Twistors and the $AdS_5 \times S^5$ Superstring

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$$Z^I = (\lambda^\alpha, \mu^{\dot\alpha}, \eta^J), \quad Y_I = (\bar\mu_\alpha, \bar\lambda_{\dot\alpha}, \bar\eta_J)$$

Ferber 1978

$$S = \int dz \, Y_I \dot{Z}^I + a(Y_I Z^I)$$

NB, 2004

$$\to S = \int d^2 z \, Y_I \underbrace{\bar{\nabla}}_{\bar\partial + \bar A} Z^I + \cancel{\text{current algebra}} \to \text{scattering amplitudes}$$

$N=4$ $D=4$ sYM

Y $($ $Z(\sigma)$

$$\frac{Tr(FFFX)}{\text{free sYM}} \to \text{closed string states } AdS_5 \times S^5 \text{ at } r=0$$

$g_{YM}=0$

2103.10446 PSU(2,2|4)

Gaberdiel
Gopakumar

$z^I_0, z^I_{-1}, \ldots$
$y_{-1\,I}, \ldots$
? AdS$_5 \times S^5$ at $r=0$?

$N=(2,2)$ worldsheet + NS-NS flux

$PSU(2|2) \times PSU(2|2)$

$AdS_3 \times S^3$ at $r=0$

WZW model

Hybrid formalism

(z, y)

scattering amp's

Kharkov Sorokin, Tkach, Volkov, Zheltukhin '89

$D=2$ $N=1$ scft, $D=10$ susy

$m = 0..9$
$\alpha = 1..16$

$$\mathbb{X}^m = x^m + \kappa \Psi^m, \quad \Theta^\alpha = \theta^\alpha + \kappa \Lambda^\alpha$$

$$D = \frac{\partial}{\partial \kappa} + \kappa \frac{\partial}{\partial z}$$

$$D\mathbb{X}^m = D\Theta^\alpha \gamma^m_{\alpha\beta} \Theta^\beta$$

$$\Rightarrow \partial x^m - \partial\theta \gamma^m \theta = \Lambda \gamma^m \Lambda, \quad \Psi^m = \Lambda \gamma^m \theta$$

$\delta\theta^\alpha = \varepsilon^\alpha$ $\delta x^m = \theta \gamma \varepsilon$

Solve for $\Lambda\gamma^m\Lambda$, $\Psi^m \to x^m, \theta^\alpha, \lambda^\alpha$

Breaks manifest $d=2$ susy

Preserves $d=10$ susy

$d=10$ pure spinor $\lambda\gamma^m\lambda = 0$

$$S = \int d^2z \left(\partial X^m \bar\partial X_m + p_\alpha \bar\partial \theta^\alpha + \omega_\alpha \bar\partial \lambda^\alpha\right)$$

$$Q_{RNS} + \underbrace{\int dz\, \Lambda^\alpha p_\alpha}_{\text{non-minimal term}} \to Q_{PS} = \int dz\, \lambda^\alpha \underbrace{\left(p_\alpha - \partial X^m (\gamma_m \theta)_\alpha\right)}_{= d_\alpha}$$

$\Lambda^\alpha \to \lambda^\alpha$, $(\kappa\bar\lambda)^\alpha$

NB
2106.04448

Solve for (X^m, Ψ^m)

$$\mathbb{X}^m = x^m + \kappa\Psi^m + \bar\kappa\bar\Psi^m + \kappa\bar\kappa F^m$$

$$\to \Theta^\alpha = \theta^\alpha + \kappa\Lambda^\alpha + \bar\kappa\bar\Lambda^\alpha + \kappa\bar\kappa f^\alpha$$

$$D = \frac{\partial}{\partial\kappa} + \kappa\frac{\partial}{\partial z}$$

$$\bar D = \frac{\partial}{\partial\bar\kappa} + \bar\kappa\frac{\partial}{\partial\bar z}$$

Flat space: $S = \int d^2z\, D\mathbb{X}^m \bar D \mathbb{X}_m$

$AdS_5 \times S^5$:

$PSU(2,2|4)$

$\Theta^R{}_J \,, \quad \hat{\Theta}^J{}_R$

$\delta \Theta^R{}_J = \epsilon^R{}_J + \Theta^R{}_K \hat{\epsilon}^K{}_S \Theta^S{}_J$

$\delta \hat{\Theta}^J{}_R = \hat{\epsilon}^J{}_R + \hat{\epsilon}^J{}_S \Theta^S{}_K \hat{\Theta}^K{}_R$

$+ \hat{\Theta}^J{}_S \Theta^S{}_K \hat{\epsilon}^K{}_R$

$$\left(\begin{array}{c|c} SU(2,2) & q^R{}_J \\ \hline \hat{q}^J{}_R & SU(4) \end{array}\right)$$

$R = 1 \ldots 4$

$J = 1 \ldots 4$

$$\longrightarrow S = r^2 \int d^2 z \left(D\Theta^R{}_J \bar{D} \hat{\Theta}^J{}_R + D\Theta^R{}_K \hat{\Theta}^K{}_S \bar{D}\Theta^S{}_L \hat{\Theta}^L{}_R \right)$$

$$S = r^2 \int d^2 z \left(\partial \Theta^R{}_S \bar{\partial} \hat{\Theta}^J{}_R + \ldots + \Lambda^R{}_J \bar{\partial} \hat{\Lambda}^J{}_R + \bar{\Lambda}^R{}_J \partial \bar{\hat{\Lambda}}^J{}_R \right.$$
$$\left. + \overline{\Lambda \hat{\Lambda} \bar{\Lambda} \bar{\hat{\Lambda}}} \right)$$

bosonic Gross-Neveu model with $d=2$ conf. invariance

integrable

$N=1$ superconf $G = \underline{G^+} + G^-$

$(\Lambda^R{}_J, \hat{\Lambda}^J{}_R) \to \Lambda^A \quad A = 1 \ldots 32$

$\frac{1}{2}$ 32 comp ~~d=12~~ spinor $d=(10,2)$

$$SO(4,2)\times SO(6) \subset SO(10,2)$$

$\Lambda \to U$ conf wt 0

$\Lambda \to V$ 1

$\underline{U}\gamma^{MN}U = 0 \quad M,N = 0,\dots 11$

~~d=12~~ $d=(10,2)$ pure spinor

16 comp's

$$S = \int d^2z \left(V\bar\partial U + \hat V \partial \hat U + \dots \right)$$

$G^+ \to Q_{\text{pure spinor}}$ of $AdS_5 \times S^5$ superstring

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Flat space limit

$r \to \infty, \quad \Theta^\alpha \to \frac{1}{\sqrt{r}}\Theta^\alpha$

$$r^2 \int d^2z \left(D\Theta \bar D \hat\Theta + \Theta D \hat\Theta \Theta \bar D \hat\Theta \right) \to \int d^2z \left(r D\Theta \bar D \hat\Theta + \Theta D\hat\Theta \Theta \bar D \hat\Theta \right)$$

$$= \int d^2 z \left(\cdots \Phi\, D\, \Theta + \frac{1}{r}\, \cancel{\Phi}\, \Phi + \text{quartic} \right)$$

$\boxed{r \to \infty}$ $\qquad \Theta^\alpha \to \frac{1}{r}\, \Theta^\alpha$

free action $\qquad \to \int d^2 z \left(D\Theta\, \bar{D}\, \bar{\Theta} \right)$

$D = (10,2)$ Lorentz inv. $\qquad \int d^2 z \left(\partial \Theta^R_\sigma\, \hat{\partial}\, \Theta + \psi^R_\sigma\, \bar{\partial} z + \hat{\psi}\, \partial \hat{z} \right)$
