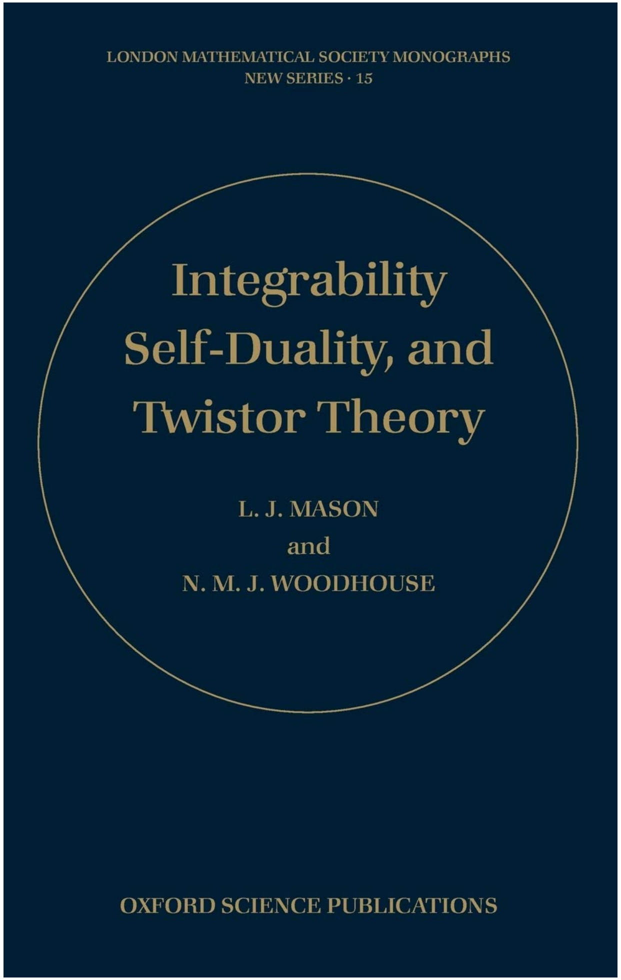
Twistors, Integrability and 4d Chern-Simons Theory

David Skinner (DAMTP)

Twistor Theory and Beyond
27/09/21

based on 2011.04638 with R. Bittleston and wip with R. Bittleston & A. Sharma

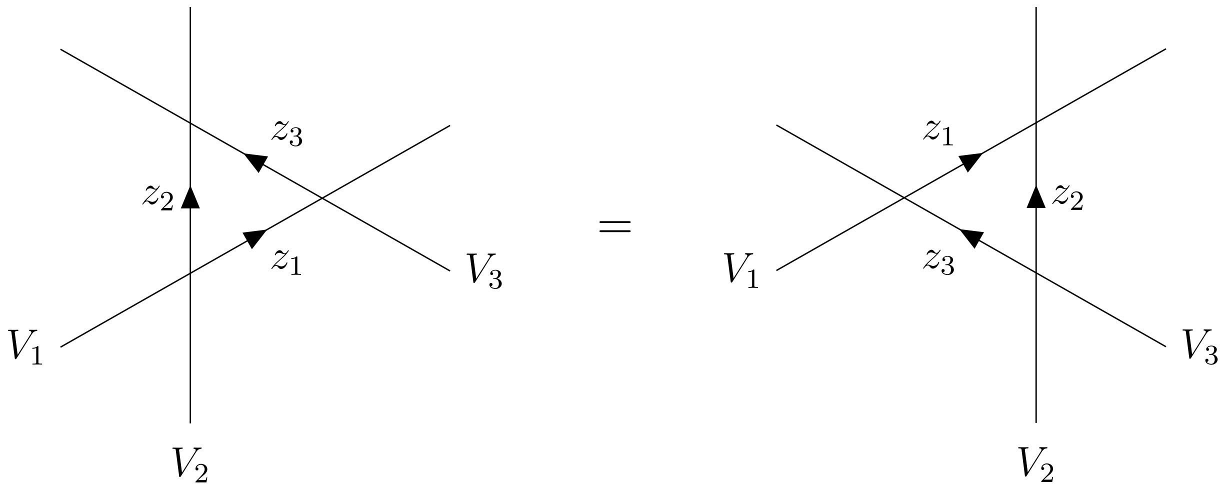


- Many integrable systems arise as symmetry reductions of the ASDYM equations $F = - \star F$
- depends on choices of $H \subset \mathrm{Conf}_4$, gauge group G, trivialisation, and various other data
- closely related to the magic of the twistor construction

In important cases, these integrable systems are the classical equations of motion of a quantum field theory which is also integrable

- A key object in 1+1 dimensional QFT is the 2-2 scattering matrix $R(z - z') : V \otimes V' \to V \otimes V'$

- The theory is integrable if R obeys the *Yang-Baxter equation*



$$R_{12}(z_{12})R_{13}(z_{13})R_{23}(z_{23}) = R_{23}(z_{23})R_{13}(z_{13})R_{12}(z_{12})$$

- The YBE entails $\sim (\dim V)^6$ equations, so is greatly over-determined

What does twistor theory know of *quantum* integrability?

The YBE resembles the 3$^{\mathrm{rd}}$ Reidemeister move of knot theory, however

- no notion of crossing 'over/under' in YBE
- no spectral parameter in knot theory

Motivated by this, Costello, Witten & Yamazaki introduced a variant of Chern-Simons theory defined on a four manifold of the form $\Sigma \times C$

$$S[A] = \frac{1}{2\pi i} \int_{\Sigma \times C} \omega \wedge \mathrm{tr}\left(A \wedge dA + \frac{2}{3} A \wedge A \wedge A \right)$$

- ω is a meromorphic (1,0)-form on C
- A is a partial connection on a principal G-bundle over $\Sigma \times C$

Periods of ω not naturally quantized, so theory only makes sense perturbatively

- poles of ω $\Leftrightarrow$ $\hbar \to 0$
- zeros of ω $\Leftrightarrow$ $\hbar \to \infty$

Thus, at least naïvely, $(C, \omega) = (\mathbb{C}, dz)$, $(\mathbb{C}^*, dz/z)$, (E, dz)

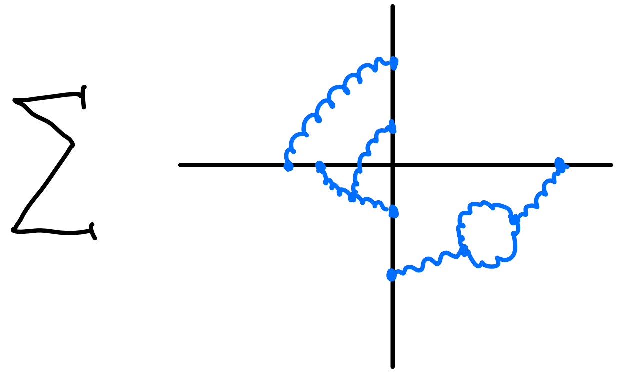
As usual in gauge theory, the basic operators are Wilson lines

$$W_V[\gamma] = P\exp\left(\int \gamma^* A\right)_V$$

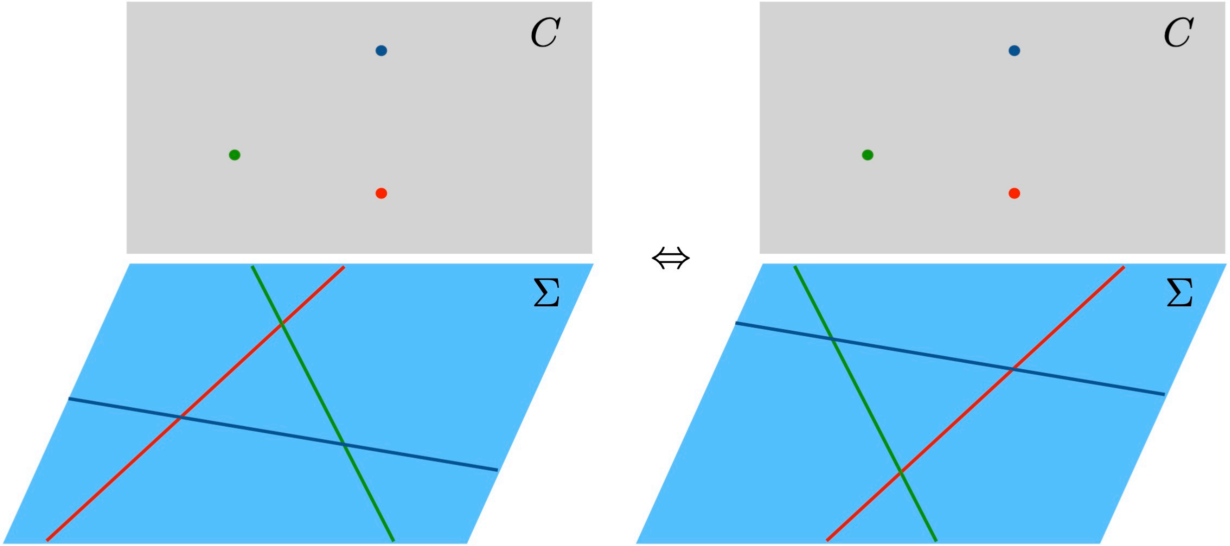
- only have a partial connection, so $\gamma \subset \Sigma$ at a point $z \in C$
- at least in simplest case, V is a rep$^{\mathrm{n}}$ of $\mathfrak{g}$

The R-matrix arises from the QFT correlator of two such line operators

$$\langle W_V[\gamma]\, W_{V'}[\gamma']\rangle =$$



The R-matrix we obtain manifestly obeys the YBE, because nothing special happens on $\Sigma \times C$ when passing between the pictures

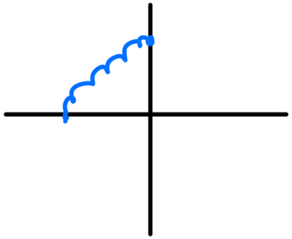


The R-matrices we produce are local to each crossing

- 4d CS is IR free, but admits unique BV quantization [Costello]
- away from Σ-crossings, propagators between different line operators are suppressed as we scale up the metric on Σ
- topological invariance in Σ then implies 4d Chern-Simons is ultra-local in Σ (but only holomorphic in C)

For example, perturbatively $R(z-z') = 1 + \hbar r(z-z') + \mathcal{O}(\hbar^2)$ where the leading-order contribution r comes from a single propagator

$$r(z-z') = \int_{\gamma\times\gamma'} \langle A_V(w)\, A_{V'}(w')\rangle = \frac{t_V \otimes t_{V'}}{z-z'}$$



- the YBE constrains r to obey the *classical Yang-Baxter equation*

$$[r_{12}(z_{12}), r_{13}(z_{13})] + [r_{12}(z_{12}), r_{23}(z_{23})] + [r_{13}(z_{13}), r_{23}(z_{23})] = 0$$

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- the explicit r found above is the basic rational solution found by Belavin & Drinfeld
- under mild assumptions, $\exists!$ completion of such an r to an R that solves the YBE to all orders in $\hbar$ (normalized by quantum determinant)

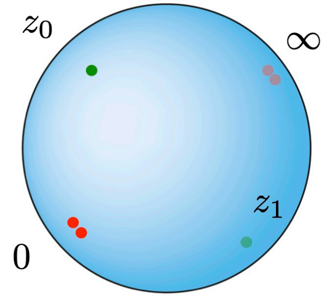
More recently, Costello & Yamazaki showed that including surface operators leads to a wide class of integrable field theories

- a particularly interesting class involves no additional fields, but allows ω to have both zeros and poles
- for the kinetic term to be elliptic, at least some components of $A|_\Sigma$ must vanish at poles of ω. Similarly, we should allow poles in $A|_\Sigma$ where ω has a zero

Such boundary conditions are sometimes called *disorder operators*. One can also include *order operators* which involve new fields living on surfaces $S_i = \Sigma \times \{z_i\}$

For example, pick $C = \mathbb{CP}^1$ with

$$\omega = \frac{(z - z_0)(z - z_1)}{z^2}\, dz$$



The required boundary conditions break topological invariance along Σ

- we still need $A|_\Sigma = 0$ at $z = 0, \infty$ but must now also allow

$$A_w \sim \frac{a_w}{z - z_0} \qquad\qquad A_{\bar{w}} \sim \frac{a_{\bar{w}}}{z - z_1}$$

  where w is a complex coordinate on $\Sigma \cong \mathbb{C}$

- varying the action, $\mathrm{tr}(\delta A_\Sigma \wedge A_\Sigma)$ has at most a first order pole, compensated by the zeros in ω

With these choices, 4d CS theory is equivalent (at least classically) to

$$S = \frac{z_0 - z_1}{8\pi} \int_\Sigma \mathrm{tr}(J \wedge \star J) + \frac{z_1 + z_0}{12\pi} \int_{\Sigma \times [0,1]} \mathrm{tr}(\tilde{J} \wedge \tilde{J} \wedge \tilde{J})$$

where J is built from the connection [Costello, Yamazaki 19]

There are striking similarities between this story and the twistor construction of integrable systems:

- both involve gauge theory
- in each case, the spectral parameter is incorporated as part of the geometry

Introducing a meromorphic (3,0)-form, with consequent boundary conditions on the (partial) connection, is also natural in twistor space

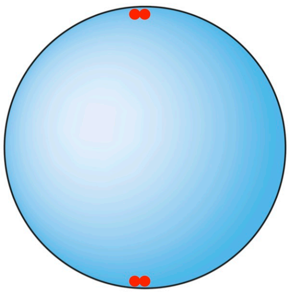
- Penrose-Ward transform interprets solutions of ASDYM as holomorphic bundles $E \to \mathbb{PT}'$ (obeying certain mild conditions)

On a CY 3-fold, these would arise as solutions to eom of holomorphic Chern-Simons theory

$$S[\mathcal{A}] = \frac{1}{2\pi i} \int \Omega \wedge \mathrm{tr}\left(\mathcal{A}\,\bar{\partial}\mathcal{A} + \frac{2}{3}\mathcal{A} \wedge \mathcal{A} \wedge \mathcal{A}\right)$$

- on twistor space, any (3,0)-form Ω necessarily has poles, so we'll need to impose boundary (or divisor) conditions on $\mathcal{A}$ [Costello]
- compare to open B-model on $\mathbb{CP}^{3|4}$ (CY supermanifold), yielding $\mathcal{N} = 4$ supersymmetric ASDYM [Witten]

A simple example is to let Ω have a double pole on each of two planes



Fix these to be $\langle \alpha\, \pi \rangle = 0$ and $\langle \hat{\alpha}\, \pi \rangle = 0$, corresponding to antipodal points on the $\mathbb{CP}^1 \ni [\pi_{\dot{\alpha}}]$

- this form is preserved by $U(2) \subset SO(4)$
- the two planes intersect at the twistor line at ∞

Poles in Ω lead to boundary terms when varying the action

$$2\pi i\, \delta S = \int_{\mathbb{PT}'} \Omega \wedge \mathrm{tr}(\delta\mathcal{A} \wedge \mathcal{F}) + \int_{\mathbb{PT}'} \bar{\partial}\Omega \wedge \mathrm{tr}(\delta\mathcal{A} \wedge \mathcal{A})$$

We eliminate these by requiring $\mathcal{A}$ vanishes to first order on each of $\langle \alpha\, \pi \rangle = 0$ and $\langle \pi\, \hat{\alpha} \rangle = 0$, with similar restrictions on gauge transformations

- compare trivializations using $h(x, \pi)$ obeying $(\bar{\partial} + \mathcal{A})_X h = 0$ and $h(x, \alpha) = 0$. Then gauge-invariant information is $\sigma(x) = h(x, \hat{\alpha})$
- there's a natural global $G \times G$ action $\sigma \mapsto g^{-1} \sigma \tilde{g}$

To reduce to $\mathbb{R}^4$, write $\mathcal{A} = \mathcal{A}|_X + \bar{e}^\alpha \mathcal{A}_\alpha = -h^{-1}\bar{\partial} h + \langle \pi\alpha\rangle\langle\pi\hat{\alpha}\rangle\, \bar{e}^\alpha \phi_\alpha$

- $h(x,\pi) = 1$ in a nbhd of α and $h(x,\pi) = \sigma(x)$ in a nbhd of $\hat{\alpha}$
- $\bar{e}^\alpha$ are a basis of (0,1)-forms along fibres of $\mathcal{O}(1) + \mathcal{O}(1) \to \mathbb{CP}^1$

The hCS action becomes

$$\int D^3Z \wedge \bar{e}^\alpha \wedge \bar{e}_\alpha \wedge \mathrm{tr}\left[\phi^\beta \bar{D}\phi_\beta - \frac{2}{\langle\pi\alpha\rangle\langle\pi\hat{\alpha}\rangle}\phi^\alpha \bar{\partial}_\alpha(h^{-1}\bar{\partial} h)\right]$$

$$= \int D^3Z \wedge \bar{e}^\alpha \wedge \bar{e}_\alpha \wedge \mathrm{tr}\left[\Phi^\beta \bar{D}\Phi_\beta - \bar{D}^{-1}\left(\frac{\bar{\partial}^\alpha(h^{-1}\bar{\partial} h)}{\langle\pi'\alpha\rangle\langle\pi'\hat{\alpha}\rangle}\right)\bar{\partial}_\alpha(h^{-1}\bar{\partial} h)\right]$$

- $\bar{D} = (\bar{\partial} + [h^{-1}\bar{\partial} h,\ \ ])_X$ is the covariant $\bar{\partial}$-operator on a twistor line
- $\bar{\partial}_\alpha$ is the $\bar{\partial}$-operator along the fibres of $\mathbb{PT} \to \mathbb{CP}^1$
- $\Phi_\beta = \phi_\beta - \bar{D}^{-1}\left(\bar{\partial}_\beta(h^{-1}\bar{\partial} h)/(\langle\pi'\alpha\rangle\langle\pi'\hat{\alpha}\rangle)\right)$

Integrating out the Φ_β gives $1/\sqrt{\det(\bar{D}_X)^2}$ (similar determinants arise from the ghosts)

The remaining term depends only on h, hence σ. The integrals over each twistor line can be performed explicitly, yielding the 4d WZW action [Donaldson; Nair,Schiff]

$$S_k[\sigma] = \frac{1}{2}\int_{\mathbb{C}^2} \mathrm{tr}(J\wedge\star J) + \frac{1}{3}\int_{\mathbb{C}^2\times[0,1]} k\wedge \mathrm{tr}(\tilde{J}\wedge\tilde{J}\wedge\tilde{J})$$

- $J = -d\sigma\,\sigma^{-1}$; similarly $\tilde{J} = -d\tilde{\sigma}\,\tilde{\sigma}^{-1}$ for a homotopy $\tilde{\sigma}$ from σ to 1
- k is the Kähler form determined by $[\alpha]\in\mathbb{CP}^1$
- the field equation $k\wedge\partial(\sigma^{-1}\bar{\partial}\sigma) = 0$ is Yang's J-matrix form of the ASDYM equations

WZW_4 is not well understood, however [Losev,Moore,Nekrasov,Shatashvili]

- there's a Polyakov-Weigmann identity

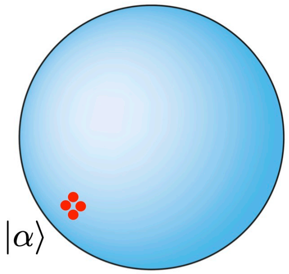
$$S_k[\sigma_1\sigma_2] = S_k[\sigma_1] + S_k[\sigma_2] + \int_{\mathbb{C}^2} k\wedge\mathrm{tr}\left(\sigma_1^{-1}\partial\sigma_1\wedge\sigma_2^{-1}\bar{\partial}\sigma_2\right)$$

  which fixes some, but not all, correlation functions
- with $\sigma = \mathrm{e}^{\pi/f_\pi}$ for a field $\pi:\mathbb{R}^4\to\mathfrak{g}$, the β-function of the coupling f_π vanishes at 1-loop

Many other actions can be generated from different choices of Ω. For example, let's pick $\Omega = D^3Z/\langle\alpha\pi\rangle^4$ which preserves $SU(2) \times B \subset SO(4)$

- $\mathcal{A}$ must now vanish to second order at $|\alpha\rangle$
- not obvious how to 'compare' trivializations, so instead fix $\mathcal{A}|_X \in H^{0,1}(\mathbb{CP}^1, \mathcal{O}(-2D_\alpha))$ to be harmonic:

$$\mathcal{A}|_X = \langle\alpha\pi\rangle^2\,\phi(x)\,\frac{\langle\hat\pi\, d\hat\pi\rangle}{\langle\pi\,\hat\pi\rangle^2}$$



This leads to a cubic action [Leznov,Mukhtarov;Parkes;Siegel]

$$S[\phi] = \int_{\mathbb{C}^2} \frac{1}{2}\mathrm{tr}(d\phi \wedge \star d\phi) + \frac{1}{3}\mu \wedge \mathrm{tr}(\phi\, d\phi \wedge d\phi)$$

whose field equation is Yang's K-matrix form of ASDYM

- $\mu = d^2x^{\alpha\beta}\alpha_\alpha\alpha_\beta$ is the (0,2)-form on $\mathbb{C}^2$ specified by $|\alpha\rangle$

If Ω has only simple poles

$$\Omega = \frac{D^3 Z}{\langle \alpha_+ \pi \rangle \langle \alpha_- \pi \rangle \langle \beta_+ \pi \rangle \langle \beta_- \pi \rangle}$$

we may impose the weaker boundary conditions

$$\mathcal{A}|_{\pi = \alpha_\pm, \beta_\pm} \in \mathfrak{l}_\pm \qquad \text{where } \mathfrak{g} = \mathfrak{l}_+ \oplus \mathfrak{l}_- \text{ is a Manin triple}$$

- gauge fixing as before gives a 'trigonometric' action in 4d, with interesting generalizations in 2d [Mason,Sparling]

Going further, allowing simple zeros in

$$\Omega = D^3 Z \, \frac{\prod_{i=1}^{n} \langle \alpha_i \pi \rangle \langle \beta_i \pi \rangle}{\prod_{j=1}^{n+2} \langle \gamma_j \pi \rangle^2}$$

means we should permit simple poles in $\mathcal{A}$

- this leads to a (complicated) theory of coupled σ-models in 4d
- not equivalent to ASDYM, but still has Lax connection and leads to known integrable 2d theories [Costello,Yamazaki]

In each of these examples, the natural operators come from twistor space

- since $\mathcal{A}$ is only a partial connection, there are no line operators
- instead, use surface operators $U_S(Z, Z') : E_{Z'} \to E_Z$ defined by

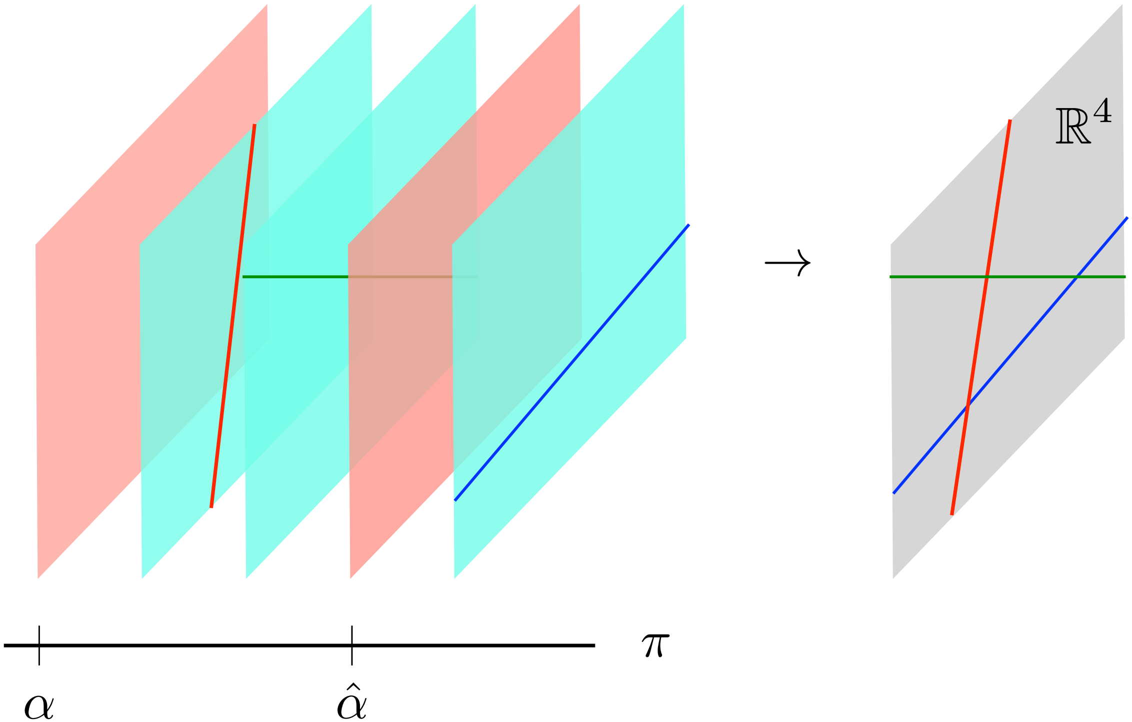
$$(\bar{\partial} + \mathcal{A})|_S U_S(Z, Z') = 0 \qquad U_S(Z, Z) = \mathrm{id}$$

where $S \subset \mathbb{PT}'$ is an (affine) complex curve

These operators have been considered before in various contexts

- they have been studied by Atiyah, Khesin, R Thomas and others in the context of holomorphic linking
- they also play a starring role in the twistor version of amplitude / Wilson loop correspondence in planar $\mathcal{N} = 4$ SYM that Lionel & I developed during visits to IHÉS and PI

Suppose S lies in the fibre of $\mathbb{PT}'$ over fixed $[\pi] \in \mathbb{CP}^1$



- such S are not real twistor lines, but project to give surfaces $\mathbb{R}^2 \subset \mathbb{R}^4$ that are holomorphic in the $\mathbb{C}$-str defined by their $\pi \in \mathbb{CP}^1/\{\alpha, \hat{\alpha}\}$

These surface operators can be moved around holomorphically within $\mathbb{PT}'$ without encountering singular configurations

We can parametrize S as $Z(s) = A + sB$ with $|B\rangle = 0$. This corresponds to the 2-surface $x(s) = |\hat{A}\rangle([A| + s[B|) + |A\rangle([\hat{A}| + \bar{s}[\hat{B}|)$ in $\mathbb{R}^4$

- if Ω has double poles on $\langle\alpha\pi\rangle = 0$ and $\langle\hat{\alpha}\pi\rangle = 0$, we must have $\mathcal{A} = \langle\alpha\pi\rangle\,\langle\hat{\alpha}\pi\rangle\,\phi(Z)$ for some $\phi(Z) \in \Omega^{0,1}(\mathbb{PT}', \mathfrak{g}(-2))$
- the leading-order contribution to the correlation function of a pair of surface operators should again come from a single mCS propagator stretched between S and S'

We may optimistically hope to obtain an r-matrix obeying a four-dimensional version of the cYBE from this procedure (wip)

This is related, not just analogous, to the previous story through symmetry reduction

- *e.g.* pick a null vector V and require the field σ of WZW_4 to be invariant under translations along V, $\bar{V}$
- WZW_4 action immediately reduces to 2d PCM

$$S_\kappa[\sigma] = \frac{1}{2}\int_{\mathbb{C}} \mathrm{tr}(J\wedge\star J) + \frac{\kappa}{3}\int_{\mathbb{C}\times[0,1]} \mathrm{tr}(\tilde{J}\wedge\tilde{J}\wedge\tilde{J})$$

with WZ coefficient $\kappa = k(V,\bar{V})$ inherited from the Kähler form

Instead of first moving to $\mathbb{R}^4$, then applying a symmetry reduction, one can apply the reduction directly in twistor space

- lift V, $\bar{V}$ to vector fields $\mathcal{V}$, $\bar{\mathcal{V}}$ on $\mathbb{PT}'$ (trivial for translations):

$$\mathcal{V} = \kappa^\alpha\mu^{\dot\alpha}\frac{\partial}{\partial x^{\alpha\dot\alpha}} \qquad \bar{\mathcal{V}} = \hat{\kappa}^\alpha\hat{\mu}^{\dot\alpha}\frac{\partial}{\partial x^{\alpha\dot\alpha}} \qquad \text{for some spinors } \mu,\ \kappa$$

- we require the twistor space meromorphic Chern-Simons field $\mathcal{A}$ obeys $\mathcal{L}_{\mathcal{V}}\mathcal{A} = 0 = \mathcal{L}_{\bar{\mathcal{V}}}\mathcal{A}$

Contracting with $\mathcal{V} \wedge \bar{\mathcal{V}}$ and using the assumed invariance of $\mathcal{A}$, the holomorphic Chern-Simons action in twistor space reduces to

$$S[A] = \frac{1}{2\pi i} \int_{\mathbb{E}^2 \times \mathbb{CP}^1} \omega \wedge \mathrm{tr}\left(A \wedge \mathrm{d}A + \frac{2}{3} A \wedge A \wedge A\right)$$

- the meromorphic (1,0)-form

$$\omega = \imath_{\mathcal{V}\wedge\bar{\mathcal{V}}}\,\Omega = \frac{\langle\pi\mu\rangle\langle\pi\hat{\mu}\rangle\,\langle\pi\, d\pi\rangle}{\langle\pi\alpha\rangle^2\langle\pi\hat{\alpha}\rangle^2}$$

  acquires zeros through contraction with $\mathcal{V}$, $\bar{\mathcal{V}}$

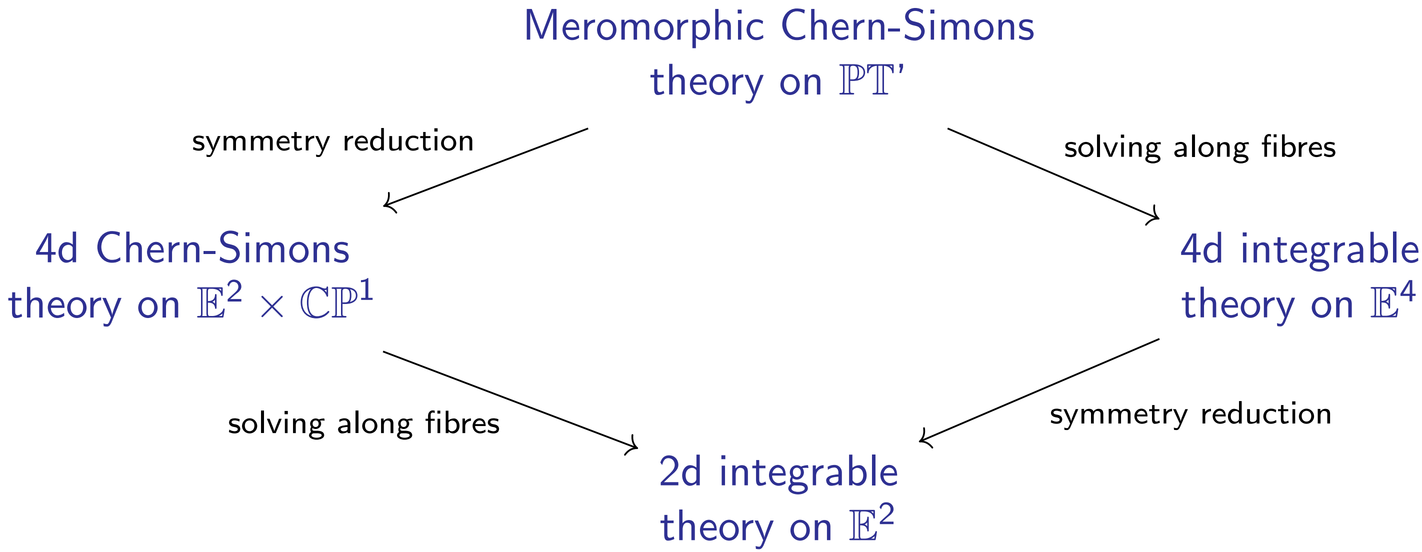
- in terms of a local coordinate $w = x^{\alpha\dot{\alpha}}\kappa_\alpha\hat{\mu}_{\dot{\alpha}} \in \mathbb{C}$, the gauge field

$$A = \mathcal{A}|_X + \frac{\langle\pi\mu\rangle}{\langle\pi\hat{\mu}\rangle}\, dw\, \imath_{\bar{\mathcal{V}}}\mathcal{A} - \frac{\langle\pi\hat{\mu}\rangle}{\langle\pi\mu\rangle}\, d\bar{w}\, \imath_{\mathcal{V}}\mathcal{A}$$

  and automatically obeys correct boundary conditions for PCM

This is exactly the 4d CS description of the 2d PCM model with WZ term

The relation may be summarized by the following commutative diagram:

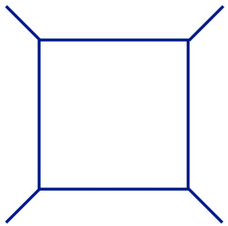


Which theory is obtained depends on the choice of Ω, the corresponding boundary conditions on $\mathcal{A}$, and the choice of symmetry reduction

- various other cases considered in [Bittleston,DS; Penna; Chen,He,Tian]

If the twistor actions could be quantized (at least perturbatively), quantum integrability of theories such as WZW_4 would be guaranteed

The theory is chiral, so has a potential gauge anomaly coming from a box diagram on $\mathbb{PT}'$



- [Costello,Li] claim that for $G = SO(8)$ this diagram can be made to cancel via a 6d version of the Green-Schwarz mechanism

The 4d theory is not anomalous, but would no longer come from a twistor progenitor at the quantum level, and there'd be no reason to expect quantum integrability

The Green-Schwarz anomaly cancellation mechanism requires coupling to gravity. There's a beautiful twistor action for (perturbative) ASD gravity [Mason,Wolf]

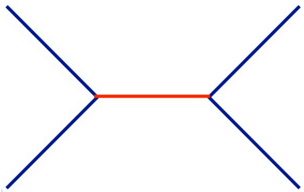
$$S[h] = \frac{1}{2\pi i}\int_{\mathbb{PT}'} \Omega' \wedge \left(h\,\bar{\partial} h + \frac{1}{3} h \wedge \{h, h\} \right)$$

- $h \in \Omega^{0,1}(\mathbb{PT}', \mathcal{O}(2))$ and $V = \{h, \cdot\,\}$ a Hamiltonian deformation of the almost $\mathbb{C}$-str of $\mathbb{PT}'$
- now choose Ω' to have 4^{th}-order poles on $\langle \pi\, \alpha \rangle = 0$ and $\langle \pi\, \hat{\alpha} \rangle = 0$

Deforming the a$\mathbb{C}$-str generates a new coupling to the gauge theory

$$S'[h, \mathcal{A}] = \frac{1}{2\pi i}\int_{\mathbb{PT}'} \Omega \wedge \mathrm{tr}\left(\mathcal{A} \wedge \{h, \mathcal{A}\} \right)$$

- this generates a new diagram which for $G = SO(8)$ can cancel the previous gauge anomaly [Costello,Li]



We expect (wip) that this twistor action is equivalent to the action

$$S[\Phi] = \int_{\mathbb{C}^2} \frac{1}{2} k \wedge \partial\Phi \wedge \bar{\partial}\Phi + \frac{1}{3}\Phi\, \partial\bar{\partial}\Phi \wedge \partial\bar{\partial}\Phi \qquad \text{[Ooguri, Vafa]}$$

where Φ is a deformation of a background (pseudo-)Kähler potential, and k the background Kähler form

- the field equation is Plebanski's 1^{st} heavenly equation for ASD Einstein metrics
- instead choosing Ω' to have a single 8^{th}-order pole should lead instead to Plebanski's 2^{nd} fundamental form

The gauge-gravity coupling modifies the WZW_4 action in the obvious way:

$$S'_k[\sigma, \Phi] = \int_{\mathbb{C}^2} \partial\bar{\partial}\Phi \wedge \mathrm{tr}(J \wedge J) + \int_{\mathbb{C}^2 \times [0,1]} \partial\bar{\partial}\Phi \wedge \mathrm{tr}(\tilde{J} \wedge \tilde{J} \wedge \tilde{J})$$

Summary

hCS theory on $\mathbb{PT}'$ with a meromorphic (3,0)-form describes a 4d theory that is integrable, at least classically

- if Ω is nowhere vanishing, the 4d theory has eom equivalent to the ASDYM eqns, but more general systems can also be obtained
- performing a symmetry reduction of the 4d theory gives an action for a lower dimensional integrable system
- performing this reduction directly in $\mathbb{PT}'$ gives the 4d CS description of this system [Costello, Yamazaki]

Realizing these equivalences at the level of the action provides a good starting point for quantization via the path integral, at least perturbatively

- many challenges remain, but encouragement from success of closely-related case of 4d Chern-Simons theory

The whole story is also very closely related to string theory

- holomorphic CS arises as the open B-model on $\mathbb{PT}'$ (though role of poles in Ω not clear)
- ASD gravity on $\mathbb{PT}'$ seems likely to be the closed B-model in the presence of a background Poisson structure. It is also closely related to $\mathcal{N} = 8$ twistor strings
- Plebanski and WZW$_4$ actions are the string field theory of $\mathcal{N} = 2$ strings [Ooguri,Vafa;Berkovits,Vafa]
- 4d CS theory is a mixed A/B-model on $C \times T^*\Sigma$

Understanding the connection between these various string theories may provide another route to quantization

Happy 61 $^{1}/_{6}$$^{\text{th}}$ Birthday!