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Reidar Hahn

EASISchool 3

Superconductivity and its applications

Dates: 28/9/2020-9/10/2020
Genoa, Italy

Cavity Design

Elliptical and non-elliptical cavities

Erk JENSEN, CERN

<https://cernbox.cern.ch/index.php/s/E4qimjeXzf67RLS>

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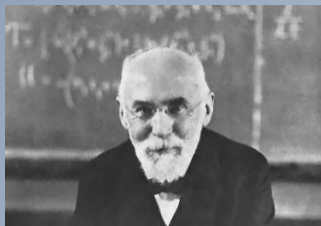


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Outline

- From waveguide to cavity
- Characterizing a cavity
- Multipactor
- Many gaps
- **Elliptical cavities**
- Higher Order Modes
- **Non-elliptical cavities**
- Power couplers
- Tuners

From Waveguide to Cavity



Hendrik A. Lorentz
1853 – 1928

Lorentz force

- A charged particle moving with velocity $\vec{v} = \frac{\vec{p}}{m \gamma}$ through an electromagnetic field in vacuum experiences the Lorentz force $\frac{d\vec{p}}{dt} = q(\vec{E} + \vec{v} \times \vec{B})$.
- The total energy of this particle is $W = \sqrt{(mc^2)^2 + (pc)^2} = \gamma mc^2$, the kinetic energy is $W_{kin} = mc^2(\gamma - 1)$.
- The role of acceleration is to increase W .
- Change of W (by differentiation):

$$WdW = c^2 \vec{p} \cdot d\vec{p} = qc^2 \vec{p} \cdot (\vec{E} + \vec{v} \times \vec{B}) dt = qc^2 \vec{p} \cdot \vec{E} dt$$
$$dW = q\vec{v} \cdot \vec{E} dt$$

Note: **Only the electric field can change the particle energy!**



James Clerk Maxwell
1831 – 1879

Maxwell's equations in vacuum

Source-free:

$$\nabla \times \vec{B} - \frac{1}{c^2} \frac{\partial}{\partial t} \vec{E} = 0 \quad \nabla \cdot \vec{B} = 0$$

$$\nabla \times \vec{E} + \frac{\partial}{\partial t} \vec{B} = 0 \quad \nabla \cdot \vec{E} = 0$$

curl (rot, $\nabla \times$) of 3rd equation and $\frac{\partial}{\partial t}$ of 1st equation:

$$\nabla \times \nabla \times \vec{E} + \frac{1}{c^2} \frac{\partial^2}{\partial t^2} \vec{E} = 0.$$

Using the vector identity $\nabla \times \nabla \times \vec{E} = \nabla \nabla \cdot \vec{E} - \nabla^2 \vec{E}$ and the 4th Maxwell equation, this yields:

$$\nabla^2 \vec{E} - \frac{1}{c^2} \frac{\partial^2}{\partial t^2} \vec{E} = 0,$$

i.e. the 4-dimensional Laplace equation.

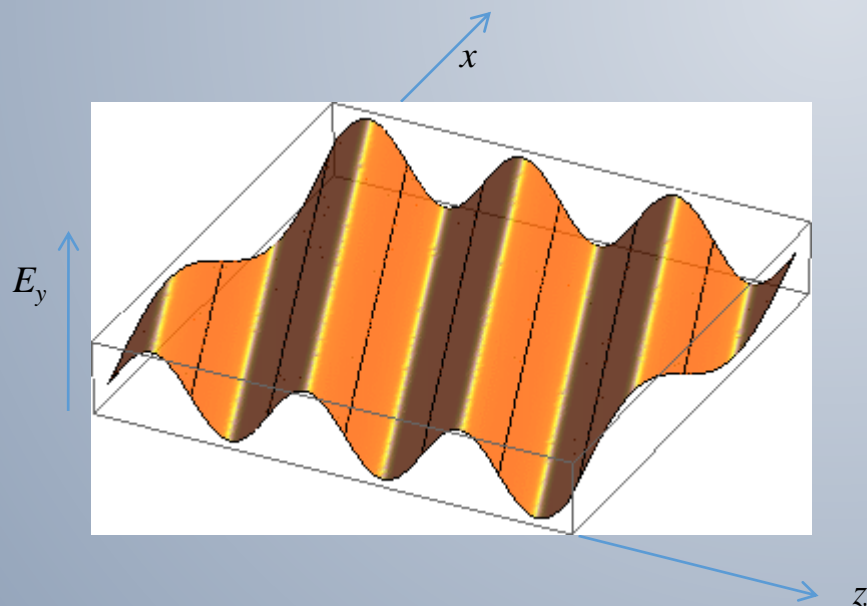


Homogeneous plane wave

$$\vec{E} \propto \vec{u}_y \cos(\omega t - \vec{k} \cdot \vec{r})$$

$$\vec{B} \propto \vec{u}_x \cos(\omega t - \vec{k} \cdot \vec{r})$$

$$\vec{k} \cdot \vec{r} = \frac{\omega}{c} (z \cos \varphi + x \sin \varphi)$$



Wave vector \vec{k} :

the direction of \vec{k} is the direction of propagation,

the length of \vec{k} is the phase shift per unit length.

\vec{k} behaves like a vector.

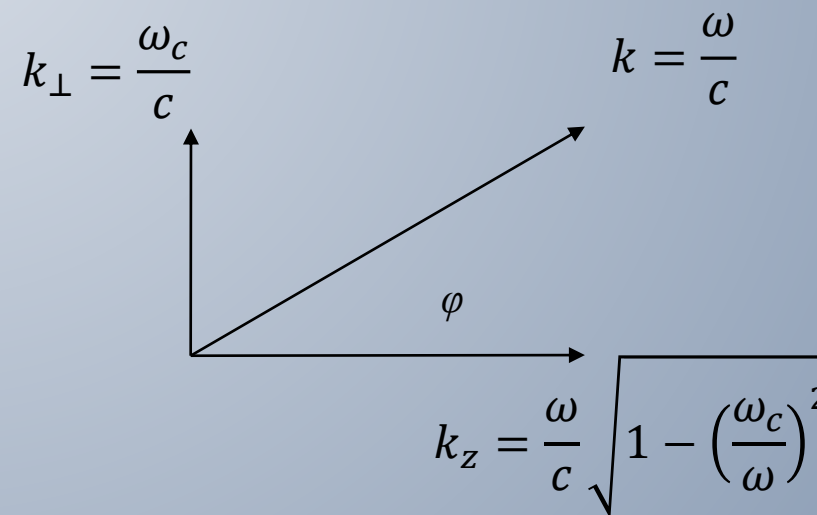
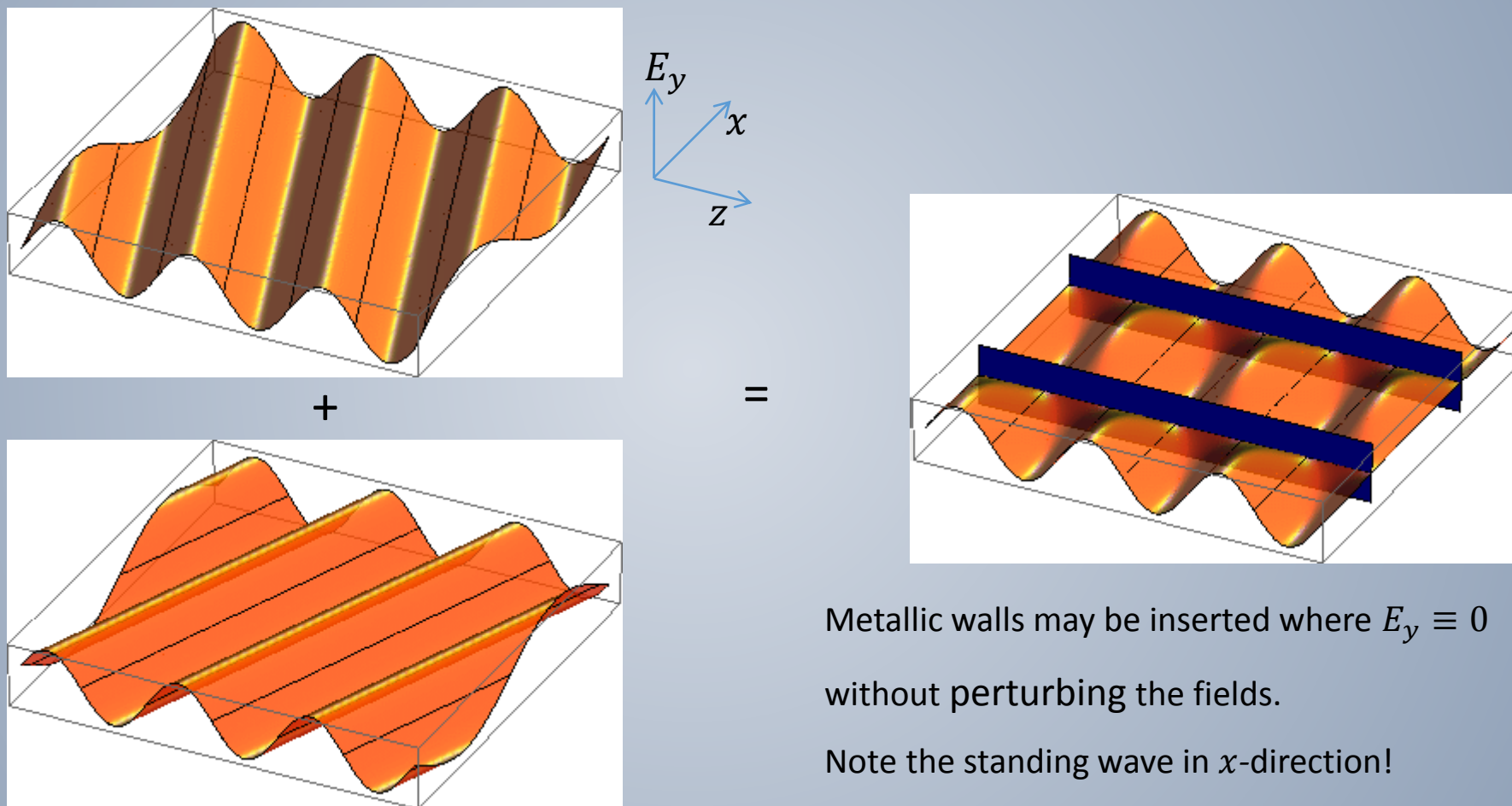




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Superposition of 2 homogeneous plane waves



This way one gets a hollow rectangular waveguide.

Metallic walls may be inserted where $E_y \equiv 0$

without perturbing the fields.

Note the standing wave in x -direction!



Rectangular waveguide

Fundamental (TE_{10} or H_{10}) mode
in a standard rectangular waveguide.

Example 1: “S-band”: 2.6 GHz ... 3.95 GHz,

Waveguide type WR284 (2.84” wide), dimensions:
72.14 mm x 34.04 mm.
cut-off: $f_c = 2.078$ GHz.

Example 2: “L-band” : 1.13 GHz ... 1.73 GHz,

Waveguide type WR650 (6.5” wide), dimensions:
165.1 mm x 82.55 mm.
cut-off: $f_c = 0.908$ GHz.

Both these pictures correspond to operation at $1.5 f_c$.

$$\text{power flow: } \frac{1}{2} \text{Re} \left\{ \iint \vec{E} \times \vec{H}^* \cdot d\vec{A} \right\}$$

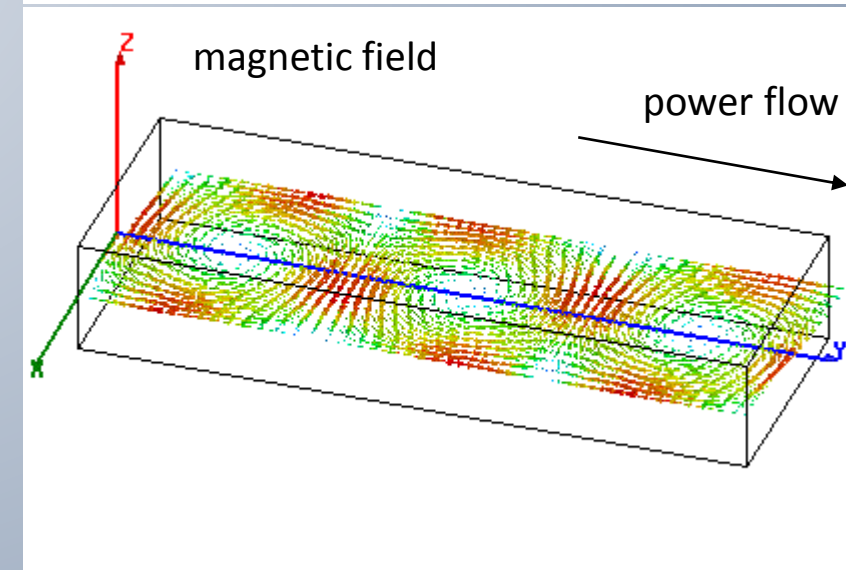
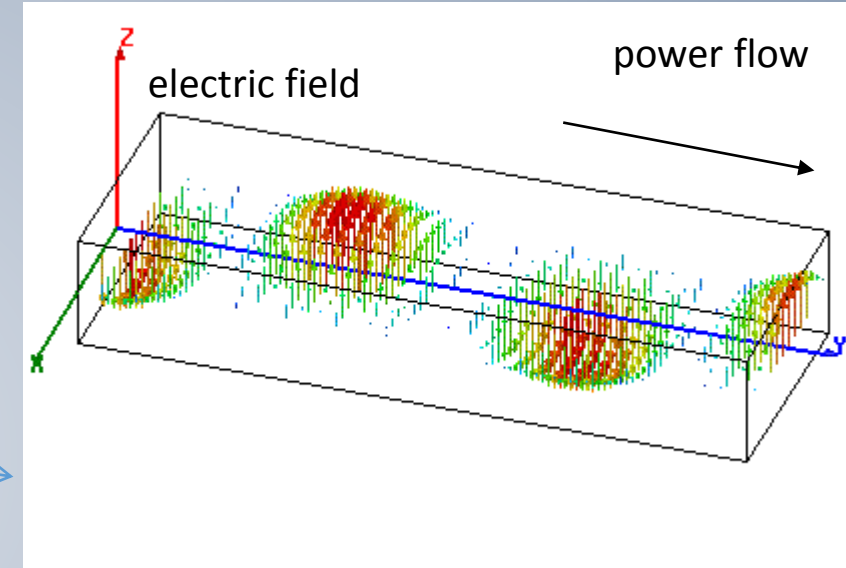
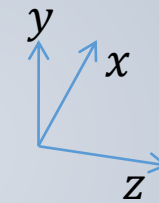




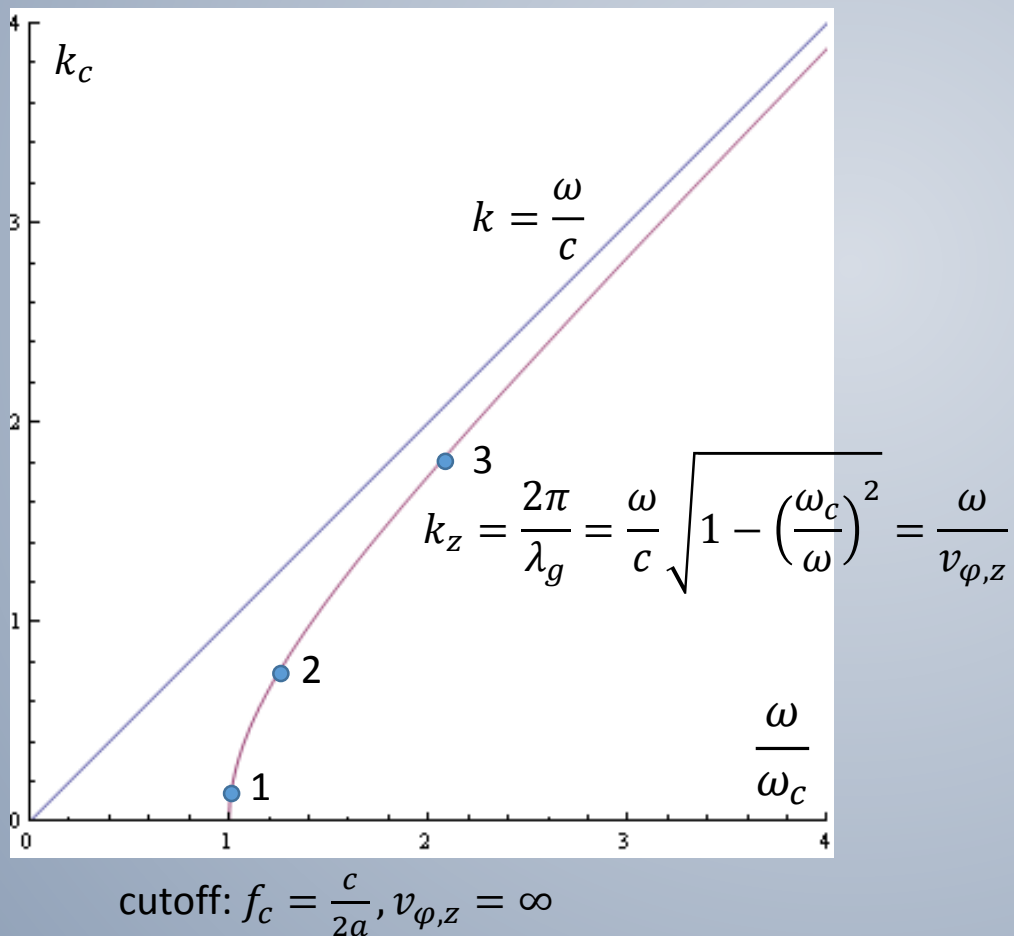
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Waveguide dispersion – phase velocity $v_{\phi,z}$

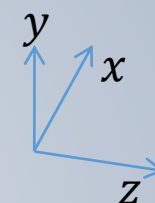
The phase velocity $v_{\phi,z}$ is the speed at which the crest (or zero-crossing) travels in z -direction.

Note on the 3 animations on the right that, at constant f , $v_{\phi,z} \propto \lambda_g$. Note also that at $f = f_c$, $v_{\phi,z} = \infty$!

With $f \rightarrow \infty$, $v_{\phi,z} \rightarrow c$!



1:
 $a = 52 \text{ mm}$,
 $f/f_c = 1.04$



2:
 $a = 72.14 \text{ mm}$,
 $f/f_c = 1.44$

3:
 $a = 144.3 \text{ mm}$,
 $f/f_c = 2.88$

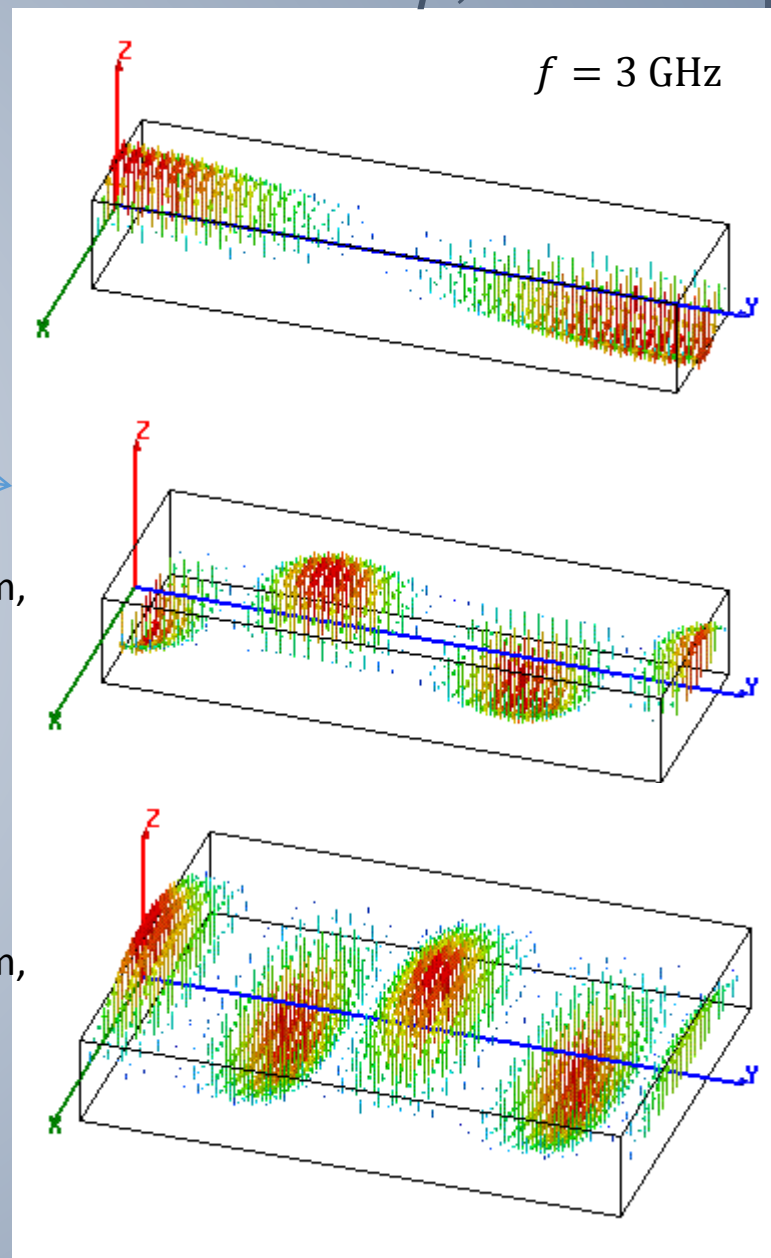




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Reidar Hahn

Summary waveguide dispersion and phase velocity:

In a **general** cylindrical waveguide:

$$k_z = \sqrt{\left(\frac{\omega}{c}\right)^2 - k_{\perp}^2} = \frac{\omega}{c} \sqrt{1 - \left(\frac{\omega_c}{\omega}\right)^2}$$

Propagation in z-direction: $\propto e^{j(\omega t - k_z z)}$

$$Z_0 = \frac{\omega \mu}{k_z} \text{ for TE, } Z_0 = \frac{k_z}{\omega \varepsilon} \text{ for TE}$$

$$k_z = \frac{2\pi}{\lambda_g}$$

Example: TE₁₀-mode in a rectangular waveguide of width a :

$$k_{\perp} = \frac{\pi}{a}$$

$$\gamma = j \sqrt{\left(\frac{\omega}{c}\right)^2 - \left(\frac{\pi}{a}\right)^2}$$

$$Z_0 = \frac{\omega \mu}{k_z}$$

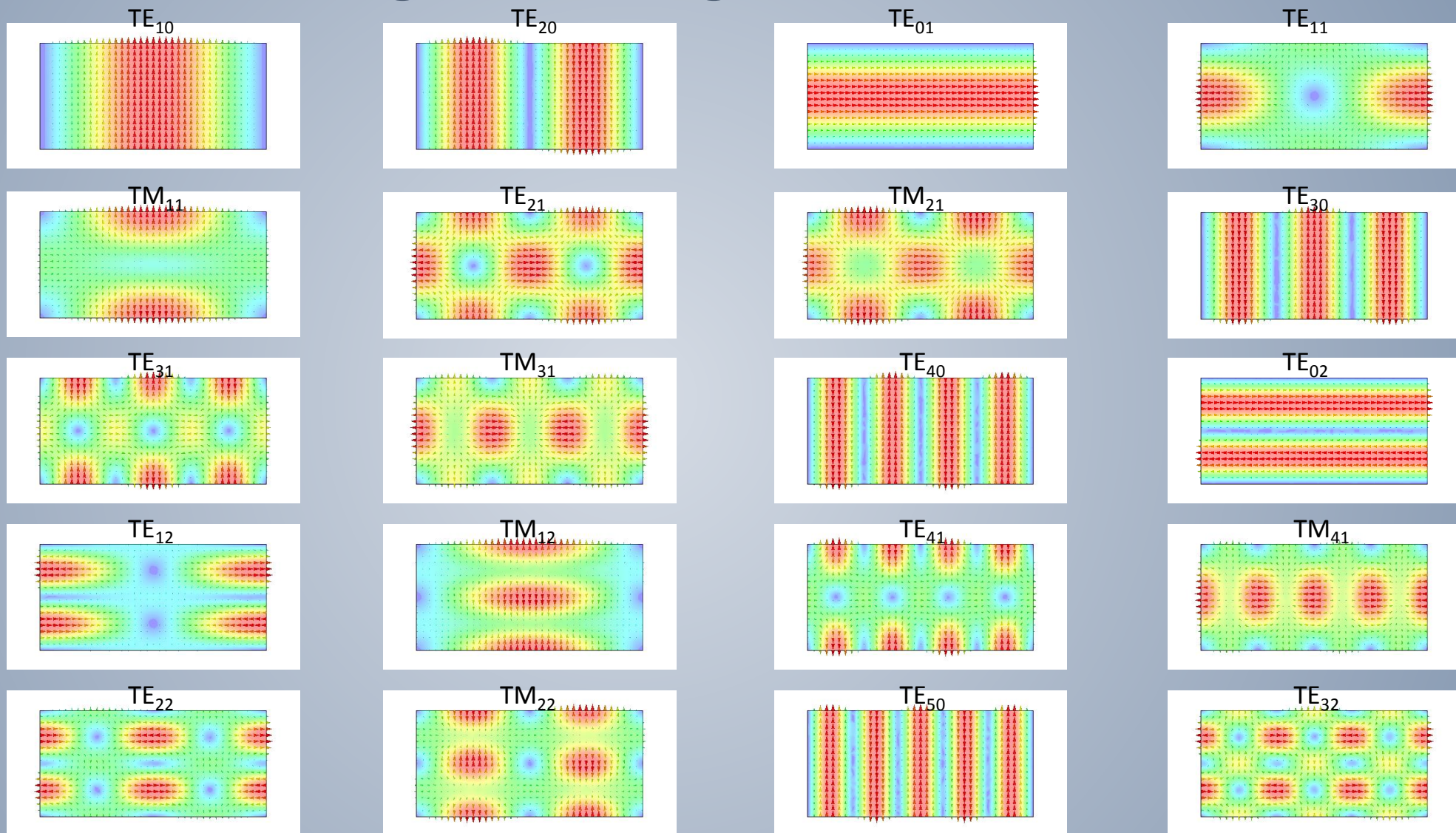
$$\lambda_{\text{cutoff}} = 2a.$$

In a hollow waveguide: phase velocity $v_{\varphi} > c$, group velocity $v_{gr} < c$, $v_{gr} \cdot v_{\varphi} = c^2$.



Photo:
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Rectangular waveguide modes



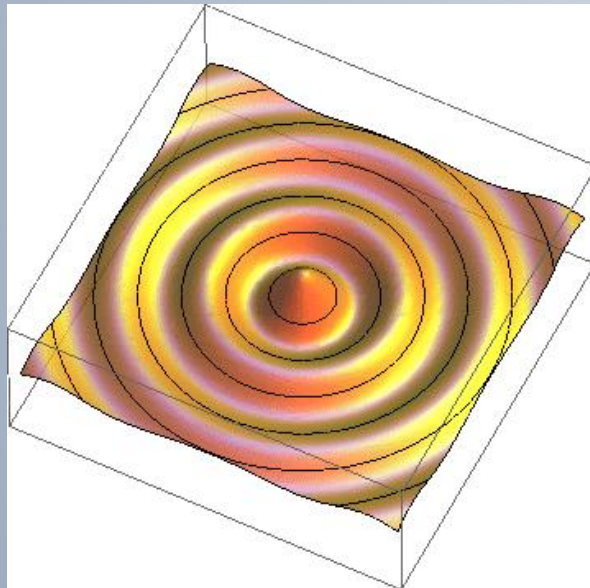
plotted: E -field



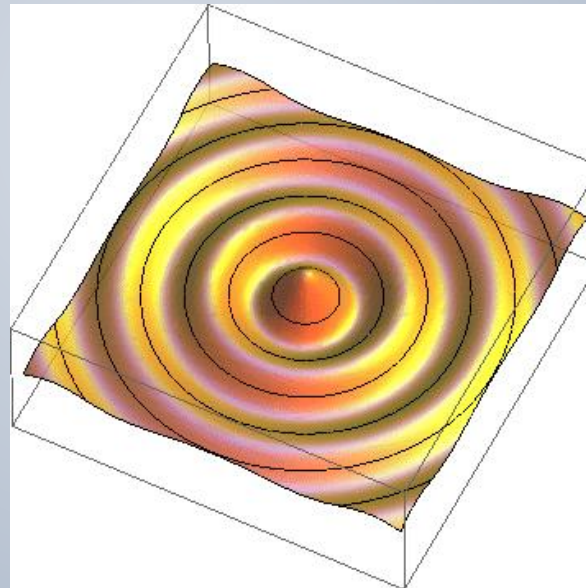
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Radial waves

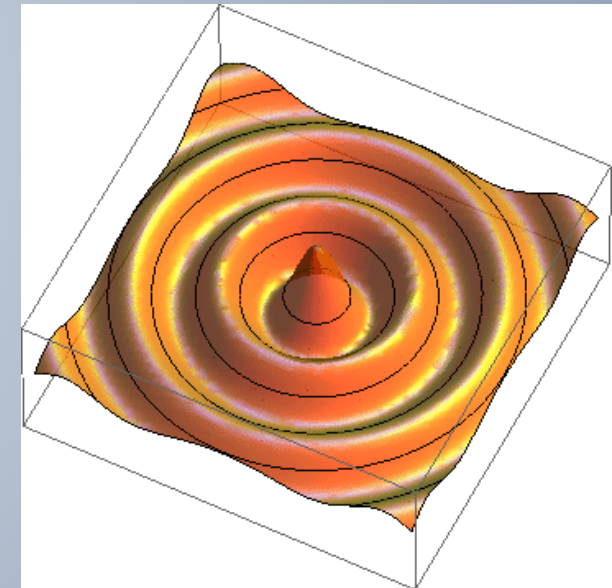
Also radial waves may be interpreted as superposition of plane waves.
The superposition of an outward and an inward radial wave can result
in the field of a round hollow waveguide.



$$E_z \propto H_n^{(2)}(k_\rho \rho) \cos(n\varphi)$$



$$E_z \propto H_n^{(1)}(k_\rho \rho) \cos(n\varphi)$$



$$E_z \propto J_n(k_\rho \rho) \cos(n\varphi)$$

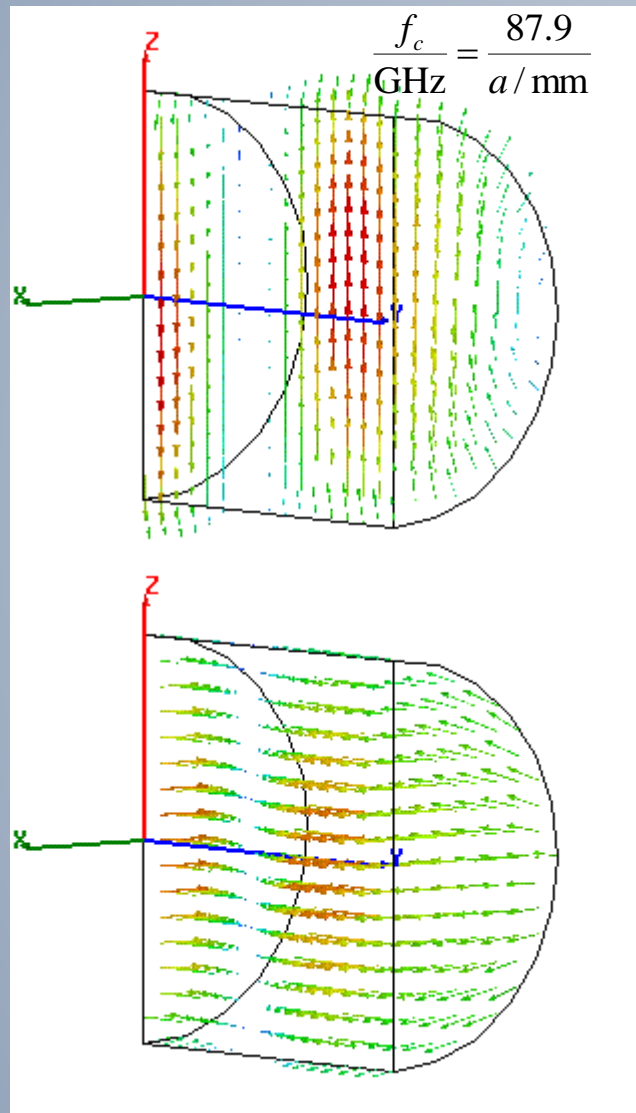


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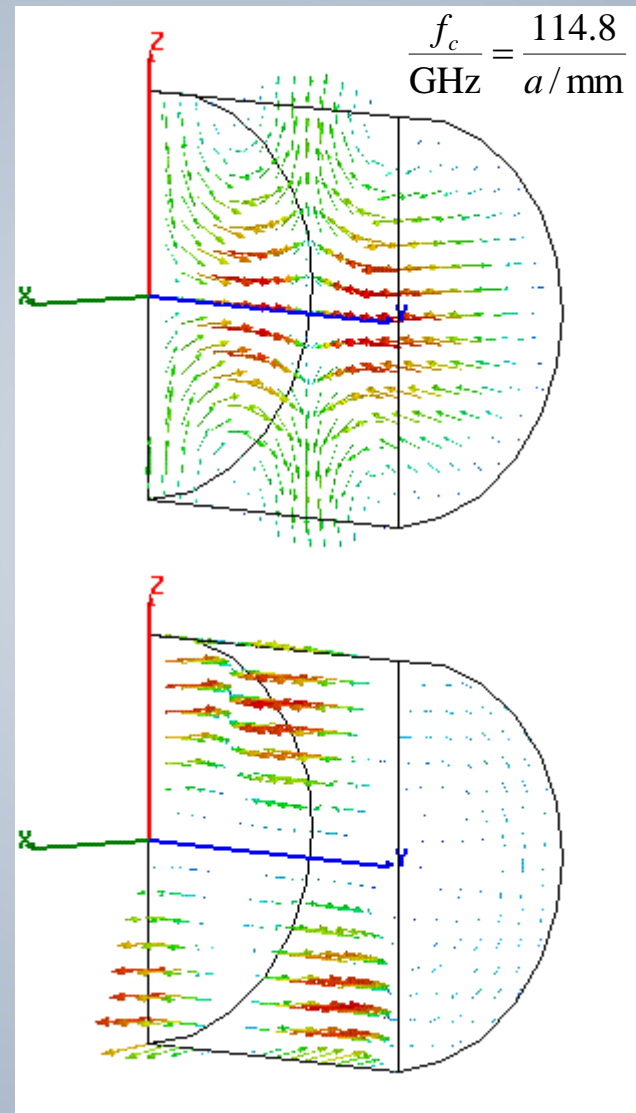
Round waveguide

$$f/f_c = 1.44$$

TE₁₁ – fundamental



TM₀₁ – axial field



TE₀₁ – low loss

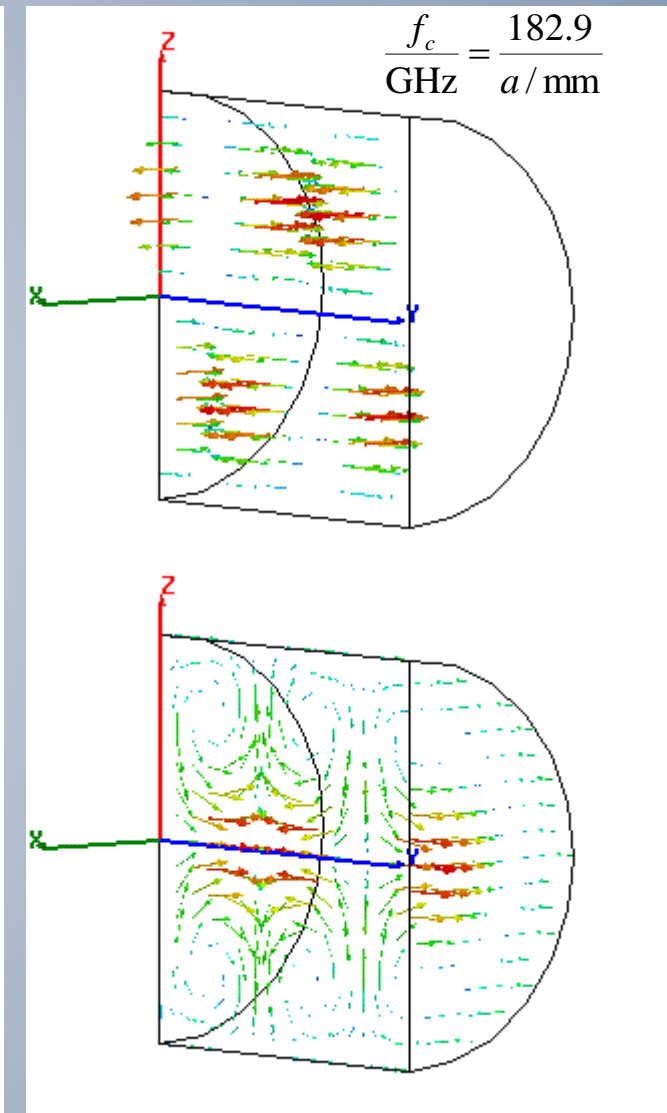
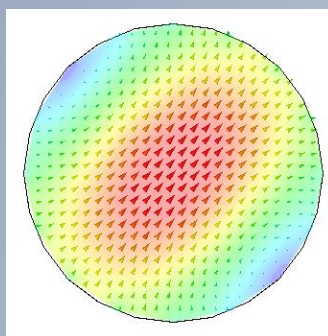


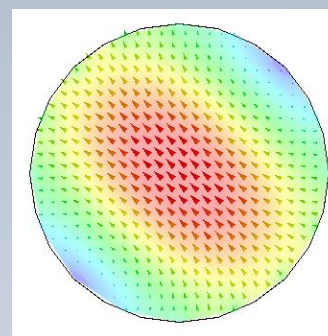


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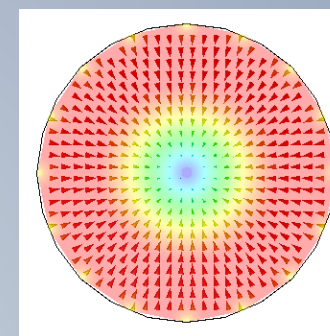
Circular waveguide modes



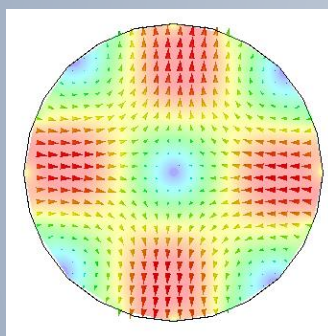
TE_{11}



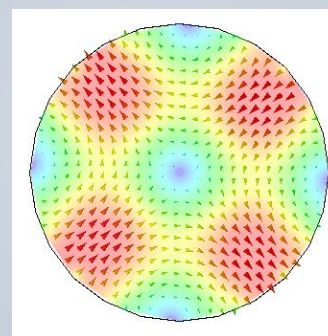
TE_{11}



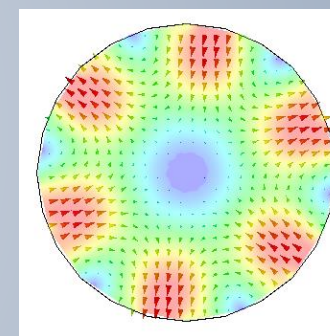
TM_{01}



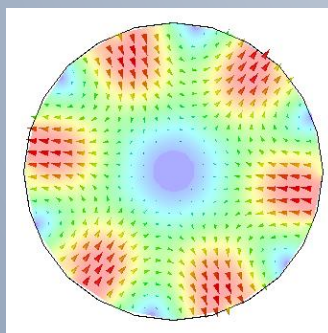
TE_{21}



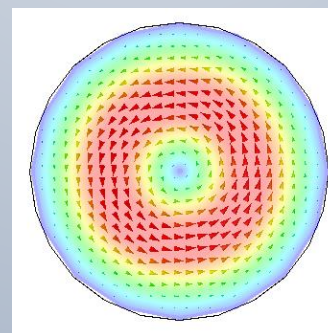
TE_{21}



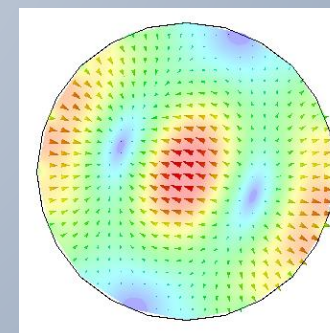
TE_{31}



TE_{31}



TE_{01}



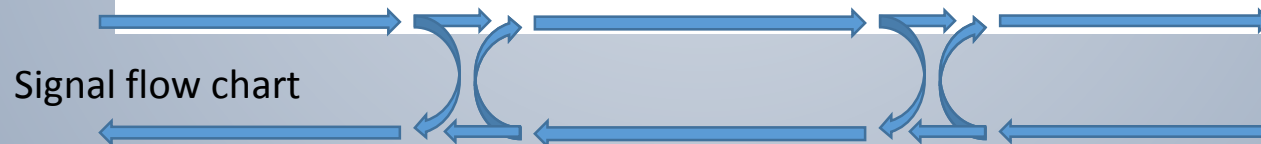
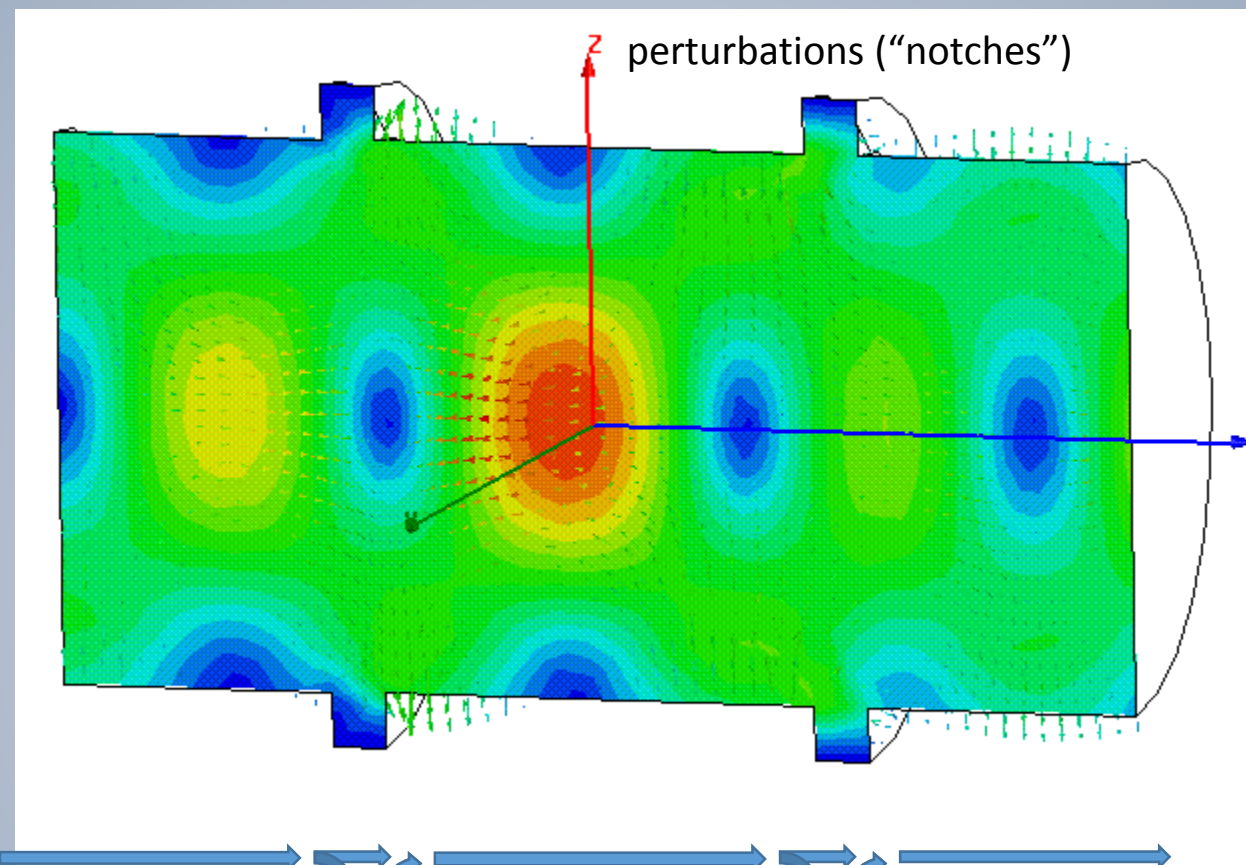
TM_{11}

plotted: E -field



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Waveguide perturbed by notches



Signal flow chart

Reflections from notches lead to a superimposed standing wave pattern.
"Trapped mode"

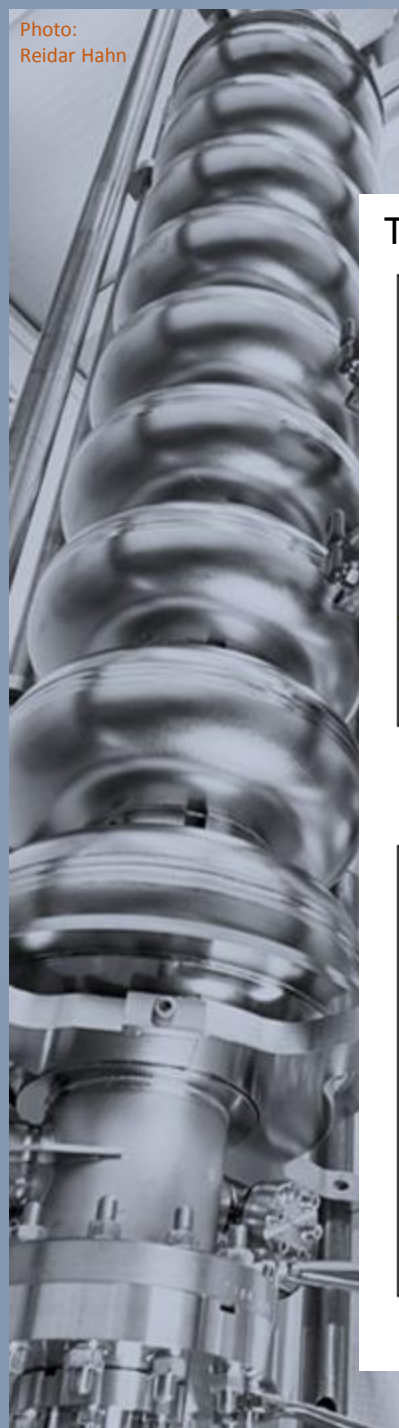
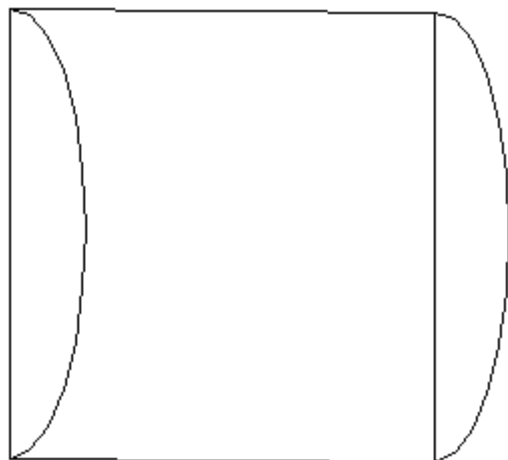
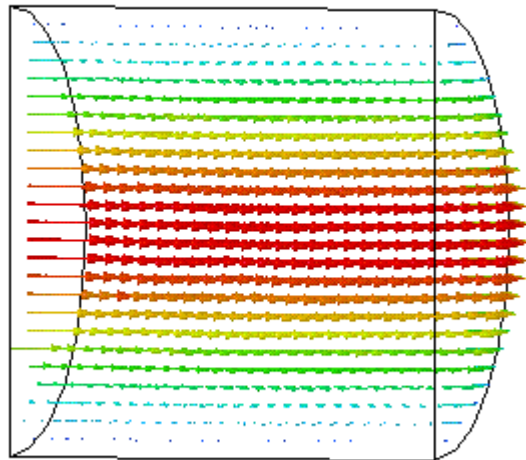


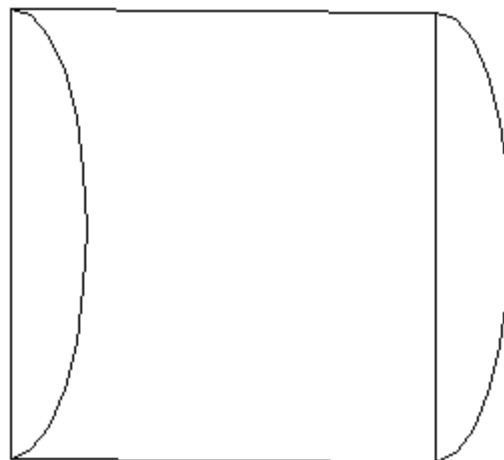
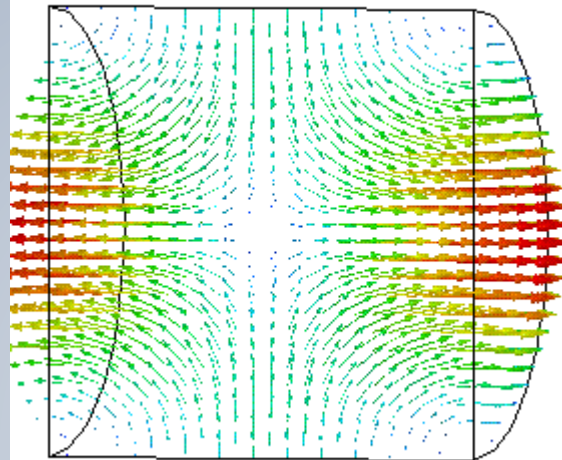
Photo:
Reidar Hahn

Short-circuited waveguide

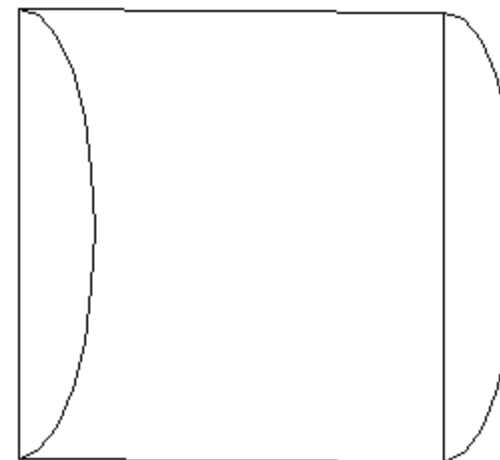
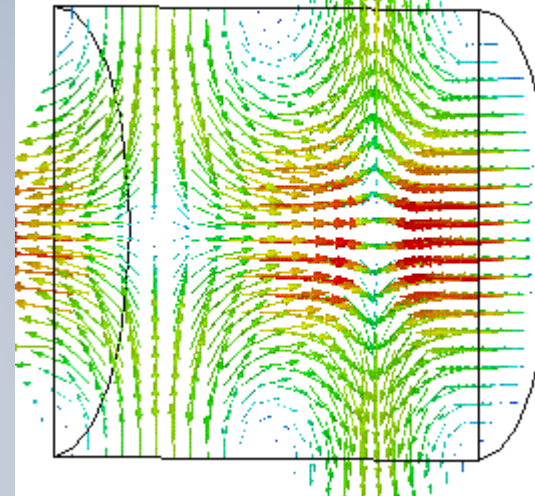
TM_{010} (no axial dependence)



TM_{011}

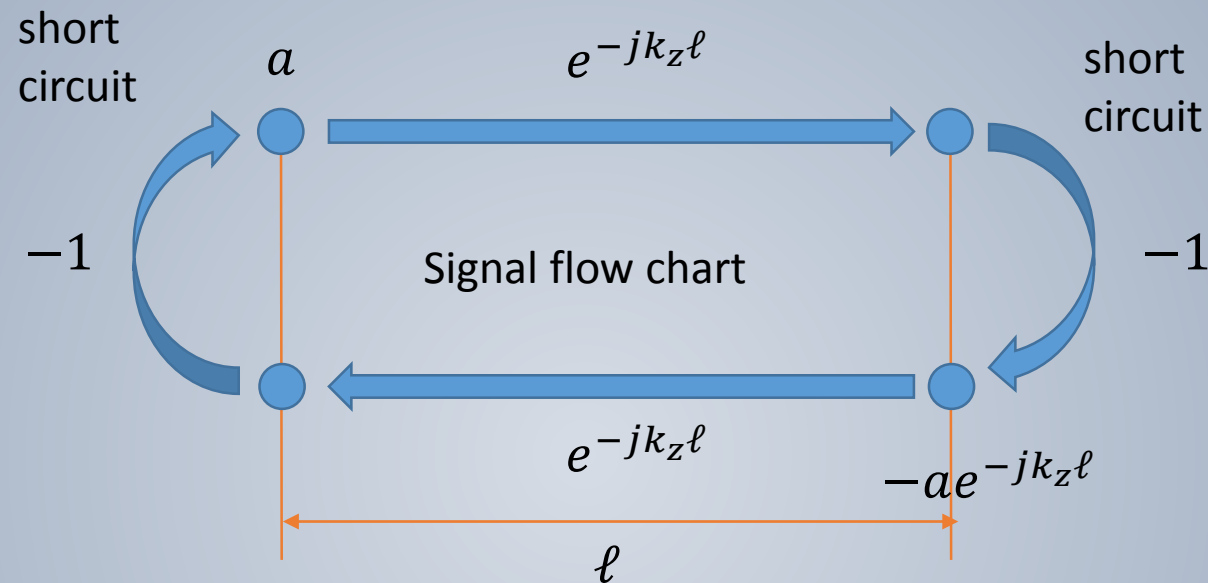


TM_{012}





Single WG mode between two shorts



Eigenvalue equation for field amplitude a :

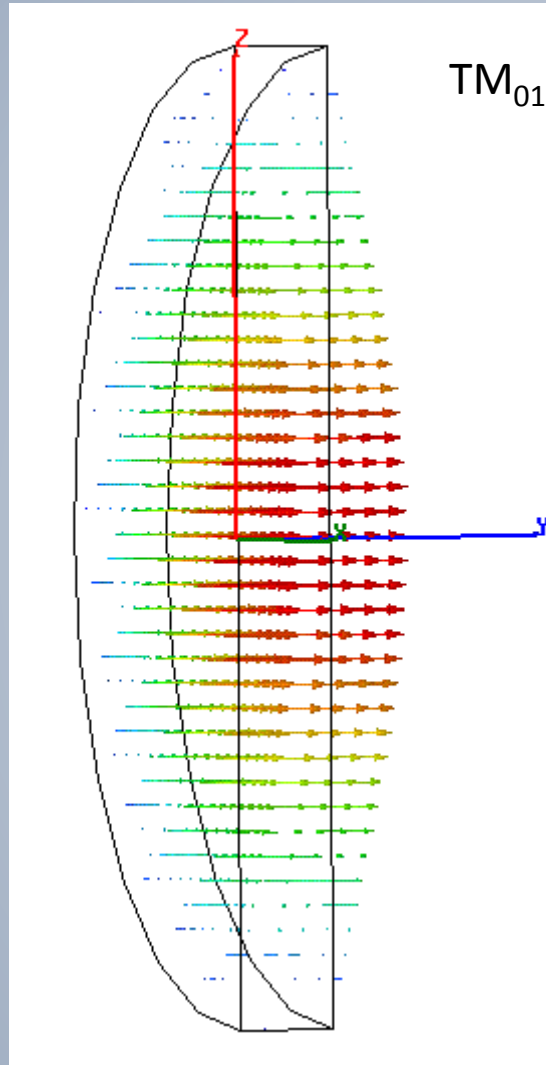
$$a = e^{-jk_z 2\ell} a$$

Non-vanishing solutions exist for $2k_z\ell = 2\pi m$:

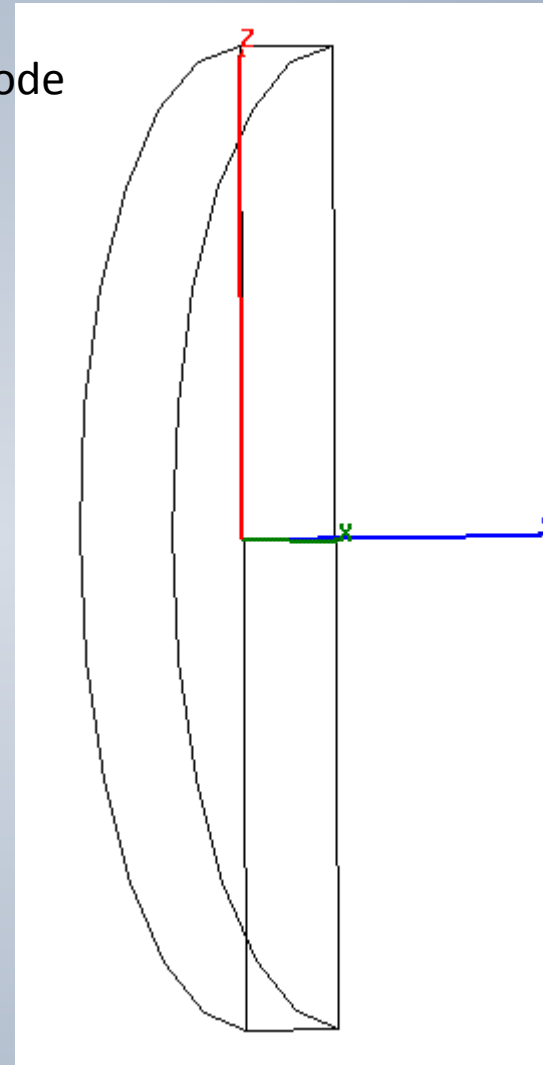
With $k_z = \frac{\omega}{c} \sqrt{1 - \left(\frac{\omega_c}{\omega}\right)^2}$, this becomes $f_0^2 = f_c^2 + \left(c \frac{m}{2\ell}\right)^2$.



Simple pillbox (only 1/2 shown)



electric field (purely axial)



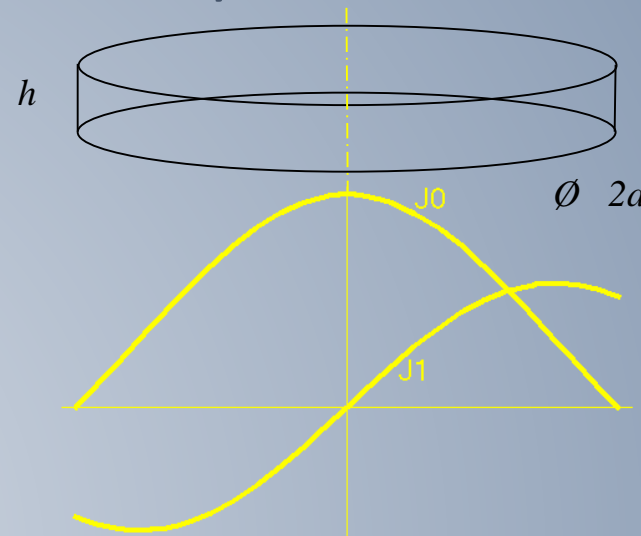
magnetic field (purely azimuthal)



Photo:
Reidar Hahn

Pillbox cavity field (w/o beam tube)

$$T(\rho, \varphi) = \sqrt{\frac{1}{\pi}} \frac{J_0\left(\frac{\chi_{01}\rho}{a}\right)}{\chi_{01} J_1\left(\frac{\chi_{01}}{a}\right)} \quad \text{with } \chi_{01} = 2.40483 \dots$$



The only non-vanishing field components :

$$E_z = \frac{1}{j\omega\epsilon} \frac{\chi_{01}}{a} \sqrt{\frac{1}{\pi}} \frac{J_0\left(\frac{\chi_{01}\rho}{a}\right)}{a J_1\left(\frac{\chi_{01}}{a}\right)}$$

$$B_\varphi = \mu_0 \sqrt{\frac{1}{\pi}} \frac{J_1\left(\frac{\chi_{01}\rho}{a}\right)}{a J_1\left(\frac{\chi_{01}}{a}\right)}$$

$$\omega_0|_{\text{pillbox}} = \frac{\chi_{01}c}{a}, \quad \eta = \sqrt{\frac{\mu_0}{\epsilon_0}} = 377 \, \Omega$$

$$Q|_{\text{pillbox}} = \frac{\sqrt{2a\eta\sigma\chi_{01}}}{2\left(1 + \frac{a}{h}\right)}$$

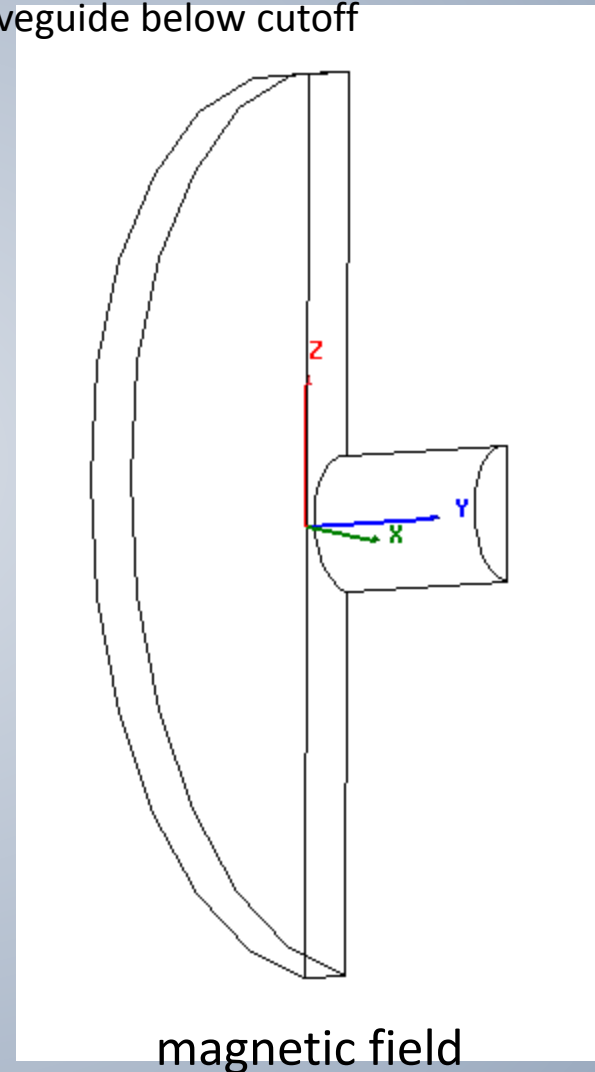
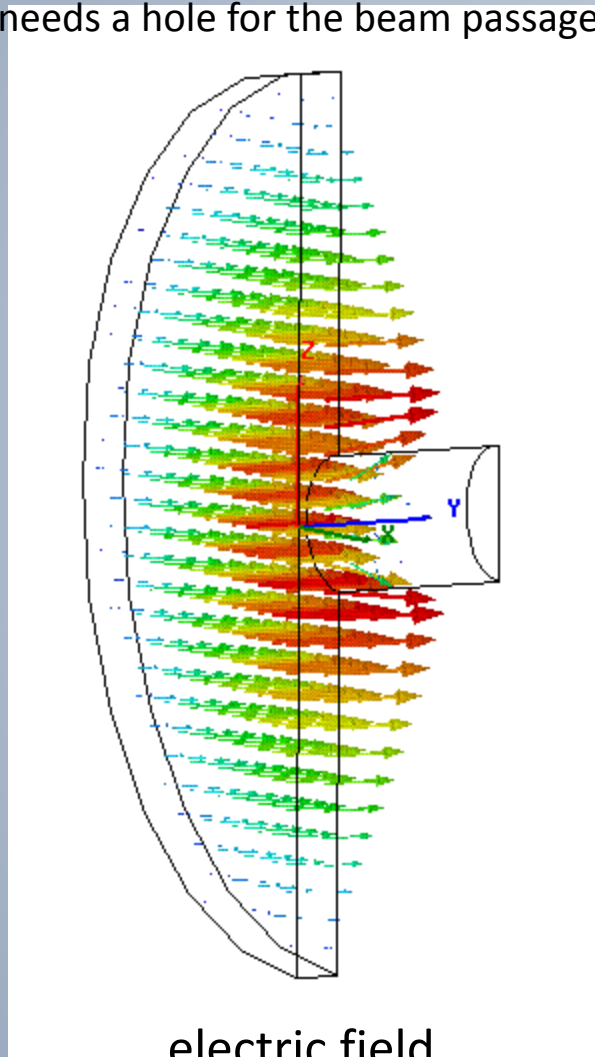
$$\frac{R}{Q}|_{\text{pillbox}} = \frac{4\eta}{\chi_{01}^3 \pi J_1^2(\chi_{01})} \frac{\sin^2\left(\frac{\chi_{01}}{2} \frac{h}{a}\right)}{h/a}$$



Pillbox with beam pipe

TM_{010} -mode (only 1/4 shown)

One needs a hole for the beam passage – circular waveguide below cutoff

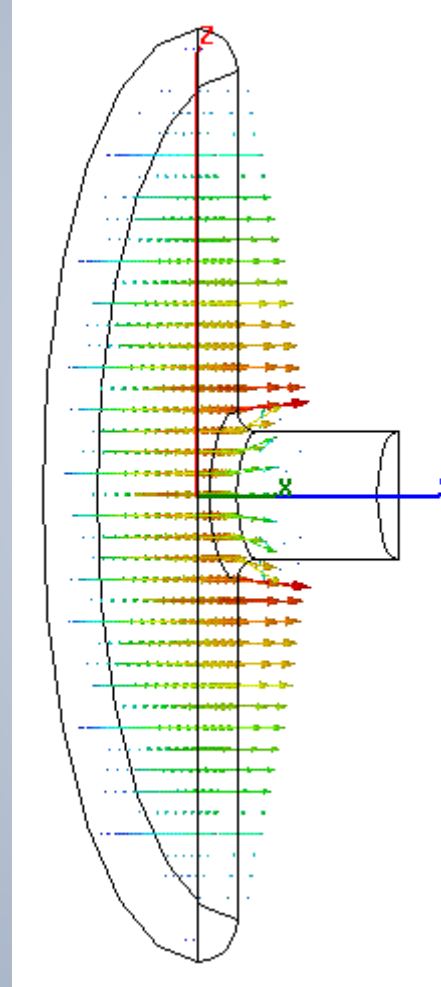




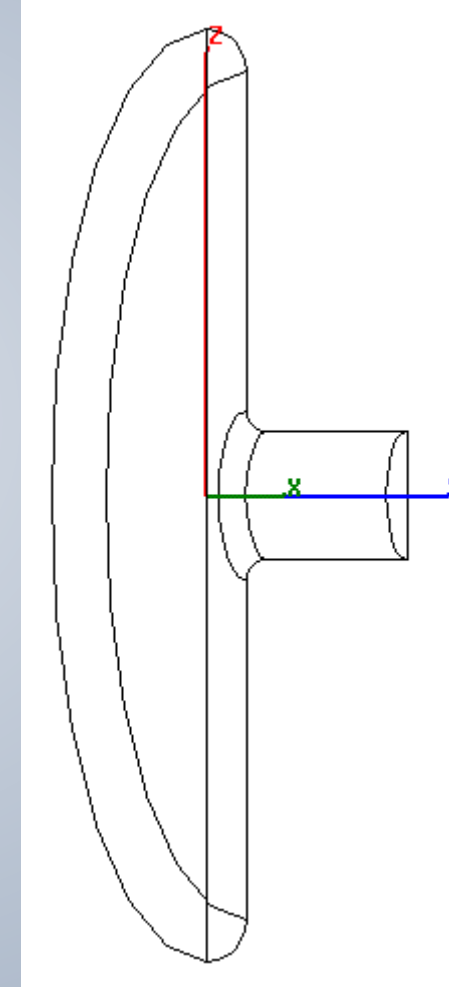
A more practical pillbox cavity

Rounding of sharp edges (to reduce field enhancement!)

TM_{010} -mode (only 1/4 shown)



electric field

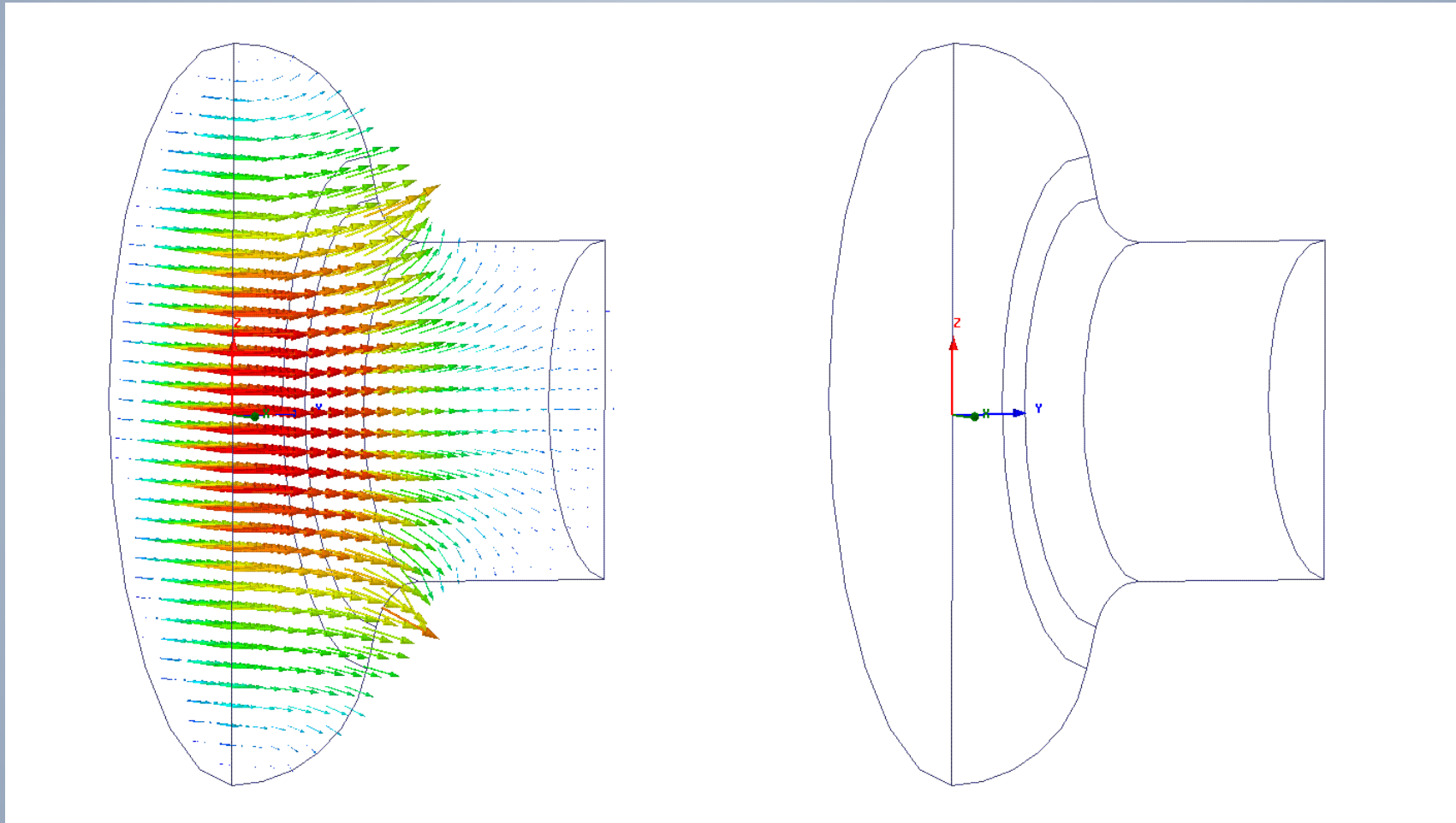


magnetic field



A (real) elliptical cavity

TM_{010} -mode (only 1/4 shown)



electric field

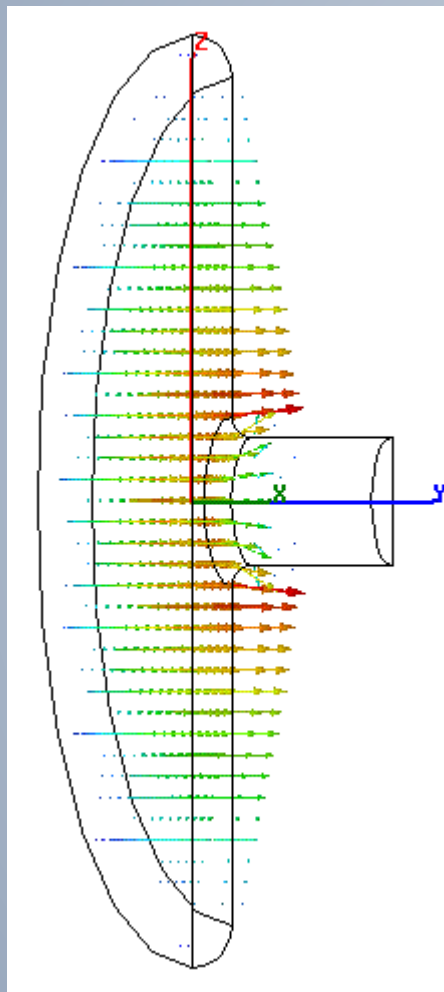
magnetic field

Characterizing a Cavity



Photo:
Reidar Hahn

Acceleration voltage and R/Q



electric field

- I define

$$V_{\text{acc}} = \int_{-\infty}^{\infty} E_z e^{j \frac{\omega}{\beta c} z} dz.$$

- The exponential factor accounts for the variation of the field while particles with velocity βc are traversing the cavity gap.
- With this definition, V_{acc} is generally complex – this becomes important with more than one gap (cell).
- For the time being we are only interested in $|V_{\text{acc}}|$.
- The square of the acceleration voltage $|V_{\text{acc}}|^2$ is proportional to the stored energy W ; the proportionality constant defines the quantity called “ R -upon- Q ”:

$$\frac{R}{Q} = \frac{|V_{\text{acc}}|^2}{2\omega_0 W}.$$

- **Attention – different definitions are used in literature!**



Transit time factor

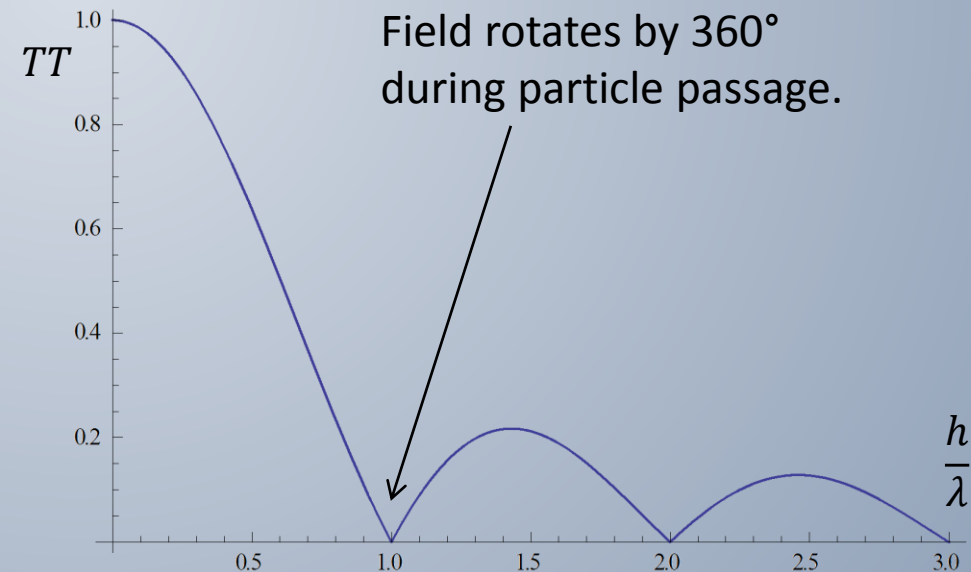
- The transit time factor is the ratio of the acceleration voltage to the (non-physical) voltage a particle with infinite velocity would see:

$$TT = \frac{|V_{acc}|}{\left| \int E_z dz \right|} = \frac{\left| \int E_z e^{j \frac{\omega}{\beta c} z} dz \right|}{\left| \int E_z dz \right|}.$$

- The transit time factor of an ideal pillbox cavity (no axial field dependence) of height (gap length) h is:

$$TT = \frac{\sin\left(\frac{\chi_{01} h}{2a}\right)}{\frac{\chi_{01} h}{2a}}$$

(remember: $\omega_0 = \frac{2\pi c}{\lambda} = \frac{\chi_{01} c}{a}$)





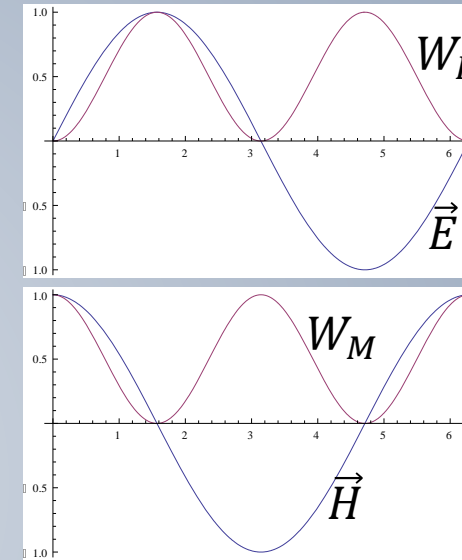
Stored energy

- The energy stored in the electric field is

$$W_E = \iiint_{\text{cavity}} \frac{\epsilon}{2} |\vec{E}|^2 dV.$$

- The energy stored in the magnetic field is

$$W_M = \iiint_{\text{cavity}} \frac{\mu}{2} |\vec{H}|^2 dV.$$

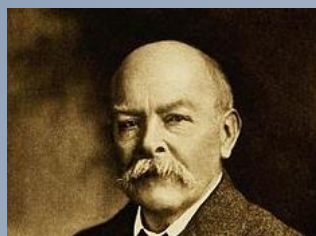


- Since \vec{E} and \vec{H} are 90° out of phase, the stored energy continuously swaps from electric energy to magnetic energy.
- On average, electric and magnetic energy must be equal.
- In steady state, the Poynting vector describes this energy flux.
- In steady state, the total energy stored (constant) is

$$W = \iiint_{\text{cavity}} \left(\frac{\epsilon}{2} |\vec{E}|^2 + \frac{\mu}{2} |\vec{H}|^2 \right) dV.$$

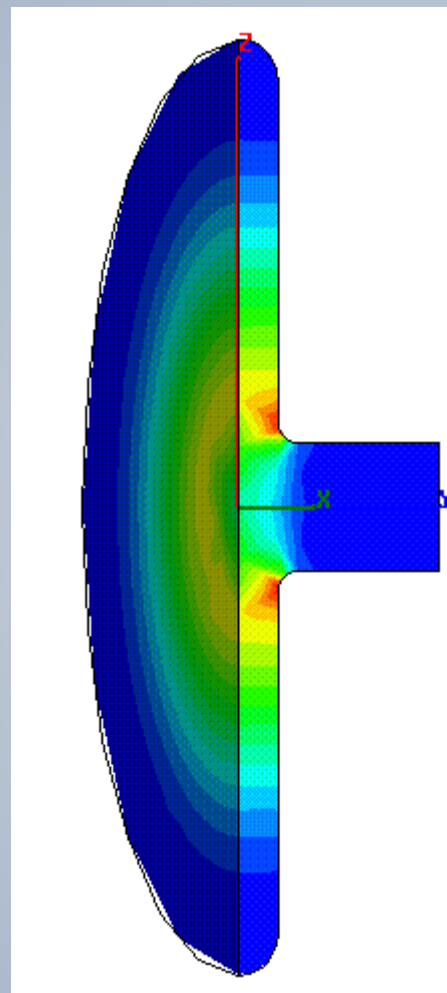


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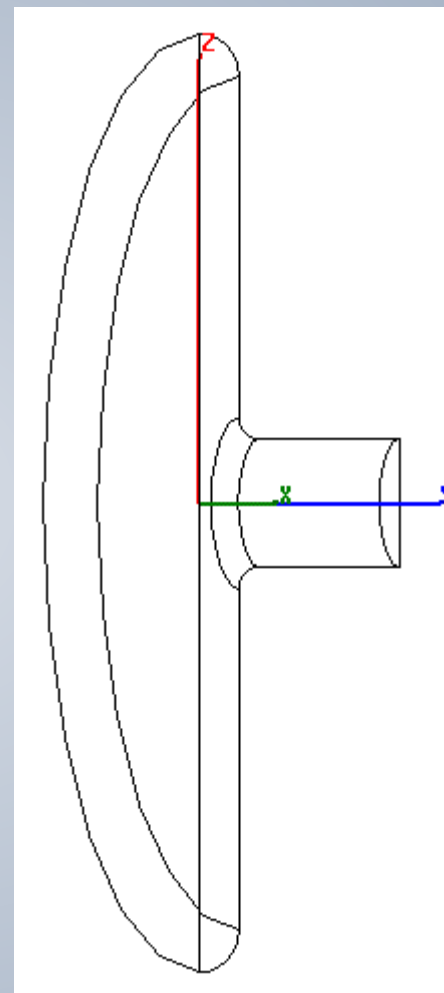


John Henry Poynting
1852 – 1914

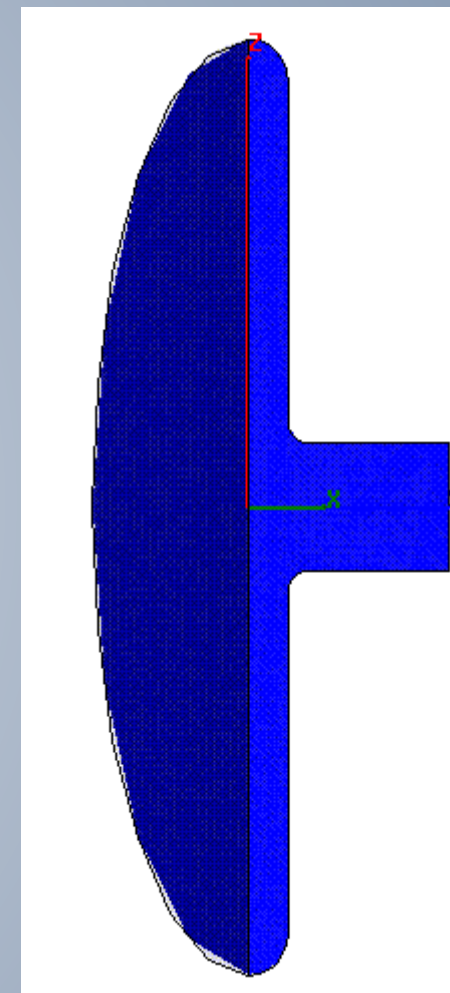
Stored energy and Poynting vector



electric field energy



Poynting vector



magnetic field energy



Wall losses & Q_0

- The losses P_{loss} are proportional to the stored energy W .
- The tangential \vec{H} on the surface is linked to a surface current $\vec{J}_A = \vec{n} \times \vec{H}$ (flowing in the skin depth $\delta = \sqrt{2 / (\omega \mu \sigma)}$).
- This surface current \vec{J}_A sees a surface resistance R_s , resulting in a local power density $R_s |H_t|^2$ flowing into the wall.
- R_s is related to skin depth δ as $\delta \sigma R_s = 1$.
 - Cu at 300 K has $\sigma \approx 5.8 \cdot 10^7 \text{ S/m}$, leading to $R_s \approx 8 \text{ m}\Omega$ at 1 GHz, scaling with $\sqrt{\omega}$.
 - Nb at 2 K has a typical $R_s \approx 10 \text{ n}\Omega$ at 1 GHz, scaling with ω^2 .
- The total wall losses result from $P_{\text{loss}} = \iint_{\text{wall}} R_s |H_t|^2 dA$.
- The cavity Q_0 (caused by wall losses) is defined as $Q_0 = \frac{\omega_0 W}{P_{\text{loss}}}$.
- Typical Q_0 values:
 - Cu at 300 K (normal-conducting): $\mathcal{O}(10^3 \dots 10^5)$, **should** improve at cryogenic T by a factor $\sqrt{\rho_{\text{RRR}}}$.
 - Nb at 2 K (superconducting): $\mathcal{O}(10^9 \dots 10^{11})$

No! Anomalous skin effect!

improves only by a factor $\approx 10!$



Shunt impedance

- Also the power loss P_{loss} is also proportional to the square of the acceleration voltage $|V_{\text{acc}}|^2$; the proportionality constant defines the “shunt impedance”

$$R = \frac{|V_{\text{acc}}|^2}{2 P_{\text{loss}}}.$$

- **Attention, also here different definitions are used!**
- Traditionally, the shunt impedance is the quantity to optimize in order to minimize the power required for a given gap voltage.
- Now the previously introduced term “ R -upon- Q ” makes sense:

$$\left(\frac{R}{Q}\right) = R/Q$$



Geometric factor

- With

$$Q_0 = \frac{\omega_0 W}{\iint_{\text{wall}} R_s |H_t|^2 dA},$$

and assuming an average surface resistance R_s , one can introduce the “geometric factor” G as

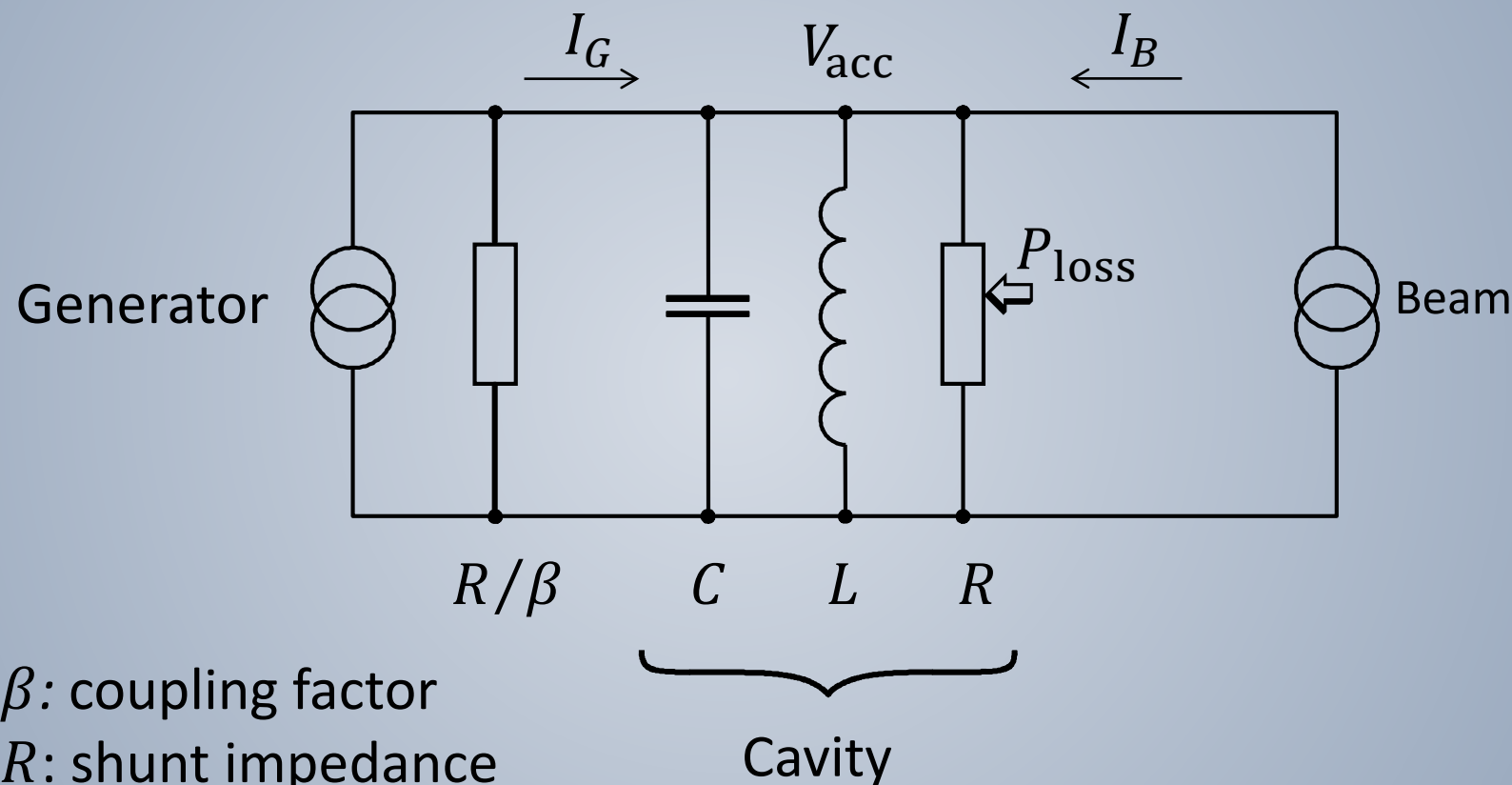
$$G = Q_0 \cdot R_s = \frac{\omega_0 W}{\iint_{\text{wall}} |H_t|^2 dA}.$$

- G has dimension Ohm, depends only on the cavity geometry (as the name suggests) and typically is $\mathcal{O}(100 \Omega)$.
- Note that $R_s \cdot R = G \cdot (R/Q)$ (dimension Ω^2 , purely geometric)
- G is only used for SC cavities.



Cavity resonator – equivalent circuit

Simplification: single mode



β : coupling factor

R : shunt impedance

$$\sqrt{L/C} = \frac{R}{Q}: R\text{-upon-}Q$$



Photo:
Reidar Hahn

Power coupling - Loaded Q

- Note that the generator inner impedance also loads the cavity – for very large Q_0 more than the cavity wall losses.
- To calculate the loaded Q (Q_L), losses have to be added:

$$\frac{1}{Q_L} = \frac{P_{\text{loss}} + P_{\text{ext}} + \dots}{\omega_0 W} = \frac{1}{Q_0} + \frac{1}{Q_{\text{ext}}} + \frac{1}{\dots}$$

- The coupling factor β is the ratio $P_{\text{ext}}/P_{\text{loss}}$.
- With β , the loaded Q can be written

$$Q_L = \frac{Q_0}{1 + \beta}$$

- For NC cavities, often $\beta = 1$ is chosen (power amplifier matched to empty cavity); for SC cavities, $\beta = \mathcal{O}(10^4 \dots 10^6)$.

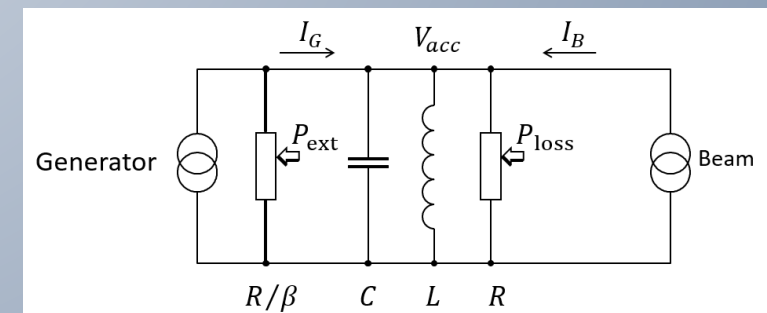
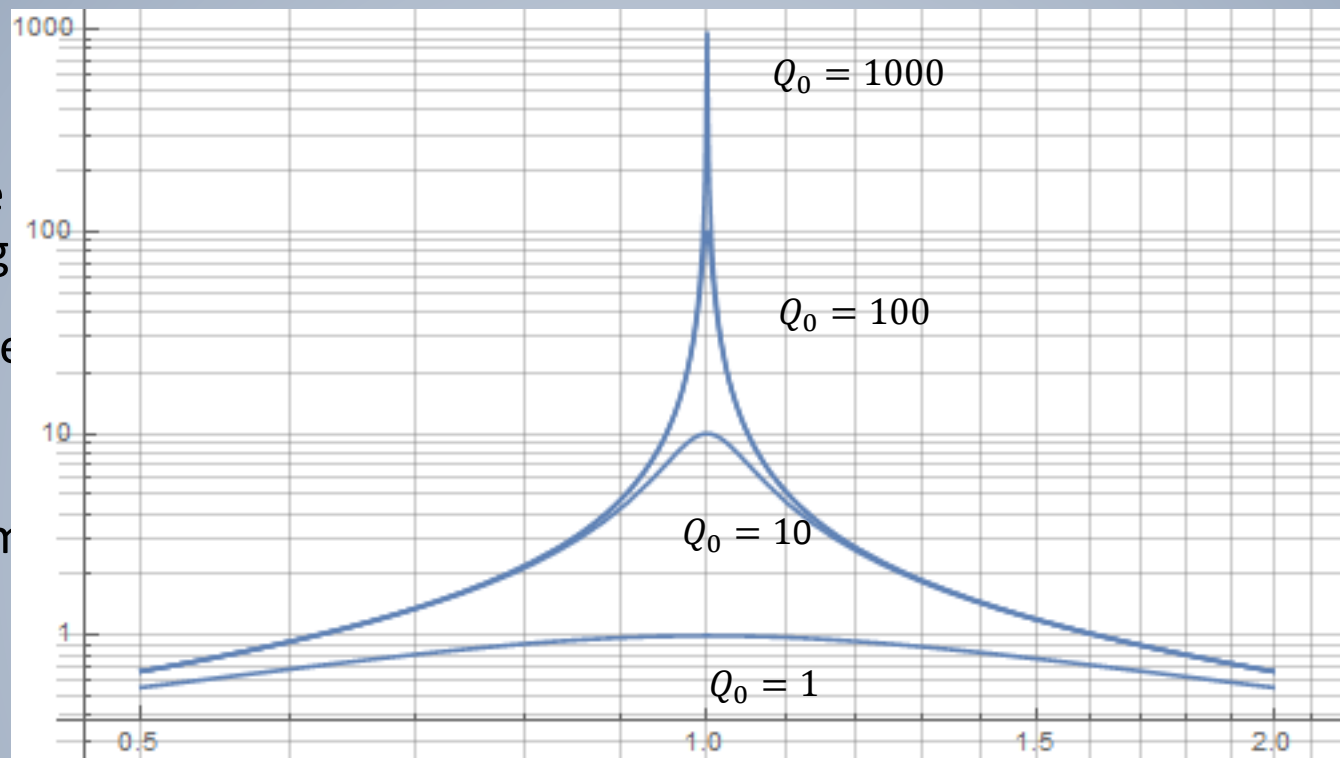




Photo:
Reidar Hahn

$$\frac{Z(\omega)}{R/Q}$$

- While voltage
- On the
- Note:
- ... to m



for the same

1 Hz!

$$\frac{\omega}{\omega_0}$$



Photo:
Reidar Hahn

Loss factor

$$k_{\text{loss}} = \frac{\omega_0}{2} \left(\frac{R}{Q} \right) = \frac{|V_{\text{acc}}|^2}{4W} = \frac{1}{2C}$$

Energy deposited by a single charge q : $k_{\text{loss}} q^2$

Voltage induced by a single charge q : $2 k_{\text{loss}} q$

$$\frac{V_{\text{acc}}}{2 k_{\text{loss}} q}$$

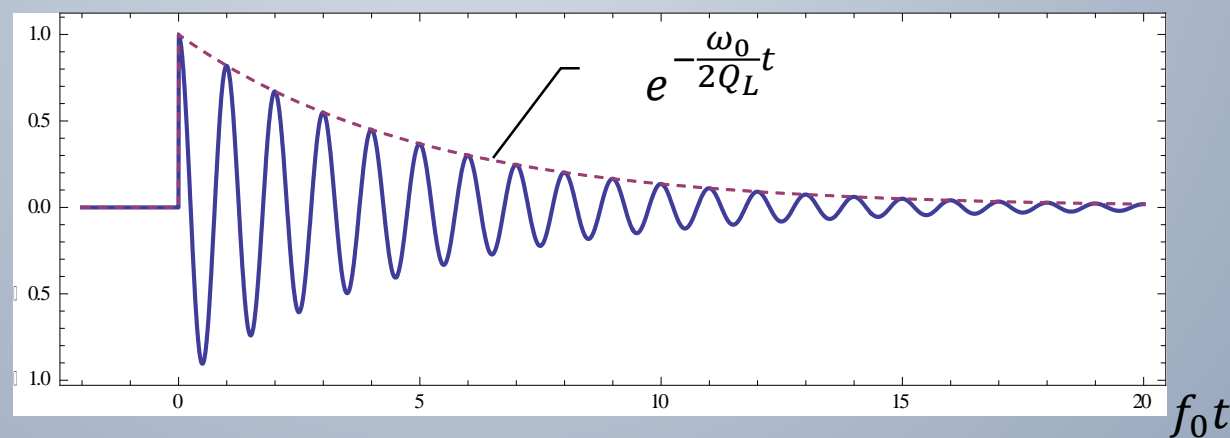
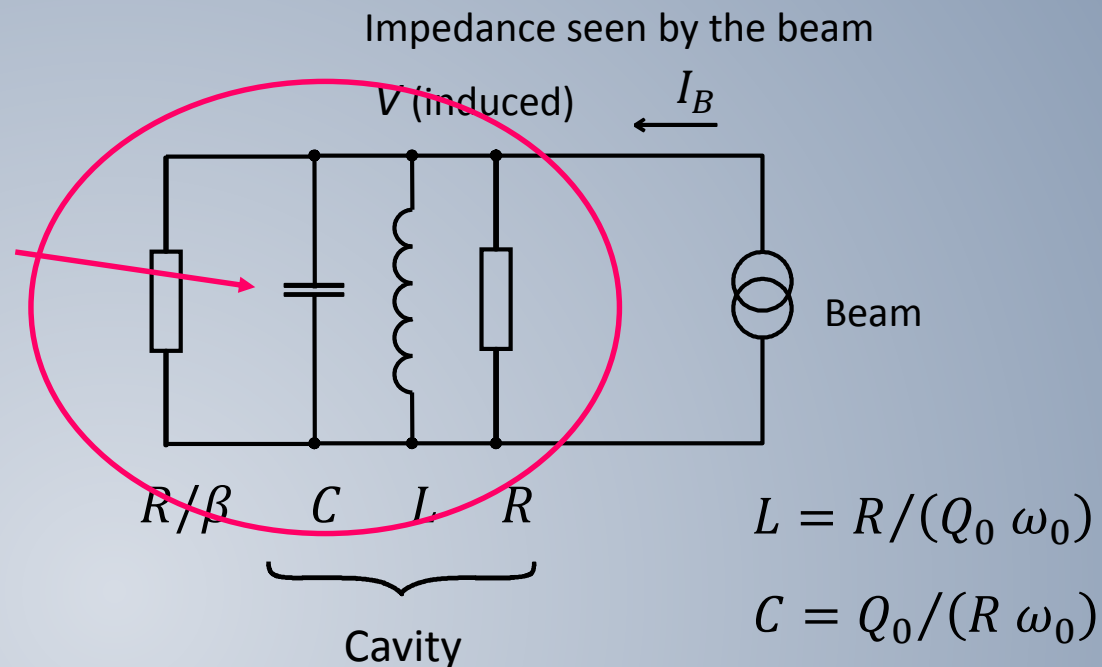
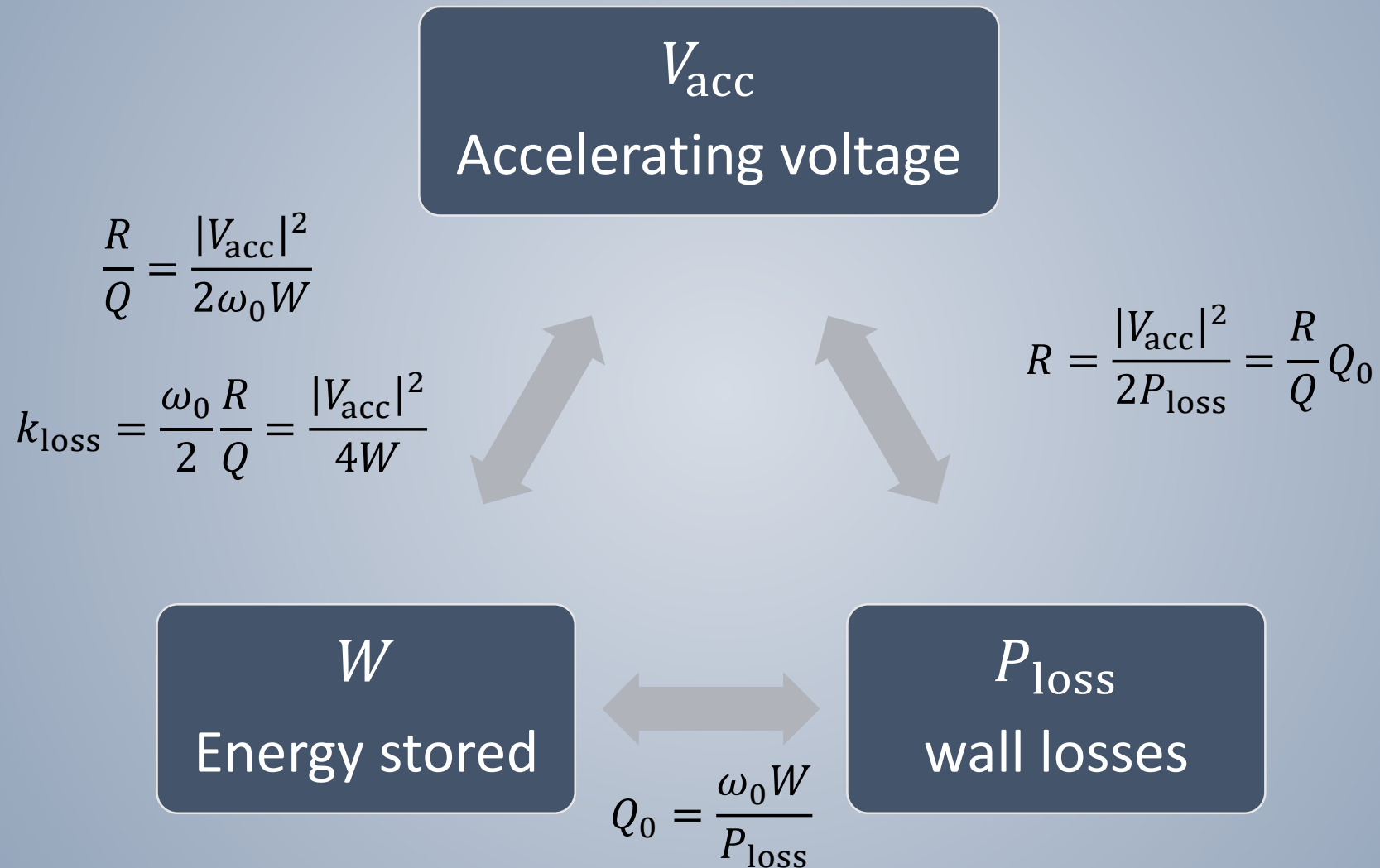




Photo:
Reidar Hahn

Summary: relations V_{acc} , W and P_{loss}

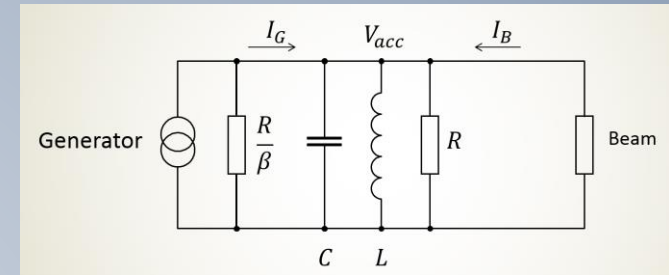
Attention – different definitions are used in literature !





Beam loading

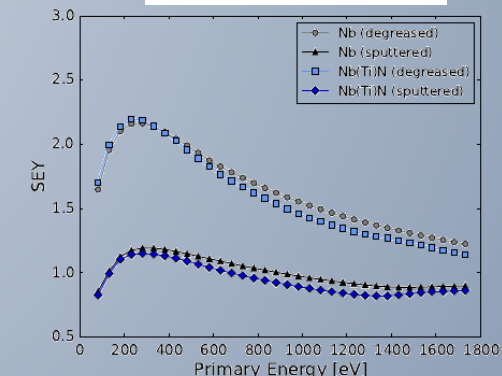
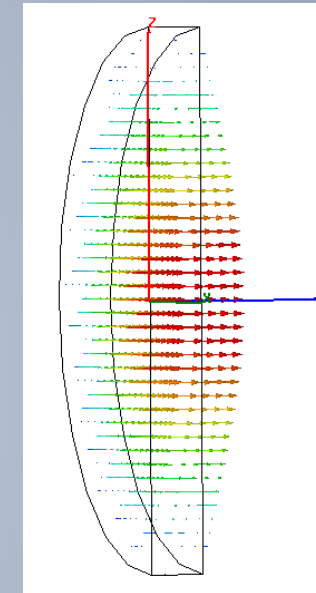
- The beam current “loads” the cavity, in the equivalent circuit this appears as an impedance in parallel to the shunt impedance.
- If the generator is matched to the unloaded cavity ($\beta = 1$), beam loading will (normally) cause the accelerating voltage to decrease.
- The power absorbed by the beam is $-\frac{1}{2} \Re\{V_{acc} I_B^*\}$.
- For high power transfer efficiency RF \rightarrow beam, beam loading must be high!
- For SC cavities (very large β), the generator is typically matched to the beam impedance!
- Variation in the beam current leads to **transient beam loading**, which requires special care!
- Often the “impedance” the beam presents is strongly reactive – this leads to a detuning of the cavity.





Multipactor

- The words “multipactor”, “to multipact” and “multipacting” are artificially composed of “multiple” “impact”.
- Multipactor describes a resonant RF phenomenon in vacuum:
 - Consider a free electron in a simple cavity – it gets accelerated by the electric field towards the wall
 - when it impacts the wall, secondary electrons will be emitted, described by the secondary emission yield (SEY)
 - in certain impact energy ranges, more than one electron is emitted for one electron impacting! So the number of electrons can increase
 - When the time for an electron from emission to impact takes exactly $\frac{1}{2}$ of the RF period, resonance occurs – with the $SEY > 1$, this leads to an avalanche increase of electrons, effectively taking all RF power at this field level, depleting the stored energy and limiting the field!
- For this simple “2-point MP”, this resonance condition is reached at $\frac{1}{4\pi} \frac{e}{m} V = (fd)^2$ or $\frac{V}{112 V} = \left(\frac{f}{\text{MHz}} \frac{d}{\text{m}} \right)^2$. There exist other resonant bands.

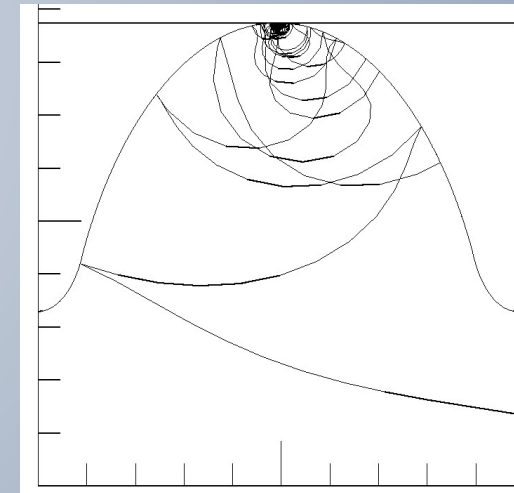


courtesy: Sarah Aull/CERN



Multipactor (contd.)

- Unfortunately, good metallic conductors (Cu, Ag, Nb) all have $SEY > 1$!
- 1-point MP occurs when the electron impact where they were emitted
- Electron trajectories can be complex since both \vec{E} and \vec{B} influence them; computer simulations allow to determine the MP bands (barriers)
- To reduce or suppress MP, a combination of the following may be considered:
 - Use materials with low SEY
 - Optimize the shape of your cavity (\rightarrow elliptical cavity)
 - Conditioning (surface altered by exposure to RF fields)
 - Coating (Ti, TiN, NEG, amorphous C ...)
 - Clearing electrode (for a superimposed DC electric field)
 - Rough surfaces

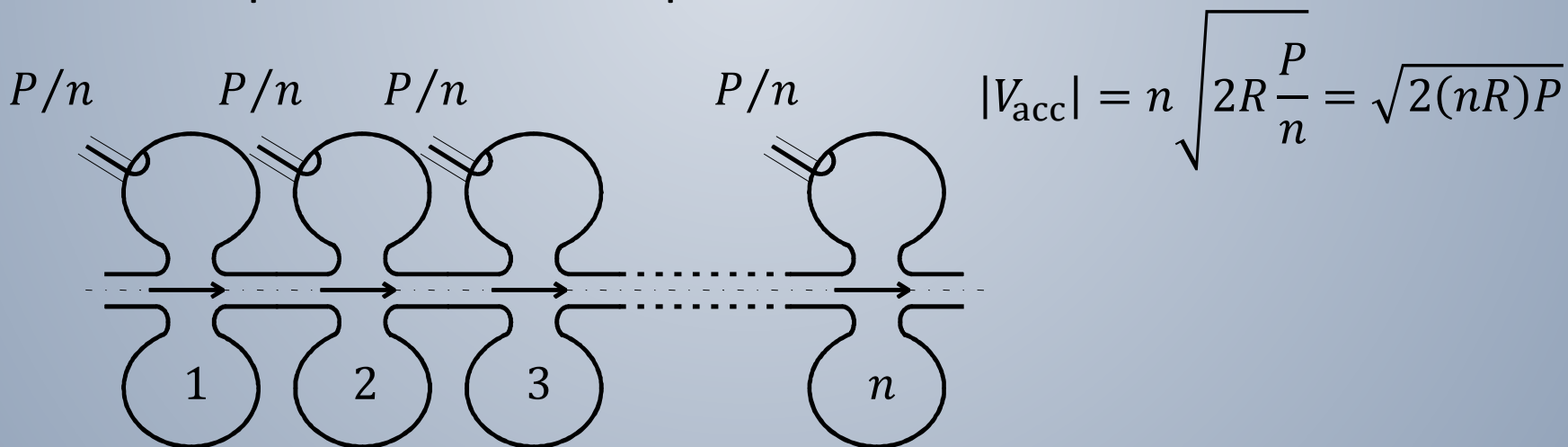


Many gaps



What do you gain with many gaps?

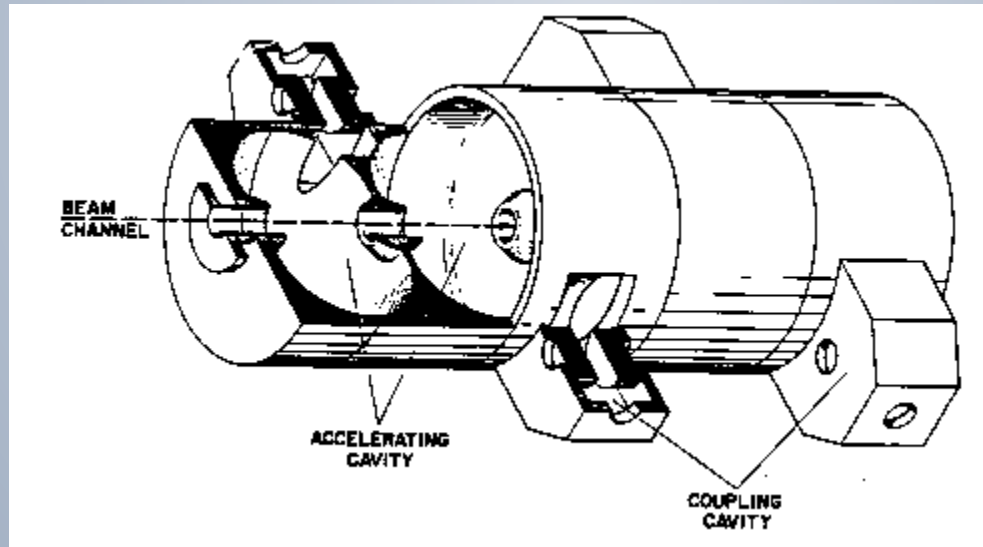
- The R/Q of a single gap cavity is limited to some 100Ω .
Now consider to distribute the available power to n identical cavities: each will receive P/n , thus produce an accelerating voltage of $\sqrt{2RP/n}$. (Attention: phase important!)
The total accelerating voltage thus increased, equivalent to a total equivalent shunt impedance of nR .





Standing wave multi-cell cavity

- Instead of distributing the power from the amplifier, one might as well couple the cavities, such that the power automatically distributes, or have a cavity with many gaps (e.g. drift tube linac).
- Coupled cavity accelerating structure (side coupled)



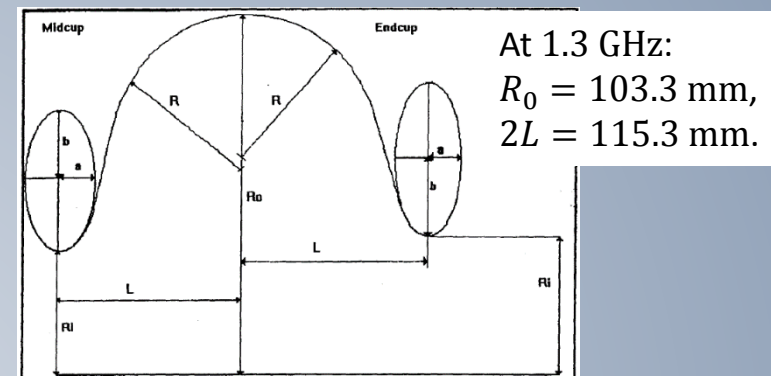
- The phase relation between gaps is important!



Photo:
Reidar Hahn

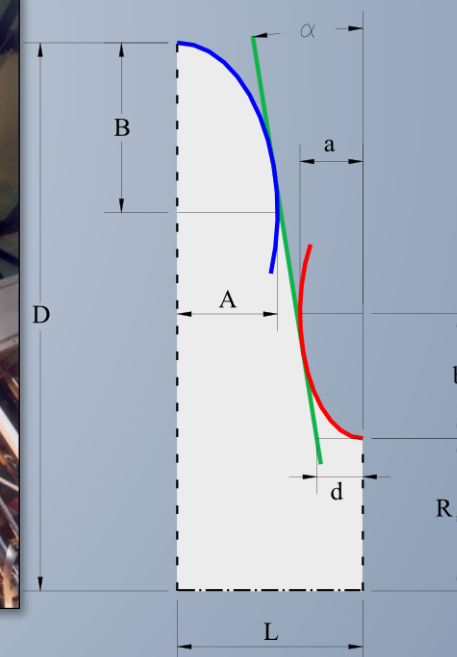
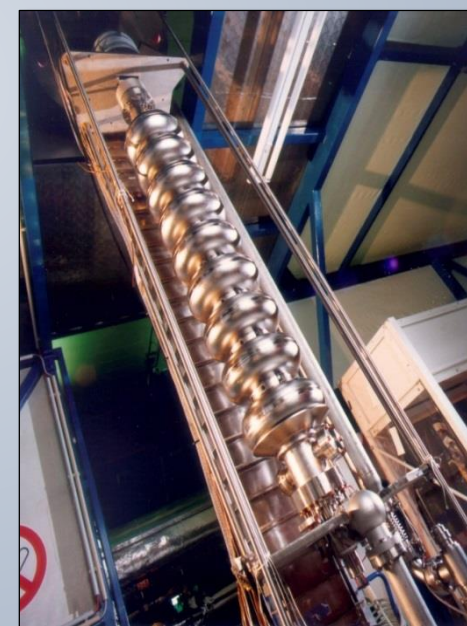
The elliptical cavity

- The elliptical shape was found as optimum compromise between
 - maximum gradient ($E_{\text{acc}}/E_{\text{surface}}$)
 - suppression of multipactor
 - mode purity
 - machinability
- A multi-cell elliptical cavity is typically operated in π -mode, i.e. cell length is exactly $\beta\lambda/2$.
- It has become de facto standard, used for ions and leptons! E.g.:
 - ILC/X-FEL: 1.3 GHz, 9-cell cavity
 - SNS (805 MHz)
 - SPL/ESS (704 MHz)
 - LHC (400 MHz)



At 1.3 GHz:
 $R_0 = 103.3$ mm,
 $2L = 115.3$ mm.

D. Proch, 1993 *)



*) : <http://accelconf.web.cern.ch/AccelConf/SRF93/papers/srf93g01.pdf>

Elliptical cavities



Elliptical cavities – the *de facto* standard for SRF

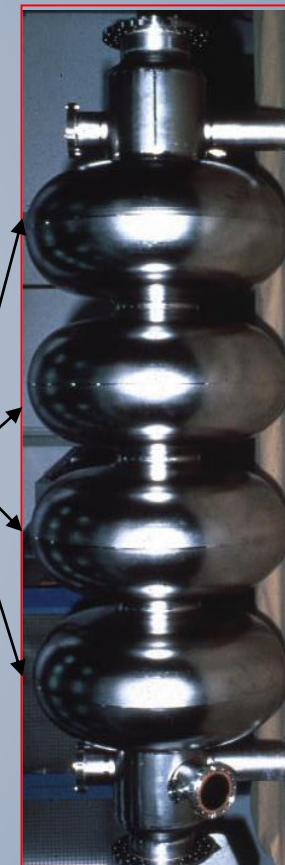
FERMI 3.9 GHz



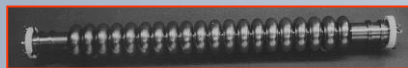
TESLA/ILC 1.3 GHz



LEP 0.352 GHz



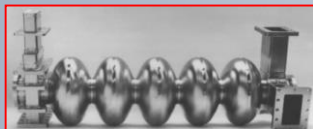
S-DALINAC 3 GHz



SNS $\beta = 0.61, 0.81, 0.805$ GHz



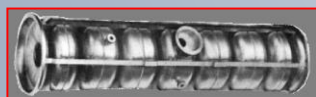
CEBAF 1.5 GHz



HERA 0.5 GHz



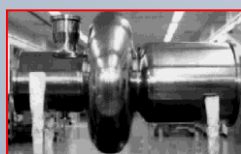
HEPL 1.3 GHz



CESR 0.5 GHz



KEK-B 0.5 GHz



TRISTAN 0.5 GHz

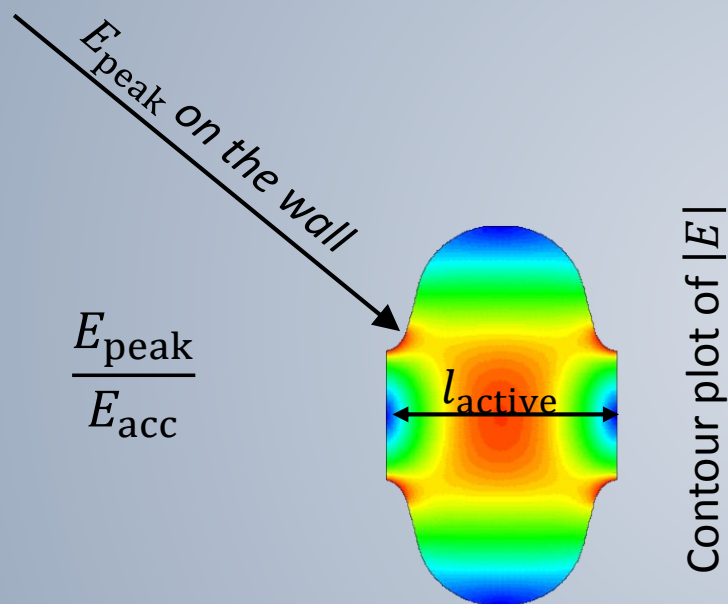


cells

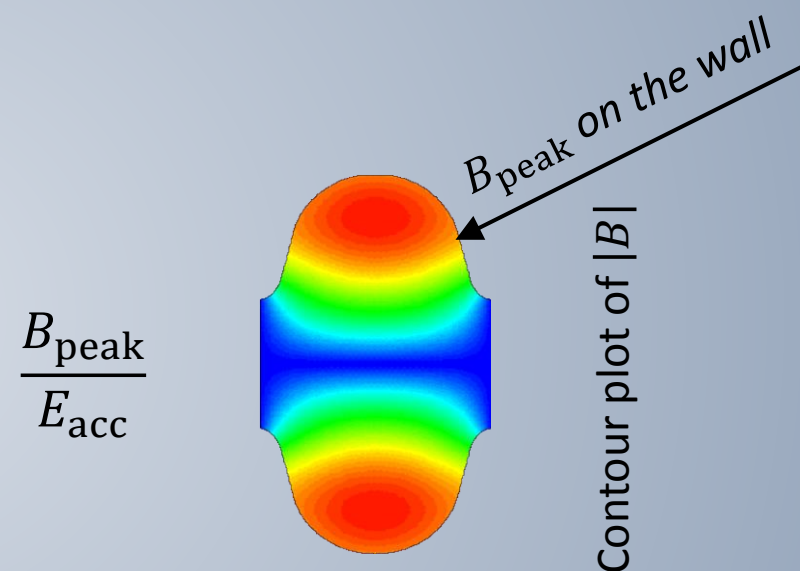


Practical RF parameters 1

- Average accelerating gradient: $E_{\text{acc}} = \frac{\sqrt{\omega W(R/Q)}}{l_{\text{active}}}$



The ratio shows sensitivity of the shape to the **field emission** of electrons.



The ratio shows limit in E_{acc} due to the breakdown of superconductivity (**quench**, Nb: ≈ 190 mT).

courtesy: Jacek Sekutovicz/DESY



Practical RF parameters 2

$$G \cdot (R/Q)$$

- Both G and R/Q are purely geometric parameters.
- Like the shunt impedance R , the product $G \cdot (R/Q)$ is a measure of the power loss for given acceleration voltage V_{acc} and surface resistance R_s .

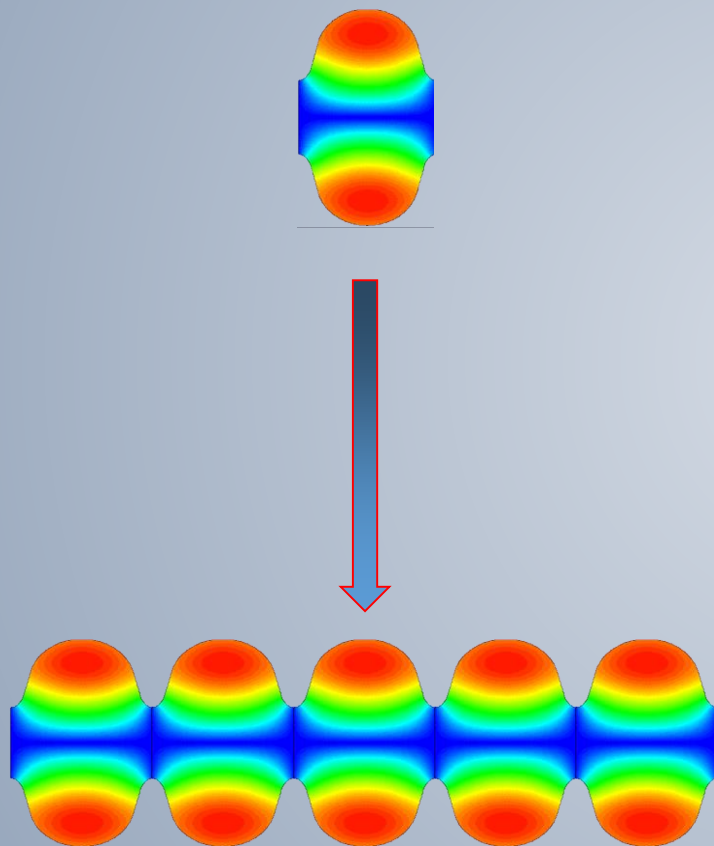
$$P_{\text{loss}} = \frac{|V_{\text{acc}}|^2 R_s}{2 \underbrace{G \cdot (R/Q)}} \quad \begin{array}{l} \text{Minimize } R_s: \\ \text{operation at lower } T, \\ \text{better surface cleanliness,} \\ \text{lower residual resistance} \end{array}$$

Optimize geometry maximizing $G \cdot (R/Q)$.

courtesy: Jacek Sekutovicz/DESY



Single-cell versus multi-cell cavities



- Advantages of single-cell cavities:
 - It is easier to manage HOM damping
 - There is no field flatness problem.
 - Input coupler transfers less power
 - They are easy for cleaning and preparation
- Advantages of multi-cell cavities:
 - much more acceleration per meter!
 - better use of the power ($R \rightarrow n R$)
 - more cost-effective for most applications

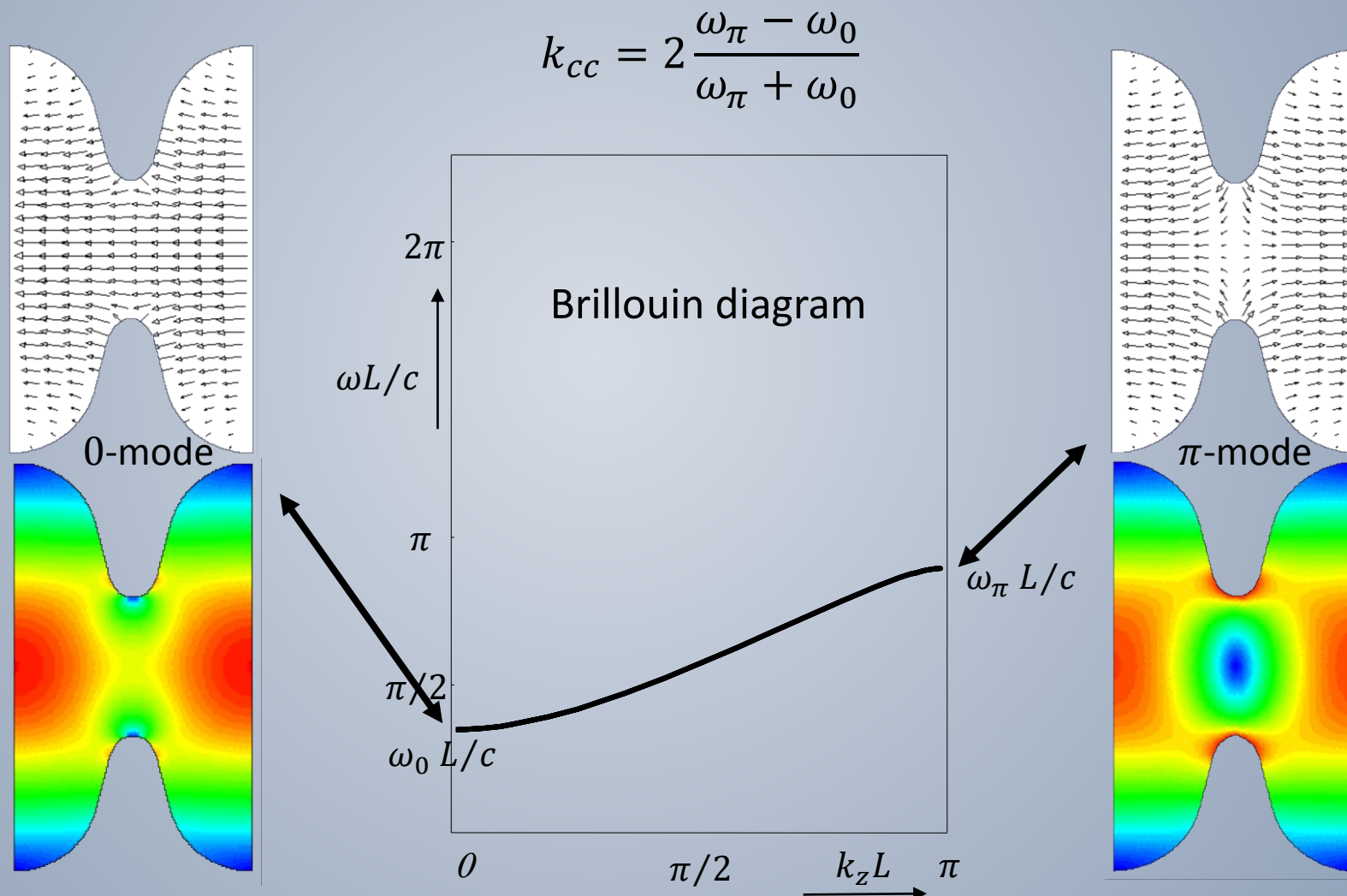
courtesy: Jacek Sekutovicz/DESY



Photo:
Reidar Hahn

Practical RF parameters 3

- **Cell-to-cell coupling** k_{cc} will determine the width of the passbands in multi-cell cavities.



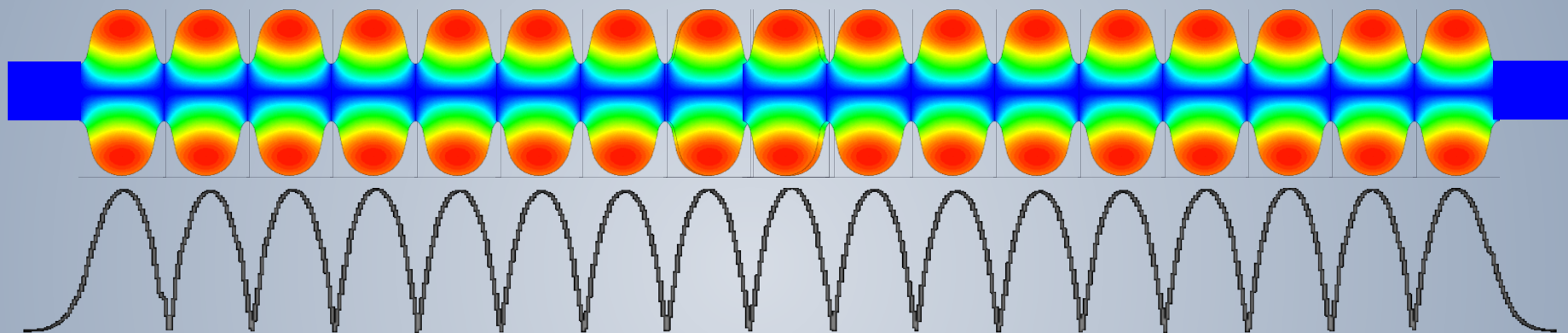
courtesy: Jacek Sekutovicz/DESY



Photo:
Reidar Hahn

Field flatness

- Field amplitude variation from cell to cell in a multi-cell structure
- Should be small for maximum acceleration.



- Field flatness sensitivity factor a_{ff} for a structure made of N cells:

$$\frac{\Delta A_i}{A_i} = a_{ff} \frac{\Delta f_i}{f_i}$$

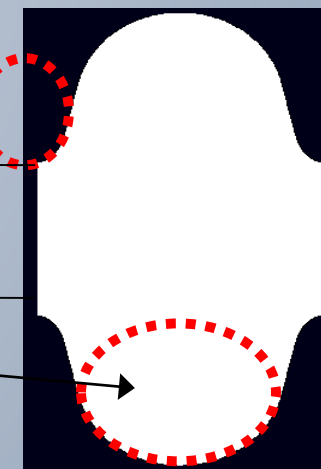
a_{ff} is related to the cell-to-cell coupling as $a_{ff} = \frac{N^2}{k_{cc}}$ and describes the sensitivity of the field flatness on the errors in individual cells.

courtesy: Jacek Sekutovicz/DESY



Criteria for Cavity Design (1)

- Here: Inner cells of multi-cell structures
- Parameters for optimization:
 - Fundamental mode: $\frac{R}{Q}$, G , $\frac{E_{\text{peak}}}{E_{\text{acc}}}$, $\frac{B_{\text{peak}}}{E_{\text{acc}}}$, k_{cc} .
 - Higher order modes: k_{\perp} , k_z .
- The elliptical cavity design has distinct advantages:
 - easy to clean (rinse)
 - little susceptible to MP – can be conditioned ...
- Geometric parameters for optimization:
 - iris ellipse half axes: a , b :
 - iris aperture radius: r_i ,
 - equator ellipse half axes: A , B
- Problem: 7 parameters to optimize, only 5 to play with – some compromise has to be found!



courtesy: Jacek Sekutovicz/DESY



Photo:
Reidar Hahn

Criteria for Cavity Design (2)

Criterion	RF parameter	Improves if	examples
high gradient operation	$E_{\text{peak}}/E_{\text{acc}}$ $B_{\text{peak}}/E_{\text{acc}}$ ↓	r_i ↓	ILC, CEBAF 12 GeV HG
low cryogenic losses	$\frac{R}{Q} \cdot G$ ↑	r_i ↓	CEBAF LL
High I_{beam}	k_{\perp}, k_z ↓	r_i ↑	B-factory RHIC cooling LHeC

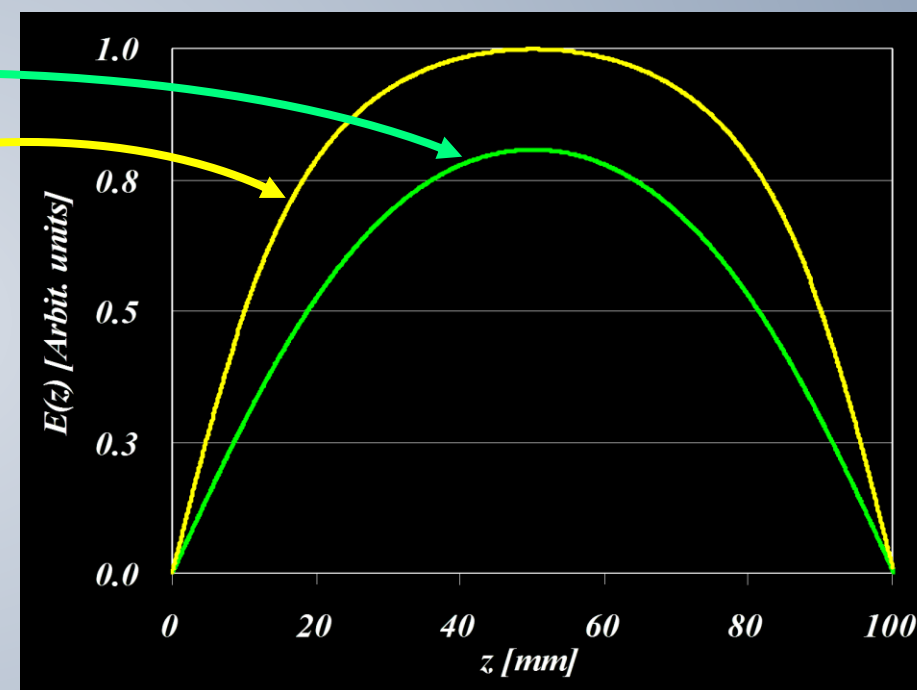
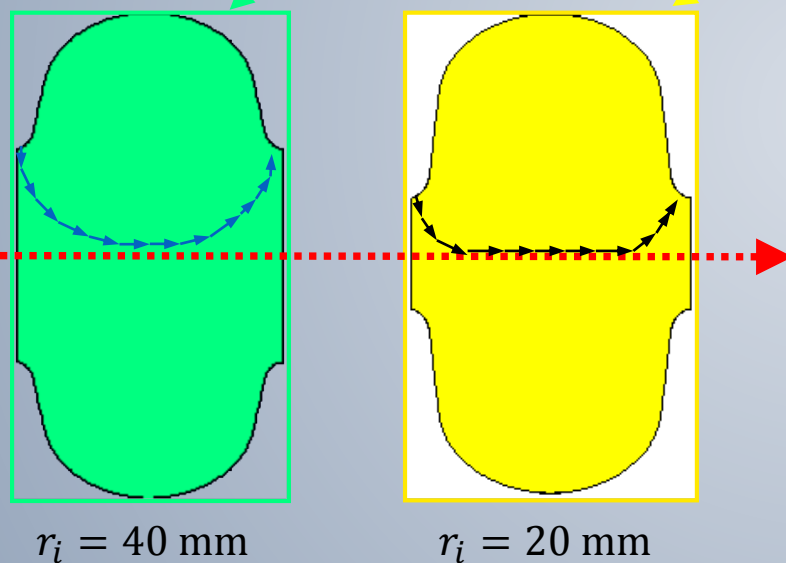
We see here that r_i is a very “powerful” variable to trim the RF-parameters of a cavity.
Of course it has to fit the aperture required for the beam!

courtesy: Jacek Sekutovicz/DESY



Effect of r_i

- Smaller r_i allows to concentrate E_z where it is needed for acceleration

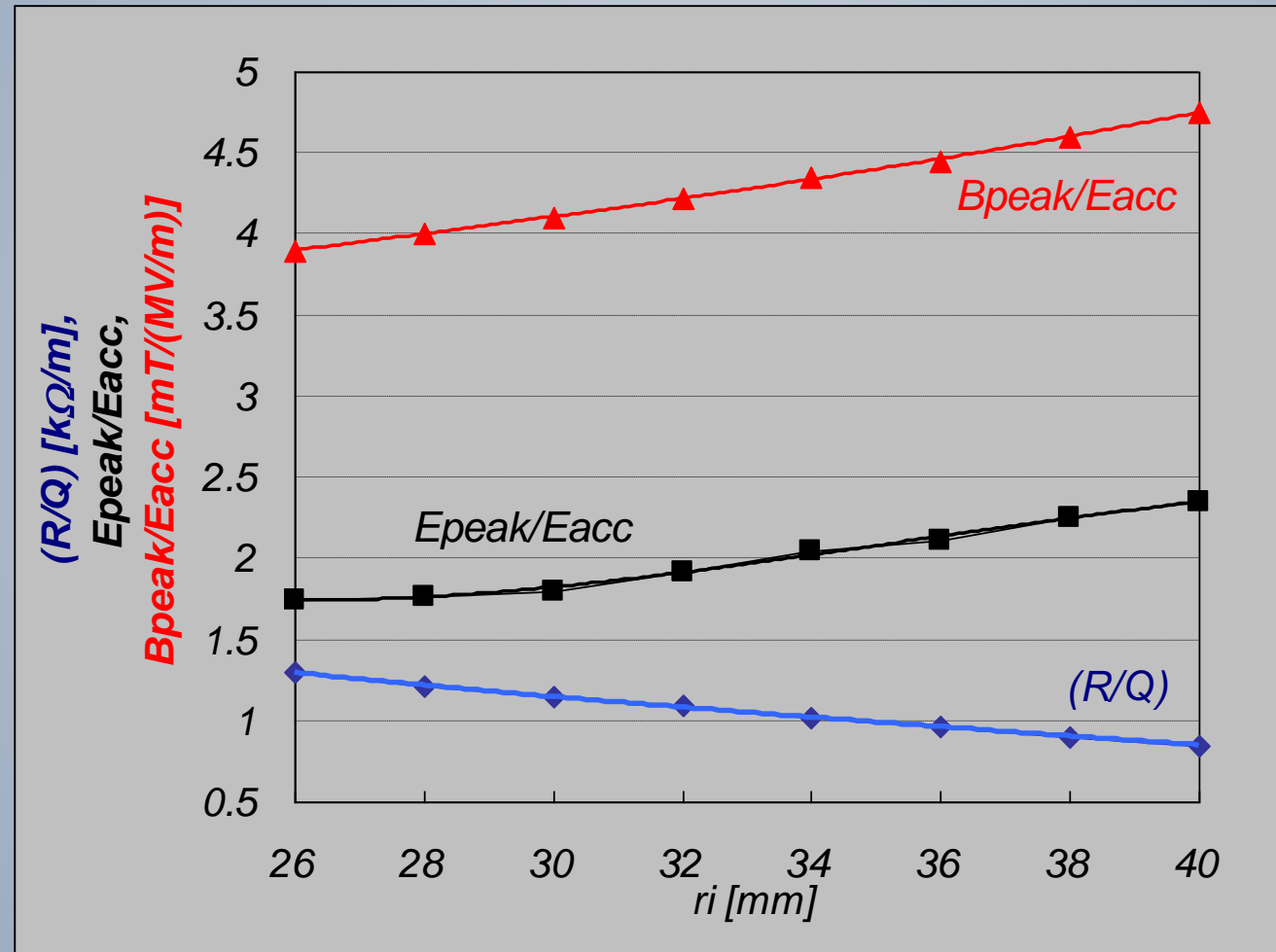


$E_z(z)$ for small and big iris radius

courtesy: Jacek Sekutovicz/DESY



Example: cell optimization at 1.5 GHz

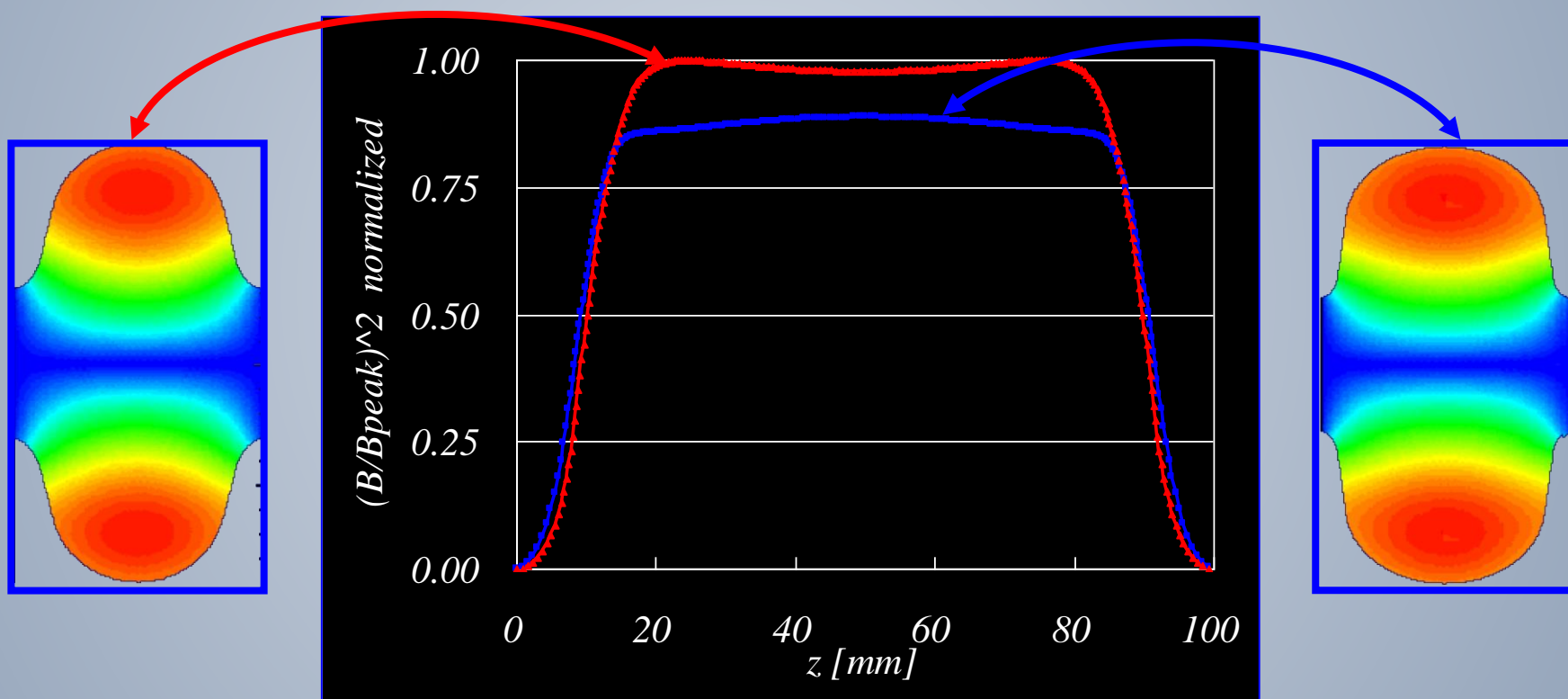


A. Mosnier, E. Haebel, SRF Workshop 1991



Equator shape optimization

- $B_{\text{peak}}/E_{\text{acc}}$ (and G) change when changing the equator shape.



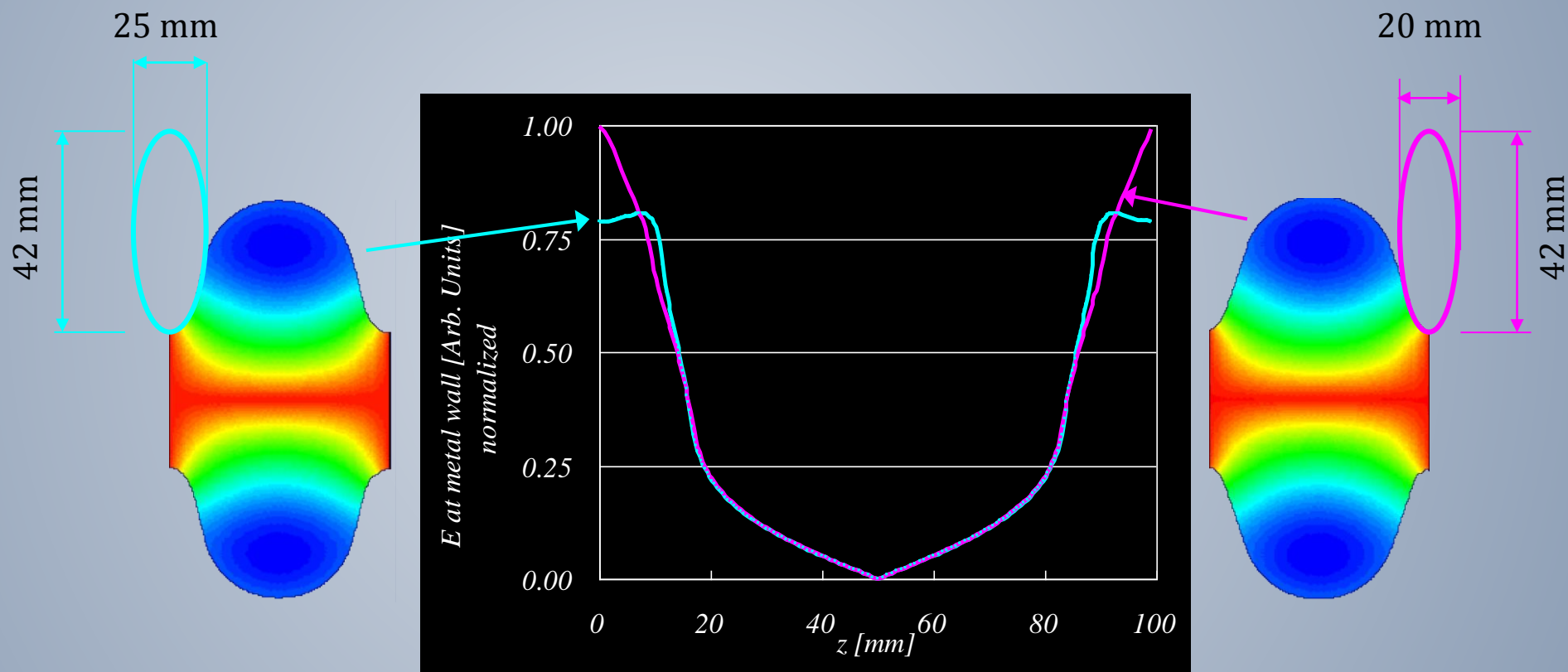
courtesy: Jacek Sekutovicz/DESY



Photo:
Reidar Hahn

Iris shape optimization

- $E_{\text{peak}}/E_{\text{acc}}$ changes with the iris shape



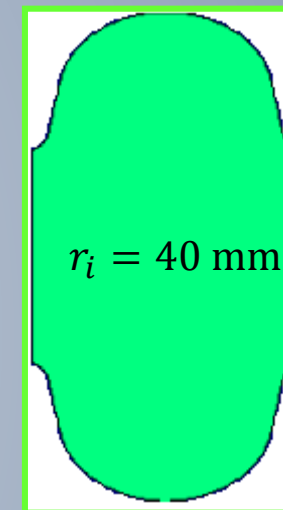
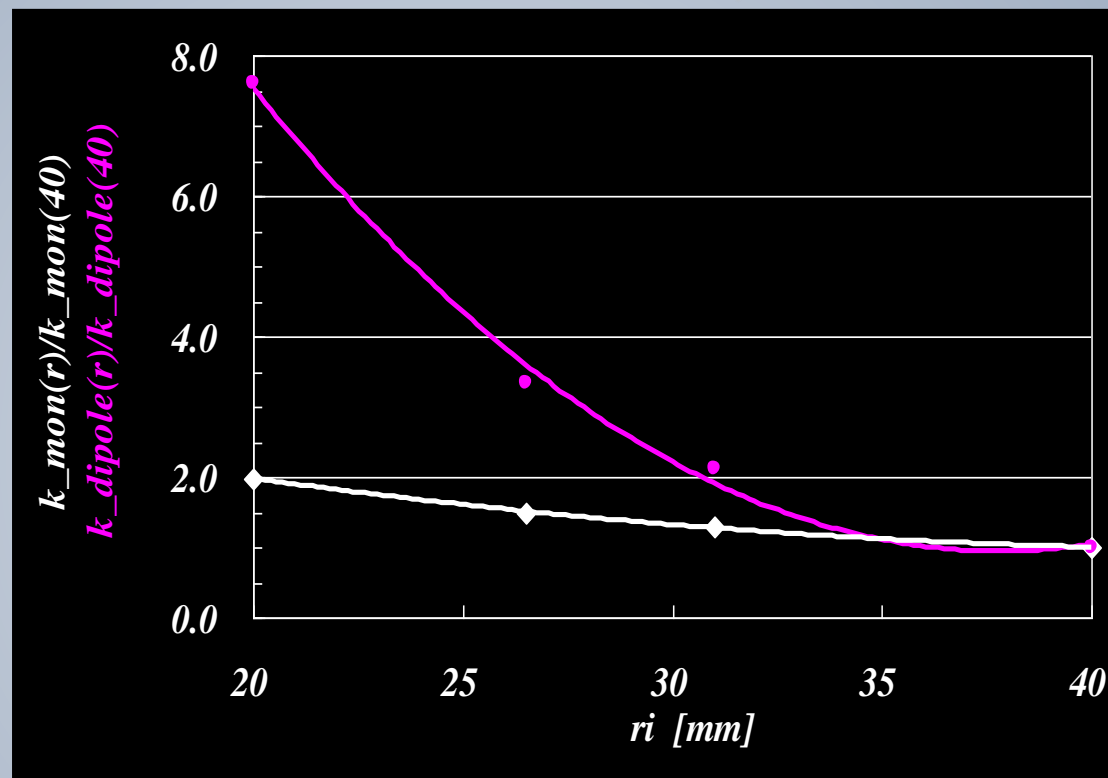
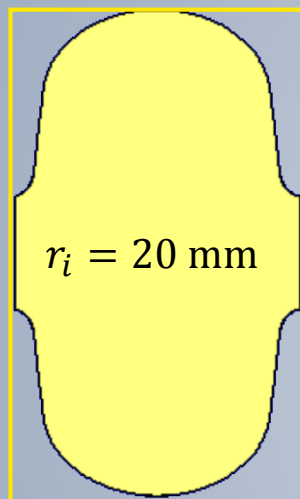
Both cells have the same: f_0 , R/Q , and r_i .

courtesy: Jacek Sekutovicz/DESY



Minimizing HOM excitation

HOMs loss factors ($k_{\text{loss},\perp}$, k_{loss})



$$R/Q = 152 \, \Omega$$

$$B_{\text{peak}}/E_{\text{acc}} = 3.5 \, \text{mT}/(\text{MV}/\text{m})$$

$$E_{\text{peak}}/E_{\text{acc}} = 1.9$$

$$R/Q = 86 \, \Omega$$

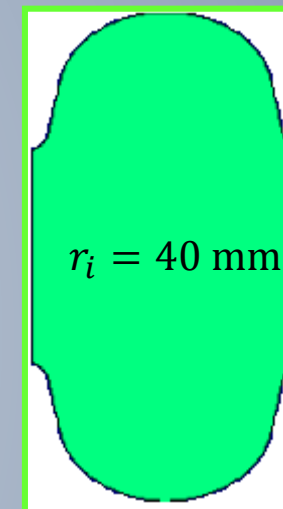
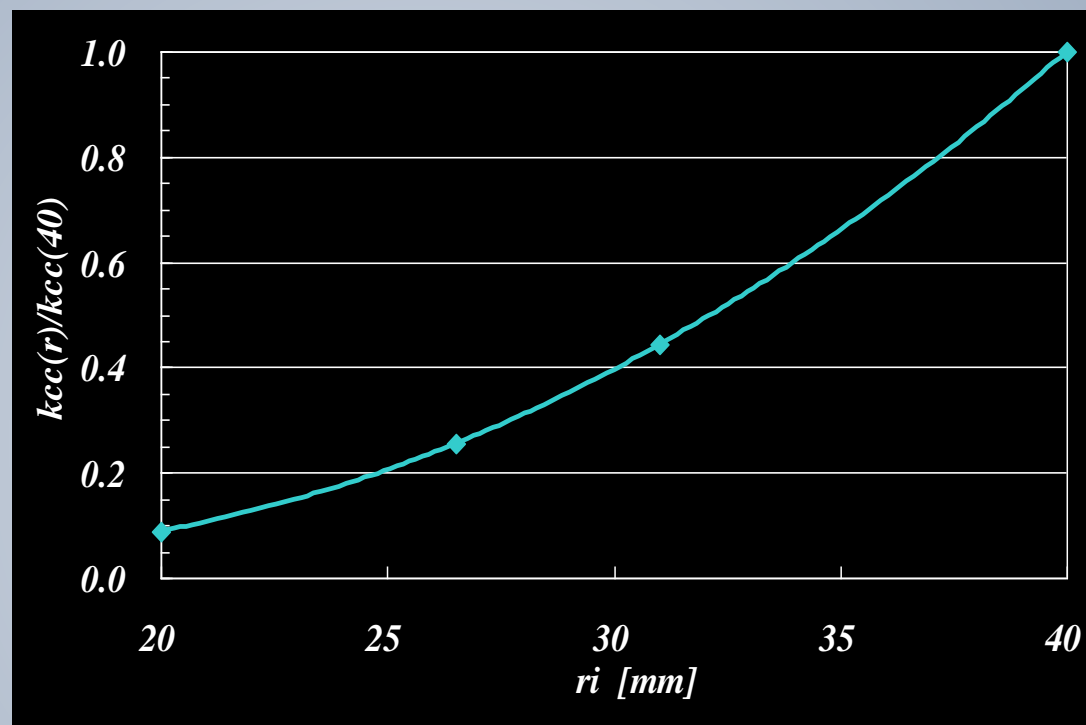
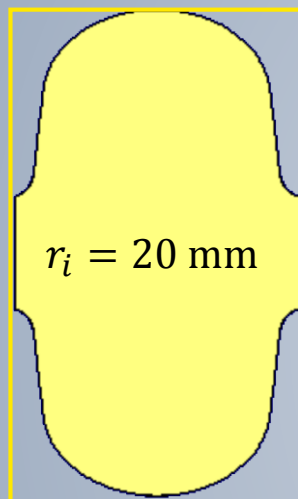
$$B_{\text{peak}}/E_{\text{acc}} = 4.6 \, \text{mT}/(\text{MV}/\text{m})$$

$$E_{\text{peak}}/E_{\text{acc}} = 3.2$$

courtesy: Jacek Sekutovicz/DESY



Cell-to-cell coupling k_{cc}



$$\begin{aligned} R/Q &= 152 \, \Omega \\ B_{\text{peak}}/E_{\text{acc}} &= 3.5 \text{ mT}/(\text{MV}/\text{m}) \\ E_{\text{peak}}/E_{\text{acc}} &= 1.9 \end{aligned}$$

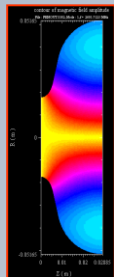
$$\begin{aligned} R/Q &= 86 \, \Omega \\ B_{\text{peak}}/E_{\text{acc}} &= 4.6 \text{ mT}/(\text{MV}/\text{m}) \\ E_{\text{peak}}/E_{\text{acc}} &= 3.2 \end{aligned}$$

courtesy: Jacek Sekutovicz/DESY

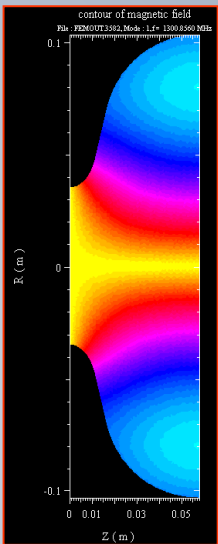


Photo:
Reidar Hahn

Scaling the frequency



$$\times 2 =$$



f_{π}	[MHz]	2600
R/Q	[Ω]	57
r/Q	[Ω/m]	2000
G	[Ω]	271

f_{π}	[MHz]	1300
R/Q	[Ω]	57
r/Q	[Ω/m]	1000
G	[Ω]	271

$$r/Q = (R/Q)/l \propto f$$

$$(\text{or } (R/Q)/\lambda = \text{const})$$

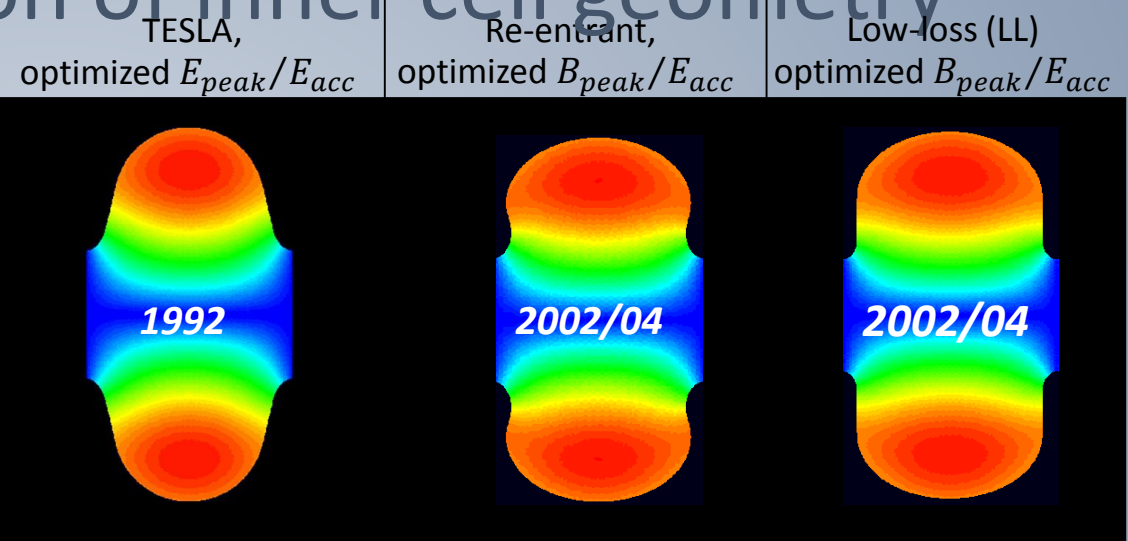
courtesy: Jacek Sekutovicz/DESY



Photo:
Reidar Hahn

Historic evolution of inner cell geometry

Example: ILC

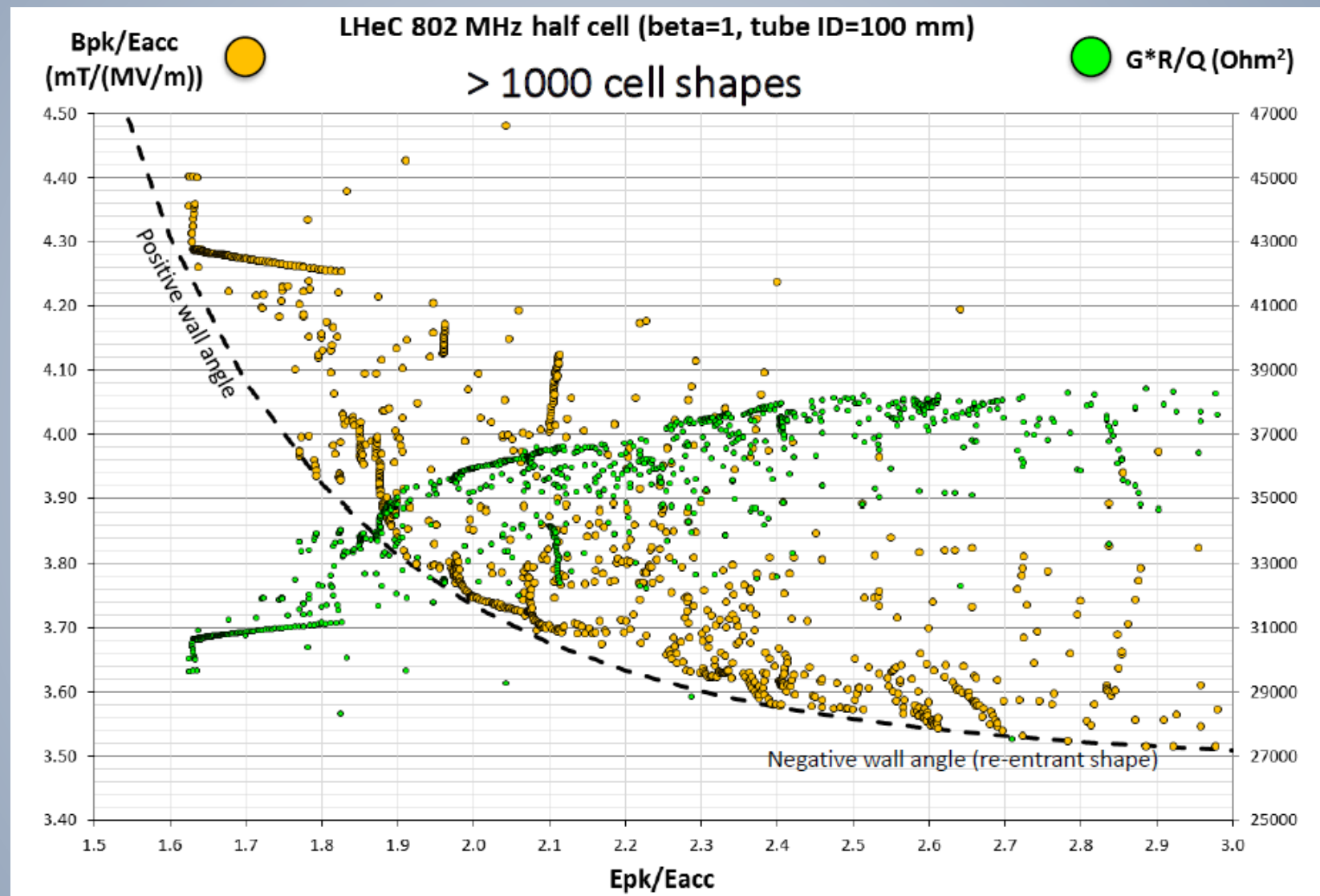


r_i	[mm]	35	30	30
k_{cc}	[%]	1.9	1.56	1.52
E_{peak}/E_{acc}	-	1.98	2.30	2.36
B_{peak}/E_{acc}	[mT/(MV/m)]	4.15	3.57	3.61
R/Q	[Ω]	113.8	135	133.7
G	[Ω]	271	284.3	284
$R/Q \cdot G$	[$\Omega^* \Omega$]	30840	38380	37970
$k_{loss,\perp} (\sigma_z = 1 \text{ mm})$	[V/pC/cm ²]	0.23	0.38	0.38
$k_{loss} (\sigma_z = 1 \text{ mm})$	[V/pC]	1.46	1.75	1.72

courtesy: Jacek Sekutovicz/DESY



Photo:
Reidar Hahn



courtesy: Frank Marhauser/JLAB

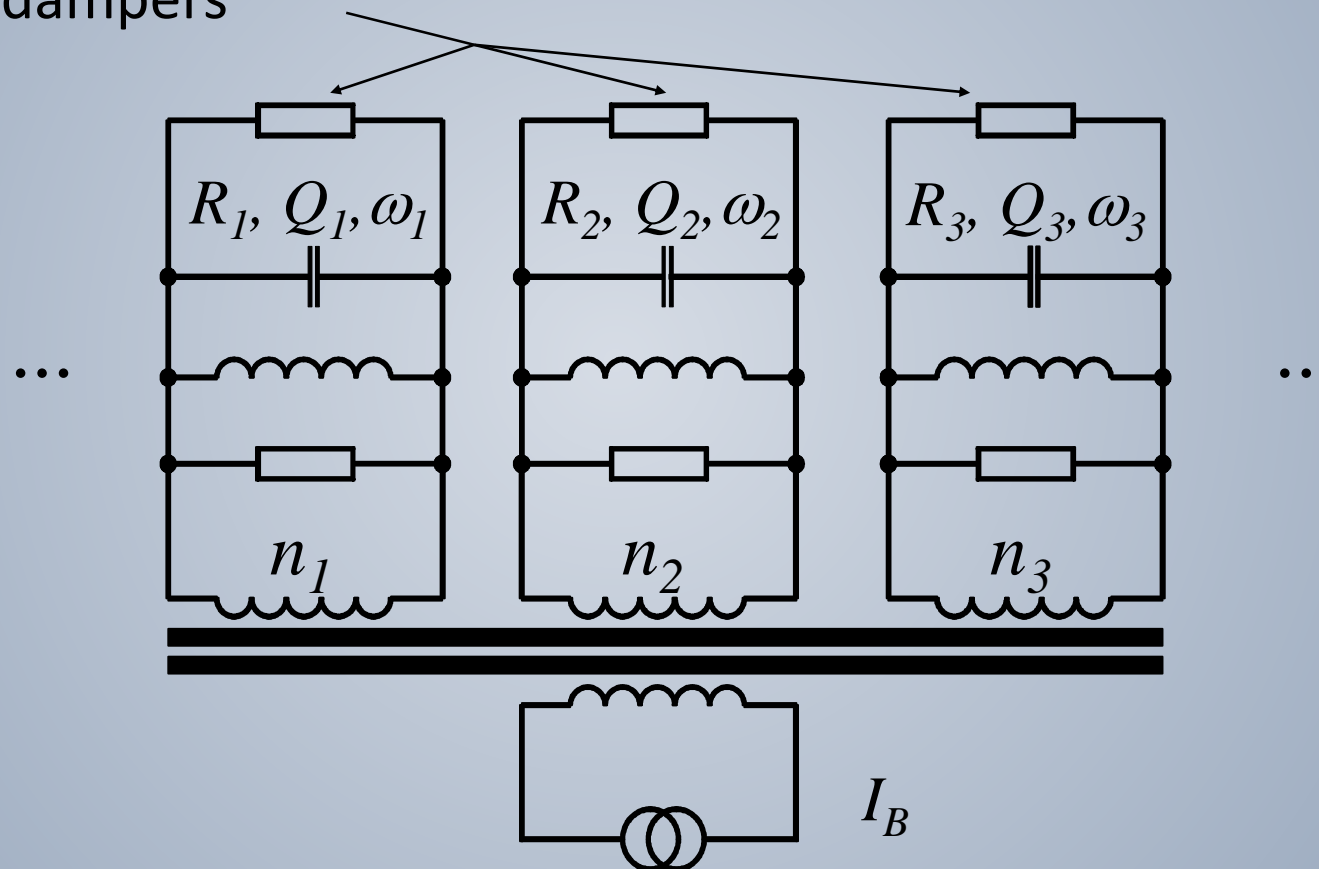
Higher Order Modes



Photo:
Reidar Hahn

Higher order modes

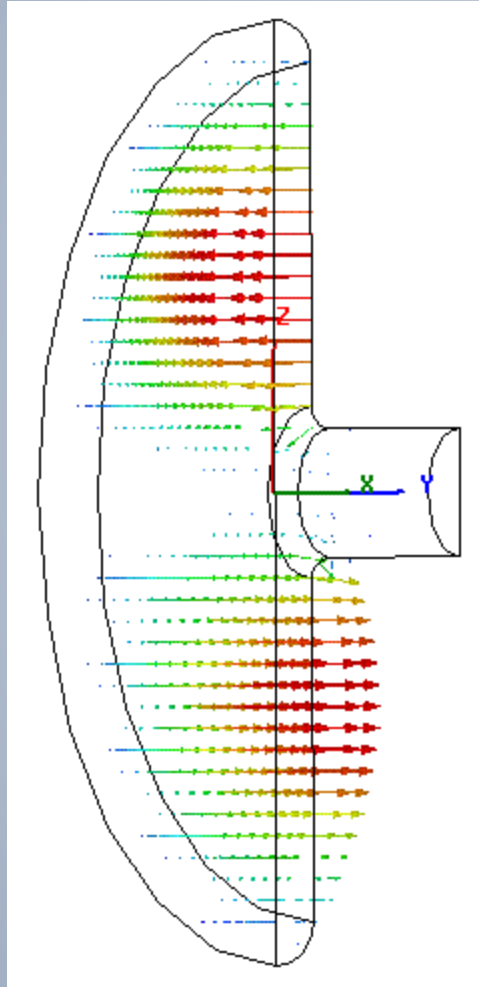
external dampers



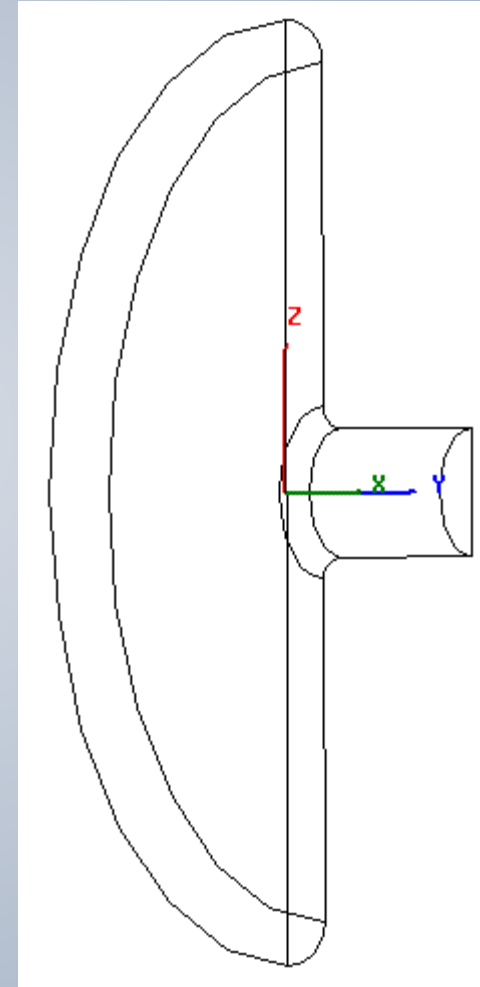


Pillbox: 1st dipole mode

TM_{110} -mode (only 1/4 shown)



electric field

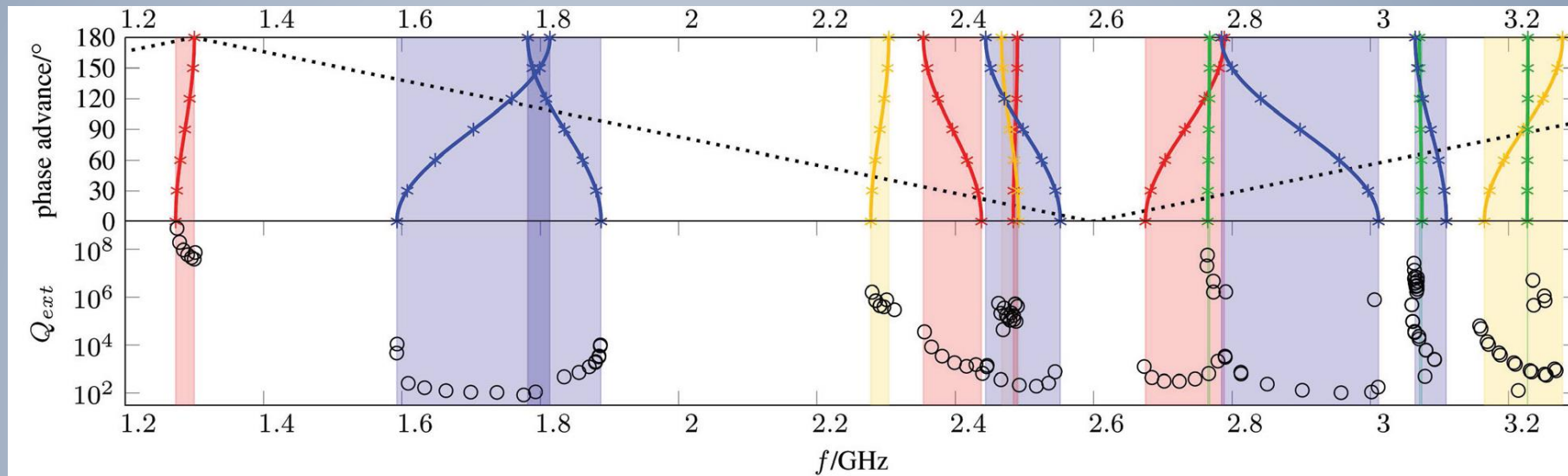


magnetic field



Photo:
Reidar Hahn

7-cell 1.3 GHz structure for

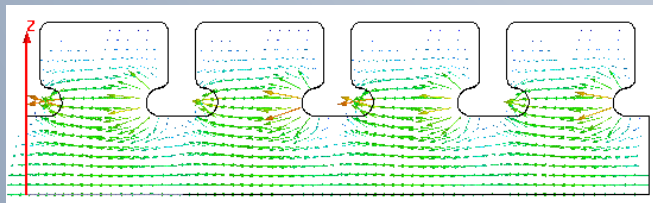


Band diagram (top) and Q-factors (bottom)

Galek et al.: IPAC2013

Reminder:

0-mode



π -mode

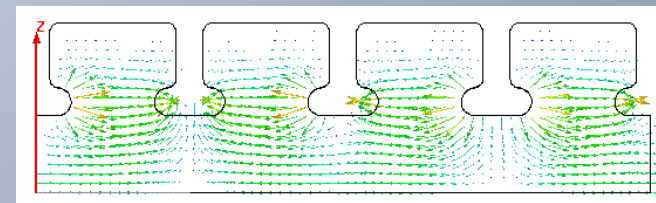
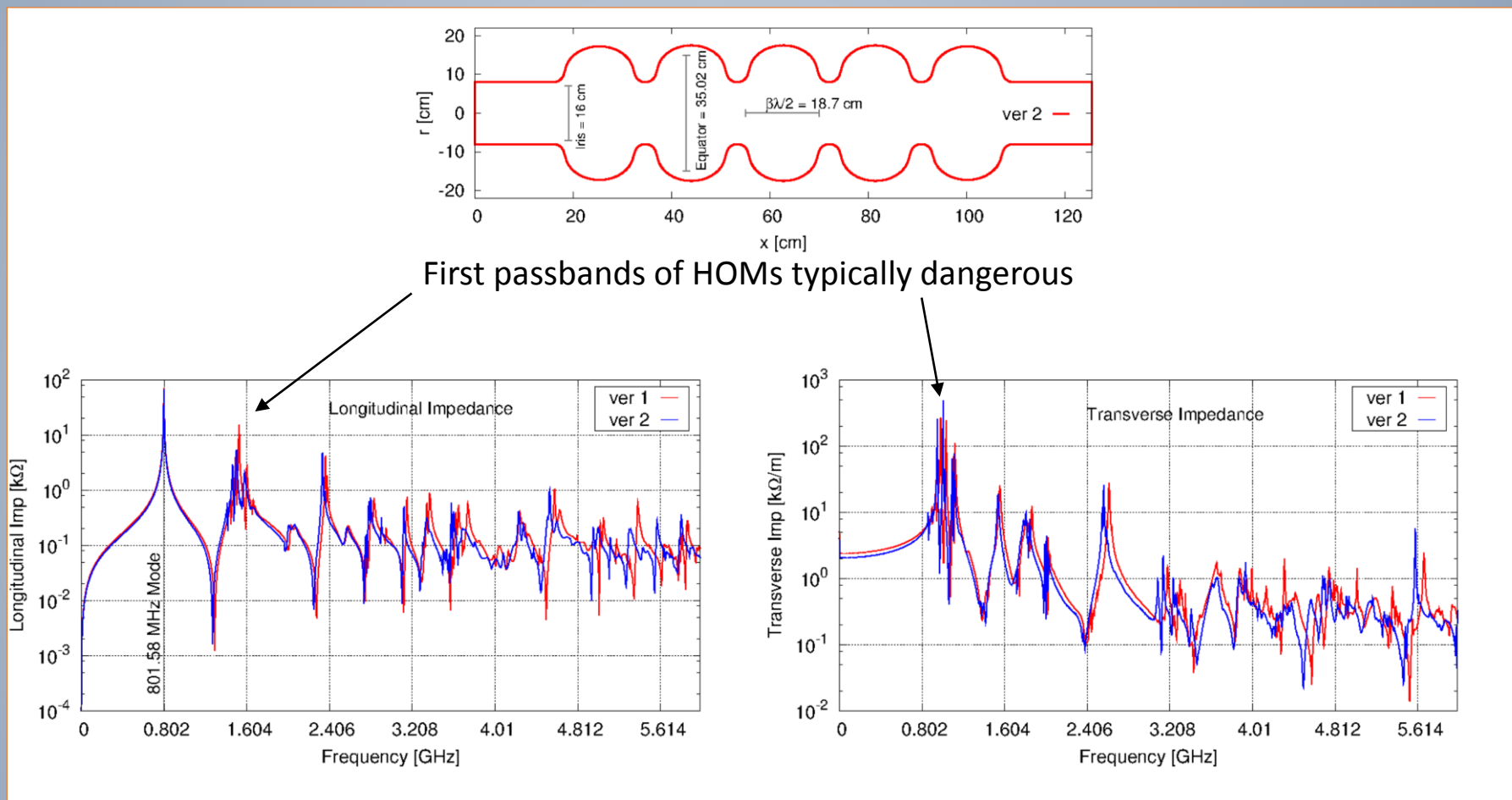




Photo:
Reidar Hahn

HOMs: Example 5-cell cavity



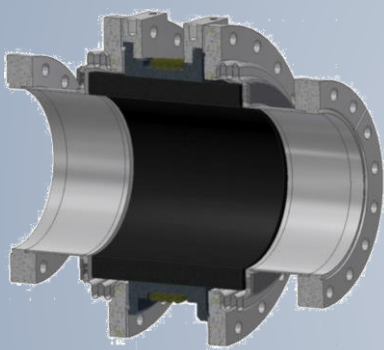
courtesy: Rama Calaga/CERN



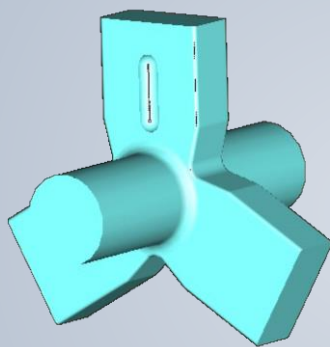
Photo:
Reidar Hahn

HOM dampers

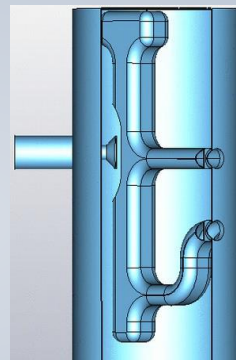
- Ferrite absorbers: broadband damper, room temperature
- Waveguides: better suited for higher frequencies (size!)
- Notch filters: narrow-band; target specific mode



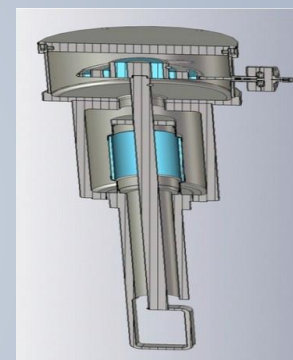
ferrite absorber



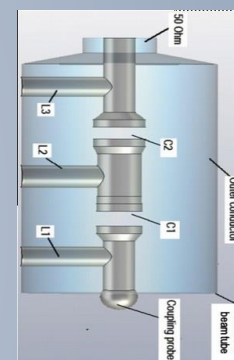
waveguides



notch filter



bandpass filter



double notch

- Multi-cell cavities require broadband dampers!

Non-elliptical cavities

Special thanks to R. Laxdal (TRIUMF) and
A. Facco (INFN & FRIB)



Photo:
Reidar Hahn

Useful relations

Protons/H- ($A = 1, Q = 1$)

$$p = \gamma\beta m_0 c = \frac{\gamma\beta E_0}{c}$$

$$E = E_{\text{kin}} + E_0 = \gamma E_0$$

$$E_{\text{kin}} = (\gamma - 1)E_0$$

$$E_{\text{kin}} = |e|V_{\text{eff}} \cos \varphi$$

Heavy Ions (A, Q)

$$p = \gamma\beta A m_0 c = \frac{\gamma\beta A E_0}{c}$$

$$E = E_{\text{kin}} + A E_0 = \gamma A E_0$$

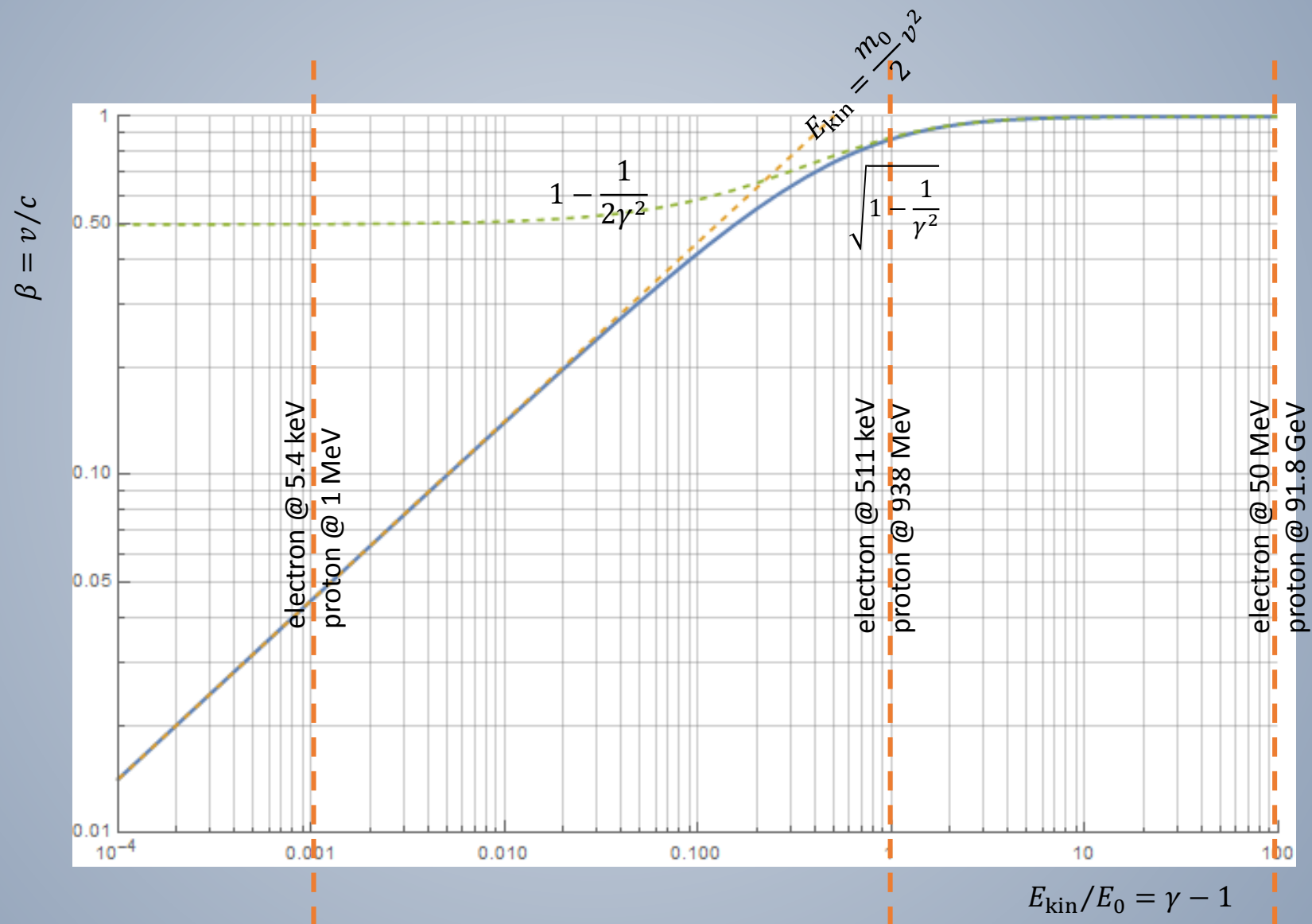
$$E_{\text{kin}}/A = (\gamma - 1)E_0$$

$$E_{\text{kin}}/A = (Q/A) V_{\text{eff}} \cos \varphi$$



Photo:
Reidar Hahn

Particle velocity vs. kinetic energy





Accelerating electrons vs. accelerating ions

Example: a 300kV DC bias is enough to get electrons going at a relativistic speed (ie $E_0=511\text{keV}$ so $\gamma=1.58$, $v/c=\beta=0.78$) – for protons a 300kV bias only produces $v/c=\beta=0.025$ – for $A=30$ $v/c=\beta=0.005$

- Electron – $0.511\text{MeV}/c^2$
 - 300kV - $\gamma=1.58$, $v/c=\beta=0.78$
 - 550MeV - $\gamma=1011$, $v/c=\beta=1$



8 gm



ARIEL 300kV e-gun

- Protons – $938\text{ MeV}/c^2$
 - 300kV - $\gamma=1.003$, $v/c=\beta=0.025$
 - 550MeV - $\gamma=1.58$, $v/c=\beta=0.78$



160 kgm



TRIUMF 500MeV cyclotron

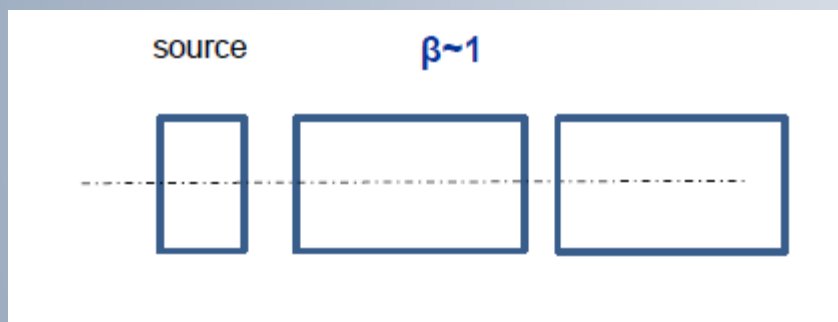


Photo:
Reidar Hahn

Accelerating electrons vs. accelerating ions

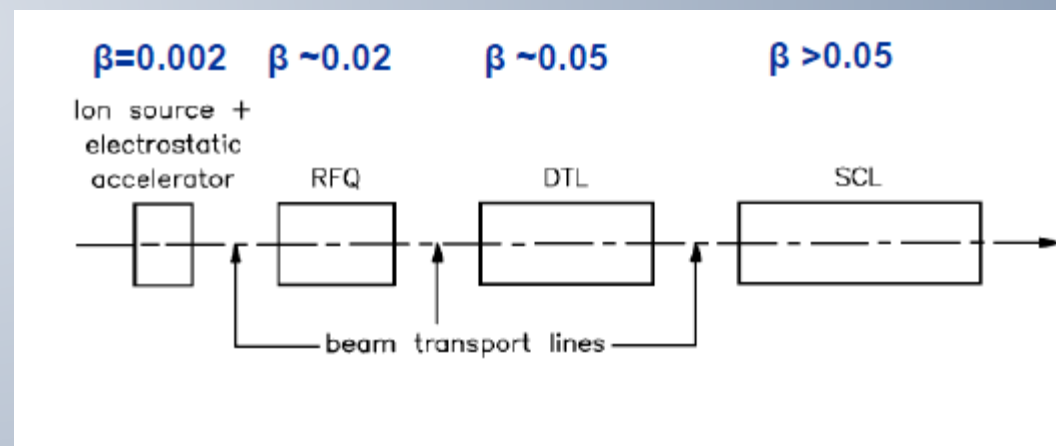
Electrons

Common building blocks – all designed for $\beta = 1$.



Ions

Various building blocks – different technologies, each optimized for a certain velocity range





Limitations of elliptical cavities

- Elliptical cavities have been designed starting at $\beta \geq 0.5$ for CW applications, for $\beta \geq 0.6$ for pulsed (SNS, ESS).
- The π -mode requires cell-to-cell distance of $\beta\lambda/2$, but outer diameter $\approx 0.9 \lambda$, i.e. at low β the cavity looks more like bellows, sensitive to LFD!

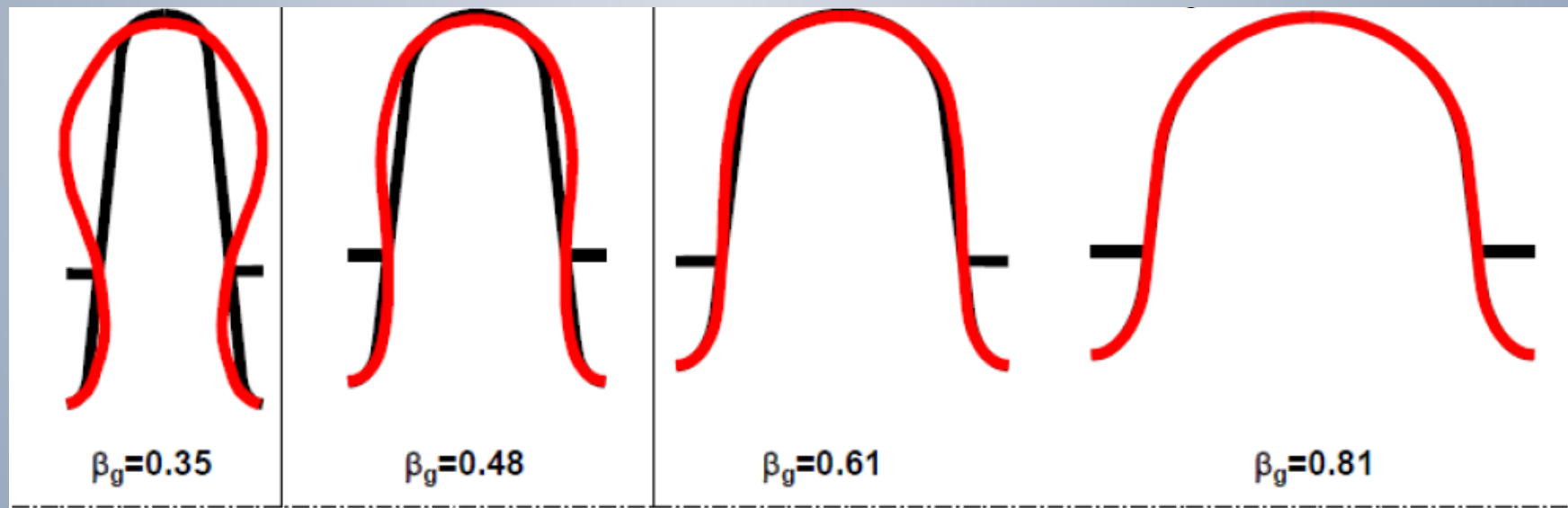
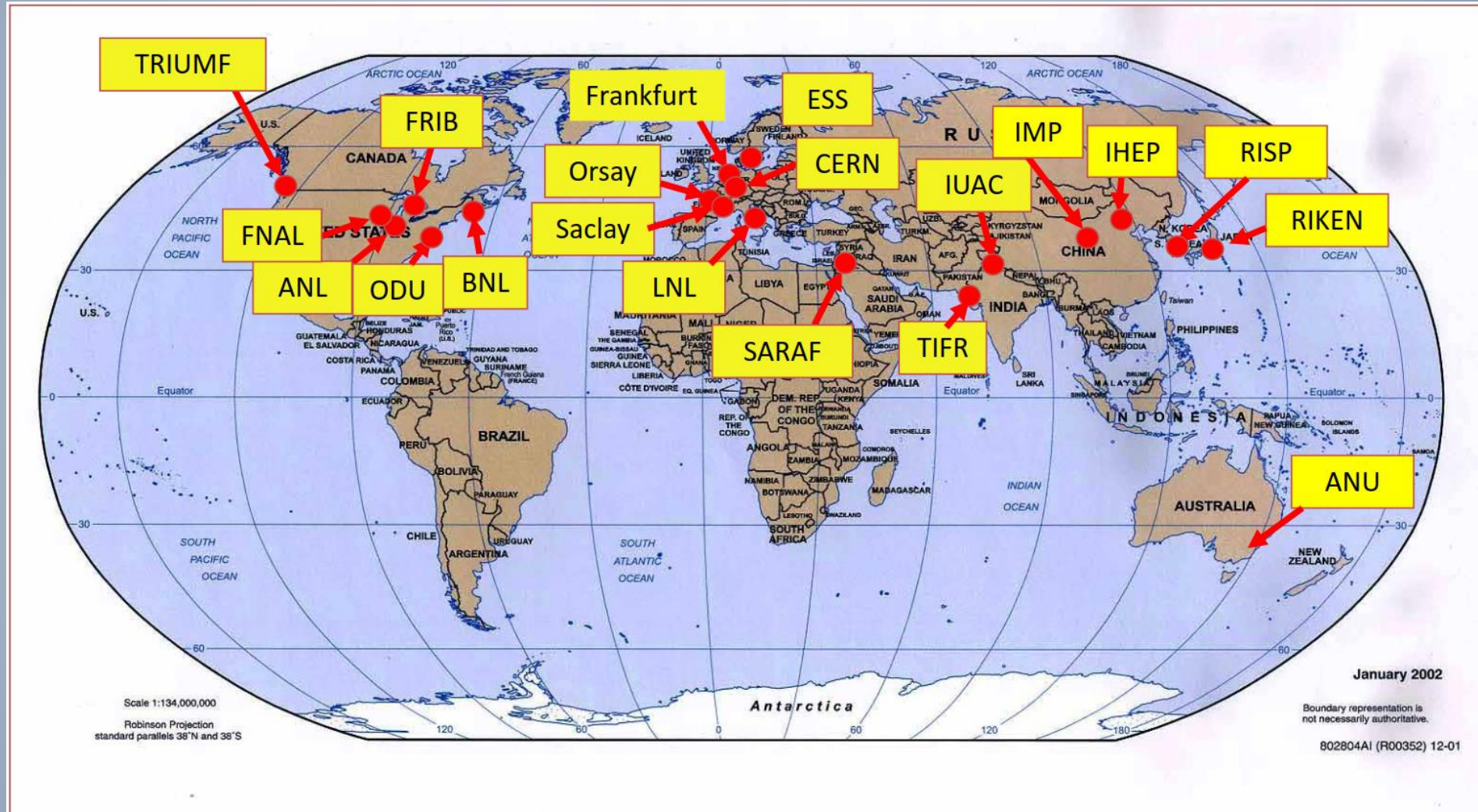




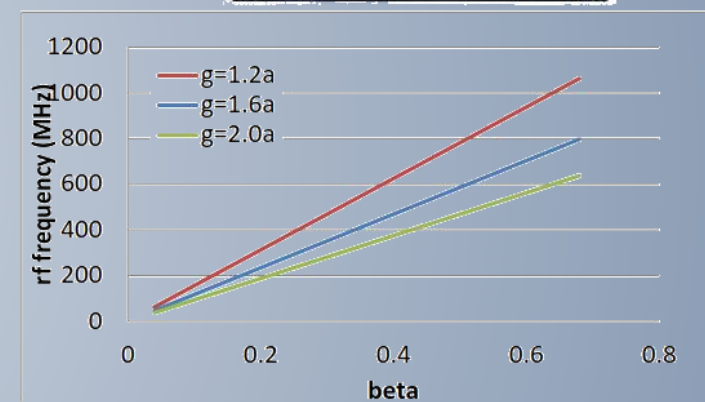
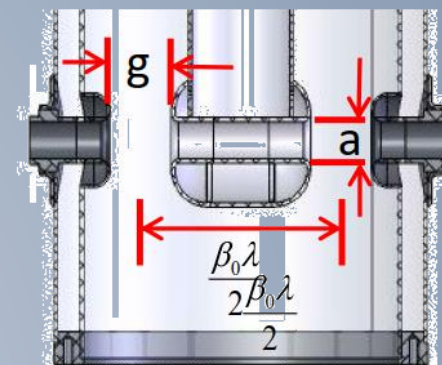
Photo:
Reidar Hahn

Non-elliptical SRF Community around the world



Resonator types for low beta acceleration

- Quarter wave resonator (QWR) $\beta \approx 0.04 \dots 0.2$
- Half wave resonator (HWR) $\beta \approx 0.1 \dots 0.5$
- Single spoke resonator (SSR) $\beta \approx 0.15 \dots 0.7$
- Multi-spoke resonator (MSR) $\beta \approx 0.06 \dots 1$
- For comparison: Elliptical cavities $\beta \approx 0.5 \dots 1$





Coaxial resonator

- Consider a coaxial geometry with grounded end plates, an inner conductor with radius a and an outer conductor with radius b .
- A standing wave occurs with E_r vanishing on the end walls at $z = 0$ and $z = d$.
- The remaining non-zero field components are

$$B_\theta = \frac{\mu_0 I_0}{\pi r} \cos\left(\frac{p\pi z}{d}\right),$$

$$E_r = -j2 \sqrt{\frac{\mu_0}{\epsilon_0}} \frac{I_0}{2\pi r} \sin\left(\frac{p\pi z}{d}\right),$$

where $\omega = \frac{p\pi c}{d}$, $p = 1, 2, 3, \dots$

- Peak voltage:

$$\hat{V}(z) = \int_a^b E_r(z) dr = \sqrt{\frac{\mu_0}{\epsilon_0}} \frac{I_0}{\pi} \ln \frac{b}{a} \sin\left(\frac{p\pi z}{d}\right)$$

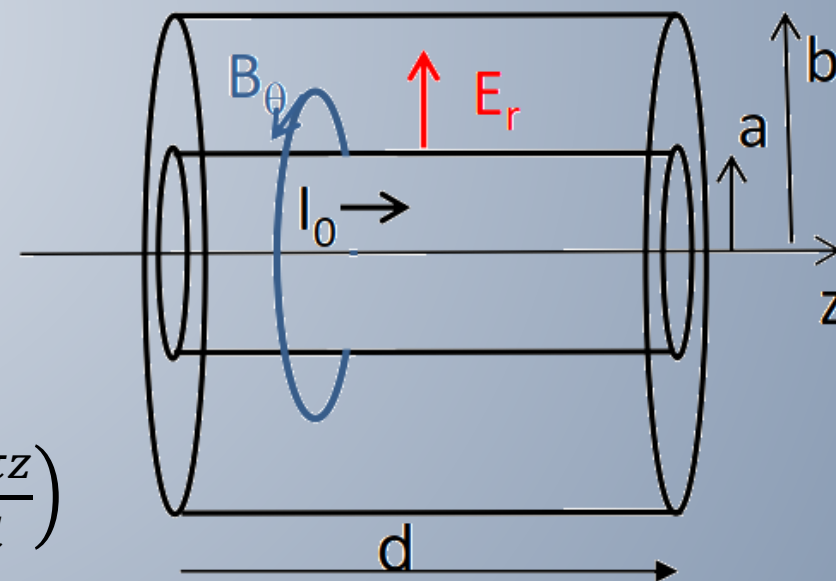
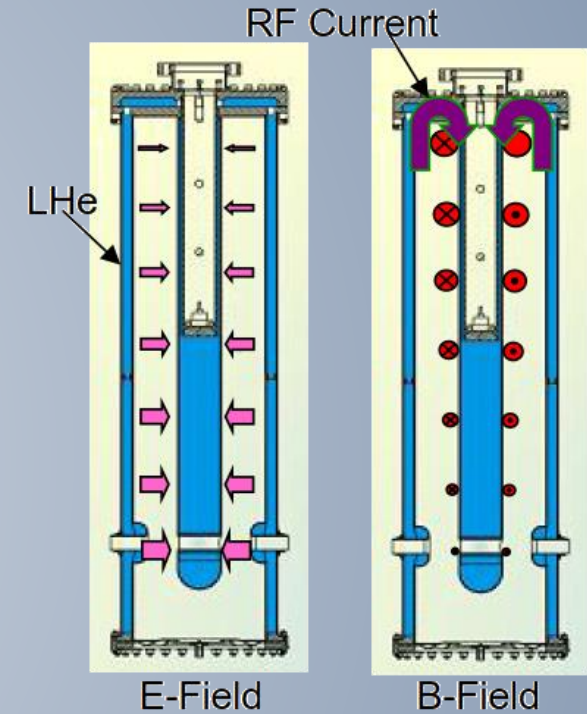
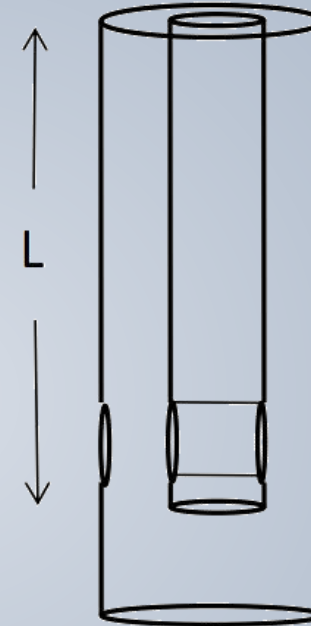




Photo:
Reidar Hahn

Quarter-wave resonator (QWR)

- The most popular coaxial TEM mode cavity is the quarter wave resonator – capacitively loaded $\lambda/4$ transmission line
- The inner conductor is open at one end with a resonant length of $(1 + 2p) \lambda/4$, $p = 0, 1, 2, \dots$
- For acceleration, $p = 0$ is chosen.
- The maximum voltage builds up on the open tip – the maximum current at the root.
- A beam tube is arranged near the end of the tip.





Half-wave resonator (HWR)

- In the HWR the beam port is at the centre of the inner conductor of a coaxial resonator, coincident with the maximum voltage for $p = 1$.
- Magnetic fields loop around the inner conductor with peak fields at the shorted ends.
- For acceleration, $p = 1$ is chosen.

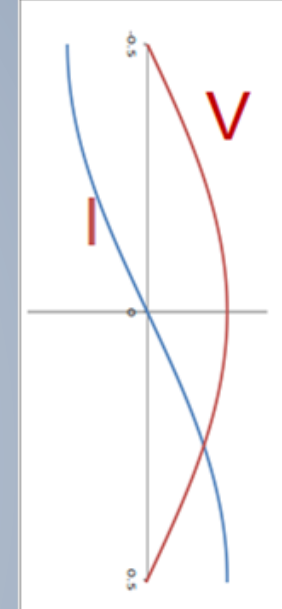
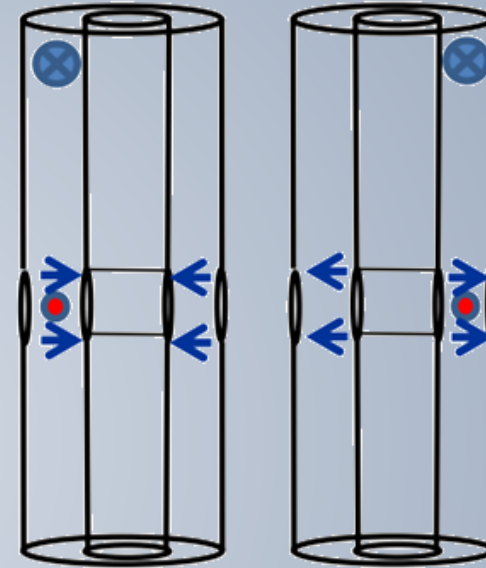




Photo:
Reidar Hahn

QWR vs. HWR

- QWR is the cavity of choice for low beta applications where a low frequency is needed
 - requires ~50% less structure compared to HWR for the same frequency – rf power loss is ~50% of HWR for same frequency and β_0 .
 - allows low frequency choice giving larger longitudinal acceptance.
 - R/Q twice that of HWR.
 - Asymmetric field pattern introduces vertical steering especially for light ions that increases with velocity – avoid use for $\beta_0 > 0.2$.
 - Less mechanically stable than HWR due to unsupported end (microphonics).
- HWR is chosen in mid velocity range ($\beta_0 > 0.2$) or where steering must be eliminated (i.e. high intensity light ion applications)
 - produces twice rf losses for the same β_0 and λ .
 - is 2x longer for the same frequency.
 - Pluses are the symmetric field pattern and increased mechanical rigidity.

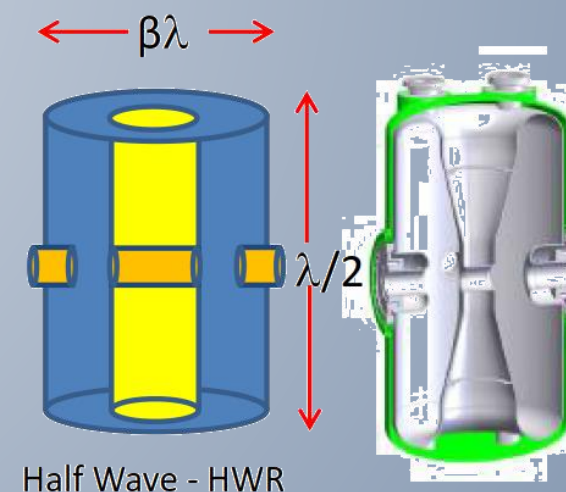
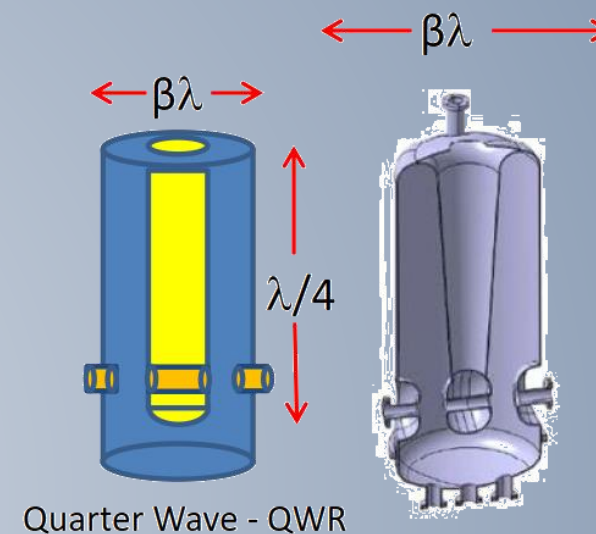
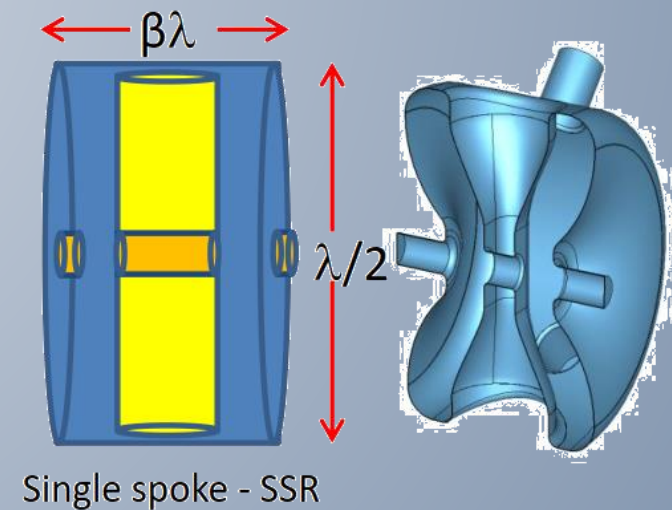
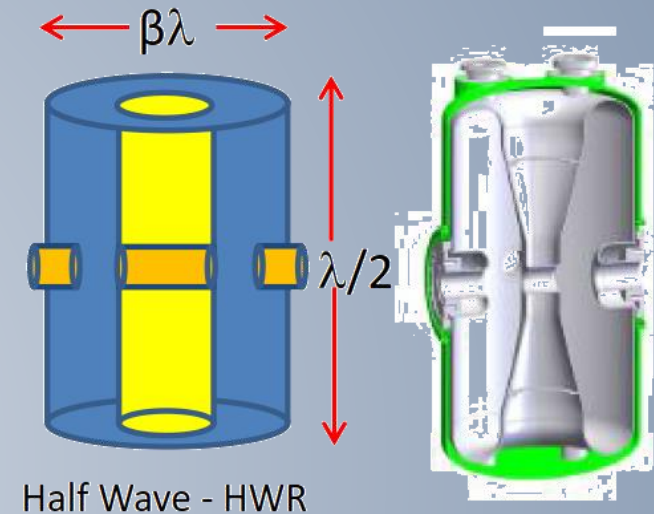




Photo:
Reidar Hahn

HWR vs. Single Spoke Resonator (SSR)

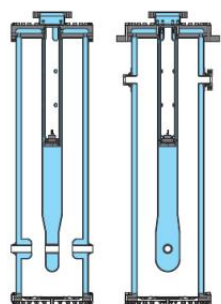
- A single spoke resonator (SSR) is another variant of the half-wave TEM mode cavity class.
- In HWR the outer conductor is coaxial with the inner conductor (with diameter $\beta_0\lambda$) while in the spoke cavities the outer cylinder is co-axial with the beam tube with diameter $\lambda/2$. It means that for $\beta_0 < 0.5$ the SSR has a larger overall physical envelop than the HWR for the same frequency.
- Thus for low beta applications ($0.1 < \beta < 0.25$) HWRs are chosen at ≈ 160 MHz, while SSRs are preferred at ≈ 320 MHz.
- The spoke geometry allows an extension along the beam path to provide multiple spokes in a single resonator giving higher effective voltage, but with a narrower transit time acceptance.



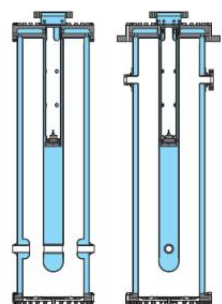


Cavity types – QWRs

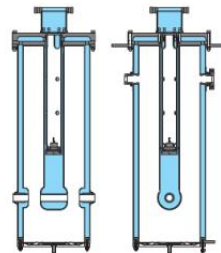
TRIUMF ISAC-II Resonators



SCB low β (5.7%)
106.08 MHz



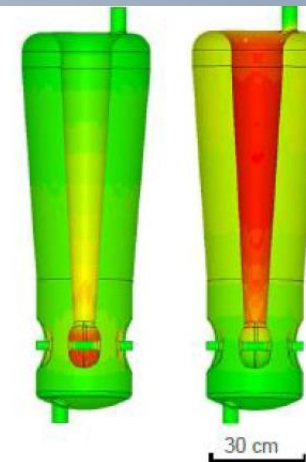
SCB medium β (7.1%)
106.08 MHz



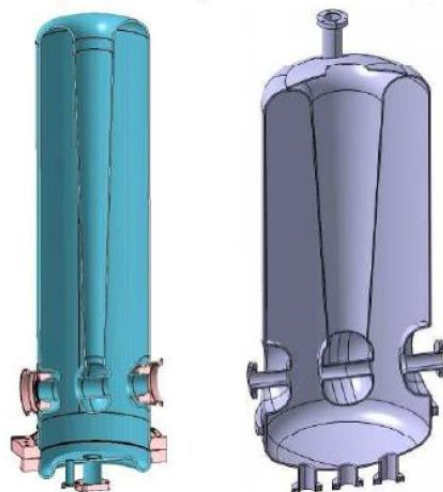
SCC high β (11%)
141.44 MHz



FRIB $\beta=0.041, 0.085$ 80.5MHz



ANL $\beta=0.077, 0.085$ 72.5MHz



Spiral-2 $\beta=0.007, 0.12$ 88.05MHz



RAON $\beta=0.047, 81.25$ MHz

Typical range:
 $0.04 < \beta < 0.2$
 $50 \text{ MHz} \leq f \leq 160 \text{ MHz}$

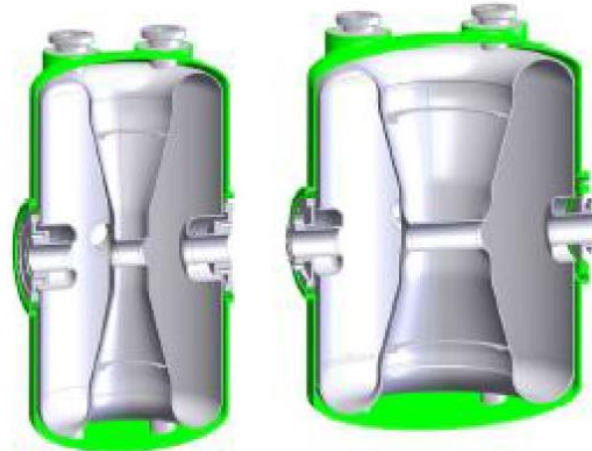


Photo:
Reidar Hahn

Cavity types – HWRs



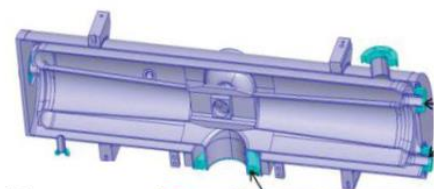
IMP $\beta=0.10$, $f=162.5\text{MHz}$



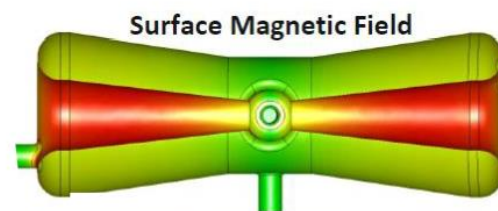
FRIB $\beta=0.29, 0.53$ $f=322\text{MHz}$



FRIB $\beta=0.29, 0.53$ $f=322\text{MHz}$



IFMIF $\beta=0.11$, $f=175\text{MHz}$



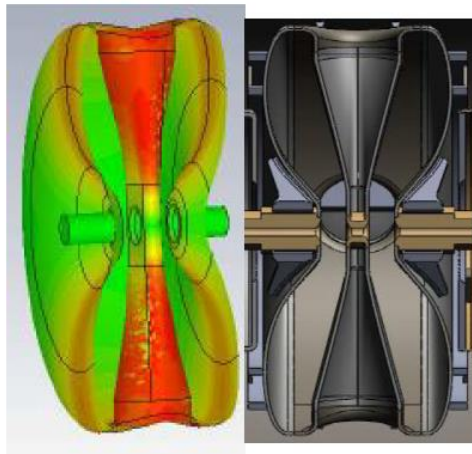
ANL $\beta=0.112$, $f=162.5\text{MHz}$

Typical range:
 $0.1 < \beta < 0.5$
 $140\text{ MHz} \leq f \leq 325\text{ MHz}$



Photo:
Reidar Hahn

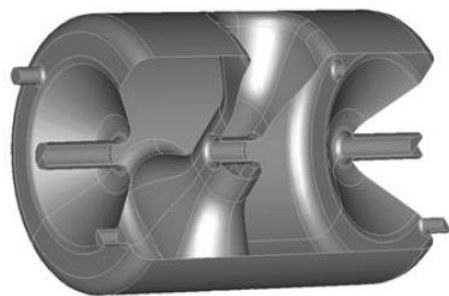
Cavity types – SSRs



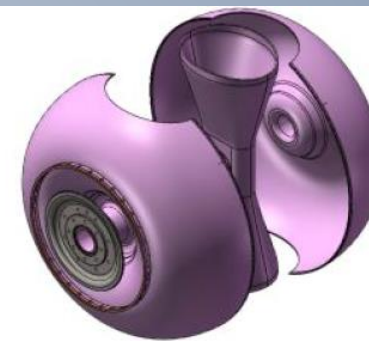
IHEP $\beta=0.12$, $f=325\text{MHz}$



FNAL $\beta=0.215$, $f=325\text{MHz}$



325 MHz, $\beta_0 = 0.82$
Single-Spoke Cavity



TRIUMF/RISP $\beta=0.3$, $f=325\text{MHz}$

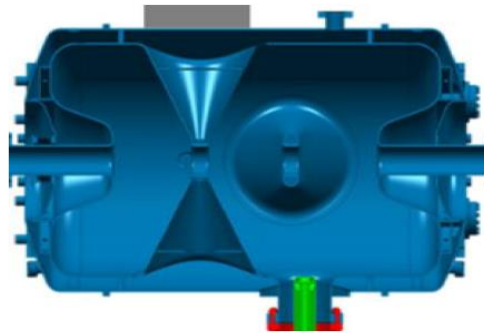


Typical range:
 $0.15 < \beta < 0.7$
 $320 \text{ MHz} \leq f \leq 700 \text{ MHz}$

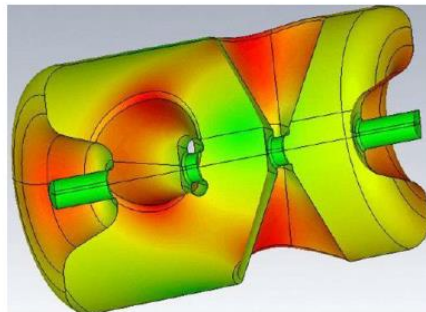


Photo:
Reidar Hahn

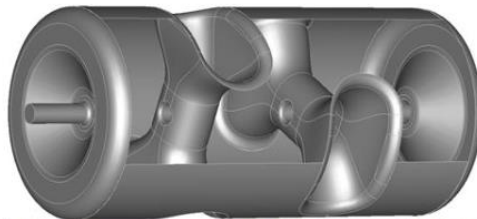
Cavity types – multi-cell



ESS/IPN $\beta=0.50$, $f=352\text{MHz}$



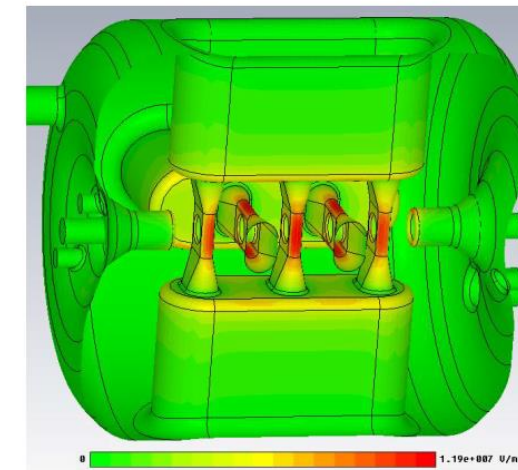
IAP 360 MHz, $\beta_0 \sim 0.1$
19 gap CH resonator



500 MHz, $\beta_0 = 1$
Double-Spoke Cavity



ANL $\beta=0.63$, $f=345\text{MHz}$

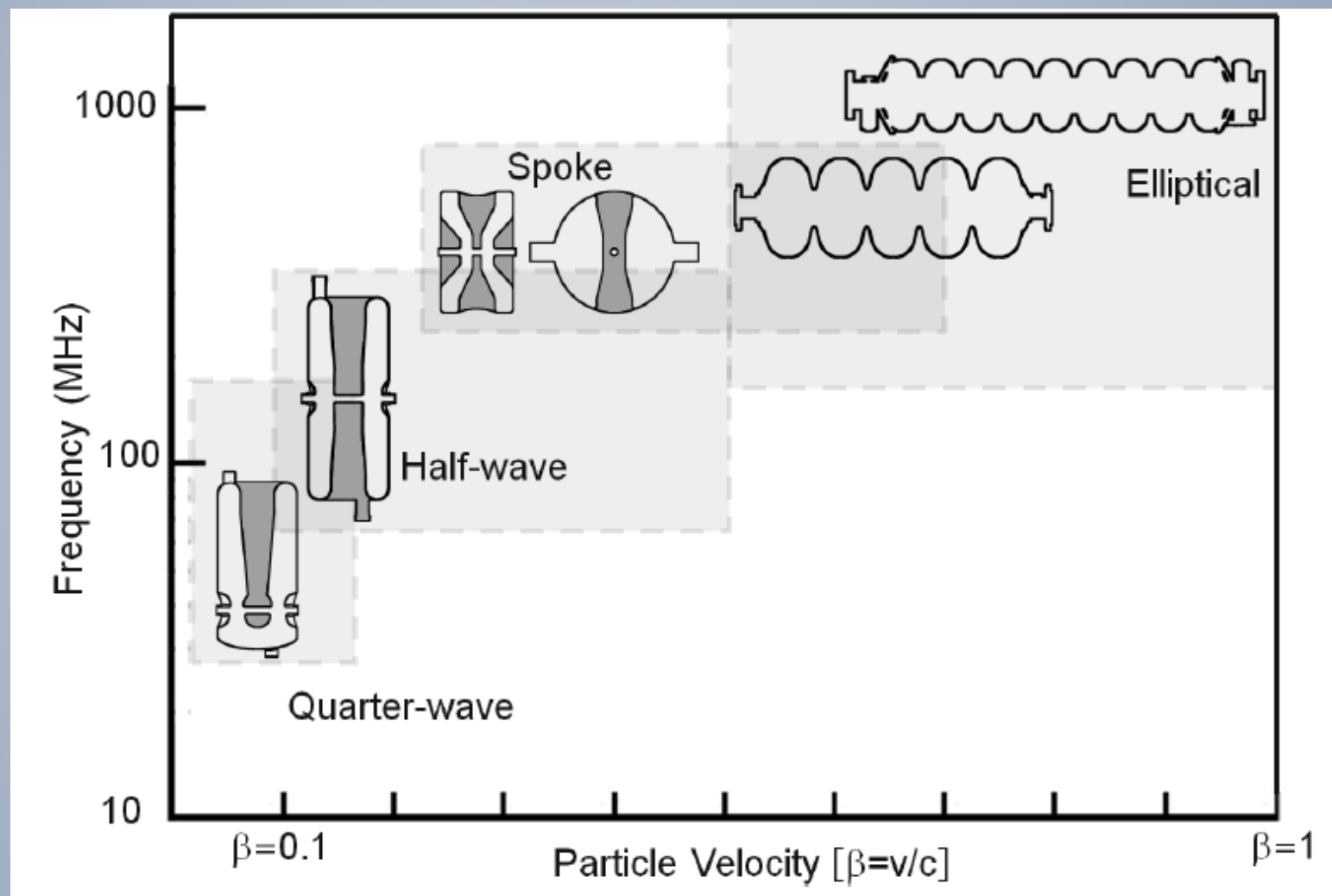


IMP CH $\beta=0.067$, $f=162.5\text{MHz}$

CH: Crossbar H-mode

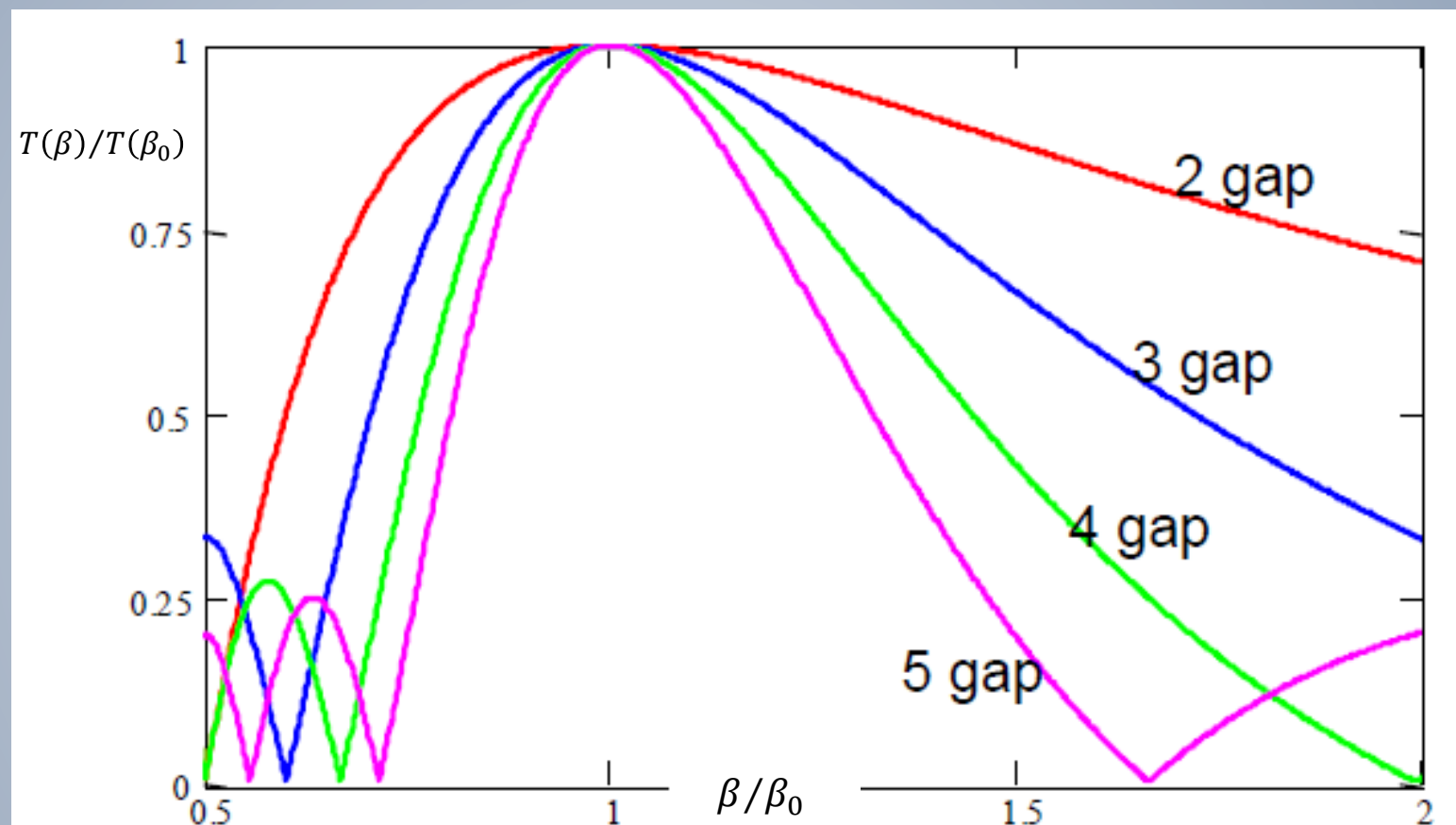


Accelerating cavity velocity/frequency chart





Transit Time factor vs. β for multiple gaps



Normalized transit time factor curves vs. normalized velocity, for cavities with different number of gap



High- β spoke cavities

- High velocity spoke cavities with $\beta > 0.8$ are being designed as alternative to elliptical cavities
- Features:
 - relatively compact
 - between 20% and 50% smaller (radially) for low- β cavities
 - for high β diameter close to TM counterparts
 - allows low frequency at reasonable size
 - mechanically stable – high shunt impedance



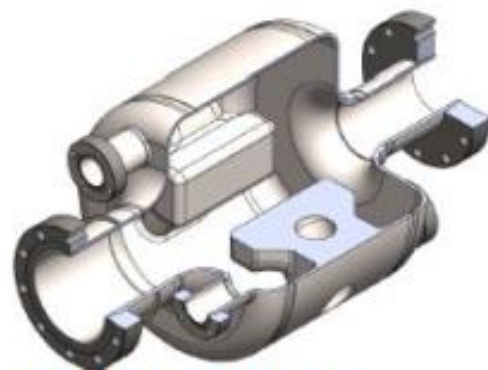
325 MHz $\beta=0.82$ Single Spoke Cavity



500 MHz $\beta=1.0$ Double Spoke Cavity

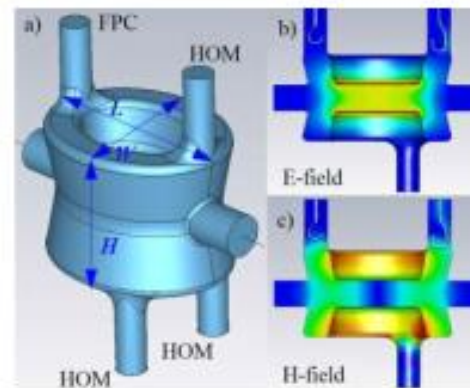


Deflecting mode cavities

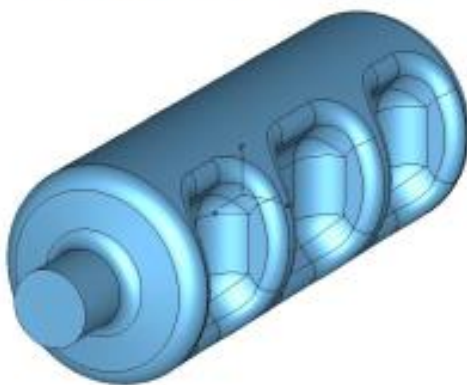


TRIUMF 650MHz

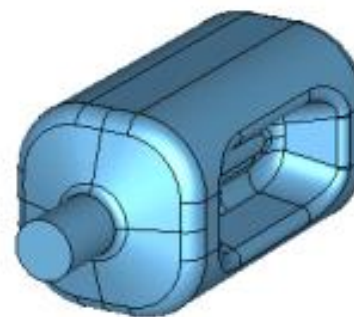
double quarter wave (DQW) – 400MHz – BNL/CERN



RFD – multi-cell – 953MHz – ODU



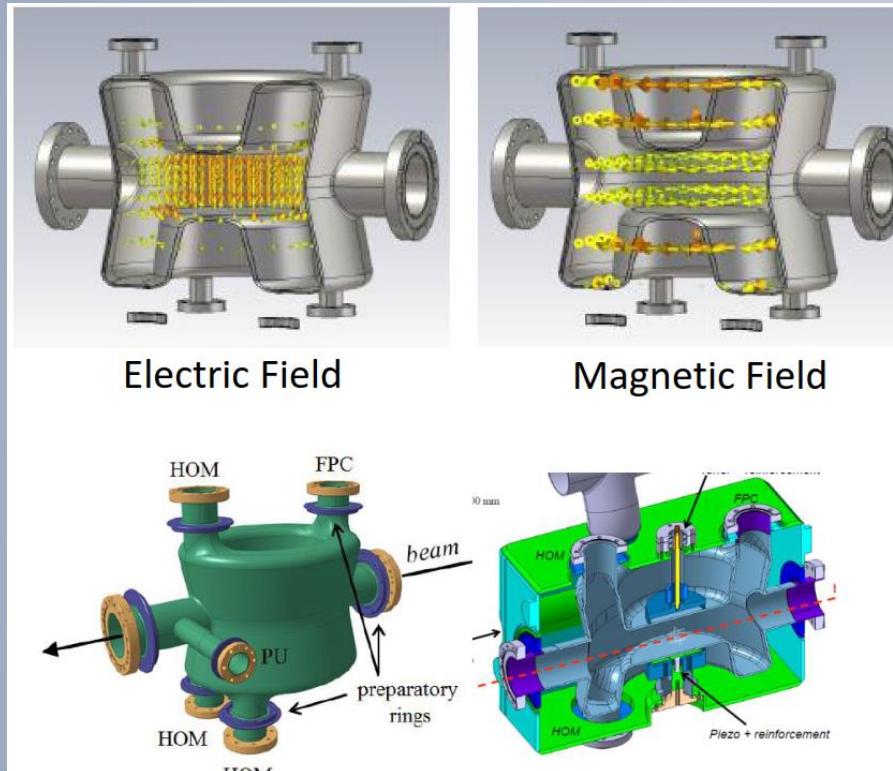
RF Dipole (RFD) – 400MHz – ODU/CERN



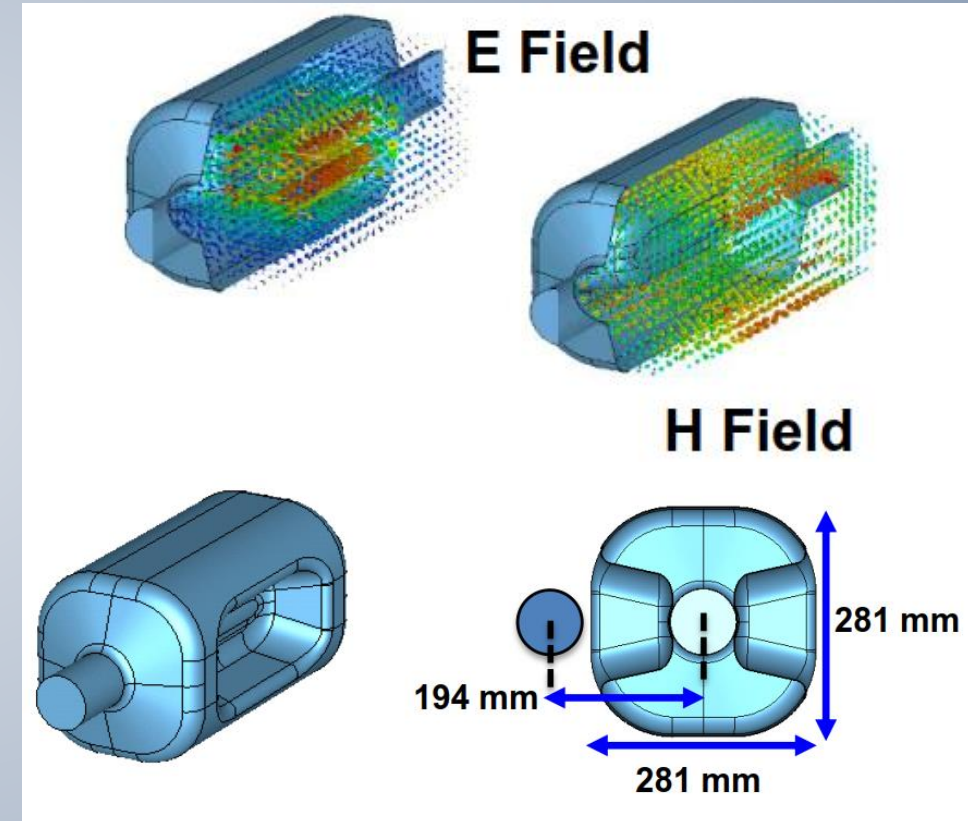


Prototype HL-LHC Crab Cavity prototypes

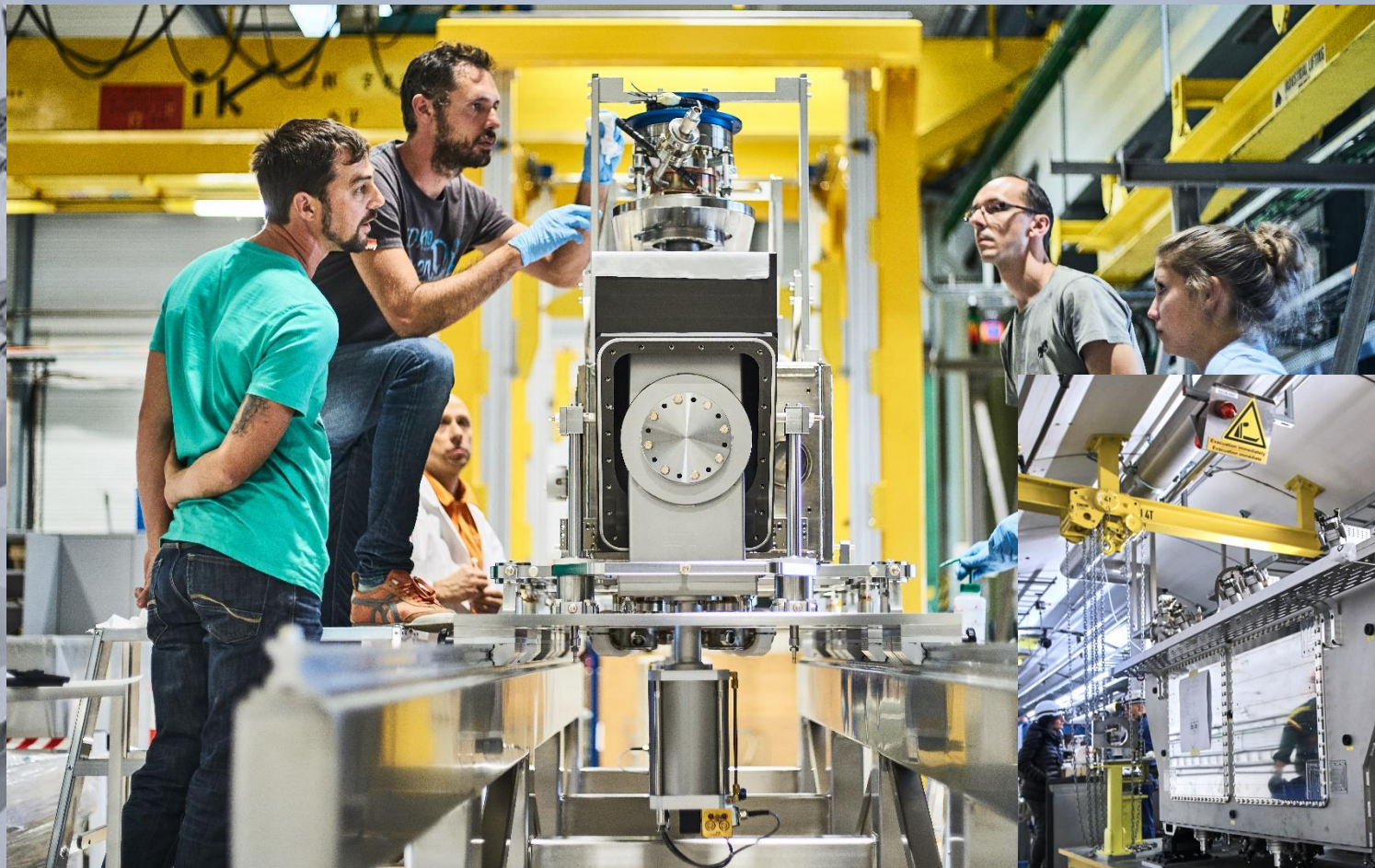
“DQW” (vertical deflection)



“RFD” (vertical deflection)



The real thing (HL-LHC Crab Cavity)



CERN-PHOTO-201708-196-10



CERN-PHOTO-2018-026-5

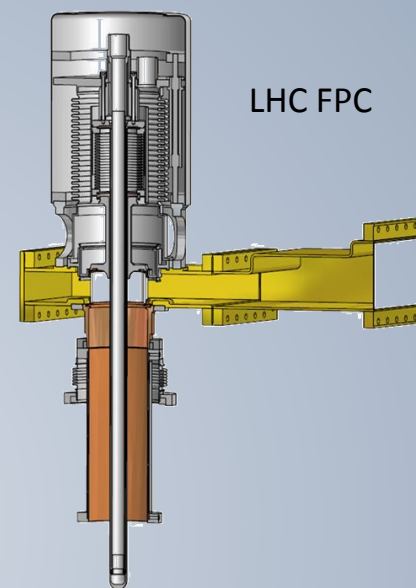
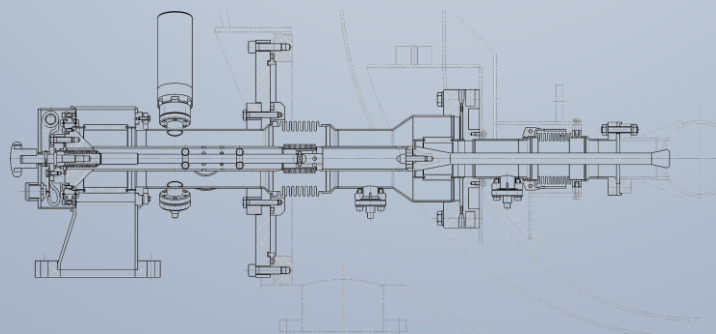
Power Couplers



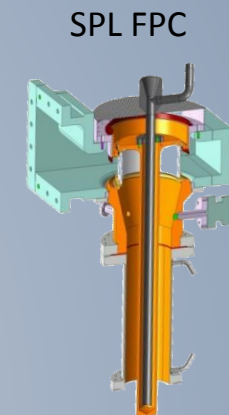
Photo:
Reidar Hahn

Fundamental Power Coupler – FPC

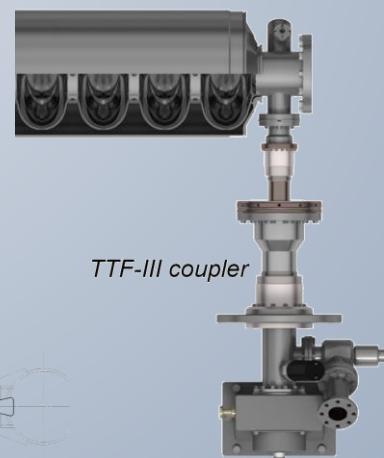
- The **Fundamental Power Coupler** is the connecting part between the RF transmission line and the RF cavity
- It is a specific piece of transmission line that also has to provide the cavity vacuum barrier.
- FPCs are amongst the most critical parts of the RF cavity system in an accelerator!
- A good RF design, a good mechanical design and a high quality fabrication are essential for an efficient and reliable operation.



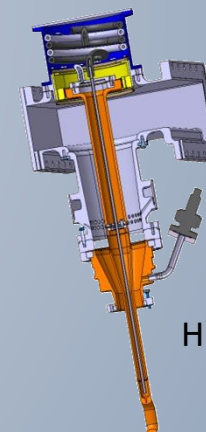
LHC FPC



SPL FPC



TTF-III coupler



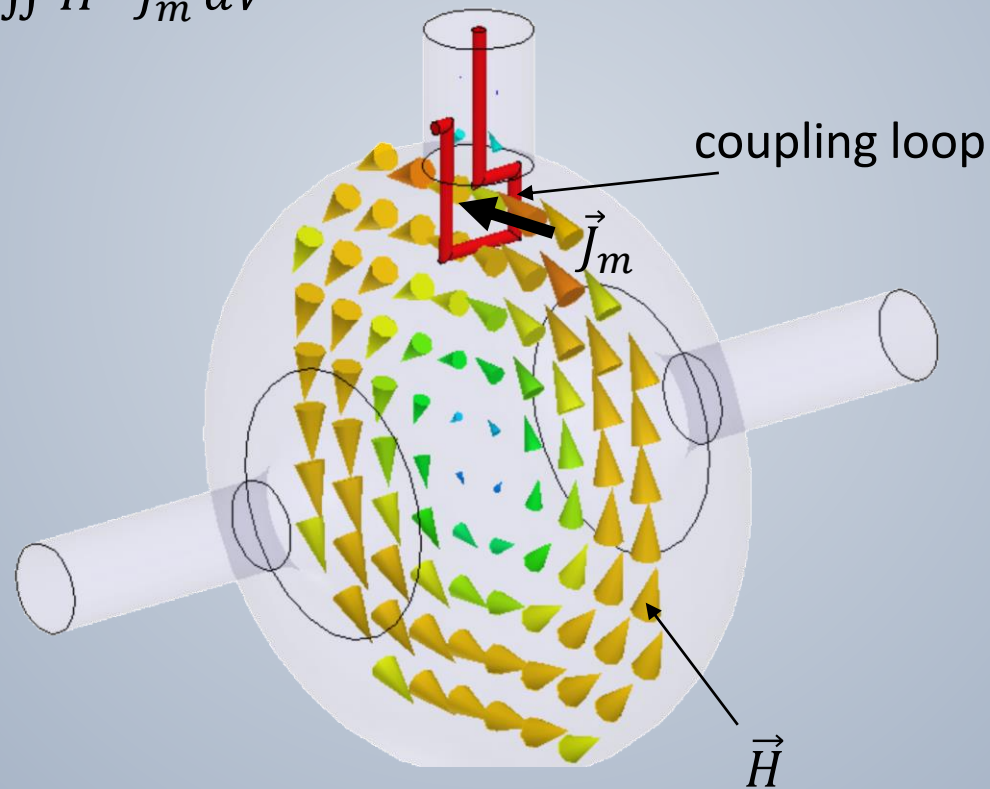
HL-LHC FPC

courtesy: Eric Montesinos/CERN



Magnetic (loop) coupling

- The magnetic field of the cavity main mode is intercepted by a coupling loop
- The coupling can be adjusted by changing the size or the orientation of the loop.
- Coupling: $\propto \iiint \vec{H} \cdot \vec{J}_m dV$

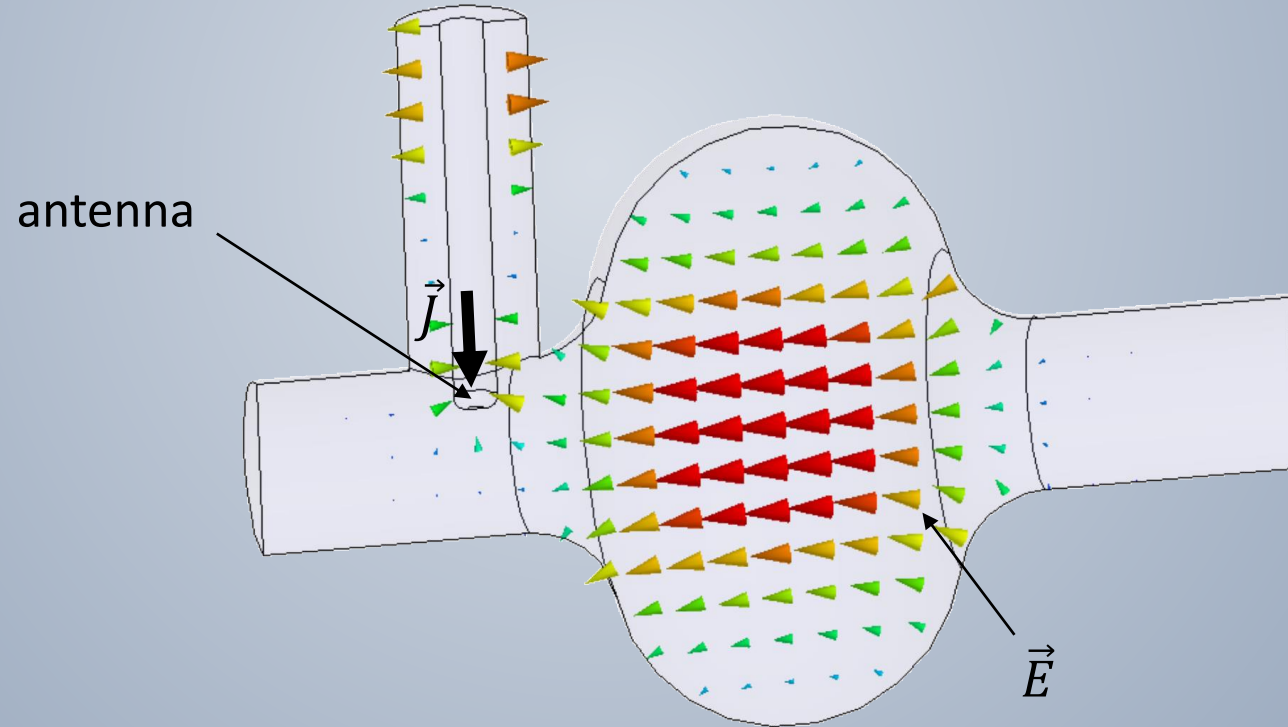


courtesy: David Alesini/INFN



Electric (antenna) coupling

- The inner conductor of the coaxial feeder line ends in an antenna penetrating into the electric field of the cavity.
- The coupling can be adjusted by varying the penetration.
- Coupling $\propto \iiint \vec{E} \cdot \vec{J} dV$



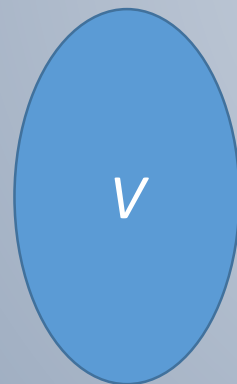
courtesy: David Alesini/INFN

Tuners



Small boundary perturbation

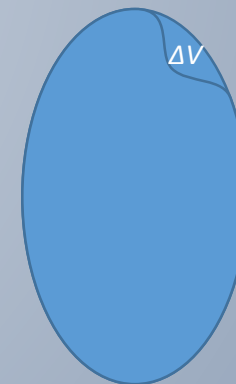
- Perturbation calculation is used to understand the basics for cavity tuning – it is used to analyse the sensitivity to (small) surface geometry perturbations.
 - This is relevant to understand the effect of fabrication tolerances.
 - Intentional surface deformation or introduced obstacles can be used to tune the cavity.
- The basic idea of the perturbation theory is use a known solution (in this case the unperturbed cavity) and assume that the deviation from it is only small. We just used this to calculate the losses (assuming H_t would be that without losses).
- The result of this calculation leads to a convenient expression for the (de)tuning:



unperturbed: ω_0

$$\frac{\omega - \omega_0}{\omega} = \frac{\iiint_{\Delta V} (\mu_0 |H_0|^2 - \varepsilon |E_0|^2) dV}{\iiint_V (\mu_0 |H_0|^2 + \varepsilon |E_0|^2) dV}$$

Slater's Theorem



perturbed: ω

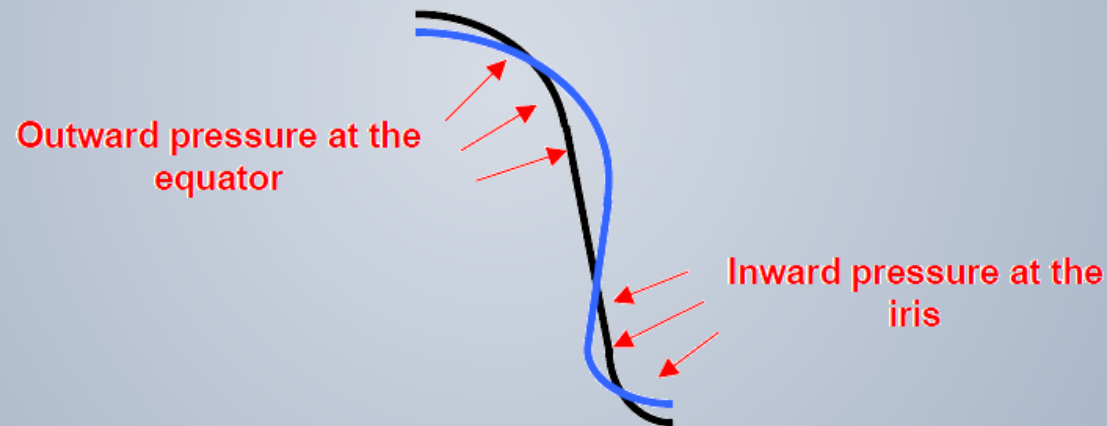


John C. Slater
1900 – 1976



Lorentz force detuning (“LFD”)

- The presence of electromagnetic fields inside the cavity lead to a mechanical pressure on the cavity.
- Radiation pressure: $P = \frac{1}{4}(\mu_0 |H|^2 - \epsilon_0 |E|^2)$
- Deformation of the cavity shape:

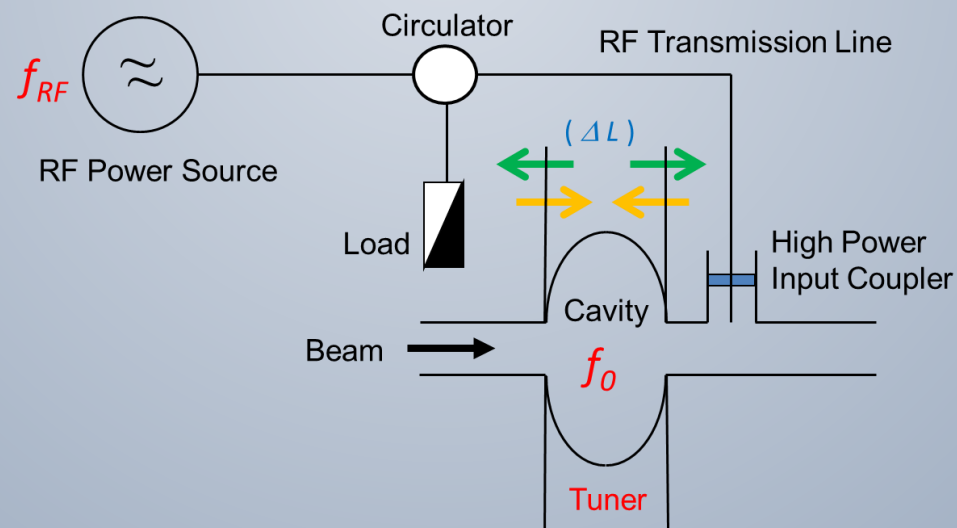


- Frequency shift: $\Delta f = K L |E_{acc}|^2$; typical: $K L \approx -(1 \dots 10) \text{Hz} / \left(\frac{\text{MV}}{\text{m}}\right)^2$
- This requires good stiffness – and the possibility to tune rapidly!



Tuner principle

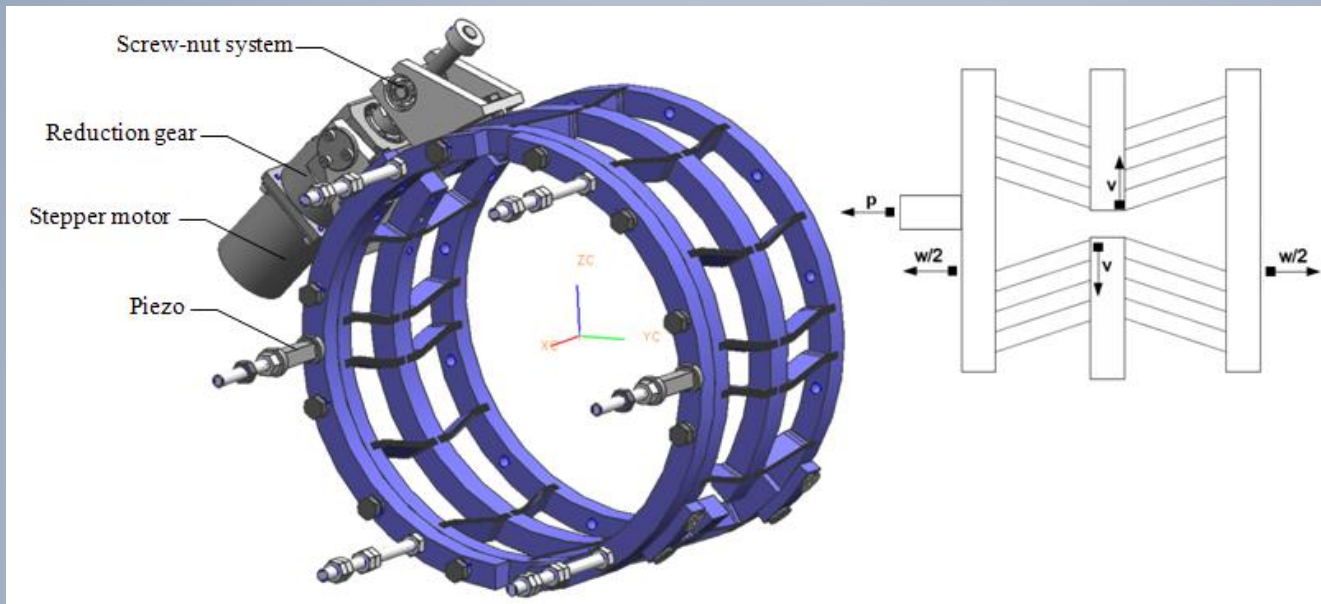
- Slow tuners:
 - compensate for mechanical tolerances,
 - realized with stepper motor drives
- Fast tuners:
 - compensate Lorentz-force detuning and reactive beam loading
 - realized with piezo crystal (lead zirconate titanate – PZT)
- Tuning of SC cavities is often realized by deforming the cavity:



courtesy: Eiji Kako/KEK



Blade tuner

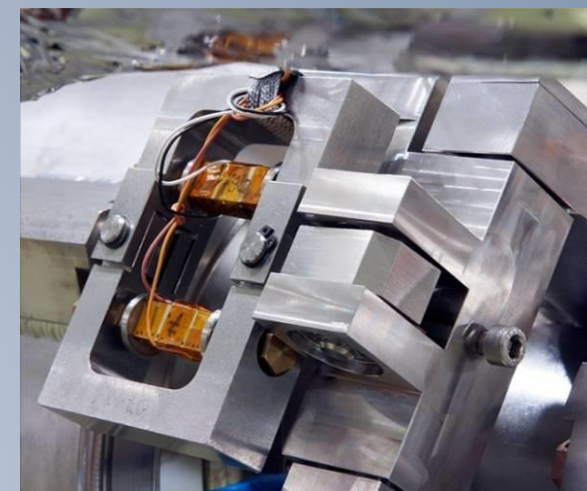
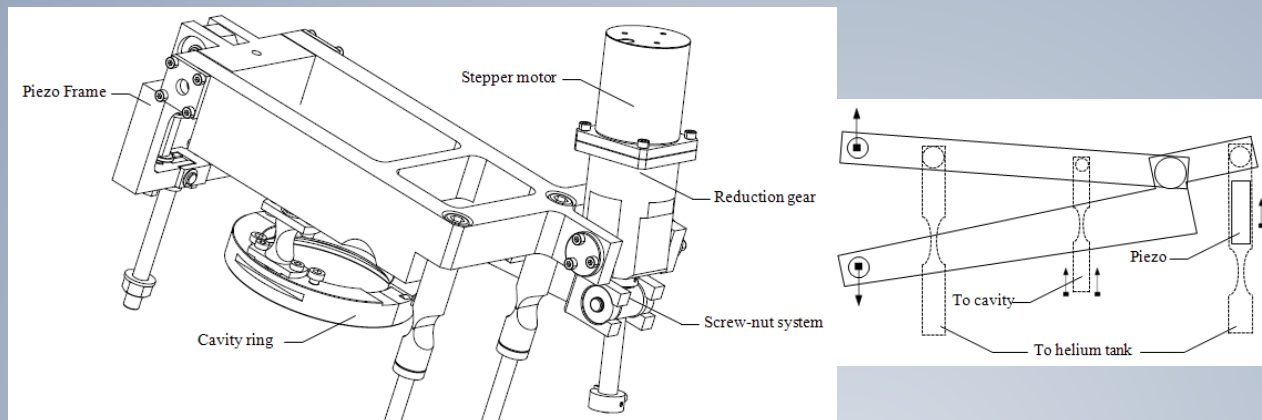


- Developed by INFN Milano
- Azimuthal motion transferred to longitudinal strain
- Zero backlash
- CuBe threaded shaft used for a screw nut system
- Stepping motor and gear combination driver
- Two piezo actuators for fast action
- All components in cold location

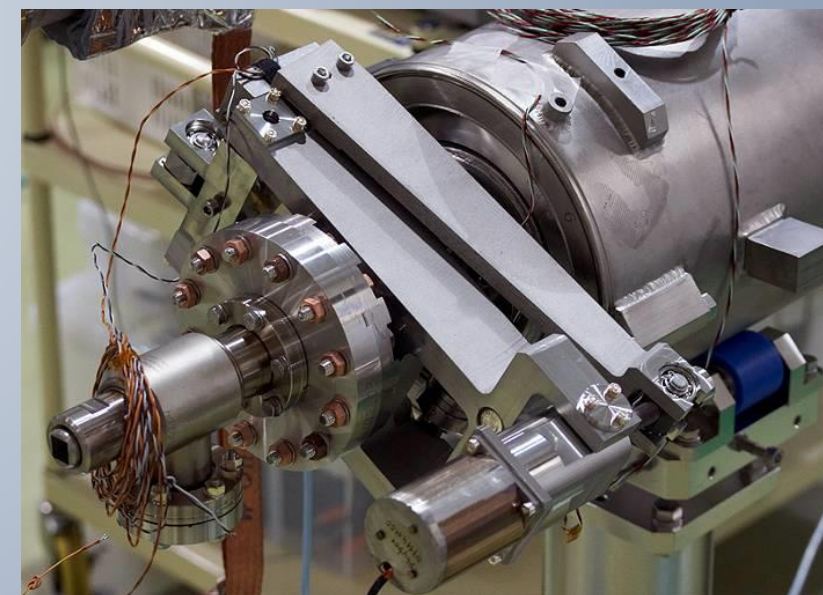


courtesy: Eiji Kako/KEK

“Saclay” lever-arm tuner



- Developed by DESY based on the Saclay design
- Double lever system (leverage 1.25)
- Cold stepping motor and gear combination
- Screw nut system
- Two piezo actuators for fast action in a preloaded frame
- All components in cold location

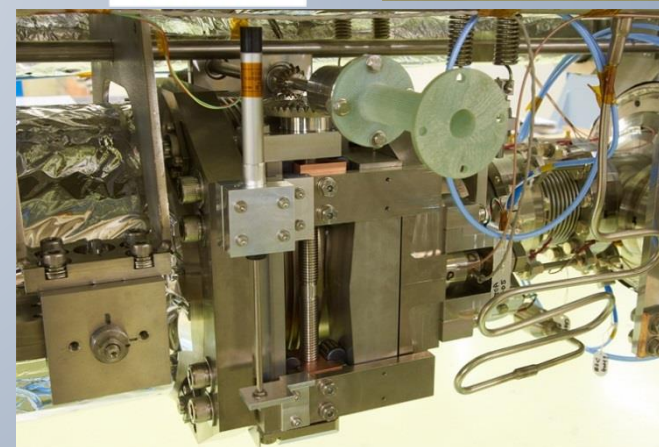
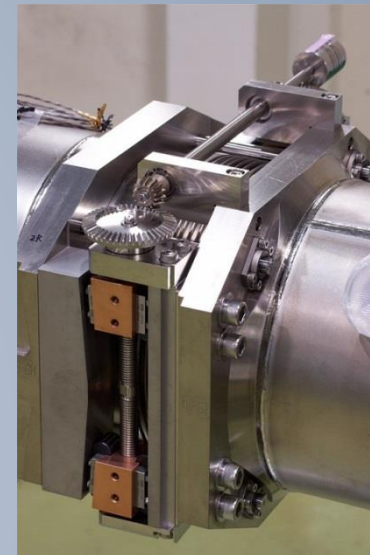
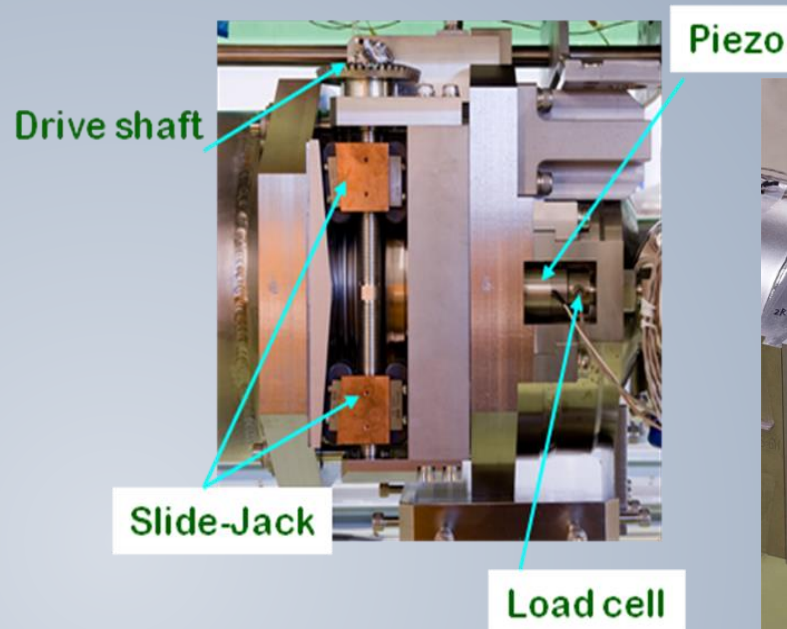
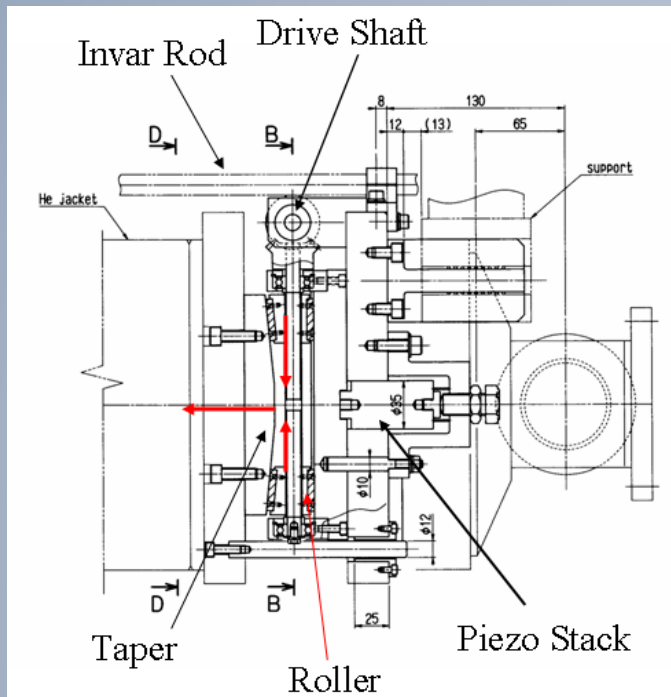


courtesy: Eiji Kako/KEK



Photo:
Reidar Hahn

Slide-jack tuner



- Developed by KEK for STF cryomodule
- Slide-jack mechanism
- Single high voltage piezo actuator for fast action
- Warm stepping motor for easy maintenance
- Access port for replacing piezo

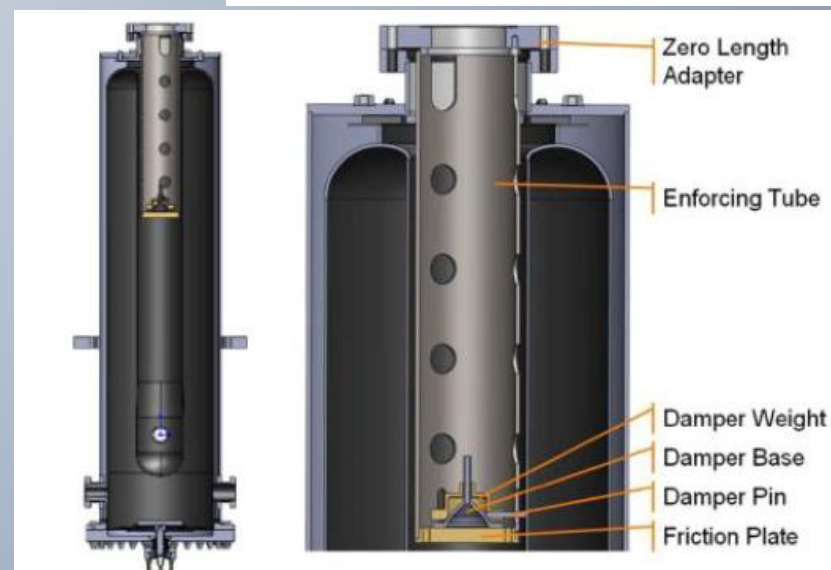
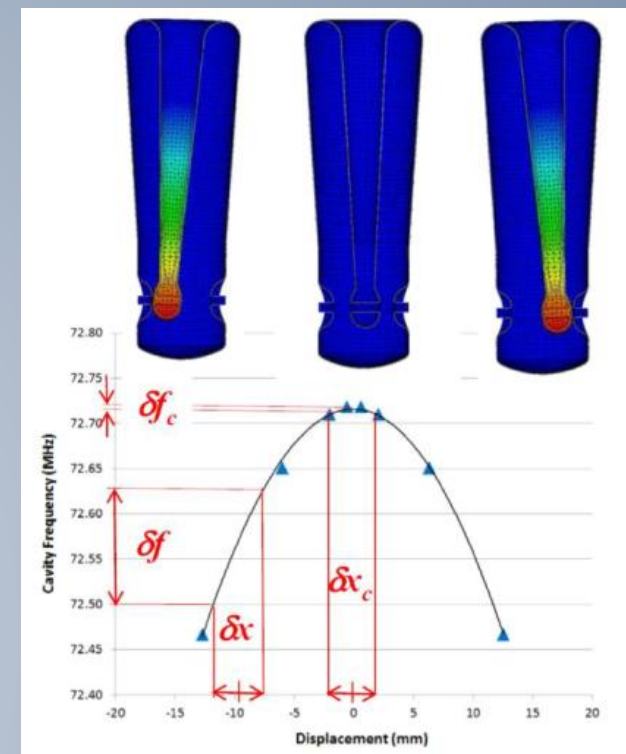
courtesy: Eiji Kako/KEK



Photo:
Reidar Hahn

Microphonics

- Driven by mechanical vibration in the environment.
- QWRs are particularly problematic due to the pendulum action of the inner conductor, which can have very low mechanical frequencies ((50 ... 100) Hz)
 - need to reduce the RMS detuning to $\ll 10\%$ of the available BW to avoid nuisance
 - the other option is to increase the BW (lower Q_L , costs power)
- Mitigation:
 - stiffening during design/manufacture
 - centring the inner conductor by plastic deformation so that $df/dx = 0$.
 - adding passive dampers
 - reduce environmental noise

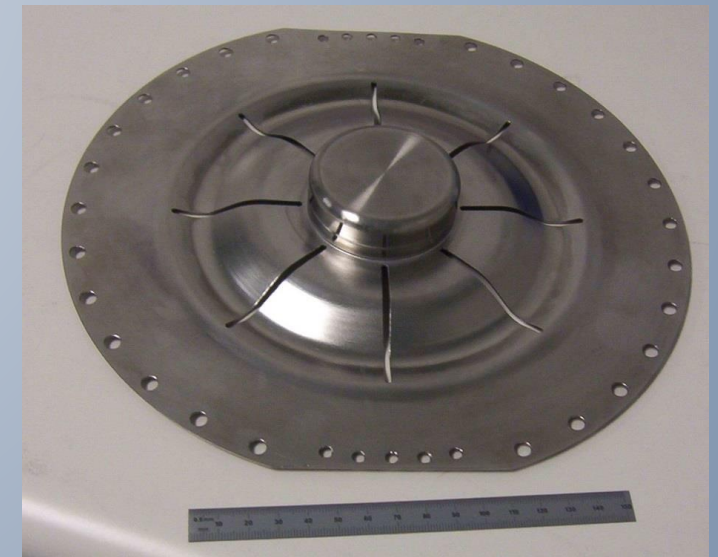
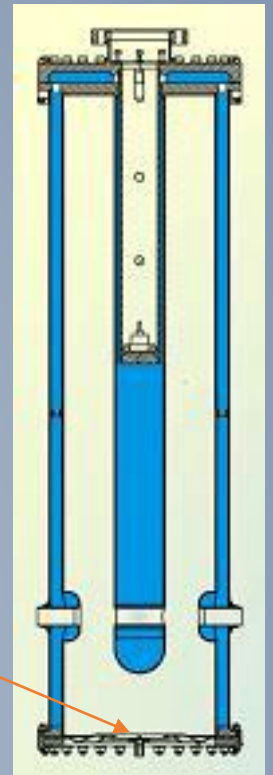




Frequency compensation/tuning

- For QWRs a tuning plate at the open end near the beam tube is generally used.
- For QWRs with removable tuning plate, a Nb puck can be welded to it – this reduces the cavity f_0 by increasing the equivalent C .
- This puck can be trimmed after final fabrication

tuning plate



Thank you very much!