SC cavity measurements

Walter Venturini Delsolaro
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Topics

- Introduction: measurements, errors, uncertainty
- Quantities of interest
    - Resonant frequency, operational bandwidth → beam requirement
    - Qo value→ heat load on cryogenics
    - Accelerating field Ea→ beam requirement
- RF basics
    - Transmission line theory; characteristic impedance, reflection coefficient
    - Standing waves, resonators
    - S parameters
    - RF components: cables and connectors, matched loads, attenuators, splitters, directional couplers
    - Measurement equipment: network analyser, spectrum analyser, power meters
- Superconducting RF cavity measurements
    - Qo and Ea measurement method(s)
    - Sources of uncertainty, evaluation and mitigation
    - Cool down precautions and effects
    - Multipacting
    - Field emission
    - Quenches
    - Q switches
    - Disentanglement of Rs “components”

Measurement

- Determining the/a value of a **measurand** (the **quantity** to be measured), implies:
  - **Definition** of the measurand
  - Definition of the measurement **unit** (the value of the measurand is a real number expressing the ratio between the measurand and the unit)
  - Definition of the **principle of measurement** (example: thermal contraction)
  - Definition of the **method** of measurement (example: by difference)
  - Definition of the measurement **procedure** (the sequence of actions to carry out the measurement)

Errors and uncertainty

- Error is an idealised concept: the “true” value of a quantity is metaphysical, sometimes even conceptually hard to define (think to the length of an object…)
- Exception: conventional true values (speed of light, Avogadro number, etc..)
- In science we believe “something is out there”, but “errors” cannot be known
- Traditionally errors are viewed as having two components: systematic and random. The distinction has to do with their behaviour in time. However:
- A “random” error in determining an influence variable or a calibration constant becomes “systematic” in the end result
- Known systematic errors should be eliminated by correction factors; these are in turn affected by “errors”
- **Uncertainty** reflects the lack of knowledge on the value of the measurand, of which the measurement results is only an estimate

Sources of uncertainty

- Incomplete definition of the measurand
- Imperfect realization of the definition
- Under sampling (non representative samples)
- Uncertainty on measurement of influence variables
- Unknown effect of all environmental conditions
- Operator bias, human error
- Errors on calibration standards
- Errors on parameters in the data reduction algorithm
- Numerical errors
- Approximations in the models
- "Unknown unknowns"

Standard uncertainty

- The standard deviation of the probability distribution of measured values around the “true” value(s)
- **Type A** and **Type B** uncertainties: the distinction refers to the method of deriving the uncertainty, conceptually they are the same, and both are accepted by the ISO norm
  - Type A is obtained with statistical procedures applied to repeated observations (**frequentist probability)**
  - Type B is obtained from an assumed probability density function based on the degree of belief that an event will occur (**subjective probability**)

Combined standard uncertainty

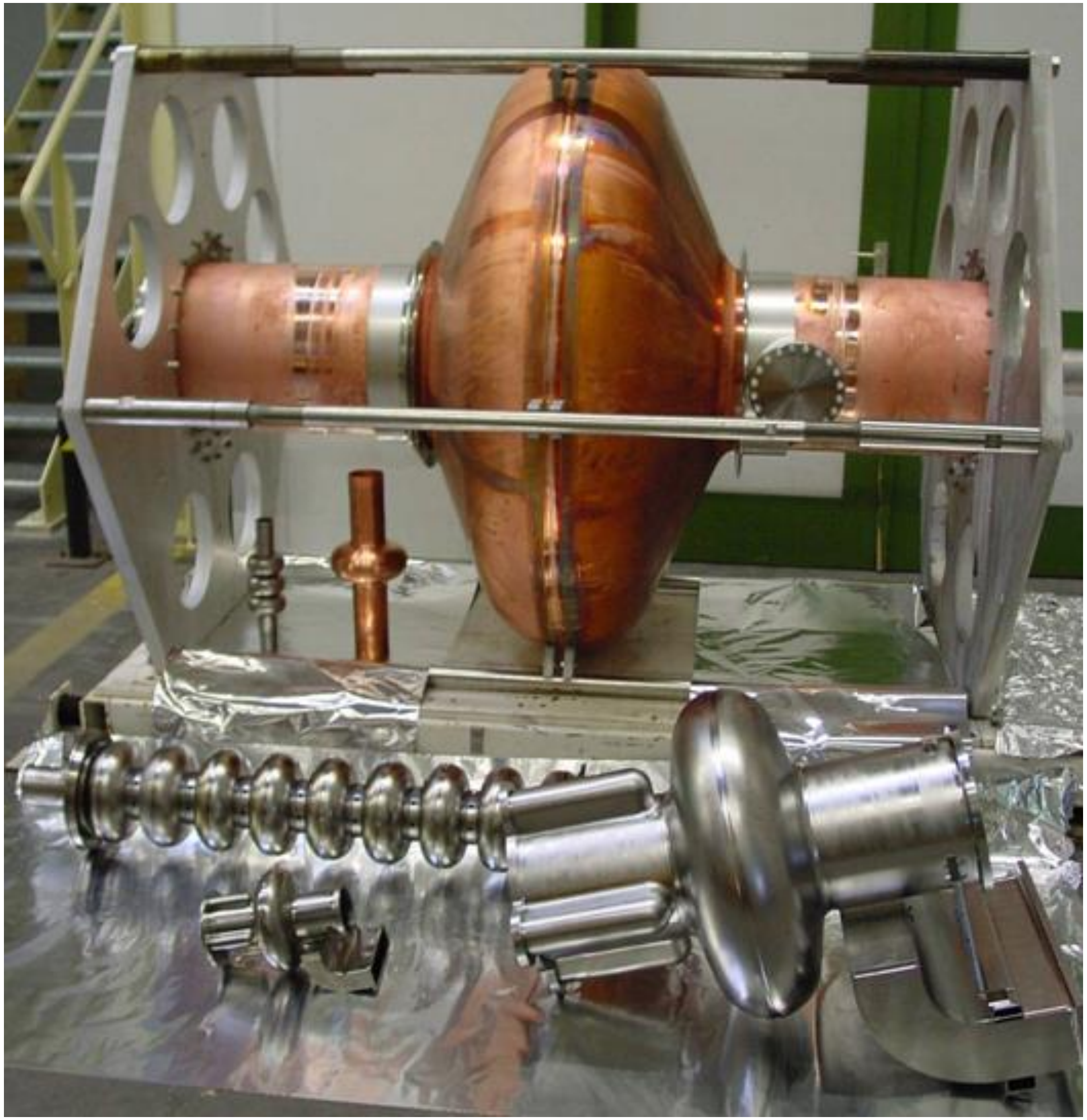
- The uncertainty on a derived quantity is assessed by the uncertainty propagation formula:

If $Y = f(X_1, X_2, \ldots X_n)$

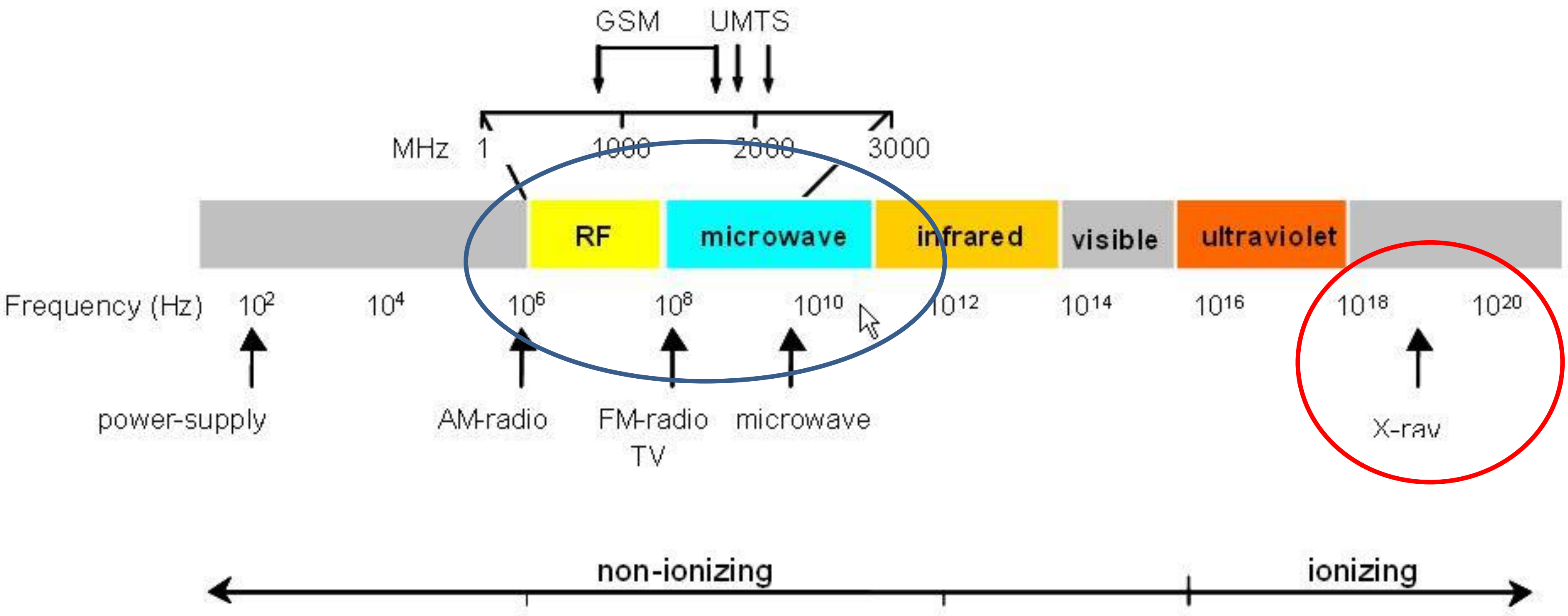
Then: $u(Y)^2 = \sum_{i=1}^{N} \left(\frac{\partial f}{\partial X_i}\right)^2 u(X_i)^2$

- This simple formula assumes no correlations between the input quantities, correlations are often present and must be accounted for
- The formula is a Taylor expansion: depending on the degree of non linearity of the model (the function f), it may be needed to include higher order terms
- Simple Monte Carlo calculations can be set up to compare with the analytical formula

Superconducting cavities



Electromagnetic spectrum



Frequency

Resonance frequency vs operating frequency

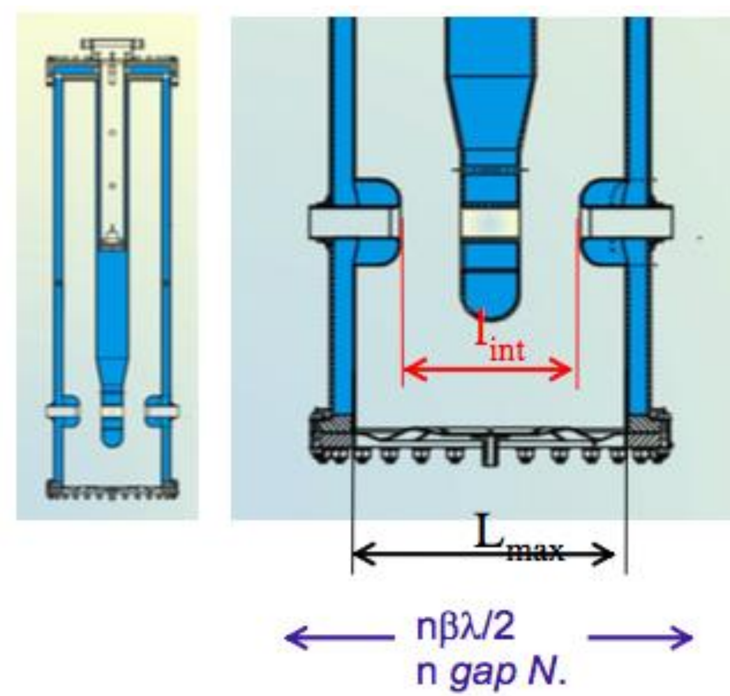
Accelerators dictate the operating frequency to the cavities, not vice versa: tuners shift the resonance frequency toward the operating frequency

Frequency control is an essential part of cavity production→ knowledge of frequency shifts all along, tuning range

Precise frequency not being critical for cavity testing, it can easily be forgotten. Keep in mind this is the simplest and yet essential “figure of merit” in cavity test

Accelerating field

- The energy gained by a charged particle traversing the cavity (on axis and with the right phase wrt to the cavity field) is the charge times an integral involving the E field over the transit
- The E field in the cavity changes along the path and in time (transit time factor).
- The accelerating field (aka gradient)is **defined** as the maximum possible voltage gain divided by a s length representing the acceleratir
- The observable from which the acc usually the power transmitted by t up antenna



Quality factor

$$Q = \frac{2\pi\ (Energy\ stored\ per\ cycle)}{Energy\ dissipated\ per\ cycle} = \frac{\omega\ (stored\ energy)}{Power\ loss}$$

Defined for every oscillator (also mechanical, optical, etc)

$$-\frac{dU}{dt} = power\ loss = \frac{\omega U}{Q}$$

The time constant of the damped oscillations is Q/ω

For linear systems (in electromagnetism – ohmic) , Q is a constant

In our cavity, the stored energy is $U = \frac{1}{2\mu 0} \int\int\int_V B^2\, dv$
And the dissipated power is $P_c = \frac{1}{2\mu 02} \int\int_S \mathrm{Rs}\, B^2\, ds$

Rs is the **surface resistance**, which encompasses the power losses due to the RF currents on the cavity walls

From Q_0(H) to R_s(H)

- Usually, using the definition of Q_0 , the surface resistance is factorized out of the integral representing the cavity dissipated power:

- $$Q_0=\frac{\omega \int\int\int_V B^2\, dv}{\int\int_S Rs\, B^2\, ds} = \frac{\Gamma}{R_s},$$

- where Γ is a geometric constant known from RF simulations.

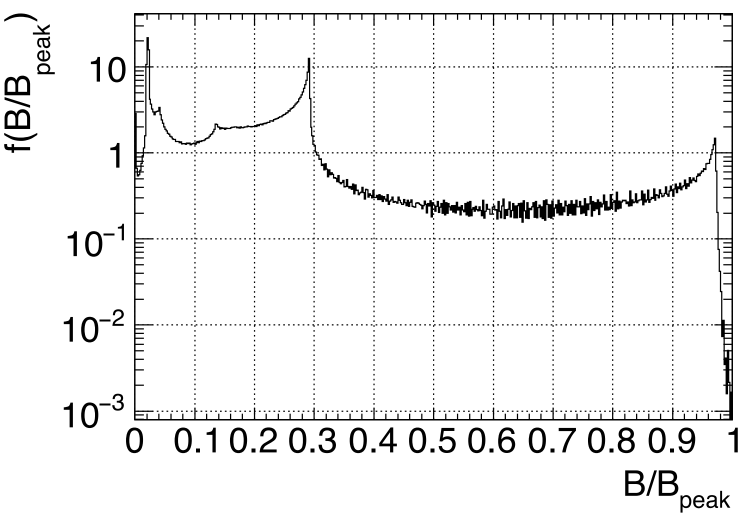
- Then $R_s(B)= \frac{\Gamma}{Q_0(B)}$. This is not correct because, if R_s(B), it can't be factorized out of the integral. Indeed, since the cavity has a field distribution from 0 to B_{peak}, R_s depends on s through B. The error is significant especially for non elliptical cavities.

- When fitting models of R_s versus B (Q slopes), it is possible to extract the exact field dependence using a numerical procedure (see for example D. Longuevergne paper at SRF13): The idea is to divide the cavity surface in parts where the field is approximately constant. Then make a change of variables in the integral from s to B, using the distribution function S(B) of the fractions of cavity surface were the magnetic field is between B and B+dB

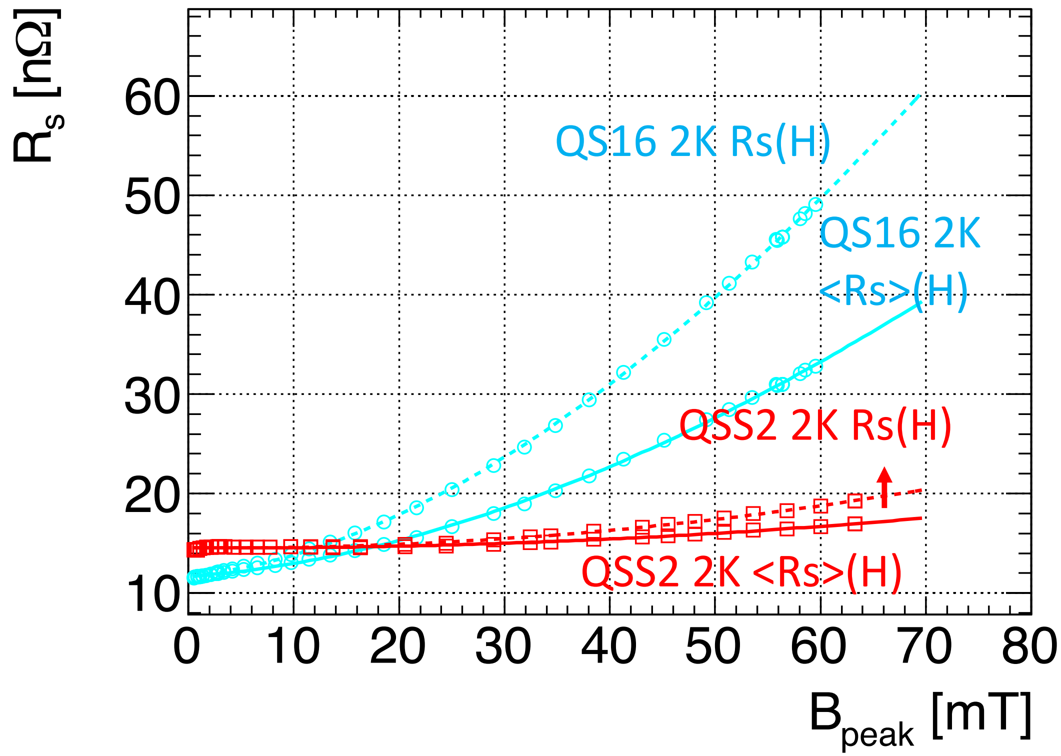
Example of application

If the local surface resistance uniform is Rs(x, y, z) = Rs[H(x, y, z)]

$$\langle R_s \rangle (B_{peak}) = \frac{\int_S R_s(x,y,z) H^2 dS}{\int_S H^2 dS} \to \frac{\int_0^{B_{peak}} R_s(H) f(H) H^2 dH}{\int_0^{B_{peak}} f(H) H^2 dH}$$



Fraction of B on the RF surface



The correction is significant when the Q slope is large, it can be neglected if the Q slope is small

How to decompose the surface resistance?

Traditionally: $R_s(T, B, \omega) = R_{BCS}(T, \omega) + R_{res}(B, \omega)$

$R_s(T \rightarrow 0) = R_{fl} + R_{res} = R_{fl} + R_{res0} + R_{res}(B)$

$R_{fl} = (R_{fl0} + R_{fl1}\ H_{rf})\ H_{ext}$

(see Physica C 351 (2001) 421-37)

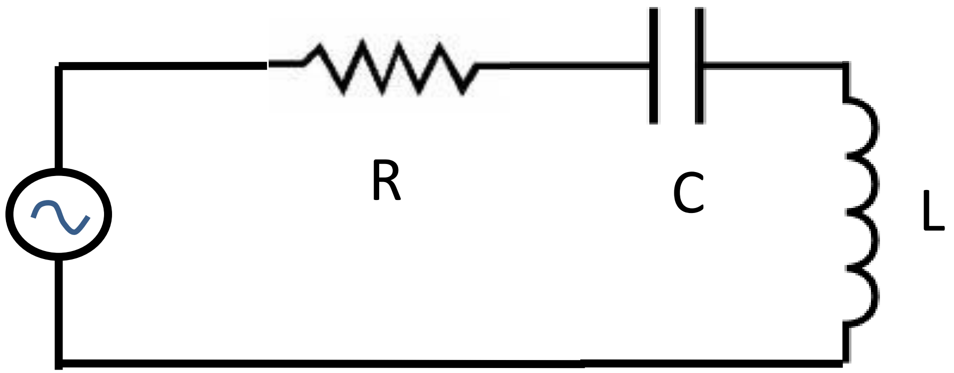
$R_{BCS}(T, \omega)$, from Matthis-Bardeen theory, is a low field approximation

→ Disentanglement at high field is not obvious!

→ Consequently, R_{res} is also only known at low field

RLC circuit, amplitude and phase responses

$$Ri + L\frac{di}{dt} + \frac{1}{C}\int idt = v(t)$$



$$v(t) = Re\{Ve^{j\omega t}\}$$

$$i(t) = Re\{Ie^{j\omega t}\}$$

$$Complex\ impedance \rightarrow Z(\omega) = \frac{V}{I} = R + j\omega L + \frac{1}{j\omega C}$$

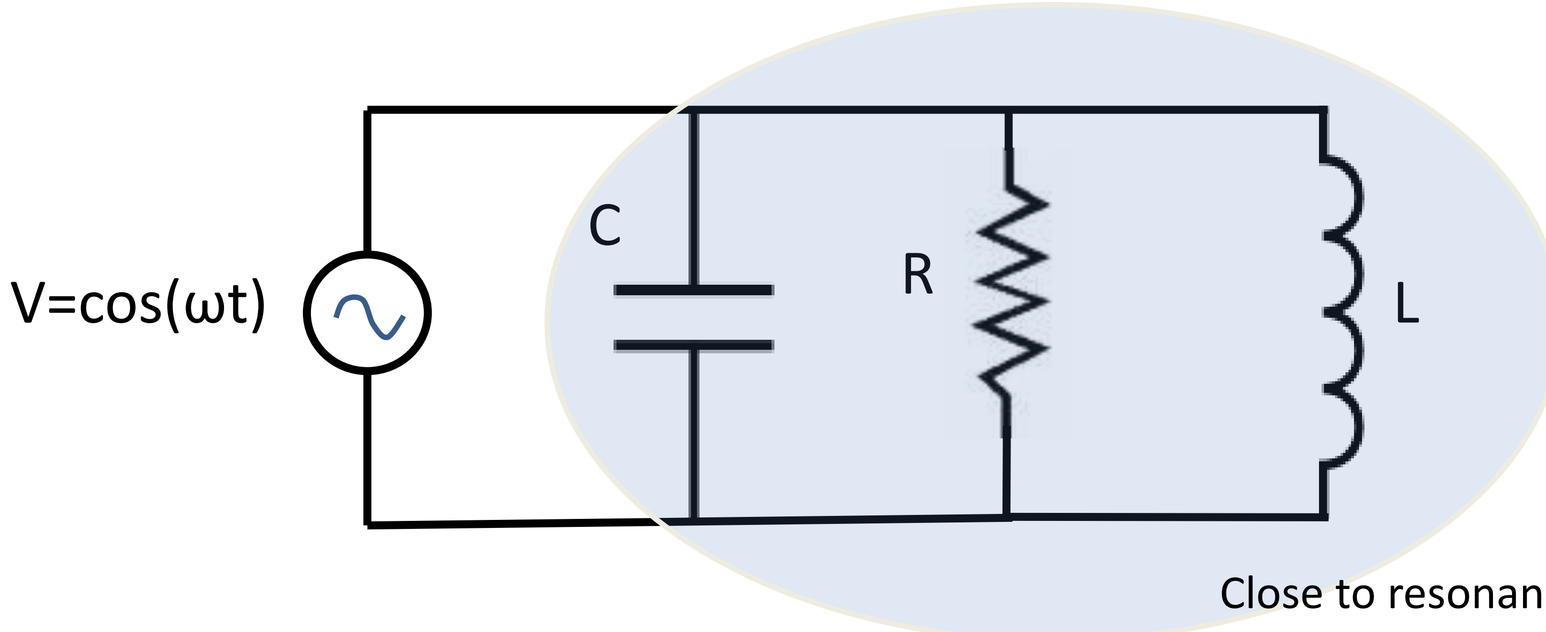
Circuit is voltage driven, the current is the response. The modulus of the current is maximum at resonance →

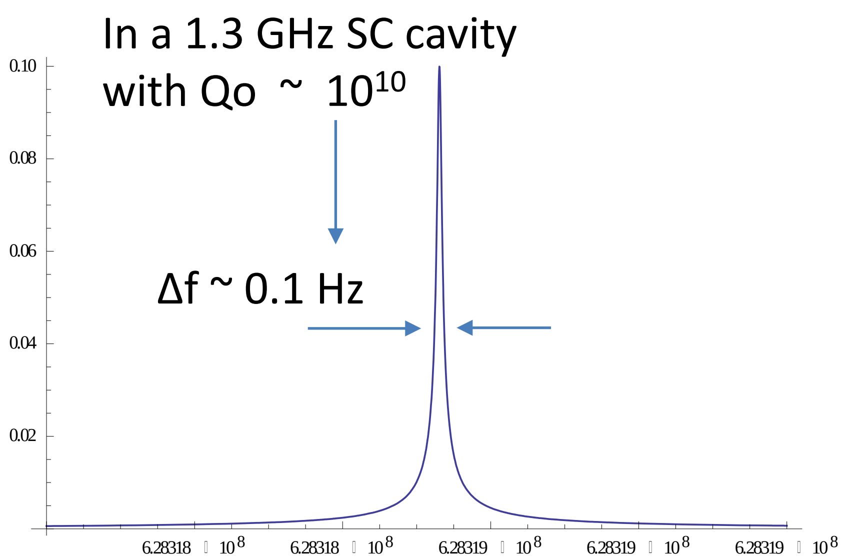
$$\omega_0 = \frac{1}{\sqrt{LC}}$$

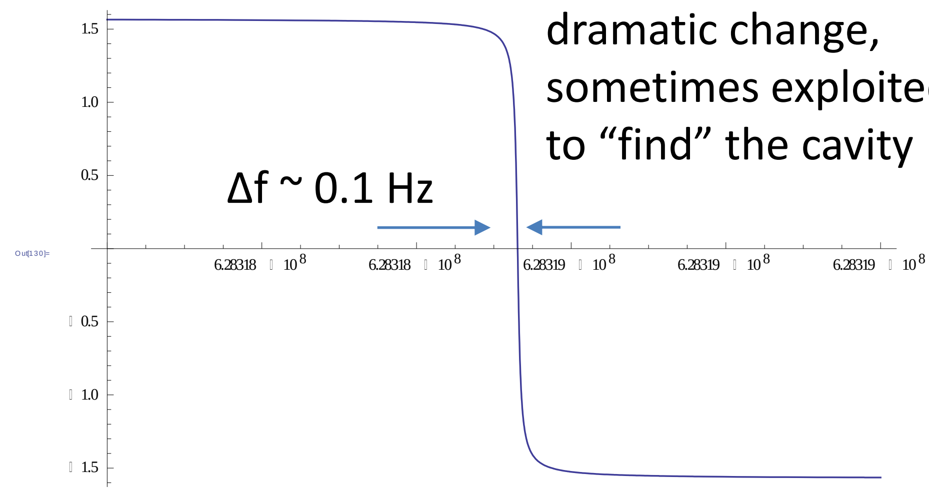
$$\omega - \omega_0 = \Delta\omega \qquad \rightarrow \ \omega^2 - {\omega_0}^2 \sim 2\omega\Delta\omega$$

$$Q = \frac{\omega_0 L}{R} \qquad \frac{\Delta\omega}{\omega_0} = \frac{BW}{2} \qquad BW = \frac{1}{Q}$$

Driven oscillator, amplitude and phase responses

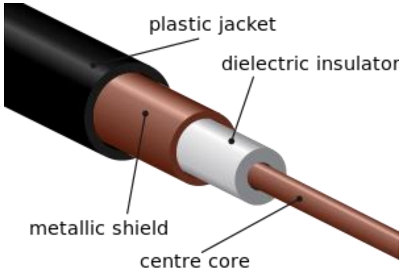




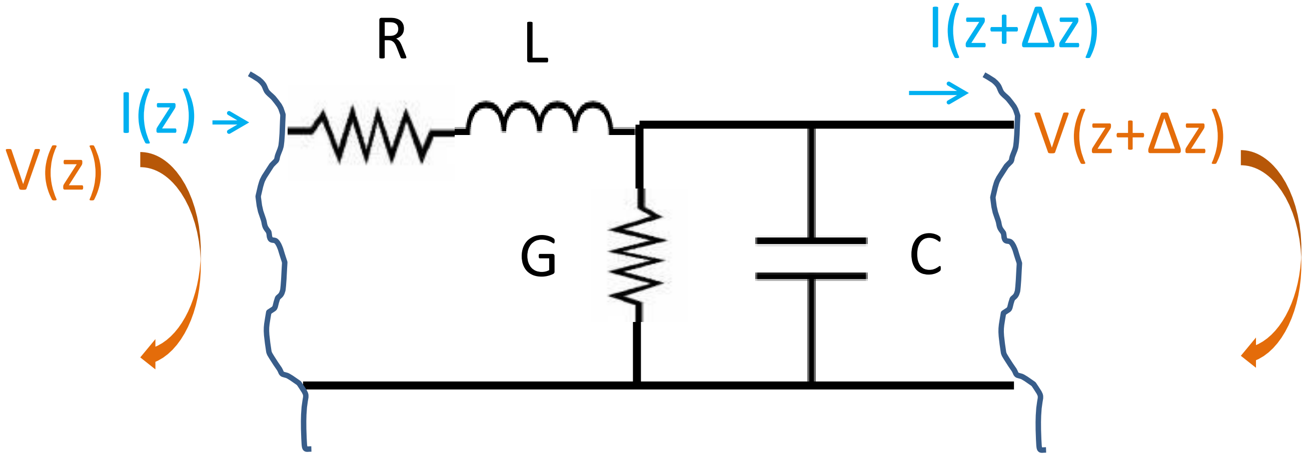


Close to resonance the phase undergoes a dramatic change, sometimes exploited to “find” the cavity

RF basics- transmission lines



- From lumped circuits to distributed parameters: telegraph equations



$(R + j\omega L)\, I(z)\, \Delta z + V(z+\Delta z) = V(z)$

$I(z) - V(z+\Delta z)\,(G + j\omega C) = I(z+\Delta z)$

$\rightarrow$

$$-\frac{\partial V(z)}{\partial z} = (R + j\omega L)\, I(z)$$

$$-\frac{\partial I(z)}{\partial z} = (G + j\omega C)\, V(z)$$

Telegraph equations

$$\frac{\partial V(z,t)}{\partial z} = -L\frac{\partial I(z,t)}{\partial t} - RI(z,t)$$

$$\frac{\partial I(z,t)}{\partial z} = -C\frac{\partial V(z,t)}{\partial t} - GV(z,t)$$

For lossless lines

$$\frac{\partial^2 V(z,t)}{\partial z^2} = -LC\frac{\partial^2 V(z,t)}{\partial t^2}$$

The same equation holds for the current: travelling voltage and current waves. In the other limit RC>>>LC wave propagation is suppressed the equation takes the form of a diffusion equation

Propagation constant

Sinusoidal oscillations:

$$-\frac{\partial \boldsymbol{V}(\boldsymbol{z})}{\partial \boldsymbol{z}} = (\boldsymbol{R} + \boldsymbol{j\omega L})\, \boldsymbol{I}(\boldsymbol{z})$$

$$-\frac{\partial \boldsymbol{I}(\boldsymbol{z})}{\partial \boldsymbol{z}} = (\boldsymbol{G} + \boldsymbol{j\omega C})\, \boldsymbol{V}(\boldsymbol{z})$$

Differentiate the first equation and plug in the second:

$$\frac{d^2V(z)}{dz^2} = k^2V(z)$$

The same equation holds for the current.

Travelling voltage and current waves
k is the complex propagation constant

$$k = \sqrt{(R + j\omega L)(G + j\omega C)} = k_r + jk_i$$

Line impedance

$$\frac{d^2V(z)}{dz^2} = k^2V(z)$$

$$V(z) = V^+e^{-kz} + V^-e^{+kz}$$

$$I(z) = I^+e^{-kz} + I^-e^{+kz}$$

Characteristic Impedance Z_0 of the line: ratio of voltage to current in the forward or in the backward wave.

$$I(z) = -\frac{1}{(R + j\omega L)}\frac{dV(z)}{dz}$$

$$I(z) = \frac{k}{(R+j\omega L)}(V^+e^{-kz} - V^-e^{+kz}) = \sqrt{\frac{(G+j\omega C)}{(R+j\omega L)}}(V^+e^{-kz} - V^-e^{+kz})$$

Impedance (cont.)

$$I(z) = \sqrt{\frac{(G+j\omega C)}{(R+j\omega L)}}\,(V^+e^{-kz} - V^-e^{+kz}) = \frac{1}{Z0}(V^+e^{-kz} - V^-e^{+kz})$$
$$= I^+e^{-kz} + I^-e^{+kz}$$

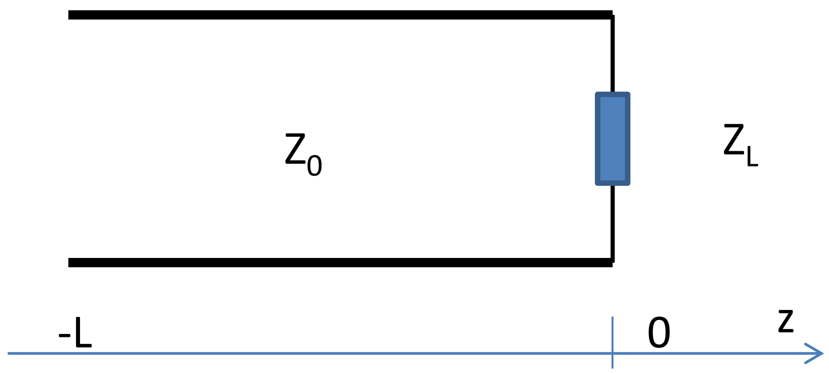
Thus: $Z_0 = \frac{(R+j\omega L)}{k} = \sqrt{\frac{(R+j\omega L)}{(G+j\omega C)}} = \frac{V^+}{I^+} = -\frac{V^-}{I^-}$

For a Lossless line (often good approximation) $Z_0 = \sqrt{\frac{L}{C}}$ is a real number.

and $k = \sqrt{(j\omega L)(j\omega C)} = -j\omega\sqrt{LC} = -\frac{j\omega}{v}$ is purely imaginary

Propagation without attenuation.

Transmission lines: reflection coefficient



The line impedance is Z_0 but at the load the ratio V/I must be Z_L so there must be a reflected wave

$$V(z) = V^{+}e^{-kz} + V^{-}e^{+kz}$$

Define the reflection coefficient as $\Gamma_0 = \frac{V^-}{V^+}$

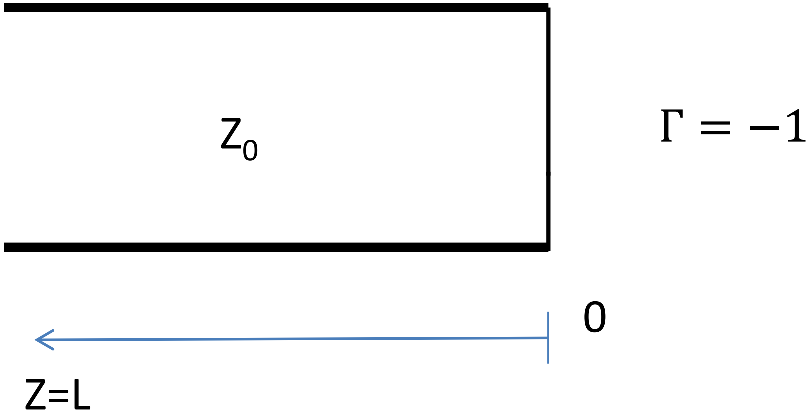
$$V(z) = V^{+}(e^{-kz} + \Gamma_0 e^{+kz})$$

$$I(z) = \frac{V^+}{Z_0}(e^{-kz} - \Gamma_0 e^{+kz})$$

$$Z(z) = Z_0 \frac{(e^{-kz}+\Gamma_0 e^{+kz})}{(e^{-kz}-\Gamma_0 e^{+kz}}$$

$$\Gamma_0 = \frac{Z_L - Z_0}{Z_L + Z_0}$$

Transmission lines: Standing waves



$V(z) = V^+(e^{-kz} + \Gamma_0 e^{+kz})$=$V^+(e^{-kz} - e^{+kz})$

Case of lossless line, k=jω/v
Change z→ -z; ω/v=β=2π/λ

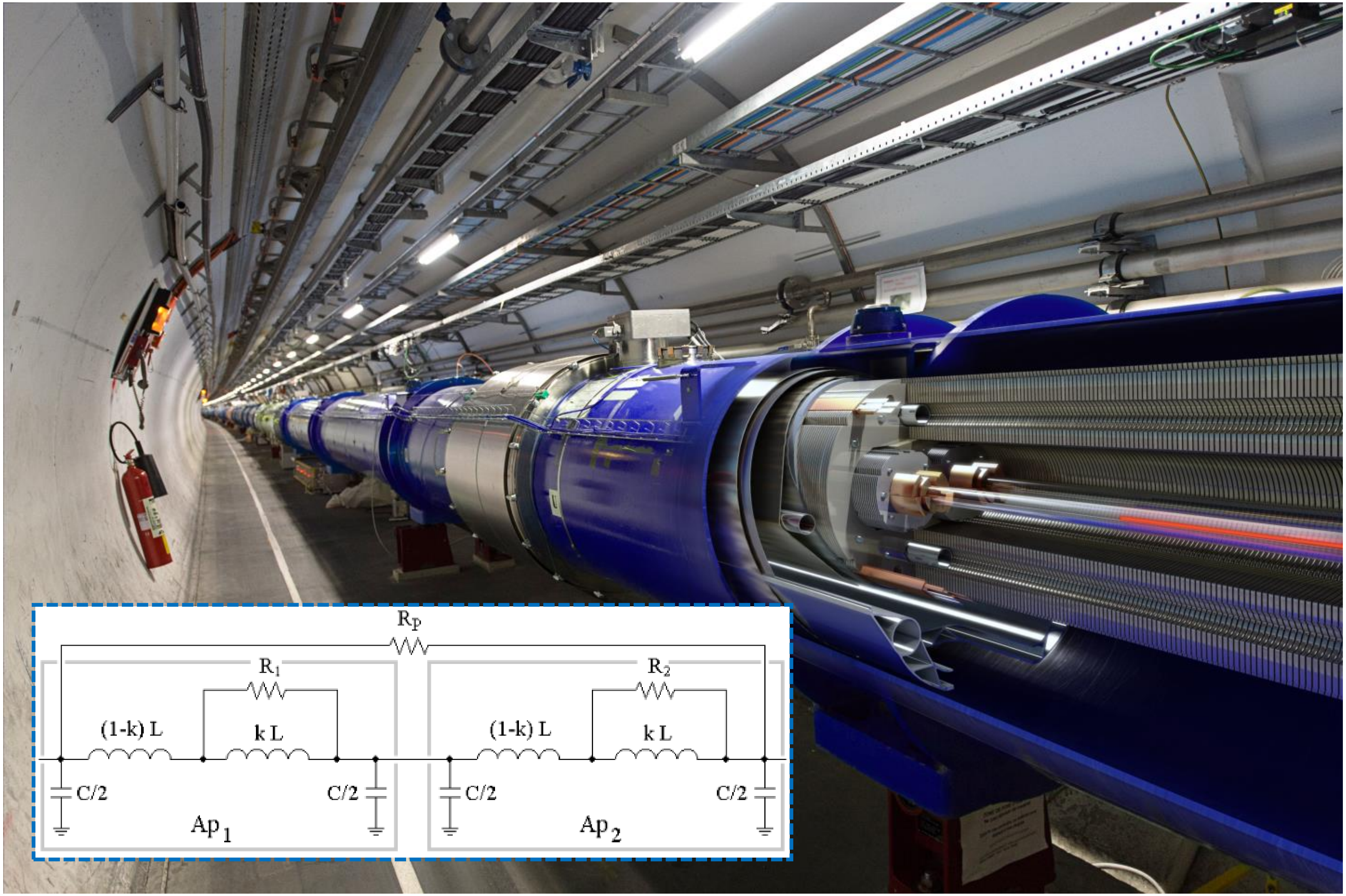
$V(z)$=$V^+(e^{+j\beta z} - e^{-j\beta z})$ ...use Euler's identity

$V(z)$=$2jV^+\sin(\beta z)$ ...go back to real: $v(z,t) = \mathrm{Re}\{\,V(z)e^{j\omega t}\}$

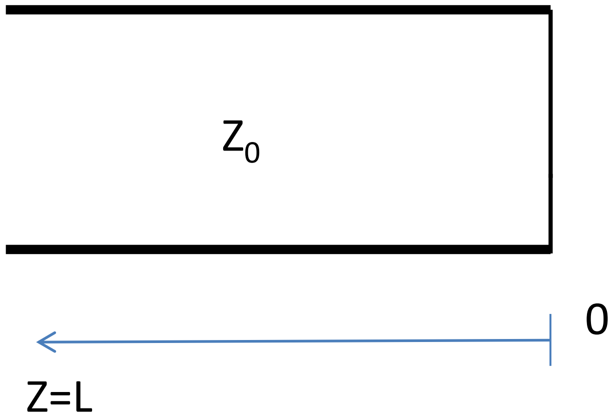
$$v(z,t) = \{2jV^+\sin(\beta z)e^{j\omega t}\} = Re\left\{2V^+\sin(\beta z)e^{j\omega t}e^{j\frac{\pi}{2}}\right\} = 2V^+\sin(\beta z)\cos(\omega t + \tfrac{\pi}{2})$$

Standing wave: time and space are decoupled, no propagation

A peculiar example of transmission line: the LHC dipole string



Standing waves, resonators



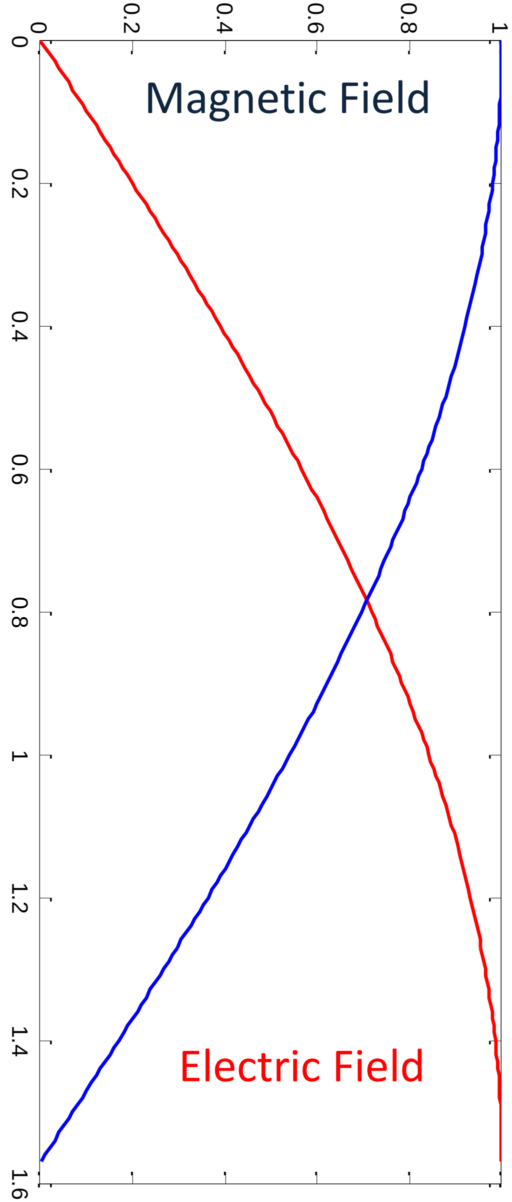
$$\Gamma = -1$$

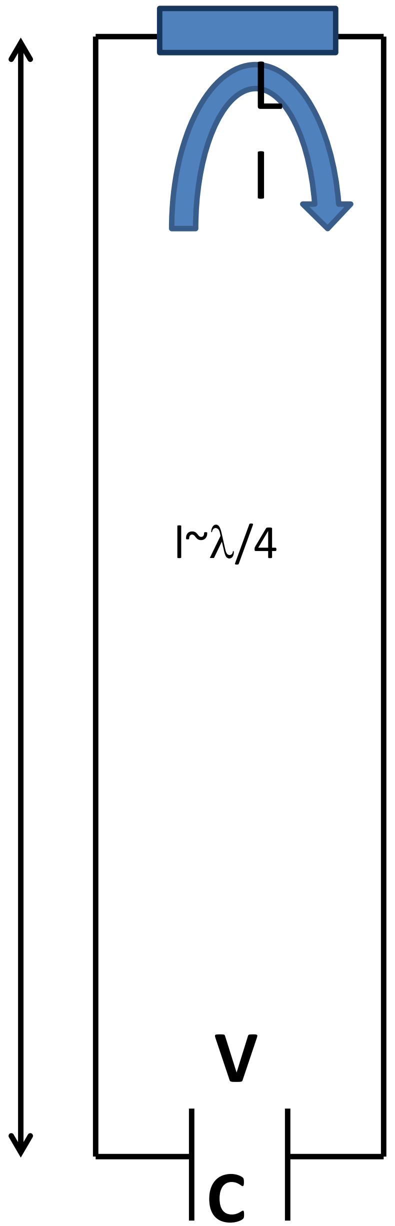
$$v(z,t) = 2V^{+} \sin\left(\frac{2\pi}{\lambda} z\right) \cos(\omega t + \frac{\pi}{2})$$

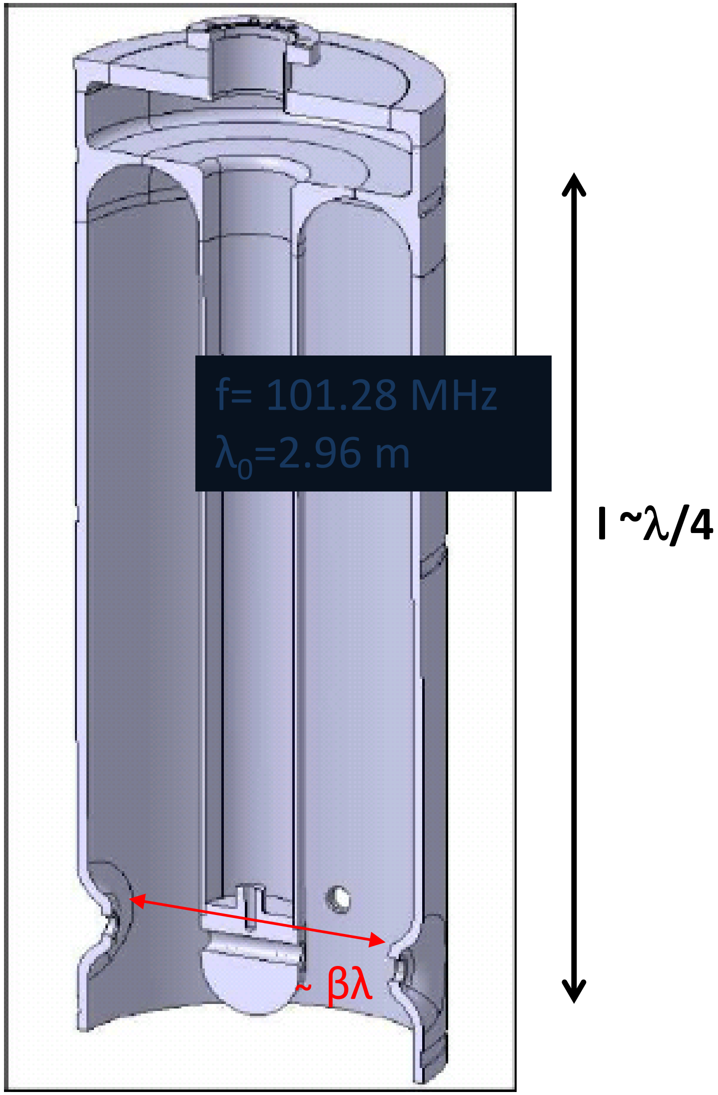
Can cut open the line at z=n $\frac{\pi}{2}$; (for n=1: half wave resonator)

Can also cut at λ/4 (quarter wave resonator)

Quarter Wave Resonator







Transformation of the reflection coefficient and standing wave ratio

Lossless line: $V(d) = V^{+}\left(e^{j\beta d} + \Gamma_0 e^{-j\beta d}\right) = V^{+} e^{j\beta d}\left(1 + \Gamma_0 e^{-j2\beta d}\right)$

$$V(d) = A(d)[1 + \Gamma(d)]$$

$\Gamma(d) = \Gamma_0 e^{-j2\beta d}$ Is the reflection coefficient at the distance d

Distance between two maxima of the reflection coefficient= λ/2

Define the standing wave ratio SWR $\text{SWR} = \frac{|V_{max}|}{|V_{min}|}$

Looking at the extremes of the modulus of V(d): $\text{SWR} = \frac{1 + \Gamma_0}{1 - \Gamma_0}$

Remember: defined only for a lossless line

S parameters for a 2 port network

- $$\begin{Bmatrix} b_1 \\ b_2 \end{Bmatrix} = \begin{bmatrix} S_{11} & S_{12} \\ S_{21} & S_{22} \end{bmatrix} \begin{Bmatrix} a_1 \\ a_2 \end{Bmatrix}$$

S_{xy}: power enters port x

Define: $a_1=V_1^+$, $b_1=V_1^-$, $a_2=V_2^+$, $b_2=V_2^-$ complex voltages

S_{11} Input port voltage reflection coefficient
S_{21} Forward voltage gain
S_{22} Output port voltage reflection coefficient
S_{12} Reverse voltage gain

$|S_{21}|$ is the scalar linear gain, that is $|V_2^- / V_1^+|$, the phase information is dropped.

The insertion loss is expressed in decibels, that is IL= $-20 \, \mathrm{Log}_{10}(|S_{21}|)$

Power units

Relative power levels are expressed in decibel

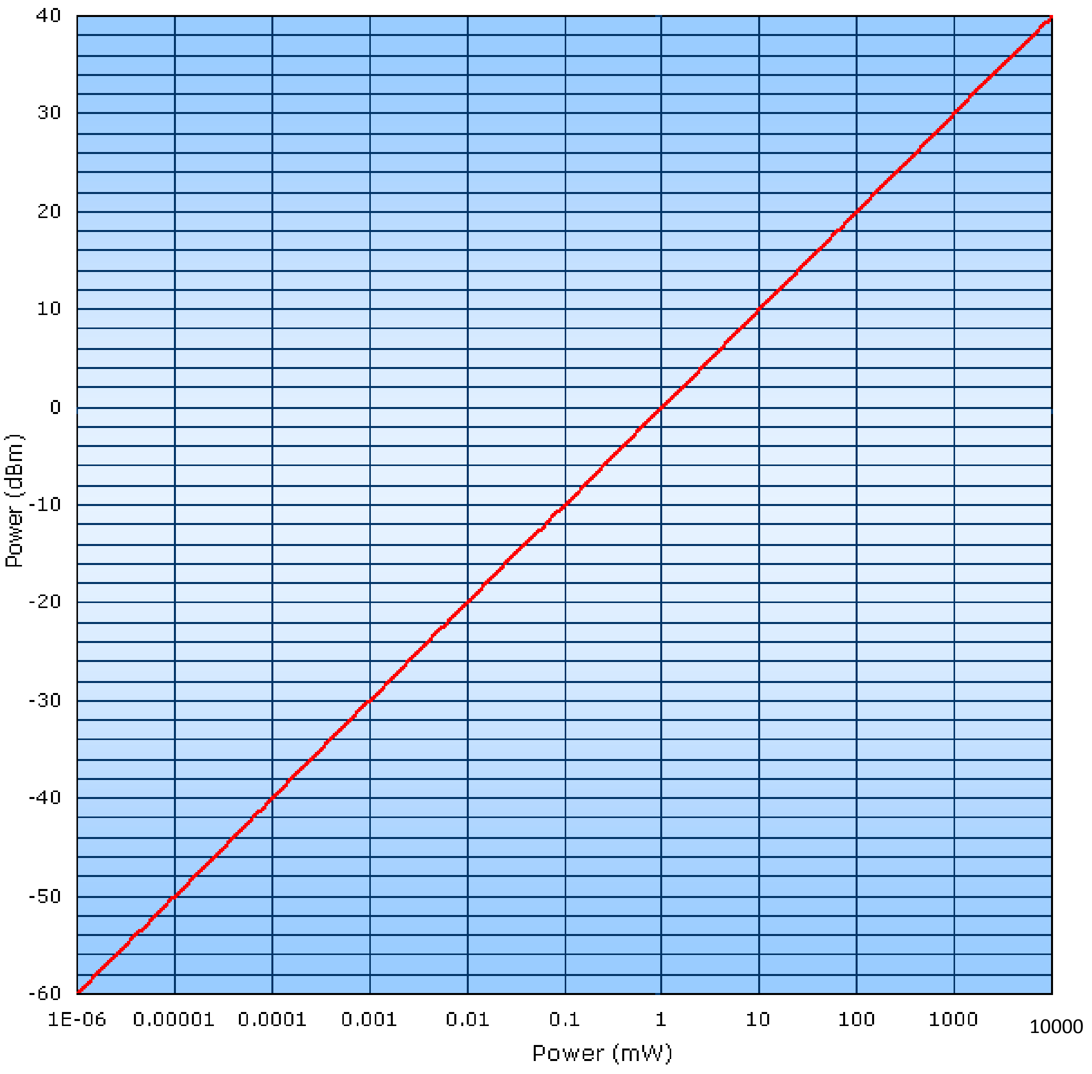
$P(dB)=10\ Log(P_1/P_2)$

Absolute power measurements often are expressed in dBm: 1 mW is taken as a power reference level

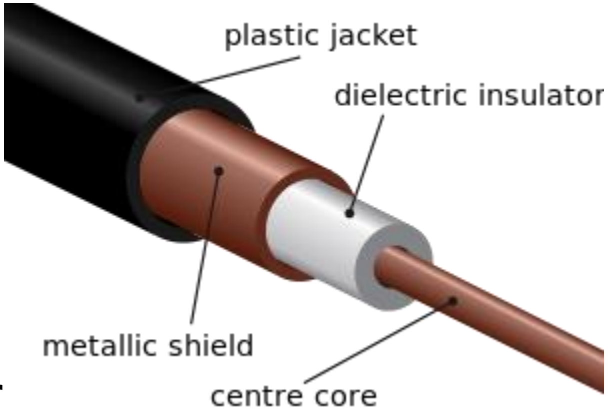
$P(dBm)=10Log(P\ in\ mW)$

$P(mW) = 10^{[P(dBm)/10]}$

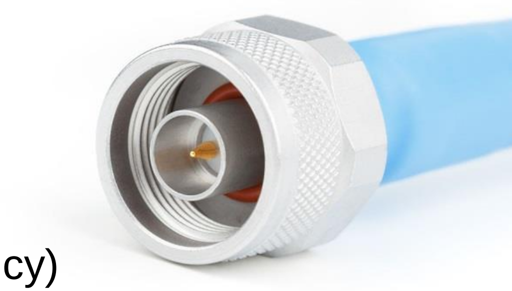
Less than 30 dBm is less than 1 W
A negative number of dBm is less than 1 mW!



RF components: cables and connectors



- **Coaxial cables: Characteristics**
  - Shunt capacitance per unit length, in **farad per meter**
  - Series inductance per unit length, in **henrys per meter**
  - Series resistance per unit length, in **ohms per meter**
  - Shunt conductance per unit length, in **siemens per meter**
  - Characteristic impedance in **ohms**
  - Attenuation (loss) per unit length, in **dB per meter**
  - Velocity of propagation, in meters per second
  - Single-mode band (in coax, TEM have zero cut-off frequency, but other modes set in at high freq.)
  - peak voltage (set by the breakdown voltage of the insulator)



- **Connectors**
- Many types: BNC, N, SMA, 7/16 DIN (rated according to max frequency)
- Decreasing the diameter of the outer conductor increases the highest usable frequency
- RF and microwave connectors are precision-made parts, easily damaged by mistreatment: as a rule, the only thing which has to be rotated is the threaded sleeve
- Connectors give important contributions to loss and mismatch on the line

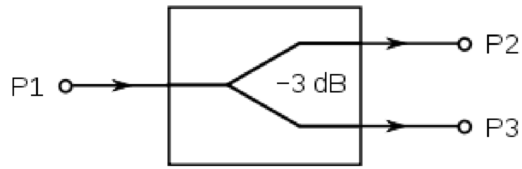
RF components: attenuators, DC stoppers, matched loads shorts, open

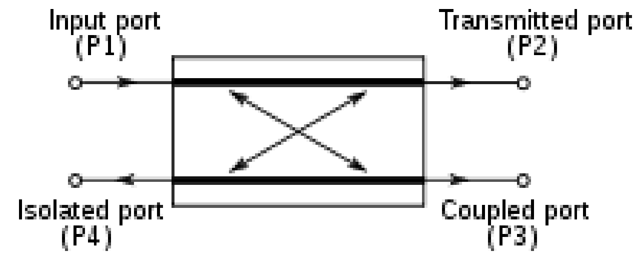
- Attenuators: reduce power (absorbing it) without distorting the waveforms.
- Characteristics**:**
  - **Attenuation** expressed in dB of relative power
  - **Frequency bandwidth**
  - **Power dissipation**
  - **SWR** for input and output ports
  - **Accuracy**
  - **Repeatability**

- DC blocks: prevent the flow of DC (and audio) power, used to protect sensitive equipment (for example power meters), or when these components are undesired
- Loads: calibrated at the characteristics impedance of the line.
- Shorts, open: mainly as calibration standards.

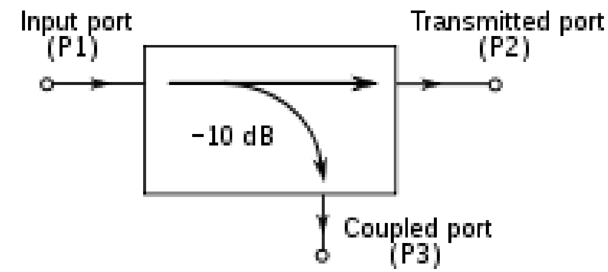
RF components: splitters, directional couplers

Power splitters (or power combiners)





Directional couplers: characteristics



Coupling factor in dB :-10 Log (P3/P1). always < -3 dB

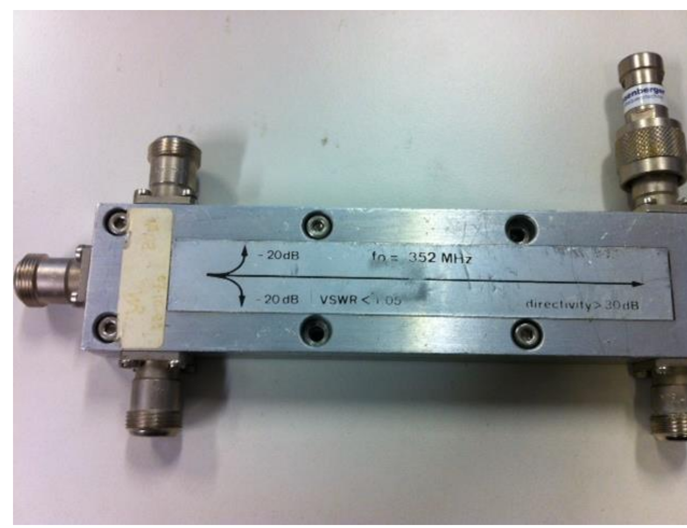
Insertion **Loss** in dB :-10 Log (P2/P1)

Isolation: -10 Log (P4/P1); -10 Log(P3/P2) (they can differ)

Directivity: -10 Log (P4/P3)= -10 Log (P4/P1)+10 Log (P3/P1)

Directivity= Isolation-coupling factor: should be as high as possible

Directivity is “narrow band”, it relies on destructive interference



Power measurements, power meters

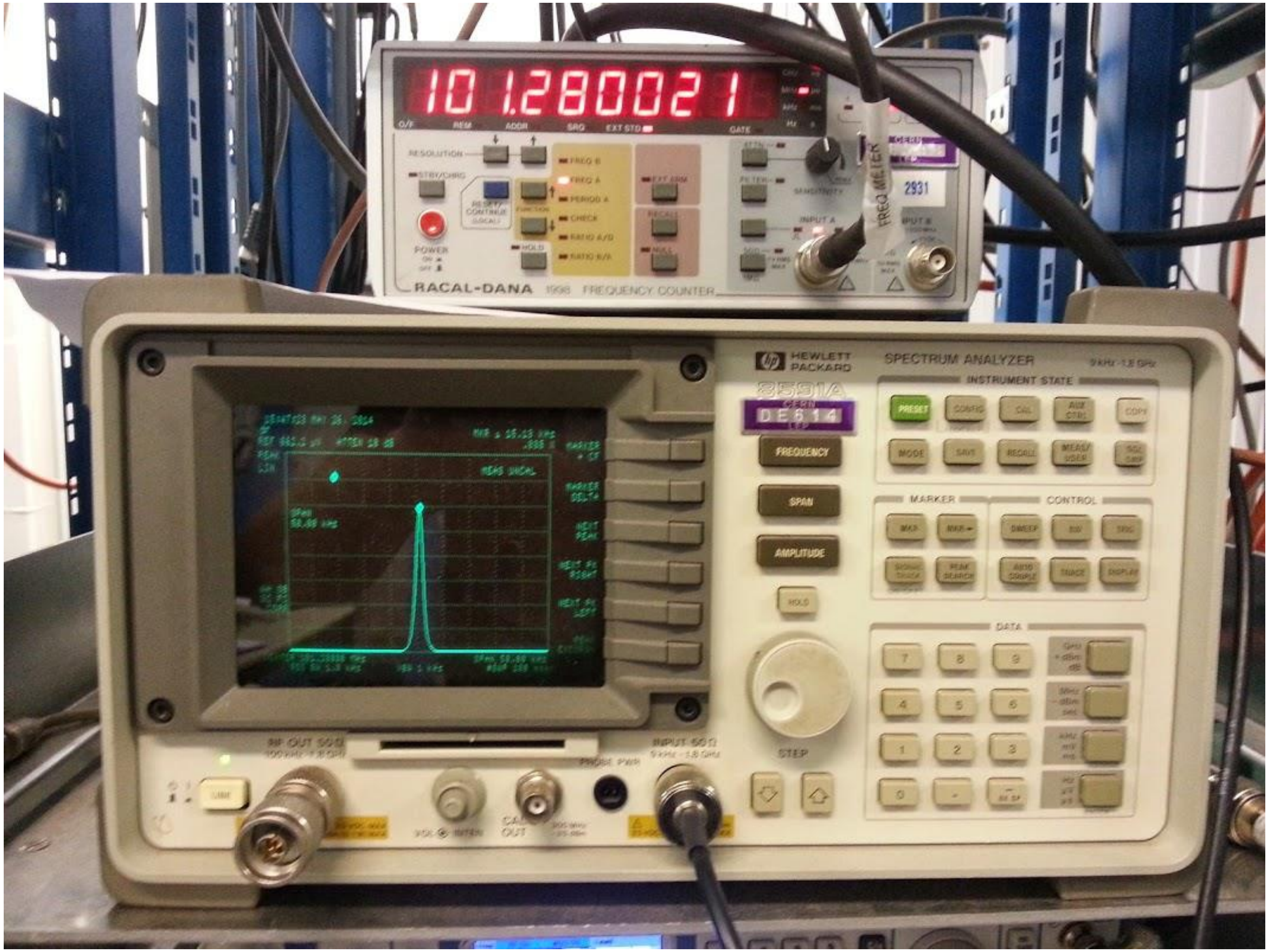
Below ~ 10 kHz power is measured by measuring V and I: it is a derived quantity

Up to 100 MHz direct measurements of voltage and current are still possible; but it is easier and more accurate to measure power directly with a power meter →

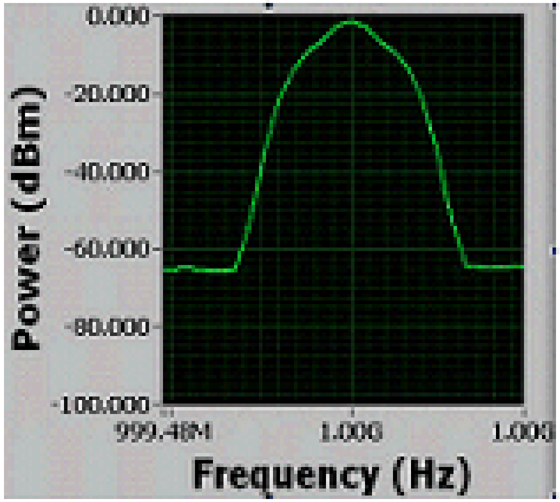
In the GHz range and beyond, power is the fundamental quantity: voltage and current are the derived quantities

Thermistors (bolometers), Thermocouples, and diode detectors are the most common sensors used to measure RF power

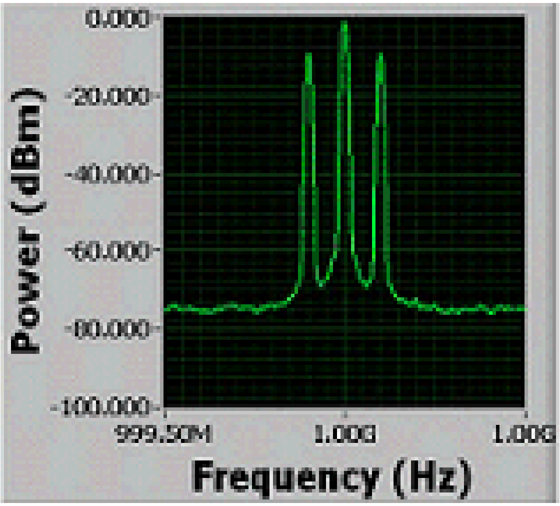
Spectrum analyzer



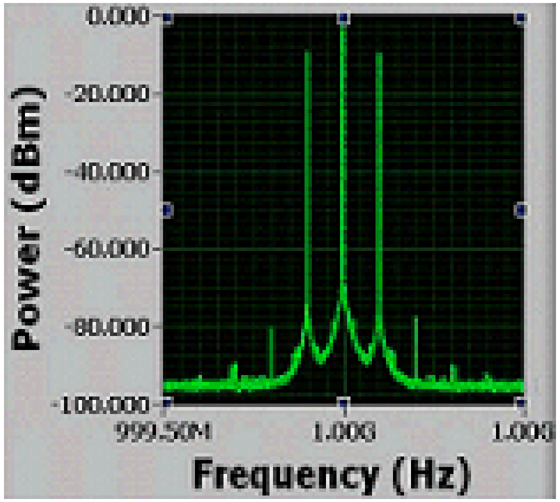
Resolution Bandwidth



RBW = 100 kHz



RBW = 10 kHz



RBW = 100 Hz

Larger RBW

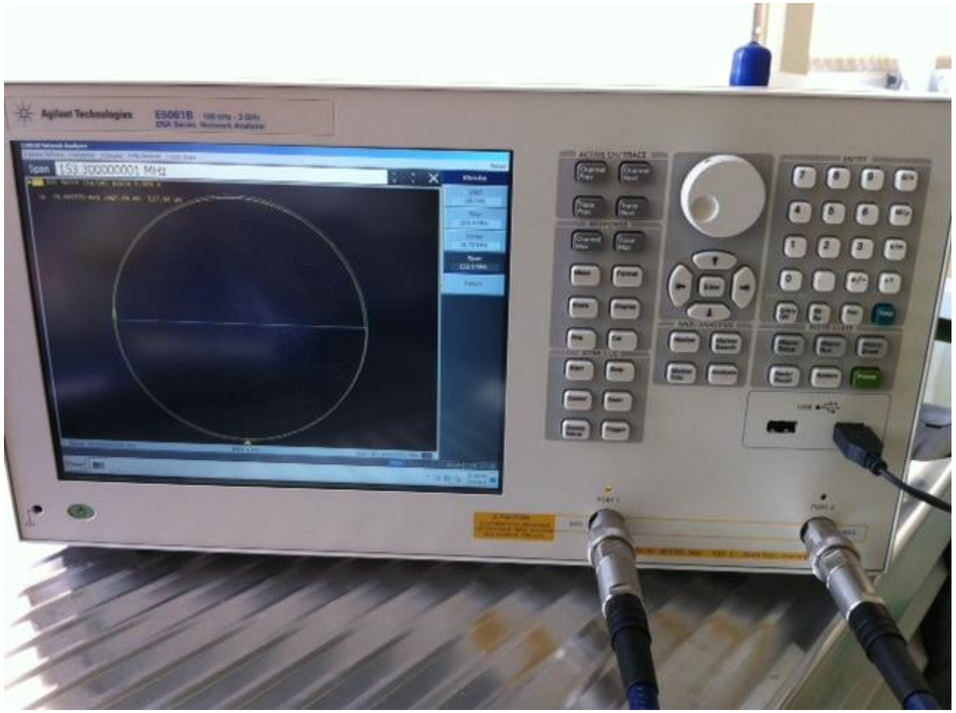
Smaller FFT size; fewer samples; requires less acquisition and computation time; often unable to resolve two closely spaced tones in a spectrum.

Smaller RBW

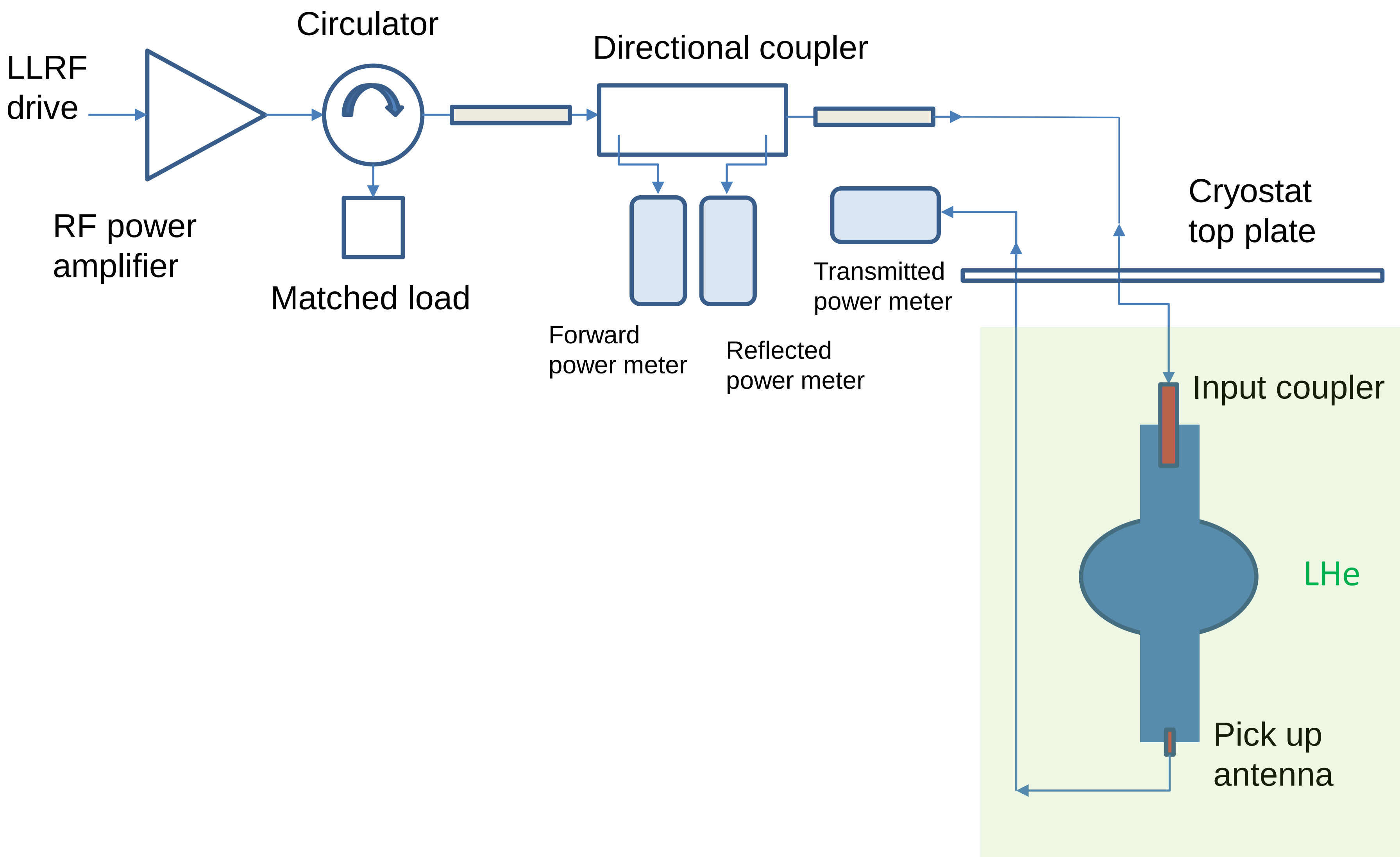
Larger FFT size; more samples; requires more acquisition and computation time; tones are easily resolved.

Network analysers

- Measure S parameters as a function of frequency
- S parameters are complex quantities
- Two types:
  - Scalar Network Analysers (**SNA**): measure amplitudes
  - Vector Network Analysers (**VNA**): measure amplitudes and phases
- VNA are the most common nowadays
- Usually two ports (4 ports and more are also available but expensive)
- With a two ports one can measure S11, S22, S12, and S21 (reflection and transmission from or through a DUT).
- Measurement output in several formats
- Require calibration performed by the user at each use on top of the periodic calibration at the factory

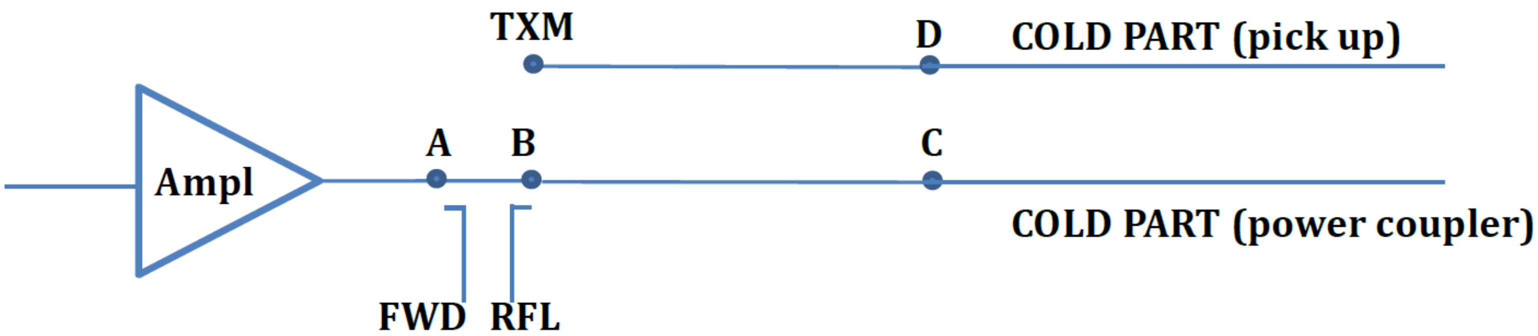


Basic measurement circuit



Attenuation measurements (cable calibrations)

The readings of the power meters on the rack must be referred to the powers at the cavity, taking into account the attenuation of the lines:

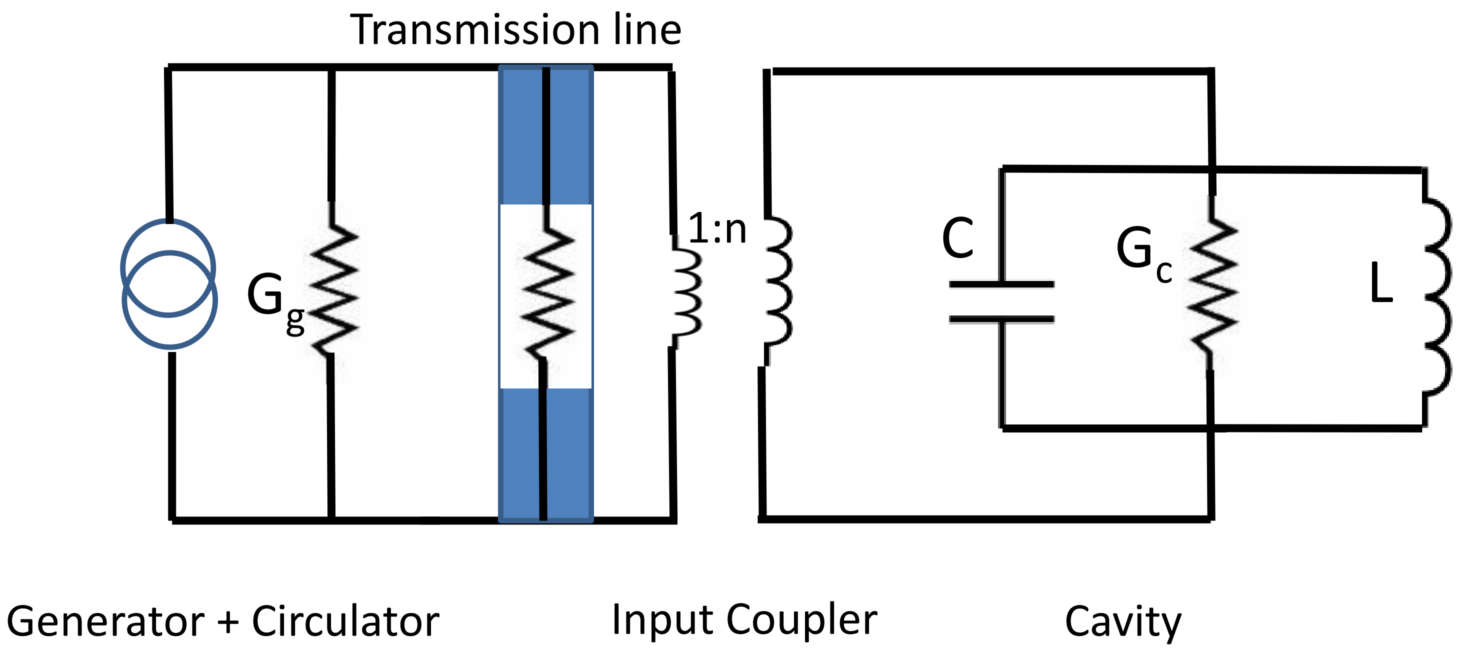


The total attenuation values are:

FWD= (A→FWD) - (A→C) - Cold part (coupler)
RFL=(B→RFL) + (B→C) + Cold part (coupler)
TXM=(D→TXM) + Cold part (antenna)

Cavity measurement analysis

- Equivalent circuit model (for one mode)



Cavity measurement analysis: undriven cavity:

$P_{tot} = P_c + P_e + P_t$; → Loaded Q: $$Q_L = \frac{\omega U}{P_{tot}}$$

$-\frac{dU}{dt} = P_{tot} = \frac{\omega U}{Q_L}$; U decreases exponentially, the time constant is $\tau_L = \frac{Q_L}{\omega}$

$\frac{P_{tot}}{\omega U} = \frac{1}{Q_L} = \frac{1}{Q_o} + \frac{1}{Q_e} + \frac{1}{Q_t}$; defines intrinsic and external Q's

Coupling parameters: $\beta_e = \frac{Q_0}{Q_e}$; : $\beta_t = \frac{Q_0}{Q_t}$

they tell how strongly the couplers interact with the cavity.

$$Q_o = Q_L(1 + \beta_e + \beta_t)$$

While external Qs only depend on geometry (overlap of field lines between cavity and coupler), the betas depend on the cavity Q.

Cavity measurement analysis: driven cavity with one coupler

- From the equivalent circuit, the reflection coefficient of the cavity seen from the transmission line can be computed:
- $\Gamma = \frac{Z_L - Z_0}{Z_L + Z_0} = \frac{1 - \frac{Y_C}{G_0}}{1 + \frac{Y_C}{G_0}}$
- The conductance seen from the generator, relative to G_0 is $\frac{1}{\beta}$ and the susceptance is $Q_e\left(\frac{\omega}{\omega_0} - \frac{\omega_0}{\omega}\right)$
- Then, the complex reflection coefficient is
- $\Gamma(\omega) = \frac{\beta - 1 - jQ_0\delta}{\beta + 1 + jQ_0\delta}$ and δ is the detuning factor $\delta = \left(\frac{\omega}{\omega_0} - \frac{\omega_0}{\omega}\right)$
- The power flowing in the cavity is $P_{in} = P_f(1 - |\Gamma|^2)$
- The reflected power can be cast in the form $P_r = (\sqrt{\frac{\omega_0 U}{Q_e}} - \sqrt{P_f})^2$
- The stored energy obeys the equation:
- $\frac{d\sqrt{U}}{dt} = \frac{1}{2\tau_L}(\sqrt{U_0} - \sqrt{U})$ → $\frac{dE}{dt} = \frac{1}{2\tau_L}(E_0 - E)$

Equilibrium value of U for a given Pf → $U_0 = \frac{4{\tau_L}^2 \omega P_f}{Q_e}$

(detailed derivations in Ref [8], p. 150)

Steady state

- from the expression of the reflected power $P_r = P_f (\frac{\beta-1}{\beta+1})^2$

- we derive $\beta = \dfrac{1 \pm \sqrt{\frac{P_r}{P_f}}}{1 \mp \sqrt{\frac{P_r}{P_f}}}$

the coupling coefficient is thus determined from the measurement in CW of the reflected and incident power provided it is known if $\beta > 1$ or $\beta < 1$

From the differential equation for E, by setting proper initial conditions, we can analyse the reflected power waveforms at RF turn on and off.

These allow to distinguish the type of coupling

RF off - transient

- $\frac{d\sqrt{U}}{dt} = \frac{1}{2\tau_L}(\sqrt{U_0} - \sqrt{U})$

- If we turn off the RF power at t=0, then U_0 =0 (the equilibrium value of the stored energy becomes zero

- $\frac{d\sqrt{U}}{dt} = -\frac{1}{2\tau_L}\sqrt{U}$,
- → the time constant for the exponential decay **of the field** is $2\tau_L$
- When measuring the time constant make sure you know if you are measuring the power or the amplitude

- Now the RF is off, so the reverse travelling power is only the emitted power.
- Therefore we can write (definition of Qe)
- P_r=P_e=$\frac{\omega U(t)}{Qe}$, U(t)=U_0 exp(-t/τ_L), and $U_0 = \frac{4{\tau_L}^2 \omega P_f}{Q_e}$ is the equilibrium value of U before the RF is turned off.

Coupling determination from RF off transient

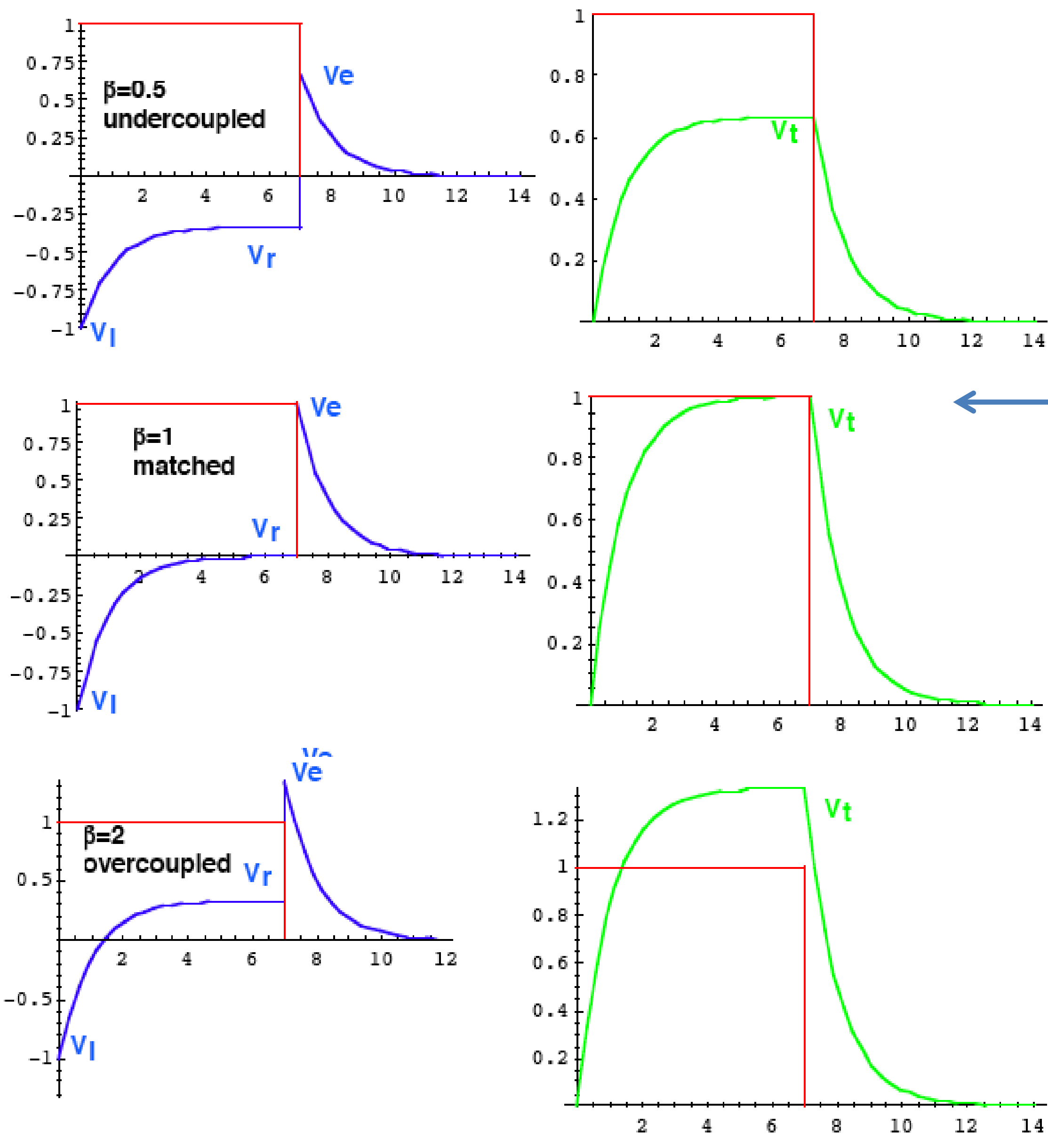
- $U_0 = \frac{4{\tau_L}^2 \omega P_f}{Q_e} = \frac{4\beta\ P_f}{(1+\beta)^2} \frac{Qo}{\omega}$

- Another way to determine the coupling factor is to compare the forward power before RF is turned off with the peak of the emitted power immediately after:

- $P_e(t = 0) = \frac{\omega Uo}{Qe} = \frac{4\beta\ P_f}{(1+\beta)^2} \frac{Qo}{Qe}$ → $\beta = \frac{1}{2\left(\sqrt{\frac{Pf}{Pe}} - 1\right)}$

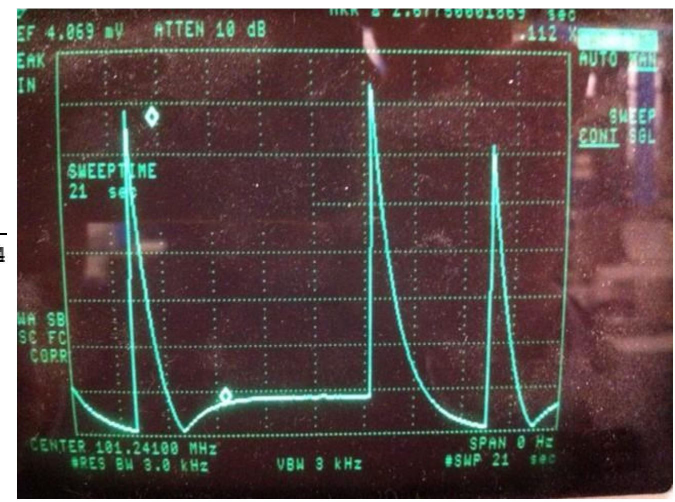
- In this case we don’t need to know beforehand if the system is under coupled or over coupled

Determination of the coupling factor



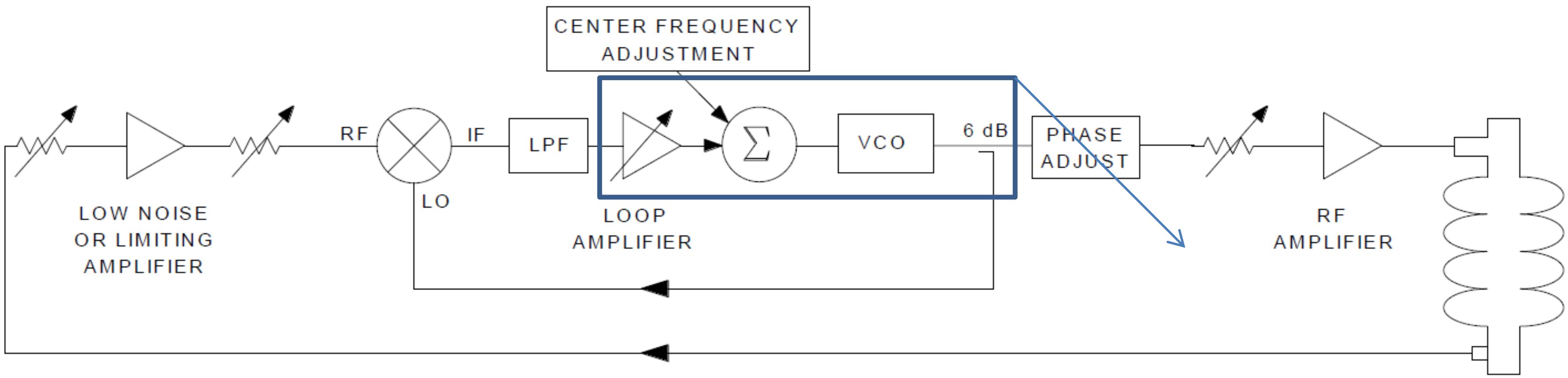
$$\beta = \frac{1-\left|\dfrac{V_r}{V_i}\right|}{1+\left|\dfrac{V_r}{V_i}\right|} = \begin{cases} \dfrac{|V_i|-|V_r|}{|V_i|+|V_r|}; \beta \le 1 \\[2ex] \dfrac{|V_i|+|V_r|}{|V_i|-|V_r|}; \beta \ge 1 \end{cases}$$

← Voltage waveforms



Reflected power from a square pulse when $\beta > 1$

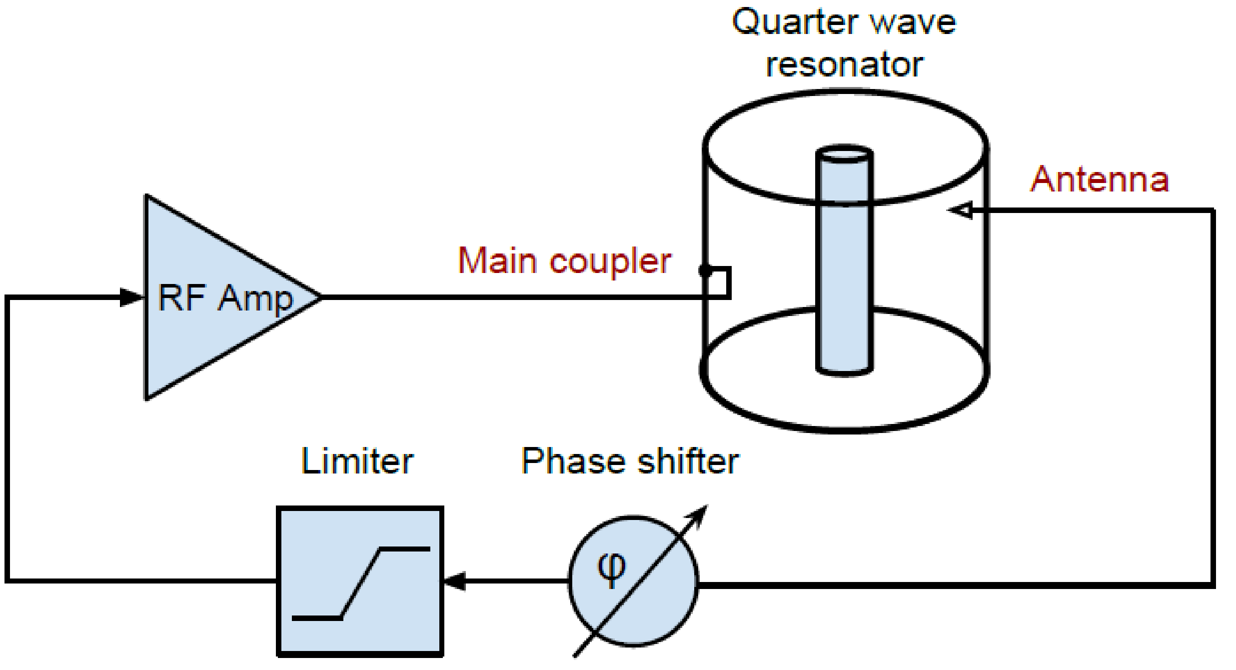
Locking the cavity: generator driven systems



Block diagram of a VCO PLL System. The encircled elements can be replaced by an RF generator which accepts frequency modulation from the DC output of the mixer (phase error-> frequency error around resonance).

From Ref. 8

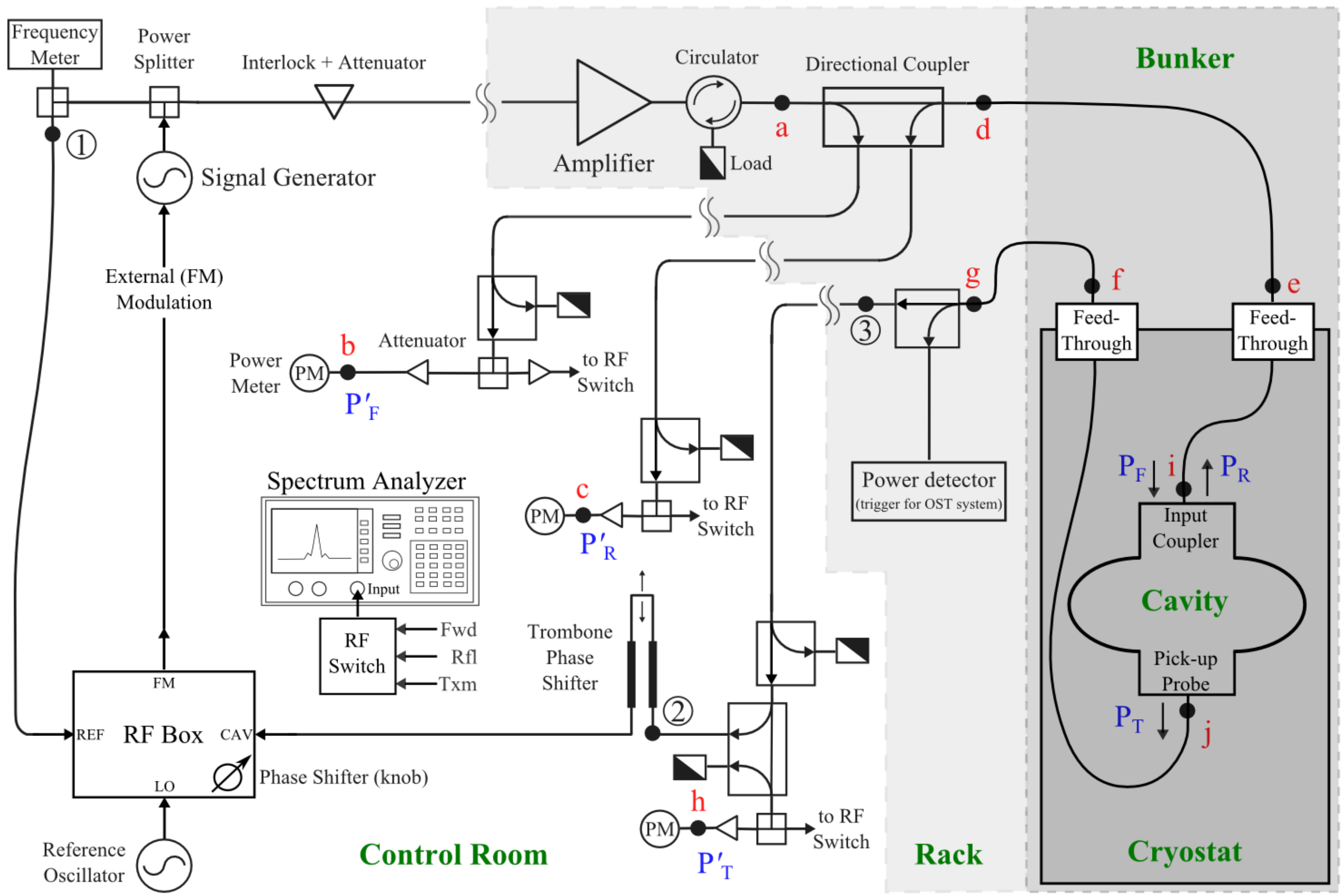
Locking the cavity: Self excited loop



High gain, positive feedback loop: the cavity selects its own resonance frequency from the noise floor and the circuits starts to oscillate. It only works with correct phase shift on the return path.

Finds and tracks the cavity resonance

Real measurement circuit



Test stand


RF Switch
Frequency Meter
Cryogenic Control System (PVSS©)
Radiation Sensor
Spectrum Analyzer
Interlock
RF Control System (LabVIEW™)
Cavity Pressure Sensor
Signal Generator
Trombone
Phase Shifter
1
Reference Oscillator
RF Box
Control PC
Bunker Control
2

Measurement procedure

1) Lock the cavity on resonance

2) Calibration point:

- Determine input coupling condition
- Measure in CW: P_i, P_r and P_t
- Measure the time constant of U
- From β and τ_L $Q_0=\omega_0 \tau_L (1+\beta)$
- From the measurement in CW, obtain the dissipated power $P_c=P_i-P_r-P_t$
- Find the stored energy by the definition of Q_0: $U_0 = \frac{P_c Q_0}{\omega}$
- Find the accelerating field $E_{acc} = k_e\sqrt{U_o}$
- Find the calibration constant of the pickup

$$K_{pu} = \frac{U_0}{P_t} = \frac{Q_{ext\ pu}}{\omega}$$

2) Q-E scan

- Increasing the power level, U is determined by the transmitted power, knowing K_{pu} hence the field is determined knowing k_e
- The corresponding value of Q is determined by the definition, knowing the stored energy and the dissipated power

MP

k_e is obtained by means of computer simulations

Warm measurement of high Q_{ext} (pickup calibration)

- The external Q of the pickup probe is essential for the measurement of E_{acc}
- As good practice it should be chosen ~ $100 \times Q_0$
- It is derived when measuring Q_0 with the decay time (calibration)
- It is good to measure it at warm as a consistency check (it is a geometric parameter)
- Q_{ext} can be determined from the reflection coefficient and the loaded Q
- A VNA can display a polar plot of the reflection coefficient, normalised to the characteristic impedance (Smith chart)
- Remember $\Gamma(\omega) = \frac{\beta - 1 - jQ_0\delta}{\beta + 1 + jQ_0\delta}$, and $\delta = \left(\frac{\omega}{\omega_0} - \frac{\omega_0}{\omega}\right)$

Warm measurement of Q_{ext} with network analyser

1[st] method (used for β not too far from 1):

1) Determine from the Smith chart if the cavity is under- or over-coupled
2) Measure the Standing Wave Ratio (SWR) on resonance

3) $\beta = \begin{cases} SWR \; if \beta > 1 \\ \frac{1}{SWR} if \beta < 1 \end{cases}$,

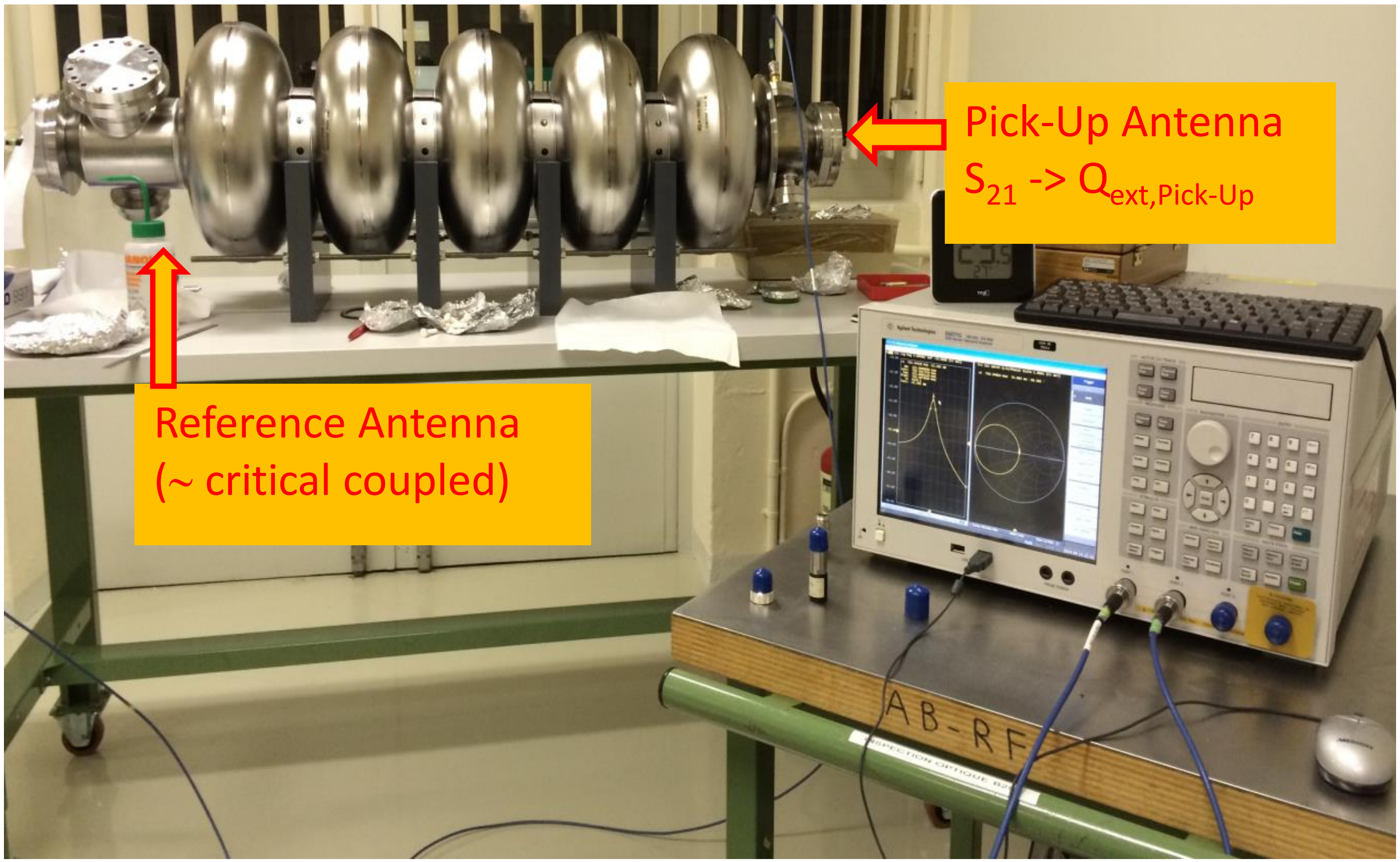
3) Determine Q_L from the 3dB bandwidth: $Q_L = \frac{f}{\Delta f}$
4) $Q_0 = (1+\beta)$,
5) $Q_{ext} = \frac{Q_0}{\beta}$

Warm measurement of high Q_{ext} with network analyser (pickup calibration)

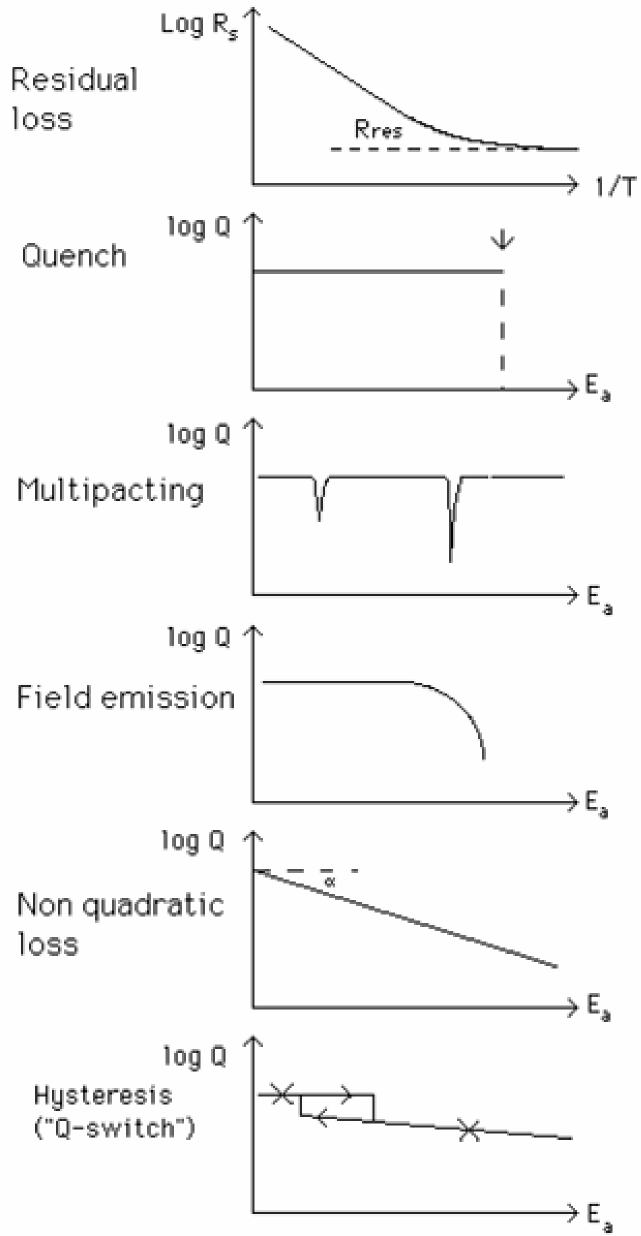
2nd method, used for pickup calibration, must use a coupler with β not too far from 1

1) Measure β from S_{11} (see method 1)
2) Measure S_{21} and Q_L
3) $\beta_{pu} = \frac{|S21|^2}{\frac{4\beta}{(1+\beta)^2} - |S21|^2}$: (Ex. derive it from $P_{in} = P_f - P_r = P_c + P_t$)
4) $Q_0 = (1 + \beta + \beta_{pu})\, Q_L$
3) $Q_{ext-pu} = \frac{Q_0}{\beta_{pu}}$

Measurement Setup



Interpretation of cavity behaviours



Fitting Q(T) data with the standard BCS expression reveals the so called "non BCS losses"

The global transition of the cavity to the normal conducting state is a catastrophic event, can happen to bulk Nb cavities in case of large beam losses

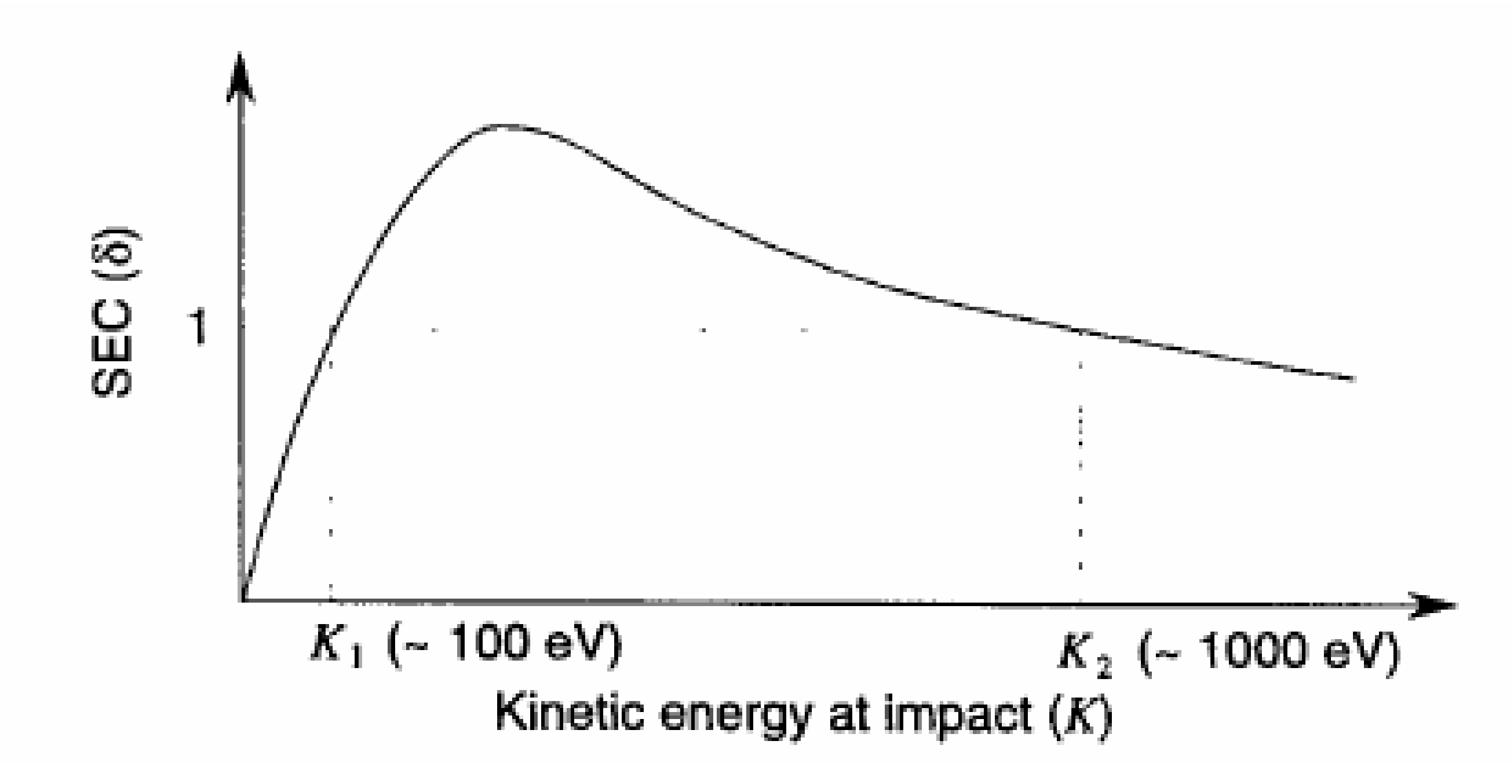
Multipacting is easily recognized by the "field lock" feature. Also is recurs at the same field levels for the same cavity geometry

Field emission most striking mark is the emission of X rays. The endpoint energy of the spectrum corresponds to the maximum kinetic e^- energy

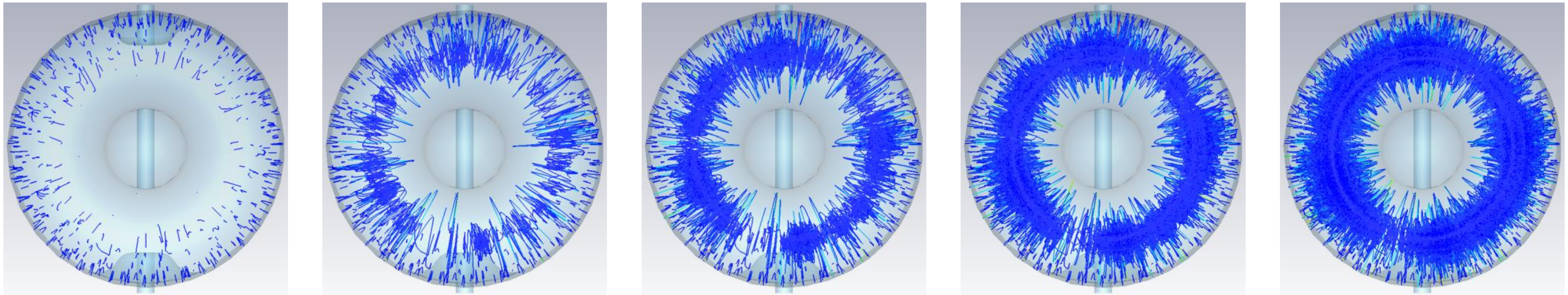
Many theories available, a lot still to understand

Q switches are small (local) quenches. The cooling power is sufficient to prevent global quench.
Typical hysteretic behaviour

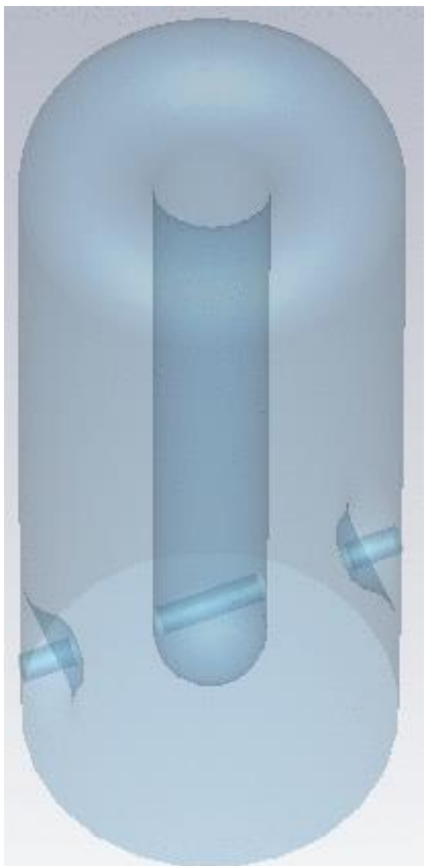
Multipacting: SEY of Nb



Multipacting (Cavity Top)



30ns 50ns 70ns 87.5ns



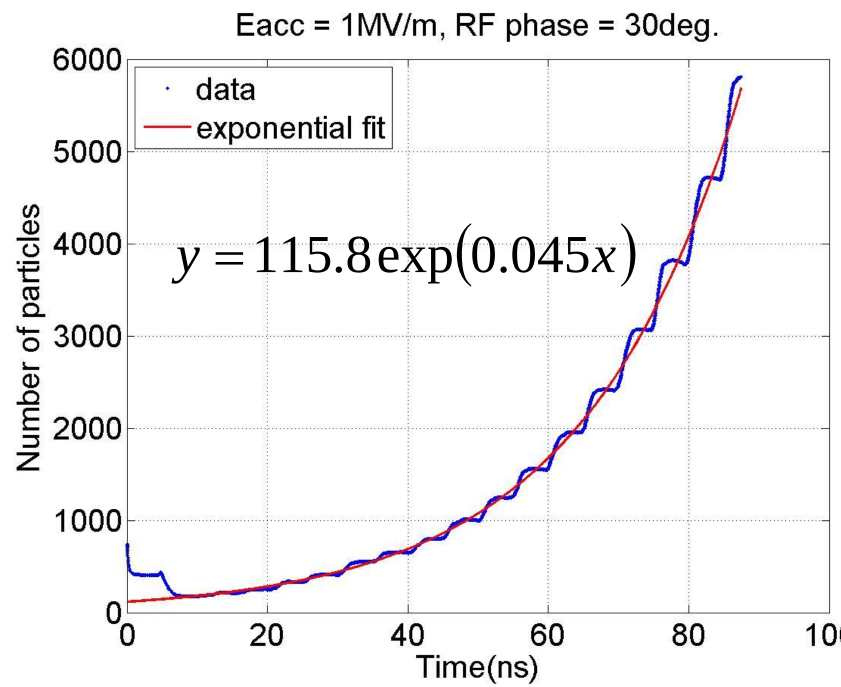
# of initial electrons: 742

Kinetic energy: 0-4eV (uniform)

Emission angle: 0-180 deg. (random)

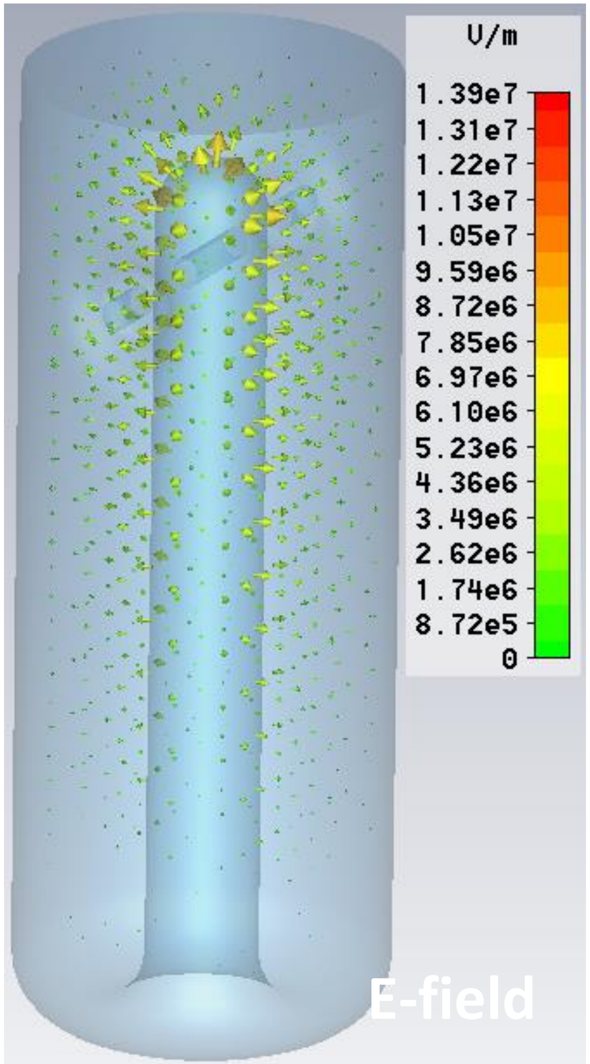
Max time steps: 4000

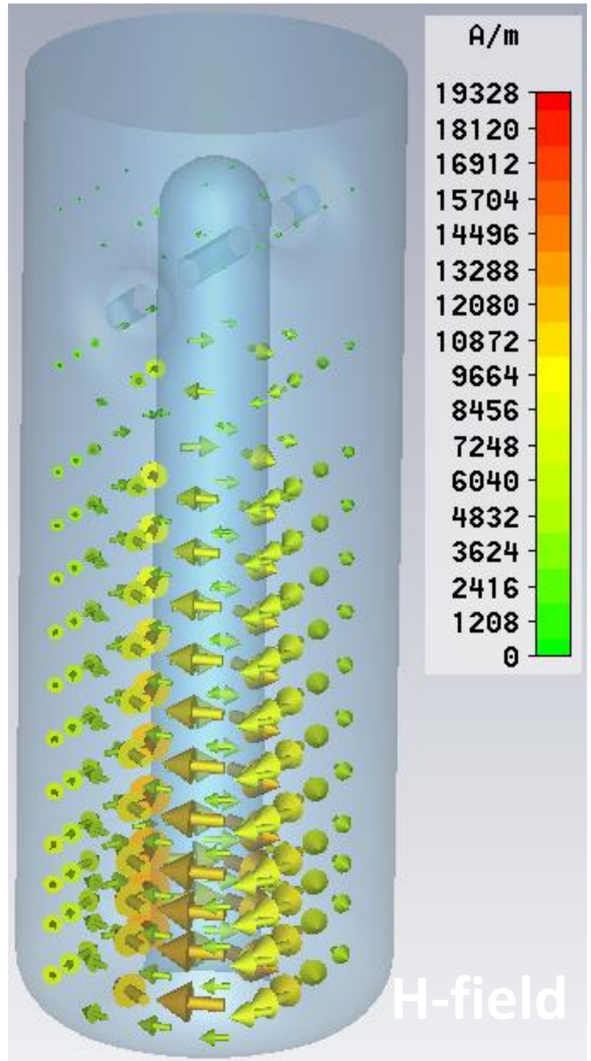
Eacc=1MV/m (30 deg)

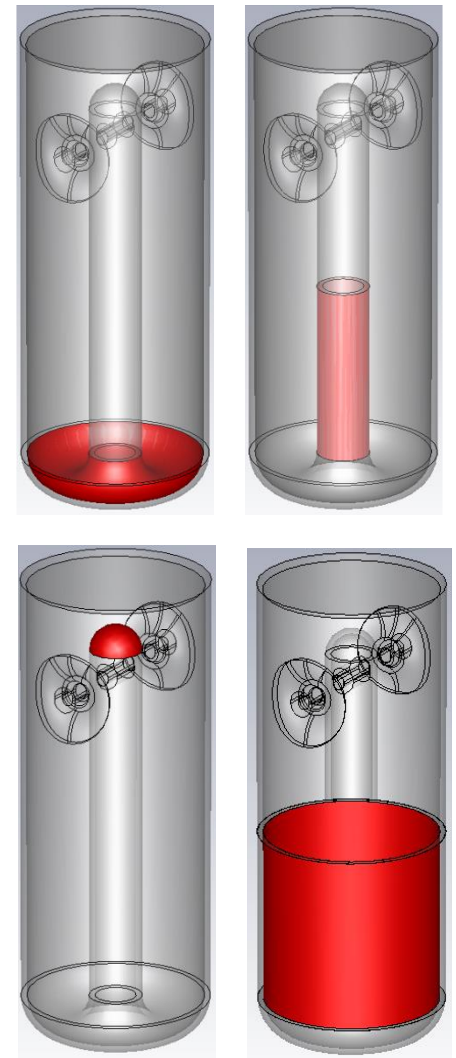


Pei Zhang

Regions of Potential Multipacting







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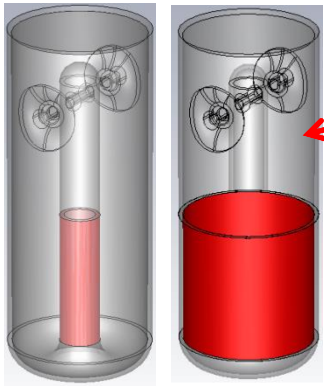
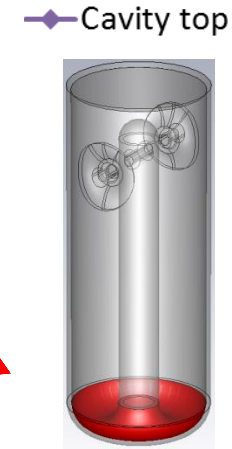
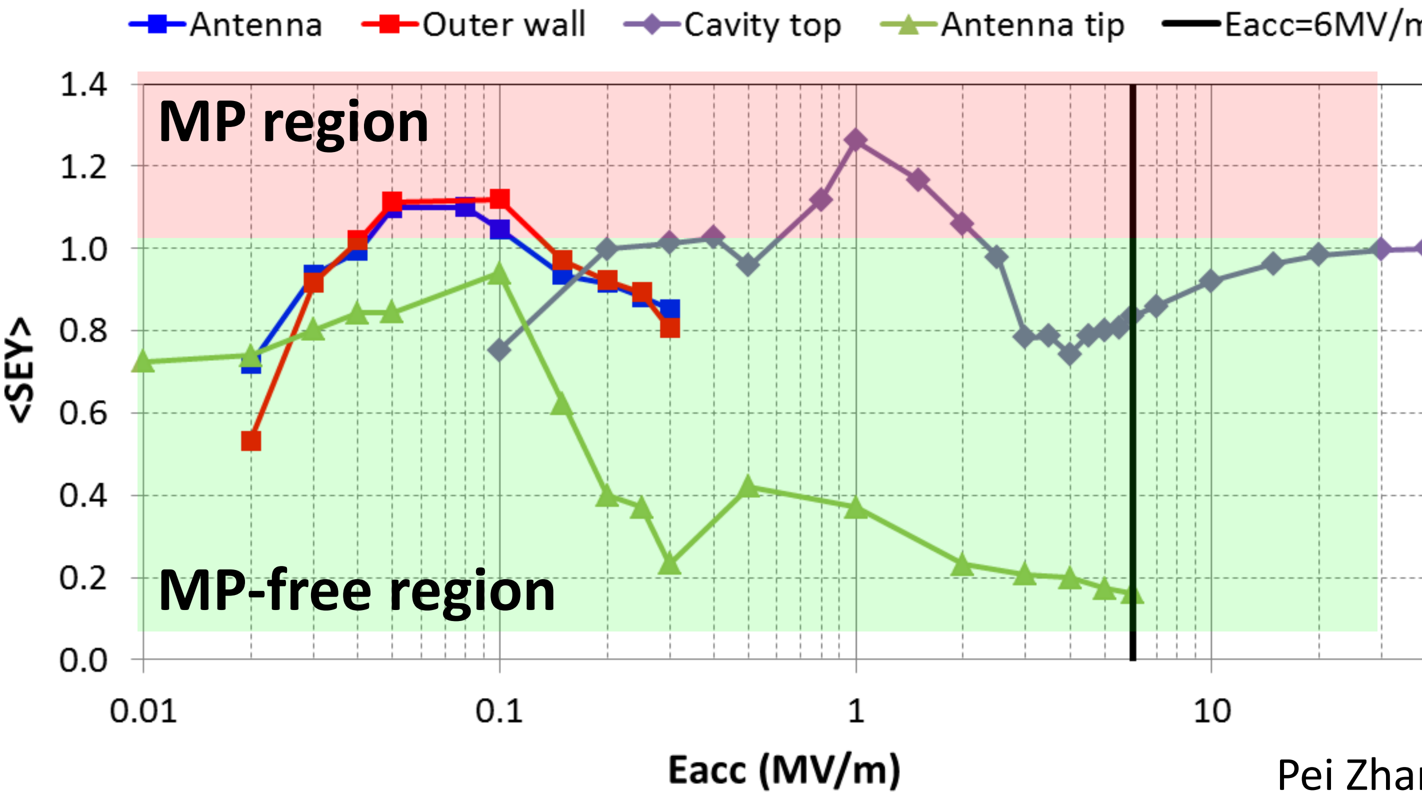
QWR multipacting Summary



- MP happens at low fields for antenna region and outer wall
  - E_{acc} = 0.05-0.1MV/m
- MP happens at higher fields for cavity top region
  - E_{acc} = 0.8-2 MV/m





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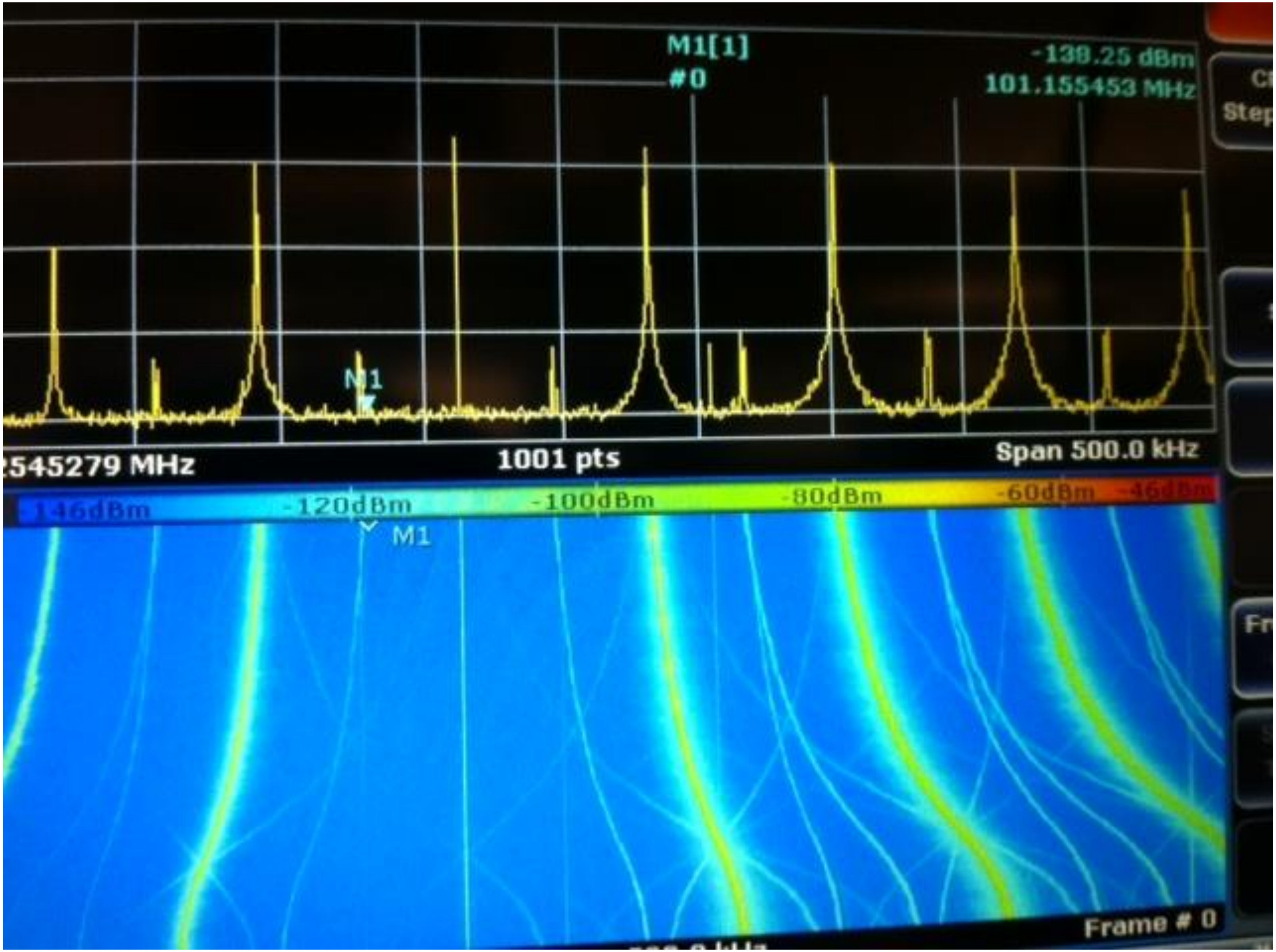
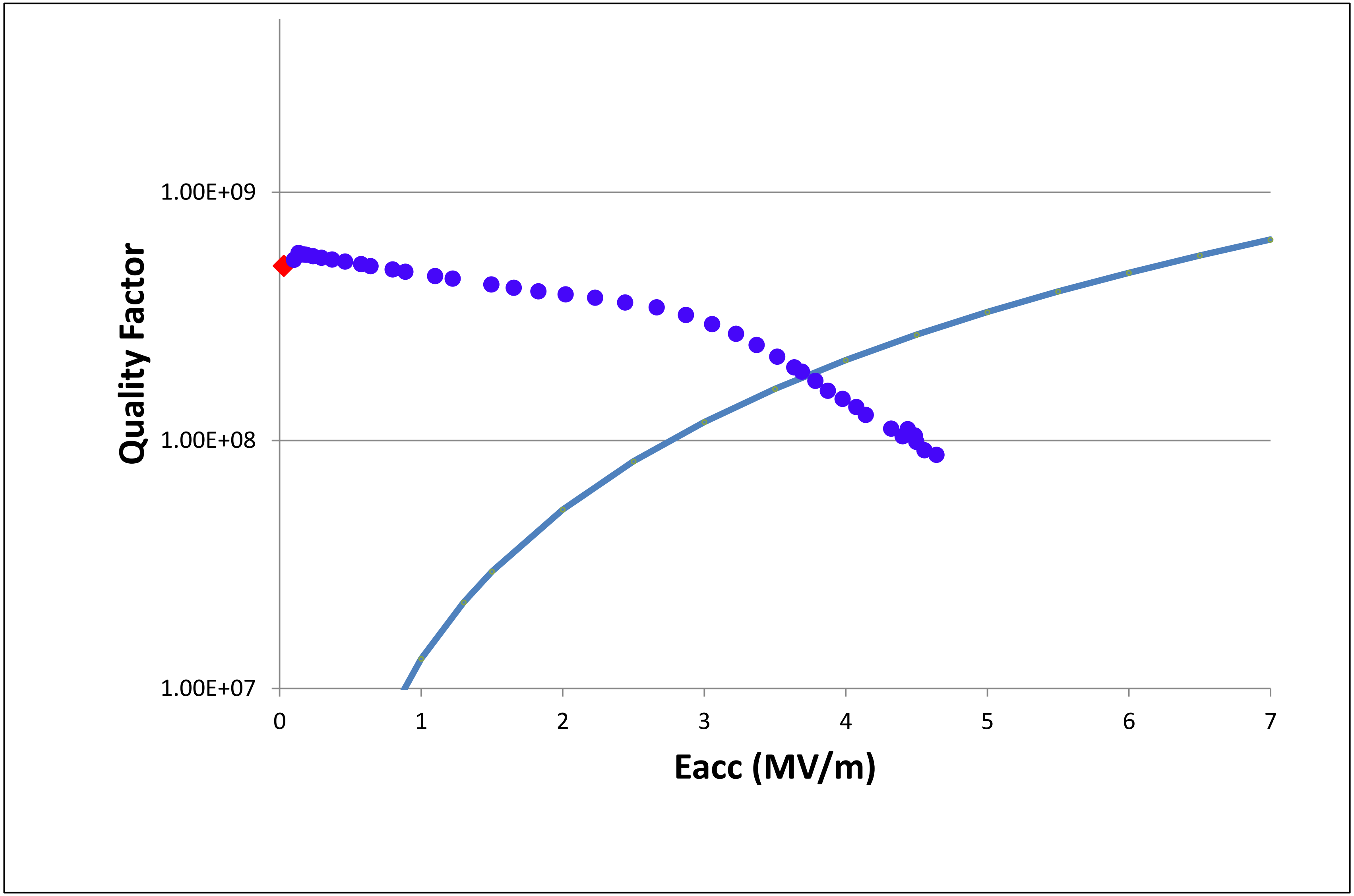
Cavity spectrum during multipacting conditioning

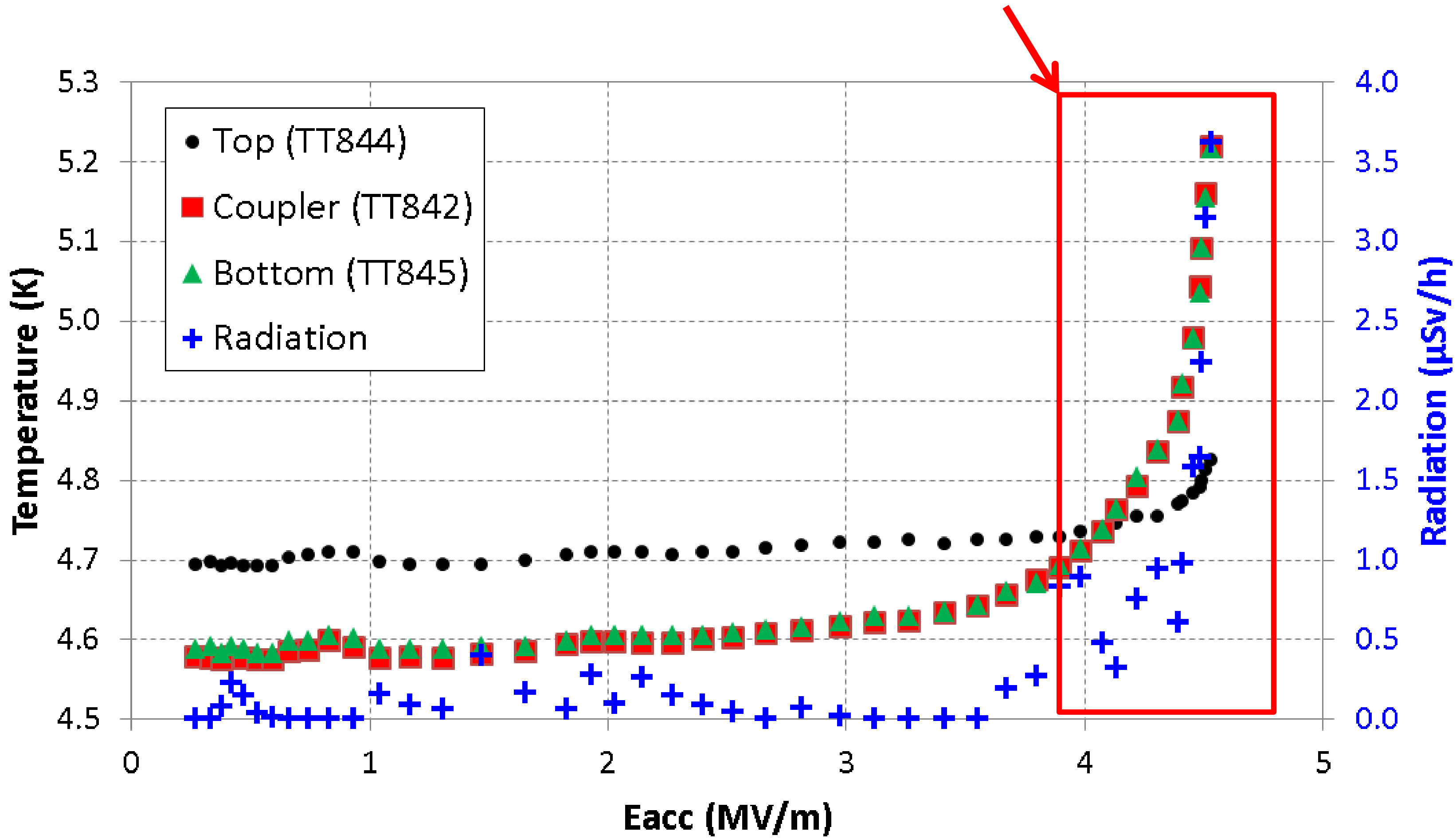


Field Emission

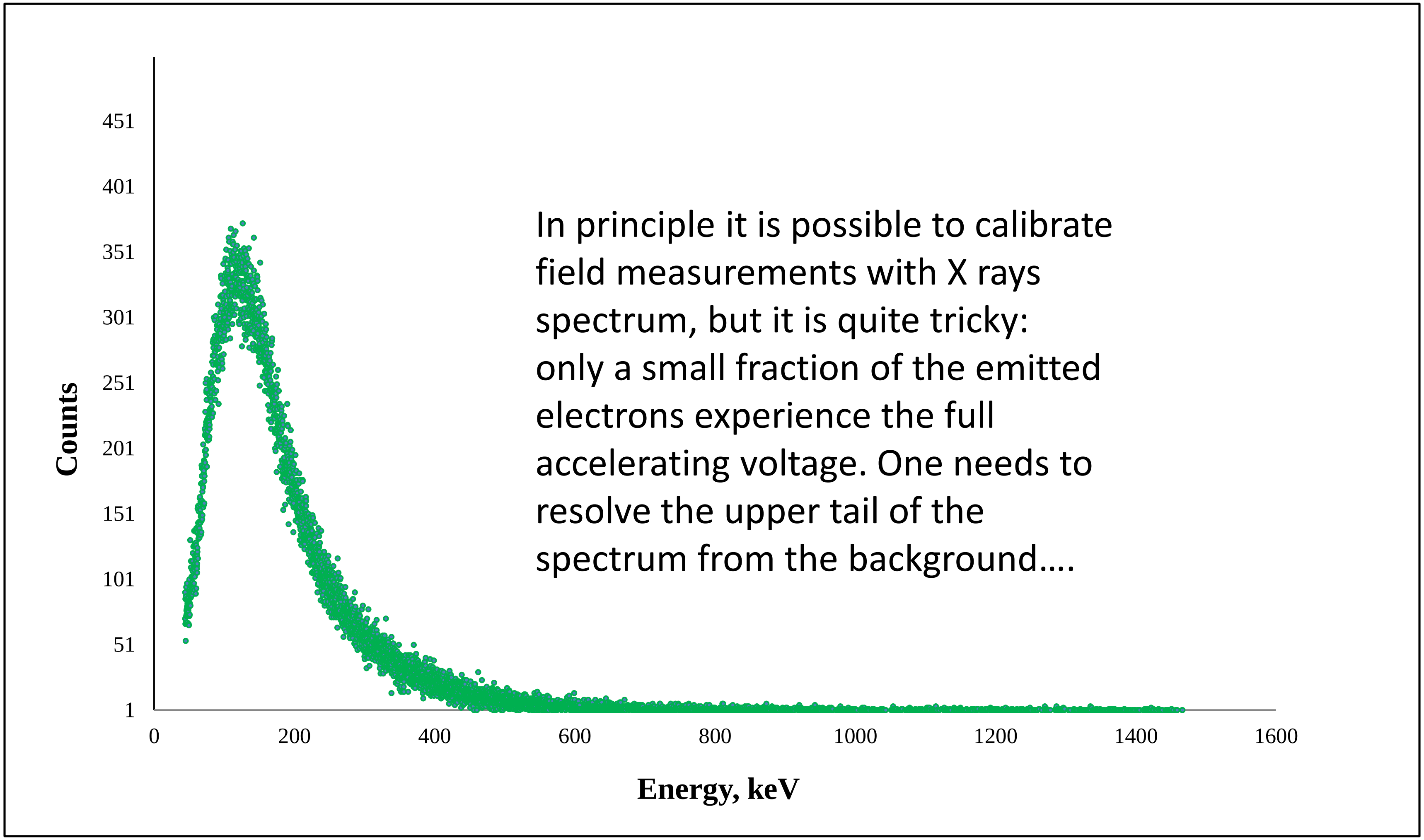


Temperature & Radiation

The cavity bottom was heated up exponentially and the radiation level went up exponentially as well, a clear indication of **field emission**.



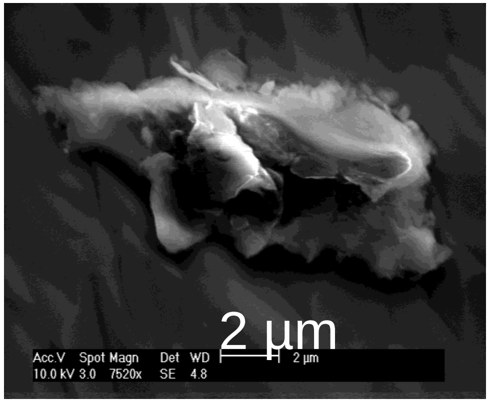
X rays spectrum (bremsstrahlung)



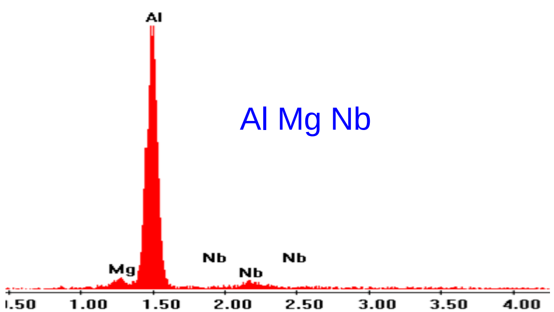
In principle it is possible to calibrate field measurements with X rays spectrum, but it is quite tricky: only a small fraction of the emitted electrons experience the full accelerating voltage. One needs to resolve the upper tail of the spectrum from the background….

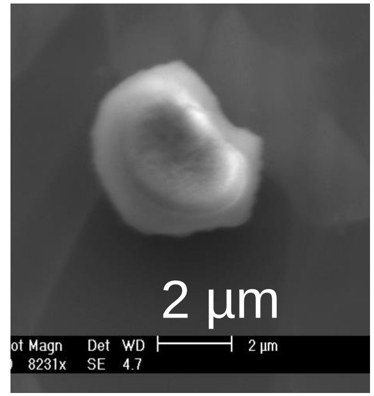
Electron field emission

- Typical particulate emitters

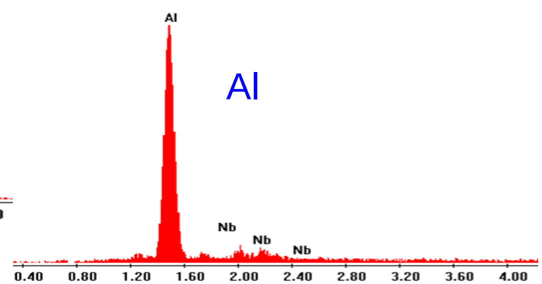


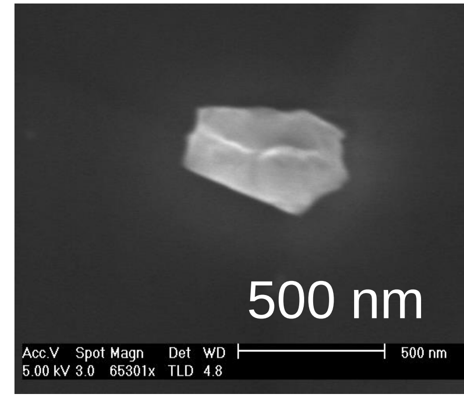
E_{on}(2nA) =140 MV/m
β= 31, S = 6.8·10^{-6} µm^2



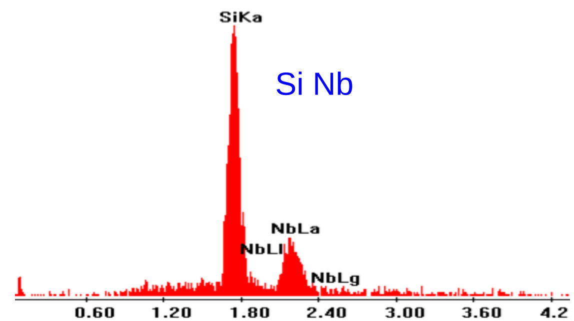


E_{on}(2nA) = 132 MV/m
β = 27, S = 7·10^{-5} µm^2





E_{on}(2nA) > 120 MV/m
β = 46, S = 6·10^{-7} µm^2

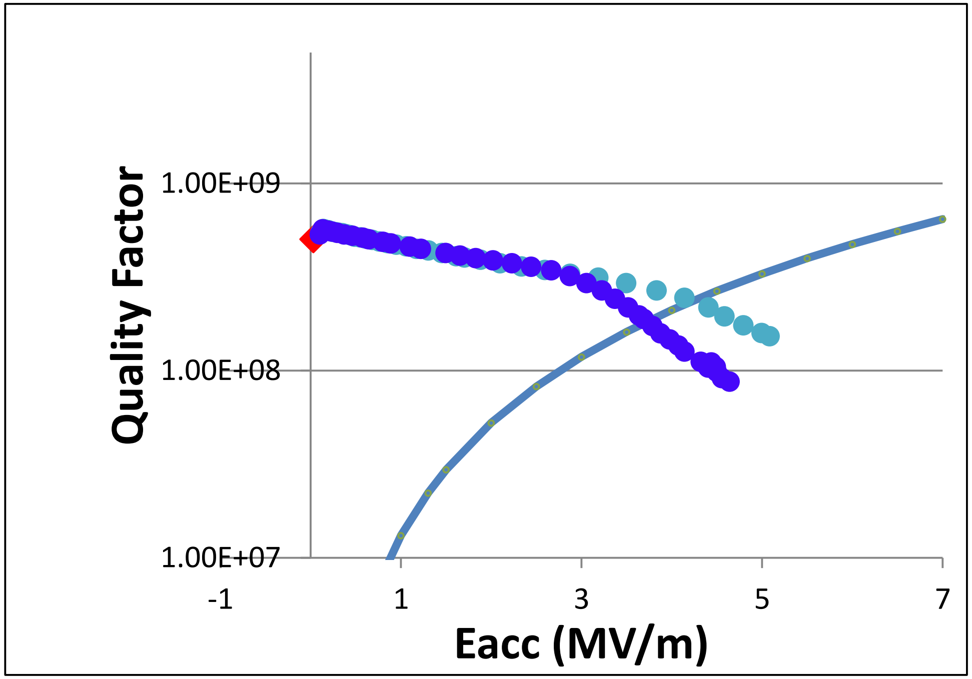


He processing

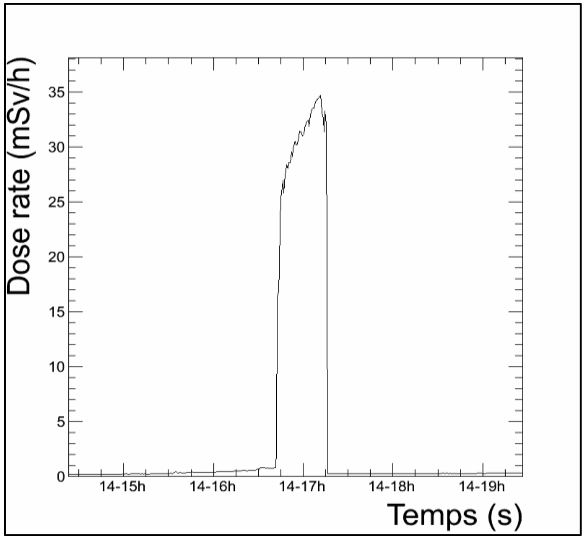
Uses RF power in ignite a low pressure plasma in the cavity filled with gaseous helium

He pressure must be high enough for this but not higher, risk of breakdown and damage to cavity and couplers

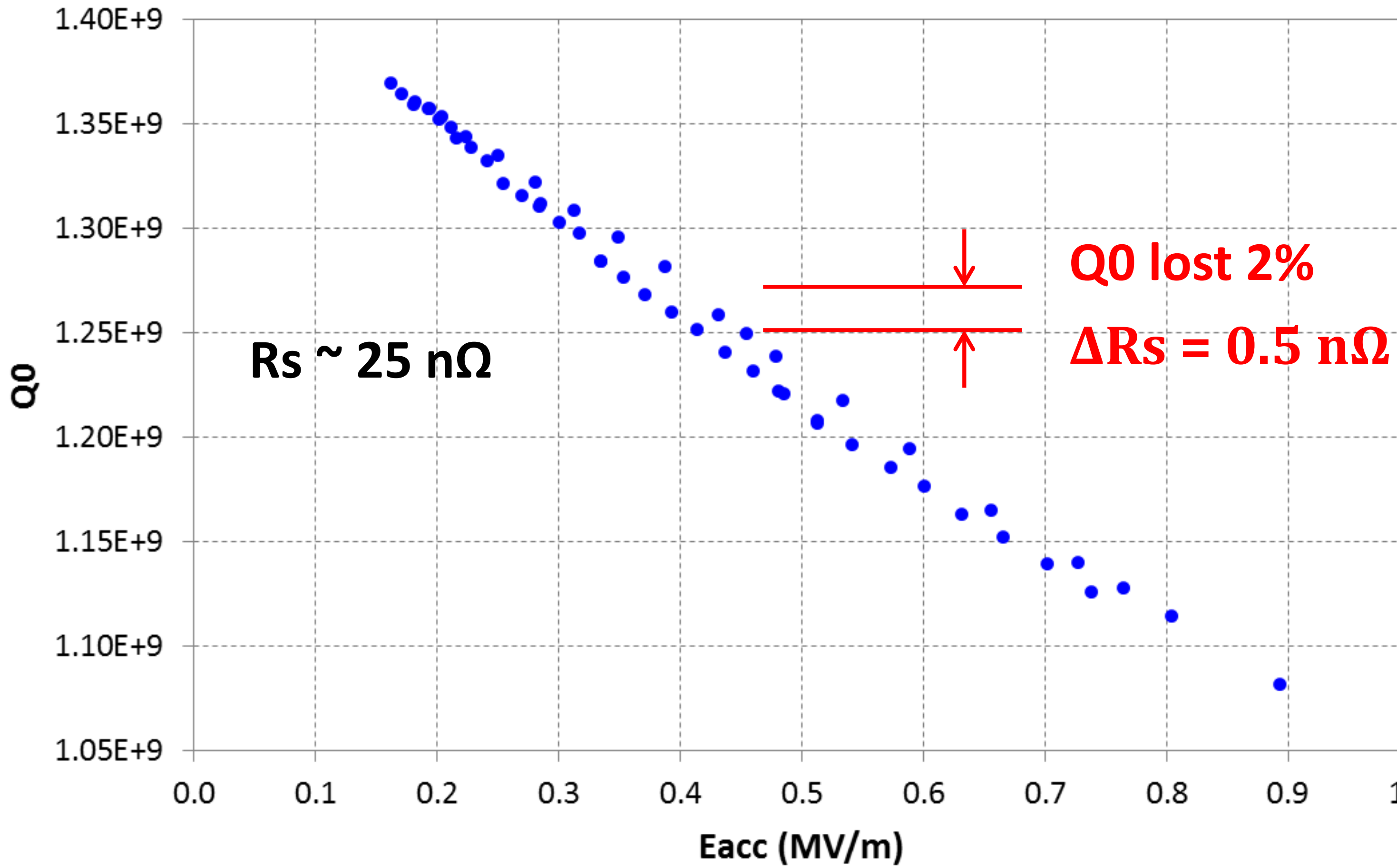
Used as a last resort against field emission when cavity can't be easily removed and rinsed with high pressure water or other chemical or mechanical treatments



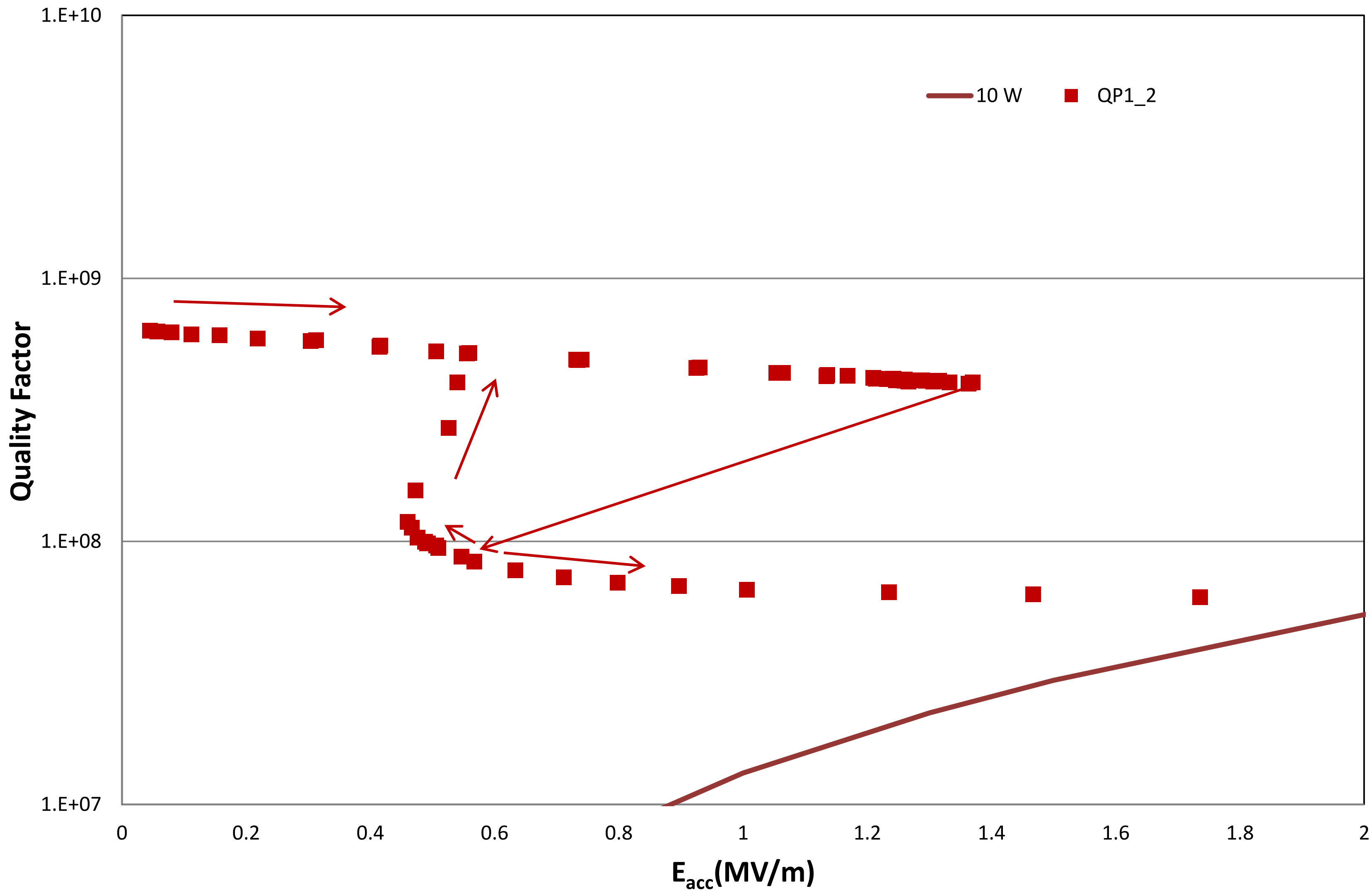
X ray dose rate data



Low Field Q switch



Example of giant Q switch
(NC transition of the tuning plate in a QWR)



Sources of uncertainty in SC cavity measurements

- Power meter accuracy and resolution
  - Determination of coupling β
  - Direct measurement of power in Q - E scans
- Cross contamination of forward and reflected power due to poor directivity of directional couplers
- Detuning
- Dependence of cable attenuations on power level
- Uncertainty on time constant measurement due to
  - Q slope (non exponential decay)
  - Multiple reflections during decay (circulator mismatch)
  - Fit error
  - Instrument accuracy
- Uncertainty on geometric constants (simulations assume perfect geometry)

Q_0 measurement uncertainties

- …from Q_o=$\omega\tau_L(1+\beta)$→

$$(\sigma_{Q_o})^2=(\frac{\partial Q_o}{\partial \beta})^2 (\sigma_\beta)^2 + (\frac{\partial Q_o}{\partial \tau_L})^2 (\sigma_{\tau_L})^2$$

under ideal conditions this leads relative uncertainties of a few %.

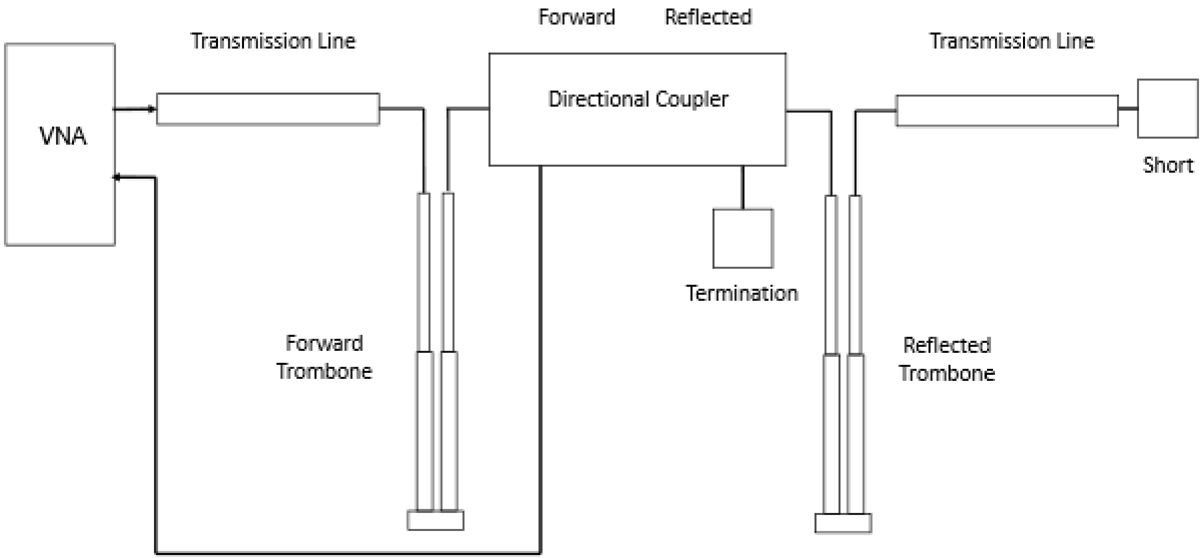
See for example: O. Melnychuk, A. Grassellino, and A. Romanenko, *Review of Scientific Instruments* 85, 124705 (2014)

However this assumes:

- perfect directivity of the directional couplers,
- perfect matching of the circulator protecting the amplifier
- no detuning during the decay measurement

These effects can be very important in some conditions

Directivity error experiment



In theory, the measurement of the forward power from the directional coupler port should not depend on the phases, but…

Picture from J.P. Holzbauer, Yu. Pishalnikov, D. A. Sergatzkov, W. Schappert, S. Smith, *Nucl. Instrum. Meth.A* 830 (2016) 22-29

Directivity error experiment

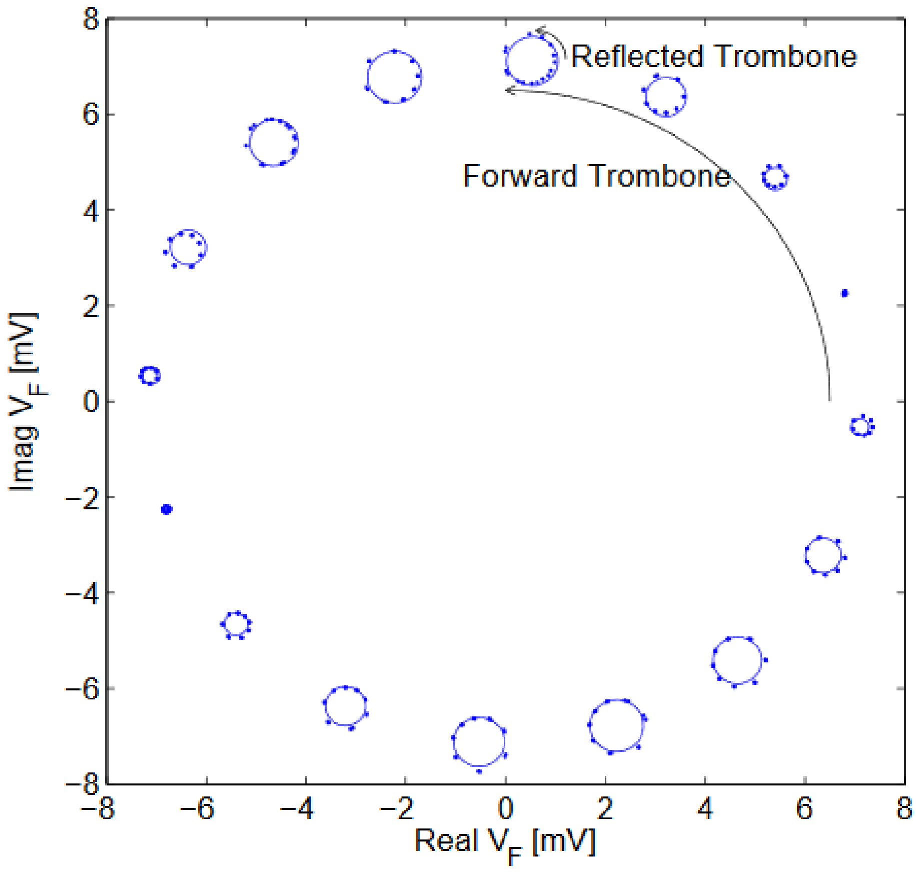


Figure 4: Direct directivity measurement including reflections from the VNA. Dots are measured values, circles are the fit as described. Measurement of an HP776D Duel Directional Coupler.

From J.P. Holzbauer, Yu. Pishalnikov, D. A. Sergatzkov, W. Schappert, S. Smith, *Nucl. Instrum. Meth.A* 830 (2016) 22-29

Bias on τ_L due circulator mismatch

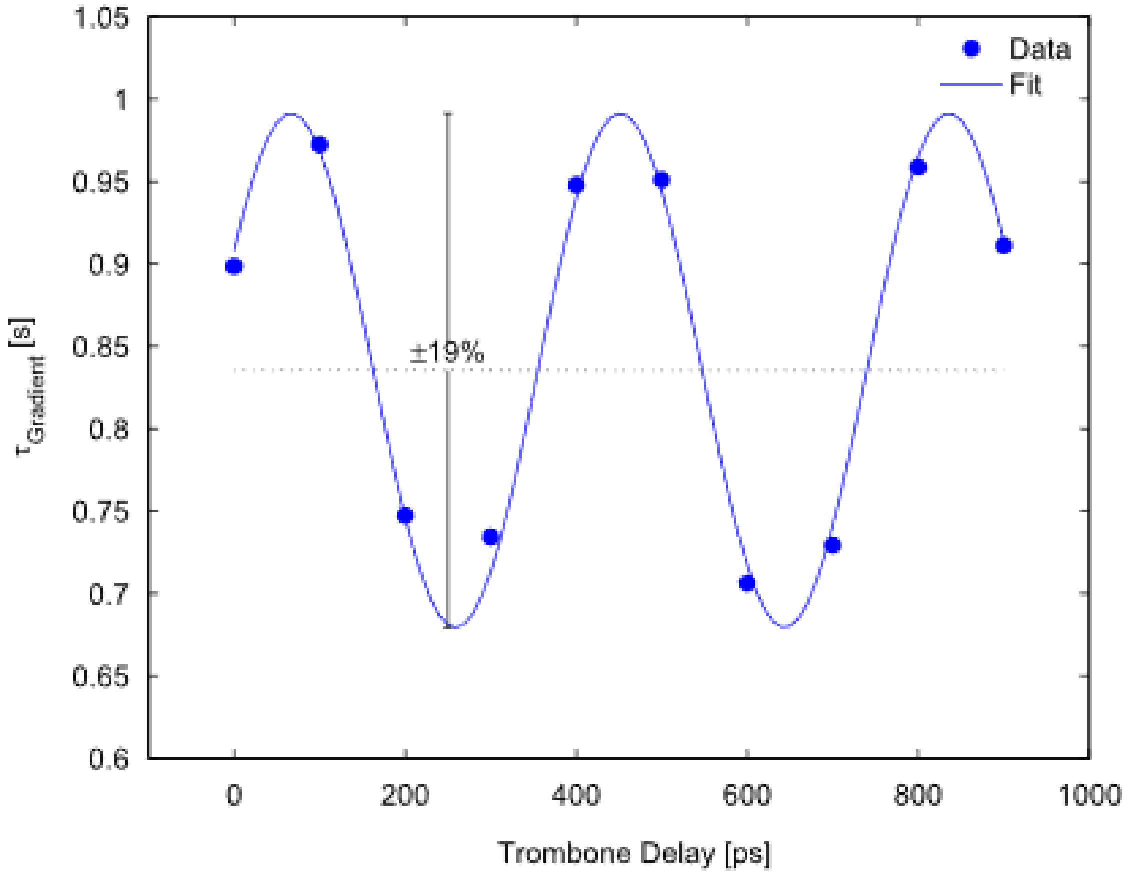
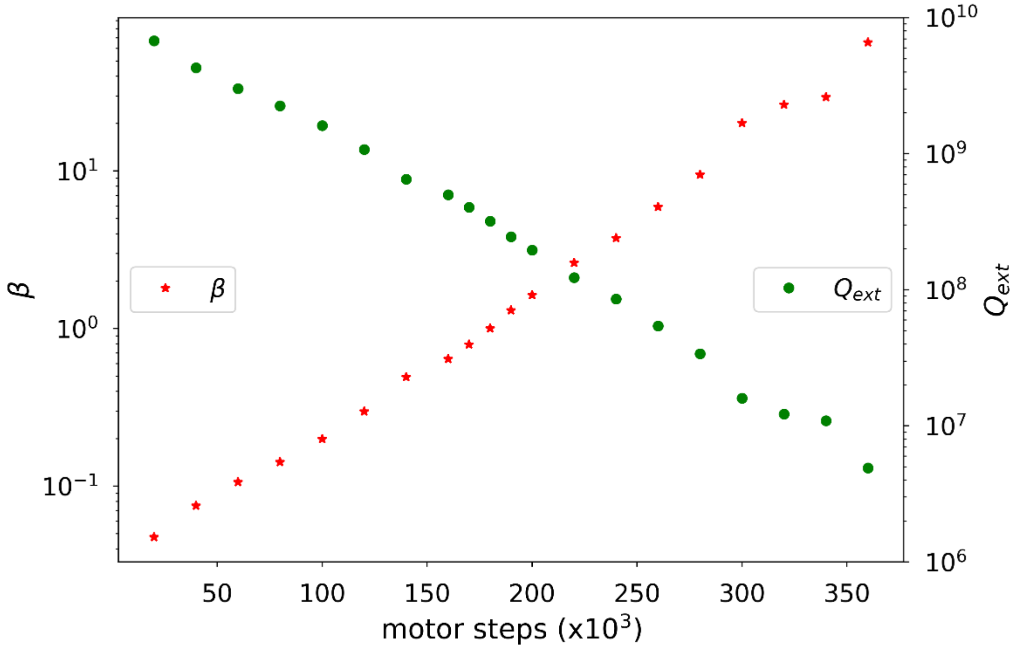


Fig. 6. Cavity decay time vs. trombone position as measured on cavity TB9ACC015, a 9-cell 1.3 GHz cavity at FNAL VTS, 7/14/2014.

Picture from J.P. Holzbauer, Yu. Pishalnikov, D. A. Sergatzkov, W. Schappert, S. Smith, *Nucl. Instrum. Meth.A* 830 (2016) 22-29

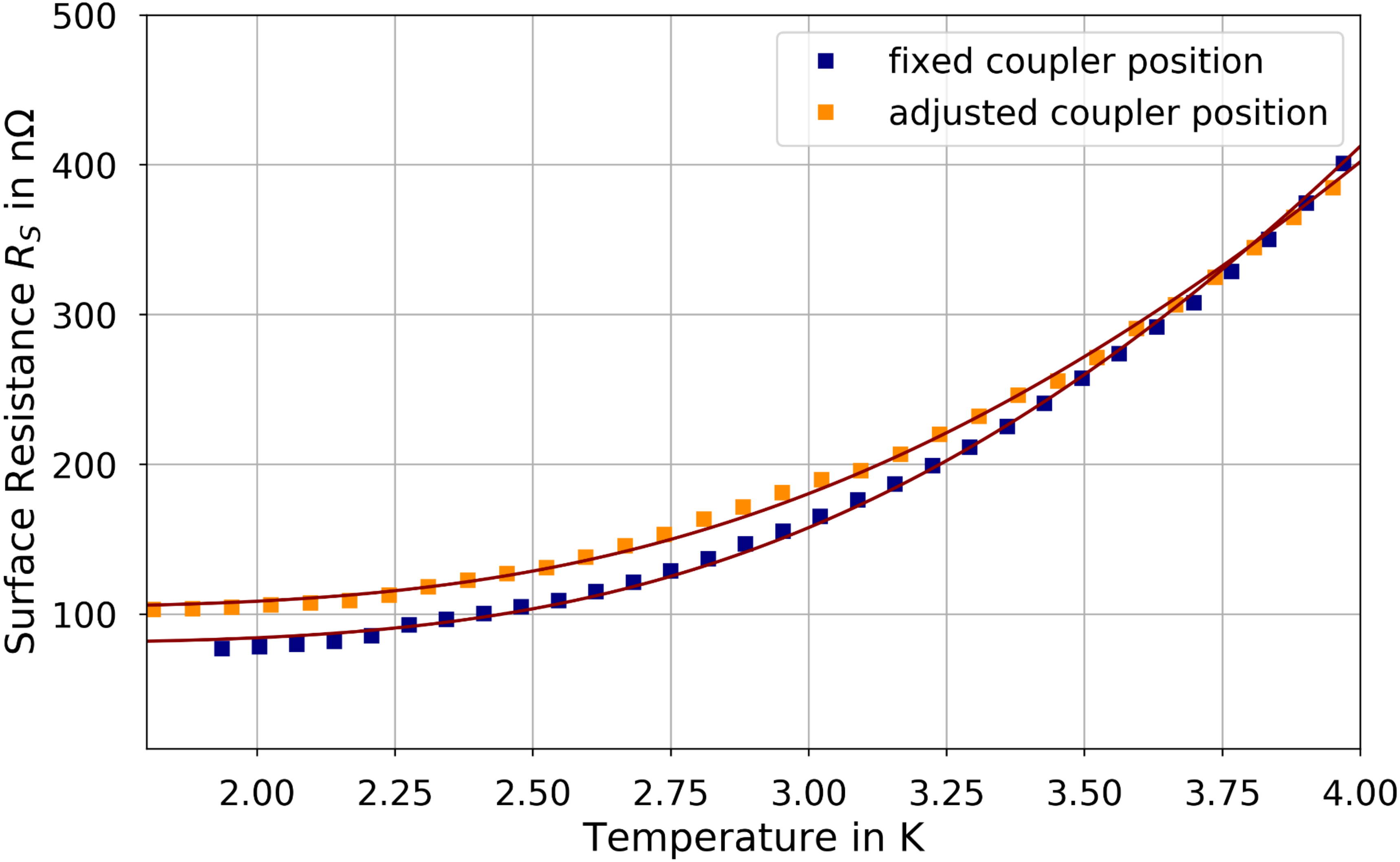
Improving measurement tools: variable coupler



1) Constant (and minimal) measurement uncertainty independent of cavity Q_0

2) Easiness of MP conditioning

Bias from reflected power



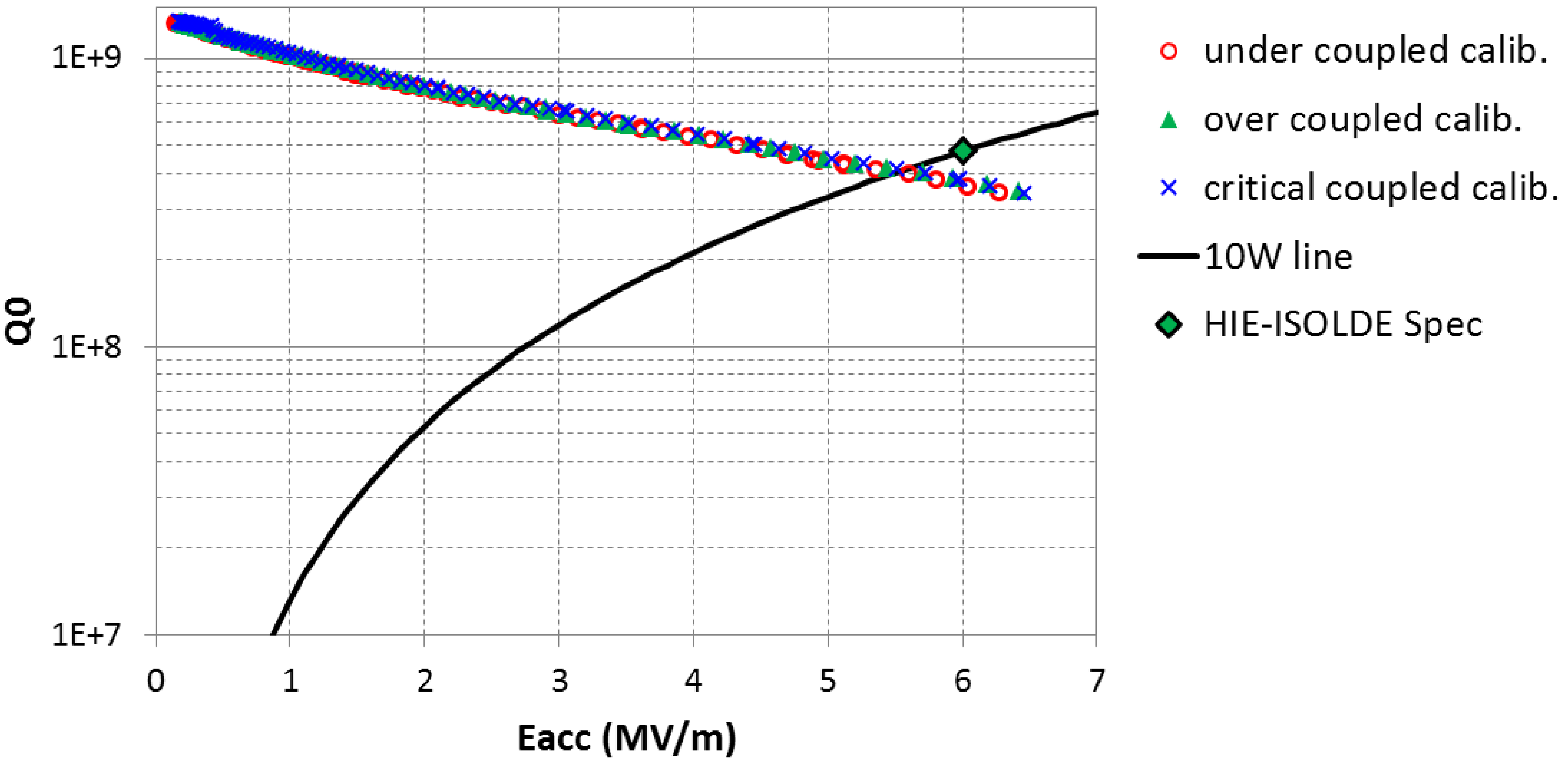
Simple cross checks

The external Q of fixed couplers should be constant at all field levels, and equal to the values found at the calibration step. It can be computed from Q_0 and β all along the Q-E scan.

with the phase locked-loop open and the generator drive frequency off resonance, Pr/Pf should stay close to unity at all power levels, and constant over a small bandwidth ($f - f_0 < \pm 1$ MHz)

Double-check the measurement

- Four different calibrations were conducted
  - over coupled, under coupled, critical coupled

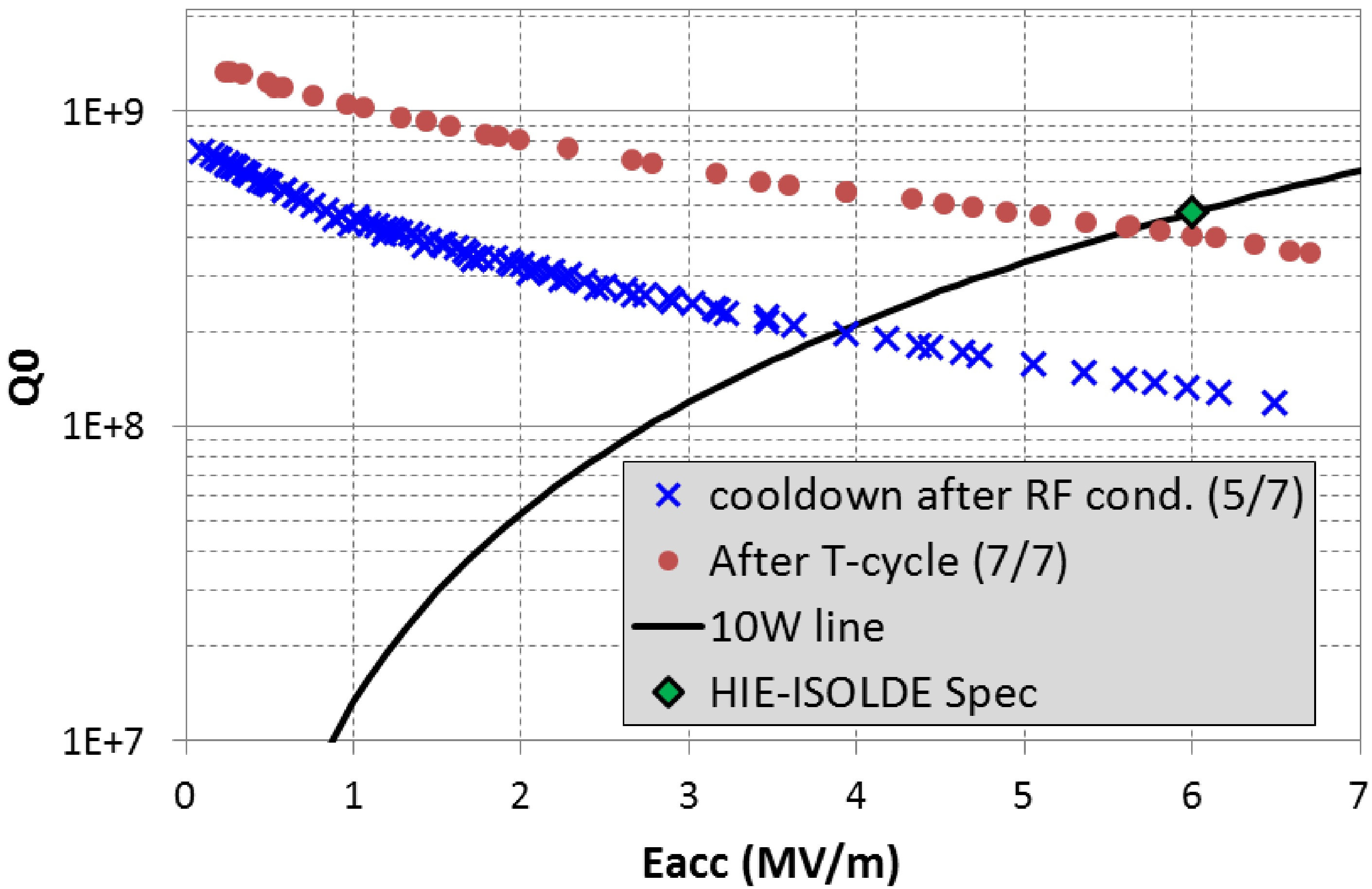


Cool down precautions

- The way the cavity is cooled down can influence the result: main effects are:
- «H disease»
- Flux trapping
- Thermo electric currents

Effects of thermal gradient when crossing Tc

- First cool down with high thermal gradient
- Thermal cycle above Tc, second cool down with more homogeneous temperature



Residual Resistance due to Flux trapping

$$R_{fl} = Bout\ \eta(l, \nabla T, ...)\ S(l, \omega, B_{RF})$$

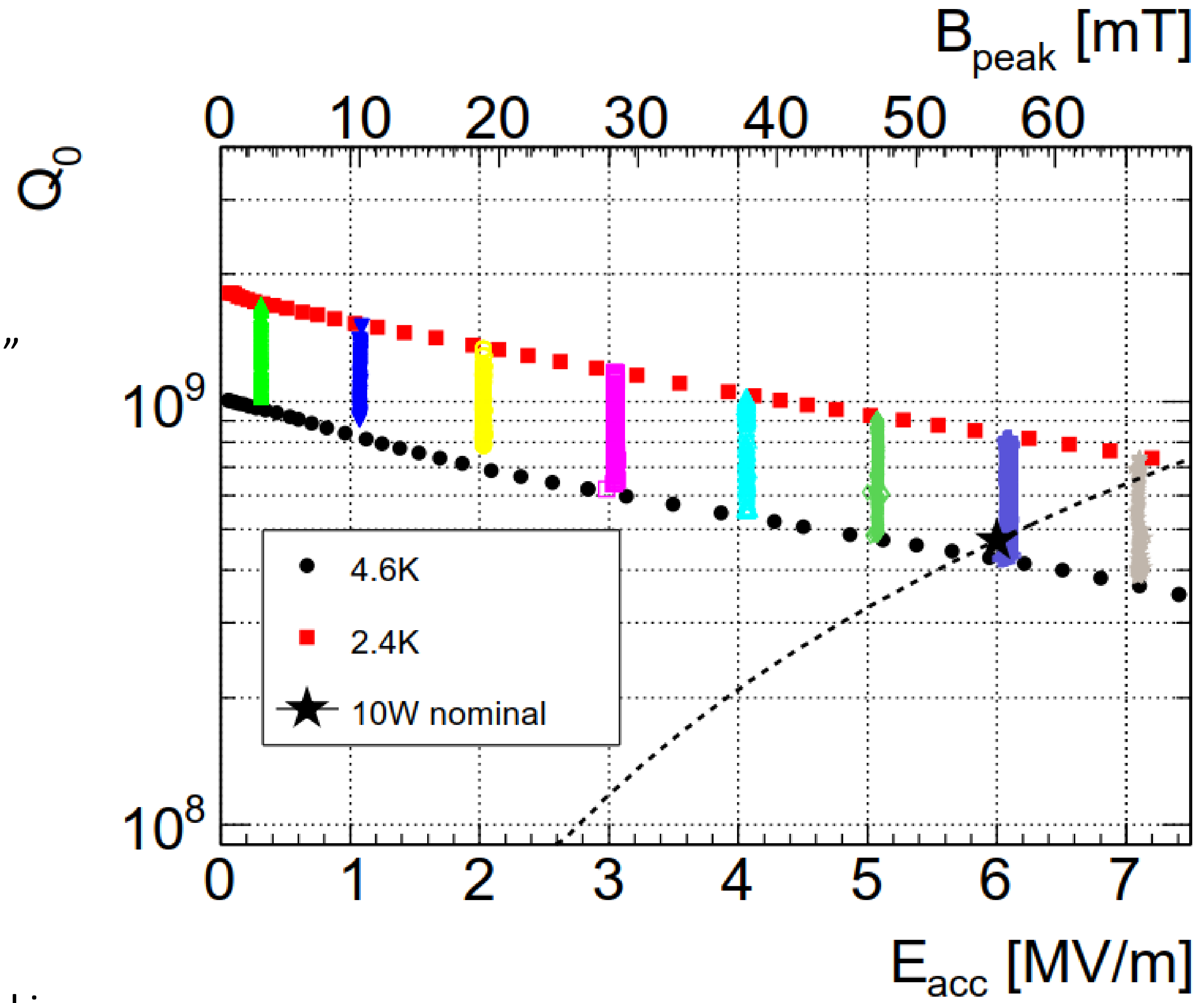
External magnetic field: B_{out} , B_a ,B_{NC} , B_{avg} , B_o

Expulsion **inefficiency**, or trapping efficiency, or trapping ratio, η_t , τ_{eq} , $B_{trapped}/B_{out}$. Related to the observable field enhancement B_{SC}/B_{NC} at cavity equator when crossing Tc (expulsion ratio, Posen ratio, r, ε_{eq})

Sensitivity (in nΩ/mG): S, R_0/B_{trap} , r_{fl}

Temperature and field Q scans

“Field cooled”



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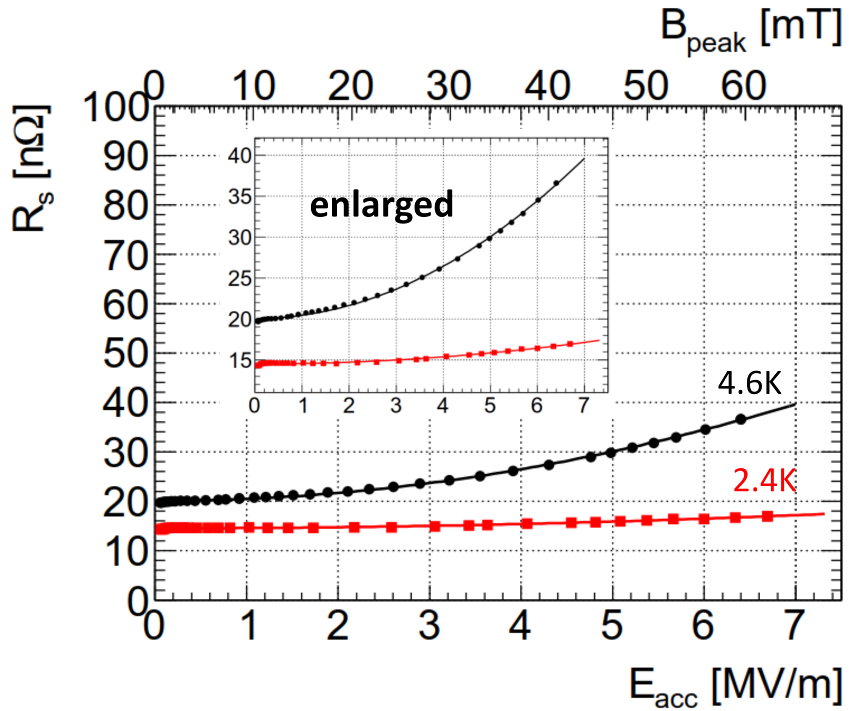
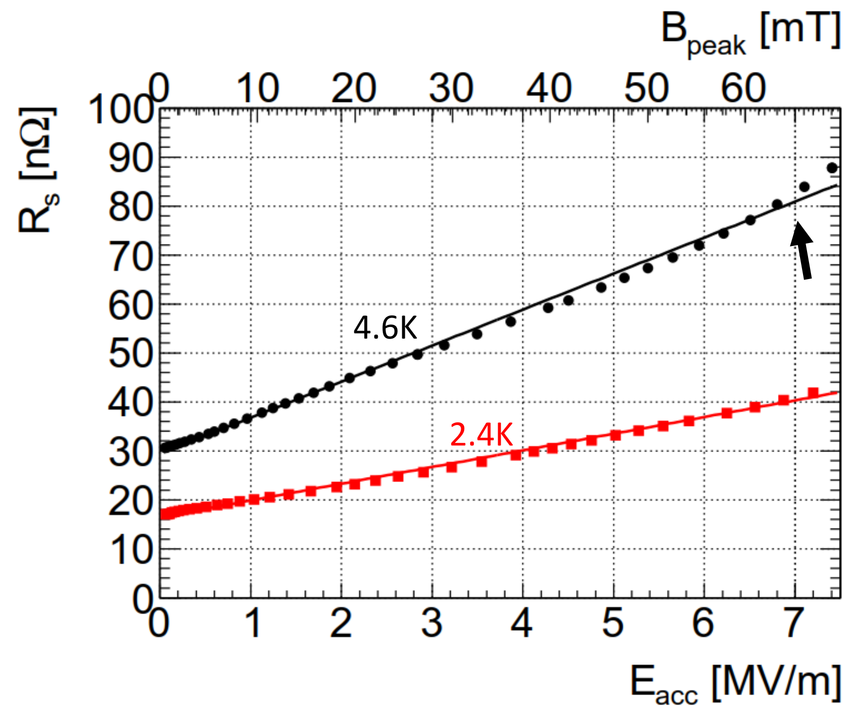
Analysis of Q slopes

B-field compensated when cavity crossed Tc



Intrinsic Q-slope is ***temperature dependent curvature*** component

B-field enhanced when cavity crossed Tc



Q-slope by trapped vortex is ***close to linear*** at low fields and ***weakly temperature dependent***

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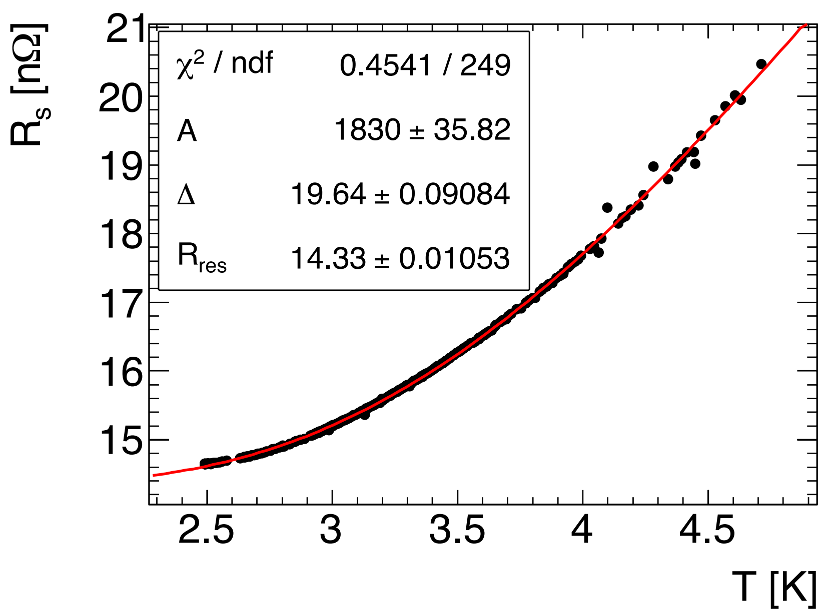
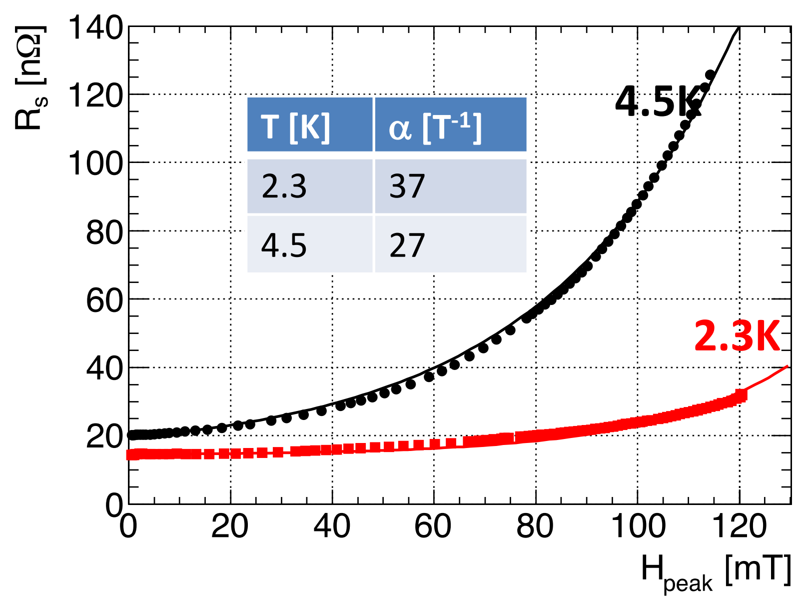
An empirically found formula to fit the non-linear Q-slope

$$R_s(T,B) = \frac{A}{T}\exp\left(-\frac{\Delta}{k_B T} + \alpha B\right) + R_{res} + R_{fl}B$$

(A, Δ, R_{res}) from R_s vs T data



Fit R_s vs H and determine α



Such an exponential dependence has been reported by others (bulk Nb and Nb/Cu)

1. R. L. Geng (Cornell) "Thermal analysis of a 200MHz Nb/Cu cavity" SRF2001
2. D. Longuevergne (IPNO) "Magnetic dependence of the enegy gap:…" SRF2013

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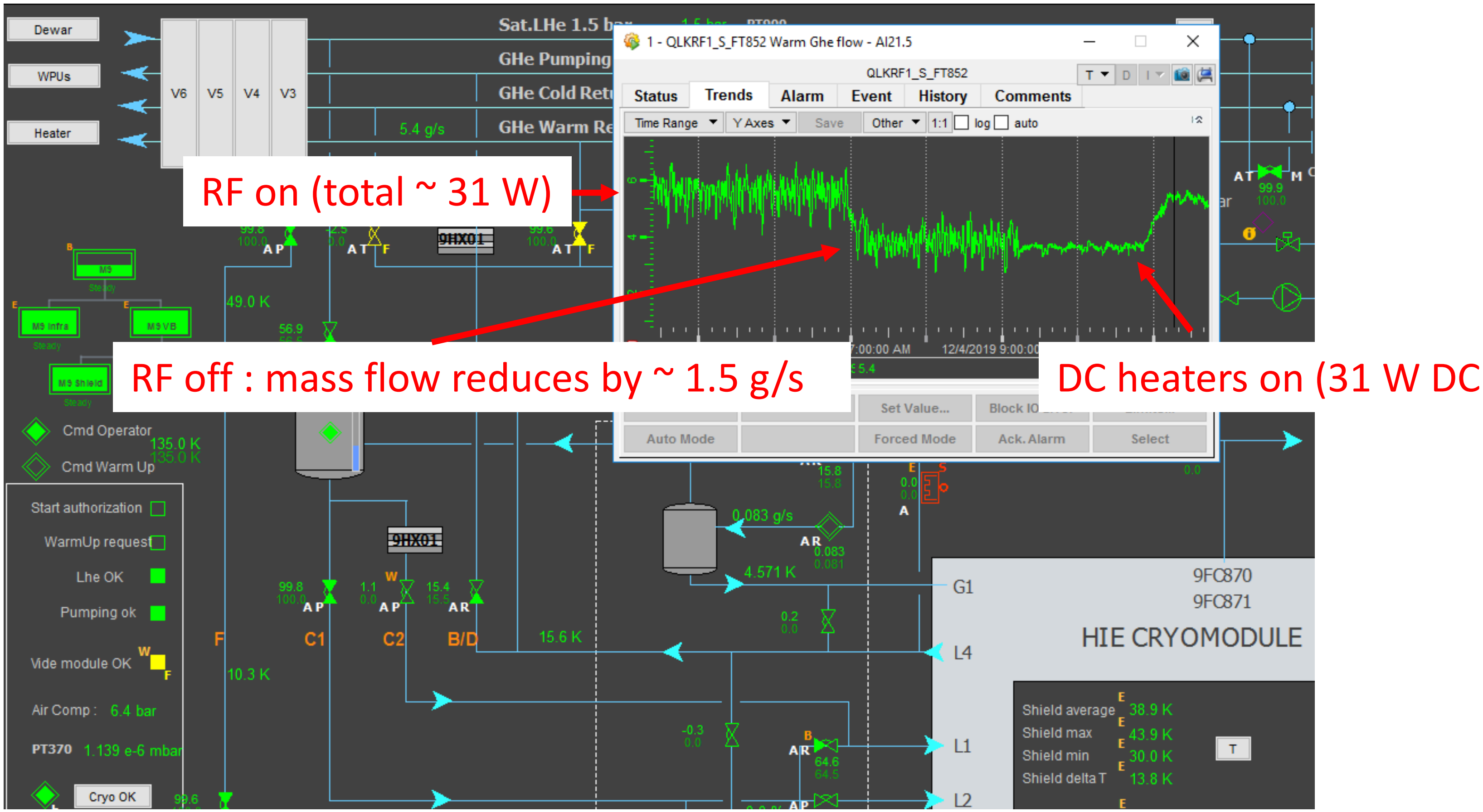
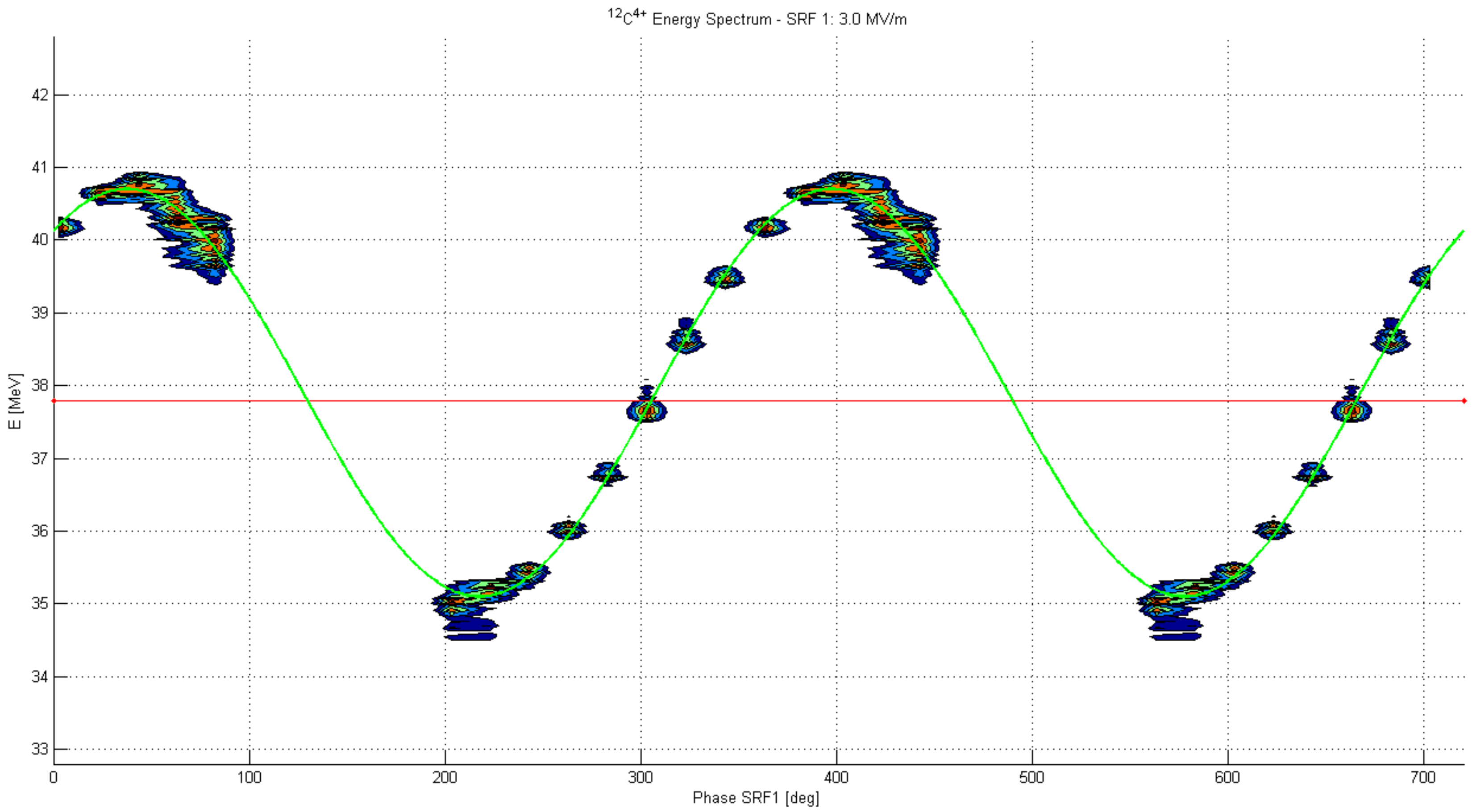
Cryomodule testing

- Cryomodules are complex units ready for installation in accelerator, the power coupler is already adapted to the beam loading (usually $\beta>>1$)
- Besides RF measurements, static heat loads and alignment are important
- Precise measurement of Q_0 with RF methods is impossible (without beam, almost all power is reflected back)
- Calorimetric methods are applied
- Field measurement are still possible (if pickup calibration was preserved in cavity assembly)

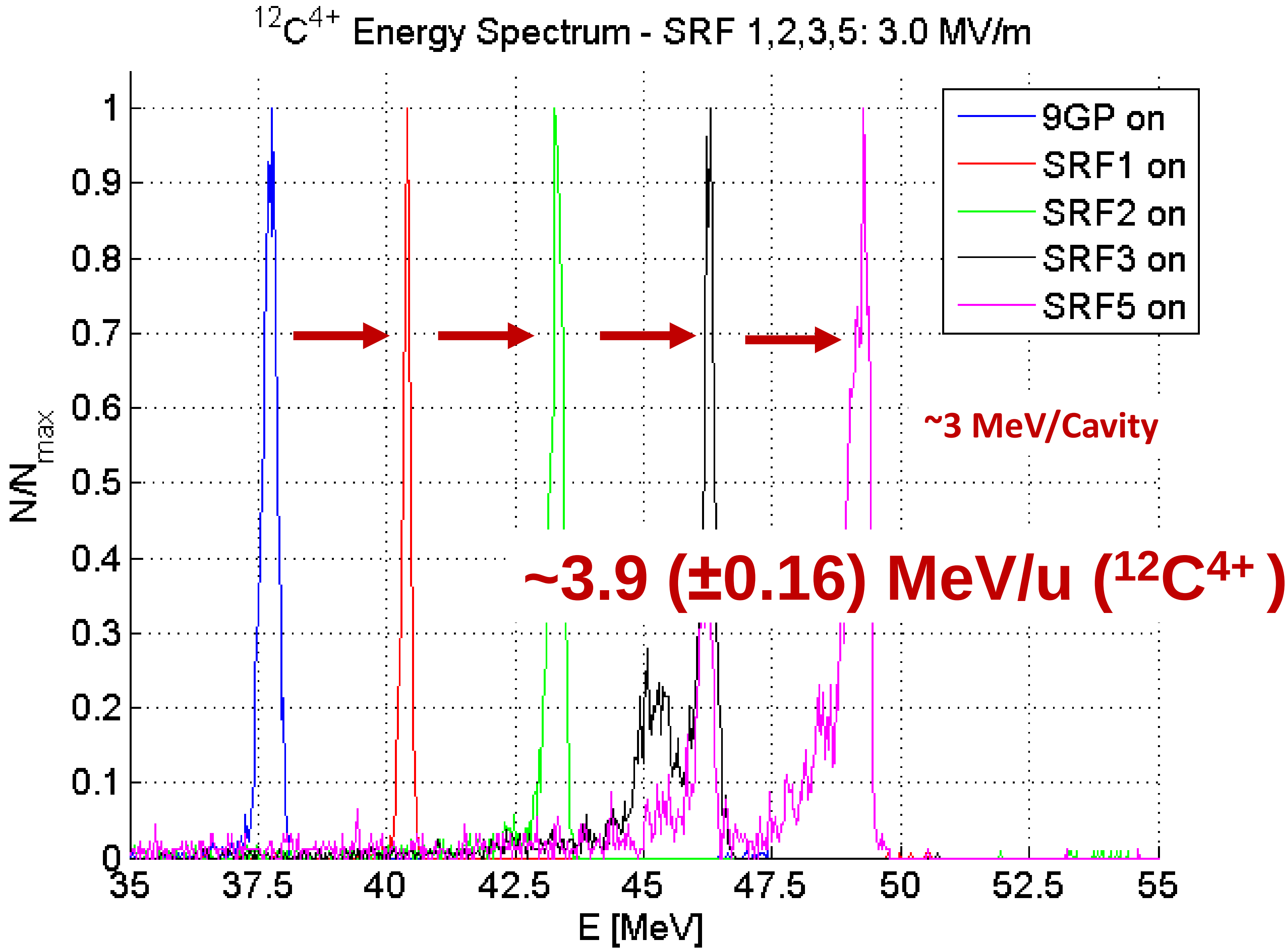
Example of dynamic heat load measurement on a cryomodule



First beam acceleration with a HIE ISOLDE superconducting QWR



Linac phasing with 4 cavities: first try



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