Open and Hidden Heavy-Flavor Transport in Medium

Ralf Rapp

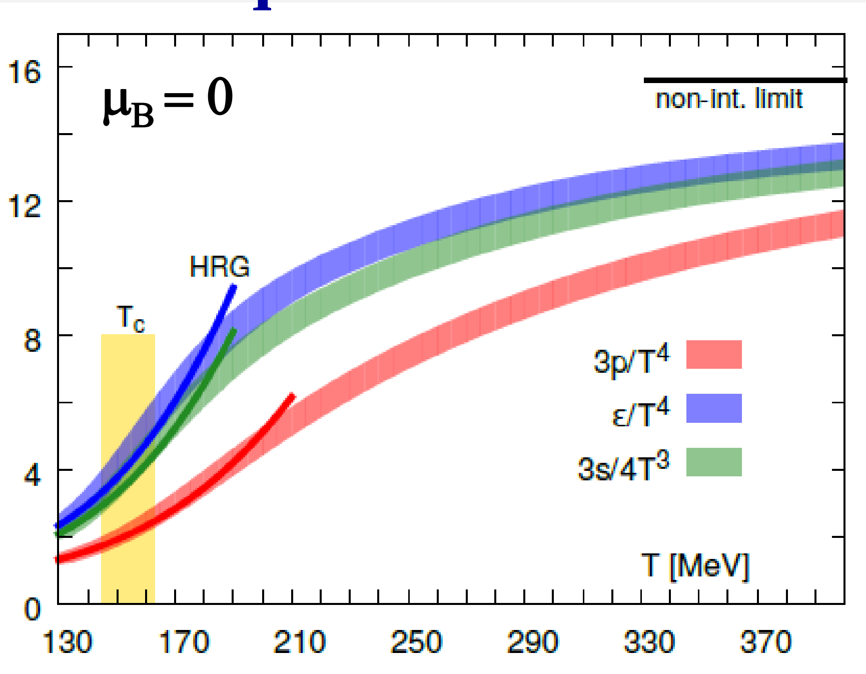
Cyclotron Institute +
Dept of Phys & Astro
Texas A&M University
College Station, TX
USA

CYCLOTRON INSTITUTE
TEXAS A&M UNIVERSITY

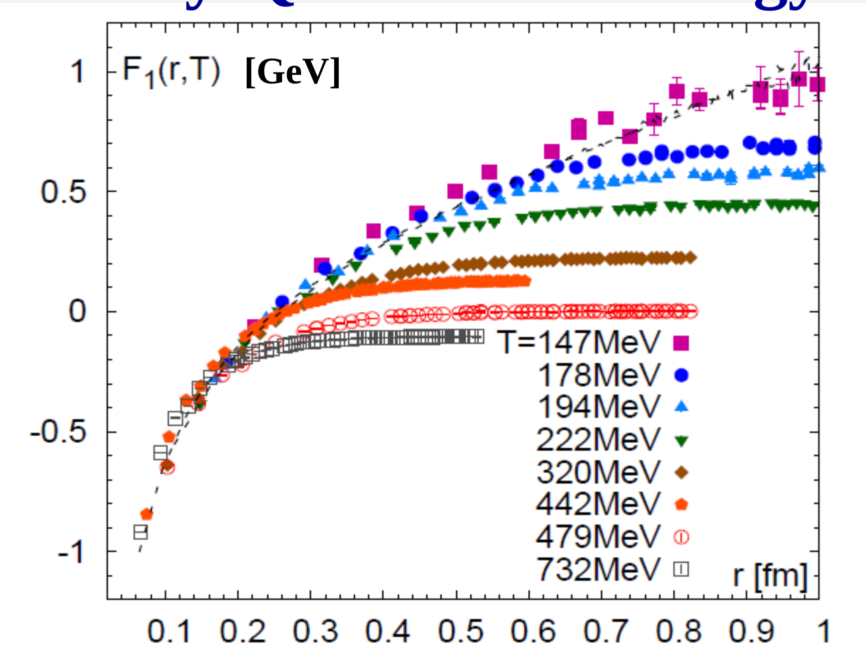
8th International Workshop on
Heavy-Flavour Production in Nuclear Collisions
Torino (Italy), July 14-16, 2022

1.) QCD Matter and In-Medium Force

Equation of State



Heavy-Quark Free Energy



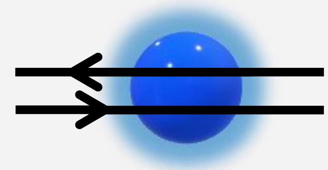
[Bazavov et al '13,'14]

- Change in dofs above T ~ 170MeV

- $F_{Q\bar{Q}} = U_{Q\bar{Q}} - T\,S_{Q\bar{Q}}$
- Non-perturbative above T_c

- **Heavy-quark diffusion and quarkonia sensitive to long-range forces in medium, probing non-perturbative QGP structure**

1.2 Quarkonia in Medium

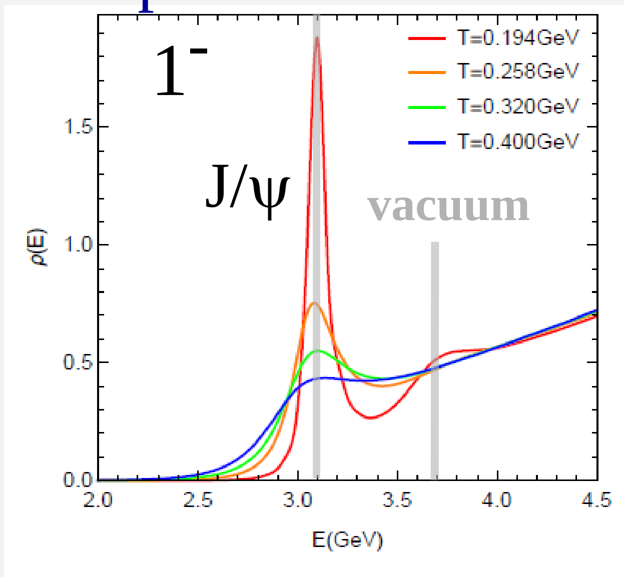


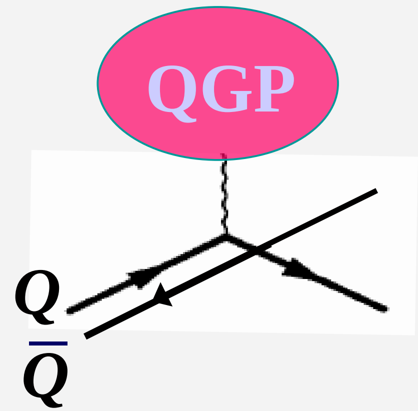
- In-medium **spectral functions**:
  - Mass / binding energy $\mathbf{E_B(p,T)}$
  - Inelastic reaction rate $\mathbf{\Gamma(p,T;E_B)}$
- **ϒ(1S)**: **color-Coulomb** force

  J/ψ, **ϒ(2S)**, …: **confining** force

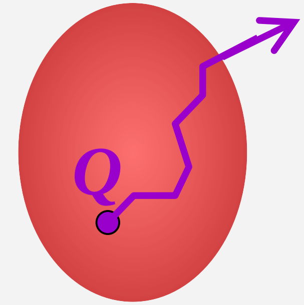
- Connection to open heavy-flavor transport

Spectral Function





1.3 Heavy-Flavor Diffusion



- **Brownian motion via elastic interactions**
  (radiation suppressed: $q_0 \sim q^2/2m_Q << q \sim T << m_Q$)

- Thermalization **delayed** by m_Q/T $\Rightarrow$ measure of **interaction strength**

- **Scattering rates:** collisional width $\leftrightarrow$ quantum effects

  $\Rightarrow$ **Implications for QGP structure**

- **Non-perturbative interactions**

- **Hadronization**: $c \rightarrow D, D_s, \Lambda_c, \ldots$ $(m_Q >> T_c)$

Outline

1.) Introduction

2.) Open Heavy-Flavor Transport

- Transport Equations + Heavy-Light Interaction
- Implications for Bulk Medium

3.) Quarkonia in Medium

- Production Models + Transport Coefficients
- Connection to Open Heavy Flavor

4.) Conclusions

2.1 Transport Equations

- **Boltzmann equation for HQ phase-space distribution** f_Q

$$\left[\frac{\partial}{\partial t} + \frac{\boldsymbol{p}}{\omega_{\boldsymbol{p}}}\frac{\partial}{\partial \boldsymbol{x}} + \boldsymbol{F}\frac{\partial}{\partial \boldsymbol{p}}\right] f_{\mathrm{Q}}(t,\boldsymbol{x},\boldsymbol{p}) = C[f_{\mathrm{Q}}]$$

- Collision term: semiclassical simulation of medium + HQ quasiparticles

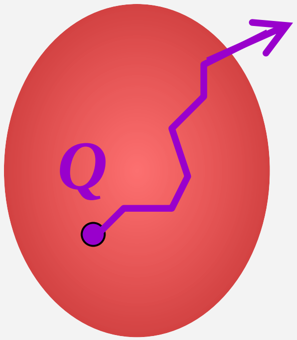
- **Fokker-Planck equation**

$$\frac{\partial}{\partial t} f_Q(t,\boldsymbol{p}) = \frac{\partial}{\partial p_i}\left\{A_i(\boldsymbol{p}) f_Q(t,\boldsymbol{p}) + \frac{\partial}{\partial p_j}[B_{ij}(\boldsymbol{p}) f_Q(t,\boldsymbol{p})]\right\}$$

- follows from Boltzmann with $\mathbf{p^2 \sim m_Q T >> q^2 \sim T^2}$ (ok for $\mathbf{m_Q/T \geq 5}$)
- **does not require quasi-particle medium**

  → well suited for **strongly coupled medium** where $\mathbf{E_{th} \leq \Gamma_{q,Q} < m_Q}$

2.2 Transport Coefficients



$$\frac{\partial}{\partial t} f_Q(t,p) = \gamma \frac{\partial}{\partial p_i}[p_i f_Q(t,p)] + D_p \Delta_{\vec{p}} f_Q(t,p)$$

Fokker-Planck

thermalization rate

$\gamma p = \int d^3q \, w_Q(q,p) \, q$

$\sim \int |T_{Qi}|^2 \, (1-\cos\theta) \, f^i \rho_i \, \rho_i'$

momentum diffusion coefficient

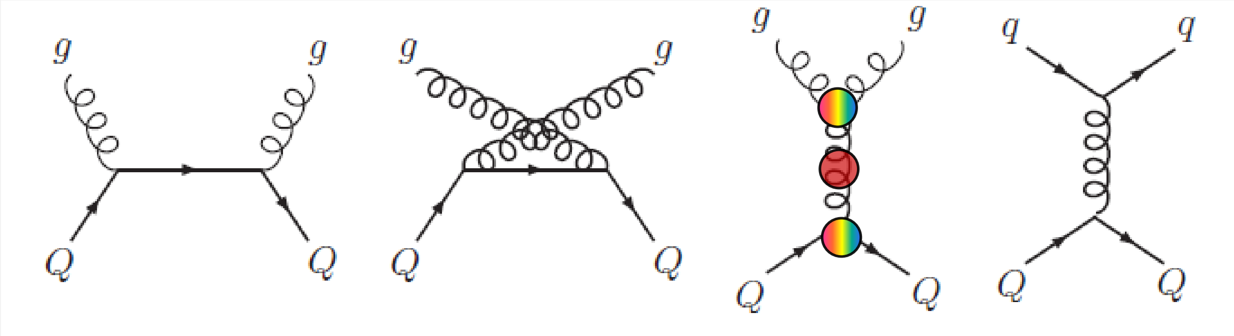
$D_p = \int d^3q \, w_Q(q,p) \, q^2$

- Relaxation time $\tau_Q = 1/\gamma$
- **Spatial diffusion constant:** $\mathcal{D}_s = T / \gamma(p=0) \, m_Q$
- Quantum effects via thermal-parton **spectral functions** $\rho_i(k_0,k)$
- Key ingredients: - **heavy-light scattering amplitude** T_{Qi}

  - **ambient medium properties**

2.3.1 Heavy-Light Interaction: Perturbative QCD

- **Leading Order**



- screened 1-gluon exchange

$$T_{Qi} \sim \frac{\alpha_s}{t - m_D^2}$$

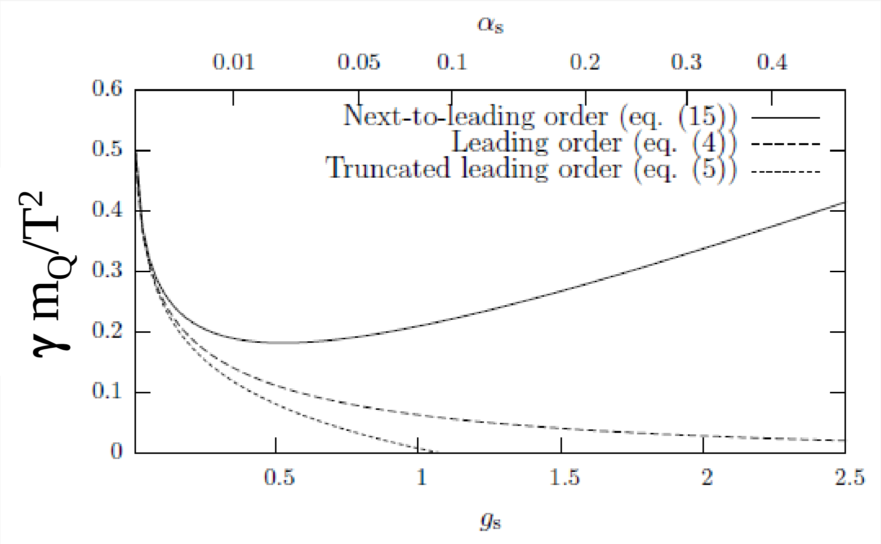
- $\tau_c \geq 20$ fm/c (T≤ 300MeV, α_s=0.4)

[Svetitsky '88, Mustafa et al '98, Molnar et al '04, AMPT, Teaney+Moore'04, Torino, Catania, PHSD, BAMPS, LBT,…]

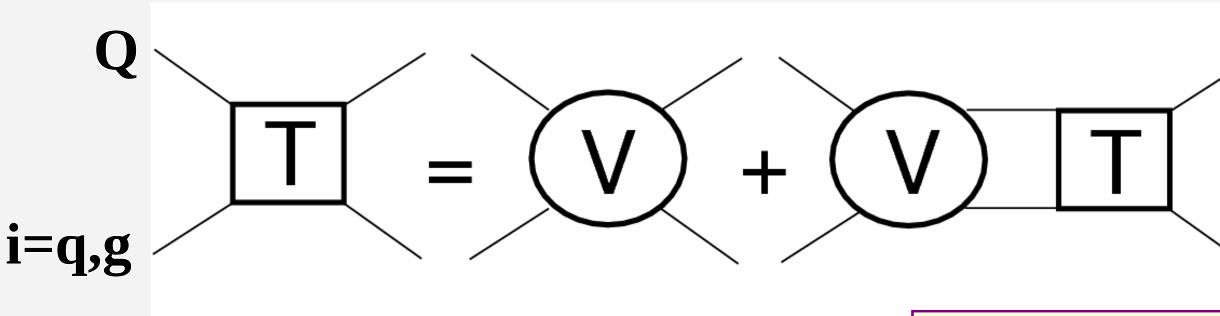
- **pQCD***: running coupling with reduced Debye mass [Nantes '08]

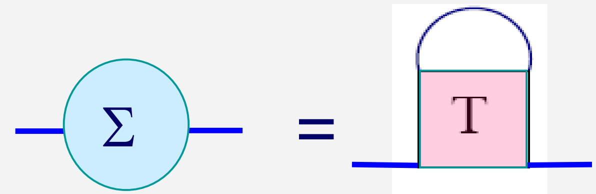
- **Next-to-Leading Order**

- bad convergence (even for α_s ~ 0.1)
- uncontrolled result [Caron-Huot+Moore '08]



2.3.2 Heavy-Light Interactions: T-Matrix

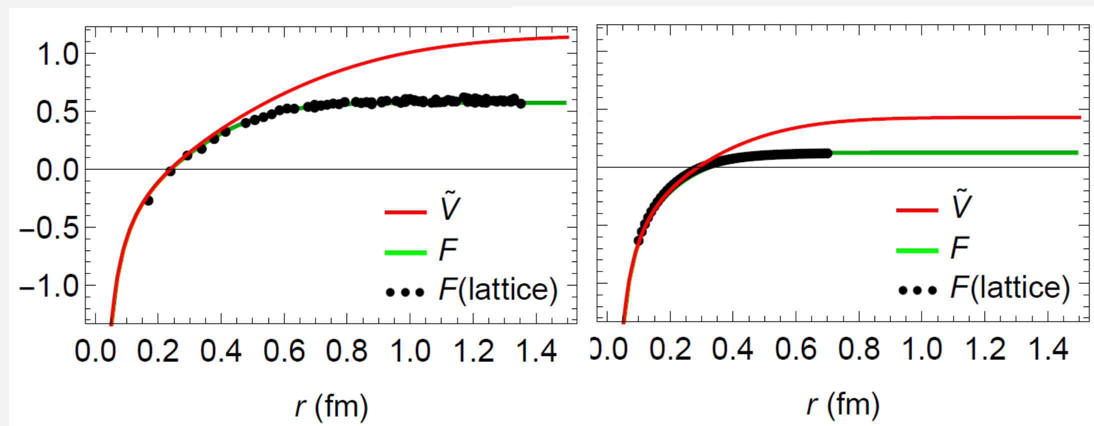


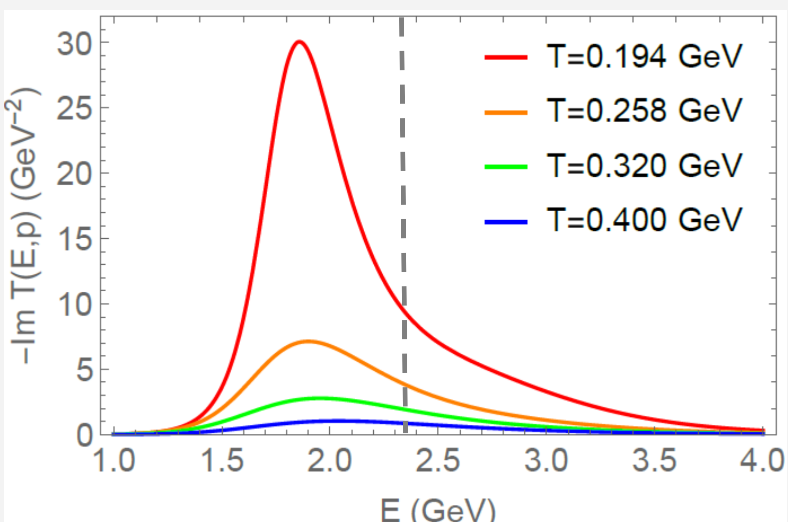


- **Strong** coupling → resummation:

$$T_{Qi} = V_{Qi} + \int V_{Qi}\, D_Q\, D_i\, T_{Qi}$$

- **Key input**: in-med **potential** V_{Qi} ,
  constrained by lattice QCD
  → "string" force above T_c





- Non-perturbative effects:
  - thermalization time $\tau_c \approx$ **3-4 fm/c**
  - **hadronization**

[TAMU '05-'18]

2.4 QGP Medium Models: Light Partons

→ essential for quantitative evaluation of transport coefficients

- Massless quarks + gluons (EoS too large)
- Perturbative quasiparticles ($\mathbf{m_{q,g} \sim gT}$, EoS for $\mathbf{T > 1.5T_c}$)
- Effective quasiparticles ($\mathbf{m_{q,g} \sim gT}$ with **g** large near $\mathbf{T_c}$)
- Dynamical quasiparticles ($\mathbf{\Gamma_{g,q} \sim 0.1\text{-}0.2\ GeV}$)
- **T**-matrix + string potential ($\mathbf{\Gamma_{g,q} \sim 0.5\ GeV}$)

- Consistency of heavy-light interaction and EoS?

2.5 Heavy-Quark Recombination

- **Instantaneous Coalescence Model (ICM)**

[Hwa '80, Likhoded et al '83, … Greco et al + Fries et al '03,…]

$$f_h(\boldsymbol{p}'_h) = \int \Big[\prod_i d\boldsymbol{p}_i f_i(\boldsymbol{p}_i)\Big] W(\{\boldsymbol{p}_i\})\delta(\boldsymbol{p}'_h - \sum_i \boldsymbol{p}_i)$$

$$W_s = g_h \frac{(2\sqrt{\pi}\sigma)^3}{V} e^{-\sigma^2 \boldsymbol{k}^2}$$

σ ~ radius of hadron **h**

- energy not conserved → challenge for chemical + thermal equilibrium

- **Resonance Recombination Model (RRM)**

[Ravagli et al '07, He et al '12]

- derived from Boltzmann equation

$$f_M(\vec{x},\vec{p}) = \frac{\gamma_M(p)}{\Gamma_M} \int \frac{d^3\vec{p}_1 d^3\vec{p}_2}{(2\pi)^3} f_q(\vec{x},\vec{p}_1) f_{\bar{q}}(\vec{x},\vec{p}_2)\, \sigma_M(s) v_{\rm rel}(\vec{p}_1,\vec{p}_2)\delta^3(\vec{p}-\vec{p}_1-\vec{p}_2)$$

- $\sigma_M(s)\, v_{rel} \sim |T_{Qi}|^2$: resonant heavy-light amplitude

→ compatible with **equilibrium limits**

Outline

1.) Introduction

2.) Open Heavy-Flavor Transport

- Transport Equations + Heavy-Light Interaction
- Implications for Bulk Medium

3.) Quarkonia in Medium

- Production Models + Transport Coefficients
- Connection to Open Heavy Flavor

4.) Conclusions

3.1 Quarkonia Transport in Heavy-Ion Collisions

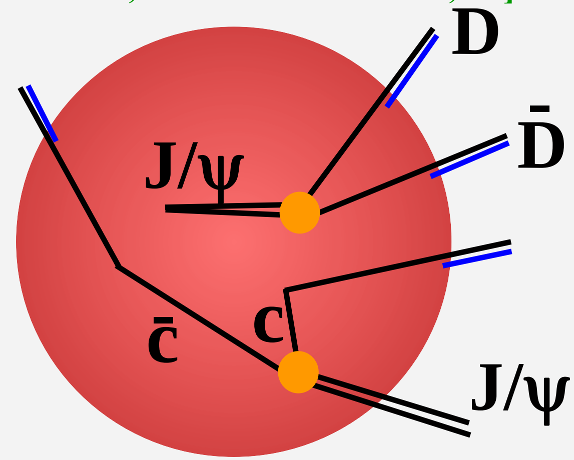
[PBM+Stachel ’00, Thews et al ’01, Grandchamp+RR ‘01, Gorenstein et al ’02, Ko et al ’02, Gossiaux et al ‘05, Zhuang et al ’05, Ferreiro et al ‘11, Strickland et al ’11, …]

- **Semi-classical**

Boltzmann eq. $\hat{p}^{\mu}\,\partial_{\mu} f^{\Psi} = -\,\Gamma_{\Psi}\, f^{\Psi} + \beta$

→ Rate eq.

$$\frac{dN_{\psi}}{dt} = -\Gamma_{\psi}\,(N_{\psi} - N_{\psi}^{eq})$$



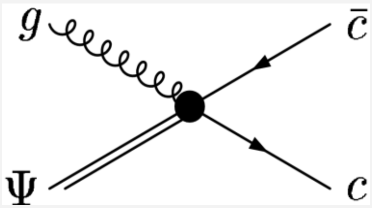
- **Transport coefficients**

- Equilibrium limit: $N_{\psi}^{eq}(E_B, T; N_{cc}) = \gamma_c^{2}\, n_{\psi}(T)\, V_{FB}$

depends on **cc** Xsec + (thermal) environment: $\sigma_{cc} \sim N_{cc} \sim \gamma_c\, n_c(m_c, T)\, V_{FB}$

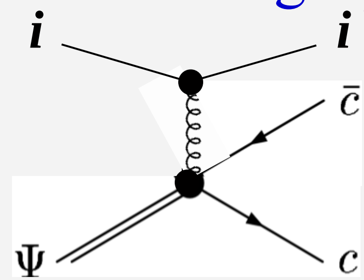
- Reaction rate: $\Gamma_{\psi}[p, E_B, T]$

“Large” binding $E_B \geq T$



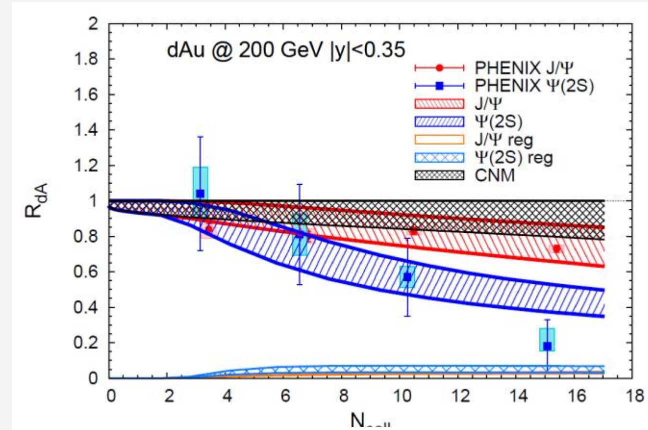
- gluo-dissociation (“singlet-to-octet”)

“Small” binding $E_B \leq m_D$

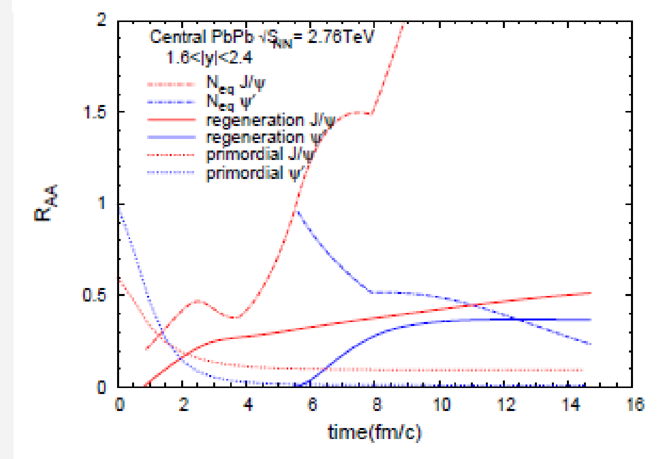


- “quasi-free”/ Landau damping

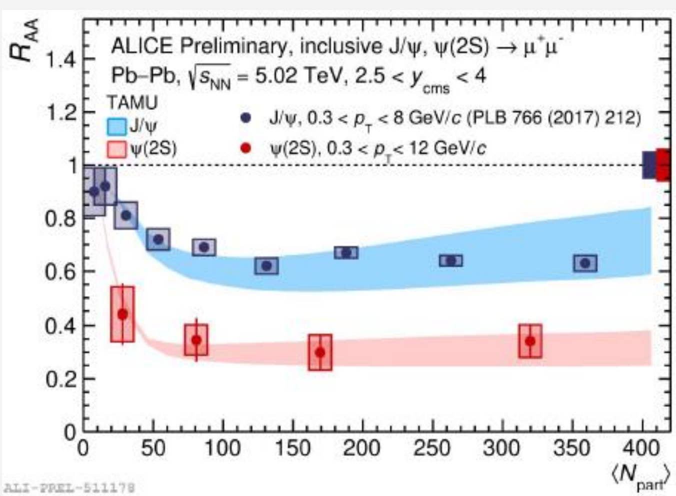
3.2 Interplay of Transport Coefficients: Charmonia

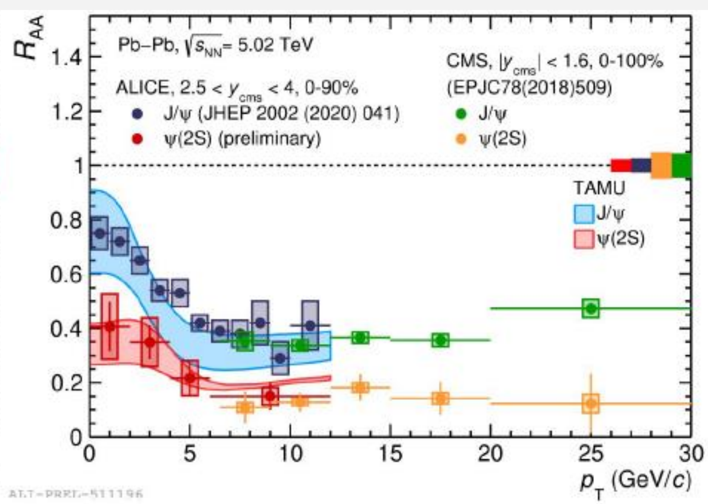


- Phenomenological constraints on Γ_ψ from **d/p-A**:
  J/ψ suppressed in QGP, **ψ(2S)** in hadronic matter



- Transport in **AA**:
  J/ψ / **ψ(2S)** near equilibrium in
  late QGP / hadronic phase
  → "**Sequential Regeneration**" **[Du+RR '15]**

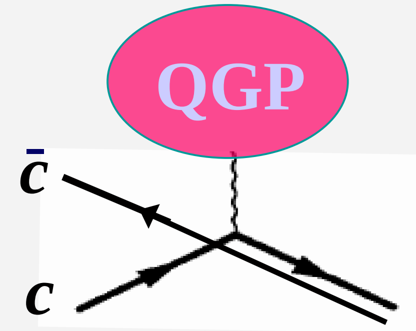




3.3 Coupling of Quarkonia + Open HF Transport

- Boltzmann:

$$\frac{\partial f_\Psi}{\partial t} + \vec{v} \cdot \nabla f_\Psi = -\alpha f_\Psi + \beta$$

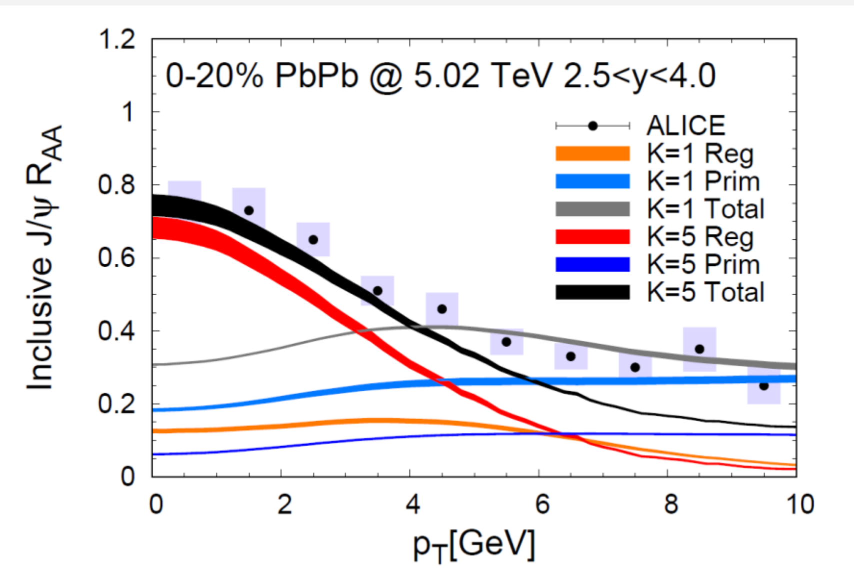


- Gain term:

$$\beta(\vec{p}, T) = \sum_i \int \mathrm{d}\Pi_\Psi \, d_{\tilde{i}} \, d_c \, d_{\bar{c}} \, \overline{|M_{\tilde{i}c\bar{c}\to i\Psi}|^2} \, [1 \pm f_i(p_i)] f_i(\tilde{p}_i) \gamma_c f_c(p_c) \gamma_c f_{\bar{c}}(p_{\bar{c}})$$

- Quasifree:

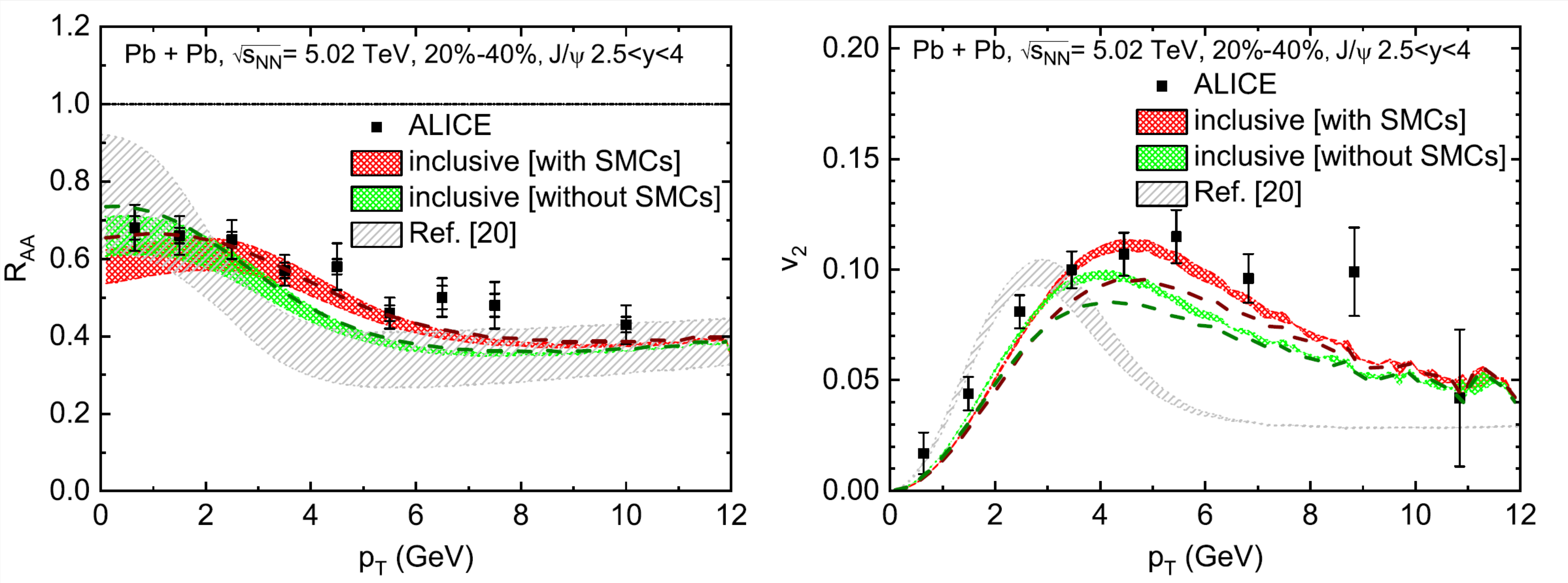
$$\overline{|M_{i\Psi\to\tilde{i}c\bar{c}}(p_i, p_\Psi, \tilde{p}_i, p_c, p_{\bar{c}})|^2} \approx 2 \overline{\left| M_{ic\to\tilde{i}c} \left(p_i, \frac{m_\Psi - m_c}{m_\Psi} p_\Psi, \tilde{p}_i, p_c \right) \right|^2} (2\pi)^3 2E_{\bar{c}} \delta^{(3)} \left(\vec{p}_{\bar{c}} - \frac{m_c}{m_\Psi} \vec{p}_\Psi \right)$$



- Simultaneous **c**-quark diffusion + charmonium kinetics (same M^2_{pQCD} * **K**)
- **J/ψ** yield + spectral shape require **K ≥ 5**
- Strong coupling for open + hidden HF

[Du+RR '22]

3.4 Charmonia - Open-HF Coupling II: Elliptic Flow

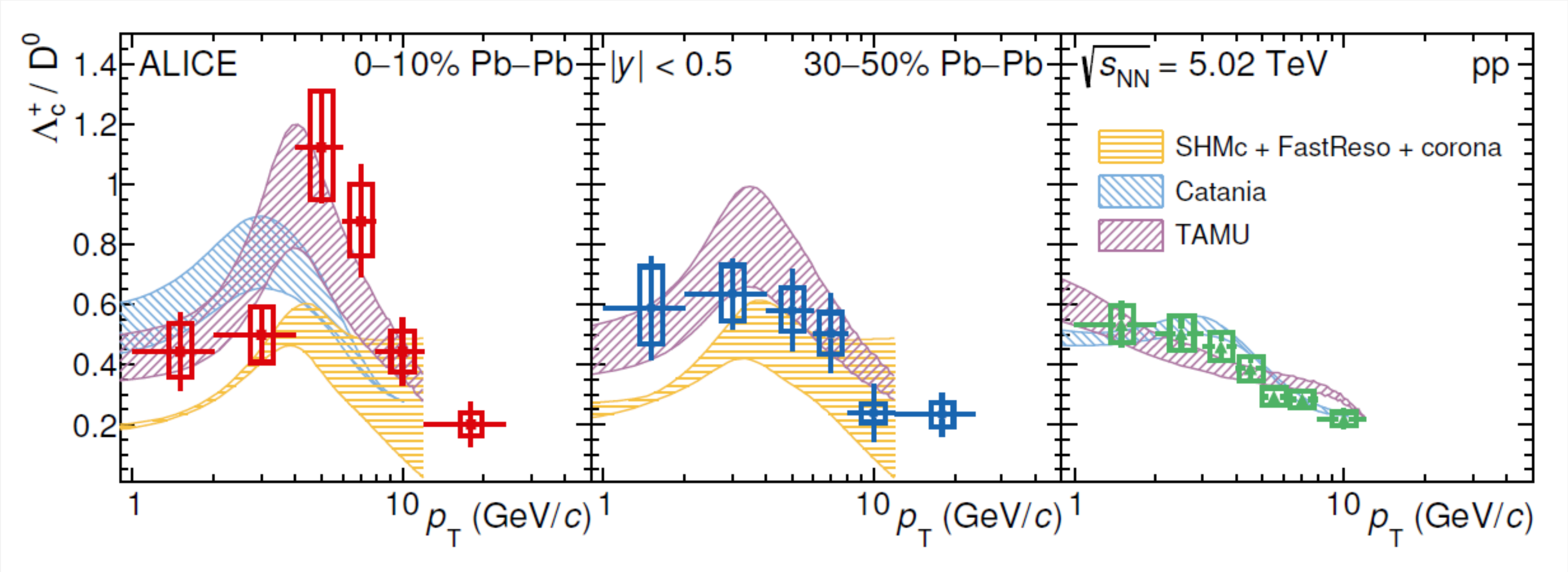


- QGP charm-quark diffusion + RRM
- Recombination reaches out to **p_T ~ 8 GeV** **[He,Wu+RR '22]**
- See also **[Villar et al '22]**

4.) Summary

- HF particles can probe non-perturbative QGP structure, by connecting lattice QCD to phenomenology (HQ free energy, correlators, susceptibilities; EoS)
- Production models need to satisfy equilibrium constraints
- Coupling of open + hidden HF sectors provides strong constraints
- Large collision rates require careful assessment of quantum effects; relations to semi-classical approaches to be understood

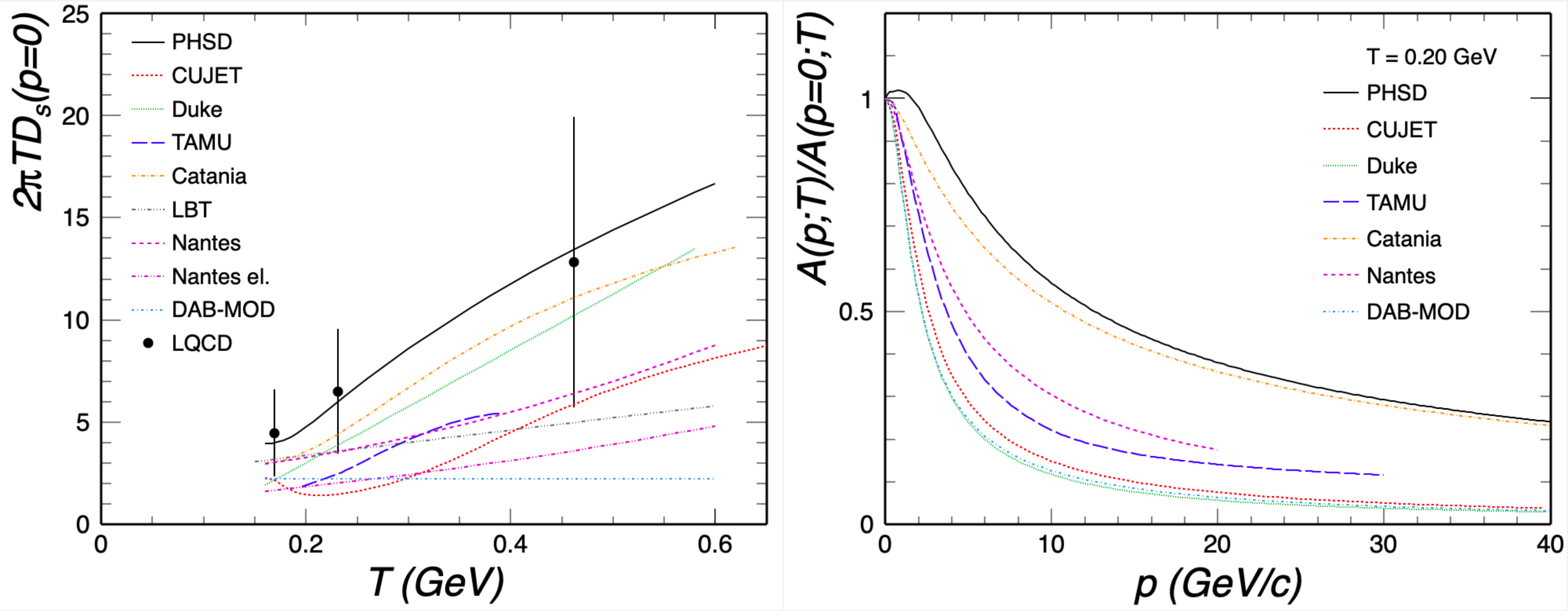
2.5 Charm-Hadron Spectra from pp to AA



- "Flow bump" vs. yield enhancement:
  control over equilibrium limit essential for interpretation

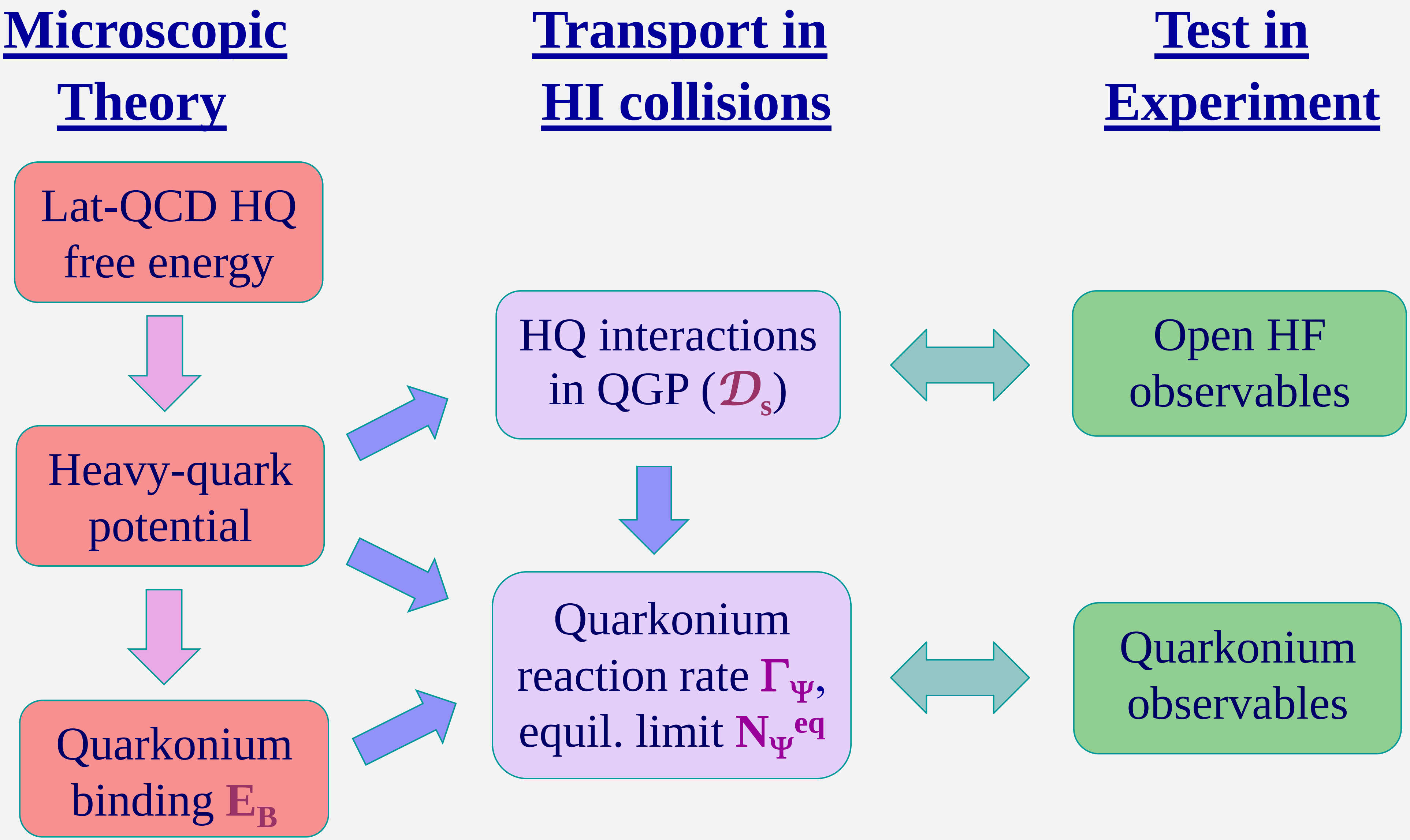
2.4 Momentum Dependence of Relaxation Rate

[ECT* HF workshop '21]



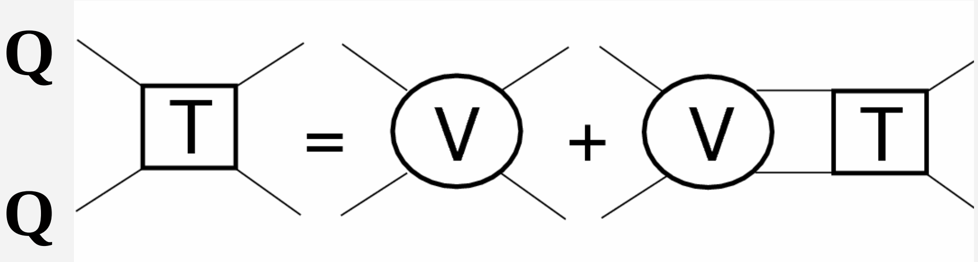
- correlation: large $\mathcal{D}_s$ ↔ weak **p**-dependence
- need handle on momentum dependence

2.2 Many-Body Theory of Heavy Flavor in QGP



2.3 Thermodynamic T-Matrix Approach

• **T-matrix equation**



In-medium potential?

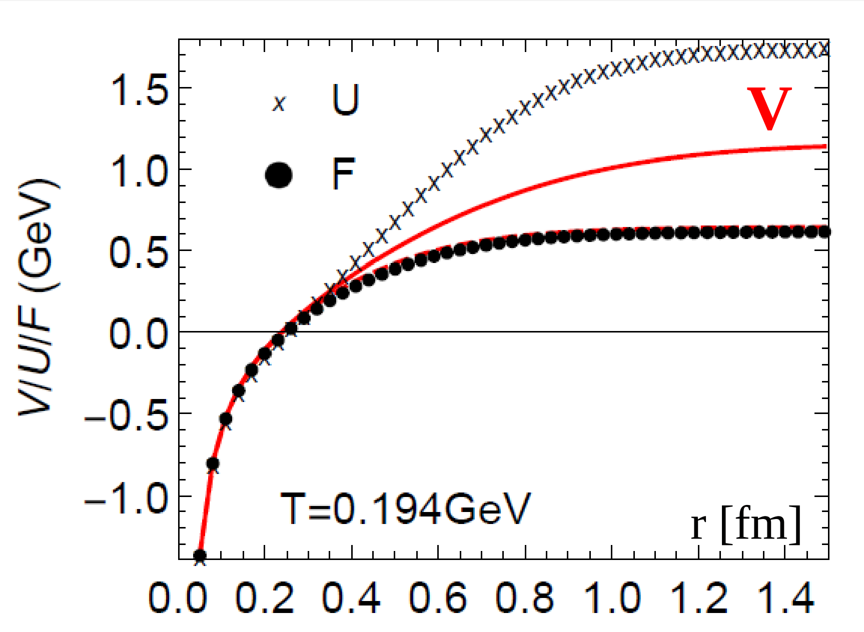
• **Free energy:**

$$F_{Q\bar{Q}}(r_1 - r_2) = -\frac{1}{\beta} \ln\left(\int_{-\infty}^{\infty} d\omega \sigma(\omega, r_1 - r_2)\, e^{-\beta\omega} \right)$$

Spectral function:

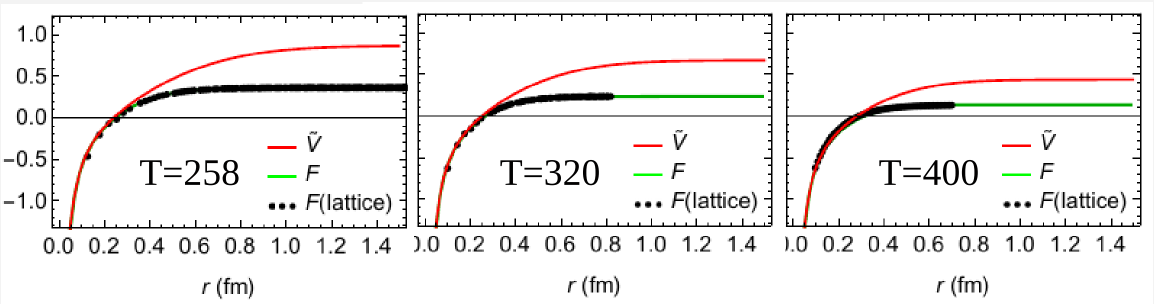
$$\sigma(\omega, r) = \frac{1}{\pi} \frac{(V+\Sigma)_I(\omega)}{(\omega - (V+\Sigma)_R)^2 + (V+\Sigma)_I^2(\omega)}$$

$\Rightarrow$ **infer in-medium potential**



• Large imaginary parts Σ_I $\Rightarrow$ **V > F**

• **Remnants of confining force above T_c!**

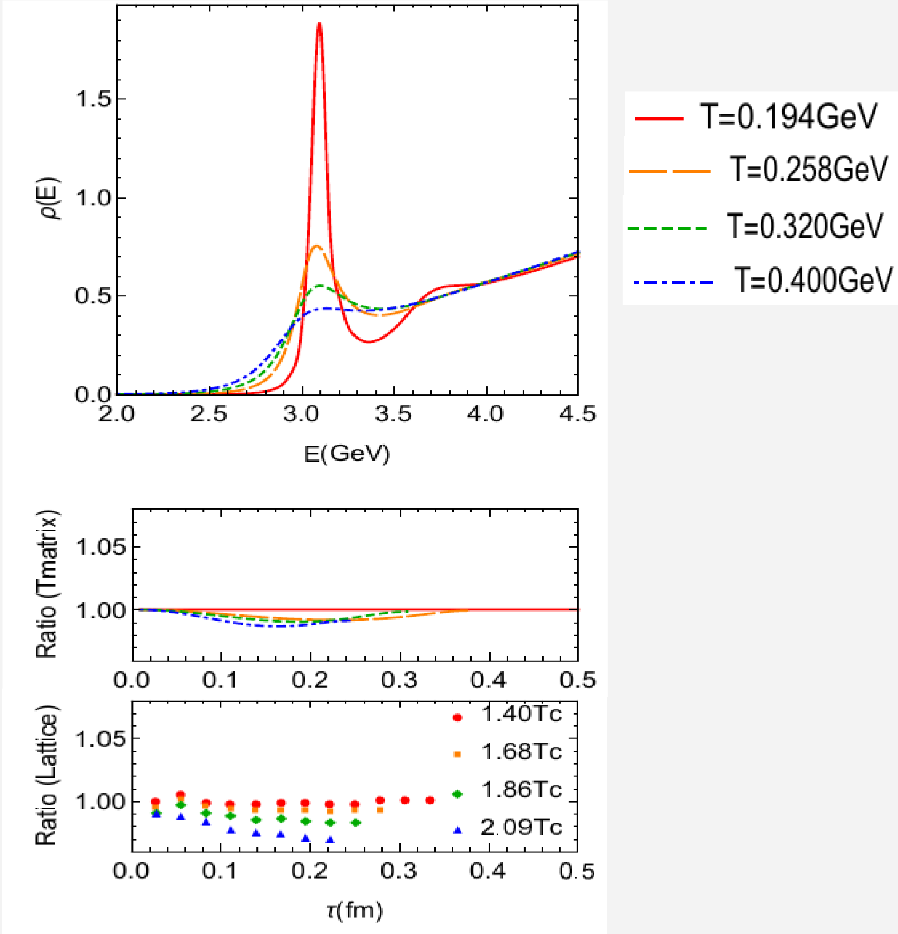


[TAMU ’05-’18]

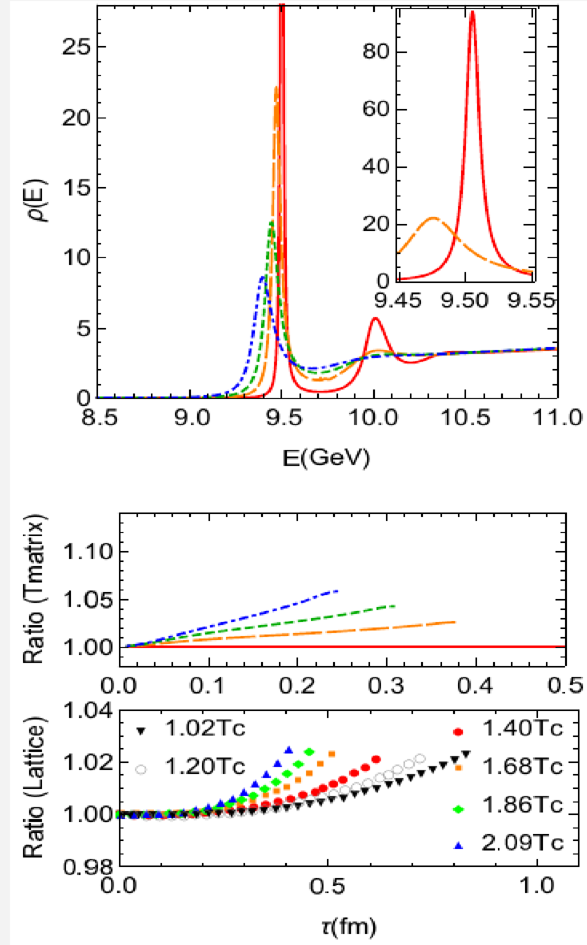
2.4 More lQCD Constraints: Euclidean Correlators

$$G_\alpha(\tau,T) = \int \frac{dE}{2\pi}\ \rho_\alpha(E,T)\ \frac{\cosh\left[E\left(\tau - 1/2T\right)\right]}{\sinh\left[E/2T\right]}$$

Charmonium



Bottomonium

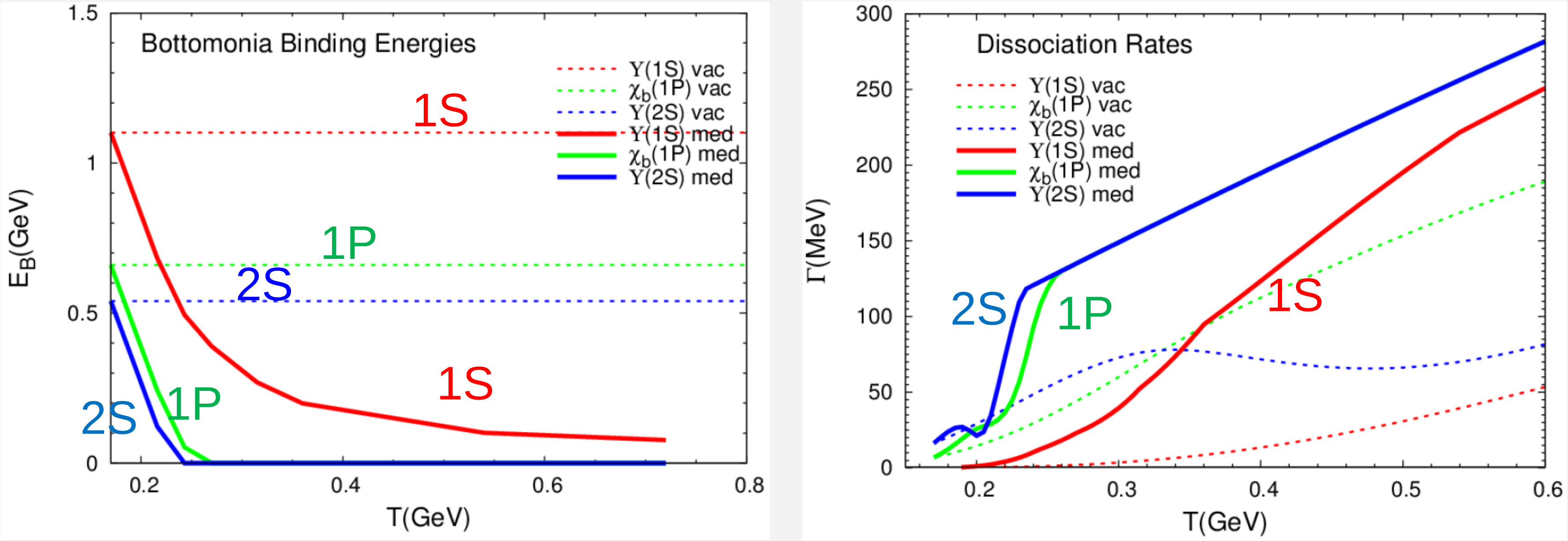


[Liu et al '18]

[Aarts et al '18]

- **J/ψ** “melts” near **T ~ 250-300 MeV**
- **ϒ(1S)** survives until **T > 400 MeV**

2.5 Binding Energies + Reaction Rates: Υ States



[Du et al '18]

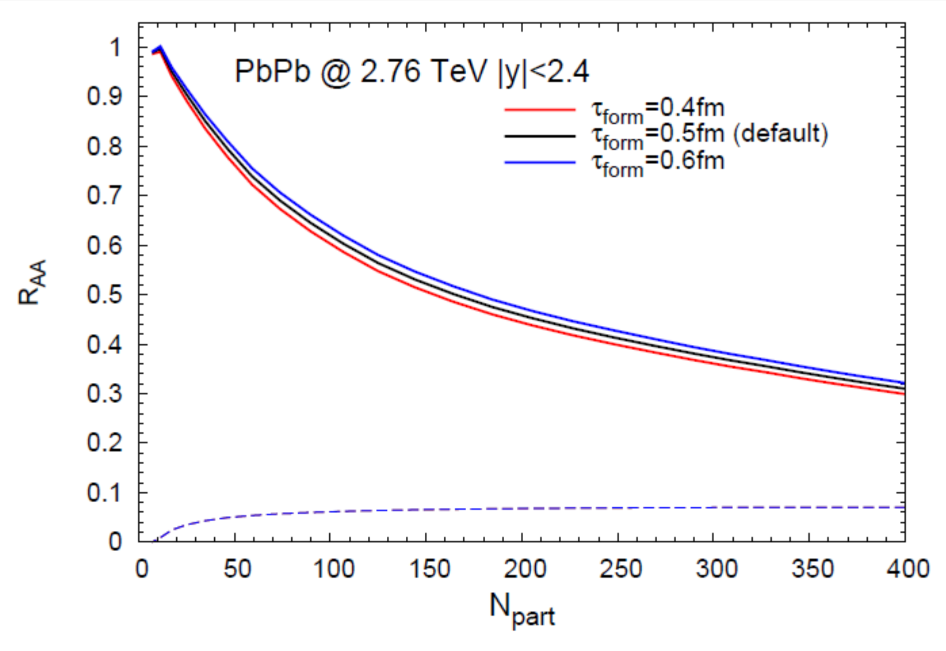
$$\Gamma_{Y}^{\rm qf}(p,T) = \sum_{p} \int \frac{{\rm d}^3 p_p}{(2\pi)^3} d_p f_p(\omega_p, T) v_{\rm rel} \sigma_{Y p \to b\bar{b}p}(s)$$

- Reduced binding "accelerates" dissociation (opens phase space)
- Dissociation rate depends on "medium" particle distribution

2.6 Quarkonium Formation Times

- Pair production time $\tau_{QQ} \leq 0.1$ fm/c
- Bound-state formation time $\tau_{form} \sim 1/E_B \sim 0.2 - 2$ fm/c
- Build-up of wave function reduces dissociation rate

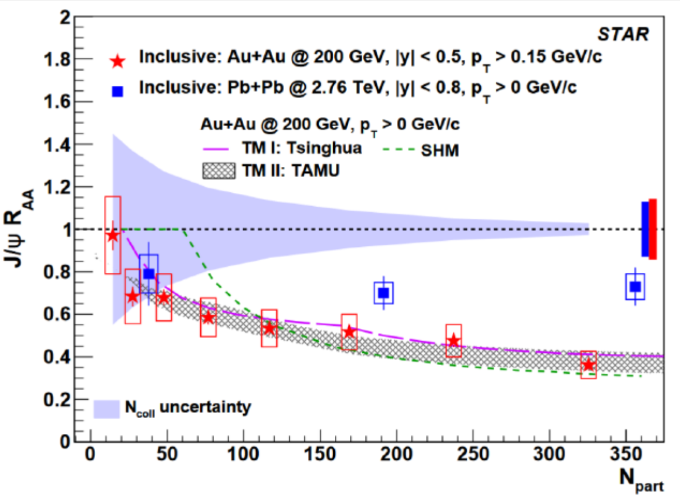
$$\alpha_Y(\vec{p}, T(\tau)) \equiv \Gamma_Y(\vec{p}, T(\tau)) \frac{\tau}{\tau_{form}} \frac{m_Y}{\sqrt{p^2 + m_Y^2}}$$



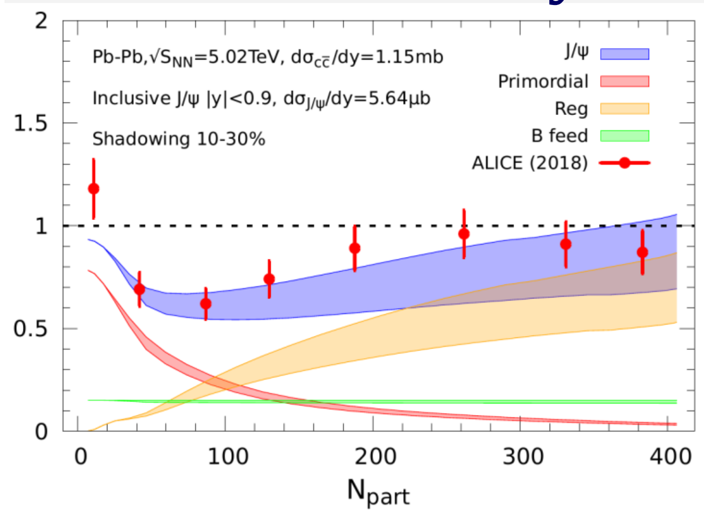
- Relatively small effect in heavy-ion collisions
- Microscopically: quantum evolution of wave package

3.1 Snapshot of Quarkonium Transport Results

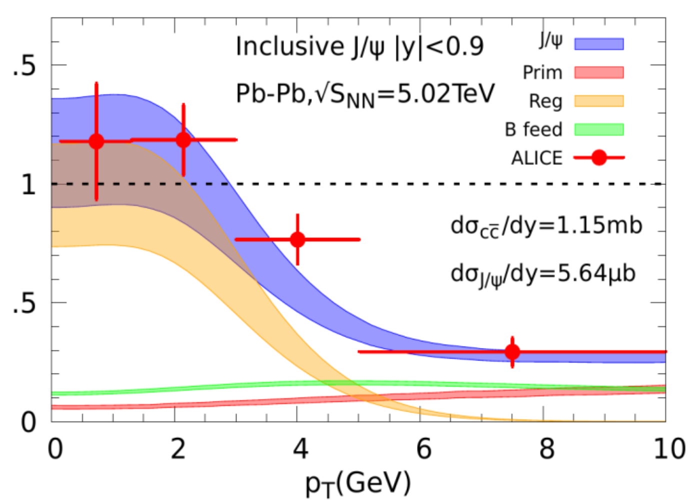
RHIC



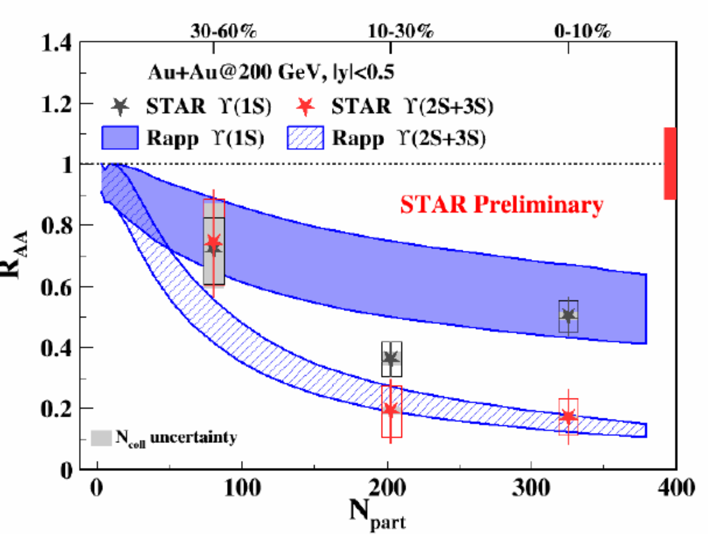
LHC: centrality

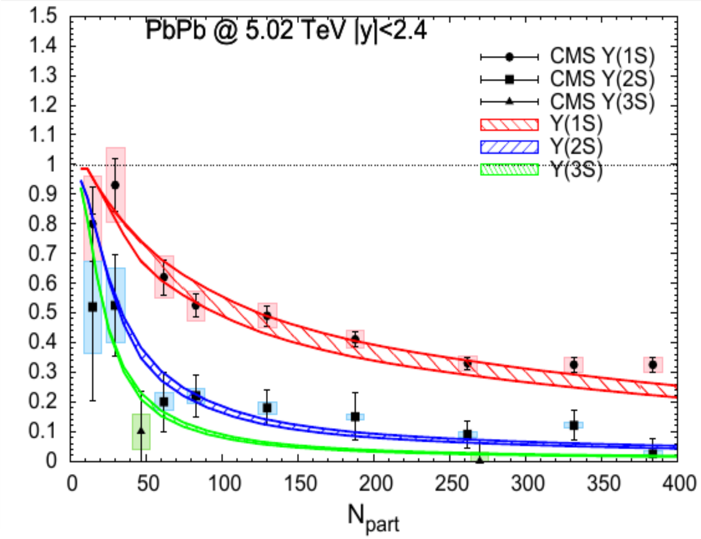


LHC: momentum



- mostly suppression
- substantial regeneration, concentrated at low p_T

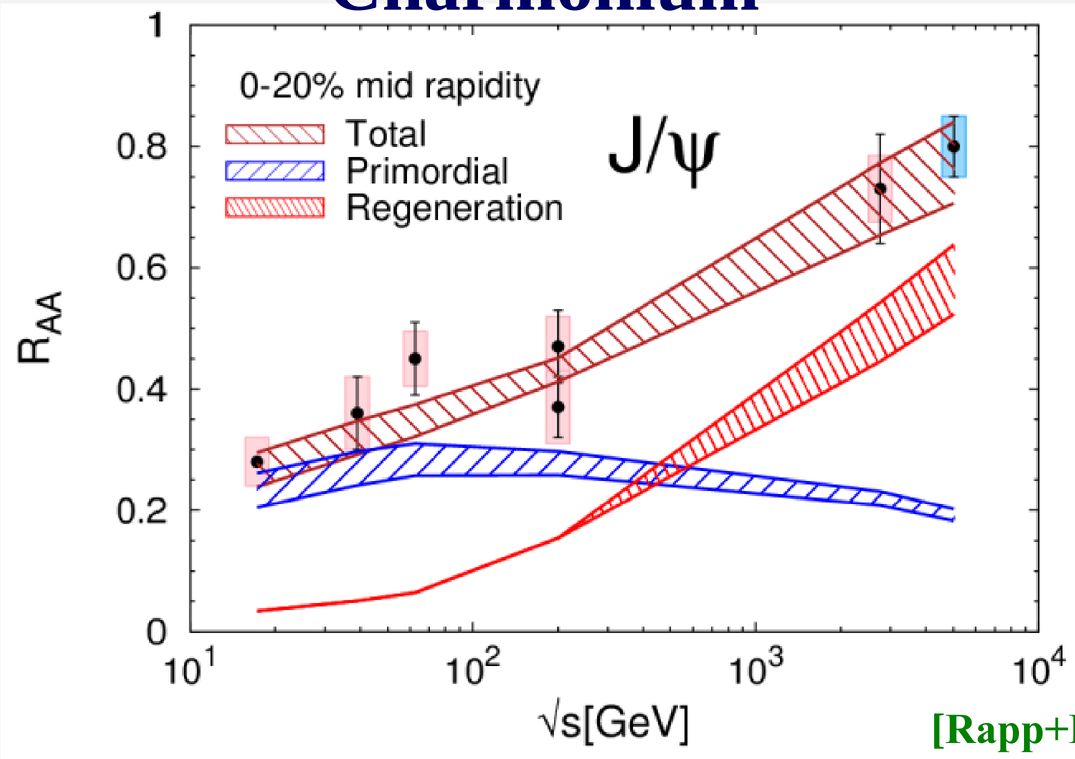


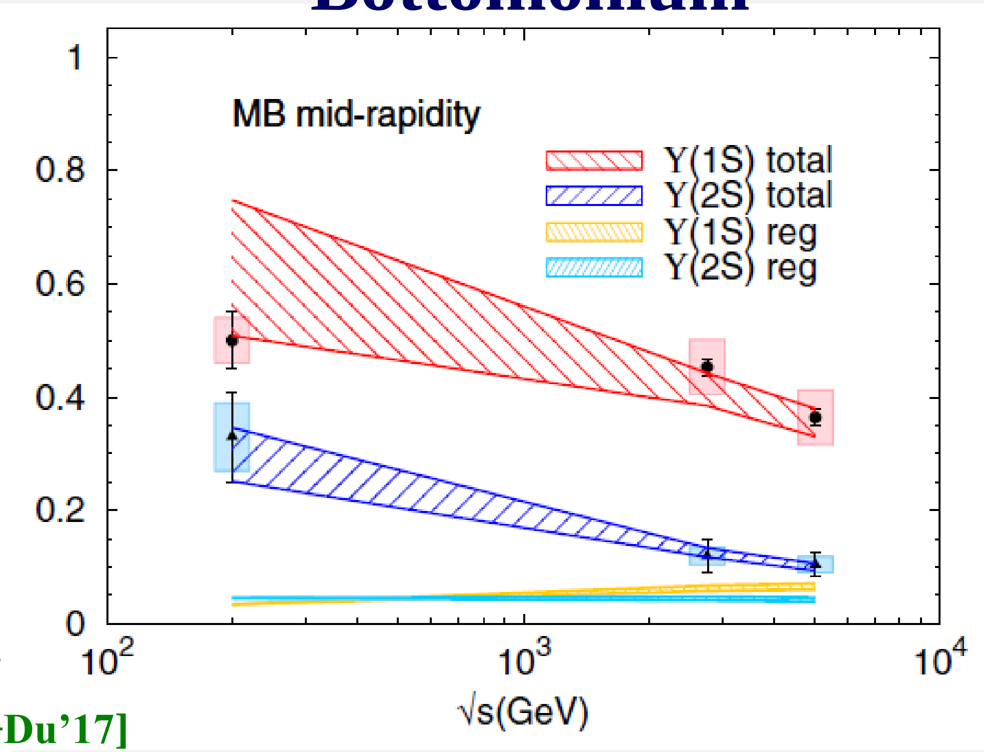


- Bottomonia mostly driven by suppression, regeneration small even at the **LHC**

3.2 Excitation Functions: SPS - RHIC - LHC

Charmonium | **Bottomonium**





[Rapp+Du'17]

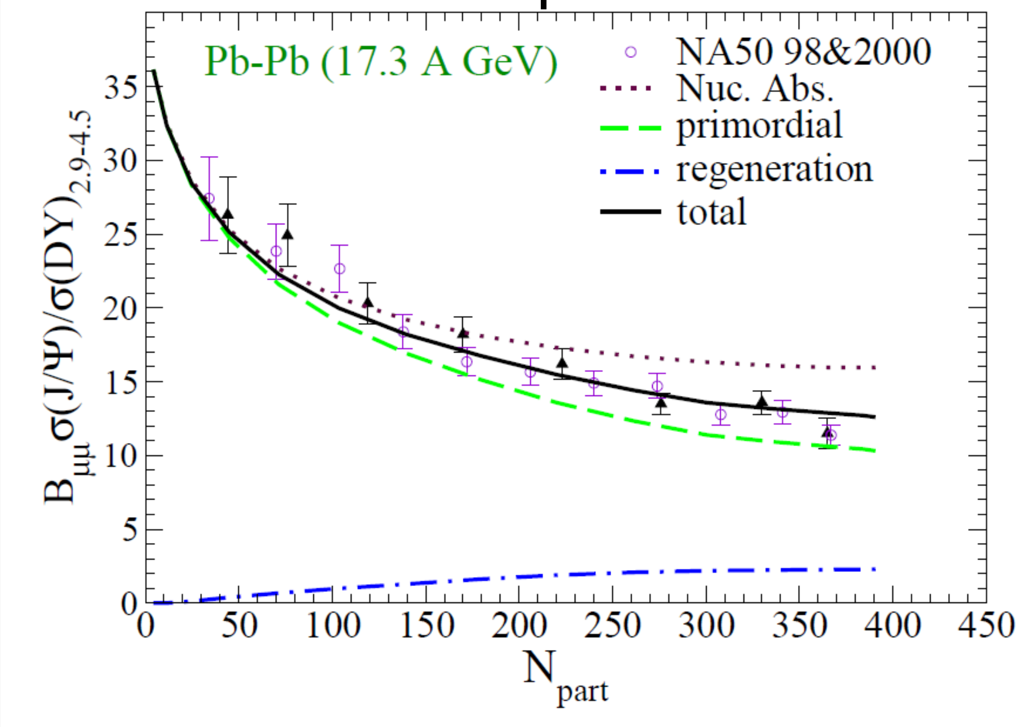
- Gradual **increase** of total **J/ψ** **R_{AA}**
- Regeneration **and** suppression increase
- Regeneration concentrated at low **p_T**

- Gradual **suppression**
- Regeneration (**N_Υ^{eq}**) small
- Qualitative difference from **J/ψ**

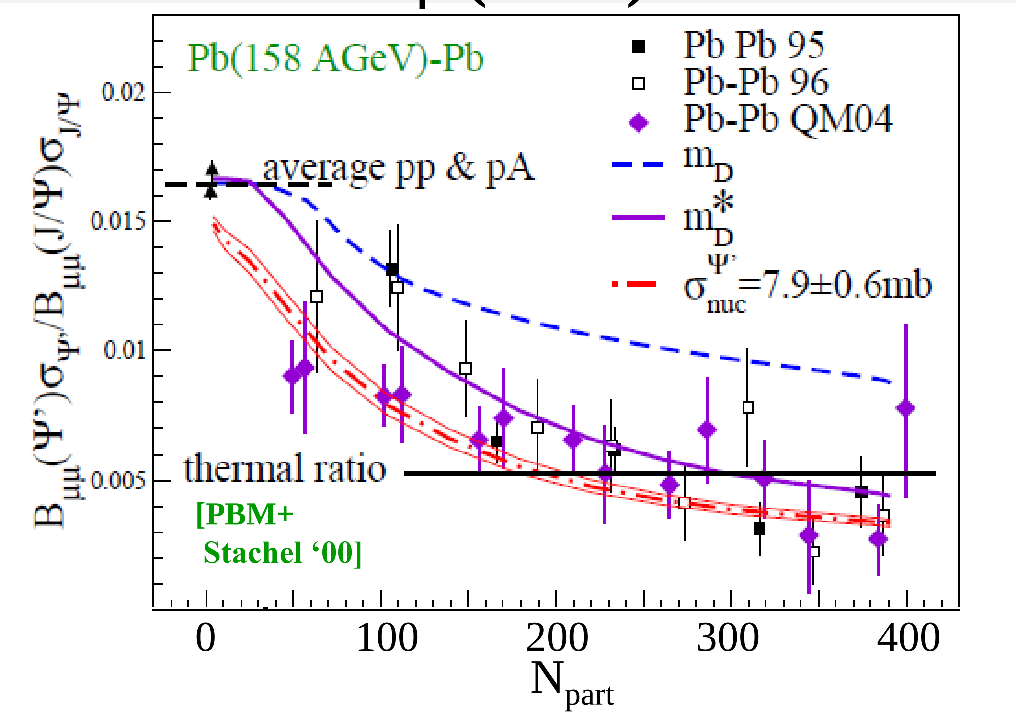
Data: NA50, PHENIX, STAR, ALICE, CMS

3.4 Charmonia at SPS Energy ($\sqrt{s}$=17 GeV)

J/ψ



ψ′(3686)



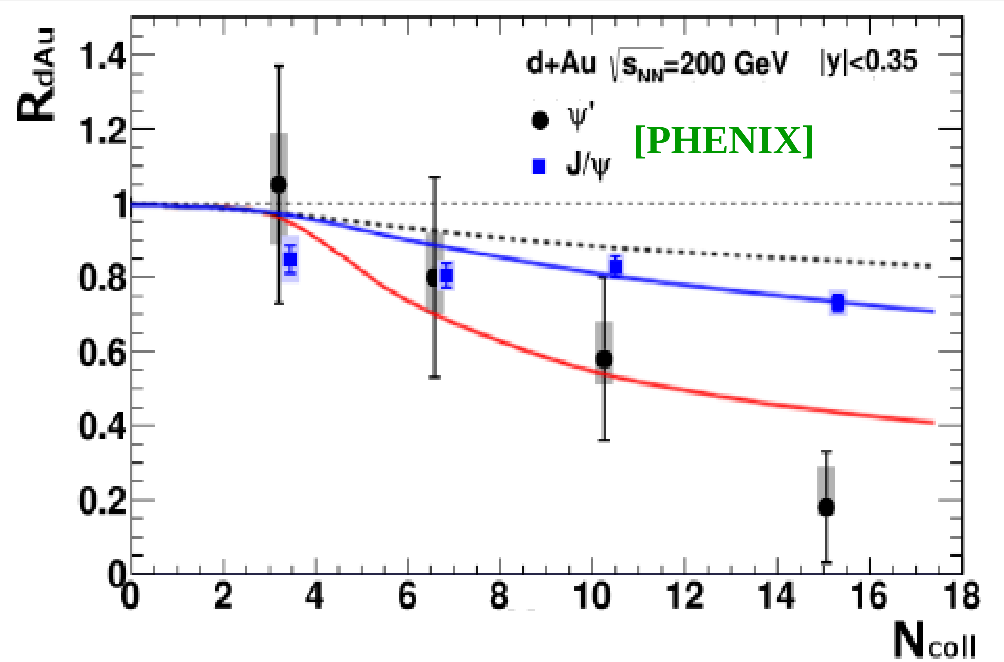
- Large contributions from initial "nuclear absorption" at low $\sqrt{s}$
- requires high-energy **N-Ψ** Xsec

- ψ′ dissociation in hadronic matter important in transport models

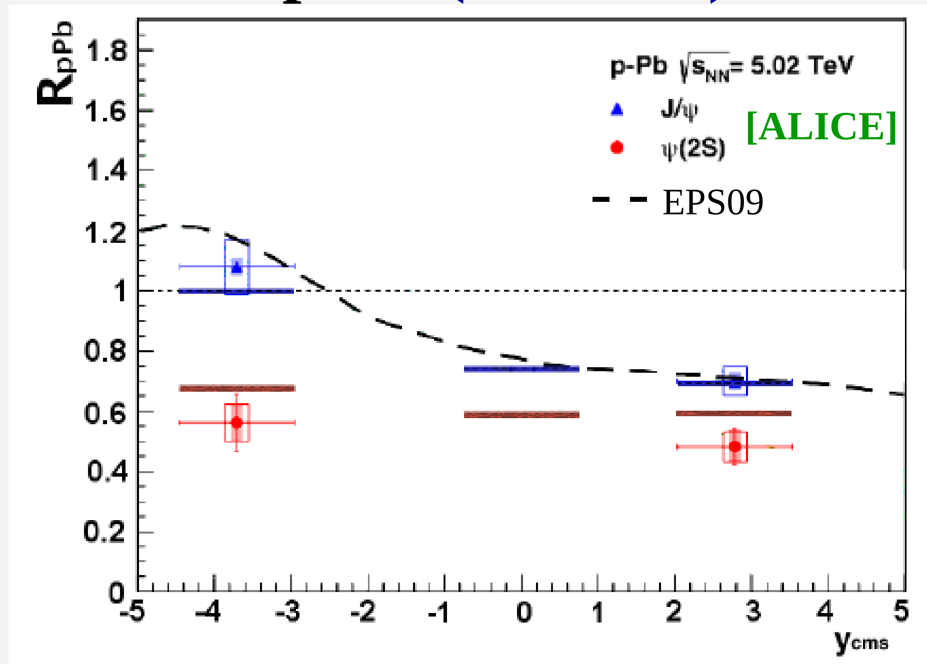
[Sorge et al '97, Grandchamp et al '03, ….]

3.5 ψ(2S) in d/p-A Collisions

d-Au (0.2TeV)



p-Pb (5.02TeV)



- noticeable ψ′ and little **J/ψ** suppression, consistent with "comovers"

[Ferreiro '15]

- supports fireball formation with:

$\tau_{FB}\ \Gamma(\psi')\ \sim 1 \quad \Rightarrow \quad \Gamma_{avg}(\psi') \sim$ **50-100 MeV** similar to thermal

$\tau_{FB}\ \Gamma(J/\psi) << 1 \quad \Rightarrow \quad \Gamma_{avg}(J/\psi) <$ **20 MeV** widths at **T≈ 200MeV**

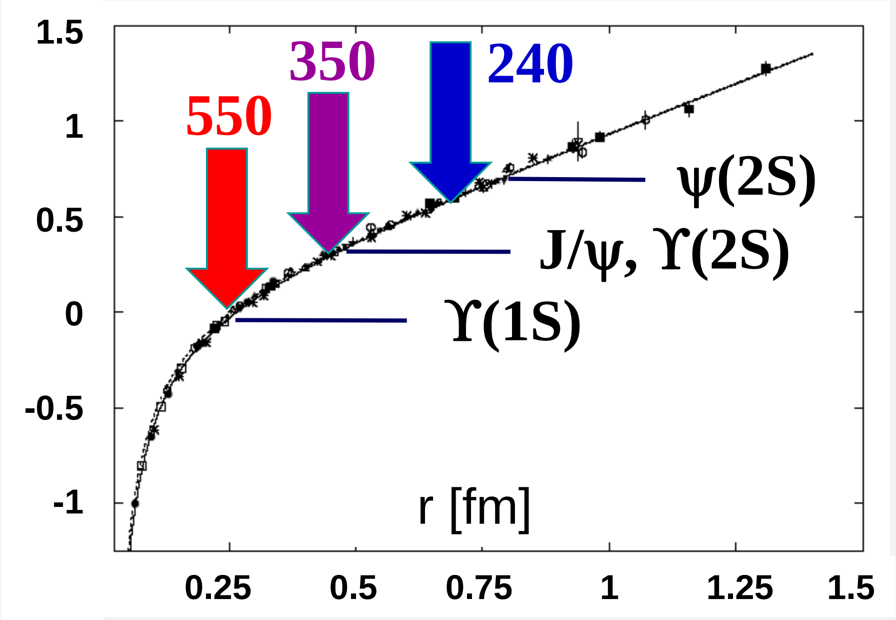
[Du et al '15]

2.3 Upshot of Quarkonium Phenomenology

Use temperature estimates from hydro/photons/dileptons to infer:

T_0^{SPS} (~240) < $T_{melt}(J/\psi,\Upsilon')$ ≤ T_0^{RHIC} (~350) < $T_{melt}(\Upsilon)$ ≤ T_0^{LHC} (~550)

- **Remnants of confining force survive at SPS** [hold **J/ψ** together]
- **Confining force screened at RHIC+LHC** [“melts” **J/ψ+ϒ(2S)**]
- **Color-Coulomb screening at LHC** [**ϒ(1S)** suppression]
- **Thermalizing charm quarks recombine at LHC** [large **J/ψ** yield]



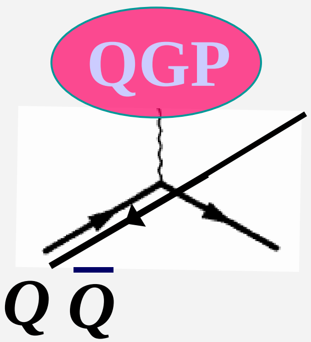
2.4 Heavy-Quark Potential Extraction from Υ Data

- Ansatz for in-med. potential $$V_{Q\bar{Q}}(r) = -\frac{4}{3}\alpha_s \frac{e^{-m_D r}}{r} + \sigma\left(r - R_{\mathrm{SB}}\right)\Theta\left(R_{\mathrm{SB}} - r\right)$$

- Parameterize **T**-dependence of **$m_D \sim \alpha T$**, **$1/R_{SB} \sim m_S(T;\beta,\gamma,\delta)$**

  $\Rightarrow$ determine in-med. binding energies **$E_\Upsilon(m_D, m_S)$**

- Compute **$\Gamma_\Upsilon\,[E_\Upsilon]$**
  - also requires HQ-medium coupling
  - perturbative: **$\alpha_s \sim 0.3$**
  - non-perturbative: **$K \geq 5$**

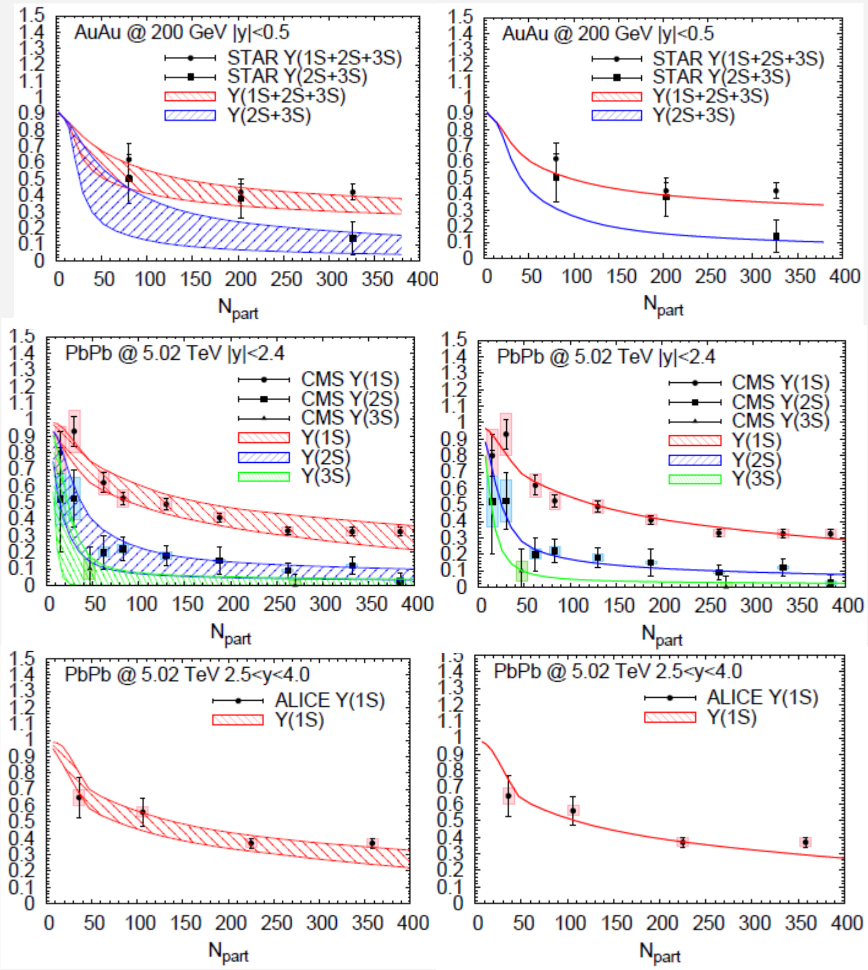


- Deploy transport approach to fit (**$\alpha,\beta,\gamma,\delta$; K**) to Υ data at **RHIC** + **LHC**

[Du,Liu+RR '19]

3.4 Statistical Extraction of Heavy-Quark Potential

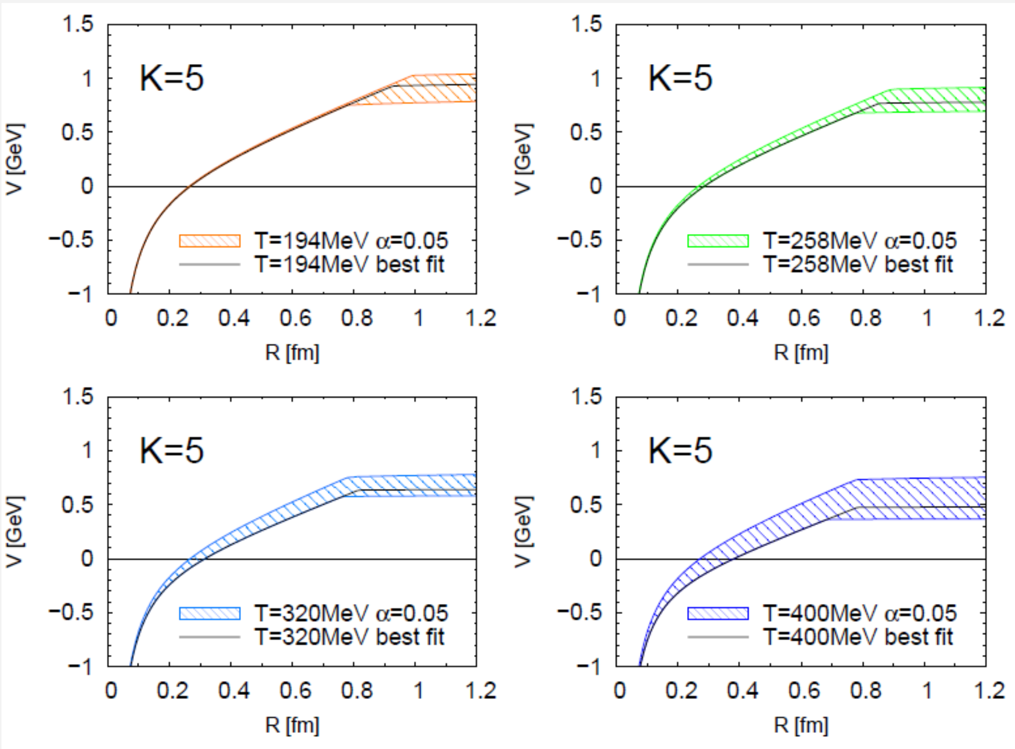
Fit Results



[Du et al '19]

- χ^2/dof ≤ 1

In-Medium Potential

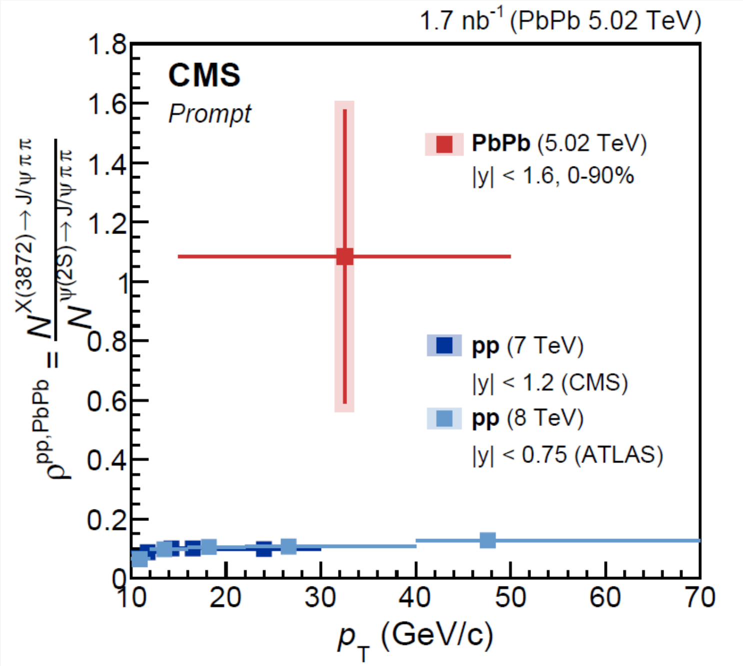


- "Strongly-coupled" solution: **remnants of confining force** survive well above T_c
- Not unique …

3.5 X(3872)

Does **X(3872)** production in heavy-ion collisions reveal **internal structure** information?

Larger size (molecule vs. tetraquark) ⇒ larger yield -- or else?



- With $\mathbf{R_{AA}^{exp}(\psi(2S)) \sim 0.14}$, $\mathbf{R_{AA}(X)}$ compatible with unity
- $\mathbf{p_T > 15GeV}$ might be outside coalescence/regeneration regime

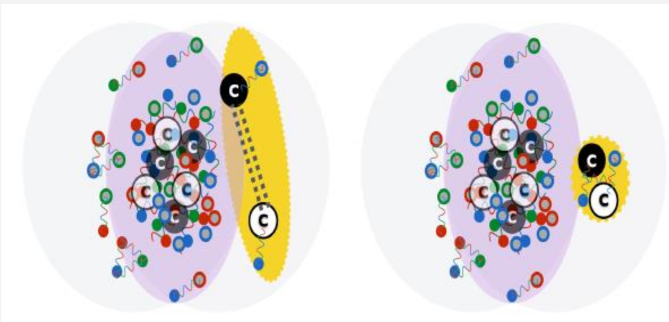
3.5.2 In-Medium X(3872) Production Models

- **Coalescence Models**: partons (f_i) projected on hadron wave fct. (W)

[Cho et al ’11, Fontoura et al. ‘19, …]

$$f_h(p'_h) = \int \left[\prod_i dp_i f_i(p_i)\right] W(\{p_i\}) \delta(p'_h - \sum_i p_i)$$

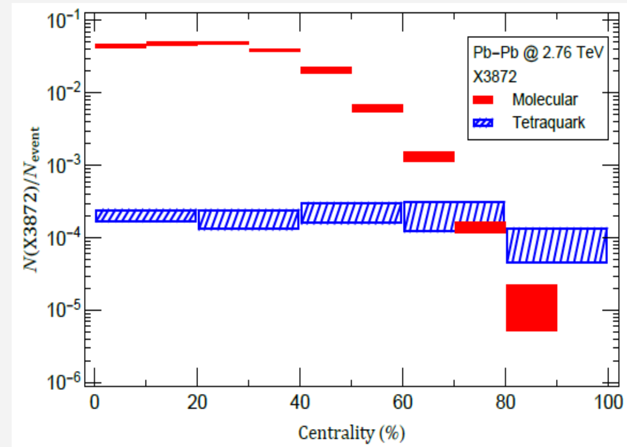
$W=W(\sigma)$ with hadron radius σ



$N_X \sim \sigma^3 \Rightarrow$

$N_{mol} >> N_{tetra}$

[H. Zhang et al ’20]



- **Transport Approach**:

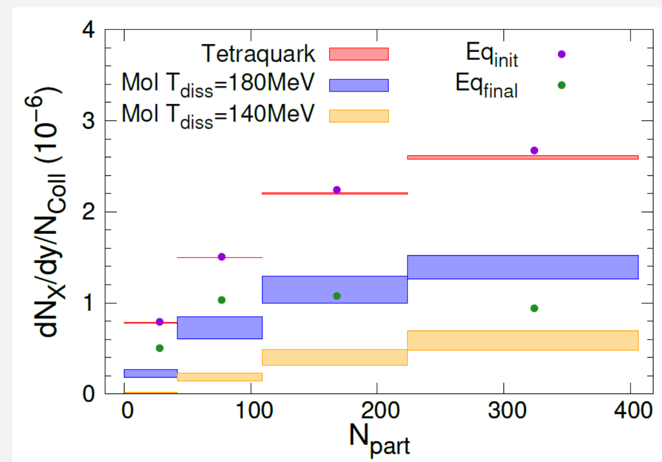
[B. Wu et al ’20]

$$\frac{dN_X}{dt} = -\Gamma_X \left(N_X - N_X^{eq}\right)$$

- Reaction rate larger for weakly bound molecule

  $\Rightarrow$ molecule freeze-out later than tetraquark

- Equilibrium limit drops with $T \Rightarrow N_{mol} < N_{tetra}$

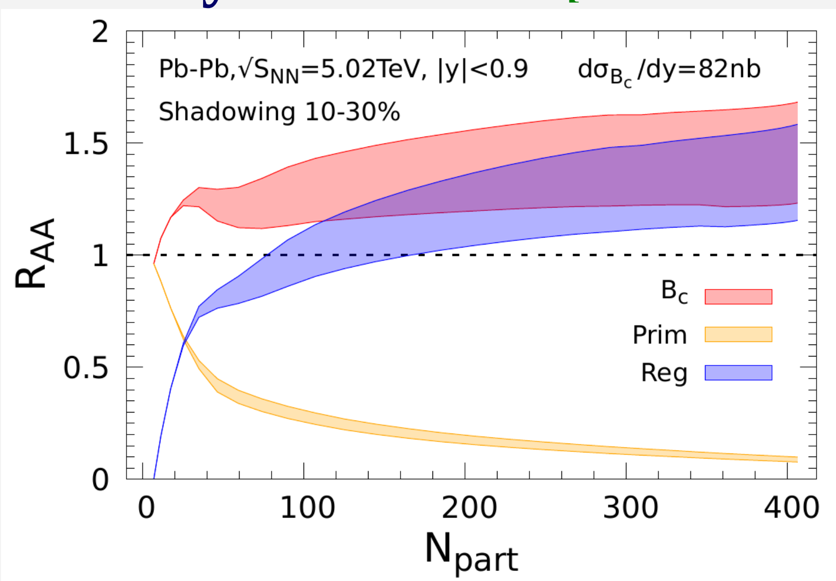


3.6 B_c(6275)

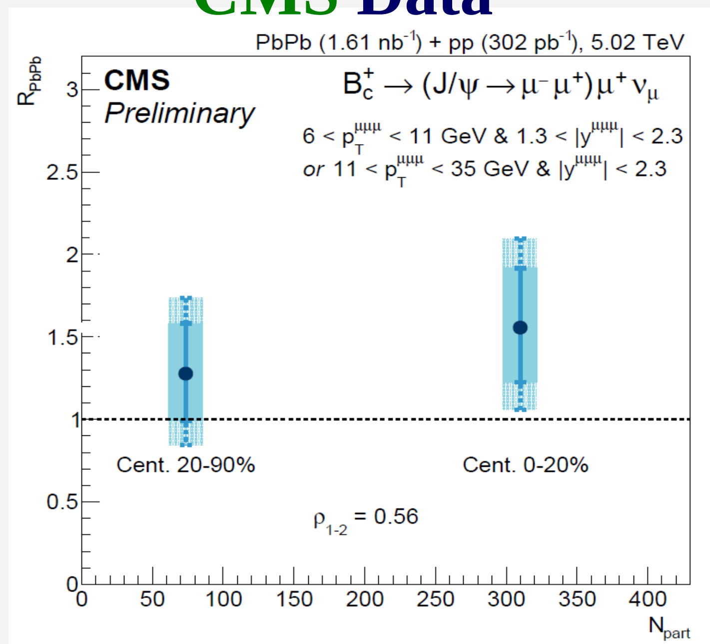
- Critical "universality test" for transport models
- Expect large yields in **AA** relative to **pp** [**$dN_c/dy \sim 30$** in **PbPb**(5TeV)]
- Need to control **pp** production cross section (denominator):

$$R_{AA}(p_T) \equiv (dN/dp_T)_{AA} \;/\; N_{coll}\,(dN/dp_T)_{pp}$$

Theory Prediction **[Wu et al. '21]**



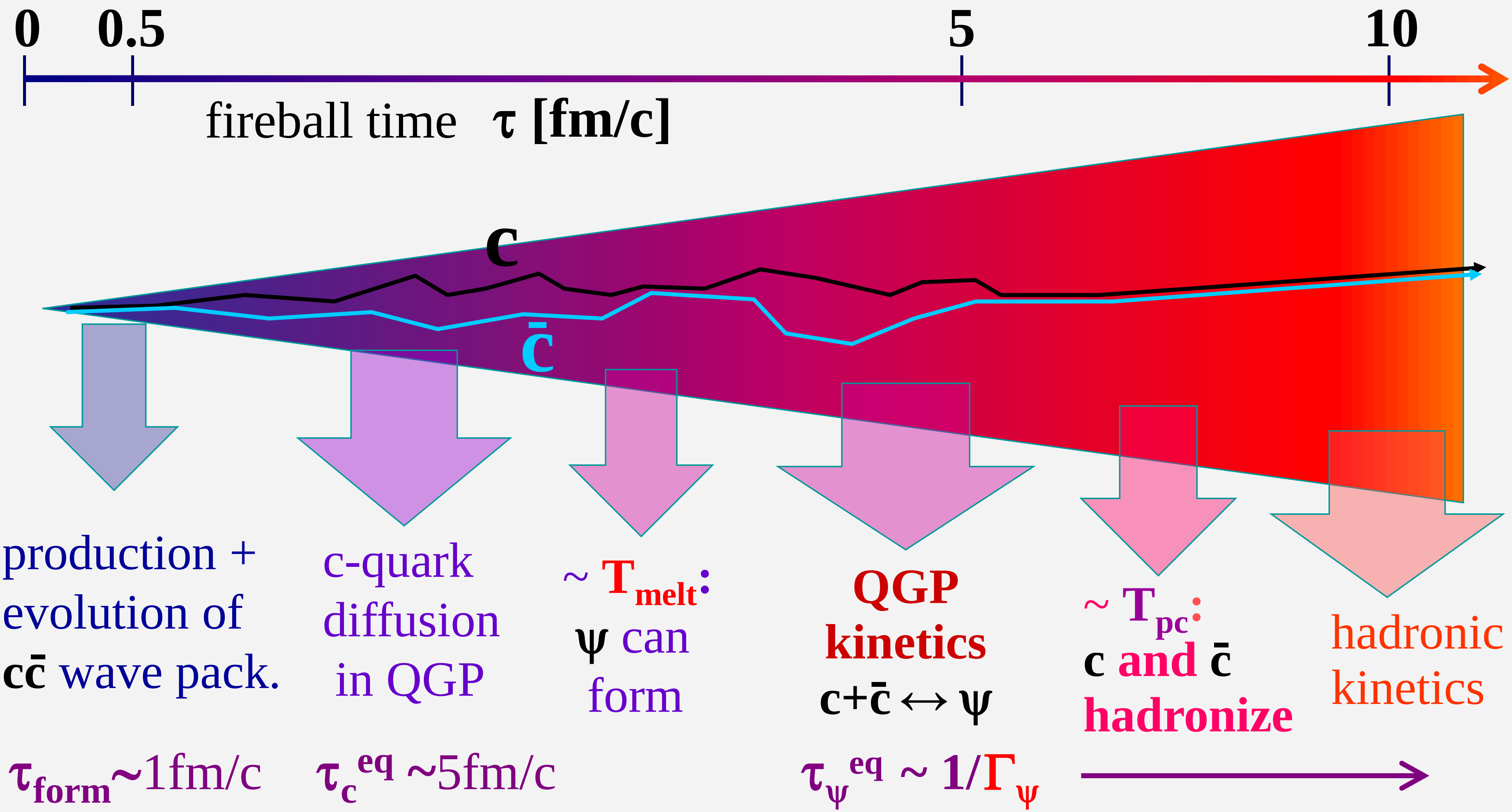
CMS Data



- Predictions appear to agree with data…

caveats: **pp** cross section, p_T range (theory: $p_T > 0$, data: $p_T^{3\mu} > 6$ GeV)

2.1 Quarkonium Transport in URHICs



[Satz et al, Capella et al, Spieles et al, PBM et al, Thews et al, Grandchamp et al, Ko et al, Zhuang et al, Zhao et al, Chaudhuri, Gossiaux et al, Young et al, Ferreiro et al, Strickland et al, Brambilla et al, …]