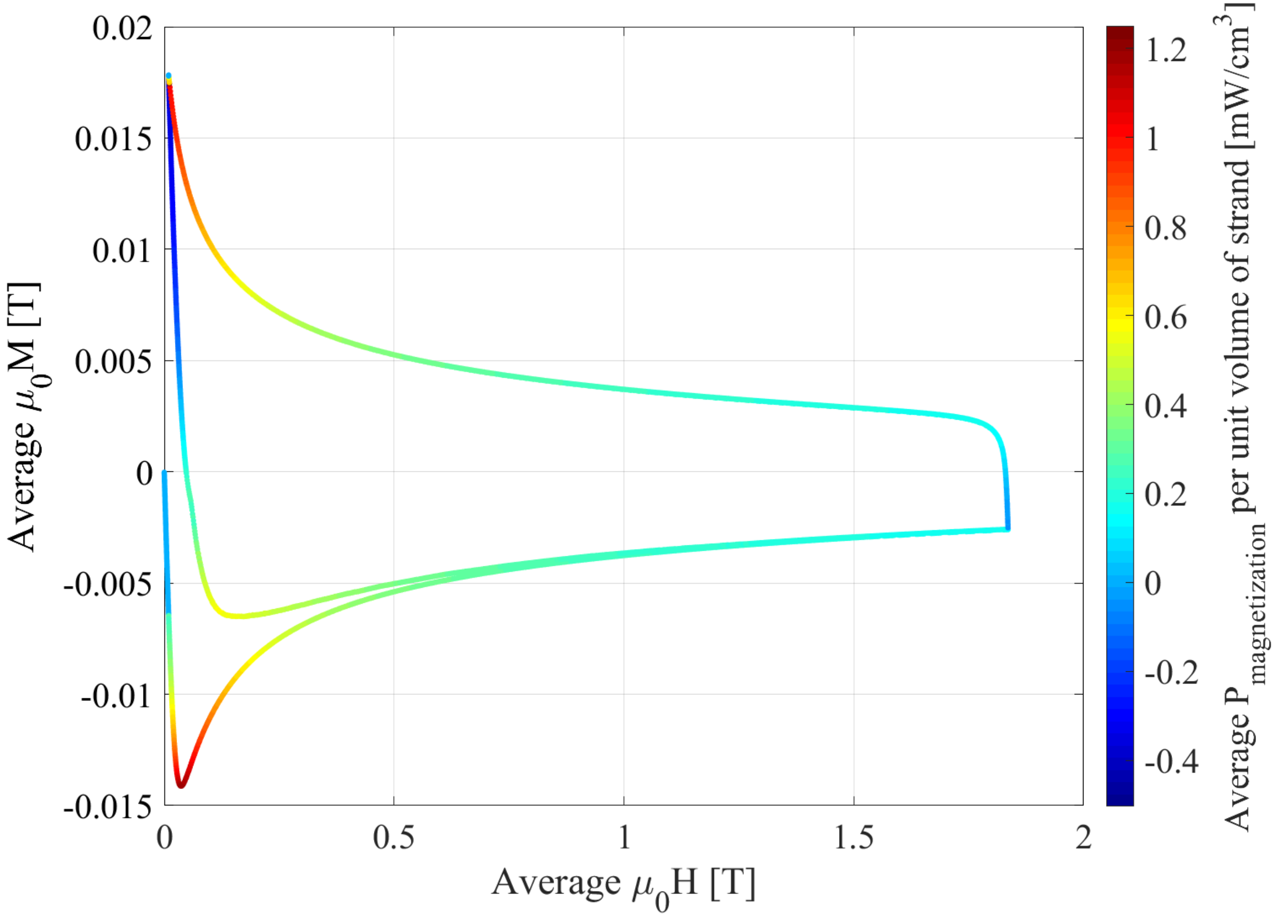
Detailed results of Cycle #2

Cycle #2

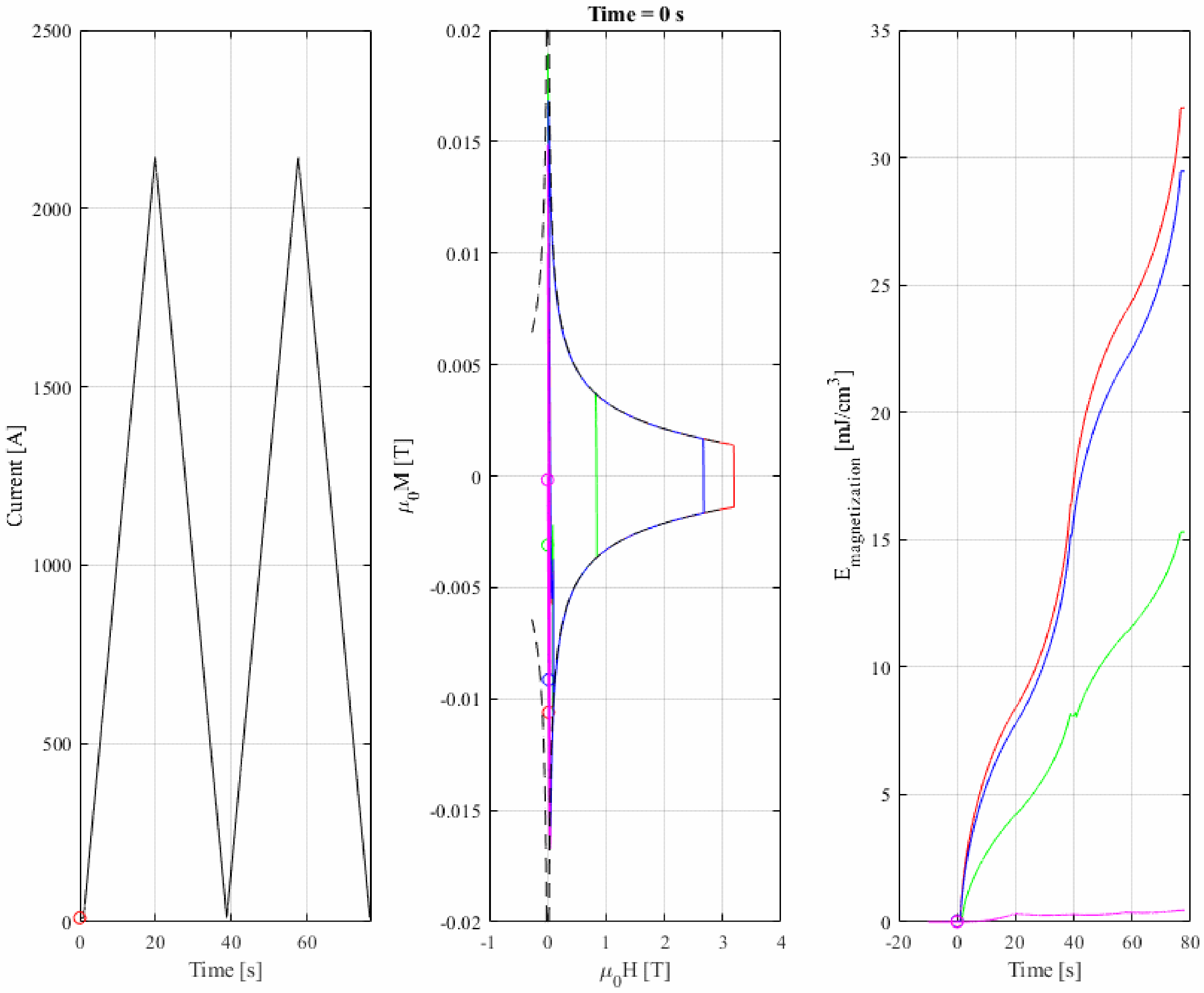
I vs time



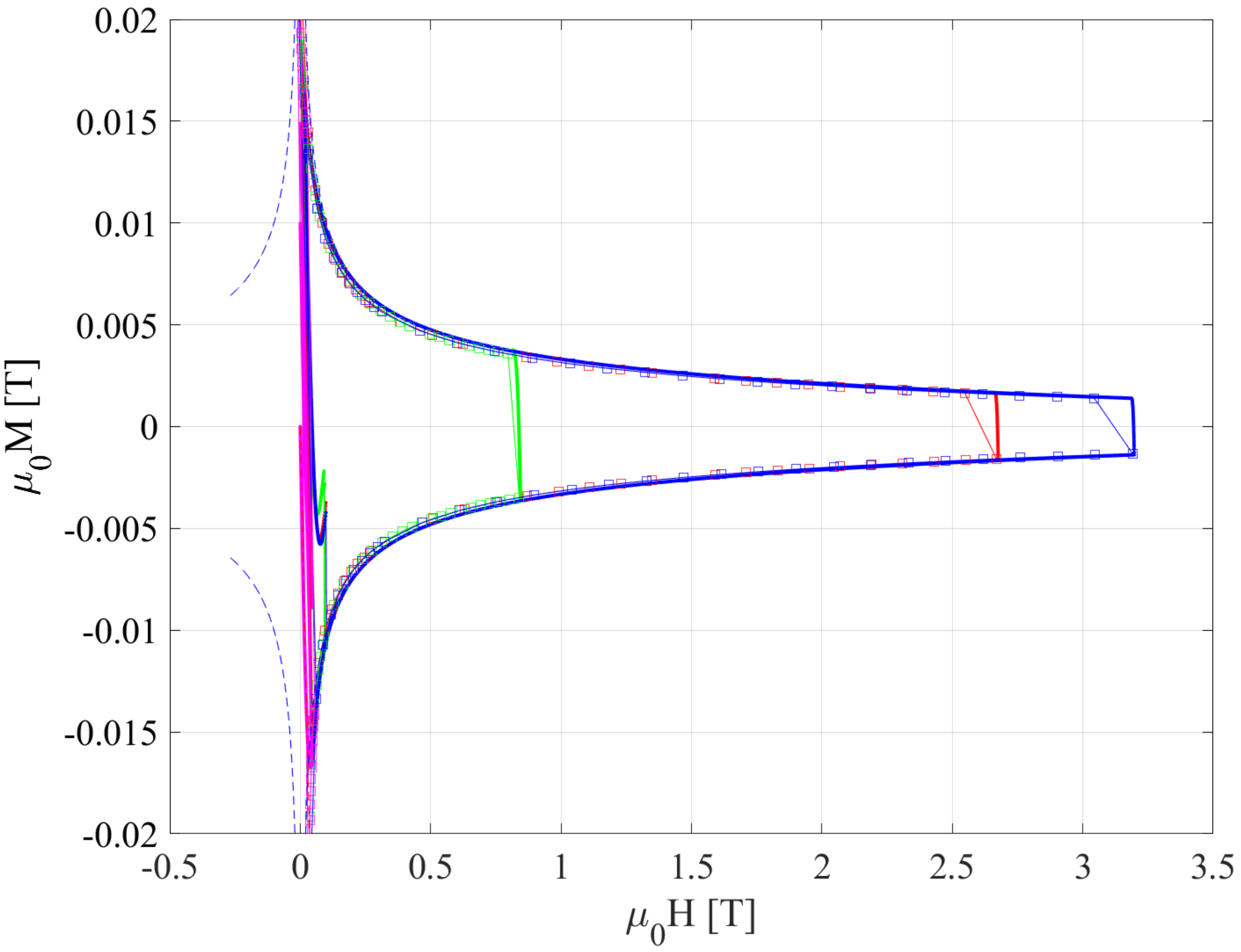
Average μ_0*M vs Average μ_0*H



Cycle #2 – Four selected strands



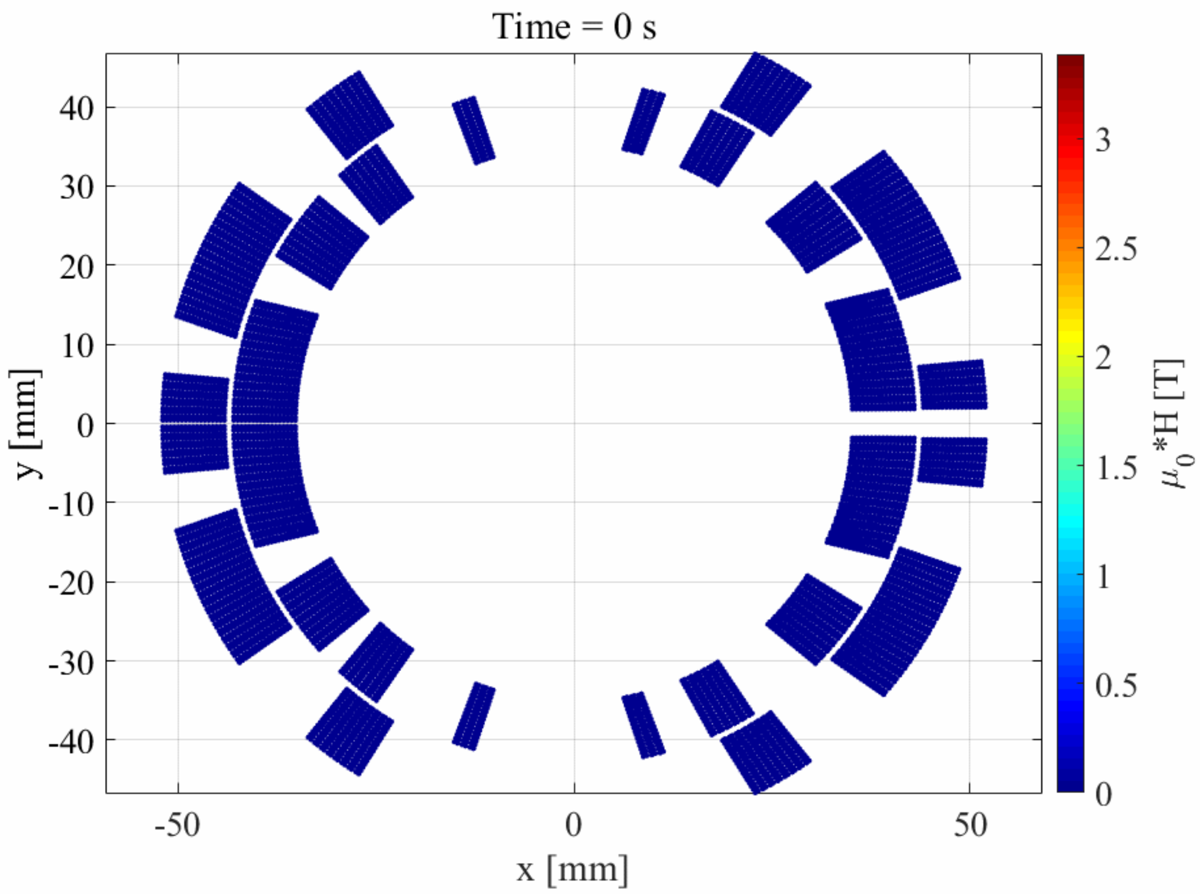
Cycle #2 – Four selected strands – Comparison with ROXIE

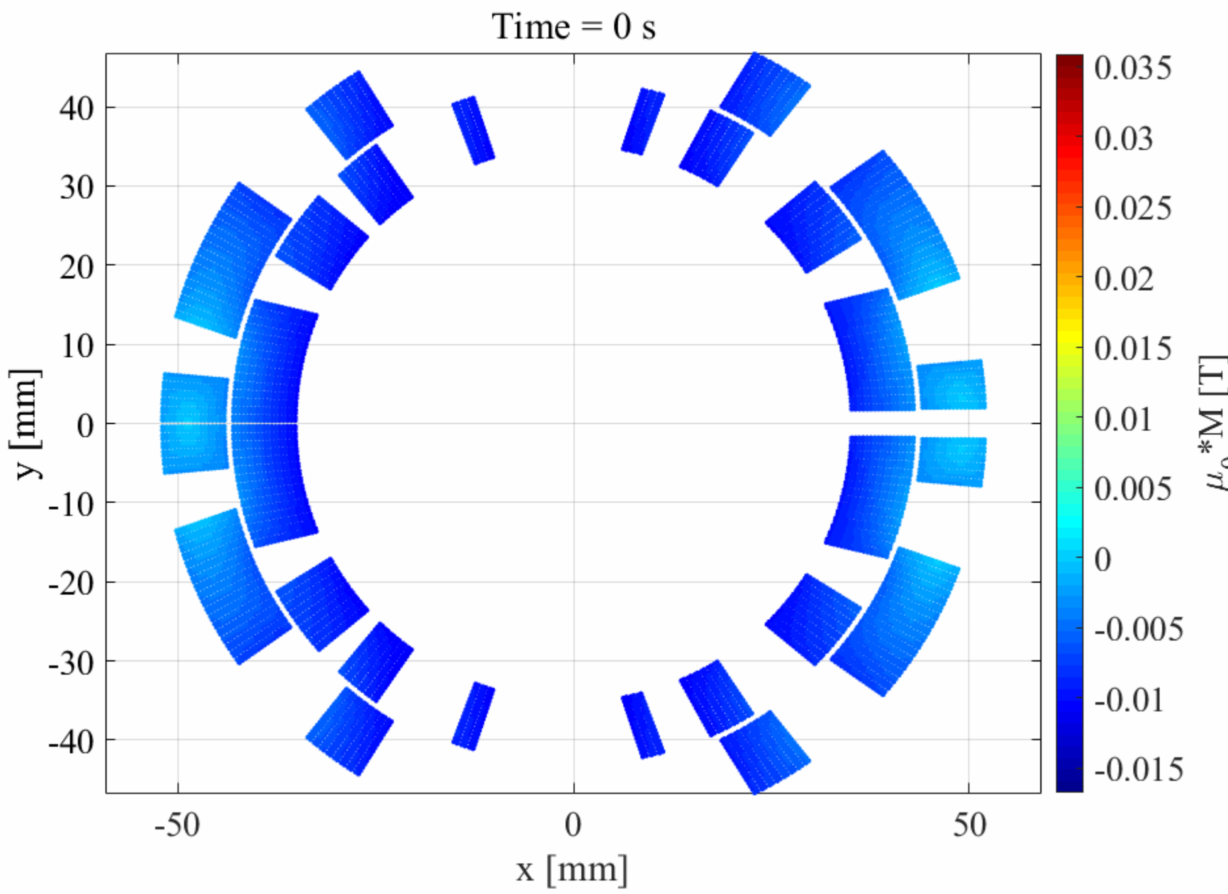


Cycle #2

μ_0*H

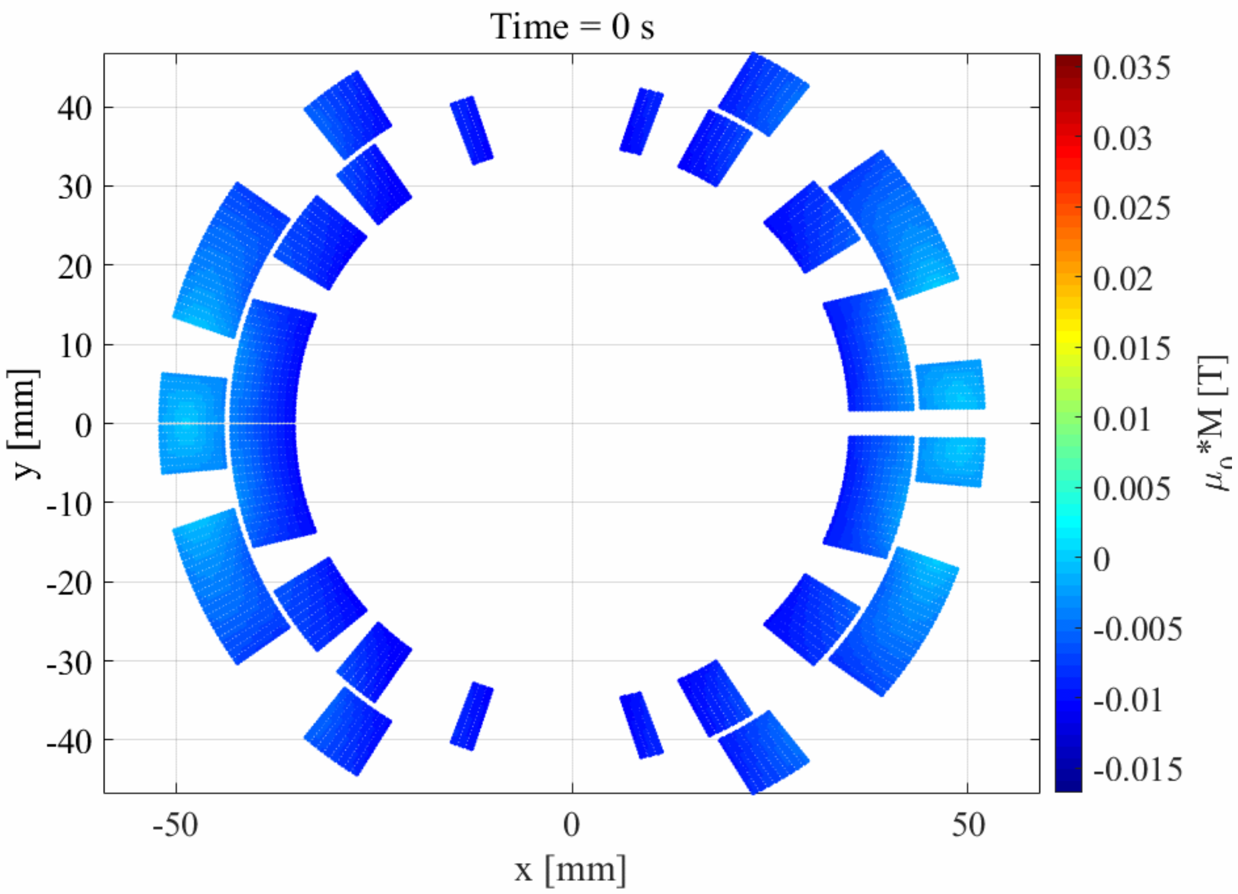
μ_0*M in the strand volume



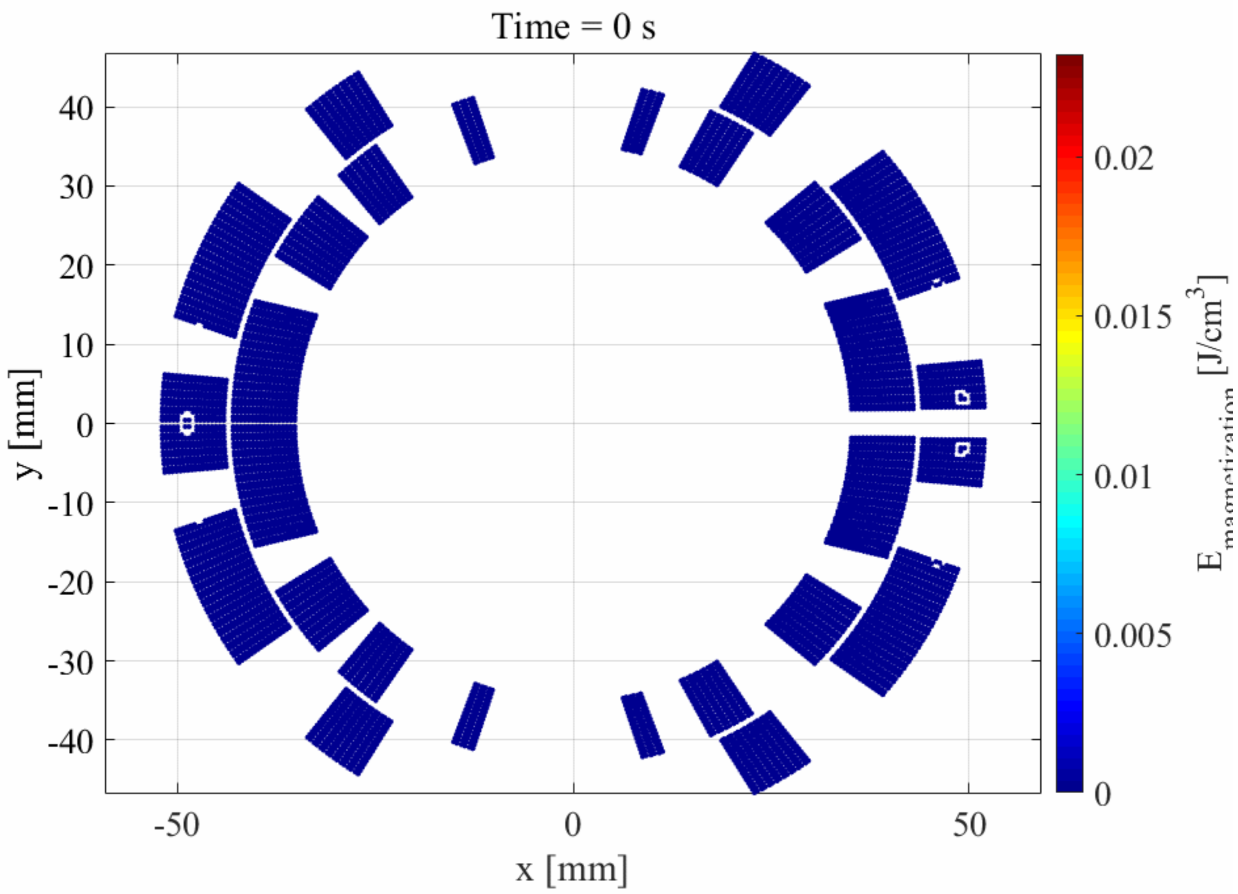


Cycle #2

μ_0*M in the strand volume

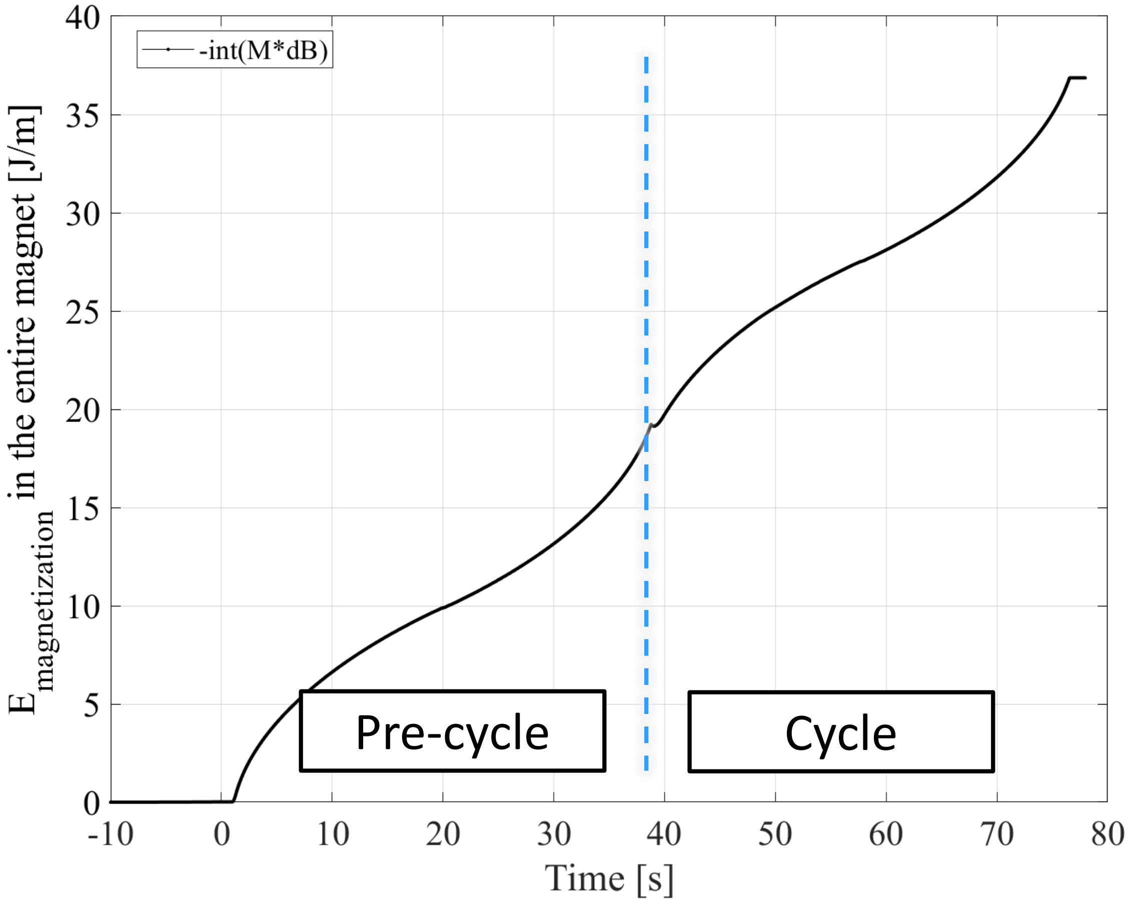


Integrated loss per unit volume

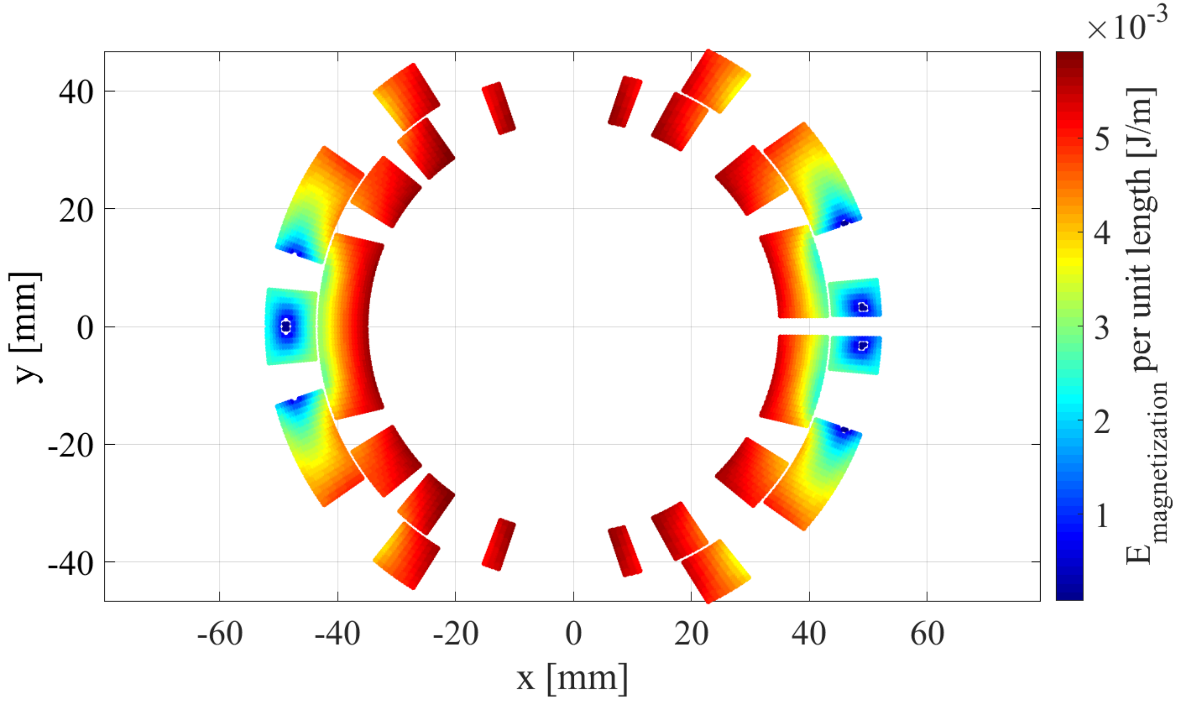


Cycle #2

Integrated loss per unit length



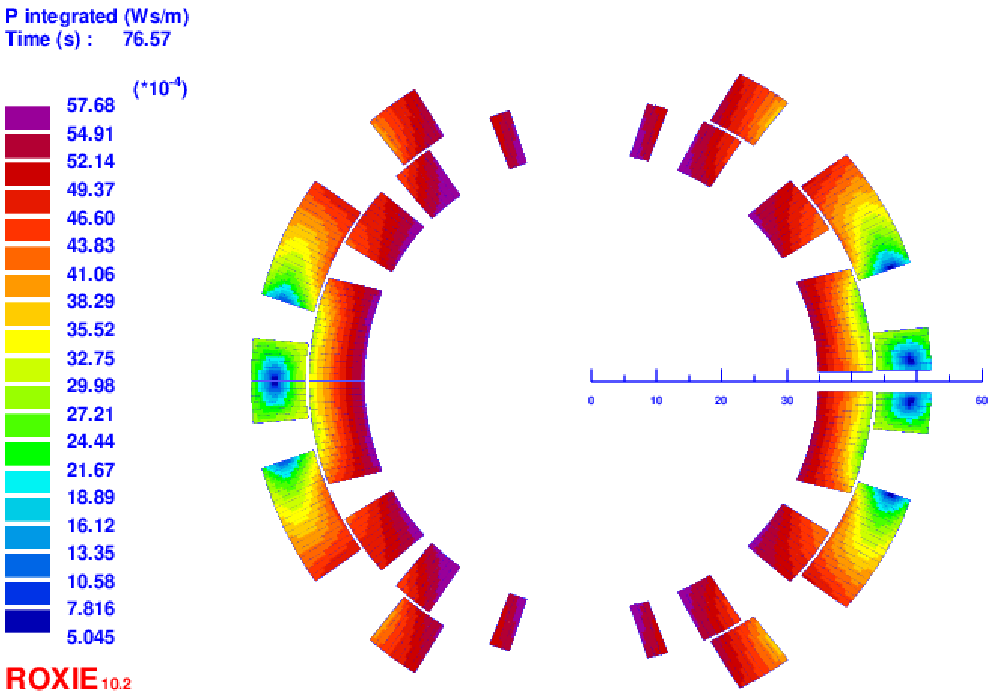
Integrated loss per unit length

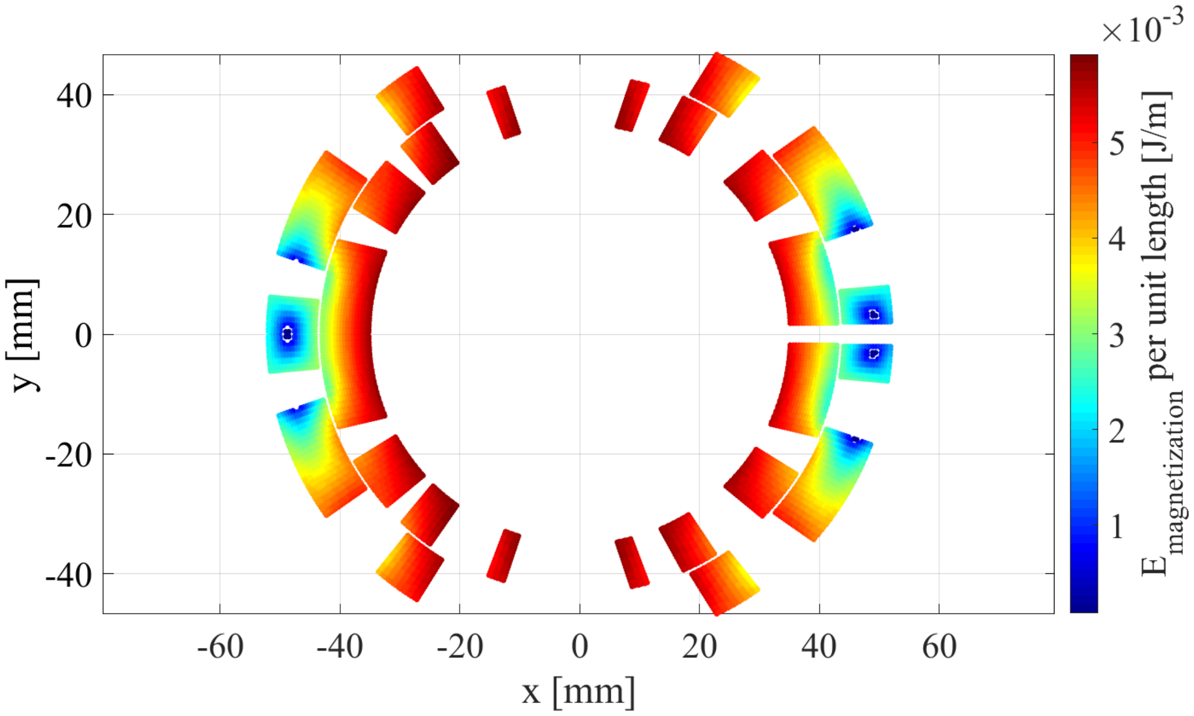


Pre-cycle + Cycle #2 – Comparison with ROXIE

Integrated loss per unit length

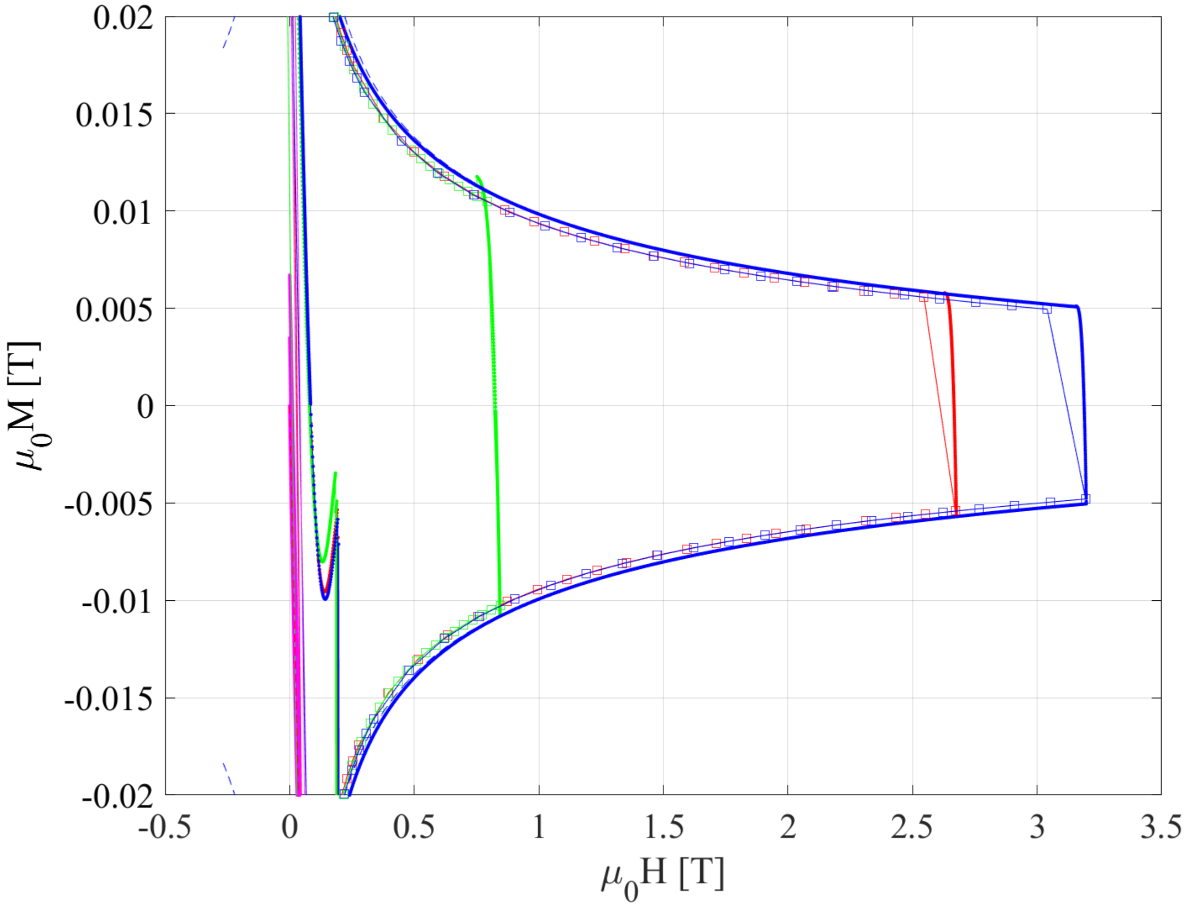
Integrated loss per unit length



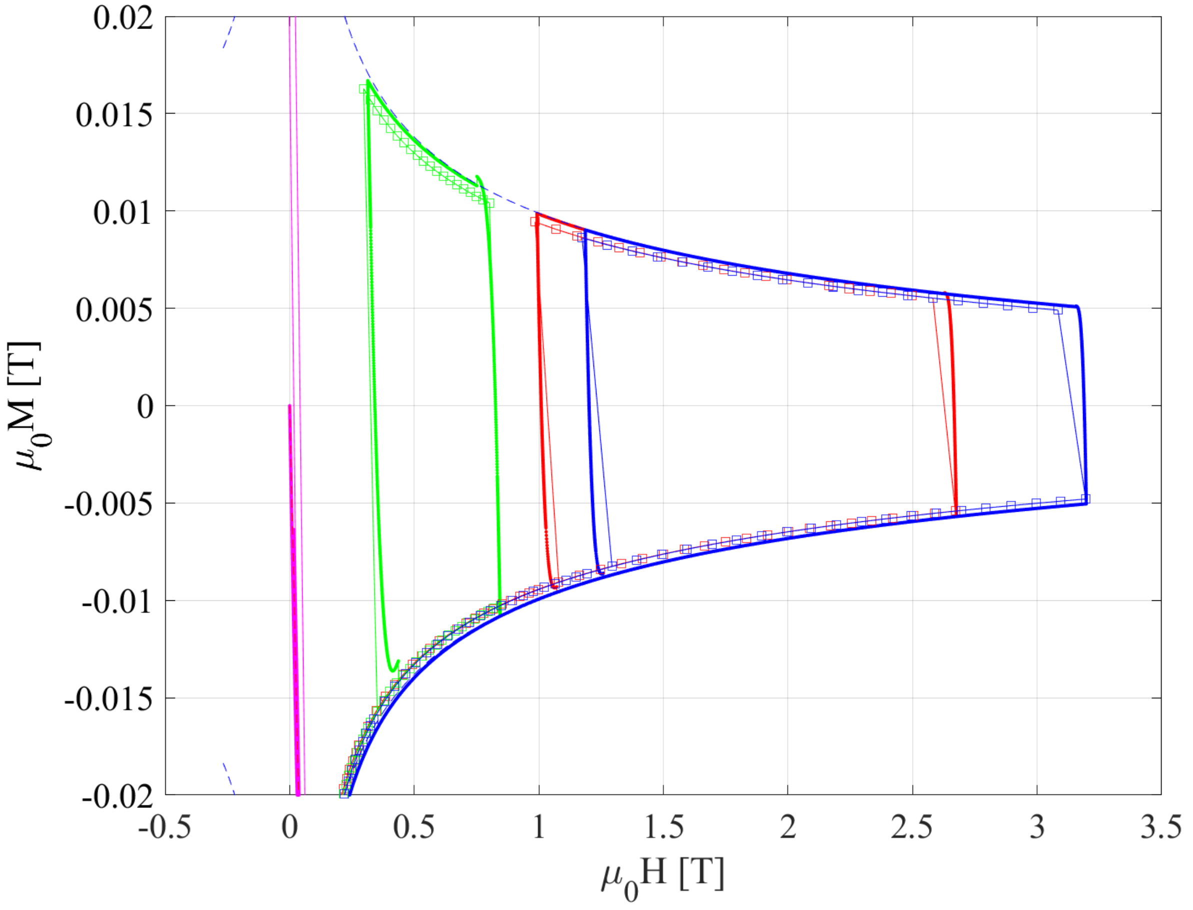


Same simulations at T=1.9 K – Comparison with ROXIE

Cycle #2



Cycle #3



More detailed derivation of the equivalent electrical circuit parameters

Magnetization stored energy and magnetization loss

In order to properly simulate the effect of magnetization on the magnet differential inductance, the energy stored as magnetization and lost as heat need both to be included in the model.

In [4], the energy exchanges are nicely described:

$Q_M = L_M + \Delta U_M$

Heat = Work + Internal energy variation

$B = \mu_0*(H+M) = \mu_0*(H_a+H_m+M)$

H_a = Applied field

H_m = Field generated by the magnetized material (aka demagnetizing field)

Energy balance (Note: they are all scalar products)

$L_M = int_{volume}(- \mu_0*(H_a*dM))$

$\Delta U_M = int_{volume}(\mu_0*(H_a*dM + M*dH_a) + \mu_0*(M*dM + M*dH_m))$ *(to be understood)*

→ $dQ_M = int_{volume}(-M*dB)$

The quantities are to be integrated over the entire space where the field is generated. For M, this is the volume of wire [not the volume of superconductor, since M is homogenized], i.e. $pi/4*d_s^{2*}l_{magnet}$

Stored/lost energy in the magnet and magnetization loops

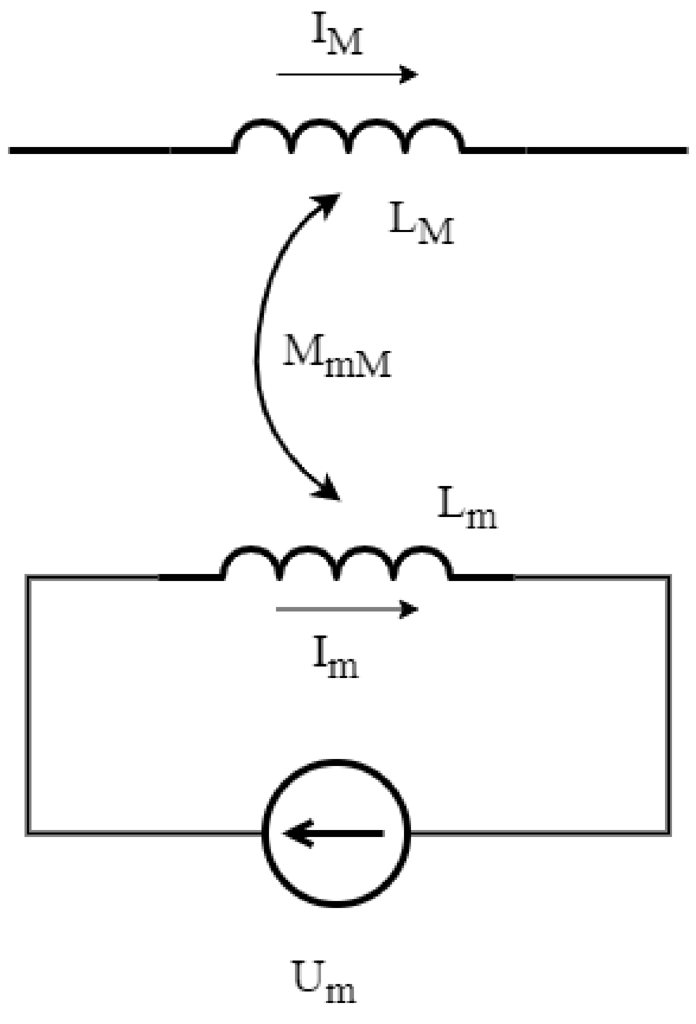
Energy stored in the magnet inductance, without magnetization effects: $int_{volume}(0.5* \mu_0*H^2))$	Energy stored in the magnet inductance, without coupled loop: $0.5*L_M*I_M^2$
Energy stored in a magnet including magnetization effects [6, and others] $E\_stored = int_{volume}(int_{B\text{-}field}(H*dB))$ $dE\_stored'''/dt = H*dB = \mu_0*(H*dH/dt + H*dM/dt)$	Energy stored in the magnet, including coupled loop: $E\_stored = 0.5*L_M*I_M^2 + 0.5*L_m*I_m^2 + M_{mM}*I_M*I_m$ $dE\_stored/dt = L_M*I_M*dI_M/dt + L_m*I_m*dI_m/dt + M_{mM}*I_M*dI_m/dt + M_{mM}*I_m*dI_M/dt$
Energy lost due to magnetization [4] $E\_magnetization = int_{volume}(int_{B\text{-}field}(M*dB))$ $dE\_magnetization'''/dt = M*dB = \mu_0*(M*dH/dt + M*dM/dt)$	Energy lost in the coupled loop, i.e. provided by the current source: $int_{time}(U_m*I_m)$ $U_m*I_m = -(M_{mM}*dI_M/dt + L_m*dI_m/dt)*I_m$
M is a function of H and its history	I_m is a function of I_M and its history
Effect of magnetization on magnet differential inductance	$L_d = U_M/(dI_M/dt)$ $= (L_M*dI_M/dt + Sum_m(M_{mM}*dI_m/dt)) /(dI_M/dt)$ $= L_M + Sum_m(M_{mM}*(dI_m/dt)/(dI_M/dt))$

Determination of 3 equivalent circuit parameters

$H = f_{mag} * I_M$ (f_{mag} in units of [1/m]. Reminder: f_{mag} [1/m] ≡ f_{mag} [T/A]/μ_0
$M = I_m/d_s$ (as proposed in [10])
L_M = Magnet self-inductance (without magnetization effects)

Current source defined to satisfy
→ $I_m = M * d_s$
Note that M depends on H and its history, so I_m depends on I_M and its history

Magnetization loss in the volume of one wire
$int_{vol,SC}(-M*dB/dt) = -\mu_0 * int_{vol,SC}(M*dH/dt + M*dM/dt)$
$= -\mu_0 * (I_m/d_s * f_{mag} * dI_M/dt + I_m/d_s\text{^}2 * dI_m/dt) * V_{wire}$
$= -\mu_0 * I_m (f_{mag}/d_s * dI_M/dt + 1/d_s\text{^}2 * dI_m/dt) * (\pi/4 * d_s\text{^}2 * l_{magnet})$
must correspond to:
$U_m * I_m = -(M_{mM} * dI_M/dt + L_m * dI_m/dt) * I_m$
→ $L_m = \mu_0 * \pi/4 * l_{magnet}$ *1/8? volume integral to double-check*
→ $M_{mM} = \mu_0 * \pi/4 * l_{magnet} * d_s * f_{mag}$



Bonus: Maximum effect of magnetization on differential inductance

It is possible to calculate analytically the maximum effect of magnetization on differential inductance.

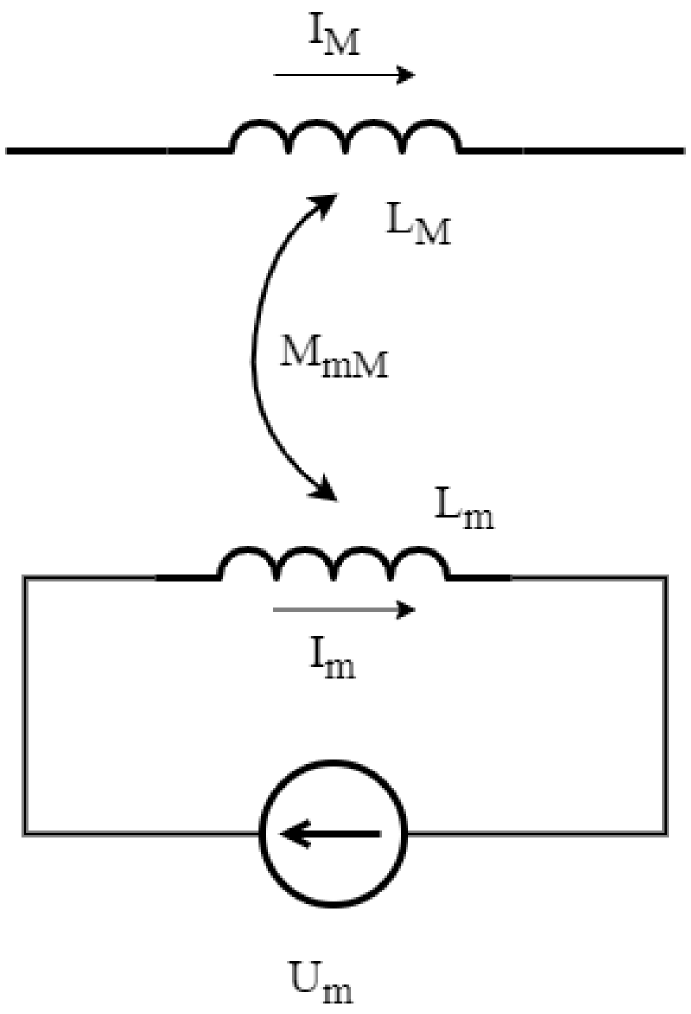
$L_d = U_M/(dI_M/dt) = (L_M*dI_M/dt + Sum_m(M_{mM}*dI_m/dt)) /(dI_M/dt)$
$= L_M + Sum_m(M_{mM}*(dI_m/dt)/(dI_M/dt))$

$M_{mM} = \mu_0*\pi/4*l_{magnet}*d_s*f_{mag}$

Maximum $(dI_m/dt)/(dI_M/dt)$ is obtained when all strands are fully saturated and $(H-H_{last})$ is inverted:
$|(dI_m/dt)/(dI_M/dt)|_{max} = |dM/dH|_{max} = -2 *f_{SC}$

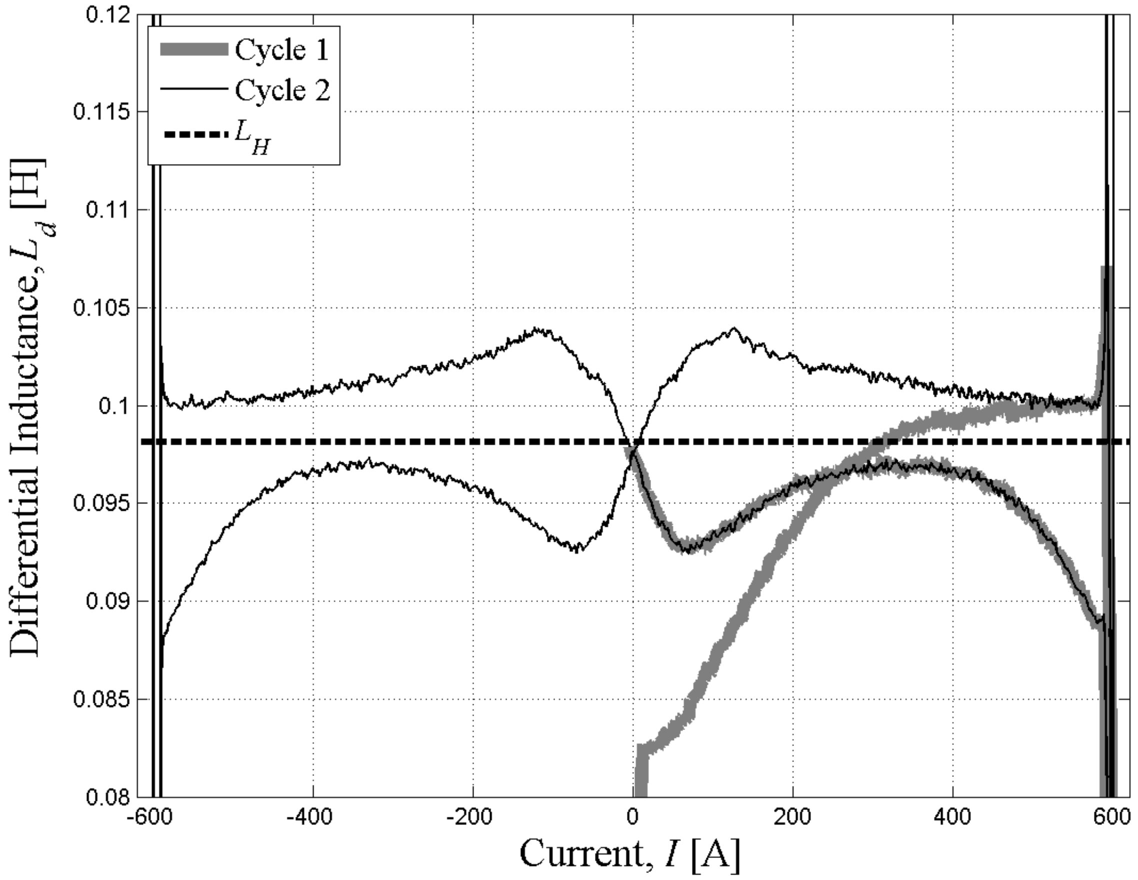
$L_{d,min} = L_M + Sum_m(-\mu_0*\pi/2*l_{magnet}*d_s*f_{SC}*f_{mag})$

For the MB magnet, the maximum L_d reduction is ~31.2 mH (L_M ~100 mH)

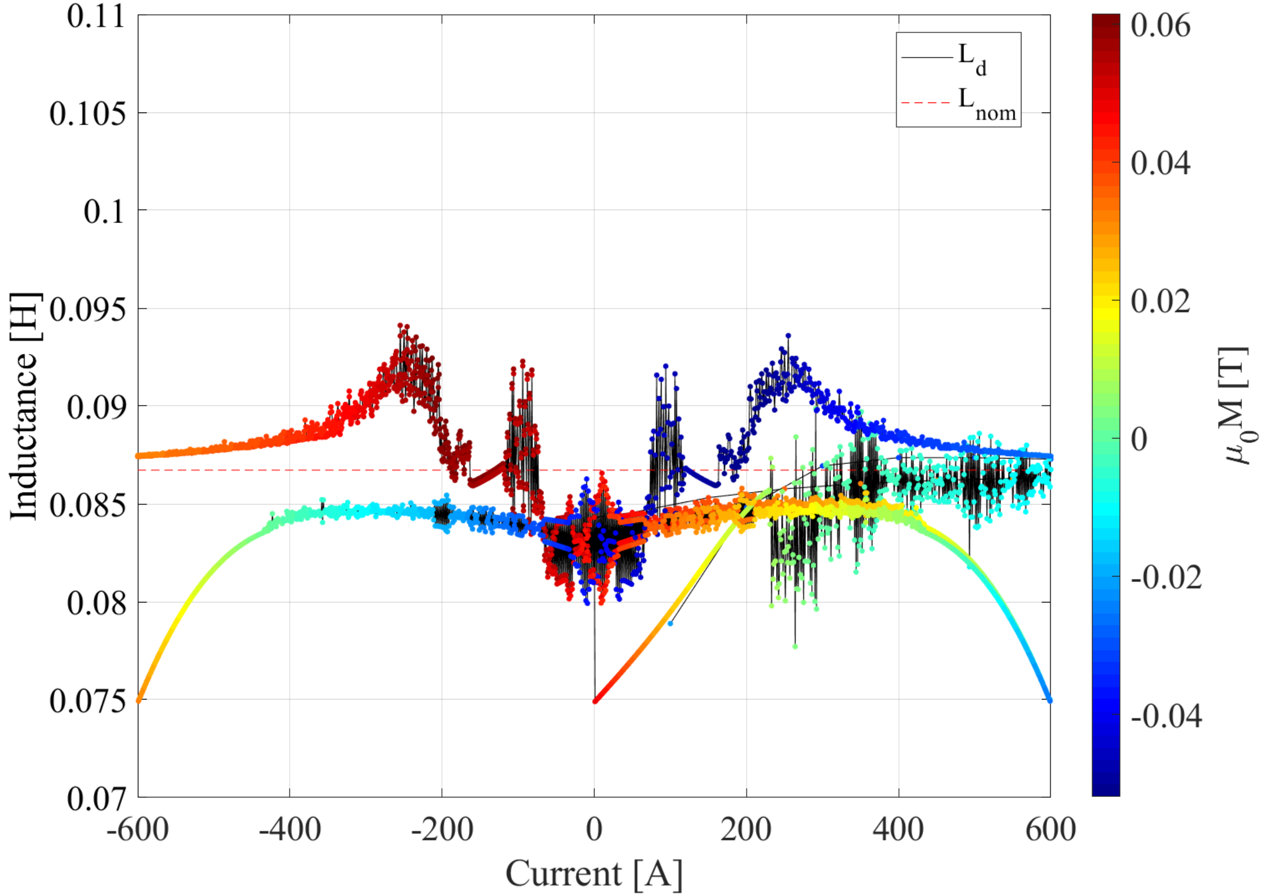


Effect of magnetization on magnet differential inductance - PRELIMINARY

MB – Experimental [11]
Differential inductance vs Current

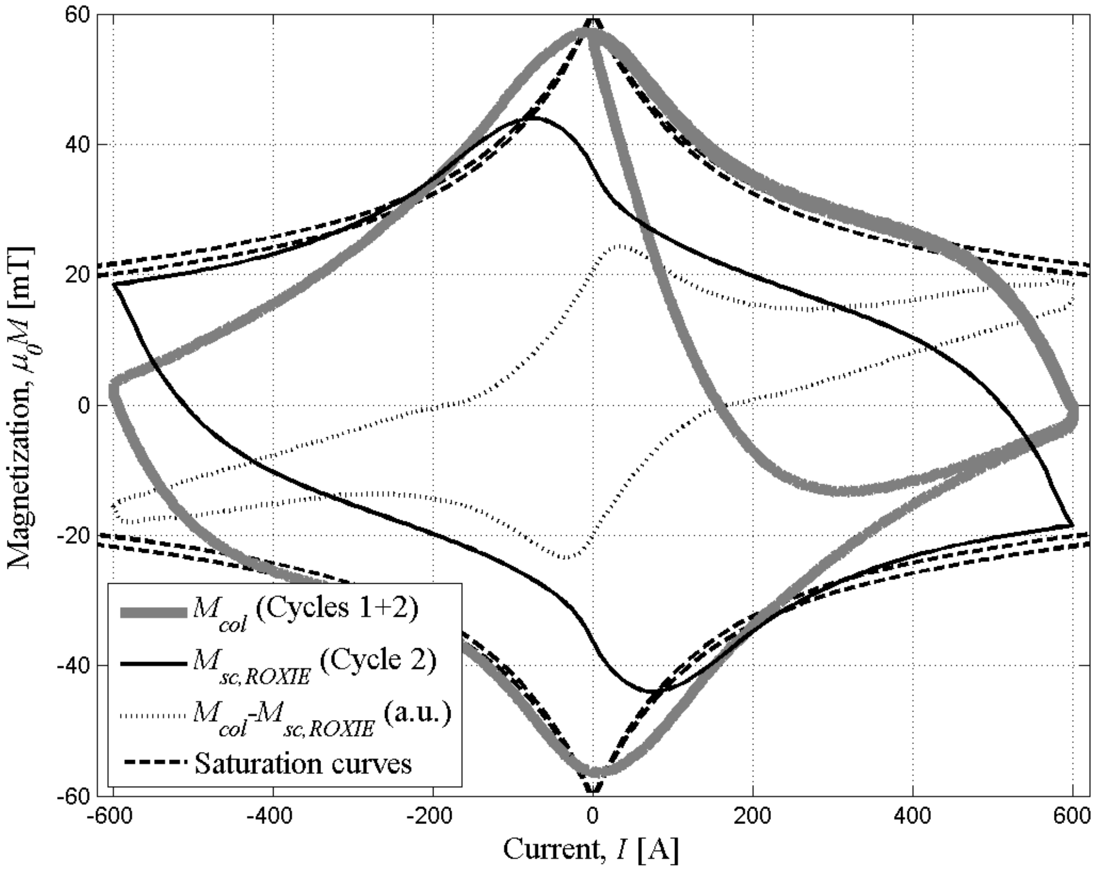


MB – Simulated
Differential inductance vs Current

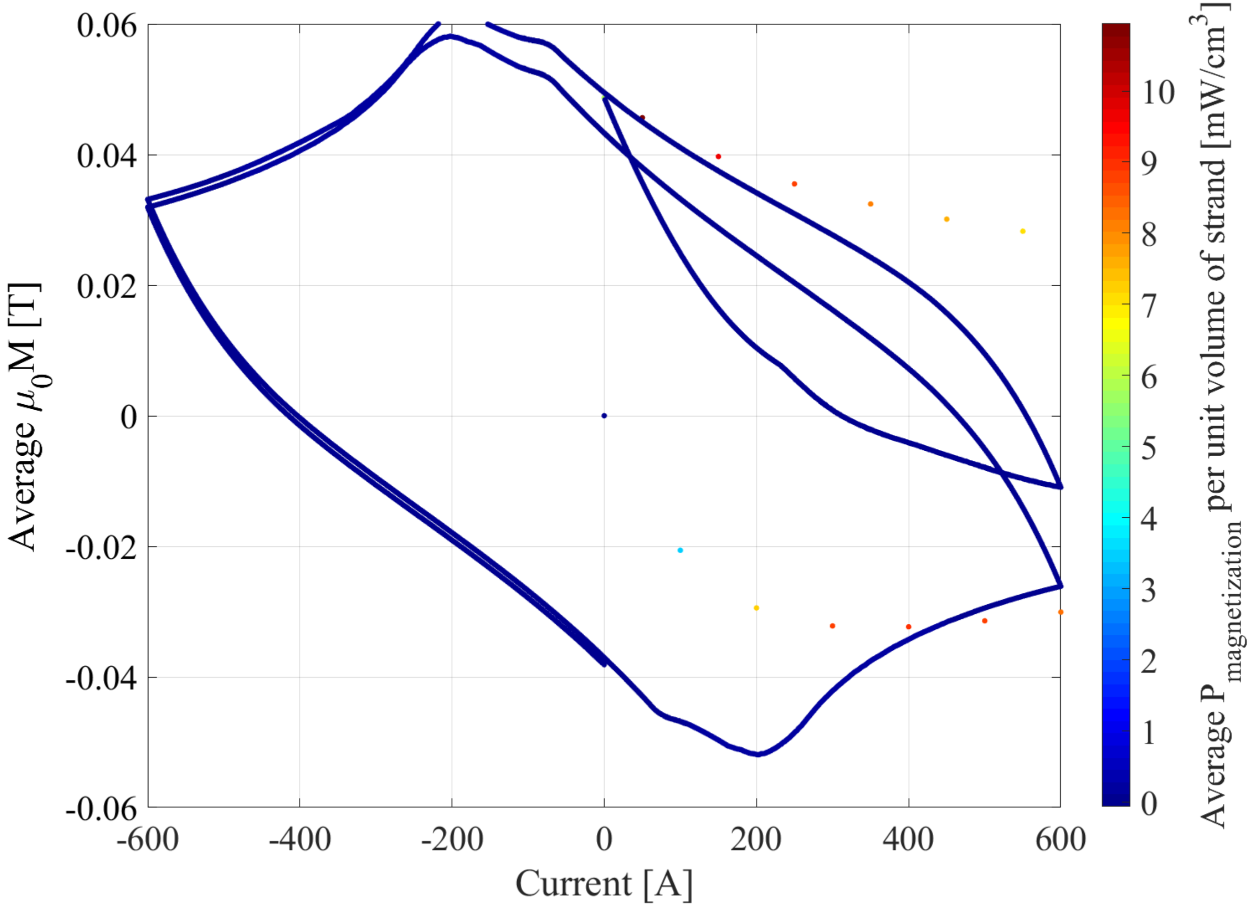


Effect of magnetization on magnet differential inductance - PRELIMINARY

MB – Experimental [11]
Average $\mu_0 M$ vs Current



MB – Simulated
Average $\mu_0 M$ vs Current



Note: Jc(T,B) fit not adjusted