Resummation in `artemide`

LHC EW precision sub-group meeting (pT W/Z benchmarking)

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Introduction

`ARTEMIDE`
is made to explore **TMD factorization** and **TMD distributions**.

Specific points

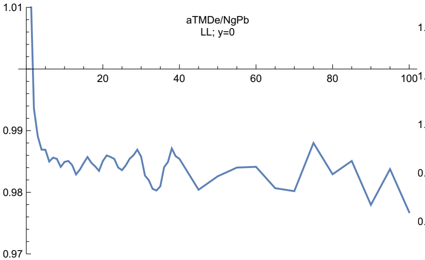
- Variety of principle processes: DY, SIDIS, SIDIS→jet (*ee*-annihilation to be added)
- Variety of sub-subprocesses: targets, polarizations, etc. (constantly updating list)
- Special emphasis of non-perturbative structure → full flexibly to define a model (b^* different kind of locality, f_{NP}'s of arbitrary form, RAD of arbitrary form, etc.)
- Modular structure
- Perturbative parts up to best known
- ζ-prescription as a main universality concept

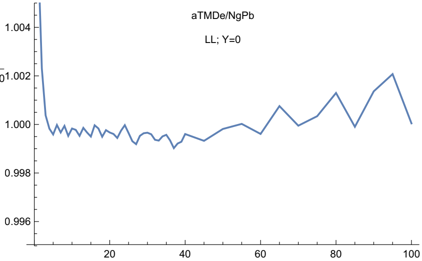
`artemide` is also exploration code (all switchable)

- Different resummation versions
- Power corrections: target mass corrections, kinematic corrections
- Large-x resummation (in progress)
- PDF variations
- ...

On earlier discrepancies

- Mistake: wrong type of resummation has been used (see next slides)
- Bug: There was a bug in the code for α_{QED} (incorrect determination of boundary condition $\to -1.5\%$ of common normalization) FIXED!





Resummation formalisms

$$\left(\frac{d\sigma}{dq_T}\right)_{\text{res.}} \overset{q_T-\text{res.}}{\underset{\text{PB}}{\propto}} e^{2S}\left[f_1 \otimes \mathcal{H} \otimes f_2\right]$$

$$\overset{\text{TMD}}{\propto} \boxed{H \times F_1 \times F_2} + \mathcal{O}\left[\left(\frac{q_T}{Q}\right)^m\right]$$

$$\overset{\text{SCET}}{\propto} H \times B_1 \times B_2 \times S$$

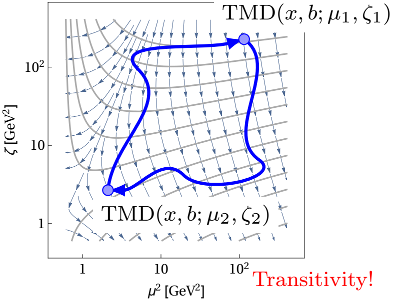
$$\boxed{H \times F_1 \times R \times F_2}$$

three non-perturbative functions

- Each non-peturbative function has its own definition in terms of operators
- Each non-peturbative function responsible for different degree of freedom

$$\mu^2 \frac{dF(x,b;\mu,\zeta)}{d\mu^2} = \frac{\gamma_F(\mu,\zeta)}{2} F(x,b;\mu,\zeta), \qquad \zeta \frac{dF(x,b;\mu,\zeta)}{d\zeta} = -\mathcal{D}(\mu,b) F(x,b;\mu,\zeta)$$

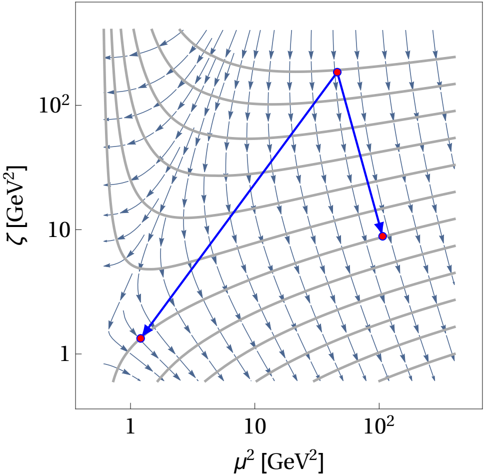
2D evolution



Transitivity!

$$R[(\mu_1,\zeta_1) \to (\mu_2,\zeta_2)] = \exp \int_P \left(\frac{d\mu}{\mu} \gamma_F - \frac{d\zeta}{\zeta} \mathcal{D} \right)$$

TMD distribution is not defined by a scale (μ, ζ)
It is defined by an equipotential line.



The scaling is defined by
~~a difference between scales~~
a difference between potentials

Evolution factor to both points
is the same
although the scales are
different by $10^2\,\mathrm{GeV}^2$

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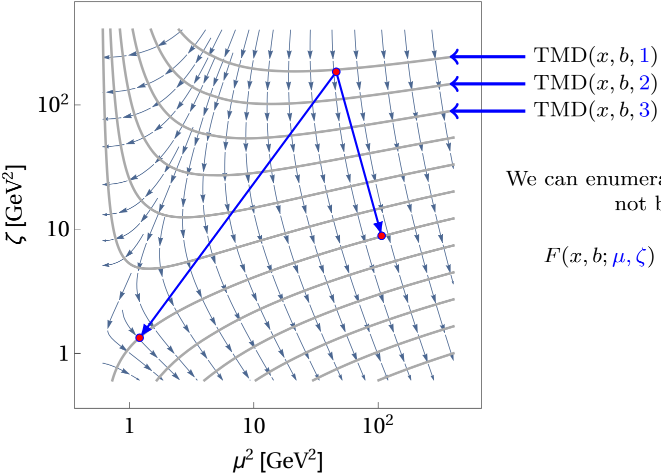
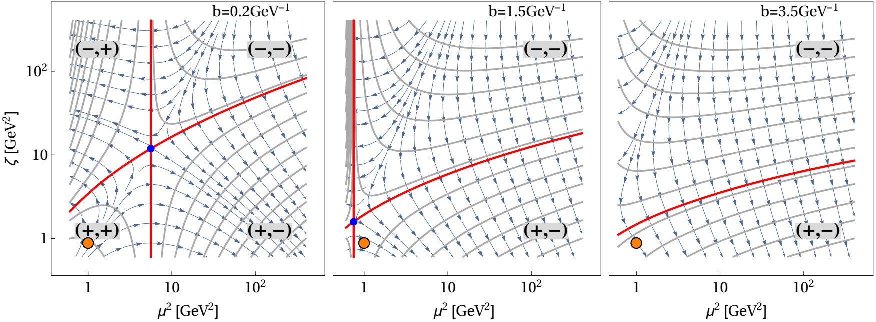
TMD distributions on the same equipotential line are equivalent.



We can enumerate them by a lines not by (μ, ζ)

$$F(x, b; \mu, \zeta) \rightarrow F(z, b; \text{line})$$

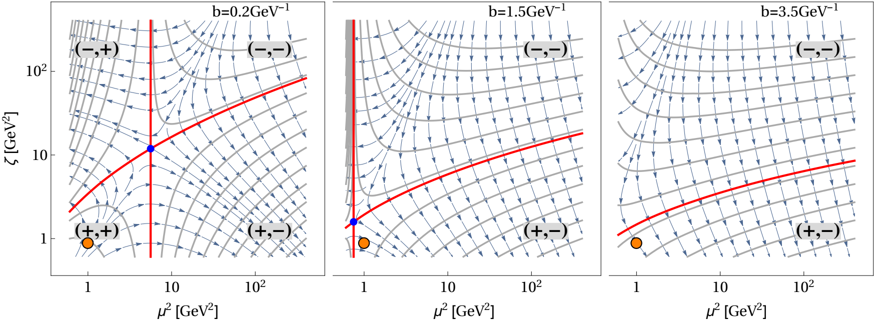
Boundary condition problem
How to select same equipotential line for each values of b?



Standard solution: fix scale

If one set $(\mu^2, \zeta) = (1, 1)\text{GeV}^2$ (f.i.);
$\Rightarrow$ this point belongs to different equipotentials at different values of b
$\Rightarrow$ $F(x, b; 1, 1)$ contains admixture of $\mathcal{D}$ at different b
another example CS-scheme $\mu = \zeta = 1/b^*(b)$

Boundary condition problem
How to select same equipotential line for each values of b?

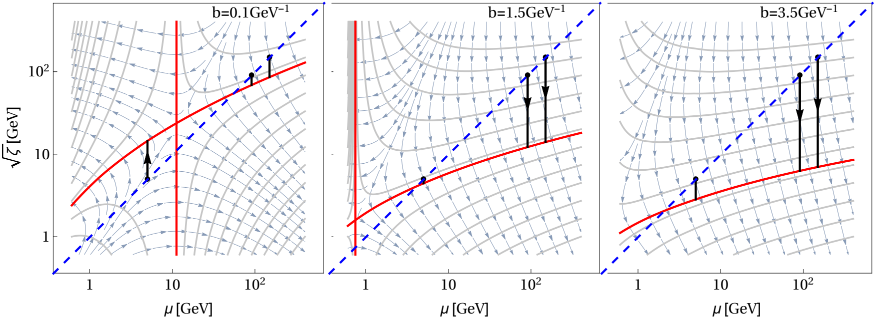


Better solution: fix line

Select an equipotential line and follow it at all values of b
Problem: Lines are discontinuous and difficult to define
Solution: Take the line that passes though saddle point (unique, continues, simple)

If the equipotential line is know evolution is simple

$$F(x,b;\mu,\zeta)=\left(\frac{\zeta}{\zeta_\mu(b)}\right)^{-\mathcal{D}(b,\mu)}F(x,b)$$



How to find equipotential line?

- Let $\zeta_\mu(b) = \zeta(b,\mu)$ be equipotential line

$$\Gamma_{\text{cusp}}(\mu)\ln\left(\frac{\mu^2}{\zeta_\mu(b)}\right) - \gamma_V(\mu) = 2\mathcal{D}(\mu,b)\frac{d\ln\zeta_\mu}{d\ln\mu^2}$$

- Special boundary condition

$$\zeta_{\mu_0}(b) = \zeta_0, \qquad \text{where} \qquad \mathcal{D}(b,\mu_0) = 0, \qquad \gamma_F(\mu_0,\zeta_0) = 0.$$

Does it helpful? Not really...

- The saddle point goes below Λ_{QCD} are large b
- $\mathcal{D}$ is generically unknown $\Rightarrow$ for each version of $\mathcal{D}$ solve SP, and solve equation $\Rightarrow$ numerically expansive

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Exact solution for arbitrary $\mathcal{D}$
$(\mu,\zeta) \to (a_s,\mathcal{D})$

- Let $\zeta_\mu(b) = \zeta(b,\mu)$ be equipotential line

$$\zeta_\mu(b) = \mu^2 \exp\left(-\frac{g(a_s,\mathcal{D})}{\mathcal{D}}\right)$$

$$2\mathcal{D} + 2\beta(a_s)\frac{\partial g(a_s,\mathcal{D})}{\partial a_s} - \Gamma_{\text{cusp}}(a_s)\frac{\partial g(a_s,\mathcal{D})}{\partial \mathcal{D}} + \gamma_V(a_s) = 0$$

- Special boundary condition

$$g(a_s,0) = 0$$

Does it helpful? Extremely!!

- Has solution for arbitrary values of $\mathcal{D}$
- Does not care about Landau pole (automatic analytic continuation)
- Can be solved exactly (if needed)
- Has very good perturbative convergence since $(\mu = Q)$ at arbitrary b
- Very fast numerically (just algebraic function)

Exact solution for arbitrary $\mathcal{D}$
$(\mu,\zeta)\to(a_s,\mathcal{D})$

- Let $\zeta_\mu(b)=\zeta(b,\mu)$ be equipotential line

$$\zeta_\mu(b)=\mu^2\exp\left(-\frac{g(a_s,\mathcal{D})}{\mathcal{D}}\right)$$

- LO+NLO ($p=2\beta_0\mathcal{D}/\Gamma_0$)

$$g(a_s,\mathcal{D})=\frac{2\beta_0^2}{a_s\Gamma_0}\Big[\Big(e^{-p}-1+p\Big)+a_s\frac{\beta_1}{\beta_0}\Big(e^{-p}-1+p-\frac{p^2}{2}+...\Big)\Big]$$

see 1912.06532 appendix C, for all formulas up to N^{3}LO

Does it helpful? Extremely!!

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- Can be solved exactly (if needed)
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In perturbative regime reproduces resummation (up to a given order in a_s)

$$\begin{aligned} F(x,b) &= C(x,b;\mu_{\text{OPE}}) \otimes f(x,\mu_{\text{OPE}}) \\ &= [\delta(\bar{x}) + a_s(P(x)\mathbf{L}_\mu + 2\bar{x} - \zeta_2\delta(\bar{x})) + ...] \otimes f(x,\mu_{\text{OPE}}) \end{aligned}$$

- Different expression for perturbative matching (no double logs!) (effect x-dependence at NNLO)
- Free parameter μ_{OPE}, independent on the scales of process

$$\mathcal{D}(b,\mu) = a_s \frac{\Gamma_0}{2}\mathbf{L}_\mu + a_s^2(...) + ...$$

$$\zeta_\mu(b) = C_0 \frac{\mu}{b} \exp\left(\frac{\gamma_1}{\Gamma_0} + a_s^2(...) + ...\right)$$

Collect together reexpand get the same expression.

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In **artemide** a TMD is defined on special equipotential line

$$d\sigma \simeq H(Q)\int dbe^{ibq_T}\left(\frac{Q^2}{\zeta_Q(b)[\mathcal{D}]}\right)^{-2\mathcal{D}(Q,b)} F_1(x_1,b)F_2(x_2,b)$$

- Three non-perturbative functions; each responsible for single variable (and b)
- In comparison to "standard CSS" matching some terms are exponentiated (whose that are related to $\mathcal{D}$)
- It gives different numerics at large-b/small-Q (even if evolution is taken the same)

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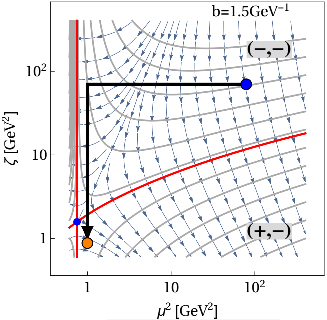
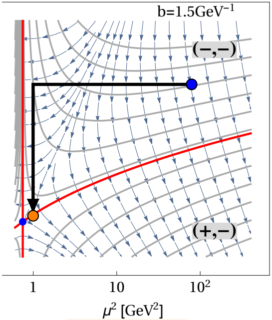
$$\mu_{\rm i}^2 = \zeta_i = Q^2$$

canonic CSS
NangaParbat



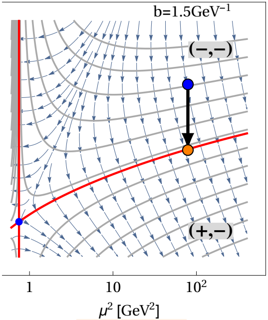
$$\mu_{\rm f}^2 = \zeta_f \simeq \frac{C_0}{b^*(b)}$$

artemide
CSS-like



$$\mu_{\rm f}^2 \simeq \frac{C_0}{b^*(b)}$$
$$\zeta = \zeta_{\mu_f}(b)$$

artemide
default



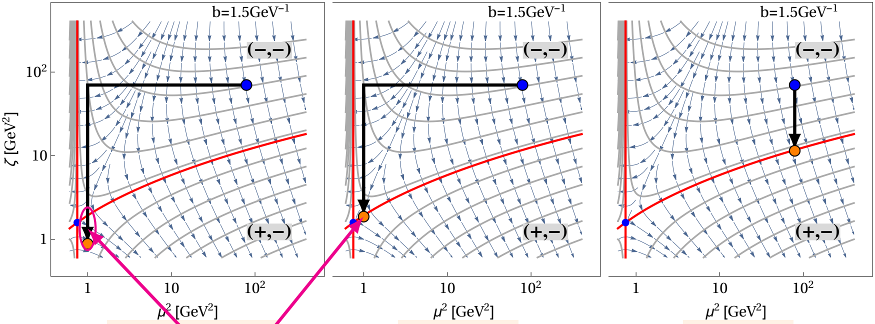
$$\mu_{\rm f}^2 = Q$$
$$\zeta = \zeta_Q(b)$$

$$R = \exp\left[\int \frac{d\mu}{\mu}\left(\Gamma \ln\left(\frac{\mu^2}{\zeta_f}\right) - \gamma_V\right)\right]\left(\frac{\zeta_f}{\zeta_i}\right)^{-\mathcal{D}}$$

$$R = \left(\frac{\zeta_f}{\zeta_\mu(b)}\right)^{-\mathcal{D}}$$

$$\mu_{\rm i}^2 = \zeta_i = Q^2$$

canonic CSS `NangaParbat`	`artemide` CSS-like	`artemide` default



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$$\mu_{\rm f}^2 = Q$$
$$\zeta = \zeta_Q(b)$$

$$\Delta = \left(\frac{\zeta_{\mu_i}(b)b^*}{C_0}\right)^{-\mathcal{D}}$$

Part of matching coefficient
leads to different definition of "unprimed" orders
($\Delta \equiv 1$ at LL)

$$R = \left(\frac{\zeta_f}{\zeta_\mu(b)}\right)^{-\mathcal{D}}$$

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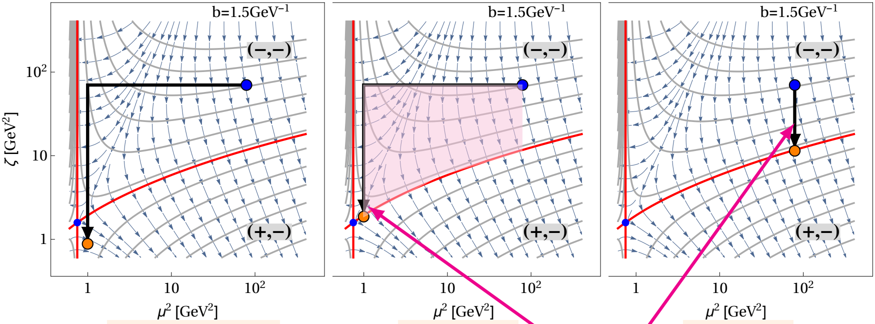
$$\mu_{\rm i}^2 = \zeta_i = Q^2$$

canonic CSS **NangaParbat**	**artemide** CSS-like	**artemide** default



$$\mu_{\rm f}^2 = \zeta_f \simeq \frac{C_0}{b^*(b)}$$

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$$\zeta = \zeta_{\mu_f}(b)$$

$$\mu_{\rm f}^2 = Q$$
$$\zeta = \zeta_Q(b)$$

$$\Delta = \exp \int_C \left(\frac{d\mu}{\mu}\gamma_F - \frac{d\zeta}{\zeta}\mathcal{D}\right)$$
$$\sim a_s^{n+1} \times \text{area}$$
decrese with the increase of order

$$R = \exp\left[\int \frac{d\mu}{\mu}\left(\Gamma \ln\left(\frac{\mu^2}{\zeta_f}\right) - \gamma_V\right)\right]\left(\frac{\zeta_f}{\zeta_i} \quad \mathcal{D}\right.$$

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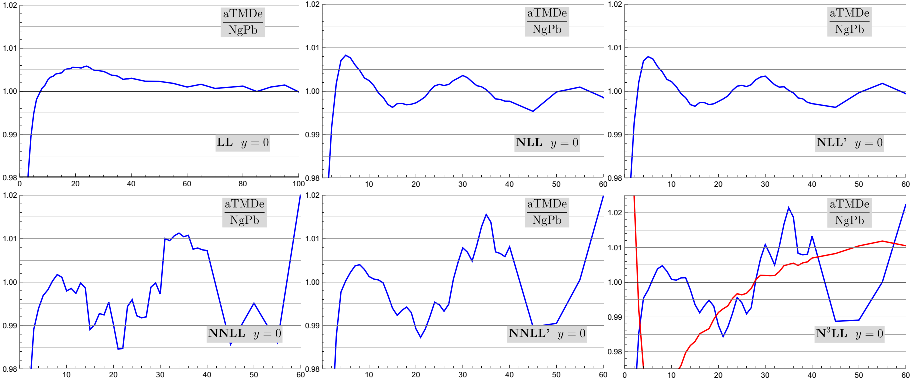
$Q = M_Z,\ y = 0$



- α_s and N_f-scheme is different (artemide=(LHA,variable), NangaParbat=(self,$N_f = 5$))
- CSS-like code in artemide is not well-optimized (noisy)
- $f_{NP} = 1$ is badly convergent (noise at large-q_T)

$$Q = M_Z,\ y = 0$$