

The statistical hadronization model for heavy quarks

A. Andronic - University of Münster



-
- The statistical model and the thermal fits
 - The charm quarks
 - The beauty quarks

Andronic, Braun-Munzinger, Redlich, Stachel, [Nature 561 \(2018\) 321](#)

...+ Köhler, Mazeliauskas, Vislavicius, [JHEP 07 \(2021\) 035](#)



The statistical (thermal) model

grand canonical partition function for specie (hadron) i :

$$\ln Z_i = \frac{V g_i}{2\pi^2} \int_0^\infty \pm p^2 dp \ln[1 \pm \exp(-(E_i - \mu_i)/T)]$$

$g_i = (2J_i + 1)$ spin degeneracy factor; T temperature;

$E_i = \sqrt{p^2 + m_i^2}$ total energy; (+) for fermions (-) for bosons

$\mu_i = \mu_B B_i + \mu_{I_3} I_{3i} + \mu_S S_i + \mu_C C_i$ chemical potentials

μ ensure conservation (on average) of quantum numbers, fixed by “initial conditions”

i) isospin: $\sum_i n_i I_{3i} / \sum_i n_i B_i = I_3^{tot} / N_B^{tot}$, $N_B^{tot} \sim \mu_B$

I_3^{tot} , N_B^{tot} isospin and baryon number of the system (=0 at high energies)

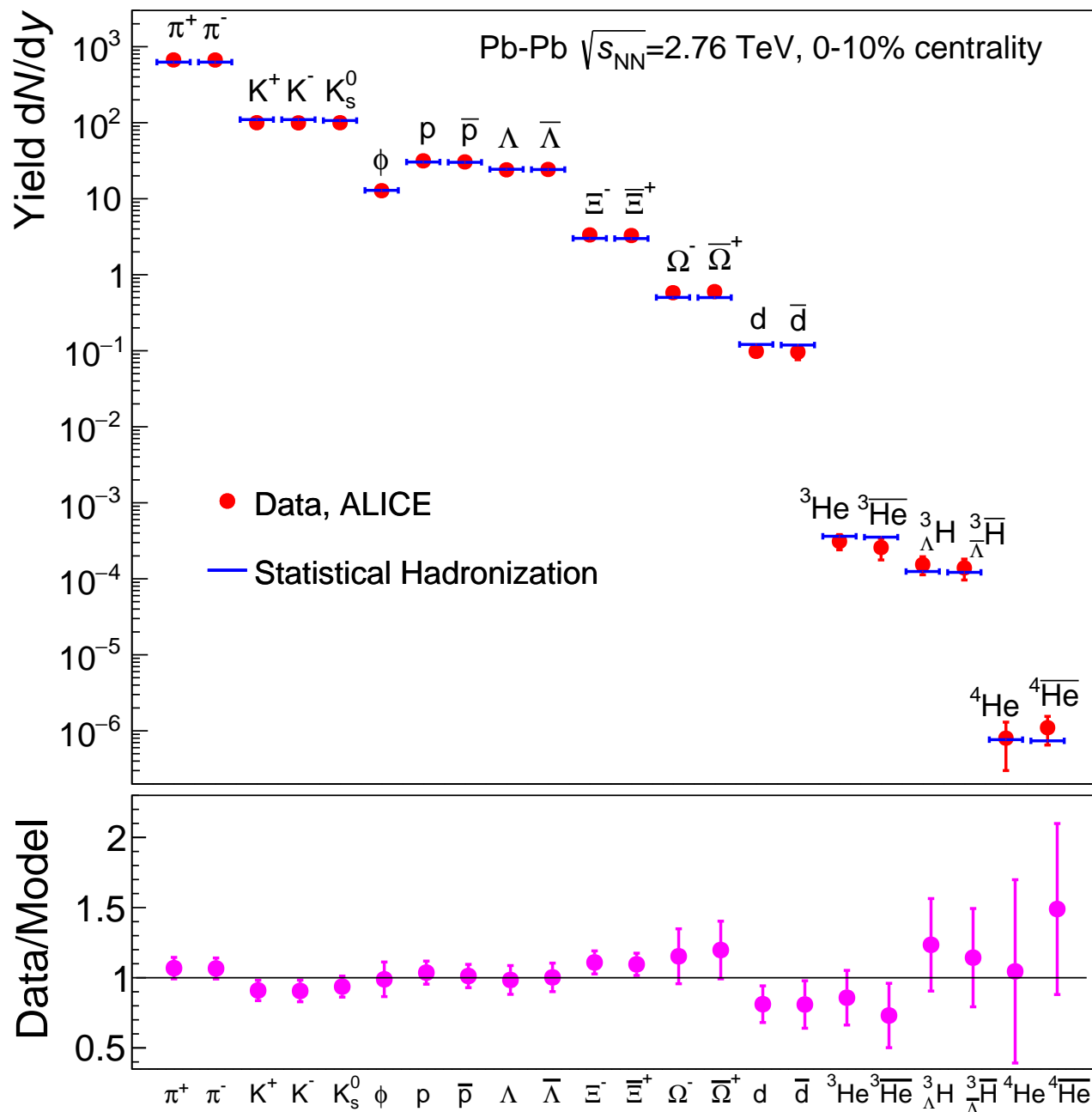
ii) strangeness: $\sum_i n_i S_i = 0$

iii) charm: $\sum_i n_i C_i = 0$.

Thermal fit – LHC, Pb–Pb, 0-10%

A. Andronic

3



matter and antimatter produced in equal amounts

$$T_{CF} = 156.6 \pm 1.7 \text{ MeV}$$

$$\mu_B = 0.7 \pm 3.8 \text{ MeV}$$

$$V_{\Delta y=1} = 4175 \pm 380 \text{ fm}^3$$

$$\chi^2/N_{df} = 16.7/19$$

S-matrix treatment

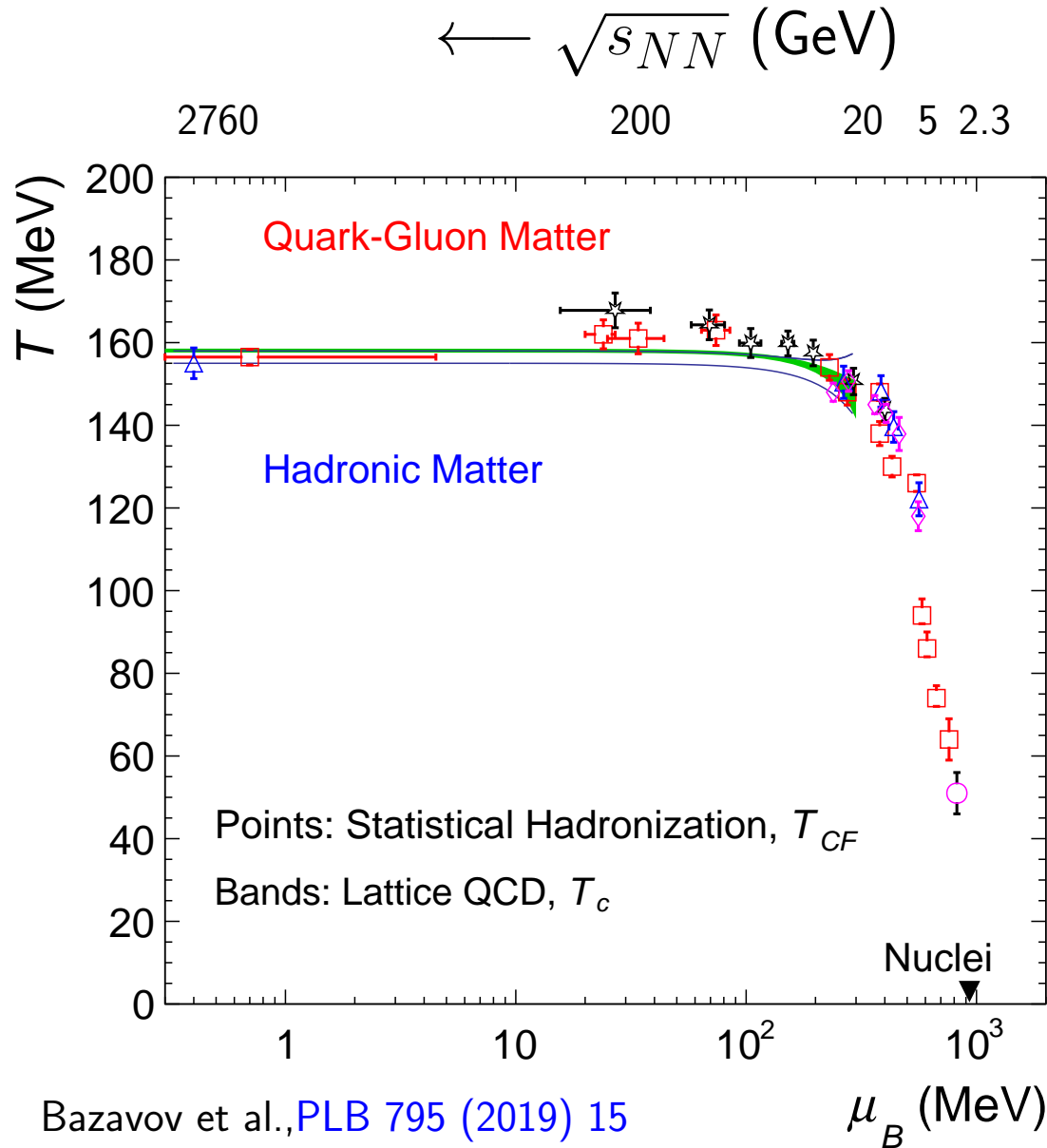
remarkably, loosely-bound objects are also well described (${}^3_{\Lambda}\text{H}$ with 25% B.R.)

hadronization as bags of quarks and gluons?

The phase diagram of QCD

A. Andronic

4



at LHC, remarkable “coincidence” with Lattice QCD results

at LHC ($\mu_B \simeq 0$): purely-produced (anti)matter ($m = E/c^2$), as in the Early Universe

$\mu_B > 0$: more matter, from “remnants” of the colliding nuclei

$\mu_B \gtrsim 400$ MeV: *the critical point awaiting discovery*

(RHIC BES / FAIR)

Bazavov et al., [PLB 795 \(2019\) 15](#)

Borsanyi et al., [PLB 370 \(2014\) 99](#)

see refs. in [Nature 561 \(2018\) 321](#)

points: independent analyses of same data → “model/code uncert.” are small

SHM for charm (SHMc)

A. Andronic

5

pQCD production, "throw in": $N_{c\bar{c}} = 9.6 \rightarrow g_c = 30.1$ ($I_1/I_0 = 0.974$)

LHC, central collisions

assume:

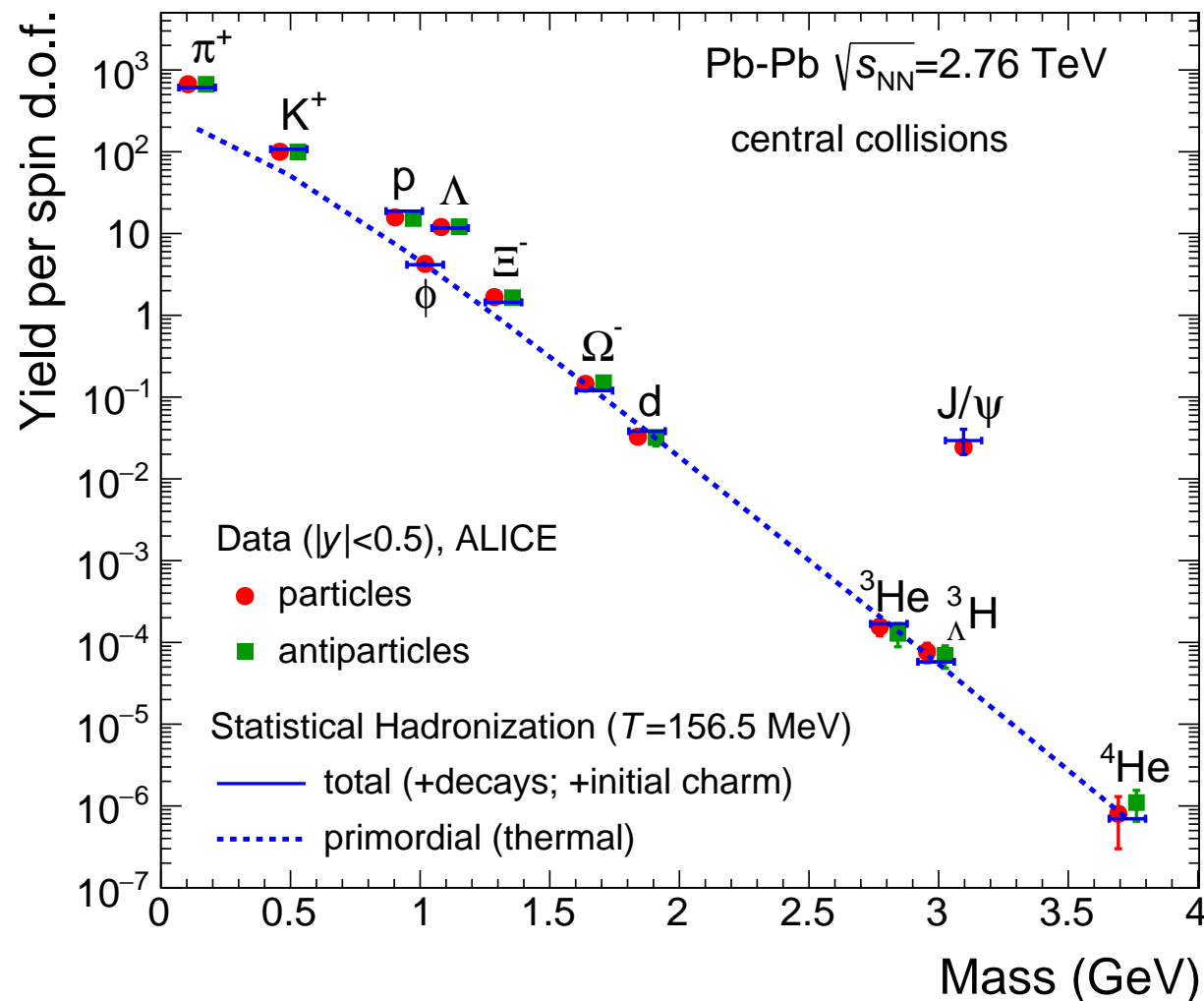
- full thermalization of c, \bar{c}
("mobility" in $V \simeq 4000 \text{ fm}^3$)

- full color screening
(Matsui-Satz)

Braun-Munzinger, Stachel, [PLB 490 \(2000\) 196](#)

Model predicts all charm
chemistry ($\psi(2S), X(3872)$)

π, K^\pm, K^0 from charm included in the thermal fit
(0.7%, 2.9%, 3.1% for $T=156.5 \text{ MeV}$)



[PLB 797 \(2019\) 134836](#)

SHMc: method and inputs

Braun-Munzinger, Stachel, [PLB 490 \(2000\) 196](#), [NPA 690 \(2001\) 119](#)

- Thermal model calculation (grand canonical) T, μ_B : $\rightarrow n_X^{th}$
- $N_{c\bar{c}}^{dir} = \frac{1}{2}g_c V (\sum_i n_{D_i}^{th} + n_{\Lambda_i}^{th}) + g_c^2 V (\sum_i n_{\psi_i}^{th} + n_{\chi_i}^{th})$
- $N_{c\bar{c}} \ll 1 \rightarrow$ Canonical (Cleymans, Redlich, Suhonen, Z. Phys. C51 (1991) 137):

Gorenstein, Kostyuk, Stöcker, Greiner, [PLB 509 \(2001\) 277](#)

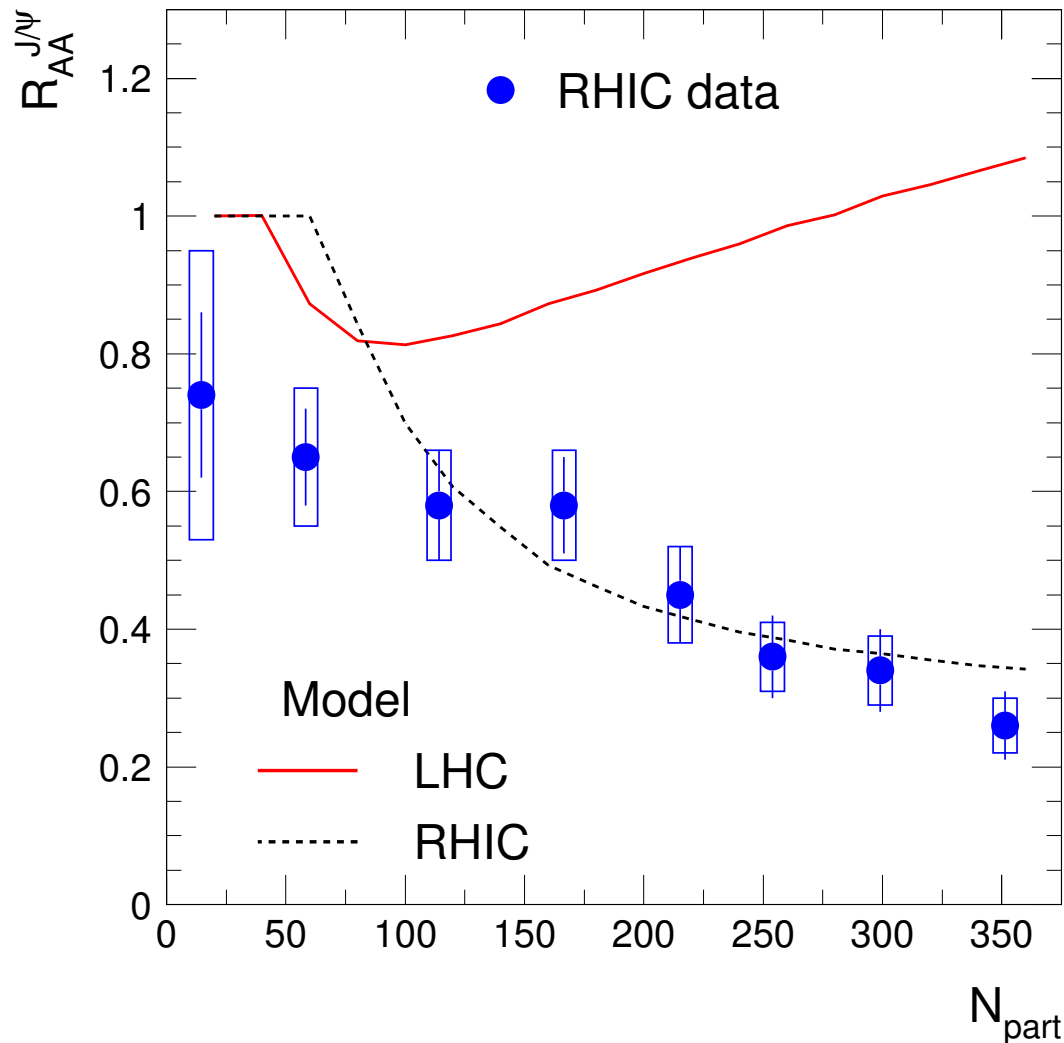
$$N_{c\bar{c}}^{dir} = \frac{1}{2}g_c N_{oc}^{th} \frac{I_1(g_c N_{oc}^{th})}{I_0(g_c N_{oc}^{th})} + g_c^2 N_{c\bar{c}}^{th} \rightarrow g_c(N_{part}) \text{ (charm fugacity)}$$

$$\text{Outcome: } N_D = g_c V n_D^{th} I_1/I_0 \quad N_{J/\psi} = g_c^2 V n_{J/\psi}^{th}$$

$$\text{Inputs: } T, \mu_B, \quad V_{\Delta y=1} (= (dN_{ch}^{exp}/dy)/n_{ch}^{th}), \quad N_{c\bar{c}}^{dir} \text{ (exp. or pQCD)}$$

Assumed minimal volume for QGP: $V_{QGP}^{min} = 200 \text{ fm}^3$

High hopes for charmonium at the LHC



$$R_{AA}^{J/\psi} = \frac{dN_{J/\psi}^{AuAu}/dy}{N_{coll} \cdot dN_{J/\psi}^{pp}/dy}$$

- "suppression" at RHIC
- "enhancement" at LHC

$$N_{J/\psi} \sim (N_{c\bar{c}}^{dir})^2$$

What is so different at LHC?
(compared to RHIC)

$\sigma_{c\bar{c}}$: $\sim 10x$, Volume: $2.5x$

PLB 652 (2007) 259

this is a generic prediction of SHM ...was confirmed by LHC data
RHIC warrants another look ($\sigma_{c\bar{c}}$ lowish, at that time)

High hopes for charmonium at the LHC

A. Andronic

8

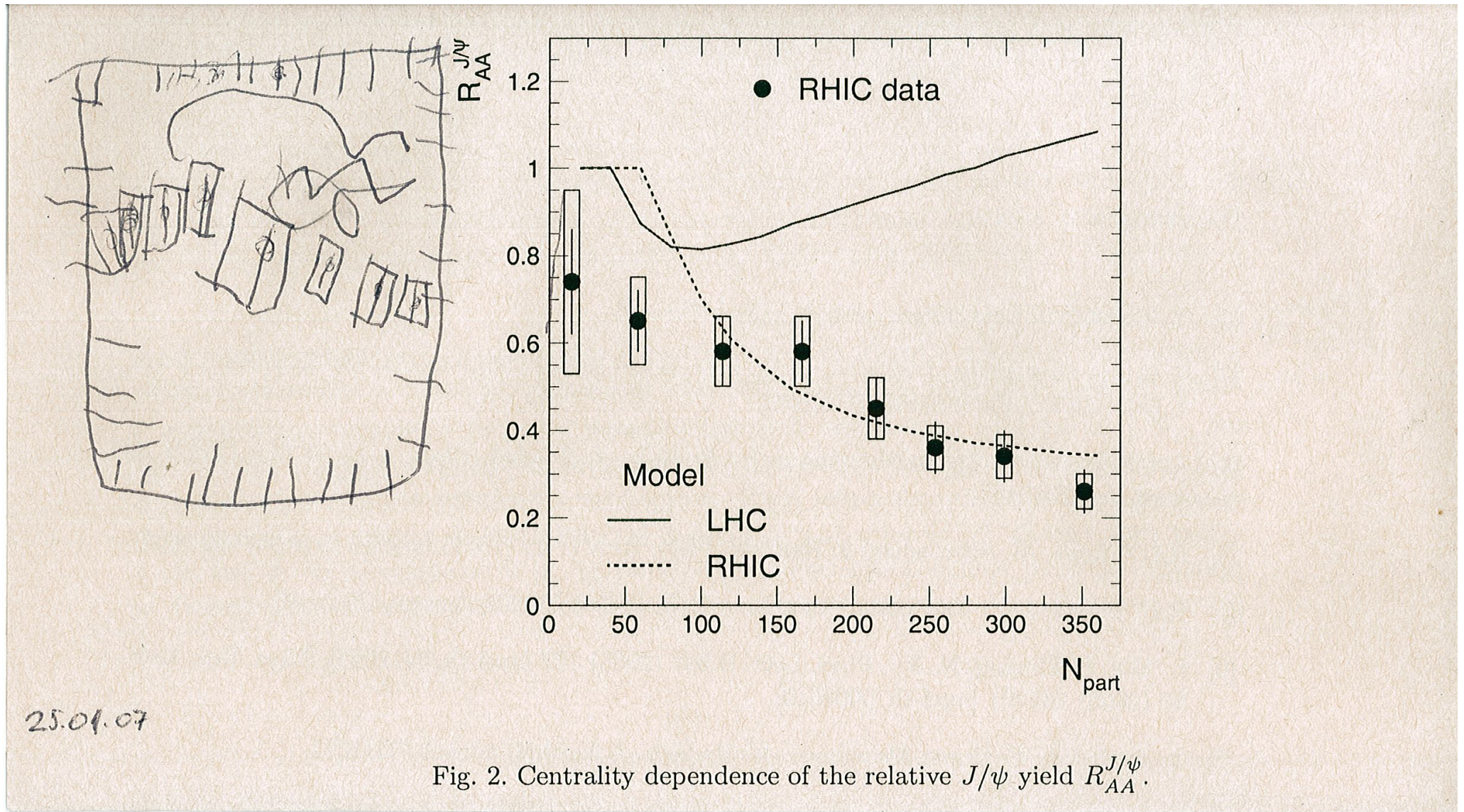


Fig. 2. Centrality dependence of the relative J/ψ yield $R_{AA}^{J/\psi}$.

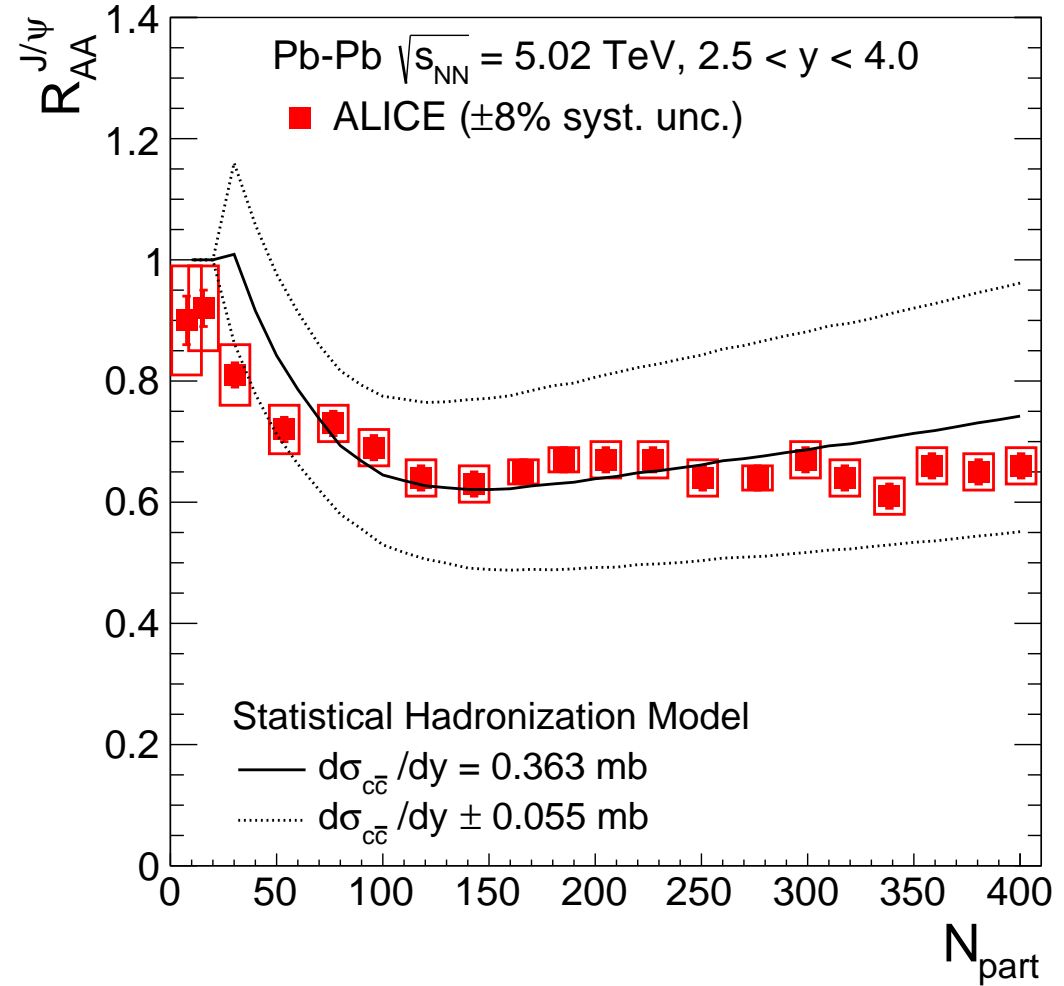
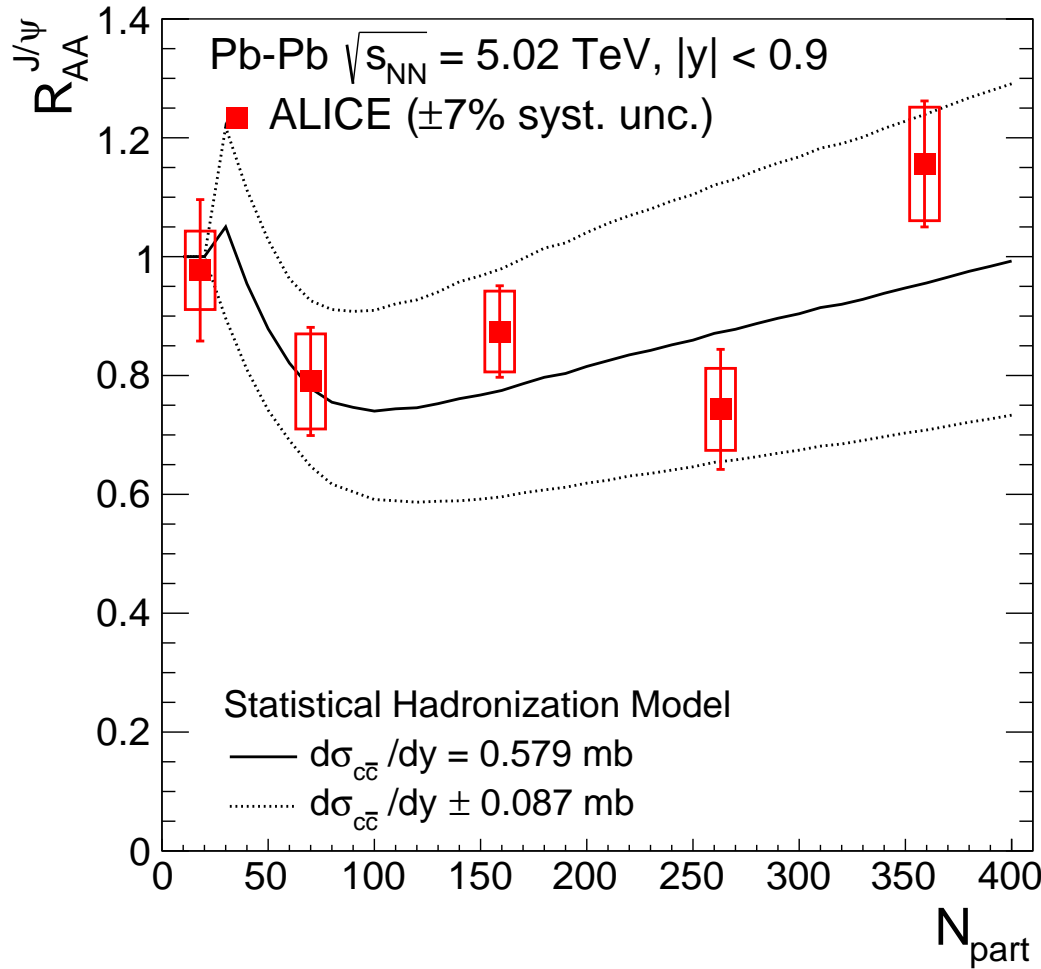
...was impressing a 3-years old girl ...who described better data than model

SHMc and charmonium data at the LHC

A. Andronic

9

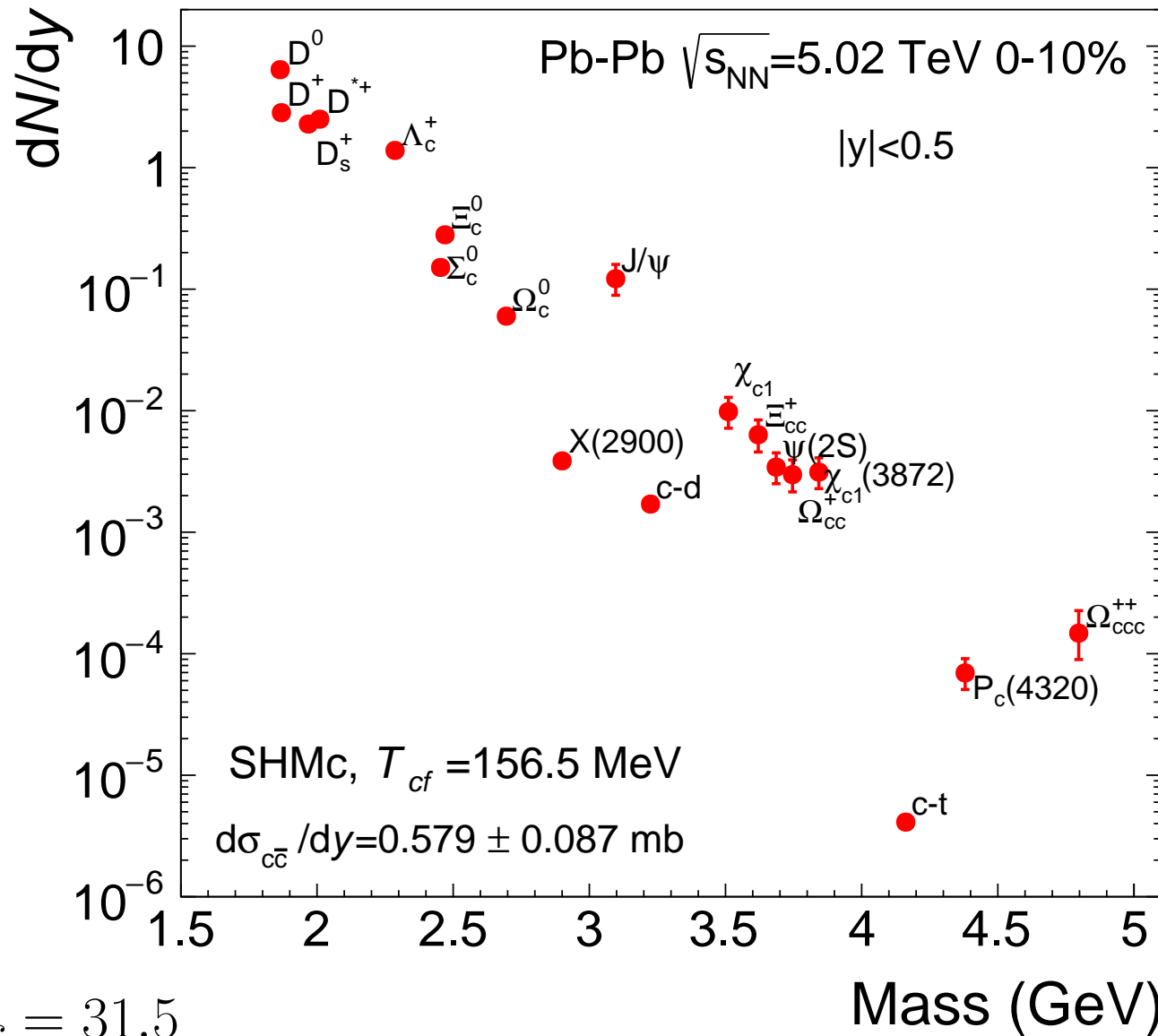
full thermalization of c quarks in QGP, hadronization at chemical freeze-out



$d\sigma_{c\bar{c}}/dy$ via normalization to D^0 in Pb-Pb 0-10%, ALICE, [arXiv:2110.09420](https://arxiv.org/abs/2110.09420)

$dN/dy = 6.82 \pm 1.03$ ($|y| < 0.5$; FONLL for $y=2.5-4$; assuming hadronization fractions in data as in SHMc)

SHMc: the full charm zoo



$$\frac{dN_{c\bar{c}}}{dy} = 13.8$$

$$\rightarrow g_c = 31.5$$

$$T_{cc}^+ \simeq 0.9 \cdot \chi_{c1}(3872)$$

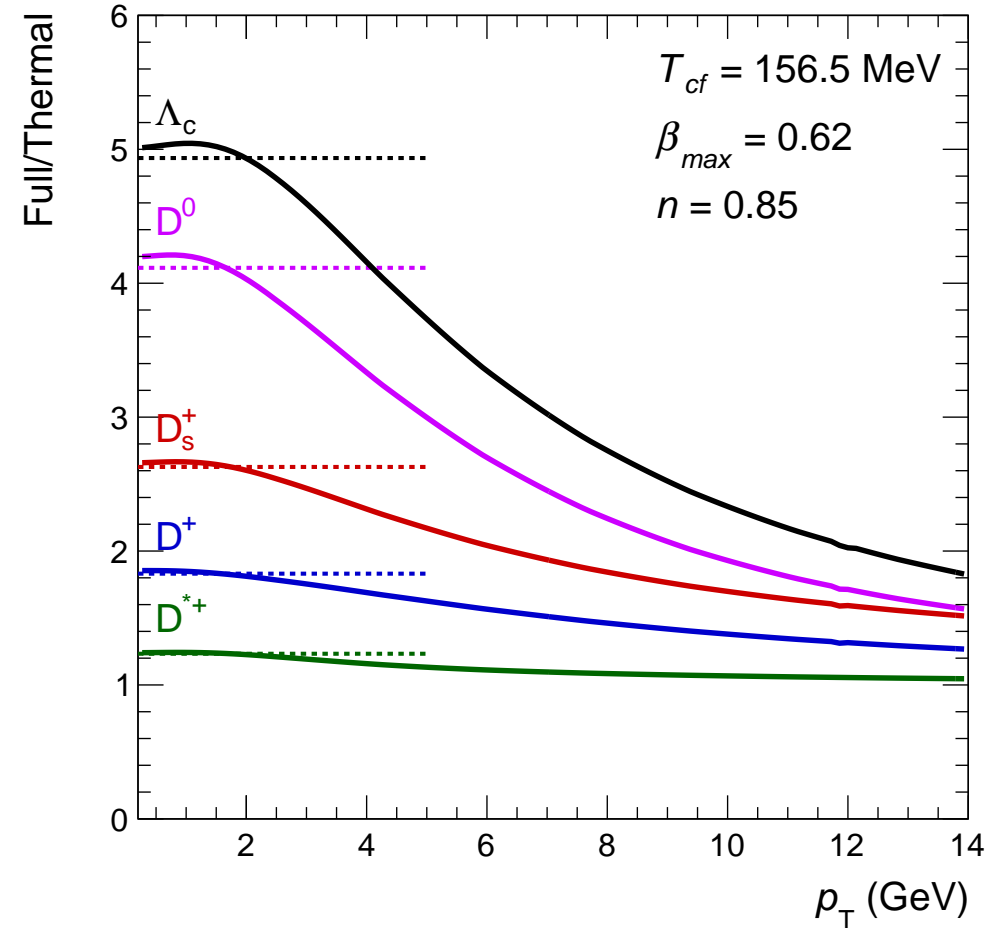
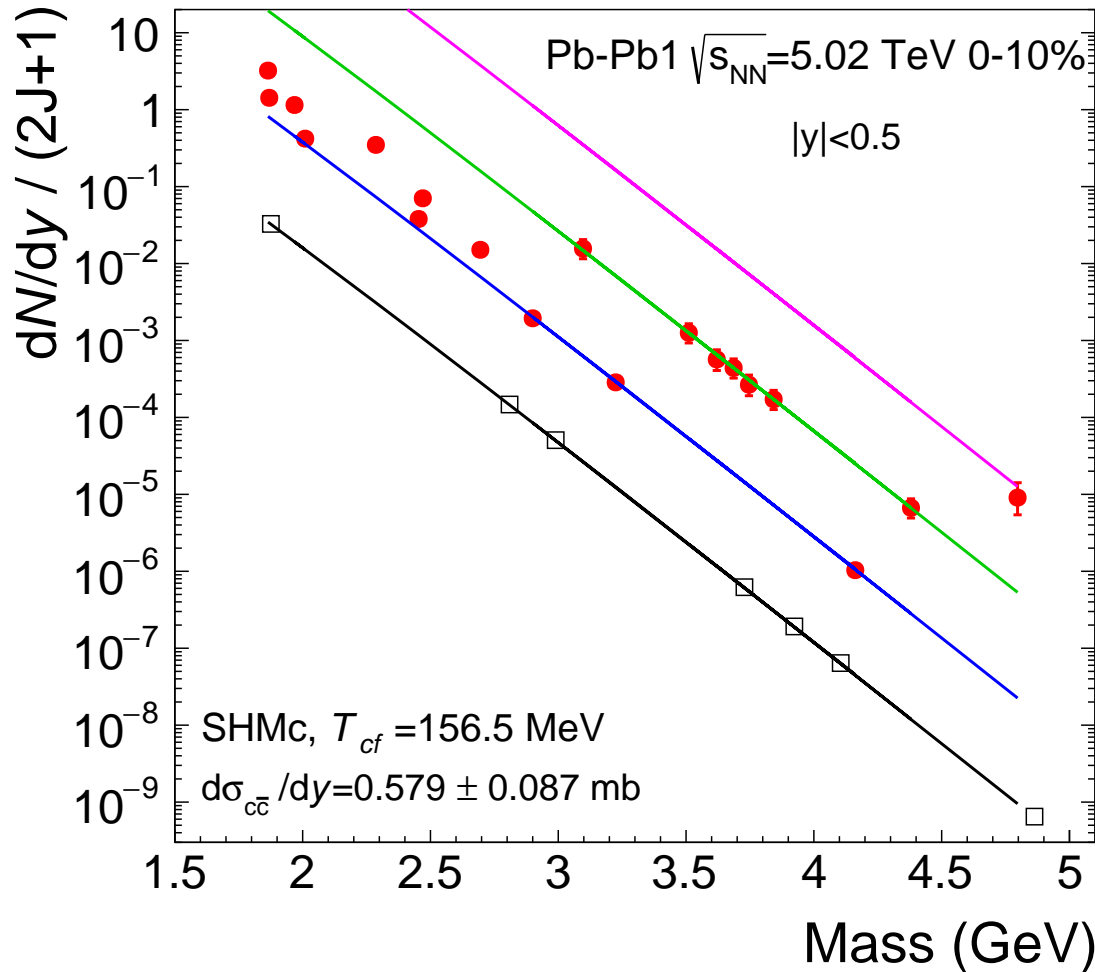
$$X(6900) \sim 10^{-8}$$

The power of the model: predicting the full suite of charmed hadrons

Full charm predictions for the LHC

A. Andronic

11



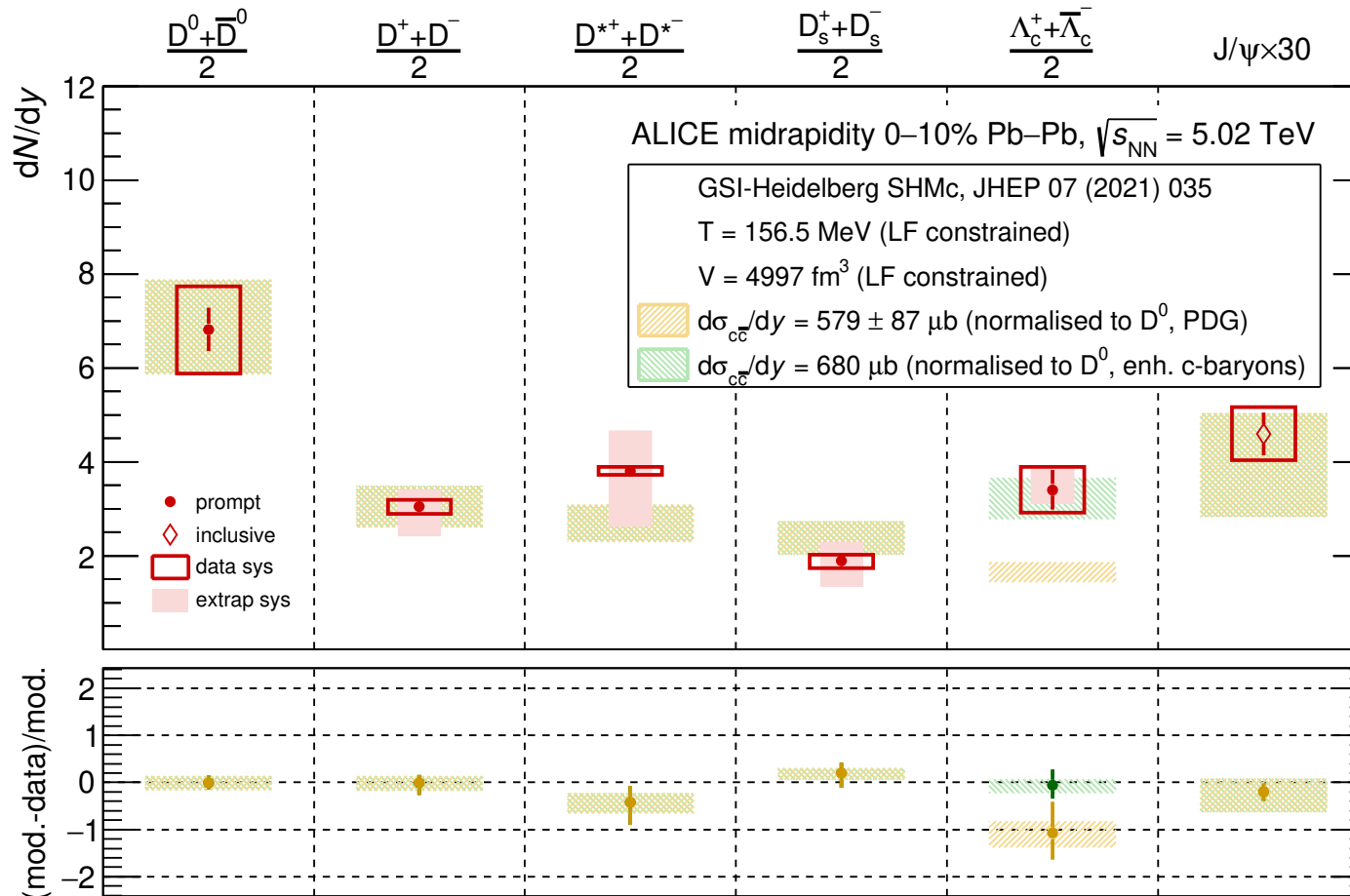
Charm-hadron spectrum as in PDG: 55 c-mesons, 74 c-baryons (part.+antipart.)

...large, but may not be complete

Modified charm-hadron spectrum

...ad-hoc: *tripled* the excited charm-baryon states, enhanced $d\sigma_{c\bar{c}}/dy$ by 19%

RQM: He,Rapp, [PLB 795 \(2019\) 117](#); LQCD, Bazavov et al., [PLB 737 \(2014\) 210](#)



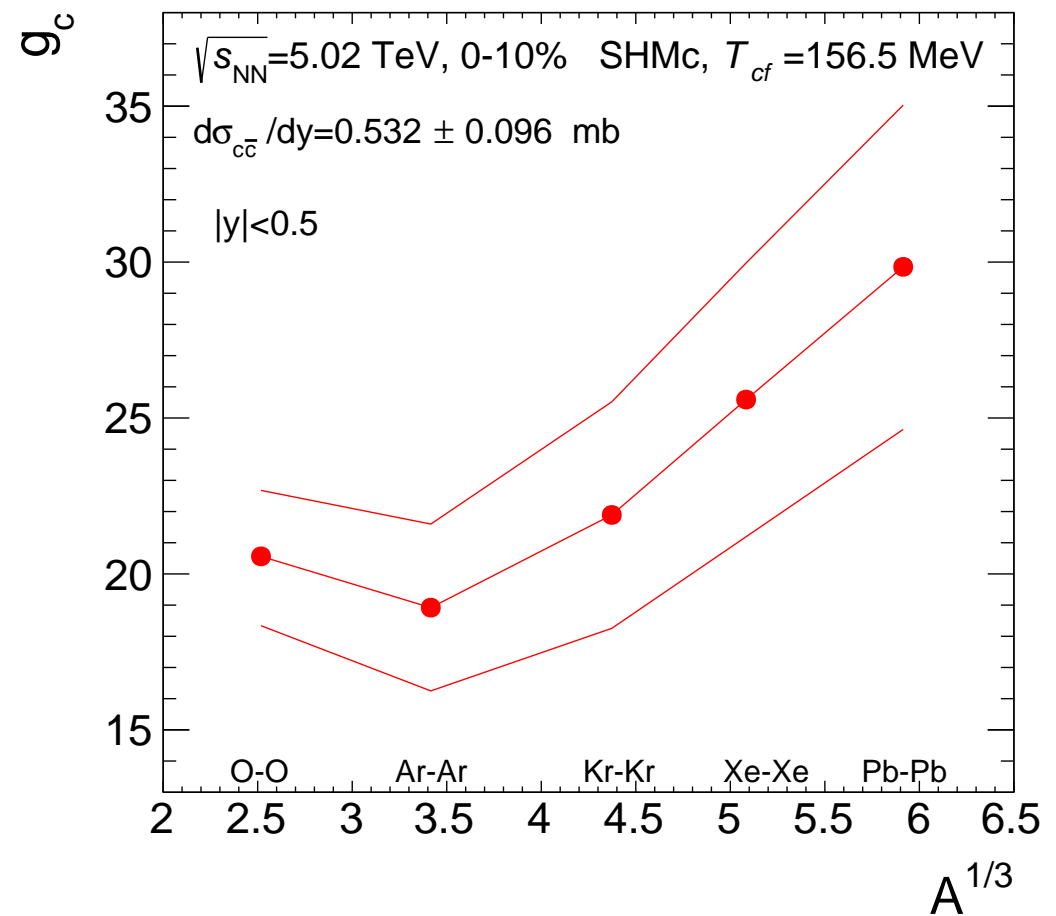
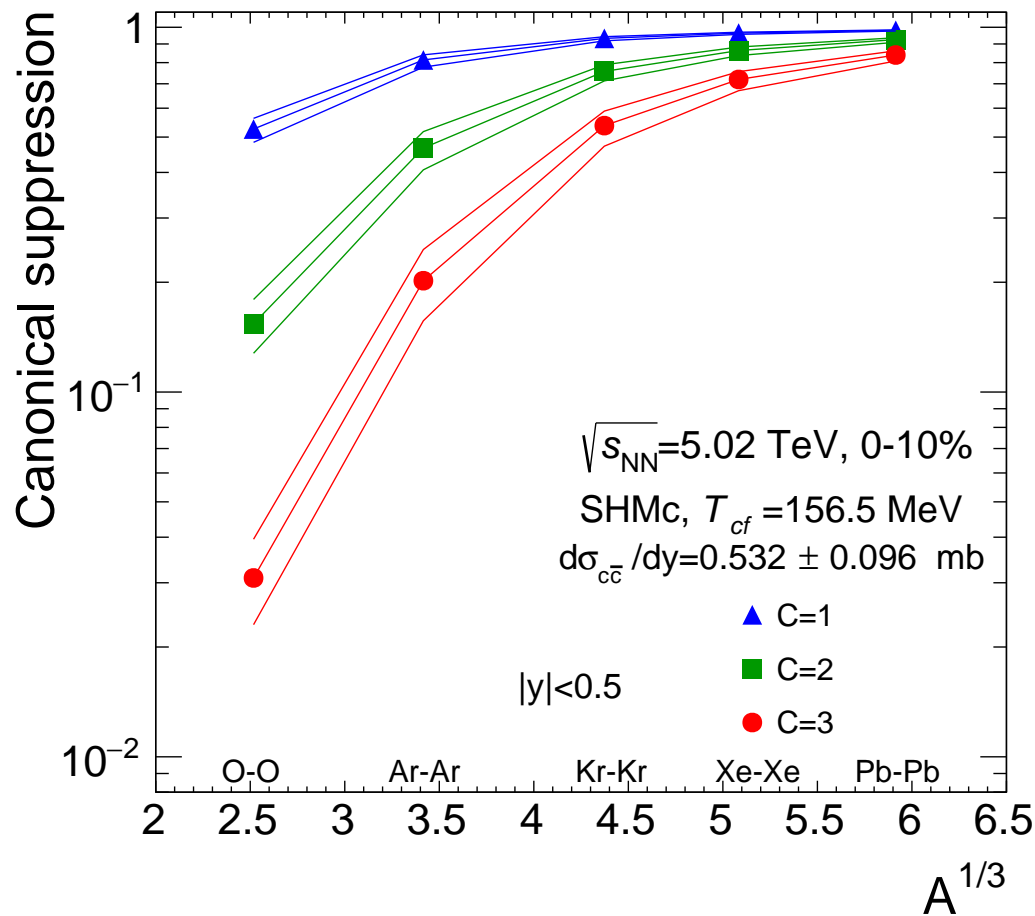
ALICE (QM'22 talk, L. Vermunt)

leaves the mesonic sector unaffected, for the commensurately larger $\sigma_{c\bar{c}}$

SHMc: system dependence (central, 0-10%)

A. Andronic

13

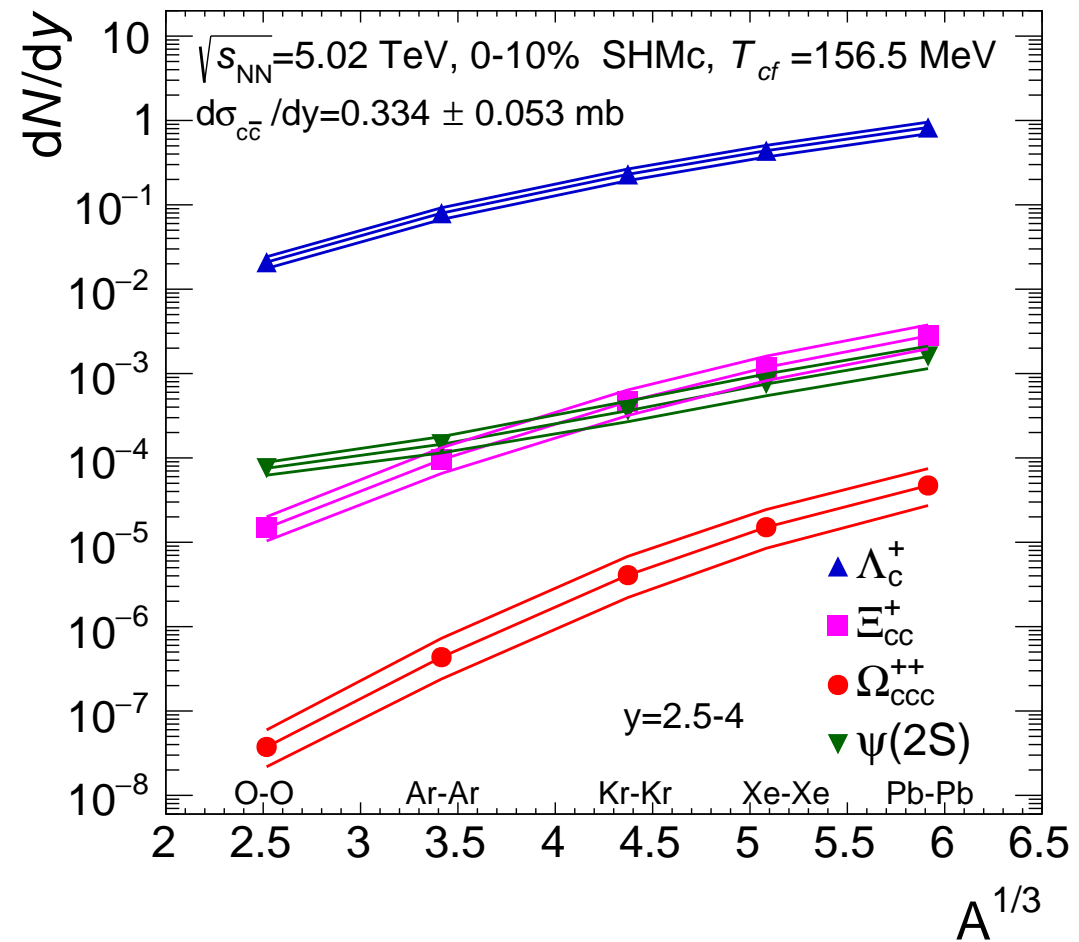
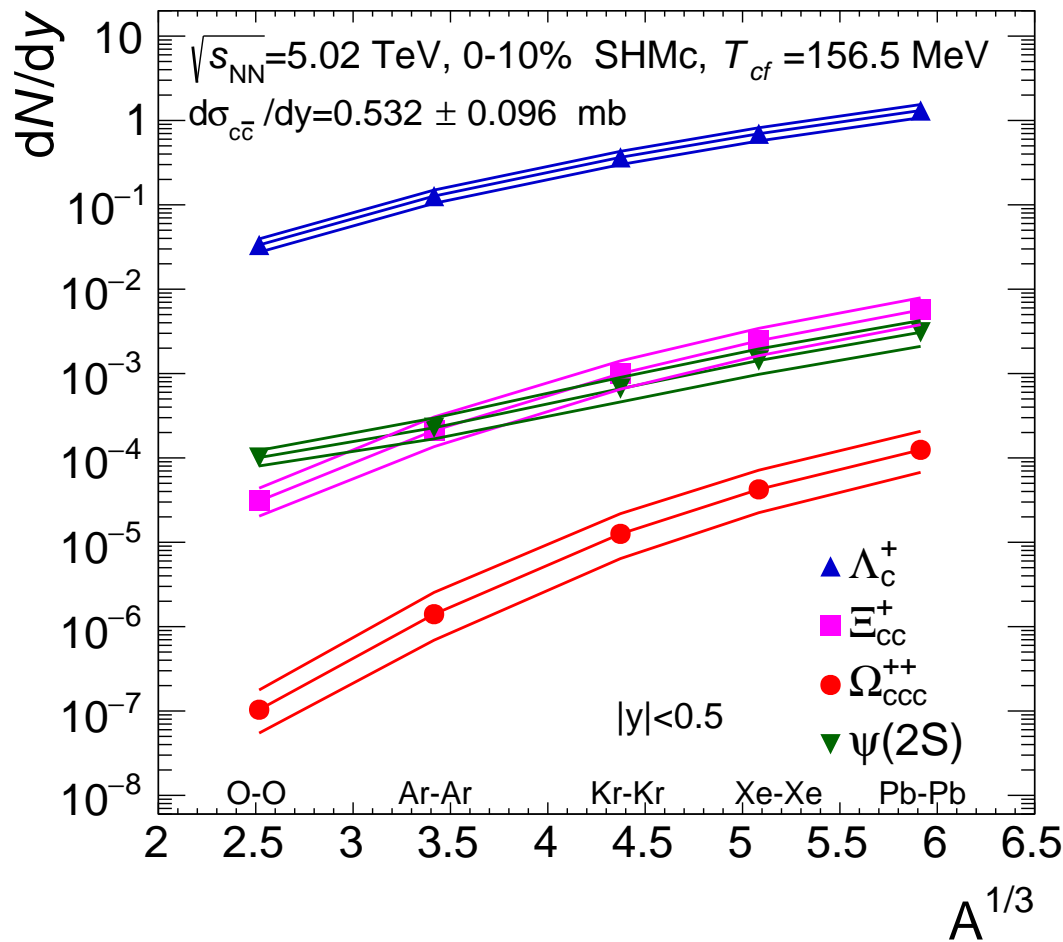


Strong canonical suppression for light systems (for multi-charm hadrons)

SHMc: system dependence (central, 0-10%)

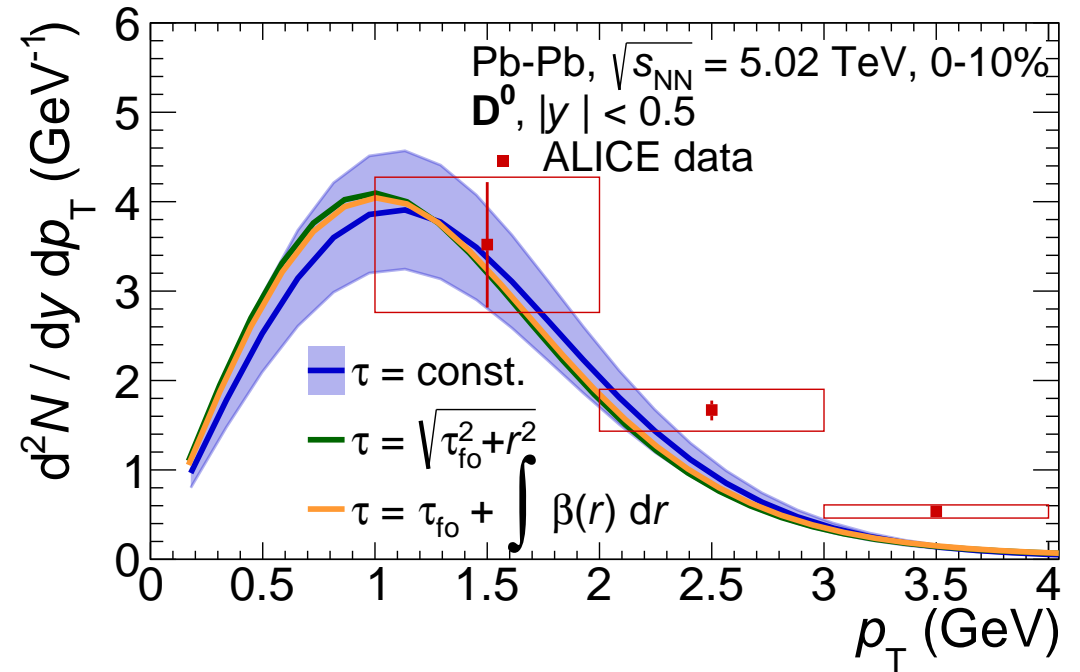
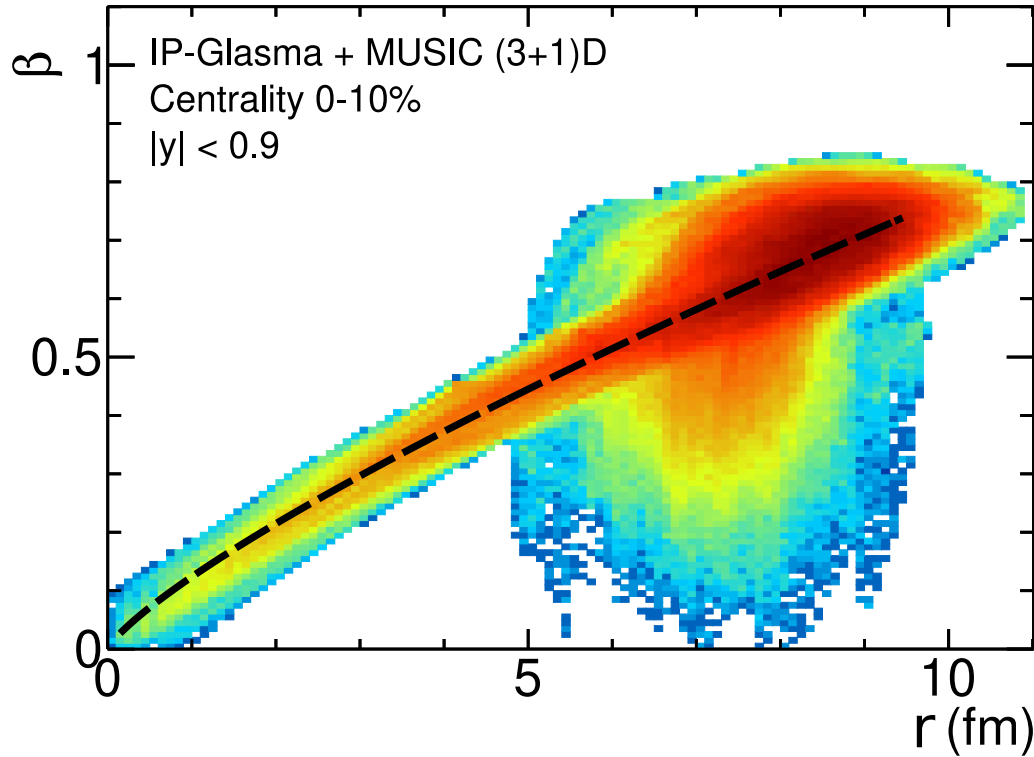
A. Andronic

14



SHMc: p_T dependence

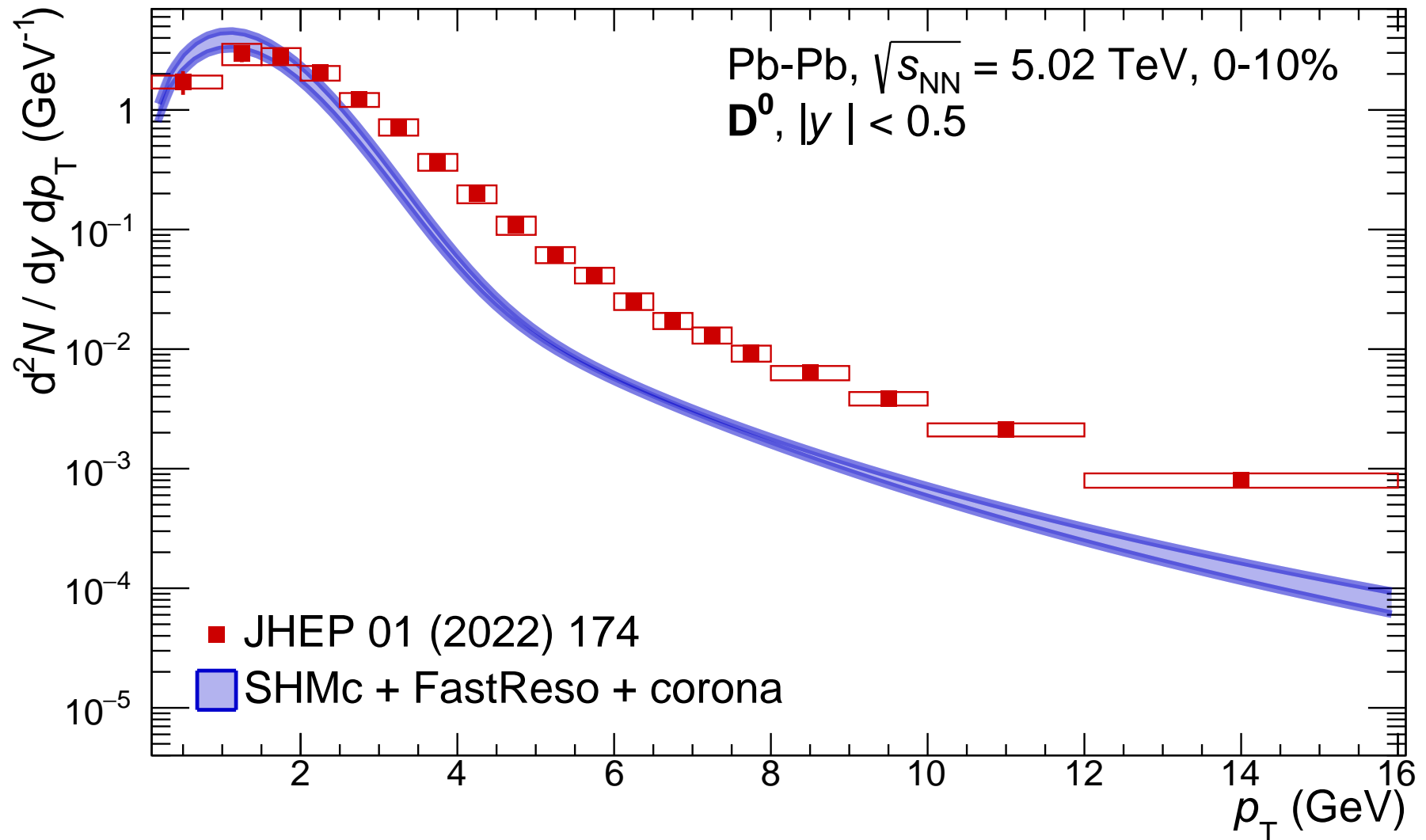
full hydrodynamic flow (MUSIC(3+1)D, IP-Glasma; parametrized via blast-wave



$$\frac{d^2N}{2\pi p_T dp_T dy} = \frac{2J+1}{(2\pi)^3} \int d\sigma_\mu p^\mu f(p) = \frac{2J+1}{(2\pi)^3} \int_0^{r_m} dr \tau(r) r \left[K_1^{\text{eq}} - \frac{\partial \tau}{\partial r} K_2^{\text{eq}} \right]$$

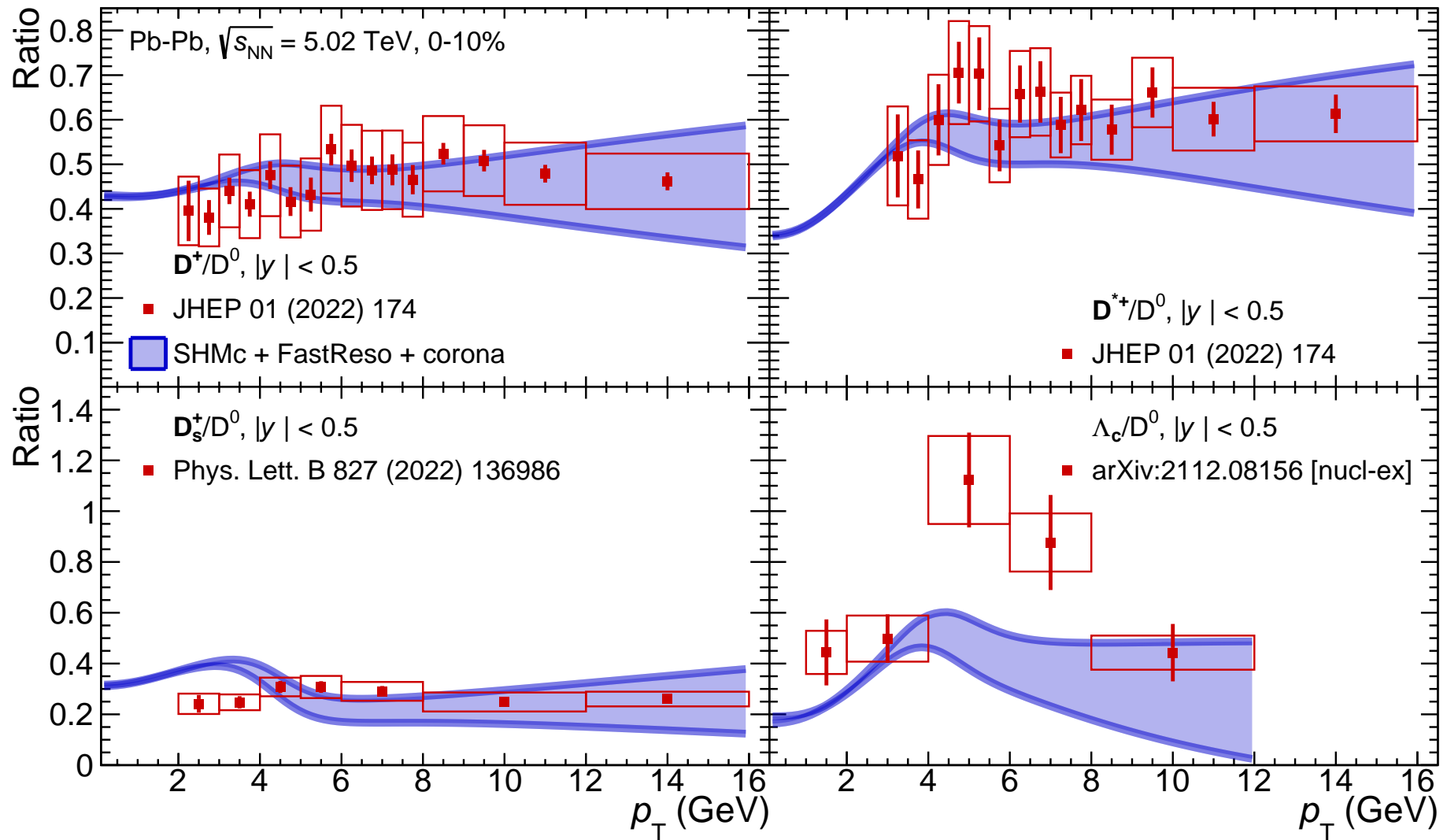
$$K_1^{\text{eq}}(p_T, u^r) = 4\pi m_T I_0 \left(\frac{p_T u^r}{T} \right) K_1 \left(\frac{m_T u^r}{T} \right), \quad K_2^{\text{eq}}(p_T, u^r) = 4\pi p_T I_1 \left(\frac{p_T u^r}{T} \right) K_0 \left(\frac{m_T u^r}{T} \right)$$

SHMc: p_T dependence



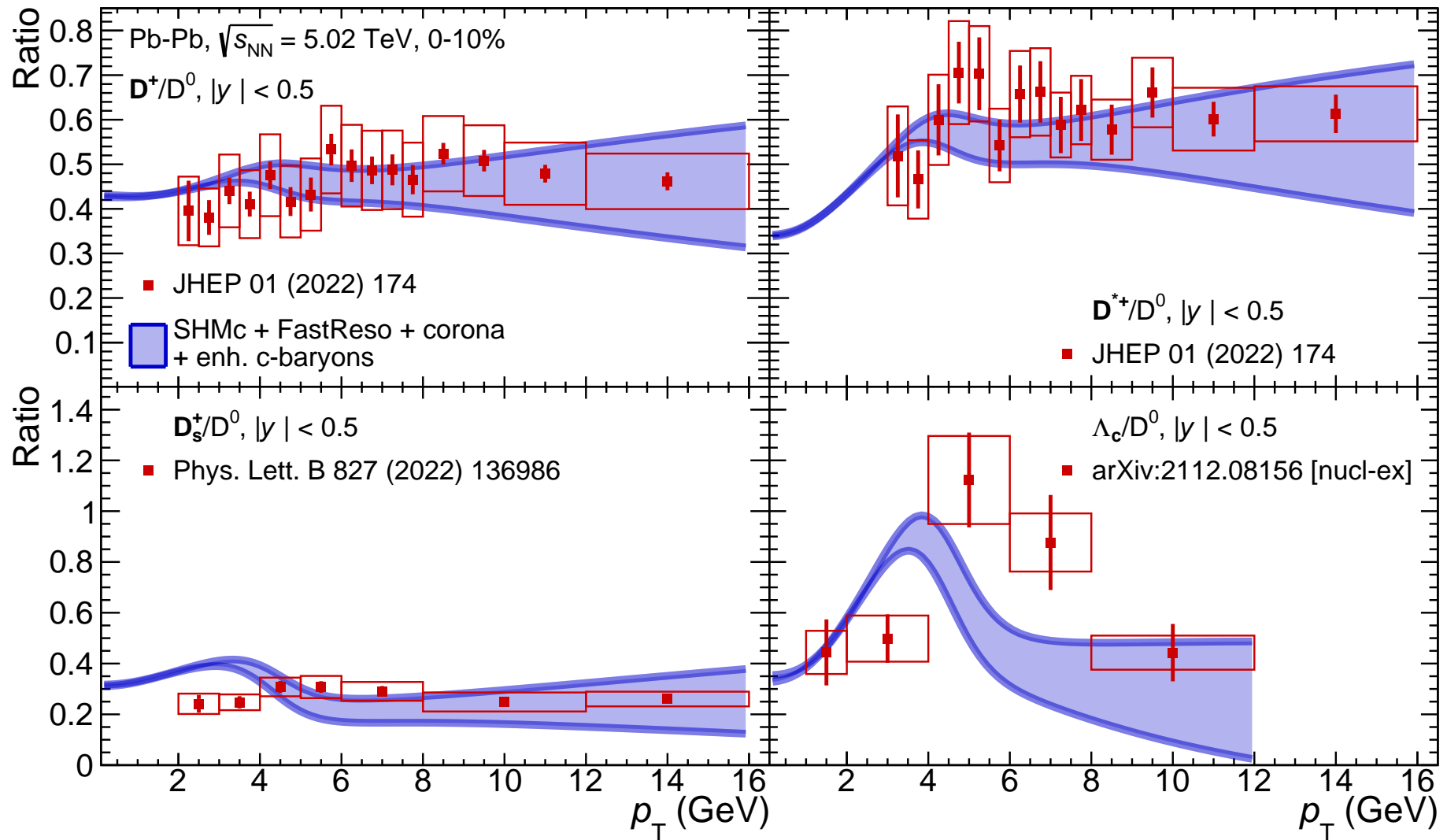
SHMc: low p_T ; high p_T : only nuclear-corona contribution (incl. uncert.)

Ratios to D^0



Charm-hadron spectrum: PDG

Ratios to D^0

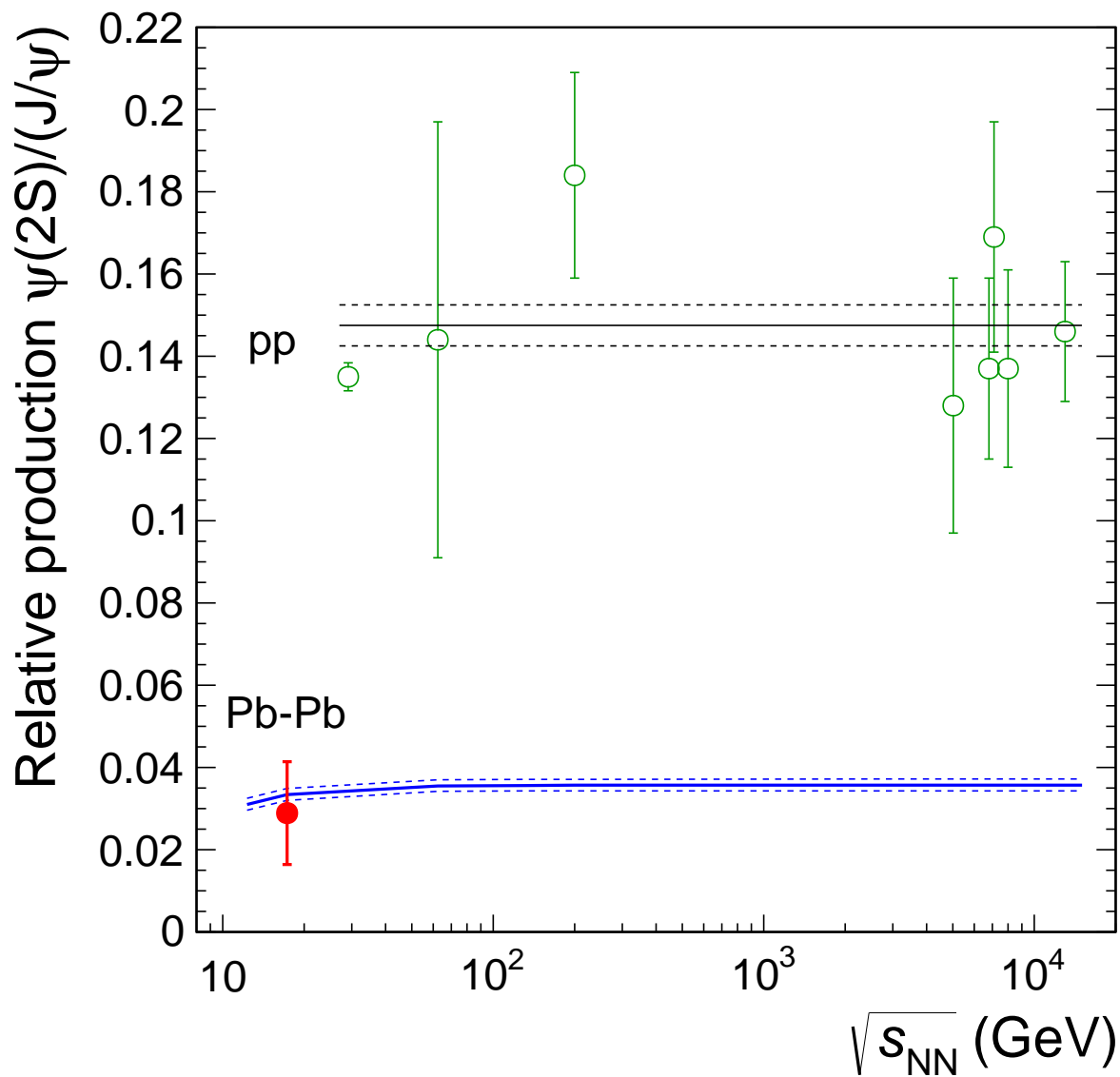


Charm-hadron spectrum: enhanced c-baryons (tripled excited states)

IF charm thermalizes (fully) also at lower energies

A. Andronic

19



...SHMc is easy to be extended to lower energies

AA, Braun-Munzinger, Redlich, Stachel, [PLB 659 \(2008\) 149](#)

..litmus test: $\psi(2S)$ ($+v_2$, R_{AA})

SHMc works (was proposed) at SPS

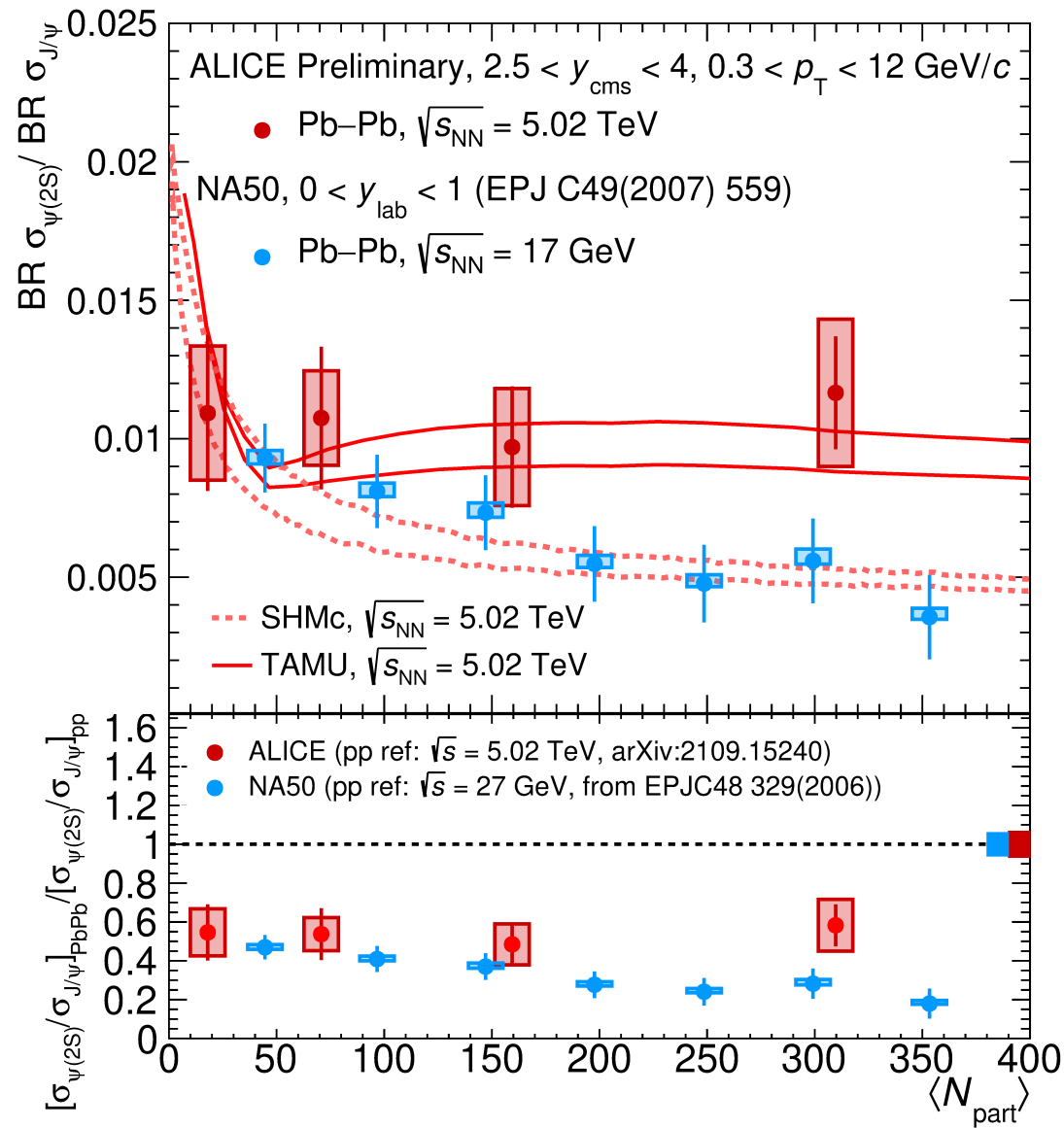
Braun-Munzinger, Stachel, [PLB 490 \(2000\) 196](#)

for D, stat. hadronization is a simpler act, may be at work in pp and in e^+e^-

[PLB 678 \(2009\) 350](#)

The measurement in Pb-Pb at LHC is a central goal for Run 3,4 ([YR, WG5 HL-LHC](#))

$\psi(2S)/J/\psi$ at the LHC



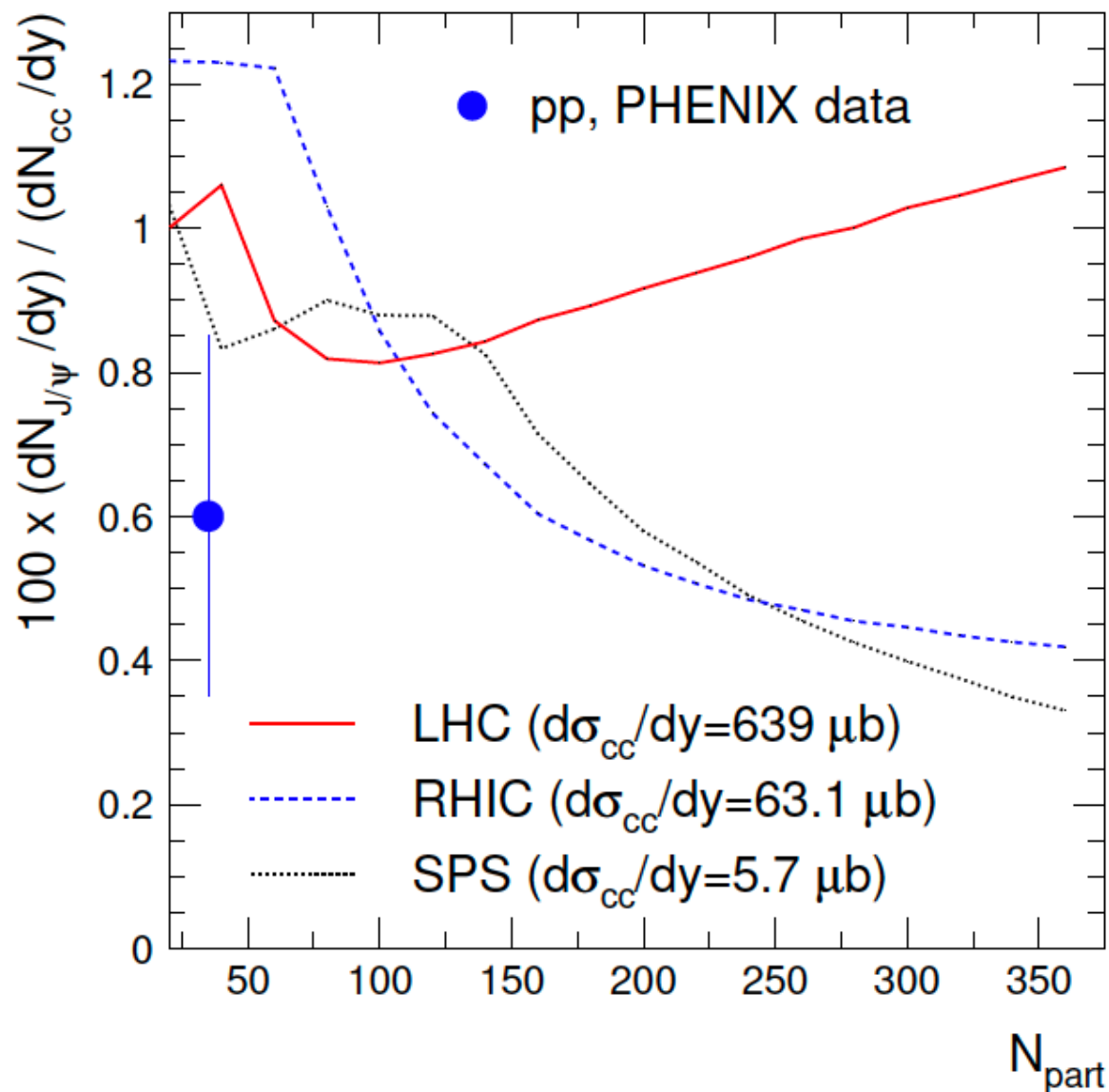
In SHMc uncertainty only due to nuclear-corona
 ($\sigma_{c\bar{c}}$ cancels out completely)

J/ψ production relative to charm

A. Andronic

21

...an observable with similar features as R_{AA}



NPA 789 (2007) 334

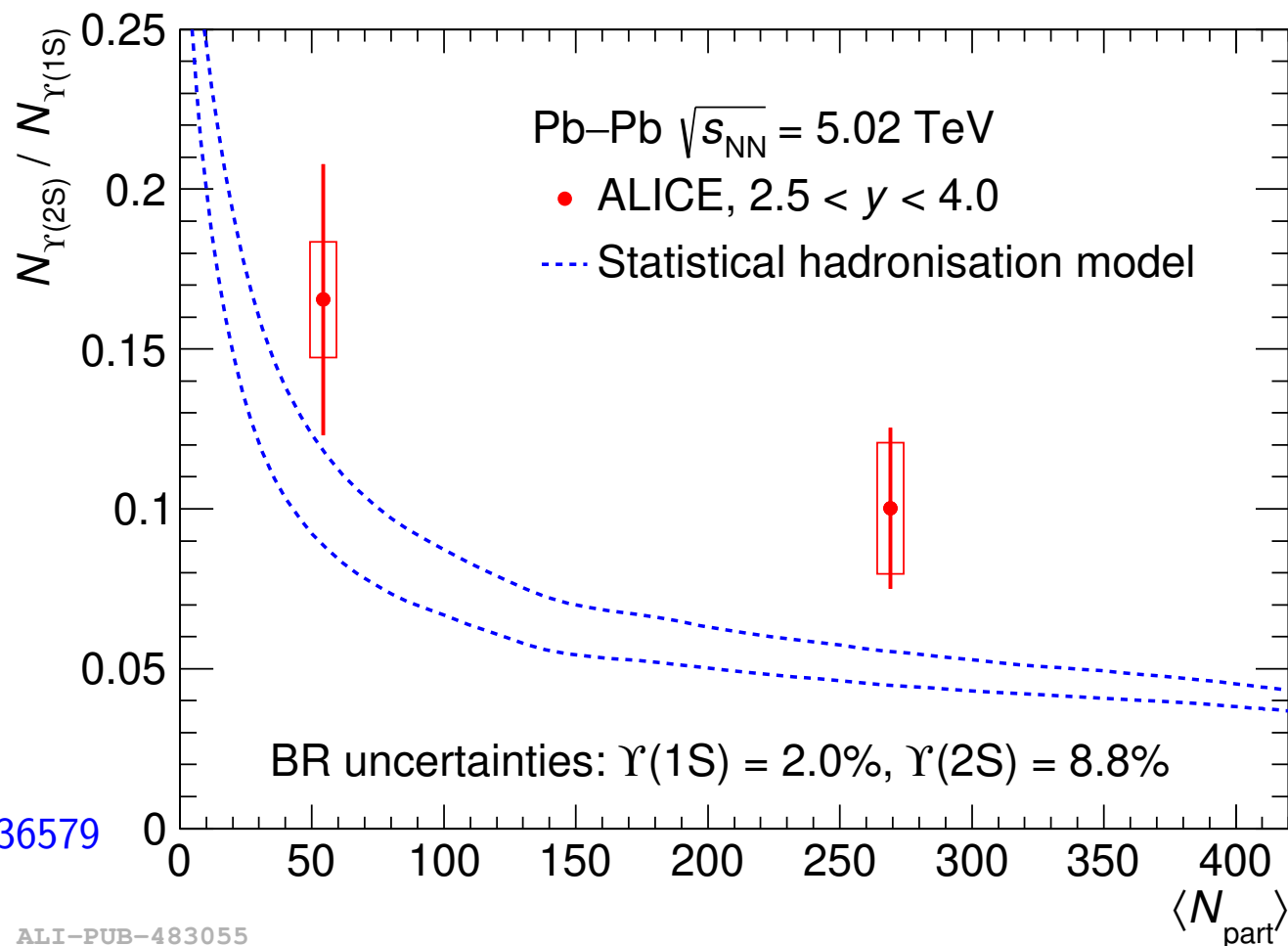
(see also: Satz, [Adv.HEP 2013 \(2013\) 242918](#))

- similar values at RHIC and SPS
...with differences in fine details
...determined by canonical suppression of open charm
same features in RHIC BES data (talk, K.Smith)
- enhancement-like at LHC
can. suppr. lifted, quadratic term dominant

...and on to beauty (at the LHC)

Beauty is expected (R_{AA} , v_2 data) to be less thermalized compared to charm

The beauty-hadron spectrum is less well known (PDG: 48 b-mes, 46 b-bar total)



ALICE,

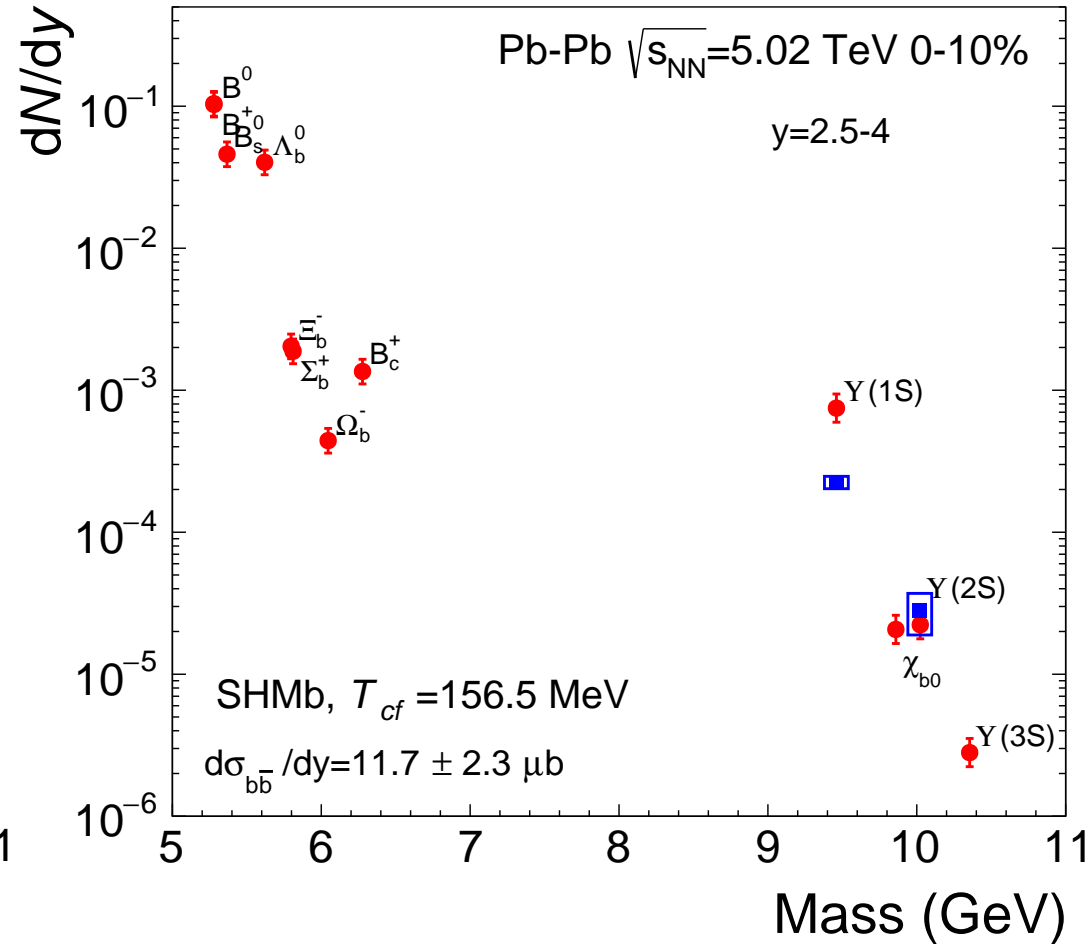
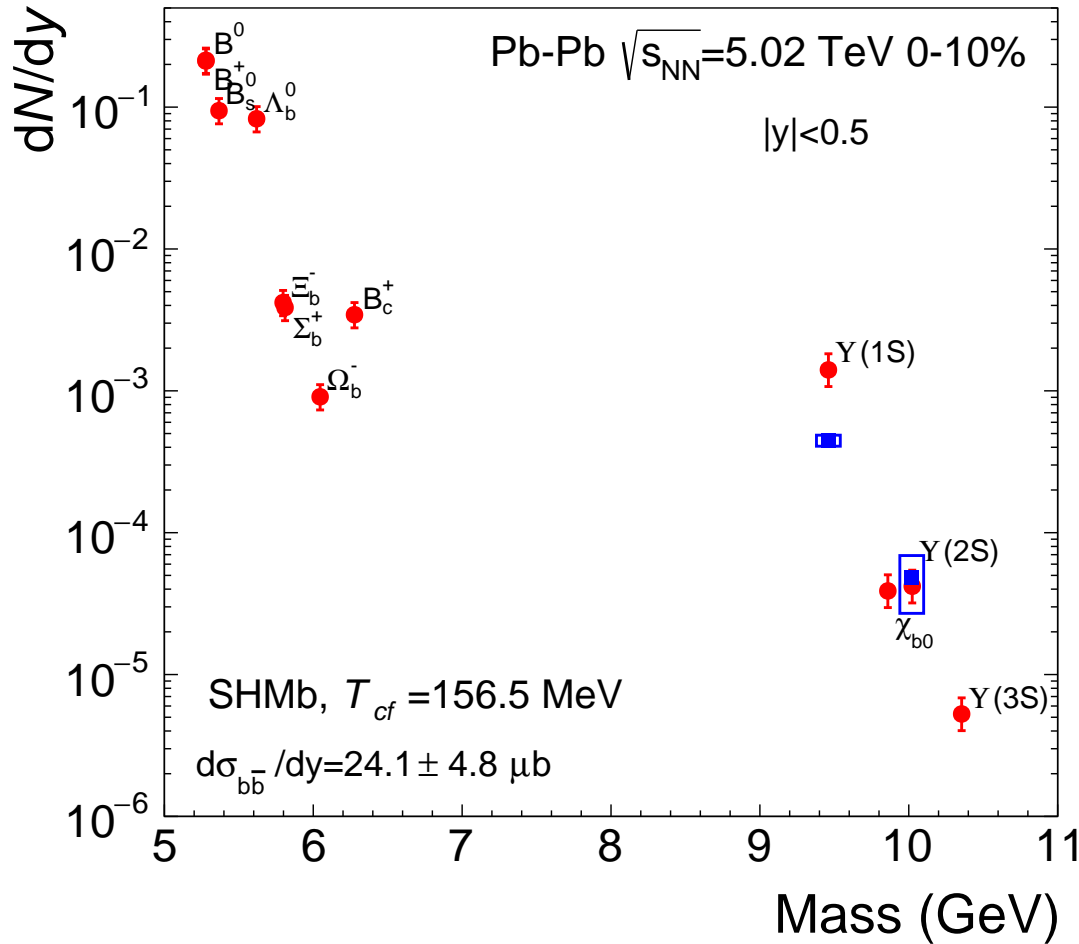
PLB 822 (2021) 136579

ALI-PUB-483055

Color screening may not destroy all Υ mesons in QGP

Uncertainty band determined by nuclear-corona

The limiting case: full beauty thermalization



$$g_b = 1.05 \cdot 10^9 \quad \left(\frac{dN_{bb\bar{b}}}{dy} = 0.57 \right)$$

$$B_c : 3.44 \cdot 10^{-3}$$

$$g_b = 0.86 \cdot 10^9 \quad \left(\frac{dN_{bb\bar{b}}}{dy} = 0.28 \right)$$

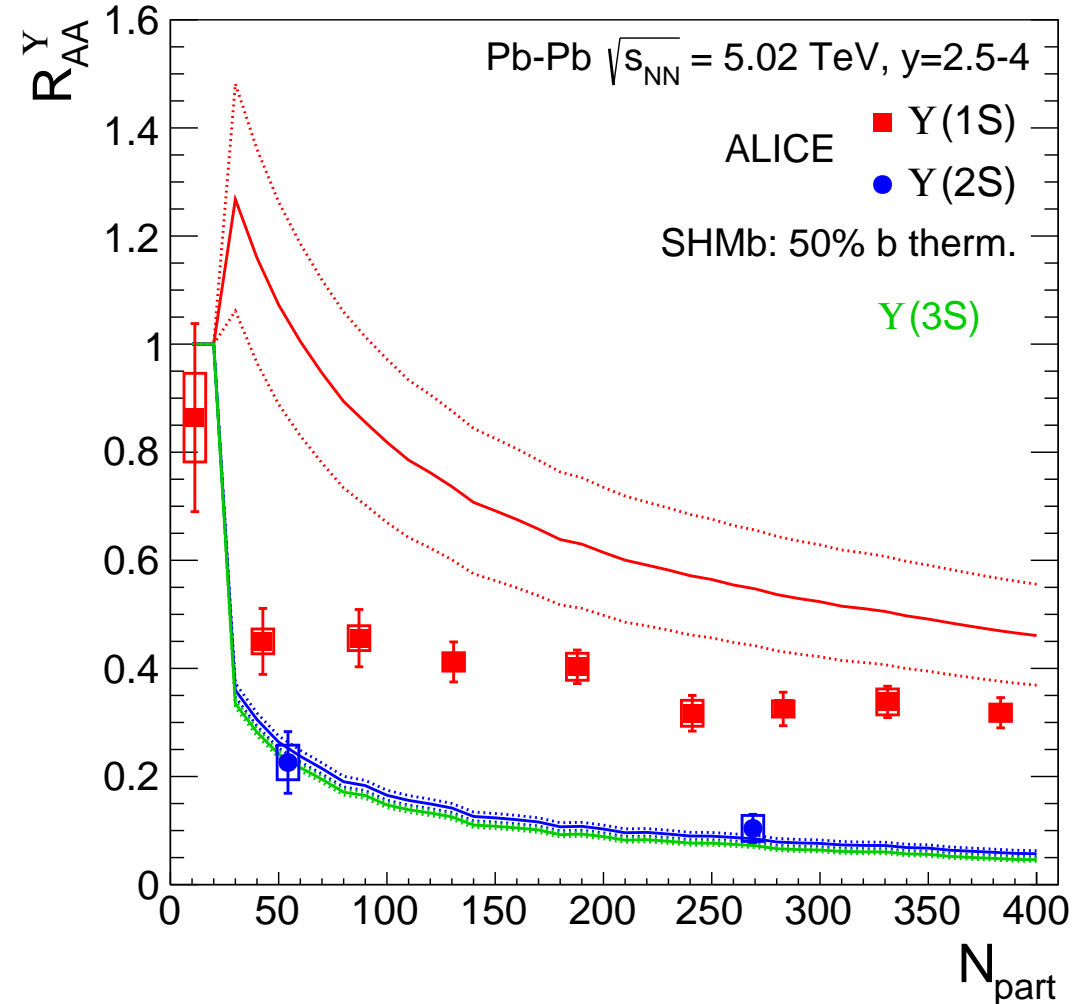
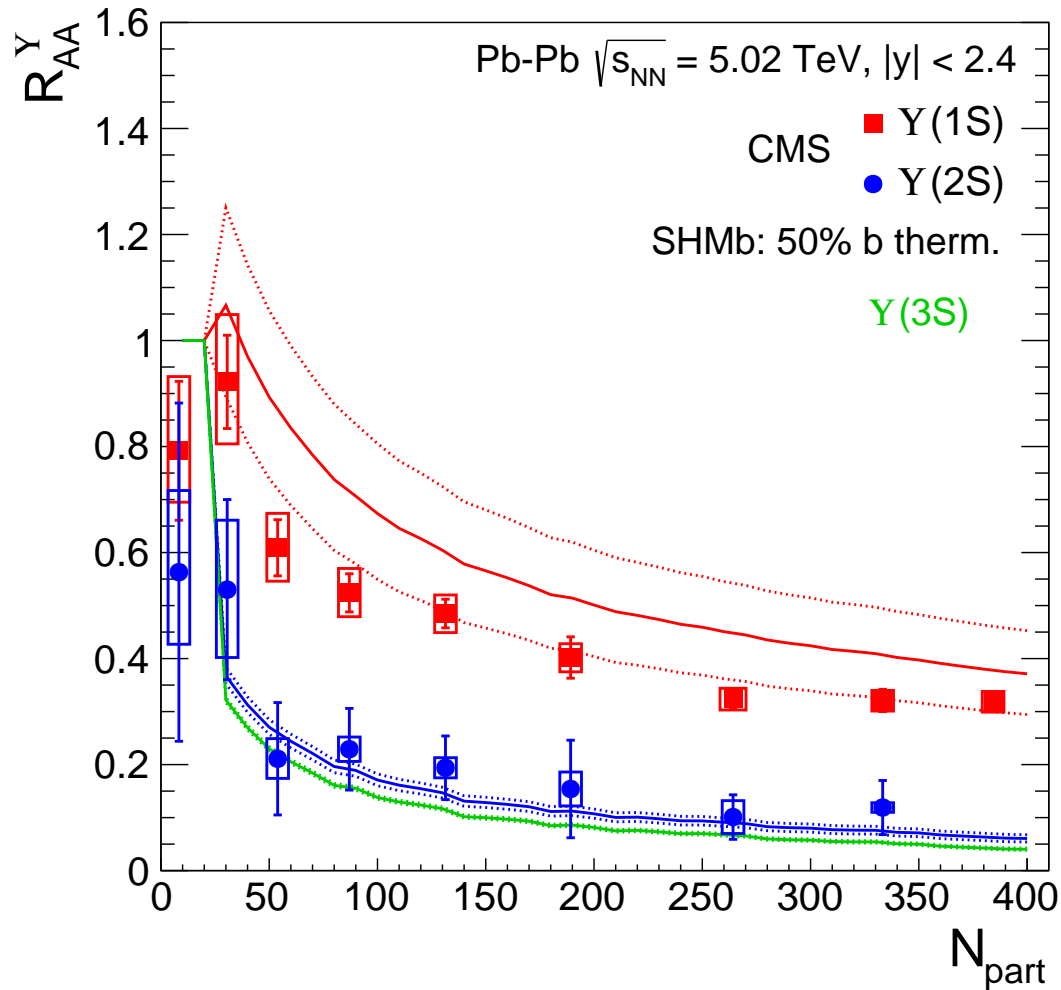
$$B_c : 1.36 \cdot 10^{-3}$$

Blue: Υ data (CMS, ALICE): calc. based on R_{AA} and pp (would be nice to include in publications dN/dy)

R_{AA} , 50% $b\bar{b}$ thermalized

A. Andronic

24



CMS, PRL 120 (2018) 142301

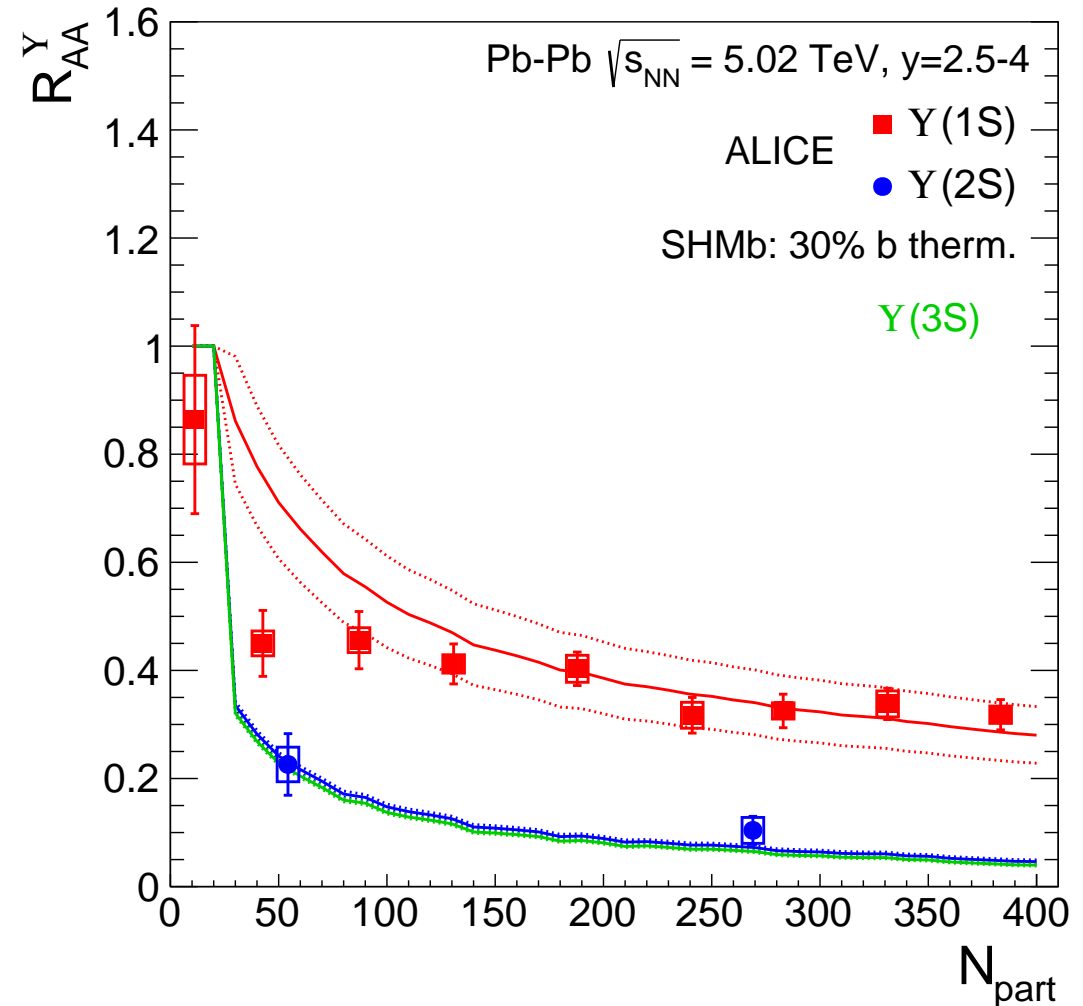
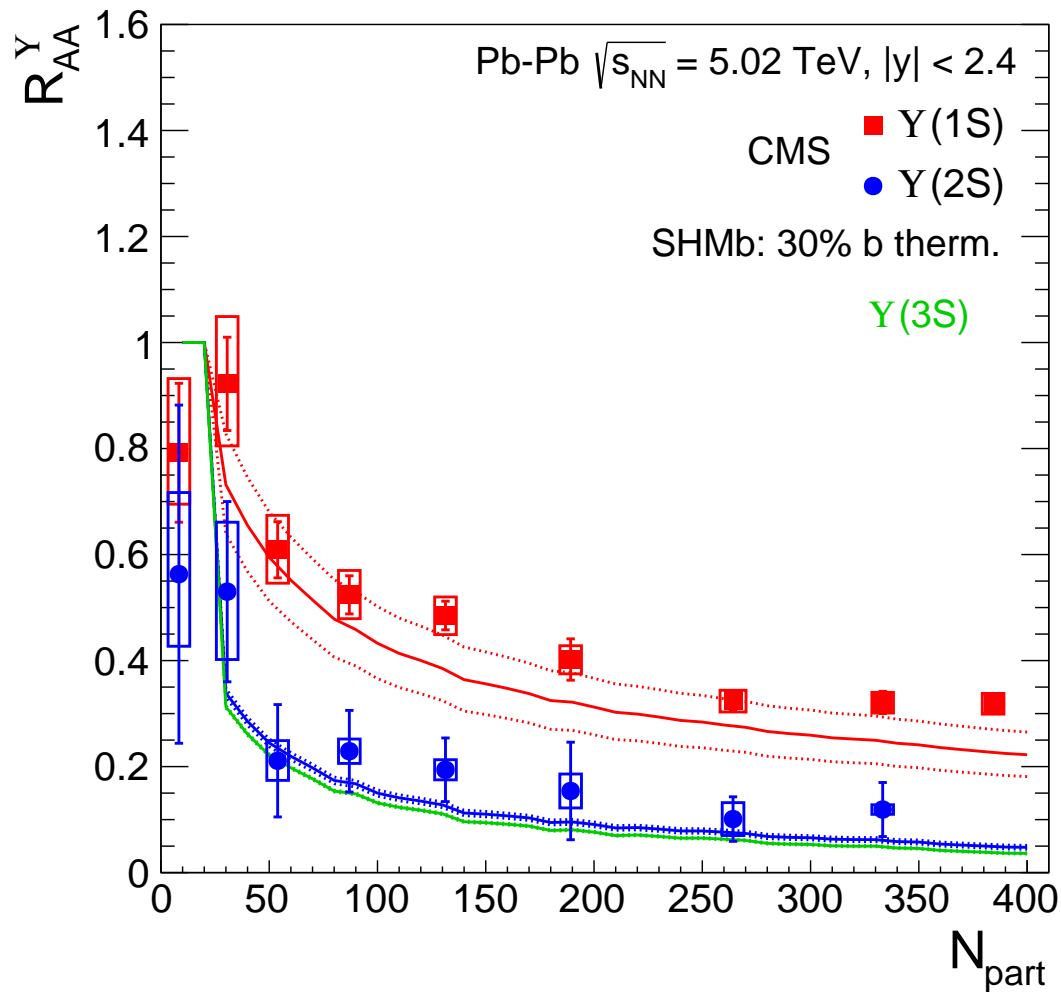
ALICE, PLB 822 (2021) 136579

What does non-thermalized beauty produce? (no room for it in SHMb)

R_{AA} , 30% $b\bar{b}$ thermalized

A. Andronic

25



CMS, PRL 120 (2018) 142301

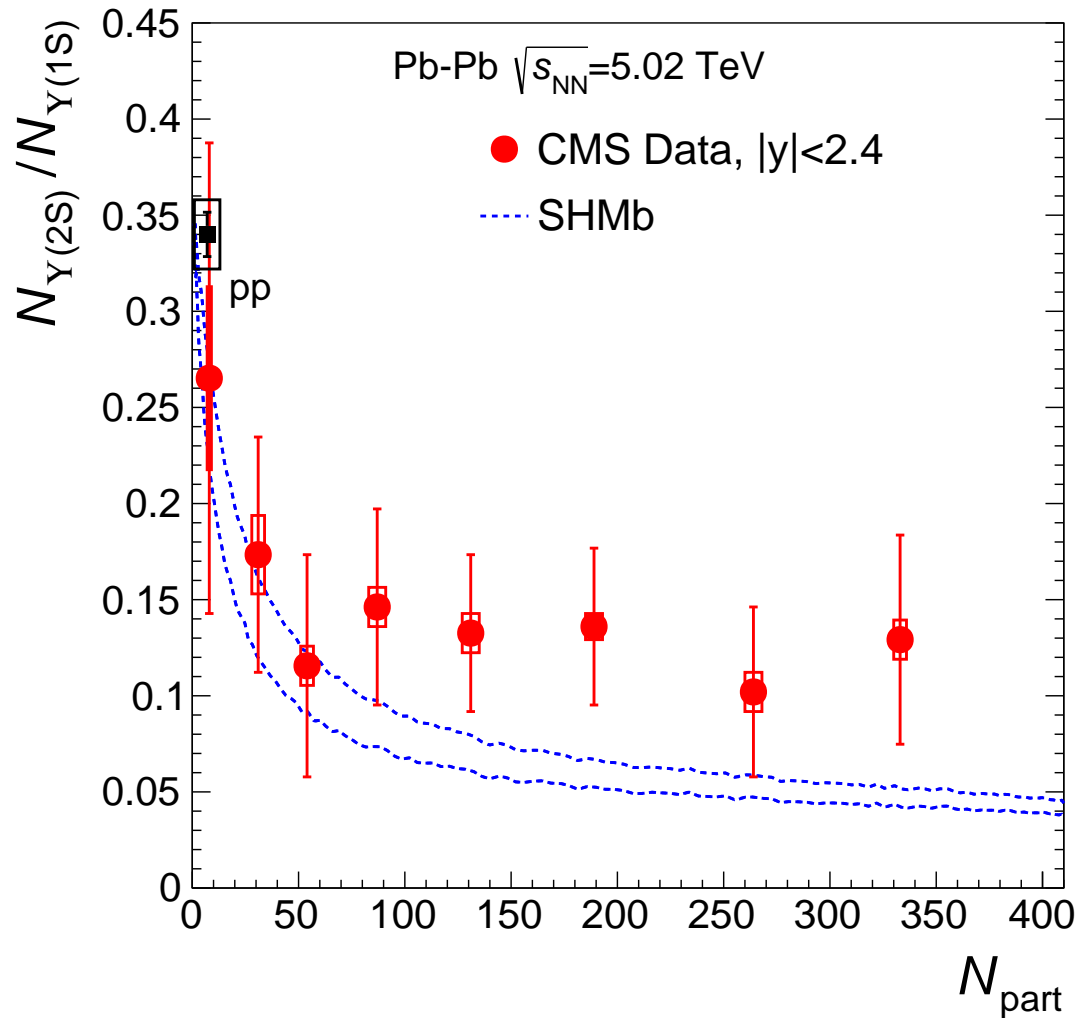
ALICE, PLB 822 (2021) 136579

What does non-thermalized beauty produce? (no room for it in SHMb)

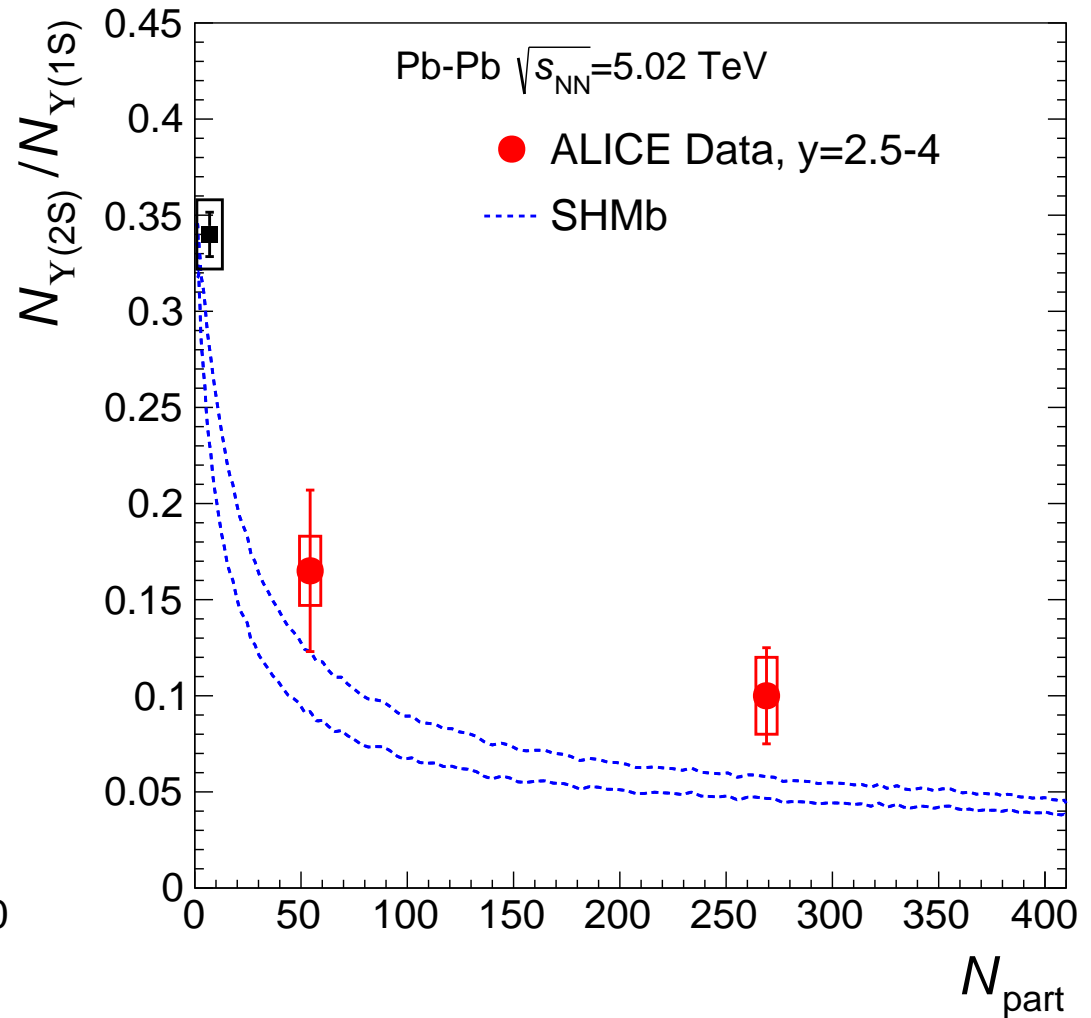
$\Upsilon(2S)/\Upsilon(1S)$ ratio (100% b thermalization)

A. Andronic

26



CMS, [PRL 120 \(2018\) 142301](#)



ALICE, [PLB 822 \(2021\) 136579](#)

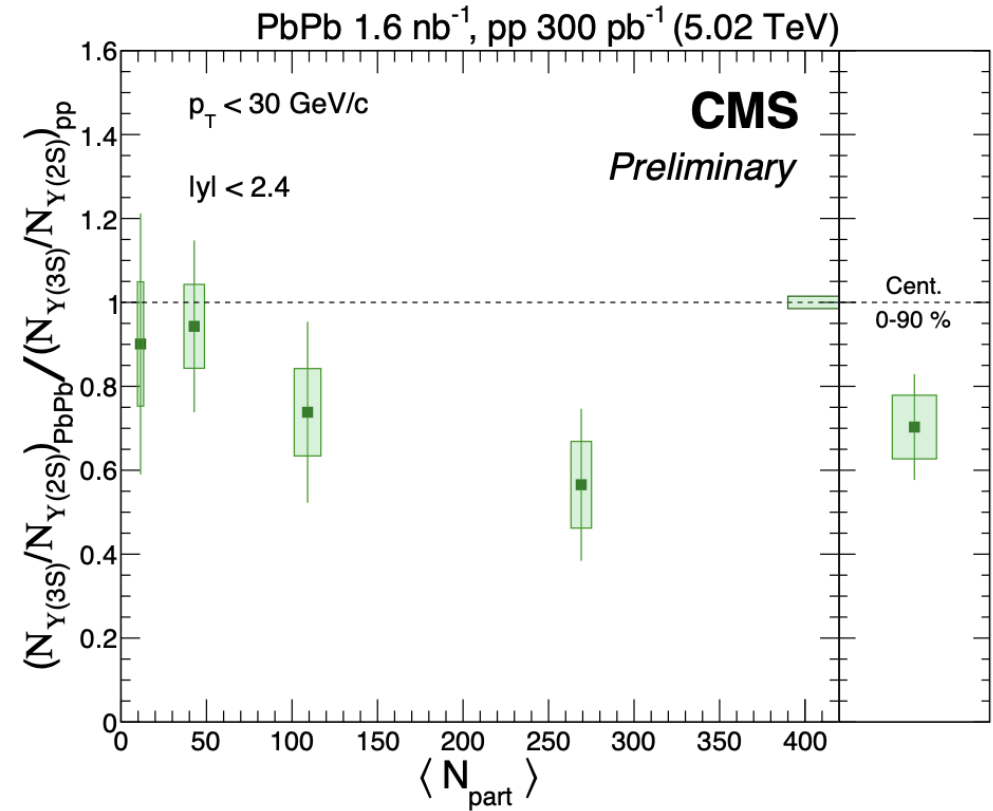
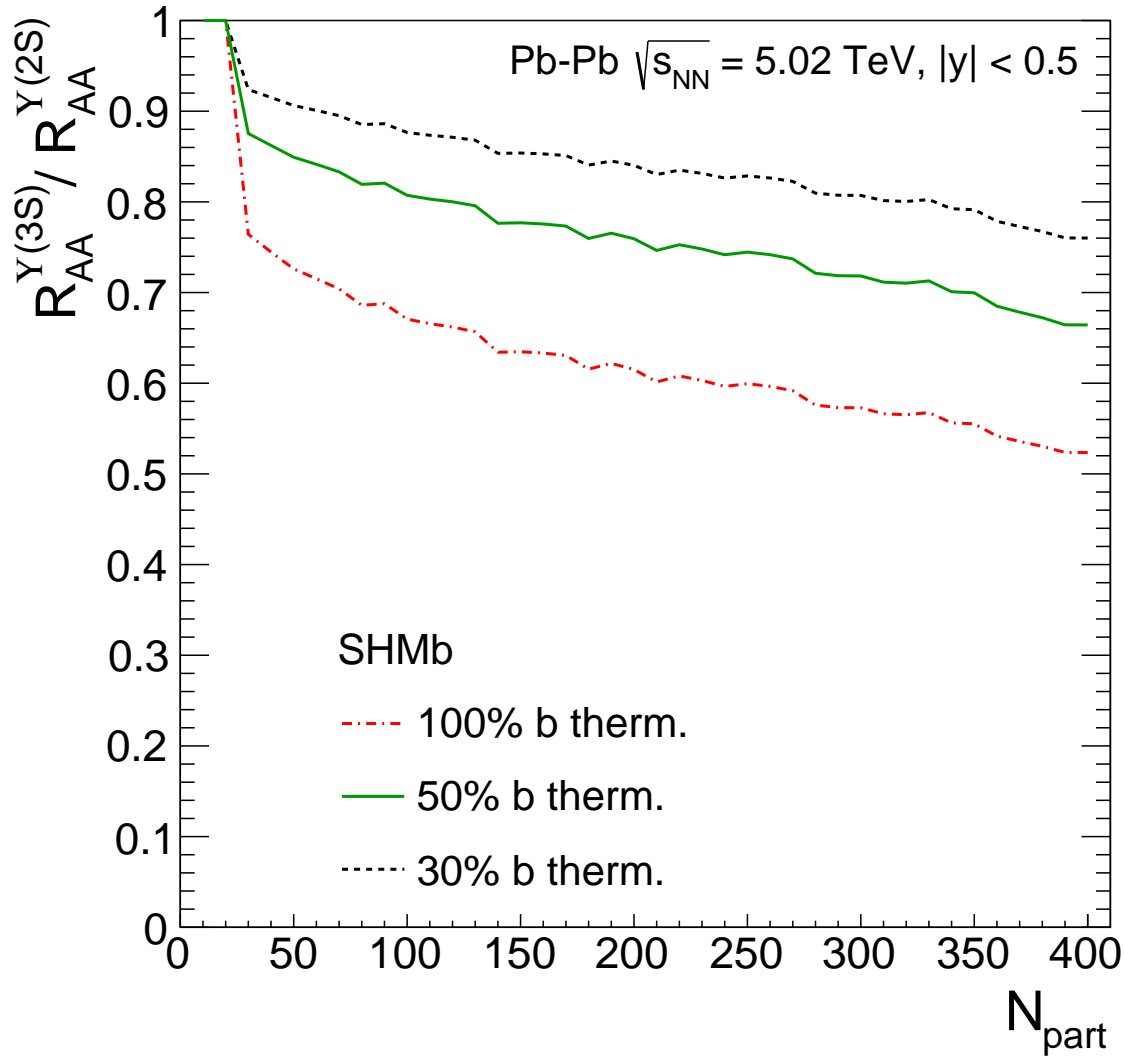
ALICE pp: $\Upsilon(2S)/\Upsilon(1S) = 0.5 \pm 0.1$, [arXiv:2109.15240](#)

SHMb uncert.: nuclear-corona (fraction)

$\Upsilon(3S)/\Upsilon(2S) R_{AA}$ ratio

A. Andronic

27



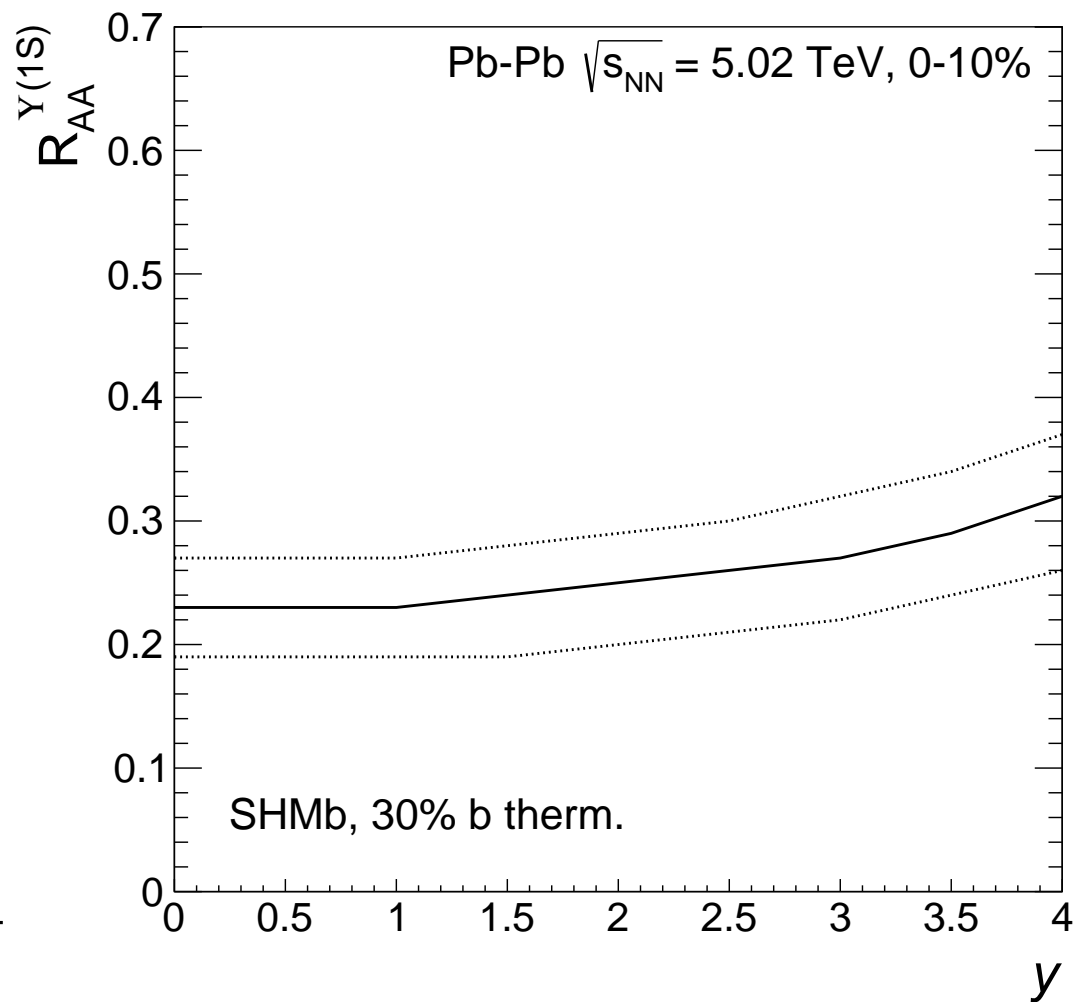
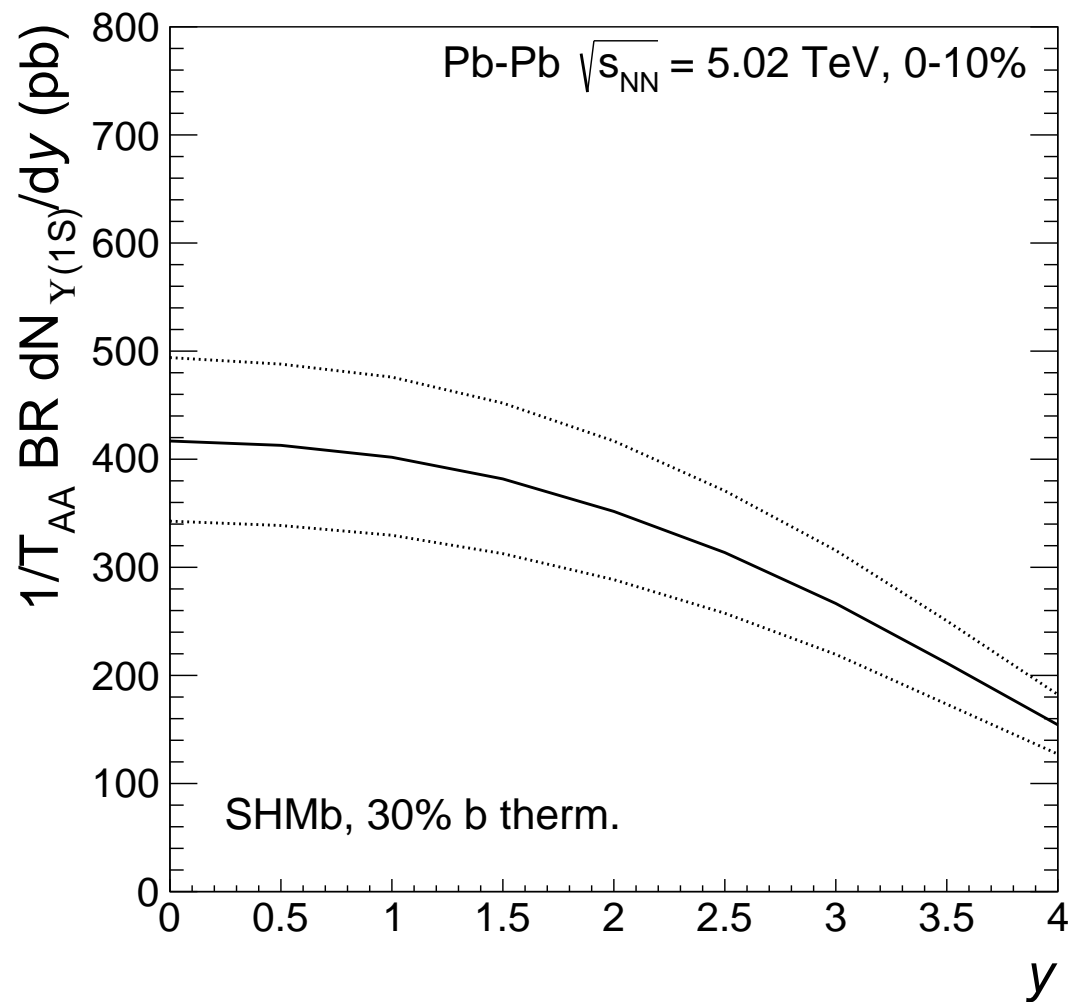
CMS, QM'22, [HIN-21-007](#)

The $\Upsilon(3S)/\Upsilon(2S) R_{AA}$ ratio is quite sensitive to the degree of b thermalization

Rapidity dependence $\Upsilon(1S)$, 30% $b\bar{b}$ thermalized

A. Andronic

28

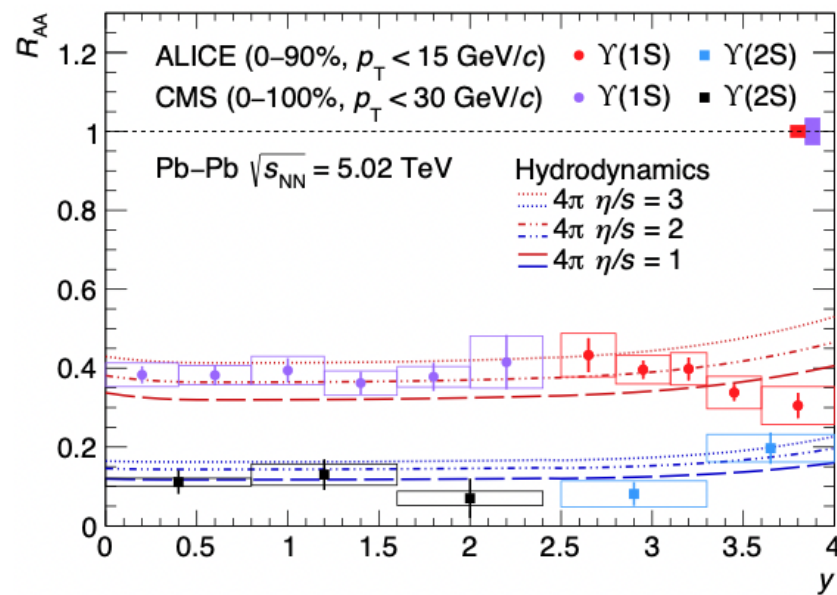
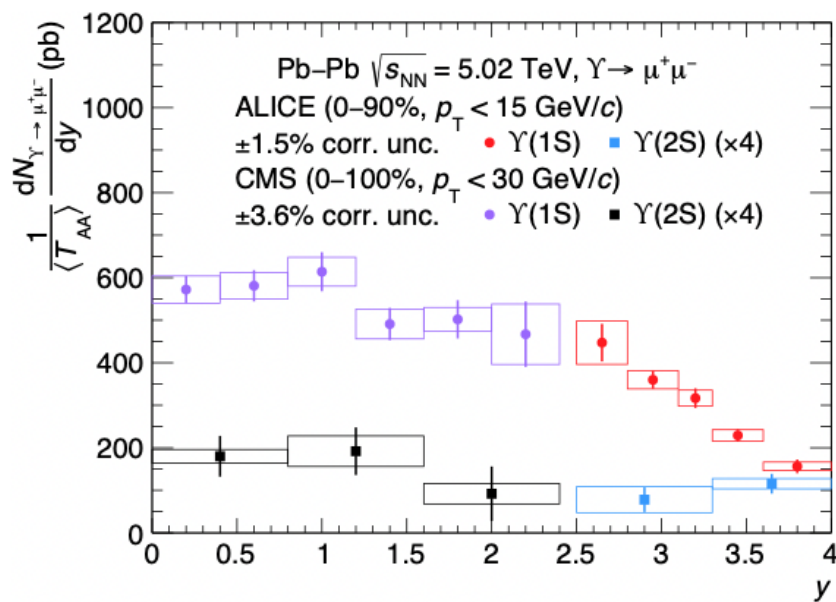
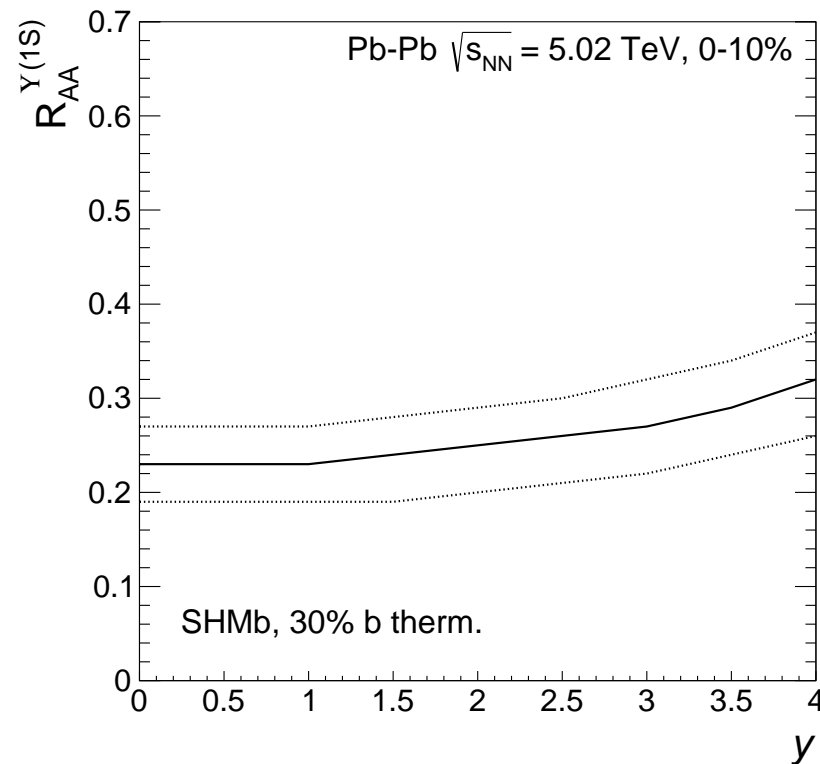
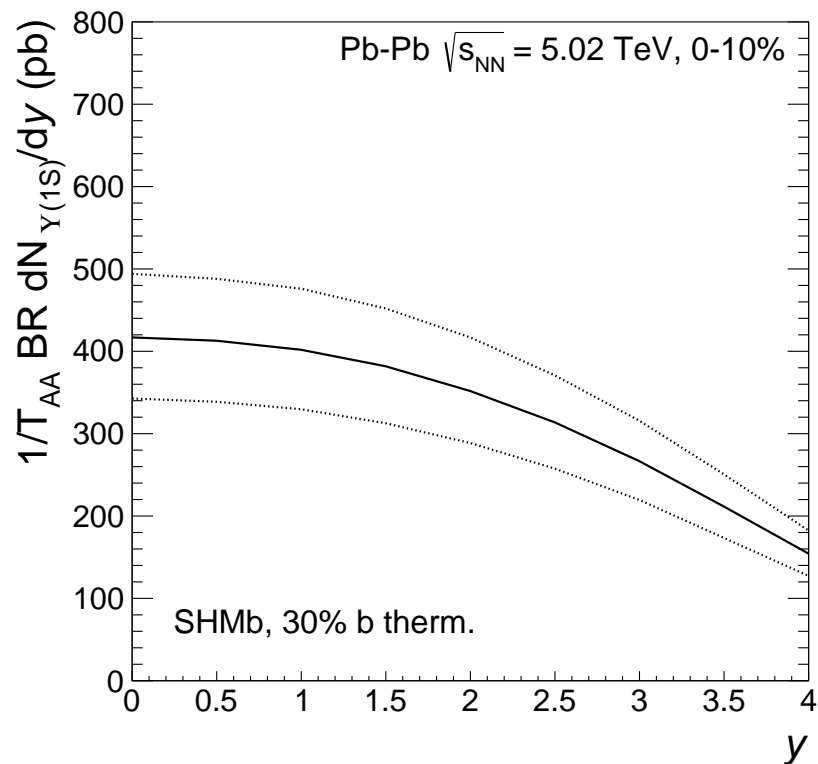


Data available only centrality-integrated ...is 0-10% (or 0-20%) doable?

Rapidity dependence $\Upsilon(1S)$, 30% $b\bar{b}$ thermalized

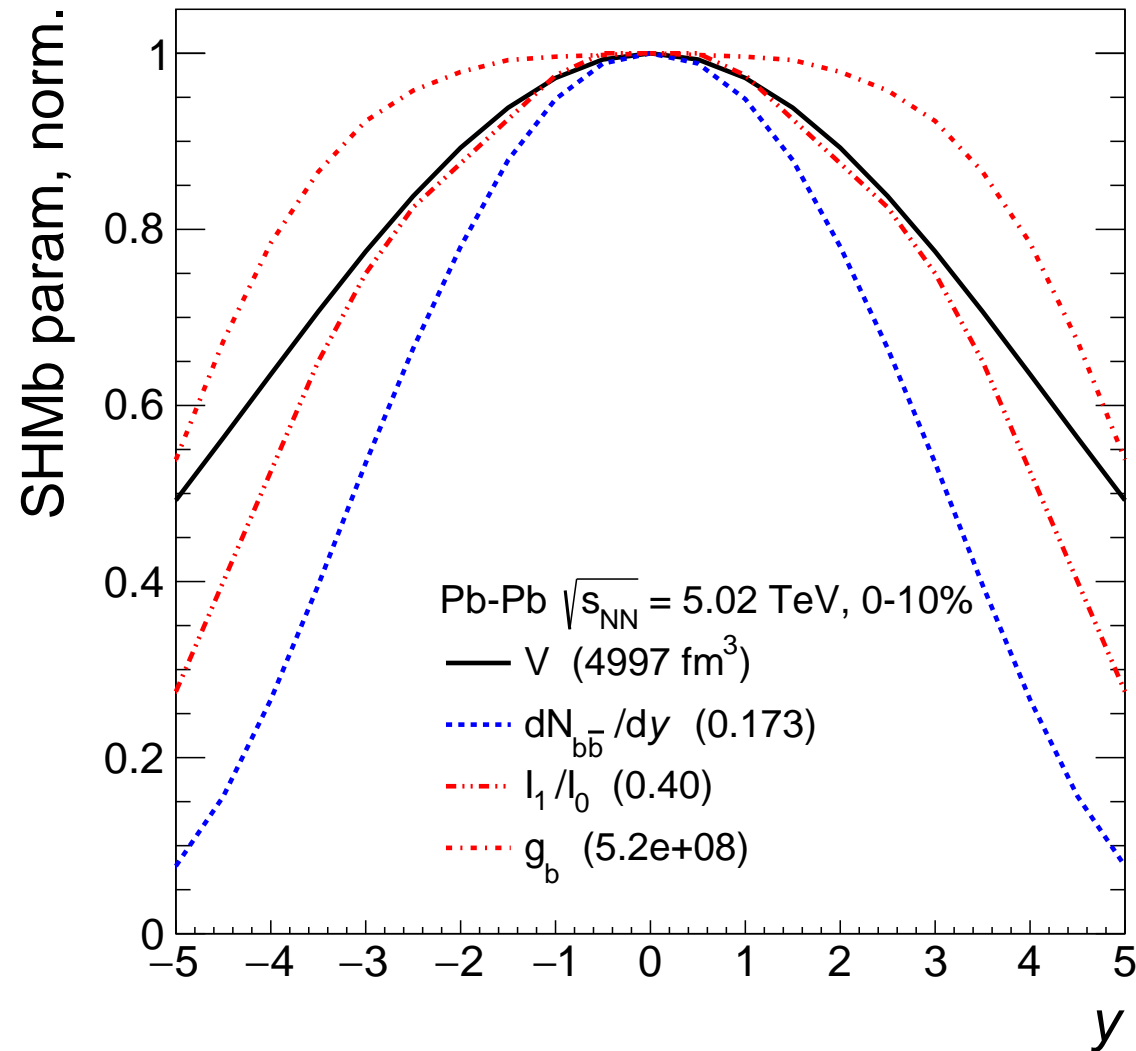
A. Andronic

29



SHMb parameters

NB: none is a free parameter, except the 30% b thermalization fraction



Summary / Conclusions: charm

In the (our) statistical hadronization model:

- The hadronization is a rapid process in which all quark flavors take part concurrently
- All charmonium and open charm states are generated exclusively at hadronization (chemical freeze-out) ...full color screening

The model is very successful in reproducing the J/ψ and open charm data
A handle for hadronization T with a mass scale well above T

"The competition":

the kinetic model, continuous J/ψ destruction and (re)generation in QGP

(only up to 2/3 of the J/ψ yield (LHC, central collisions) originates from deconfined c and \bar{c} quarks)

Discriminating the two pictures implies providing an answer to fundamental questions related to the fate of hadrons in a hot deconfined medium.

A precision ($\pm 10\%$) measurement of $d\sigma_{c\bar{c}}/dy$ in Pb-Pb (Au-Au) collisions needed for a stringent test
(within reach with the upgraded detectors at the LHC and RHIC)

Summary / Conclusions: beauty

- Full beauty thermalization seems not realized in nature
 - ...with 30-50% of beauty quarks fully thermalized we can explain the Υ data
 - ...but this fraction is (significantly) dependent on the b-hadron spectrum
- What does non/partially-thermalized beauty produce?
 - no Υ because strong coupling with the medium destroys the $b\bar{b}$ correlation?
 - ...related: is there non-screened bottomonium at all? (...or maybe just $\Upsilon(1S)$?)
- Another difficulty: $R_{AA}^{Y(1S)}(p_T)$ is flat (we would predict a bump), v_2 is small

similar to Reygers et al., [PRC 101 \(2020\) 064905](#)

forthcoming LHC data will (hopefully) clarify these questions (...in a while)

inspite (because:) of its simplicity SHM provides for all more sophisticated models a meaningful (powerful) limit to check (worth even if not fully realized in nature:)

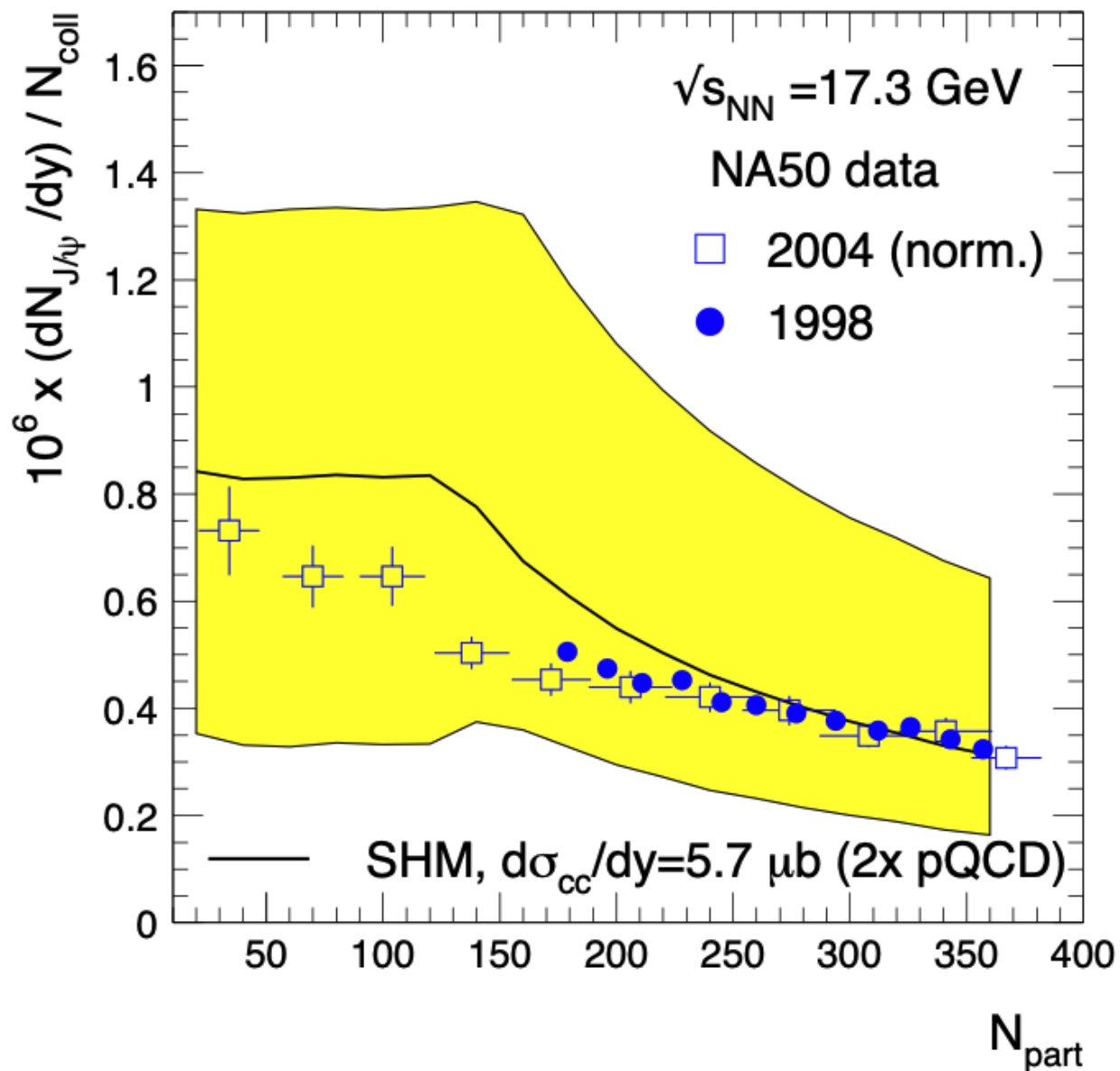
Extra slides

A. Andronic

J/ψ production at SPS and SHMc

A. Andronic

34



NPA 789 (2007) 334

Trend determined by canonical suppression of open charm

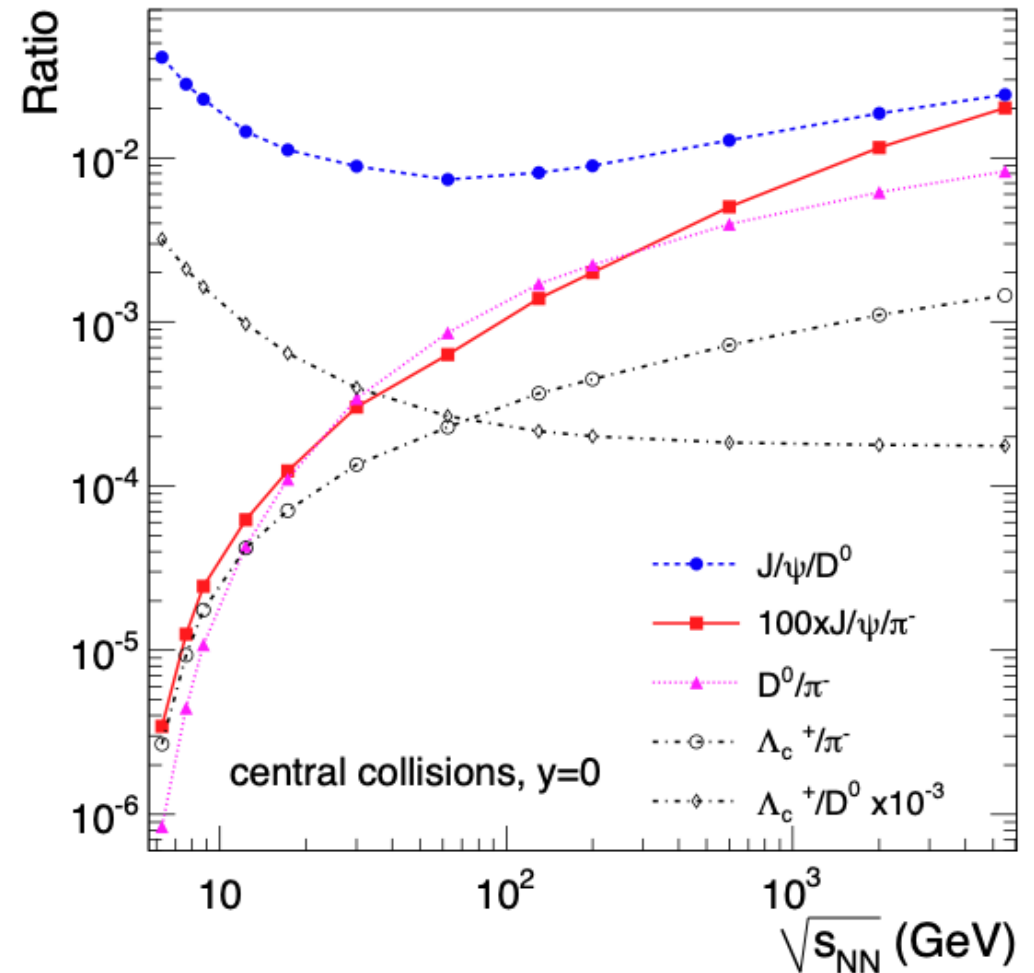
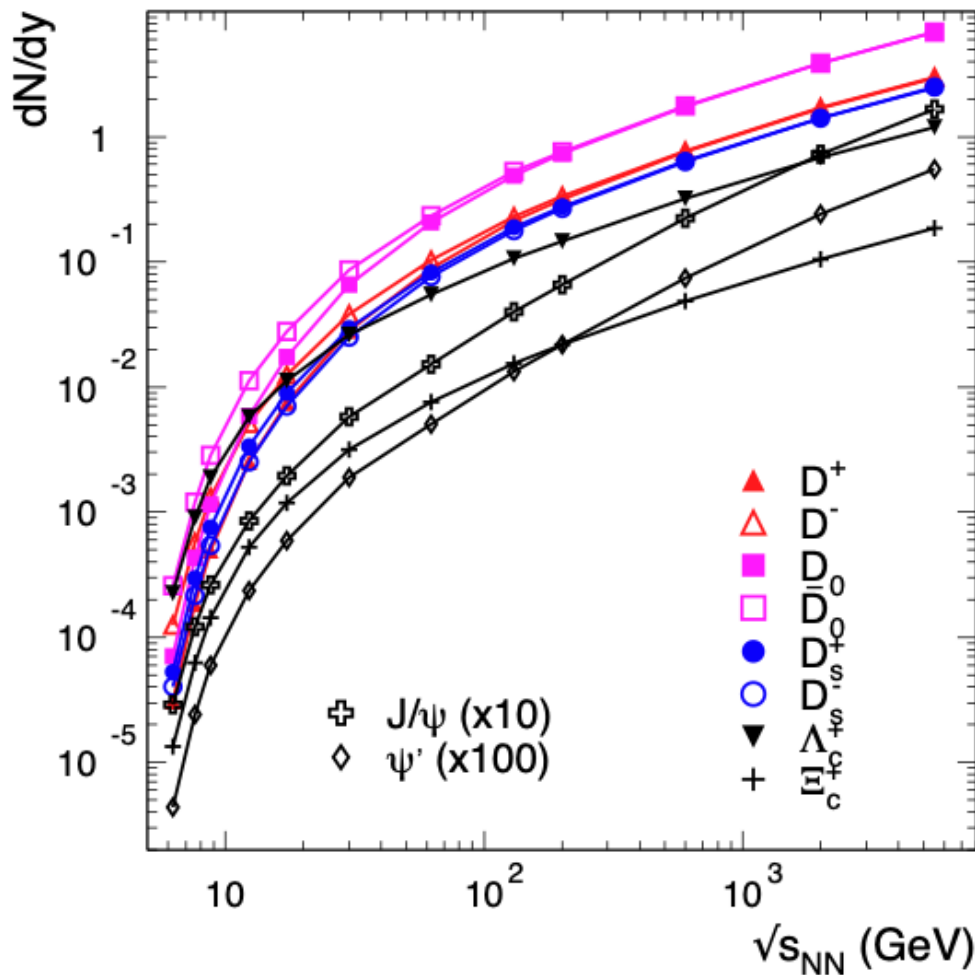
Model uncertainties large ($d\sigma_{c\bar{c}}/dy$)

...constraining the model needs a good measurement of $d\sigma_{c\bar{c}}/dy$

Charm chemistry at lower energies

A. Andronic

35

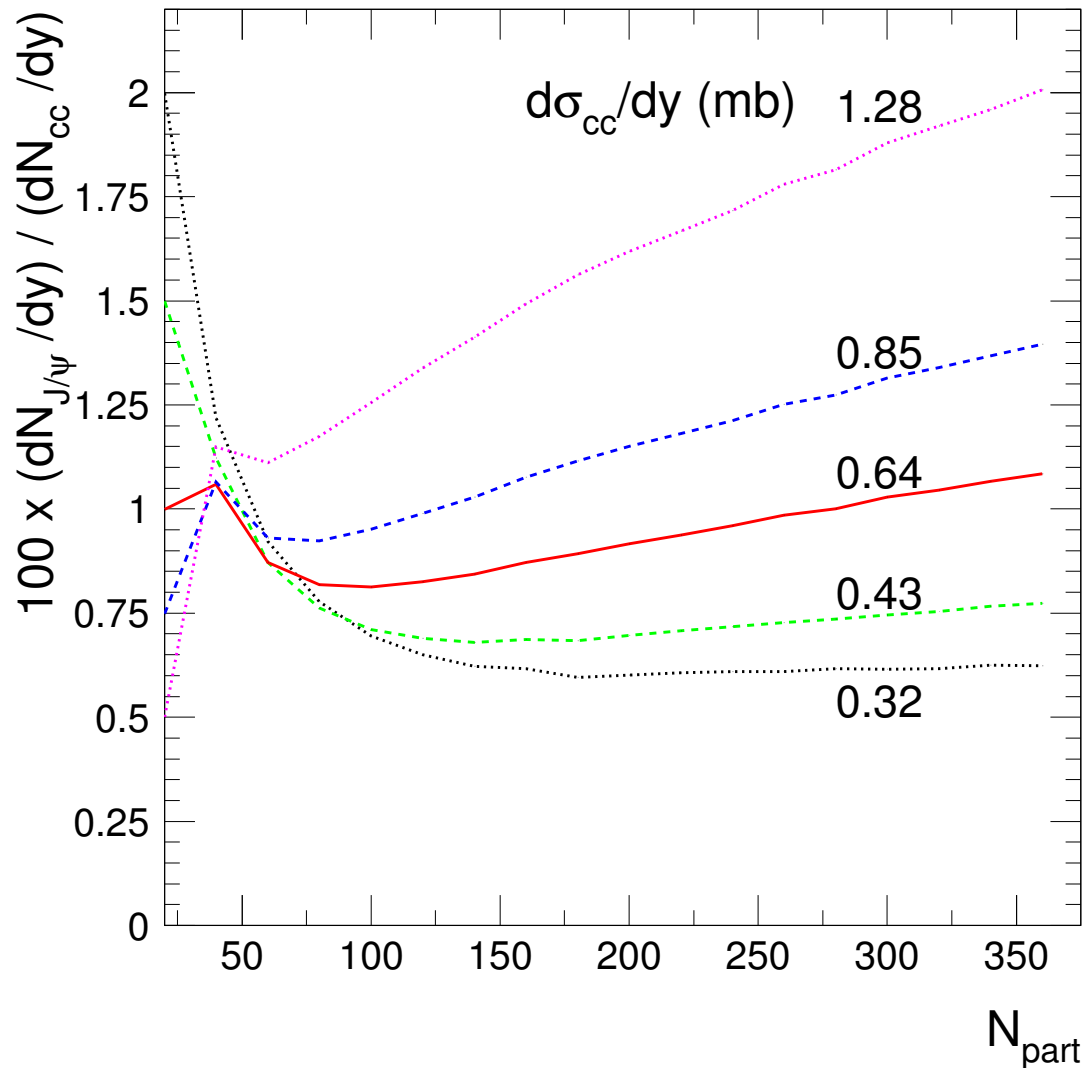


AA, Braun-Munzinger, Redlich, Stachel, [JPG 37 \(2010\) 094014](#)

NB: uncertainties from $\sigma_{c\bar{c}}$ are large

There are non-monotonic ratios ($J/\psi/D$), determined by canonical suppression

SHM and charmonium production at the LHC



$$\frac{dN_{J/\psi}^{AA}/dy}{dN_{c\bar{c}}^{AA}/dy}$$

(“proxy” for R_{AA})

- “enhancement” at the LHC

$$N_{J/\psi} \sim (N_{c\bar{c}}^{dir})^2$$

canonical suppression (mostly)
lifted, quadratic term dominant

it can be more dramatic at FCC

AA et al., in N. Armesto et al., “Last Call...”, [JPG 35 \(2008\) 054001](#)

this was for $\sqrt{s_{NN}} = 5.5$ TeV ... but is a generic prediction of the model

The charm production cross section at the LHC

- LHCb $p_T < 8 \text{ GeV}/c$, $2.0 < y < 4.5$

pp 5 TeV, [JHEP 06 \(2017\) 147](#)

$$\sigma(c\bar{c}) = 1193 \pm 3(\text{stat}) \pm 67(\text{syst}) \pm 58(\text{frag}) \mu\text{b}, \quad \rightarrow \frac{d\sigma_{c\bar{c}}}{dy} = 0.477 \pm 0.036 \text{ mb}$$

pp 7 TeV, [NPB 871 \(2013\) 1](#)

$$\sigma(c\bar{c}) = 1419 \pm 12(\text{stat}) \pm 116(\text{syst}) \pm 65(\text{frag}) \mu\text{b}$$

$$\Lambda_c/D^0 = 0.140 \pm 0.045$$

- ALICE, $|y| < 0.5$, $p_T > 0 \text{ GeV}/c$

pp 5 TeV, [arXiv:2105.06335](#), [arXiv:2102.13601](#) :

$$\frac{d\sigma_{c\bar{c}}}{dy} = 1.165 \pm 0.044(\text{stat}) \pm 0.065(\text{syst})_{-0.038}^{+0.098}(\text{extr}) \pm 0.043(\text{BR}) \pm 0.042(\text{RS}) \pm 0.024(\text{lumi}) \text{ mb}$$

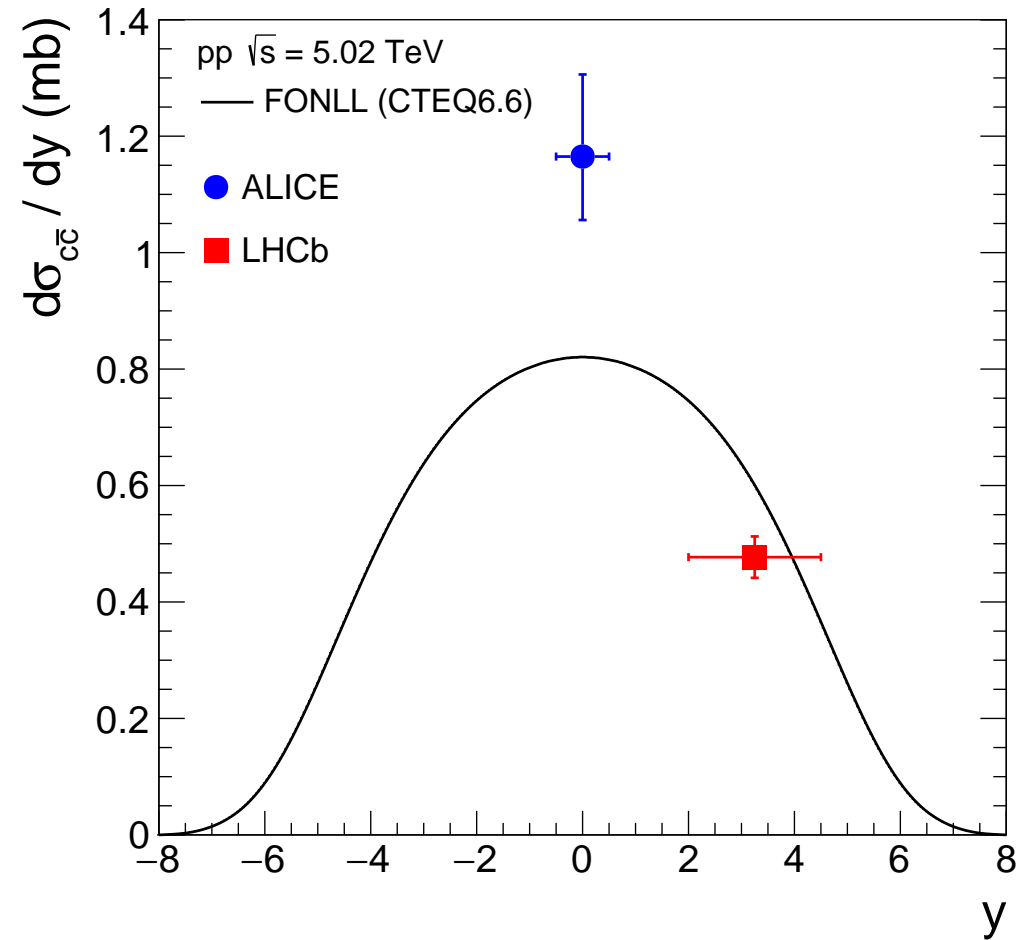
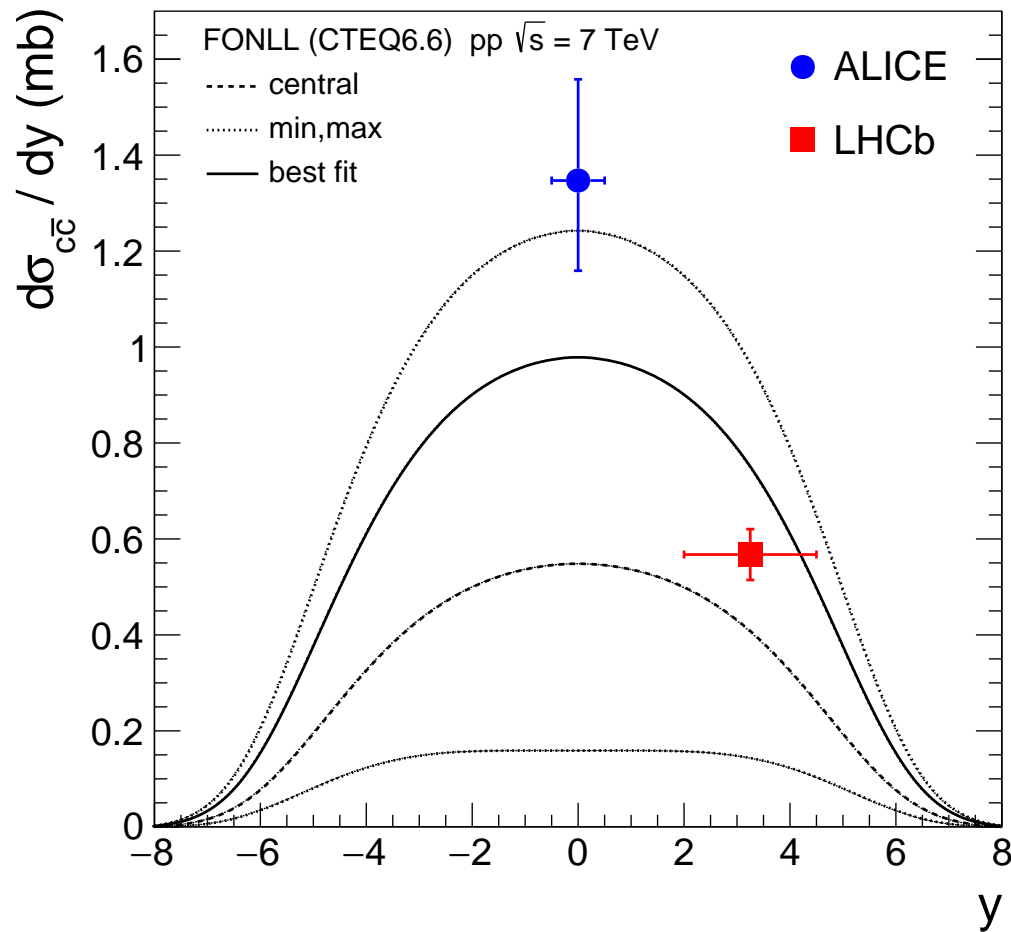
$$\Lambda_c/D^0 = 0.51 \pm 0.06$$

pp 7 TeV, [JHEP 04 \(2018\) 108](#)

$$\frac{d\sigma_{c\bar{c}}}{dy} = 1.347 \pm 0.097(\text{stat}) \pm 0.104(\text{syst})_{-0.105}^{+0.142}(\text{FF}) \pm 0.011(\text{BR}) \pm 0.044(\text{RS}) \pm 0.047(\text{lumi}) \text{ mb}$$

Data in comparison to FONLL

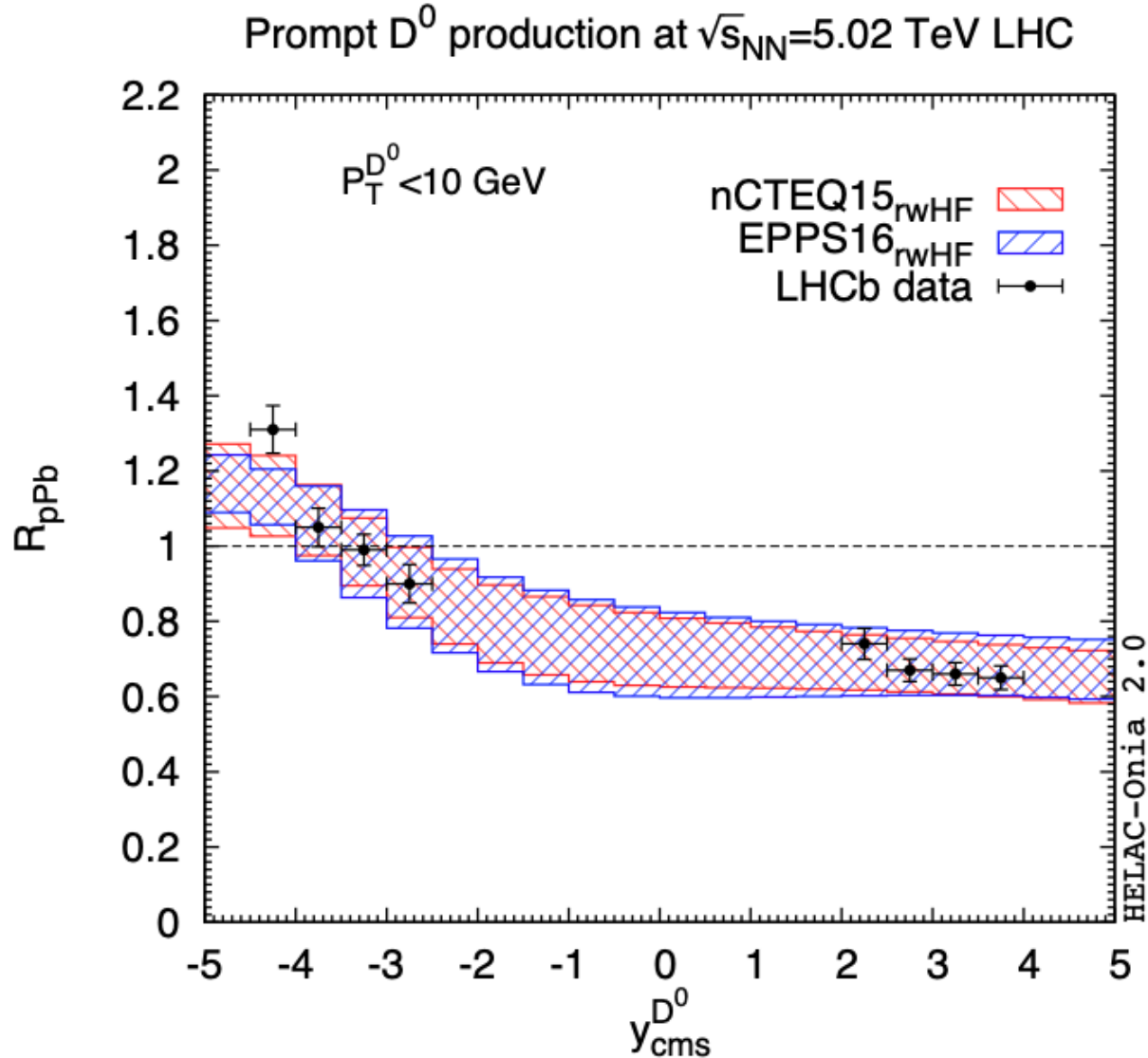
... p_T -integrated ($p_T > 0$); FONLL web interface



R_{pPb} and shadowing

A. Andronic

39



weighted nPDFs (90% CL):

Kusina et al, [PRD 104 \(2021\) 014010](#)

Data: LHCb, [JHEP 10 \(2017\) 090](#)

ALICE: 0.960 ± 0.086 , [JHEP 12 \(2019\) 092](#)

$y=0$:

$$R_{pPb} = 0.73 \pm 0.067 (1\sigma)$$

$$\Rightarrow S_{PbPb} = 0.53 \pm 0.097$$

in SHMc (earlier value):

$$S_{PbPb} = 0.65 \pm 0.12$$

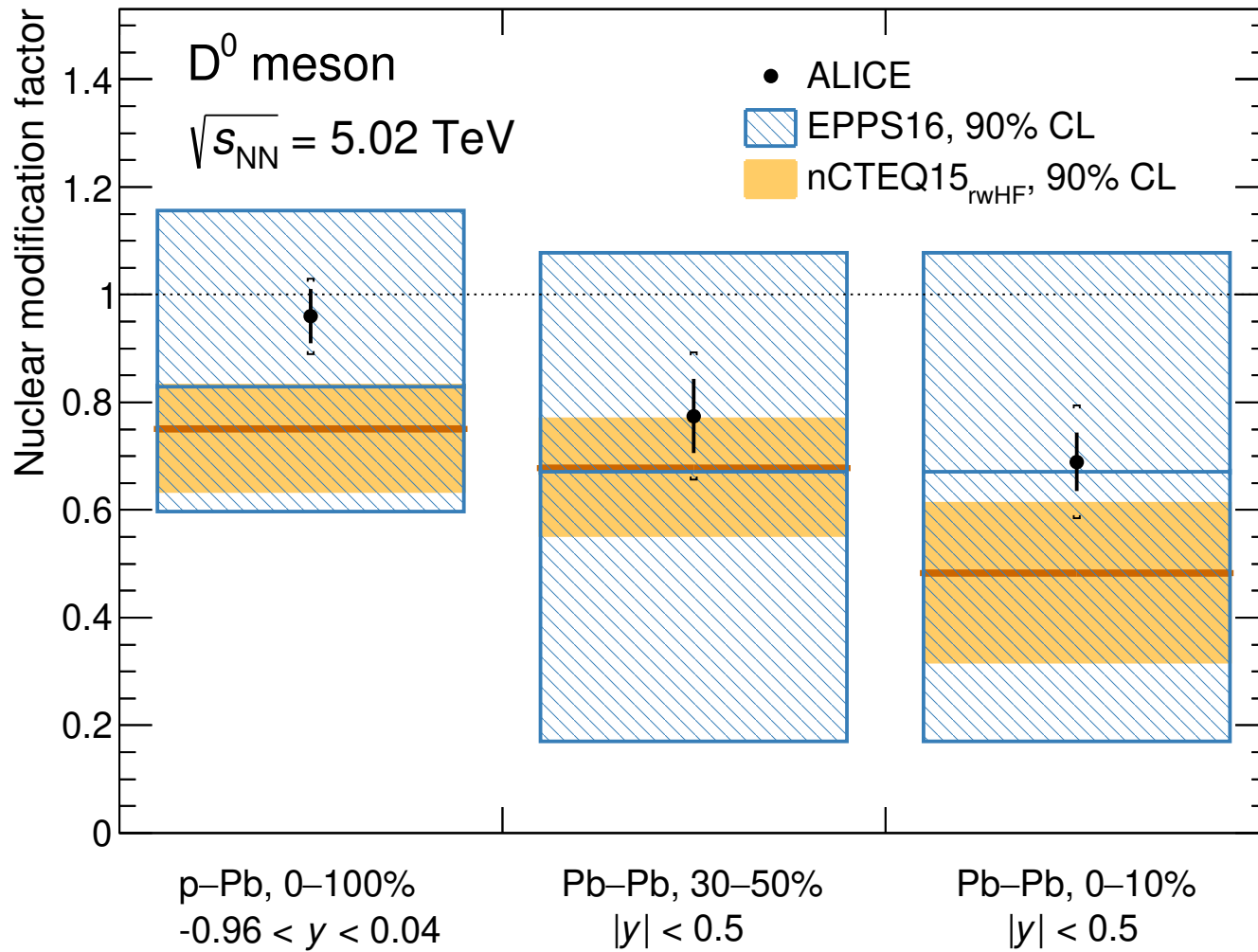
assumed shadowing in Pb–Pb

$$S_{PbPb} = R_{pPb}(-y) \cdot R_{pPb}(y)$$

$$\Rightarrow \frac{d\sigma_{c\bar{c}}}{dy} = 0.532 \pm 0.096 \text{ mb for } y=0 \text{ (Pb–Pb at } \sqrt{s_{NN}} = 5.02 \text{ TeV, SHMc)}$$

$$0.334 \pm 0.053 \text{ for } y=2.5-4$$

A fresh ALICE shadowing in Pb-Pb



0-10%

Data (stat+sys):

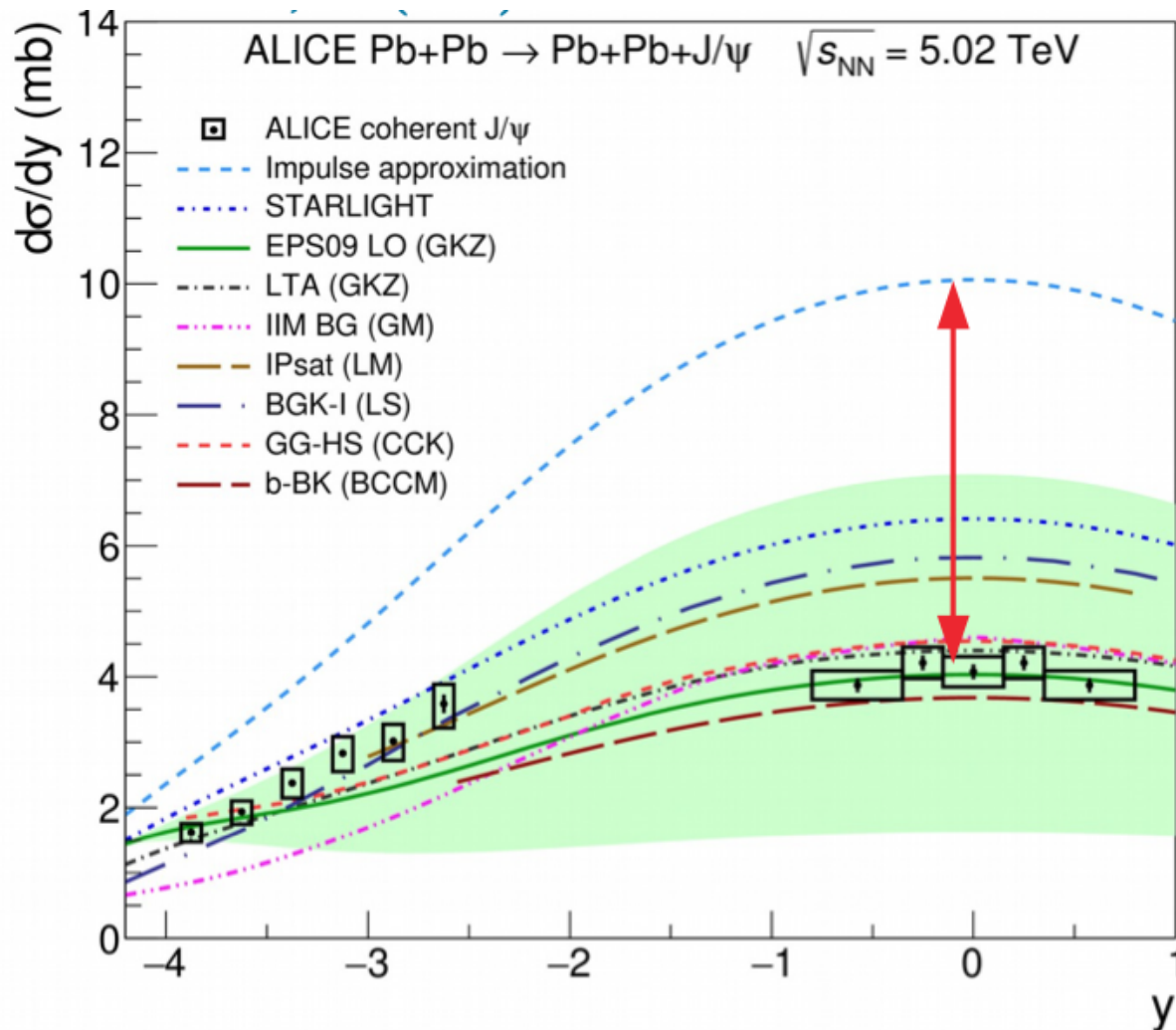
$$S_{PbPb} = 0.69 \pm 0.11$$

nCTEQ15_{rWHF} (1σ):

$$S_{PbPb} = 0.47 \pm 0.09$$

(centrality-dependent)

Another argument for large shadowing in Pb-Pb



J/ψ photoproduction

$$S_{PbPb} = \frac{d\sigma/dy(data)}{d\sigma/dy(imp.approx.)}$$

$$S_{PbPb} = 0.42 \pm 0.04$$

$$(x_{Bj} \simeq (0.3 - 1.4) \cdot 10^{-4})$$

ALI-PUB-482756

ALICE, EPJC 81 (2021) 712

Impulse approximation: STARLIGHT, no nuclear effects except coherence

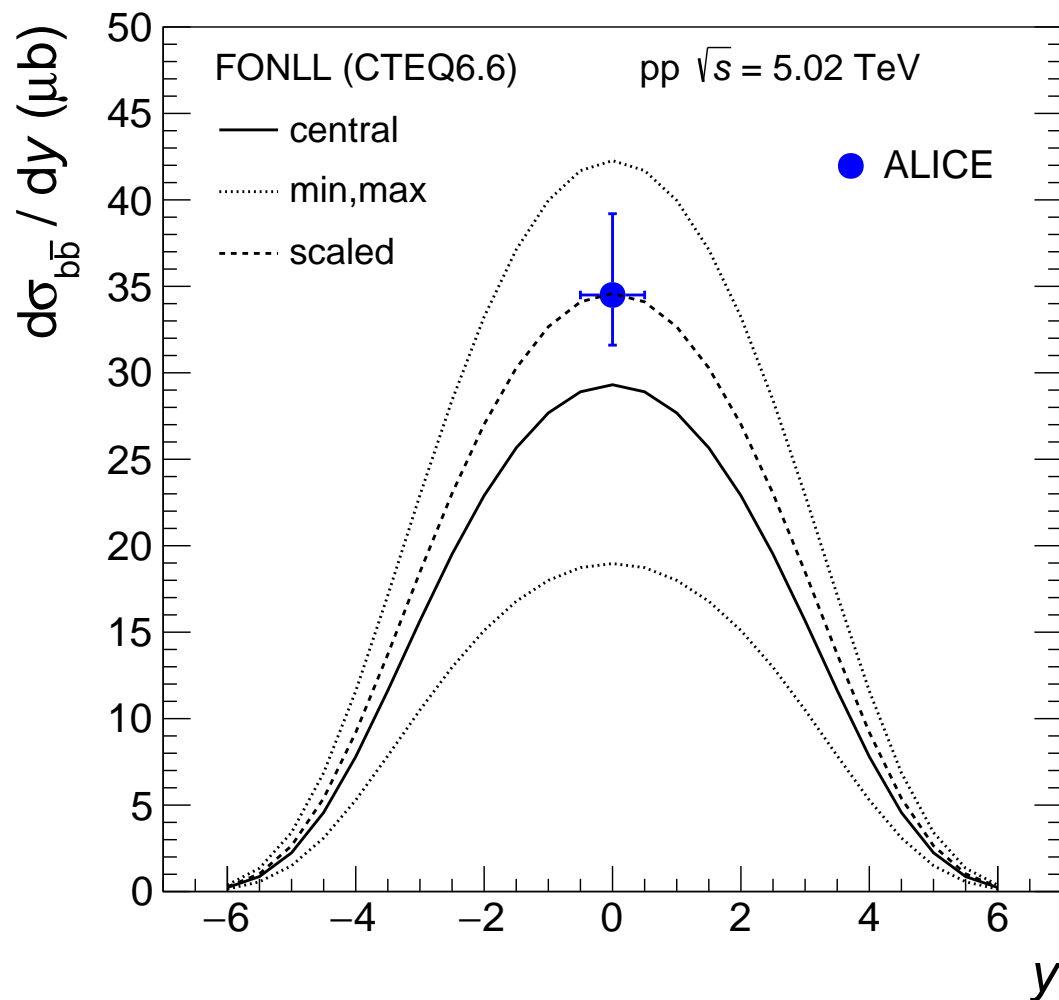
Caveat: there are sophistications, see Eskola et al. [2203.11613](#)

The beauty production cross section $d\sigma_{b\bar{b}}/dy$

A. Andronic

42

ALICE, [JHEP 05 \(2021\) 220](#): $\frac{d\sigma_{b\bar{b}}}{dy} = 34.5 \pm 2.4(stat)_{-2.9}^{+4.7}(tot.syst)\mu b$



[FONLL](#), Cacciari et al., [JHEP 1210 \(2012\)](#)

scaling for $y=2.5-4$

Shadowing: 0.7, independent on y

$$y = 0 : \frac{dN_{b\bar{b}}}{dy} = 0.57$$

$\pm 20\%$ total uncertainty

All produced anew ...no Υ state survives in QGP (extreme for $\Upsilon(1S)$?)

Core(SHMb)-corona with pp data input CMS, [PLB 790 \(2019\) 270](#), ALICE, [arXiv:2109.15240](#)

