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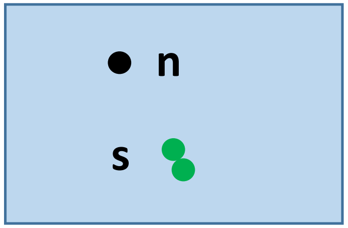
INFN Sezione di Napoli

EASISchool 3, Genova Sept. 28, 2020

Ruggero Vaglio

BASIC PRINCIPLES OF RF SUPERCONDUCTIVITY

Two –fluid model

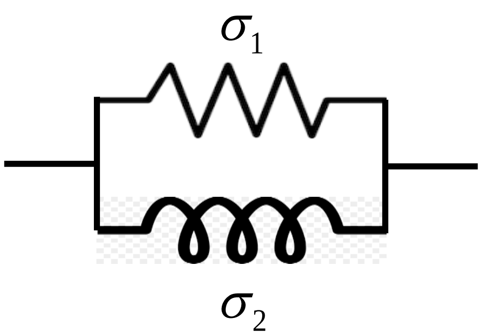


$$\vec{J}_n = \sigma_1 \vec{E} \qquad \sigma_1 = n_n e \mu_n = \frac{n_n e^2 \tau_r}{m}$$

$$\omega \tau_r << 1 \Rightarrow f << 100 GHz$$

$$m_s \frac{d\vec{v}_s}{dt} = q_s \vec{E} \Rightarrow \frac{d\vec{J}_s}{dt} = \frac{n_s q_s^2}{m} \vec{E} \qquad \begin{cases} \vec{E} = \vec{E}_o e^{i\omega t} \\ \vec{J}_s = \vec{J}_{so} e^{i\omega t} \end{cases}$$

$$\vec{J}_s = -i \frac{n_s q_s^2}{m_s \omega} \vec{E} = -i \sigma_2 \vec{E} \qquad \sigma_2 = \frac{n_s q_s^2}{m_s \omega} = \frac{1}{\mu_o \omega \lambda_L^2}$$

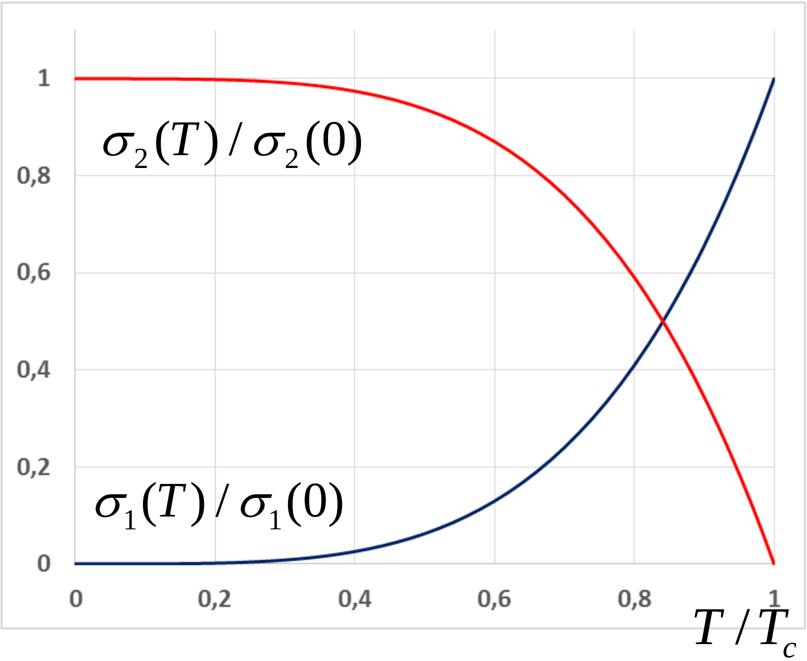


$$\vec{J} = \vec{J}_n + \vec{J}_s = (\sigma_1 - i\sigma_2) \vec{E}$$

«local limit»

$$\sigma_1(T) = \sigma_1(0)\left(\frac{T}{T_c}\right)^4$$ (frequency independent)

$$\sigma_2(T) = \frac{1}{\mu_o \omega \lambda_L^2(T)} = \sigma_2(0)\left[1-\left(\frac{T}{T_c}\right)^4\right] \quad \propto 1/\omega \qquad \lambda_L(T) = \sqrt{\frac{m_e}{\mu_o n_s(T) e^2}}$$



$$T < T_c/2 \longrightarrow \sigma_1 << \sigma_2$$

BCS (Matthis-Bardeen theory) :

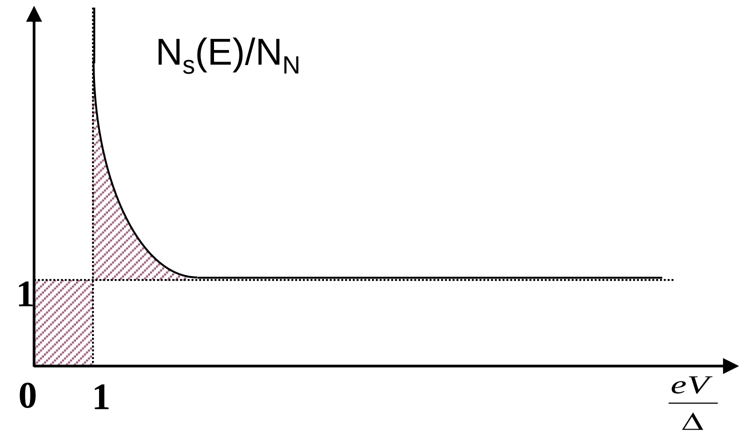
$$\frac{\sigma_1(\omega,T)}{\sigma_n(\omega,T)} = \frac{2}{\hbar\omega}\int_{\Delta}^{\infty}\left[f(E) - f(E+\hbar\omega)\right]g(E)\,dE +$$

$$+\frac{1}{\hbar\omega}\int_{[\Delta-\hbar\omega]}^{-\Delta}\left[2 - f(E+\hbar\omega)\right]g(E)\,dE$$

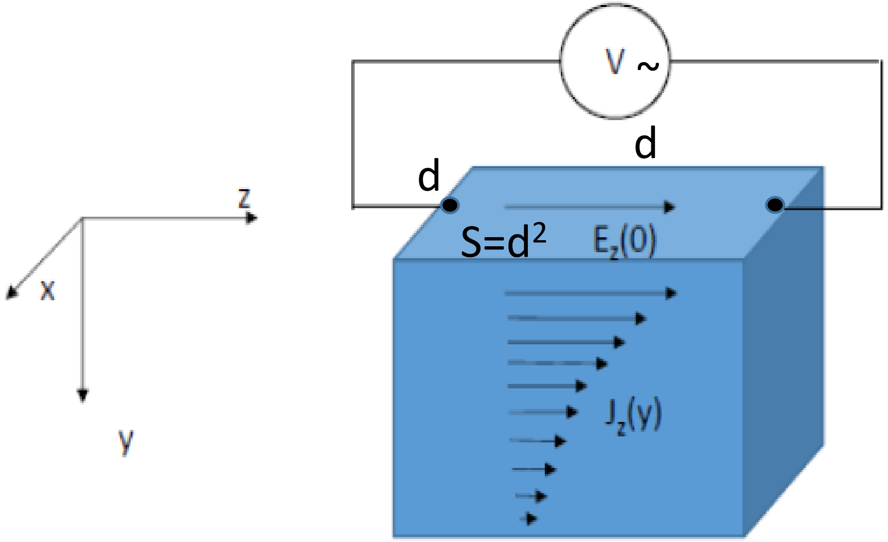
$$\frac{\sigma_2(\omega,T)}{\sigma_n(\omega,T)} = \frac{1}{\hbar\omega}\int_{[\Delta-\hbar\omega,-\Delta]}^{\Delta}\left[1-2f(E+\hbar\omega)\right]g(E)\,dE$$

$$f(E) = \frac{1}{1+e^{E/K_BT}} \qquad g(E) = \frac{E^2+\Delta^2+\hbar\omega E}{(E^2-\Delta^2)^{1/2}\left[(E+\hbar\omega)^2-\Delta^2\right]^{1/2}}$$

$$N_S(E) = \frac{N_n|E|}{\sqrt{E^2-\Delta^2}}$$



Surface impedance



$$Z_s = \frac{V}{I}$$

;

$$V = dE_z(0)x; I = d\int_0^{+\infty} J_z(y)dy$$

$$Z_s = R_s + iX_s = \frac{E_z(0)}{\int_0^{+\infty} J_z(y)\,dy} = \frac{E_z(0)}{H_x(0)}$$

$$\overline{P_{tot}(t)} = \frac{1}{2}R_s I^2 = \frac{1}{2}R_s d^2 H_x^2$$

$$\bar{P}/S = P_{rf} = \frac{1}{2}R_s H_{rf}^2$$

Normal metals (Ohm law+Maxwell equations):

$$Z_n = R_n + iX_n = \sqrt{\frac{\mu_o \omega}{2\sigma_n}}(1+i)$$

$$(R_n = X_n \propto \sqrt{\omega})$$

$$\delta = \sqrt{\frac{2}{\omega \mu_o \sigma_n}}$$

Superconductors (local limit):

$$Z_s = R_s + iX_s = \sqrt{\frac{\mu_o \omega}{2(\sigma_1 - i\sigma_2)}}(1+i)$$

Thin films :

$$Z_s = Z_{s,\infty} \coth(ikd)$$

d : film thikness

$$k = \sqrt{\frac{\mu_o \omega \sigma}{2}}(1-i)$$

Superconductors:

$$T < T_c / 2 \longrightarrow \sigma_1 << \sigma_2$$

$$R_s = \frac{R_n}{\sqrt{2}} \frac{\sigma_1 / \sigma_n}{(\sigma_2 / \sigma_n)^{3/2}} = \frac{1}{2\lambda_L} \frac{\sigma_1}{\sigma_2^2}$$

$$X_s = \frac{X_n \sqrt{2}}{(\sigma_2 / \sigma_n)^{1/2}} = \mu_o \omega \lambda \propto \omega$$

Two fluid model:

$$R_s = \frac{R_n}{\sqrt{2}} \frac{\sigma_1 / \sigma_n}{(\sigma_2 / \sigma_n)^{3/2}} = A\rho^{1/2}\omega^2 \frac{(T/T_c)^\alpha}{\left[1-(T/T_c)^\alpha\right]^\beta}$$

$$R_s \propto \omega^2$$

$$(\alpha = 4, \beta = 3/2)$$

HTS superconductors :

$$\alpha = 2, \beta = 0.3$$

BCS :

Low frequency limit : $\hbar\omega << 2\Delta$

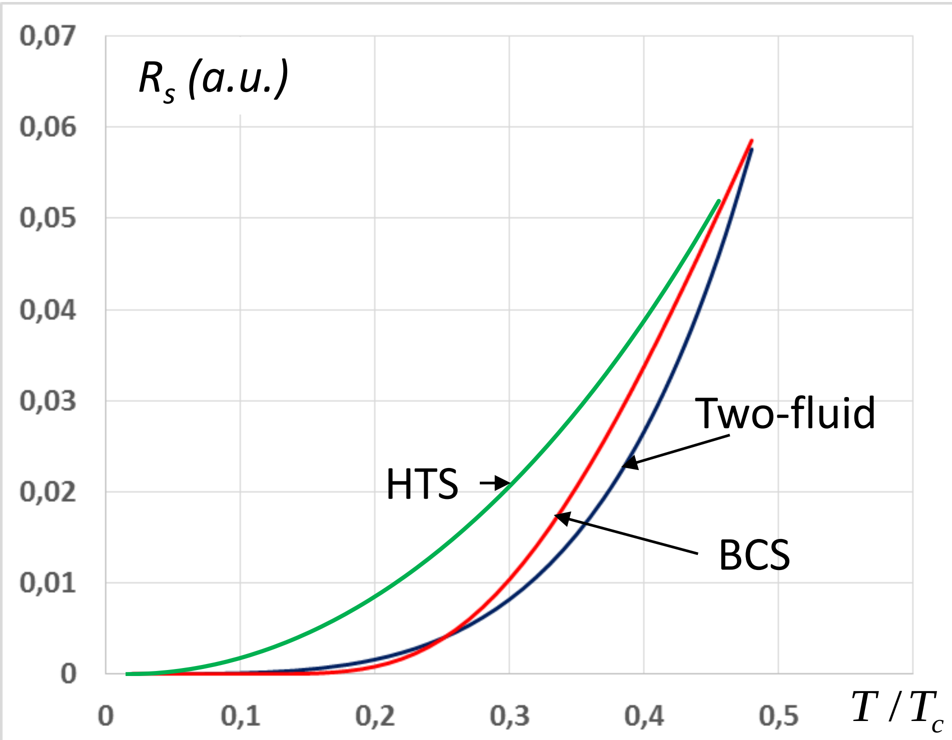
$$\begin{cases} \dfrac{\sigma_1}{\sigma_n} \cong \dfrac{2\Delta(T)}{K_B T} \dfrac{\exp\left[\Delta(T)/K_B T\right]}{\left\{1+\exp\left[\Delta(T)/K_B T\right]\right\}^2} \ln\left[\Delta(T)/\hbar\omega\right] \\ \dfrac{\sigma_2}{\sigma_n} \cong \dfrac{\pi\Delta}{\hbar\omega} \tanh\dfrac{\Delta}{2K_B T} \end{cases}$$

Low temperature : $T < T_c/2$

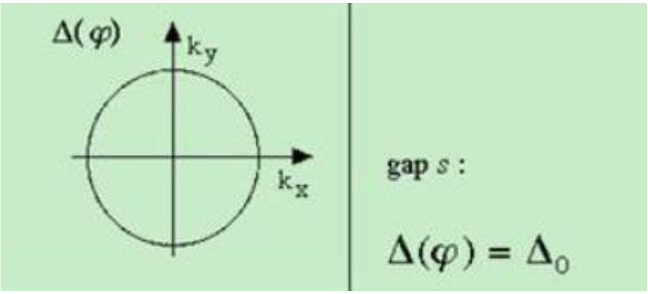
$$R_s^{BCS} = \frac{R_n}{\sqrt{2}} \frac{\sigma_1/\sigma_n}{(\sigma_2/\sigma_n)^{3/2}} = \frac{A\rho_n^{1/2}\omega^{\alpha}}{T} \exp\left(-\frac{\Delta_o}{K_B T}\right)$$

$$\alpha \cong 1.7 \div 2$$

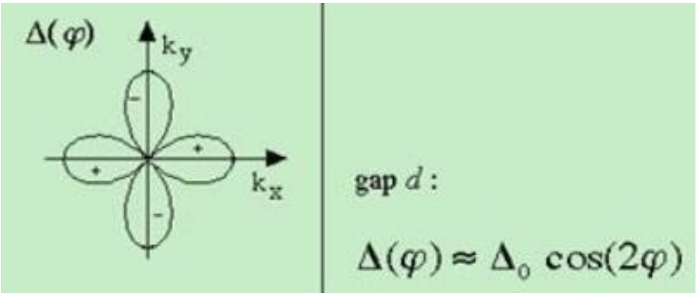
$$\frac{\Delta_0}{K_B T_C} = 1.76$$



0,07
R_s (a.u.)
0,06
0,05
0,04
0,03
0,02
0,01
0
0
0,1
0,2
0,3
0,4
0,5
T / T_c
Two-fluid
HTS
BCS

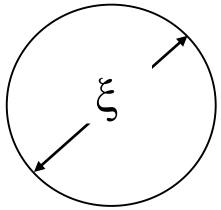


BCS
$\Delta(\varphi)$
k_y
k_x
gap s :
$\Delta(\varphi) = \Delta_0$



HTS
$\Delta(\varphi)$
k_y
k_x
gap d :
$\Delta(\varphi) \approx \Delta_0 \cos(2\varphi)$

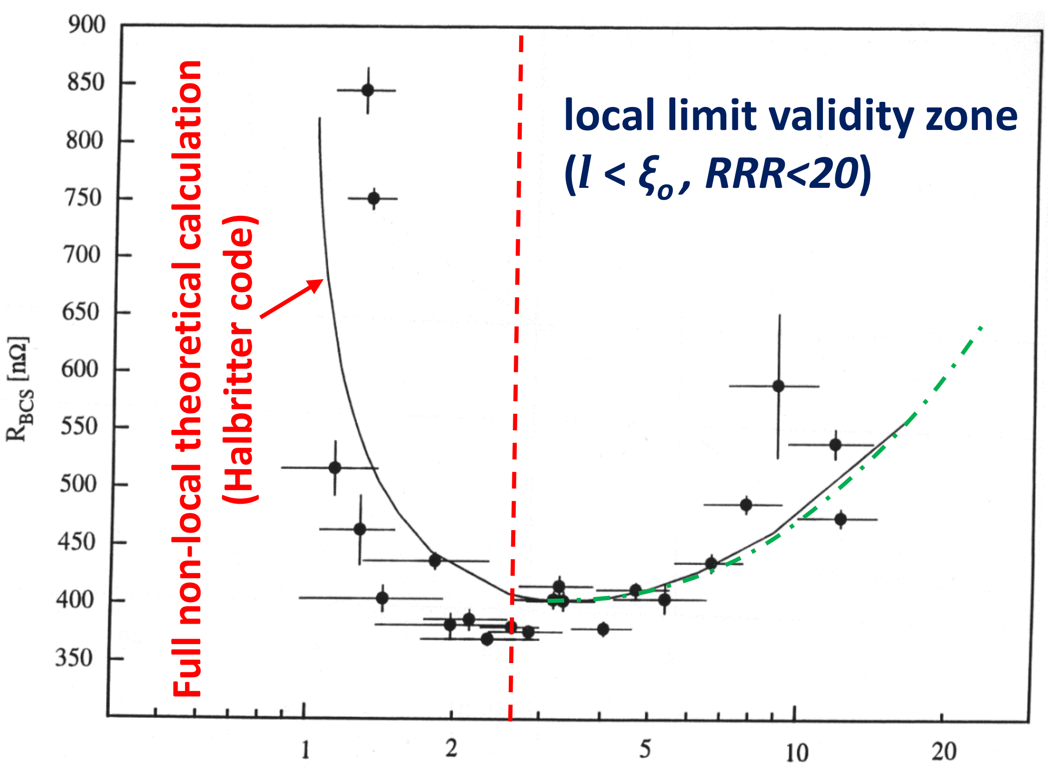
Non-local theory (Pippard 1950)



$$\frac{1}{\xi} = \frac{1}{l} + \frac{1}{\xi_0}$$

$$\vec{J}_s(\vec{r}) = -\frac{3}{4\pi\xi_0\lambda}\int \frac{\vec{R}\left(\vec{R}\cdot\vec{A}(r')\right)}{R^4} e^{-R/\xi} dr' \qquad \begin{cases} \vec{E} = -\dfrac{\partial \vec{A}}{\partial t} \\ \vec{R} = \vec{r} - \vec{r}' \end{cases}$$

The local limit applies always well enough in superconducting compounds and alloys (*l* small compared to λ , «local dirthy limit)) and HTS (ξ_o small compared to λ, «local clean limit»), but not for high purity bulk Nb !



Nb, T=4.2K, f=1.5GHz
Benvenuti et al, Physica C
Physica C 316 ,153 (1999)

Setting : $\dfrac{\sigma_2}{\sigma_n} = f\,\dfrac{\pi\Delta}{\hbar\omega}\tanh\dfrac{\Delta}{2K_BT}$

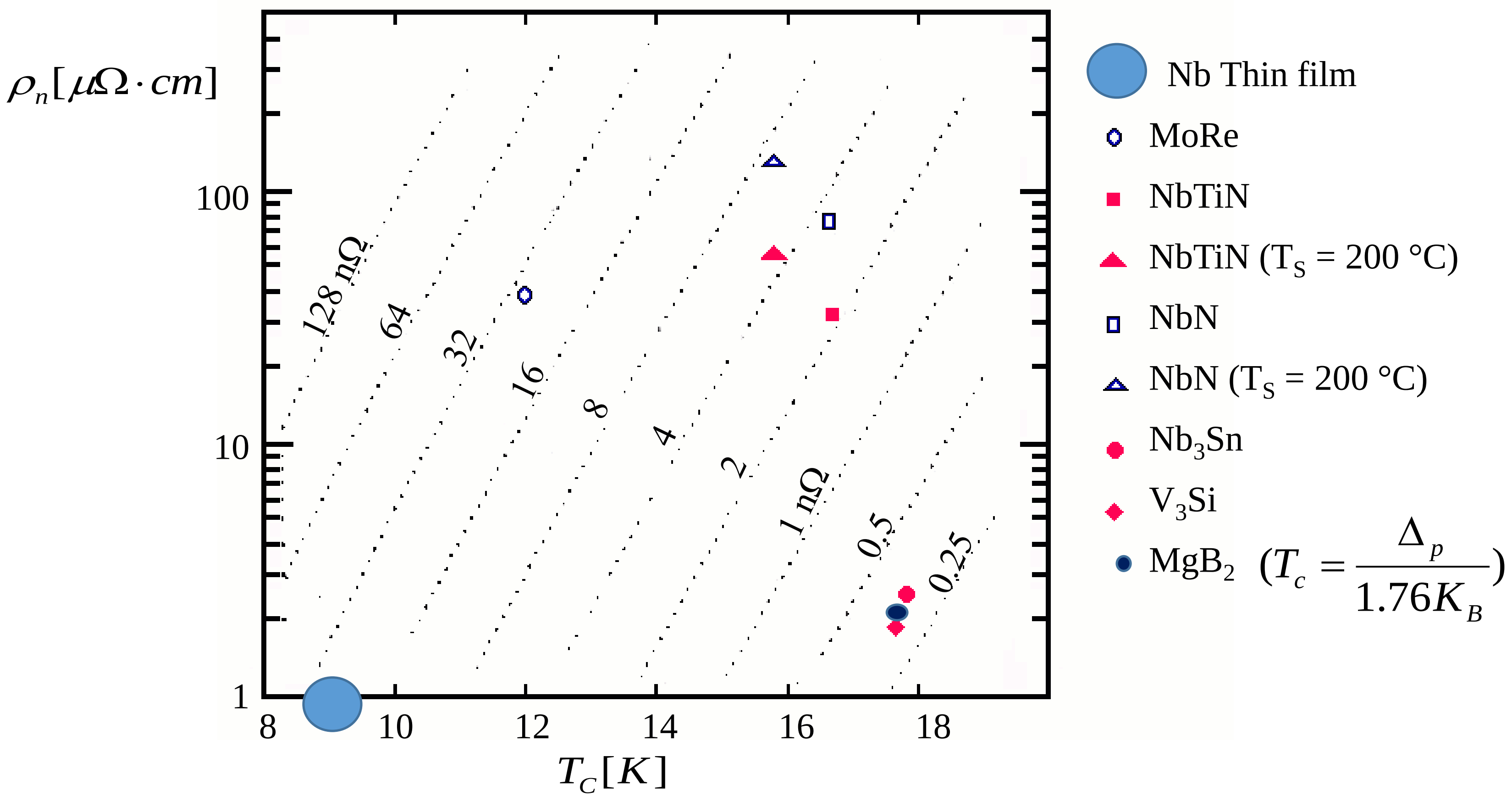
with : $f = 1/(1 + l/\xi_o)$

extends the validity of the BCS low frequency, low temperature,dirthy limit approximate formulas up to

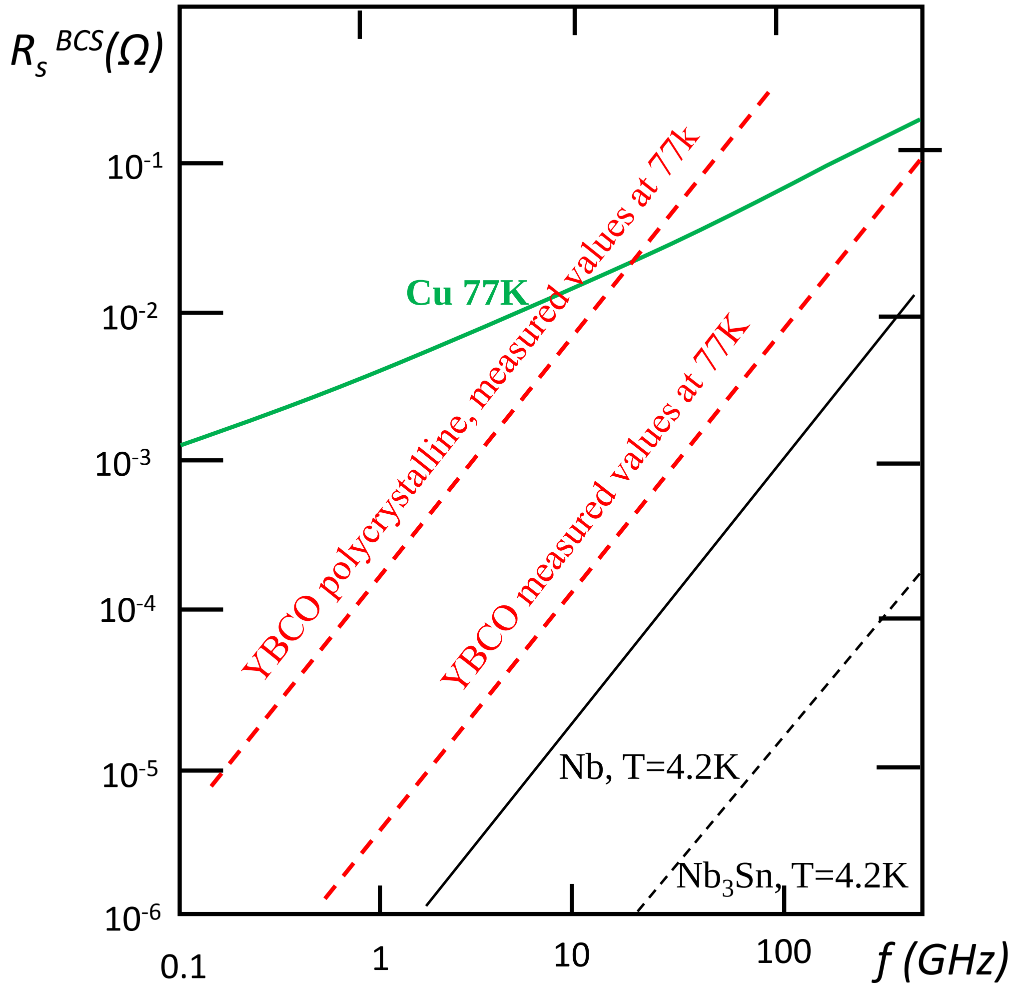
$l \cong \xi_o$ - · - · -

R_s^{BCS} for different superconducting materials in the local limit

Lines of equal R_s^{BCS} in the *(ρ_n, T_C)* plane at T = 4.2 K and *f* = 0.5 GHz (V. Palmieri, Thesis , 1985)



R_s^{BCS} frequency dependence



Normal metals : $R_n \propto \sqrt{\omega}$

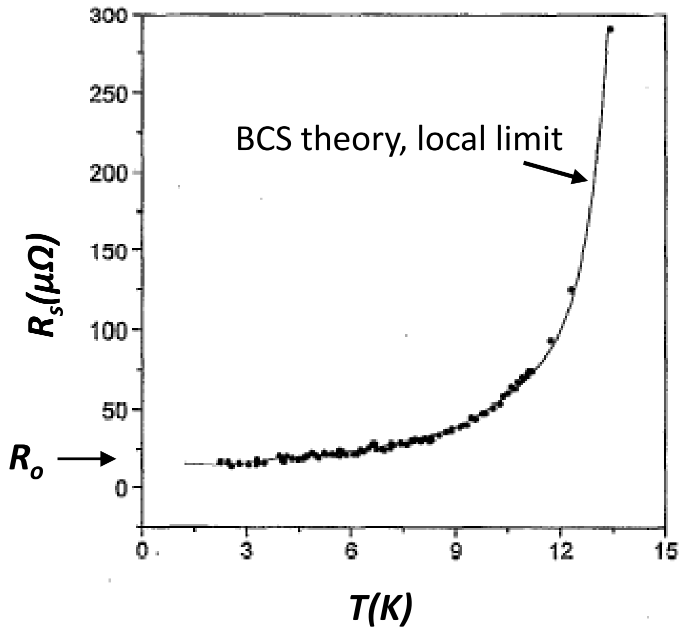
Superconductors : $R_s \propto \omega^{\alpha}$

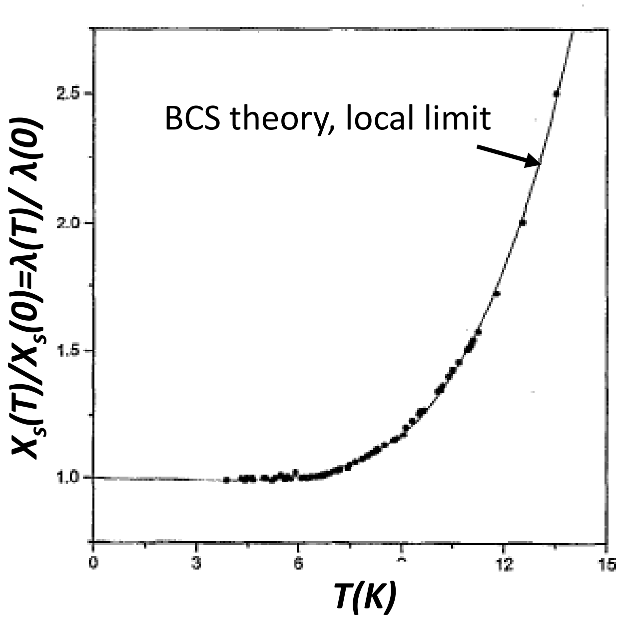
$\alpha \cong 1.7 \div 2$ BCS

$\alpha = 2$ two fluid

Measurements of the surface impedance temperture dependence :

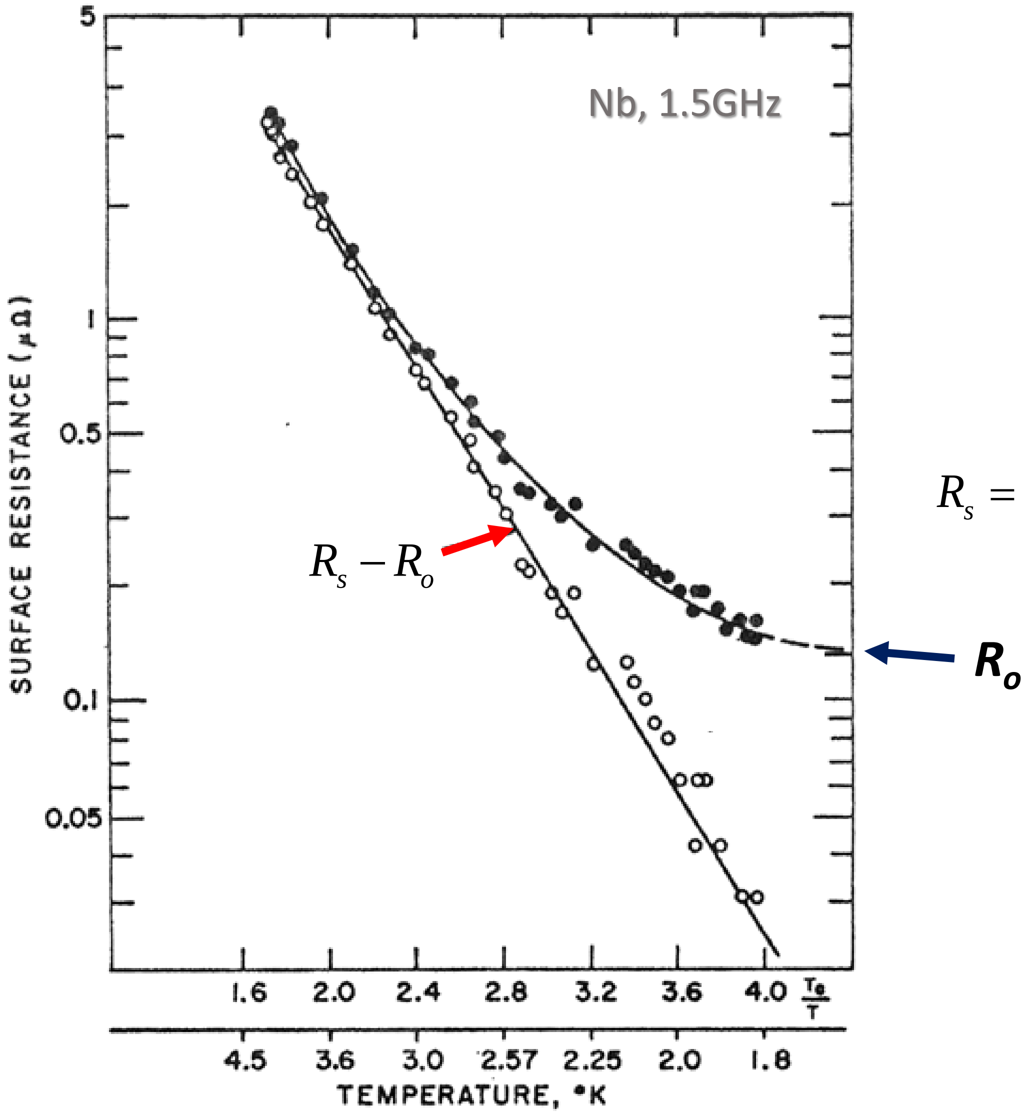
V_3Si (BCS)





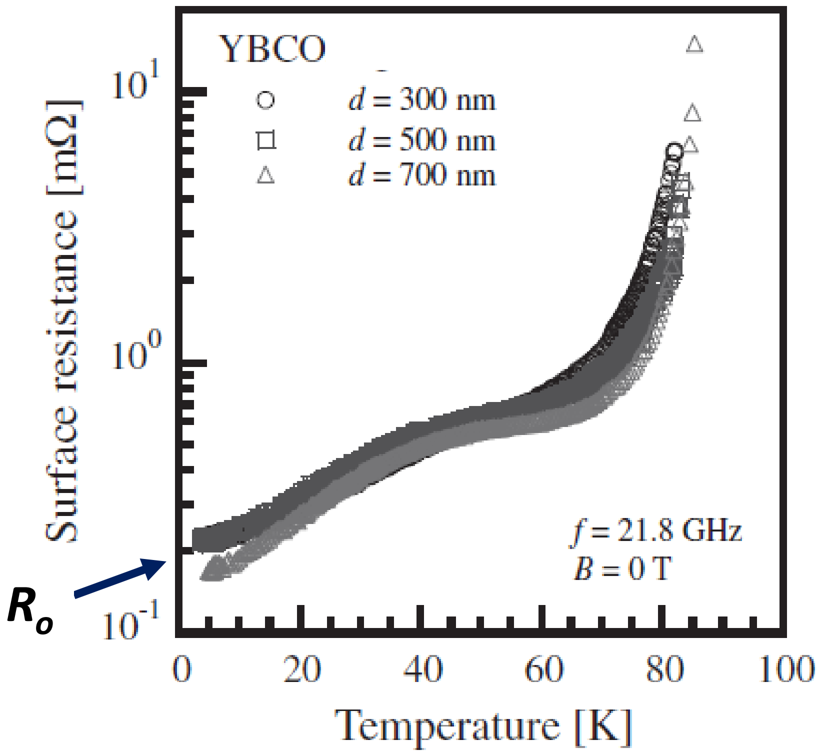
A.Andreone, A.Cassinese, A. DiChiara, M.Salluzzo , R. Vaglio, JAP 78,1862 (1995).

Surface impedance measurements (Nb film)



$$R_s = R_s^{BCS} = \frac{A\rho_n^{1/2}\omega^{1.9}}{T}\exp\left(-\frac{\Delta_o}{K_B T}\right) + R_o$$

Surface resistance measurements (YBCO film)



Honma et al, Physica C 484, 46 (2013)

Fig. 2. Temperature dependence of R_s of YBCO thin films with thicknesses of 300, 500 and 700 nm in a zero magnetic field at 21.8 GHz.

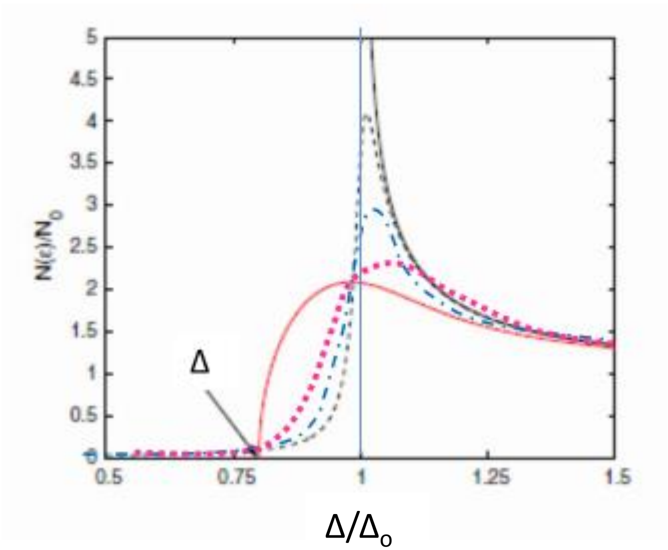
BCS surface resistance dependence on the rf field amplitude

$$R_s^{BCS}(T, B_{rf}) = \frac{A\rho_n^{1/2}\omega^{\alpha}}{T}\exp\left[-\frac{\Delta_o(B_{rf})}{K_B T}\right]$$

The following phenomenological expression appears to be approximately valid :
(up to intermediate rf fields)

$$\Delta_o(B_{rf}) = \Delta_o(0) - \alpha B_{rf}$$

There is no full consensus on the theoretical interpretation. One approach is to perform the Matthis-Bardeen BCS calculations using the full DOS as modified in a current-carrying state :



Xiao et al, Physica C 490, 26 (2013)

For very pure samples negative α values have been observed and can be justified within this model

Residual resistivity possible origins :

1) Grain boundaries (in policrystalline materials)

2) Trapped magnetic flux

3) Defects due to localized «normal» or magnetic inclusions

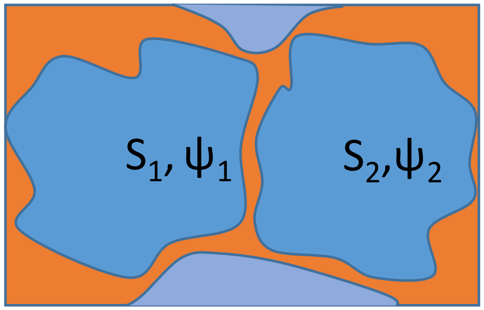
4) ………………………………..

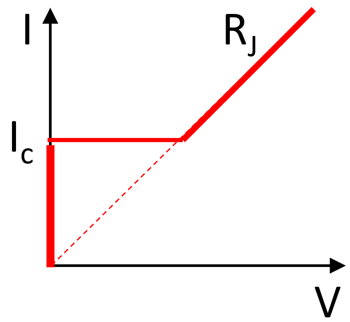
$$R_o = R_{ogb} + R_{ofl} + R_{on} + \ldots\ldots$$

Grain boundaries (polycrystalline materials)

Weak superconducting links : Josephson effect

$\Downarrow$



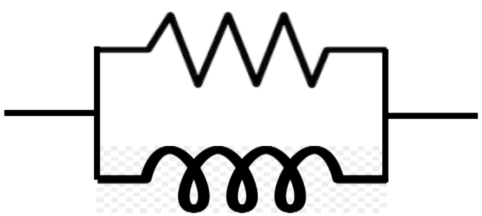


$$I_c R_J \cong const$$

$$\begin{cases} I = I_c \sin\phi \\ \dfrac{\partial\varphi}{\partial t} = \dfrac{2e}{\hbar}V \end{cases} \qquad (\varphi = \varphi_2 - \varphi_1)$$

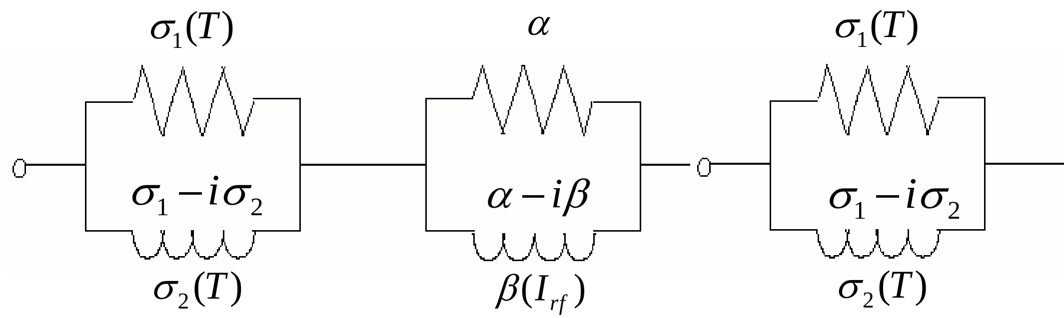
$$I(t) = I_c \sin\left(\frac{2e}{\hbar}\int_0^t V(t)dt\right) \Rightarrow V = \frac{d}{dt}\left[L_J(I)I\right]$$

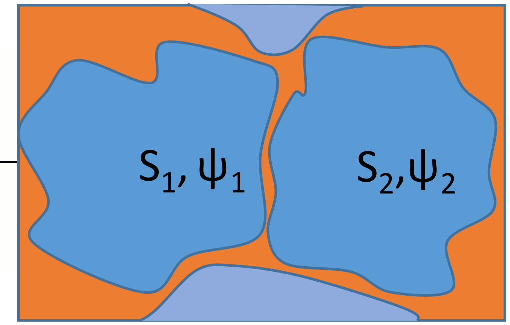
$$\begin{cases} L_J(I) = L_{Jo}F(I_{rf}) \\ F(I_{rf}) = \dfrac{\sin^{-1}(I_{rf}/I_c)}{I_{rf}/I_c} \end{cases} \qquad L_{Jo} = \frac{\hbar}{2eI_c}$$



$$\begin{cases} \alpha = \dfrac{1}{R_J a} = \text{cost.} \\ \beta(I_{rf}) = \dfrac{1}{\omega L_J(I_{rf})a} \end{cases}$$

Non-linear Josephson inductance





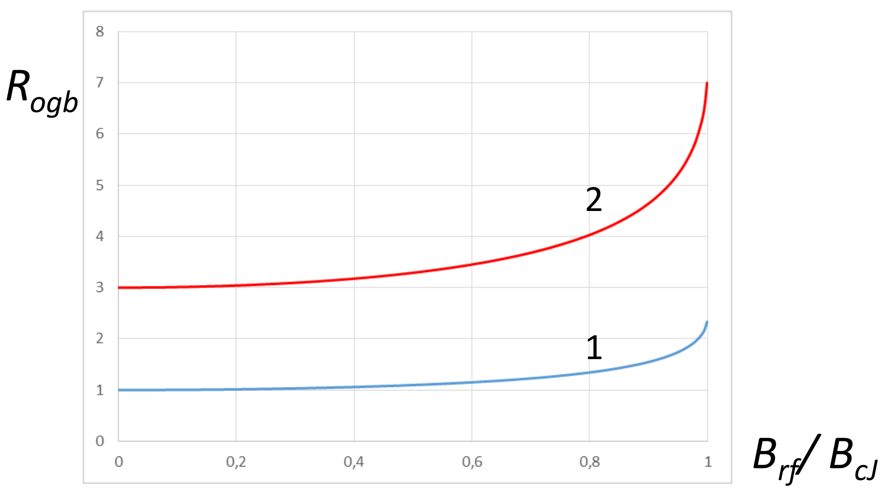
$$R_S = R_{s1} + R_{ogb}$$

$$\left.\begin{array}{l}\sigma_1 \ll \sigma_2 \\ \alpha \ll \beta\end{array}\right\} \rightarrow \begin{cases} R_S = \dfrac{1}{2\lambda}\left[\dfrac{\sigma_1}{\sigma_2^2}\left(1+\dfrac{\alpha^2}{\beta^2}\right)+\dfrac{\alpha}{\beta^2}\right] \\ X_S = \mu\omega\lambda_{eff} \end{cases} \rightarrow$$

$$R_{s1}(B_{rf},T) = \frac{1}{2\lambda}\frac{\sigma_1}{\sigma_2^2}\left(1+\frac{\alpha^2}{\beta^2}\right) \cong R_s^{BCS}(T)$$

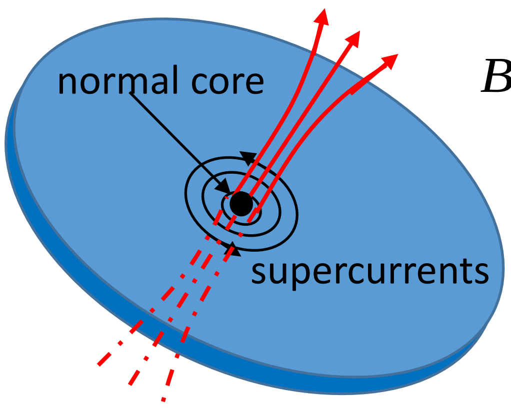
$$\lambda_{eff}^2 = \lambda^2 + \lambda_J^2 = \left[\frac{1}{\mu_o\omega}\left(\frac{1}{\sigma_2}+\frac{1}{\beta}\right)\right]$$

$$R_{ogb} = \frac{1}{2\lambda}\frac{\alpha}{\beta^2(I_{rf})} \propto \omega^2$$



Vaglio et al, Physical Review B 43, 6128 (1991)

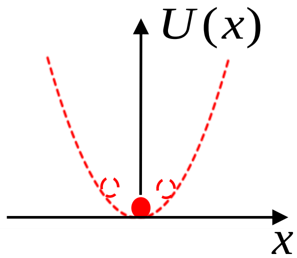
Trapped magnetic flux



$$B_o = n_{tr}\phi_o$$

$$U(x) = \frac{1}{2}kx^2$$

$$(J_{rf} << J_c)$$



$$f_p = -kx$$

$$m\ddot{x} + \eta\dot{x} + kx = J_{rf}\phi_o$$

Gittleman and Rosenblum: Phys Rev. Lett. 16, 734 (1966)

m : fluxon mass per unit lenght ~ 0

$$\eta = \frac{\phi_o B_{c2}}{\rho_n}$$: fluxon viscosity per unit lenght

$$J_{rf} = J_{rfo}e^{i\omega t}, \; \dot{x} = v = v_o e^{i\omega t}$$

$$v_o = \frac{J_{rfo}\phi_o}{\eta}\left(\frac{\omega^2}{\omega^2+\omega_o^2} + i\frac{\omega\omega_o}{\omega^2+\omega_o^2}\right)$$

$$\left(\omega_o = \frac{k}{\eta}\right)$$

«depinning frequency»

$$\vec{J}_{rf} = (\sigma_1 - i\sigma_2)(\vec{E}_{rf} - \vec{v} \times \vec{B}_o)$$ + Maxwell equations

General expression for the surface impedance of a superconductor in presence of rigidly oscillating vortices , with the rf current perpendicular to the magnetic field

$$R_s = R_n \sqrt{\sqrt{a^2 + b^2} - b}$$

Surface resistance

$$X_s = R_n \sqrt{\sqrt{a^2 + b^2} + b}$$

Surface reactance

$$a = \frac{\sigma_1/\sigma_n}{(\sigma_2/\sigma_n)^2} + \frac{B_o}{B_{c2}} \alpha(\omega)$$

$$b = \frac{1}{\sigma_2/\sigma_n} + \frac{B_o}{B_{c2}} \beta(\omega)$$

$$\alpha(\omega) = \frac{\omega^2}{\omega^2 + \omega_o^2}$$

$$\beta(\omega) = \frac{\omega\omega_o}{\omega^2 + \omega_o^2}$$

S. Calatroni, R.Vaglio : IEEE Trans. Superconductivity 2017

For small fields, i.e. for a small number of trapped vortices :

$$R_s \cong R_n \frac{a}{\sqrt{2b}}$$

$$\begin{cases} a = \dfrac{\sigma_1/\sigma_n}{(\sigma_2/\sigma_n)^2} + \dfrac{B_o}{B_{c2}}\alpha(\omega) \\ b = \dfrac{1}{\sigma_2/\sigma_n} \end{cases}$$

$$R_s \cong \frac{R_n}{\sqrt{2}} \frac{\sigma_1/\sigma_n}{(\sigma_2/\sigma_n)^{3/2}} + \frac{R_n}{\sqrt{2}} \sqrt{\sigma_2/\sigma_n} \frac{B_0}{B_{c2}} \alpha(\omega)$$

$$R_s \cong R_{sBCS} + R_{ofl} \quad \text{with} \quad R_{ofl} = \frac{\rho_n}{2\lambda} \frac{B_0}{B_{c2}} \frac{\omega^2}{\omega^2 + \omega_o^2}$$

The observed rf field dependence can be due to a nonlinear pinning force, Calatroni, Vaglio, PRB Acc. & Beams 22, 022001 (2019)

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Conclusions

- Rf properties of materials are well characterized by their surface impedance. Superconductors present a non zero surface resistance but lower that any other material at low-intermediate frequencies
- The rf properties of "conventional" superconductors are very well described by the BCS theory, but for the presence of a "residual" (temperature independent) term R_o. The rf field dependence is a still open theoretical issue
- For most BCS superconductors of practical interest the local limit applies, where The BCS equations take a simple form at low frequency and $T<T_c/2$ (only for high purity, high RRR bulk Nb this is not true)
- HTS are well described by a modified 2-fluid model. Also in this case a significant residual term is present a low temperatures.
- The residual term R_o can have different origins and different frequency and rf field dependence. Some of the possible mechanisms (weak grain boundaries, trapped flux and others) are well theoretically understood.