Refractive index for speckles mask

SM, 15/1/2020
• Our case:
  - big particles: $x = ka \ll 1$ where $a$ radius of particle, $k = \frac{2\pi}{\lambda}$ Low refractive index: $\delta \ll 1$ where $n = 1 - \delta + i\beta$

• This is the so-called ‘anomalous diffraction’ case. Scattering is result of:
  - Reflection and refraction of light inside the particle (negligible)
  - Diffraction from particle shape (dominant)

• Purpose is to calculate the scattering efficiency as a function of the refractive index. All equations are taken from Van de Hulst, “Light scattering by small particles”
Extinction and scattering function (single particle)

- **Extinction** is the optical power removed from transmitted by scattering and absorption. In our case:
  \[ Q_{\text{ext}} \approx \frac{1}{2} \rho^2 \]
  Where \( \rho = 2x\delta \). \( \rho \) is the max variation of optical path (“ray” crossing the centre of the particle). Under the assumption that absorption is negligible (\( \delta \gg \beta \)) this is entirely due to scattering.

- **Scattering function**. Our case is at the border between the anomalous diffraction case (very big particles) and the Rayleigh-Gans approximation (very low \( \delta \)). In this case the scattering field amplitude is
  \[ S(\theta) = i\rho k^2 a^2 \left( \frac{\pi}{2z^3} \right)^{1/2} J_{3/2}(z) \]
  Where \( z = \theta x = \theta ka \) That is linearly proportional to \( \rho = 2x\delta \).
Comparison with pure diffraction
Comparison with pure diffraction
Comparison with pure diffraction
Comparison with pure diffraction
Comparison with pure diffraction
Mask vs particles

- Exploiting Babinet’s principle, a spherical particle of radius $a$ and refractive index $\delta$ is equivalent to (except sign of field amplitude) a hole of same radius in slab of refractive index $\delta$.

- For $N$ particles, scattering amplitudes (and extinction) add linearly ($S_{tot}(\theta) = NS(\theta)$). In the case of a mask, amplitude proportional to the number of holes per unit surface.

In order to maximize the extinction (=scattering), the mask should have a maximum of $\rho = 2kd\delta$, where $d$ is the thickness. That is:

\[
\text{max thickness, max } \delta \text{ and max } N
\]
A few numbers

<table>
<thead>
<tr>
<th>material</th>
<th>$\delta @ 12$ keV</th>
<th>$\rho$</th>
<th>N</th>
</tr>
</thead>
<tbody>
<tr>
<td>500nm SiO2 particles</td>
<td>3.18E-06</td>
<td>4.8E-02</td>
<td>Order 1E06 (200x200 um2)</td>
</tr>
<tr>
<td>Water</td>
<td>1.6E-06</td>
<td>N/A</td>
<td>N/A</td>
</tr>
<tr>
<td>Si, 10 um</td>
<td>3.39E-06</td>
<td>2.1</td>
<td>Order 1E03</td>
</tr>
<tr>
<td>Au, 10 um</td>
<td>1.79E-05</td>
<td>10.9</td>
<td></td>
</tr>
<tr>
<td>Air</td>
<td>? I assume 0 (n=1)</td>
<td>N/A</td>
<td>N/A</td>
</tr>
</tbody>
</table>

- Increase of signal per scatterer (x40 increase with 500 nm holes in 10 um Si slab). For $|S(\theta)|^2$?

- However, surface density of scatterers WAY lower for mask. As a result, overall signal might be lower for mask. This can be compensated by increase of exposure time (static pattern).