

# Diffusion of heavy quarks in the early stages of high energy nuclear collisions

Marco Ruggieri

*School of Nuclear Science and Technology  
Lanzhou University, Lanzhou, China  
ruggieri@lzu.edu.cn*

## Collaborators

Pooja  
Dana Avramescu  
David Muller  
Santosh K. Das  
Vincenzo Greco  
Virgil Baran  
JunHong Liu  
YiFeng Sun  
Lucia Oliva



# Plan of the talk

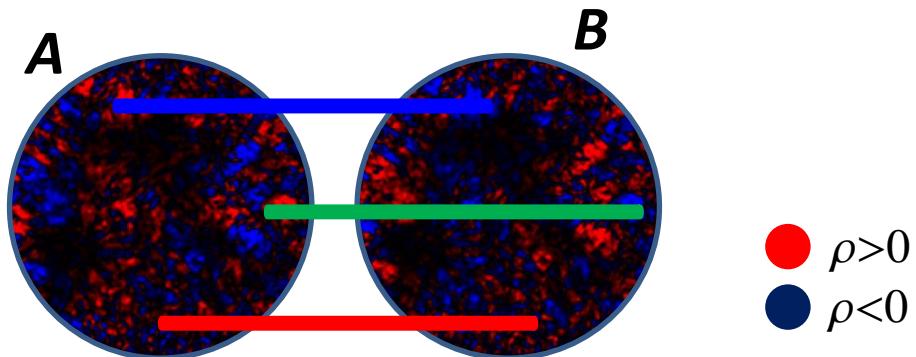


- ❖ *Early stages of heavy ion collisions: evolving Glasma (Ev-Glasma)*
- ❖ *Heavy quarks diffusion: momentum broadening,  $R_{AA}$ ,  $v_2$*
- ❖ *Conclusions and Outlook*



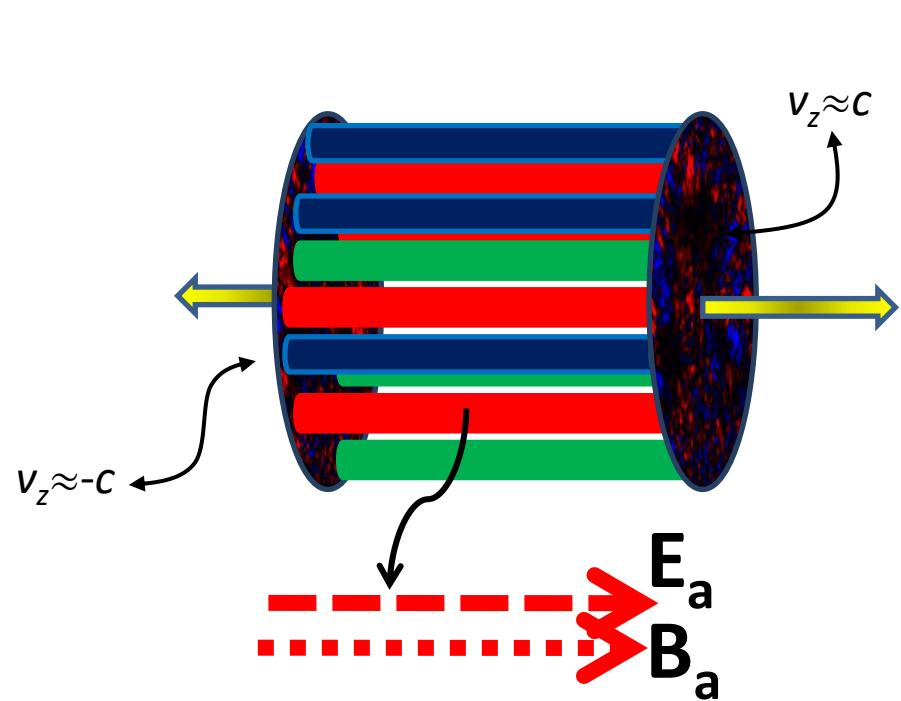
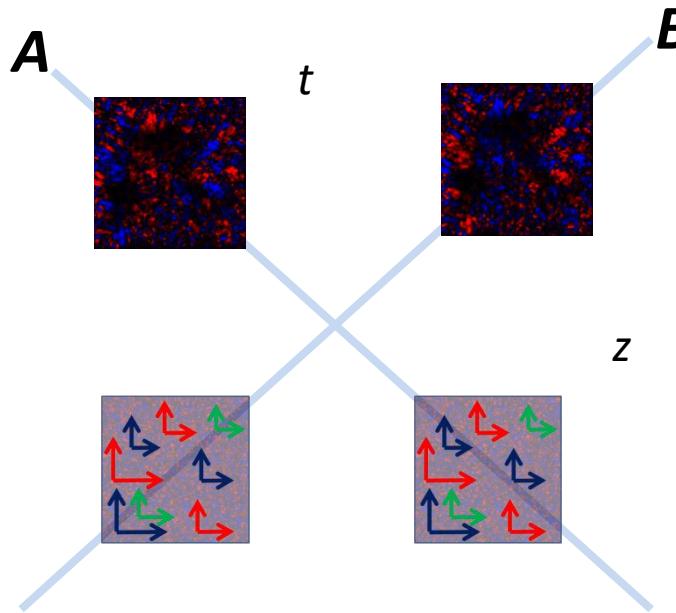
# Glasma: the initial condition of RHICs

Dense gluons **interact** and form two sets of opposite **effective color charges** on the light cone of the two nuclei.



$$\begin{array}{l} \textcolor{red}{\bullet} \quad \rho > 0 \\ \textcolor{blue}{\bullet} \quad \rho < 0 \end{array}$$

$$\begin{aligned} \nabla \cdot \mathbf{E} &= \rho_E \\ \nabla \cdot \mathbf{B} &= \rho_B \end{aligned}$$



# The MV model of color sources

- Fast (large momentum) partons

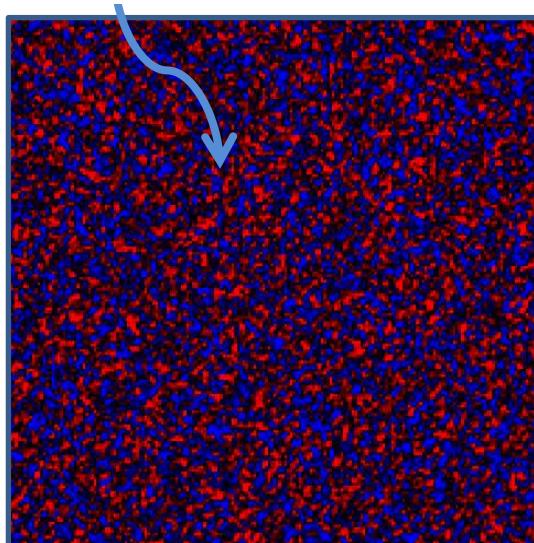
Their dynamics in the Lab frame is slowed down due to time dilation: **static sources of color fields**.

## Model of static sources (MV model)

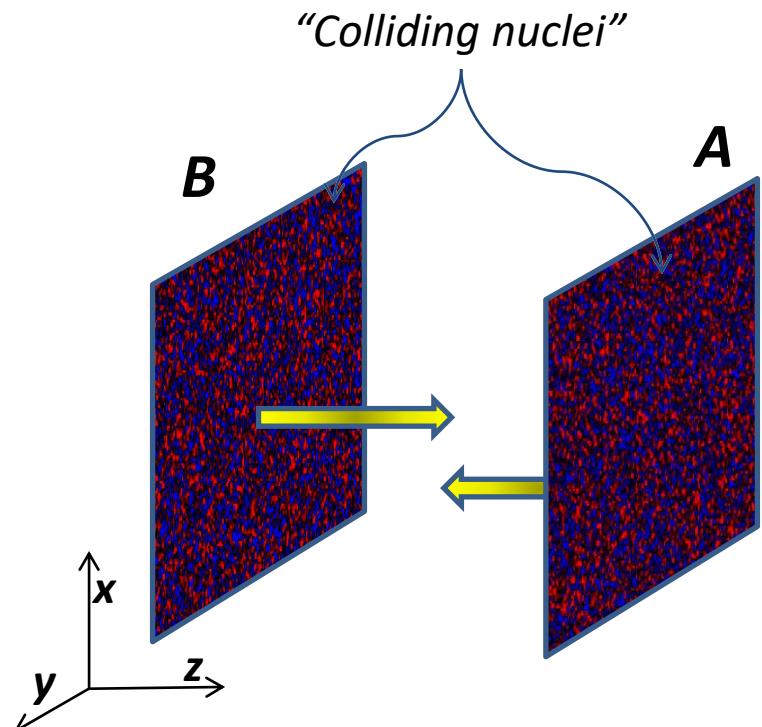
Uncorrelated color density fluctuations on the two nuclei.

$$\langle \rho^a(x_T) \rangle = 0,$$

$$\langle \rho_A^a(x_T) \rho_A^b(y_T) \rangle = (g\mu_A)^2 \delta^{ab} \delta^{(2)}(x_T - y_T)$$



●  $\rho > 0$   
●  $\rho < 0$



$g^2\mu \approx Q_s$ : saturation scale

*From the correlators of Wilson lines:*

$$g^2\mu = O(Q_s)$$

Lappi (2008)

$$Q_s \approx 1 - 3 \text{ GeV}$$

McLerran and Venugopalan (1996)  
Kovchegov (1996)

# Classical Yang-Mills equations

Due to the large density the gluon field behaves like a classical field:

*Dynamics is governed by classical EoMs, namely the classical Yang-Mills (CYM) equations.*

$$(D^\mu F_{\mu\nu})^a = 0,$$

$$\begin{aligned} \partial_\tau E_i &= \frac{1}{\tau} \mathcal{D}_\eta F_{\eta i} + \tau \mathcal{D}_j F_{ji}, & E_i &= \tau \partial_\tau A_i, \\ \partial_\tau E_\eta &= \frac{1}{\tau} \mathcal{D}_j F_{j\eta}, & E_\eta &= \frac{1}{\tau} \partial_\tau A_\eta. \end{aligned}$$

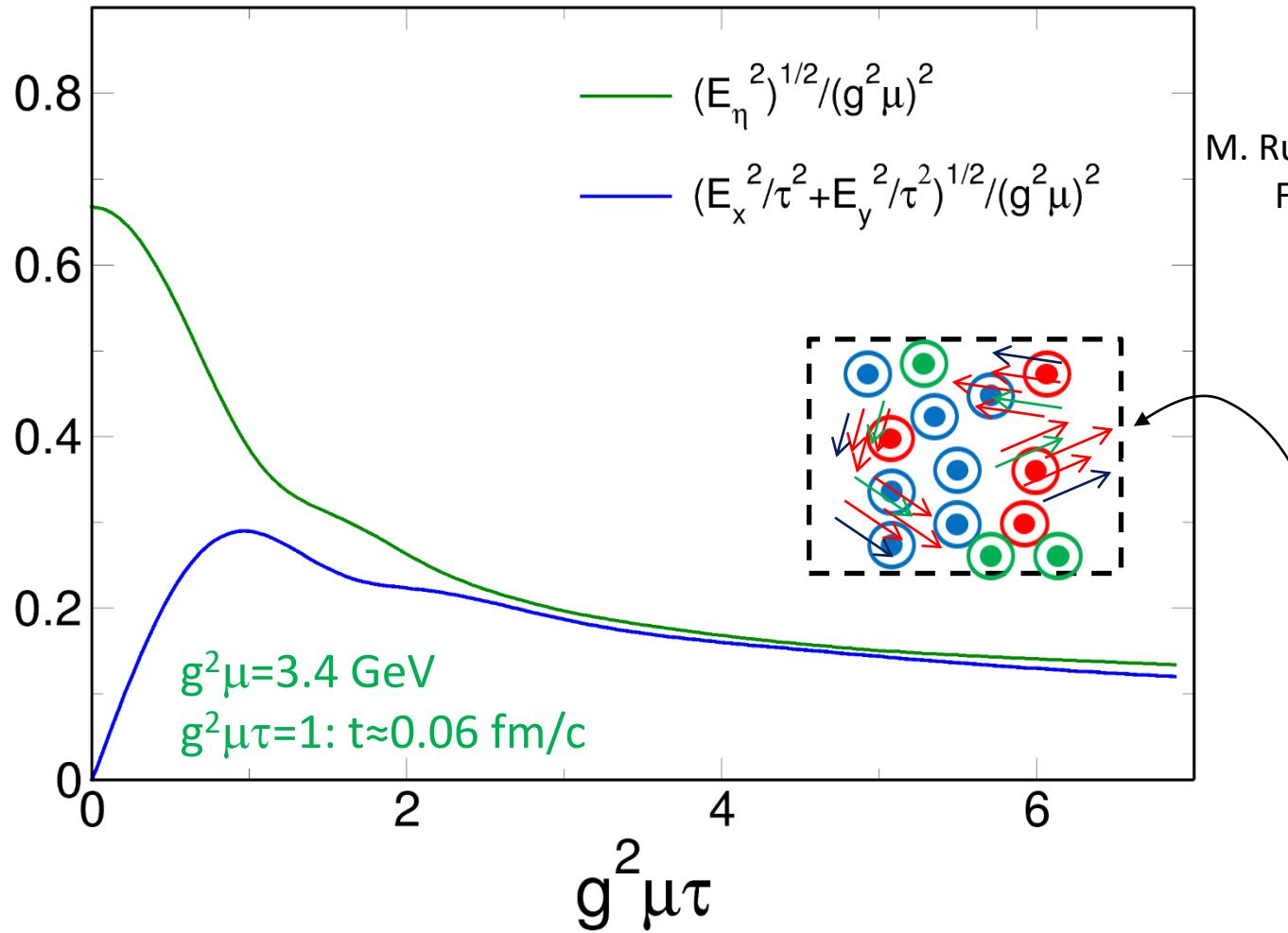
$$\begin{aligned} \tau &= \sqrt{t^2 - z^2} \\ \eta &= \frac{1}{2} \log \left( \frac{t+z}{t-z} \right) \end{aligned}$$

and the **chromo-magnetic field** is defined as

$$B_i = -\epsilon^{ij} F_{j\eta} \quad B_\eta = -\frac{1}{2} \epsilon^{ij} F_{ij}$$

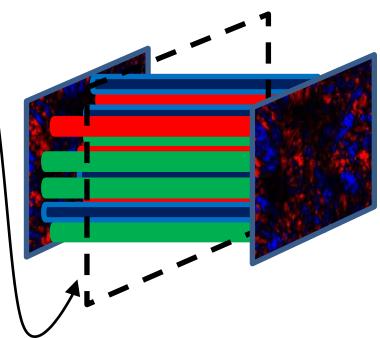
*Evolution of the system is studied assuming the Glasma initial condition, and evolving this condition by virtue of the CYM equations.*

# Ev-Glasma: color fields



See also

- M. Ruggieri *et al.* (2019)
- M. Ruggieri and S. K. Das (2018)
- Fukushima and Gelis (2011)
- Ohnishi *et al.* (2011)
- Lappi and McLerran (2006)



$$\left. \frac{dE_a^x}{dt} \right|_{t=0^+} = \partial_y B_z^a + f_{abc} A_y^b B_z^c$$

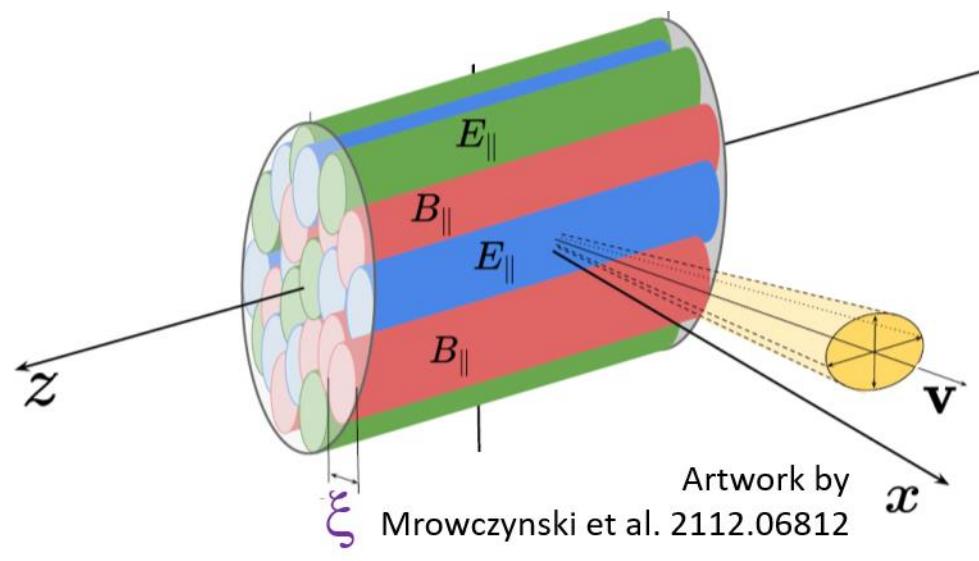
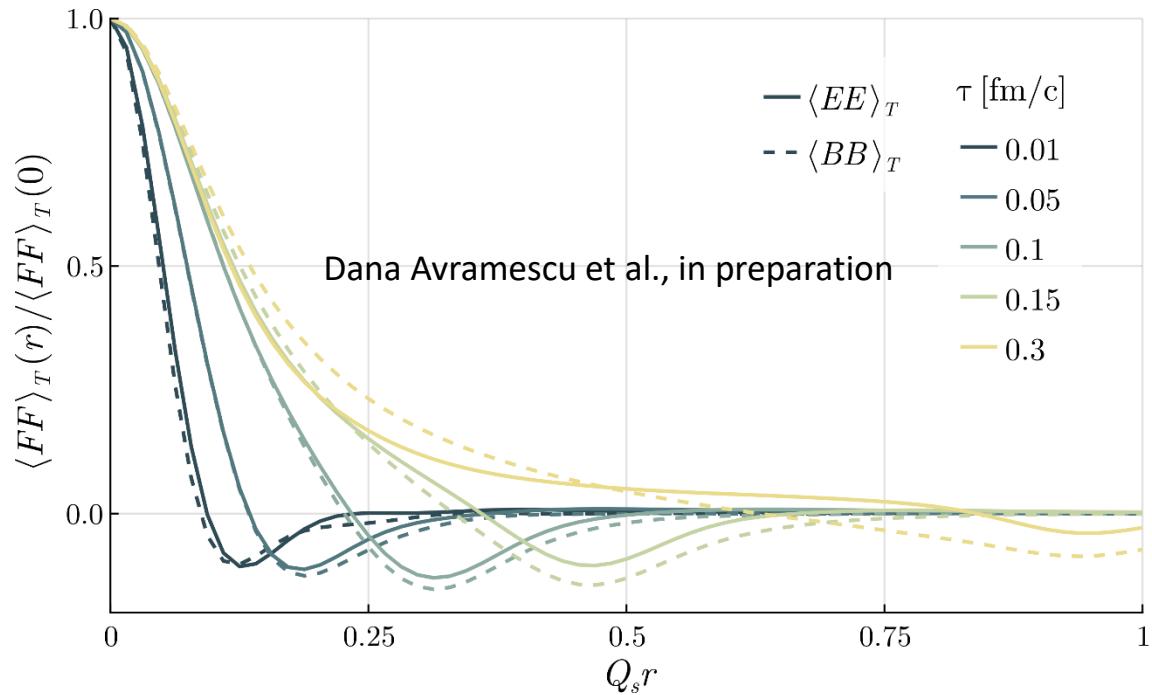
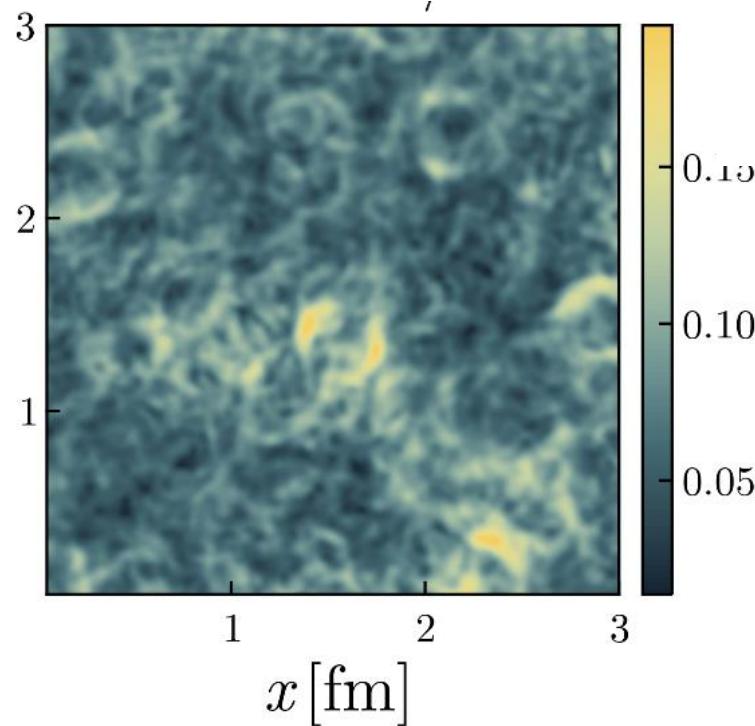
*Formation time of transverse fields:*  
 $Q_s \tau \approx 1$  namely  $\tau \approx 0.1 \text{ fm}/c$

# Correlator of transverse ( $E_x, E_y, B_x, B_y$ ) color fields

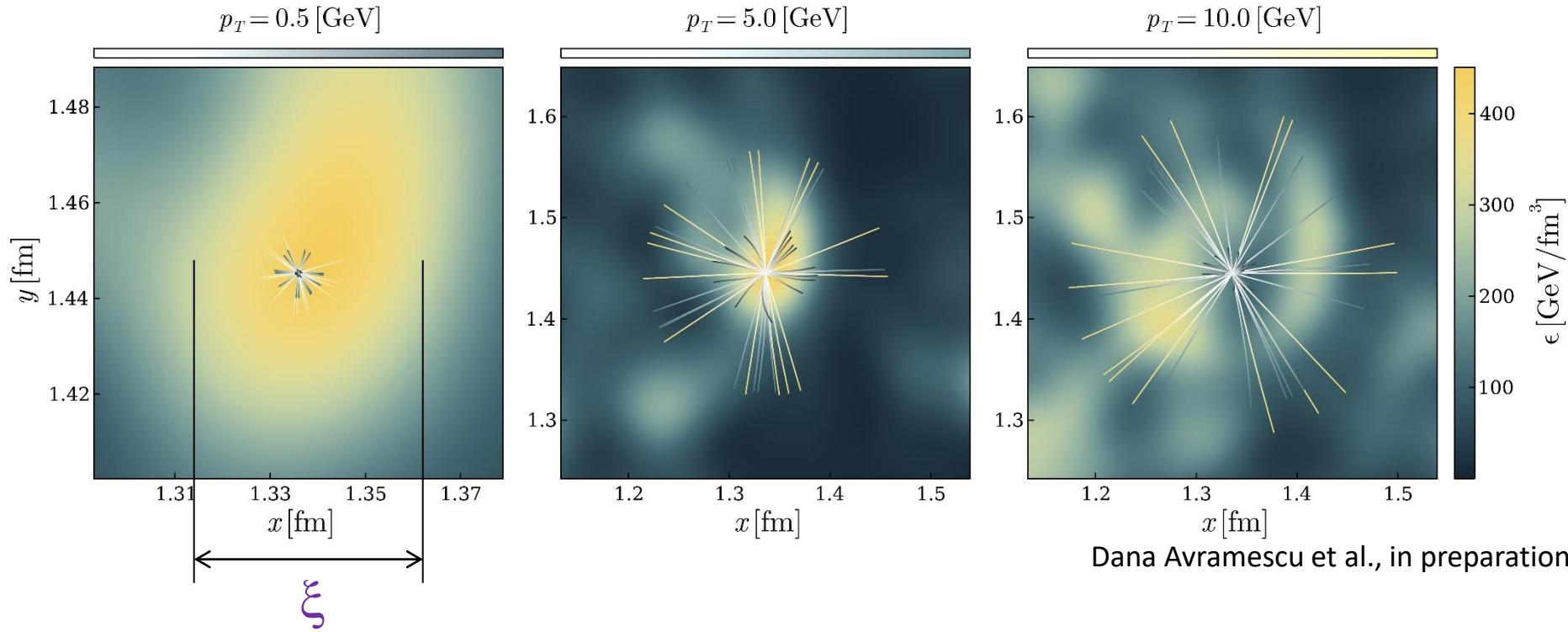
$$\langle FF \rangle_T \propto e^{-r/\xi}$$

$\xi$ : correlation length

Fields arrange in  
correlation domains,  
(aka flux tubes, filaments) of  
transverse area  $\approx \xi^2 = O(1/Q_s^2)$



# Color charges in the filaments

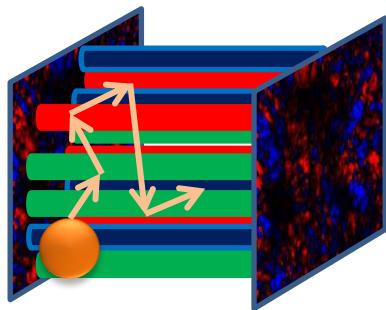


Dana Avramescu et al., in preparation

*Slow color charges spend some time within one single filament: diffusion in a coherent field, rather than in a random medium.*

*The force exerted on these charges is time-correlated.*

# Heavy quarks as probes of the evolving Glasma



$$t_{\text{formation}} \approx \frac{1}{2m_c} \approx 0.06 \text{ fm/c}$$



*HQs can probe the very early evolution of the Glasma fields*

Equations of motion of c-quarks [Wong (1979); Heinz (1985)]:

$$\frac{dx_i}{dt} = \frac{p_i}{E} \quad E = \sqrt{\mathbf{p}^2 + m^2}$$

$$v \equiv \frac{\mathbf{p}}{E} \quad (\text{Relativistic}) \text{ Velocity}$$

$$E \frac{dp_i}{dt} = g Q_a F_{i\nu}^a p^\nu$$

$$\frac{d\mathbf{p}}{dt} = q\mathbf{E} + q(\mathbf{v} \times \mathbf{B}) \quad \text{Lorentz force}$$

$$E \frac{dQ_a}{dt} = -g Q_c \epsilon^{cba} \mathbf{A}_b \cdot \mathbf{p}$$

$$D_\mu J_a^\mu = 0 \quad \text{Gauge-invariant conservation of the color current carried by heavy quarks + gluons}$$

$$J_a^\mu = \bar{c} \gamma^\mu T_a c$$

$$Q_a = \langle T_a \rangle$$

*Correspondence between the color charges and the SU(3)-color generators, valid in the Classical Limit*

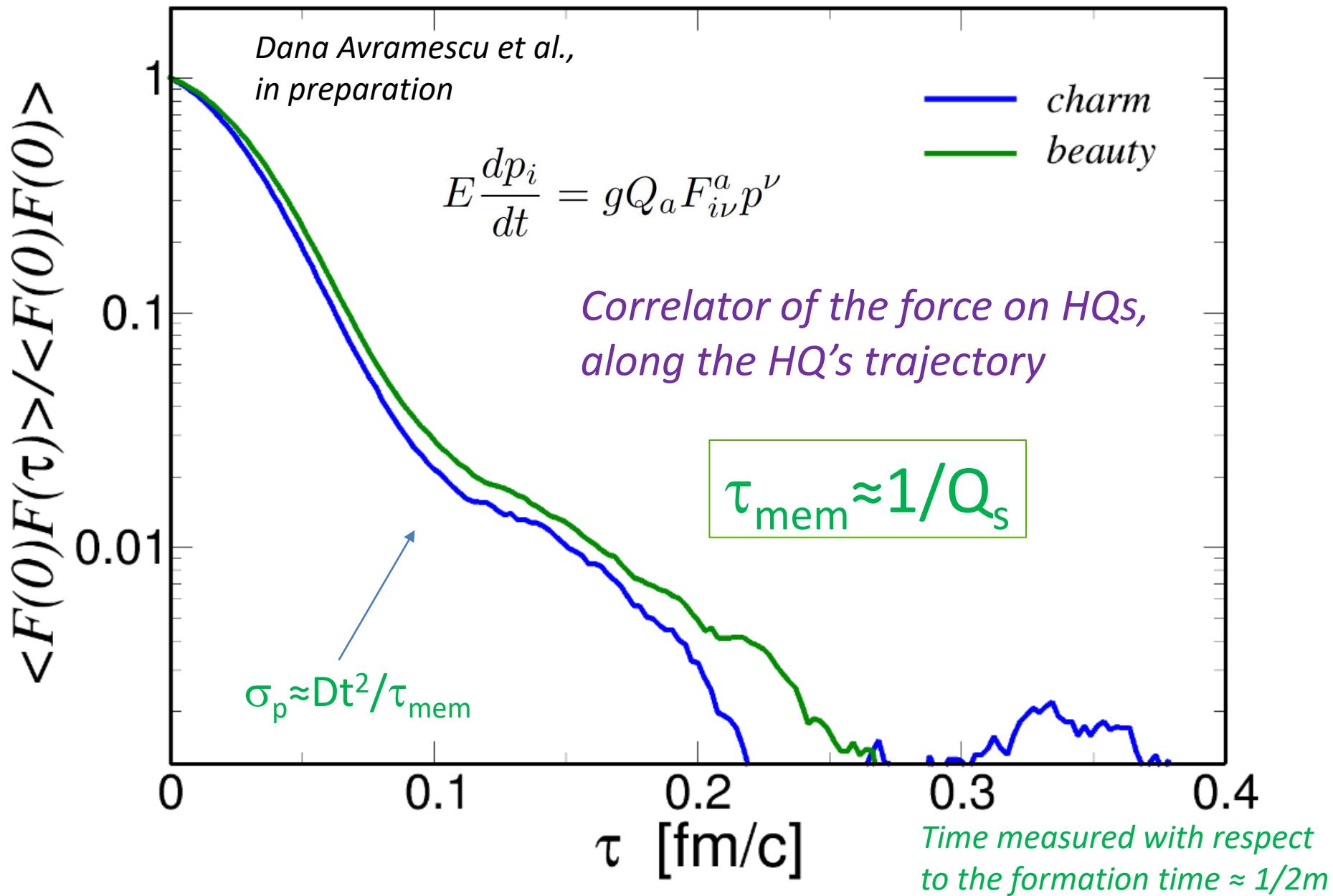
Rotation in color space

$$Q_a Q_a$$

$$d_{abc} Q_a Q_b Q_c$$

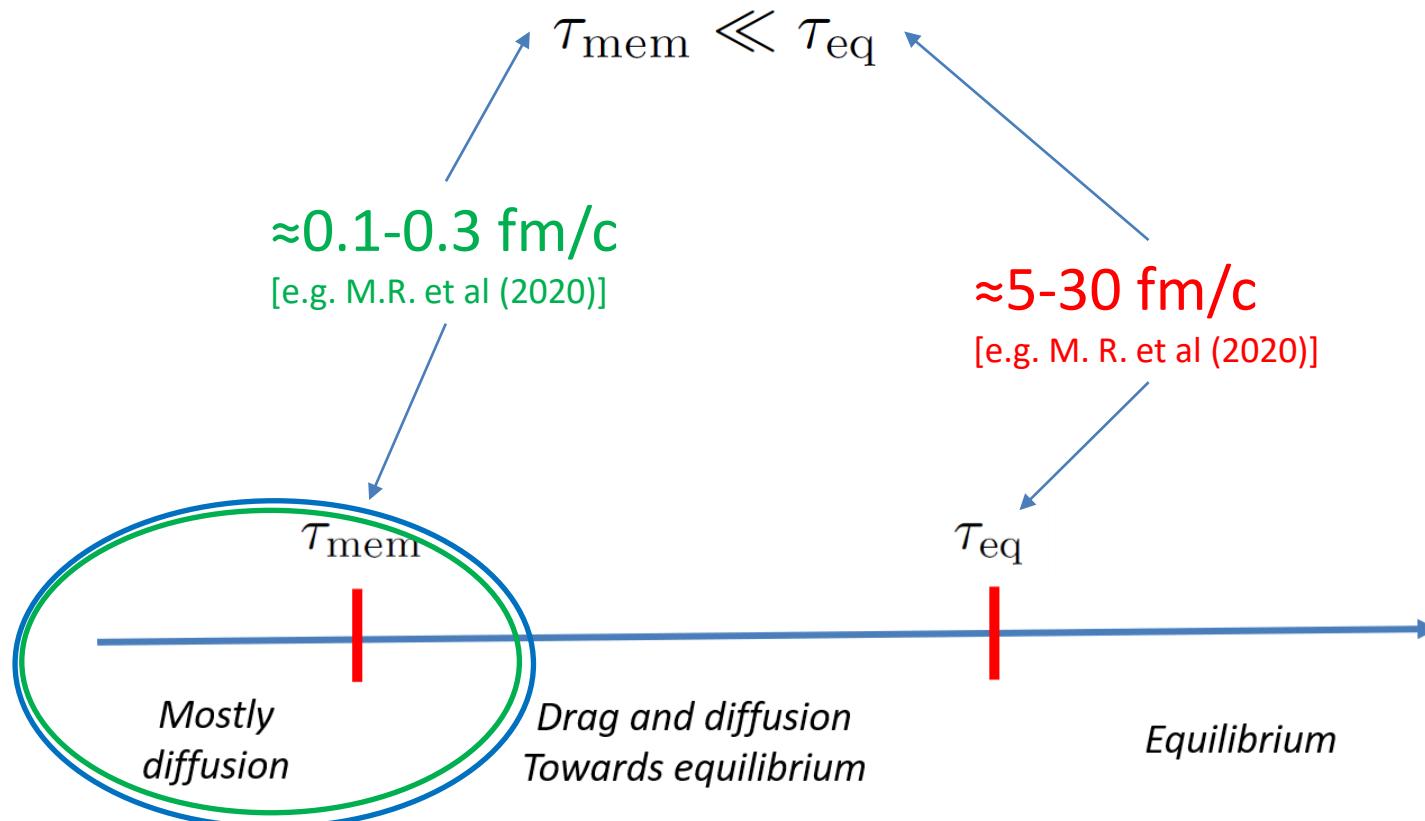
*Conserved during the evolution*

# Memory for the HQs diffusion in EvGlasma

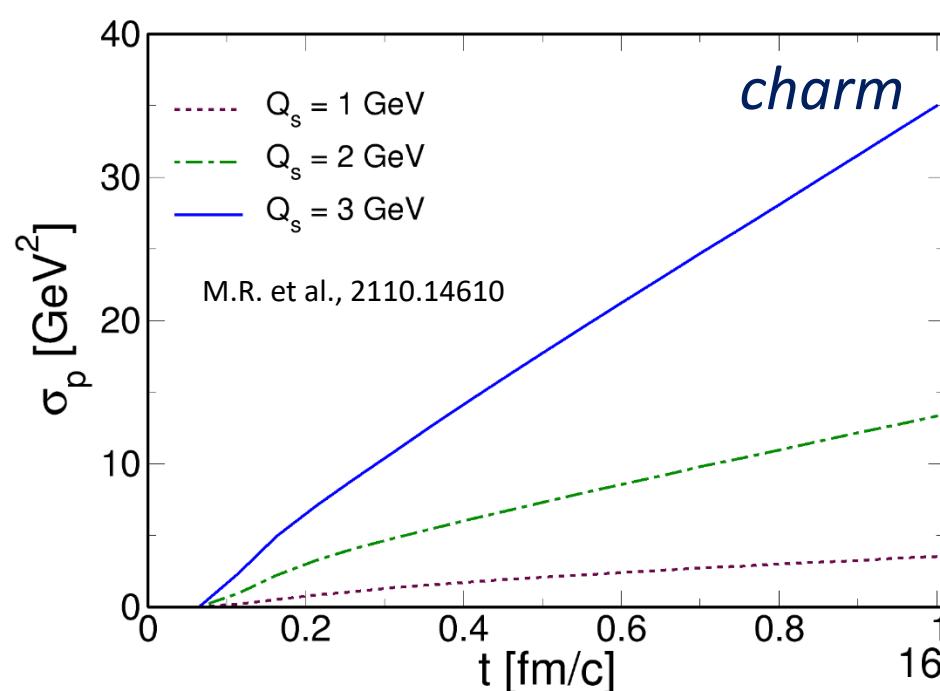


# Heavy quarks in Glasma: diffusion-dominated motion

EvGlasma lifetime  $\approx$  QGP thermalization time  $\approx 0.3\text{-}0.6 \text{ fm}/c$



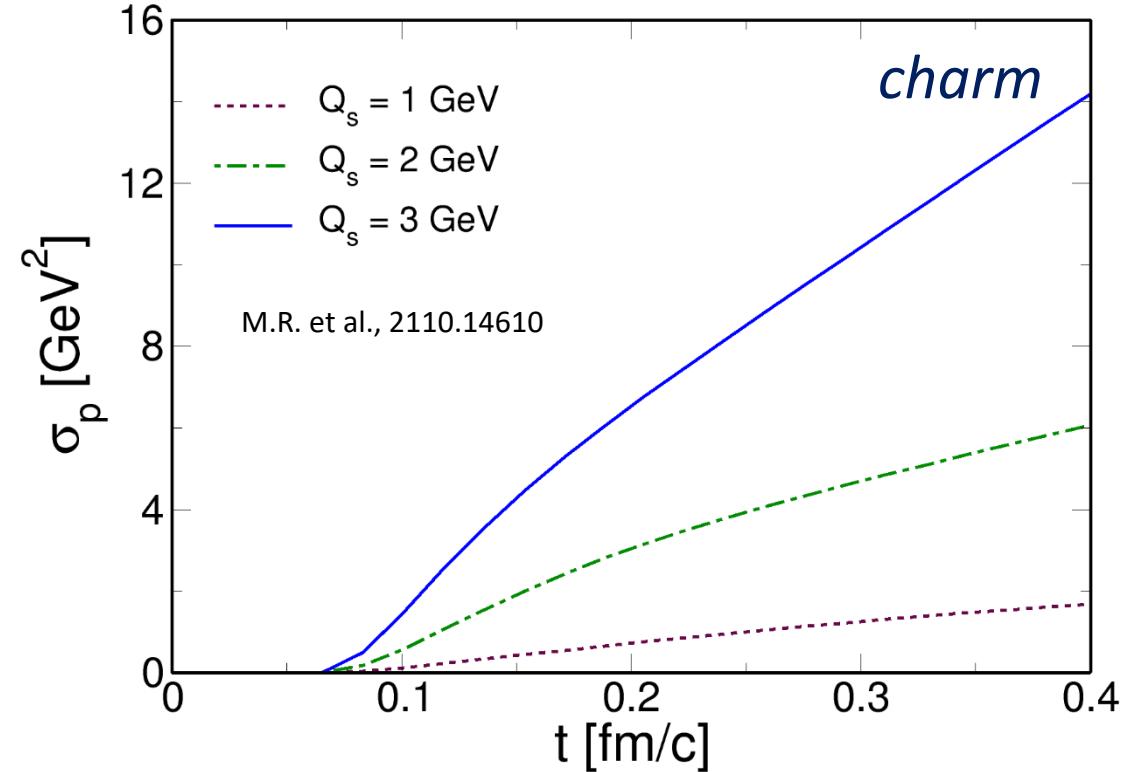
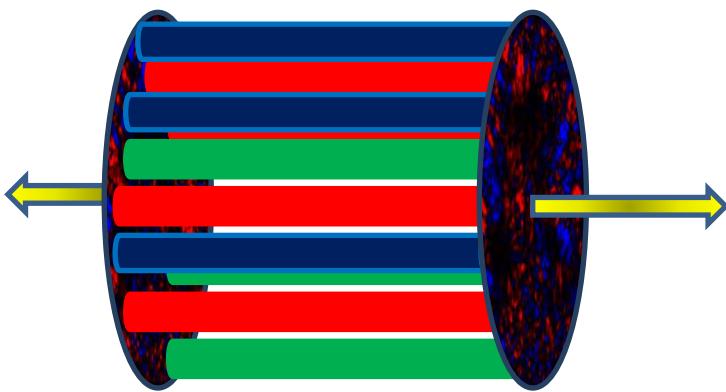
- *Diffusion dominates because memory time is much smaller than the equilibration time*
- *Memory leads to nonlinear evolution of  $\sigma_p$*



# Momentum broadening

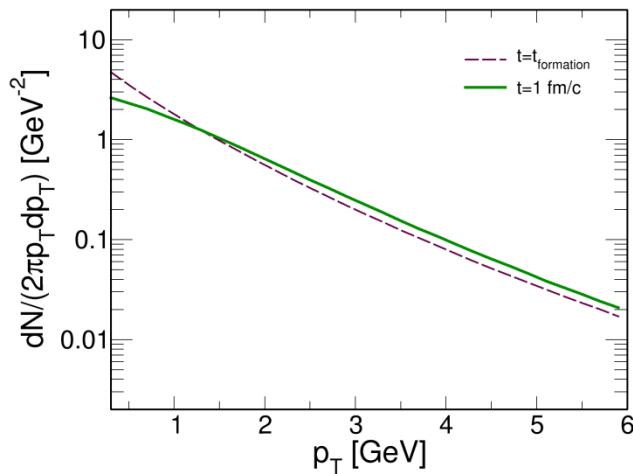
*Effect of memory can be read as diffusion within a coherent flux tube as well.*

- Early time:  $\sigma_p \approx D t^2 / \tau_{\text{mem}}$
- Later time:  $\sigma_p \approx 2 D t$



# Impact on RAA

p-Pb @ 5.02 TeV



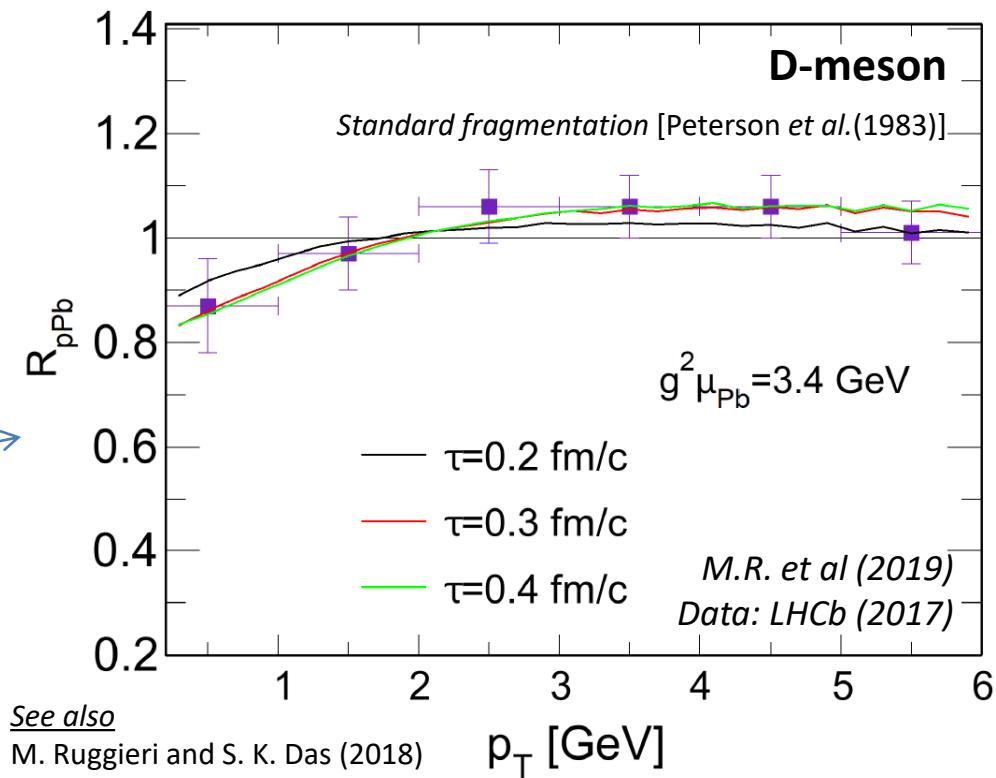
$$R_{\text{pPb}} = \frac{(dN/d^2p_T)_{\text{final}}}{(dN/d^2p_T)_{\text{pQCD}}}$$

$$R_{\text{pPb}} \neq 1$$

*Interaction with the fields created by the collision*

## Initial distribution

*From perturbative QCD, aka **prompt** [Scardina et al. (2017)]*

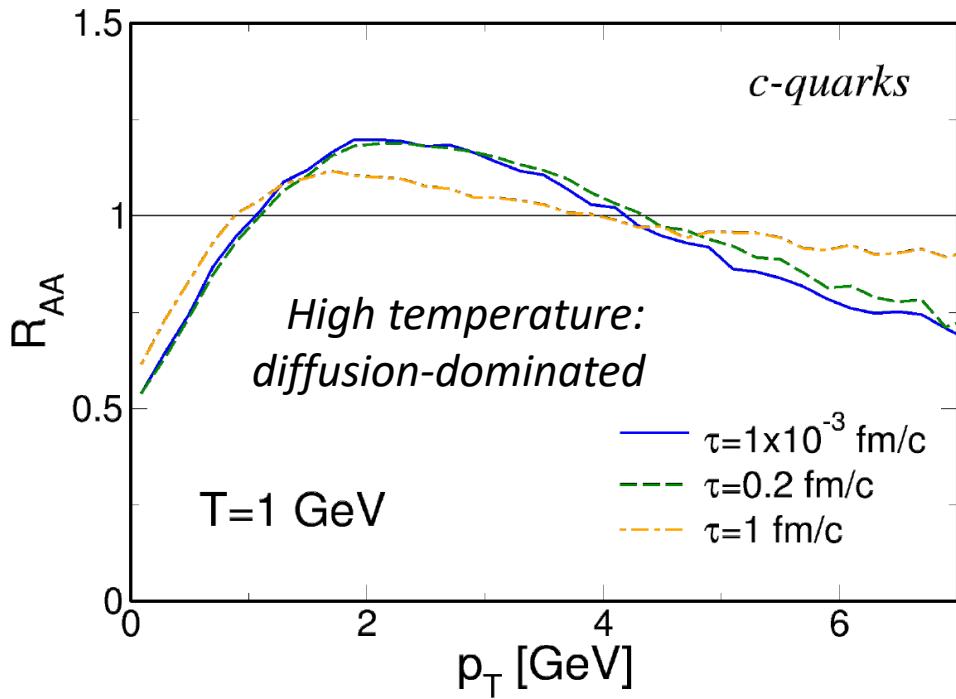
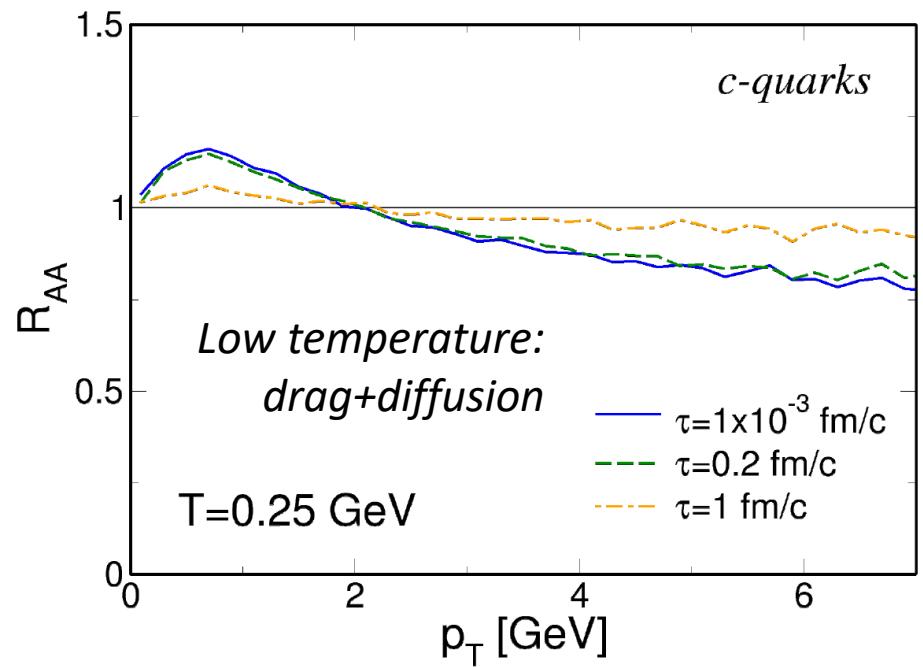


*Initial spectrum is tilted by the combined effect of*

- *Diffusion-dominated evolution*
- *Memory that slows down momentum broadening*



# Impact on RAA

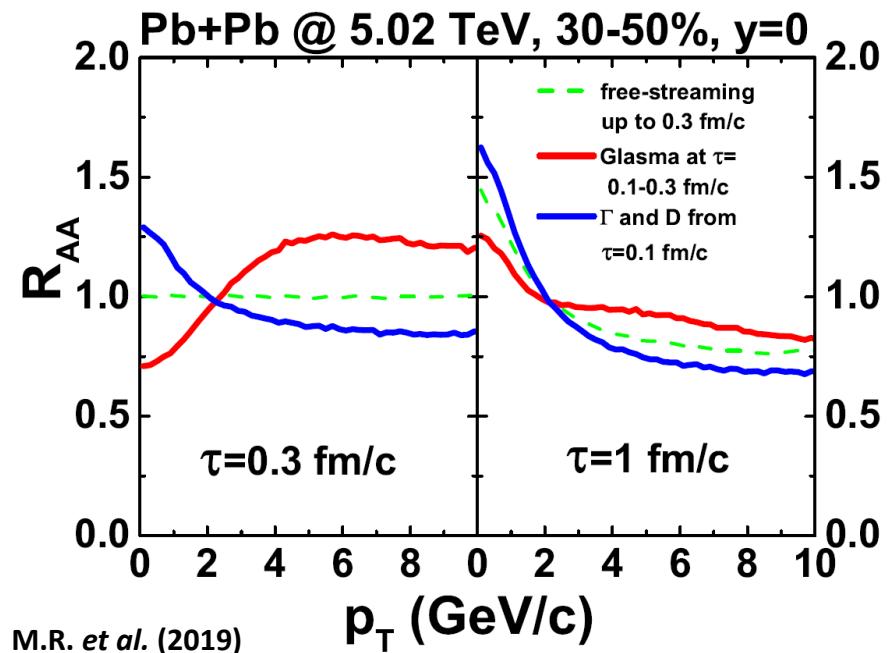


*Initial spectrum is tilted by the combined effect of*

- *Diffusion-dominated evolution*
- *Memory slowing down momentum broadening*

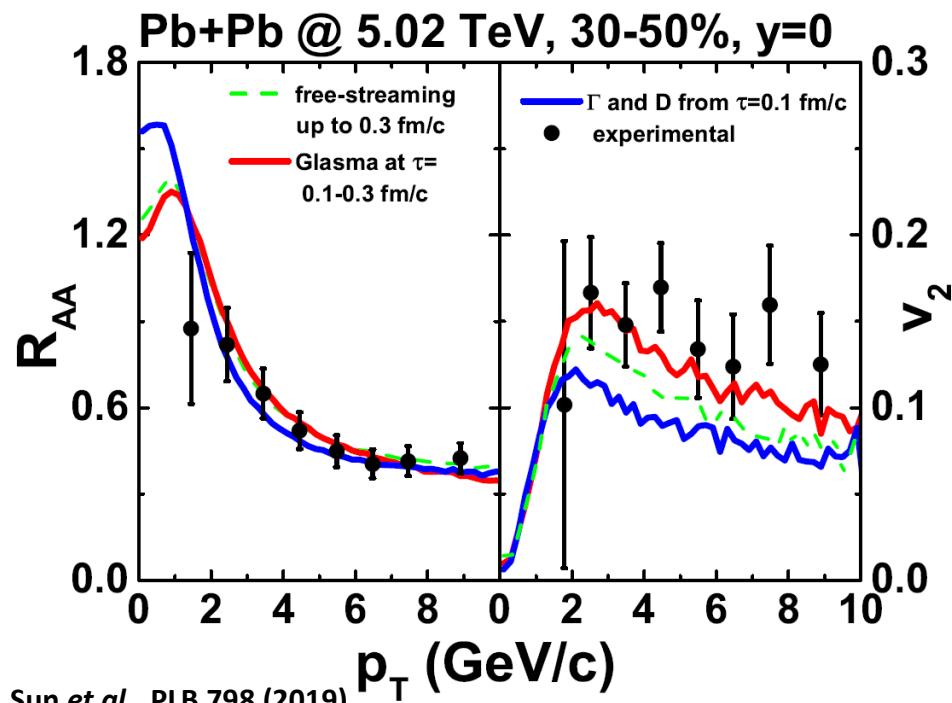


# The effect on the elliptic flow in Pb-Pb



*Diffusion in the early stage  
helps to describe simultaneously  
the RAA and the  $v_2$ .*

- *Diffusion in the early stage*
- *Evolution in the QGP*



# Conclusions and Outlook



- Borrowing the CGC picture, the early stage of high energy nuclear collisions is described by a set of dense, gluon filaments called Glasma.
- Diffusion of HQs in the early stage of high energy collisions is affected by the strong fields: coherence/memory effects are substantial
- **Potential effect on observables ( $R_{AA}$ ,  $v_n$ ,  $pA$  and AA, 2-p correlators)**
- **Smooth connection to kinetic theory in the early stage**
  - Relation to quasi-particle models
  - Running coupling in the kinetic equations
  - Initial state fluctuations
- **Longitudinal diffusion: superdiffusion?**
- **Fluctuation-Dissipation theorem(?)**
- **Polarization of  $c$ - $\bar{c}$  and  $b$ - $\bar{b}$  pairs**

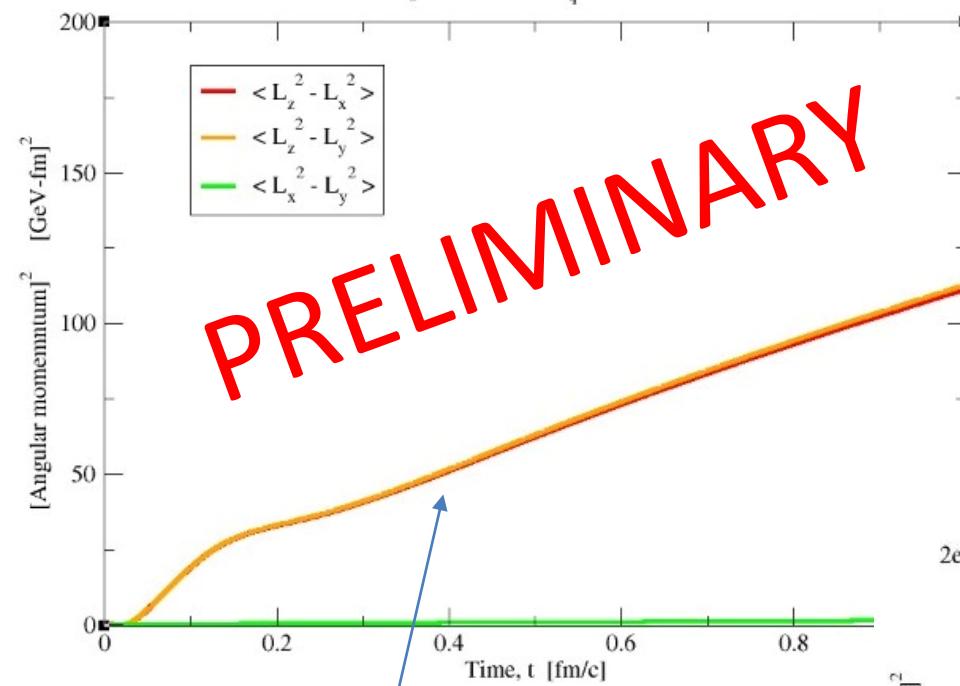
Thank you for your attention!



# Polarization(?)

Non-Relativistic (with spin)

$Q_s = 3 \text{ GeV}$     $m_q = 30 \text{ GeV}$



*Angular momentum  
anisotropy*

*Spin unaffected*

$$\begin{aligned} \frac{dP^i}{dt} &= -\frac{g}{m} F_a^{ij} Q_a P^j + g F_a^{i0} Q_a - \frac{g}{m} Q_c (\mathcal{D}^i B_c^j) S^j, \\ \frac{dQ_a}{dt} &= \frac{g}{m} f_{abc} A_b^j P^j Q_c + \frac{g}{m} f_{abc} S^j B_b^j Q_c, \\ \frac{dS^i}{dt} &= -\frac{g Q_a}{m} F_a^{ik} S^k, \\ \frac{dL^i}{dt} &= \varepsilon_{ijk} x^j \frac{dP^k}{dt}. \end{aligned}$$

