SQUIDs

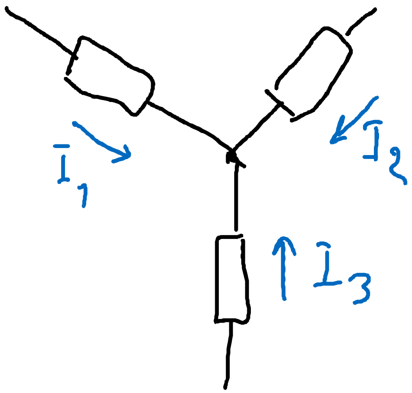
EASISchool 3, Genova, Italy

Mikko Kiviranta

Overview

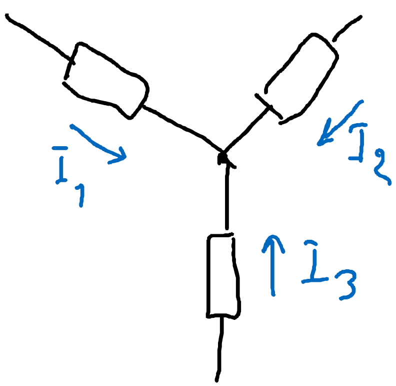
- Kirchoff laws with superconductors
- Weak link - flux quantization
- One-junction rf-SQUID - MISSING
- Two-junction dc-SQUID from SET – SQUID dualism
- Overdamped dc-SQUID, analytic solution
- Realistic dc SQUID characteristics
- A detour to 3-junction interferometer
- Different bias conditions
- Input coil, magnetometers and gradiometers
- SQUID readout, Flux-locked loop

Kirchoffs 1. law



$$\sum_n I_n = 0$$

Kirchoffs 1. law



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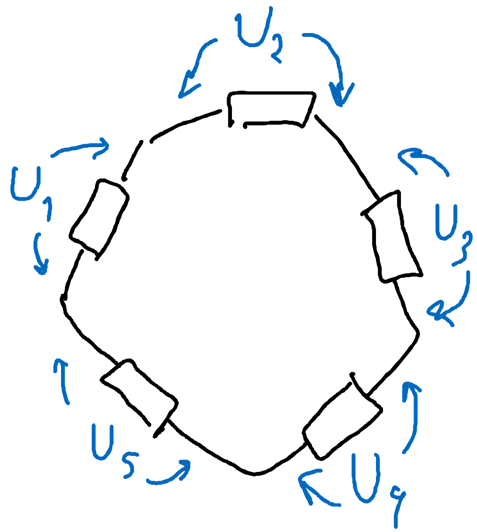
Kirchoffs 2. law



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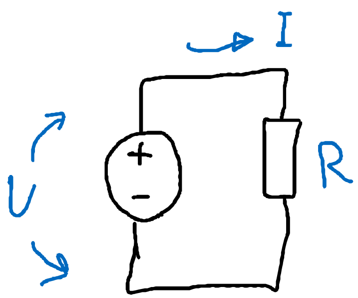
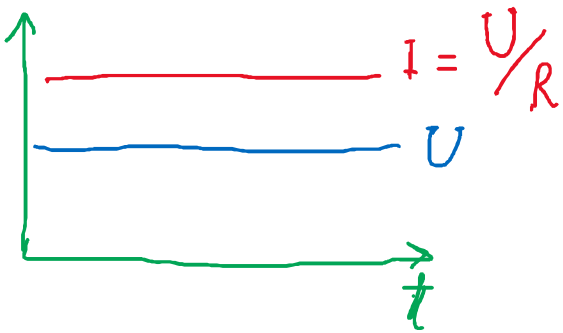
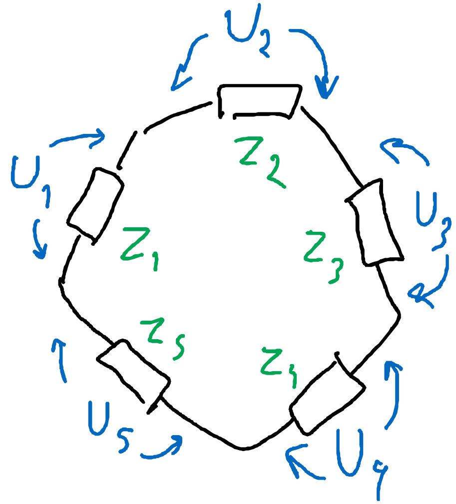
Ohms law





Kirchoffs 2. law



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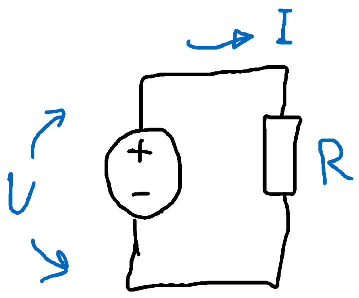
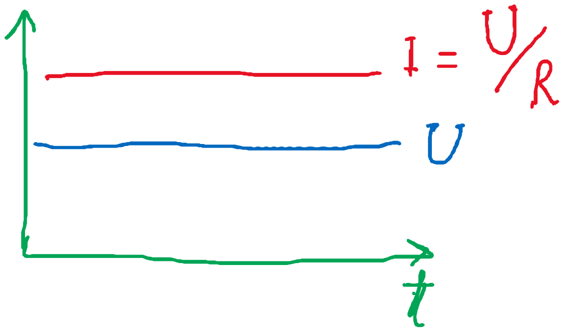
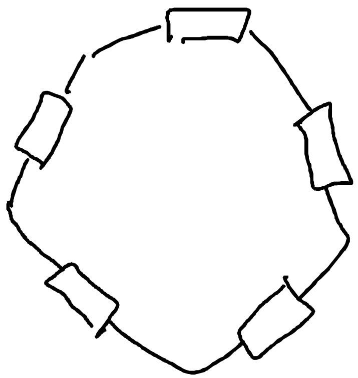
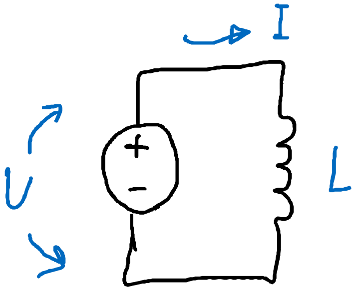


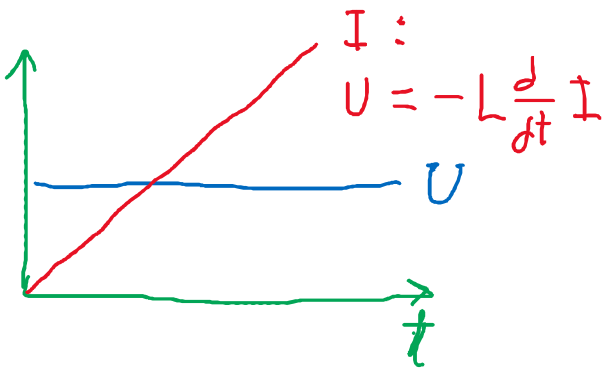
Superconductor

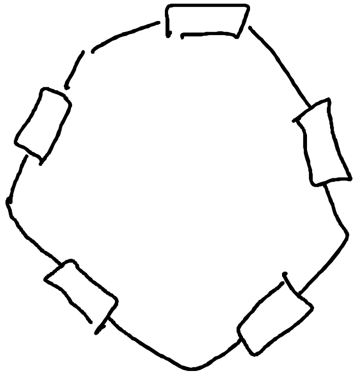
R ≟ 0 ?

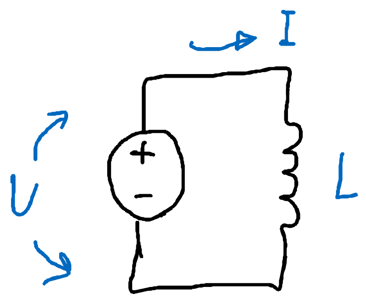


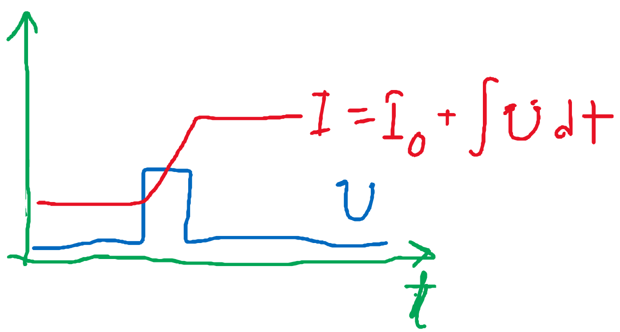


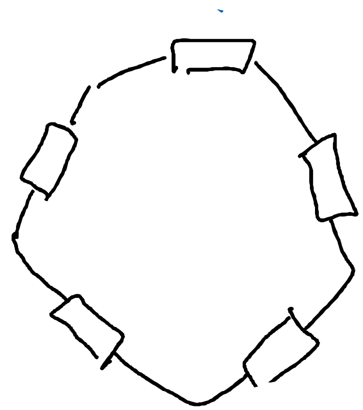

I
U
+
−
L

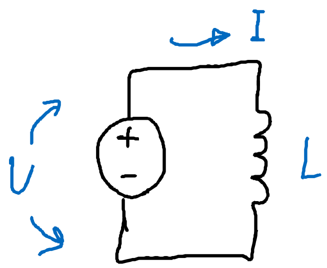

I :
U = −L d/dt I
U
t

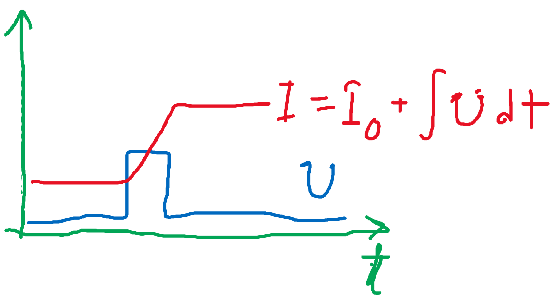



I
U
L

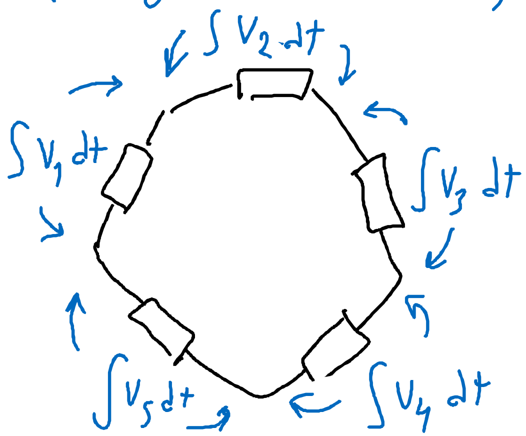

I = I0 + ∫U dt
U
t







Kirchoffs 2. law
(integrated version)

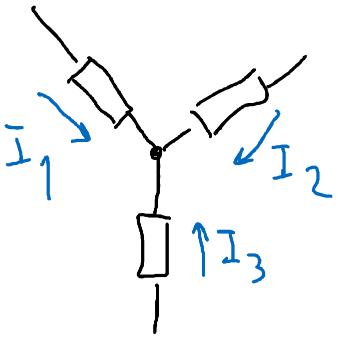


Integral of voltage

$$\int V(t)\,dt = \Phi$$

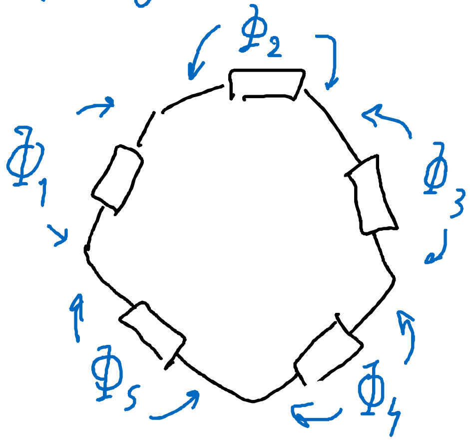
is called flux

Kirchoffs 1. law



$$\sum_n I_n = 0$$

Kirchoffs 2. law (integrated version)

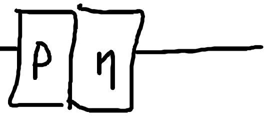


$$\sum_n \Phi_n = 0$$

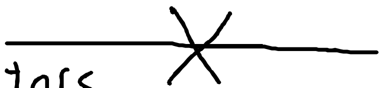
Linear circuits are boring

Non-linear

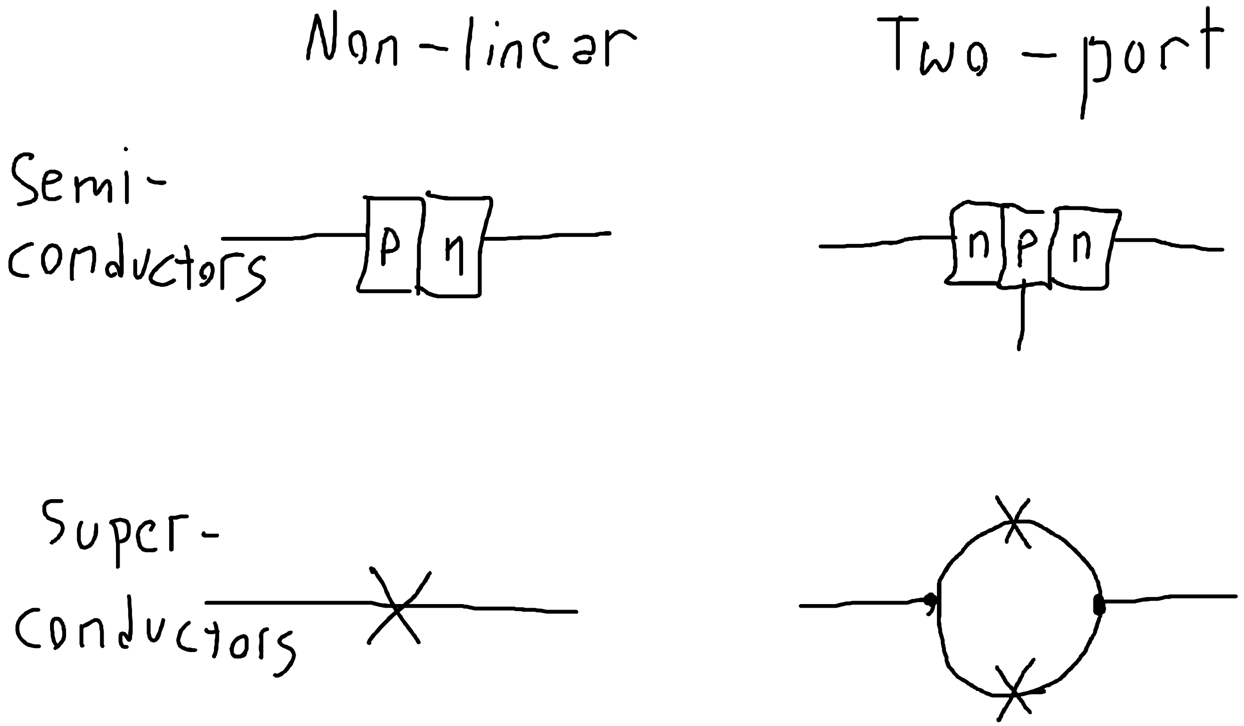
Semi-conductors



Super-conductors



Linear circuits are boring



From Josephson relations

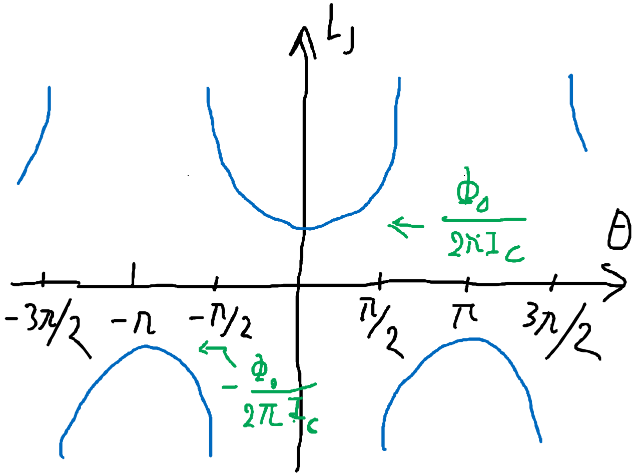
$$\begin{cases} I = I_c \sin\theta \\ U = \frac{\Phi_0}{2\pi}\frac{d}{dt}\theta \end{cases}$$

$$\Rightarrow U = \frac{\Phi_0}{2\pi}\cdot\frac{1}{\sqrt{1 - I^2/I_c^2}}\cdot\frac{d}{dt}I$$

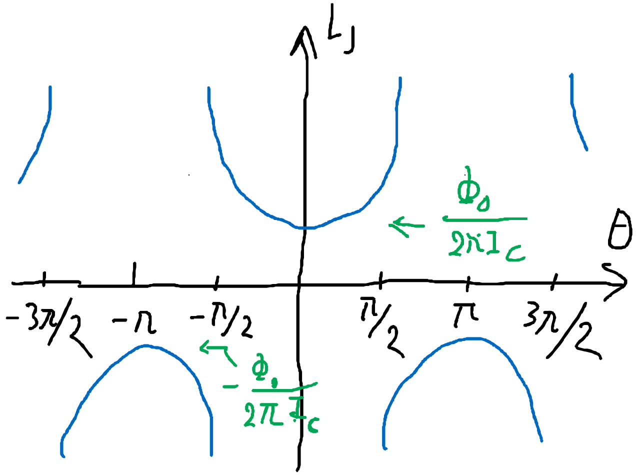
← Derivative of arc sin

$$U = \underbrace{\frac{\Phi_0}{2\pi}\frac{1}{I_c\cos\theta}}_{L_J}\frac{d}{dt}I$$

L_J: Josephson inductance



$$U = \underbrace{\frac{\Phi_0}{2\pi}\frac{1}{I_c\cos\theta}}_{L_J\text{: Josephson inductance}}\frac{d}{dt}I$$



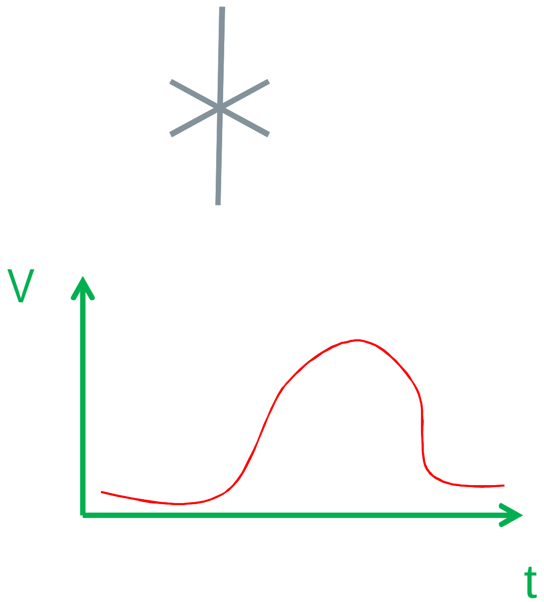
$$U = \underbrace{\frac{\Phi_0}{2\pi}\frac{1}{I_c \cos\theta}}_{L_J:\ \text{Josephson inductance}} \frac{d}{dt} I$$

Work done on the junction

$$\Delta E = W = \int U \cdot I \, d\theta = -\frac{I_c \Phi_0}{2\pi} \cos\theta$$

is finite however

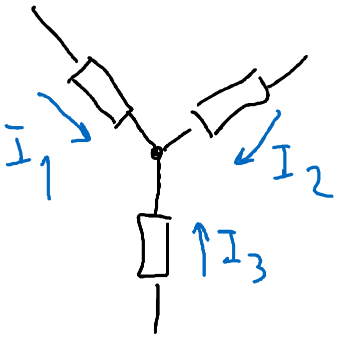
Phase slip, a.k.a SFQ



- When quantum phase advances by 2-pi integral of voltage across junction is always 2.07 picosecond-millivolts.
- Such a pulse creates in any inductor the flux of 2.07 microampere-nanohenrys

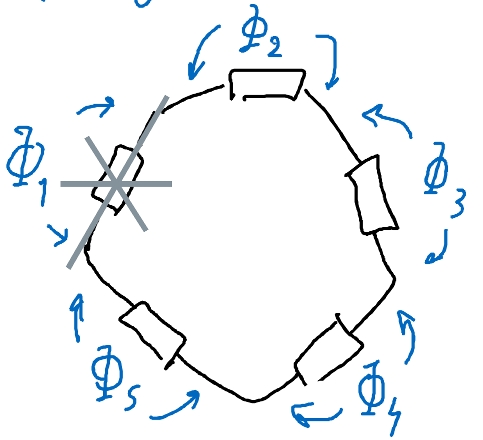
Version with weak links

Kirchhoffs 1. law



$$\sum_n I_n = 0$$

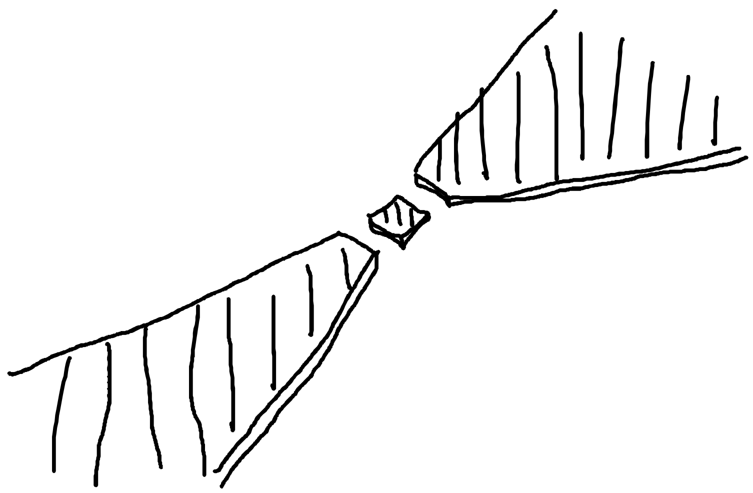
Kirchhoffs 2. law (integrated version)

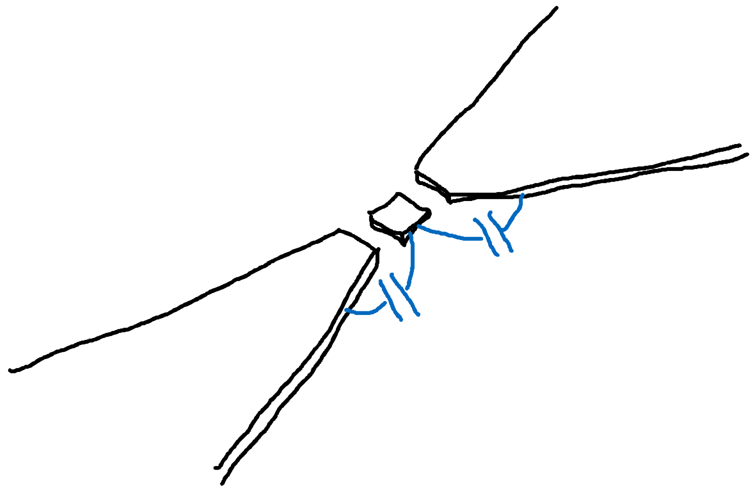


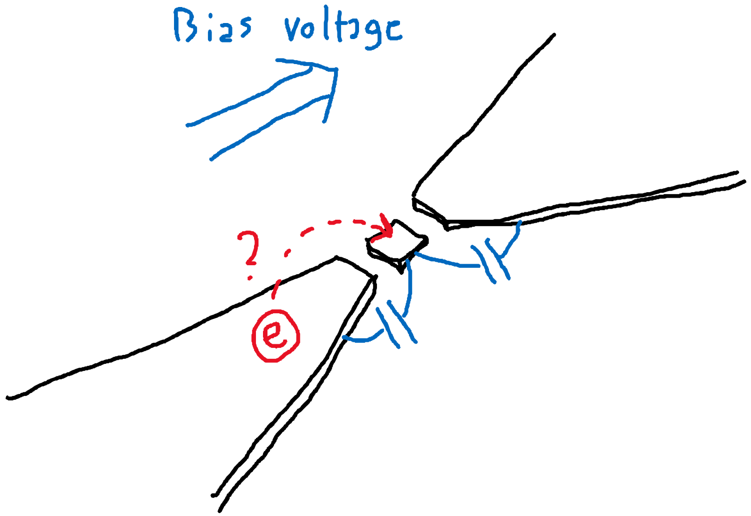
$$\sum_n \Phi_n = n\Phi_0$$

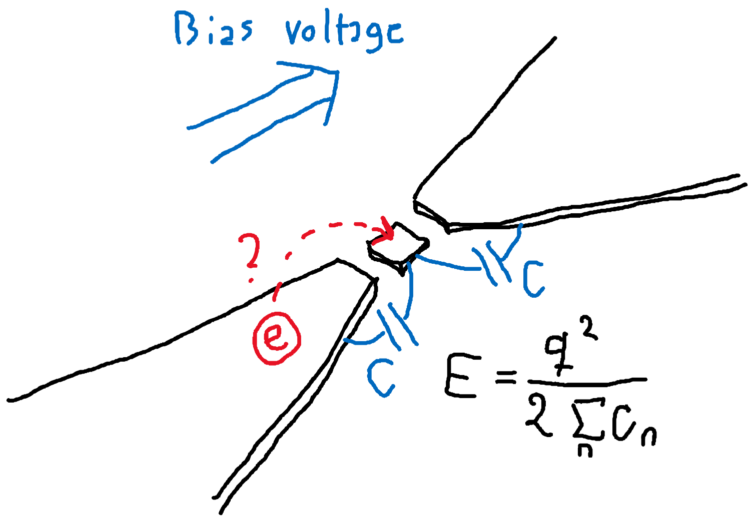
Single-junction SQUID

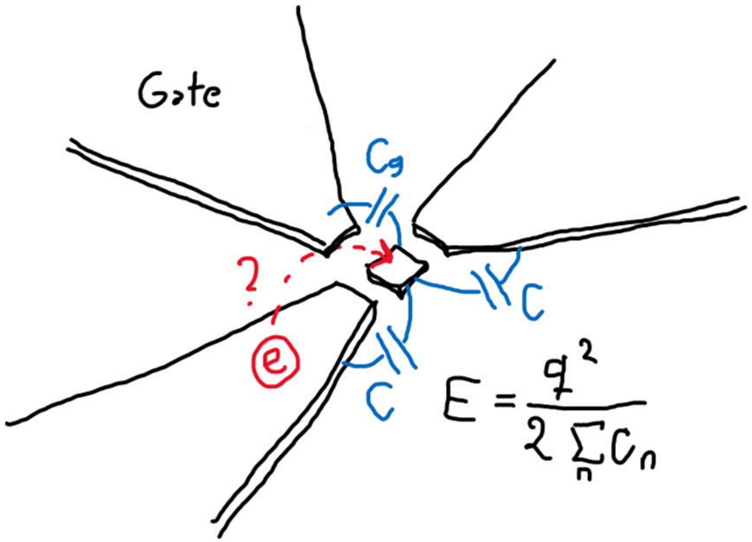
Single electron transistor



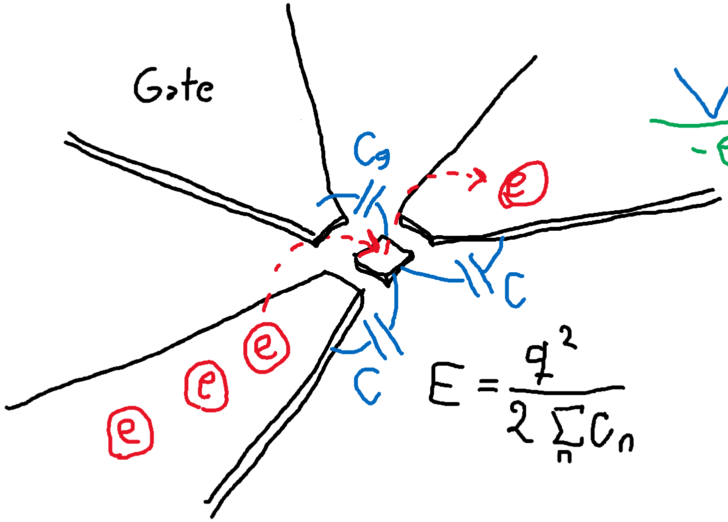


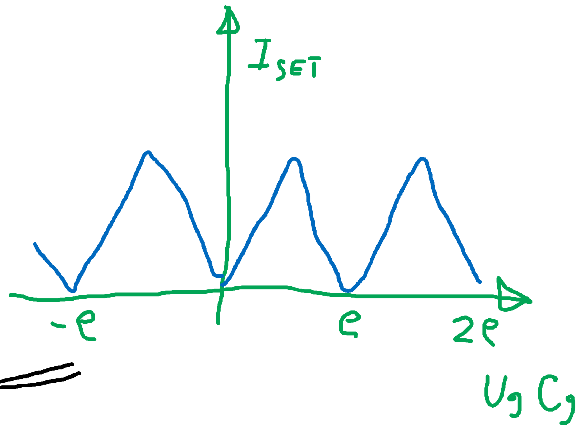

Bias voltage
?
e


Bias voltage
?
e
C
C
$E = \frac{q^2}{2 \sum_n C_n}$

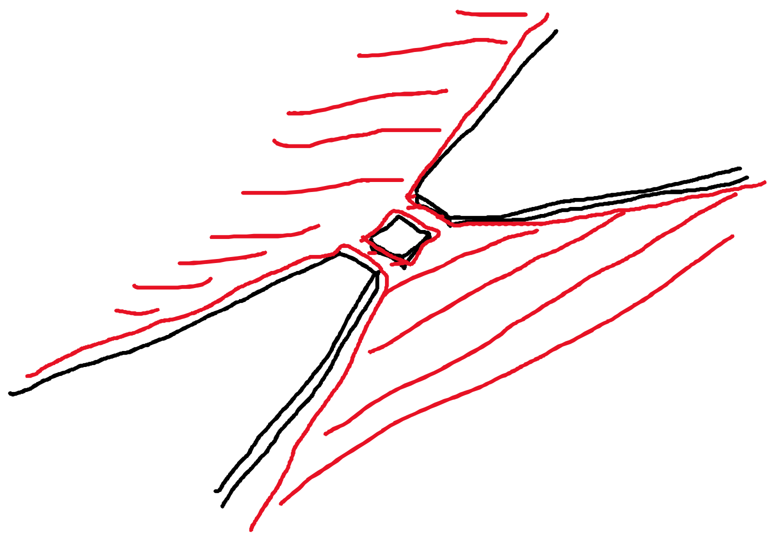


Gate
C_g
?
e
C
C
$E = \frac{q^2}{2\sum_n C_n}$

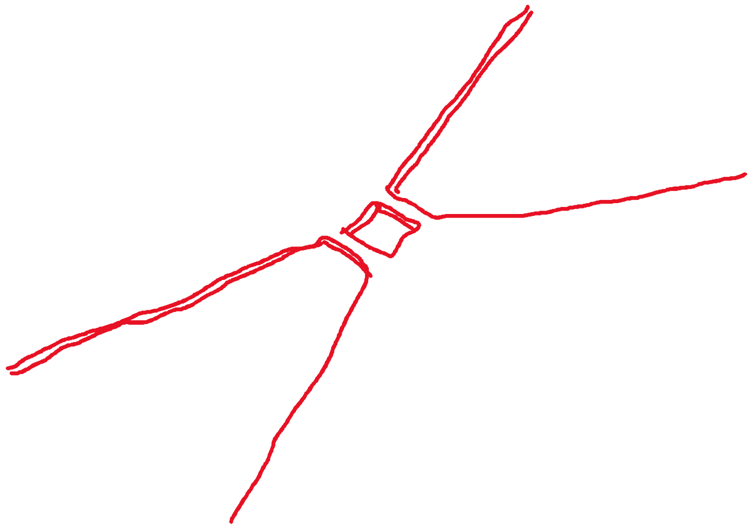

Gate
C_g
C
C
$E = \frac{q^2}{2\sum_n C_n}$
e
e
e
e


I_{SET}
-e
e
2e
$U_g C_g$

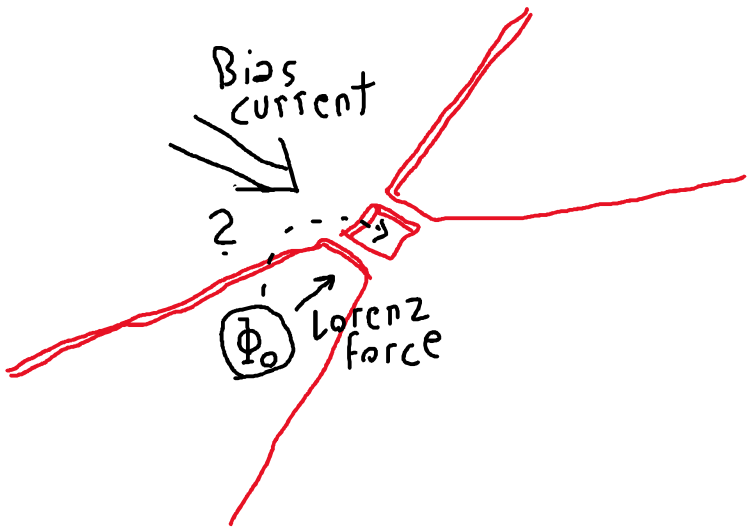
Electrostatic dual of a SET



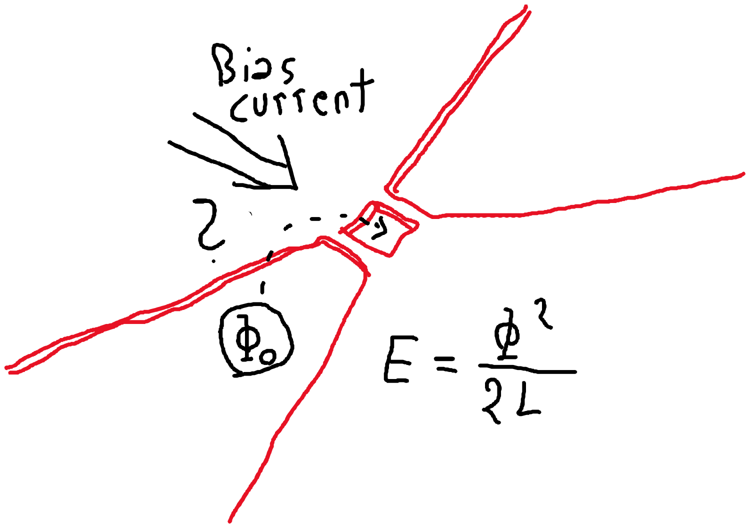
Electrostatic dual of a SET



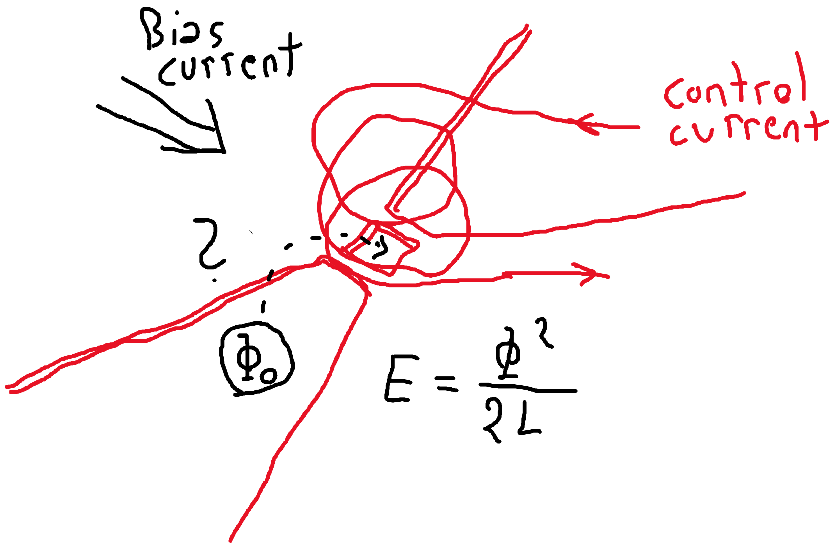
Electrostatic dual of a SET



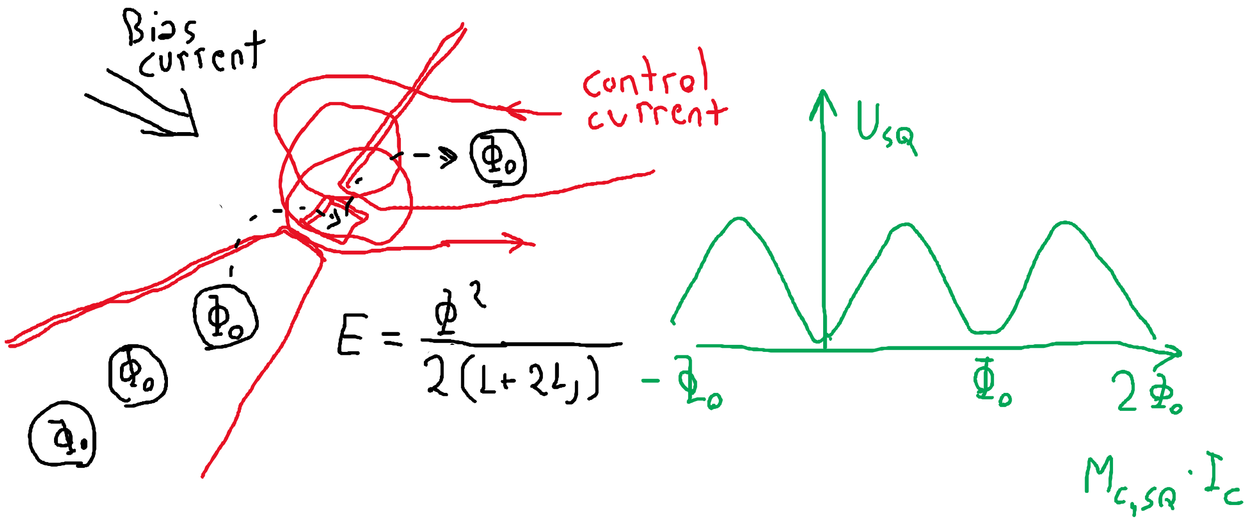
Electrostatic dual of a SET



Electrostatic dual of a SET

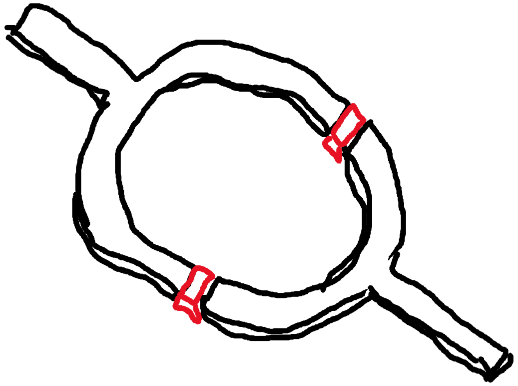


Electrostatic dual of a SET = dc SQUID



Likharev 1987, Guichard and Hekking 2010

Lets take dc SQUID more quantitatively



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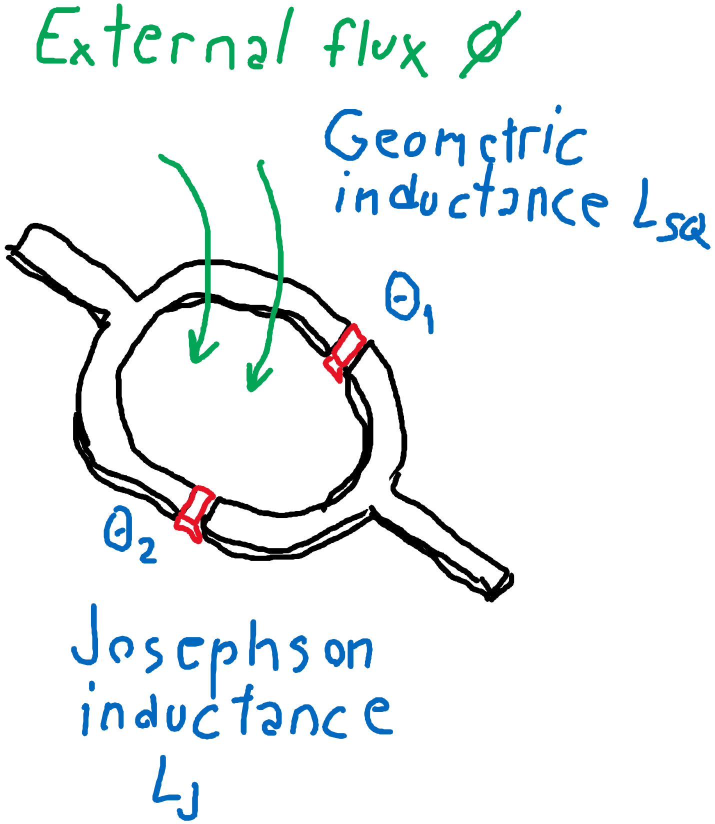
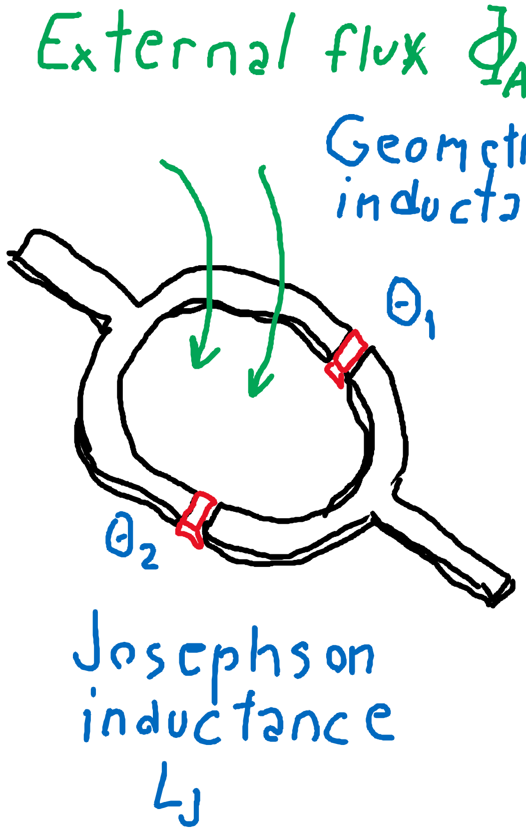
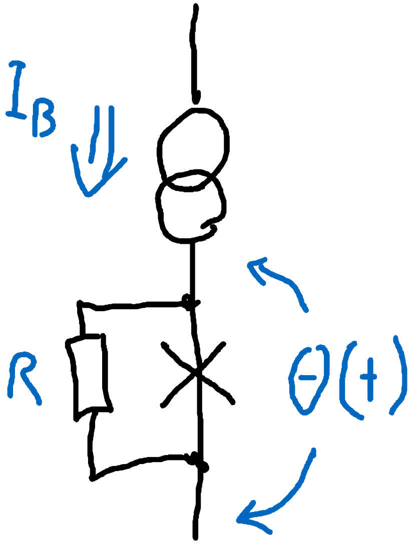




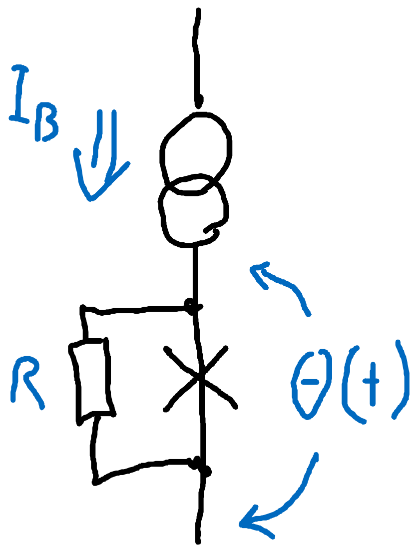
Limit $L_{SQ} \ll L_J$:

Equation of motion for $\frac{1}{2}(\theta_1 + \theta_2)$ is the same as the single JJ whose crit. current is not I_c but $|\cos(\pi \Phi_A)| \cdot 2 I_c$

Equation of motion of single J-junction



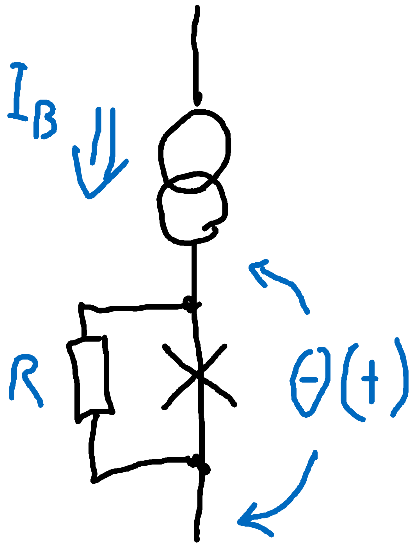
Equation of motion of single J-junction



Dimensionless units

$i = I/I_c \qquad u = U/RI_c \qquad \tau = t\,\dfrac{2\pi R I_c}{\Phi_0}$

Equation of motion of single J-junction



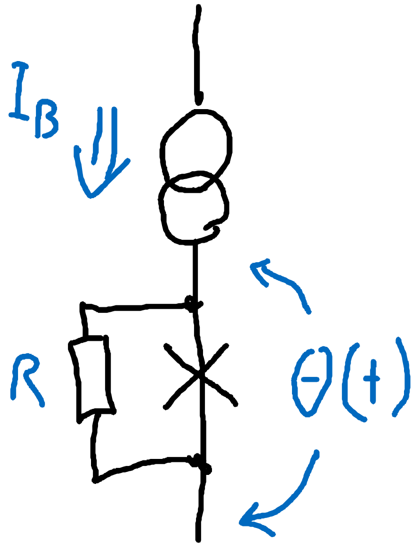
Dimensionless units

$$i = I/I_c \qquad u = U/RI_c \qquad \tau = t\,\frac{2\pi R I_c}{\Phi_0}$$

$$\frac{d}{dt}\Theta = i_B - \sin\Theta$$

Dimensionless units

$i = I / I_c \qquad u = U / R I_c \qquad \tau = t \, \frac{2\pi R I_c}{\Phi_0}$

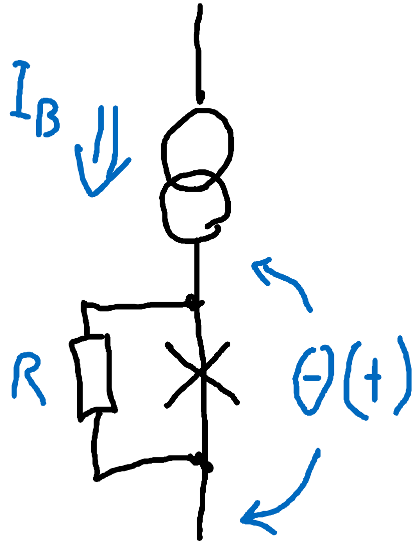


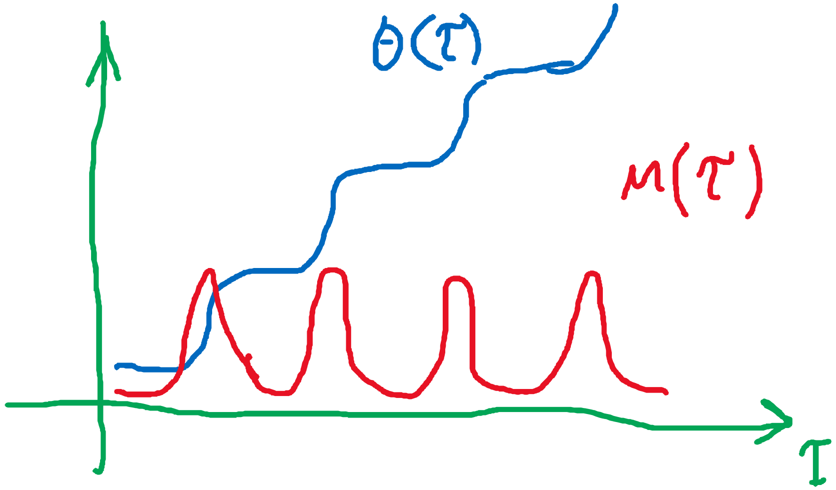
$$\frac{d}{dt} \theta = i_B - \sin\theta$$

Asmalazov-Larkin (or Maple):

$$\theta(t) = 2\tan^{-1}\left[\frac{\sqrt{i_B^2 - 1}}{i_B + 1} \tan\left(\frac{1}{2}\sqrt{i_B^2 - 1}\, t\right)\right] + \frac{\pi}{2}$$

$$u = \frac{d}{d\tau}\theta = \frac{i_B^2 - 1}{i_B + \cos\left(\sqrt{i_B^2 - 1} \cdot t\right)}$$





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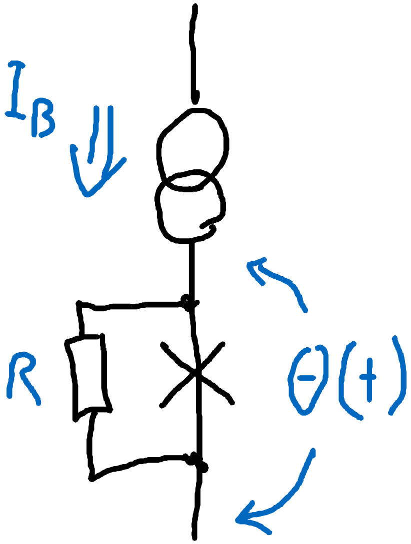
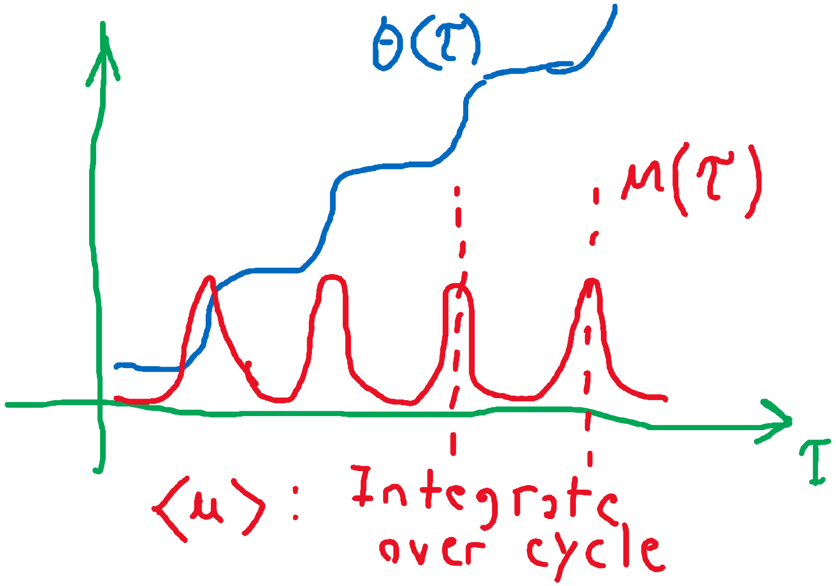
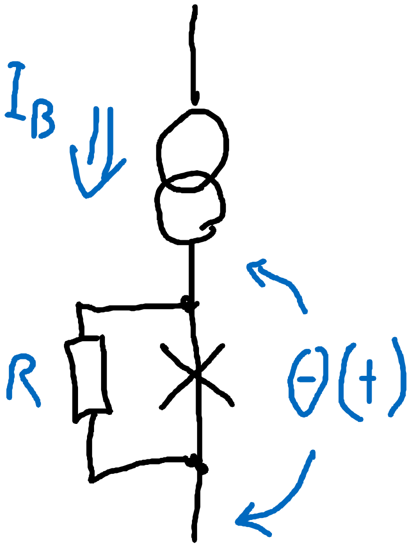
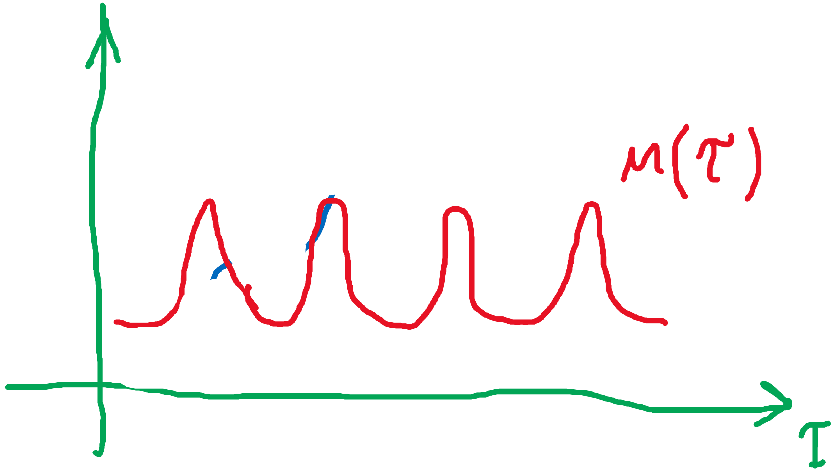


I_B
R
Θ(t)

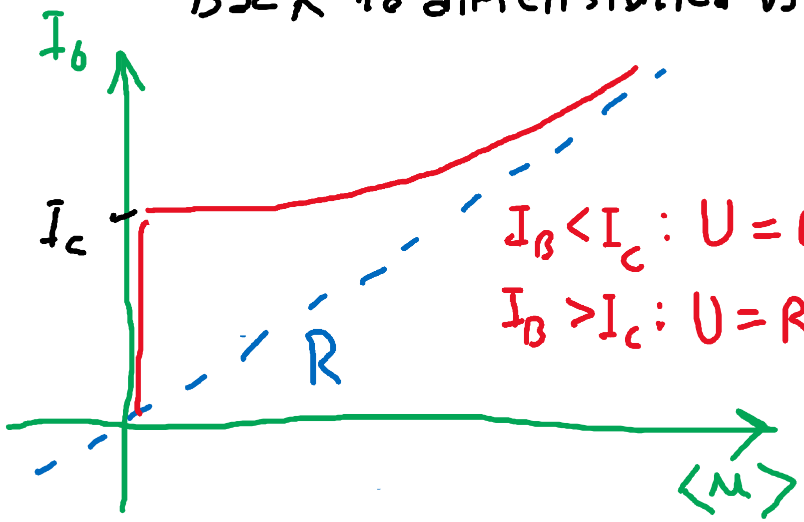


θ(τ)
u(τ)
τ
⟨u⟩: Integrate over cycle





Back to dimensioned values



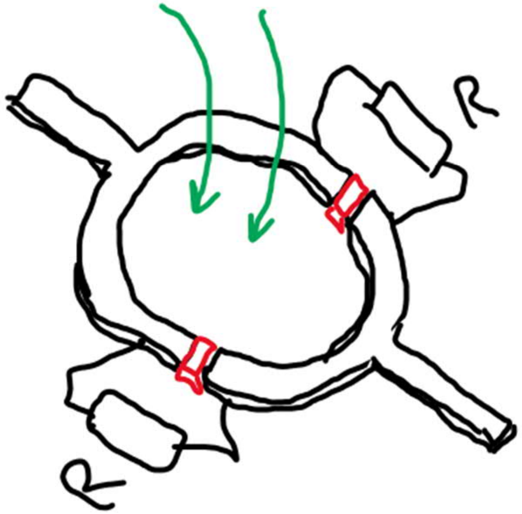
$I_B < I_c : U = 0$

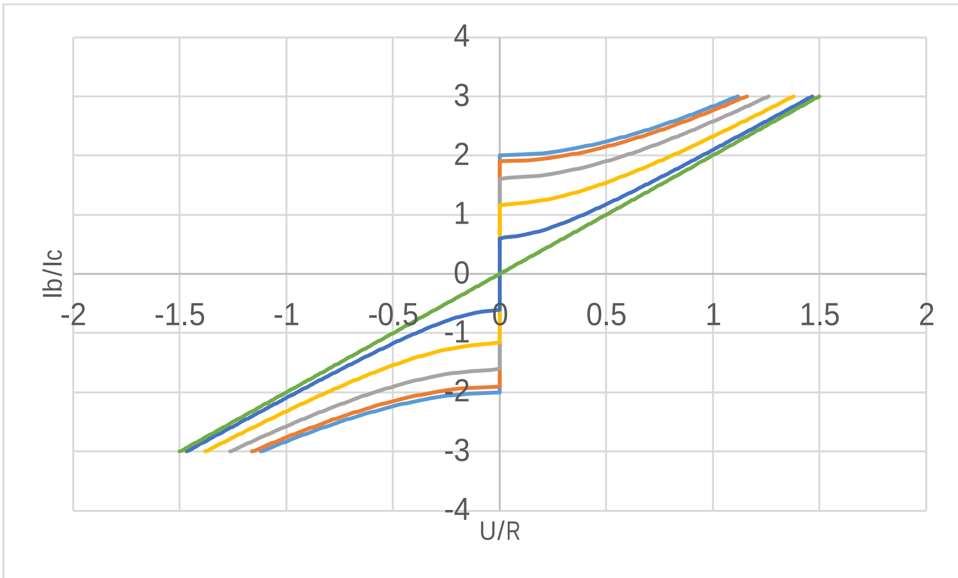
$I_B > I_c : U = RI\sqrt{1 - I_c^2/I^2}$

DC SQUID is like single J-junction with flux-controllable critical current

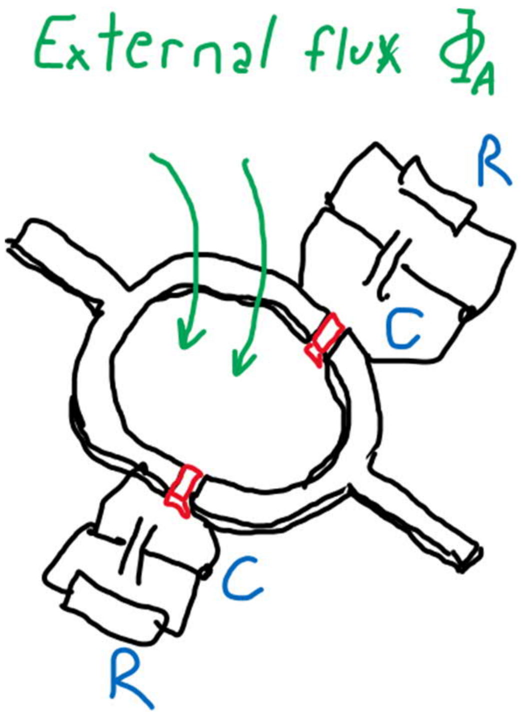
External flux Φ_A

$$U = \frac{RI}{2}\sqrt{1-\left(\frac{2I_c}{I}\cos\frac{\pi\Phi_A}{\Phi_0}\right)^2}$$





Added complication: junction capacitance

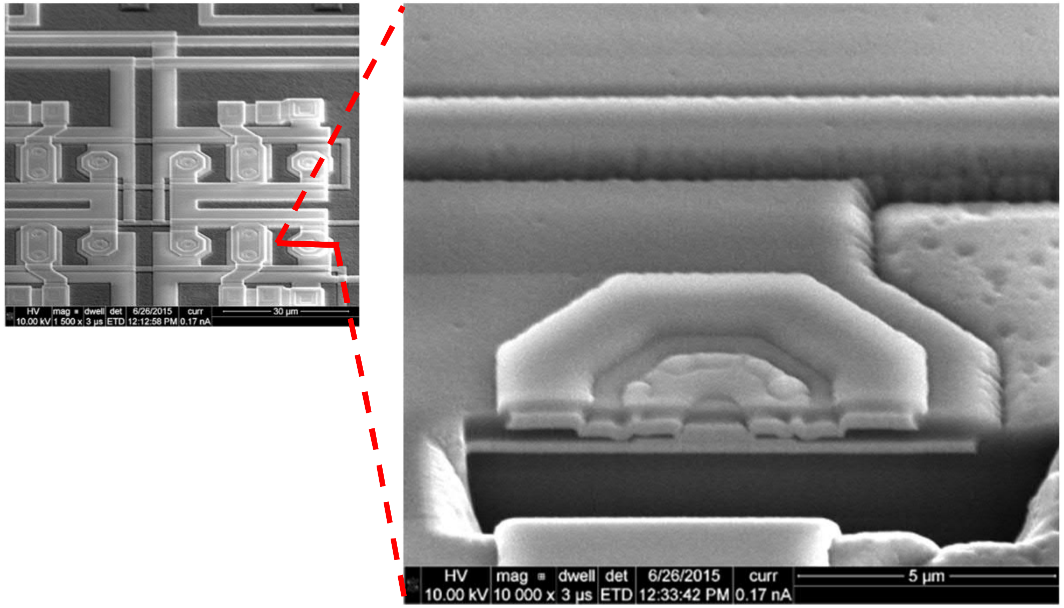


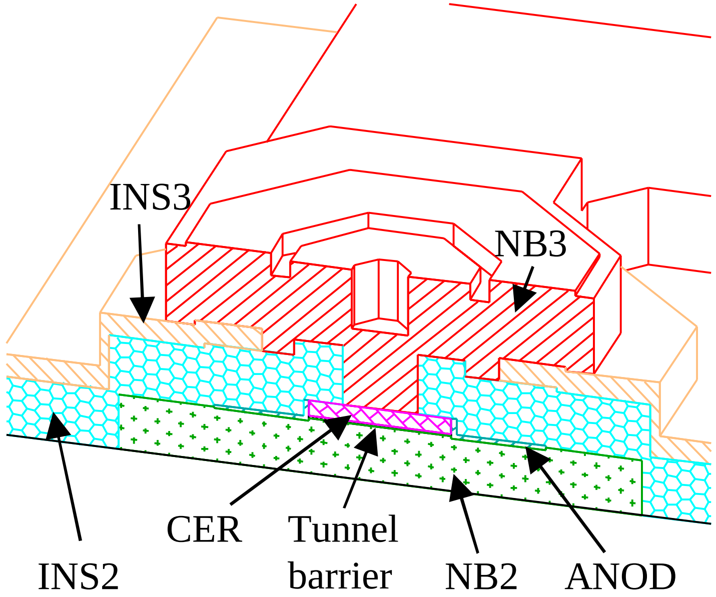
- Makes characteristics hysteretic
- For single-valued response must add sufficiently small shunt resistors
- Johnson noise of the resistors is the main source of noise in dc SQUIDs

$$\beta_C = \frac{2\pi I_C C_J R_S^2}{\Phi_0} < 0.7$$

H. H. Zappe 1972, doi:10.1063/1.1662354

Added complications: junction capacitance





Phenomena due to junction capacitance

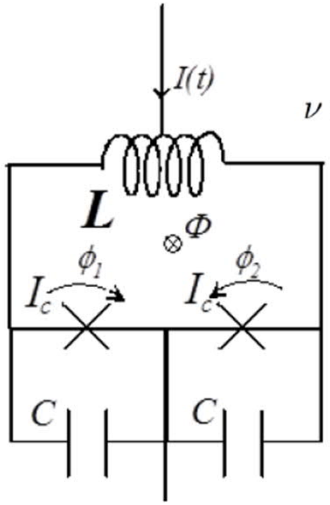
Josephson inductance forms an LC resonator
Inductance depends on oscillation amplitude -> anharmonicity

- Dynamical equations of DC SQUID.

$$\beta_c \frac{d^2\nu}{d\tau^2} + \frac{d\nu}{d\tau} + \sin\nu\cos\phi = i$$

$$\beta_c \frac{d^2\phi}{d\tau^2} + \frac{d\phi}{d\tau} + \cos\nu\sin\phi + \frac{2}{\beta_L}(\phi - \phi_a) = 0$$

External flux



$$\nu = \frac{\delta_1 + \delta_2}{2} \qquad \phi = \frac{\delta_1 - \delta_2}{2}$$

$$\beta_L = \frac{2\pi L I_c}{\Phi_0}$$

$$U(\nu, \phi) = \frac{1}{\beta_L}(\phi\text{-}\phi_a)^2 - i\nu - \cos\nu\cos\phi$$

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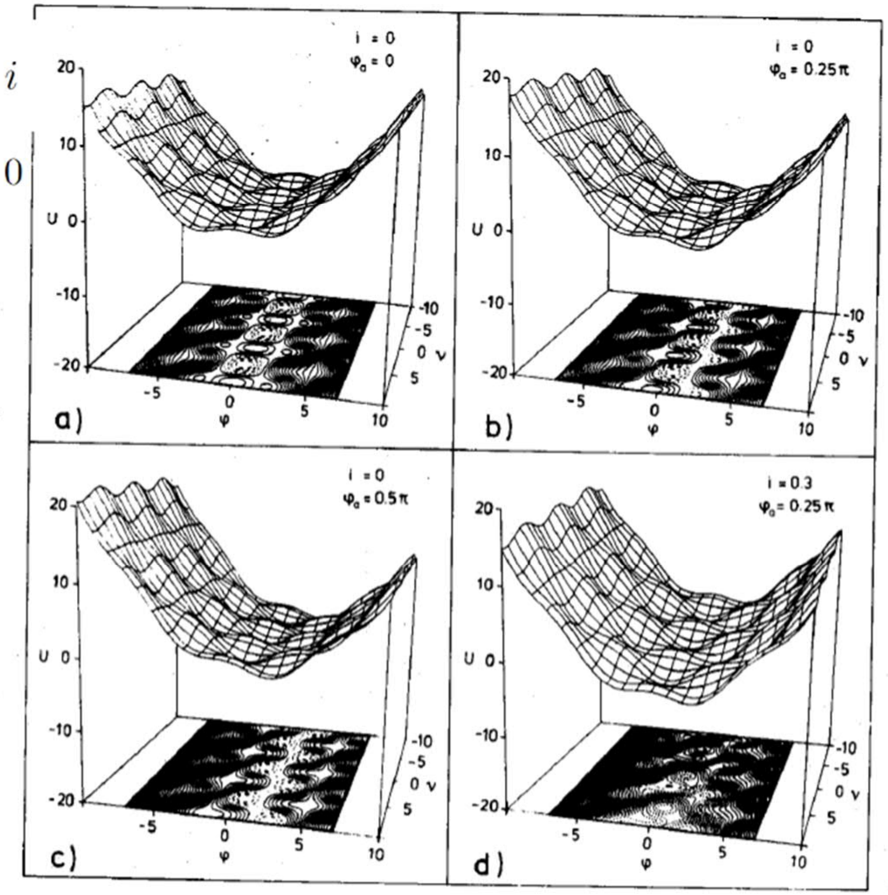
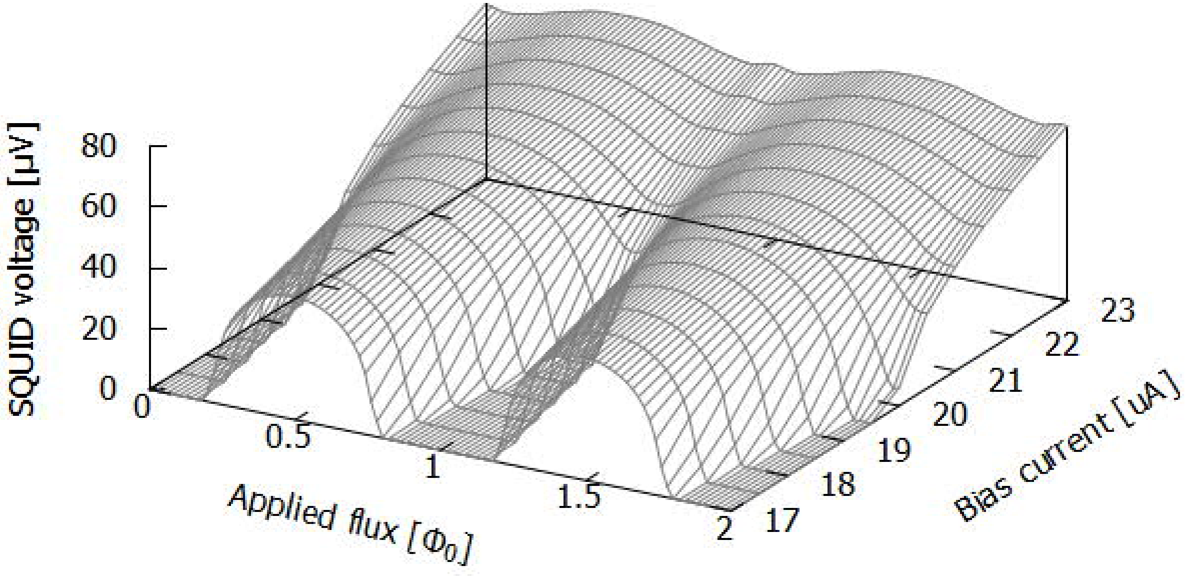
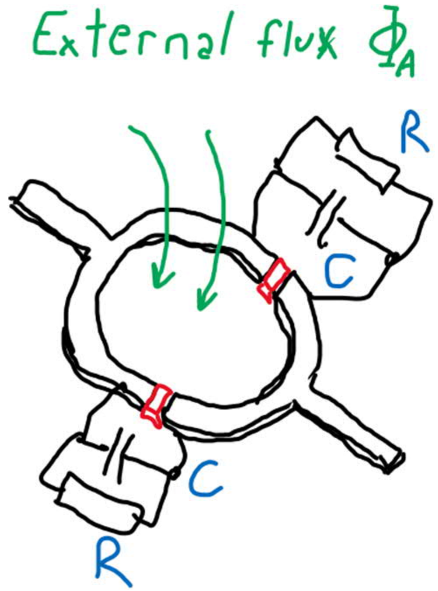


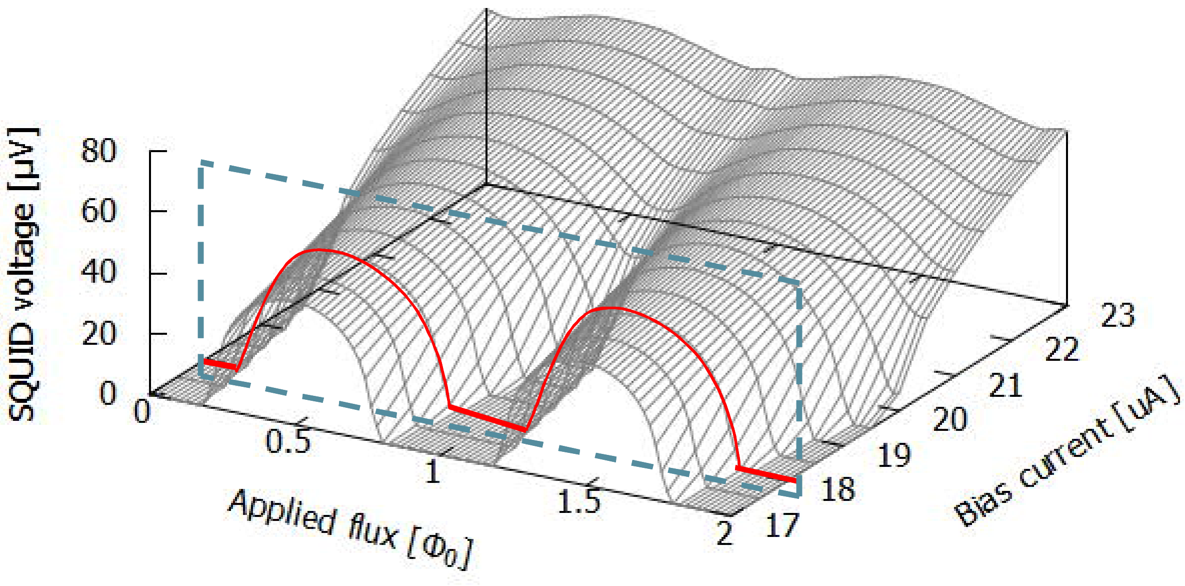
Realistic dc SQUID characteristics

$$\beta_C = \frac{2\pi I_C C_J R_S^2}{\Phi_0} = 0.7 \qquad \beta_L = \frac{2\pi L_{SQ} I_C}{\Phi_0} = 1.0$$

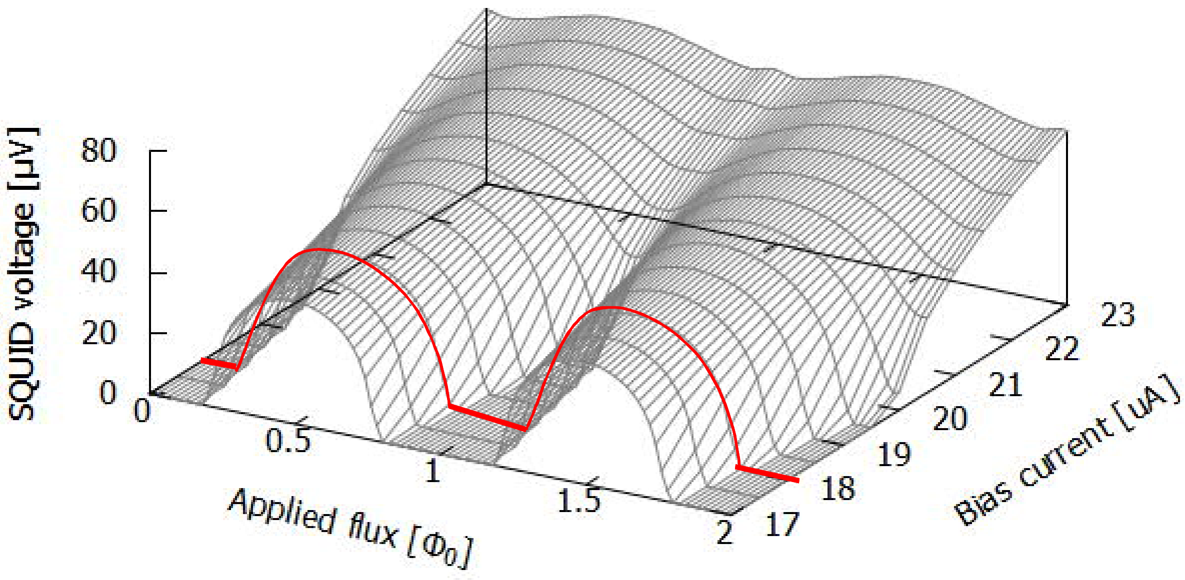




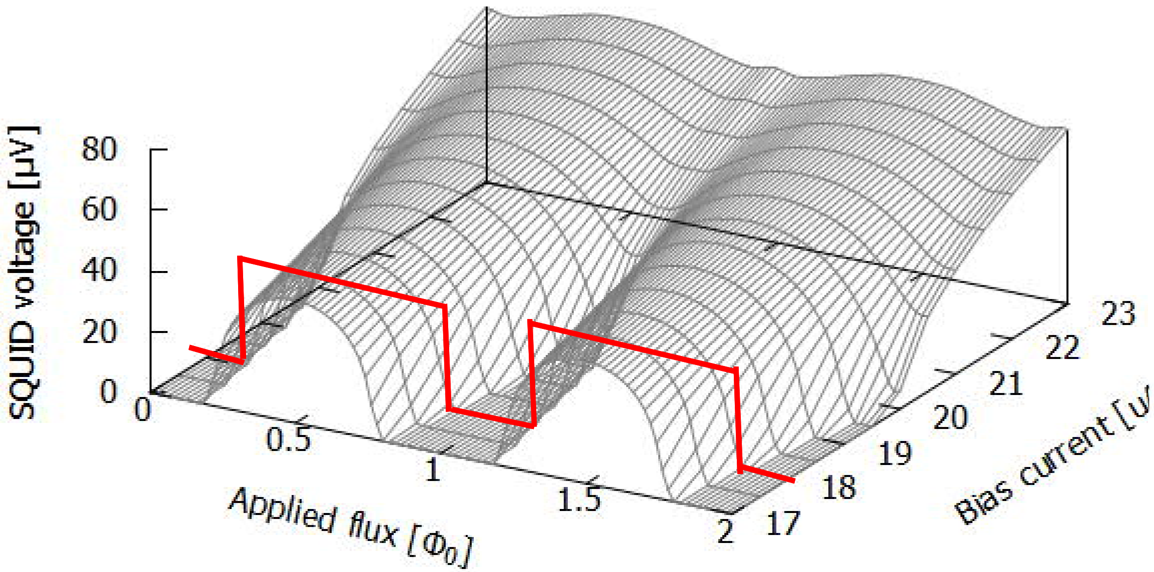
Current bias



Current bias, $I_B < 2I_C$



Detour: 3-junction interferometer - Zappe switch



H. H. Zappe, IEEE Transactions on Magnetics, 1977

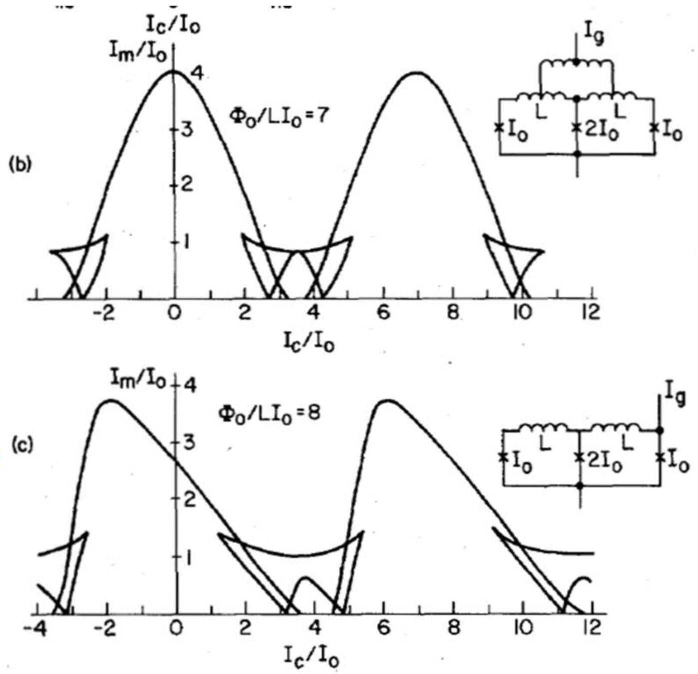
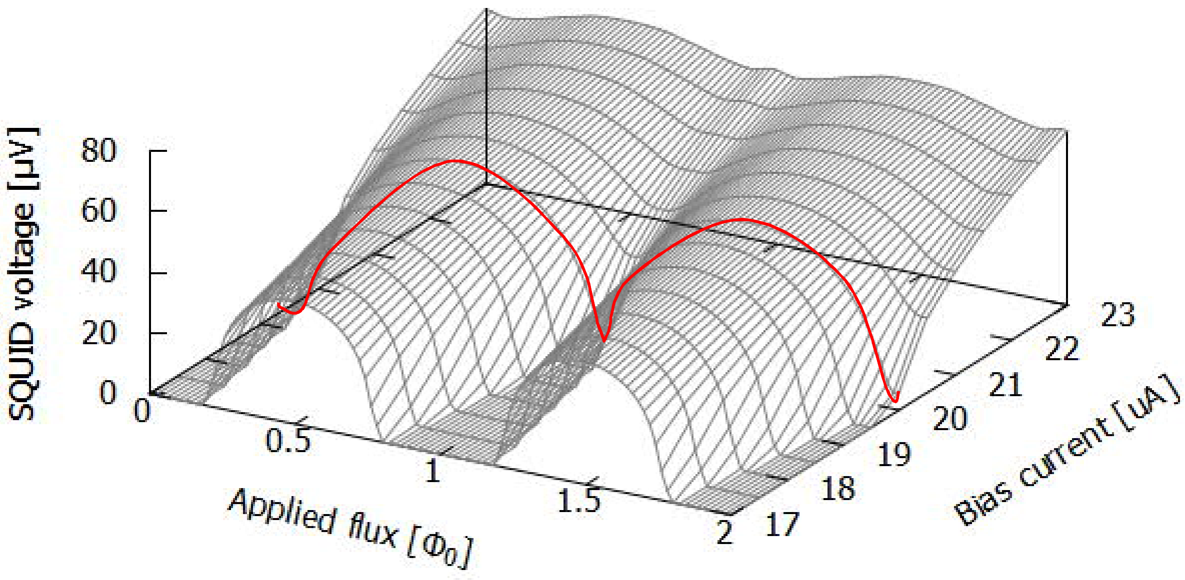
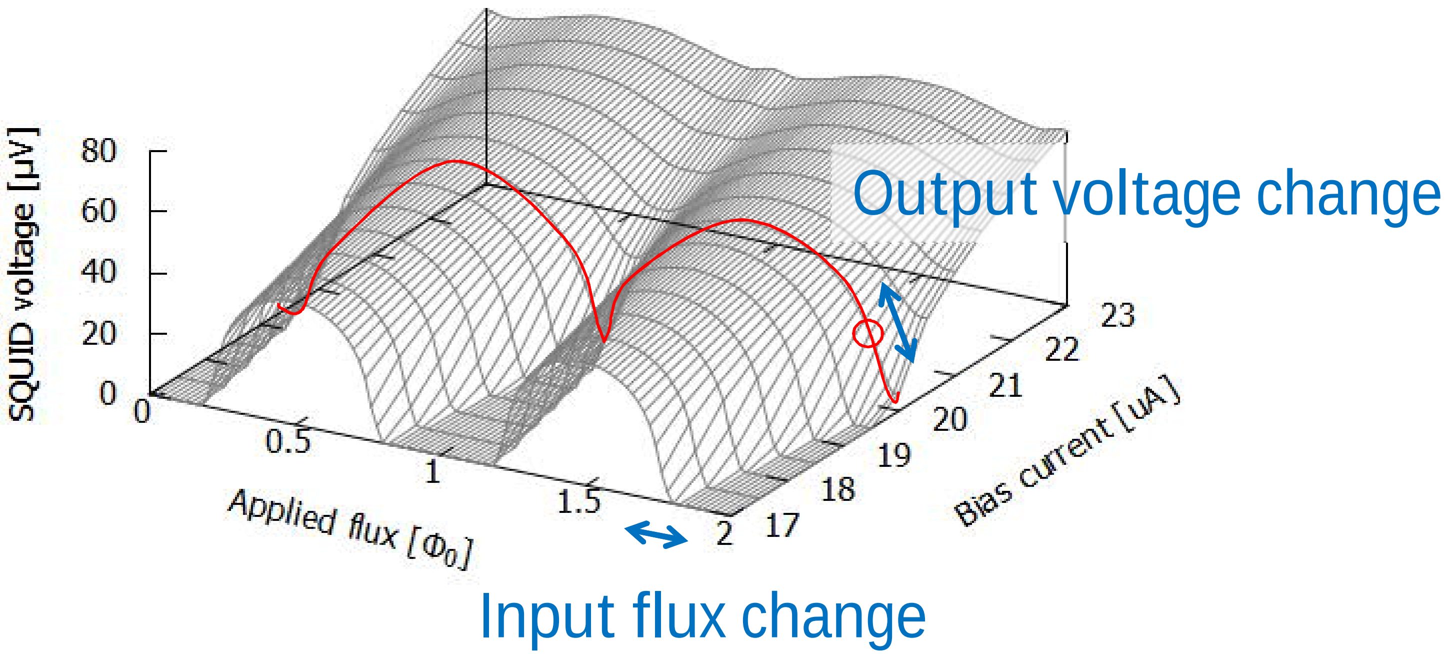


Fig. 5. Threshold curves of quantum interference devices. (a) is a two junction device. (b) and (c) are three junction interferometers with symmetric and asymmetric supply current feed respectively.

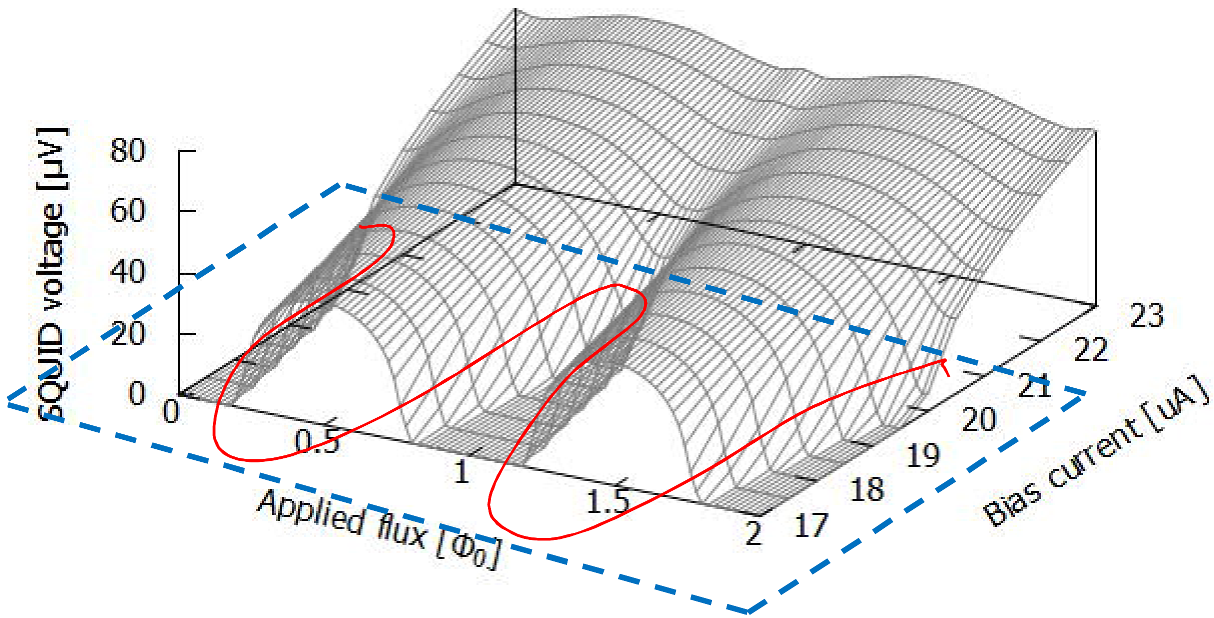
Current bias, $I_B > 2I_C$



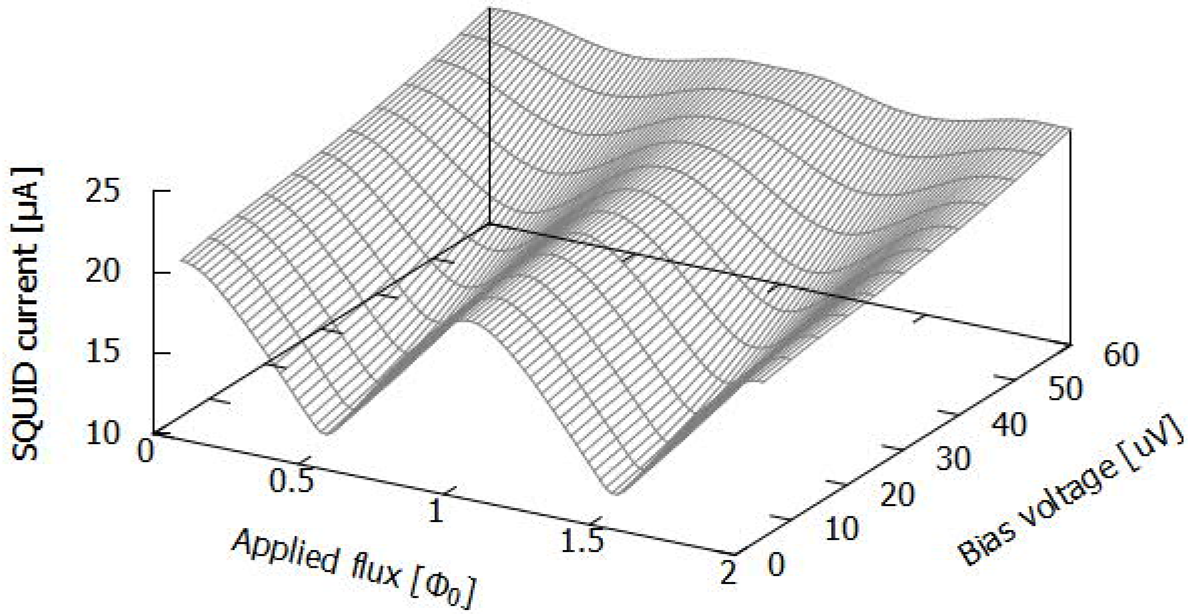
Current bias, $I_B > 2I_C$



Voltage bias



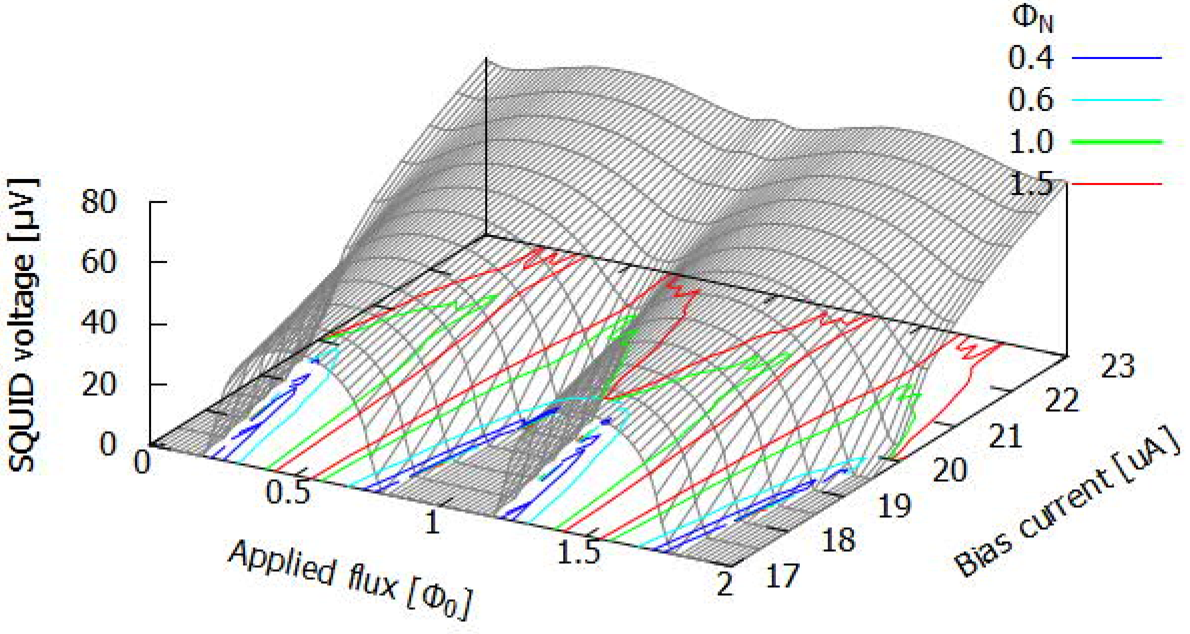
Voltage biased SQUID characteristics



Flux noise (with current bias)

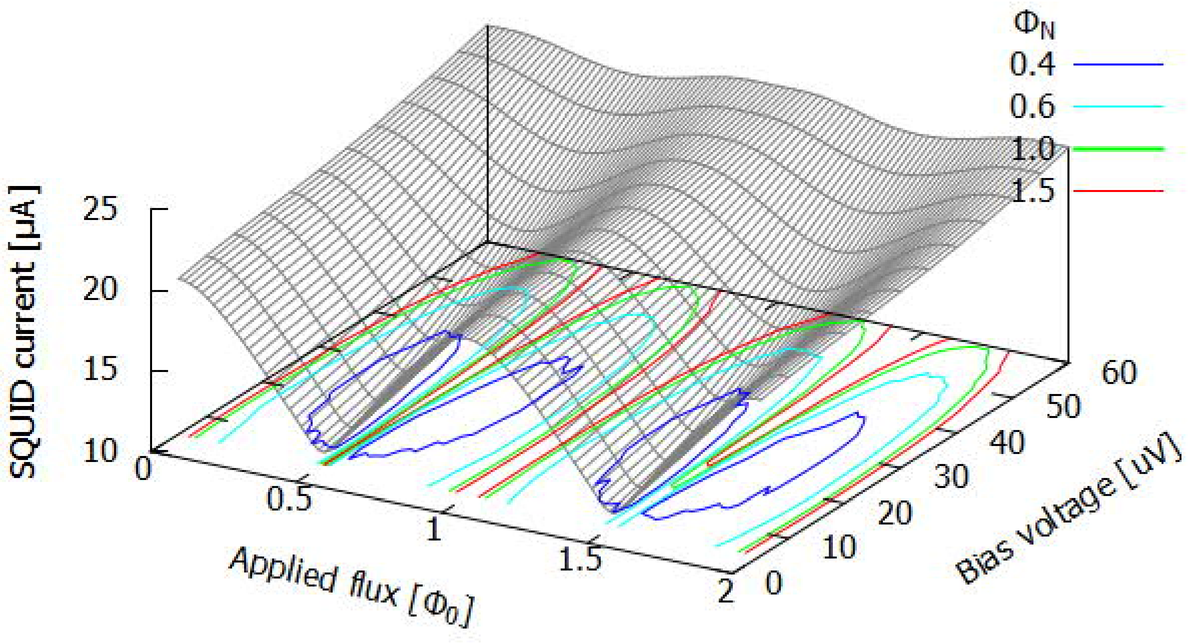
VTT

In units of $\mu\Phi_0$ / $Hz^{1/2}$

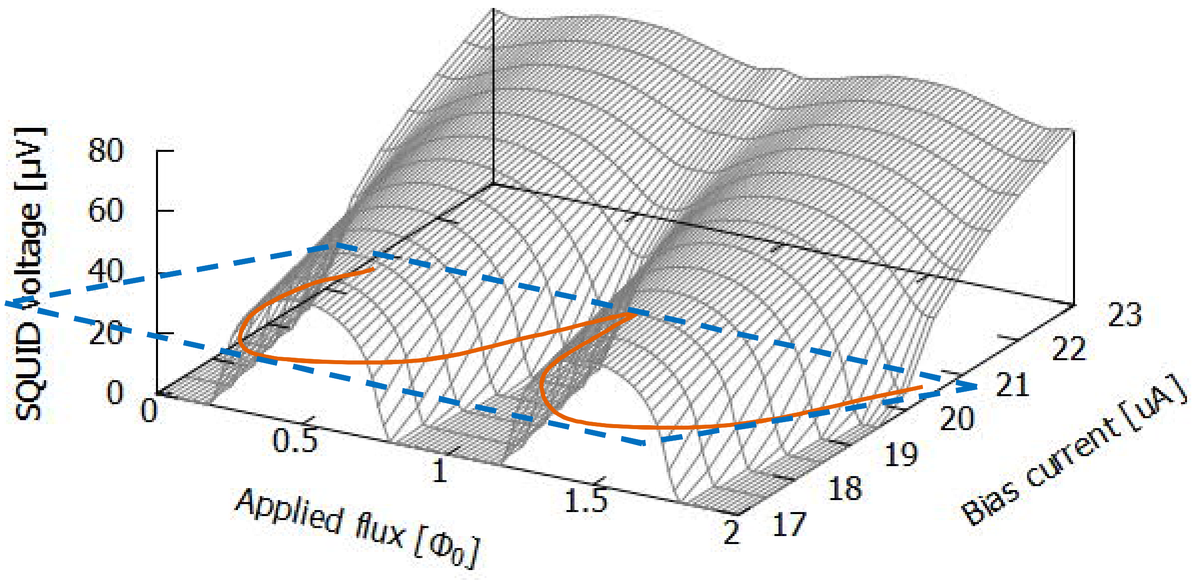


Flux noise (with voltage bias)

In units of $\mu\Phi_0$ / $Hz^{1/2}$

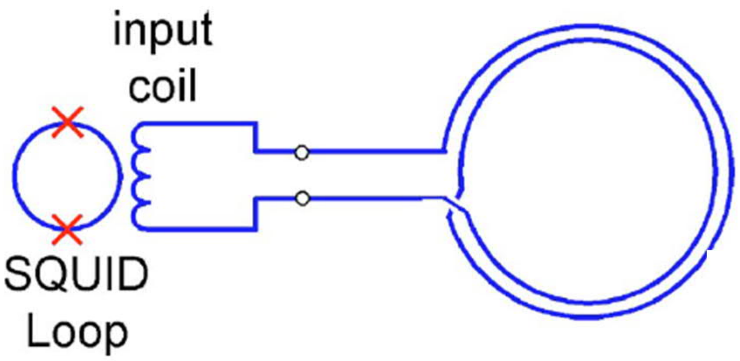


Matched bias

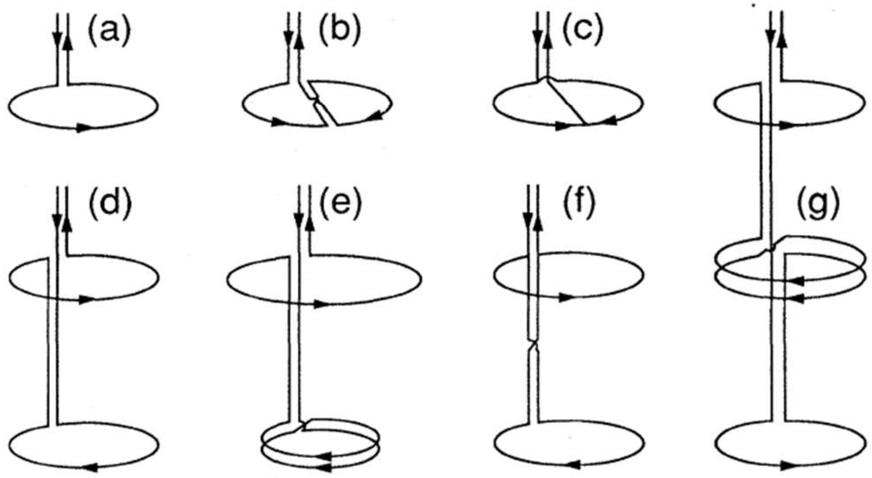


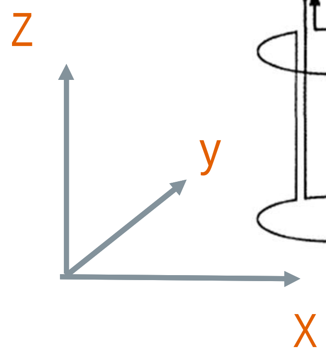
Load line:
0 Ω – voltage bias
∞ Ω – current bias
SQUID R_D – matched bias

Dc SQUID as a magnetometer

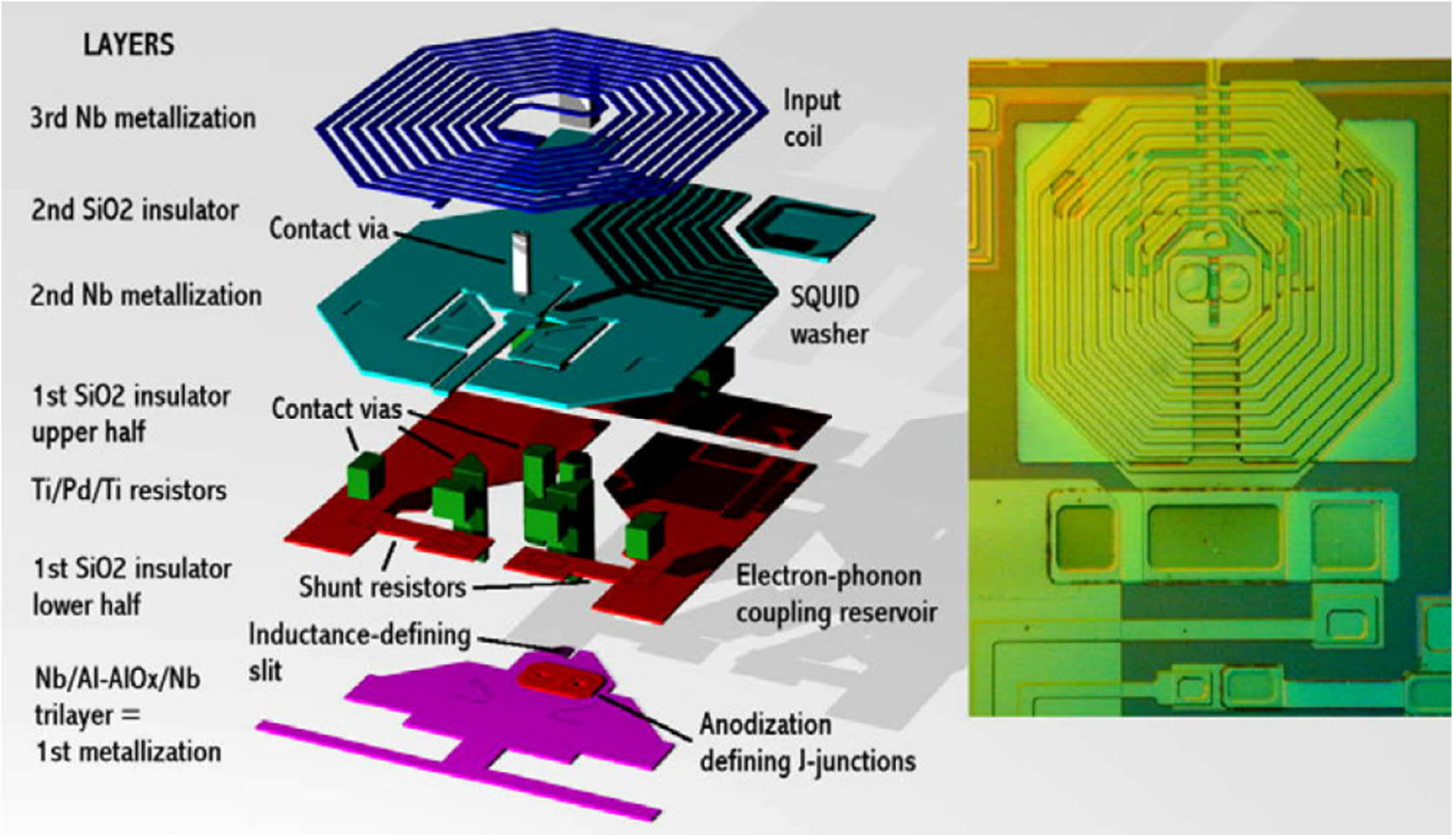


Various pickup loops: magnetometers and gradiometers



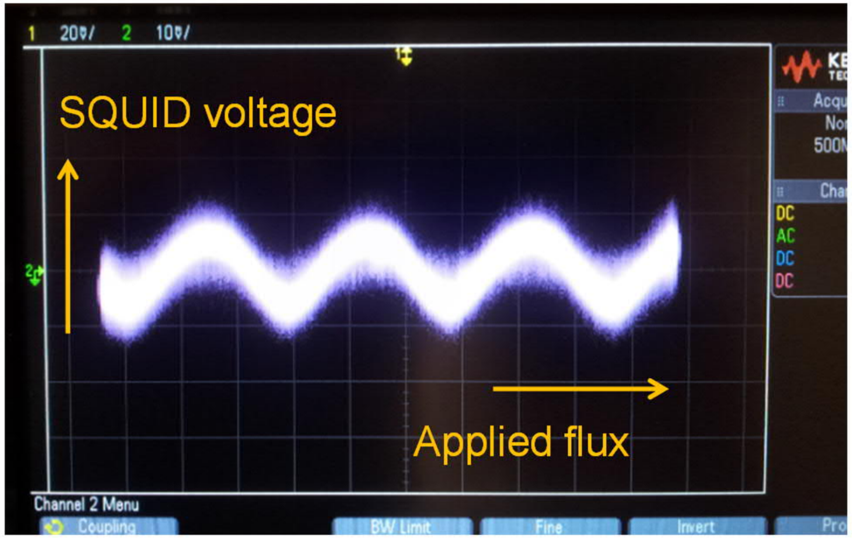


The thin-film stack of a washer SQUID

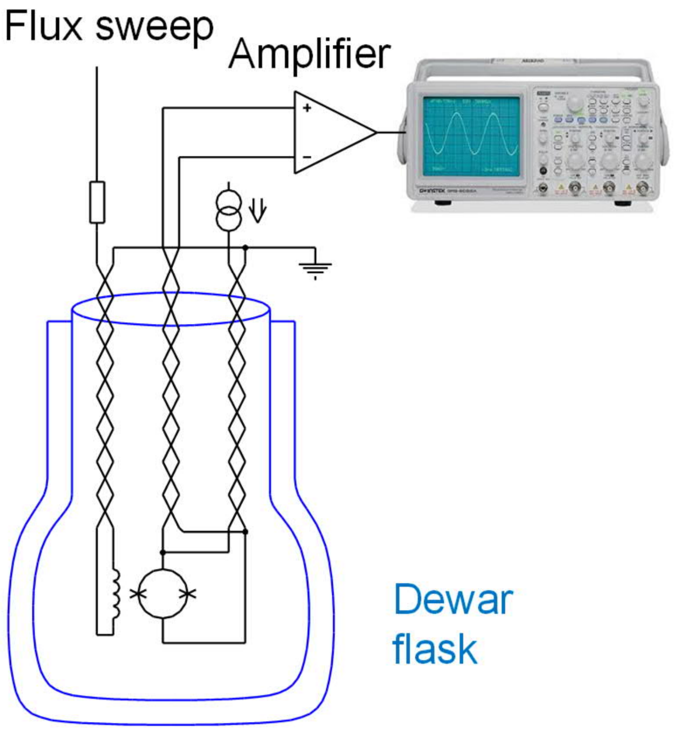


SQUID readout in practice

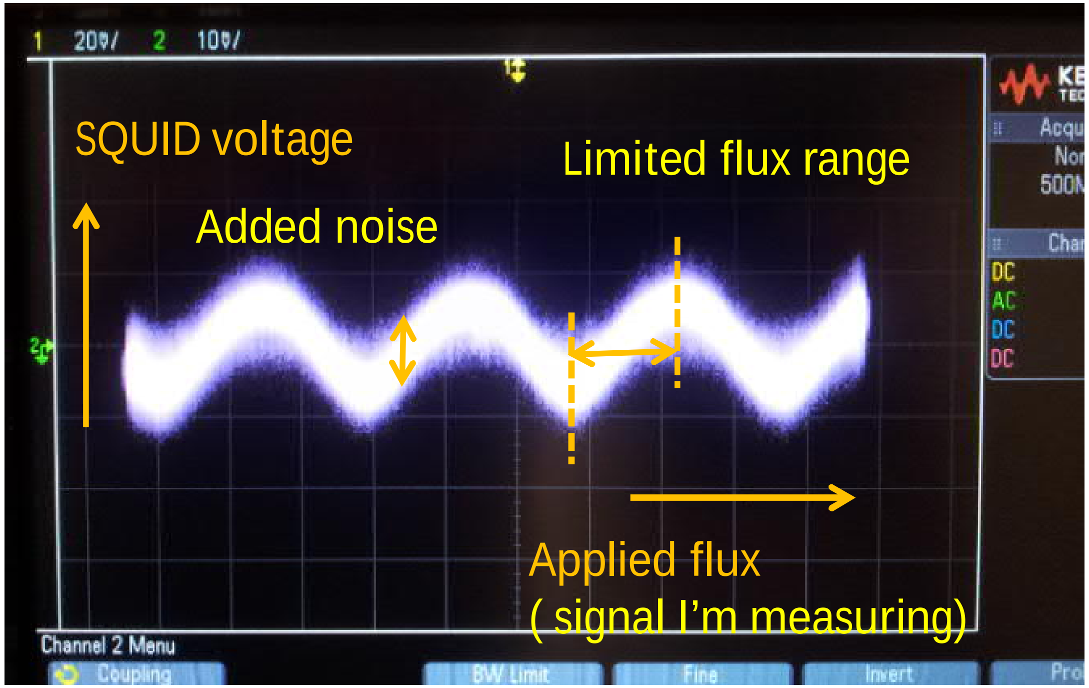
What a SQUID flux characteristics may look like in a real experiment



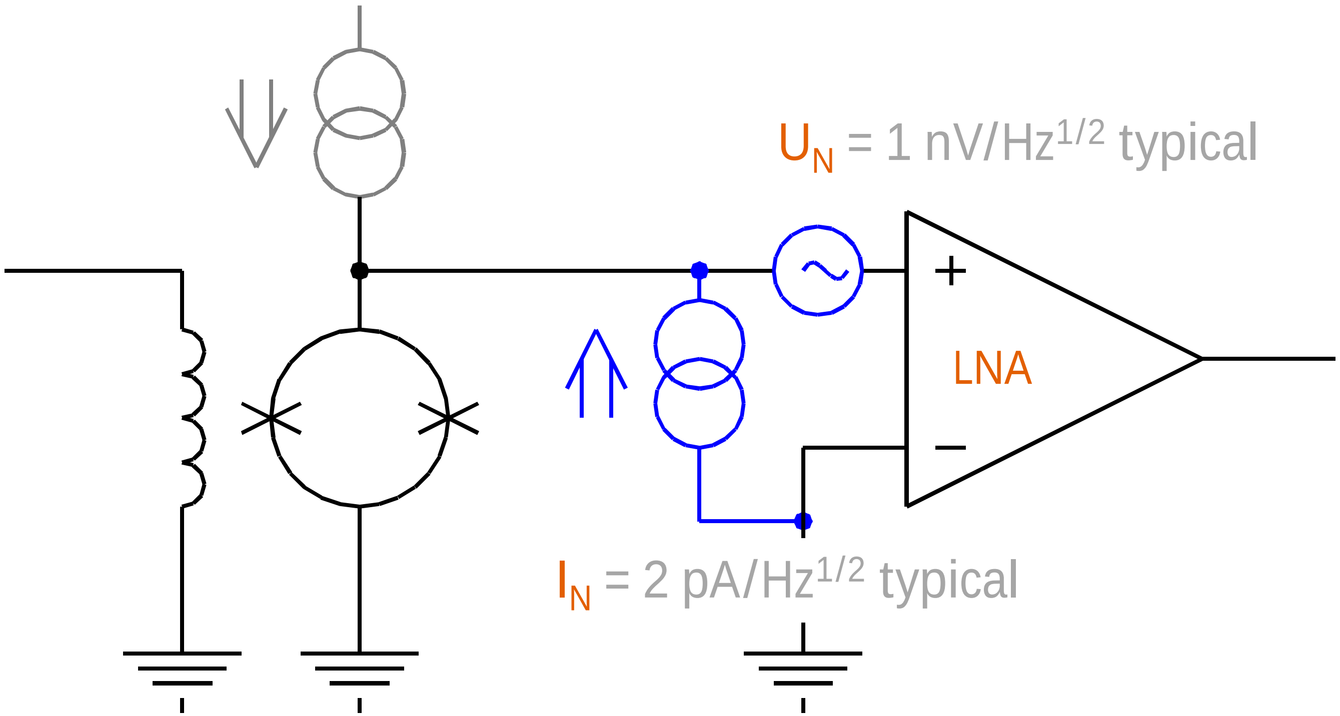
20151214_132039.jpg



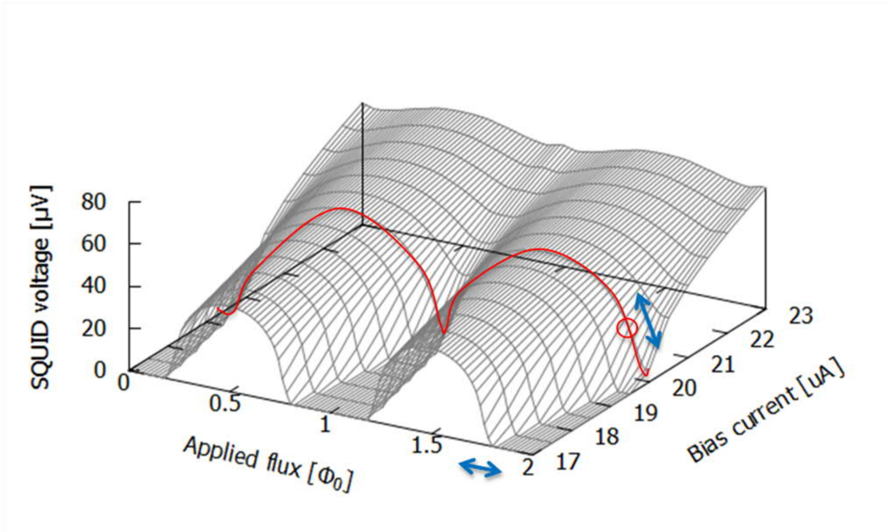
Dc SQUID readout in practice

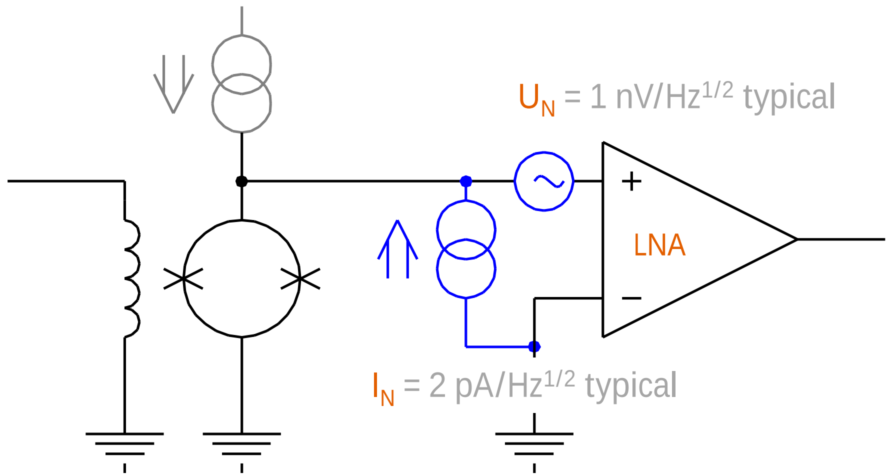


Low Noise Amplifier (LNA) noise



Voltage noise at SQUID output





SQUID $\Delta V \approx 60\ \mu V$

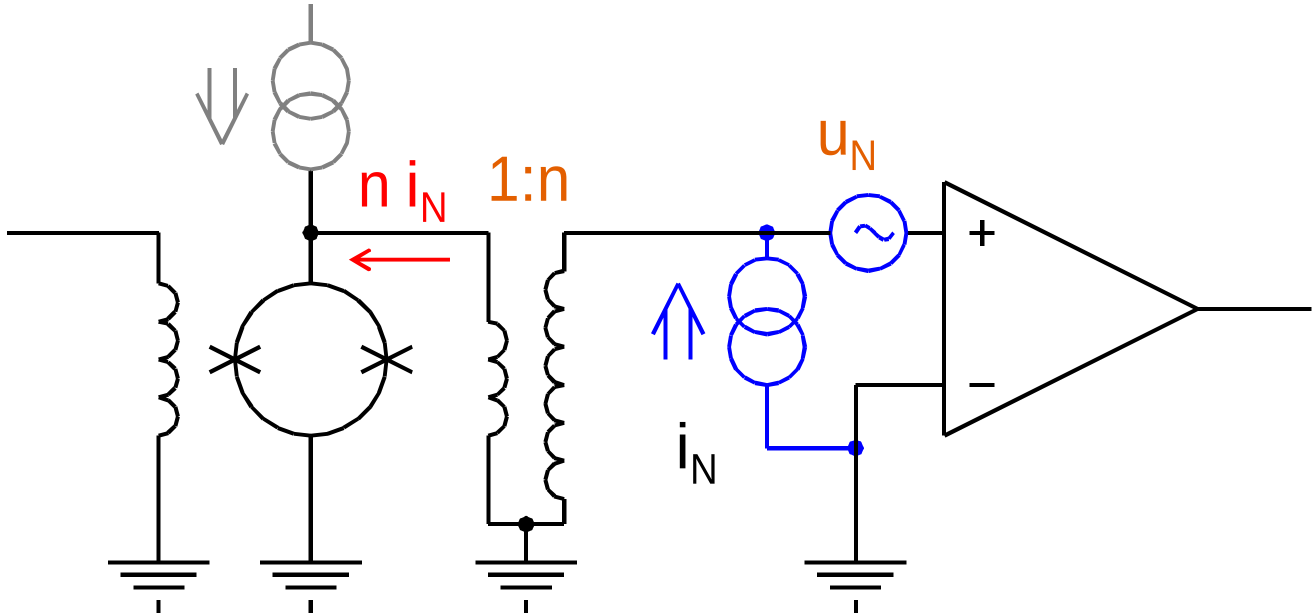
$dV/d\Phi \approx 200\ \mu V/\Phi_0$

SQUID flux noise $\Phi_N \approx 0.5\ \mu\Phi_0$ / Hz$^{1/2}$

$\Rightarrow$ Voltage noise at SQUID output 0.1 nV / Hz$^{1/2}$

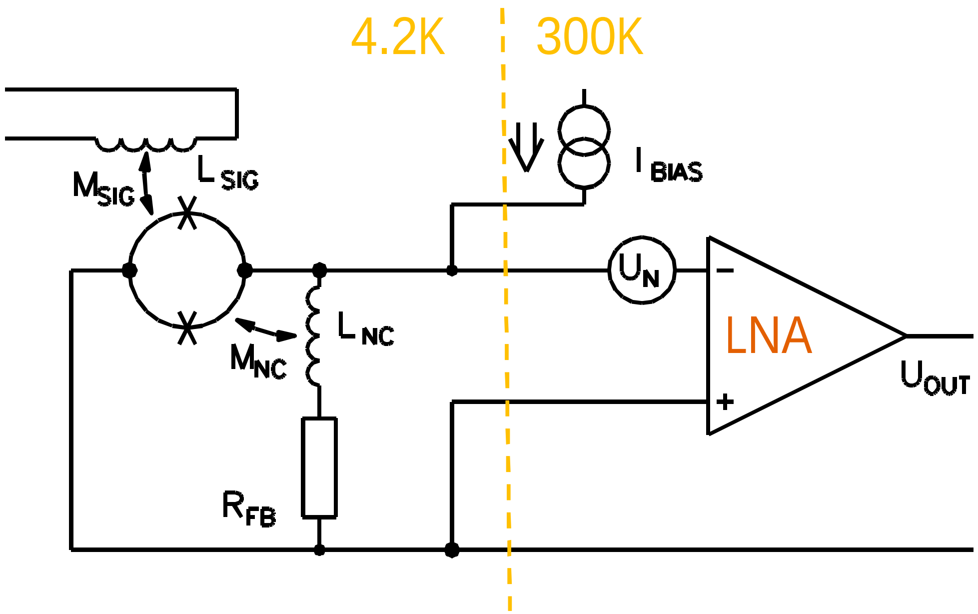
$\Rightarrow$ LNA noise dominates (always, actually...)

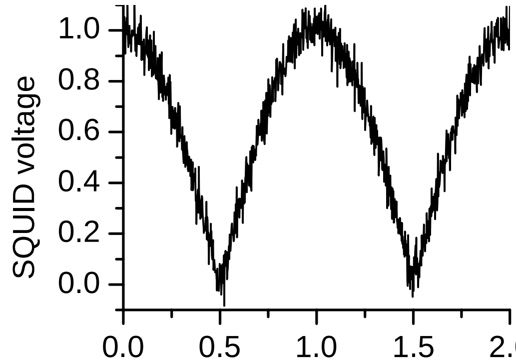
Use transformer, to increase dV/dΦ ?



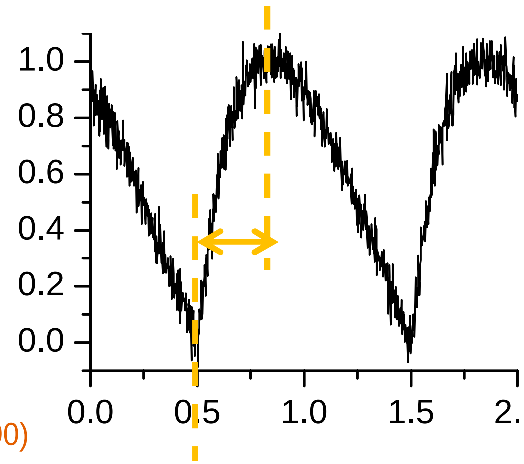
- Must chop SQUID signal (transformer does not pass DC)
- Current noise contribution i_N increases
- Feasible at low frequency, and using FET LNA.
- Was used in old days, not very practical today

Positive feedback





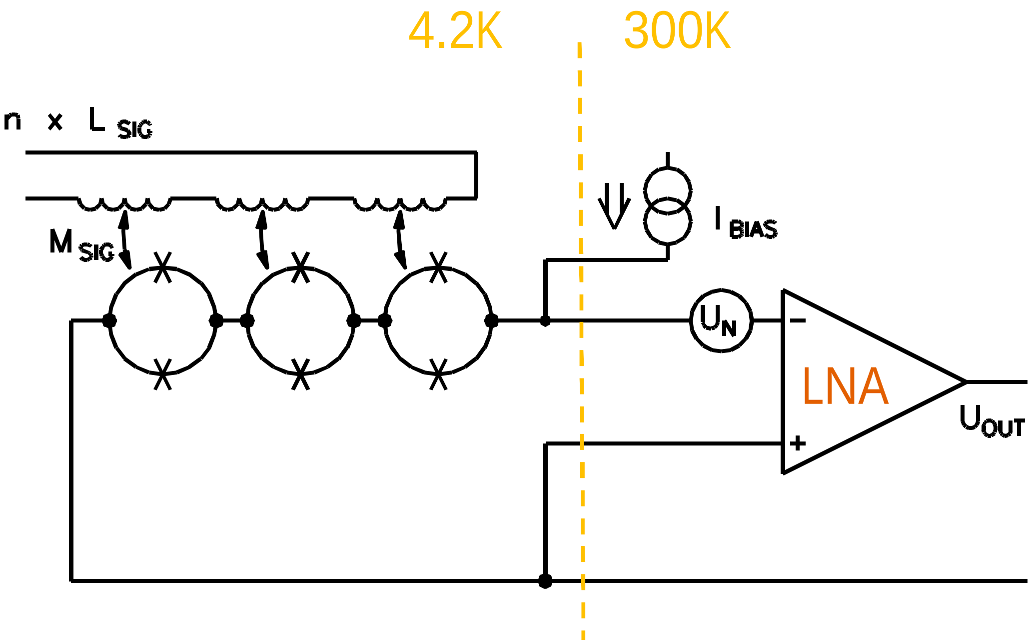
Plain current-biased SQUID



- Part of the amplified flux signal is fed back as additional flux
- Narrowed flux operating range, adverse effect on input inductance L_{SIG} (if L_{NC} and L_{SIG} coupled)

Drung et al, APL 57, 406 (1990)

SQUID arrays



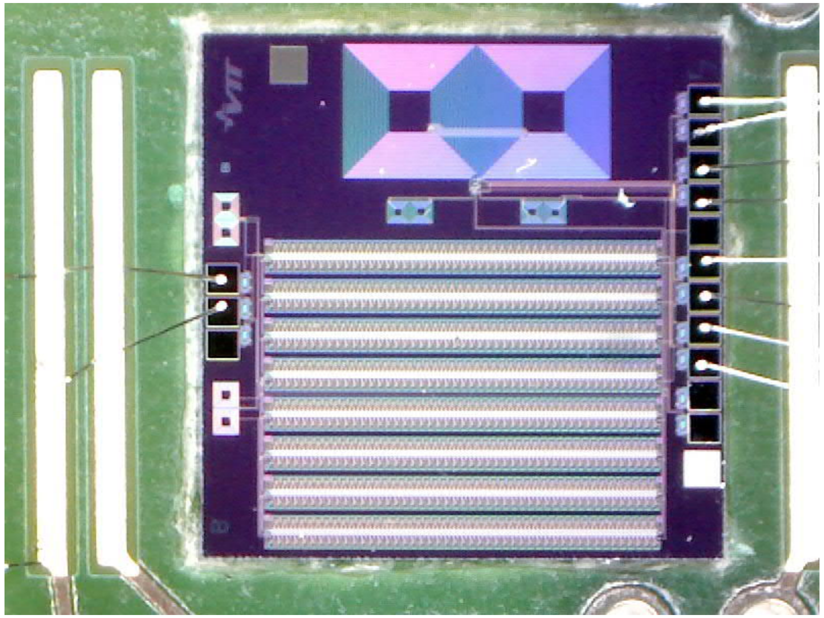
Welty and Martinis, IEEE Tran. Magn. 27 2924 (1991)

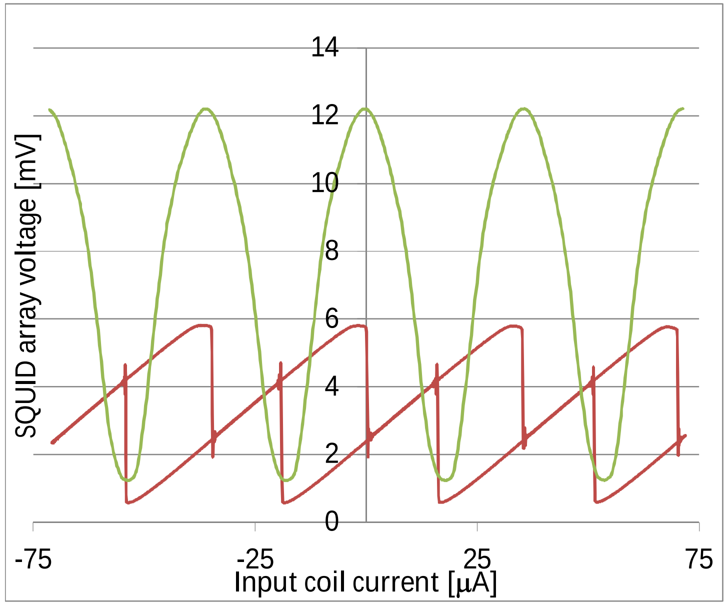
- N times the $dV/d\Phi$ of a single constituent SQUID
- Recall that LNA –limited flux noise $\Phi_N = u_N / (dV/d\Phi)$

  $\Rightarrow$ improves ~ N
- SQUID-limited flux noise improves ~ $\sqrt{N}$

  $\Rightarrow$ natural dynamic range improves ~ $\sqrt{N}$
  (DR: ratio of Φ_N to $\Phi_0/2$)
- Coherent SQUID operation is a must !

An example of a SQUID array

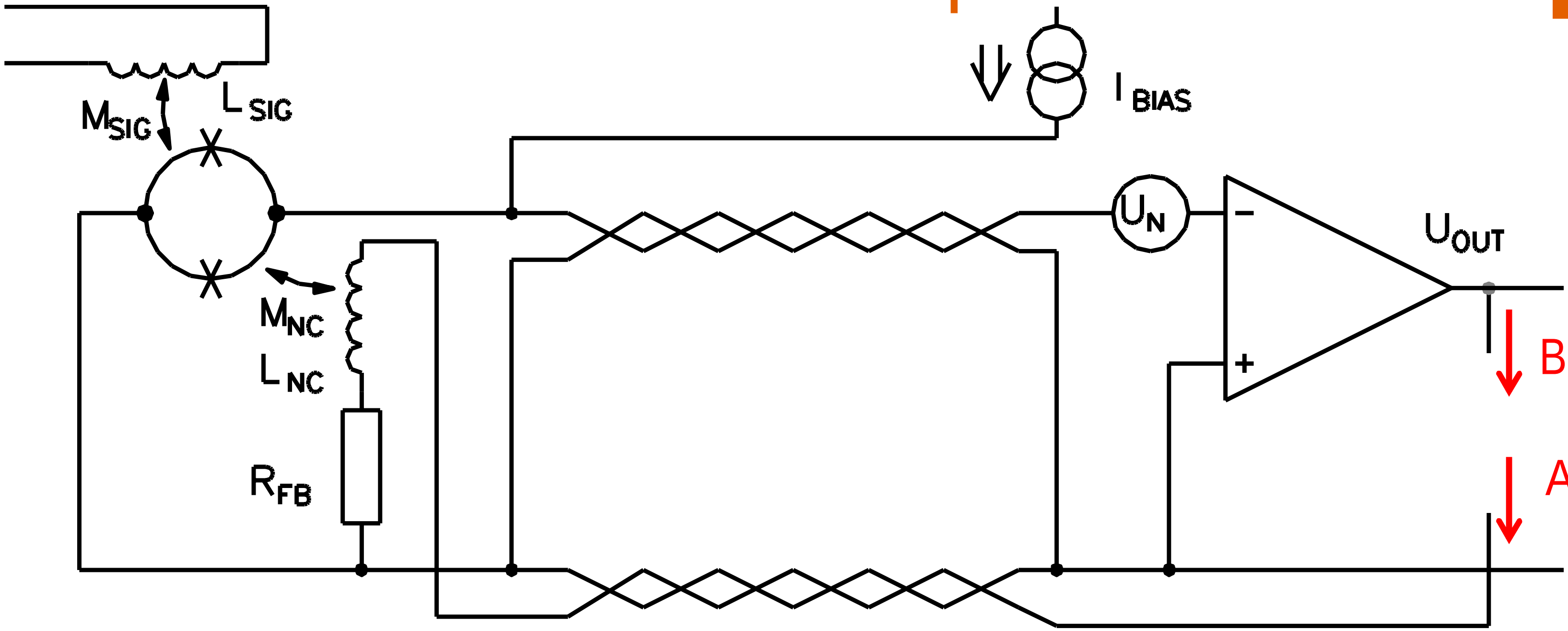




- 184-series 4-parallel SQUID array

$\Delta V \approx 11$ mV, $dV/d\Phi \approx 20$ mV/Φ_0

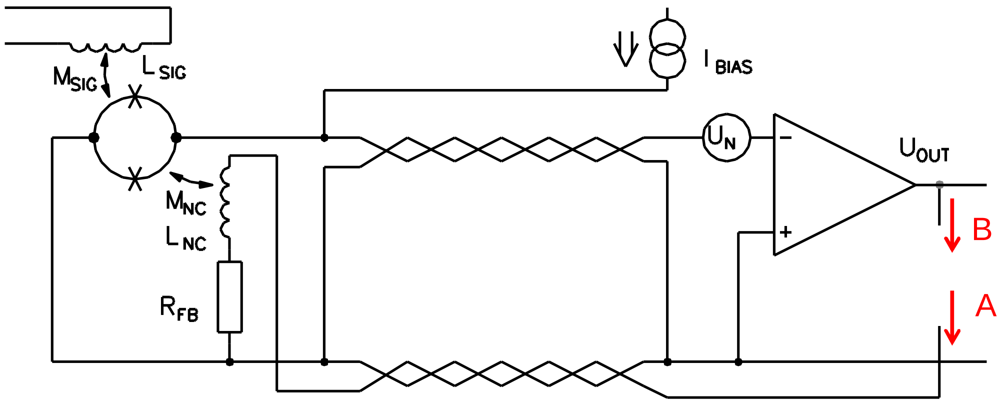
Flux locked loop

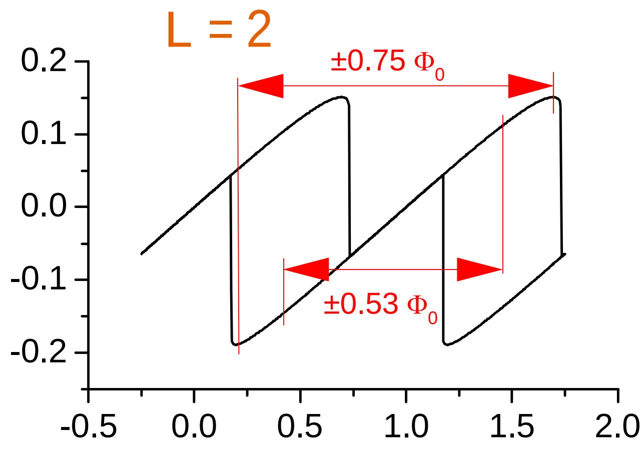


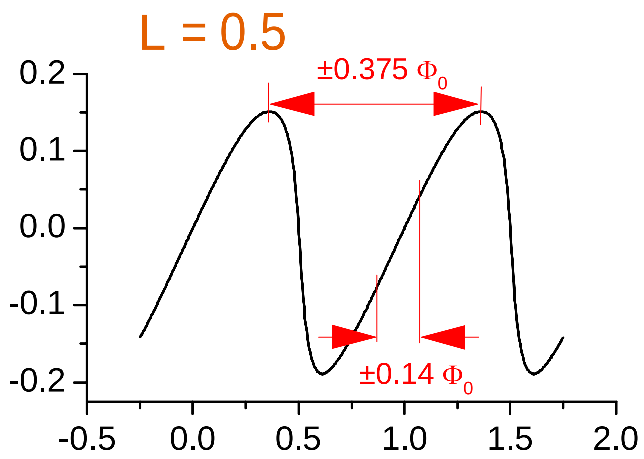
- Cut the loop open (any location will do)
- Inject test signal A
- Measure response B
- The ratio B/A is the loop gain L

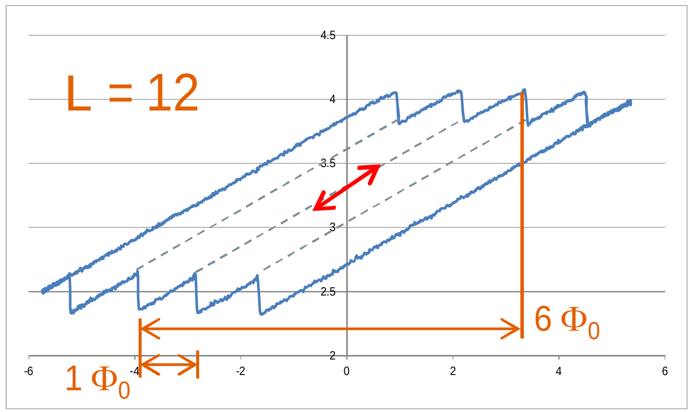
- Needs a loop filter A→B (for stability)
- Just put an integrator there (for more, see http://stacks.iop.org/SUST/16/1320)

Flux locked loop

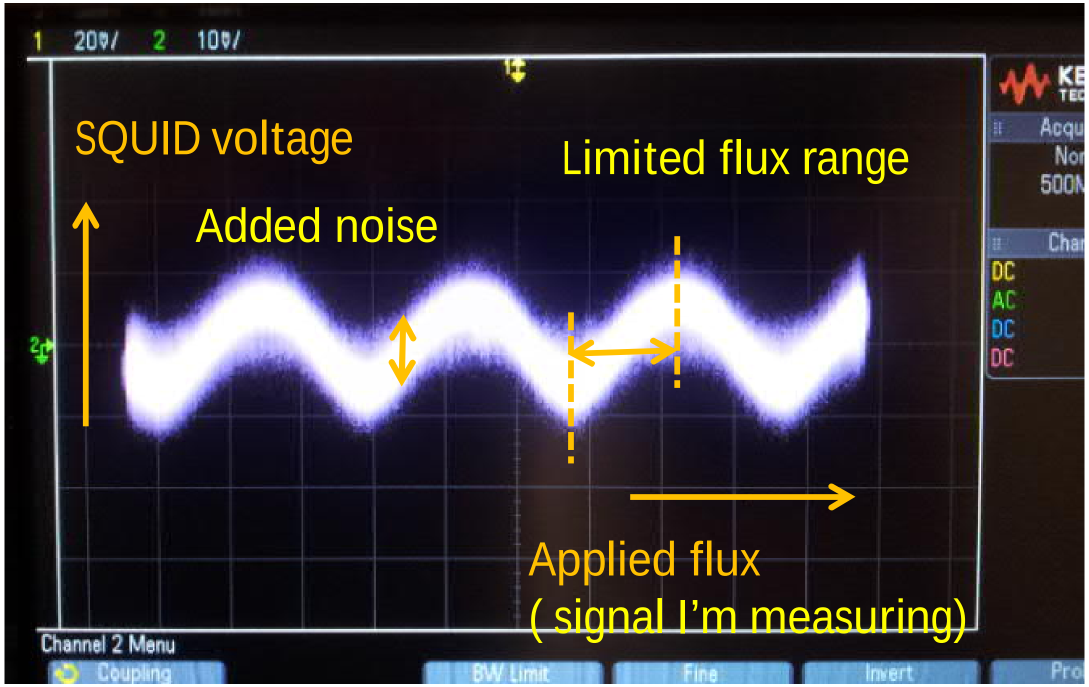








With positive feedback (or array SQUID) and FLL we have mitigated the problems



Thank you for listening

- Kirchoff laws with superconductors
- Weak link - flux quantization
- One-junction rf-SQUID - MISSING
- Two-junction dc-SQUID from SET – SQUID dualism
- Overdamped dc-SQUID, analytic solution
- Realistic dc SQUID characteristics
- A detour to 3-junction interferometer
- Different bias conditions
- Input coil, magnetometers and gradiometers
- SQUID readout, Flux-locked loop