Emittance Exchange in MICE

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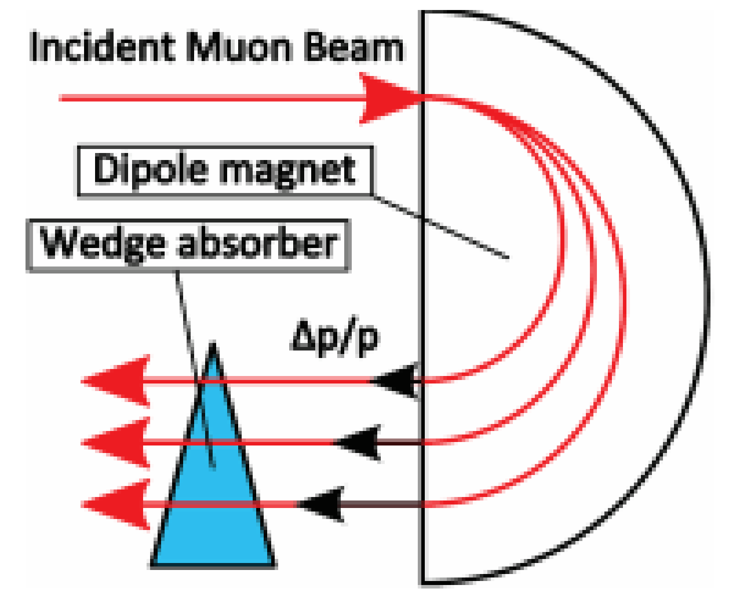
Aims



- Demonstrate Emittance Exchange and Reverse Emittance Exchange in the Wedge using MICE data
- Emittance Exchange can be demonstrated by looking at the change in phase space density of the particle selection before and after having passed through a Wedge absorber
- Emittance Exchange is shown by a decreased transverse phase space density (x, px, y, py) and increased longitudinal phase space density (z, pz), (and vice versa for Reverse Emittance Exchange)
- Can use a number of techniques to calculate phase space density: KDE, KNN, Voronoi Tessellations, etc.
- MICE beam only has a small natural dispersion → Use beam reweighing techniques to select beams with desired dispersion

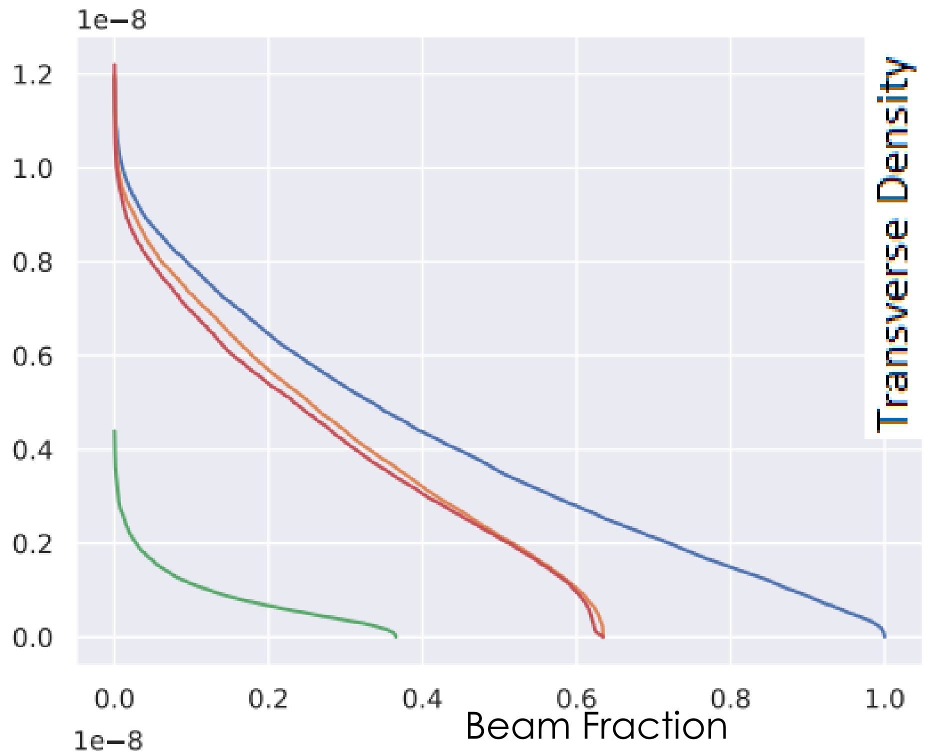
Previously

- Showed change in transverse phase-space density plots for various absorbers in two different ways. Both are however biased.

Case 1: Biased by Transmission Losses

- Cooling seen when the transverse downstream phase space density is greater than the upstream density.
- Bias is introduced by the missing particles being excluded from the downstream phase space volume calculation i.e comparing different volumes
- The current normalization doesn’t account for the change in the particle distribution function.

Case 2: Biased by surviving beam particles



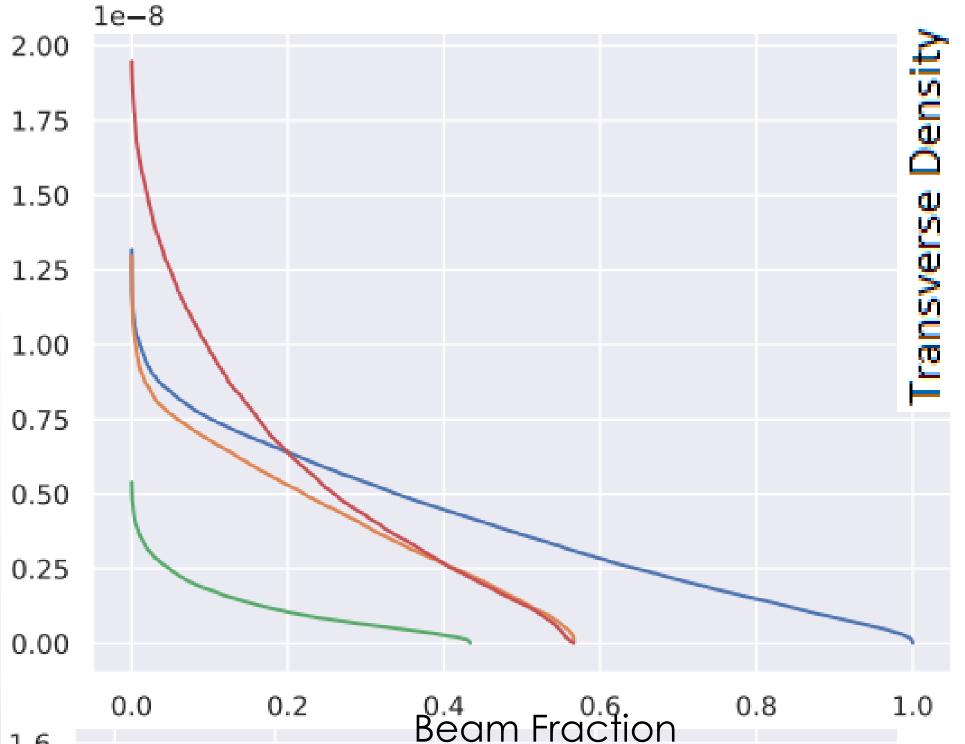
Fraction of beam above certain density

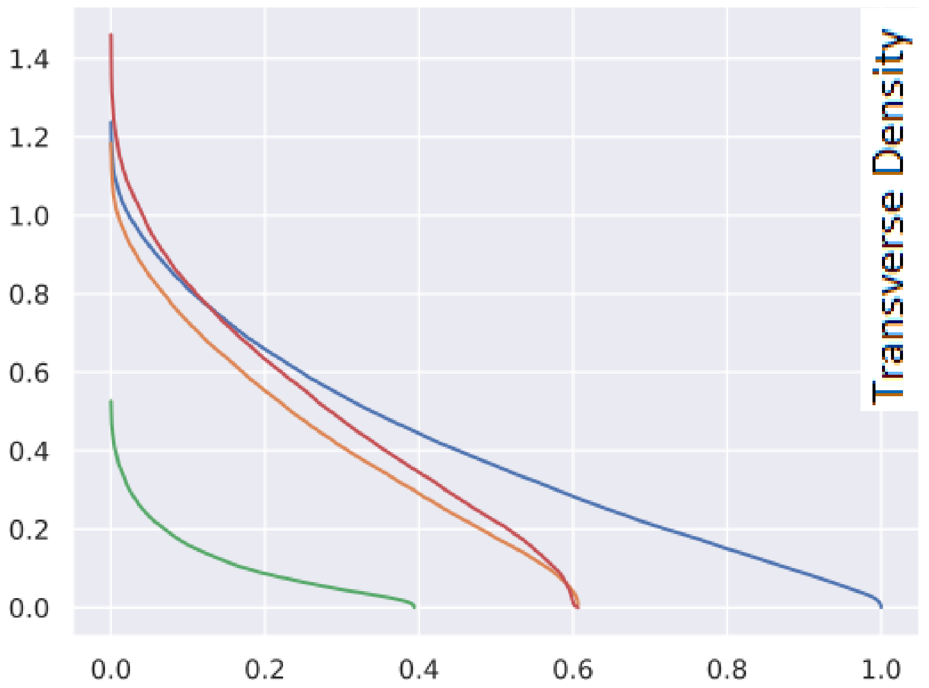
Top Left: No absorber

Top Right: Wedge

Bottom Left: LiH

Bottom Right: LH2



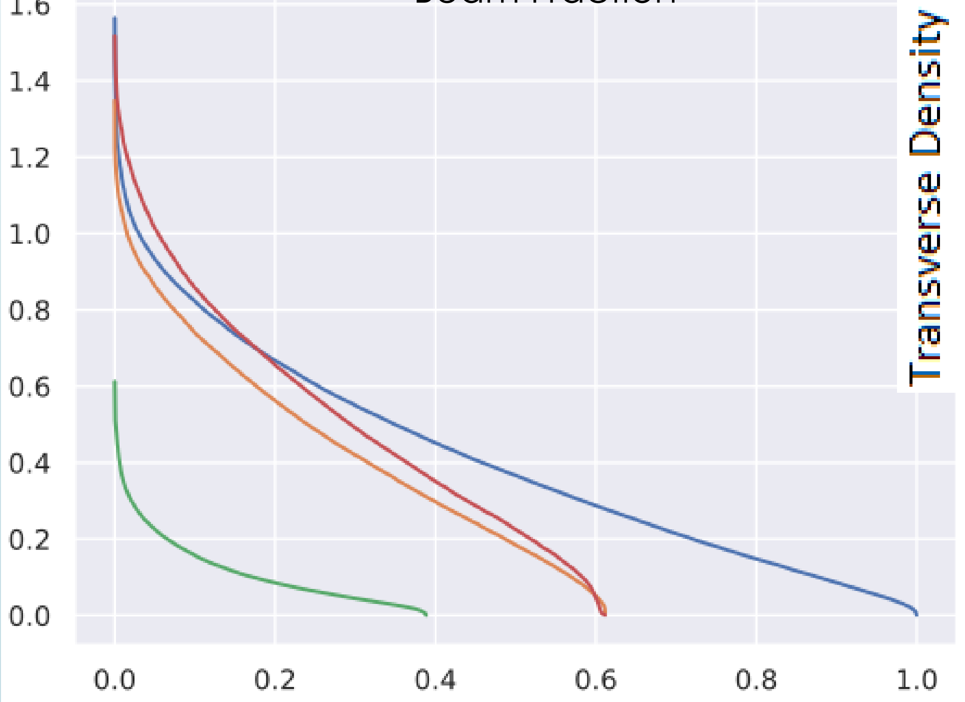


Blue – Full Upstream Sample

Red – Full Downstream Sample

Orange – Upstream Sample which made it Downstream

Green – Upstream Sample which doesn’t make it downstream



Previously

- Showed change in transverse phase-space density plots for various absorbers in two different ways. Both are however biased.

Case 1: Biased by Transmission Losses

Case 2: Biased by surviving beam particles

- The ratio of the downstream to upstream densities is a constant for the flat/no absorber case (expected when comparing same volumes)
- Lost particles are however excluded. Biased as it excludes some of the heating aspect

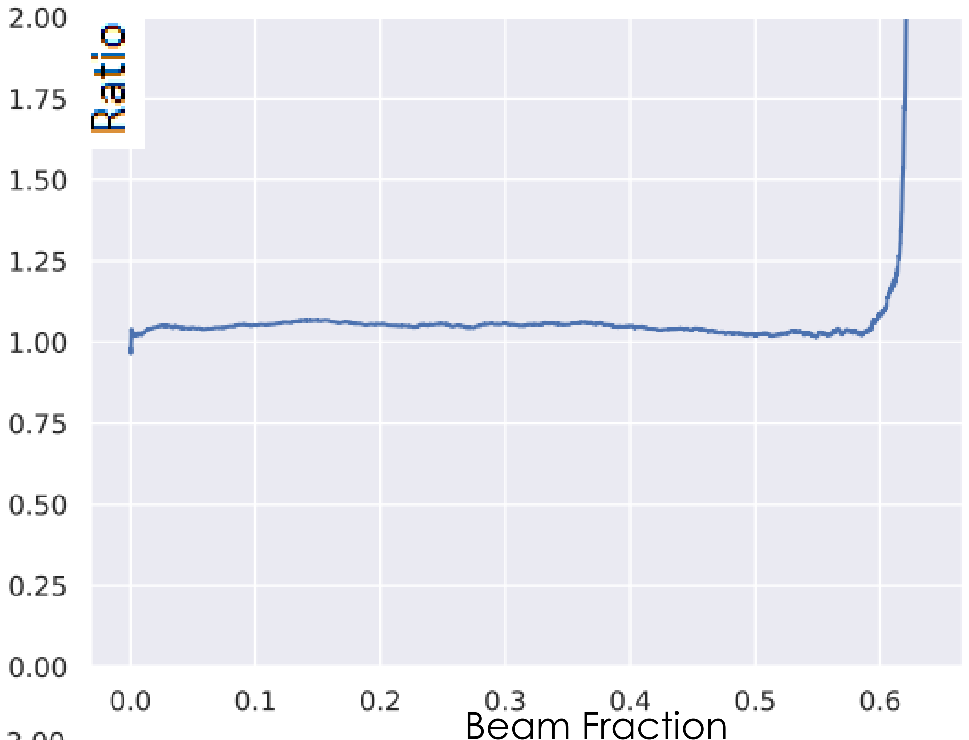
Ratio of the Downstream density to the Upstream density which makes it downstream

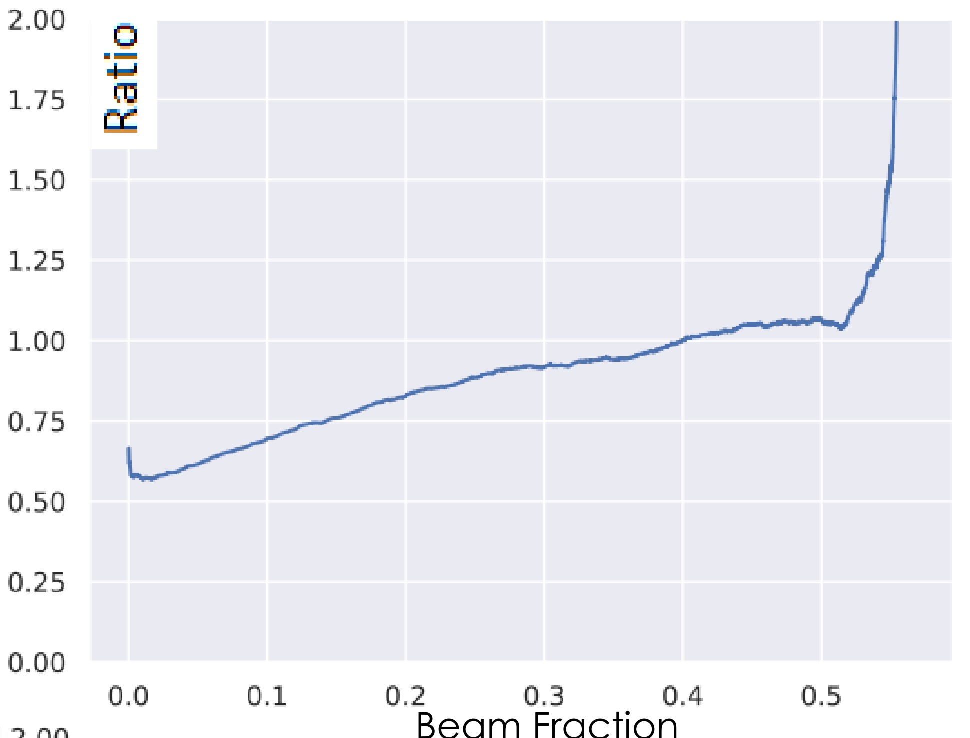
Top Left: No absorber

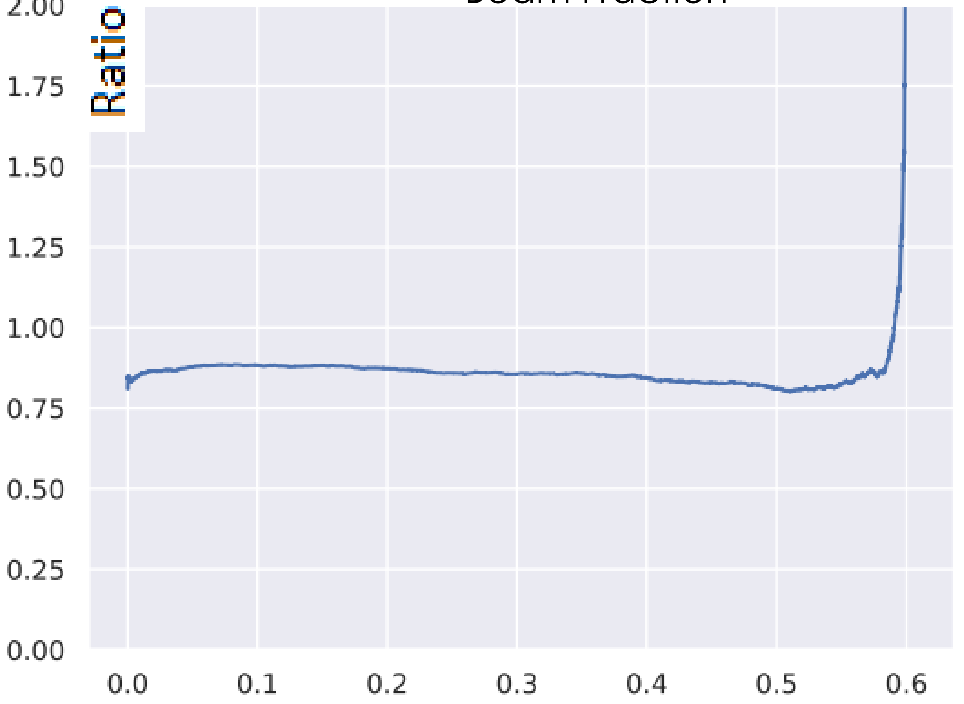
Top Right: Wedge

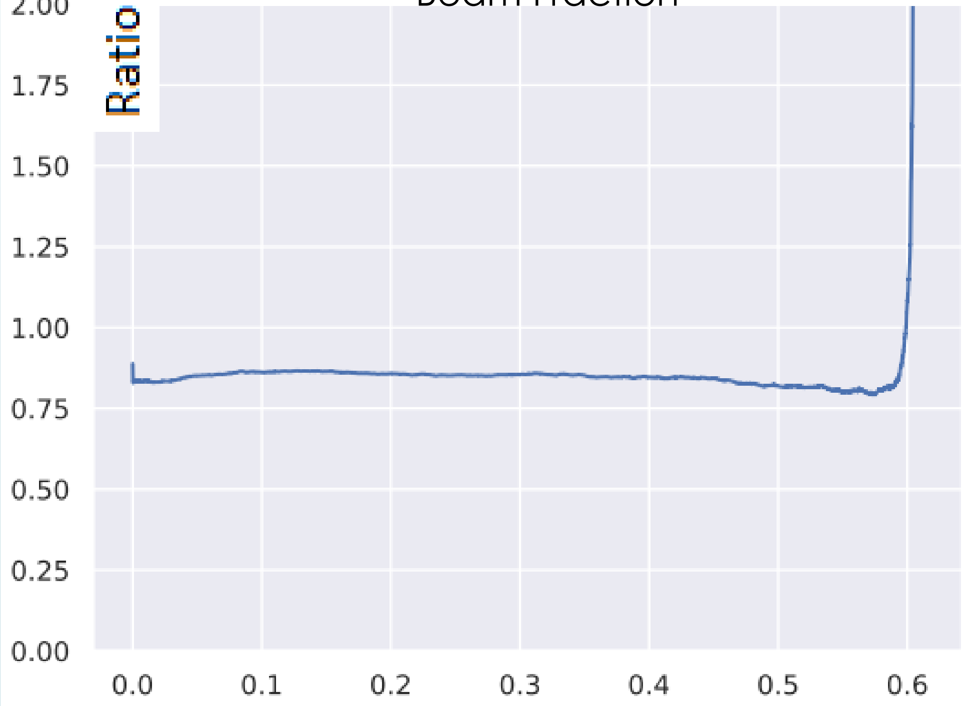
Bottom Left: LiH

Bottom Right: LH2









Ratio above one indicates heating while a ratio below one indicates cooling.

Transmission limits the beam to approximately 60% of the full upstream sample.

The min and max are limited by low sample size and scraping respectively

Transmission losses (Recall)

- Liouville’s theorem only applies to the same particles (or to system with the same particle distribution function). I.e the volume remains the same and the change in the covariance matrix can be described in a conserved manner.
- Transmission losses and subsequent change in particle distribution function can be described by the change it has on the covariance matrix (subscript 1: Full Upstream sample, 2: Upstream which makes it downstream, 3: Upstream which goes missing)

$$\begin{aligned}\Sigma_1 &= \frac{N_2^3}{(N_2+N_3)^3}\Sigma_2 + \frac{N_3^3}{(N_2+N_3)^3}\Sigma_3 \\ &+ \sum_1^{N_2}\Big(N_2N_3(P_i-\bar{P}_2)(P_i-\bar{P}_3) + N_2N_3(P_i-\bar{P}_3)(P_i-\bar{P}_2) + N_3^2(P_i-\bar{P}_3)(P_i-\bar{P}_3)\Big)/(N_2+N_3)^3 \\ &+ \sum_1^{N_3}\Big(N_2N_3(P_i-\bar{P}_2)(P_i-\bar{P}_3) + N_2N_3(P_i-\bar{P}_3)(P_i-\bar{P}_2) + N_2^2(P_i-\bar{P}_2)(P_i-\bar{P}_2)\Big)/(N_2+N_3)^3\end{aligned}$$

- For the case of a symmetric absorber this can be simplified to

$$N_1\Sigma_1 = N_2\Sigma_2 + N_3\Sigma_3$$

The determinant of a matrix (Recall)

- The determinant of a matrix can be separated into parts using:

$$|\Sigma_1| = \sum_{i=0}^{n} \Gamma_n^i \left| {}^{\Sigma_2}\!/\!{}_{\Sigma_3}{}^{i} \right| = |\Sigma_2| + |\Sigma_3| + \sum_{i=1}^{n-1} \Gamma_n^i \left| {}^{\Sigma_2}\!/\!{}_{\Sigma_3}{}^{i} \right|$$

Where Γ_n^i represents substituting all combinations of i^{th} lines from Σ_2 by the same lines in Σ_3 and taking the subsequent determinant of the new matrix

- For the symmetric case (LiH, LH2 and no absorber) the previous and above substitutions could be made to compare the upstream and downstream densities. Due to the asymmetry this cannot be done for the wedge and requires further derivation for the asymmetric case.

Potential next step (Recall)

- The missing data downstream is inaccessible, however the upstream sample which makes it downstream can be compared to the downstream sample
- The transport, M, of a covariance matrix from upstream to downstream can be given by:

$$\Sigma_{down} = \langle X_{down}\tilde{X}_{down}\rangle = \langle M X_{up}\tilde{M}\tilde{X}_{up}\rangle = M\langle X_{up}\tilde{X}_{up}\rangle\tilde{M} = M\Sigma_{up}\tilde{M}$$

- The determinant is given by:

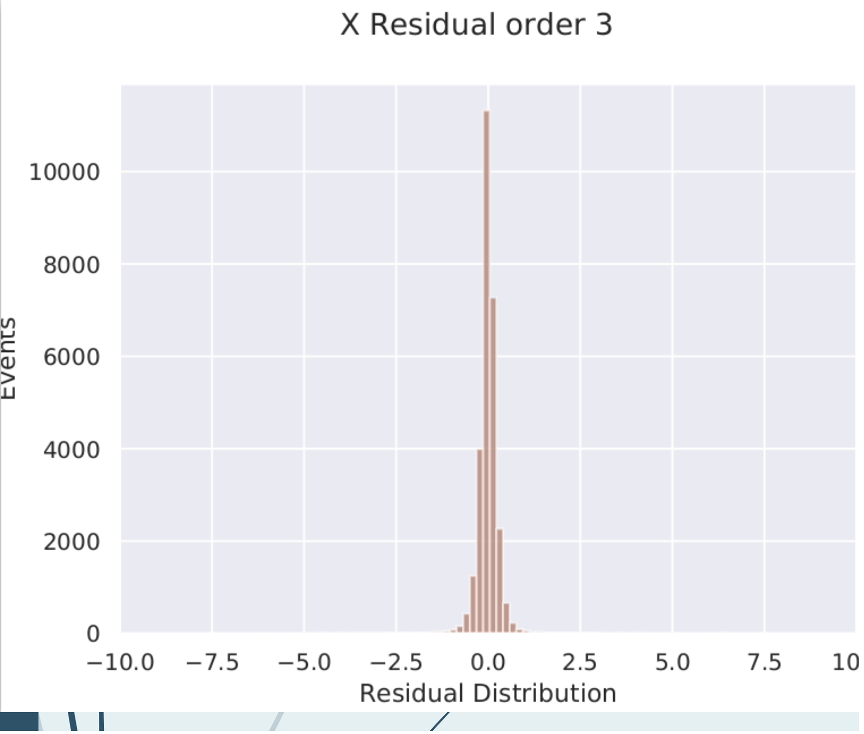
$$|\Sigma_{down}| = \left|M\Sigma_{up}\tilde{M}\right| = |M|^2\left|\Sigma_{up}\right| = \left|\Sigma_{up}\right|$$

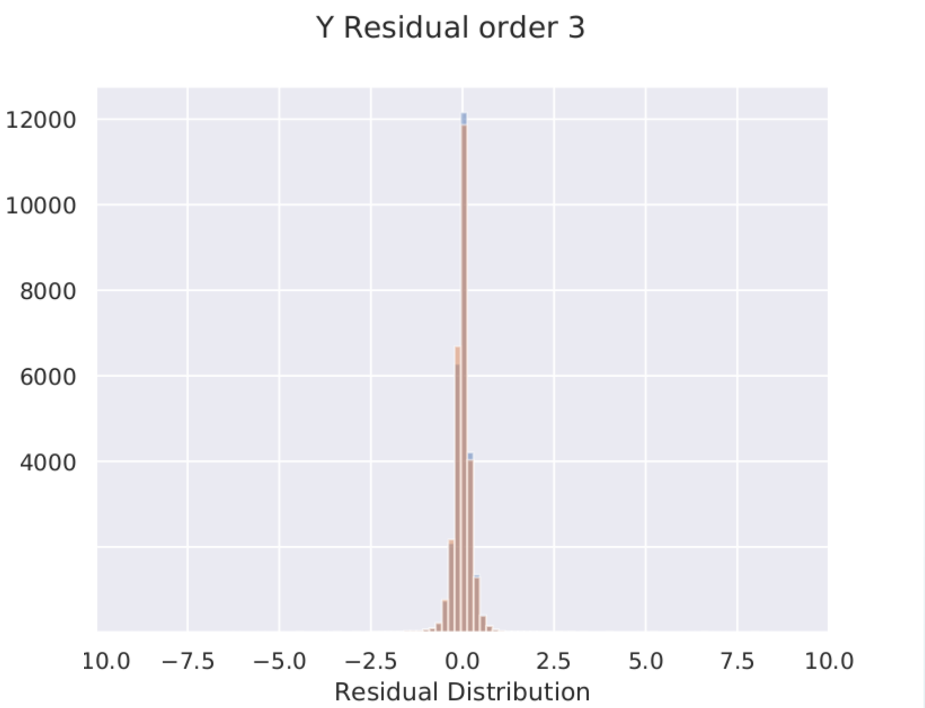
- The transfer matrix M has been previously investigated by Sophie Middleton and Chris Rogers
- A potential investigation would be to investigate the change in R for different fraction sizes of the beam. If stable it could be used to investigate the missing data downstream to see if it is due to scraping and magnet misalignment affects and nothing else

Sample case from TKU S2 to TKU S1

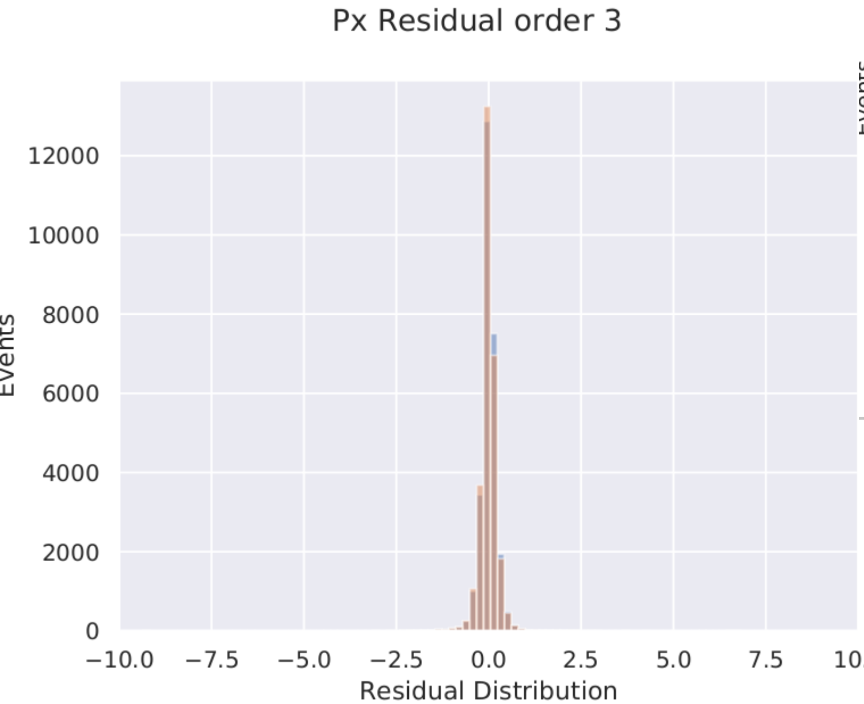
- Last analysis meeting, showed plots for a third order transfer matrix from TKU S2 to TKU S1 excluding ~1% of highly scattered particles, decays, etc, i.e. highly deviating particles.
- Applied transfer matrix to independent sample, and showed residuals from through position
- Residuals were on par with width of scintillating fibre

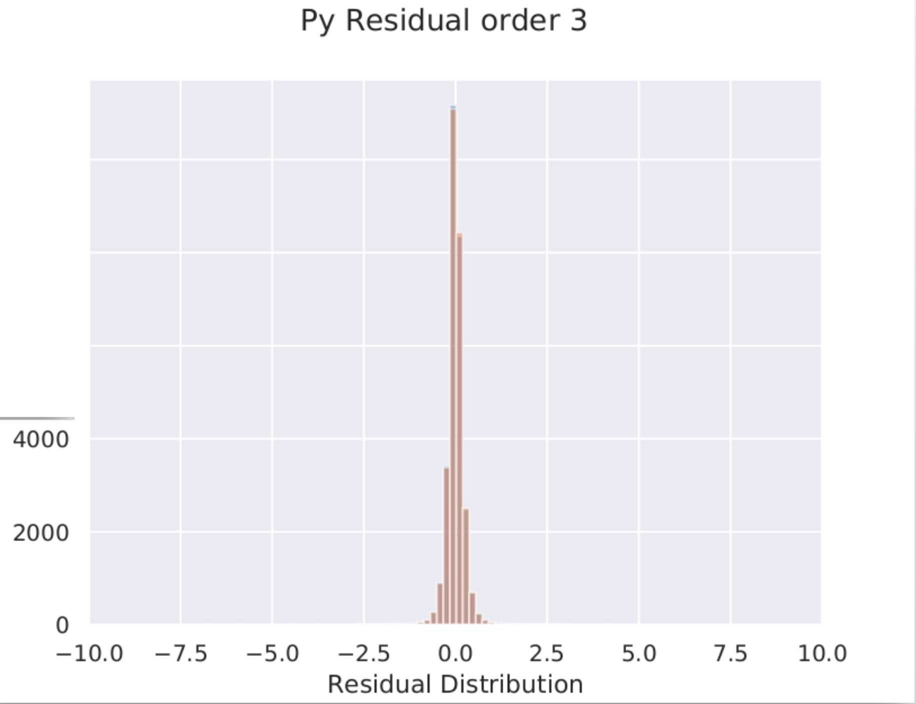
- Idea is to extend this for further distances and determine performance of transfer matrix from upstream to downstream.


X Residual order 3


Y Residual order 3


Pz Residual order 3


Px Residual order 3


Py Residual order 3

Concerns

- Advised results are too optimal due to Kalman actually pulling the spacepoints to desired location. Transfer matrix working too optimally by default.
- Not sure I agree (yet), as trackpoints should not be pulled beyond fibre width (and perhaps Gaussian like), although there may be inherent biases in trackpoint calculation

- Began investigating spacepoints and trackpoints
- Transfer matrix should apply on spacepoints just as on trackpoints.
- Became concerned about Kalman implamantation as it is supposedly highly sensitive to the seed position, and the Pz discrepancy. Transfer matrix will be compromised by wrong Pz, but likely only a larger error.

Trackpoints and Spacepoints

- Trackpoints are in a global reference frame
- Spacepoints are in a local reference frame
- Local coordinates are transformed to global coordinates by taking account of tracker misalignments
- Residuals between local Spacepoints and Global Trackpoints should be straight lines of each tracker misalignment

- **Residual between Global Spacepoints and Global Trackpoints at each station should be random unless there is an inherent bias**

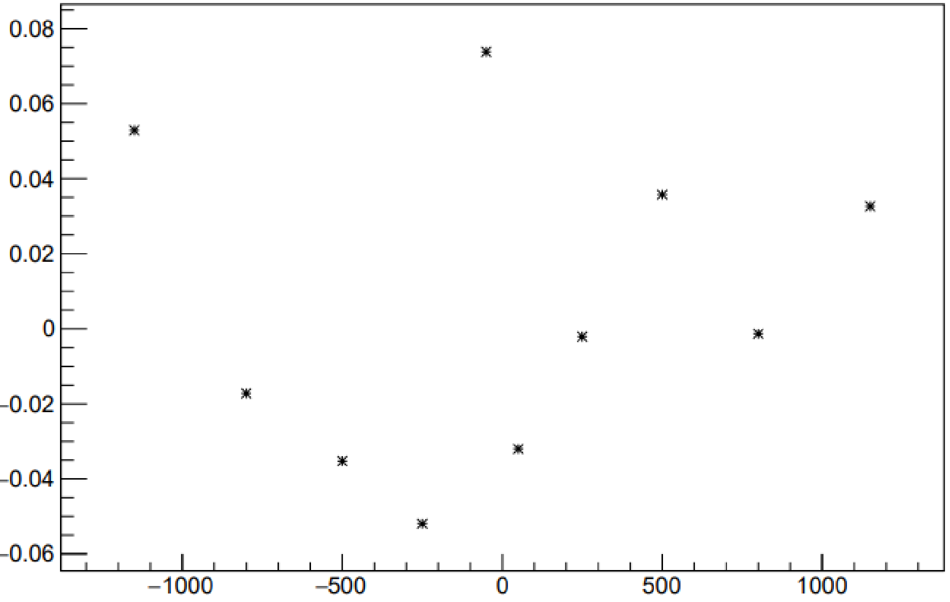
X Residual Global

X Axis: Local Station Coordinates (mm)
Y Axis: Residual (mm)

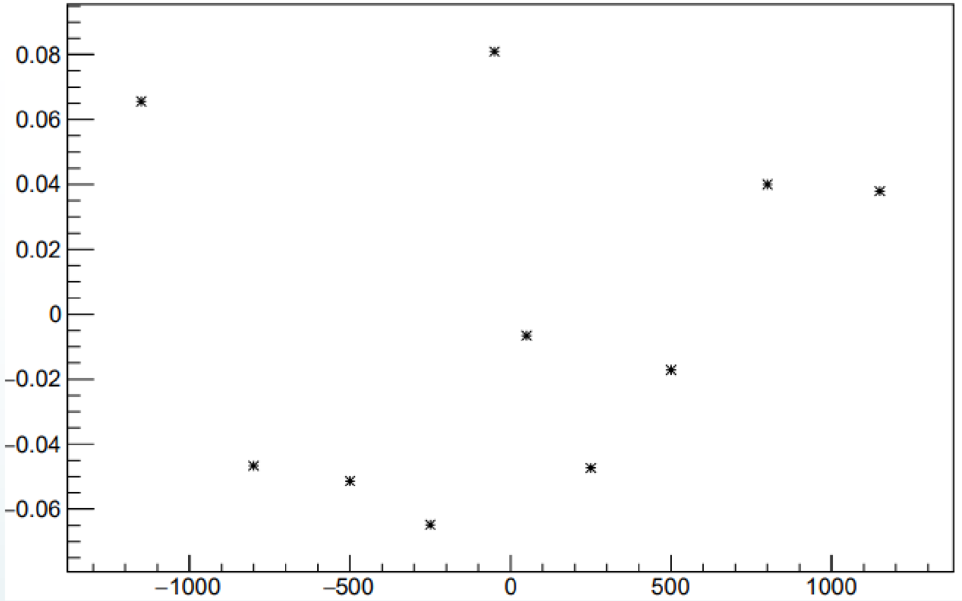
Top Left: No absorber
Top Right: Wedge
Bottom Left: LiH
Bottom Right: LH2

X Residual is between Global X position Track point and Global X position Space point (+/- 50 mm Offset introduced for TKU and TKD respectively)

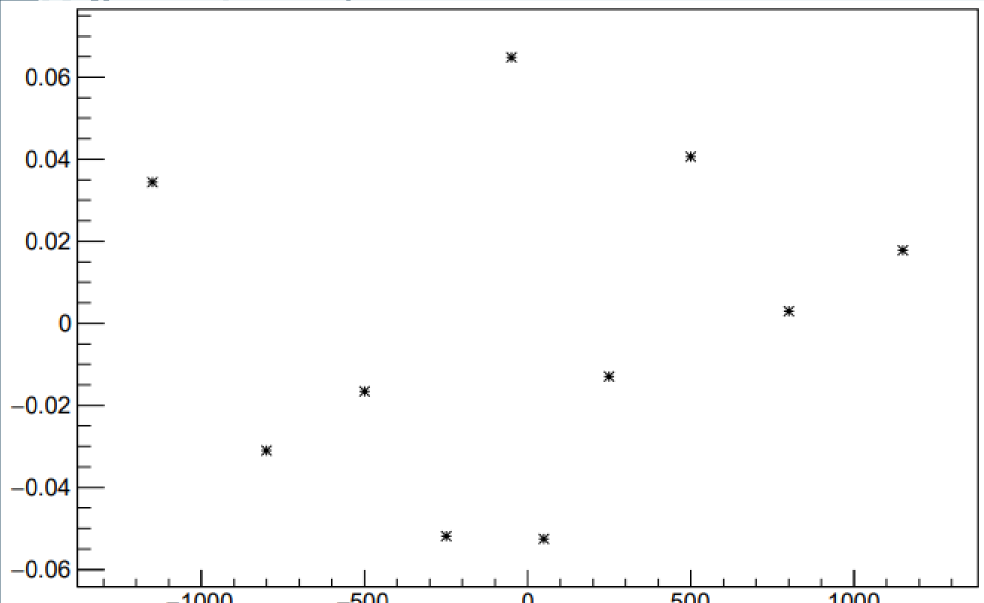
If there is no inherent bias, they should be randomly distributed

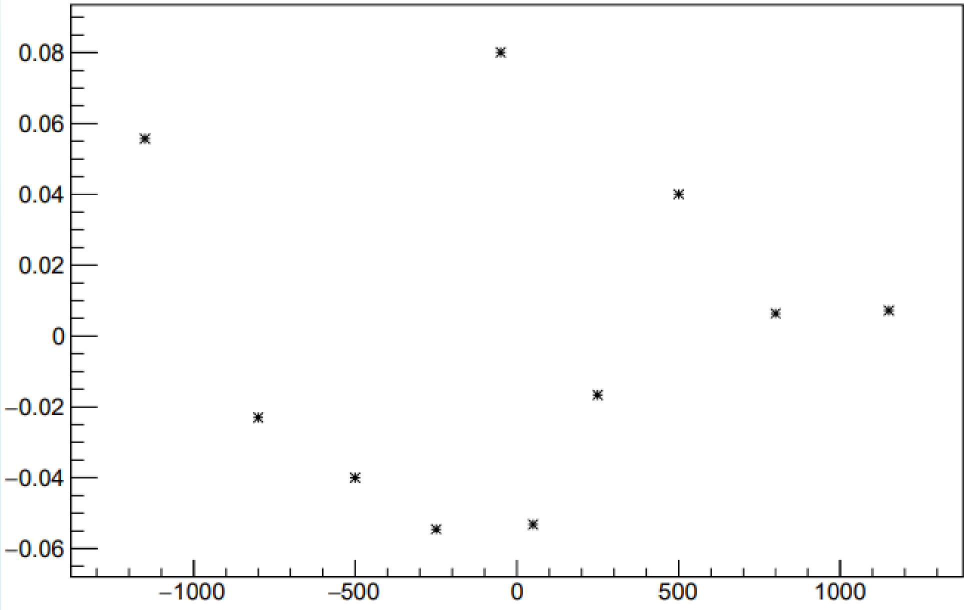


TKU TKD



TKU TKD





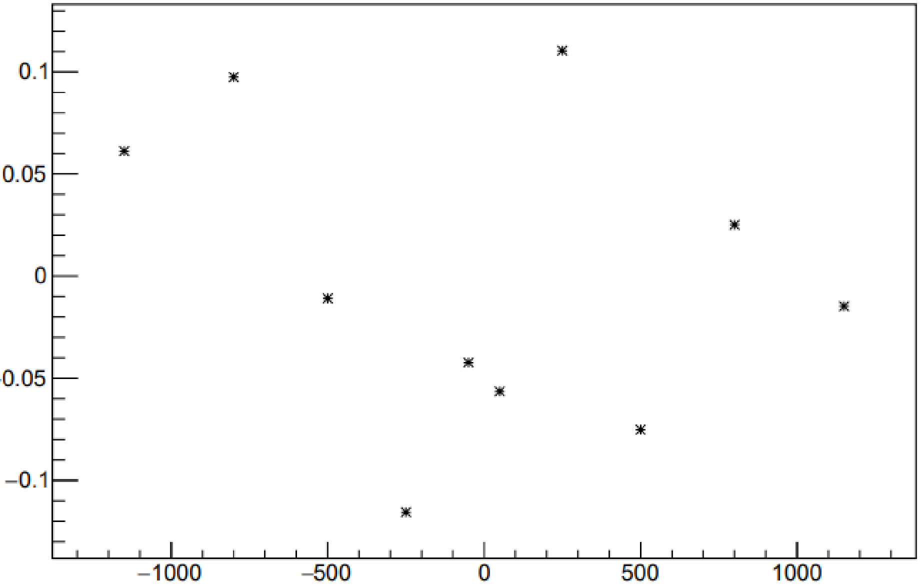
Y Residual Global

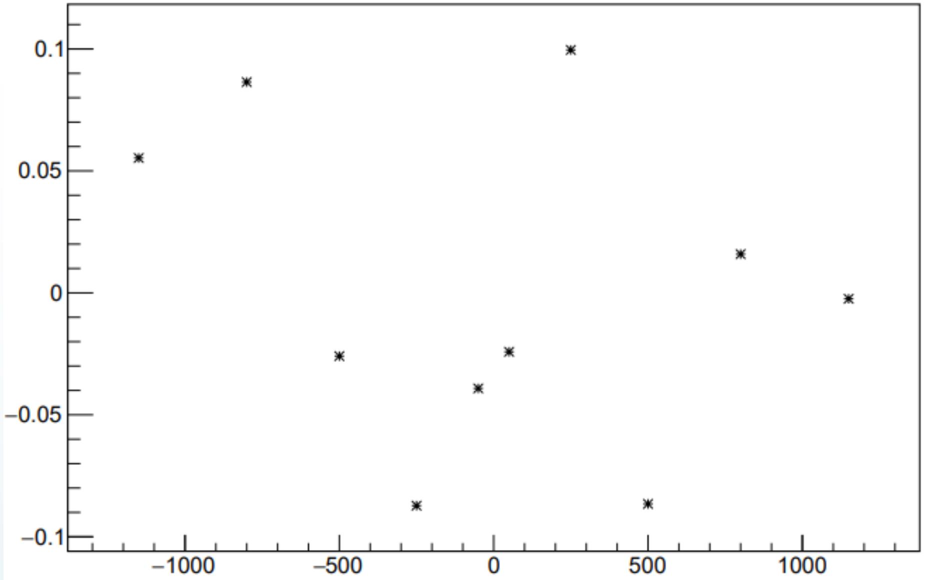
X Axis: Local Station Coordinates (mm)
Y Axis: Residual (mm)

Top Left: No absorber
Top Right: Wedge
Bottom Left: LiH
Bottom Right: LH2

Y Residual is between Global Y position Track point and Global Y position Space point (+/- 50 mm Offset introduced for TKU and TKD respectively)

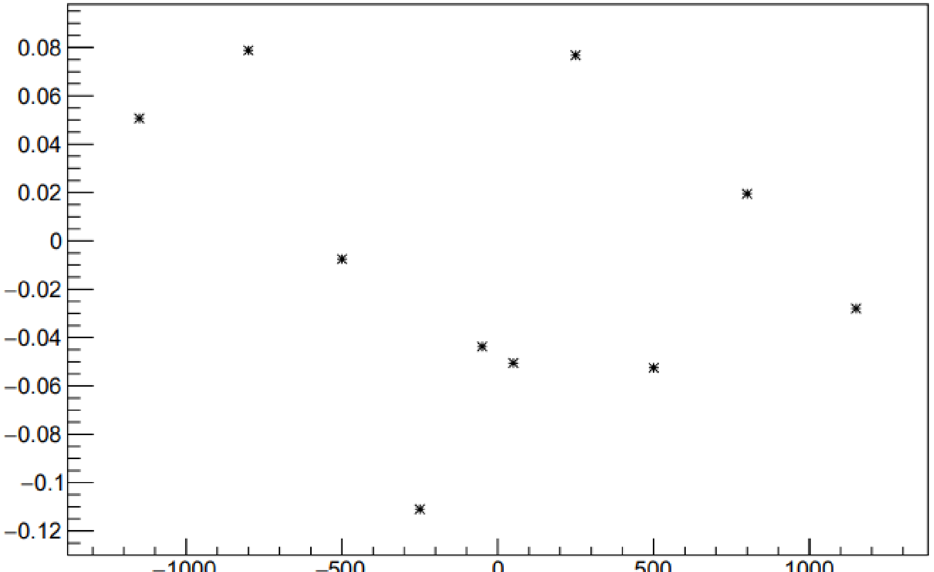
If there is no inherent bias, they should be randomly distributed

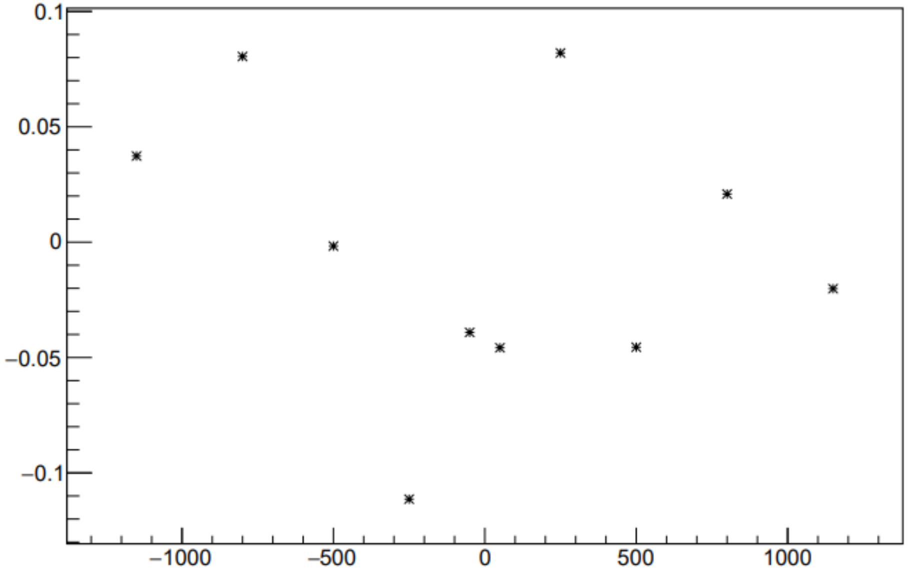




TKU TKD

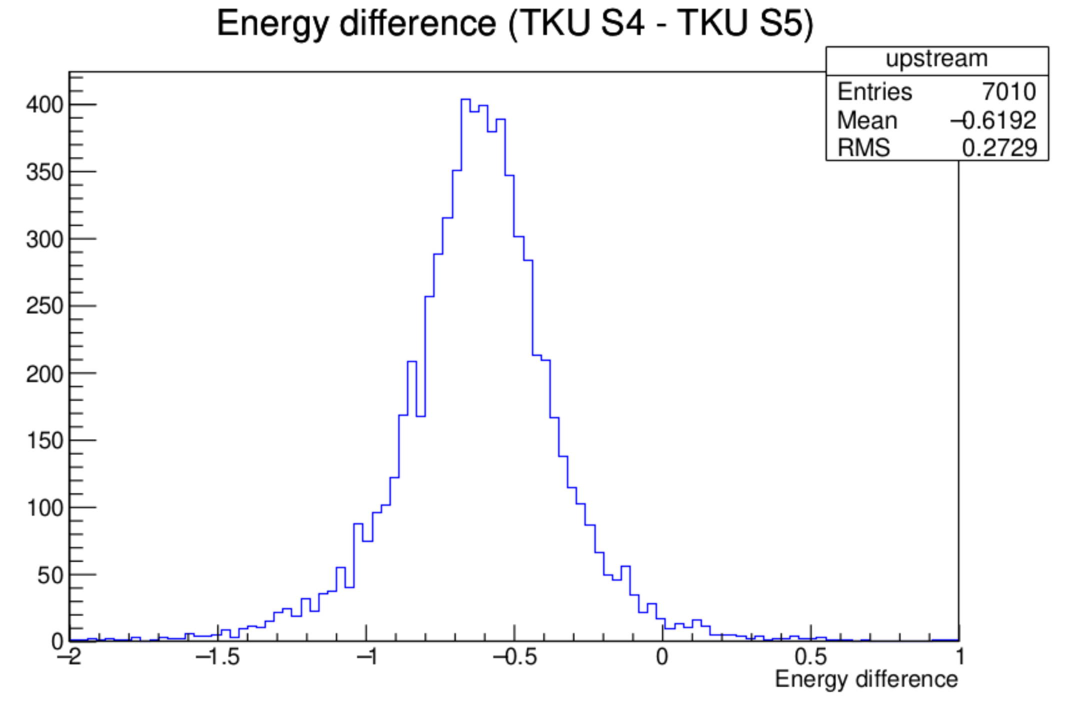
TKU TKD

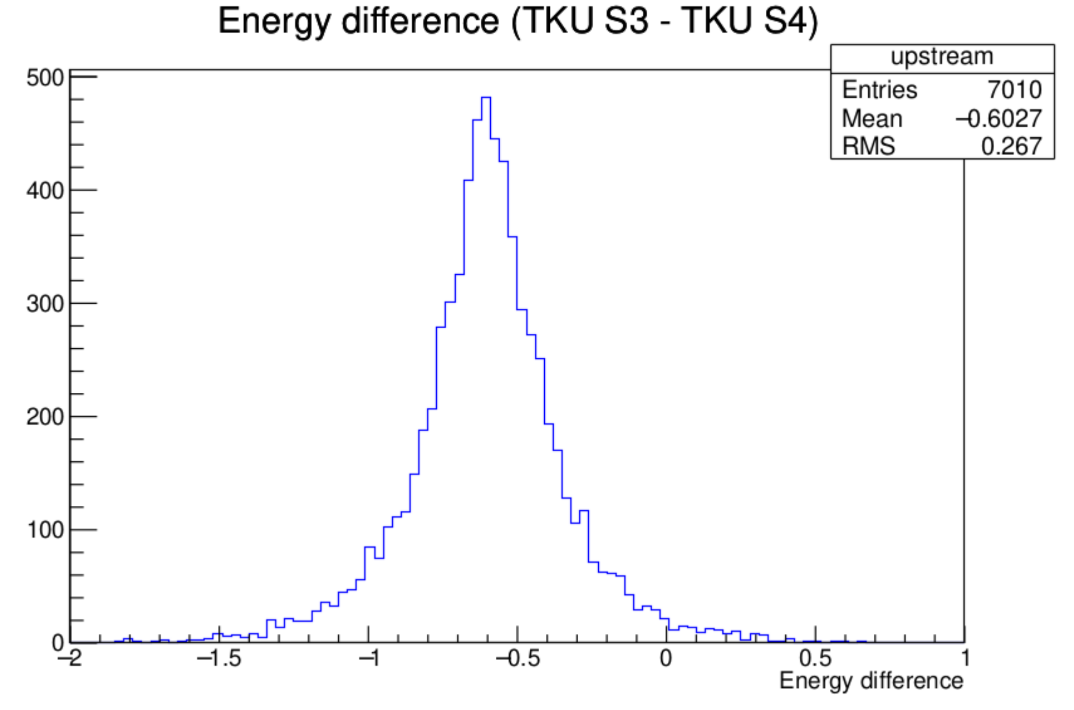


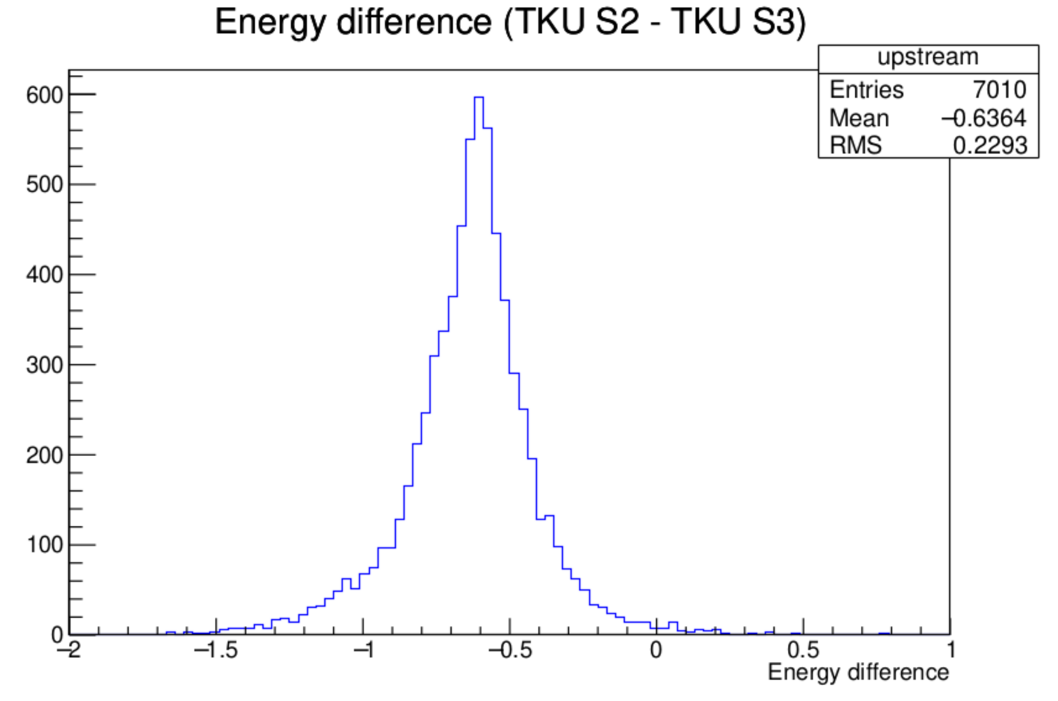


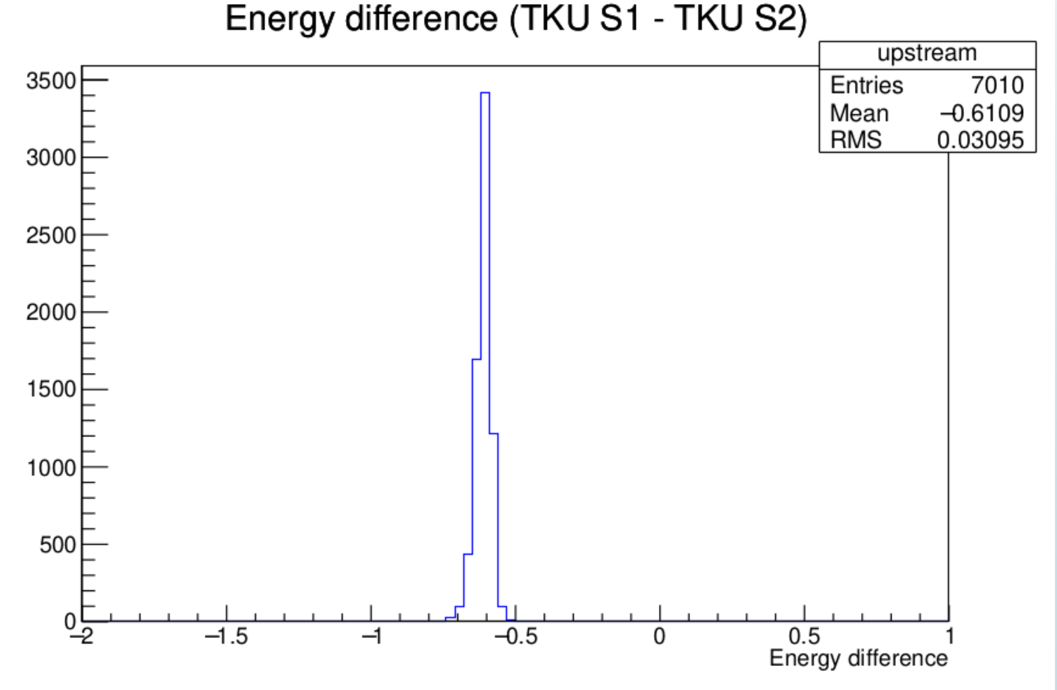
Energy Loss at the stations

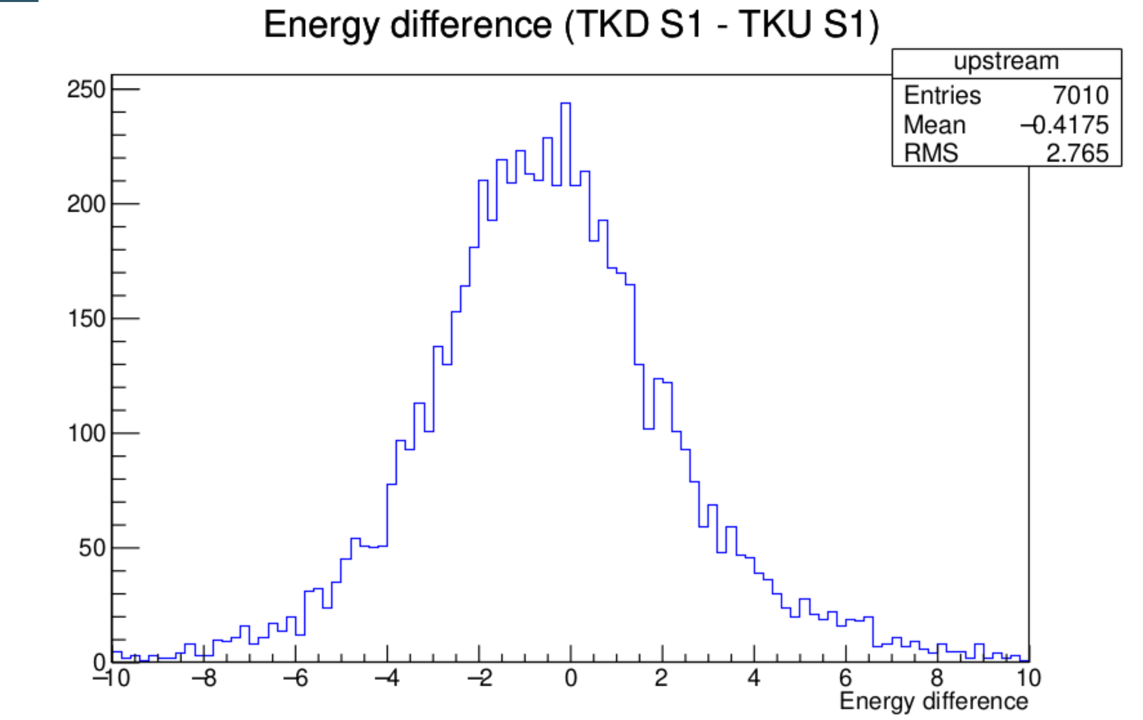
- Energy Loss through the stations is expected to be small, so that the mean energy loss and RMS at each station should be similar.
- This is not the case in the reconstruction
- While the mean is very similar, the RMS is not
- Either side of the absorber, the RMS Energy Loss is smallest between two innermost stations and increases between stations as one moves away from the absorber
- Some of the difference could be explained by the larger dz between stations further away from the absorber
- The difference in RMS between S1 and S2, and the other stations may be due to an inherent bias in the Reconstruction/Kalman Filter

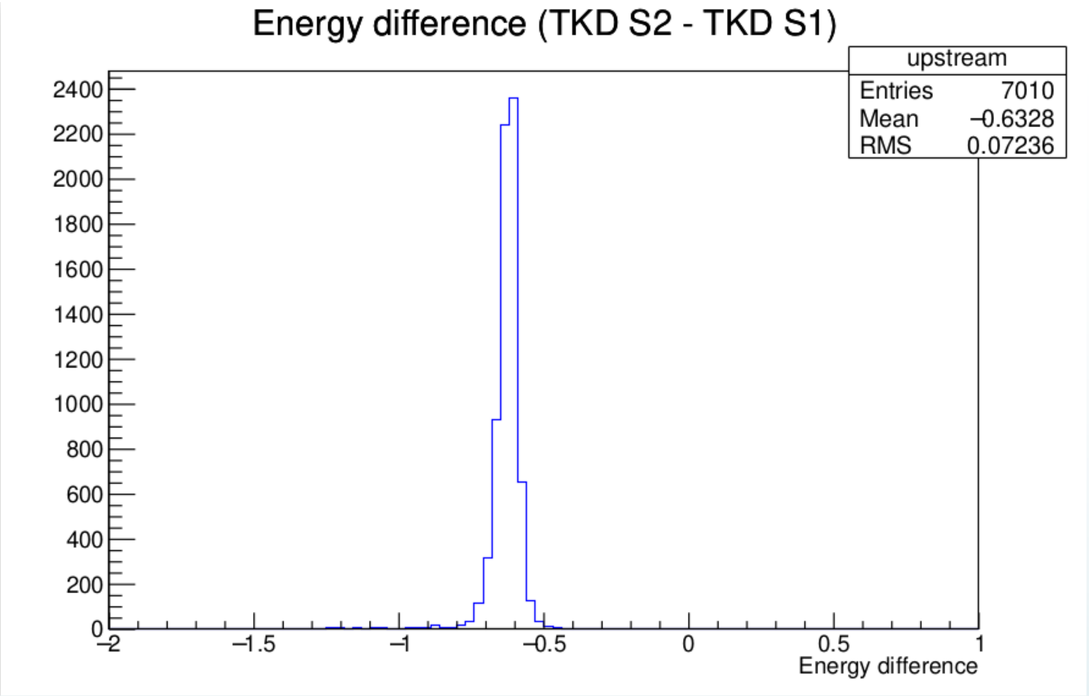

Energy difference (TKU S4 - TKU S5)
upstream
Entries 7010
Mean −0.6192
RMS 0.2729
400
350
300
250
200
150
100
50
0
−2
−1.5
−1
−0.5
0
0.5
1
Energy difference

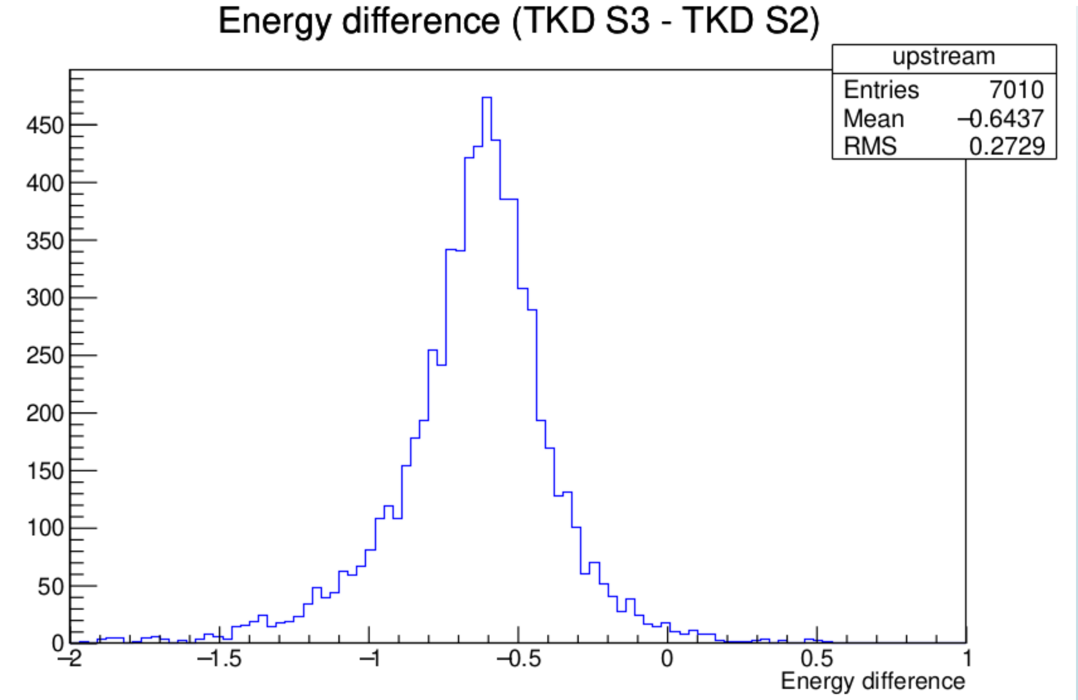

Energy difference (TKU S3 - TKU S4)
upstream
Entries 7010
Mean −0.6027
RMS 0.267
500
400
300
200
100
0
−2
−1.5
−1
−0.5
0
0.5
1
Energy difference

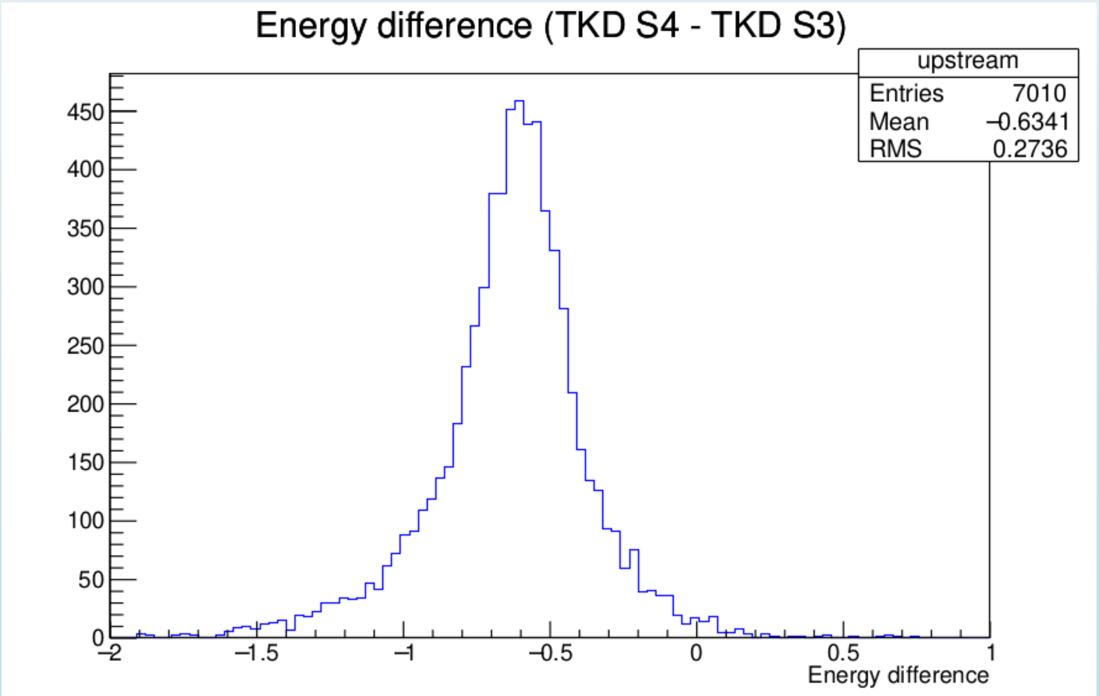

Energy difference (TKU S2 - TKU S3)
upstream
Entries 7010
Mean −0.6364
RMS 0.2293
600
500
400
300
200
100
0
−2
−1.5
−1
−0.5
0
0.5
1
Energy difference


Energy difference (TKU S1 - TKU S2)
upstream
Entries 7010
Mean −0.6109
RMS 0.03095
3500
3000
2500
2000
1500
1000
500
0
−2
−1.5
−1
−0.5
0
0.5
1
Energy difference


Energy difference (TKD S1 - TKU S1)

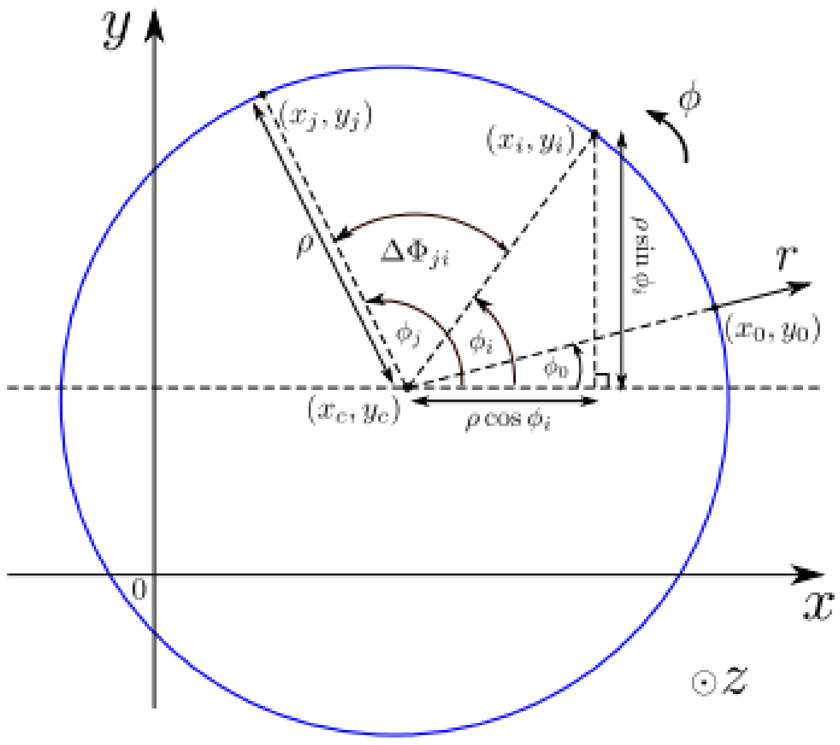

Energy difference (TKD S2 - TKD S1)

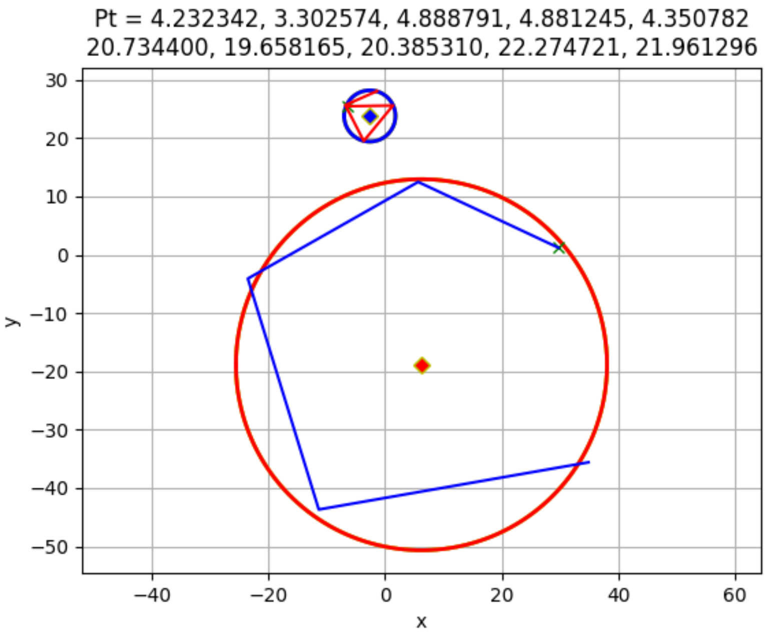

Energy difference (TKD S3 - TKD S2)


Energy difference (TKD S4 - TKD S3)

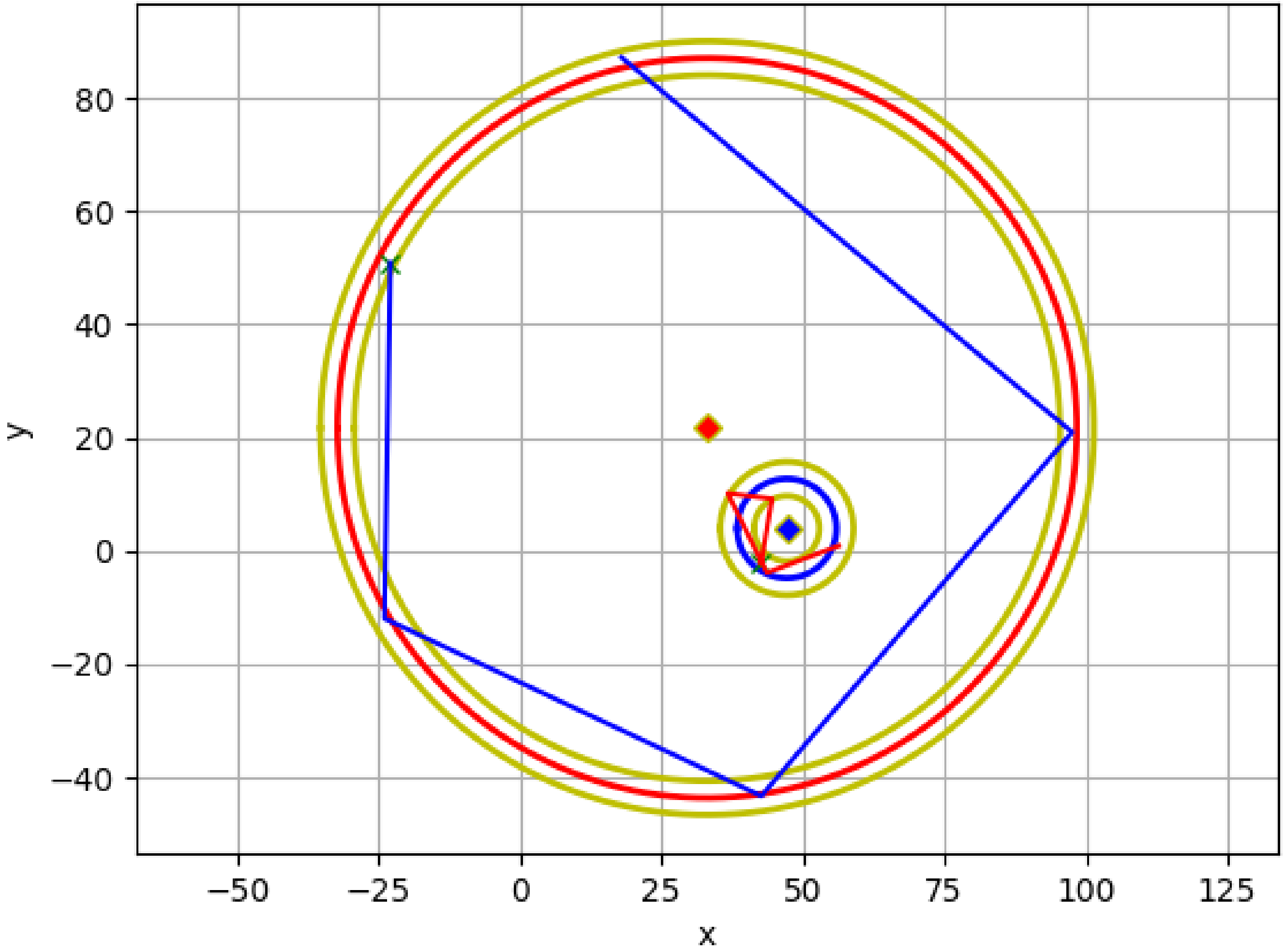
Circle Fit of spacepoints

- Currently spacepoints are fitted to a circle, accepted if chi-squared are small enough
- A straight line is also made in s-z plane, accepted if it passes Roadcut
- Radius of circle determines transverse momentum i.e. ~ $p_t = cBQR$
- Longitudinal momentum determined through $^{p_z}/_{p_t} = {}^{\Delta z}/_{R\Delta\varphi}$
- For circle fit R and p_t don't change until Kalman does its smoothing. Therefore p_z is determined mostly by the phase advance until it is Kalman smoothed
- Kalman is sensitive to the seed position, so the question is how the seed position is determined and used (haven't figured it out yet)




Pt = 4.232342, 3.302574, 4.888791, 4.881245, 4.350782
20.734400, 19.658165, 20.385310, 22.274721, 21.961296

Does it Fit?



- Left shows a large radius upstream track and a low radius downstream track
- Red and blue circles show fit to each of the 5 points
- Yellow circles are +/- 3 mm change in radius from centre.
- To see how well the 5 track points fit a circle fit, will look at the number of particles that deviate a certain distance from the circle
- Strictness of radius cut, determines which candidates are accepted
- Low radius particle in this case has managed to fit a circle to the hits, as it has passed the radius cut, without being particularly a circular path

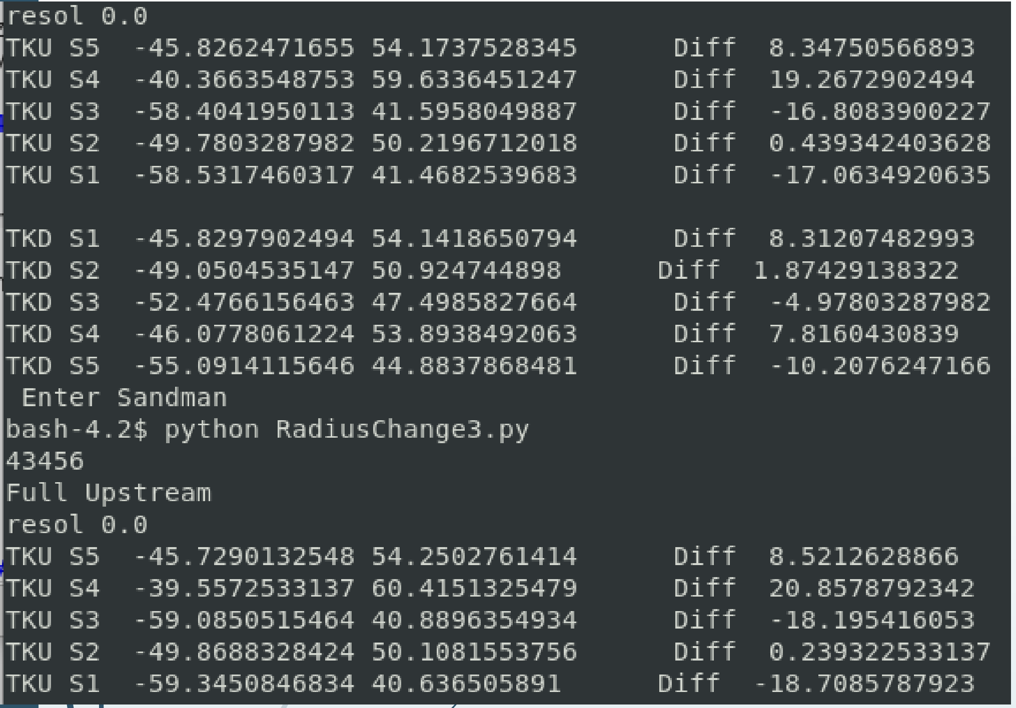
No absorber

Particles inside or outside the bounding yellow circle lines with a radius of:

Top Left: +/- 0 mm
Top Right: +/- 1 mm
Bottom Left: +/- 2 mm
Bottom Right +/- 3 mm

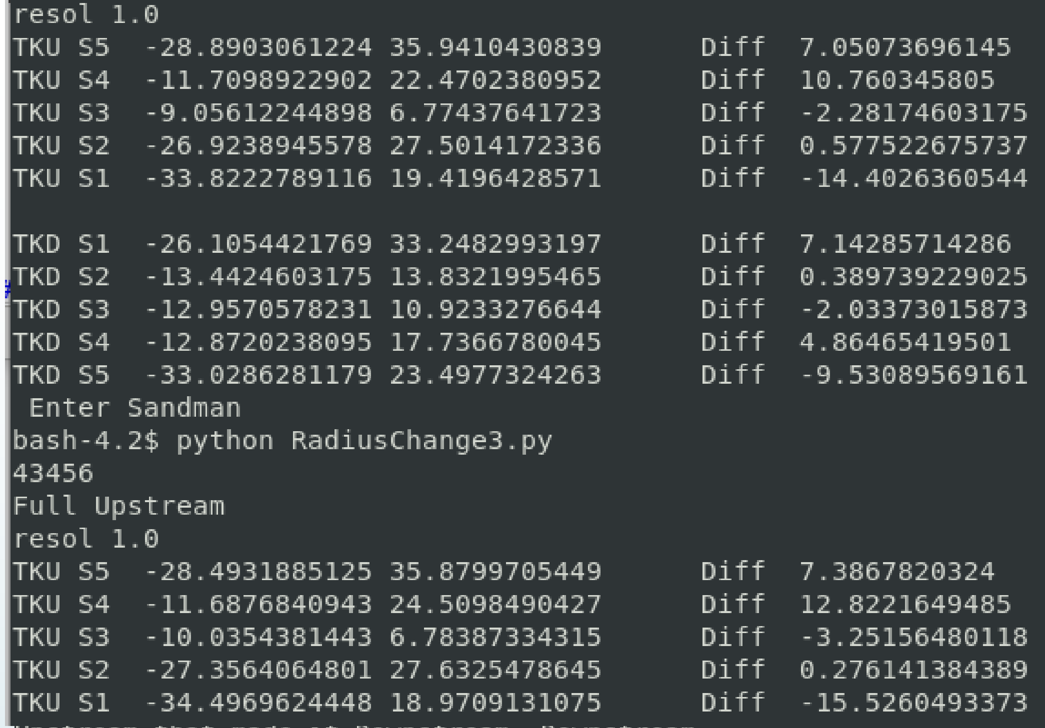
Particles start outside the circle and spiral inwards on average

However, large number of particles deviate significantly from circle fit line



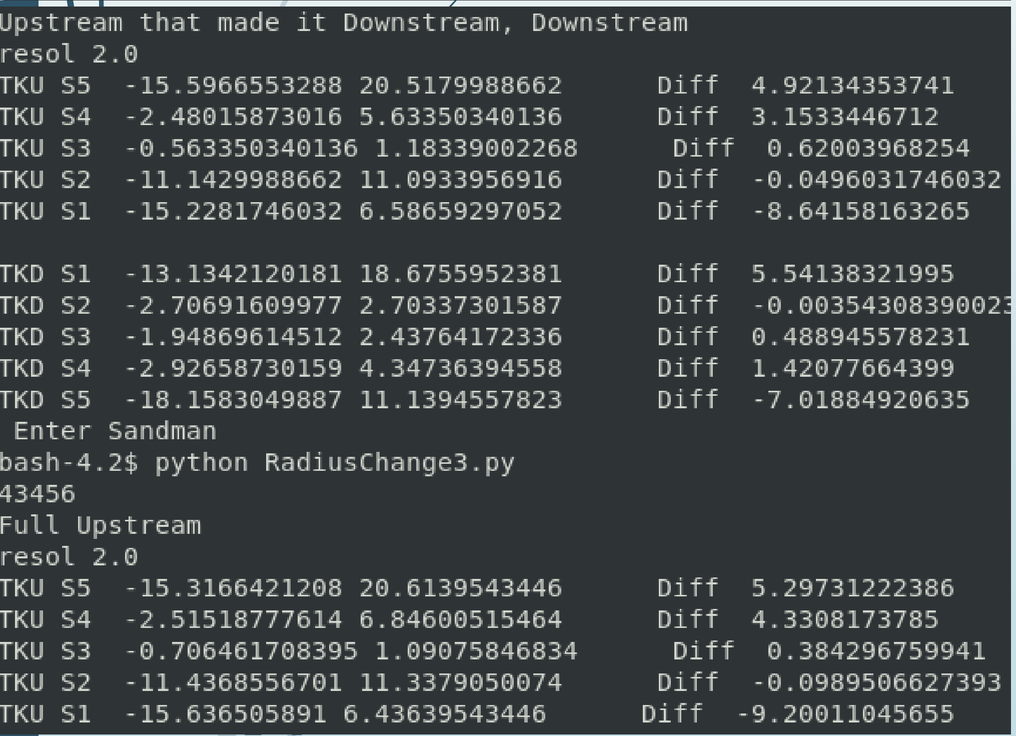
```
resol 0.0
TKU S5  -45.8262471655 54.1737528345      Diff  8.34750566893
TKU S4  -40.3663548753 59.6336451247      Diff  19.2672902494
TKU S3  -58.4041950113 41.5958049887      Diff  -16.8083900227
TKU S2  -49.7803287982 50.2196712018      Diff  0.439342403628
TKU S1  -58.5317460317 41.4682539683      Diff  -17.0634920635

TKD S1  -45.8297902494 54.1418650794      Diff  8.31207482993
TKD S2  -49.0504535147 50.924744898      Diff  1.87429138322
TKD S3  -52.4766156463 47.4985827664      Diff  -4.97803287982
TKD S4  -46.0778061224 53.8938492063      Diff  7.8160430839
TKD S5  -55.0914115646 44.8837868481      Diff  -10.2076247166
 Enter Sandman
bash-4.2$ python RadiusChange3.py
43456
Full Upstream
resol 0.0
TKU S5  -45.7290132548 54.2502761414      Diff  8.5212628866
TKU S4  -39.5572533137 60.4151325479      Diff  20.8578792342
TKU S3  -59.0850515464 40.8896354934      Diff  -18.195416053
TKU S2  -49.8688328424 50.1081553756      Diff  0.239322533137
TKU S1  -59.3450846834 40.636505891      Diff  -18.7085787923
```



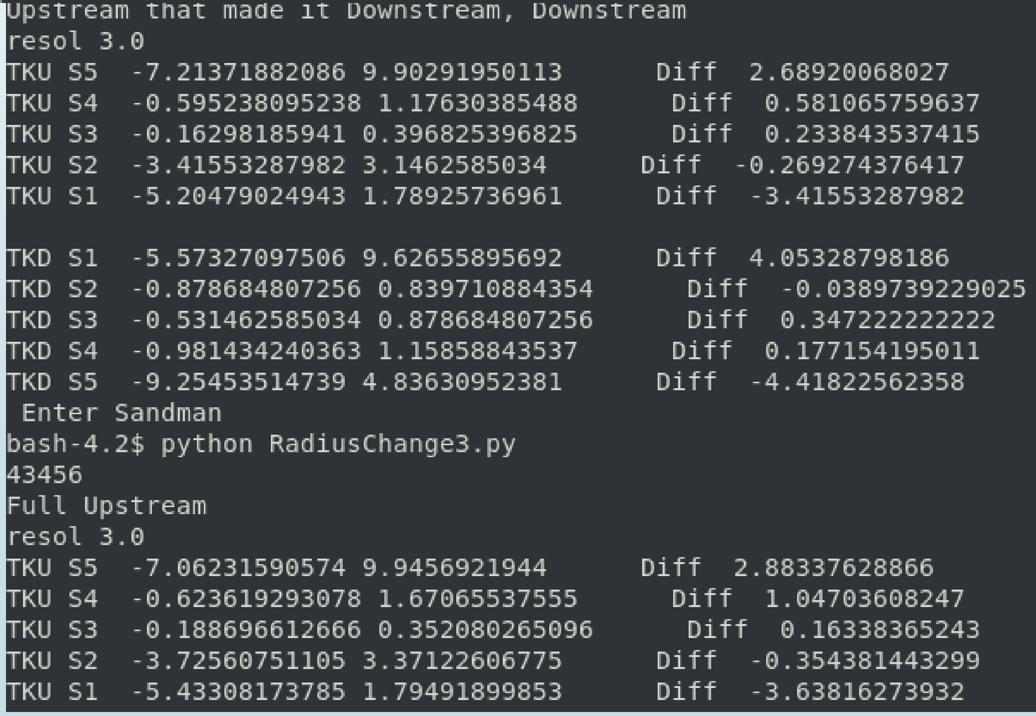
```
resol 1.0
TKU S5  -28.8903061224 35.9410430839      Diff  7.05073696145
TKU S4  -11.7098922902 22.4702380952      Diff  10.760345805
TKU S3  -9.05612244898 6.77437641723      Diff  -2.28174603175
TKU S2  -26.9238945578 27.5014172336      Diff  0.577522675737
TKU S1  -33.8222789116 19.4196428571      Diff  -14.4026360544

TKD S1  -26.1054421769 33.2482993197      Diff  7.14285714286
TKD S2  -13.4424603175 13.8321995465      Diff  0.389739229025
TKD S3  -12.9570578231 10.9233276644      Diff  -2.03373015873
TKD S4  -12.8720238095 17.7366780045      Diff  4.86465419501
TKD S5  -33.0286281179 23.4977324263      Diff  -9.53089569161
 Enter Sandman
bash-4.2$ python RadiusChange3.py
43456
Full Upstream
resol 1.0
TKU S5  -28.4931885125 35.8799705449      Diff  7.3867820324
TKU S4  -11.6876840943 24.5098490427      Diff  12.8221649485
TKU S3  -10.0354381443 6.78387334315      Diff  -3.2515648011 8
TKU S2  -27.3564064801 27.6325478645      Diff  0.276141384389
TKU S1  -34.4969624448 18.9709131075      Diff  -15.5260493373
```



```
Upstream that made it Downstream, Downstream
resol 2.0
TKU S5  -15.5966553288 20.5179988662      Diff  4.92134353741
TKU S4  -2.48015873016 5.63350340136      Diff  3.1533446712
TKU S3  -0.563350340136 1.18339002268      Diff  0.62003968254
TKU S2  -11.1429988662 11.0933956916      Diff  -0.0496031746032
TKU S1  -15.2281746032 6.58659297052      Diff  -8.64158163265

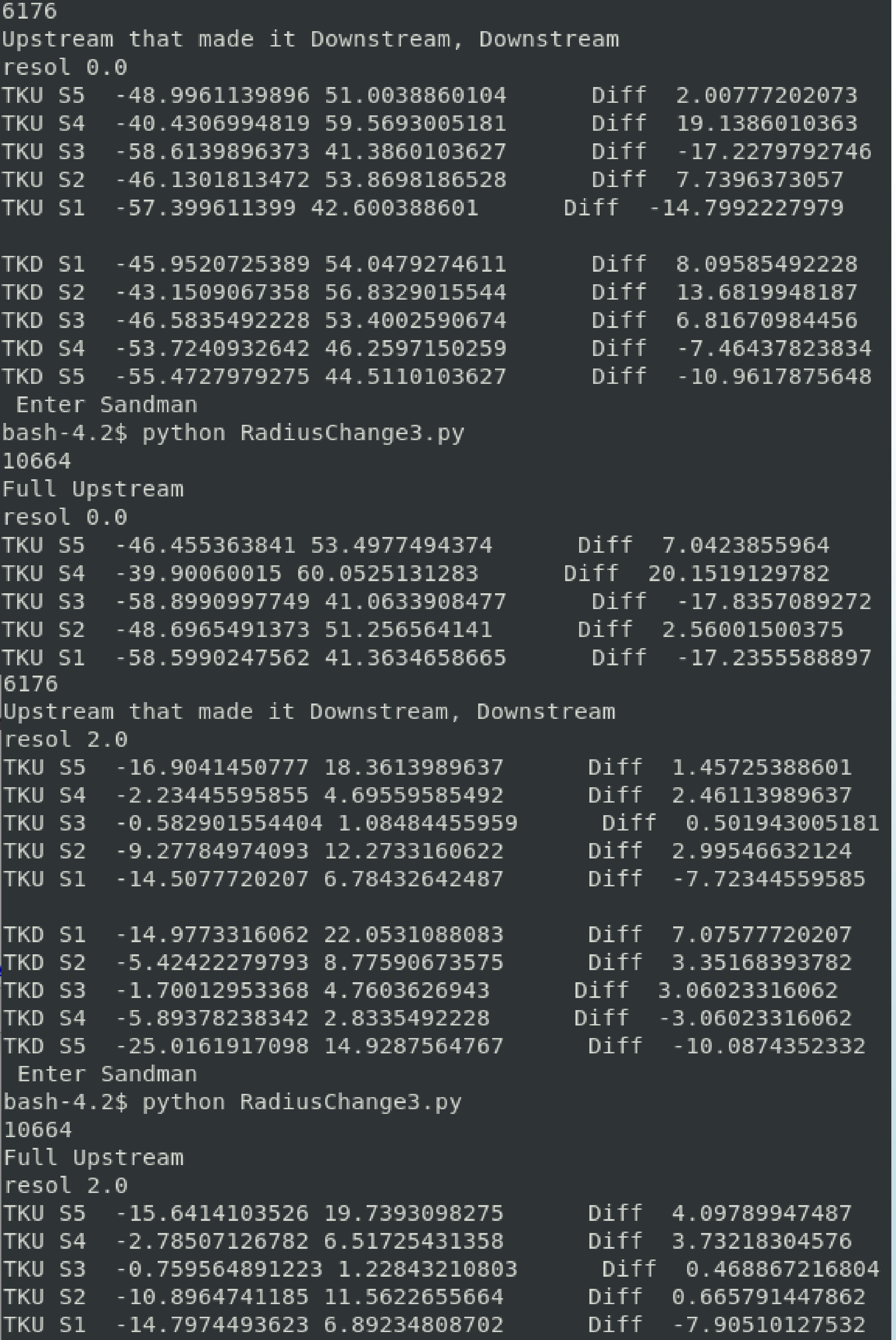
TKD S1  -13.1342120181 18.6755952381      Diff  5.54138321995
TKD S2  -2.70691609977 2.70337301587      Diff  -0.00354308390023
TKD S3  -1.94869614512 2.43764172336      Diff  0.488945578231
TKD S4  -2.92658730159 4.34736394558      Diff  1.42077664399
TKD S5  -18.1583049887 11.1394557823      Diff  -7.01884920635
 Enter Sandman
bash-4.2$ python RadiusChange3.py
43456
Full Upstream
resol 2.0
TKU S5  -15.3166421208 20.6139543446      Diff  5.29731222386
TKU S4  -2.51518777614 6.84600515464      Diff  4.3308173785
TKU S3  -0.706461708395 1.09075846834      Diff  0.384296759941
TKU S2  -11.4368556701 11.3379050074      Diff  -0.0989506627393
TKU S1  -15.636505891 6.43639543446      Diff  -9.20011045655
```



```
Upstream that made it Downstream, Downstream
resol 3.0
TKU S5  -7.21371882086 9.90291950113      Diff  2.68920068027
TKU S4  -0.595238095238 1.17630385488      Diff  0.581065759637
TKU S3  -0.16298185941 0.396825396825      Diff  0.233843537415
TKU S2  -3.41553287982 3.1462585034      Diff  -0.269274376417
TKU S1  -5.20479024943 1.78925736961      Diff  -3.41553287982

TKD S1  -5.57327097506 9.62655895692      Diff  4.05328798186
TKD S2  -0.878684807256 0.839710884354      Diff  -0.0389739229025
TKD S3  -0.531462585034 0.878684807256      Diff  0.347222222222
TKD S4  -0.981434240363 1.15858843537      Diff  0.177154195011
TKD S5  -9.25453514739 4.83630952381      Diff  -4.41822562358
 Enter Sandman
bash-4.2$ python RadiusChange3.py
43456
Full Upstream
resol 3.0
TKU S5  -7.06231590574 9.9456921944      Diff  2.88337628866
TKU S4  -0.623619293078 1.67065537555      Diff  1.04703608247
TKU S3  -0.188696612666 0.352080265096      Diff  0.16338365243
TKU S2  -3.72560751105 3.37122606775      Diff  -0.354381443299
TKU S1  -5.43308173785 1.79491899853      Diff  -3.63816273932
```

Wedge



```
6176
Upstream that made it Downstream, Downstream
resol 0.0
TKU S5  -48.9961139896 51.0038860104      Diff  2.00777202073
TKU S4  -40.4306994819 59.5693005181      Diff  19.1386010363
TKU S3  -58.6139896373 41.3860103627      Diff  -17.2279792746
TKU S2  -46.1301813472 53.8698186528      Diff  7.7396373057
TKU S1  -57.399611399 42.600388601       Diff  -14.7992227979

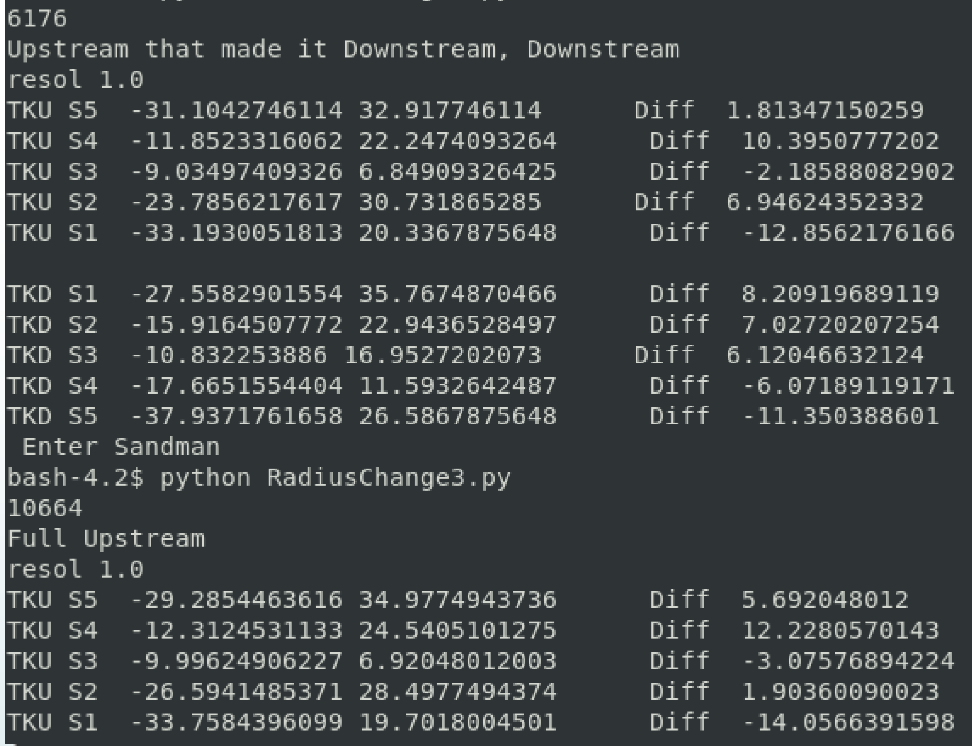
TKD S1  -45.9520725389 54.0479274611      Diff  8.09585492228
TKD S2  -43.1509067358 56.8329015544      Diff  13.6819948187
TKD S3  -46.5835492228 53.4002590674      Diff  6.81670984456
TKD S4  -53.7240932642 46.2597150259      Diff  -7.46437823834
TKD S5  -55.4727979275 44.5110103627      Diff  -10.9617875648
 Enter Sandman
bash-4.2$ python RadiusChange3.py
10664
Full Upstream
resol 0.0
TKU S5  -46.455363841 53.4977494374      Diff  7.0423855964
TKU S4  -39.90060015 60.0525131283      Diff  20.1519129782
TKU S3  -58.8990997749 41.0633908477      Diff  -17.8357089272
TKU S2  -48.6965491373 51.256564141      Diff  2.56001500375
TKU S1  -58.5990247562 41.3634658665      Diff  -17.2355588897
6176
Upstream that made it Downstream, Downstream
resol 2.0
TKU S5  -16.9041450777 18.3613989637      Diff  1.45725388601
TKU S4  -2.23445595855 4.69559585492      Diff  2.46113989637
TKU S3  -0.582901554404 1.08484455959      Diff  0.501943005181
TKU S2  -9.27784974093 12.2733160622      Diff  2.99546632124
TKU S1  -14.5077720207 6.78432642487      Diff  -7.72344559585

TKD S1  -14.9773316062 22.0531088083      Diff  7.07577720207
TKD S2  -5.42422279793 8.77590673575      Diff  3.35168393782
TKD S3  -1.70012953368 4.7603626943      Diff  3.06023316062
TKD S4  -5.89378238342 2.8335492228      Diff  -3.06023316062
TKD S5  -25.0161917098 14.9287564767      Diff  -10.0874352332
 Enter Sandman
bash-4.2$ python RadiusChange3.py
10664
Full Upstream
resol 2.0
TKU S5  -15.6414103526 19.7393098275      Diff  4.09789947487
TKU S4  -2.78507126782 6.51725431358      Diff  3.73218304576
TKU S3  -0.759564891223 1.22843210803      Diff  0.468867216804
TKU S2  -10.8964741185 11.5622655664      Diff  0.665791447862
TKU S1  -14.7974493623 6.89234808702      Diff  -7.90510127532
```

Particles inside or outside the bounding yellow circle lines with a radius of:

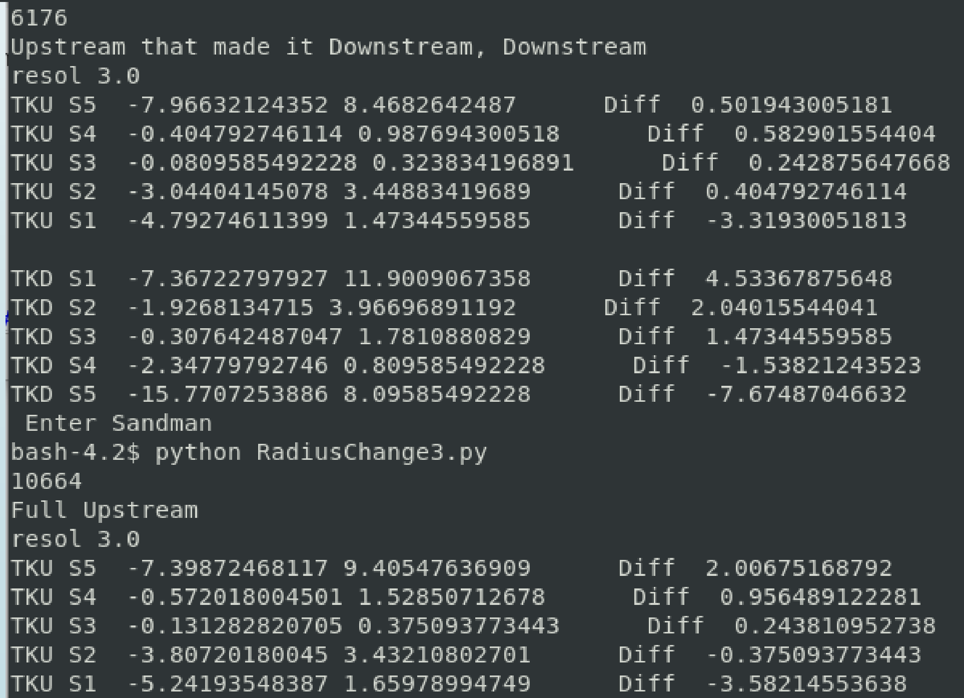
Top Left: +/- 0 mm
Top Right: +/- 1 mm
Bottom Left: +/- 2 mm
Bottom Right +/- 3 mm

Changing the absorber appears to have no effect on the particles in the tracker



```
6176
Upstream that made it Downstream, Downstream
resol 1.0
TKU S5  -31.1042746114 32.917746114      Diff  1.81347150259
TKU S4  -11.8523316062 22.2474093264      Diff  10.3950777202
TKU S3  -9.03497409326 6.84909326425      Diff  -2.18588082902
TKU S2  -23.7856217617 30.731865285      Diff  6.94624352332
TKU S1  -33.1930051813 20.3367875648      Diff  -12.8562176166

TKD S1  -27.5582901554 35.7674870466      Diff  8.20919689119
TKD S2  -15.9164507772 22.9436528497      Diff  7.02720207254
TKD S3  -10.832253886 16.9527202073      Diff  6.12046632124
TKD S4  -17.6651554404 11.5932642487      Diff  -6.07189119171
TKD S5  -37.9371761658 26.5867875648      Diff  -11.350388601
 Enter Sandman
bash-4.2$ python RadiusChange3.py
10664
Full Upstream
resol 1.0
TKU S5  -29.2854463616 34.9774943736      Diff  5.692048012
TKU S4  -12.3124531133 24.5405101275      Diff  12.2280570143
TKU S3  -9.99624906227 6.92048012003      Diff  -3.07576894224
TKU S2  -26.5941485371 28.4977494374      Diff  1.90360090023
TKU S1  -33.7584396099 19.7018004501      Diff  -14.0566391598
```



```
6176
Upstream that made it Downstream, Downstream
resol 3.0
TKU S5  -7.96632124352 8.4682642487      Diff  0.501943005181
TKU S4  -0.404792746114 0.987694300518      Diff  0.582901554404
TKU S3  -0.0809585492228 0.323834196891      Diff  0.242875647668
TKU S2  -3.04404145078 3.44883419689      Diff  0.404792746114
TKU S1  -4.79274611399 1.47344559585      Diff  -3.31930051813

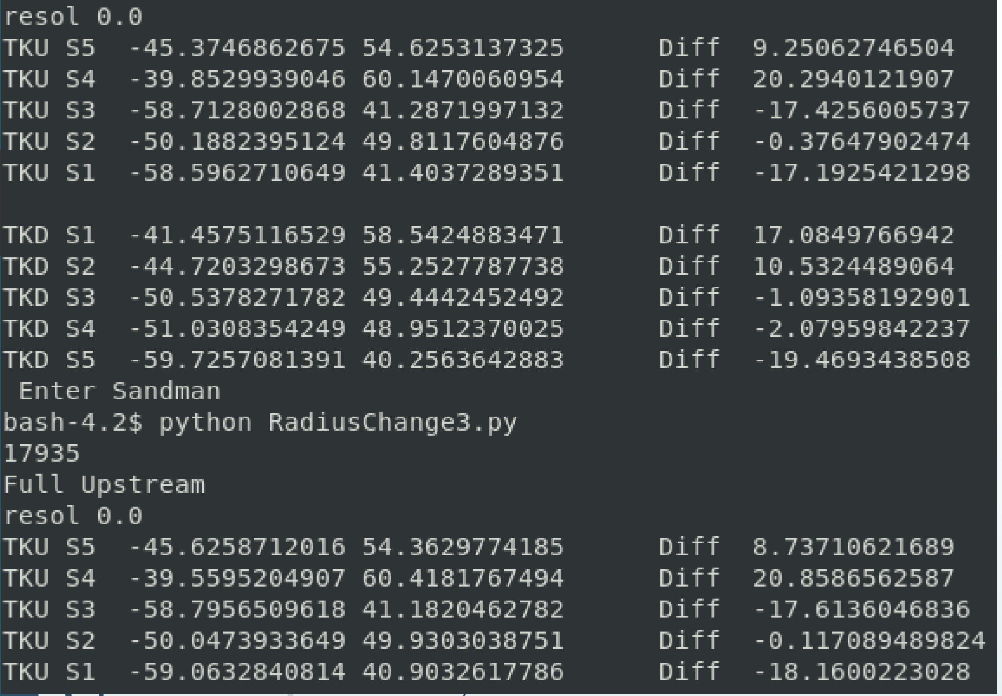
TKD S1  -7.36722797927 11.9009067358      Diff  4.53367875648
TKD S2  -1.9268134715 3.96696891192      Diff  2.04015544041
TKD S3  -0.307642487047 1.7810880829      Diff  1.47344559585
TKD S4  -2.34779792746 0.809585492228      Diff  -1.53821243523
TKD S5  -15.7707253886 8.09585492228      Diff  -7.67487046632
 Enter Sandman
bash-4.2$ python RadiusChange3.py
10664
Full Upstream
resol 3.0
TKU S5  -7.39872468117 9.40547636909      Diff  2.00675168792
TKU S4  -0.572018004501 1.52850712678      Diff  0.956489122281
TKU S3  -0.131282820705 0.375093773443      Diff  0.243810952738
TKU S2  -3.80720180045 3.43210802701      Diff  -0.375093773443
TKU S1  -5.24193548387 1.65978994749      Diff  -3.58214553638
```

LiH

Particles inside or outside the bounding yellow circle lines with a radius of:

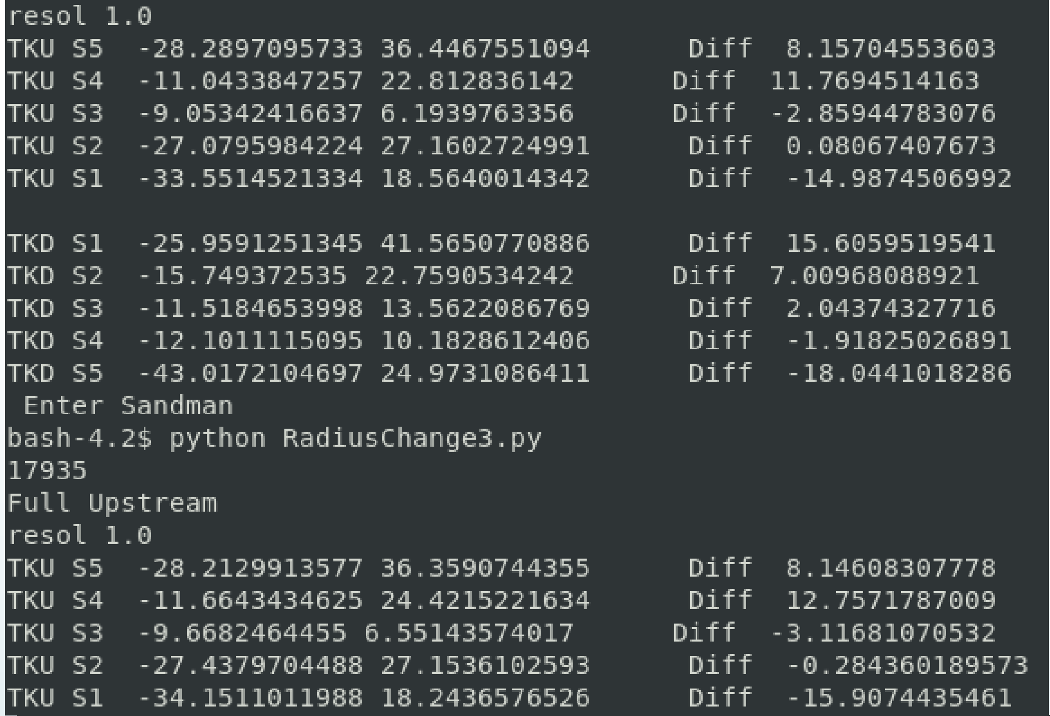
Top Left: +/- 0 mm
Top Right: +/- 1 mm
Bottom Left: +/- 2 mm
Bottom Right +/- 3 mm

Looking at the full Upstream sample, or the Upstream sample that made it downstream also appears to have no effect



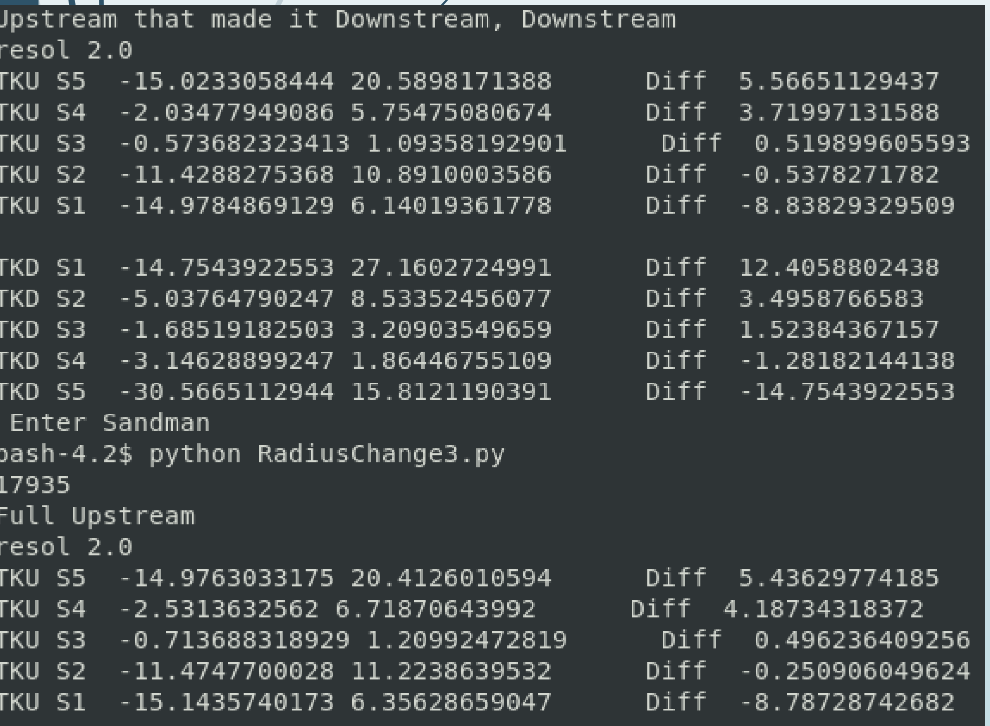
```
resol 0.0
TKU S5  -45.3746862675 54.6253137325      Diff  9.25062746504
TKU S4  -39.8529939046 60.1470060954      Diff  20.2940121907
TKU S3  -58.7128002868 41.2871997132      Diff  -17.4256005737
TKU S2  -50.1882395124 49.8117604876      Diff  -0.37647902474
TKU S1  -58.5962710649 41.4037289351      Diff  -17.1925421298

TKD S1  -41.4575116529 58.5424883471      Diff  17.0849766942
TKD S2  -44.7203298673 55.2527787738      Diff  10.5324489064
TKD S3  -50.5378271782 49.4442452492      Diff  -1.09358192901
TKD S4  -51.0308354249 48.9512370025      Diff  -2.07959842237
TKD S5  -59.7257081391 40.2563642883      Diff  -19.4693438508
 Enter Sandman
bash-4.2$ python RadiusChange3.py
17935
Full Upstream
resol 0.0
TKU S5  -45.6258712016 54.3629774185      Diff  8.73710621689
TKU S4  -39.5595204907 60.4181767494      Diff  20.8586562587
TKU S3  -58.7956509618 41.1820462782      Diff  -17.6136046836
TKU S2  -50.0473933649 49.9303038751      Diff  -0.117089489824
TKU S1  -59.0632840814 40.9032617786      Diff  -18.1600223028
```



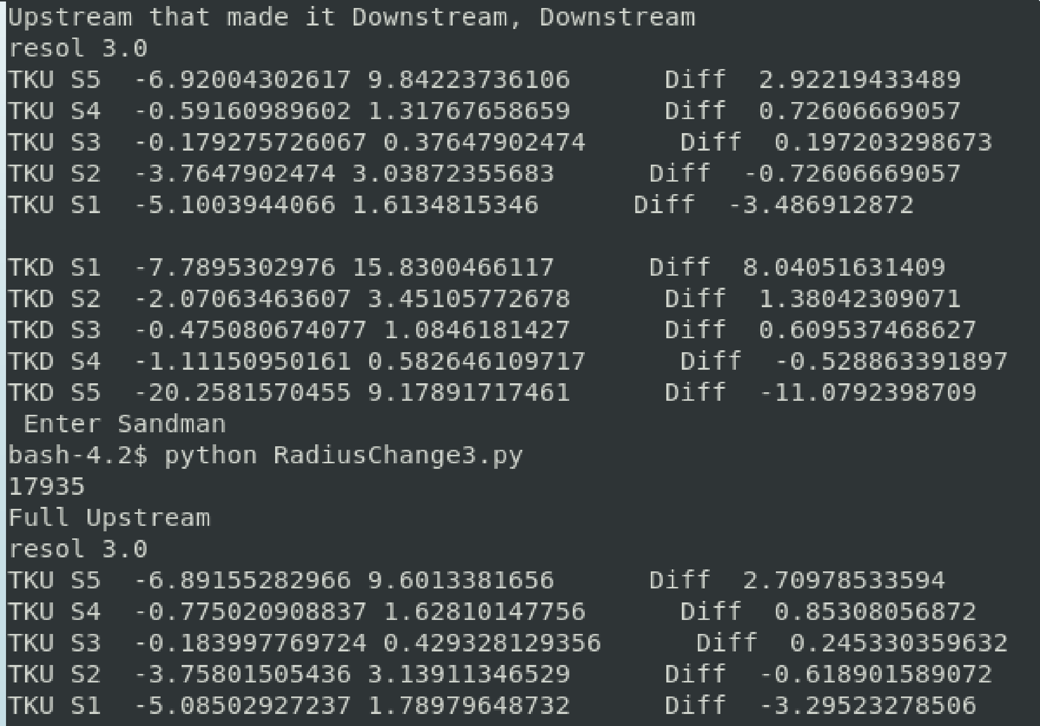
```
resol 1.0
TKU S5  -28.2897095733 36.4467551094      Diff  8.15704553603
TKU S4  -11.0433847257 22.812836142      Diff  11.7694514163
TKU S3  -9.05342416637 6.1939763356      Diff  -2.85944783076
TKU S2  -27.0795984224 27.1602724991      Diff  0.08067407673
TKU S1  -33.5514521334 18.5640014342      Diff  -14.9874506992

TKD S1  -25.9591251345 41.5650770886      Diff  15.6059519541
TKD S2  -15.749372535 22.7590534242      Diff  7.00968088921
TKD S3  -11.5184653998 13.5622086769      Diff  2.04374327716
TKD S4  -12.1011115095 10.1828612406      Diff  -1.91825026891
TKD S5  -43.0172104697 24.9731086411      Diff  -18.0441018286
 Enter Sandman
bash-4.2$ python RadiusChange3.py
17935
Full Upstream
resol 1.0
TKU S5  -28.2129913577 36.3590744355      Diff  8.14608307778
TKU S4  -11.6643434625 24.4215221634      Diff  12.7571787009
TKU S3  -9.6682464455 6.55143574017      Diff  -3.11681070532
TKU S2  -27.4379704488 27.1536102593      Diff  -0.284360189573
TKU S1  -34.1511011988 18.2436576526      Diff  -15.9074435461
```



```
Upstream that made it Downstream, Downstream
resol 2.0
TKU S5  -15.0233058444 20.5898171388      Diff  5.56651129437
TKU S4  -2.03477949086 5.75475080674      Diff  3.71997131588
TKU S3  -0.573682323413 1.09358192901      Diff  0.519899605593
TKU S2  -11.4288275368 10.8910003586      Diff  -0.5378271782
TKU S1  -14.9784869129 6.14019361778      Diff  -8.83829329509

TKD S1  -14.7543922553 27.1602724991      Diff  12.4058802438
TKD S2  -5.03764790247 8.53352456077      Diff  3.4958766583
TKD S3  -1.68519182503 3.20903549659      Diff  1.52384367157
TKD S4  -3.14628899247 1.86446755109      Diff  -1.28182144138
TKD S5  -30.5665112944 15.8121190391      Diff  -14.7543922553
 Enter Sandman
bash-4.2$ python RadiusChange3.py
17935
Full Upstream
resol 2.0
TKU S5  -14.9763033175 20.4126010594      Diff  5.43629774185
TKU S4  -2.5313632562 6.71870643992      Diff  4.18734318372
TKU S3  -0.713688318929 1.20992472819      Diff  0.496236409256
TKU S2  -11.4747700028 11.2238639532      Diff  -0.250906049624
TKU S1  -15.1435740173 6.35628659047      Diff  -8.78728742682
```



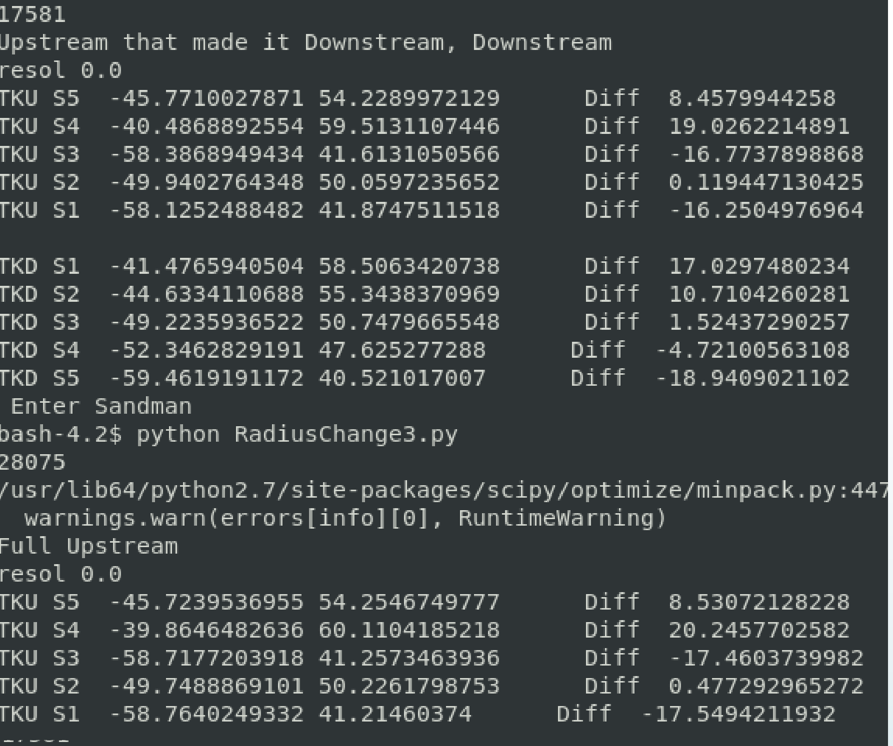
```
Upstream that made it Downstream, Downstream
resol 3.0
TKU S5  -6.92004302617 9.84223736106      Diff  2.92219433489
TKU S4  -0.59160989602 1.31767658659      Diff  0.72606669057
TKU S3  -0.179275726067 0.37647902474      Diff  0.197203298673
TKU S2  -3.7647902474 3.03872355683      Diff  -0.72606669057
TKU S1  -5.1003944066 1.6134815346      Diff  -3.486912872

TKD S1  -7.7895302976 15.8300466117      Diff  8.04051631409
TKD S2  -2.07063463607 3.45105772678      Diff  1.38042309071
TKD S3  -0.475080674077 1.0846181427      Diff  0.609537468627
TKD S4  -1.11150950161 0.582646109717      Diff  -0.528863391897
TKD S5  -20.2581570455 9.17891717461      Diff  -11.0792398709
 Enter Sandman
bash-4.2$ python RadiusChange3.py
17935
Full Upstream
resol 3.0
TKU S5  -6.89155282966 9.6013381656      Diff  2.70978533594
TKU S4  -0.775020908837 1.62810147756      Diff  0.85308056872
TKU S3  -0.183997769724 0.429328129356      Diff  0.245330359632
TKU S2  -3.75801505436 3.13911346529      Diff  -0.618901589072
TKU S1  -5.08502927237 1.78979648732      Diff  -3.29523278506
```

IH2

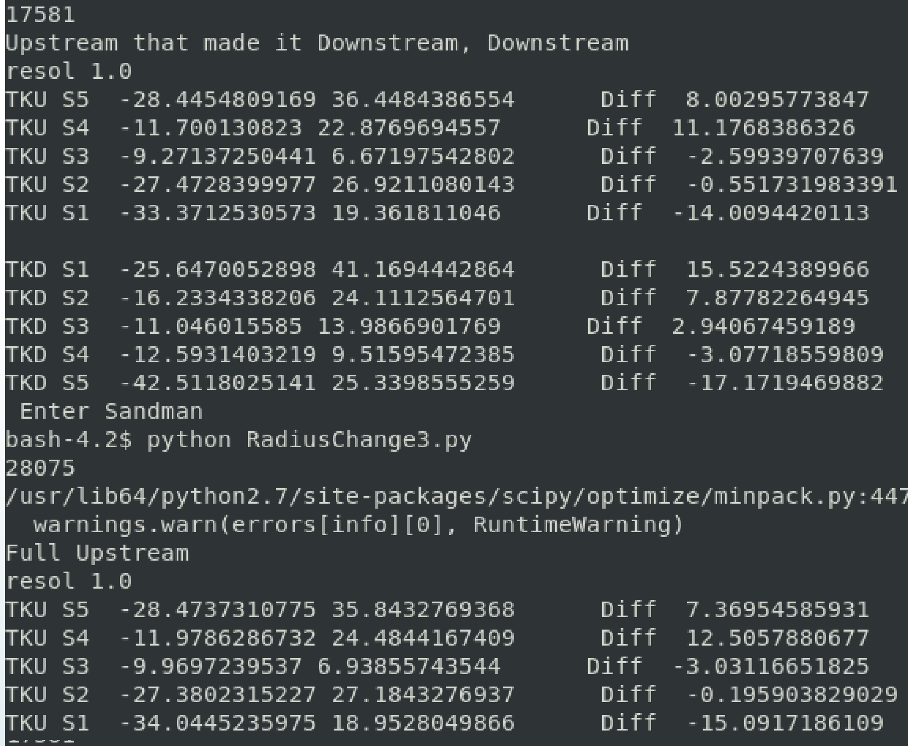
Particles inside or outside the bounding yellow circle lines with a radius of:

Top Left: +/- 0 mm
Top Right: +/- 1 mm
Bottom Left: +/- 2 mm
Bottom Right +/- 3 mm



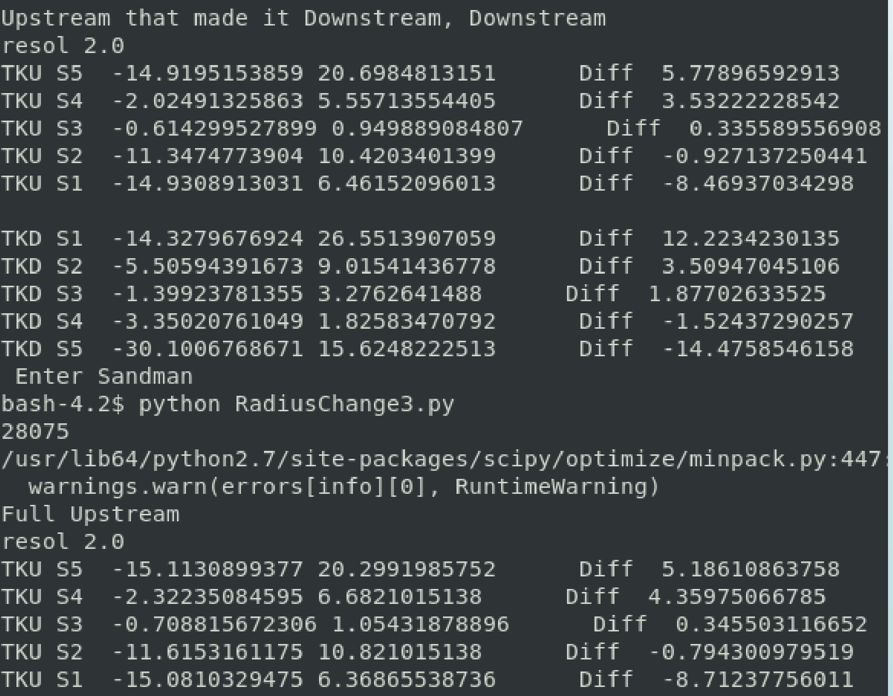
```
17581
Upstream that made it Downstream, Downstream
resol 0.0
TKU S5  -45.7710027871 54.2289972129      Diff  8.4579944258
TKU S4  -40.4868892554 59.5131107446      Diff  19.0262214891
TKU S3  -58.3868949434 41.6131050566      Diff  -16.7737898868
TKU S2  -49.9402764348 50.0597235652      Diff  0.119447130425
TKU S1  -58.1252488482 41.8747511518      Diff  -16.2504976964

TKD S1  -41.4765940504 58.5063420738      Diff  17.0297480234
TKD S2  -44.6334110688 55.3438370969      Diff  10.7104260281
TKD S3  -49.2235936522 50.7479665548      Diff  1.52437290257
TKD S4  -52.3462829191 47.625277288      Diff  -4.72100563108
TKD S5  -59.4619191172 40.521017007      Diff  -18.9409021102
 Enter Sandman
bash-4.2$ python RadiusChange3.py
28075
/usr/lib64/python2.7/site-packages/scipy/optimize/minpack.py:447
  warnings.warn(errors[info][0], RuntimeWarning)
Full Upstream
resol 0.0
TKU S5  -45.7239536955 54.2546749777      Diff  8.53072128228
TKU S4  -39.8646482636 60.1104185218      Diff  20.2457702582
TKU S3  -58.7177203918 41.2573463936      Diff  -17.4603739982
TKU S2  -49.7488869101 50.2261798753      Diff  0.477292965272
TKU S1  -58.7640249332 41.21460374      Diff  -17.5494211932
```



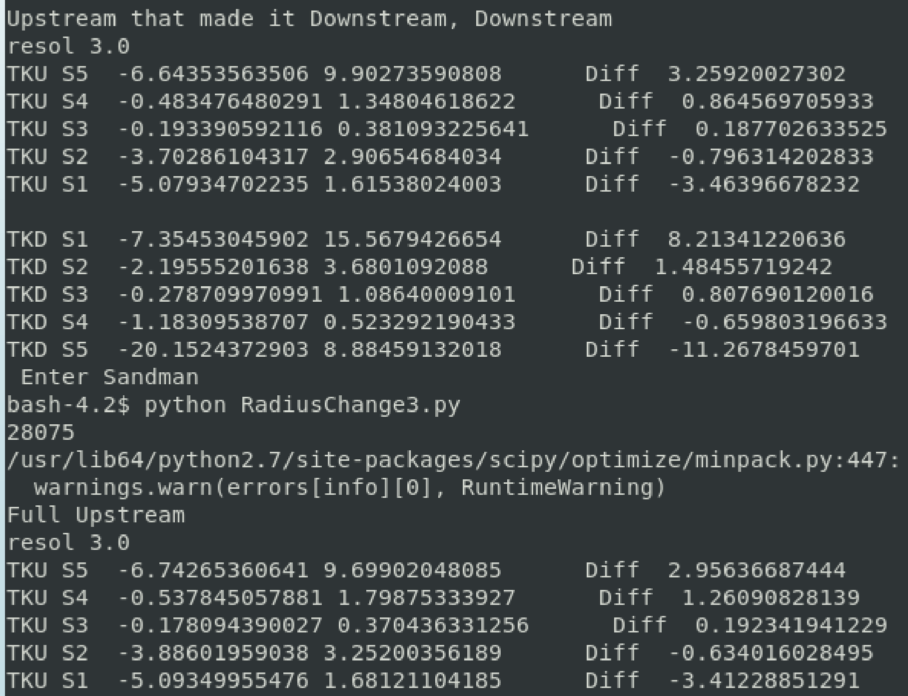
```
17581
Upstream that made it Downstream, Downstream
resol 1.0
TKU S5  -28.4454809169 36.4484386554      Diff  8.00295773847
TKU S4  -11.700130823 22.8769694557      Diff  11.1768386326
TKU S3  -9.27137250441 6.67197542802      Diff  -2.59939707639
TKU S2  -27.4728399977 26.9211080143      Diff  -0.551731983391
TKU S1  -33.3712530573 19.361811046      Diff  -14.0094420113

TKD S1  -25.6470052898 41.1694442864      Diff  15.5224389966
TKD S2  -16.2334338206 24.1112564701      Diff  7.87782264945
TKD S3  -11.046015585 13.9866901769      Diff  2.94067459189
TKD S4  -12.5931403219 9.51595472385      Diff  -3.07718559809
TKD S5  -42.5118025141 25.3398555259      Diff  -17.1719469882
 Enter Sandman
bash-4.2$ python RadiusChange3.py
28075
/usr/lib64/python2.7/site-packages/scipy/optimize/minpack.py:447
  warnings.warn(errors[info][0], RuntimeWarning)
Full Upstream
resol 1.0
TKU S5  -28.4737310775 35.8432769368      Diff  7.36954585931
TKU S4  -11.9786286732 24.4844167409      Diff  12.5057880677
TKU S3  -9.9697239537 6.93855743544      Diff  -3.03116651825
TKU S2  -27.3802315227 27.1843276937      Diff  -0.195903829029
TKU S1  -34.0445235975 18.9528049866      Diff  -15.0917186109
```



```
Upstream that made it Downstream, Downstream
resol 2.0
TKU S5  -14.9195153859 20.6984813151      Diff  5.77896592913
TKU S4  -2.02491325863 5.55713554405      Diff  3.5322228542
TKU S3  -0.614299527899 0.949889084807      Diff  0.335589556908
TKU S2  -11.3474773904 10.4203401399      Diff  -0.927137250441
TKU S1  -14.9308913031 6.46152096013      Diff  -8.46937034298

TKD S1  -14.3279676924 26.5513907059      Diff  12.2234230135
TKD S2  -5.50594391673 9.01541436778      Diff  3.50947045106
TKD S3  -1.39923781355 3.2762641488      Diff  1.87702633525
TKD S4  -3.35020761049 1.82583470792      Diff  -1.52437290257
TKD S5  -30.1006768671 15.6248222513      Diff  -14.4758546158
 Enter Sandman
bash-4.2$ python RadiusChange3.py
28075
/usr/lib64/python2.7/site-packages/scipy/optimize/minpack.py:447
  warnings.warn(errors[info][0], RuntimeWarning)
Full Upstream
resol 2.0
TKU S5  -15.1130899377 20.2991985752      Diff  5.18610863758
TKU S4  -2.32235084595 6.6821015138      Diff  4.35975066785
TKU S3  -0.708815672306 1.05431878896      Diff  0.345503116652
TKU S2  -11.6153161175 10.821015138      Diff  -0.794300979519
TKU S1  -15.0810329475 6.36865538736      Diff  -8.71237756011
```



```
Upstream that made it Downstream, Downstream
resol 3.0
TKU S5  -6.64353563506 9.90273590808      Diff  3.25920027302
TKU S4  -0.483476480291 1.34804618622      Diff  0.864569705933
TKU S3  -0.193390592116 0.381093225641      Diff  0.187702633525
TKU S2  -3.70286104317 2.90654684034      Diff  -0.796314202833
TKU S1  -5.07934702235 1.61538024003      Diff  -3.46396678232

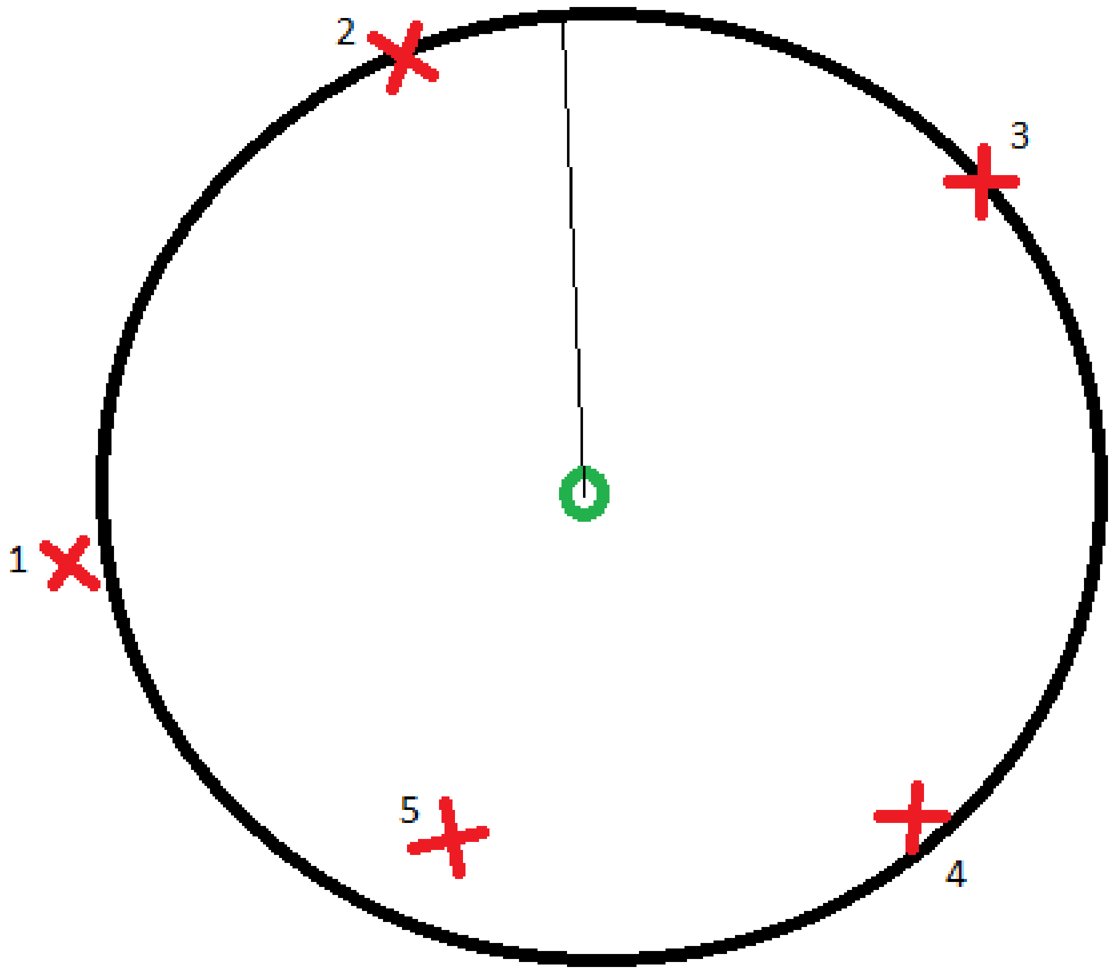
TKD S1  -7.35453045902 15.5679426654      Diff  8.21341220636
TKD S2  -2.19555201638 3.6801092088      Diff  1.48455719242
TKD S3  -0.278709970991 1.08640009101      Diff  0.807690120016
TKD S4  -1.18309538707 0.523292190433      Diff  -0.659803196633
TKD S5  -20.1524372903 8.88459132018      Diff  -11.2678459701
 Enter Sandman
bash-4.2$ python RadiusChange3.py
28075
/usr/lib64/python2.7/site-packages/scipy/optimize/minpack.py:447:
  warnings.warn(errors[info][0], RuntimeWarning)
Full Upstream
resol 3.0
TKU S5  -6.74265360641 9.69902048085      Diff  2.95636687444
TKU S4  -0.537845057881 1.79875333927      Diff  1.26090828139
TKU S3  -0.178094390027 0.370436331256      Diff  0.192341941229
TKU S2  -3.88601959038 3.25200356189      Diff  -0.634016028495
TKU S1  -5.09349955476 1.68121104185      Diff  -3.41228851291
```

Path of particle in ideal solenoid

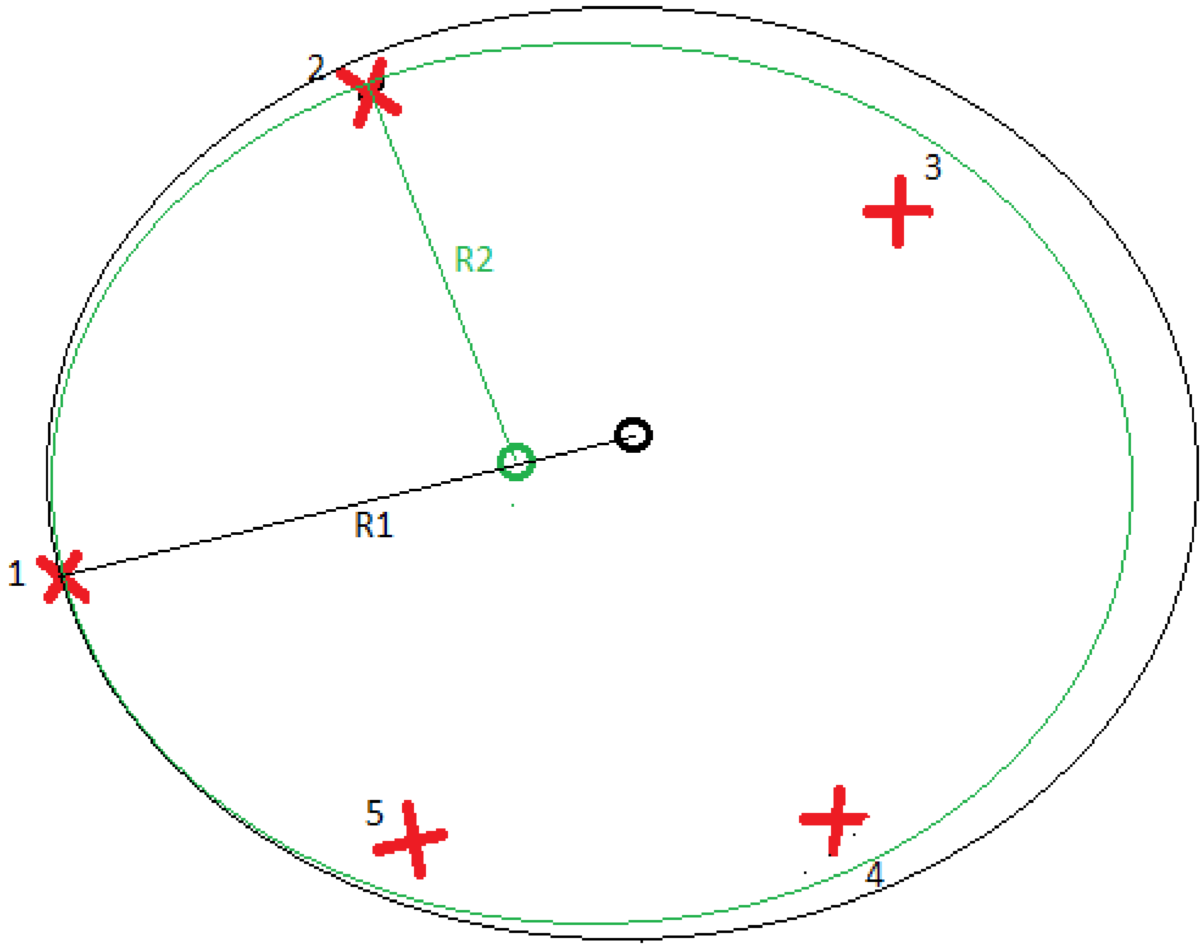
- If there is no Energy Loss, then the particle will follow a constant radius path
- If there is a constant Energy Loss with no scattering, then the particle will spiral towards a centre with radius $r = a\varphi$, where φ is the turning angle and a is angle of the polar slope (between tangent and polar circle, dictates expansion of spiral).
- dE/dx is fairly constant through the stations as the Energy Loss is small (or as implemented by MAUS)
- In MICE we have 5 stations per tracker. Between stations the particles follow a helical path (with no Energy Loss, assume perfect vacuum) and are deviated at the station.
- At the station, Energy Loss occurs, and the particle is deviated to a lower radius path but remains tangential to the circle centre unless scattered.
- This in turn creates a new circle centre along the radial path. The radius change is proportional to the Energy Loss.

Exaggerated case – not to any scale

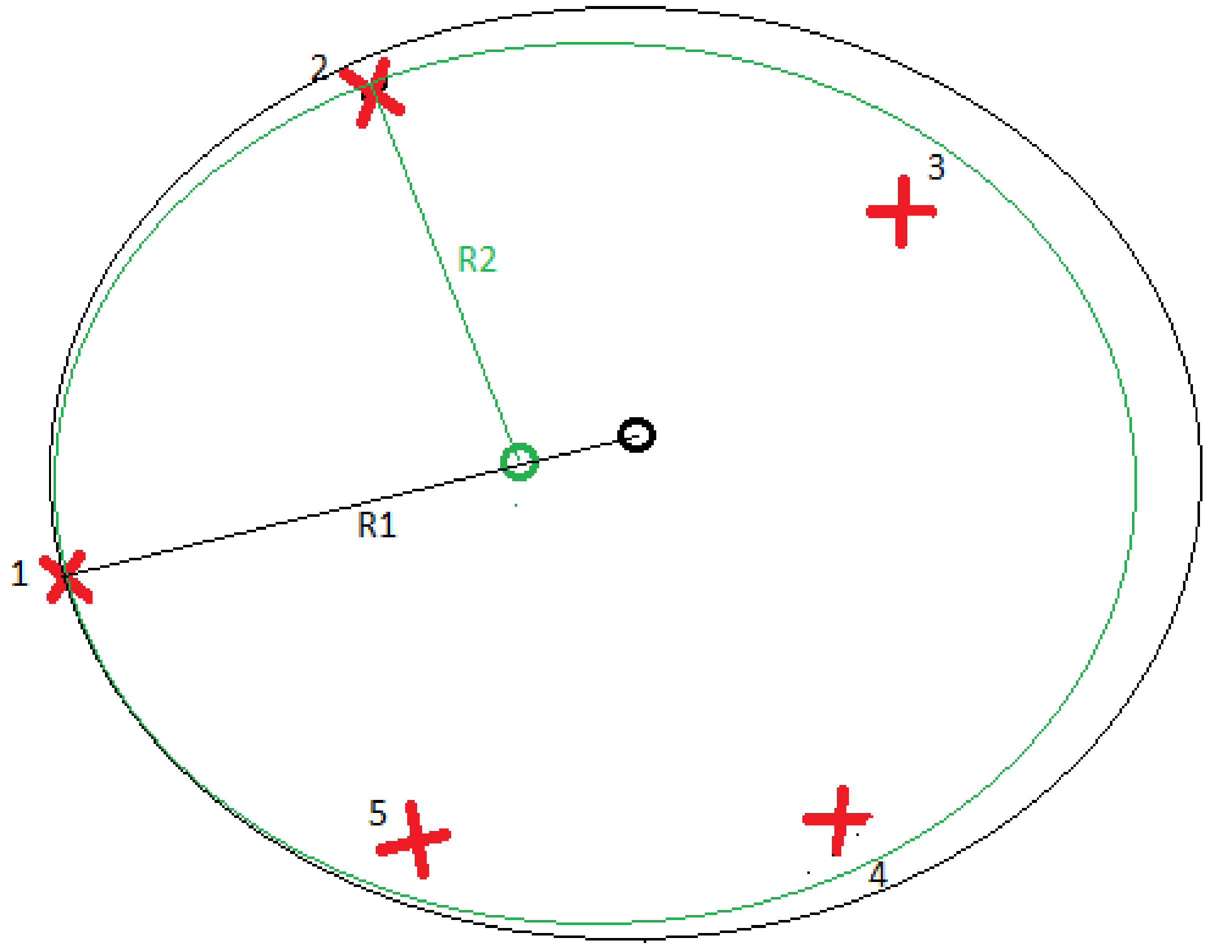
Circle fit radius of five stations

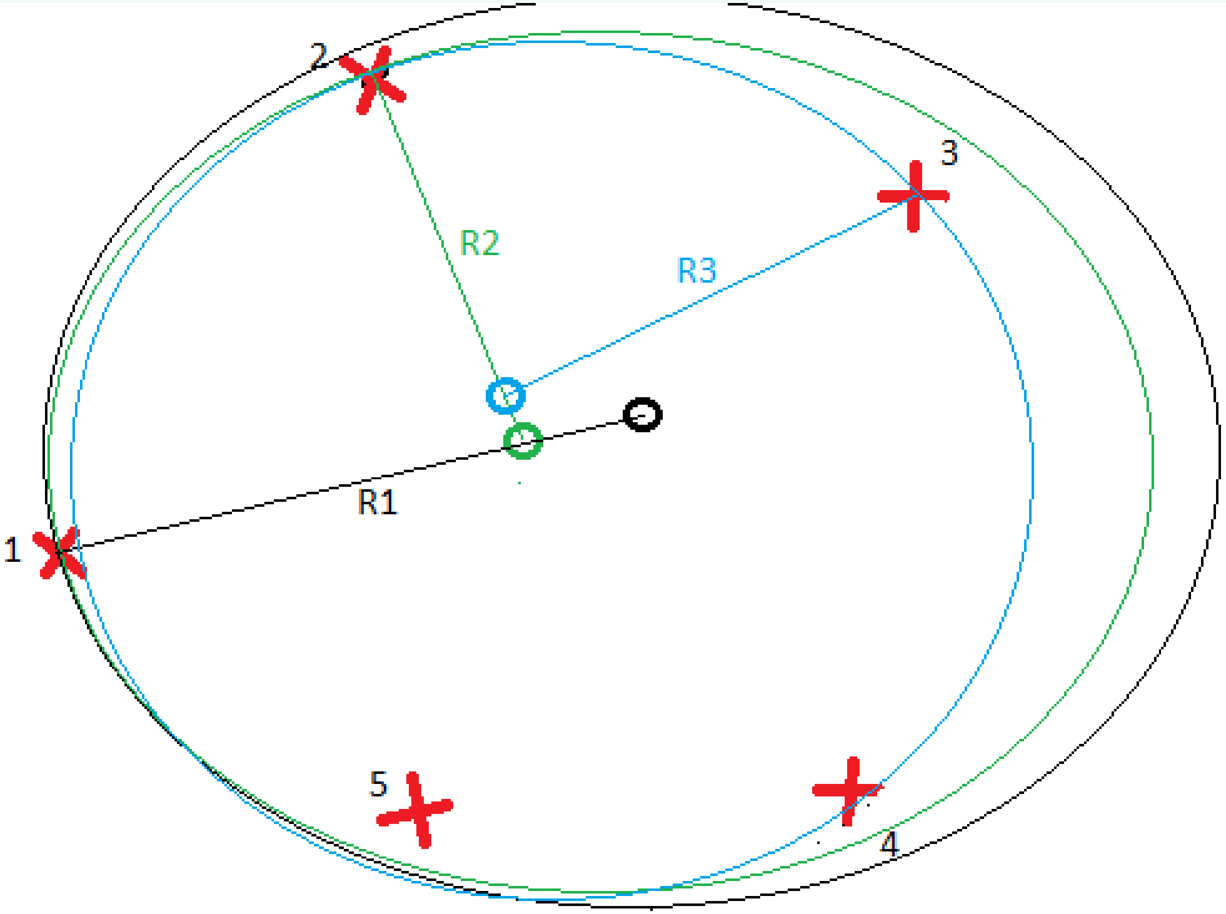


R1 true radius of initial particle
R2 true radius of particle after Energy Loss through 1st station, with new centre

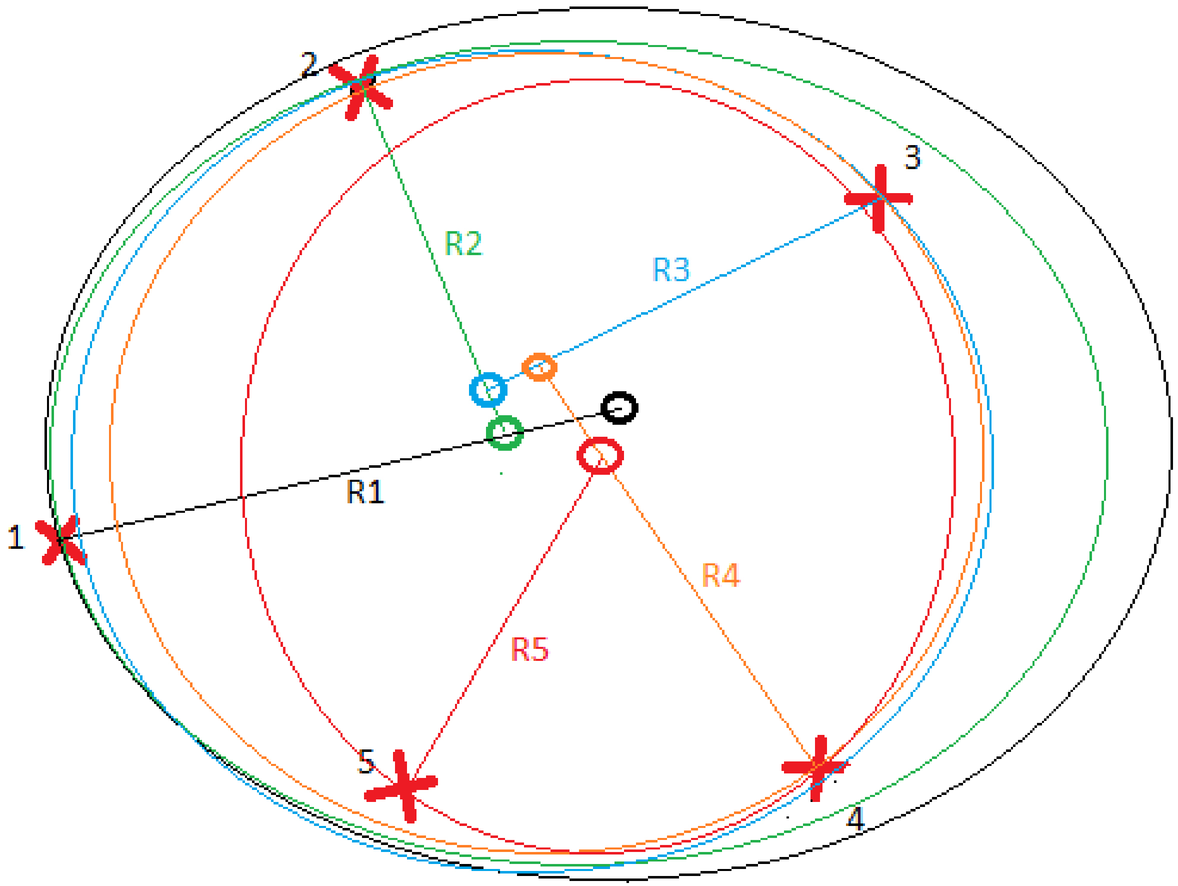


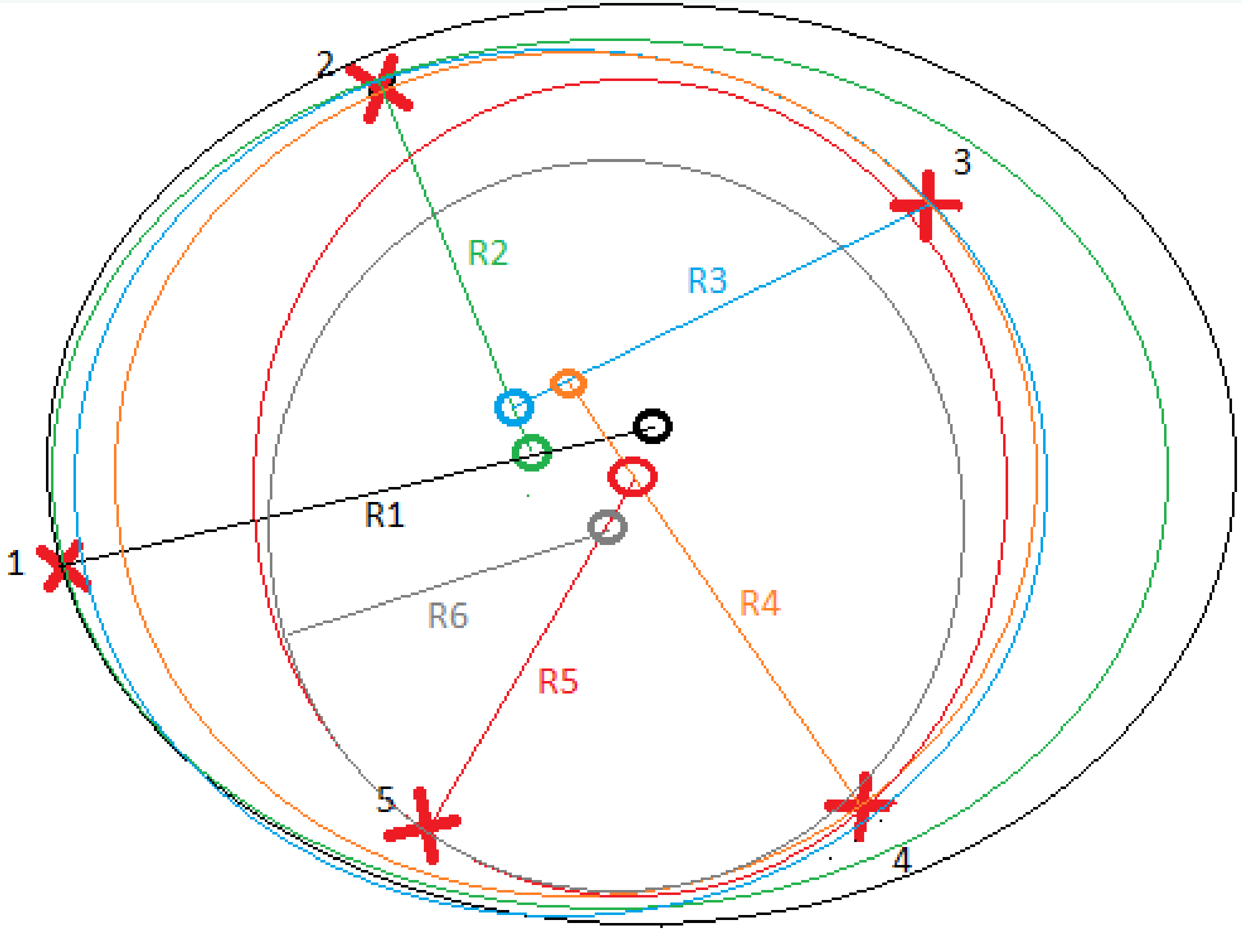
Before Station 2 to after Station 2





Before Station 5 to after Station 5





What affect does it have on Pt and Pz

- $p_t = cBQR$
- c, B and Q are constant (should be), so transverse momentum changes by radius loss
- A particle loses approximately 0.6 MeV per station, so ~ 3 MeV per tracker, which for a 140 MeV particle is ~2%
- Therefore the radius from start to finish reduces by 2%
- For a high radius particle, e.g. 100mm, this radius reduction would be more than a few widths of fibres, leading to a poor qui-squared value for the circle fit and thus being excluded

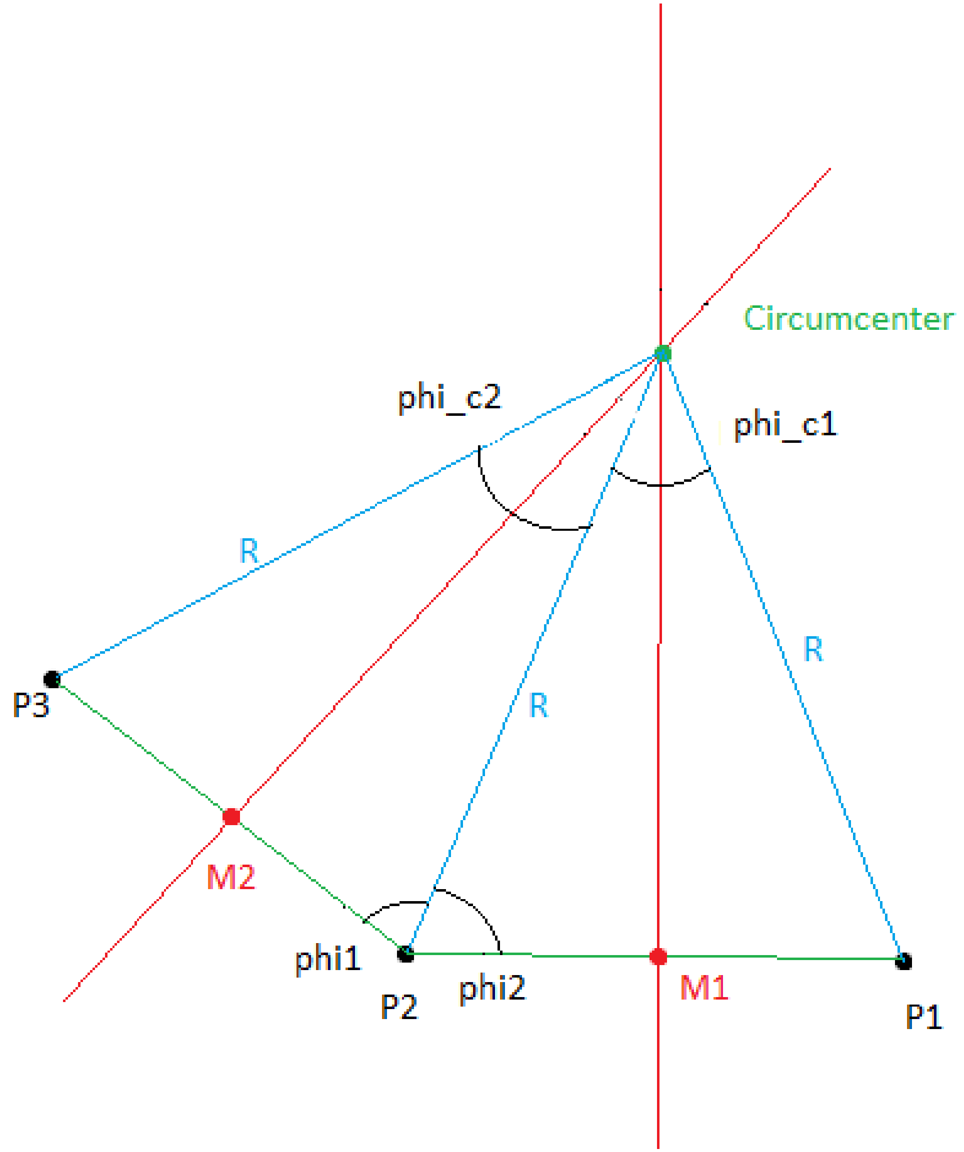
What effect does it have on Pt and Pz

- z-s plane
- Another qui-squared cut is made in the z-s plane, if the fit in the z-s plane fits a straight line.
- $z = \frac{dz}{ds} s - s_0 \ with \ s = R\varphi$, however if the radius is not constant, or not the appropriate radius (wrong circle centre), then the phase advance will be wrong.
- Should have straight line between stations in s-z plane, however a small deviation at each station. That deviation should be similar at each station (i.e. angle change)
- A too strict straight line qui-squared cut may exclude valid particles, but more importantly:

$$^{p_z}/_{p_t} = {}^{\Delta z}/_{R\Delta\varphi}$$

- The p_t to R ratio should be fairly constant and thus p_z is heavily influenced by the phase advance.
- If the movement of circle centre isn't accounted for, then will have the wrong phase advance angle

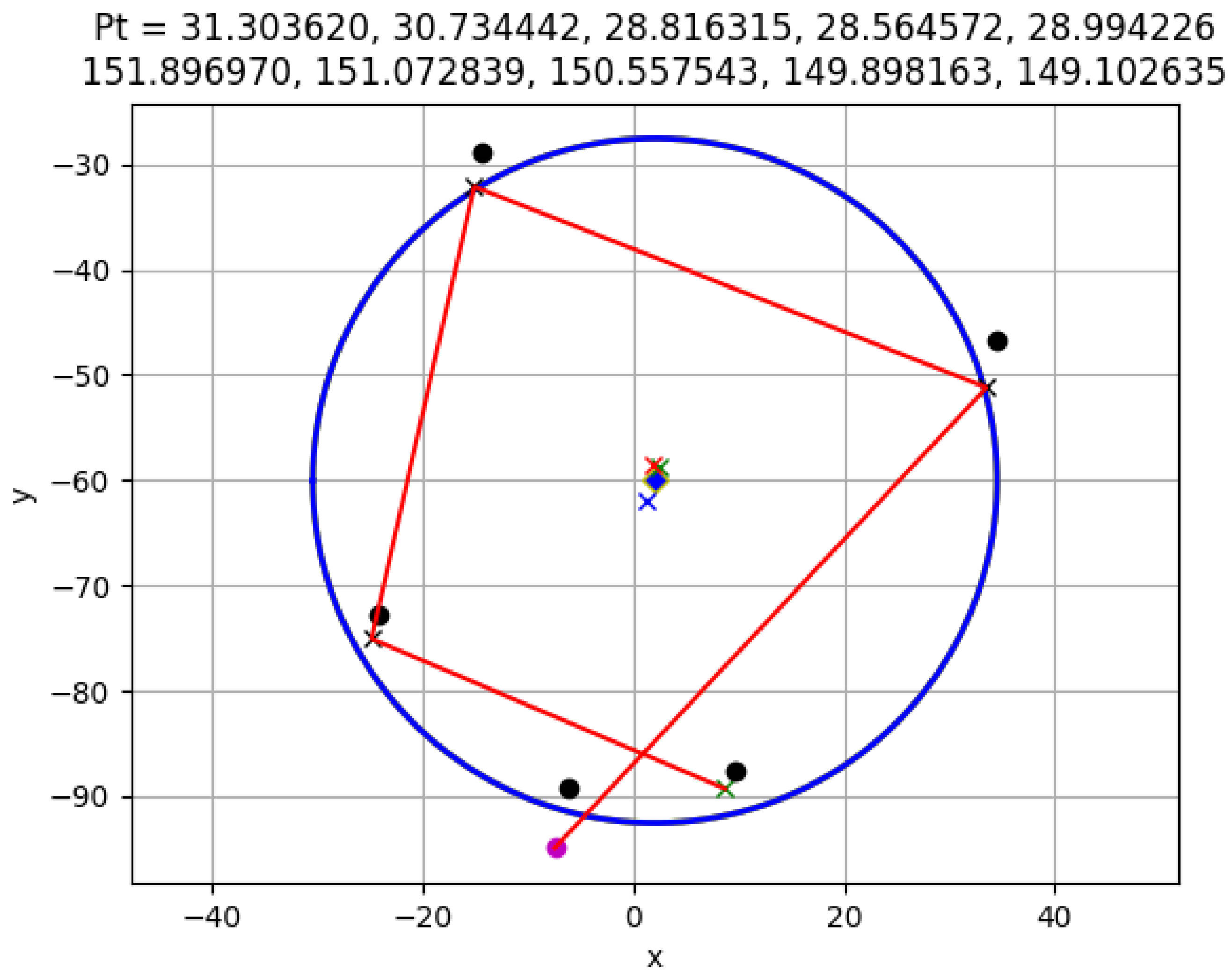
Circle for 3 points (No Energy Loss)



- For any 3 points a circle can be found
- Circumcentre for those 3 points found by the intersection of tangential midpoint lines

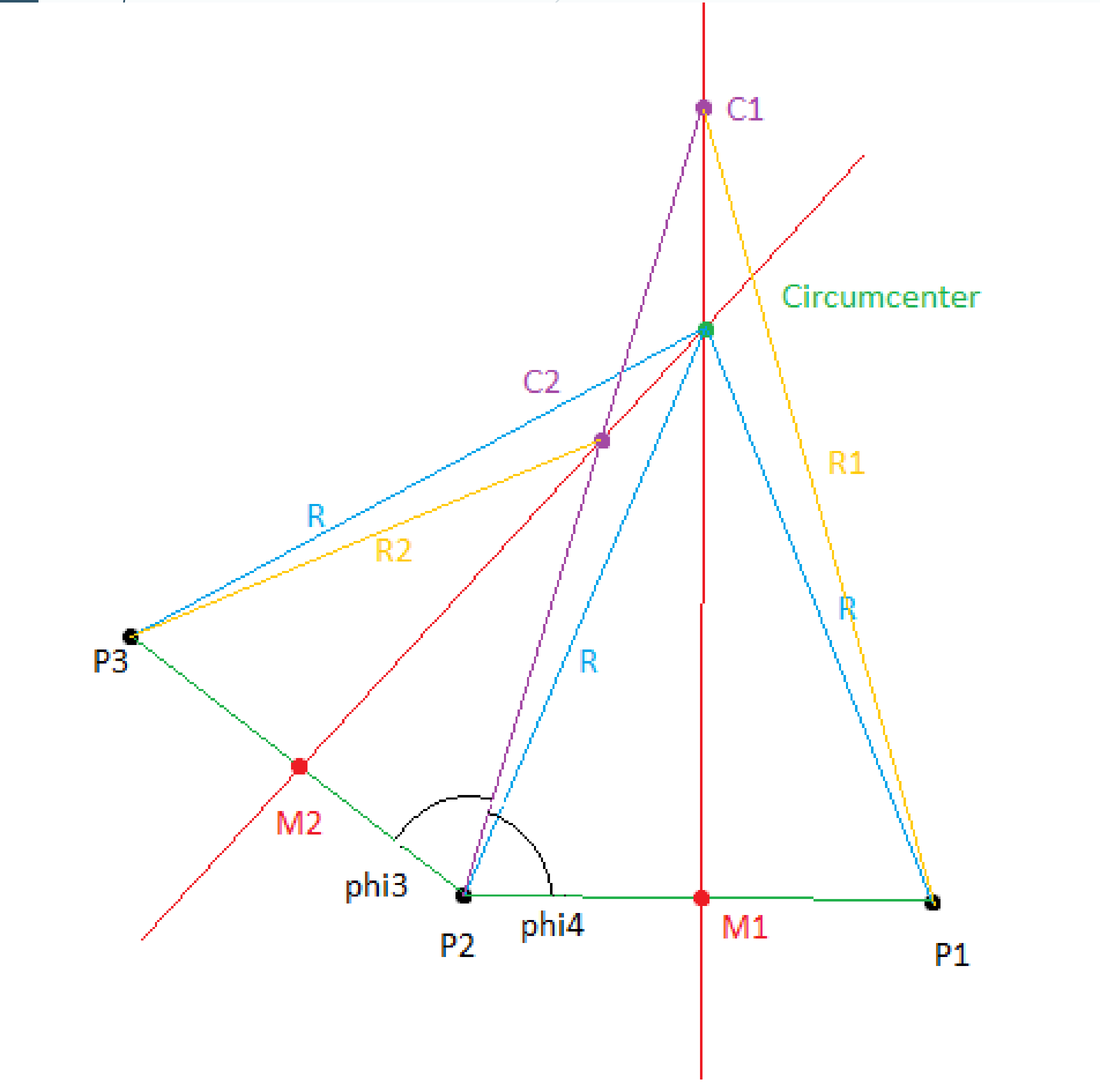
- For 5 points, this can be repeated for each set of consecutive 3 points (In the no Energy Loss case it can be for any 3 points)
- I.e. Find the circumcentre for points 1,2,3 and 2,3,4 and 3,4,5
- If No Energy loss then the 3 circumcentres should match

Example case

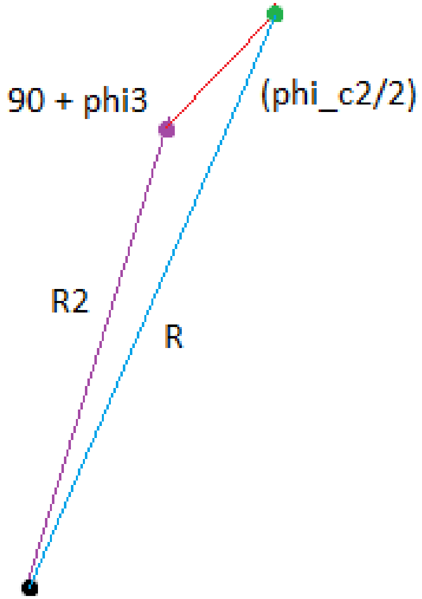


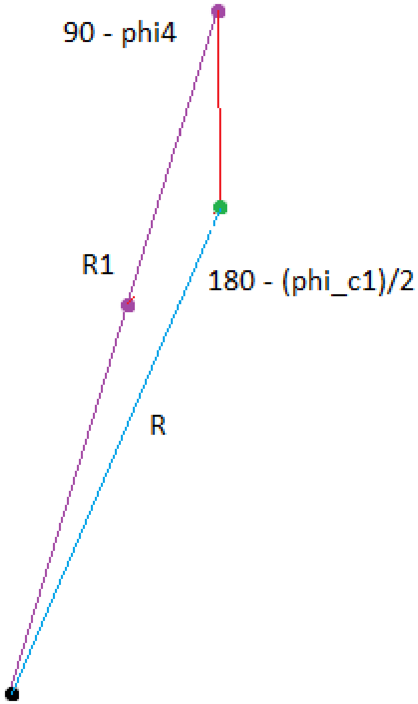
- Purple point is most upstream point with the following x marks hits in the following stations (local reference frame).
- Blue Diamond and circle is the circle fit to those five points
- Blue, red and green are the circumcentre for each three consecutive points assuming no energy loss
- Circumcentres shift slightly due to energy loss, this leads to slightly incorrect calculation of seed Pt and Pz
- Will try to introduce Energy Loss and match parameters between 3 consecutive circles
- Black points are in global reference frame, as well as showing the trackpoint Pt and Pz

Circle for 3 points (with Energy Loss)



- For no Energy Loss at Station 2, the green point is the centre of the three points.
- Energy Loss changes the radial path taken by the particle.
- For the same three hits, the particle must have started at a higher radius path and can to a lower radius path (assuming ionization acts uniformly)
- For points 1 and 2, they still share the same radius, and thus the new centre must still remain on their tangential midpoint line.
- The same respectively happens for points 2 and 3.
- At station 2 the radial paths overlap, where the energy loss from the higher radius path to the lower radius path can be given in terms of some parameter, alpha.
- This parameter, alpha, can be minimized for three consecutive circles to match radii, pt, pz, ds/dz, ds^2/dz^2, etc.





Solving for alpha (Energy Loss at a station)

$$R_2 = \alpha R_1$$

- Let $R_2 = \varepsilon_2 R$ and $R_1 = \varepsilon_1 R$, i.e. $\alpha = {}^{\varepsilon_2}/_{\varepsilon_1}$ and use Sin rule to solve left triangles

$$\frac{\sin(90-\theta_4)}{R} = \frac{\sin(180-\theta_{c1}/2)}{R_1} \qquad \text{and} \qquad \frac{\sin(\theta_{c2}/2)}{R_2} = \frac{\sin(90+\theta_3)}{R}$$

- Using $\theta_1 + \theta_2 = \theta_3 + \theta_4$ and $\theta_1 = 90 - \theta_{c2}/2$, $\theta_2 = 90 - \theta_{c1}/2$
- $\alpha = {}^{\varepsilon_2}/_{\varepsilon_1} = \frac{\sin(\theta_{c2}/2)}{\sin(90+\theta_3)} \frac{\sin(90-\theta_4)}{\sin(180-\theta_{c1}/2)} =$

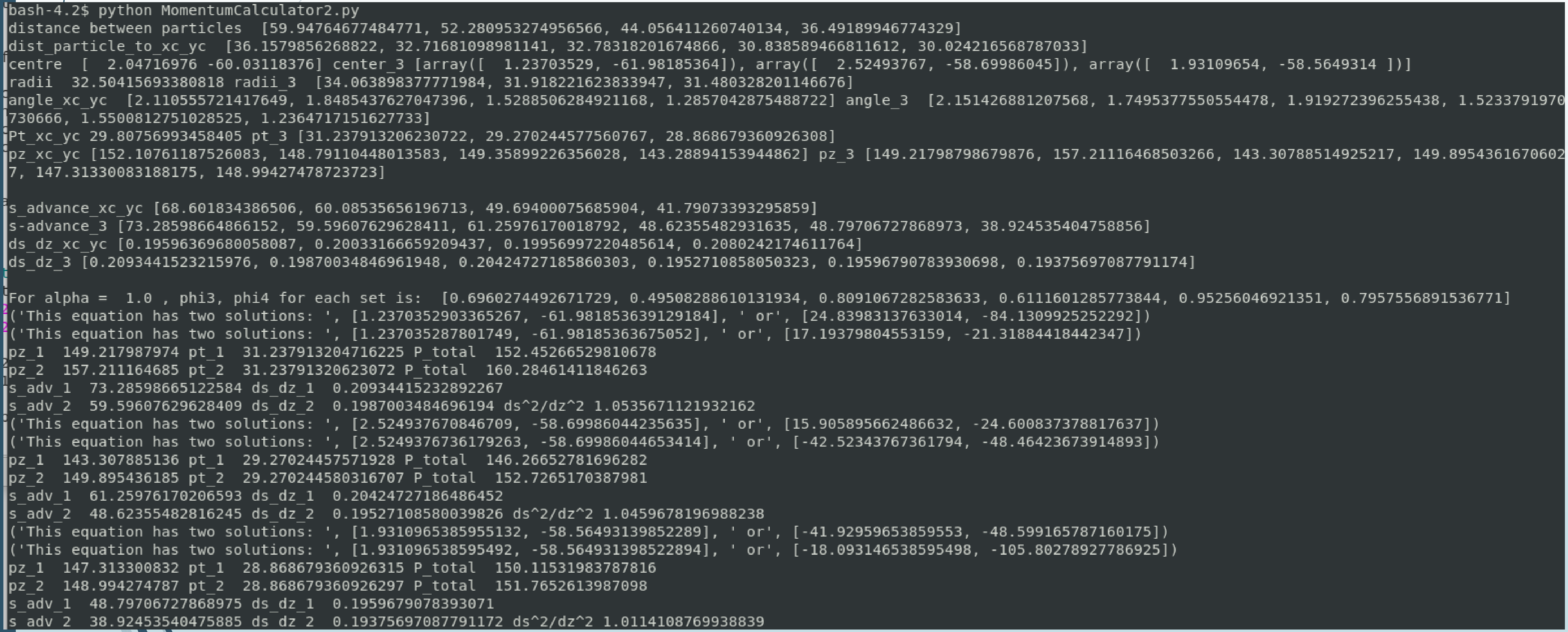
$$= \frac{\sin\left(\frac{\theta_{c2}}{2}\right)}{\sin\left(\frac{\theta_{c1}}{2}\right)}\left(-\cos\left(\frac{\theta_{c1}}{2}+\frac{\theta_{c2}}{2}\right) + \sin\left(\frac{\theta_{c1}}{2}+\frac{\theta_{c2}}{2}\right)\tan(\theta_3)\right)$$

- Alpha effectively changes the opening angle (θ_3) made by the radial path at station 2. It can be more effective writing it in terms of θ_3

$$\theta_3 = \frac{\alpha \sin(\frac{\theta_{c1}}{2})}{\sin\left(\frac{\theta_{c2}}{2}\right)\sin(\frac{\theta_{c1}}{2}+\frac{\theta_{c2}}{2})} + \frac{1}{\tan(\frac{\theta_{c1}}{2}+\frac{\theta_{c2}}{2})}$$

- Currently I am matching and minimizing parameters between three consecutive circles.
- Can then see if it changes/improves pz discrepancy, Energy Loss in cooling channel

Alpha = 1.0



```
bash-4.2$ python MomentumCalculator2.py
distance between particles  [59.94764677484771, 52.280953274956566, 44.056411260740134, 36.49189946774329]
dist_particle_to_xc_yc  [36.1579856268822, 32.71681098981141, 32.78318201674866, 30.838589466811612, 30.024216568787033]
centre  [  2.04716976 -60.03118376] center_3 [array([  1.23703529, -61.98185364]), array([  2.52493767, -58.69986045]), array([  1.93109654, -58.5649314 ])]
radii  32.50415693380818 radii_3  [34.063898377771984, 31.918221623833947, 31.480328201146676]
angle_xc_yc  [2.110555721417649, 1.8485437627047396, 1.5288506284921168, 1.2857042875488722] angle_3  [2.151426881207568, 1.7495377550554478, 1.919272396255438, 1.5233791970
730666, 1.5500812751028525, 1.2364717151627733]
Pt_xc_yc 29.80756993458405 pt_3 [31.237913206230722, 29.270244577560767, 28.868679360926308]
pz_xc_yc [152.10761187526083, 148.79110448013583, 149.35899226356028, 143.28894153944862] pz_3 [149.21798798679876, 157.21116468503266, 143.30788514925217, 149.8954361670602
7, 147.31330083188175, 148.99427478723723]

s_advance_xc_yc [68.601834386506, 60.08535656196713, 49.69400075685904, 41.79073393295859]
s-advance_3 [73.28598664866152, 59.59607629628411, 61.25976170018792, 48.62355482931635, 48.79706727868973, 38.924535404758856]
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For alpha =  1.0 , phi3, phi4 for each set is:  [0.6960274492671729, 0.49508288610131934, 0.8091067282583633, 0.6111601285773844, 0.95256046921351, 0.7957556891536771]
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pz_1  149.217987974 pt_1  31.237913204716225 P_total  152.45266529810678
pz_2  157.211164685 pt_2  31.23791320623072 P_total  160.28461411846263
s_adv_1  73.28598665122584 ds_dz_1  0.20934415232892267
s_adv_2  59.59607629628409 ds_dz_2  0.1987003484696194 ds^2/dz^2 1.0535671121932162
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pz_1  143.307885136 pt_1  29.27024457571928 P_total  146.26652781696282
pz_2  149.895436185 pt_2  29.270244580316707 P_total  152.7265170387981
s_adv_1  61.25976170206593 ds_dz_1  0.20424727186486452
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pz_1  147.313300832 pt_1  28.868679360926315 P_total  150.11531983787816
pz_2  148.994274787 pt_2  28.868679360926297 P_total  151.7652613987098
s_adv_1  48.79706727868975 ds_dz_1  0.1959679078393071
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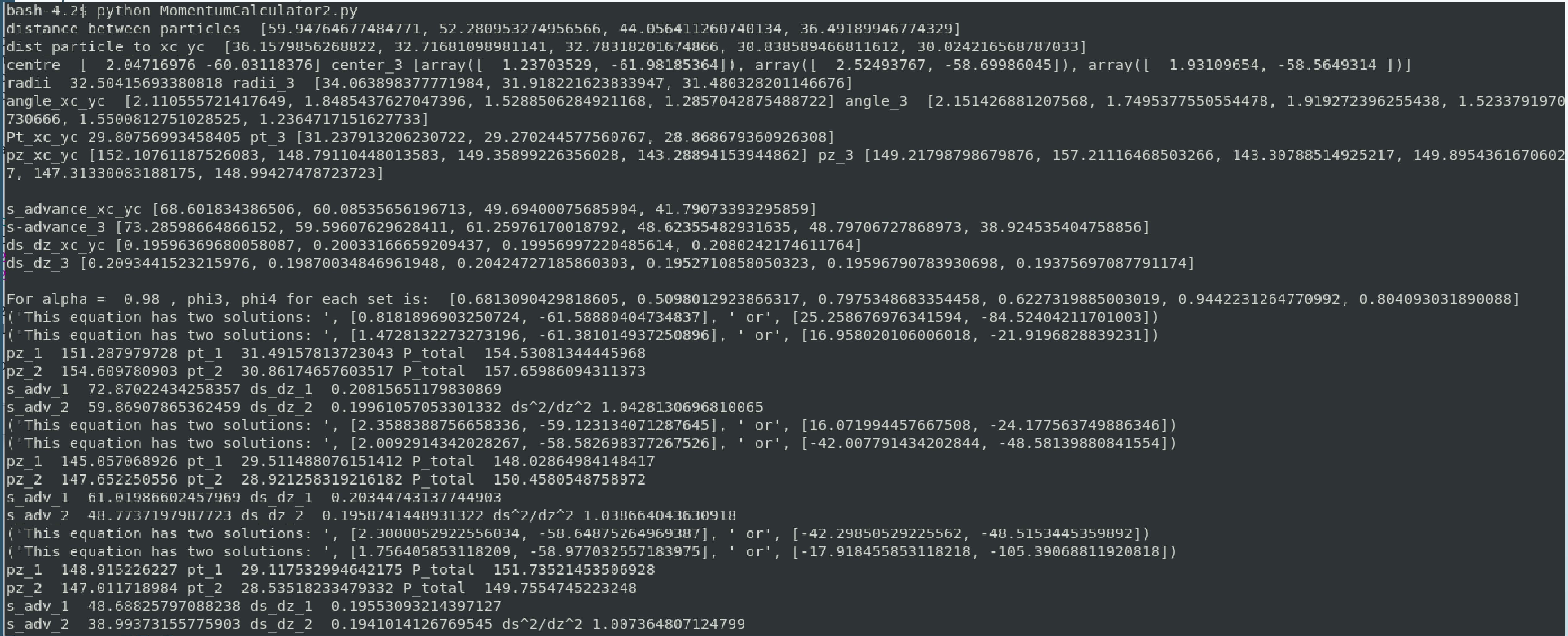
Alpha = 0.98



```
bash-4.2$ python MomentumCalculator2.py
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centre  [  2.04716976 -60.03118376] center_3 [array([  1.23703529, -61.98185364]), array([  2.52493767, -58.69986045]), array([  1.93109654, -58.5649314 ])]
radii  32.50415693380818 radii_3  [34.063898377771984, 31.918221623833947, 31.480328201146676]
angle_xc_yc  [2.110555721417649, 1.8485437627047396, 1.5288506284921168, 1.2857042875488722] angle_3  [2.151426881207568, 1.7495377550554478, 1.919272396255438, 1.5233791970
730666, 1.5500812751028525, 1.2364717151627733]
Pt_xc_yc 29.80756993458405 pt_3 [31.237913206230722, 29.270244577560767, 28.868679360926308]
pz_xc_yc [152.1076118752608 3, 148.79110448013583, 149.35899226356028, 143.28894153944862] pz_3 [149.21798798679876, 157.21116468503266, 143.30788514925217, 149.8954361670602
7, 147.31330083188175, 148.99427478723723]

s_advance_xc_yc [68.601834386506, 60.08535656196713, 49.69400075685904, 41.79073393295859]
s-advance_3 [73.28598664866152, 59.59607629628411, 61.25976170018792, 48.62355482931635, 48.79706727868973, 38.924535404758856]
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ds_dz_3 [0.2093441523215976, 0.19870034846961948, 0.20424727185860303, 0.1952710858050323, 0.19596790783930698, 0.19375697087791174]

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pz_1  151.287979728 pt_1  31.49157813723043 P_total  154.53081344445968
pz_2  154.609780903 pt_2  30.86174657603517 P_total  157.65986094311373
s_adv_1  72.87022434258357 ds_dz_1  0.20815651179830869
s_adv_2  59.86907865362459 ds_dz_2  0.19961057053301332 ds^2/dz^2 1.0428130696810065
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pz_1  145.057068926 pt_1  29.511488076151412 P_total  148.02864984148417
pz_2  147.652250556 pt_2  28.921258319216182 P_total  150.4580548758972
s_adv_1  61.01986602457969 ds_dz_1  0.20344743137744903
s_adv_2  48.7737197987723 ds_dz_2  0.1958741448931322 ds^2/dz^2 1.038664043630918
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pz_1  148.915226227 pt_1  29.117532994642175 P_total  151.73521453506928
pz_2  147.011718984 pt_2  28.53518233479332 P_total  149.7554745223248
s_adv_1  48.68825797088238 ds_dz_1  0.19553093214397127
s_adv_2  38.99373155775903 ds_dz_2  0.1941014126769545 ds^2/dz^2 1.007364807124799
```

Next Steps

- Currently I am matching and minimizing parameters between three consecutive circles.
- Alpha will change between stations
- Can get a distribution of alpha which should look like the Energy Loss distribution for going through tracker material
- Can then see if it changes/improves pz discrepancy, Energy Loss in cooling channel

- There are changes between runs. E.g. misalignments and movement. Need to consider for transfer matrix approach

Conclusion

- Transmission losses heavily bias cooling results as the particle distribution function of the remaining sample is heavily changed.
- Particle losses occur at both low and high density
- In the limit of full transmission, changes in the volume occupied have a smaller effect
- To eliminate the bias in the particle distribution function, will try to use a transfer matrix approach to approximate what the downstream particle distribution function look like. Can be tested in reverse on the Upstream sample.

- Transfer Matrix will have heavy correlations as in reality we only have x, y and z with everything else derived from there
- Need to ensure Momenta are correct to eliminate biases from the density calculations

THE END

Extra Slides

Trackpoints and Spacepoints

- Trackpoints are in a global reference frame
- Spacepoints are in a local reference frame
- Local coordinates are transformed to global coordinates by taking account of tracker misalignments
- **Residuals between local Spacepoints and Global Trackpoints should be straight lines of each tracker misalignment**

- Residual between Global Spacepoints and Global Trackpoints at each station should be random unless there is an inherent bias

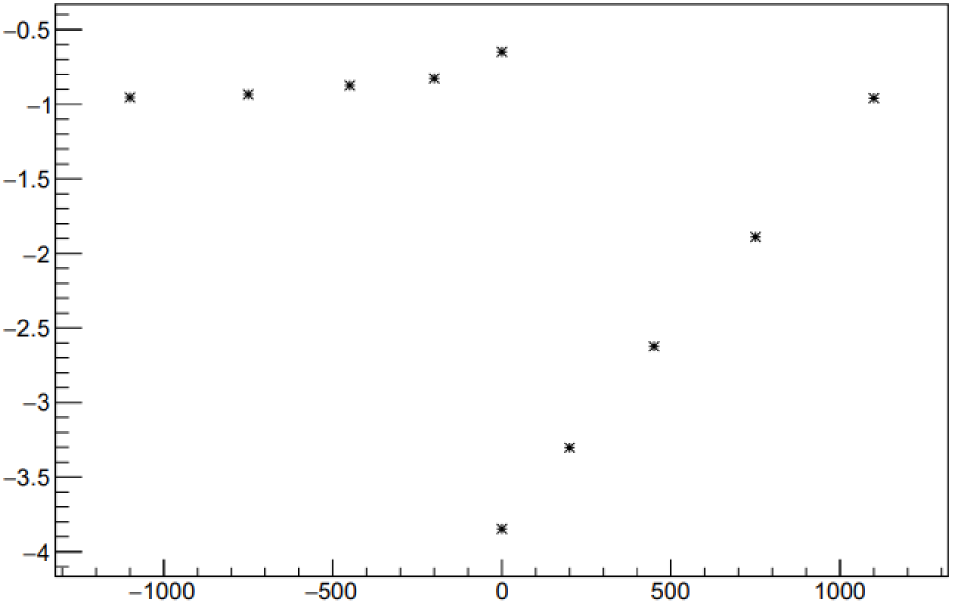
X Residual

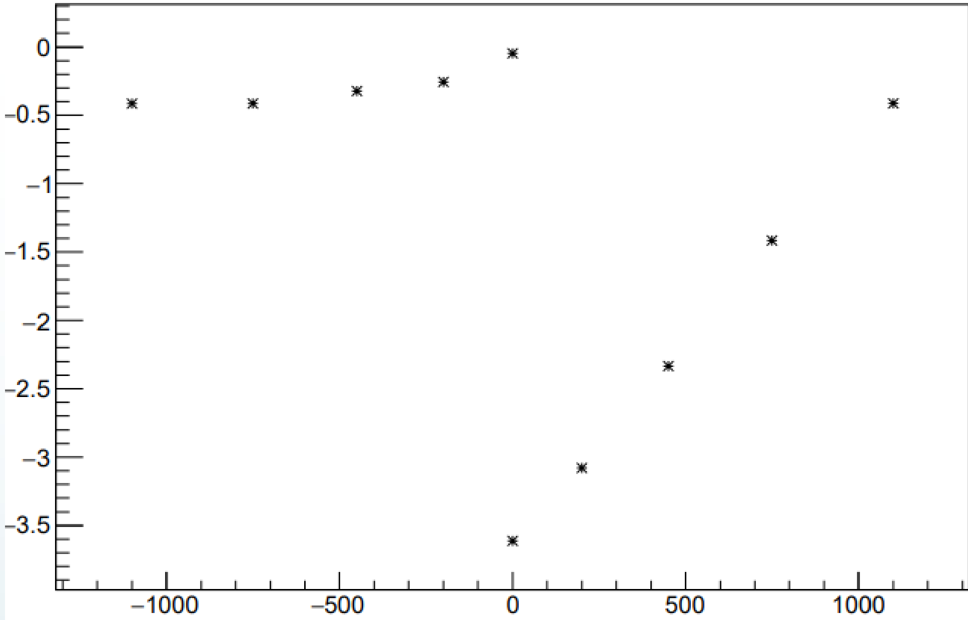
X Axis: Local Station Coordinates (mm)
Y Axis: Residual (mm)

Top Left: No absorber
Top Right: Wedge
Bottom Left: LiH
Bottom Right: LH2

X Residual is between Global X position Track point and local X position Space point

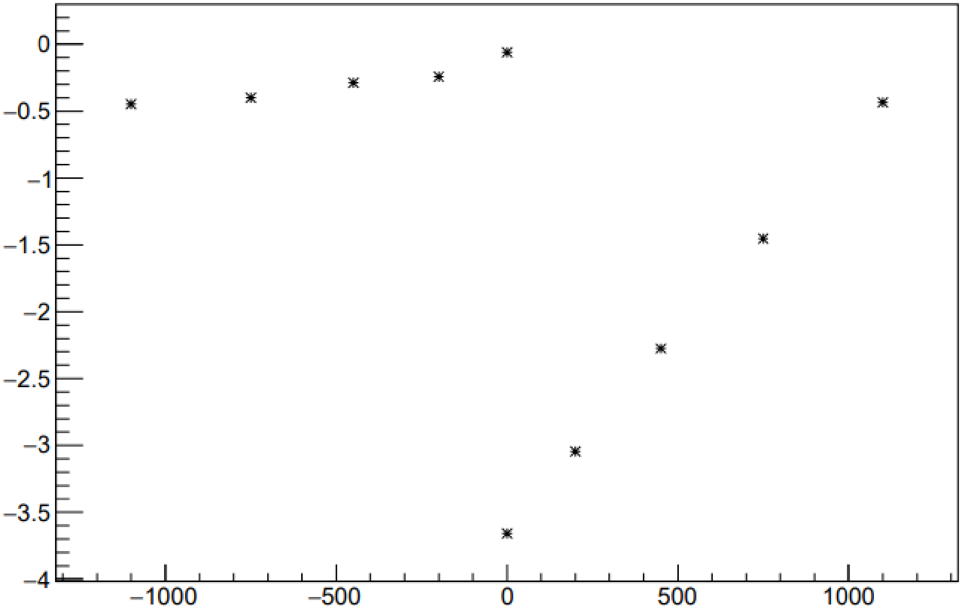
Residual should simply show the input Tracker misalignment. Should it be equal?

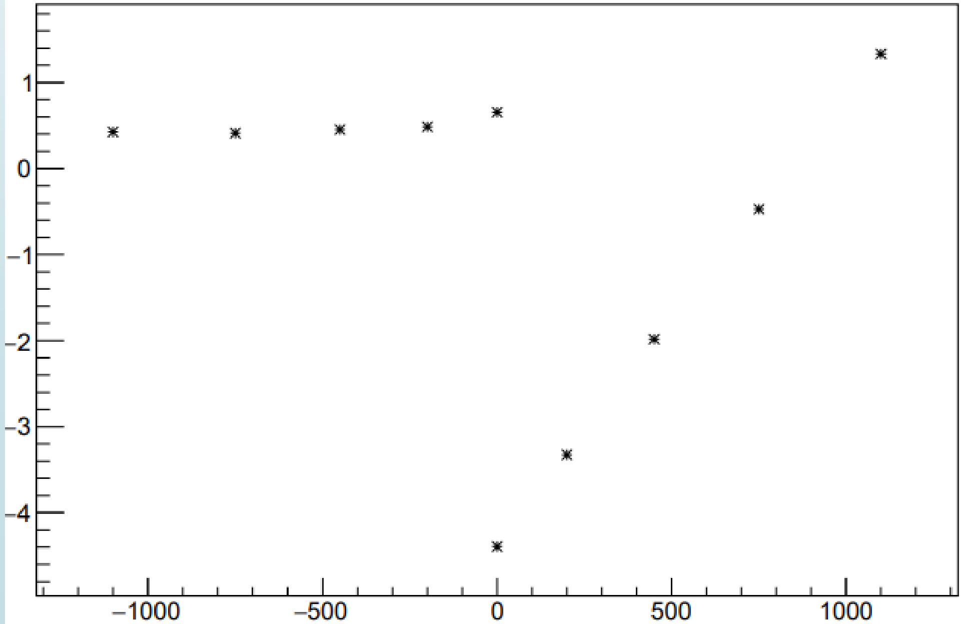




TKU TKD

TKU TKD





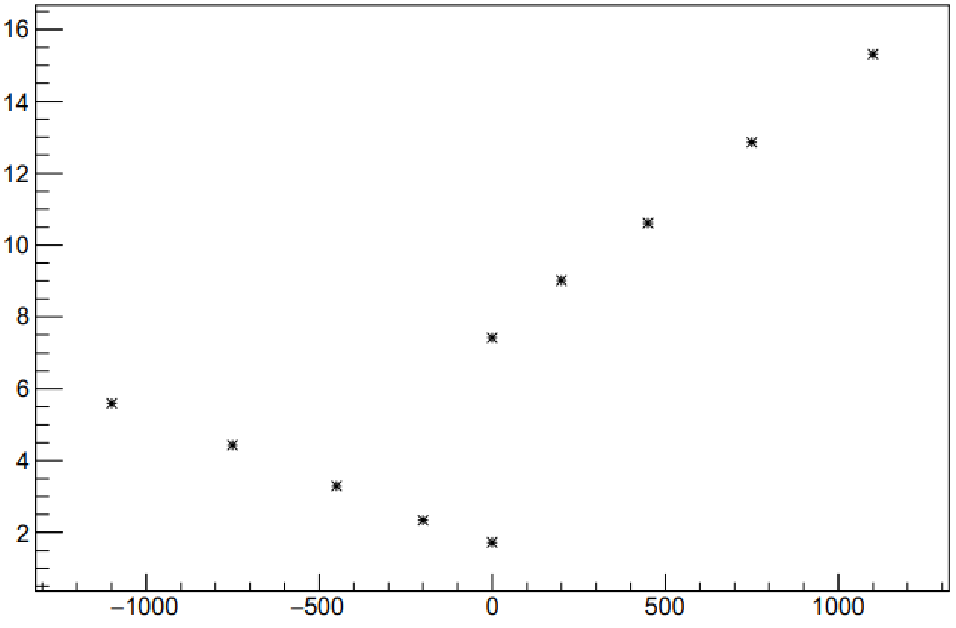
Y Residual

X Axis: Local Station Coordinates (mm)
Y Axis: Residual (mm)

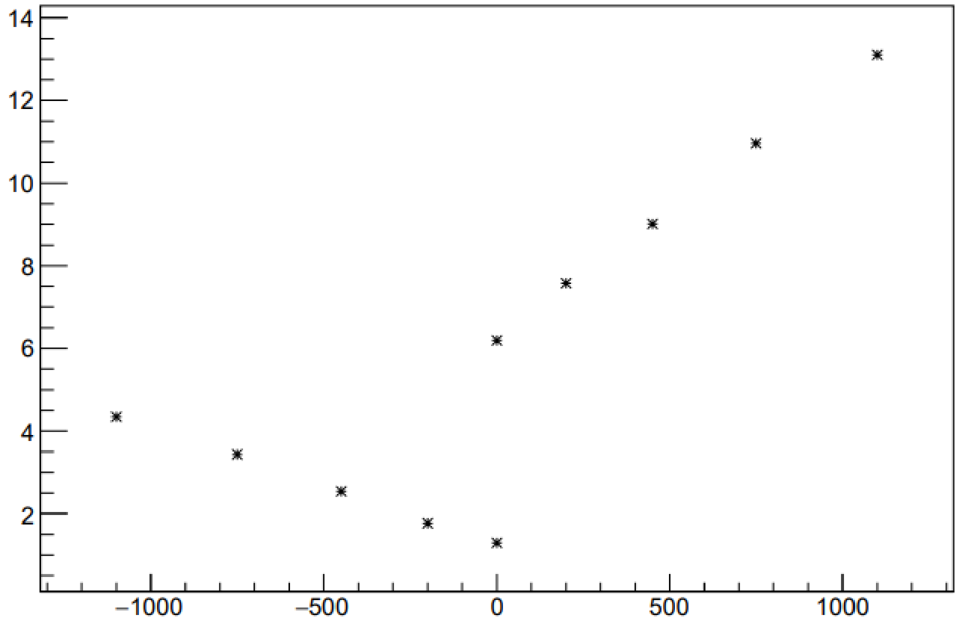
Top Left: No absorber
Top Right: Wedge
Bottom Left: LiH
Bottom Right: LH2

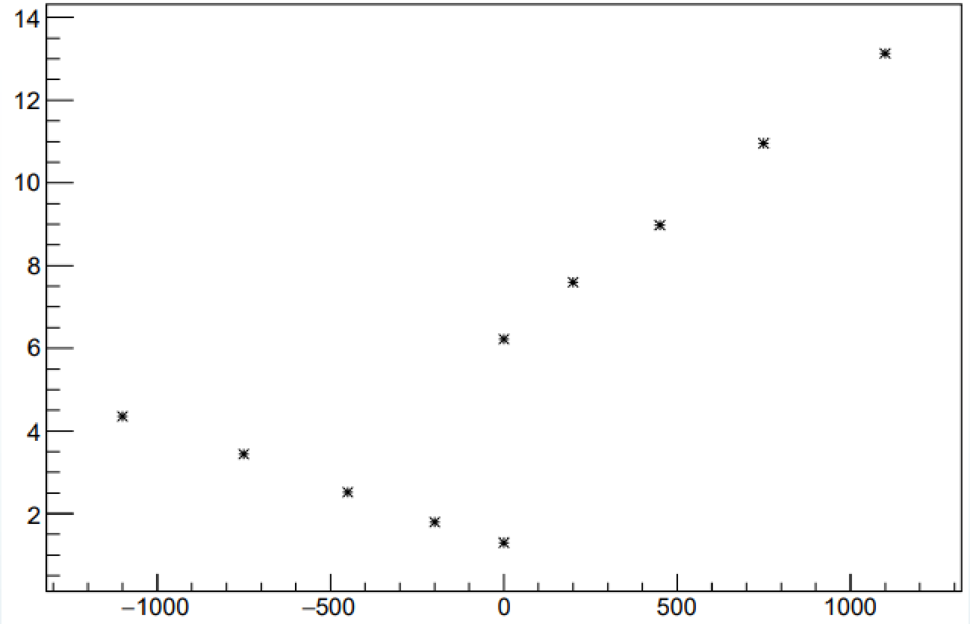
Y Residual is between Global Y position Track point and local Y position Space point

Residual should simply show the input Tracker misalignment. Should it be equal?

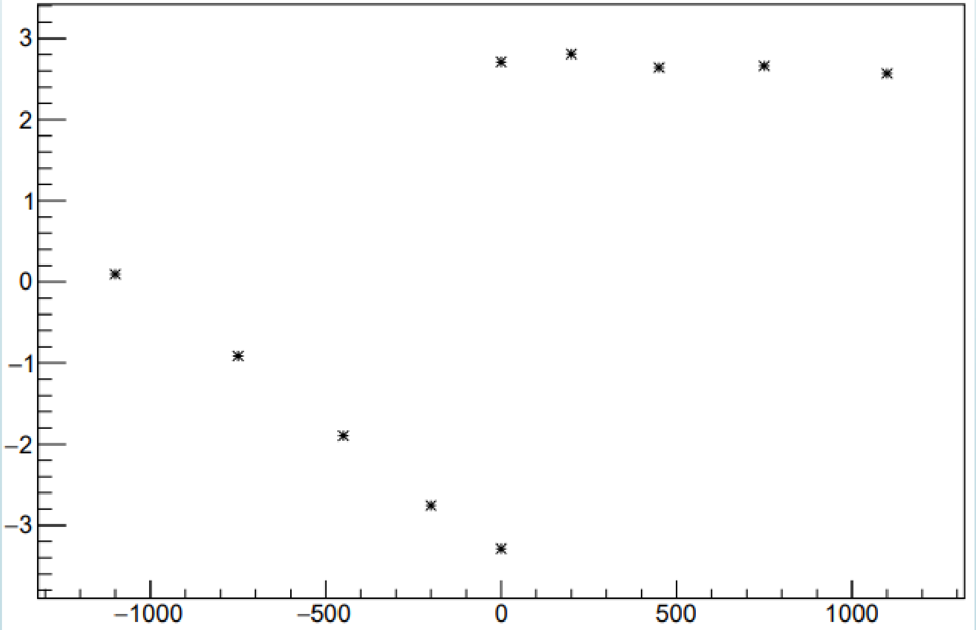


TKU TKD





TKU TKD



Emittance in Experiments

- Emittance measurements can be biased
- The scraping of the beam on the aperture can give a false cooling effect
- Non-linearities can give rise to a false heating effect. The emittance of the beam has increased due to the non-linearities but the phase space volume hasn't changed size
- To see cooling, one can look at the change in phase-space volume or the change in density of that volume before and after it has gone through some material

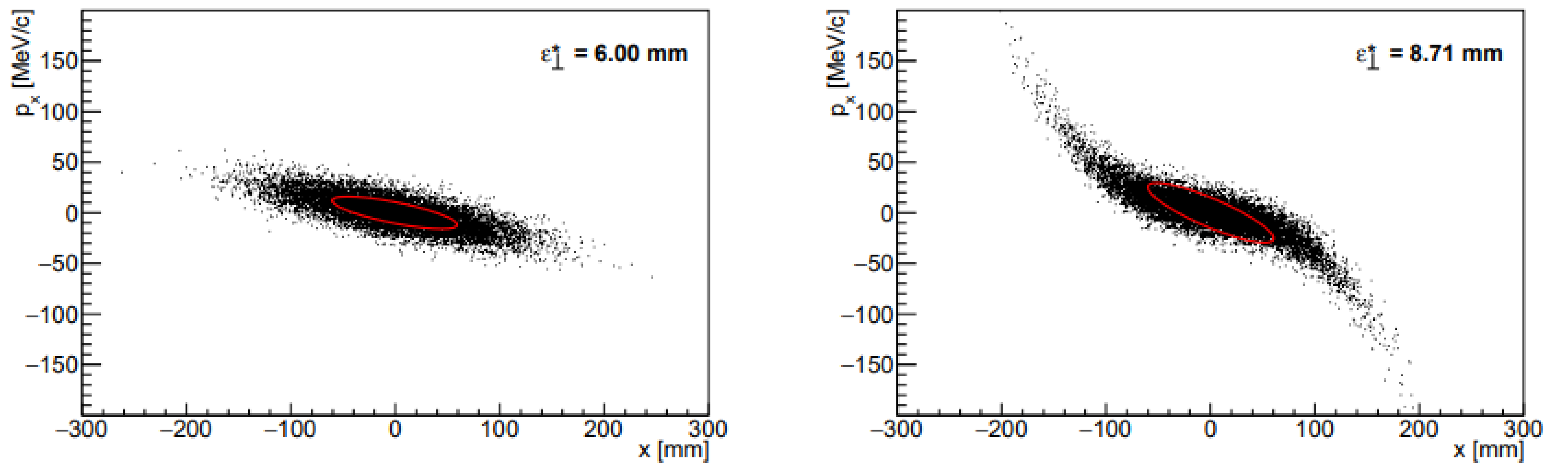
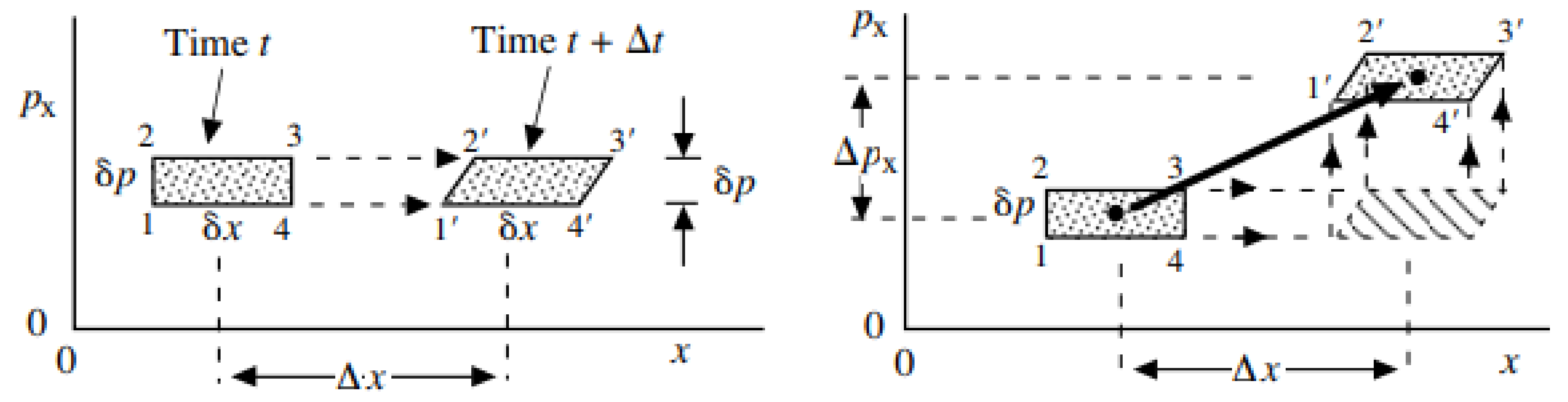


Figure 6.6: Scatter plot of a beam ($\epsilon_i = 6\,\mathrm{mm}$, $\langle p_z \rangle = 140\,\mathrm{MeV}/c$ and $\beta_\perp = 800\,\mathrm{mm}$) after transport through a linear focusing lens of $f = 5\,\mathrm{mm}^{-1}$ (left) and a similar nonlinear lens with $C_\alpha = 10^{-4}\,\mathrm{mm}^{-2}$ (right). The red curve is the RMS ellipse.

Phase Space Volume and Density

- Take an arbitrary phase space volume upstream of the absorber and count the number of particles in that volume. Take the same volume downstream and count the number of particles in that volume. If it has changed then heating or cooling has taken place
- The problem is what does that phase space volume actually look like downstream as it has changed in shape due to differing momenta of particles in the beam and the magnetic forces of the cooling channel
- Transmission losses also need to be accounted for in an unbiased way



Liouville’s theorem

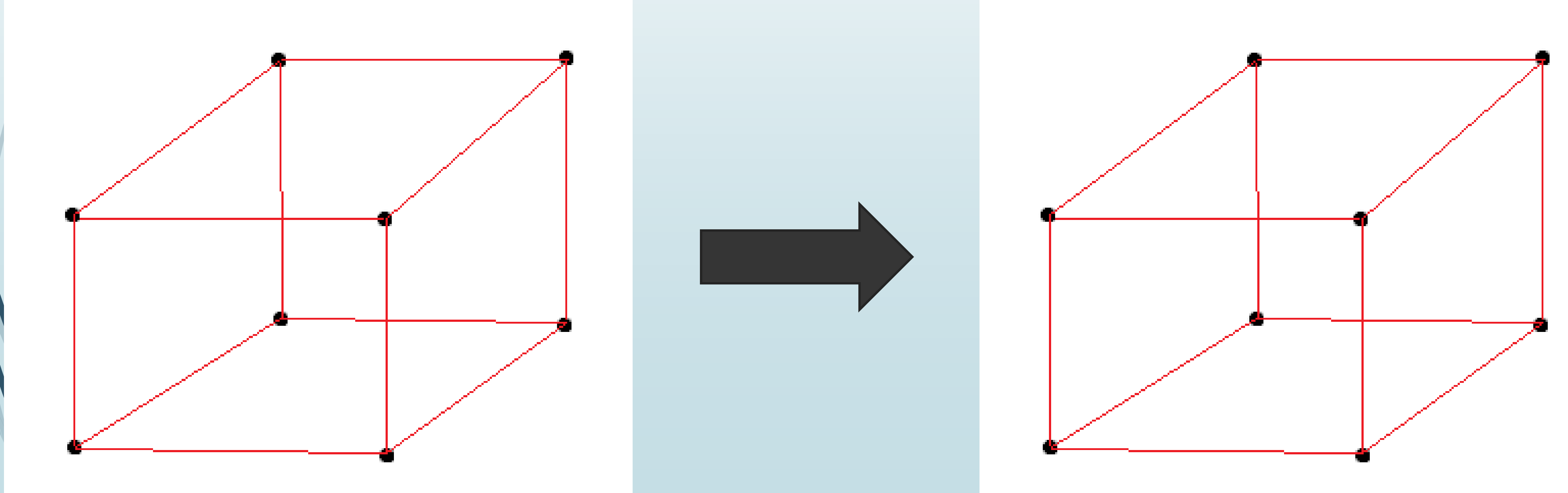
- A particle beam can be described by the distribution of the particles in the beam also known as the phase space density $\rho(x, y, z, p_x, p_y, p_z)$.
- Liouville's theorem states that the density of particles in phase space is a constant i.e. ${}^{d\rho}/_{dt} = 0$ (providing there are no dissipative forces)
- The number of particles in a phase-space volume is then given by:

$$N = \int \rho\left(x, y, z, p_x, p_y, p_z\right) dx dy dz dp_x dp_y dp_z = \int \rho dV$$

- The phase-space density is directly related to the phase space volume
- The phase-space density can be calculated in a number of ways using density estimation techniques such as Kernel Density Estimation (KDE), the k-Nearest Neighbour Approach (KNN) plus many more
- Phase Space Density Estimation is a non-parametric technique to estimate the underlying probability density, the probability that a particle will be realized at a particular phase space density

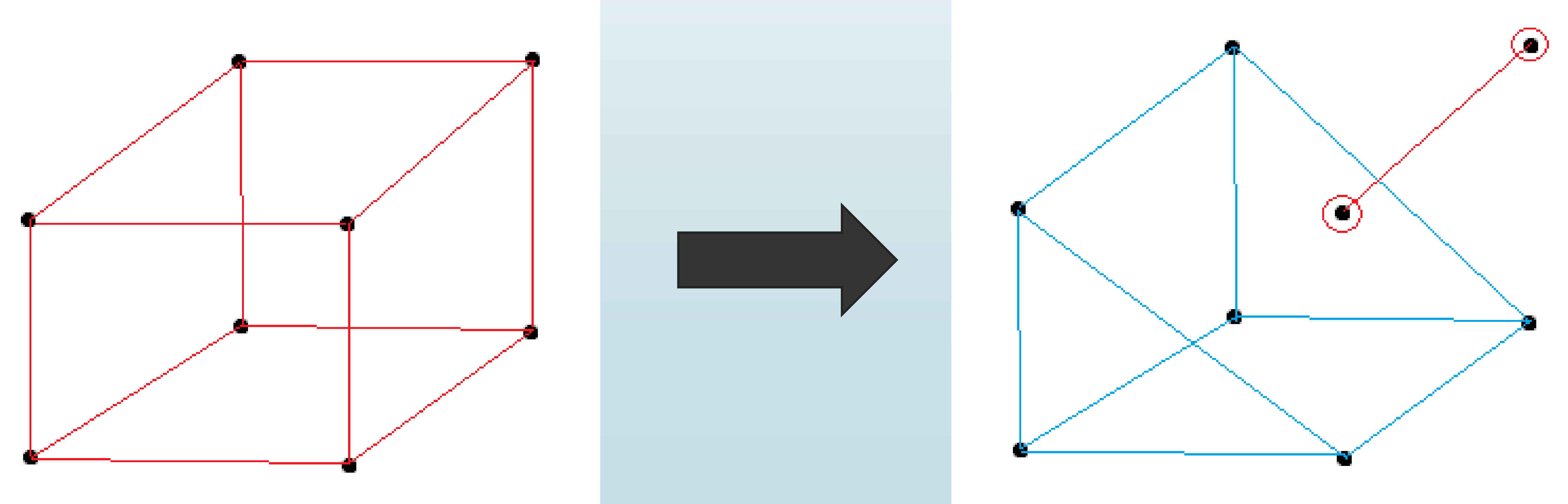
Transmission effects – extreme example

- Imagine phase space distribution given by 8 points arranged in a cube separated by a 1 unit distance, giving a 1 unit volume.
- The system is sent through a magnetic system with no dissipative forces. The points may have changed location, but the 1 unit volume is preserved.



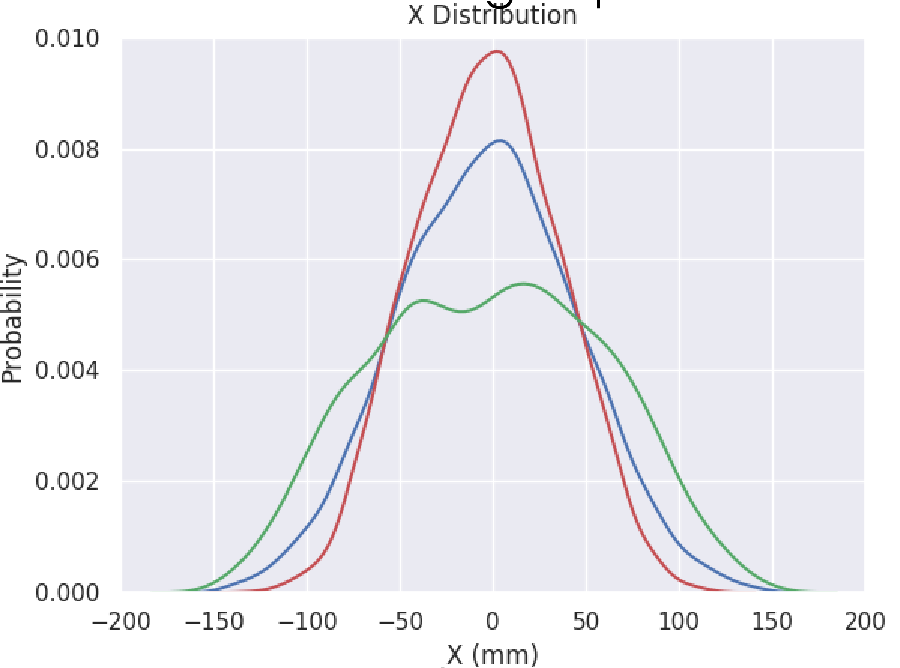
Transmission effects – extreme example

- The eight particles are again put through a magnetic system which has an aperture (acts as a dissipative force), resulting in a loss of two particles.
- The volume of the remaining 6 particles is 0.5 unit volume.
- If one were to normalize the downstream sample by the sample size, one would artificially increase the density (which is wrong). For transmission losses, the change is particle distribution is important.

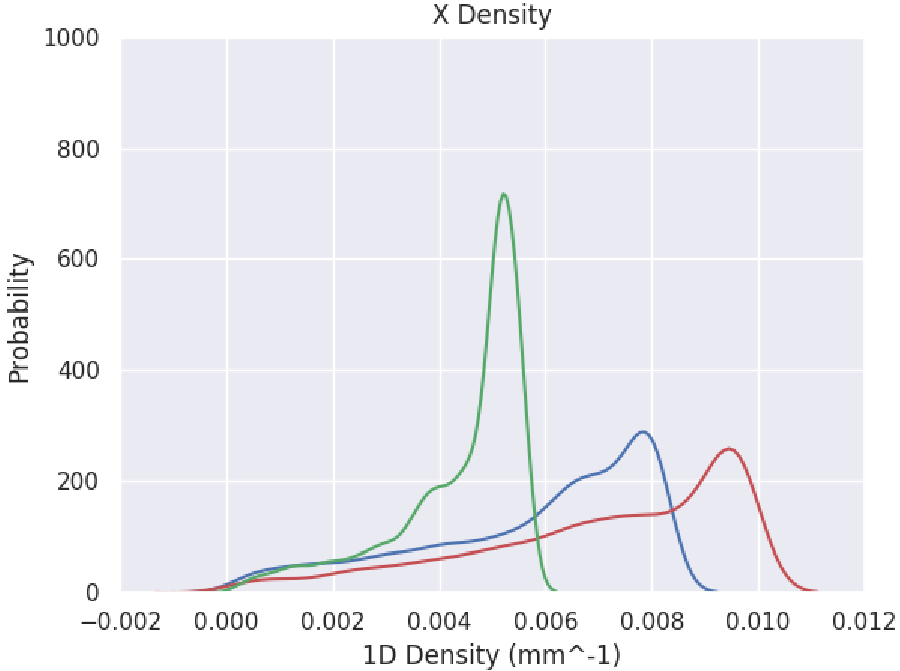


No Wedge (left) and Wedge (right)

No Wedge Upstream



No Wedge Upstream



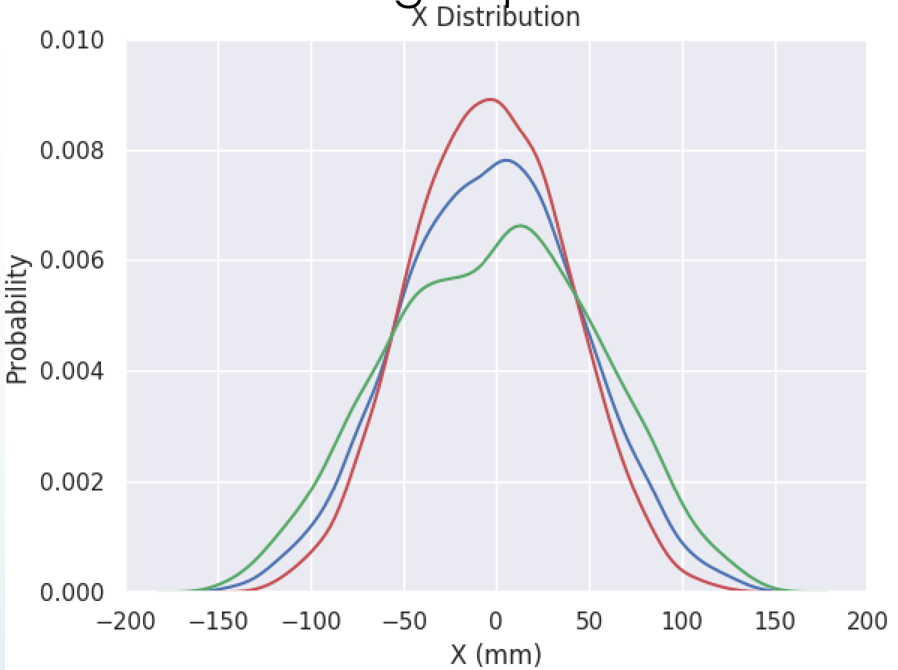
X Distribution (Top) and Density (Bottom)

Blue – Full Upstream Sample
Red – Upstream Sample which makes it Downstream
Green – Upstream Sample which does not make it Downstream

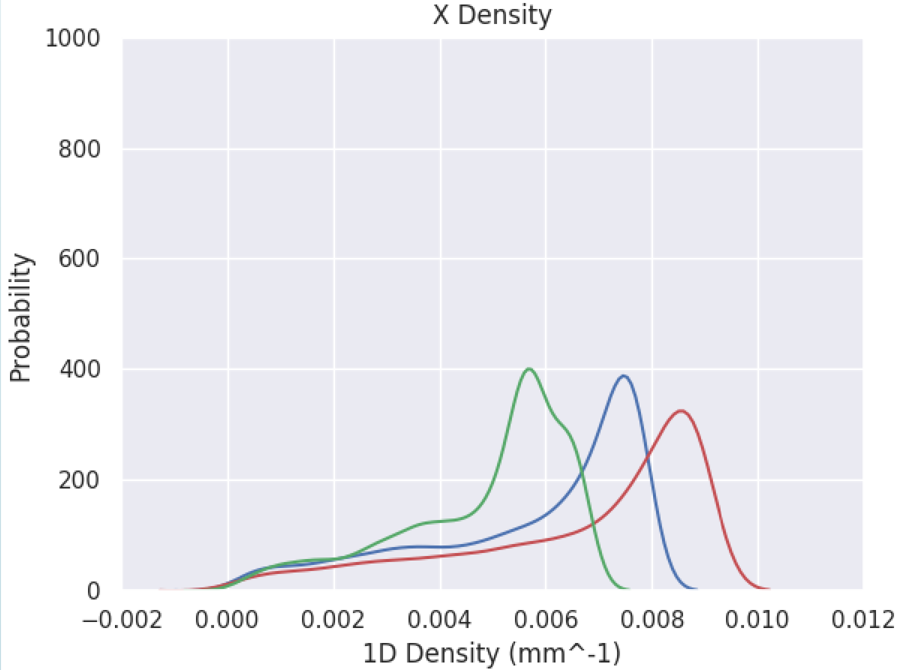
Small preference for larger magnitude x not to make it downstream

Wedge case shows slight directional bias as well. The Wedge does not transmit up to 15% of particles that would have made it downstream otherwise.

Wedge Upstream



Wedge Upstream



Tanaz (left) vs Francois (right)
6-140 LiH analysis

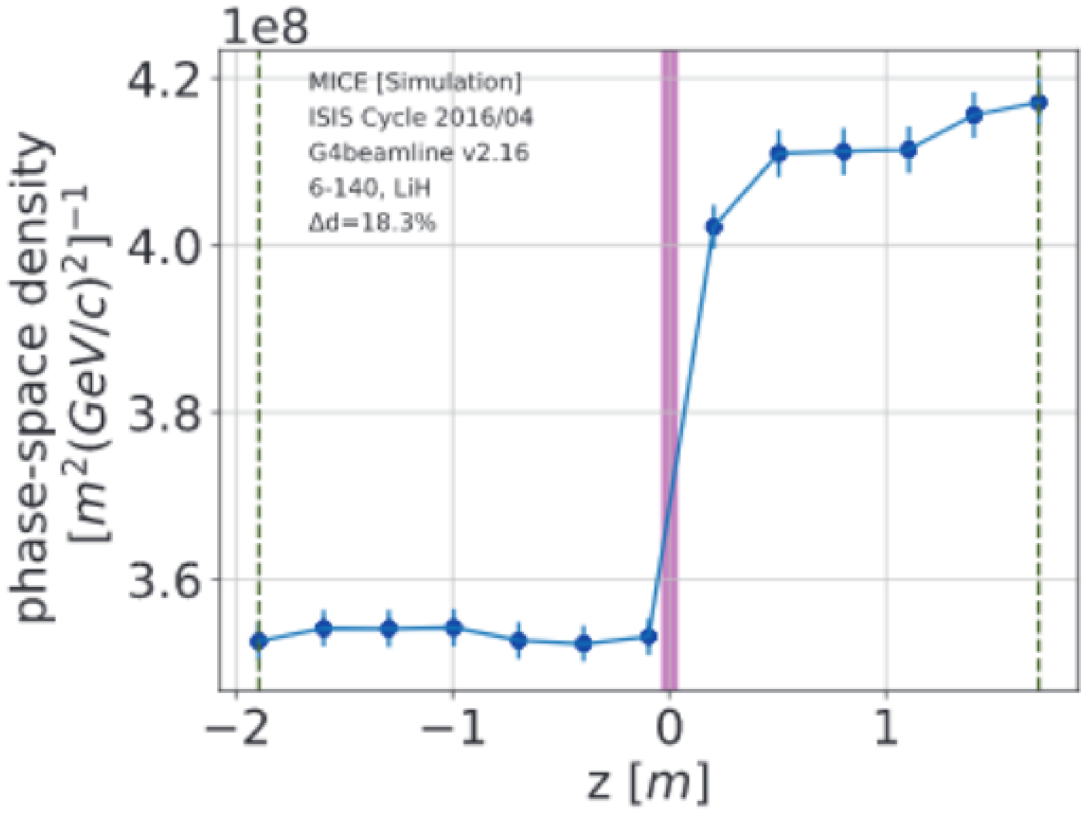
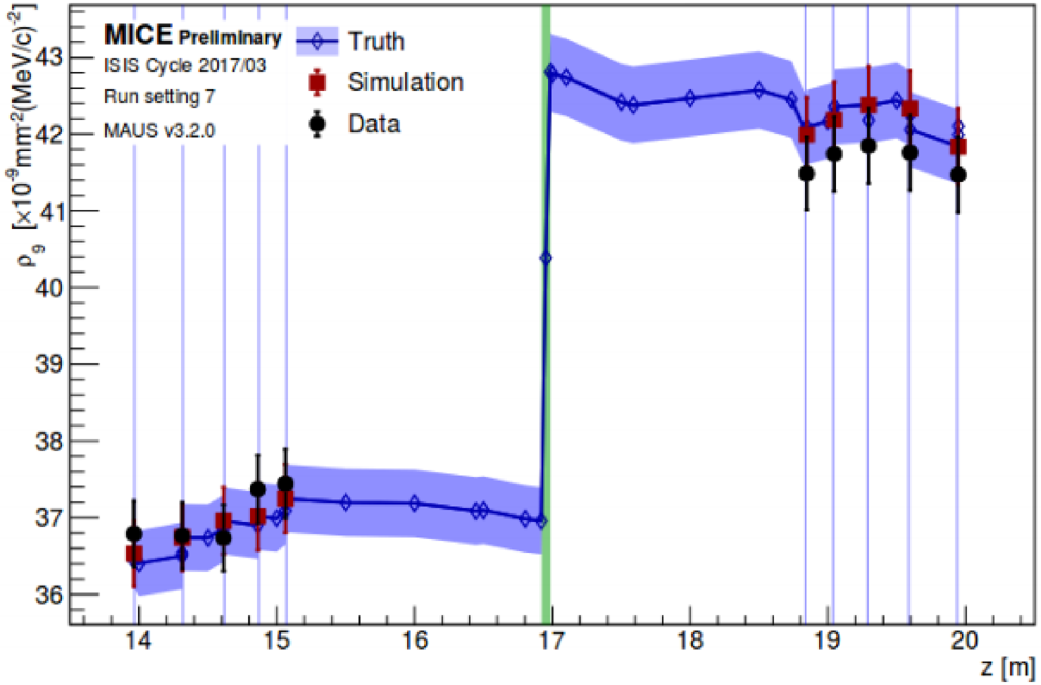


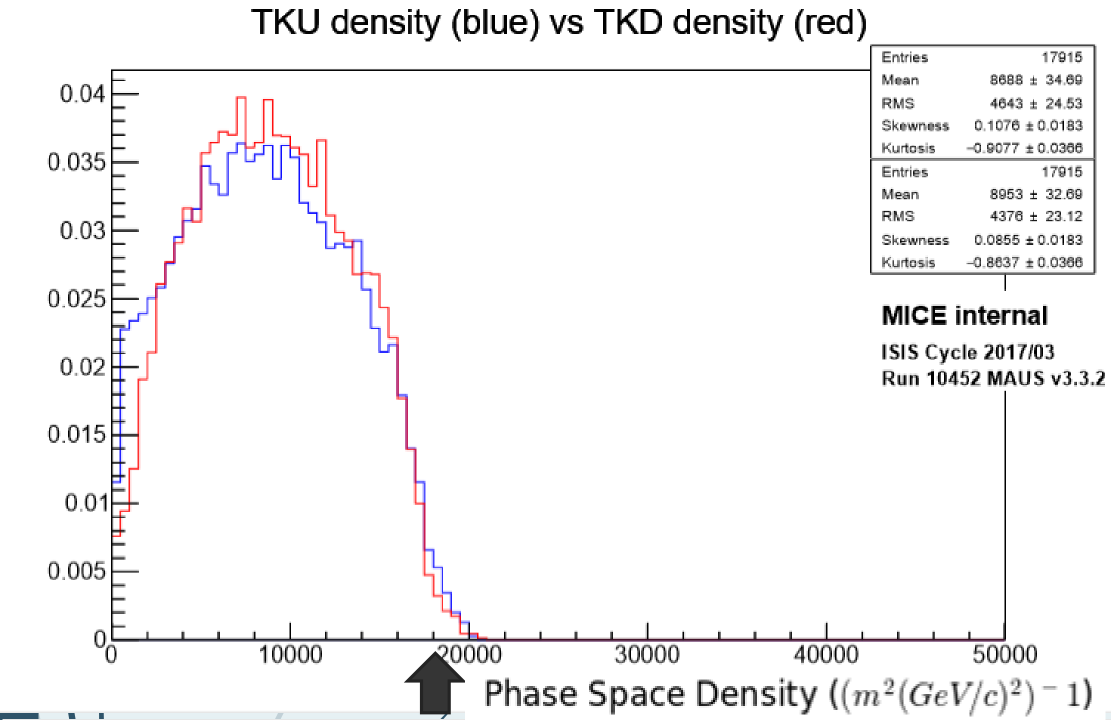
Figure 4: Evolution of the core phase-space density for the 6 – 140 beam setting.

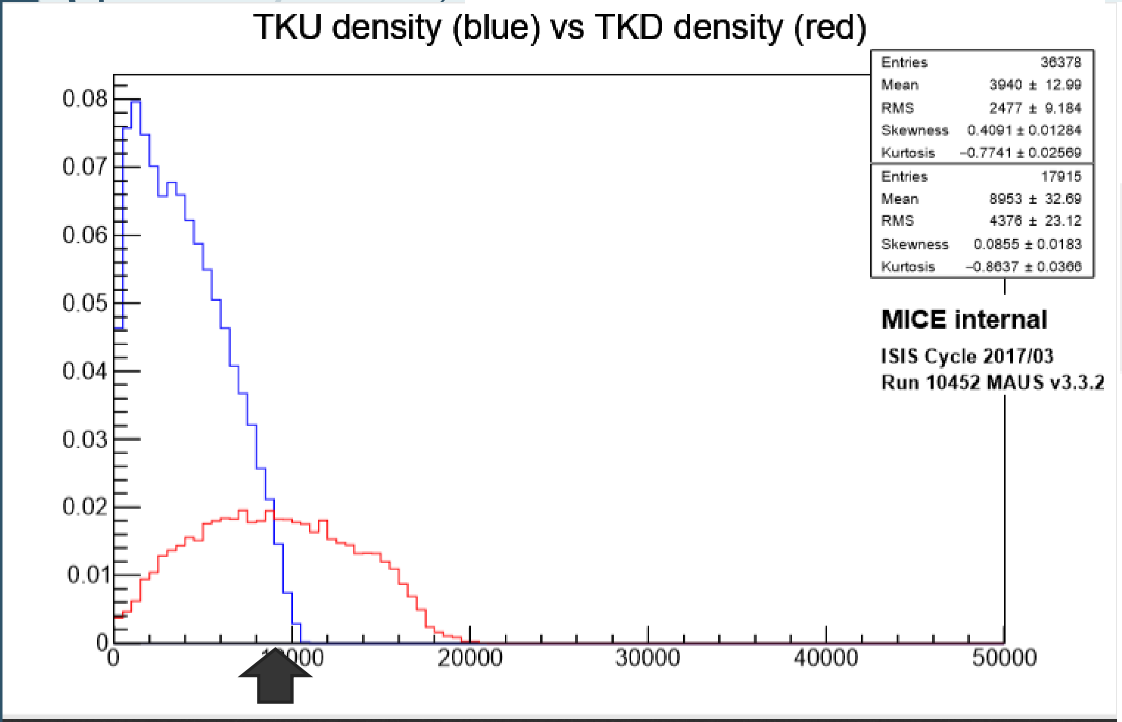
(9%) Contour density evolution (kNN)
6-mm 140-MeV/c beam – LiH – flip

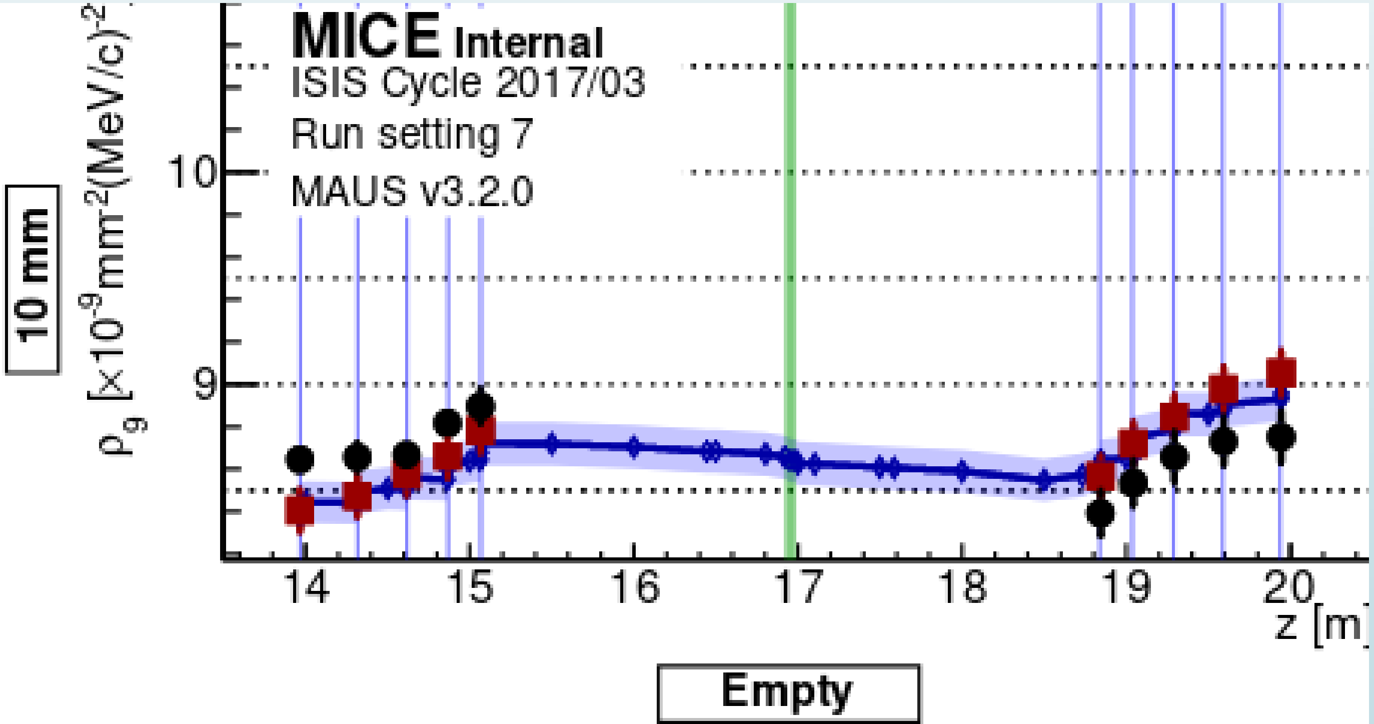


Me (left) vs Francois (right) 10-140 No absorber

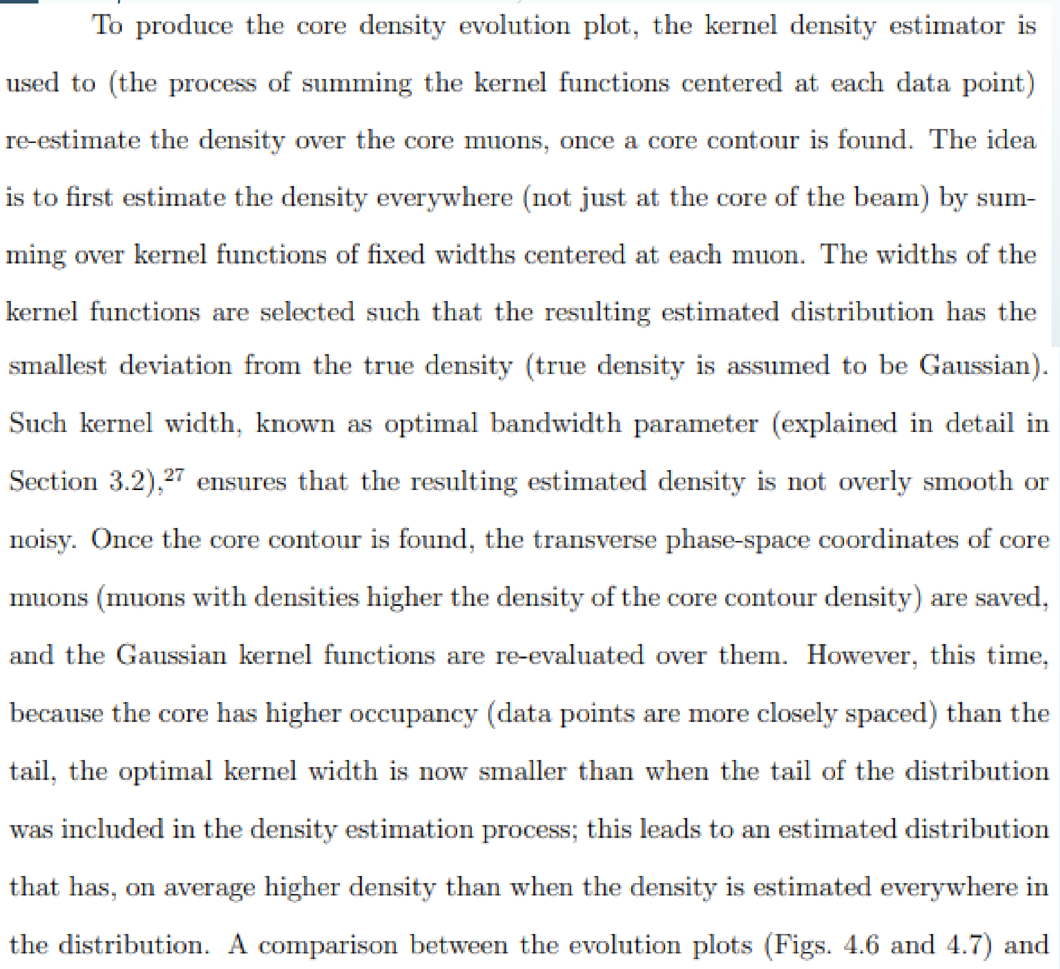
- Bottom Right: Change in density through cooling channel
- Top left: Upstream (blue) which made it downstream (red) at reference planes (100% Transmission, biased sample)
- Bottom left: Full Upstream sample (blue) vs downstream (red) (Unbiased Upstream sample, ~50-60% Transmission)







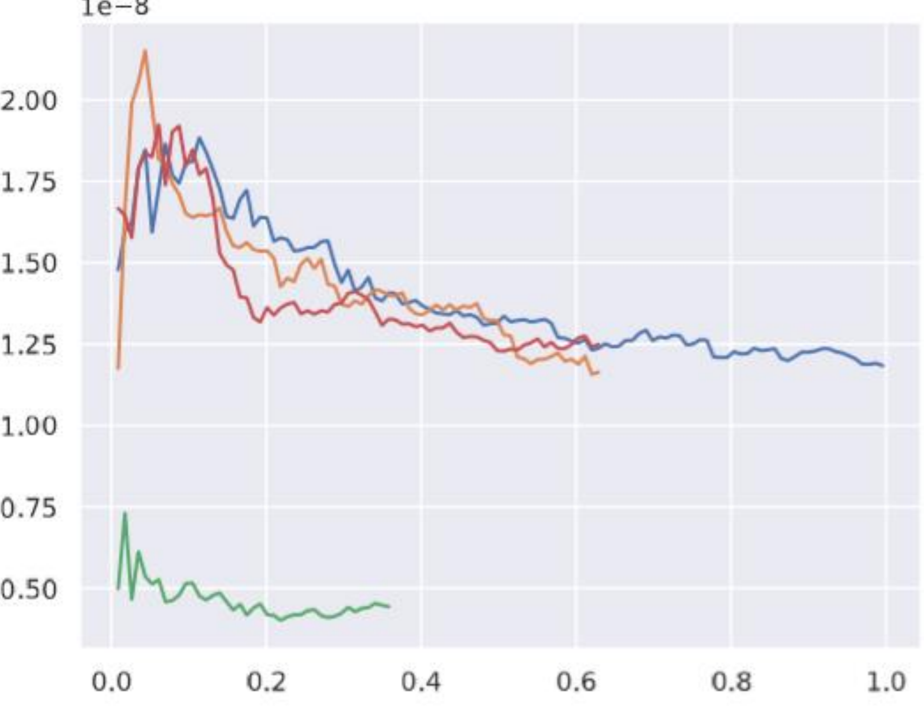
Tanaz and Francois analysis (why the numbers don't match)

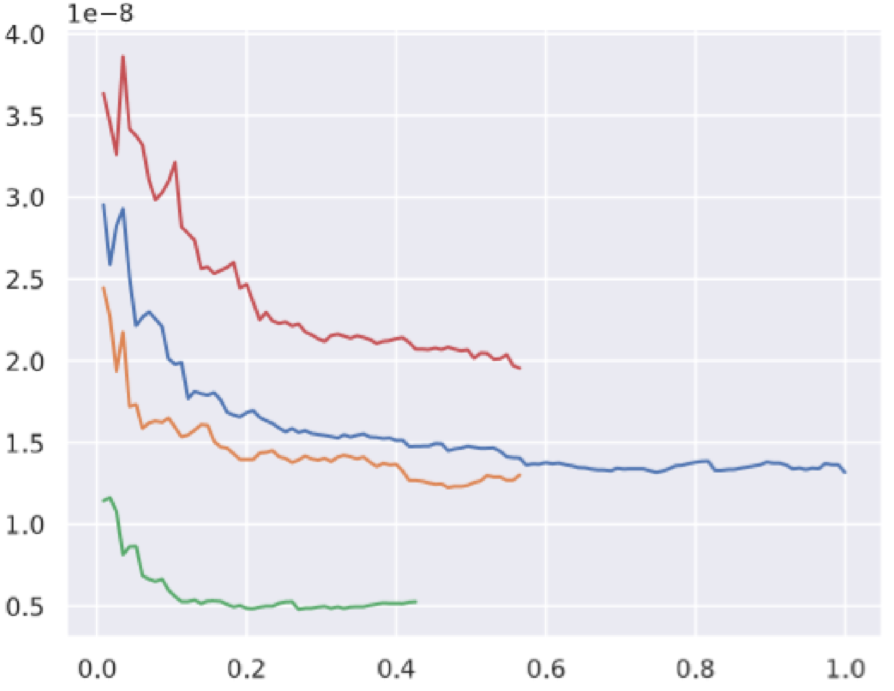


To produce the core density evolution plot, the kernel density estimator is used to (the process of summing the kernel functions centered at each data point) re-estimate the density over the core muons, once a core contour is found. The idea is to first estimate the density everywhere (not just at the core of the beam) by summing over kernel functions of fixed widths centered at each muon. The widths of the kernel functions are selected such that the resulting estimated distribution has the smallest deviation from the true density (true density is assumed to be Gaussian). Such kernel width, known as optimal bandwidth parameter (explained in detail in Section 3.2),[27] ensures that the resulting estimated density is not overly smooth or noisy. Once the core contour is found, the transverse phase-space coordinates of core muons (muons with densities higher the density of the core contour density) are saved, and the Gaussian kernel functions are re-evaluated over them. However, this time, because the core has higher occupancy (data points are more closely spaced) than the tail, the optimal kernel width is now smaller than when the tail of the distribution was included in the density estimation process; this leads to an estimated distribution that has, on average higher density than when the density is estimated everywhere in the distribution. A comparison between the evolution plots (Figs. 4.6 and 4.7) and

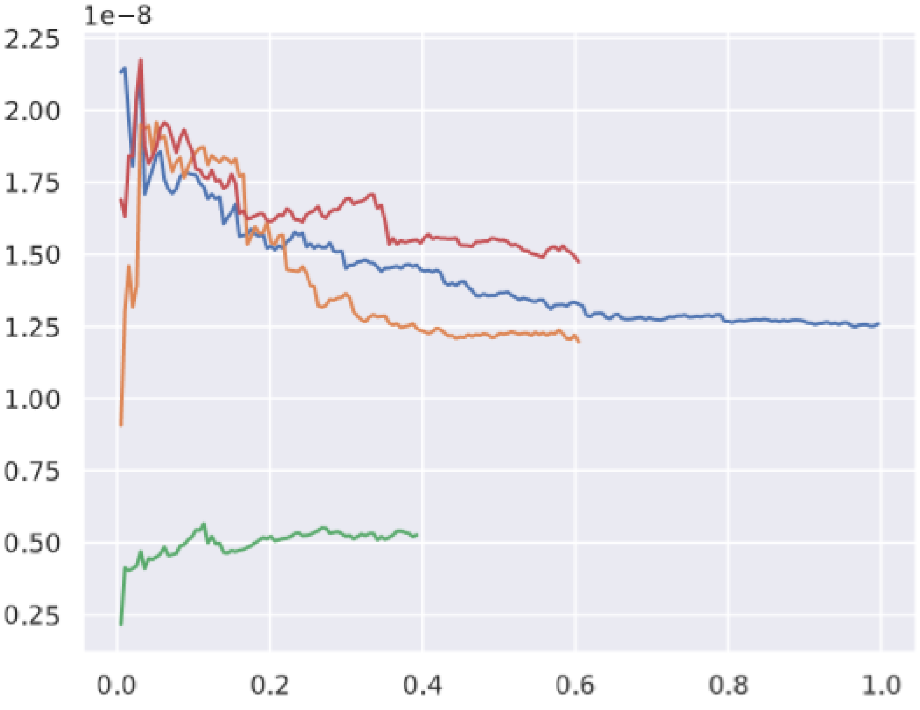
- I had agreement with Francois, difference with Tanaz
- Accounting for change in units, factor of 10,000 difference
- Tanaz and Francois results look similar bar the 10,000 difference, however, she actually does calculate the density differently:
- Tanaz finds the 9% core and isolates those particles. From those particles she recalculates the density with the remaining sample. This has changed the particle distribution, as well as the volume over which it has been calculated.
- Isolating the core can be advantageous to aid with transmission, however it appears the 9-th percentile density is calculated on the 9% core.
- ~10% for each of four dimensions would give a factor of ~10,000
- Effectively < 1% of particles are chosen, which can result in significant statistical fluctuations
- It also doesn't deal with transmission losses and if the same particles are being compared

Change in Peak density vs beam fraction

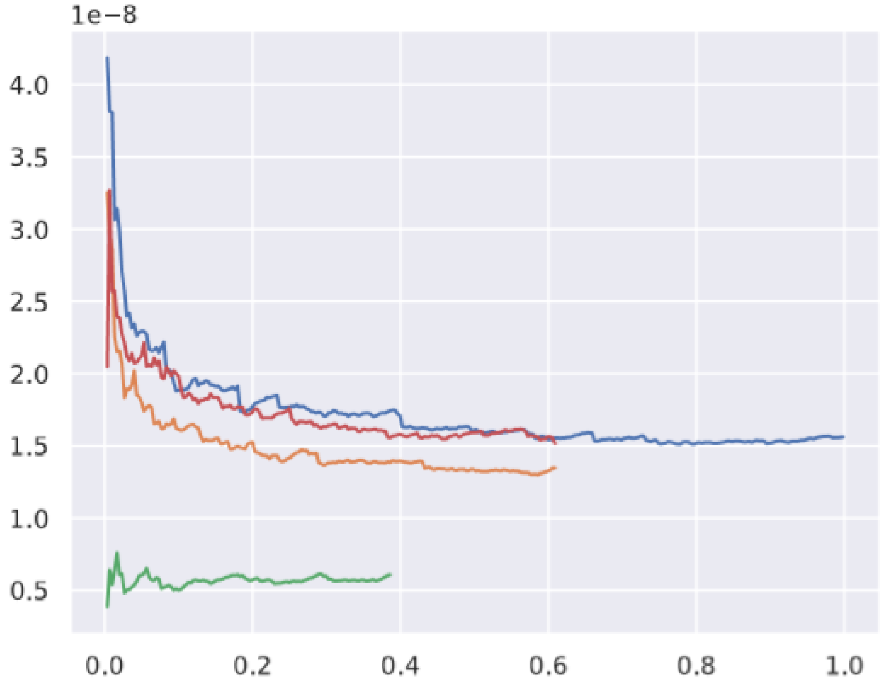


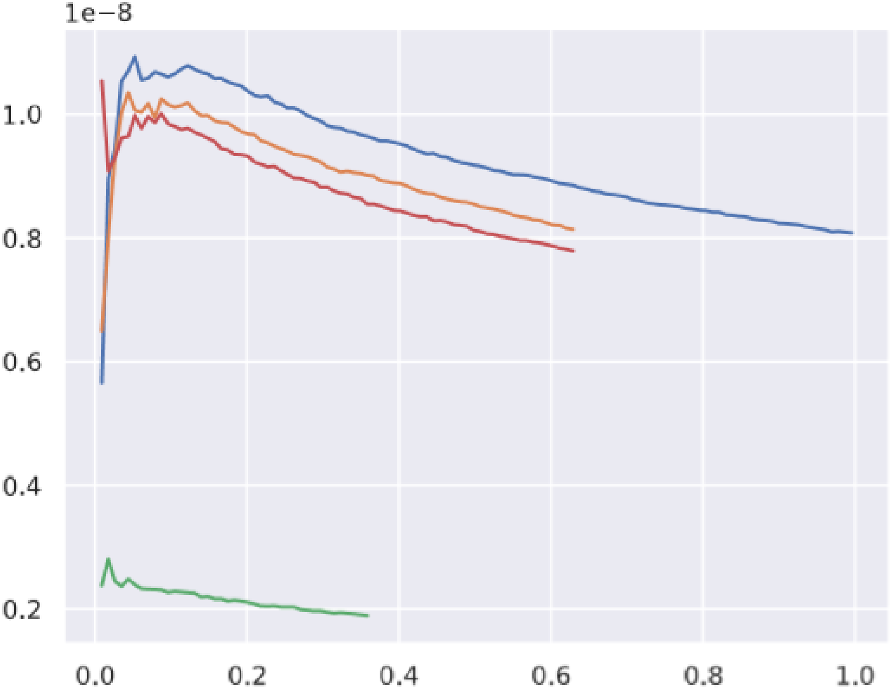


Top Left: No absorber
Top Right: Wedge
Bottom Left: LiH
Bottom Right: LH2

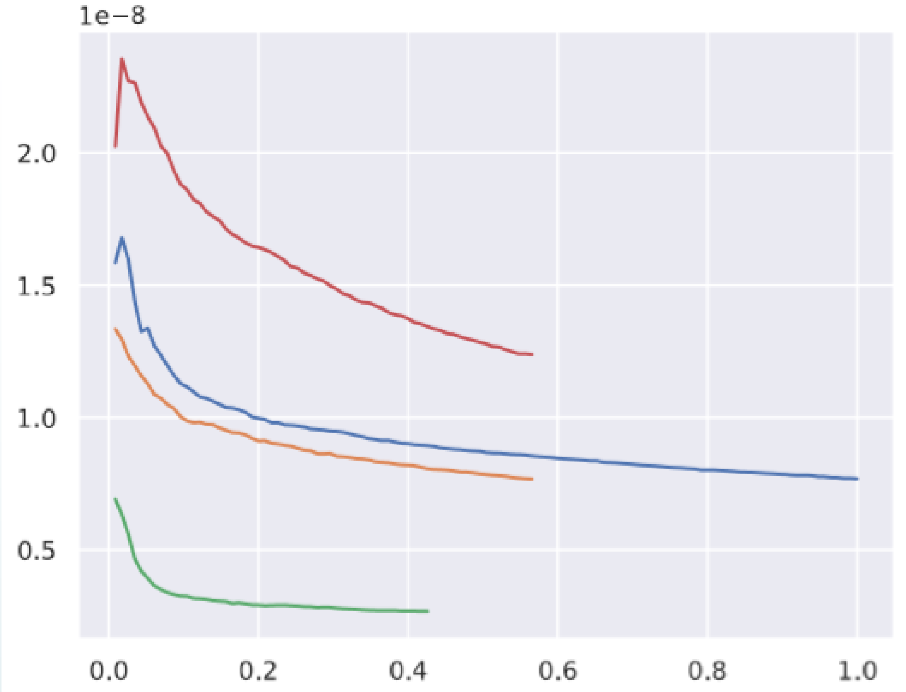


Blue – Full Upstream Sample
Red – Full Downstream Sample
Orange – Upstream Sample which made it Downstream
Green – Upstream Sample which doesn’t make it downstream

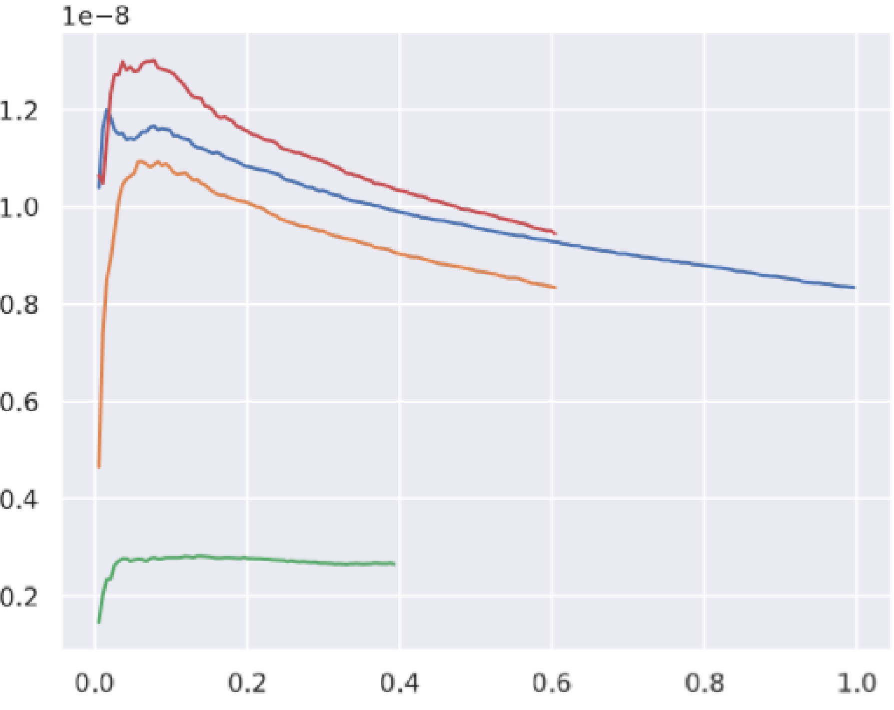




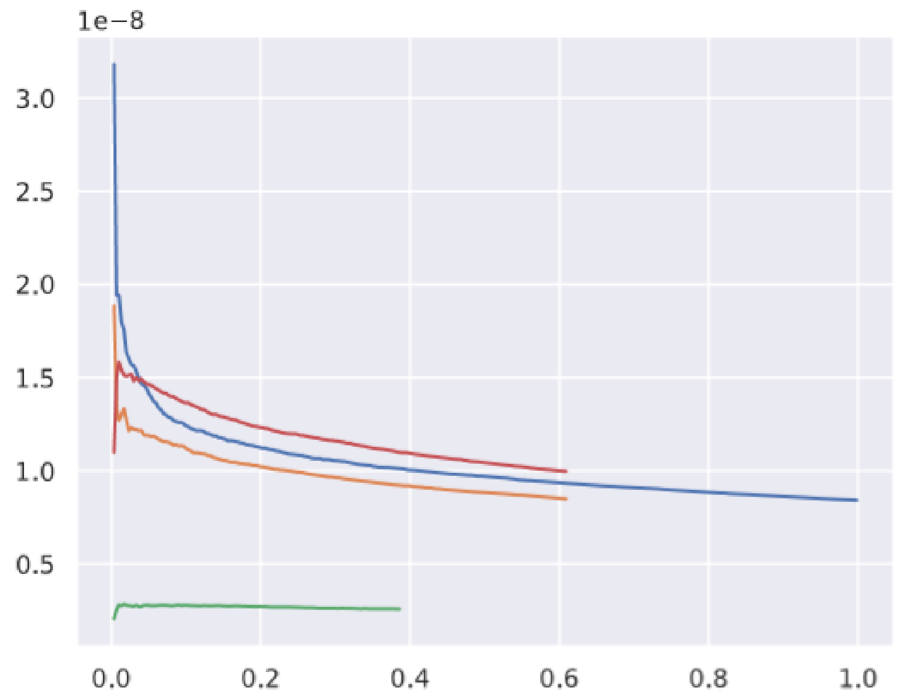
Change in 9th percentile density vs beam fraction



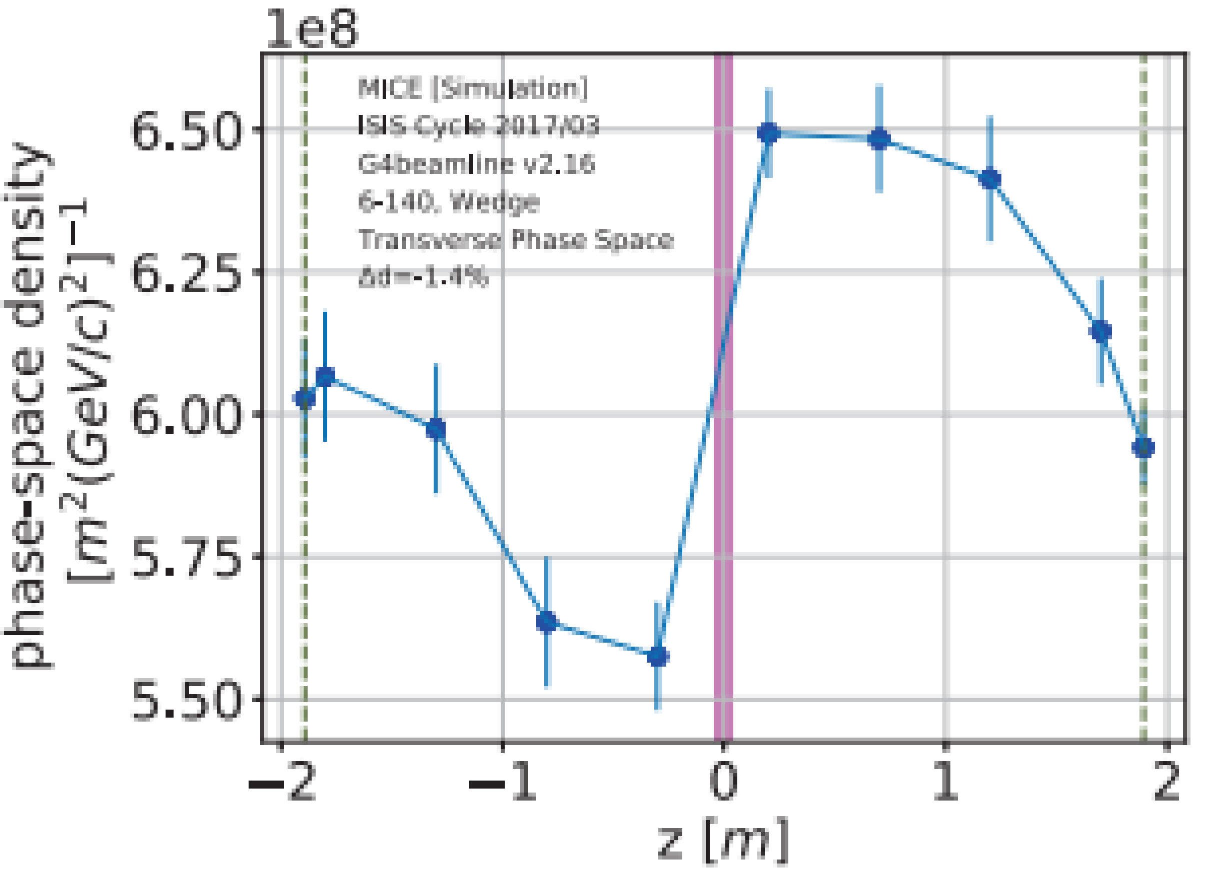
Top Left: No absorber
Top Right: Wedge
Bottom Left: LiH
Bottom Right: LH2



Blue – Full Upstream Sample
Red – Full Downstream Sample
Orange – Upstream Sample which made it Downstream
Green – Upstream Sample which doesn’t make it downstream



Tanaz’s 6-140 transverse 4D results – IPAC2018



- Tanaz 6-140 Wedge plot
- Analysis is based on comparing the reference planes where it claims a decrease in density.
- Liouville – change in density only through dissipative forces, therefore change in density should only occur across the absorber (the wedge in this case)
- Before and after the density should remain constant (for the case where transverse components can be isolated from the longitudinal components)
- However a change is seen (something has gone wrong)
- Either the transmission losses are heavily biasing the results, or the statistical errors of choosing too small a sample size haven’t been accounted for.
- In either case, Emittance Exchange can’t be claimed here

Not only low density particles are eliminated

Blue – Full Upstream Sample
Orange – Upstream Sample which makes it Downstream
Green – Upstream Sample which doesn't make it Downstream

The full upstream distribution (blue) can be divided into the upstream distribution which makes it downstream (orange) and upstream distribution which doesn't make it downstream (green) calculated over the full Upstream distribution volume.