Introduction to Weak Superconductivity

Josephson Effect:
Physics and Applications

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Outline of the lessons:

- **Basics of Josephson effect**
- **Fabrication Technology**

Basics of Josephson effect:

- **Tunneling among superconductors**
- **Josephson Equations**
- **DC and AC Josephson effect**
- **Microscopic Theory**
- **Electrodynamics of a Josephson junction**
- **Magnetic field effects**
- **RF fields effects**
- **Nonlinear waves in Josephson junctions**

A superconductor can be seen as a macroscopic quantum state with a **long range order**

$$\psi = \rho^{1/2} e^{j\varphi}$$

where ϕ is the phase common to all the particles and ρ represents, in this macroscopic picture, their actual density in the macrostate |s> :

$$\langle s|\psi^*\psi|s\rangle = |\psi|^2 = \rho$$

The electric current density can be written, in the presence of a vector potential A:

$$\mathbf{J} = \frac{e^*}{m^*}\left[\frac{j\hbar}{2}(\psi\nabla\psi^* - \psi^*\nabla\psi) - \frac{e^*}{c}\mathbf{A}|\psi|^2\right]$$

or

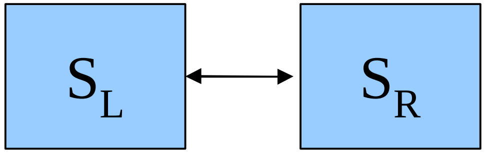
$$\mathbf{J} = \rho\frac{e}{m}\left(\hbar\nabla\varphi - \frac{2e}{c}\mathbf{A}\right)$$

The time evolution of ψ in stationary conditions obeys the usual quantum mechanical equation of the form:

$j = \sqrt{-1}$

$$j\hbar\frac{\partial\psi}{\partial t} = E\psi$$

Coupled Superconductors



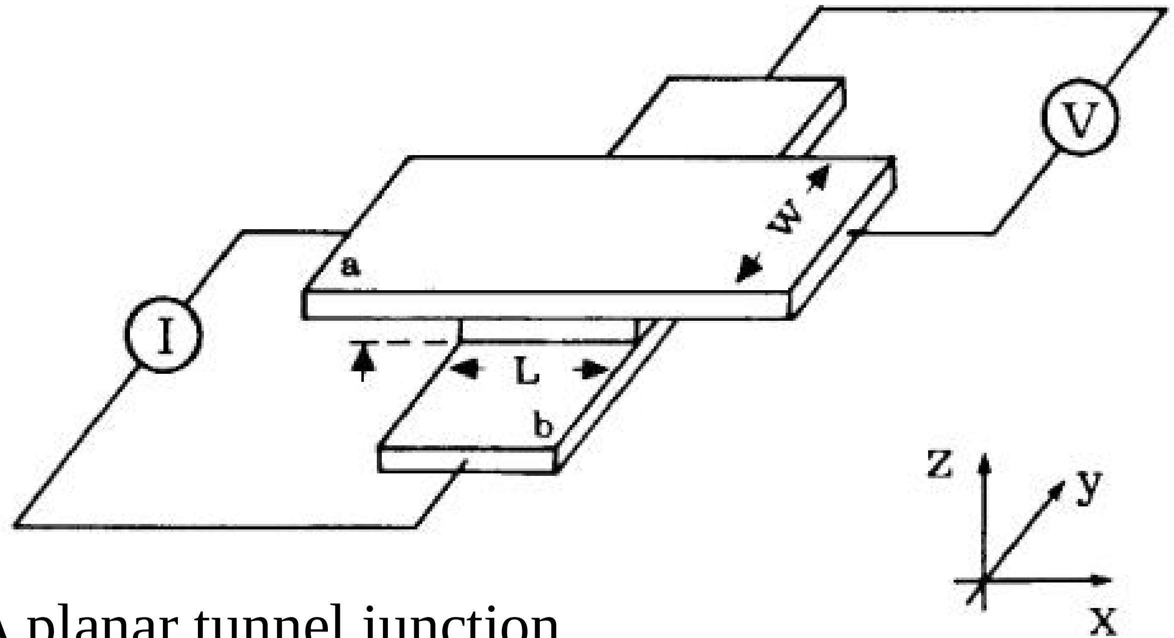
Let us now consider two superconductors S_L and S_R separated by a macroscopic distance. In this situation, the phase of the two superconductors can change independently.

As the two superconductors are moved closer, so that their separation is reduced to about 3 nm, quasiparticles can flow from one superconductor to the other by means of the tunneling effect (single electron tunneling).

If we reduce further the distance between S_L and S_R down to say 1 nm, then also Cooper pairs can flow from one superconductor to the other (Josephson tunneling). The **long range order** is "**transmitted**" across the boundary.

The whole system of the two superconductors separated by a thin dielectric barrier will behave as a single superconductor. This phenomenon is often called "weak superconductivity" (Anderson 1963) because of the much lower values of the critical parameters involved.

Quasiparticle tunneling



A planar tunnel junction

$$I_{L\to R} = \frac{2\pi}{\hbar}\int_{-\infty}^{+\infty} |\mathrm{T}|^2 N_L(E) f_L(E) N_R(E)(1-f_R(E))\, dE$$

$$I = I_{L\to R} - I_{R\to L} = \frac{2\pi}{\hbar}\int_{-\infty}^{+\infty} |\mathrm{T}|^2 N_L(E) N_R(E)[f_L(E)-f_R(E)]\, dE$$

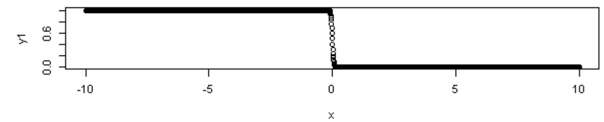
$$I = \frac{2\pi}{\hbar}|\mathrm{T}|^2 \int_{-\infty}^{+\infty} N_L(E) N_R(E+eV)[f_L(E)-f_R(E+eV)]\, dE$$

Let us consider two normal metals. Assume also N_L and N_R to be constant and equal to the densities of states at the Fermi energy level.

$$I_{NN} = \text{constant} \times \int_{-\infty}^{+\infty} [f(E) - f(E+eV)]\, dE$$

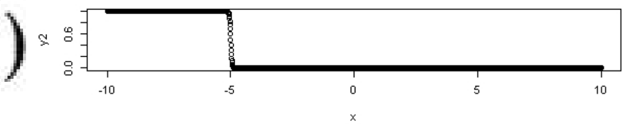
that is $\quad I_{NN} = \sigma_N V$

$f(E)$



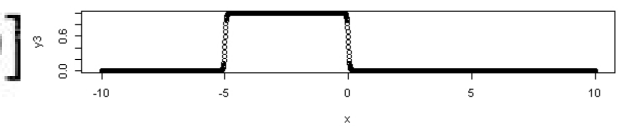
The surprising result is that the resulting I-V curve is Ohmic.

$f(E+eV)$



Most of the weldings between two wires are actually tunnel junctions

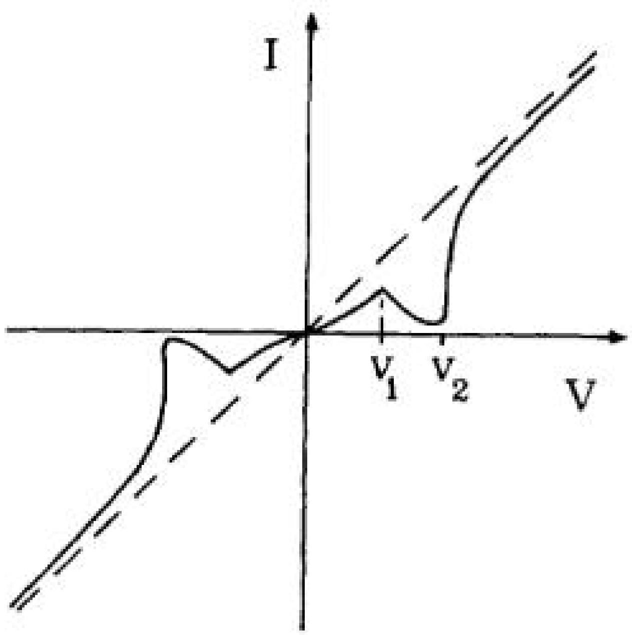
$[f(E) - f(E+eV)]$



When the two metals are in the superconducting state the situation is greatly altered. In fact the densities of states are now given by:

$$N(E) = N(0)\frac{E}{\sqrt{E^2-\Delta^2}} \qquad |E| \geqslant \Delta$$

$$N(E) = 0 \qquad |E| < \Delta$$

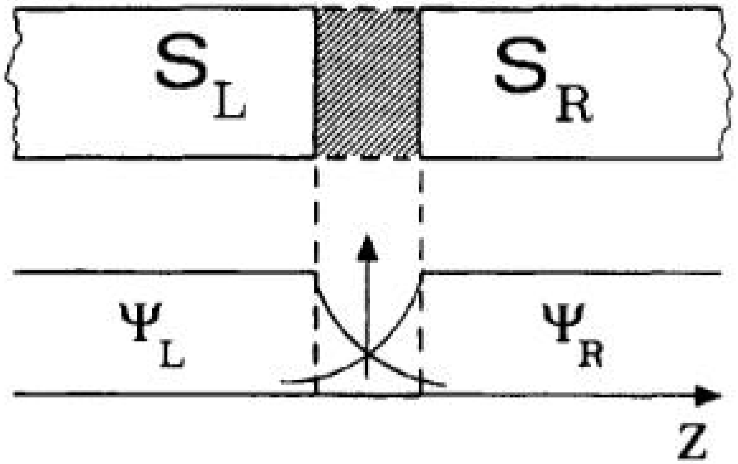


Therefore the tunneling current in a junction with the electrodes both superconductors is given by

$$I_{SS} = \text{constant} \times \int_{-\infty}^{+\infty} \frac{|E|}{|E^2-\Delta_L^2|^{1/2}} \frac{|E+eV|}{|(E+eV)^2-\Delta_R^2|^{1/2}} [f(E)-f(E+eV)]\, dE$$

The resulting I-V curve is **nonlinear** for voltages below the gap voltage

Cooper pair tunneling



a simple derivation due to Feynman is based on a weakly coupled “two level system" picture.

$$\langle L|\psi_L^*\psi_L|L\rangle = |\psi_L|^2 = \rho_L \qquad \langle R|\psi_R^*\psi_R|R\rangle = |\psi_R|^2 = \rho_R$$

$$|\psi\rangle = \psi_R|R\rangle + \psi_L|L\rangle \qquad j\hbar\frac{\partial|\psi\rangle}{\partial t} = \mathcal{H}|\psi\rangle$$

$$\mathcal{H} = \mathcal{H}_L + \mathcal{H}_R + \mathcal{H}_T \quad \mathcal{H}_L = E_L|L\rangle\langle L| \quad \mathcal{H}_R = E_R|R\rangle\langle R| \quad \mathcal{H}_T = K[|L\rangle\langle R| + |R\rangle\langle L|]$$

$$j\hbar\frac{\partial\psi_R}{\partial t} = E_R\psi_R + K\psi_L$$

$$j\hbar\frac{\partial\psi_L}{\partial t} = E_L\psi_L + K\psi_R$$

E_L and E_R are the ground state energies and K is the coupling amplitude. If we consider a d.c. voltage V across the junction the ground states are shifted by an amount eV and consequently it is E_L-E_R = 2eV.

$$j\hbar\frac{\partial\psi_R}{\partial t} = -eV\psi_R + K\psi_L$$

$$j\hbar\frac{\partial\psi_L}{\partial t} = eV\psi_L + K\psi_R$$

$$\psi_L = \rho_L^{1/2}e^{j\varphi_L} \qquad \psi_R = \rho_R^{1/2}e^{j\varphi_R}$$

$$\psi_L = \rho_L^{1/2} e^{j\varphi_L} \qquad \psi_R = \rho_R^{1/2} e^{j\varphi_R}$$

$$\frac{\partial \rho_L}{\partial t} = \frac{2}{\hbar} K \sqrt{\rho_L \rho_R} \sin\varphi \qquad \frac{\partial \varphi_L}{\partial t} = \frac{K}{\hbar} \sqrt{\frac{\rho_L}{\rho_R}} \cos\varphi + \frac{eV}{\hbar}$$

$$\varphi = \varphi_L - \varphi_R$$

$$\frac{\partial \rho_R}{\partial t} = -\frac{2}{\hbar} K \sqrt{\rho_L \rho_R} \sin\varphi \qquad \frac{\partial \varphi_R}{\partial t} = \frac{K}{\hbar} \sqrt{\frac{\rho_L}{\rho_R}} \cos\varphi - \frac{eV}{\hbar}$$

$$J \equiv \frac{\partial \rho_L}{\partial t} = -\frac{\partial \rho_R}{\partial t}$$

$$J = \frac{2K}{\hbar} \sqrt{\rho_L \rho_R} \sin\varphi \qquad \frac{\partial \varphi}{\partial t} = \frac{2eV}{\hbar}$$

The Josephson equations

Assuming ρ_L and ρ_R constant (a current source continuously replaces the pairs tunneling across the barrier)

$$J = J_1 \sin\varphi$$

$$J = J_1 \sin\varphi \qquad \frac{\partial\varphi}{\partial t} = \frac{2eV}{\hbar}$$

Assuming V=0 the phase difference ϕ results to be constant not necessarily zero, so that a finite current density with a maximum value J_l can flow through the barrier with zero voltage drop across the junction.

This is the essence of the **d.c. Josephson effect** (Josephson 1962). The first observation was made by Anderson and Rowell in 1963.

If we apply a constant voltage V=0, it follows that the phase ϕ varies in time as $\phi = \phi_0 + 2eV/\hbar\, t$ and therefore there appears an alternating current

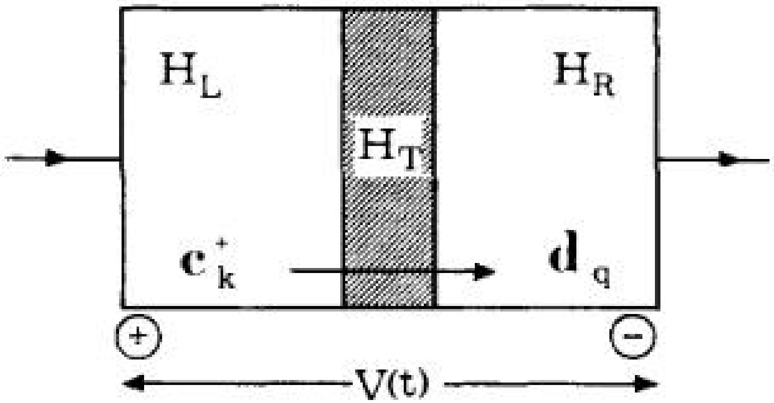
$$J = J_1 \sin\left(\varphi_0 + \frac{2e}{\hbar} V t \right)$$

with a frequency $\omega = 2\pi f = 2eV/\hbar$. This is called **a.c. Josephson effect**.

The ratio between frequency and voltage is:

$$f/V = 483.6 \text{ MHz}/\mu\text{V}$$

Microscopic theory of Josephson effect: Tunneling Hamiltonian formalism



$$\mathcal{H}=\mathcal{H}_L+\mathcal{H}_R+\mathcal{H}_T$$

$$N_L=\sum_{\mathbf{k},\sigma} c^{+}_{\mathbf{k}\sigma}c_{\mathbf{k}\sigma}; \qquad N_L=\sum_{\mathbf{q},\sigma} d^{+}_{\mathbf{q}\sigma}d_{\mathbf{q}\sigma}$$

$$\mathcal{H}_T=\sum_{\mathbf{kq}\sigma}\left[\mathrm{T}_{\mathbf{kq}}c^{+}_{\mathbf{k}\sigma}d_{\mathbf{q}\sigma}+\mathrm{T}^{*}_{\mathbf{kq}}d^{+}_{\mathbf{q}\sigma}c_{\mathbf{k}\sigma}\right] \qquad \frac{d\varphi}{dt}=\frac{2e}{\hbar}V(t)$$

$$I(V,T)=-e\langle \dot{N}_R\rangle \qquad I(t,V_0,T)=I_{qp}(V_0,T)+I_{J1}(V_0,T)\sin\varphi(t)+I_{J2}(V_0,T)\cos\varphi(t)$$

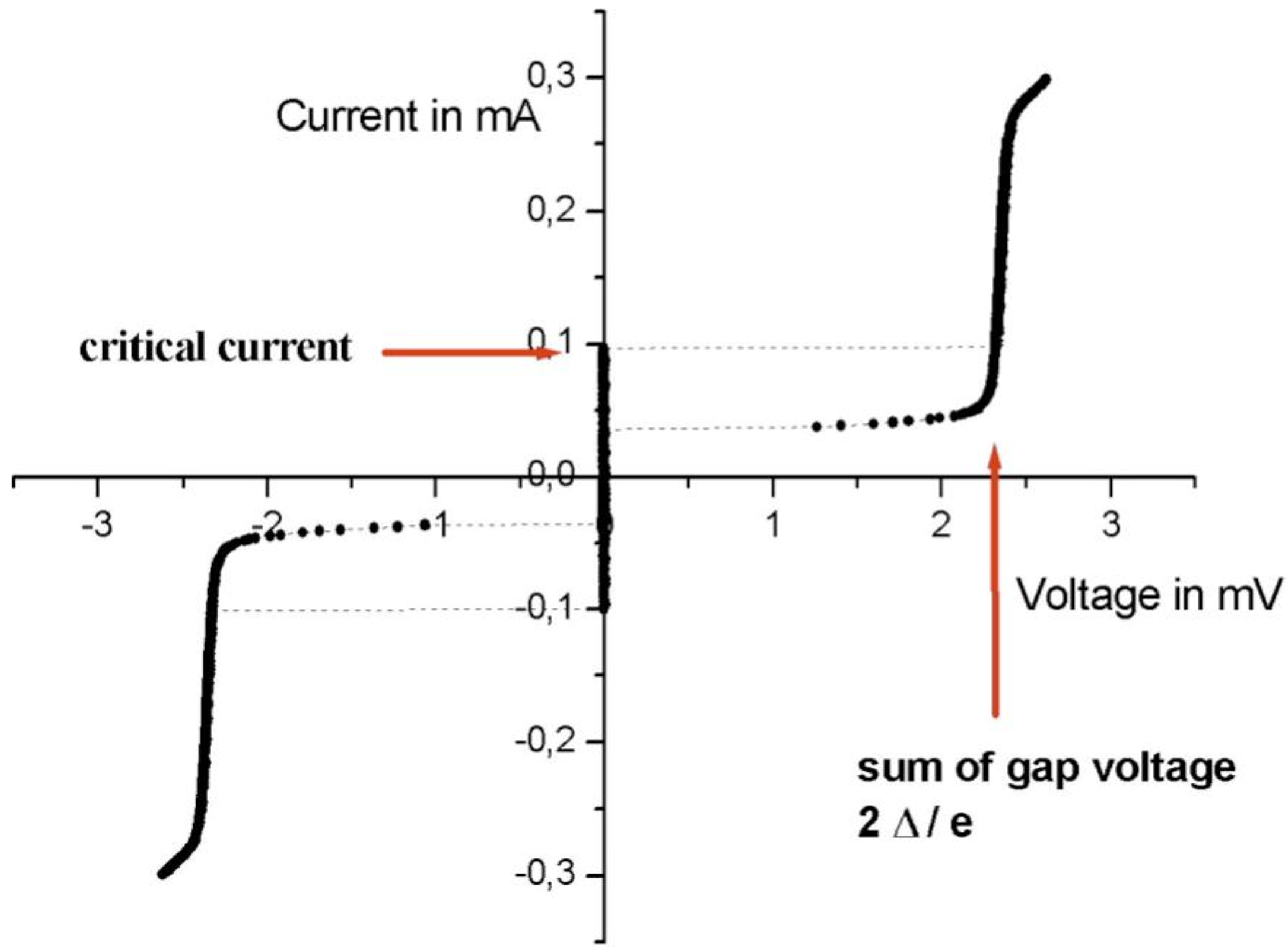
$$I_{qp_1}=\frac{\hbar}{\pi e R_N}\mathrm{P}\int_{-\infty}^{+\infty}d\omega\int_{-\infty}^{+\infty}d\omega'\,\frac{n_L(\omega)n_R(\omega')}{\omega-\omega'-\omega_0}\left[f(\omega)-f(\omega')\right]$$

$$I_{qp}=-\frac{\hbar}{eR_N}\int_{-\infty}^{+\infty}d\omega\, n_L(\omega)n_R(\omega-\omega_0)\left[f(\omega)-f(\omega-\omega_0)\right]$$

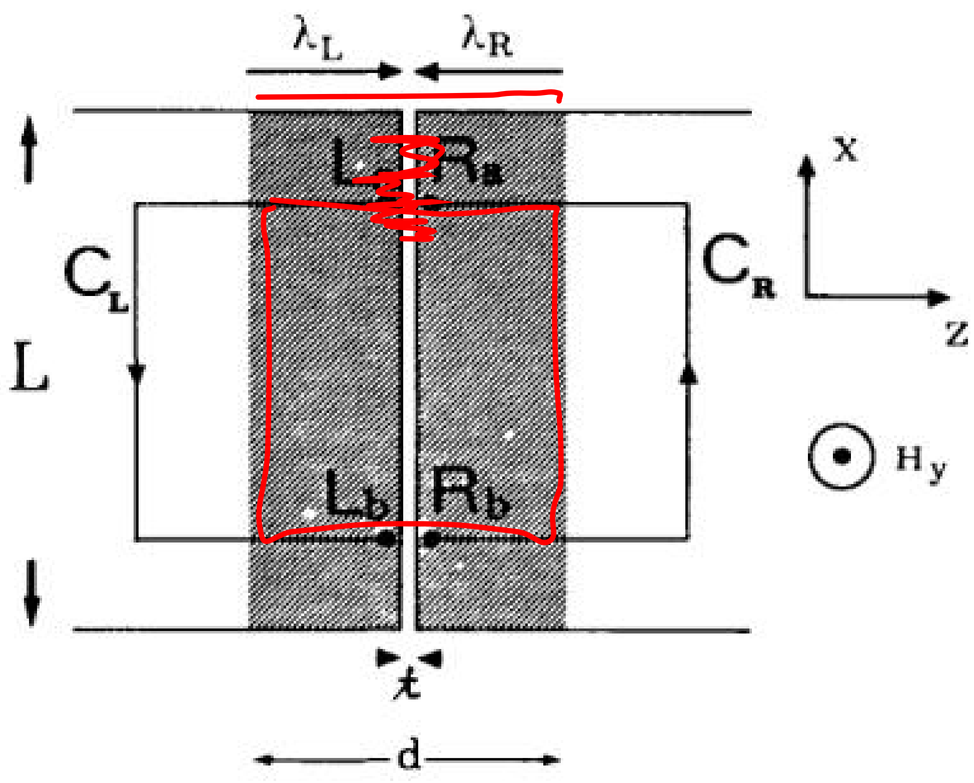
$$I_{J1}=-\frac{\hbar}{\pi e R_N}\mathrm{P}\int_{-\infty}^{+\infty}d\omega\int_{-\infty}^{+\infty}d\omega'\,\frac{p_L(\omega)p_R(\omega')}{\omega-\omega'-\omega_0}\left[f(\omega')-f(\omega)\right]$$

$$\omega_0=\frac{\omega_f}{2}=\frac{eV_0}{\hbar} \qquad I_{J2}=\frac{\hbar}{eR_N}\int_{-\infty}^{+\infty}d\omega\, p_L(\omega)p_R(\omega-\omega_0)\left[f(\omega-\omega_0)-f(\omega)\right]$$

for details see Physics and Applications of the Josephson Effect by A. Barone and G. Paternò,1982 by John Wiley & Sons, Inc., NY


Current in mA
0,3
0,2
critical current
0,1
0,0
-0,1
-0,2
-0,3
-3
-2
-1
0
1
2
3
Voltage in mV
sum of gap voltage
2 Δ / e

Magnetic field effects



$$\mathbf{J} = \rho \frac{e}{m}\left(\hbar \nabla \varphi - \frac{2e}{c}\mathbf{A}\right)$$

$$\nabla \varphi_{L,R} = \frac{2e}{\hbar c}\left(\frac{mc}{2e^2\rho}\mathbf{J}_S + \mathbf{A}\right)$$

$$\nabla \times \mathbf{A} = \mathbf{H}.$$

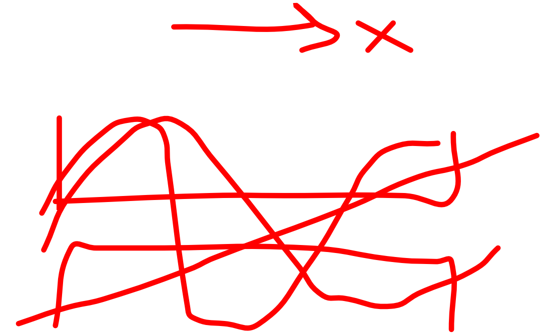
$$\varphi(x+dx) - \varphi(x) = \frac{2e}{\hbar c}\oint \mathbf{A}\cdot \mathbf{dl}$$

$$\varphi_{Ra}(x) - \varphi_{Rb}(x+dx) = \frac{2e}{\hbar c}\int_{C_R}\left(\mathbf{A} + \frac{mc}{2e^2\rho}\mathbf{J}_S\right)\cdot \mathbf{dl}$$

$$\varphi_{Lb}(x+dx) - \varphi_{La}(x) = \frac{2e}{\hbar c}\int_{C_L}\left(\mathbf{A} + \frac{mc}{2e^2\rho}\mathbf{J}_S\right)\cdot \mathbf{dl}$$

$$\oint \mathbf{A}\cdot \mathbf{dl} = H_y(\lambda_L + \lambda_R + \mathrm{t})\, dx$$

$$\frac{d\varphi}{dx} = \frac{2e}{\hbar c}(\lambda_L + \lambda_R + \mathrm{t}) H_y$$



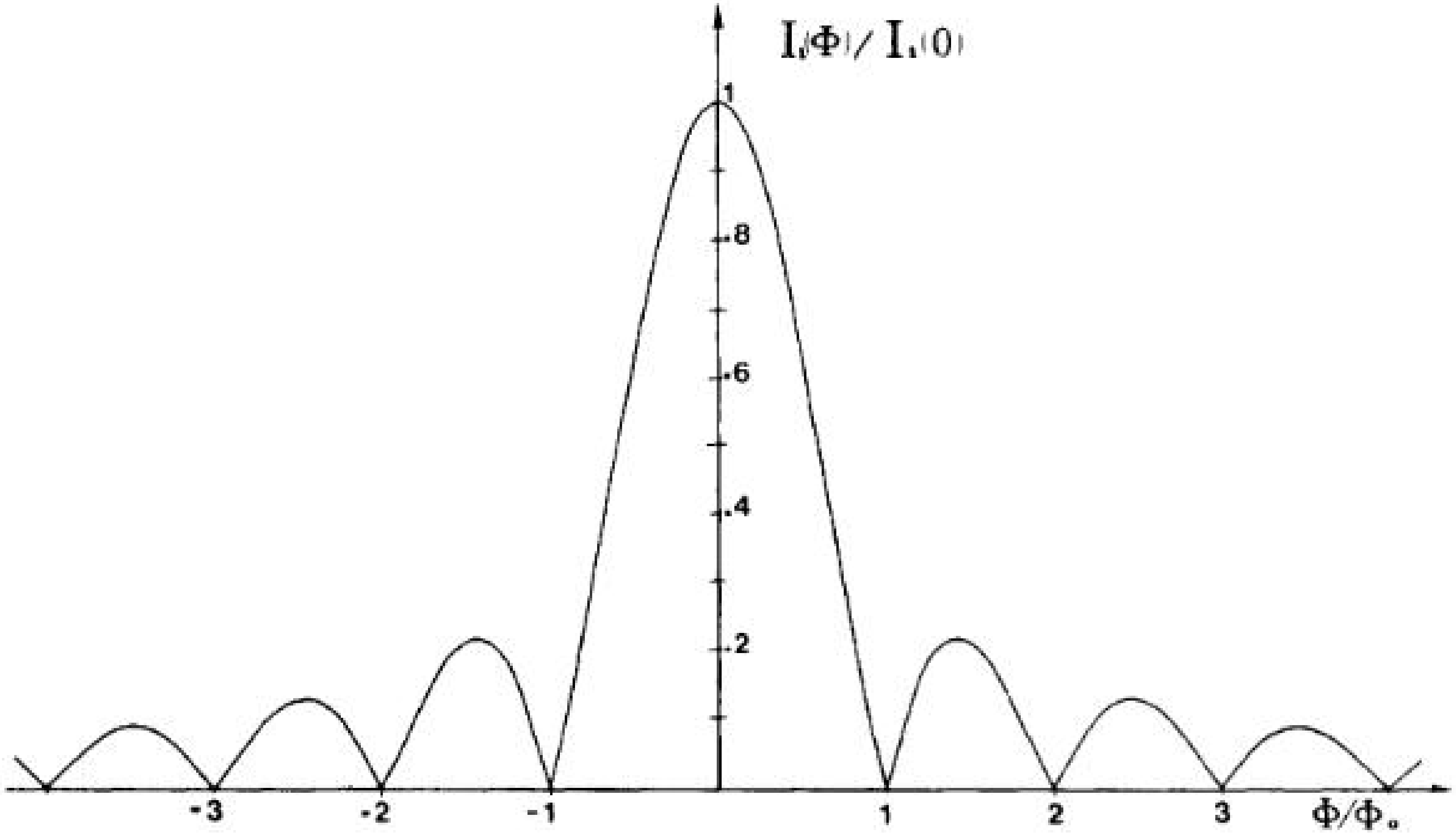
$$\frac{d\varphi}{dx} = \frac{2e}{\hbar c}(\lambda_L + \lambda_R + t)H_y$$

$$\varphi = \frac{2e}{\hbar c} d\, H_y x + \varphi_0 \qquad d = (\lambda_L + \lambda_R + t) \qquad J = J_1 \sin\left(\frac{2e}{\hbar c} d\, H_y x + \varphi_0\right)$$

this indicates that the tunneling supercurrent is **spatially modulated by the magnetic field**. Then, due to the periodic character of the expression, situations can be realized in which the net tunneling current is zero.
In particular, a rectangular junction with a uniform zero field tunneling current distribution exhibits a dependence of the maximum supercurrent on the applied magnetic field in the form of a **Fraunhofer-like diffraction pattern**.

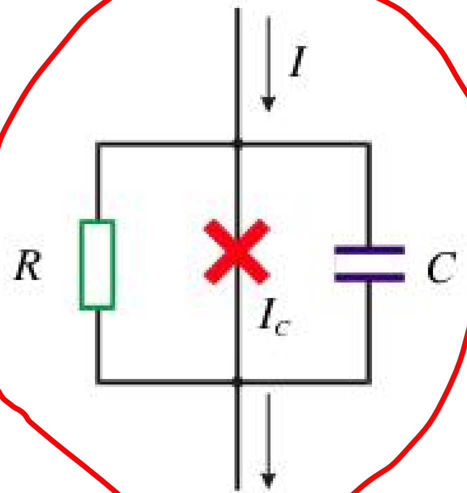
$$I_1(H) = I_1(0)\left|\frac{\sin \pi \dfrac{\Phi}{\Phi_0}}{\pi \dfrac{\Phi}{\Phi_0}}\right|$$

Φ is the total magnetic flux threading the junction
$\Phi_0 = hc/2e$ is the flux quantum = 2.07 10^{-7} G cm^2



Theoretical magnetic field dependence of the maximum Josephson current for a rectangular junction.

The resistively shunted model RSJ

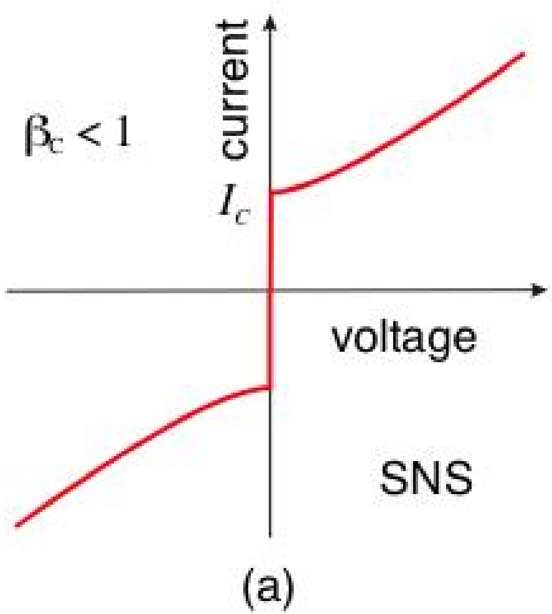


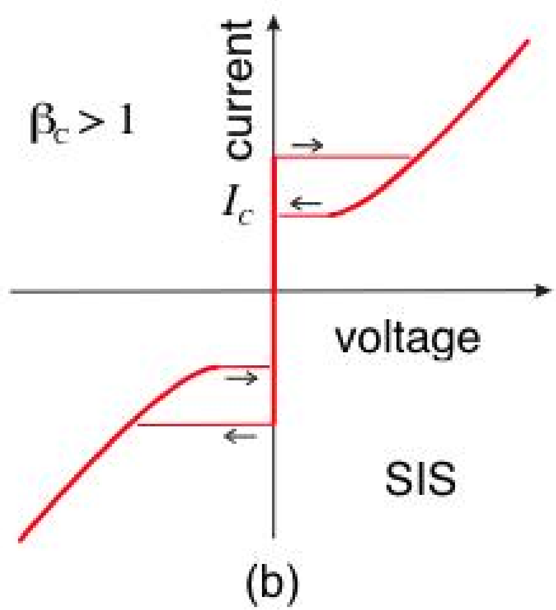
$$I = I_c \sin\varphi + \frac{V}{R} + C\frac{dV}{dt} \qquad \frac{d\varphi}{dt} \equiv \dot{\varphi} = \frac{2\pi}{\Phi_0}V$$

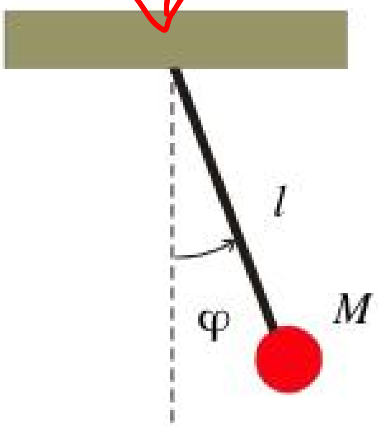
$$I = I_c \sin\varphi + \frac{\Phi_0}{2\pi R}\dot{\varphi} + \frac{\Phi_0 C}{2\pi}\ddot{\varphi} \qquad i \equiv \frac{I}{I_c}, \quad \tau = \frac{2\pi I_c R}{\Phi_0}t$$

$$\beta_c \frac{d^2\varphi}{d\tau^2} + \frac{d\varphi}{d\tau} + \sin\varphi = i \qquad \beta_c = \frac{2\pi I_c R^2 C}{\Phi_0}$$

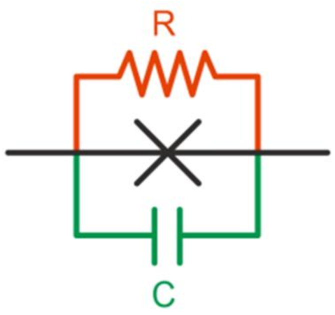
McCumber parameter





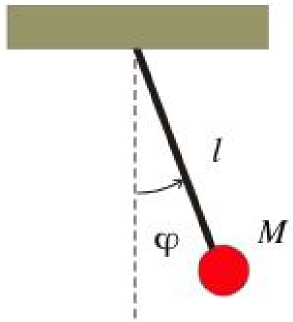


$$T = Mgl\sin\varphi + \chi\dot{\varphi} + \Theta\ddot{\varphi}$$



$$C\phi_b \frac{d^2\varphi}{dt^2} + \frac{\phi_b}{R}\frac{d\varphi}{dt} + I_0 \sin\varphi = I$$

$$\phi_b = \frac{\hbar}{2e}$$



$$T = Mgl\sin\varphi + \chi\dot{\varphi} + \Theta\ddot{\varphi}$$

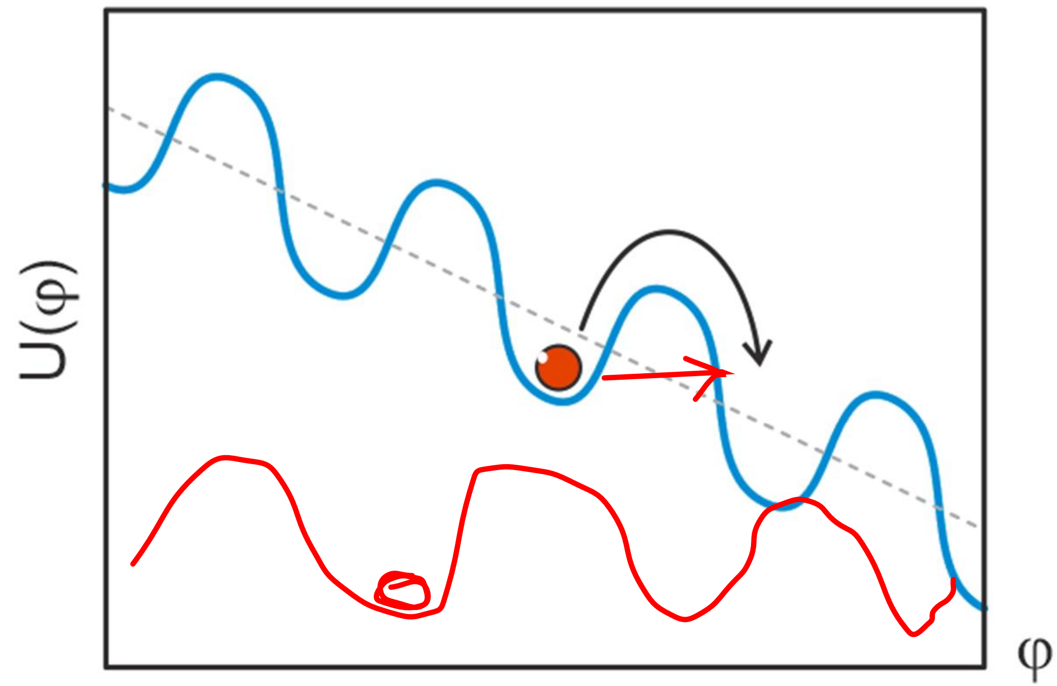
Motion equations

$$M\frac{d^2\varphi}{dt^2} + M\gamma\frac{d\varphi}{dt} = -\frac{dU}{d\varphi}$$

Effective potential

$$U = -E_j(\cos\varphi + I/I_0\,\varphi)$$

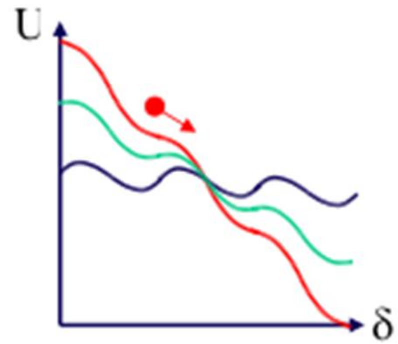
$$M = C\phi_b^2$$
$$E_j = I_0\phi_b^2$$
$$\gamma = 1/RC$$



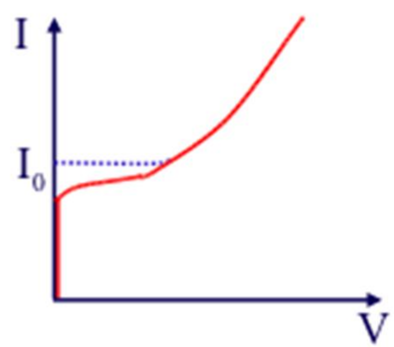
$$V = \frac{\hbar}{2e}\frac{d\varphi}{dt}$$

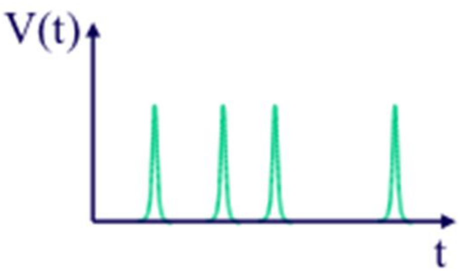
Thermal Noise in the Overdamped RSJ ($\beta_c << 1$)

Langevin Equation $$\frac{\hbar C}{2e}\ddot{\delta}+\frac{\hbar}{2eR}\dot{\delta}+I_0\sin\delta = I + I_N(t), \quad S_I(f)=\frac{4k_BT}{R}$$

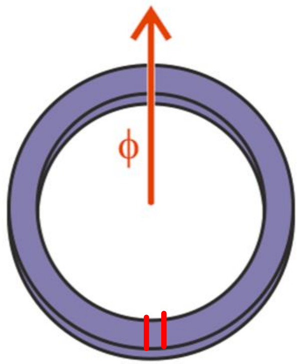


Noise rounding (Ambegaokar and Halperin)





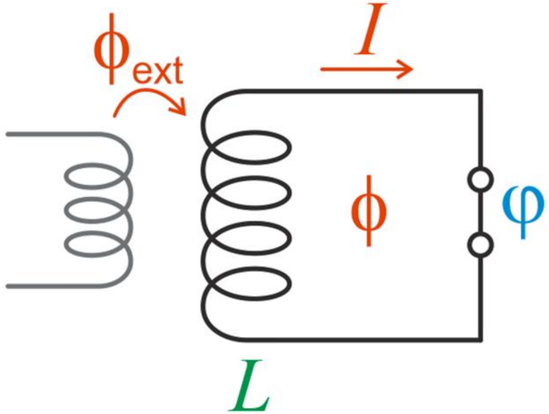
Superconducting loop



$$\phi = \phi_{ext} - LI$$

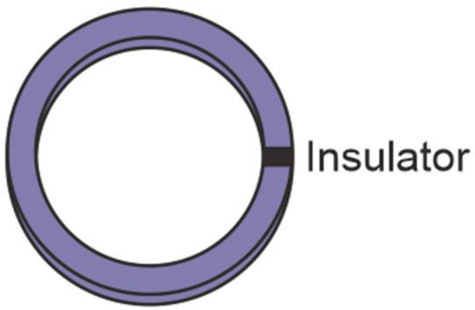
$$\varphi(x+dx) - \varphi(x) = \frac{2e}{\hbar c}\oint \mathbf{A}\cdot\mathbf{dl} \Rightarrow \phi = \frac{\hbar}{2e}\varphi$$

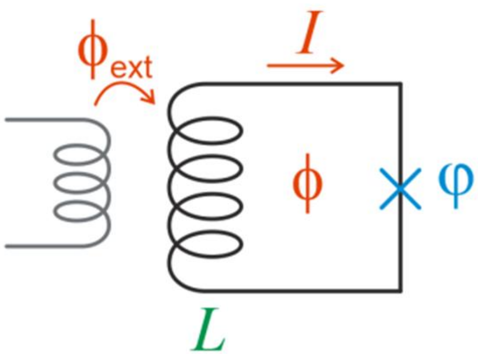
$$\varphi = 2\pi n$$

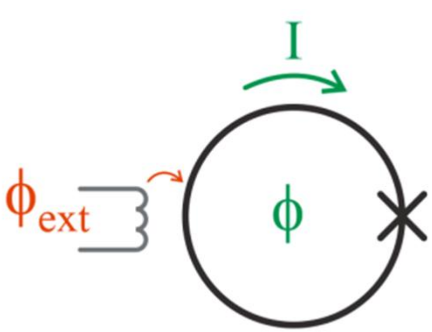


$$\phi = \phi_{ext} - LI = n\phi_0$$

A superconducting loop with one Josephson junction: the rf Superconducting Quantum Interference Device







$$C\phi_b \frac{d^2\varphi}{dt^2} + \frac{\phi_b}{R}\frac{d\varphi}{dt} + I_0 \sin\varphi = I$$

$$\phi = \phi_{ext} - LI$$

$$\phi = \frac{\hbar}{2e}\varphi$$

Motion equations

$$M\frac{d^2\varphi}{dt^2} + M\gamma\frac{d\varphi}{dt} = -\frac{dU}{d\varphi}$$

Effective potential

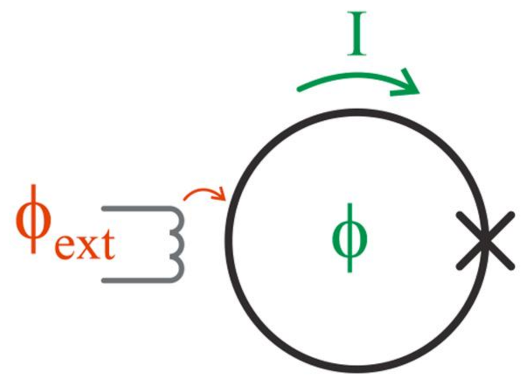
$$U = \frac{1}{2}E_L(\varphi - \phi_{ext}/\phi_0)^2 - E_j \cos\varphi$$

$$M = C\phi_b^2 \quad \gamma = 1/RC$$

$$E_j = I_0\phi_b^2 \quad E_L = \phi_b^2/L$$

courtesy of Fabio Chiarello

rf SQUID effective potential

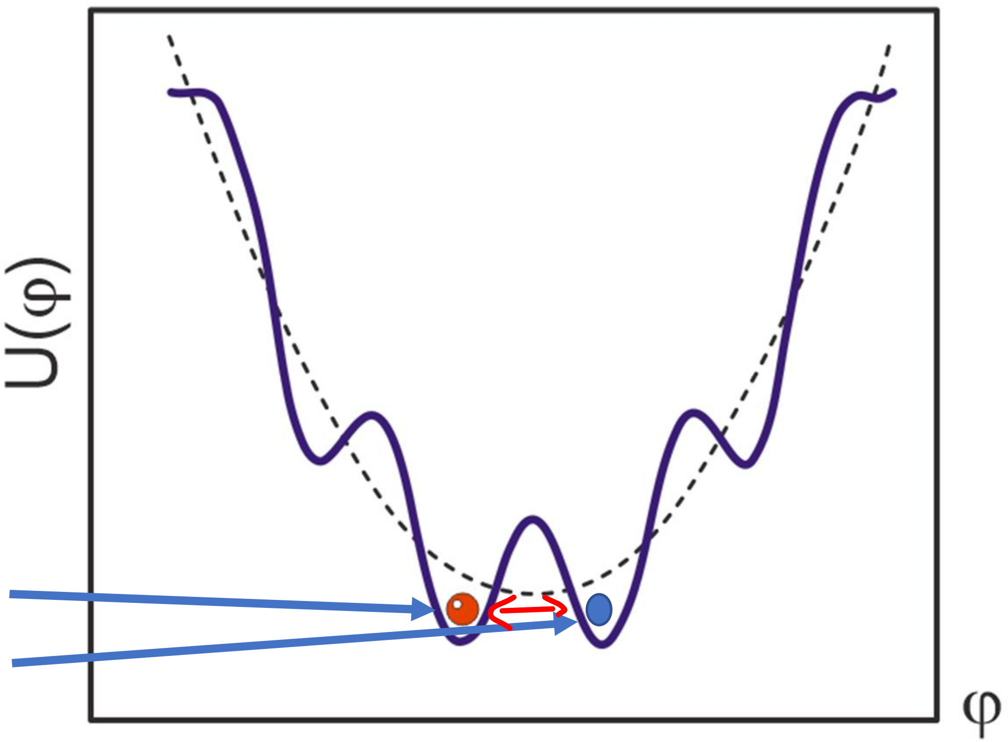


$$M\frac{d^2\varphi}{dt^2} + M\gamma\frac{d\varphi}{dt} = -\frac{dU}{d\varphi}$$

$$U = \frac{1}{2}E_L(\varphi - \phi_{ext}/\phi_0)^2 - E_j cos\varphi$$

$M = C\phi_b^2 \quad \gamma = 1/RC$

$E_j = I_0\phi_b^2 \quad E_L = \phi_b^2/L$

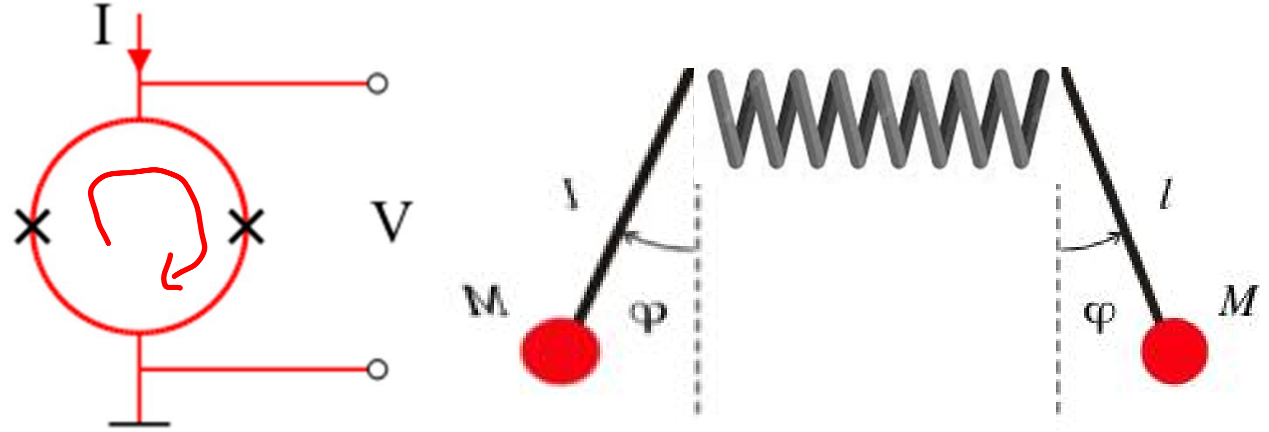


Two metastable states that can tunnel one to the other: the basis of a Qubit

The dc Superconducting Quantum Interference Device

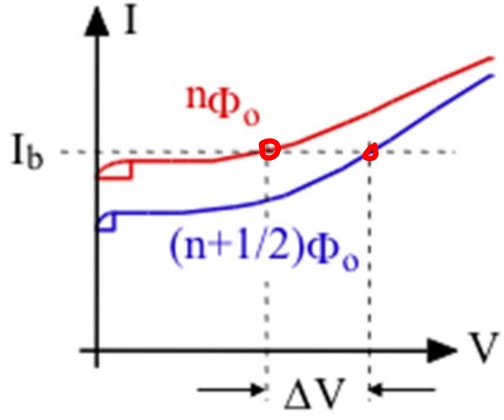
- **dc SQUID**

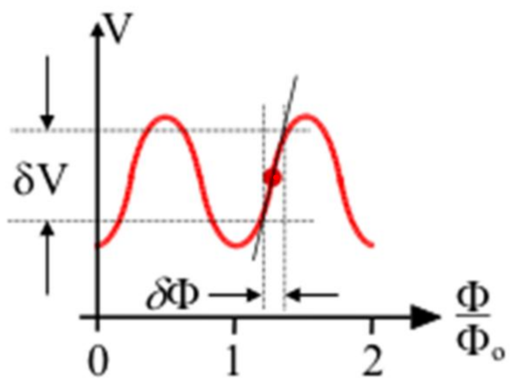
  Two Josephson junctions on a superconducting ring



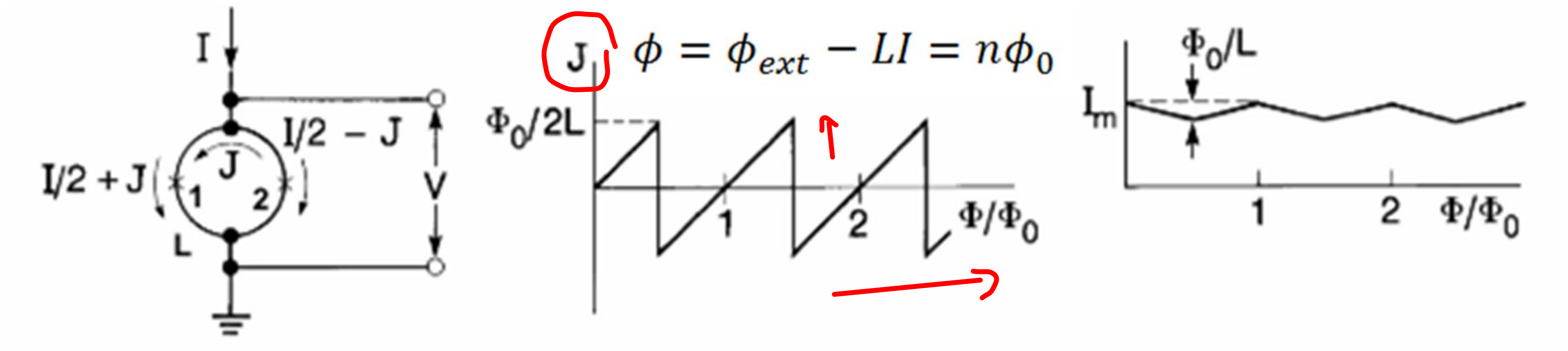
- **Current-voltage (I-V) characteristic modulated by magnetic flux Φ:**

  Period one flux quantum $\Phi_o = h/2e = 2 \times 10^{-15}$ T m^2

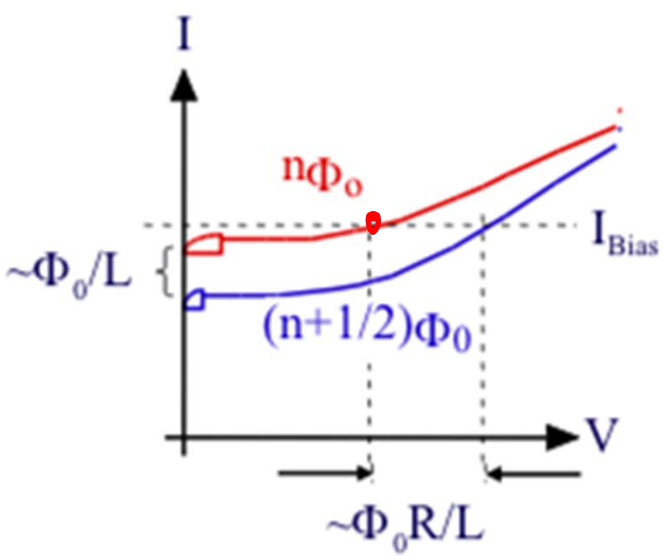




A superconducting loop with two Josephson junctions: the dc Superconducting Quantum Interference Device



$J = \Phi/L$: $I_1 = I/2 + J$, $I_2 = I/2 - J$. Since junction 1 switches at I_0, the critical current of the SQUID is reduced by 2J. At $\Phi = \Phi_0/2$, the reduction is thus Φ_0/L.



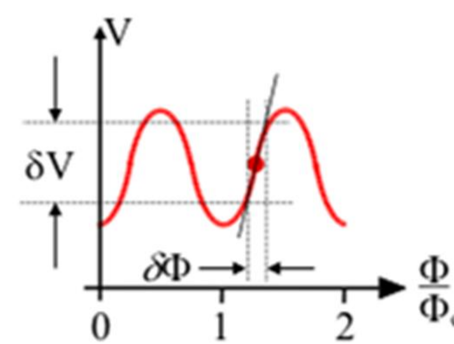
Maximum voltage swing $\sim(\Phi_0/L)R$

Flux-to-voltage transfer coefficient:

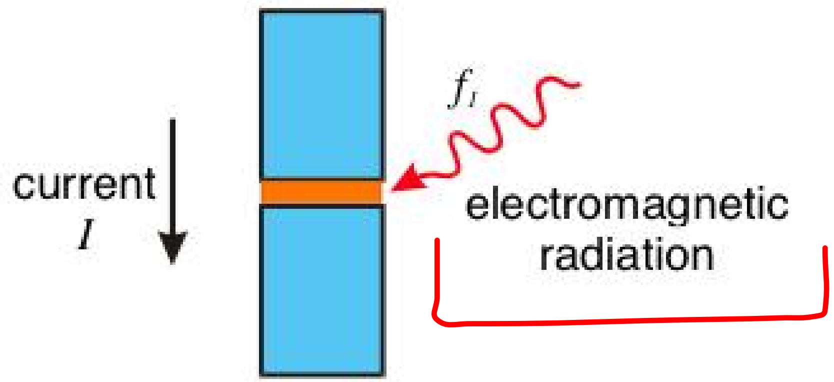
$$V_\Phi \equiv \left.\frac{\partial V}{\partial \Phi}\right|_I \sim \frac{R}{L} \quad \text{for } \Phi \sim (2n+1)\frac{\Phi_0}{4}$$

Thus $\delta V \sim (R/L)\, \delta\Phi$

1 fT



RF effects: Shapiro steps



$$V = V_0 + V_1 \cos(2\pi f_1 t)$$

$$\varphi = \int \frac{2\pi}{\Phi_0} V\,dt = \varphi_0 + \frac{2\pi}{\Phi_0} V_0 t + \frac{V_1}{\Phi_0 f_1} \cos(2\pi f_1 t)$$

$$\sin(z \sin x) = 2 \sum_{k=0}^{\infty} J_{2k+1}(z) \sin(2k+1)x$$

$$\cos(z \sin x) = J_0(z) + 2 \sum_{k=1}^{\infty} J_{2k}(z) \cos 2kx$$

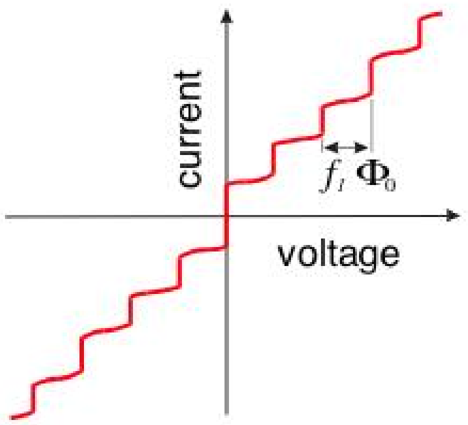
$$I_s = I_c \sin\varphi = I_c \sin[\varphi_0 + \frac{2\pi}{\Phi_0} V_0 t + \frac{V_1}{\Phi_0 f_1} \cos(2\pi f_1 t)]$$

$$= I_c \sin[\varphi_0 + \frac{2\pi}{\Phi_0} V_0 t] \cos[\frac{V_1}{\Phi_0 f_1} \cos(2\pi f_1 t)]$$

$$+ I_c \cos[\varphi_0 + \frac{2\pi}{\Phi_0} V_0 t] \sin[\frac{V_1}{\Phi_0 f_1} \cos(2\pi f_1 t)]$$

dc

$$I_s = I_c \sum_{n=0}^{\infty} (-1)^n J_n(\frac{V_1}{\Phi_0 f_1}) \sin[\varphi_0 + \frac{2\pi}{\Phi_0} V_0 t - 2\pi n f_1 t]$$



At specific voltages the current is dc $\frac{2\pi}{\Phi_0} V_0 = 2\pi n f_1$

$$V_0 = n f_1 \Phi_0,\ n = 0, \pm 1, \pm 2, \ldots$$

$$\Delta I_n \simeq I_c J_n(\frac{V_1}{\Phi_0 f_1})$$

Electrodynamics of a Josephson junction

Assuming non zero H in x and y

$$\frac{\partial \varphi}{\partial x} = \frac{2e}{\hbar c} H_y d$$

and using Maxwell equation

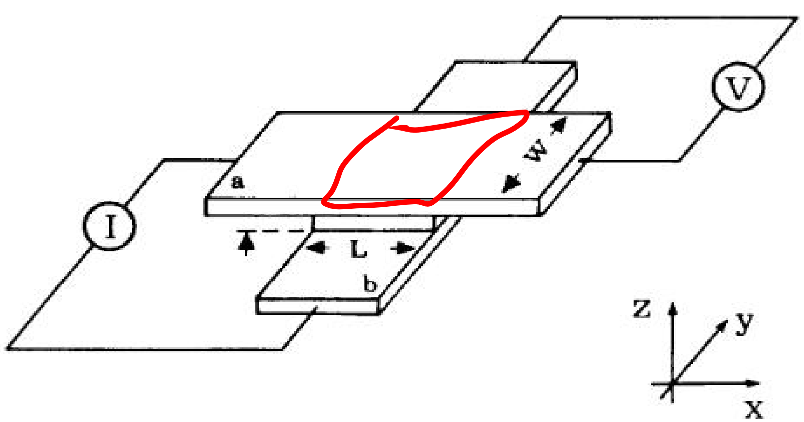
$$\frac{\partial \varphi}{\partial y} = -\frac{2e}{\hbar c} H_x d$$

$$\nabla \times \mathbf{H} = \frac{4\pi}{c}\mathbf{J} + \frac{1}{c}\frac{\partial \mathbf{D}}{\partial t}$$

$$\frac{\partial H_y}{\partial x} - \frac{\partial H_x}{\partial y} = \frac{4\pi}{c} J_z + \frac{1}{c}\frac{\partial D_z}{\partial t} \quad \Rightarrow \quad \frac{\hbar c^2}{8\pi e d}\left(\frac{\partial^2 \varphi}{\partial x^2} + \frac{\partial^2 \varphi}{\partial y^2}\right) = J_1 \sin\varphi + C\frac{dV}{dt}$$

$$C = \varepsilon_r / 4\pi \mathrm{t}$$

$$\frac{\partial^2 \varphi}{\partial x^2} + \frac{\partial^2 \varphi}{\partial y^2} - \frac{1}{\bar{c}^2}\frac{\partial^2 \varphi}{\partial t^2} = \frac{1}{\lambda_J^2}\sin\varphi$$



$$\lambda_J = \left(\frac{\hbar c^2}{8\pi e d J_1}\right)^{1/2} \qquad \bar{c} = c\left(\frac{1}{4\pi C d}\right)^{1/2} = c\left(\frac{\mathrm{t}}{\varepsilon_r d}\right)^{1/2}$$

From microscopic theory of Josephson effect, there is a dissipative term

$$J = J_1(V)\sin\varphi + [\sigma_1(V)\cos\varphi + \sigma_0(V)]V \quad \Rightarrow \quad J = J_1\sin\varphi + \sigma_0(V)V$$

$$\beta = \sigma_0 / C$$

$$\frac{\partial^2\varphi}{\partial x^2} + \frac{\partial^2\varphi}{\partial y^2} - \frac{1}{\bar{c}^2}\frac{\partial^2\varphi}{\partial t^2} - \frac{\beta}{\bar{c}^2}\frac{\partial\varphi}{\partial t} = \frac{1}{\lambda_J^2}\sin\varphi$$

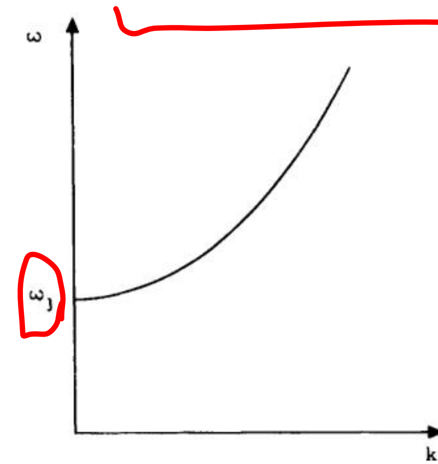
Nonlinear wave equation (sine-Gordon equation)

In the case of spatially constant ϕ $\quad \frac{d^2\varphi}{dt^2} + \omega_J^2\sin\varphi = 0 \qquad \omega_J = \bar{c}/\lambda_J$

Pendulum equation

For small amplitude travelling wave solutions of the type ϕ ~ exp[j (ωt-kx)]
It is easy to derive the dispersion relation as

$$\omega^2 = \omega_J^2 + k^2\bar{c}^2$$



Nonlinear waves in Josephson junctions

When the phase ϕ is spatially dependent
i.e. one of the junction dimensions is > λ_J

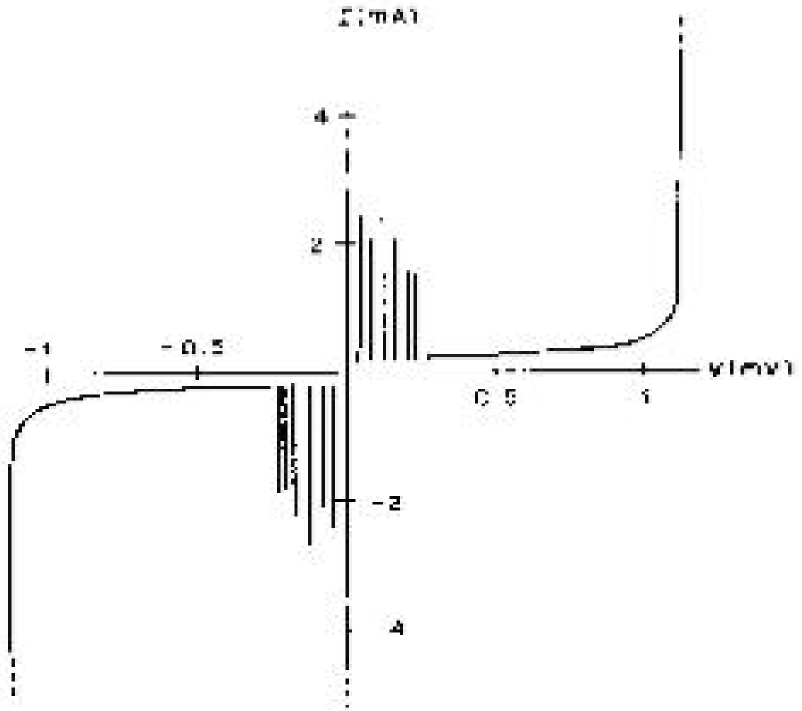
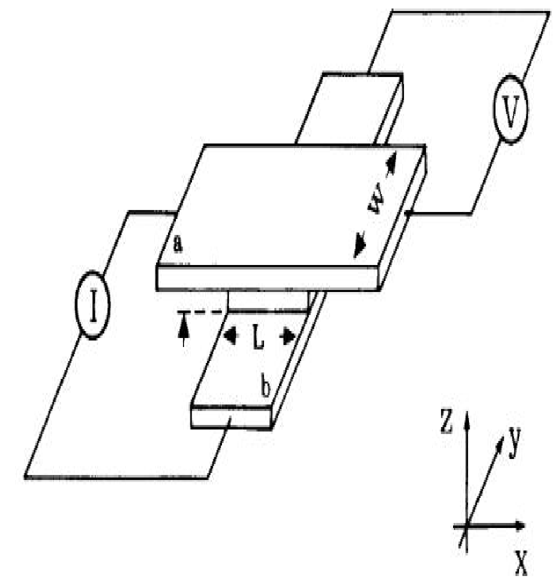


FIG. 2. The I - (V) curve of a long, narrow tunnel junction showing the vortex-propagation branches. The near symmetry in I/ indicates that the stray and applied magnetic fields are nearly zero.



T.A.Fulton and R.C.Dynes, Solid St. Commun. **12**, 57 (1972)

In the case of a long and narrow Josephson junction the wave equation becomes:

$$\varphi_{xx} - \varphi_{tt} = \sin\varphi + \alpha\varphi_t - \beta\varphi_{xxt} - \gamma$$

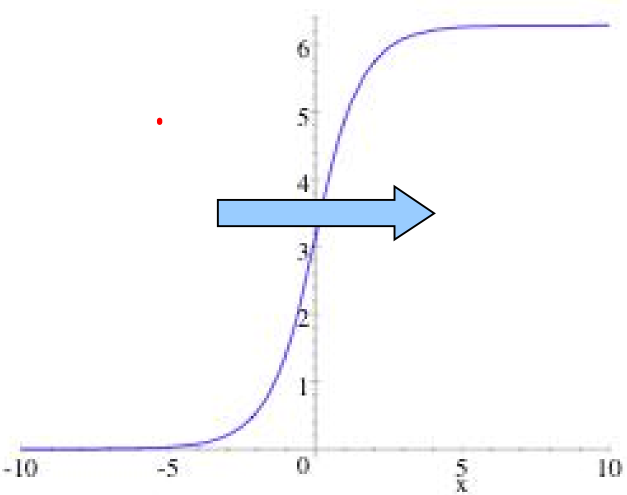
$$\frac{x}{\lambda_J} \to x \; ; \quad \frac{t}{\omega_p^{-1}} \to t \qquad \omega_p = \sqrt{\frac{2\pi j_c}{\Phi_0 C}} \qquad \lambda_J = \sqrt{\frac{\Phi_0}{2\pi j_c \mu_0 (2\lambda_L + t_0)}}$$

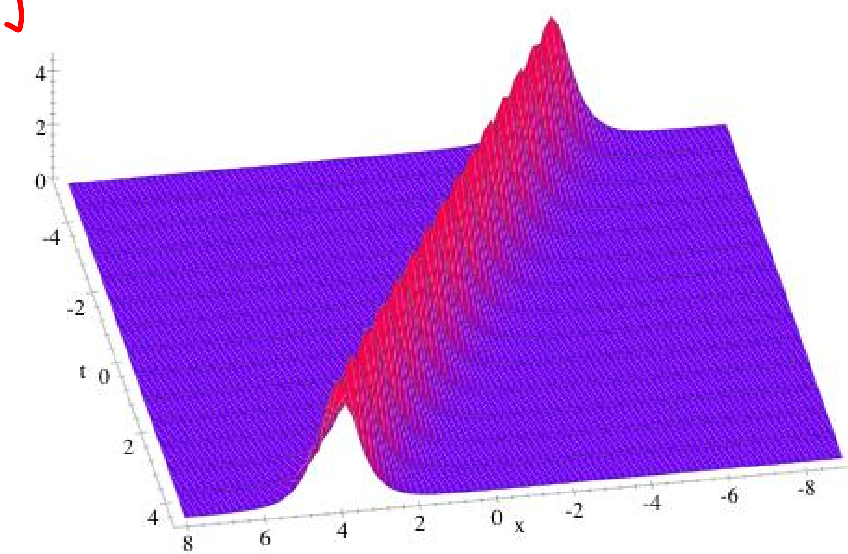
$$\bar{c} = \lambda_J \omega_p = \frac{1}{\sqrt{L'C}} = c_0 \sqrt{\frac{t_0}{\varepsilon(2\lambda_L + t_0)}} \qquad \varphi_{xx} \equiv \frac{\partial^2\varphi}{\partial x^2},\; \varphi_{tt} \equiv \frac{\partial^2\varphi}{\partial t^2},\; \varphi_t \equiv \frac{\partial\varphi}{\partial t}.$$

$$\alpha = \frac{1}{RC\omega_p} \qquad \beta = \frac{\omega_p L'}{R_s} \qquad \gamma = \frac{j_B}{j_c}$$

If α=β=γ = 0, the equation becomes the sine-Gordon equation $\varphi_{xx} - \varphi_{tt} = \sin\varphi$

$$\varphi_0(x,t) = 4\arctan[\exp\frac{x - vt - x_0}{\sqrt{1 - v^2}}]$$

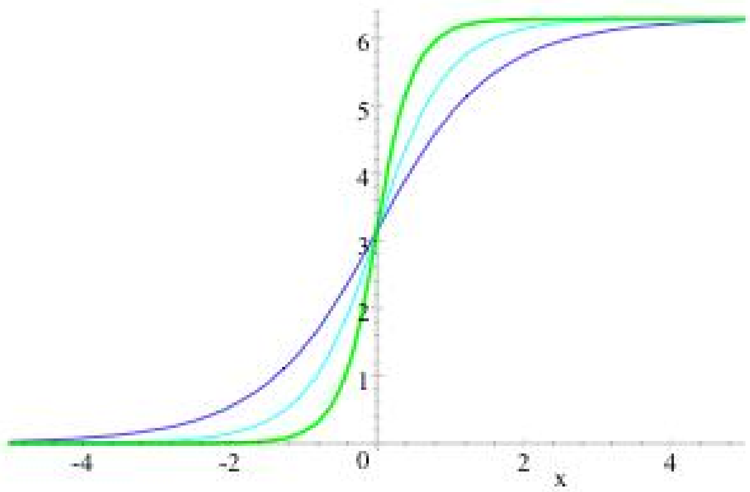




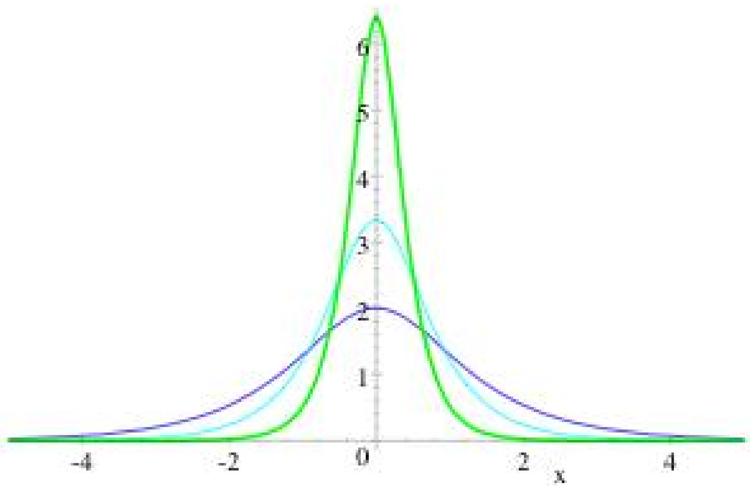
A soliton $\frac{d\varphi_0}{dx}$ moving with $v = 0.9$

The sine-Gordon equation is invariant under Lorentz transformations

$$x \to x' = \frac{x - vt}{\sqrt{1 - v^2}}, \quad t \to t' = \frac{t - x/v}{\sqrt{1 - v^2}}$$



Lorentz contraction of $\overline{\varphi_0}(x,0)$: $v = 0$ (blue), $v = 0.8$ (cyan), $v = 0.95$ (green).



Lorentz contraction of $\frac{d}{dx}[\varphi_0(x,0)]$:

as dϕ/dx is proportional to H this represents the local magnetic field (a flux quanta or fluxon)

dϕ/dt is proportional to V → the area under it is one flux quanta (RSFQ logic)

Real case: α,β, and γ > 0 the perturbed sine-Gordon equation

$$\varphi_{xx} - \varphi_{tt} = \sin\varphi + \alpha\varphi_t - \beta\varphi_{xxt} - \gamma$$

For the sine-Gordon equation we can write an Hamiltonian functional such that $dH^{SG}/dt = 0$ for each solution.

$$H^{SG} \equiv \int_{-\infty}^{\infty} \left(\tfrac{1}{2}\varphi_x^2 + \tfrac{1}{2}\varphi_t^2 + 1 - \cos\varphi\right)dx$$

Using a kink solution in the case of the perturbed SGE we have :

$$\frac{dH^{SG}(\varphi)}{dt} = \int_{-\infty}^{\infty} \left(-\alpha\varphi_t^2 - \beta\varphi_{xt}^2 + \gamma\varphi_t\right)dx$$

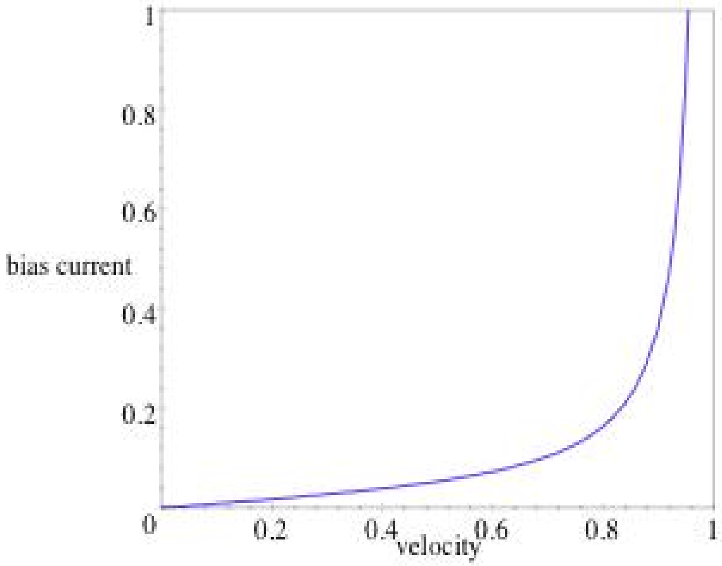
Assuming that the effect of the perturbations is just a velocity modulation of the kink

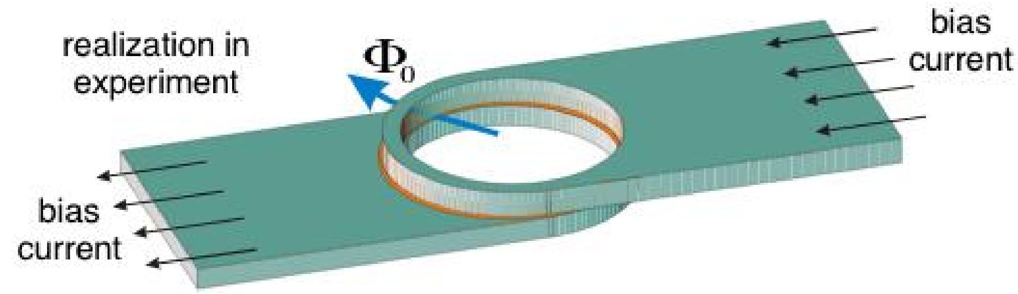
$$\frac{dv}{dt} = -\alpha v(1 - v^2) - \tfrac{1}{3}\beta v - \tfrac{1}{4}\pi\gamma(1 - v^2)^{3/2}$$

At equilibrium a constant velocity v_∞ is obtained

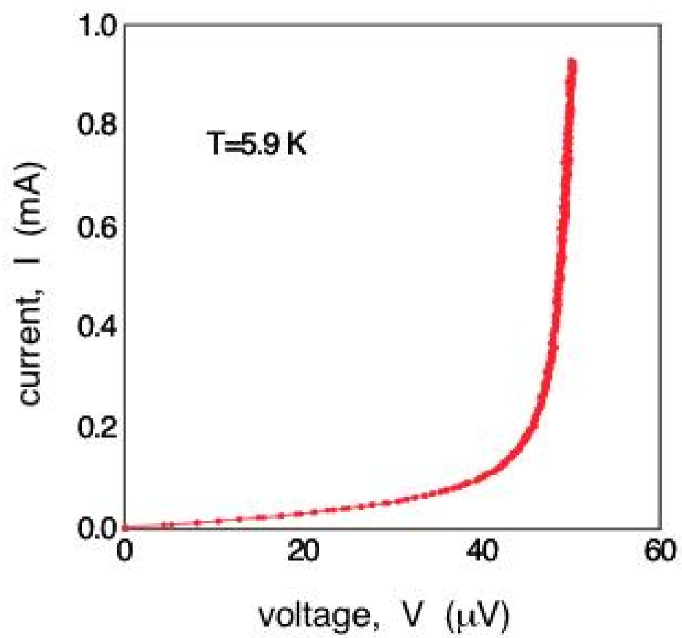
$$\gamma = \frac{4|v_\infty|}{\pi\sqrt{1 - v_\infty^2}}\left(\alpha + \frac{\beta}{3(1 - v_\infty^2)}\right)$$

Current-Velocity (Voltage) characteristics of a single fluxon





Annular junction with $n = 1$ fluxon



Experimental $\gamma(v_\infty)$ of a fluxon

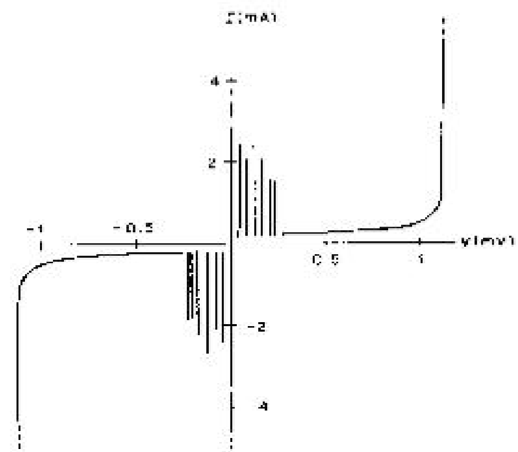
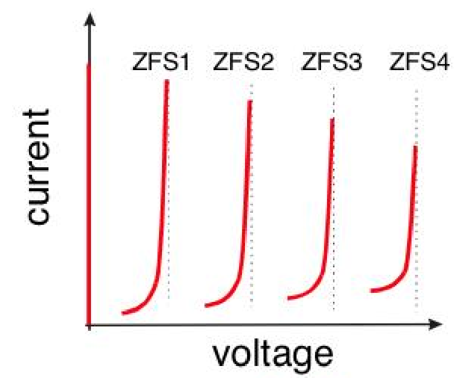


FIG. 2. The $I(V)$ curve of a long, narrow tunnel junction showing the vortex-propagation branches. The near symmetry in ΔI indicates that the stray and applied magnetic fields are nearly zero.



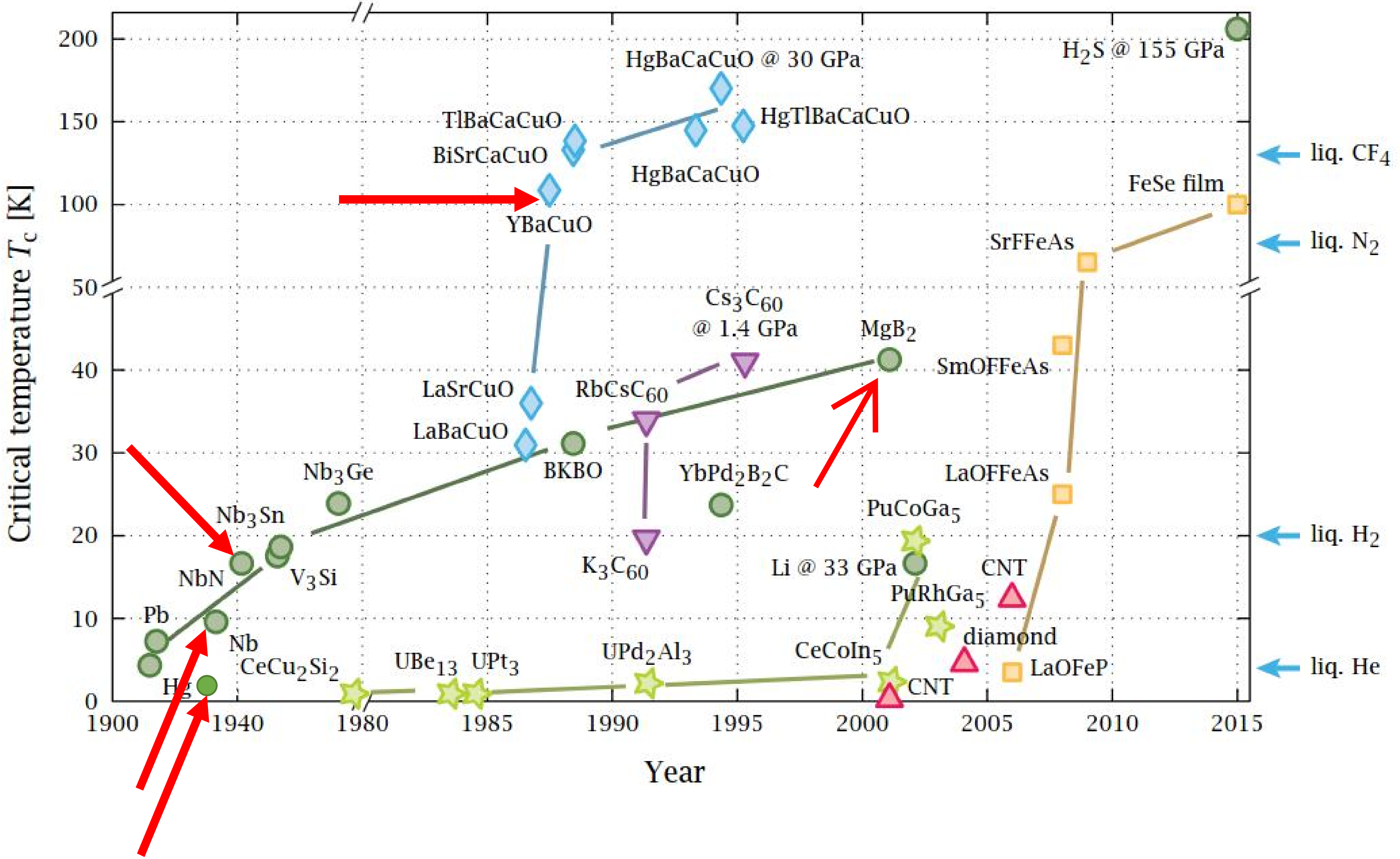
Overlap junction: $I - V$ curve

Fabrication technology for Josephson junction and related devices

Practically all electronic applications of Josephson junctions are based on the thin film technology

To realize a Josephson based circuit is necessary:

1. A technology to **grow** thin films of the wanted material (superconducting, normal , insulating)
2. A technology to **pattern** the films on a micrometric scale
3. A technology to deposit and pattern **multilayer** films of different materials
4. A technology to realize the **thin barrier** forming the Josephson junction

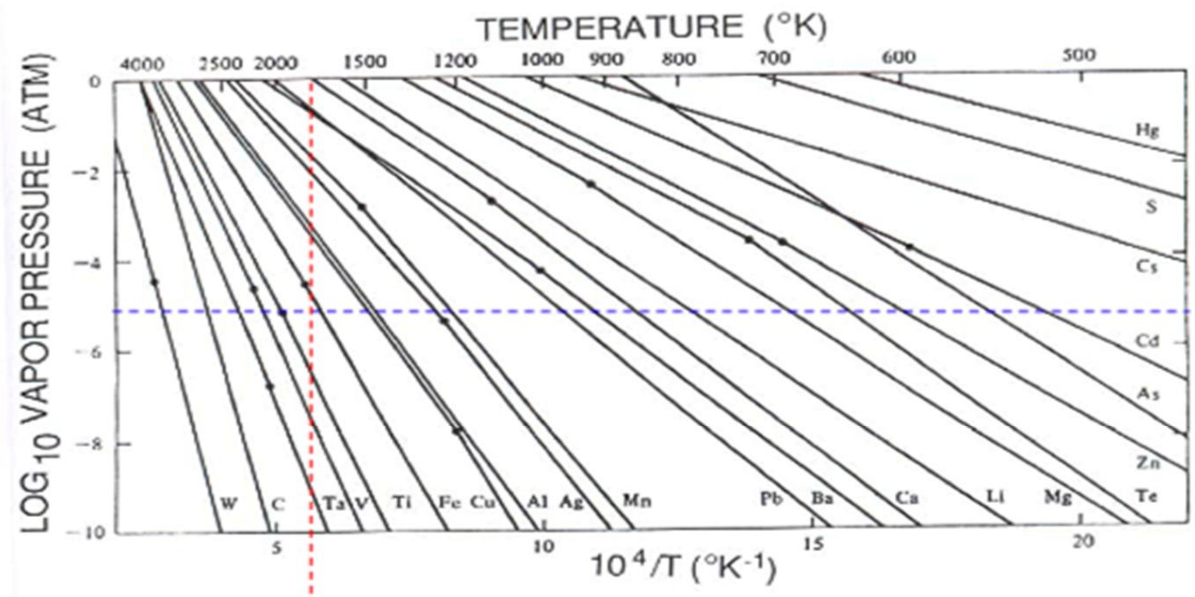


Critical temperature T_c [K]
Year
HgBaCaCuO @ 30 GPa
H$_2$S @ 155 GPa
TlBaCaCuO
HgTlBaCaCuO
BiSrCaCuO
HgBaCaCuO
liq. CF$_4$
FeSe film
YBaCuO
SrFFeAs
liq. N$_2$
Cs$_3$C$_{60}$ @ 1.4 GPa
MgB$_2$
SmOFFeAs
LaSrCuO
RbCsC$_{60}$
LaBaCuO
BKBO
Nb$_3$Ge
YbPd$_2$B$_2$C
LaOFFeAs
PuCoGa$_5$
Nb$_3$Sn
liq. H$_2$
NbN
V$_3$Si
K$_3$C$_{60}$
Li @ 33 GPa
CNT
PuRhGa$_5$
Pb
Nb
diamond
CeCu$_2$Si$_2$
UBe$_{13}$
UPt$_3$
UPd$_2$Al$_3$
CeCoIn$_5$
LaOFeP
liq. He
Hg
CNT

Technically Interesting Superconductors for SE

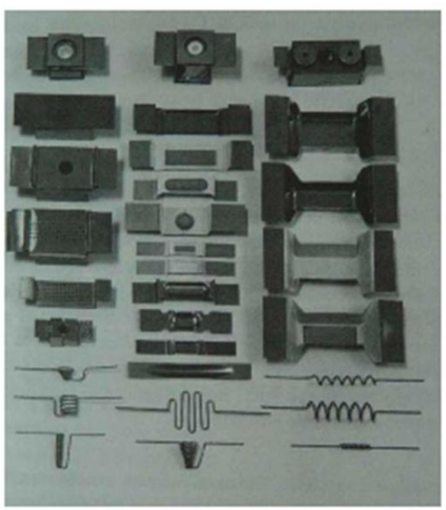
Class	Superconductor	$\lambda(0)$ [nm]	ξ_0 [nm]	T_c [K]
VLTS	Al	16	1500	1.18
	In	25	400	3.3
	Sn	28	300	3.7
LTS	Pb	28	110	7.2
	Nb	32	39	9.2
MTS	Nb_3Sn	50	6	18
	NbN	50 (200)	6	17
	BKBO	(320)	5-7	30
	MgB_2	(140)	3-5	39
HTS	YBCO	140	1.5	92
	...			

Thin film deposition techniques: evaporation

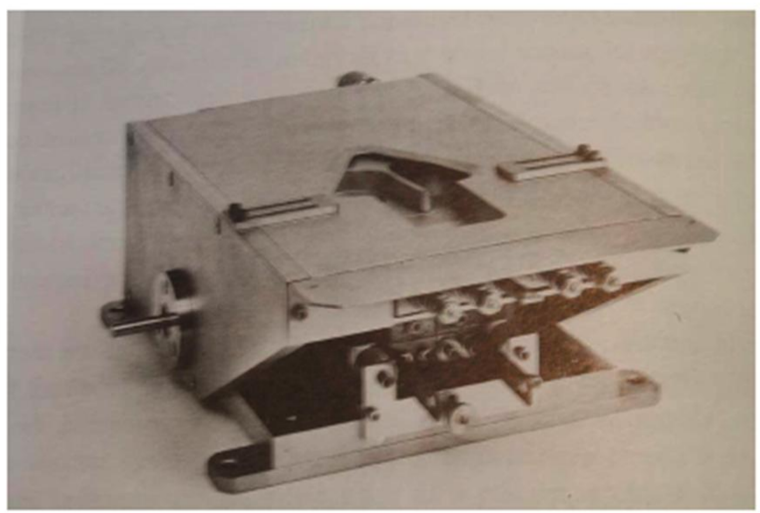


e-beam evaporation

thermal evaporation



Thermal evaporators

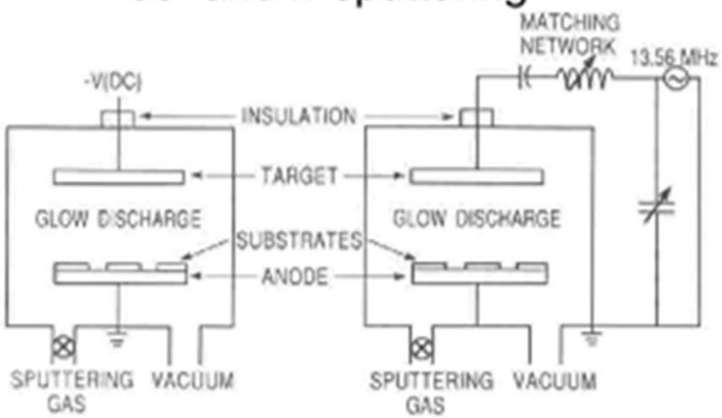


E-beam evaporator

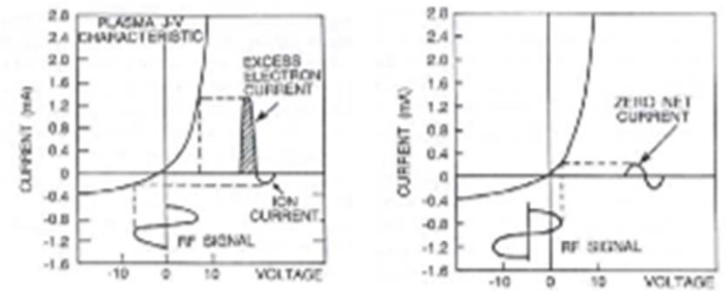
from Milton Ohring: "The Material Science of Thin Films", AP

Thin film deposition techniques: sputtering

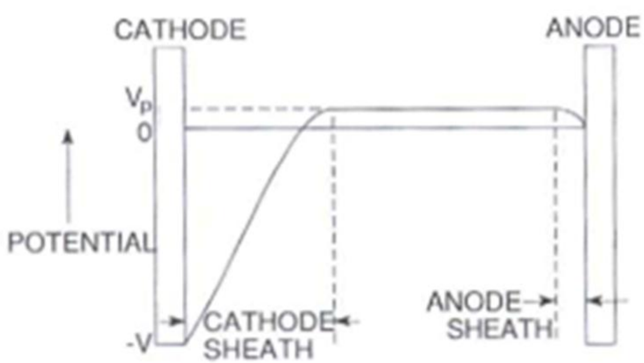
dc- and rf-sputtering

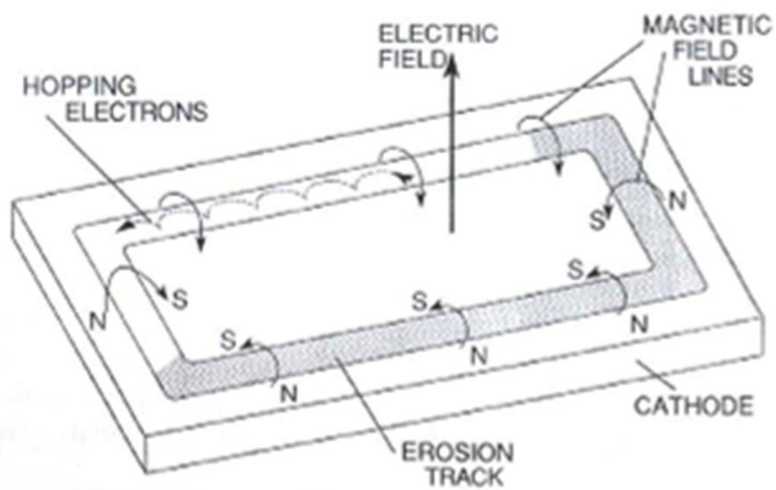


rf self-bias



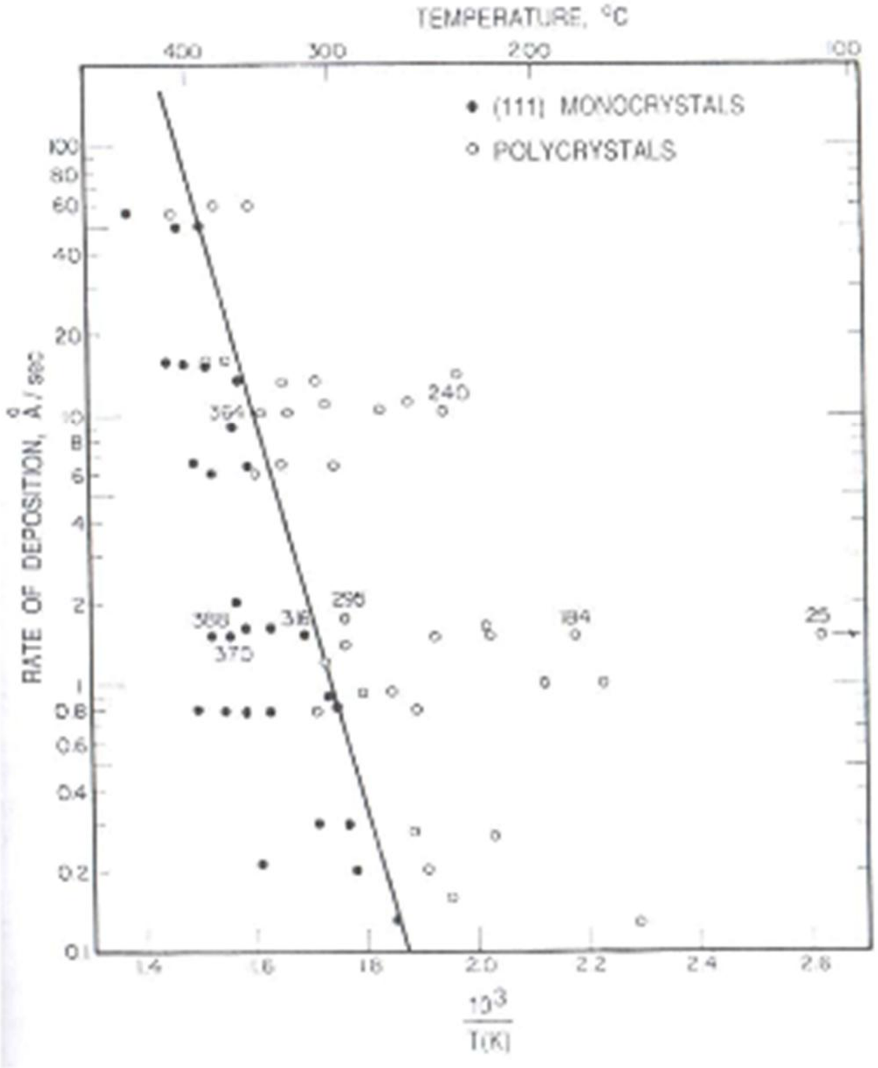
Potential

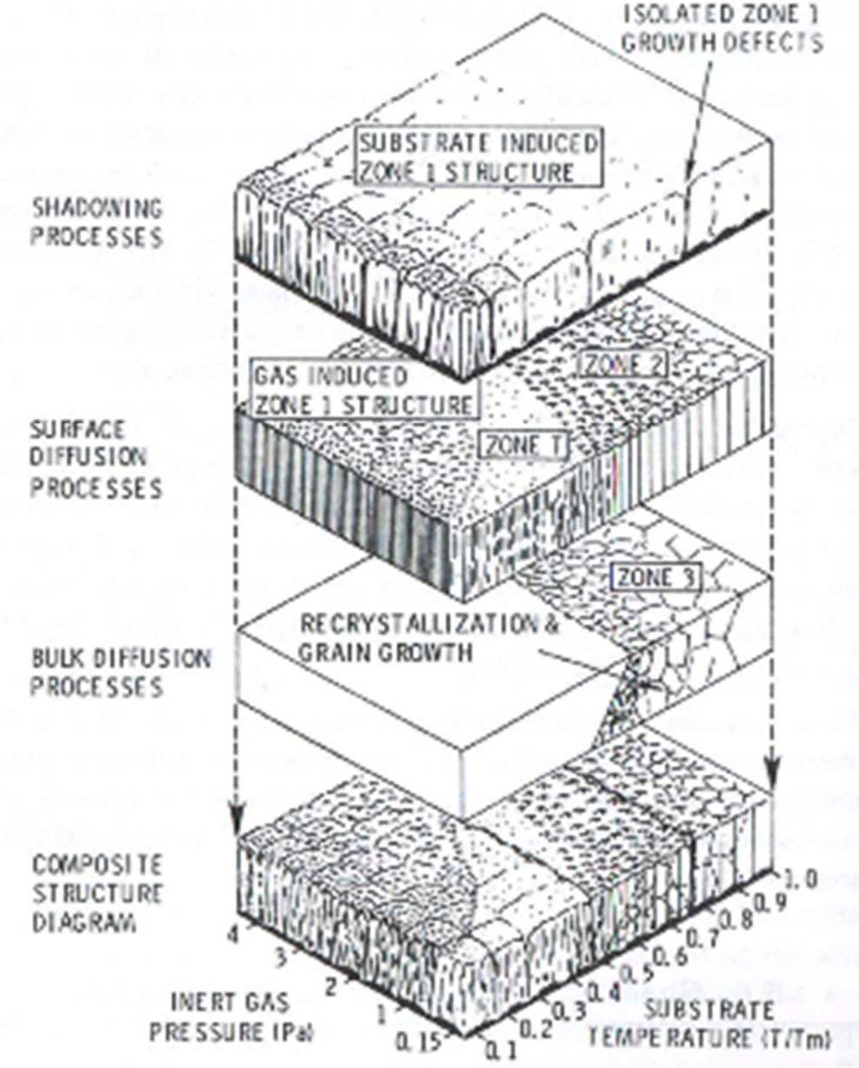




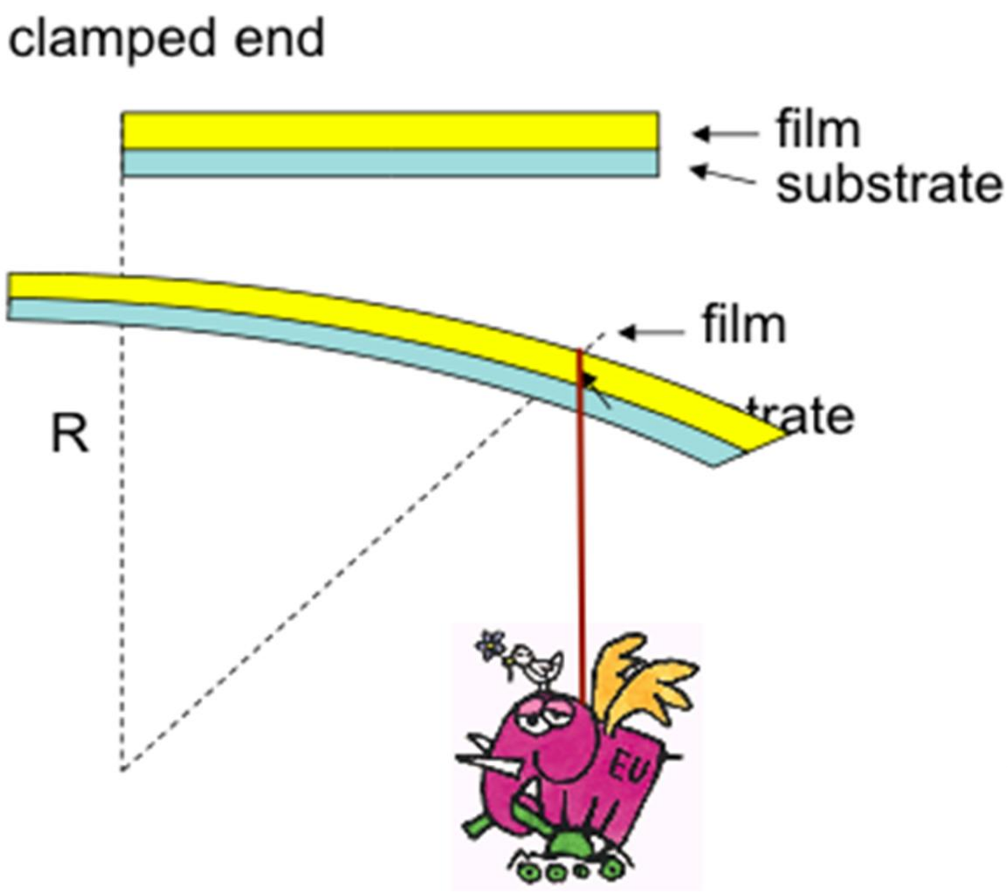
from Milton Ohring: "The Material Science of Thin Films", AP

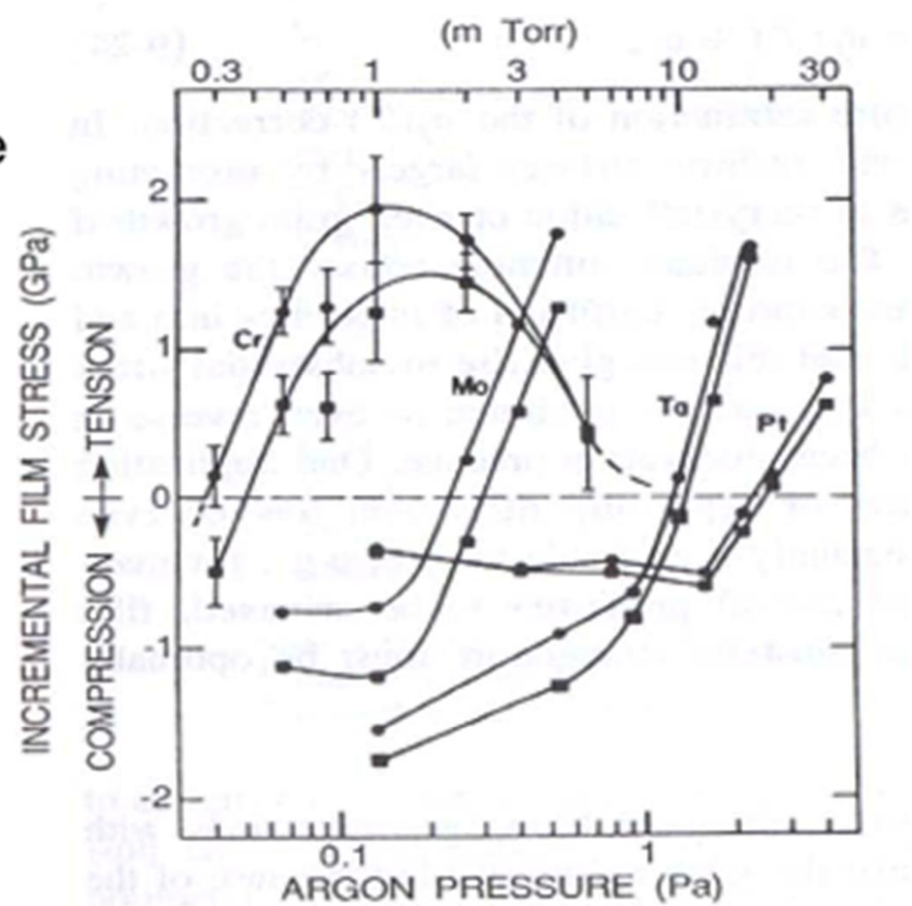
Thin film growth





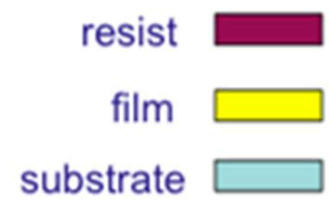
Film effects: stress



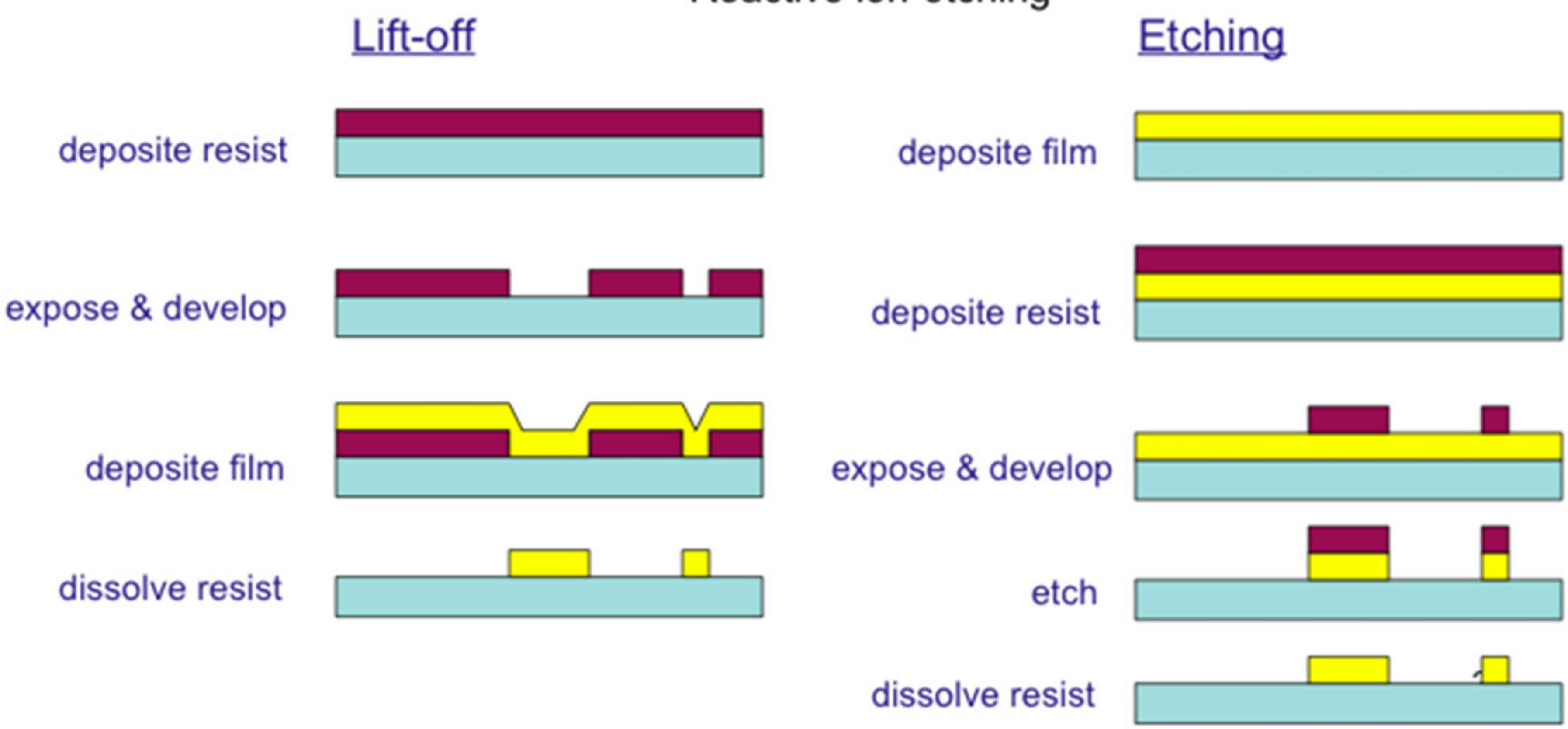


bending radius → stress

Thin film patterning



- Lift-off
- Chemical etching
- Ar ion-etching
- Reactive ion-etching



Structuring methods

Lift-off

Advantages

- ‘gentle’
- shadow techniques, e.g. for narrower lines

Disadvantages

- ‘ears’
- resist hardening

Ion-etching (Ar)

Advantages

- nearly all materials

Disadvantages

- often low etching rate
- re-deposition
- sample heating
- non-selective

Chemical etching

Advantages

- Wide range of materials
- selective

Disadvantages

- under-etching
- can attack surfaces

Reactive ion-etching

Advantages

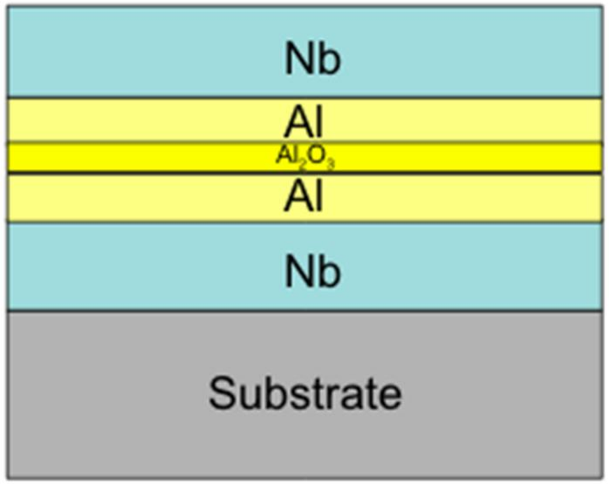
- selective
- low damage

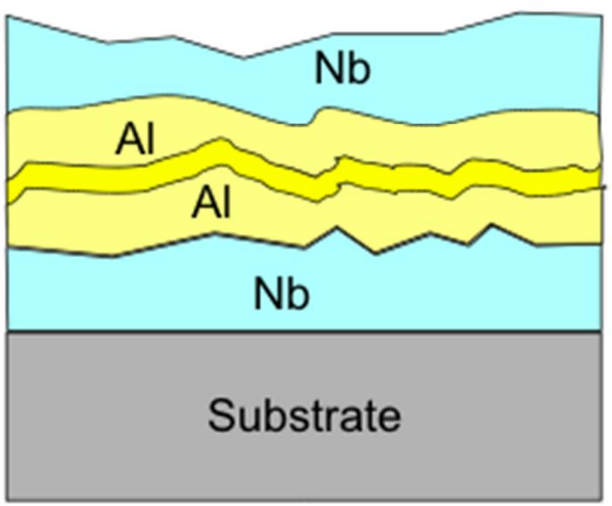
Disadvantages

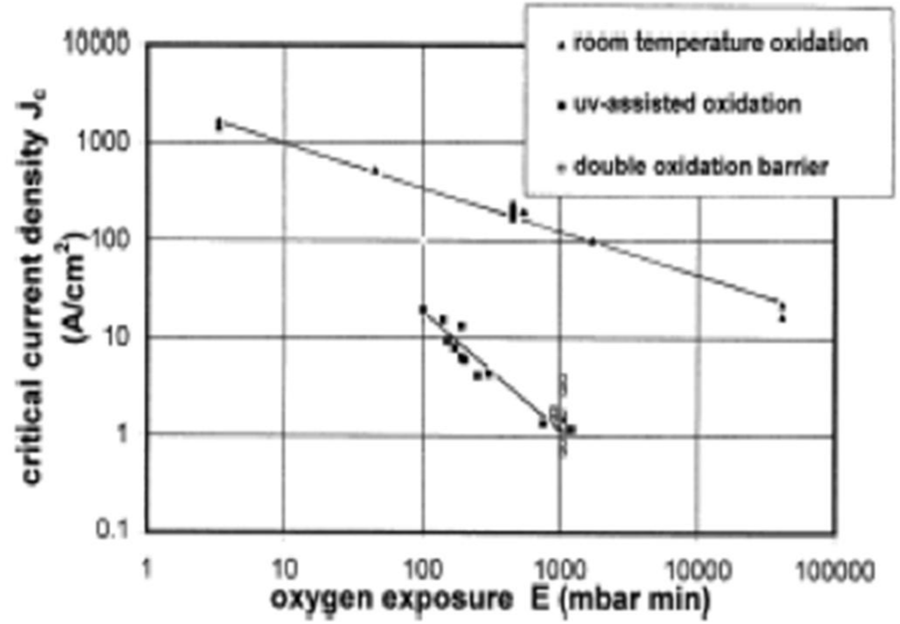
- Limited range of materials

Junction barrier formation

Nb/Al/Al2O3/Al/Nb Junctions







$$E = p_{O2} \cdot t \qquad J_c \propto E^{-n} \quad n = 0.4..0.5$$

L. Fritzsch *, H.-J. Köhler, F. Thrum, G. Wende, H.-G. Meyer
Physica C 296 (1998) 319–324

A.W. Kleinsasser, R.E. Miller, W.H. Mallison, IEEE Trans. Appl. Supercond. 5 (1995) 26.

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Critical current densities

Voltage standards for low drive frequency : (e.g. f = 10GHz) — < 3A/cm²

SQUIDs — ≈ 10..100 A/cm²

$$\text{e.g.:}\quad \frac{10\mu A}{5\mu m \cdot 5\mu m} = 40\frac{A}{cm^2}$$

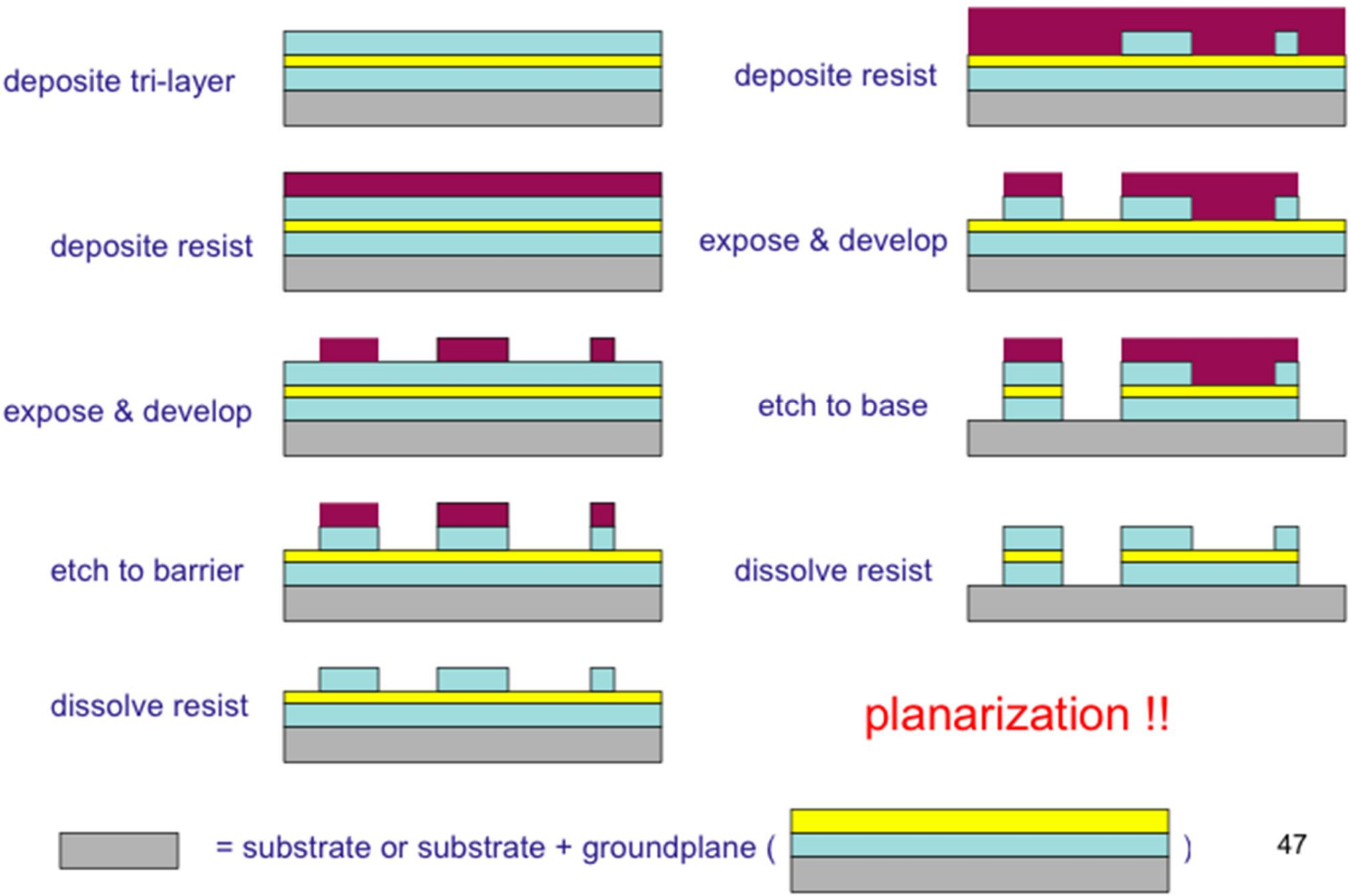
Standard digital circuits (junction size) — ≈ 2..10 kA/cm²

$$\text{e.g.:}\quad \frac{20\mu A}{1\mu m \cdot 1\mu m} = 2\frac{kA}{cm^2}$$

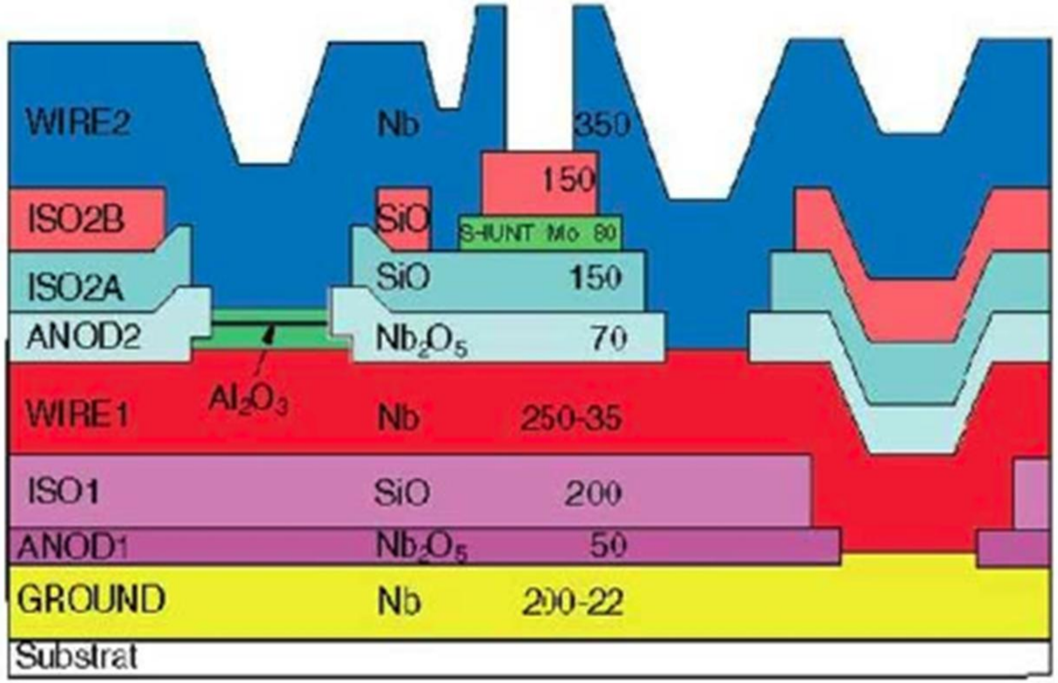
Digital circuits (sub-micron junctions) — > 100 kA/cm

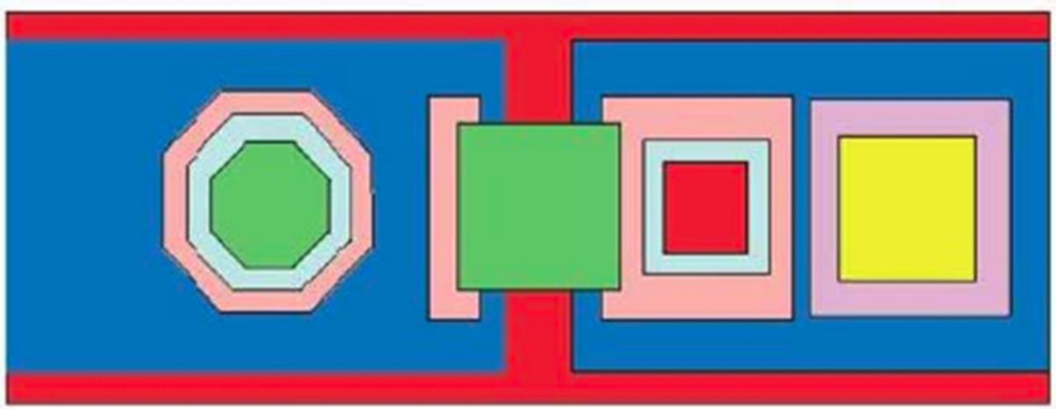
$$\text{e.g.:}\quad \frac{20\mu A}{100nm \cdot 100nm} = 200\frac{kA}{cm^2}$$

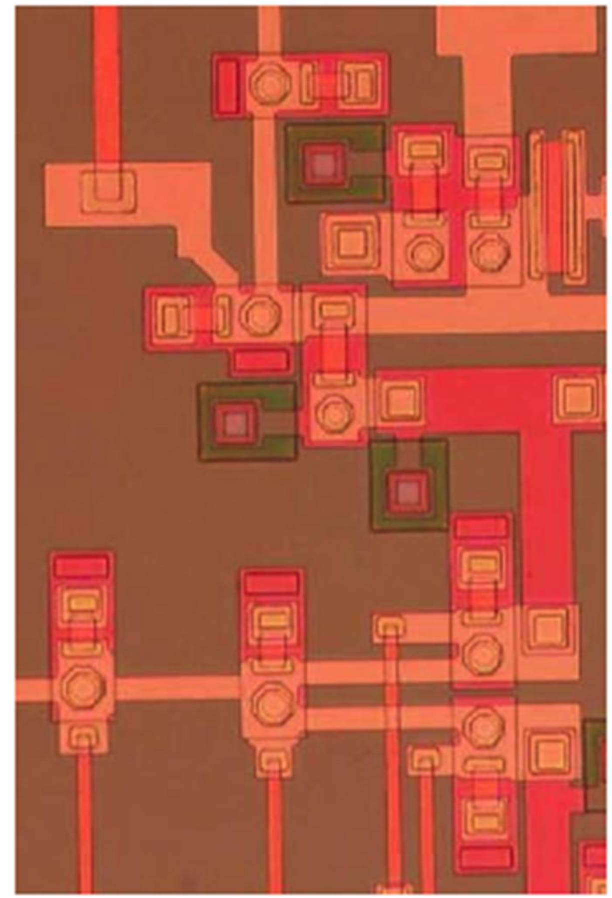
Structuring of Nb/x/Nb-sandwiches



Nb/Al/Al_2O_3/Al/Nb circuit







IPHT Jena 48

High Tc Junction Fabrication

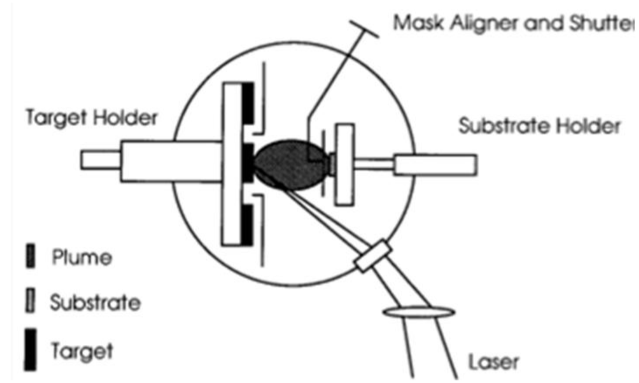
No reliable multilayer technology, must resort to single layer

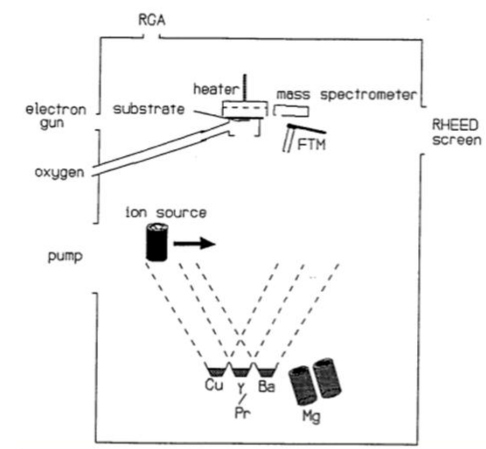
Short coherence length makes structural defects very important

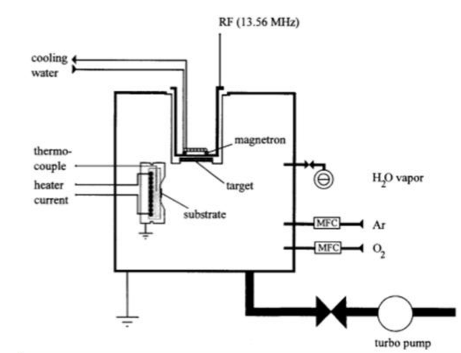
Nearly all junctions are made using $YBa_2Cu_3O_7$

Fabrication techniques used:

- Evaporation
- Laser ablation
- Sputtering
- (MOCVD)







High Tc Junction Types

- Single Layer
– Grain boundary
- Bicrystal
- Biepitaxial
- Step edge
– Damage – e.g. ion implantation, electron beam
– Proximity effect through normal metal
- Multilayer
– Grown barrier
– Interface engineered

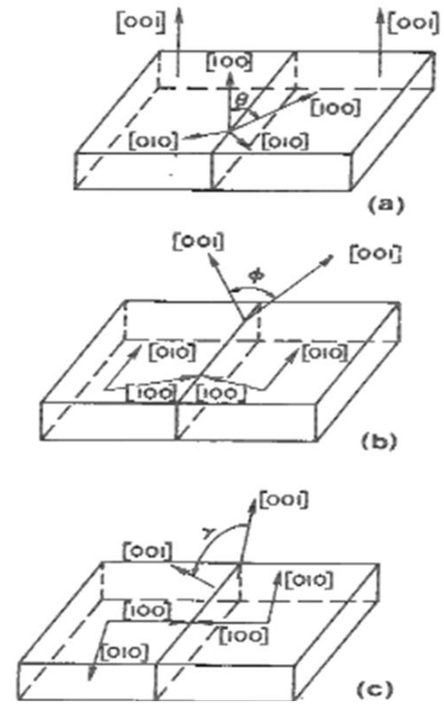
Grain Boundary Junctions

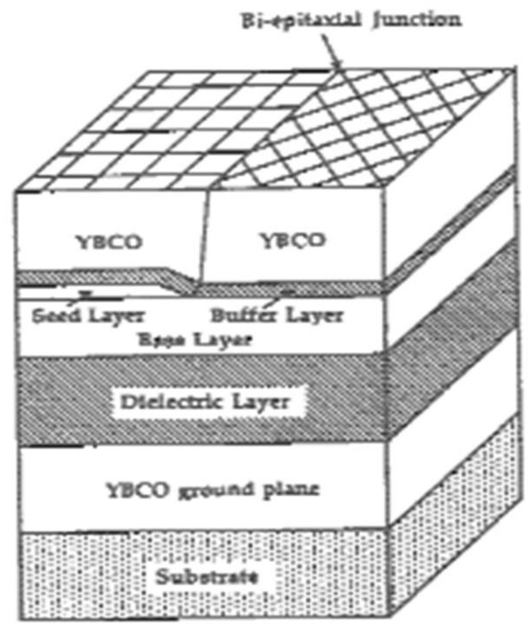
• Grain boundary junctions limit (destroy?) the critical current of any bulk material or wire that is not made with extreme care

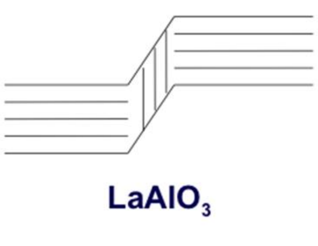
• Films on bicrystal substrates

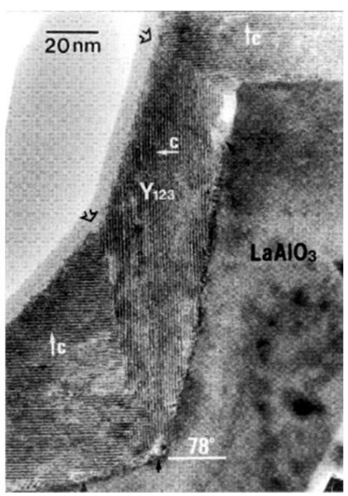
• Films on substrates with a thin patterned buffer layer –Bi-epitaxial junction

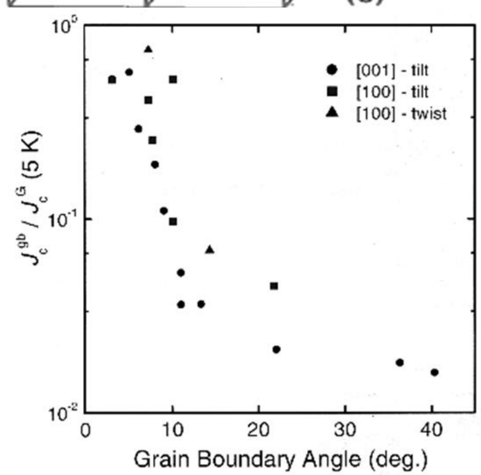
• Films grown over steps











Ramp Junctions

- **Advantages**

–Arbitrary junction positioning

–Small junction area, small capacitance,

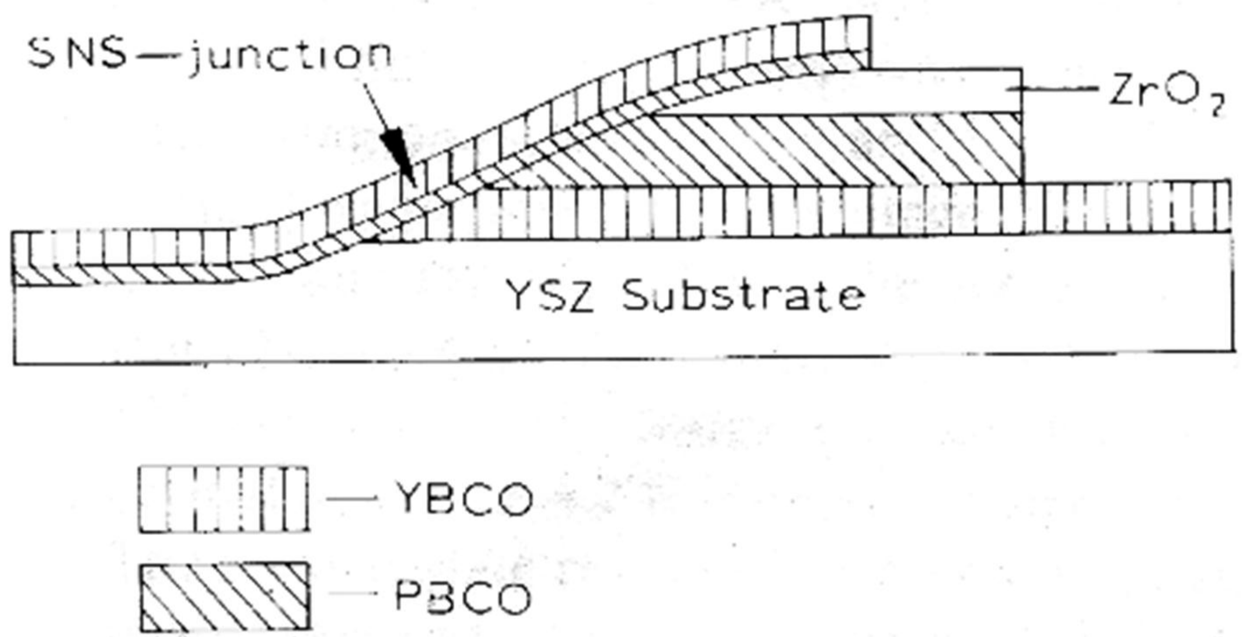
–design for high Jc which means high IcRN

–Connect to the longer coherence length a-b plane

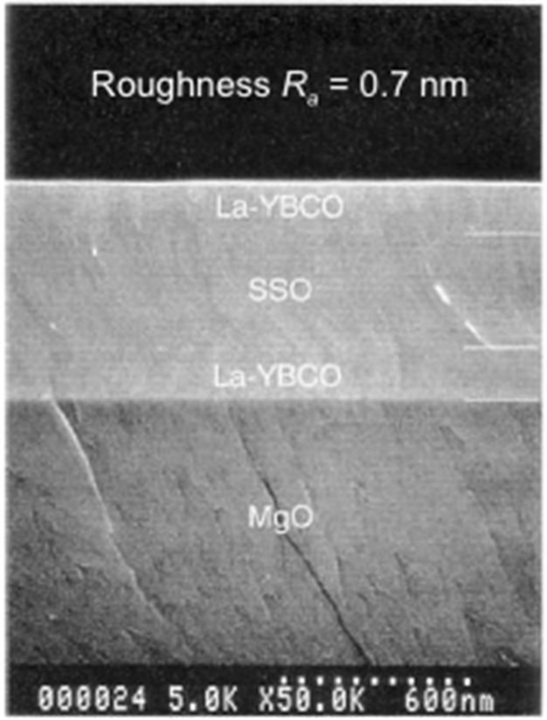
- **Disadvantages**

–Can get reduced Tc – problem for SQUIDs

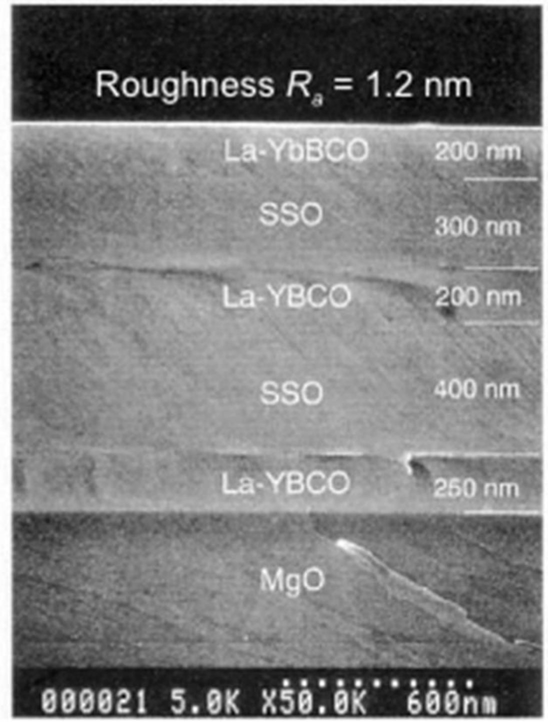
–Parasitic inductance (SFQ)



Recent results on multilayer technology at ISTEC

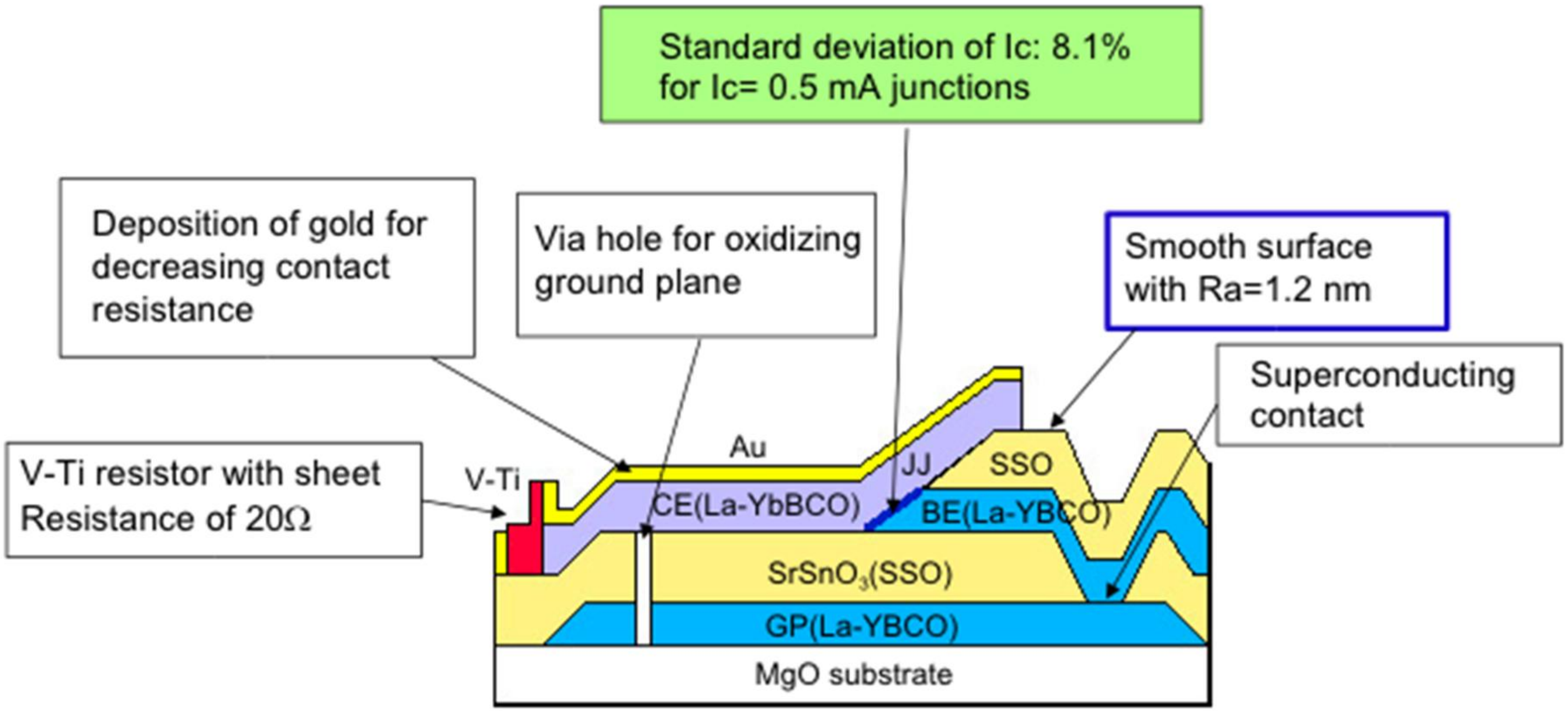


2-layer structure

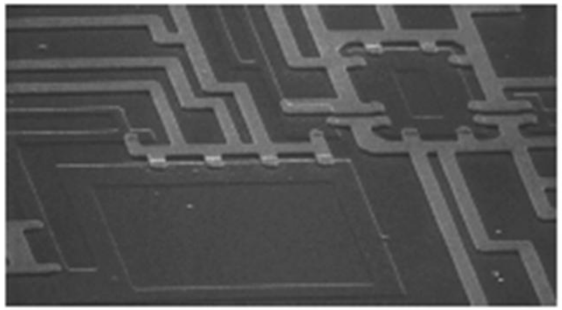


3-layer structure

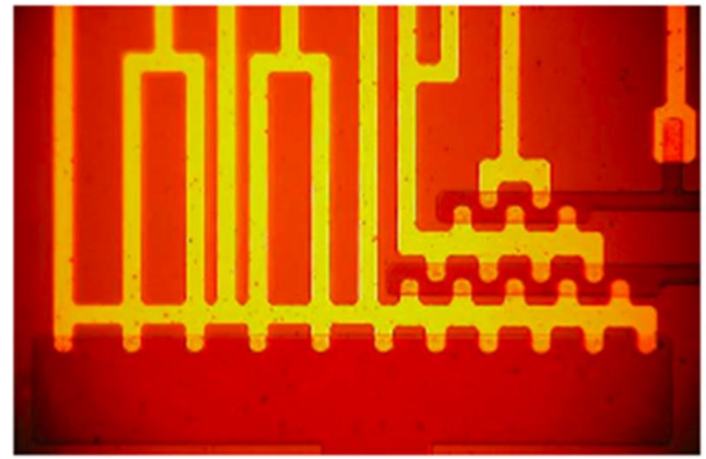
- Optimization of deposition condition with $SrSnO_3$(SSO) insulation layer
- Reproducible deposition of smooth layer with surface roughness less than 2 nm


Standard deviation of Ic: 8.1%
for Ic= 0.5 mA junctions
Deposition of gold for
decreasing contact
resistance
Via hole for oxidizing
ground plane
Smooth surface
with Ra=1.2 nm
Superconducting
contact
V-Ti resistor with sheet
Resistance of 20Ω
V-Ti
Au
JJ
SSO
CE(La-YbBCO)
BE(La-YBCO)
SrSnO3(SSO)
GP(La-YBCO)
MgO substrate

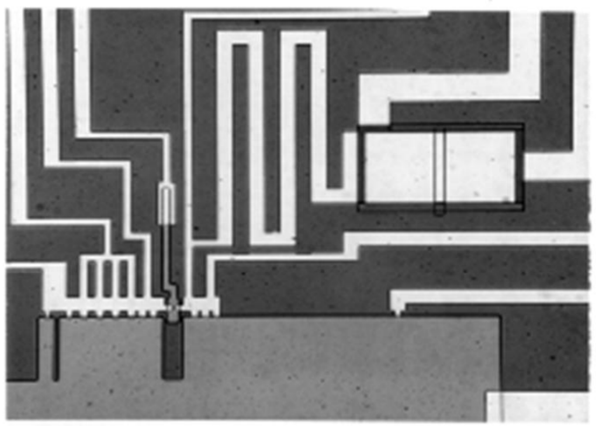
.



Ring Oscillator (21 JJ, Toshiba) 57 GHz @20K



SQUID-array interface (25 JJ, SRL)



Σ-Δ AD modulator (13 JJ, Hitachi) 100 GHz @20K

- Latch-type interface (10 JJ, Fujitsu)
  >I mV output @30 K
- Sampler circuit with JTL buffer (25 JJ, NEC)
  20 GHz signal observation @35 K
- QOS comparator (10 JJ, SRL)
  82 GHz @40 K

ISTEC

Useful references

Physics and Applications of the Josephson Effect by A. Barone and G. Games Paternò,1982 by John Wiley & Sons, Inc., NY

A.V. Ustinov, Lecture notes