



Simplified modelling for e-cloud instabilities

G. Iadarola

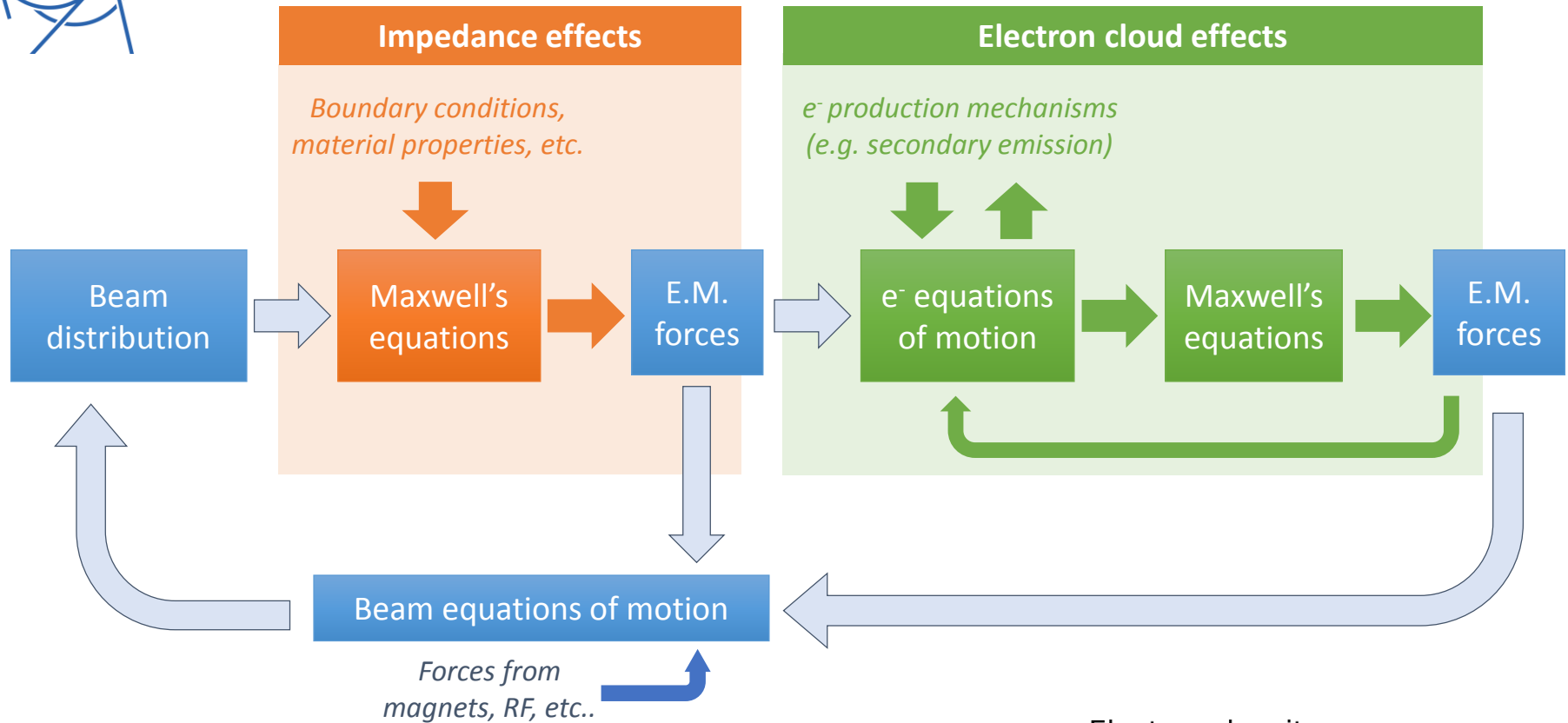
Many thanks to:

L. Mether, E. Métral, N. Mounet, G. Rumolo L. Sabato

- **Synthetic description the e-cloud/beam interaction**
 - Introduction
 - Harmonic response
 - Superposition check
- **Instability modelling**
 - Harmonic response alone
 - Harmonic response + non-linearities
 - RF voltage scan
 - e-cloud strength scan

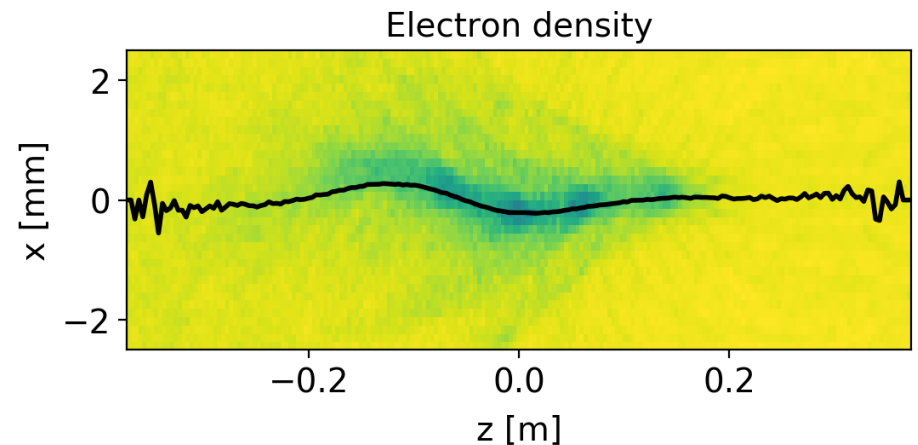


Electron cloud driven instabilities



e-cloud instabilities are studied by simulating the **coupled motion of beam and electron particles** (very heavy)

→ can we find a **simplified model** for the effect of the electrons (avoiding the full simulation)?





We choose a set of **orthogonal functions**:

$$h_m(z) = \begin{cases} A_m \cos\left(2\pi \frac{m}{2} \frac{z}{L_{\text{bkt}}}\right), & \text{if } m \text{ is even} \\ A_m \sin\left(2\pi \frac{m-1}{2} \frac{z}{L_{\text{bkt}}}\right), & \text{if } m \text{ is odd} \end{cases}$$

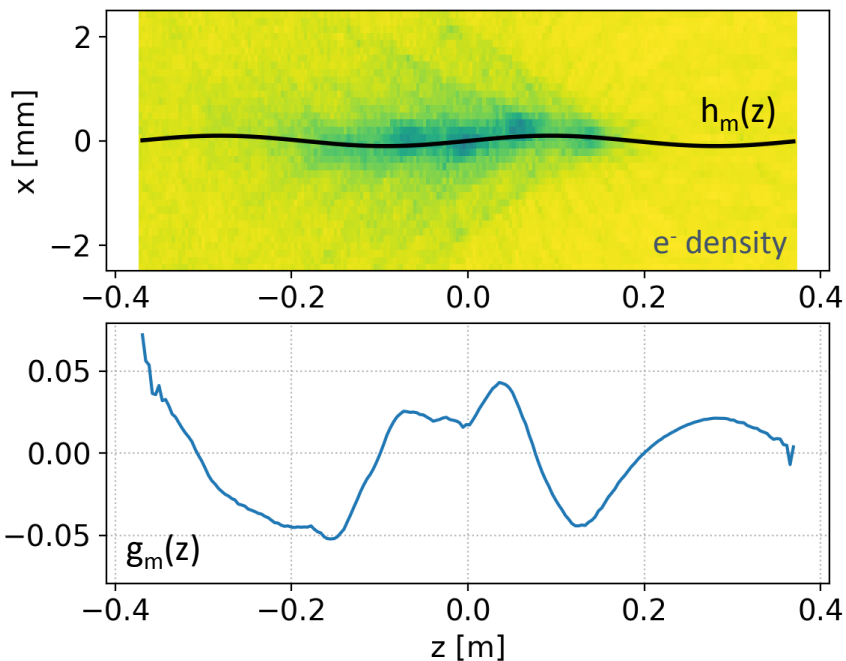
We **apply to the bunch a distortion equal to $h_m(z)$** to the horizontal position along a bunch and we call **$g_m(z)$ the resulting dipolar kick along the bunch.**

We can measure the $g_m(z)$ through **short PyELOUD simulations** (single pass)

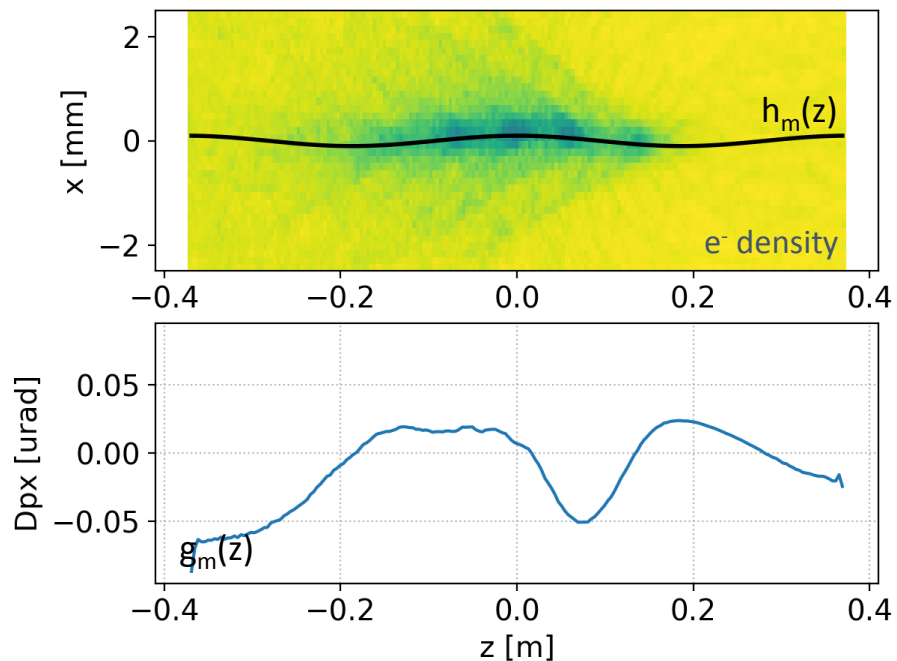


I can measure the $g_m(z)$ through short (single pass) PyELOUD simulations

n=2 sine



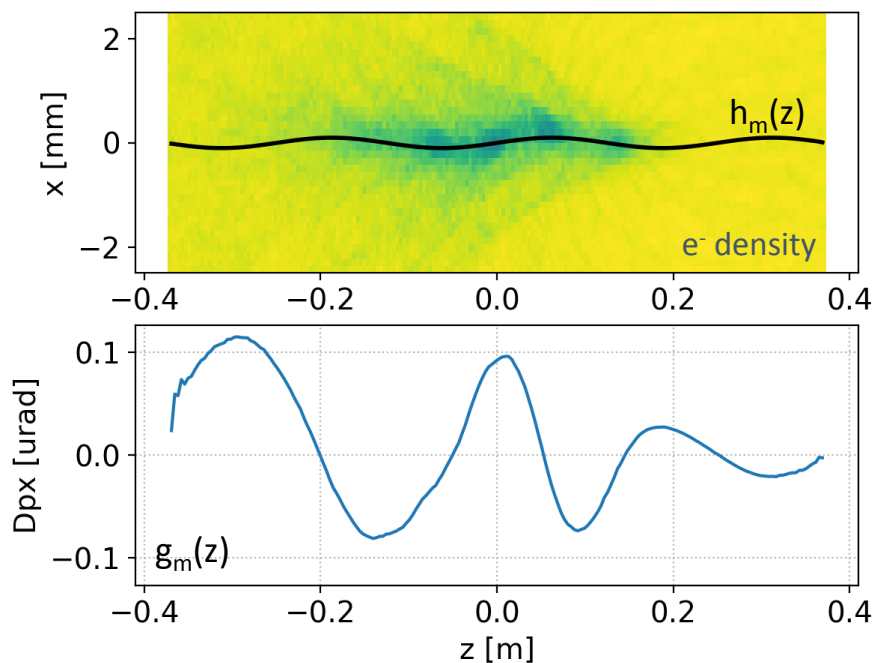
n=2 cosine



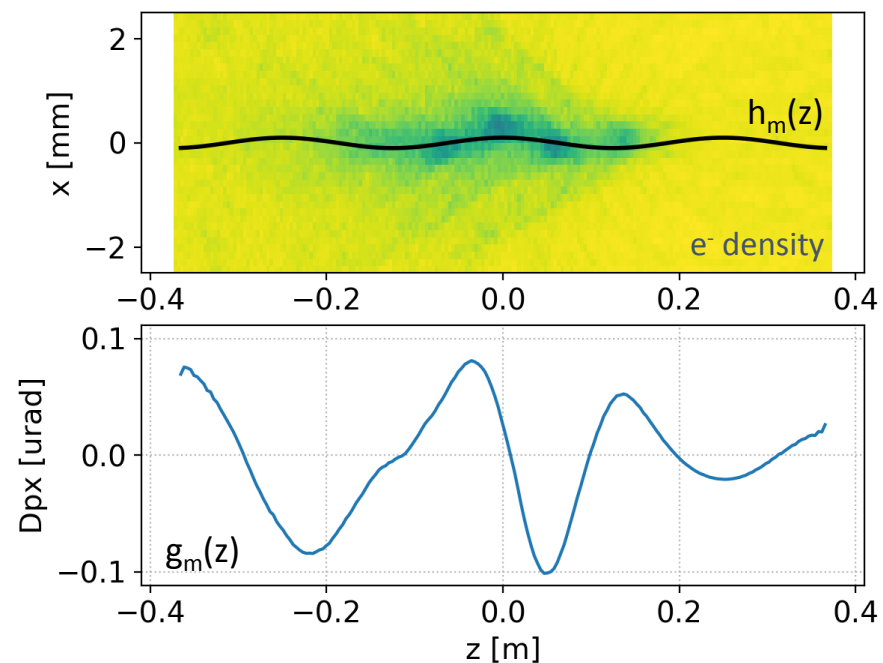


I can measure the $g_m(z)$ through short (single pass) PyECLLOUD simulations

n=3 sine



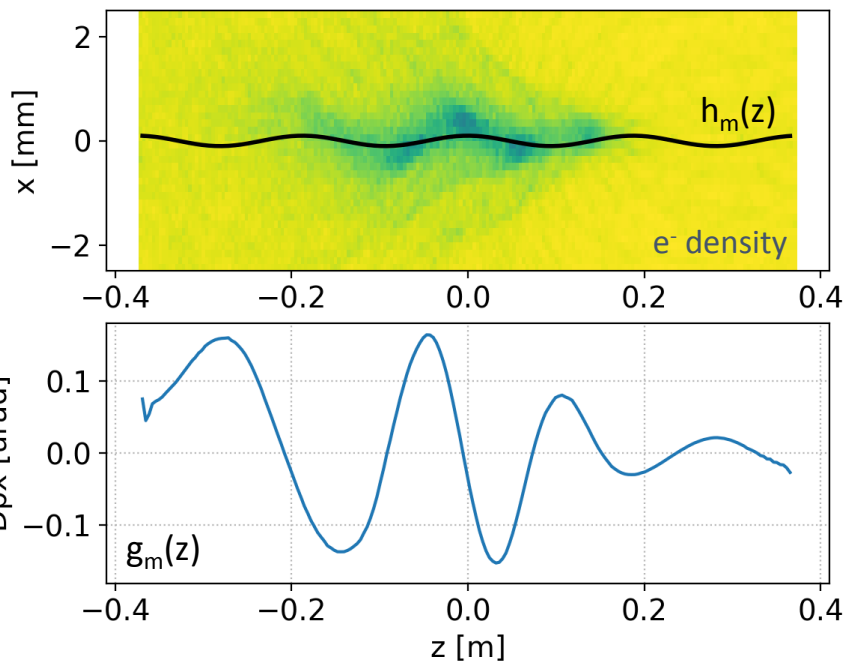
n=3 cosine



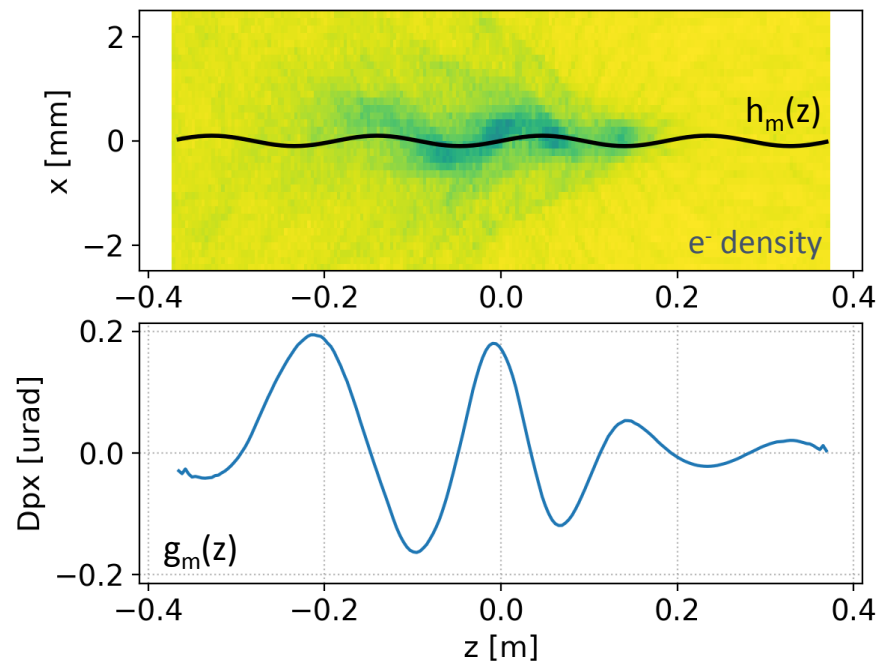


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n=4 cosine



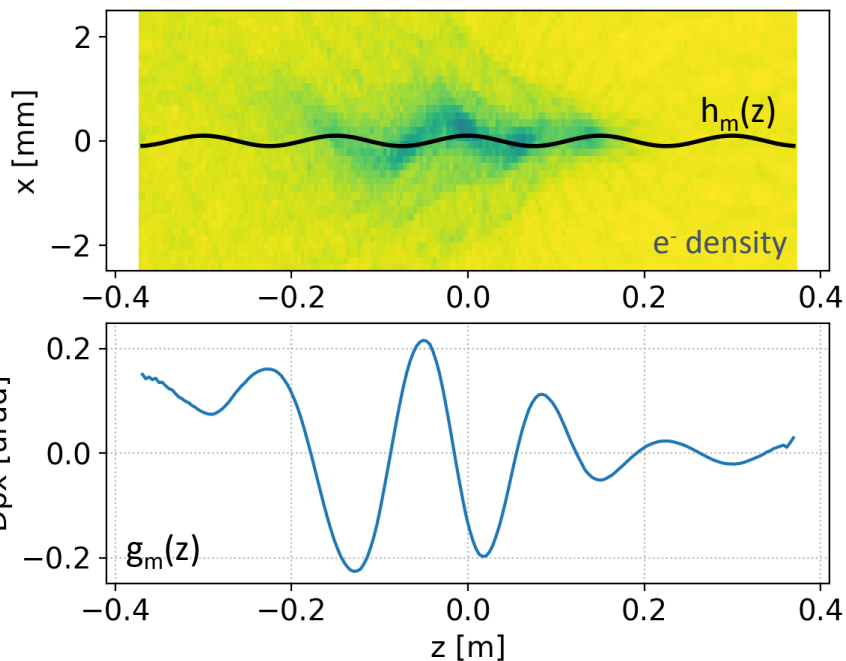
n=4 sine



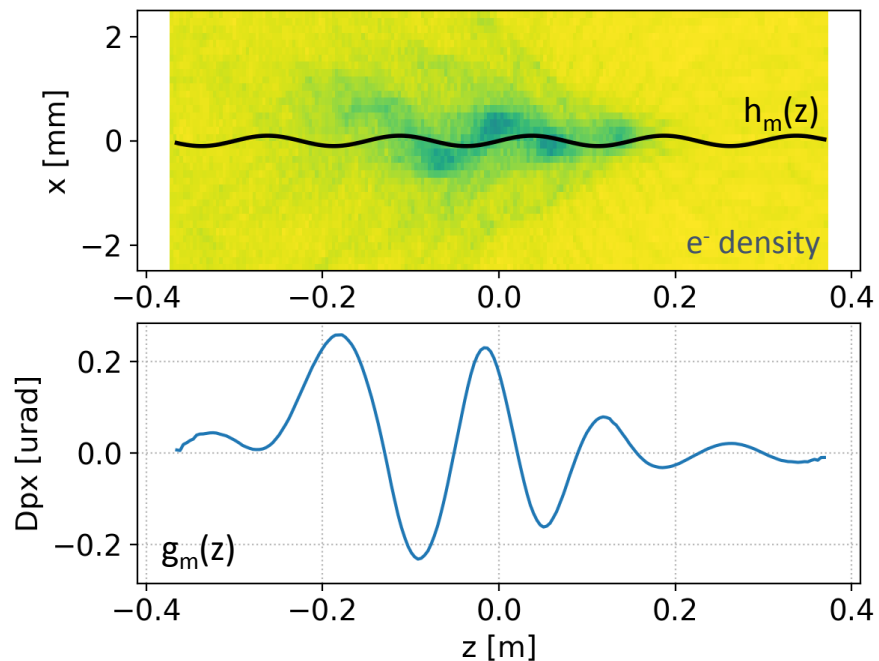


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n=5 cosine



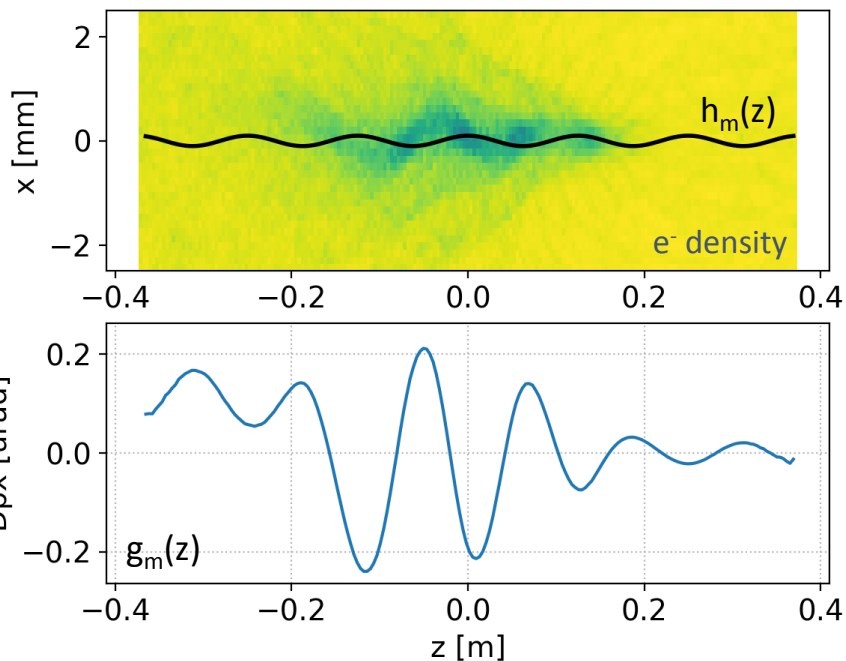
n=5 sine



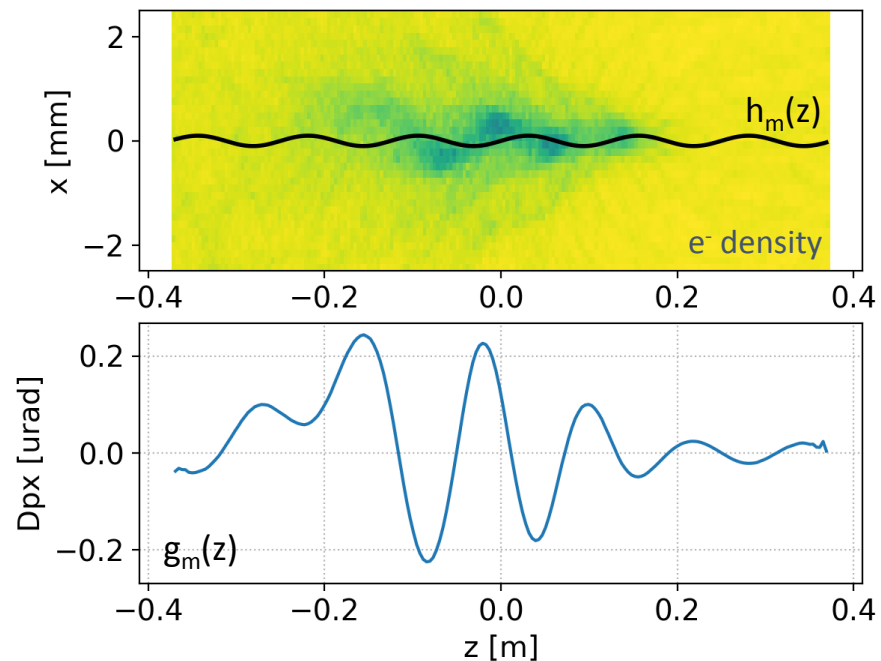


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n=6 cosine



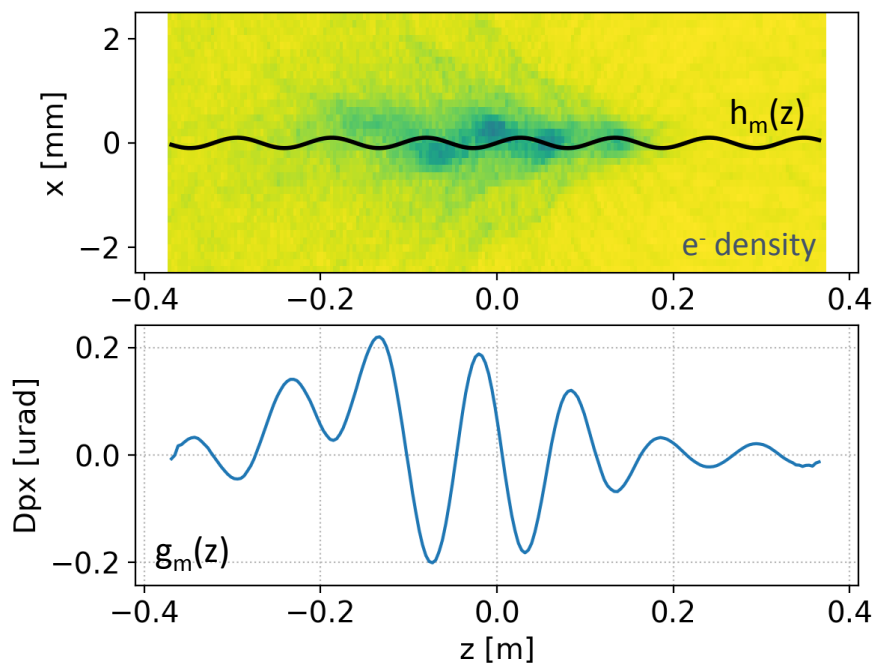
n=6 sine



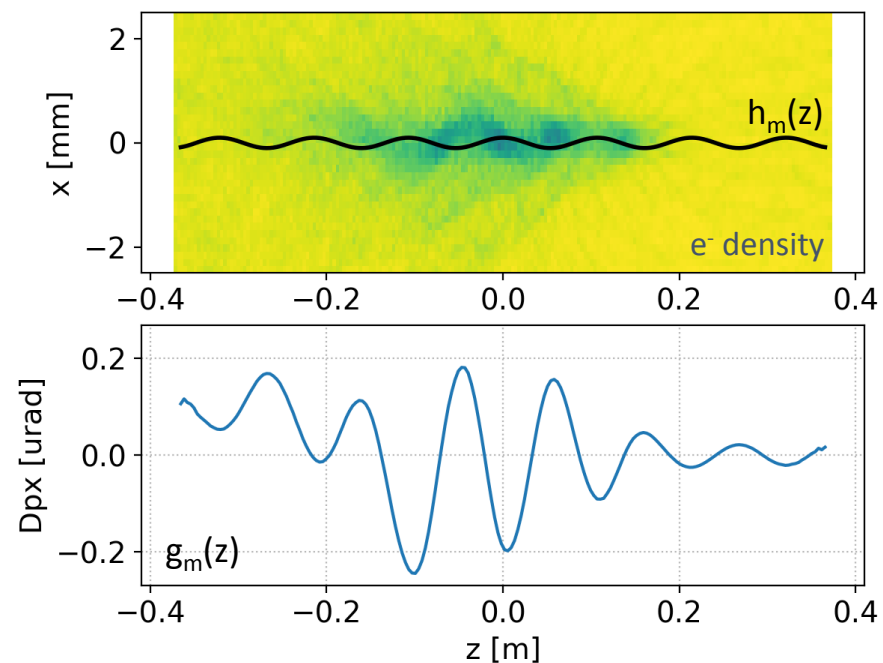


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n=7 sine



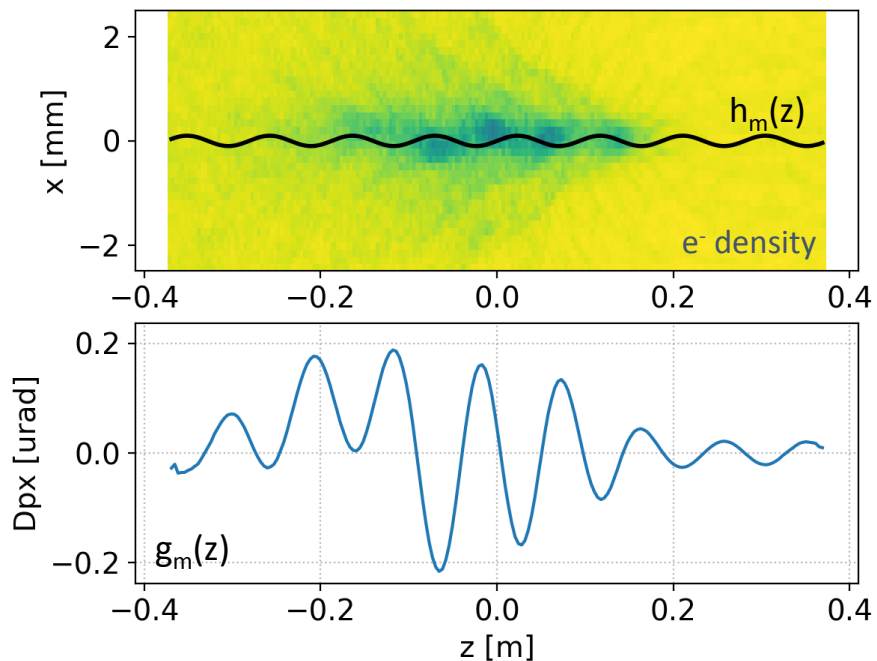
n=7 cosine



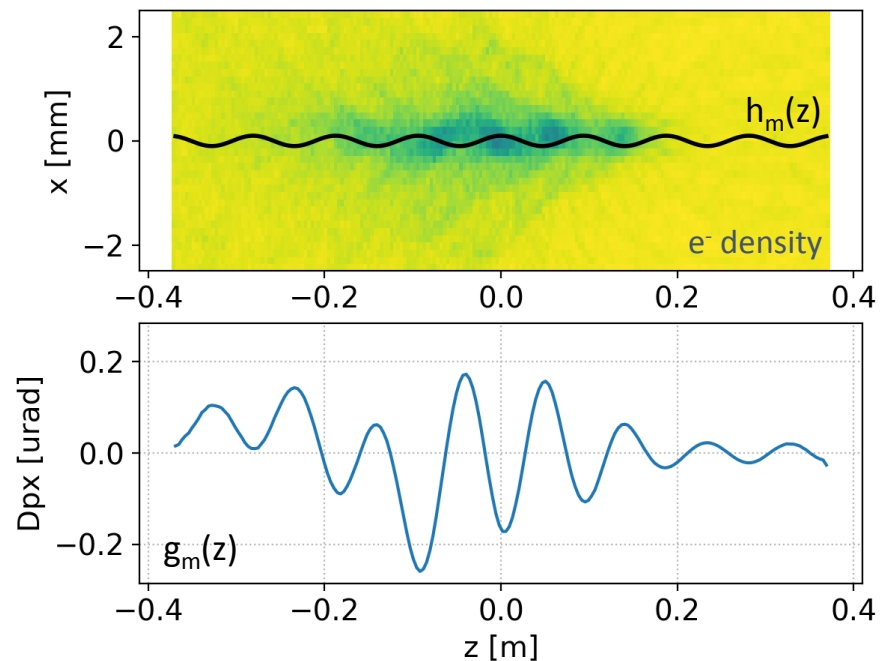


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n=8 sine



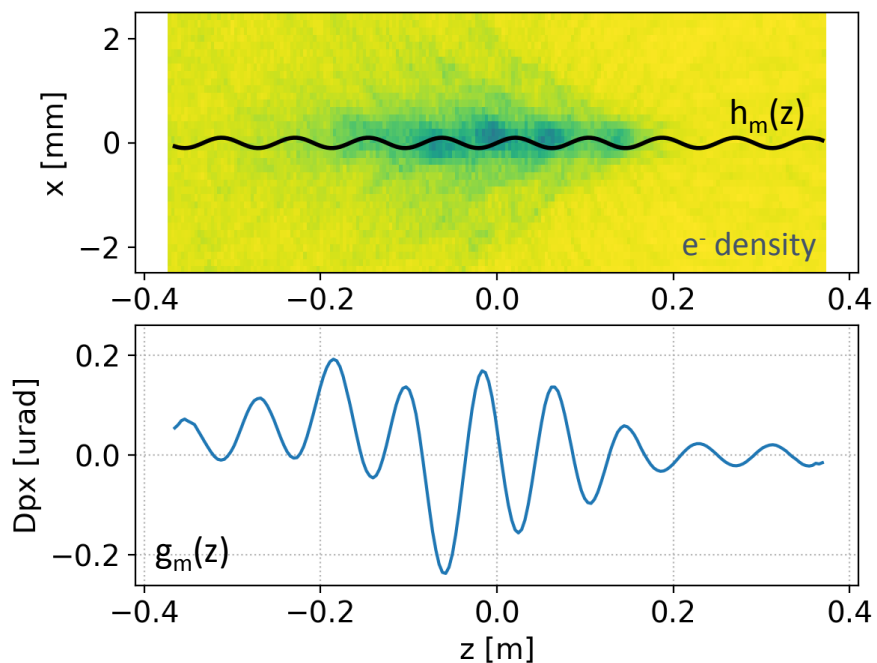
n=8 cosine



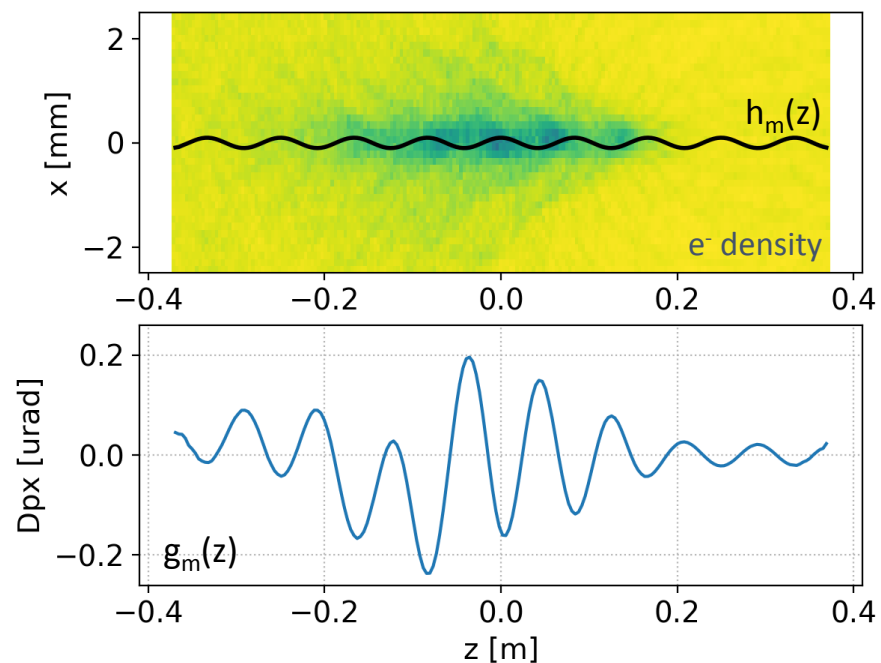


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n=9 sine



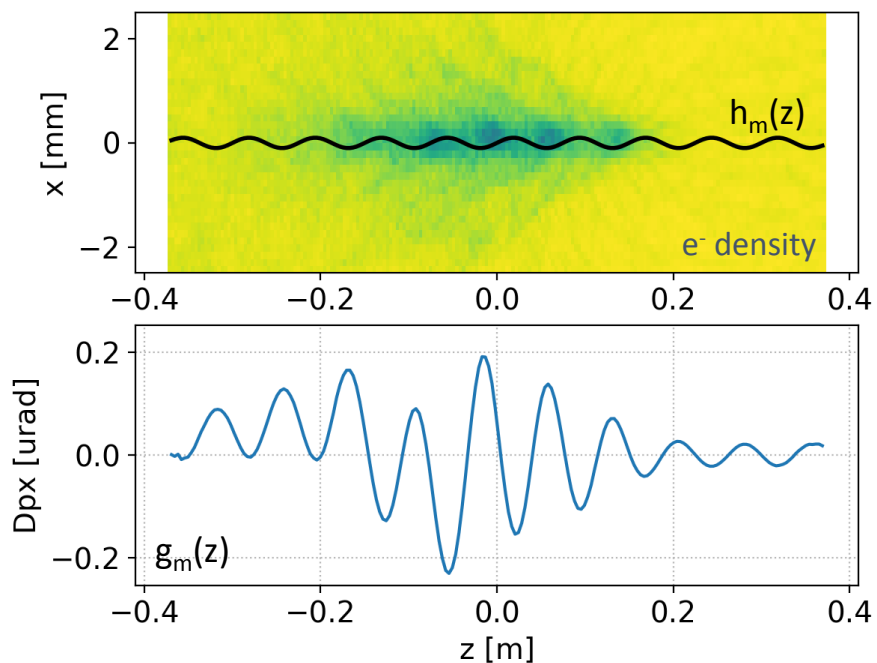
n=9 cosine



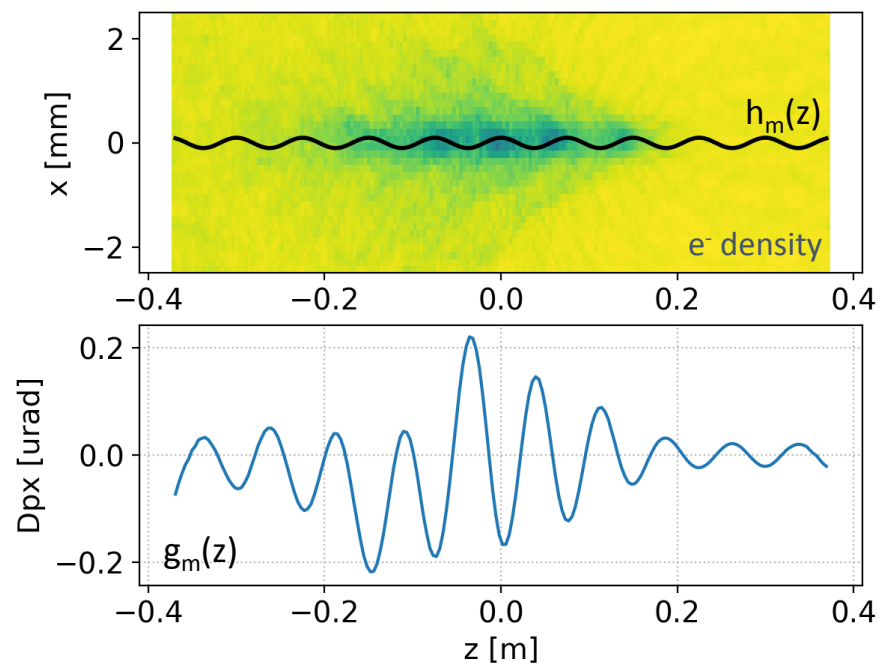


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n=10 sine



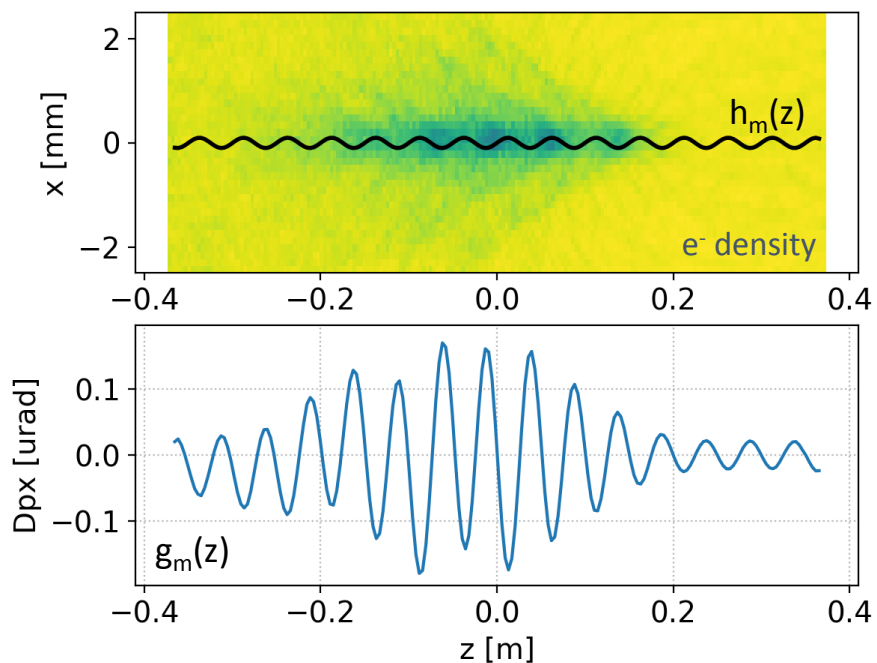
n=10 cosine



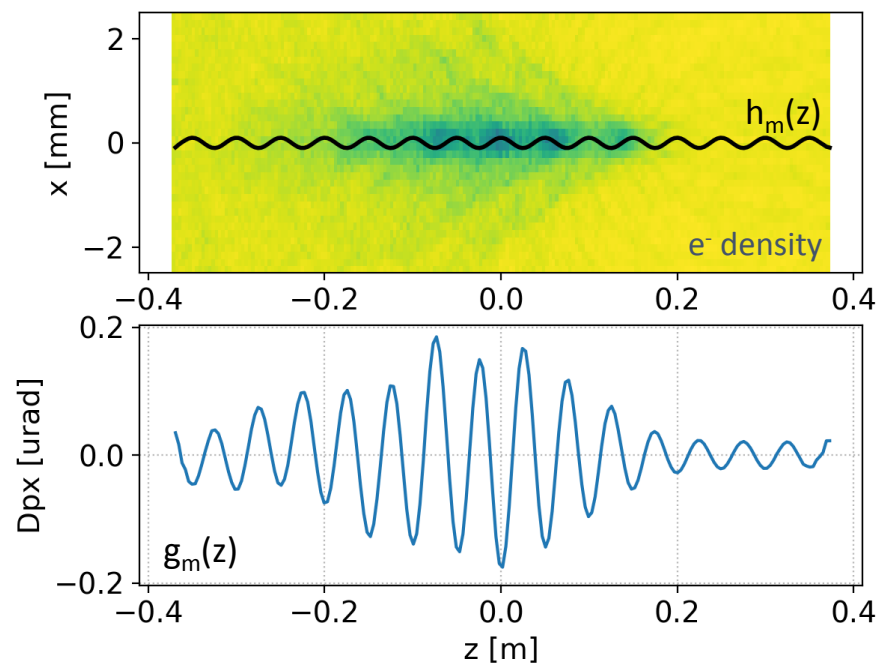


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n=15 sine



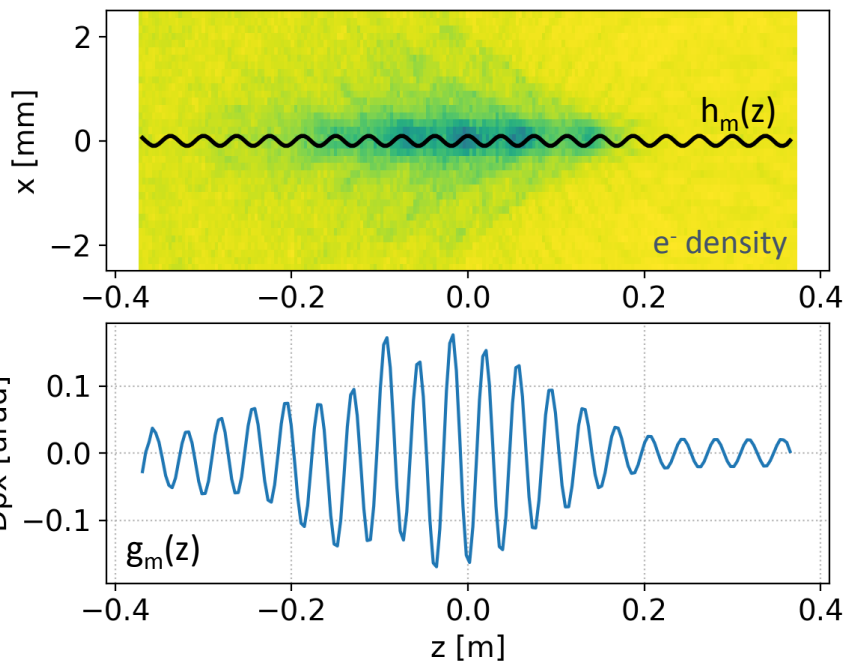
n=15 cosine



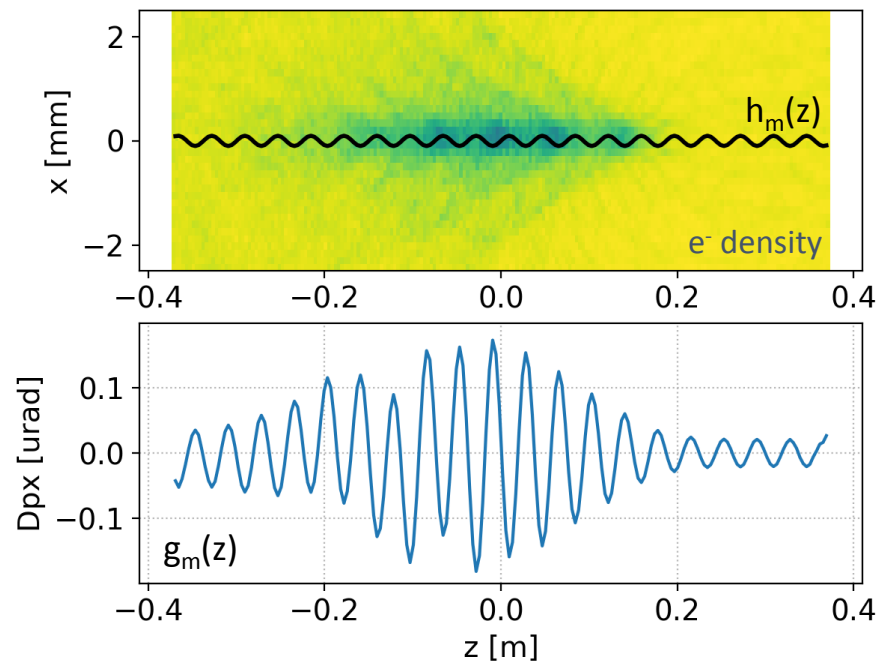


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n=20 cosine



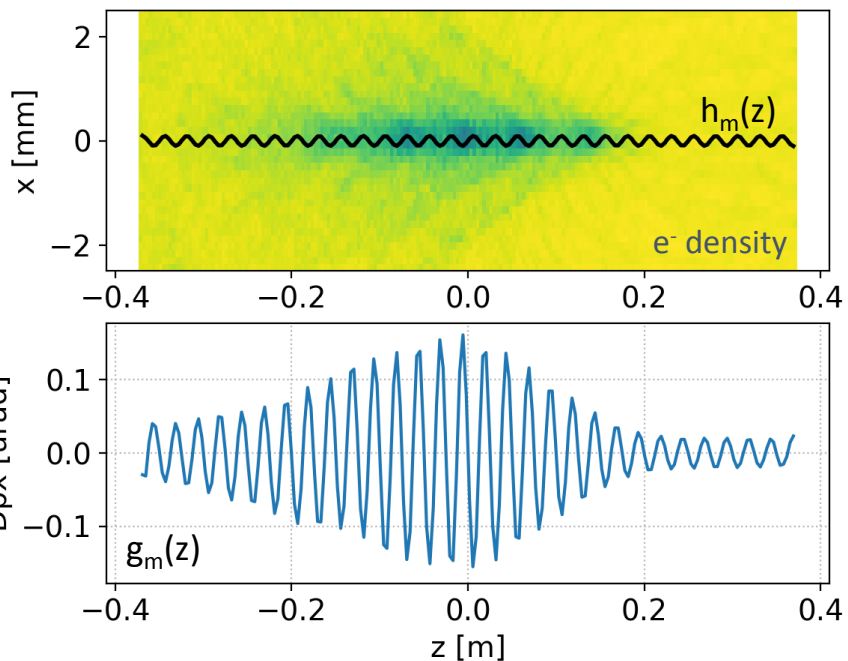
n=20 sine



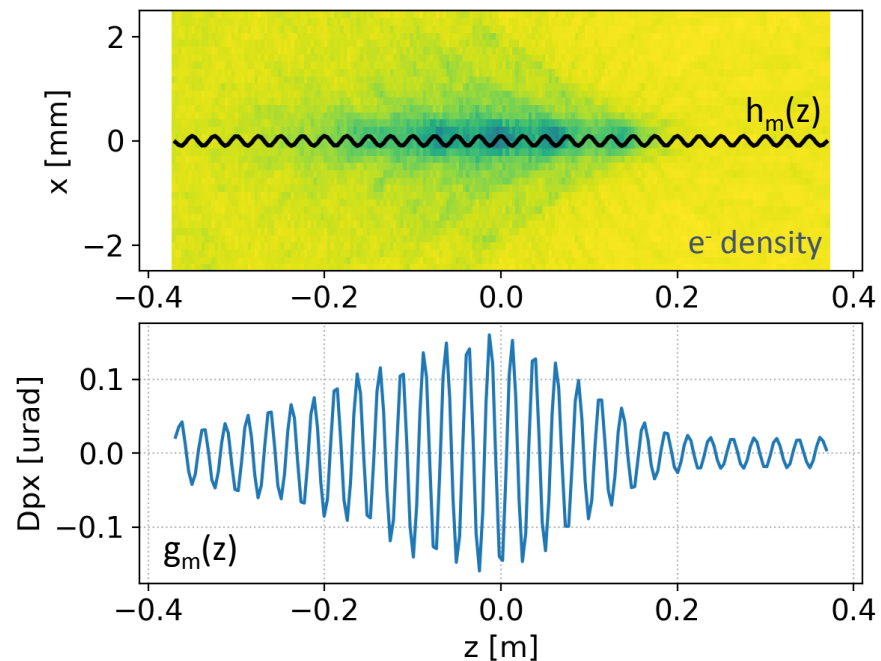


I can measure the $g_m(z)$ through short (single pass) PyELOUD simulations

n=30 sine



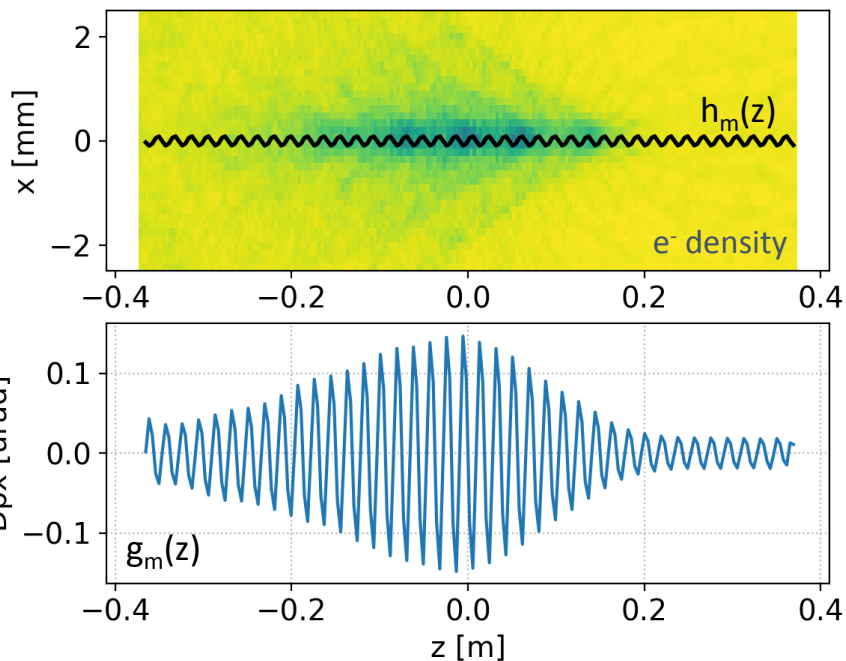
n=30 cosine



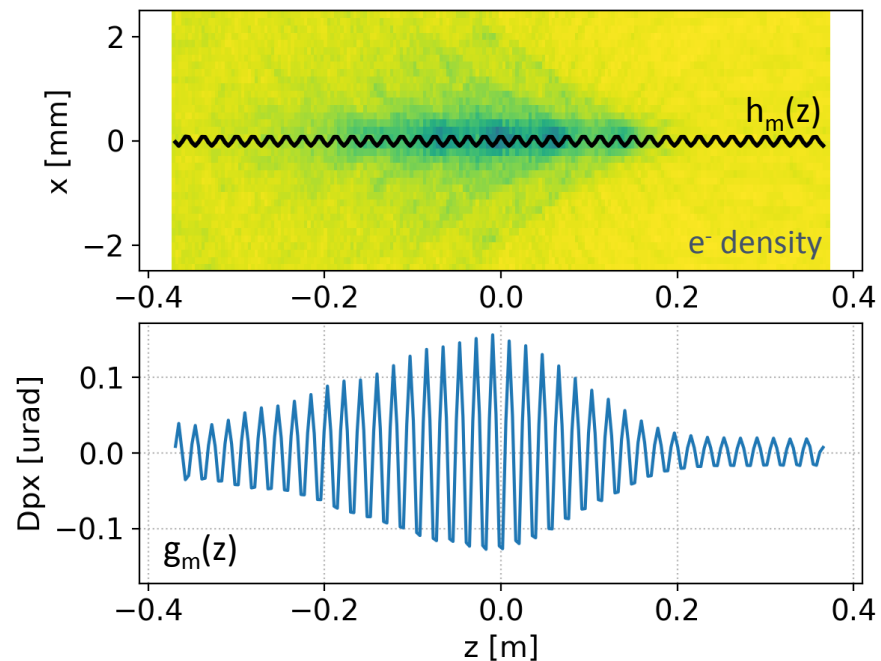


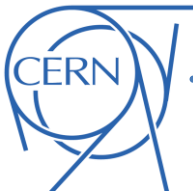
I can measure the $g_m(z)$ through short (single pass) PyELOUD simulations

n=40 sine



n=40 cosine





Superposition check



Can I use the calculated harmonic response to **predict the response to an arbitrary distortion?**

A generic distortion $x(z)$ can be decomposed as a superposition of harmonics:

$$x(z) = \sum_{m=0}^M a_m h_m(z) \quad \text{with:} \quad a_n = \int_{-L_{\text{bkt}}/2}^{L_{\text{bkt}}/2} x(z) h_m(z) dz$$

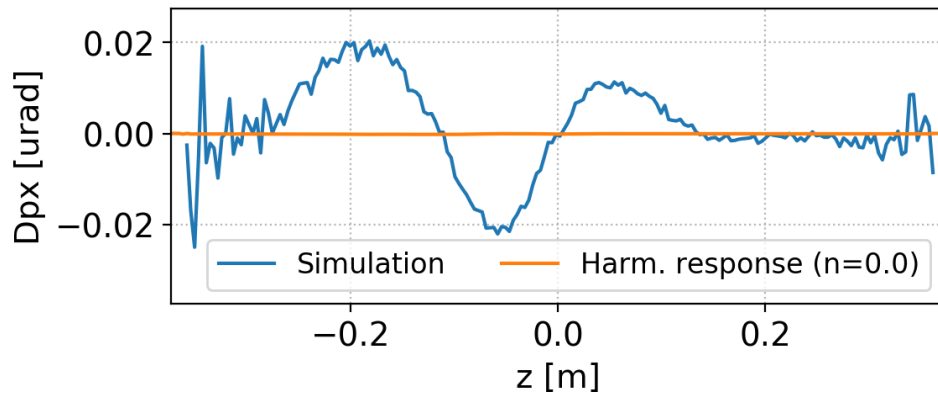
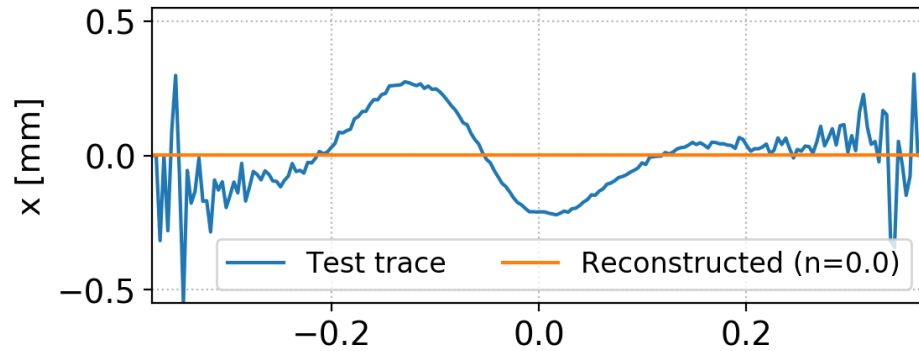
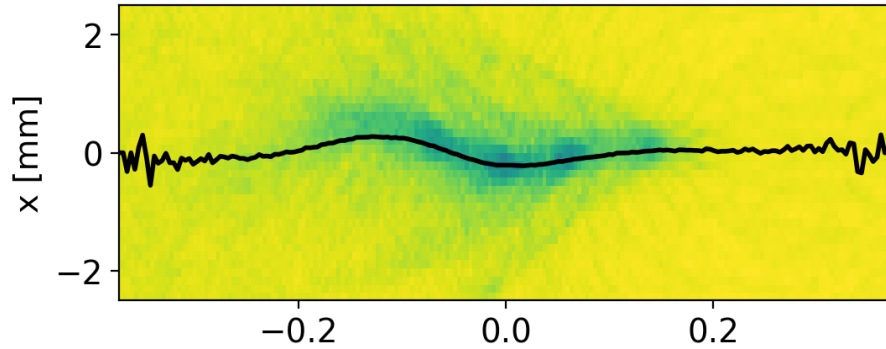
Is the system sufficiently linear to use a **linear superposition?**

$$\Delta x'(z) = \sum_{m=0}^M a_m g_m(z) = \sum_{m=0}^M g_m(z) \int_{-L_{\text{bkt}}/2}^{L_{\text{bkt}}/2} x(z') h_m(z') dz'$$



We test this hypothesis on the oscillation observed during an instability

Electron density



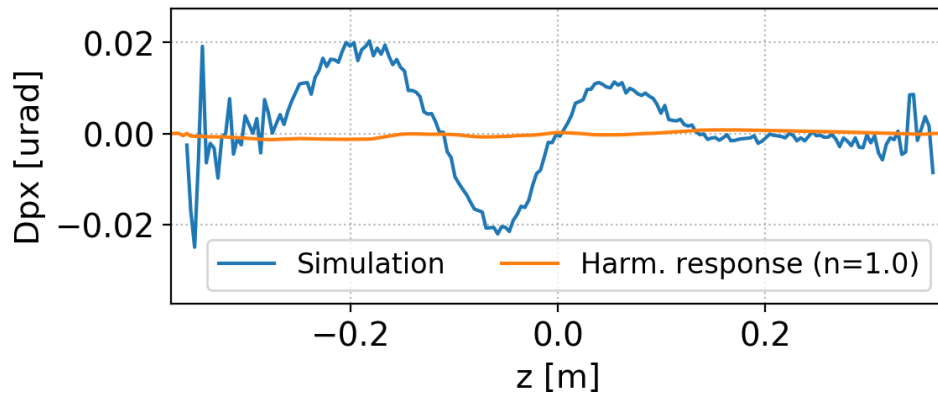
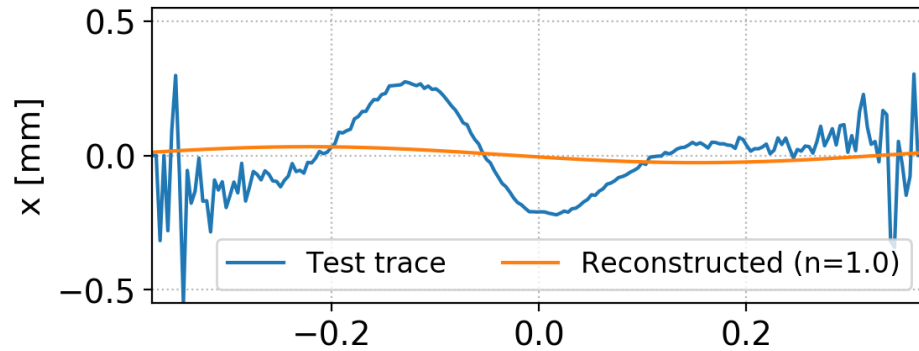
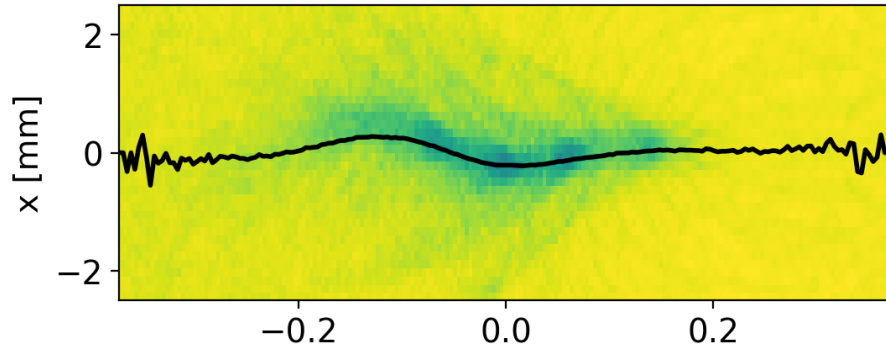
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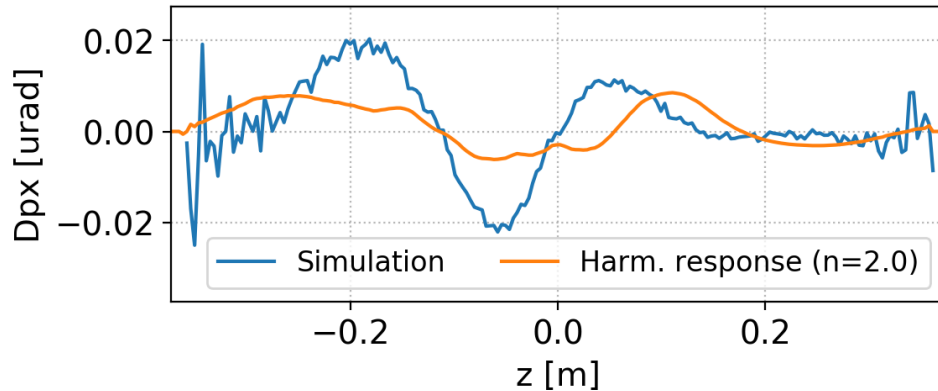
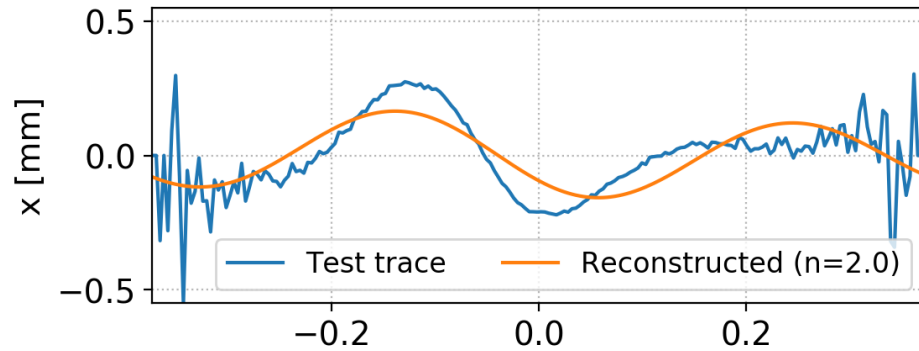
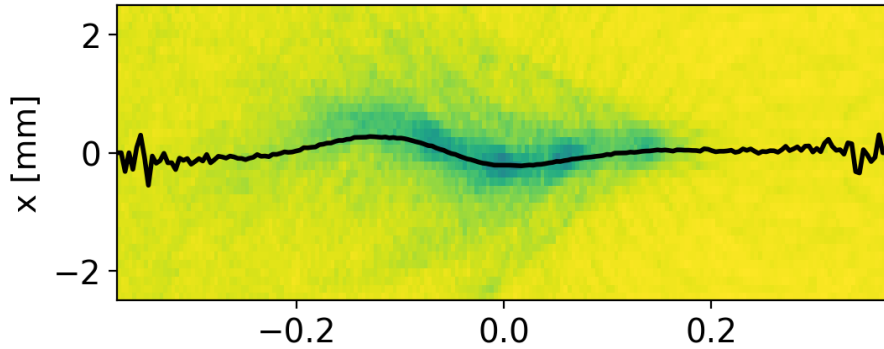
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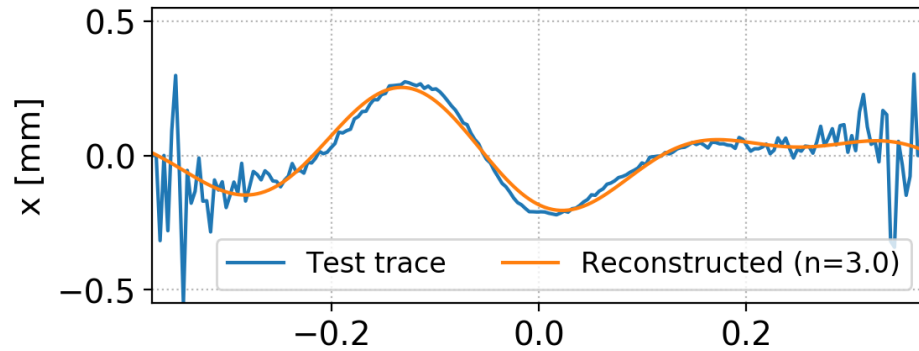
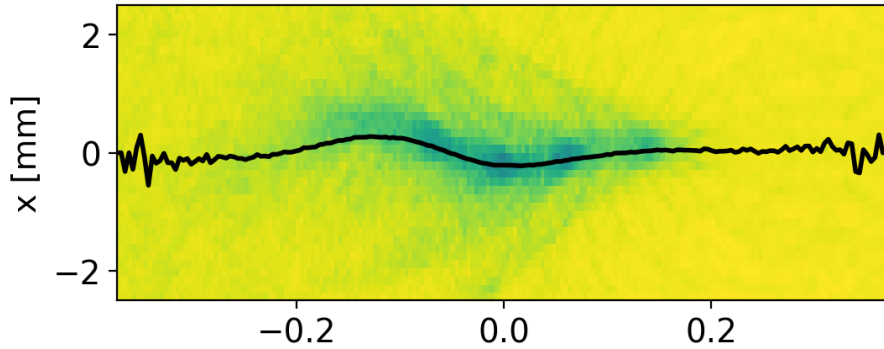
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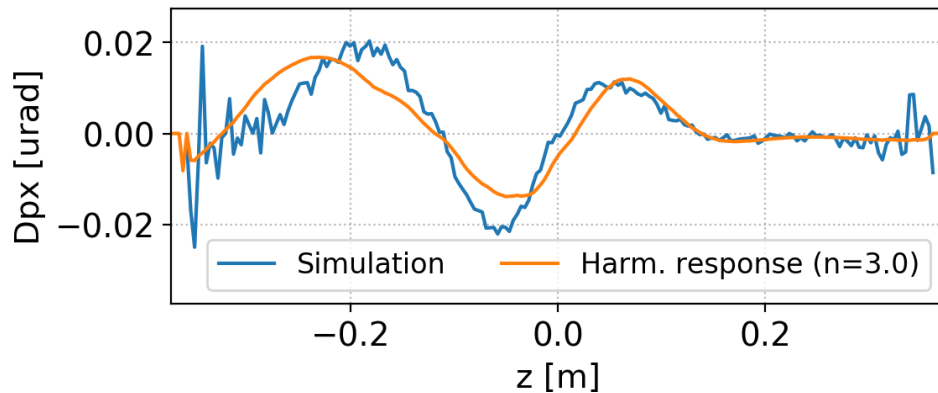


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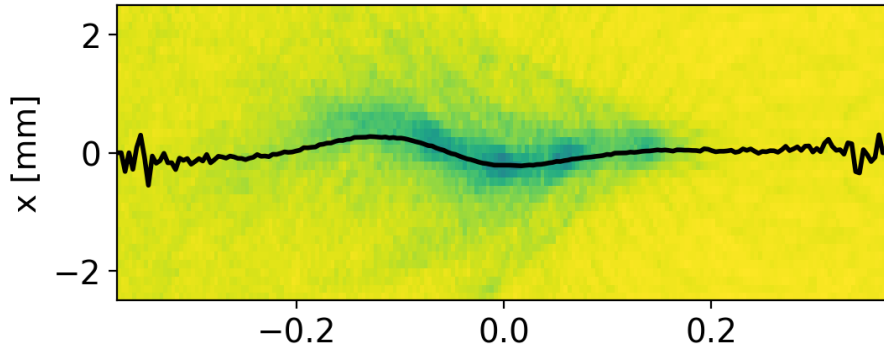


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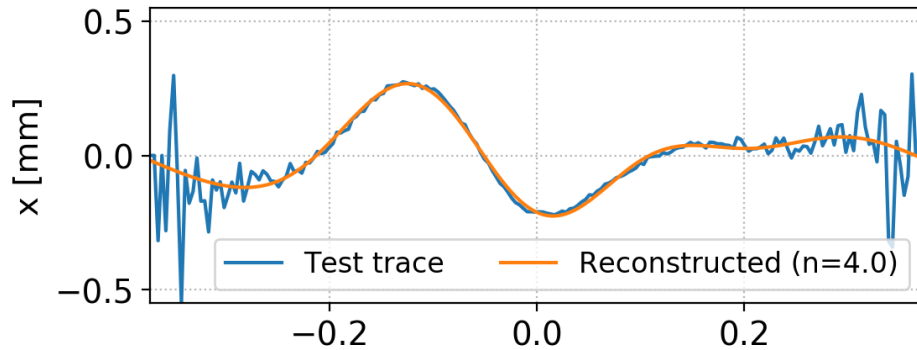


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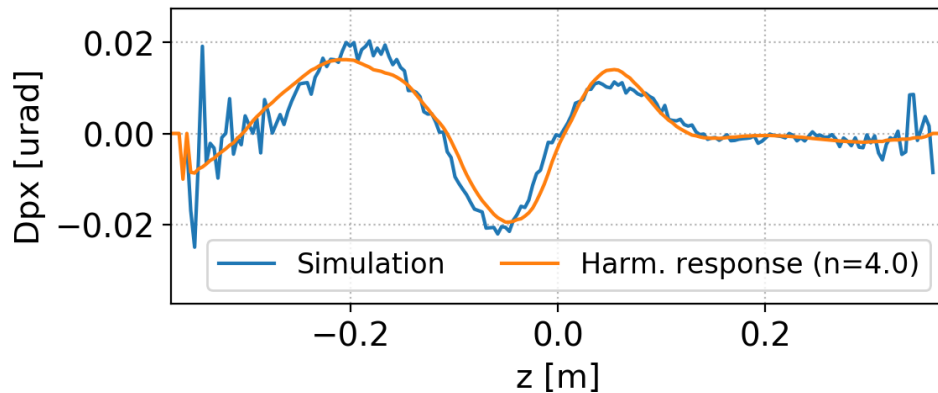
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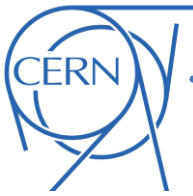
Superposition principle **works very well** and **convergence is quite fast**



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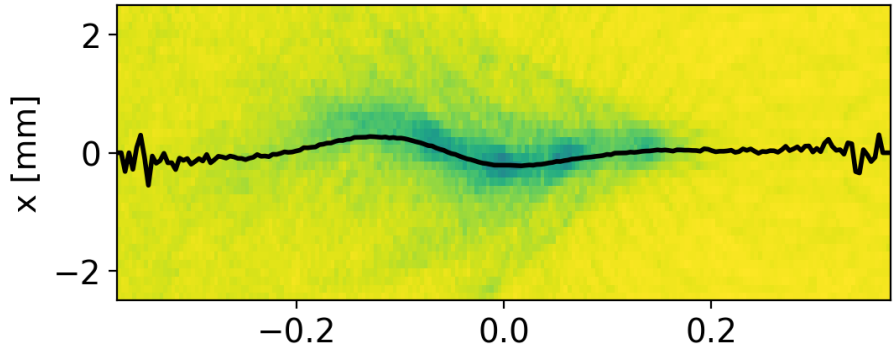


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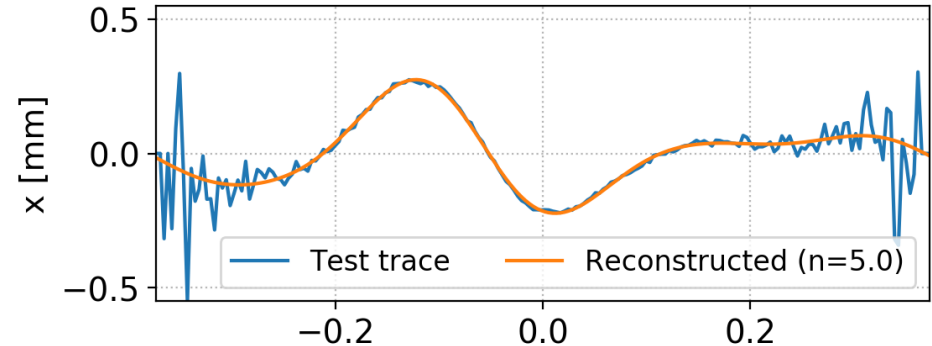


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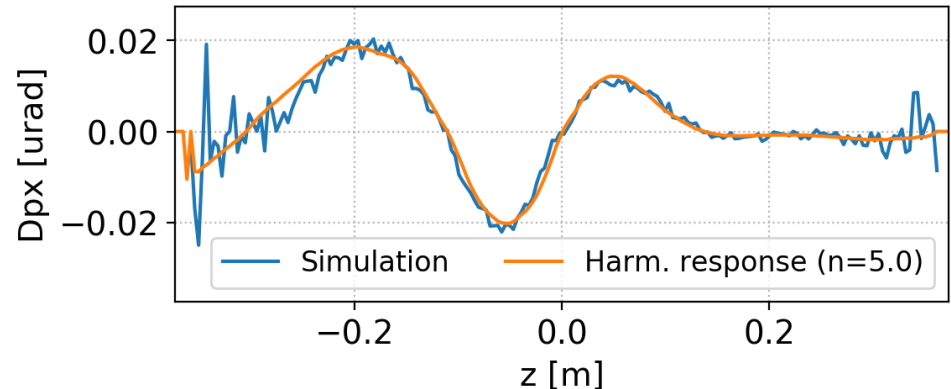
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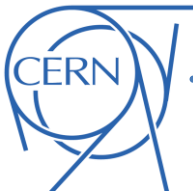
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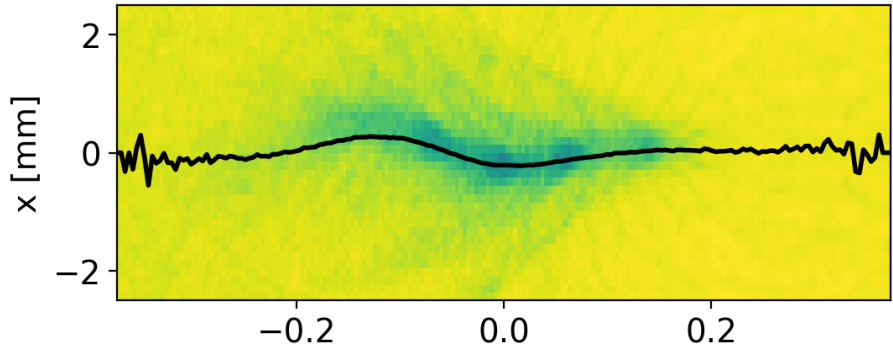


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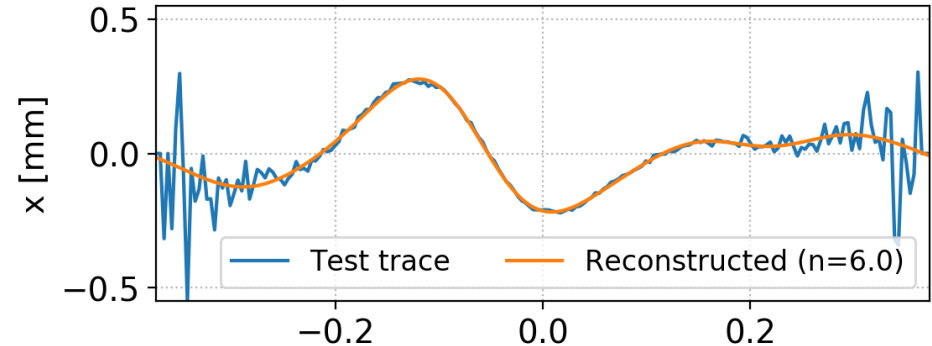


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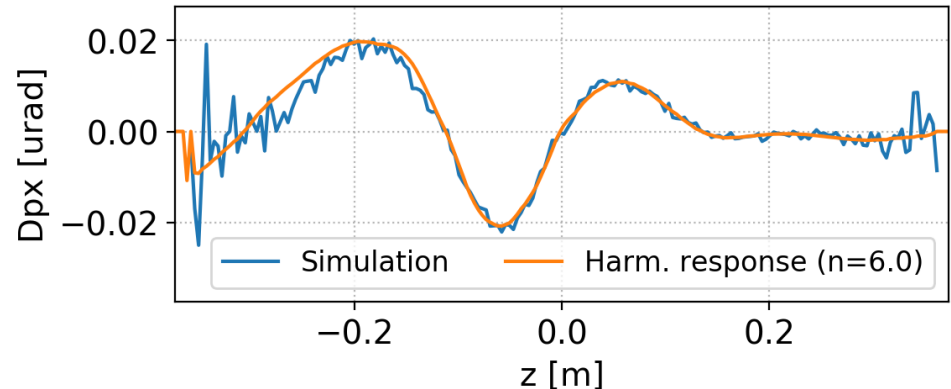
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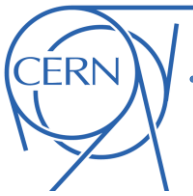
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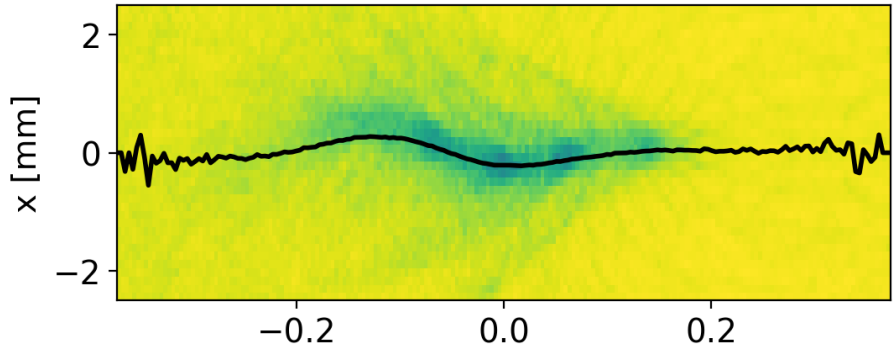


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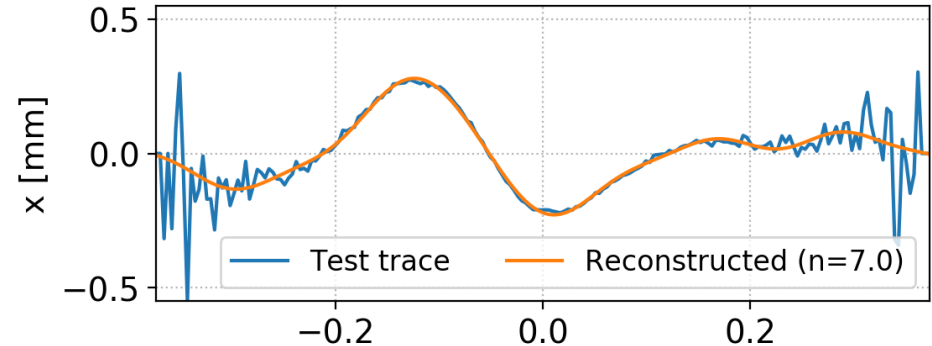


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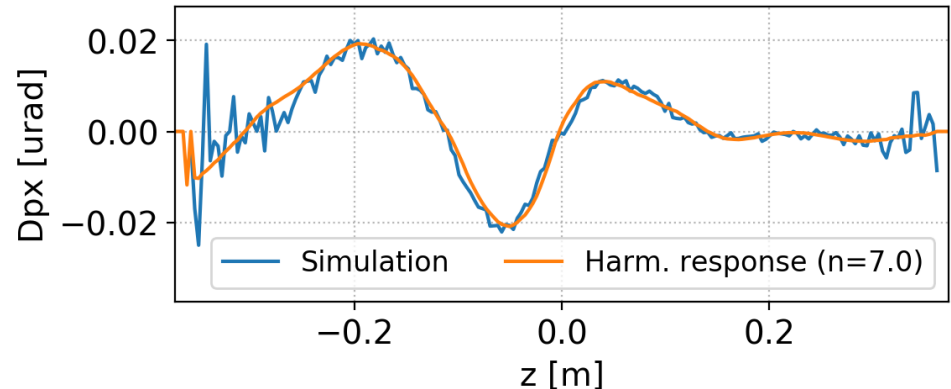
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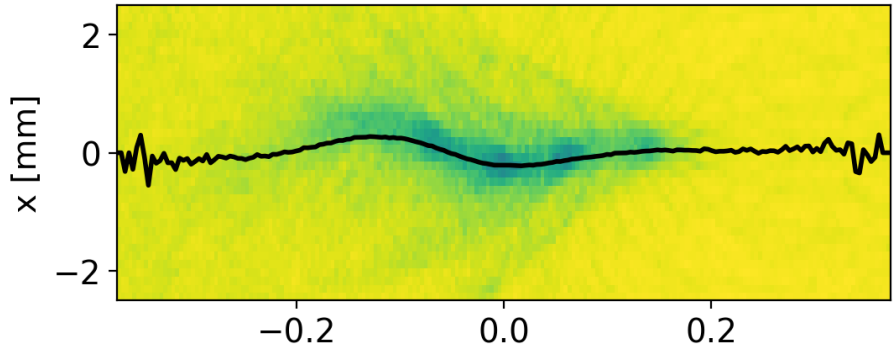


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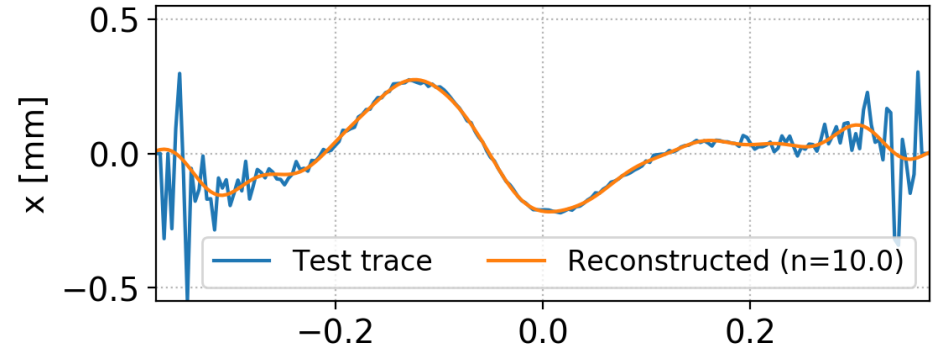


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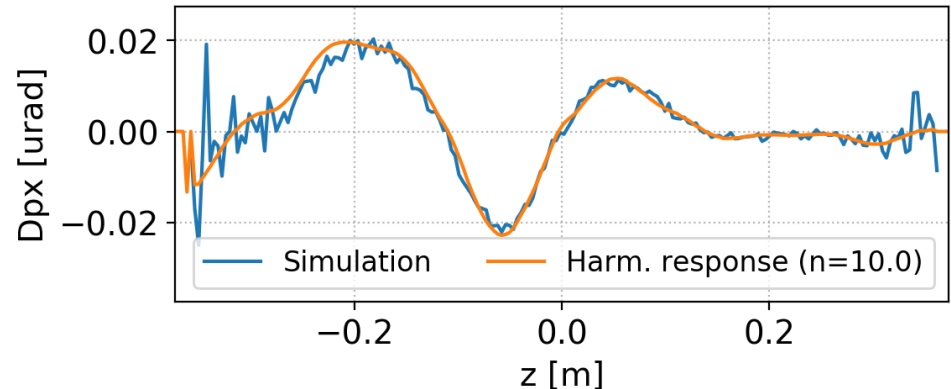
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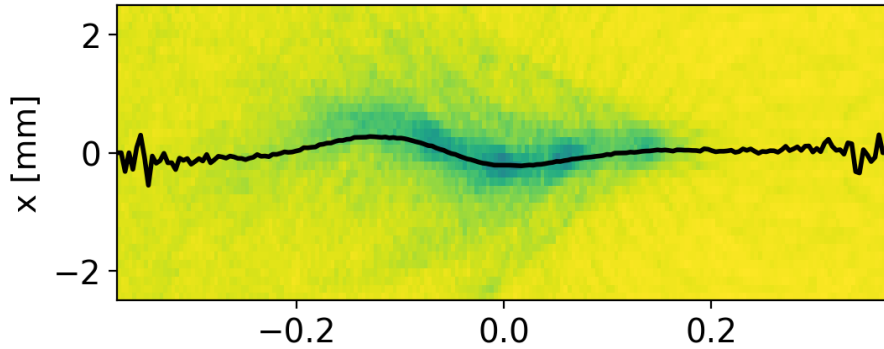


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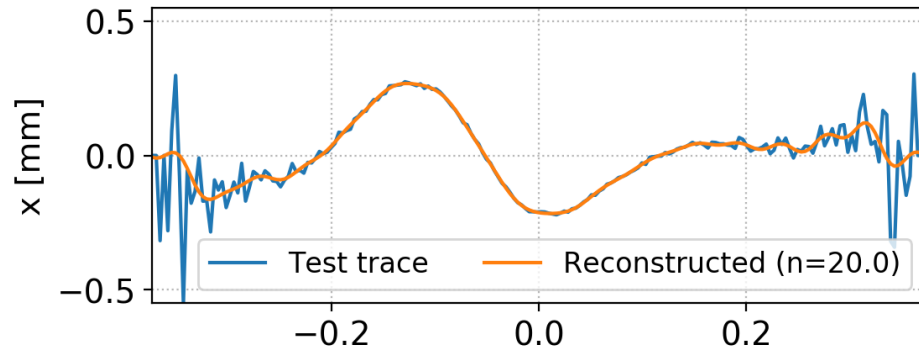


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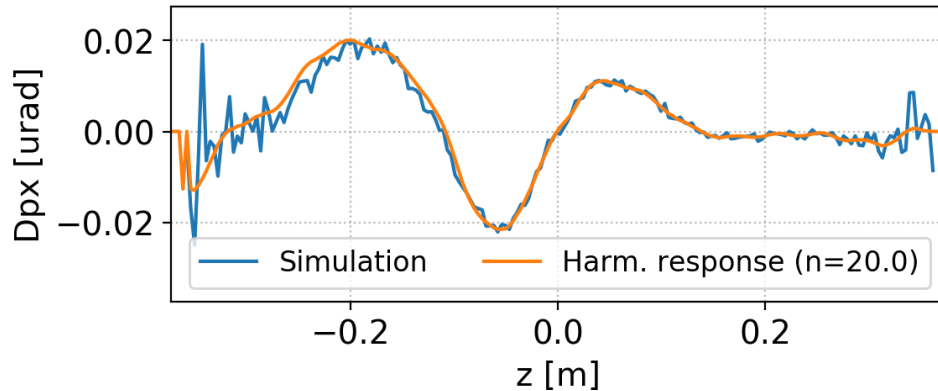
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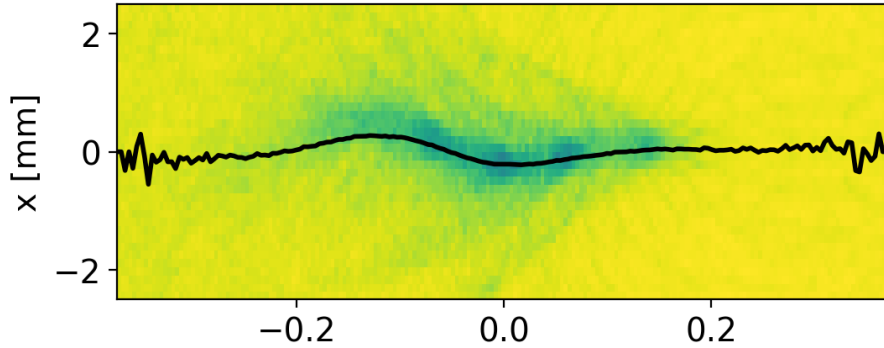


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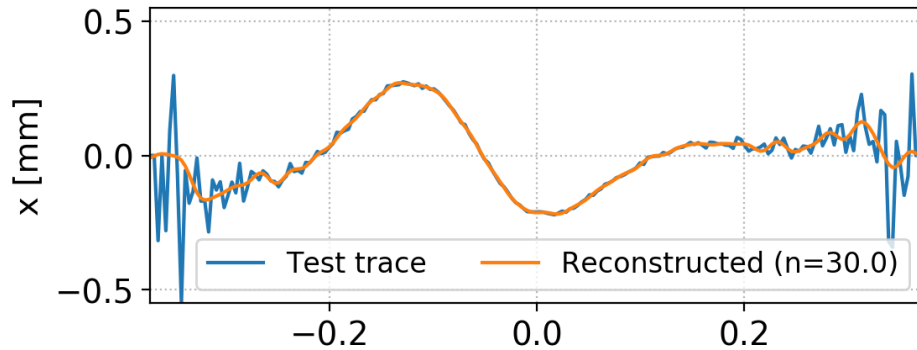


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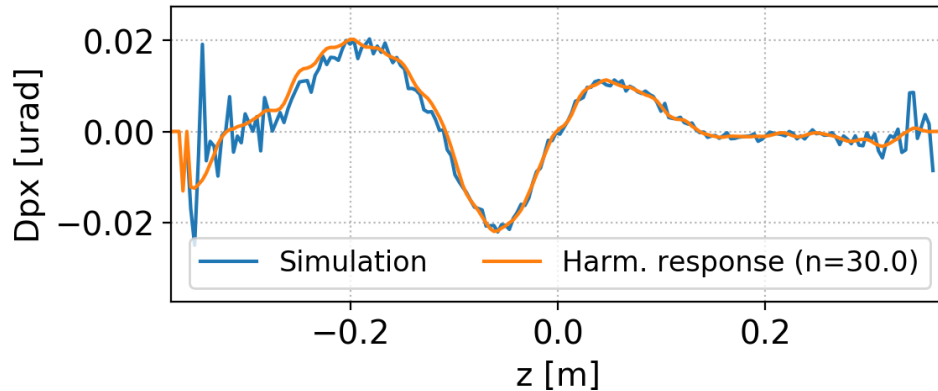
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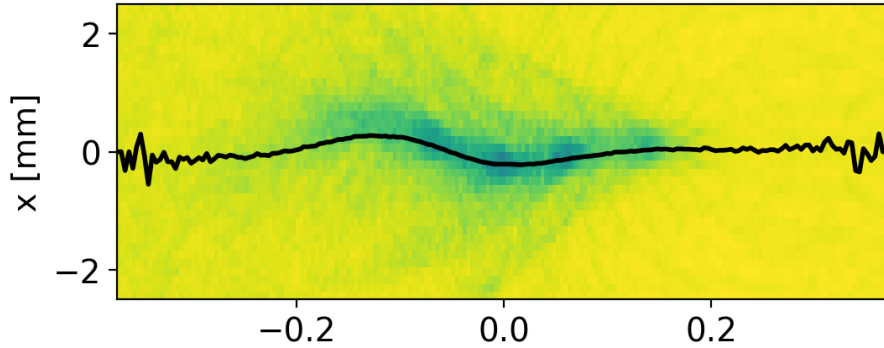


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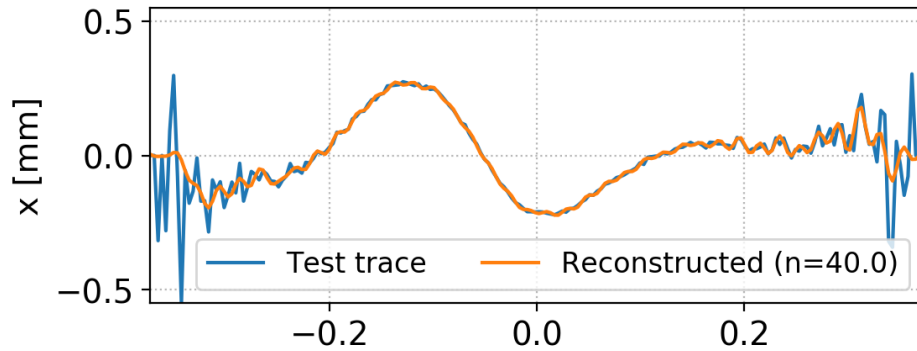


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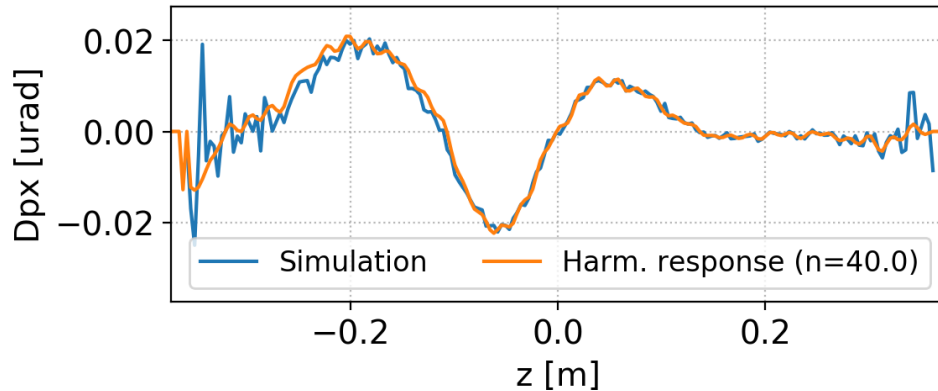
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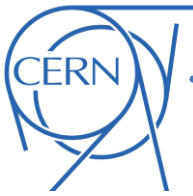
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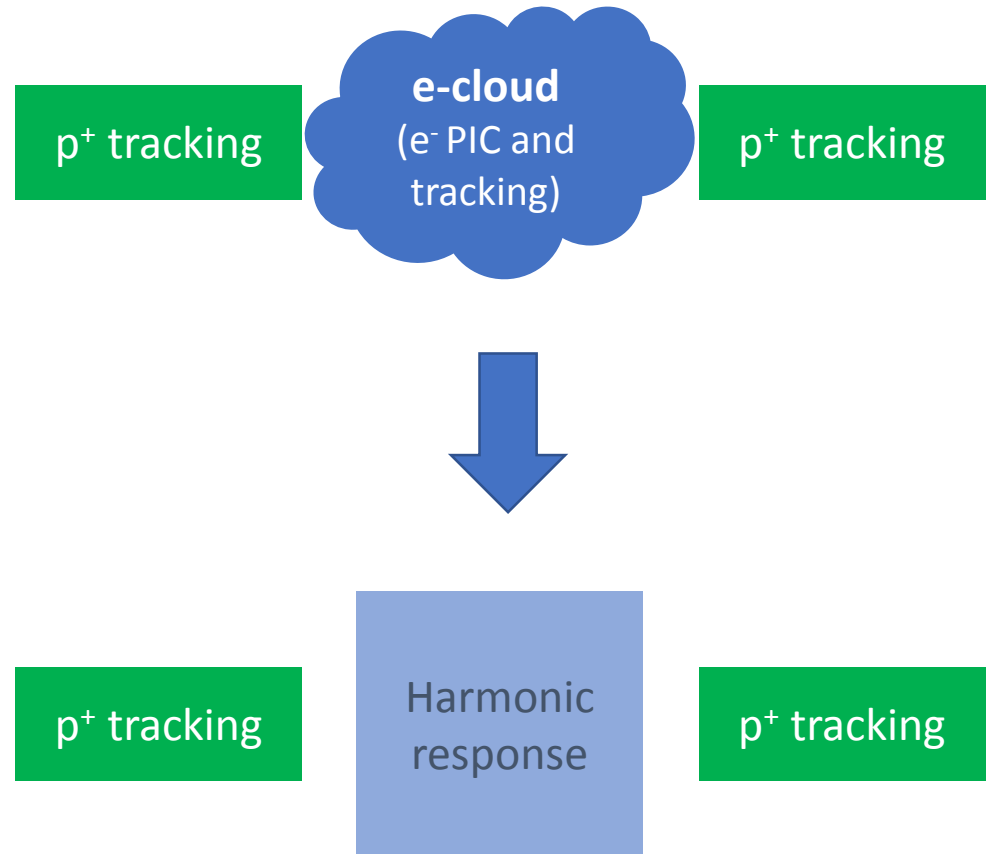


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Can we use the harmonic response to model instabilities?

In the PyHEADTAIL model we **replace the full e-cloud interactions** with the **computed harmonic response matrix**

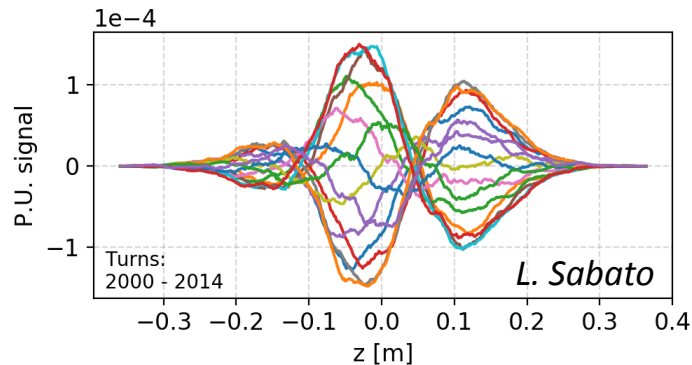
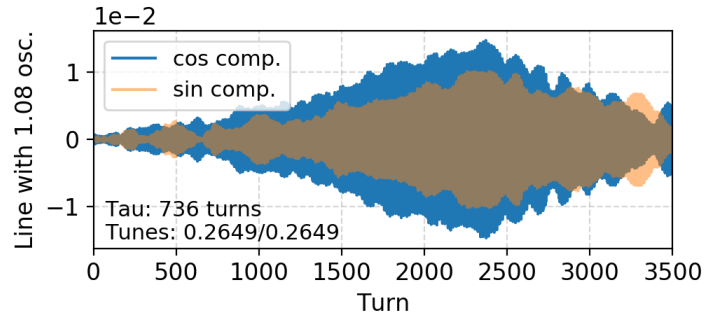
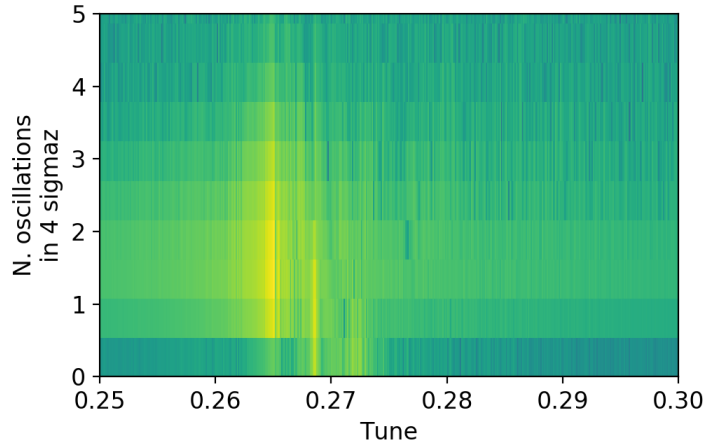




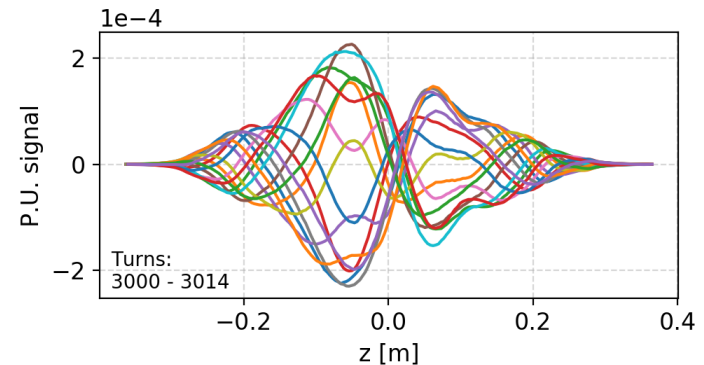
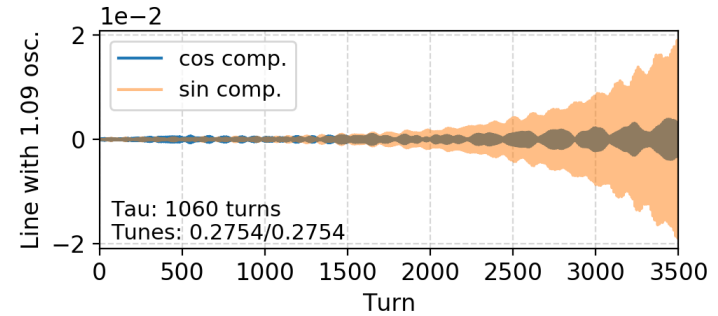
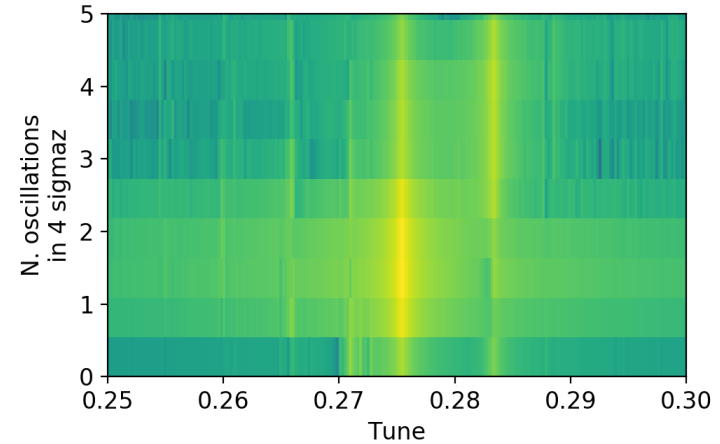
Comparison against simulations

With the matrix alone the **instability is not reproduced** (frequency is different)

e-cloud simulation



Harmonic matrix

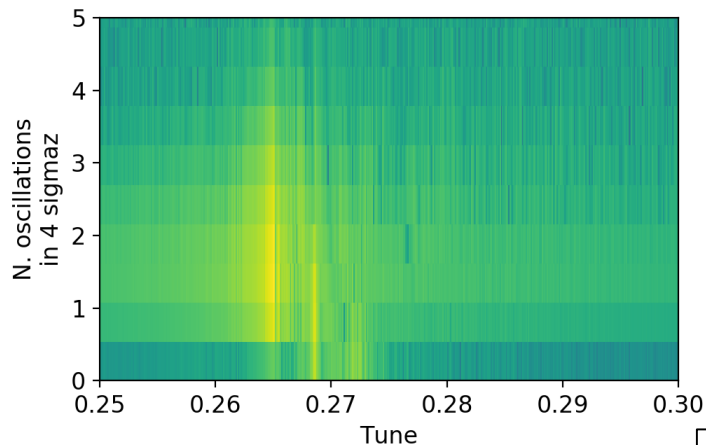




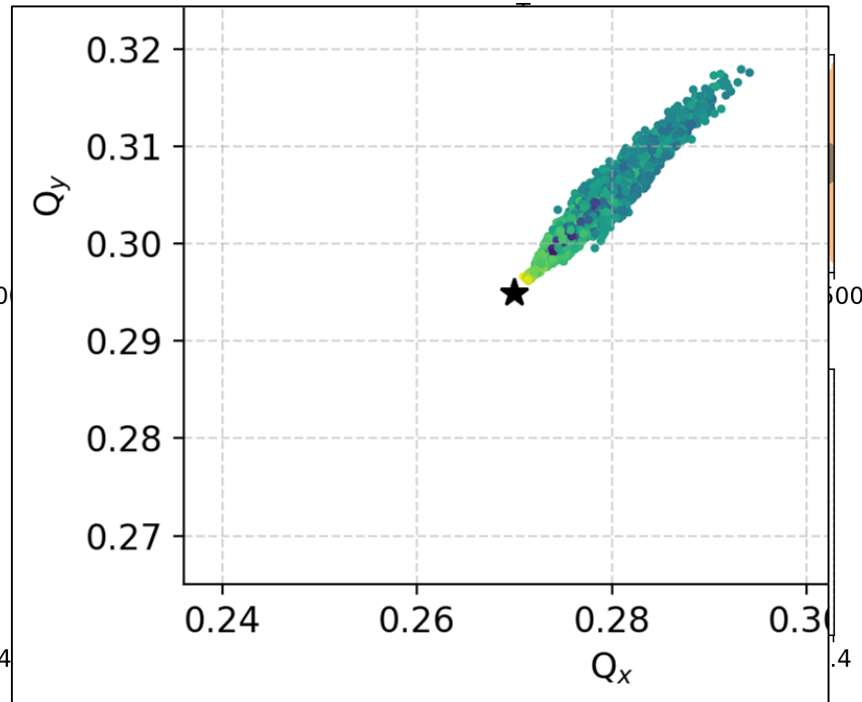
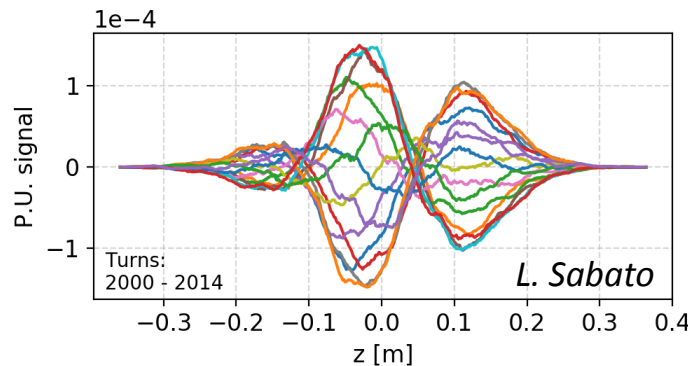
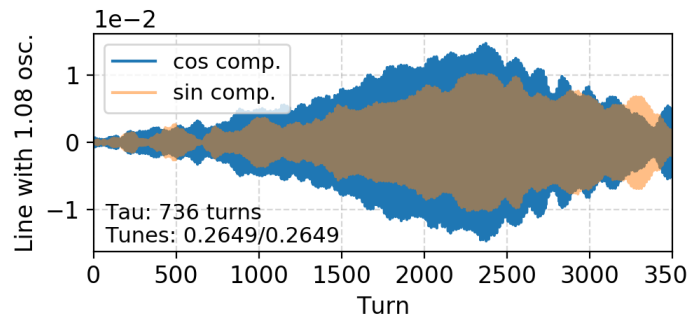
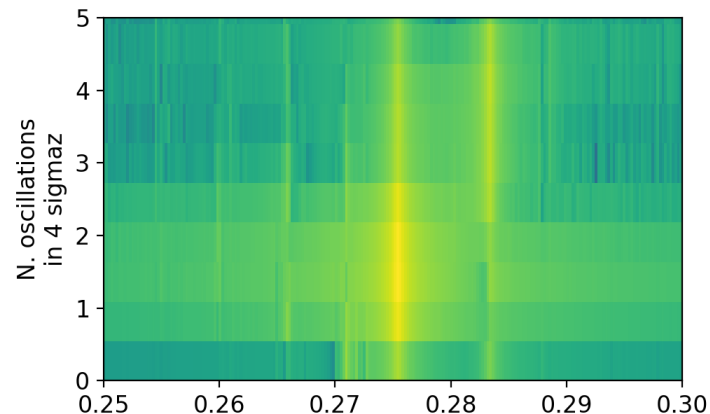
Comparison against simulations

With the matrix alone the **instability is not reproduced** (frequency is different)
→ not surprising (positive tune spread from the e-cloud is not present...)

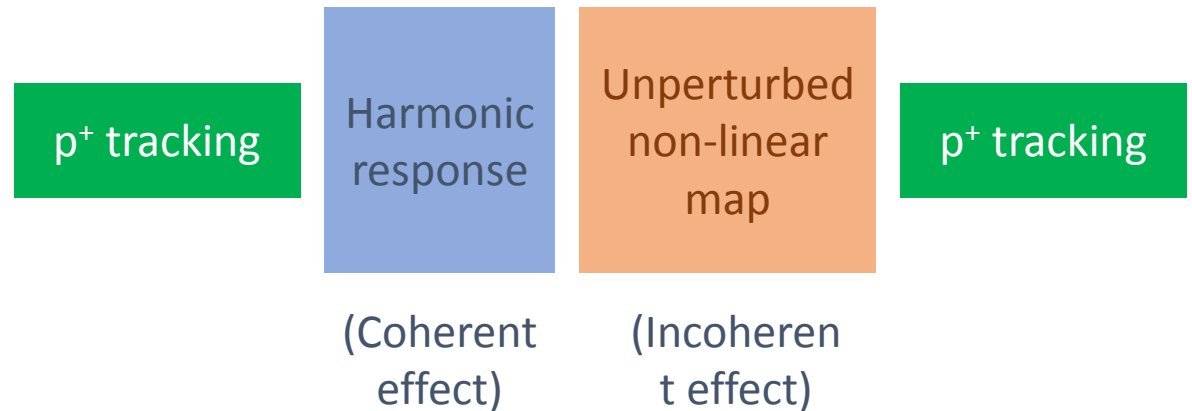
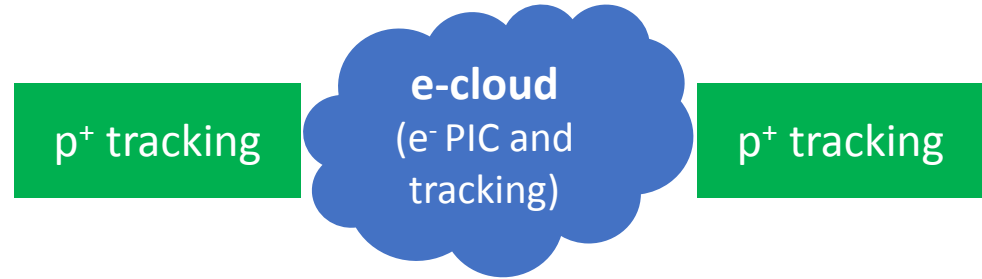
e-cloud simulation



Harmonic matrix



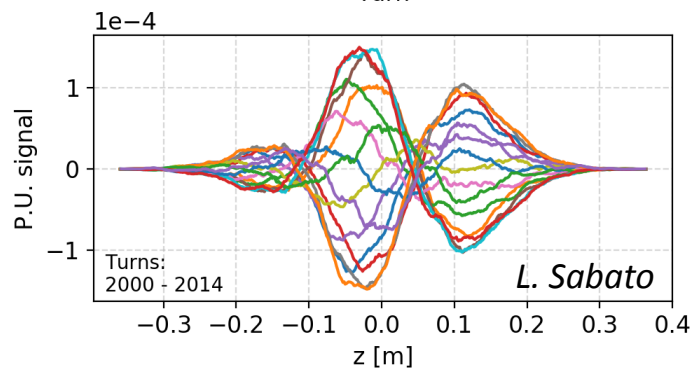
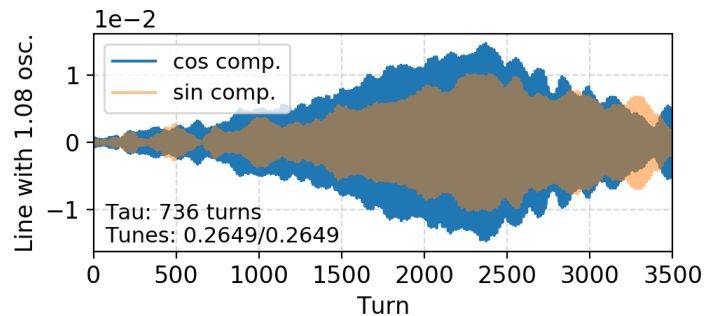
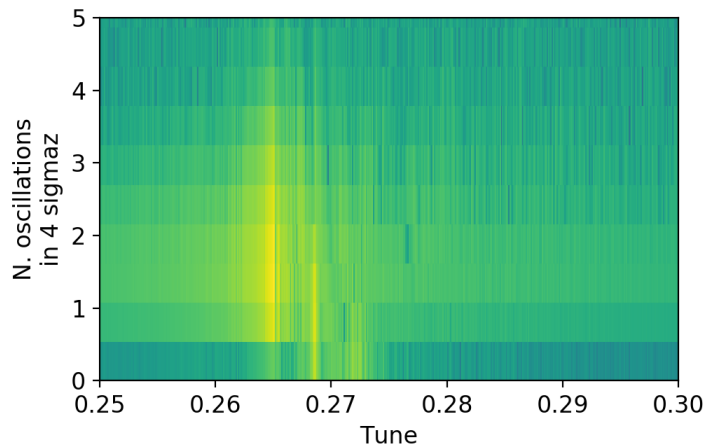
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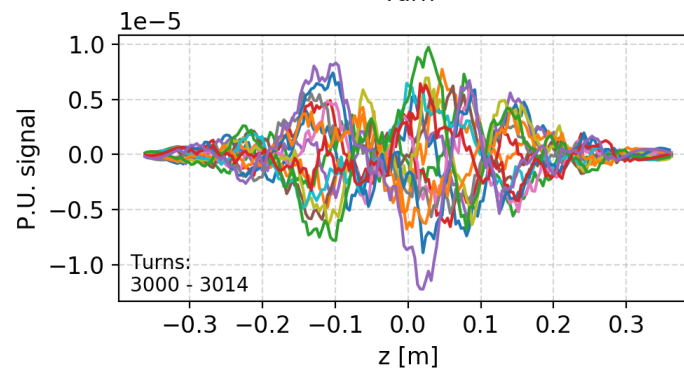
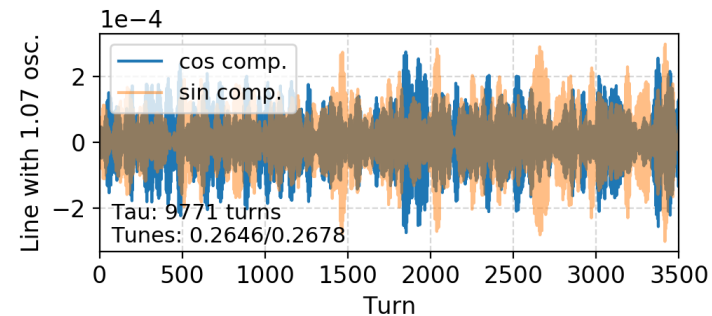
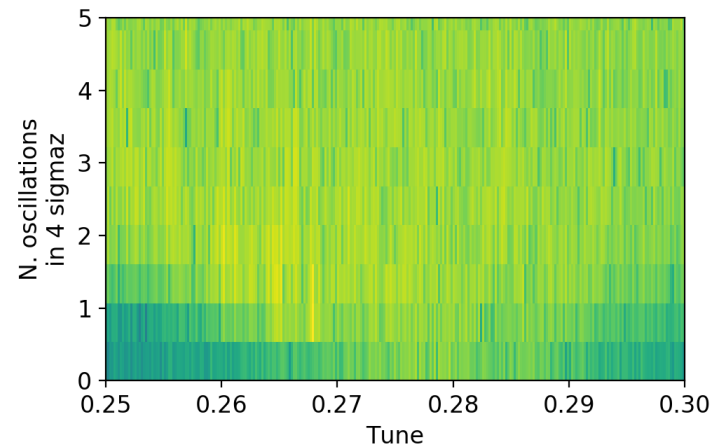
We model the single particle **non-linear detuning** of the e-cloud using the **field map from the unperturbed distribution** (re-centered around each slice → no additional dipolar kick)

With the non-linear map alone no instability is observed

e-cloud simulation



Unperturbed non-linear map

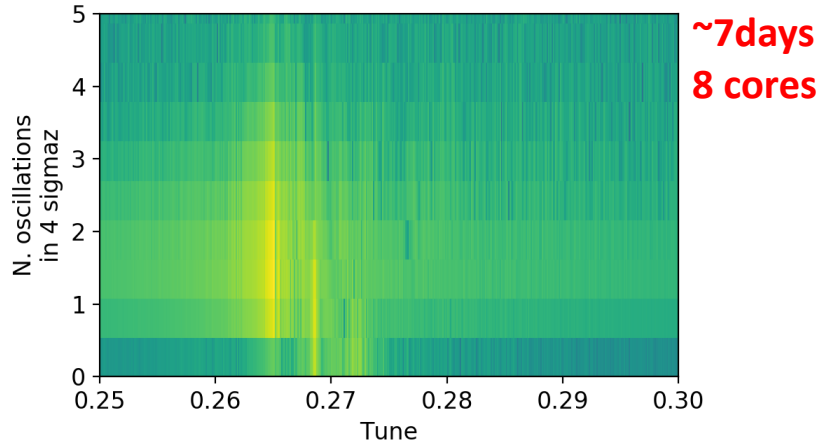




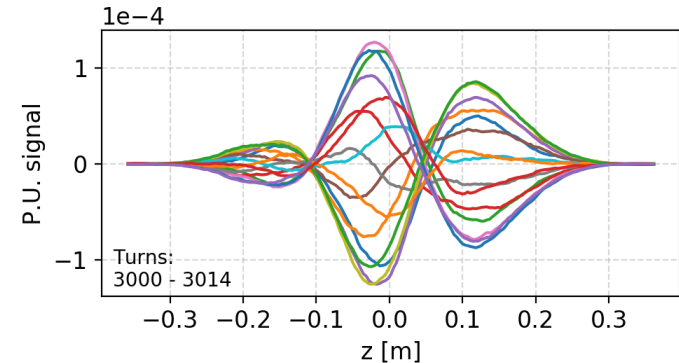
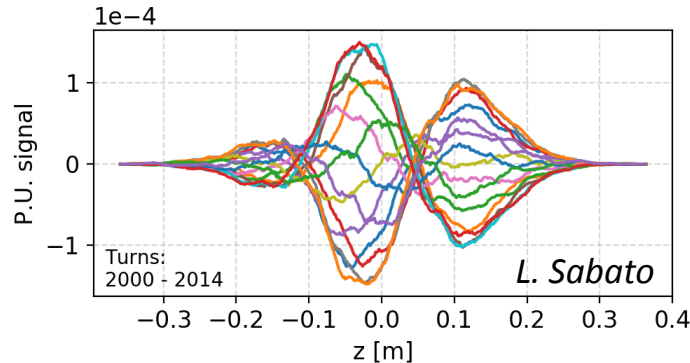
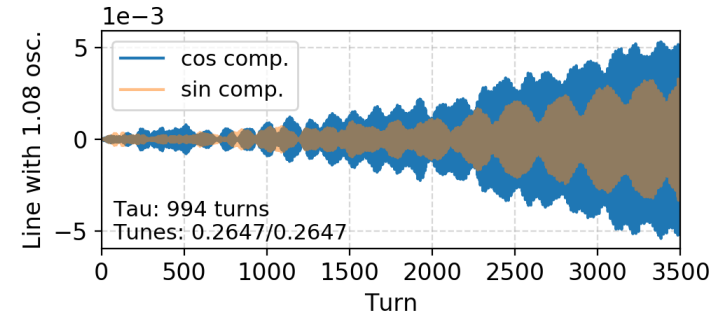
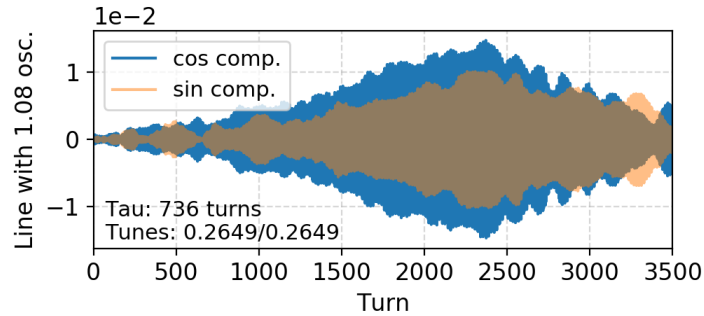
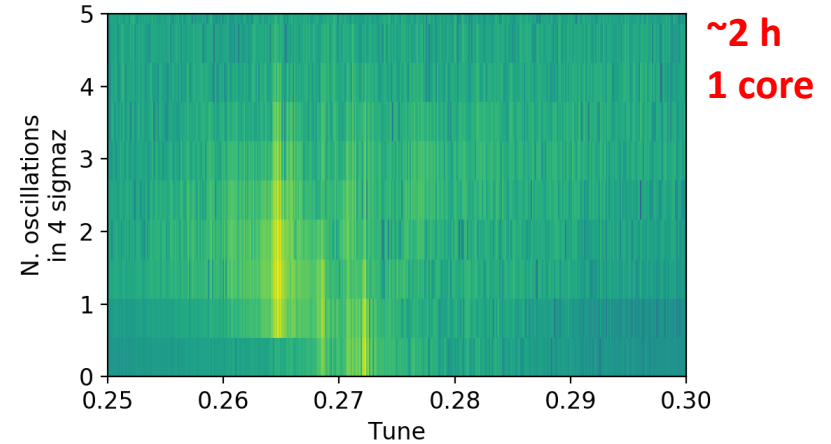
Comparison against simulations

This combined model reproduces very well the nature of the instability!

e-cloud simulation



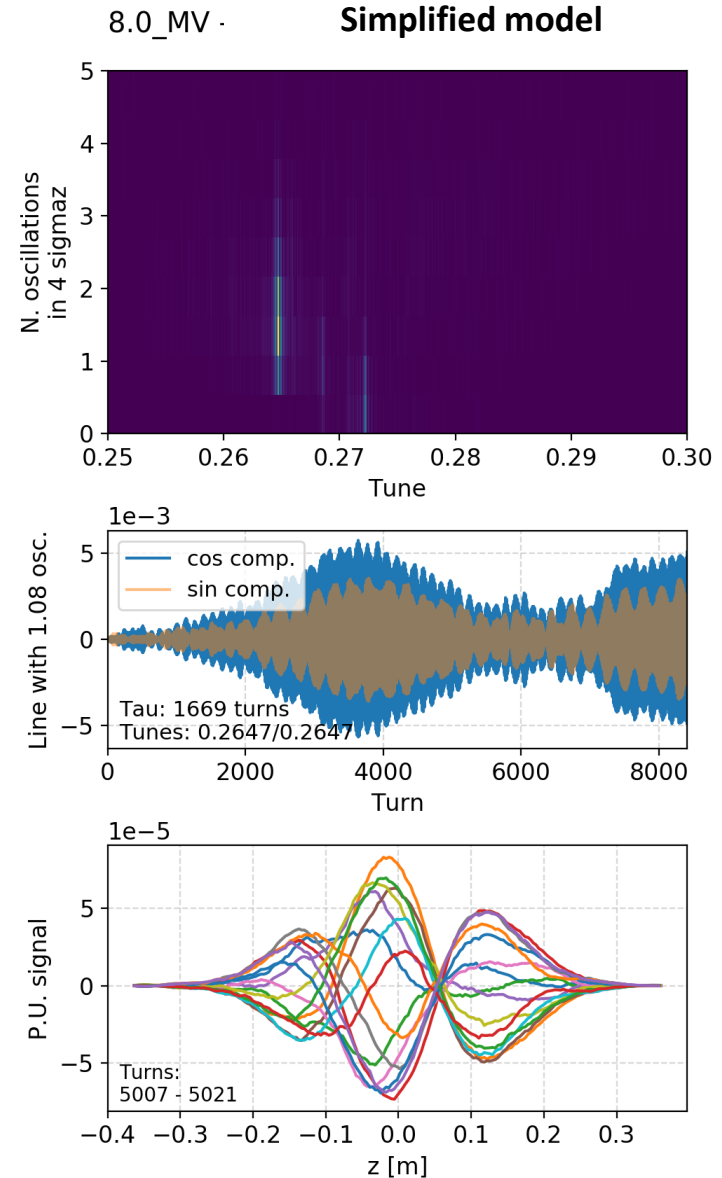
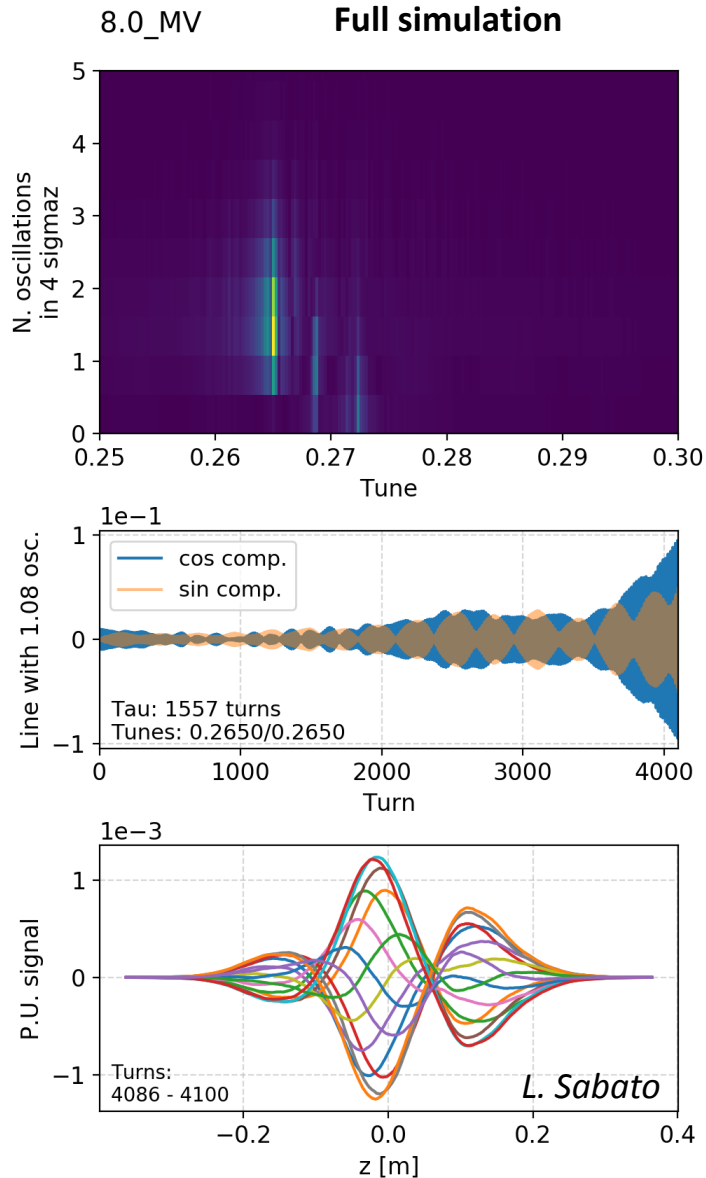
Harmonic matrix + unperturbed non-linear map



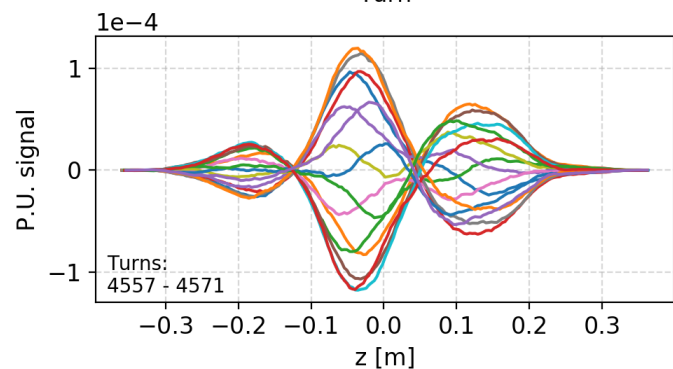
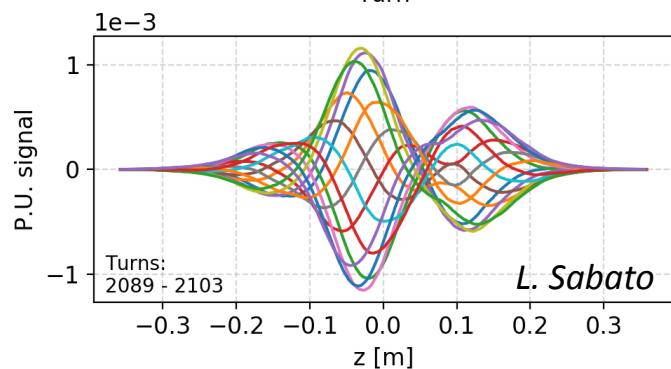
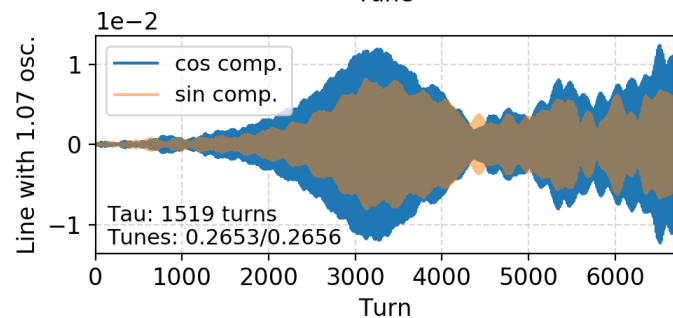
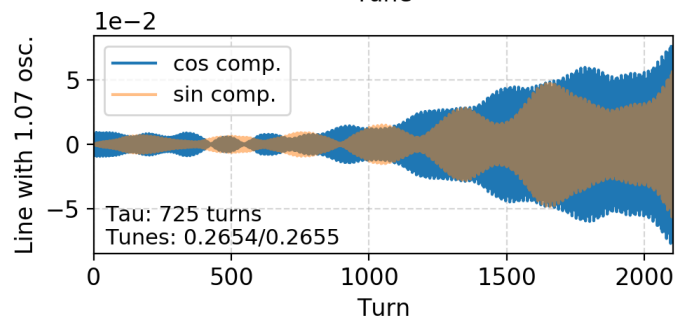
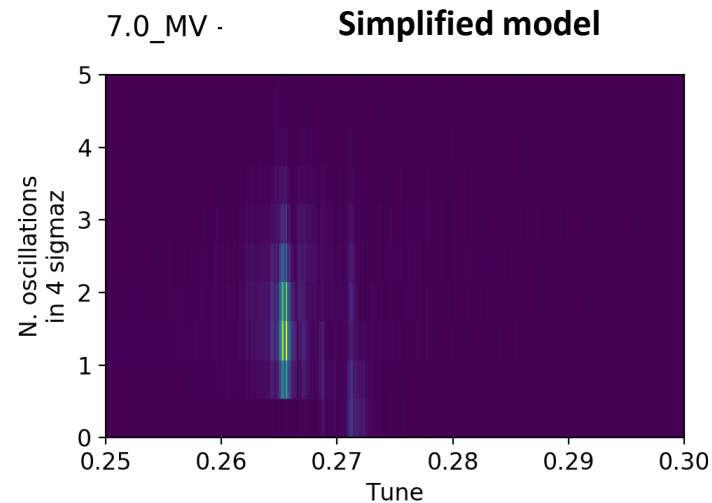
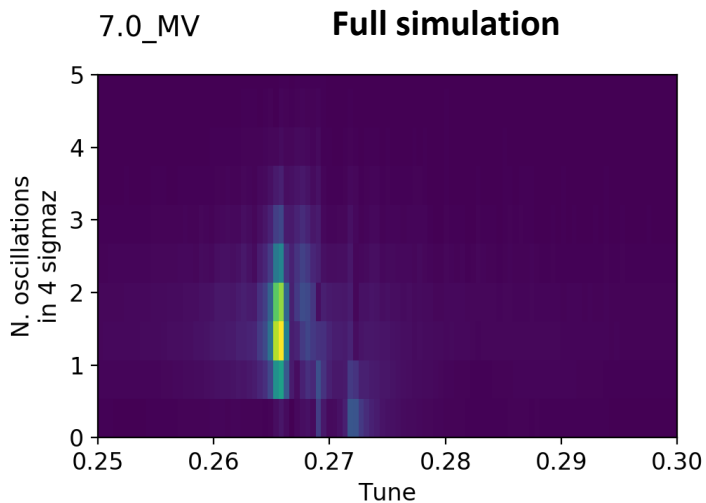


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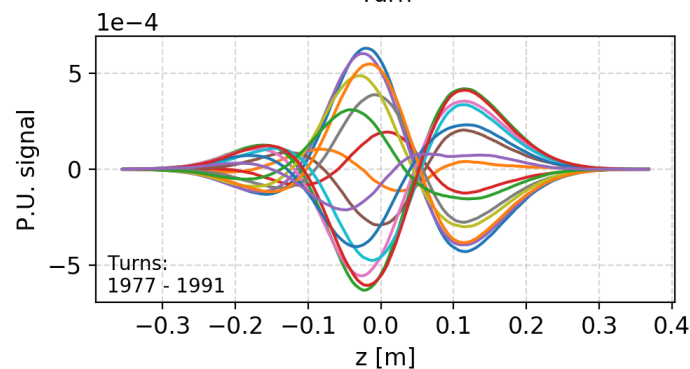
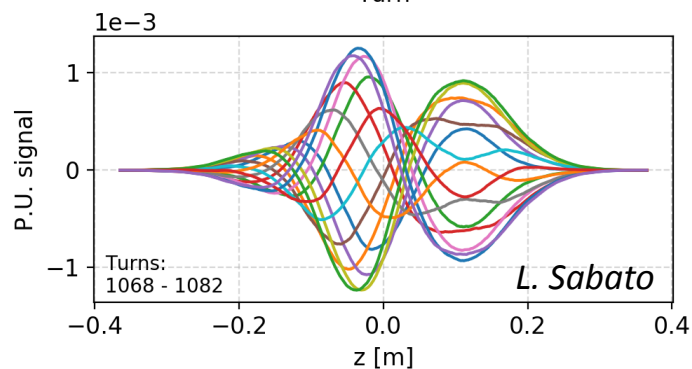
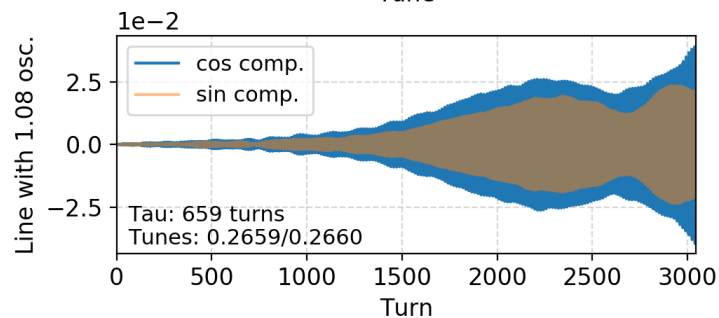
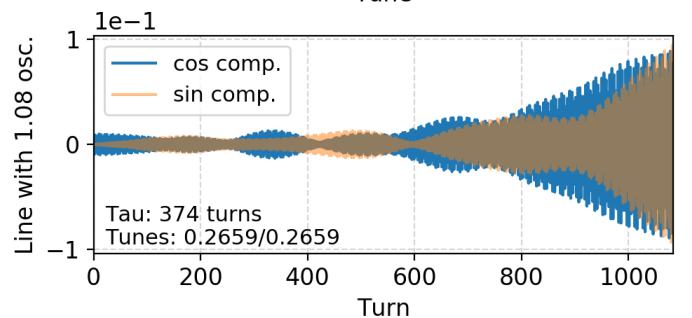
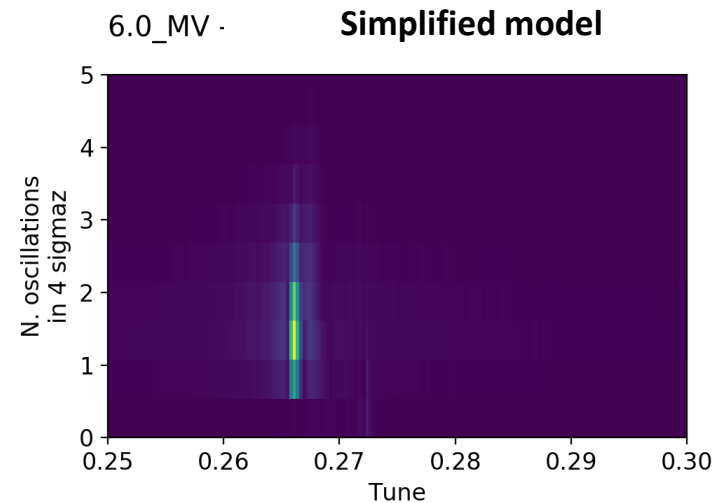
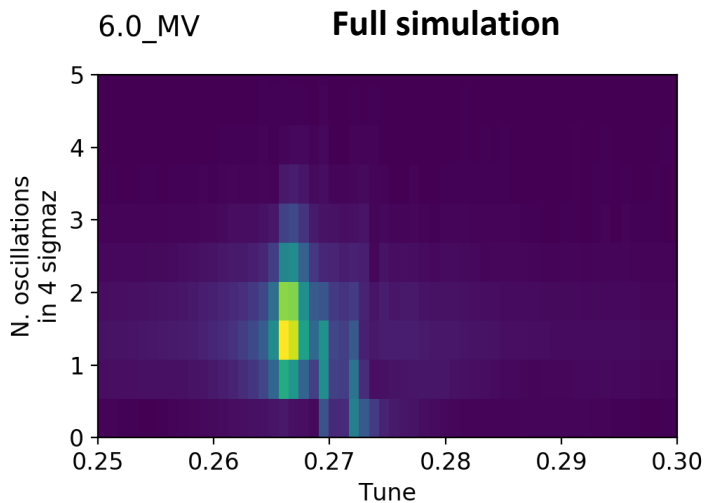
The agreement is quite good for all RF frequencies



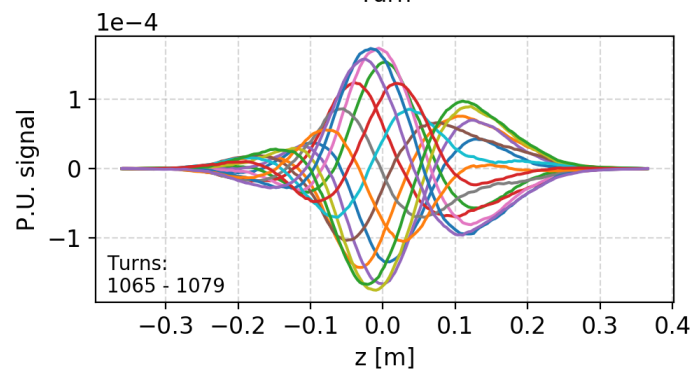
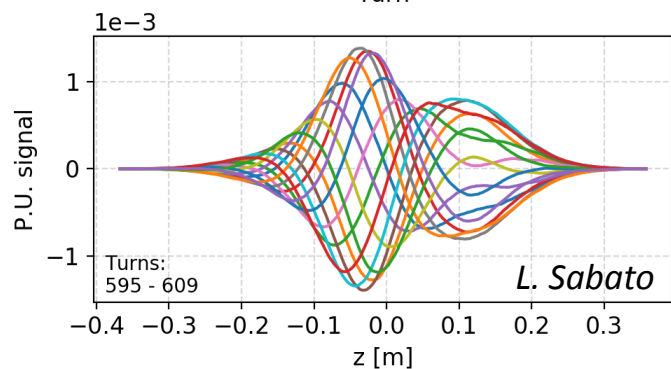
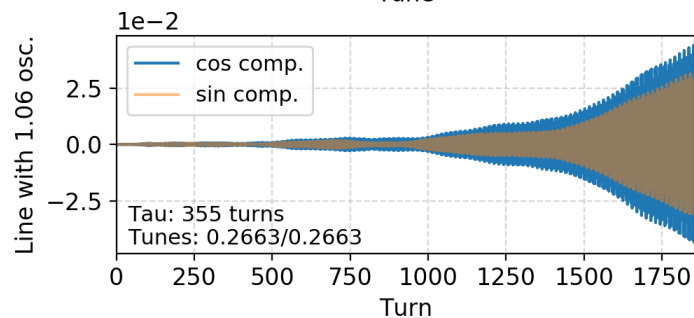
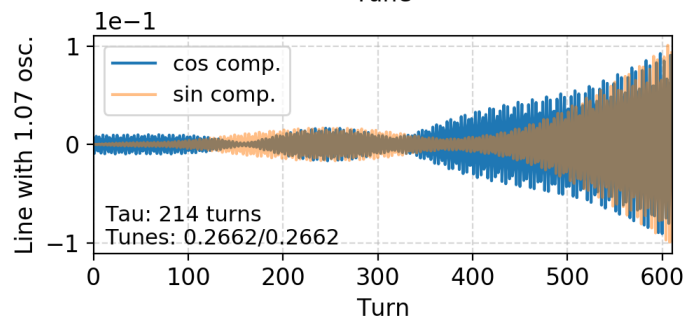
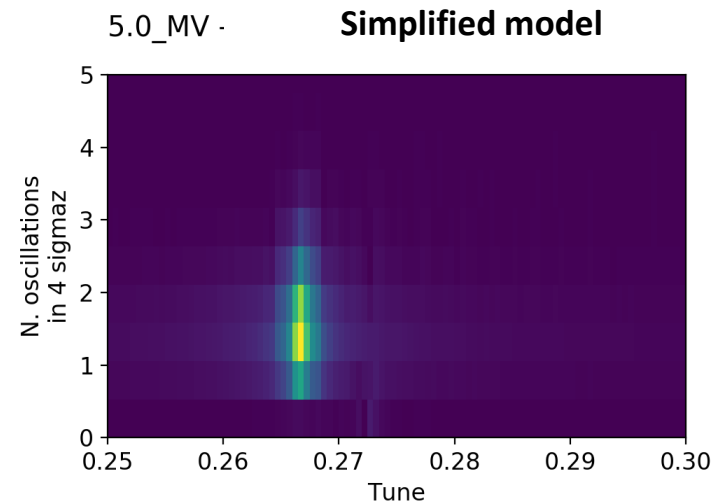
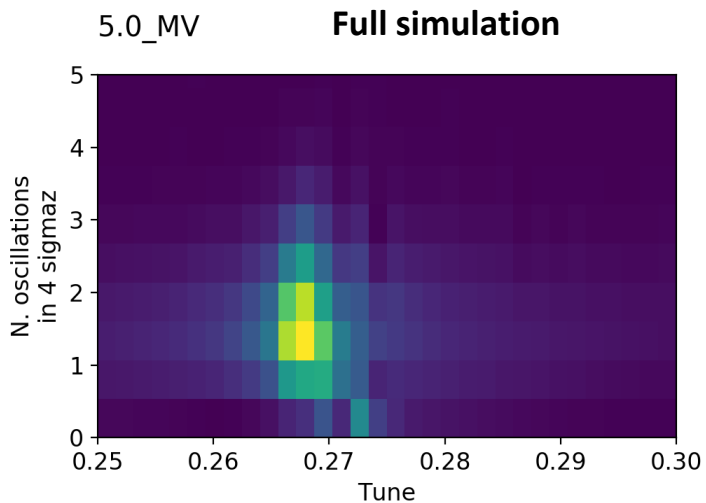
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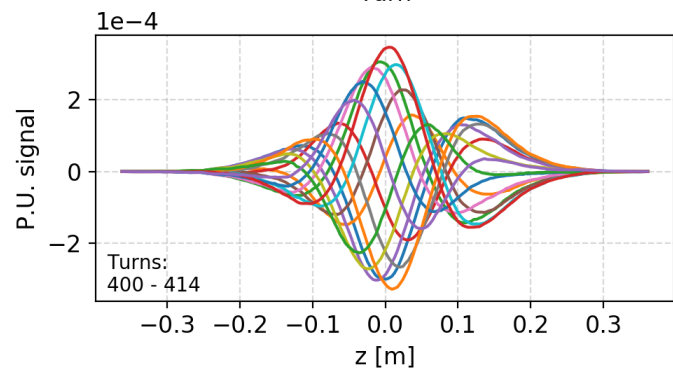
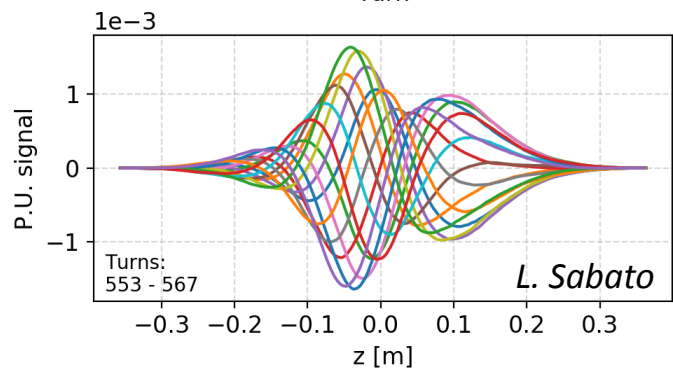
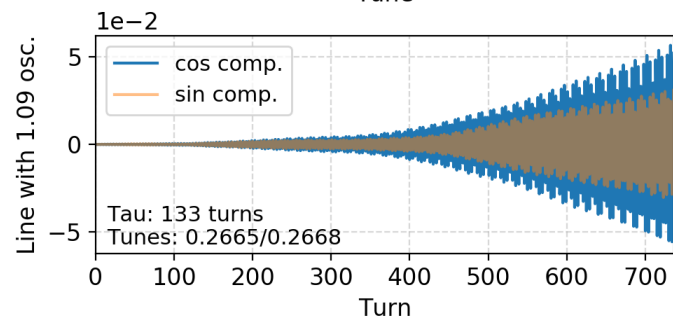
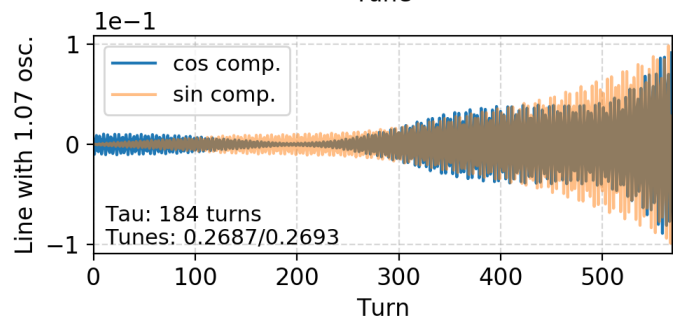
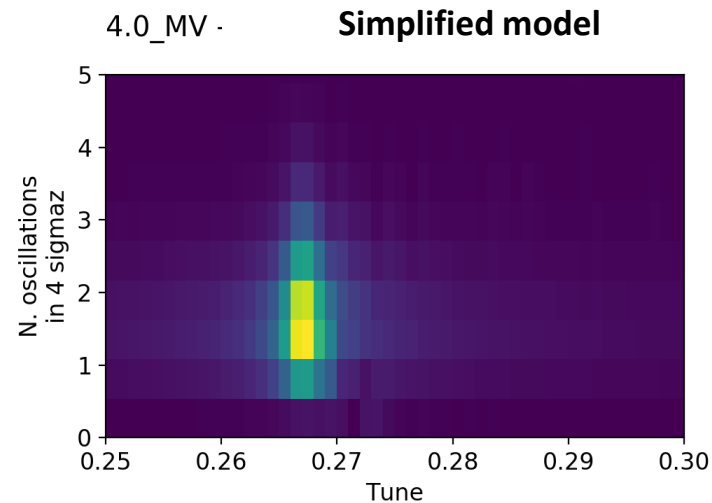
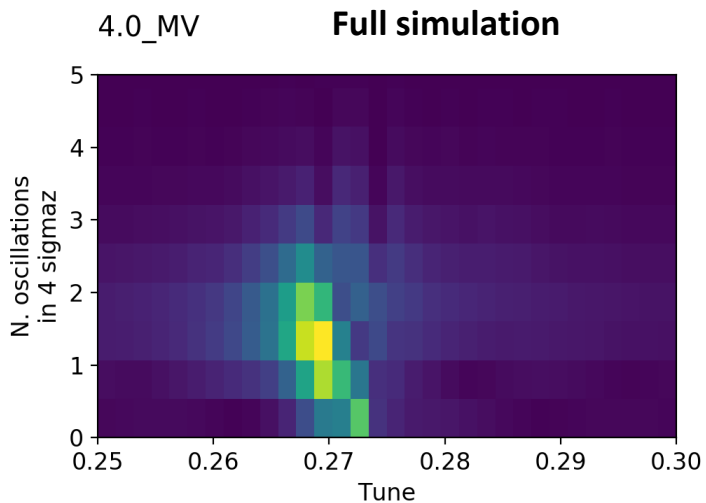
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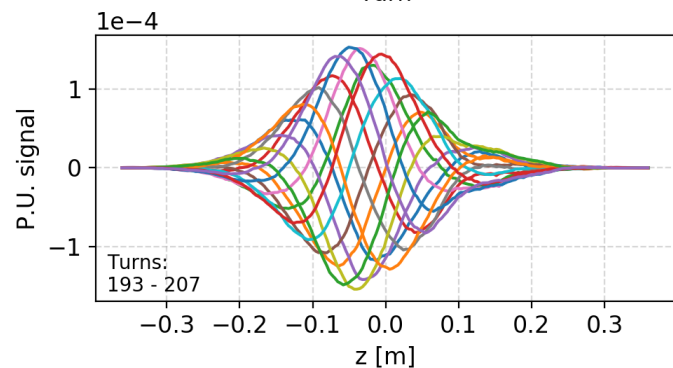
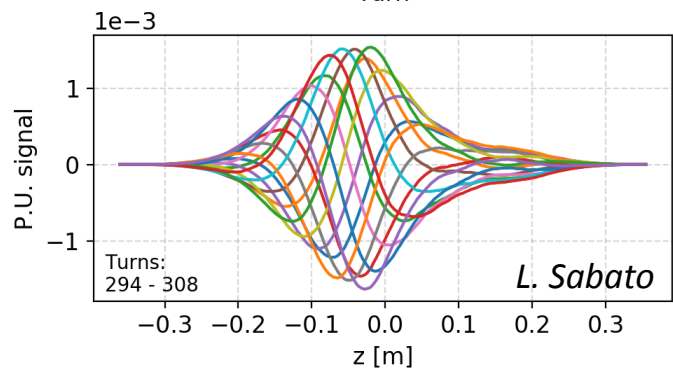
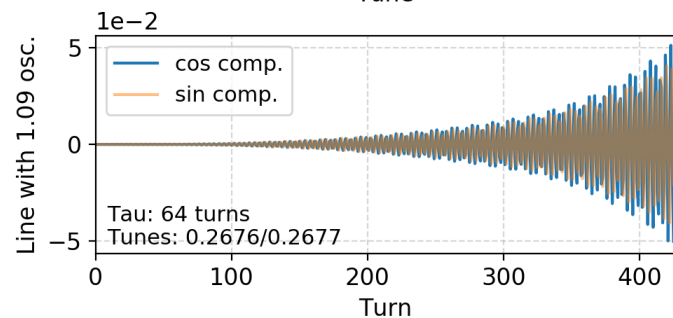
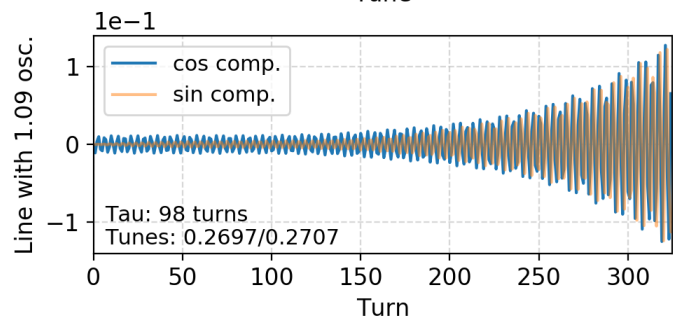
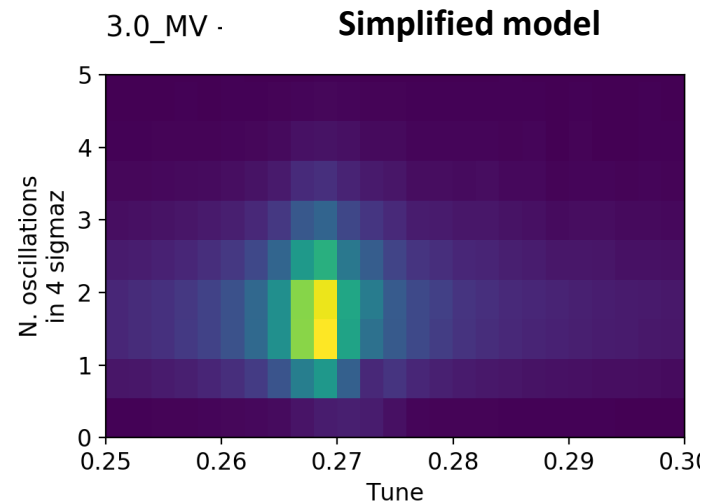
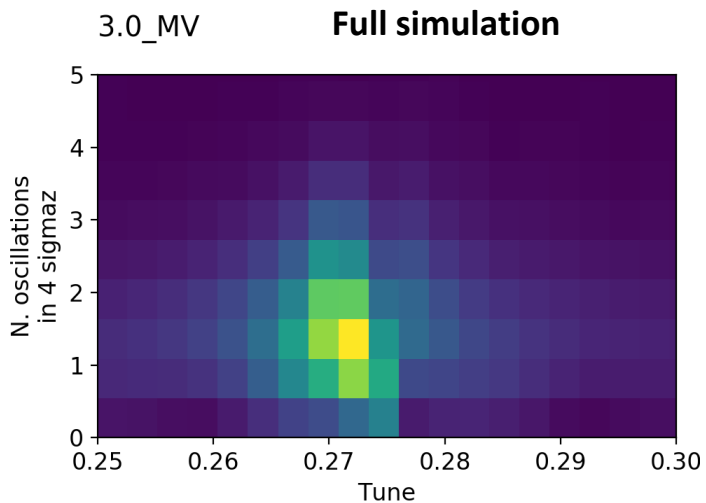
The agreement is quite good for all RF frequencies



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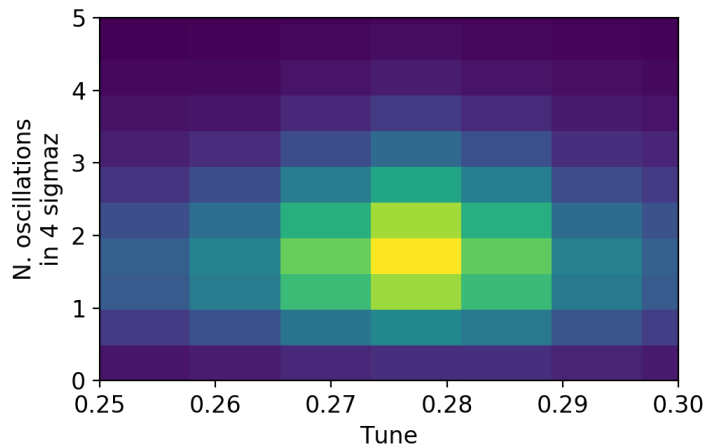
The agreement is quite good for all RF frequencies



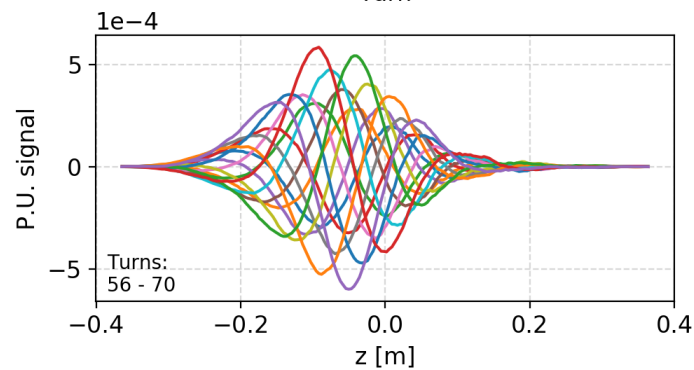
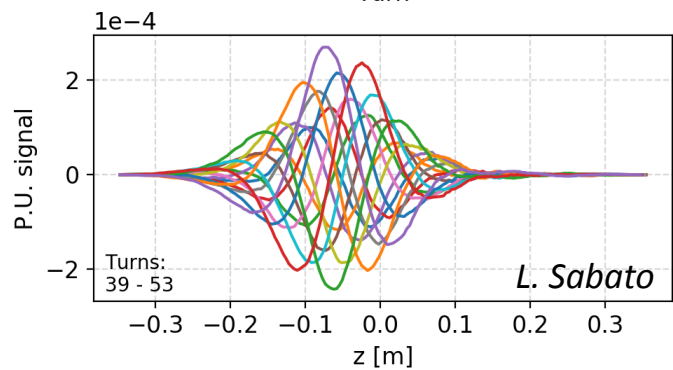
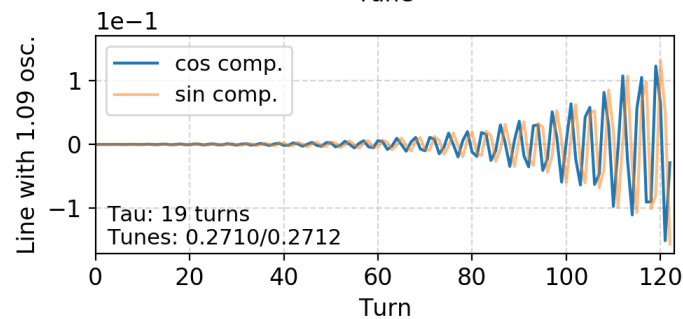
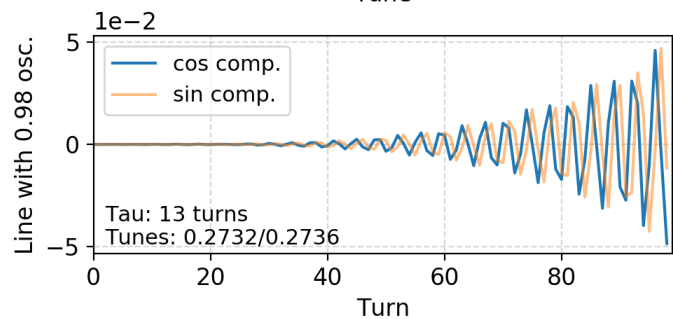
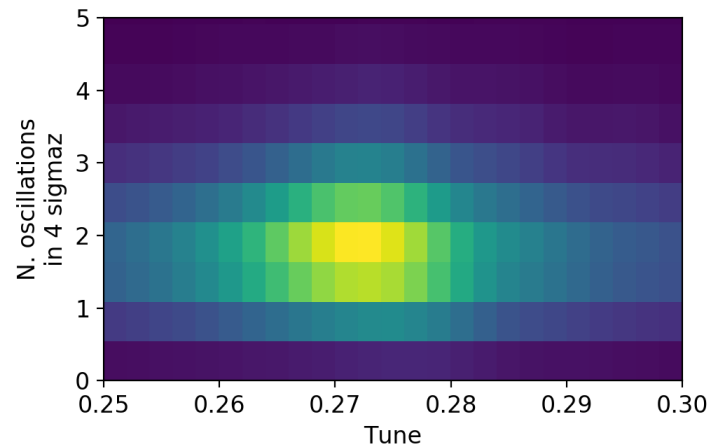


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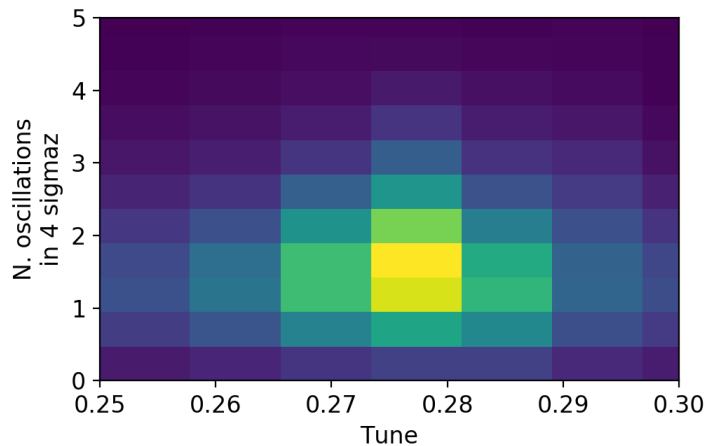
strength 2.0 **Full simulation**



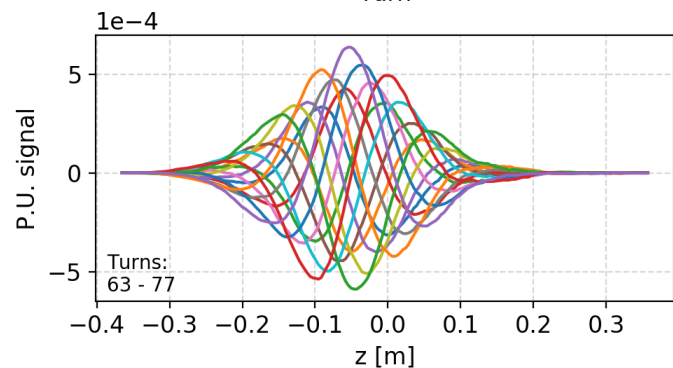
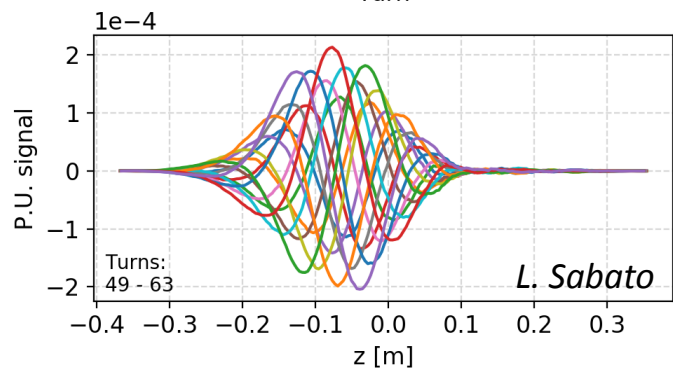
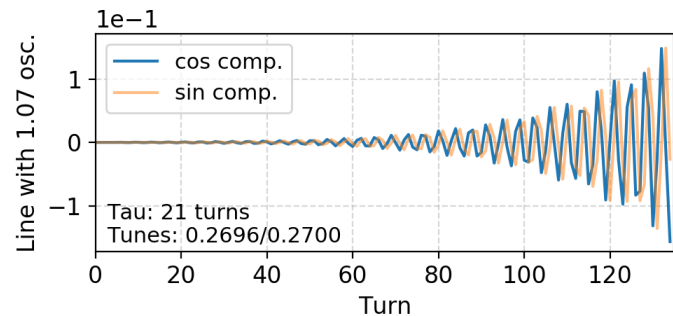
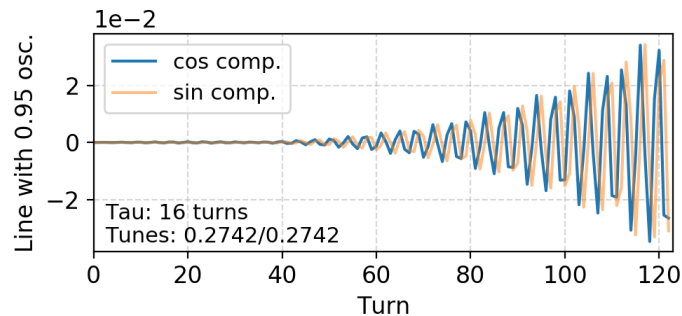
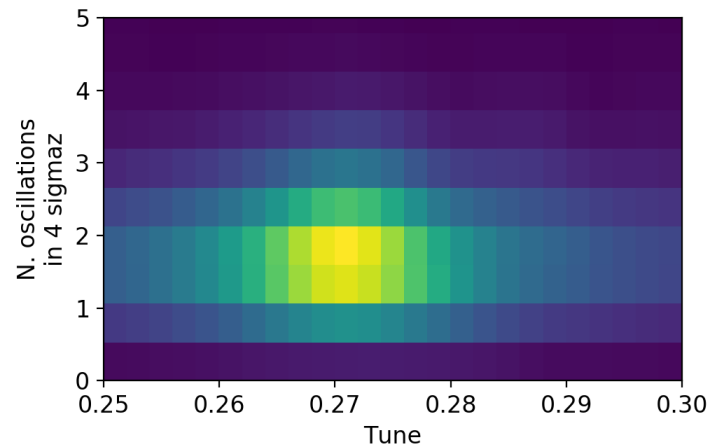
strength 2.0 - **h-matrix + unperturbed map**



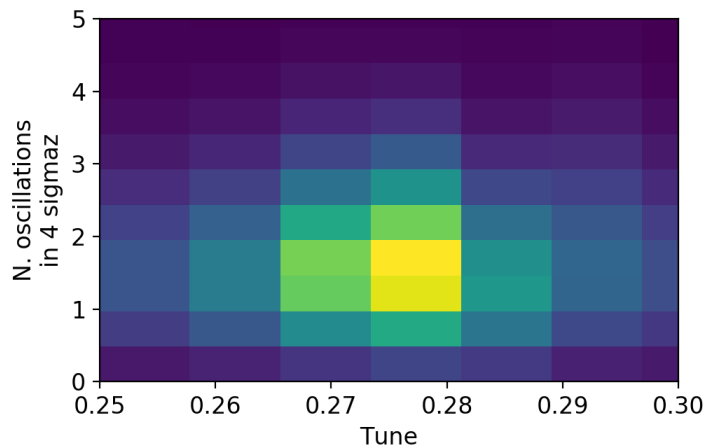
strength 1.9 **Full simulation**



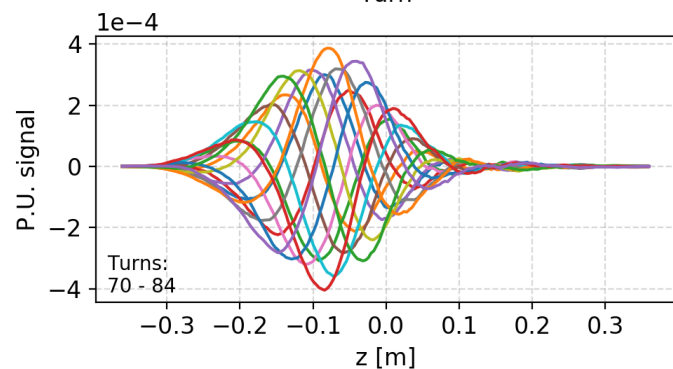
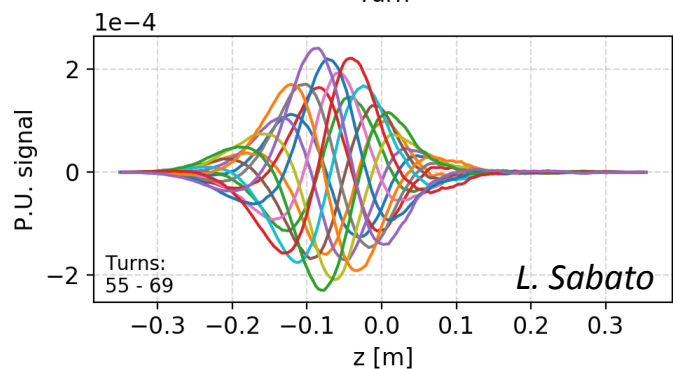
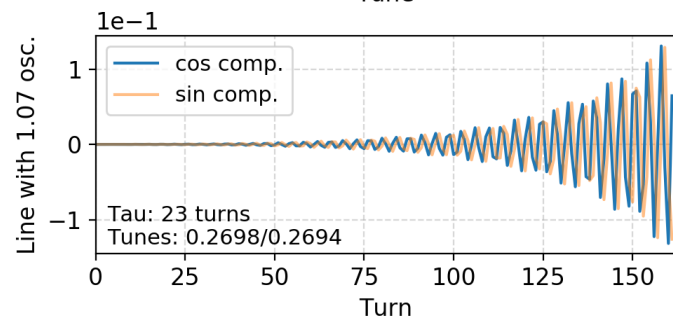
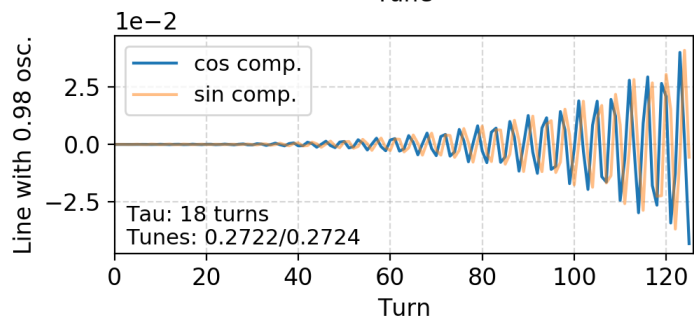
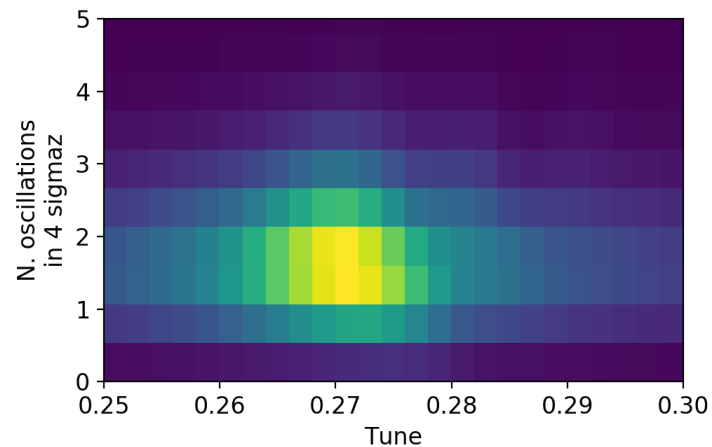
strength 1.9 - **h-matrix + unperturbed map**



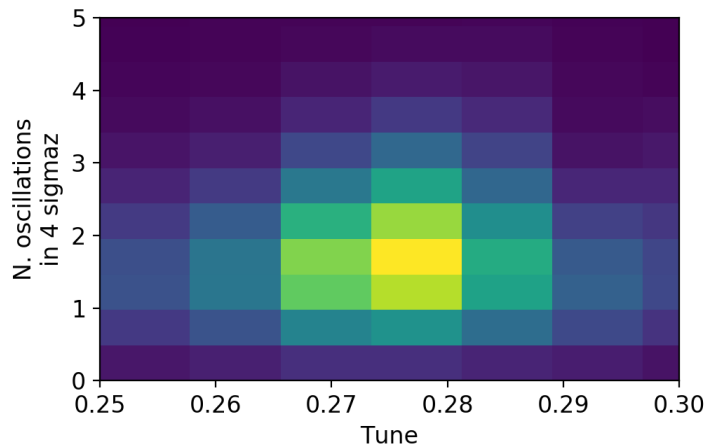
strength 1.8 **Full simulation**



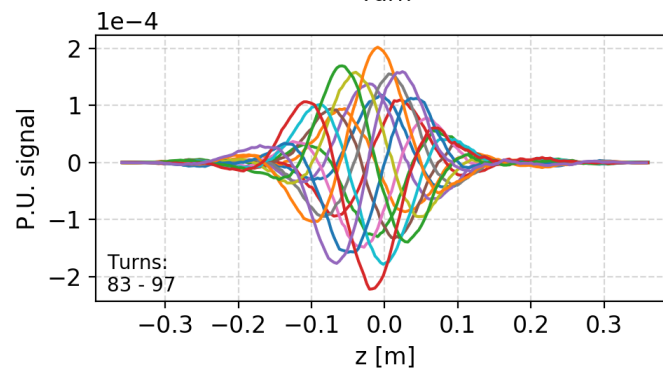
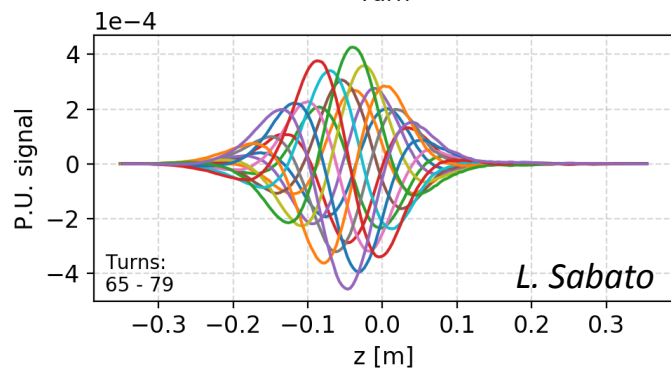
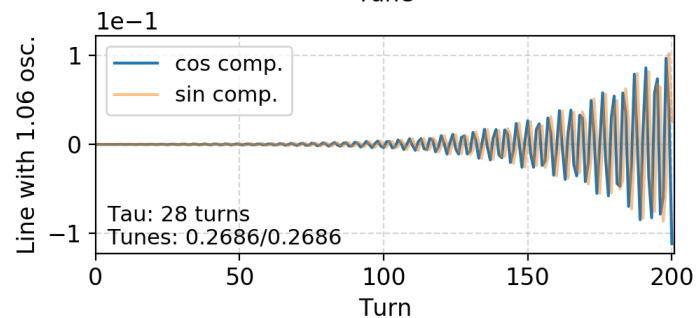
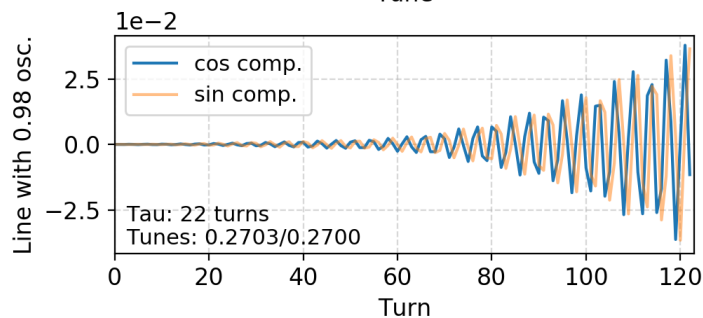
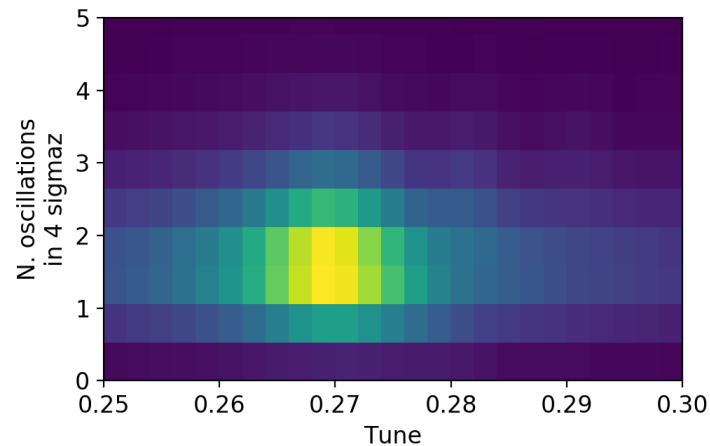
strength 1.8 - **h-matrix + unperturbed map**



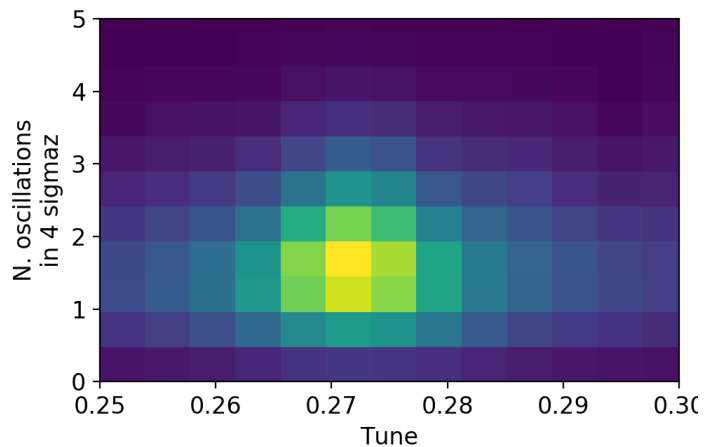
strength 1.7 **Full simulation**



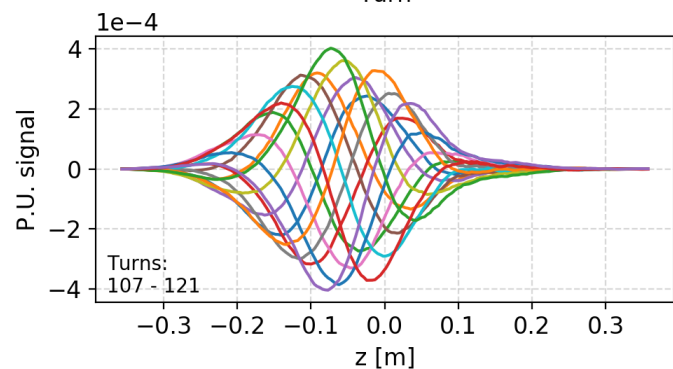
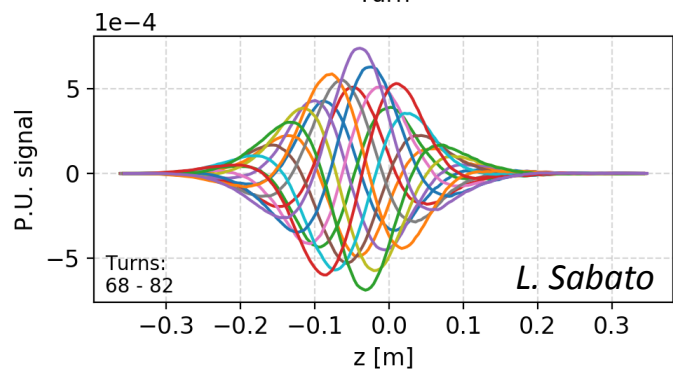
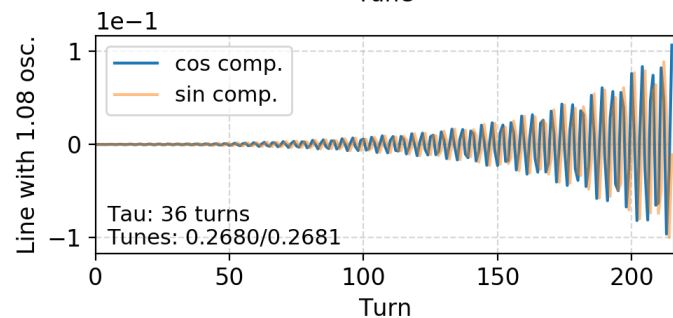
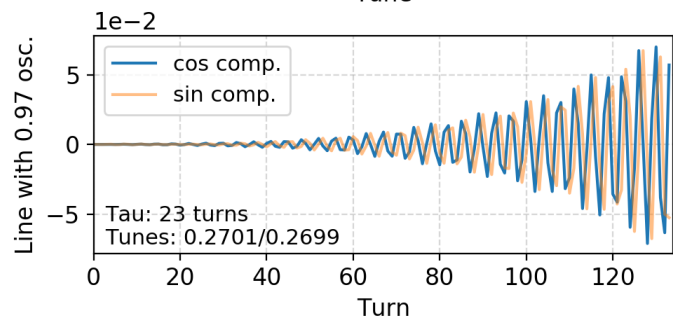
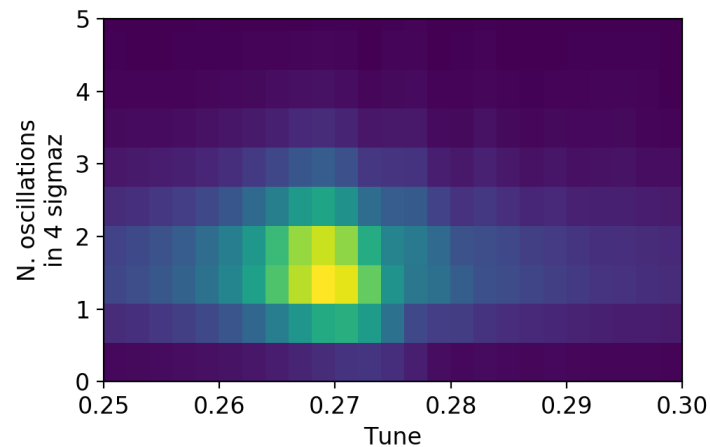
strength 1.7 - **h-matrix + unperturbed map**



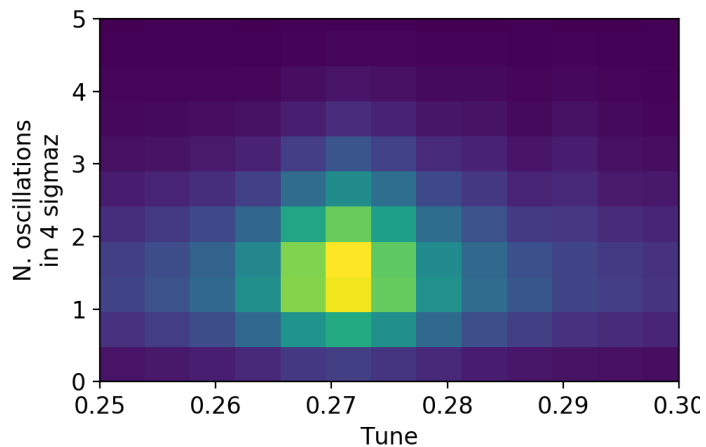
strength 1.6 **Full simulation**



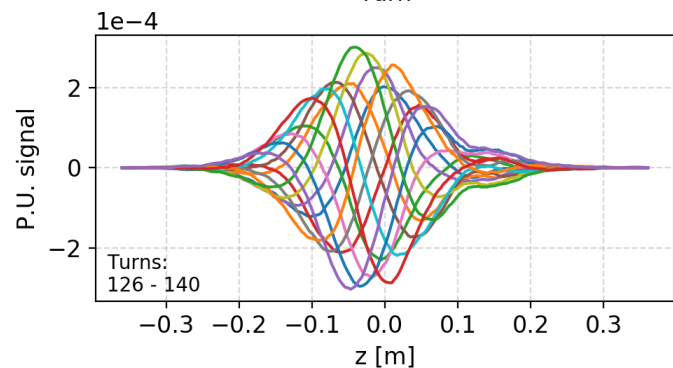
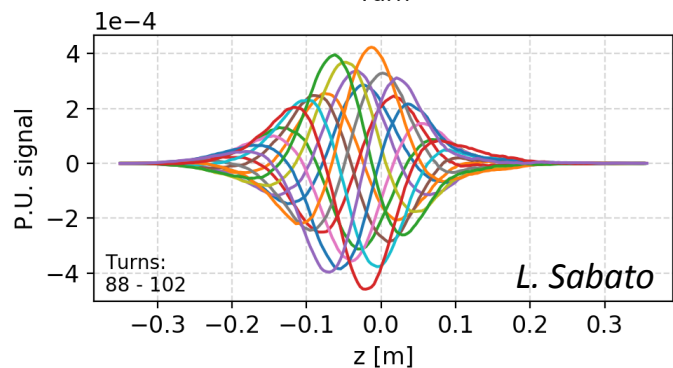
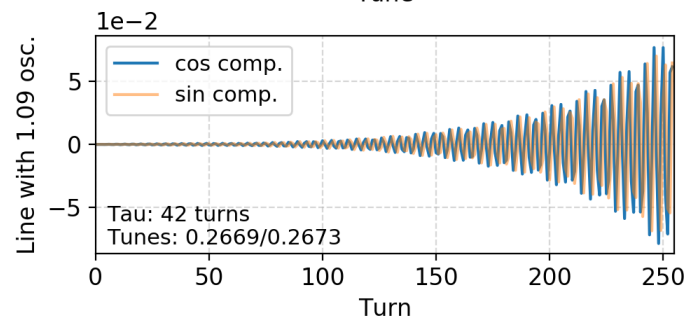
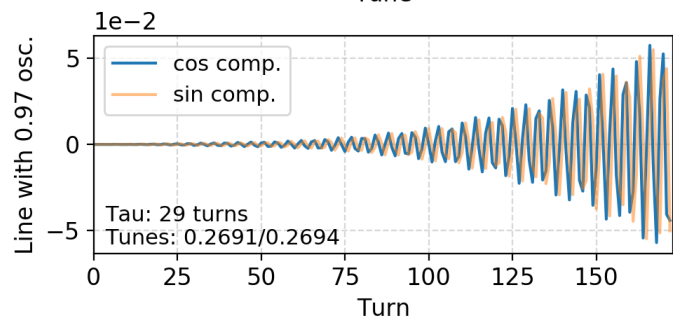
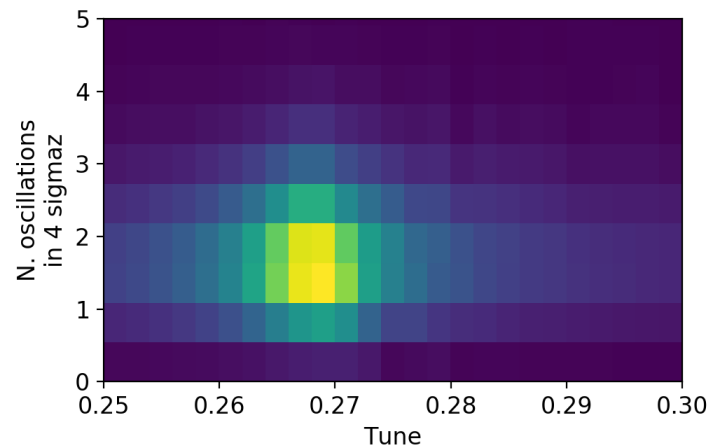
strength 1.6 - **h-matrix + unperturbed map**



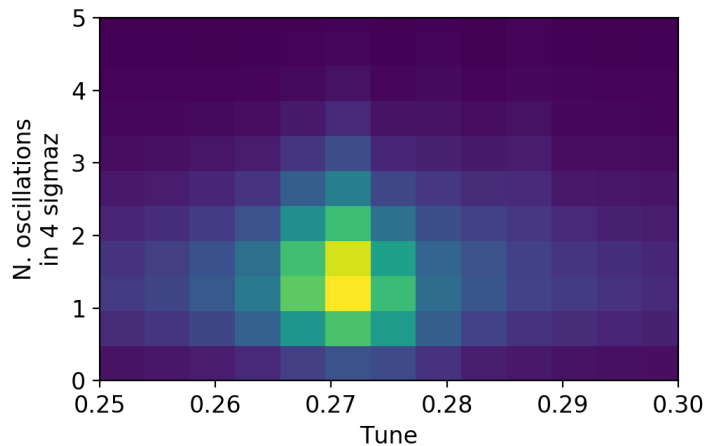
strength 1.5 **Full simulation**



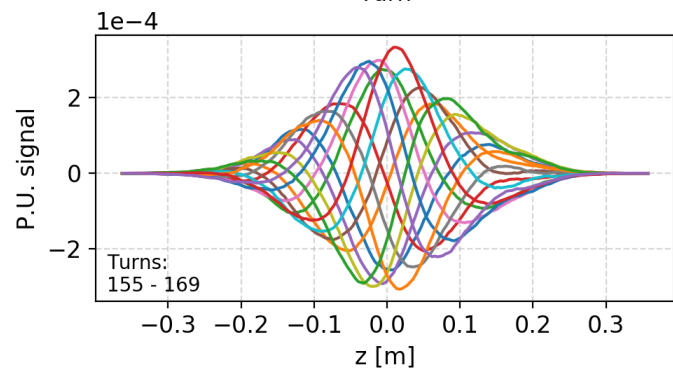
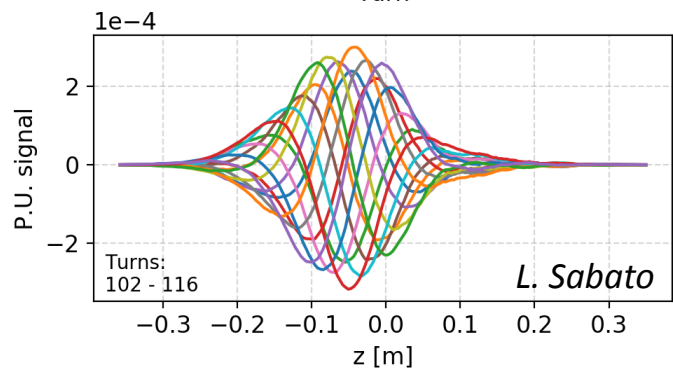
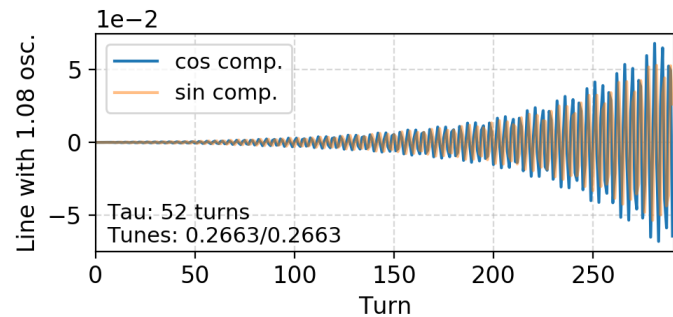
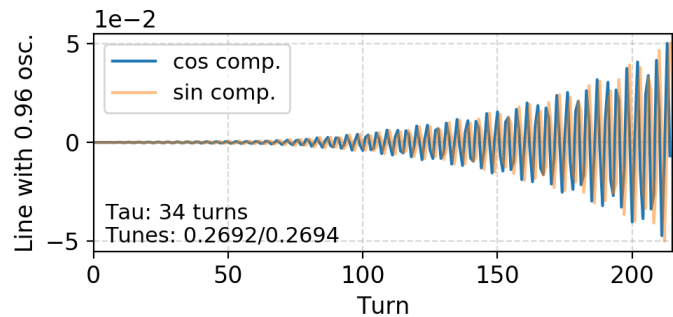
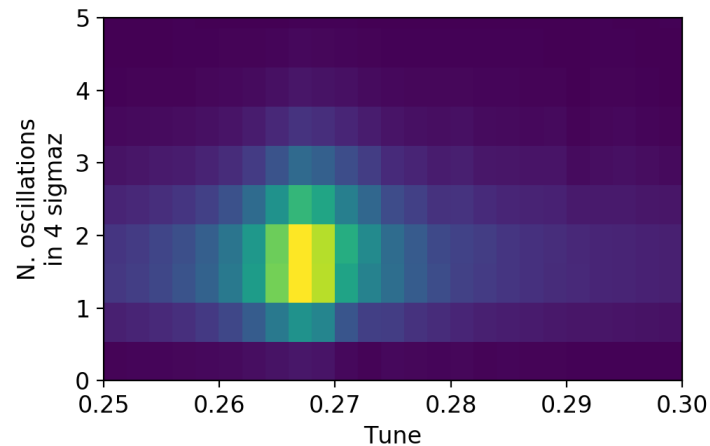
strength 1.5 - **h-matrix + unperturbed map**



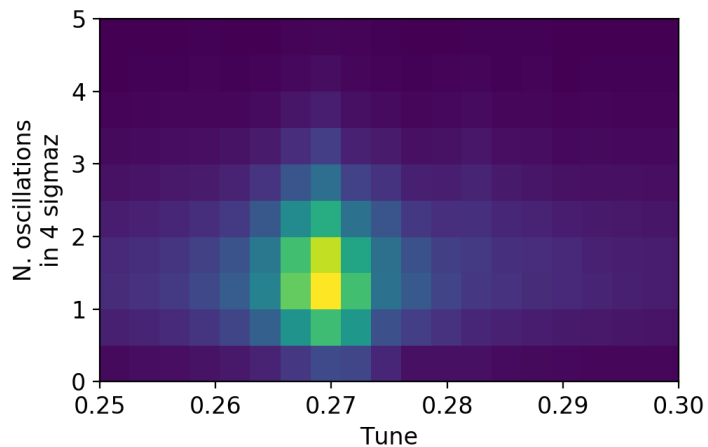
strength 1.4 **Full simulation**



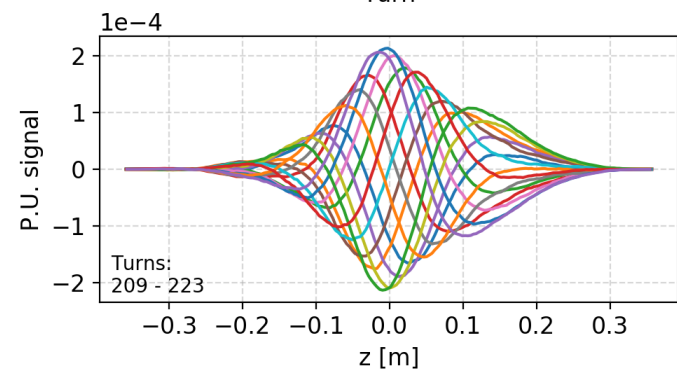
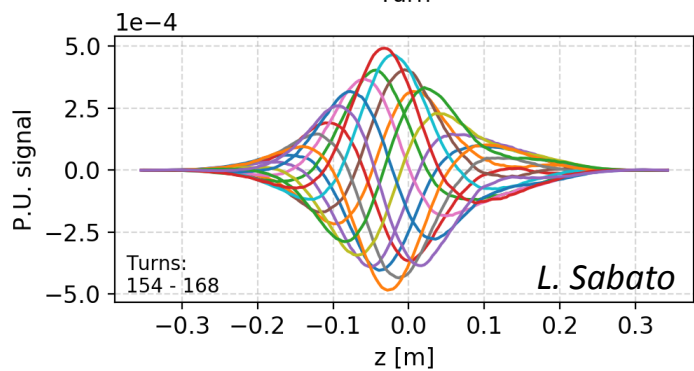
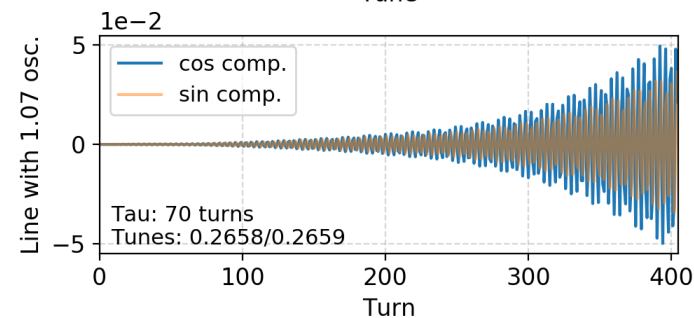
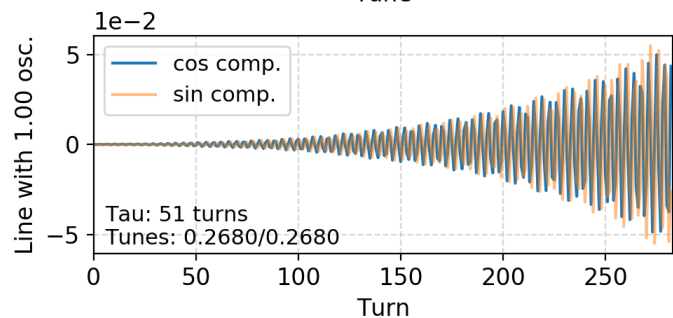
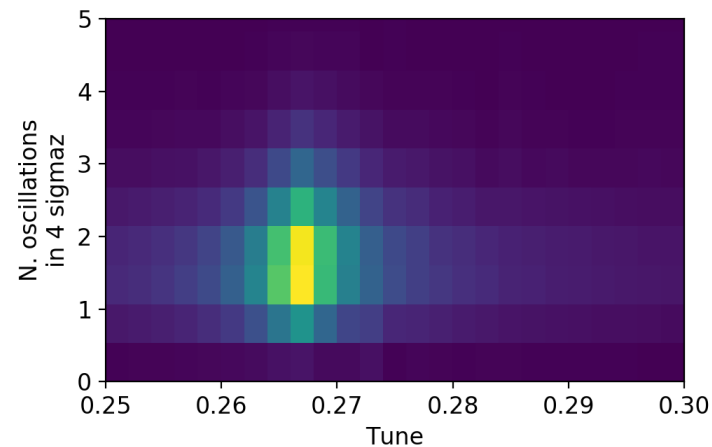
strength 1.4 - **h-matrix + unperturbed map**



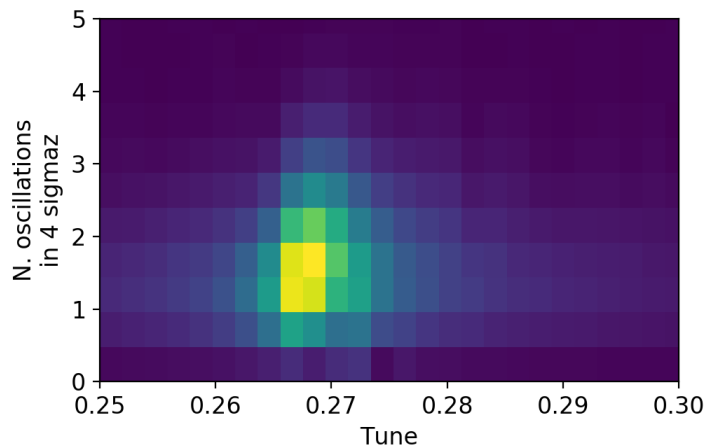
strength 1.3 **Full simulation**



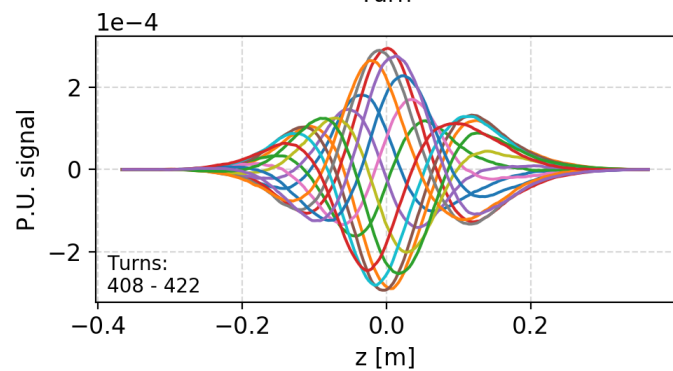
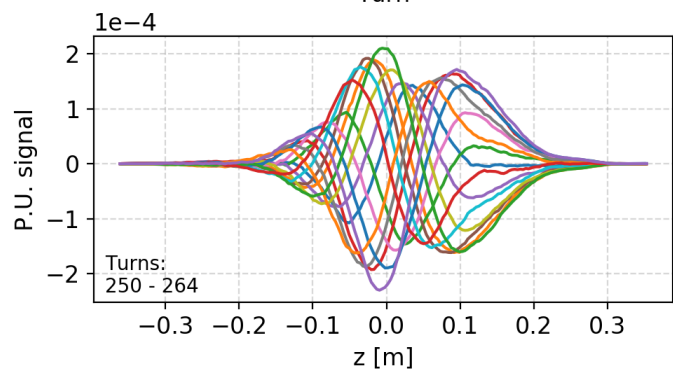
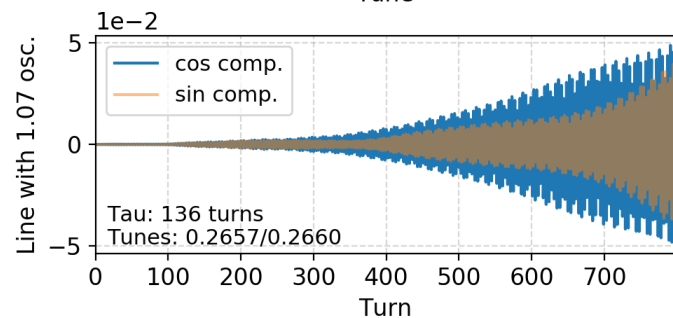
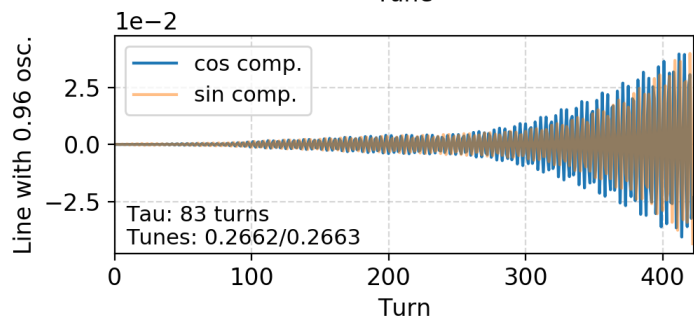
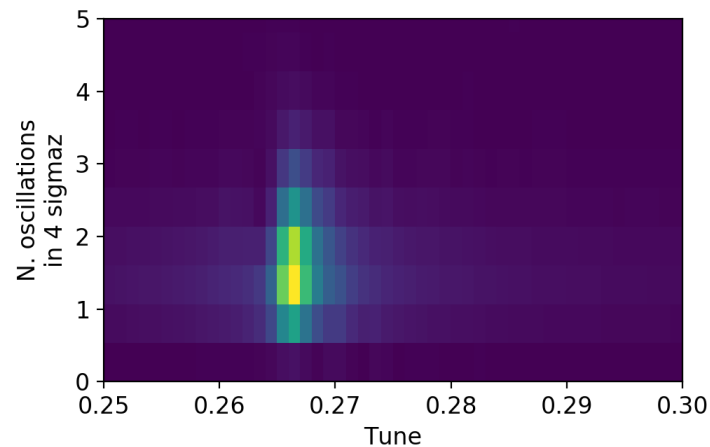
strength 1.3 - **h-matrix + unperturbed map**



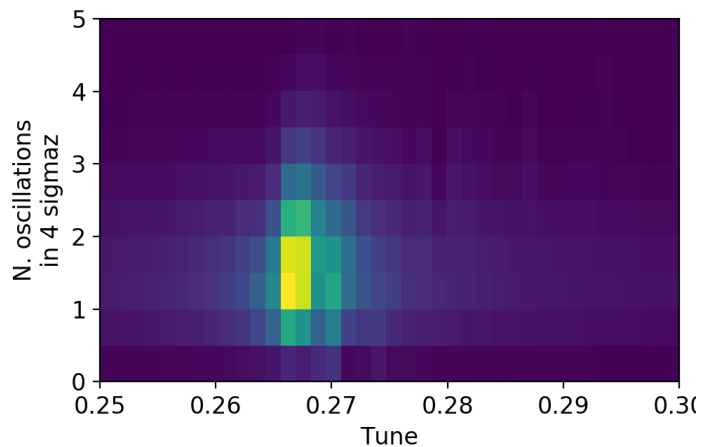
strength 1.2 **Full simulation**



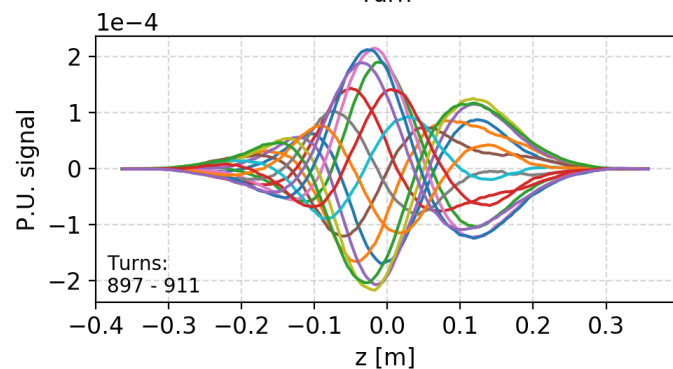
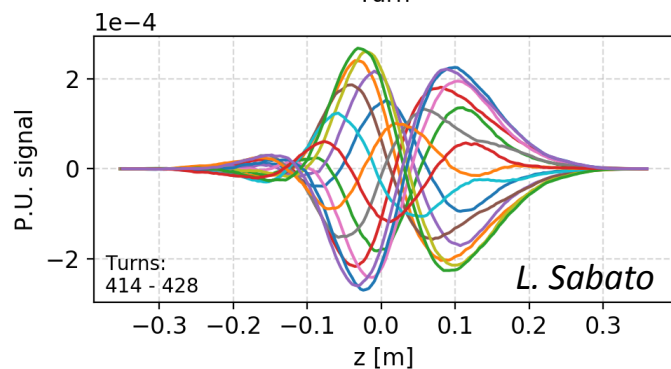
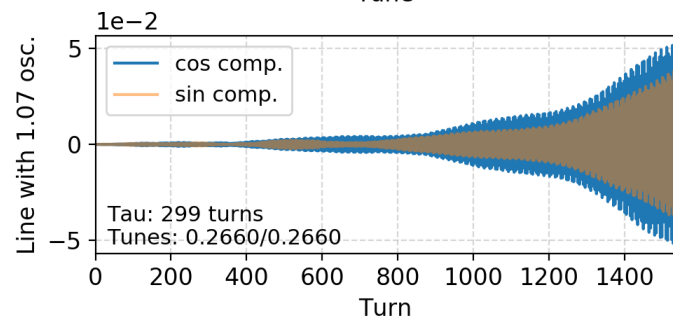
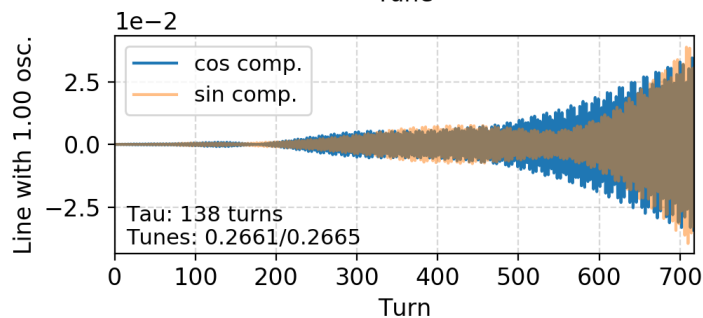
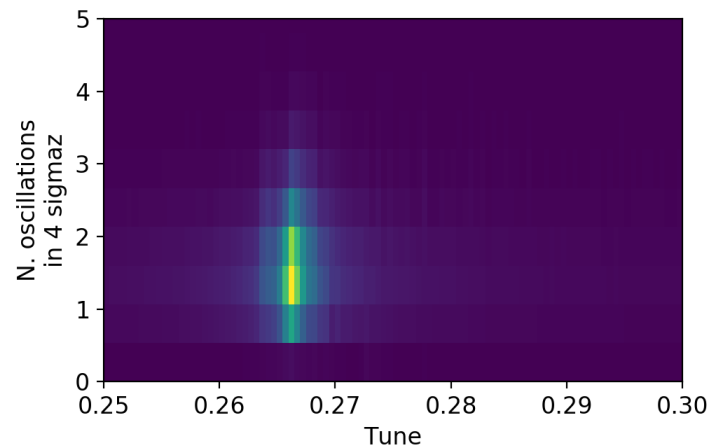
strength 1.2 - **h-matrix + unperturbed map**



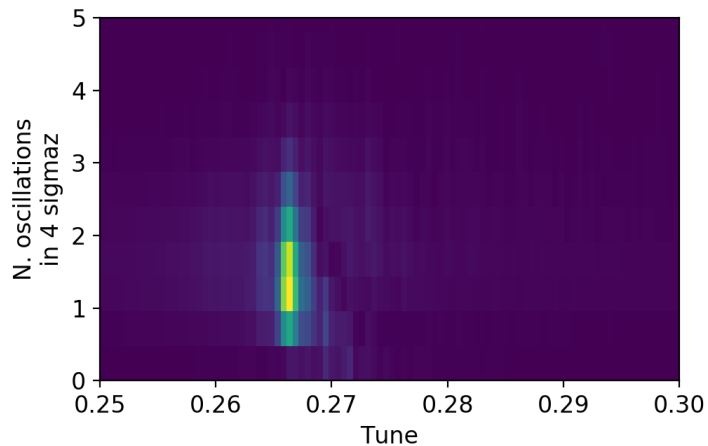
strength 1.1 **Full simulation**



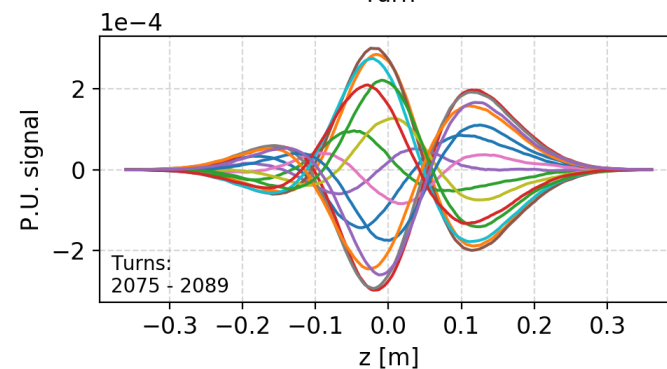
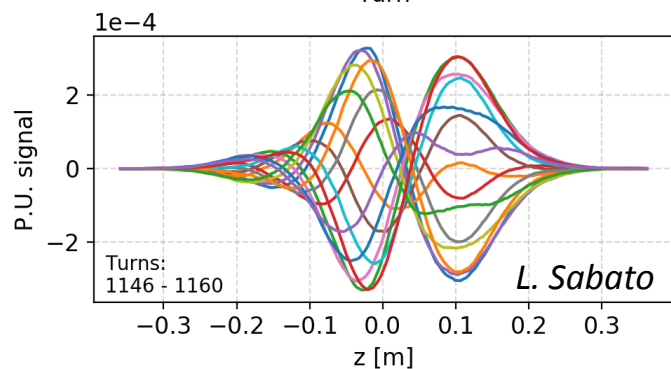
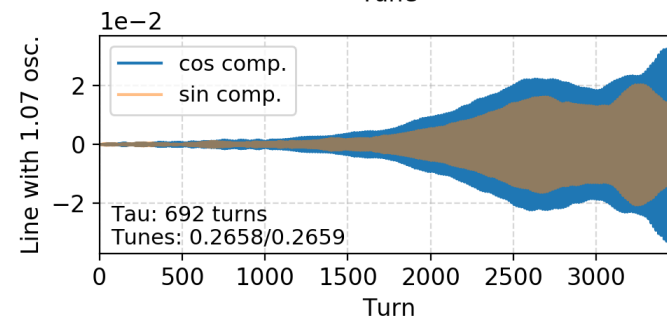
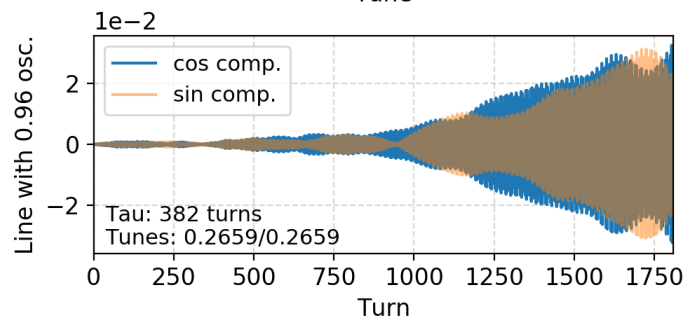
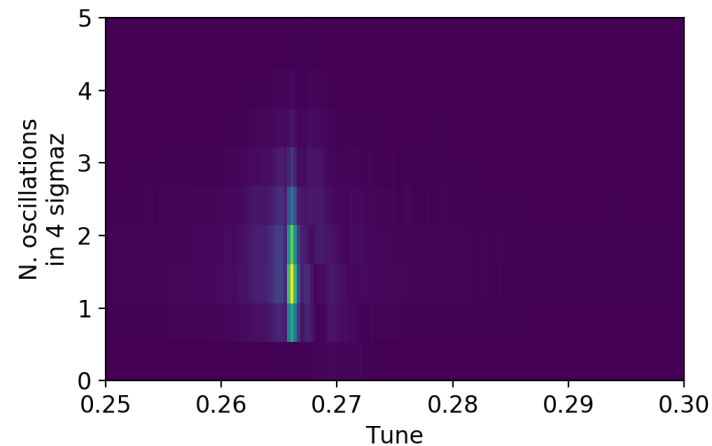
strength 1.1 - **h-matrix + unperturbed map**



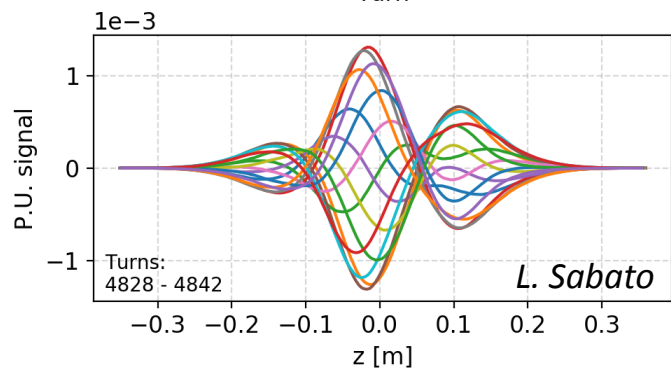
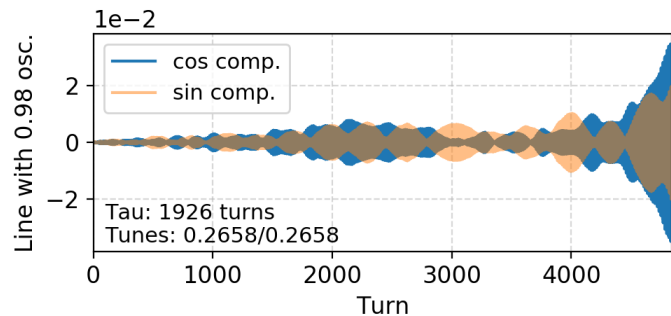
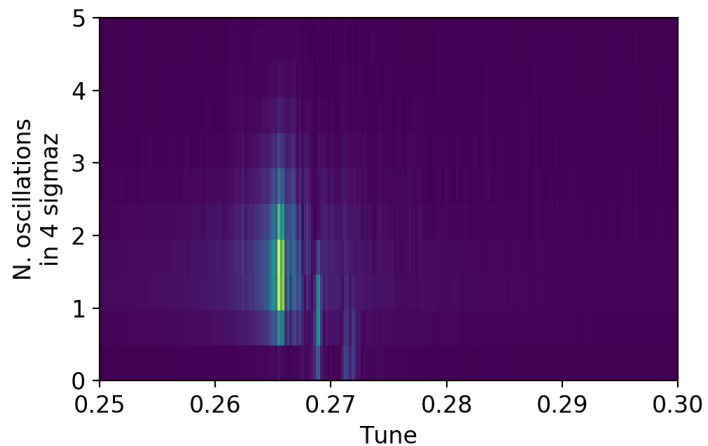
strength 1.0 **Full simulation**



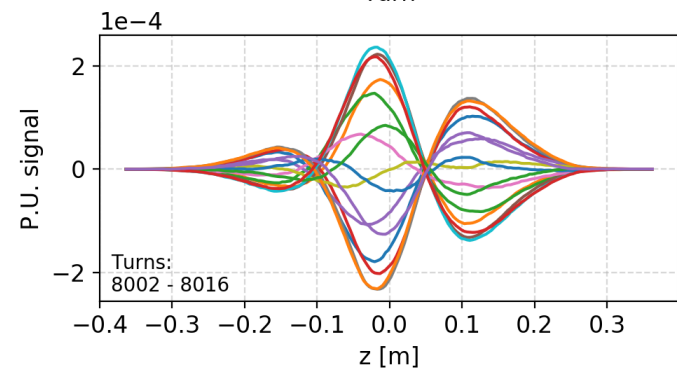
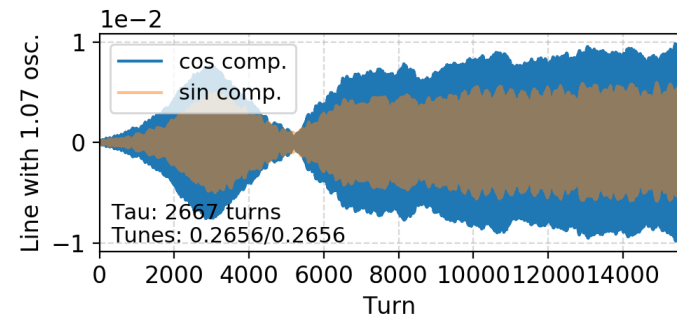
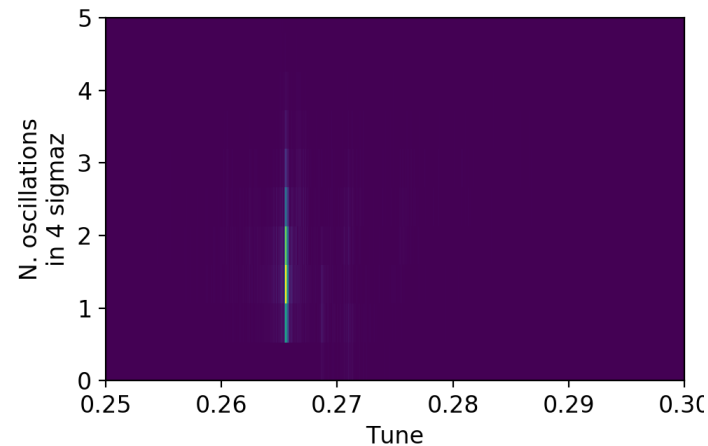
strength 1.0 - **h-matrix + unperturbed map**



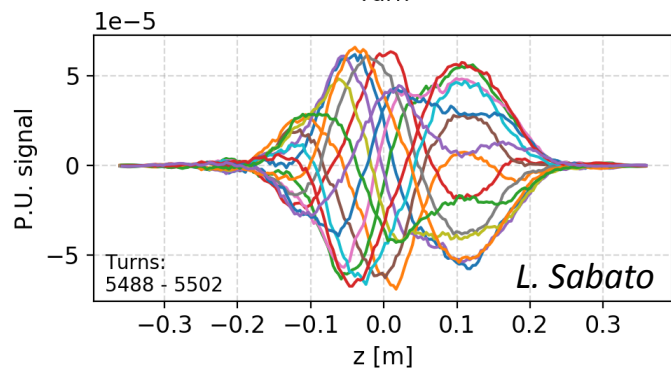
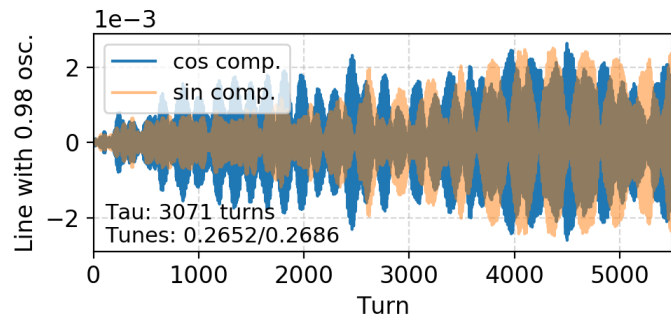
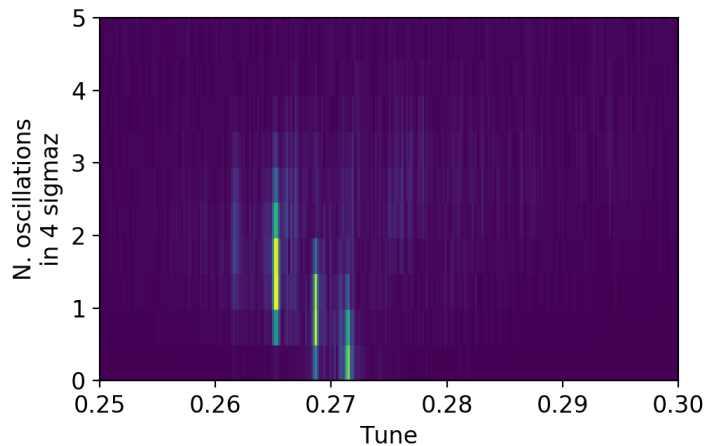
strength 0.9 **Full simulation**



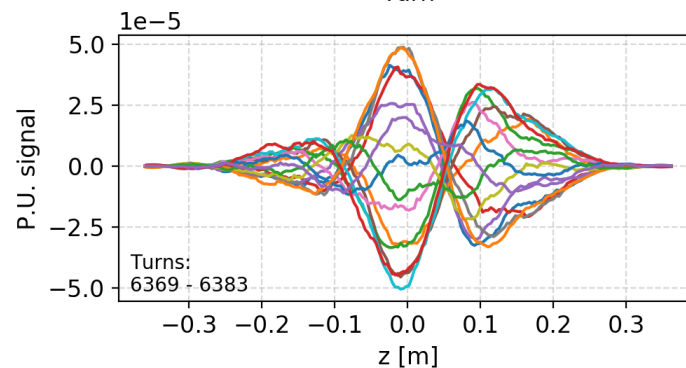
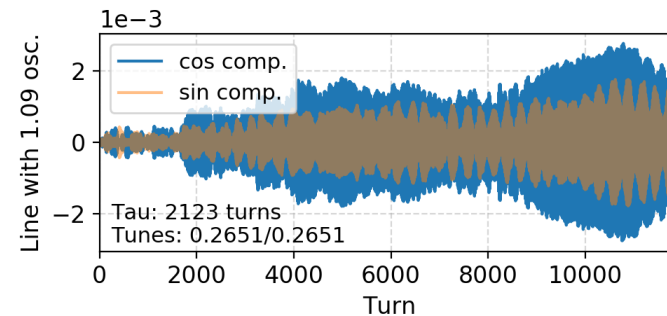
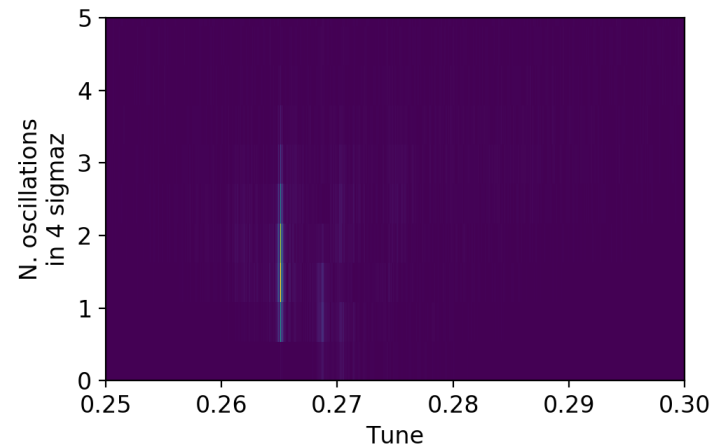
strength 0.9 - **h-matrix + unperturbed map**



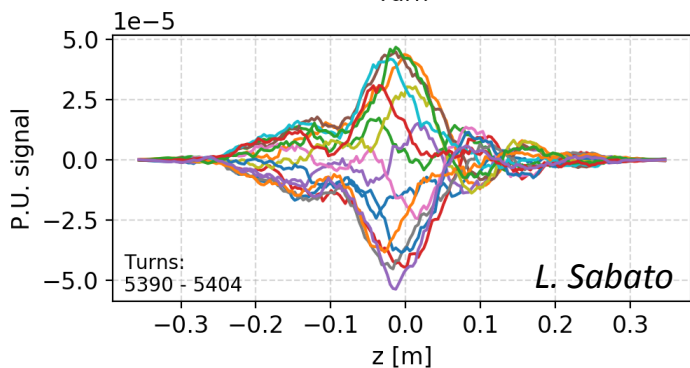
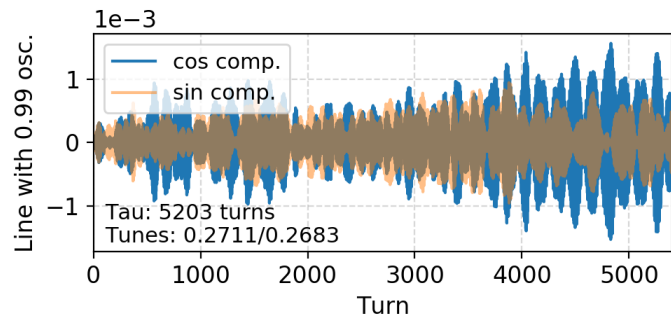
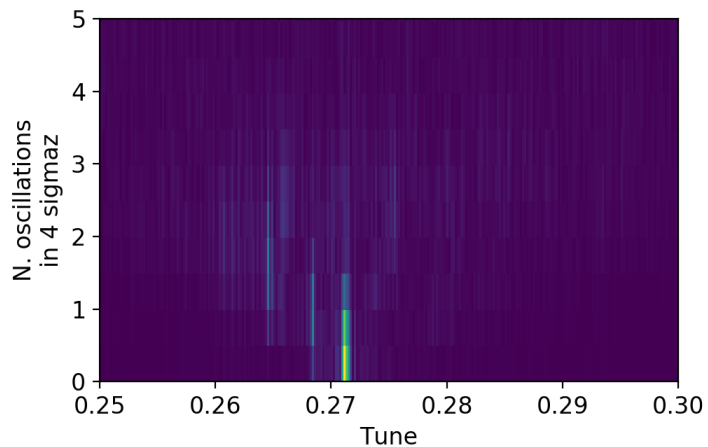
strength 0.8 **Full simulation**



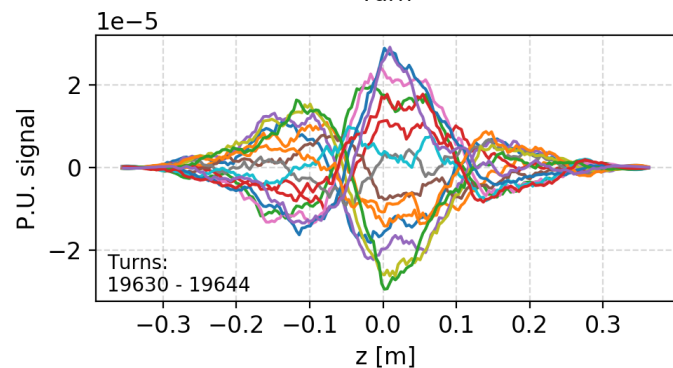
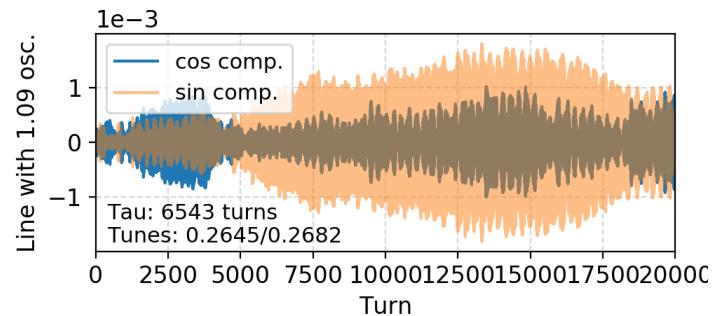
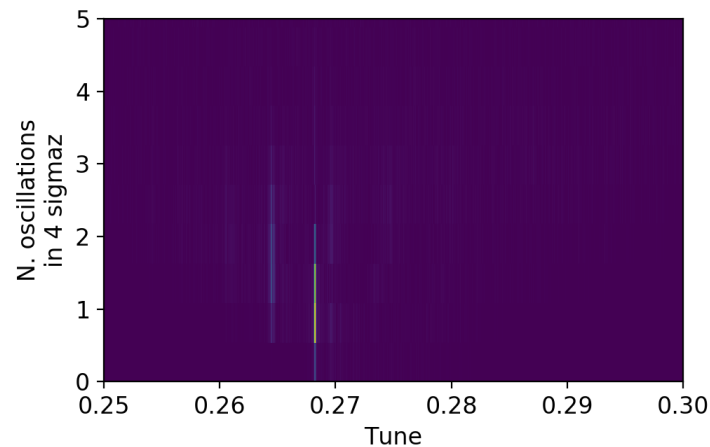
strength 0.8 - **h-matrix + unperturbed map**



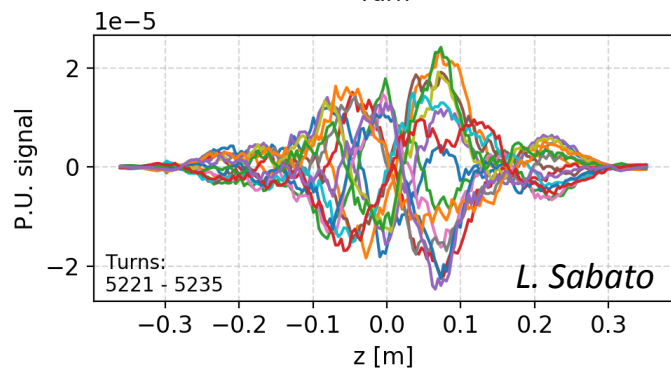
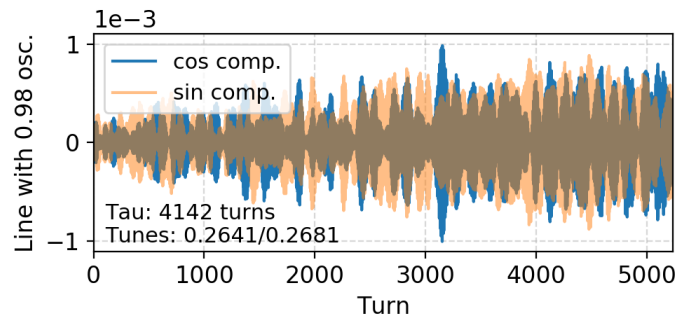
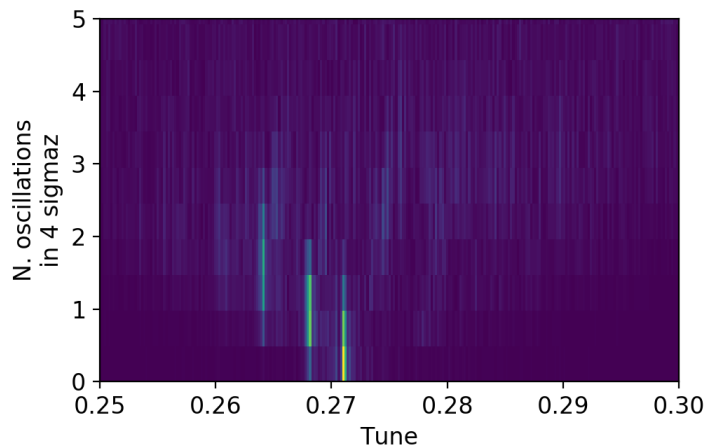
strength 0.7 **Full simulation**



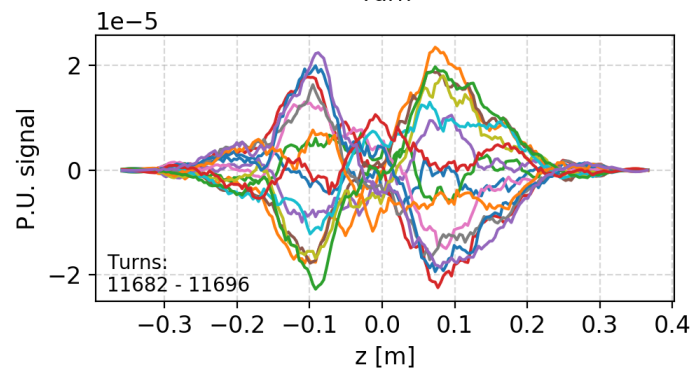
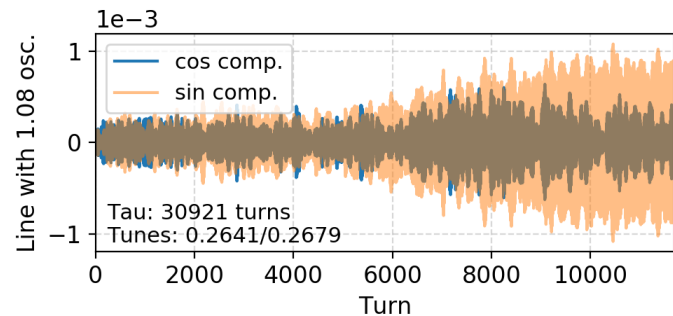
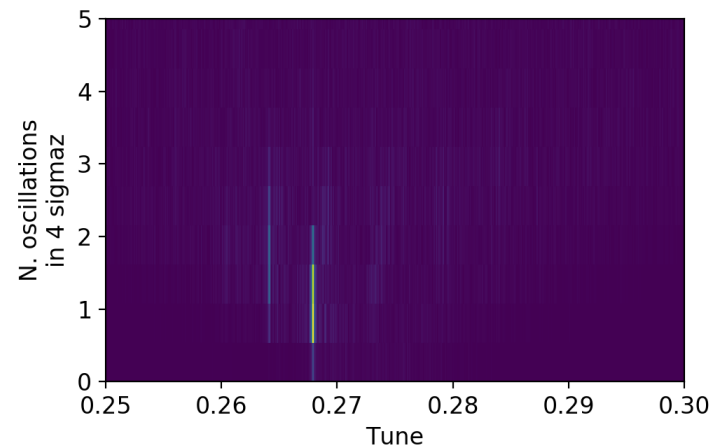
strength 0.7 - **h-matrix + unperturbed map**



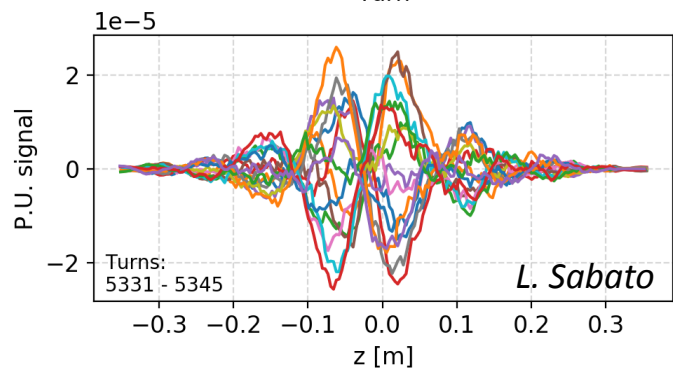
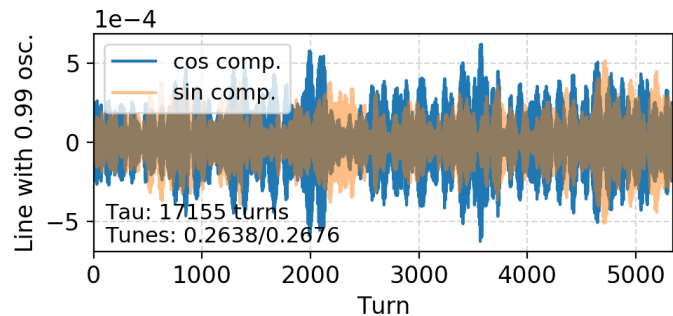
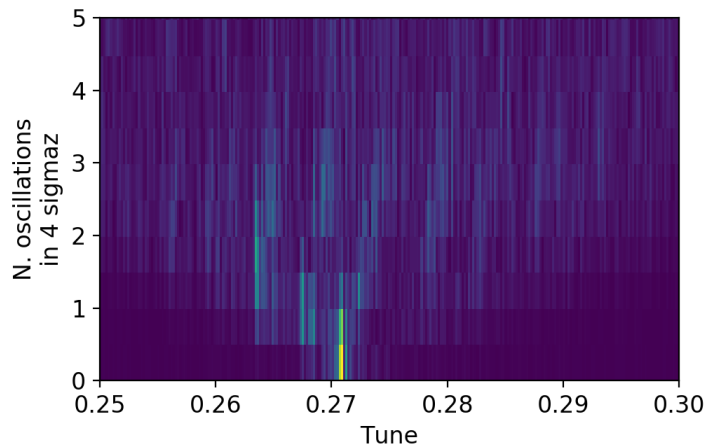
strength 0.6 - compact_forced_strength_scan



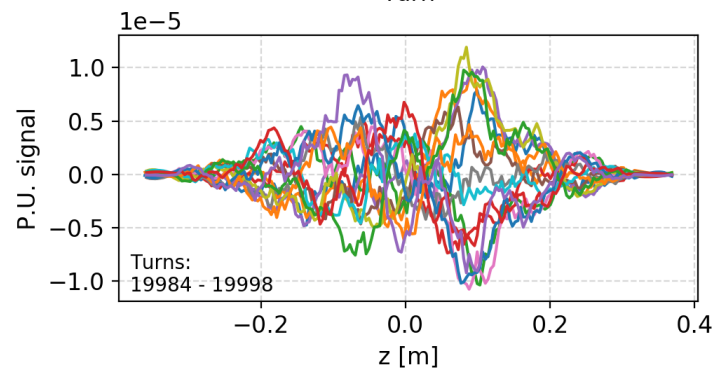
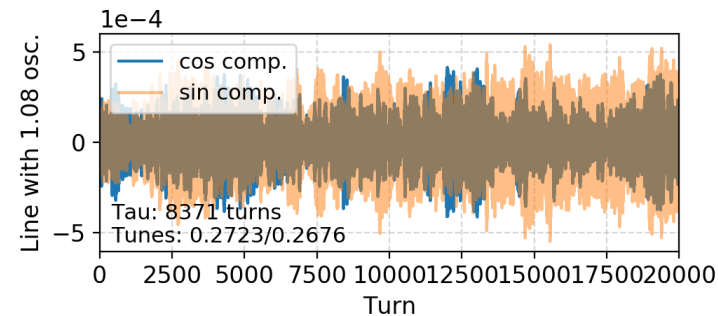
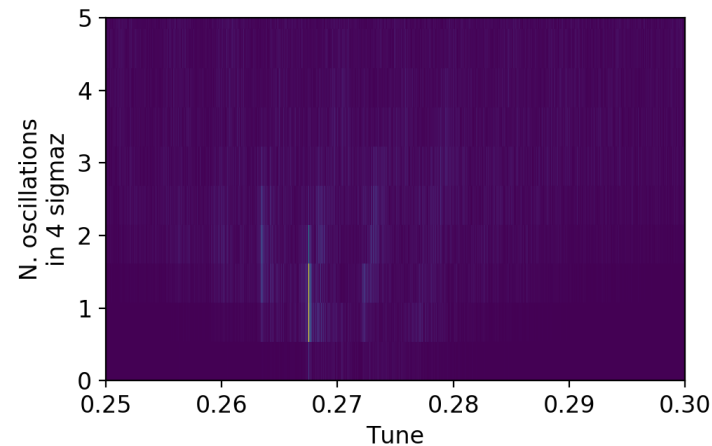
strength 0.6 - **h-matrix + unperturbed map**



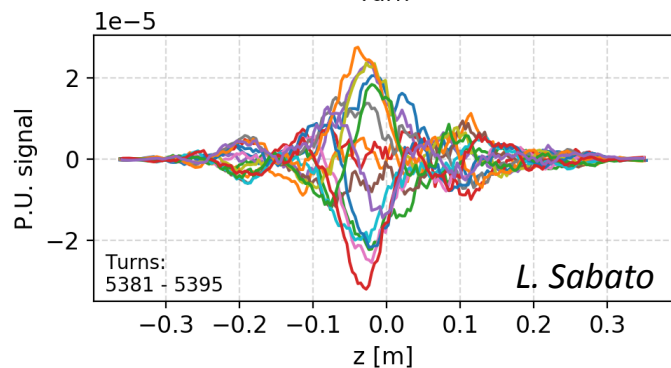
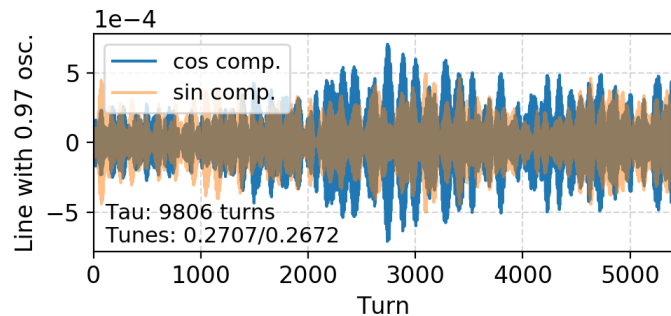
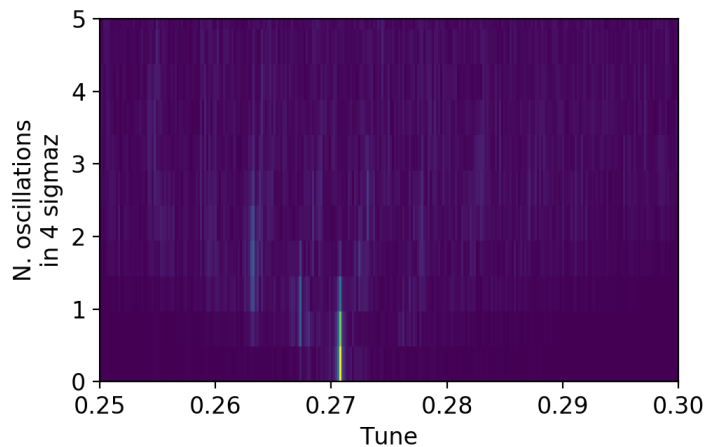
strength 0.5 **Full simulation**



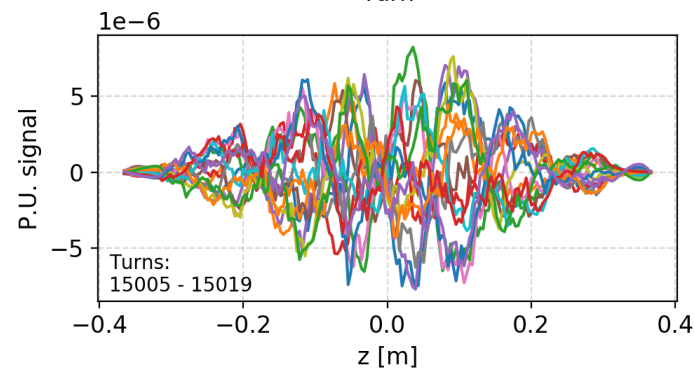
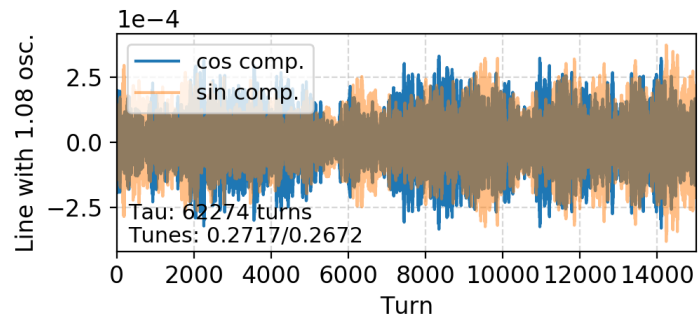
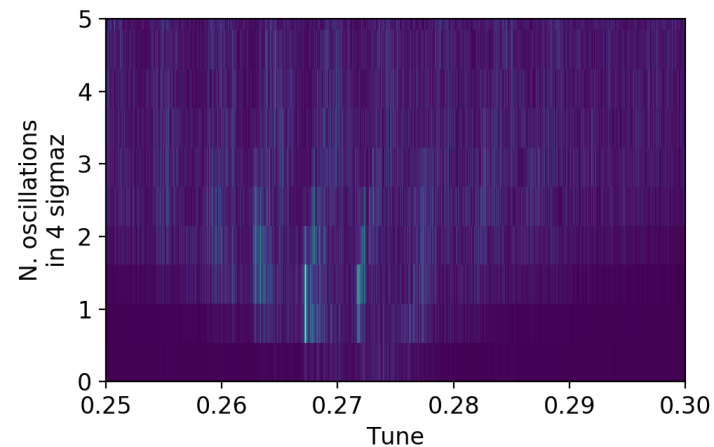
strength 0.5 - **h-matrix + unperturbed map**



strength 0.4 **Full simulation**



strength 0.4 - **h-matrix + unperturbed map**

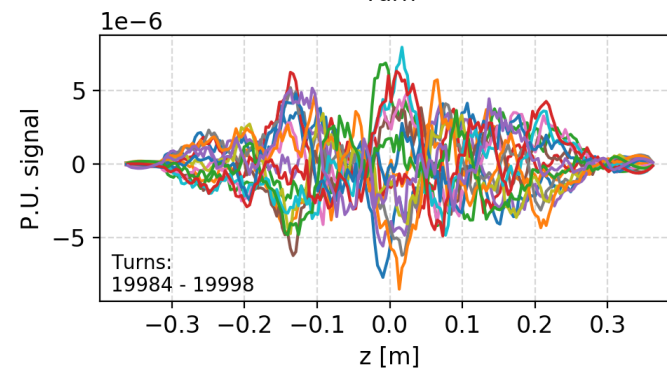
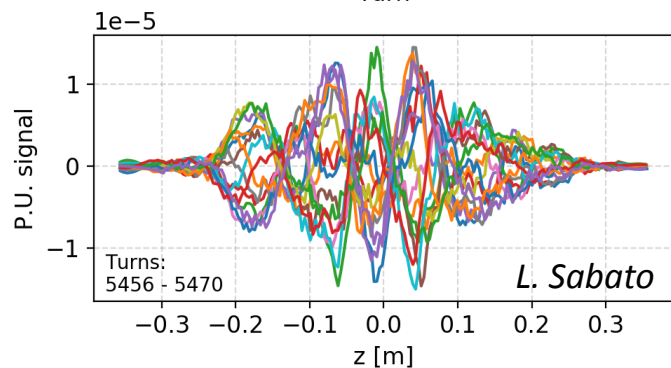
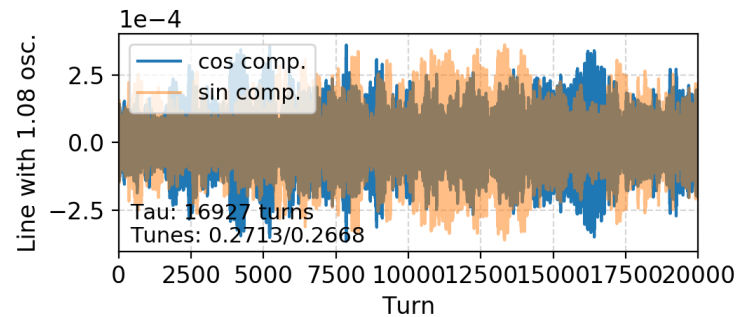
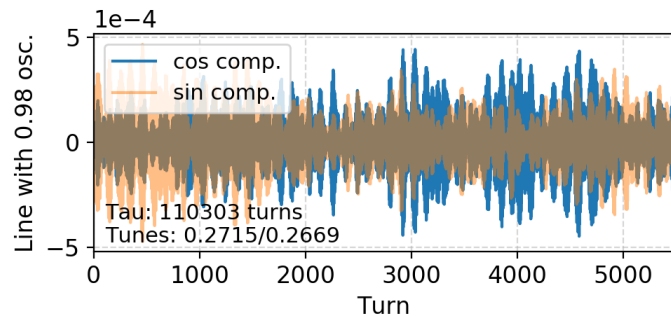
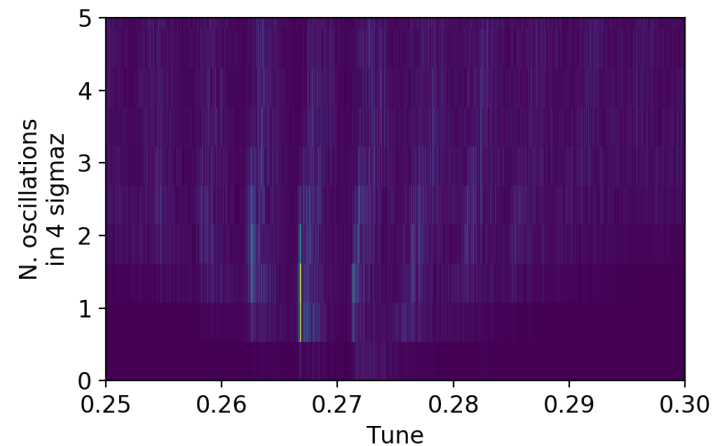
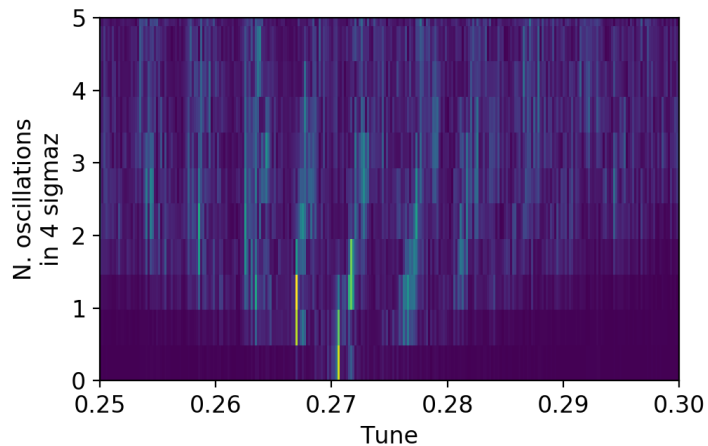


strength 0.3

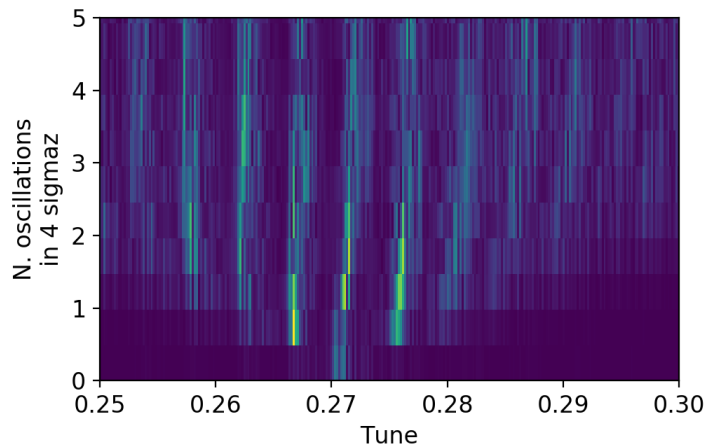
Full simulation

1

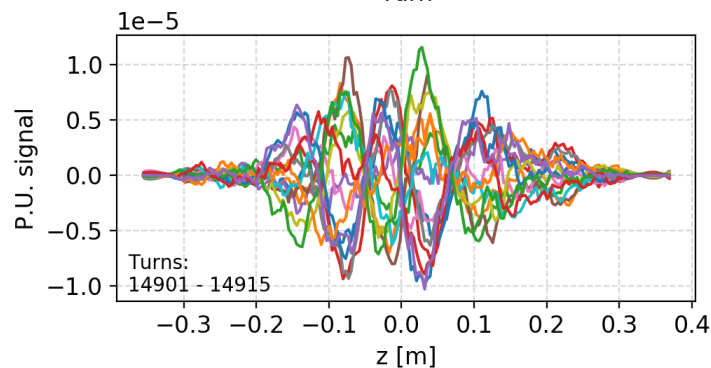
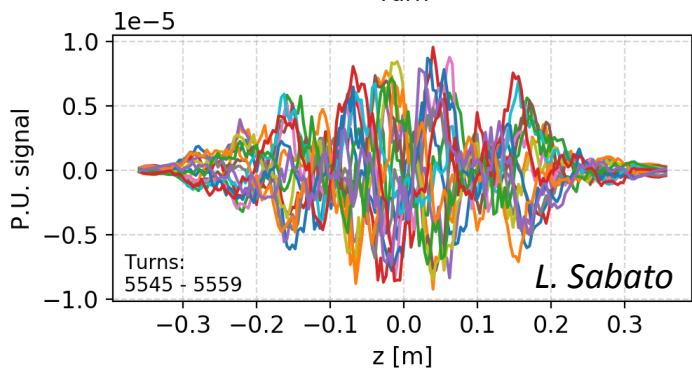
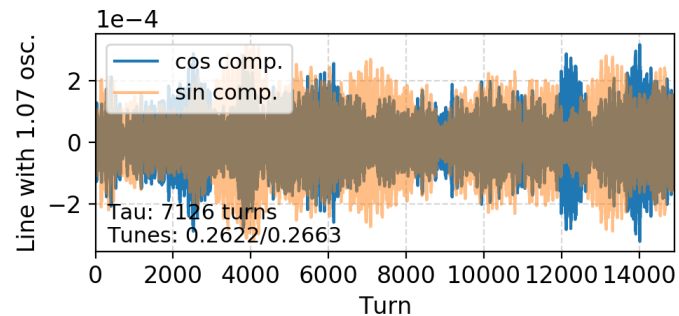
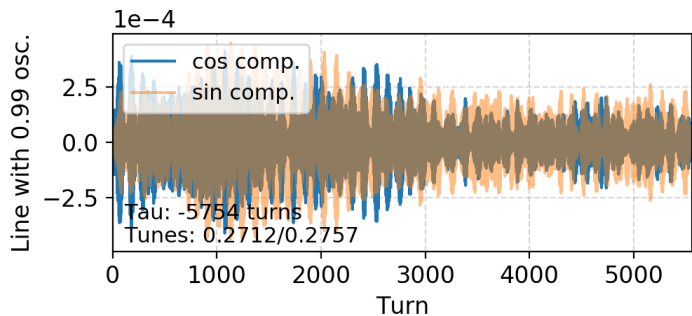
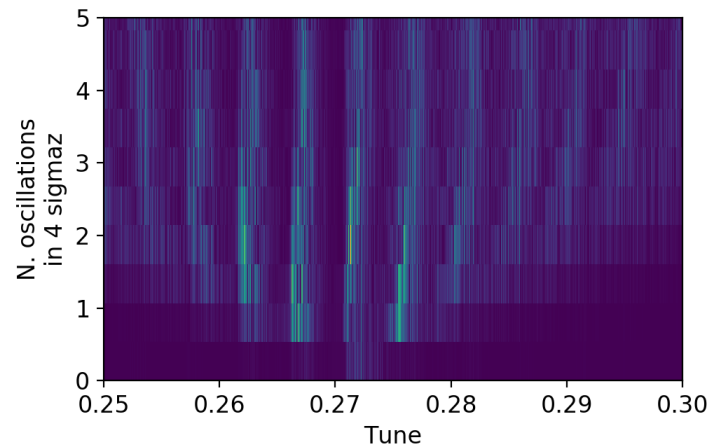
strength 0.3 - h-matrix + unperturbed map



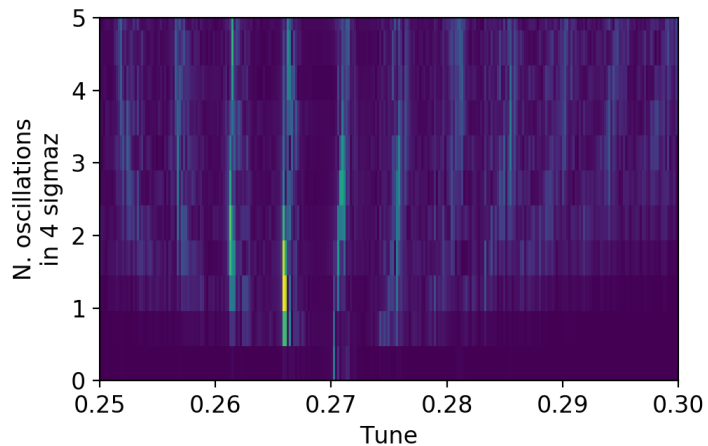
strength 0.2 **Full simulation**



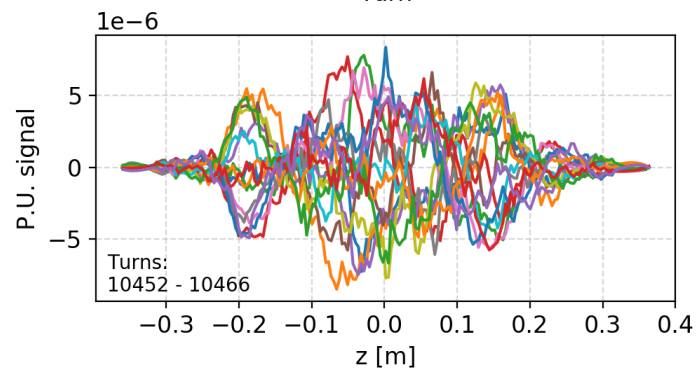
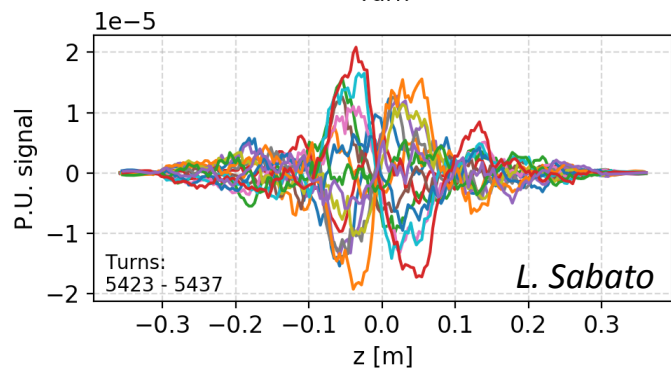
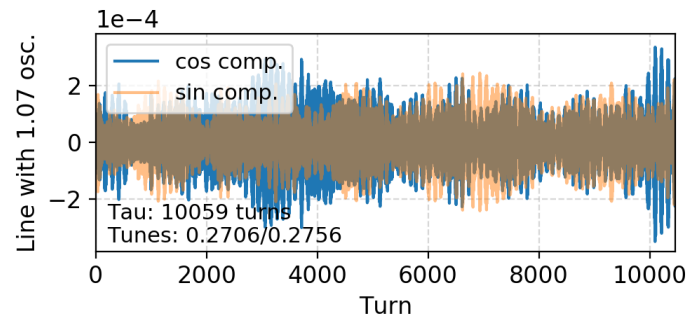
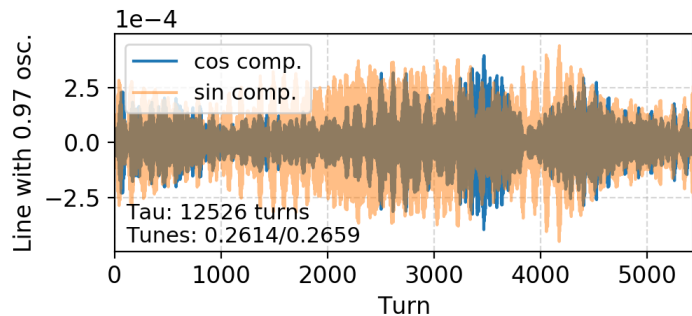
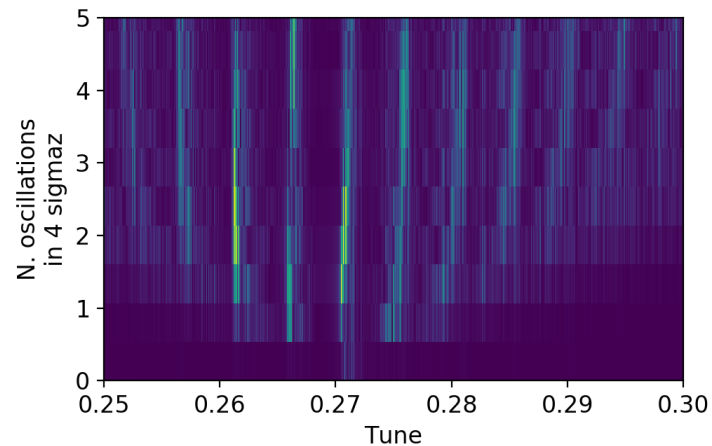
strength 0.2 - **h-matrix + unperturbed map**



strength 0.1 **Full simulation**



strength 0.1 - **h-matrix + unperturbed map**





In the investigated case (LHC quad, 450 GeV), it is possible to model **instabilities driven by e-cloud** by including:

- The dipolar kicks along the bunch through an **harmonic response**
- The **non-linearities** as seen by a stable bunch

Potential applications:

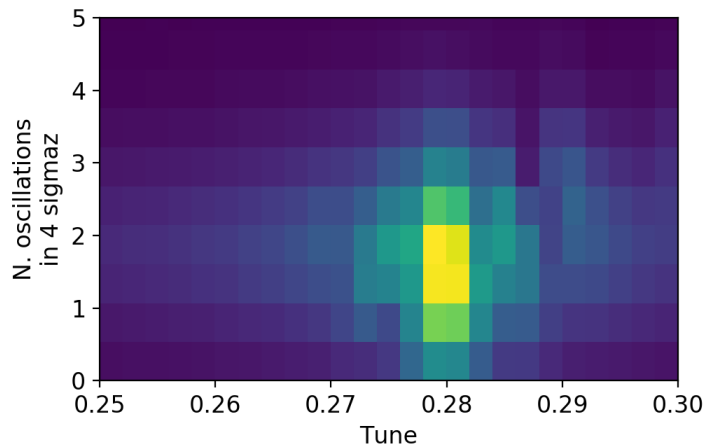
- Significant speedup for the simulations
- Modelling through Vlasov solver & dispersion integrals



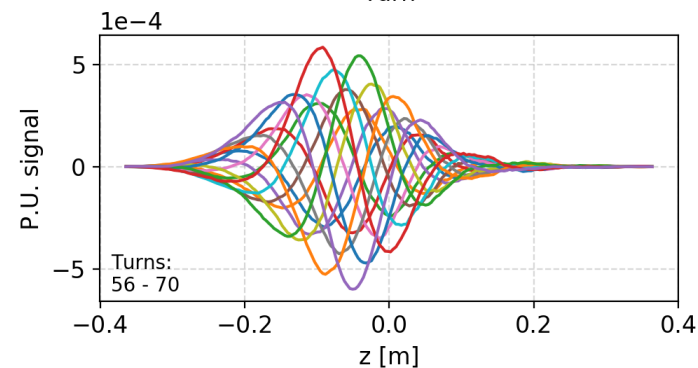
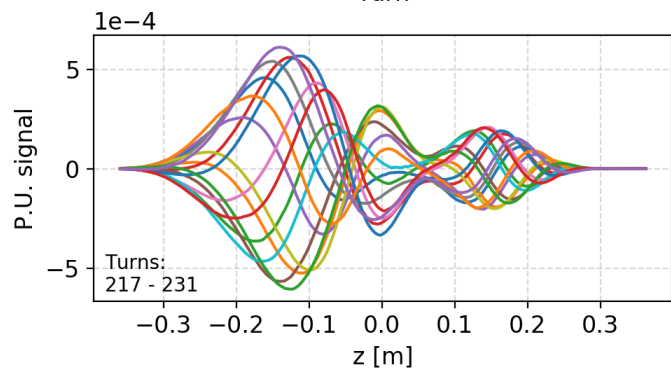
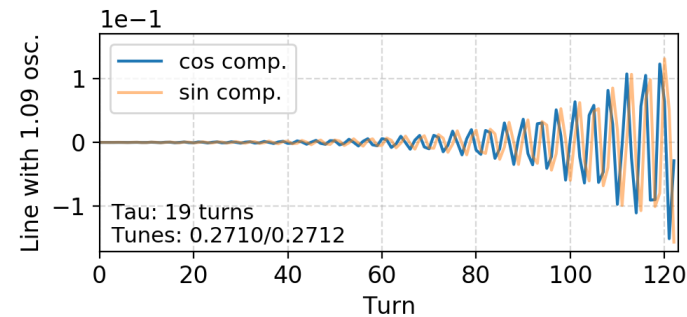
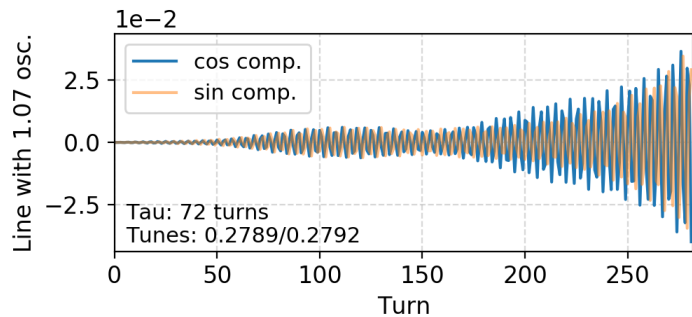
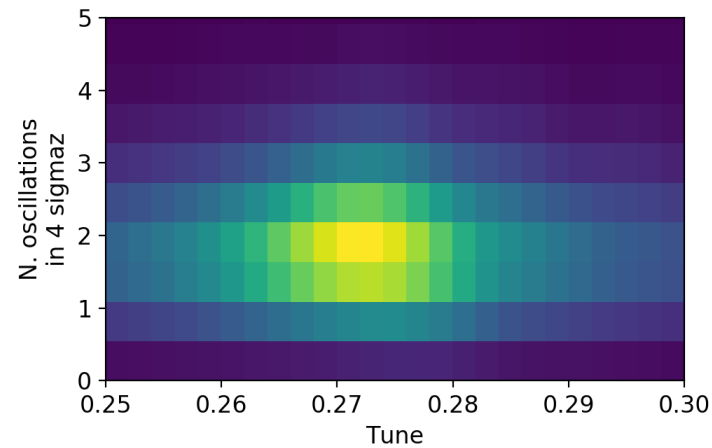
**Scanning the strength of the e-cloud
(simplified model only –
effect of non-linearities)**

For high strength the instability is faster with non-linearity!

strength 2.0 - **h-matrix only**

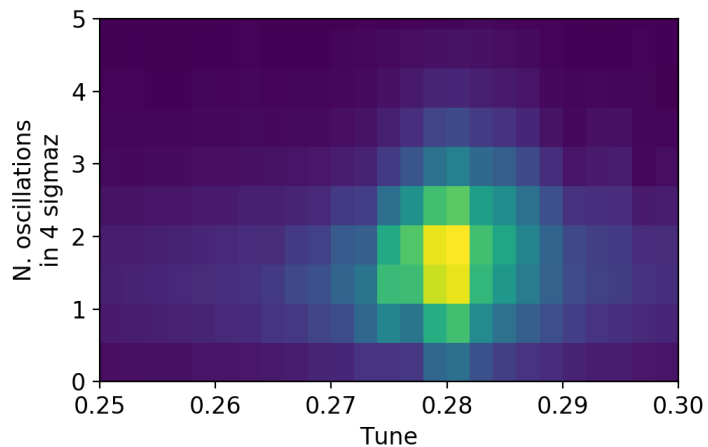


strength 2.0 - **h-matrix + unperturbed map**

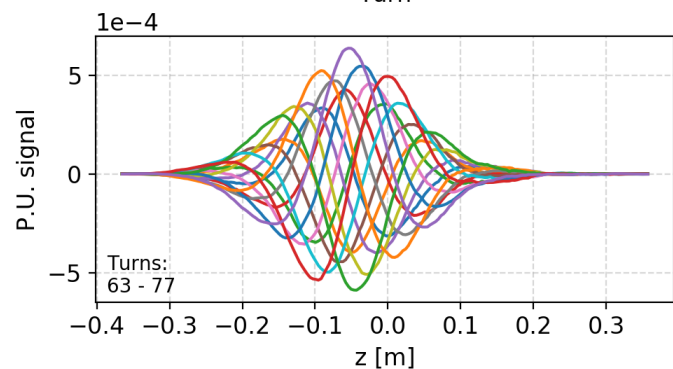
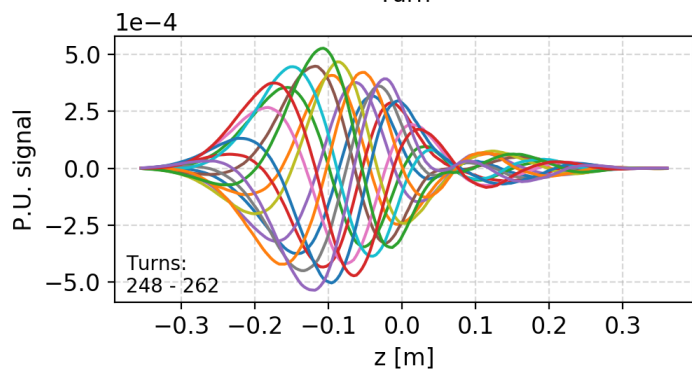
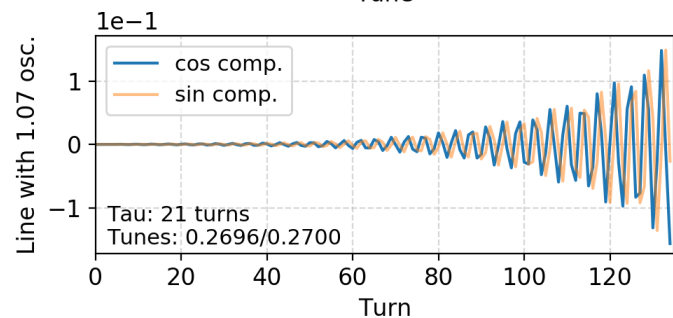
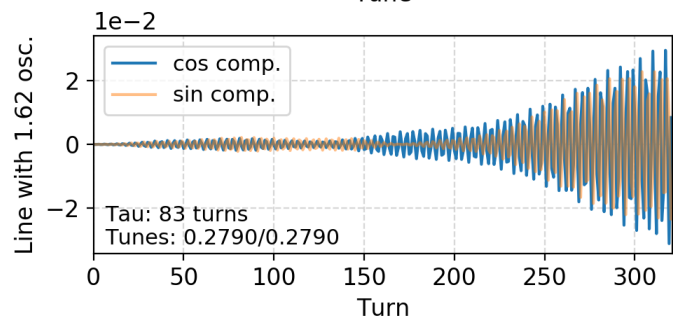
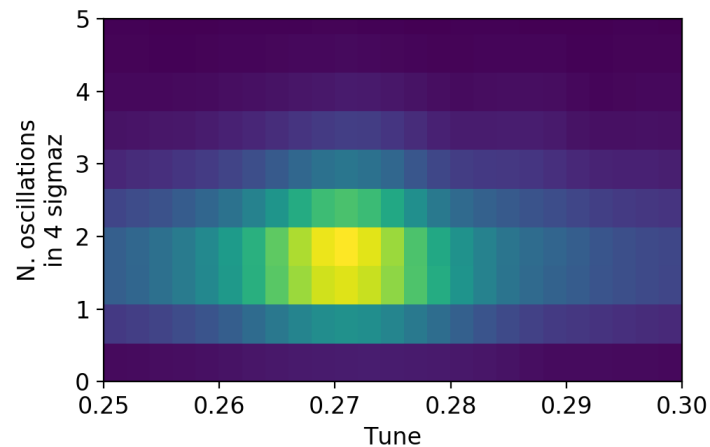


For high strength the instability is faster with non-linearity!

strength 1.9 - **h-matrix only**

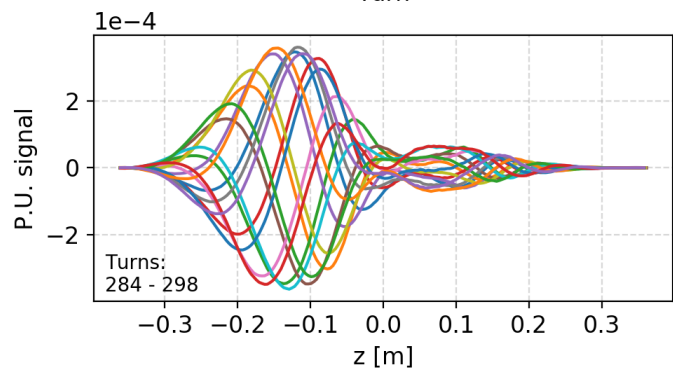
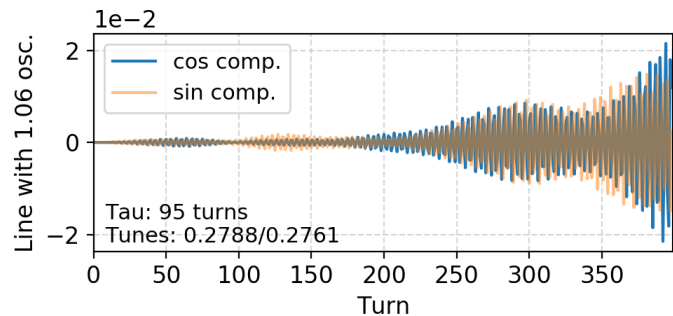
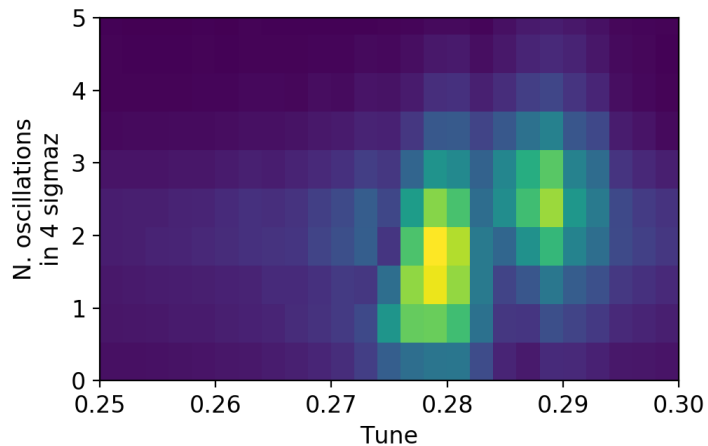


strength 1.9 - **h-matrix + unperturbed map**

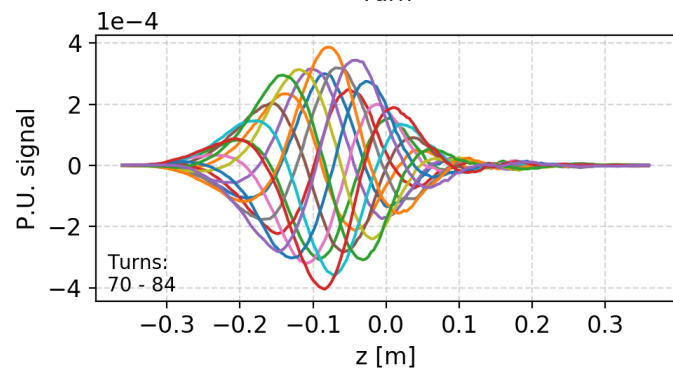
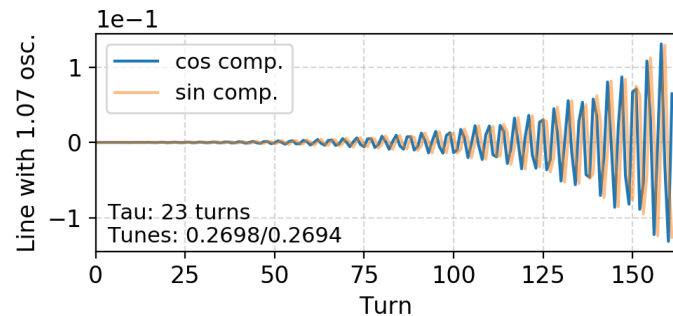
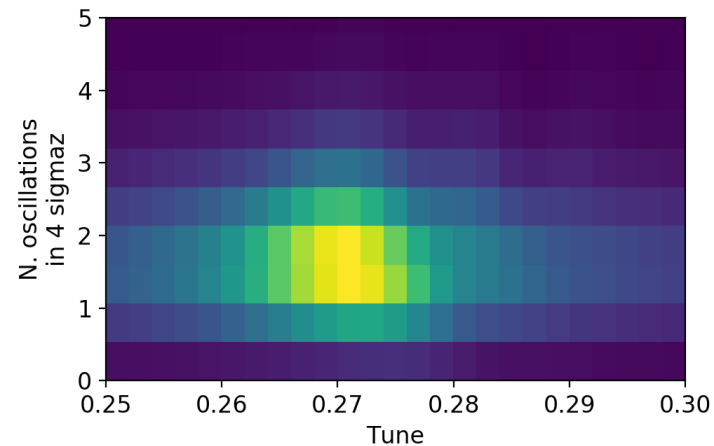


For high strength the instability is faster with non-linearity!

strength 1.8 - **h-matrix only**

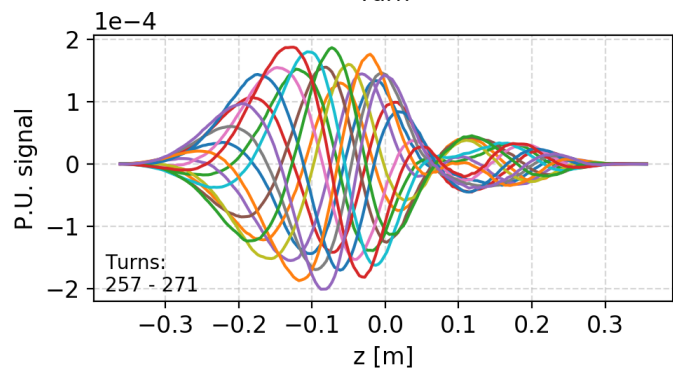
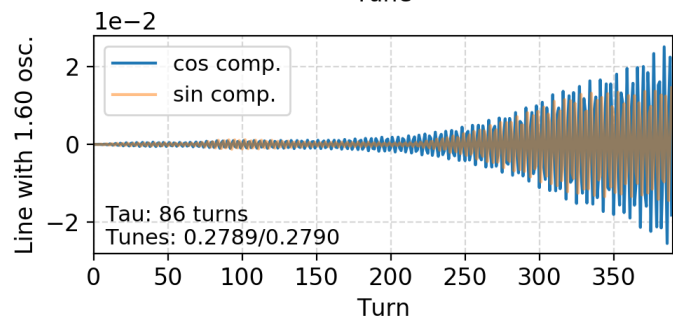
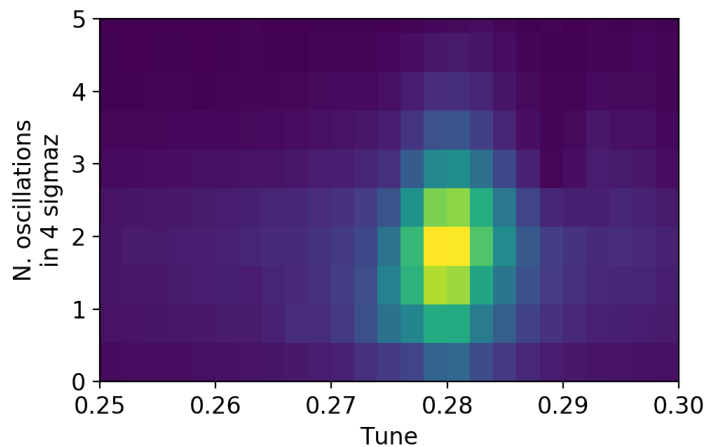


strength 1.8 - **h-matrix + unperturbed map**

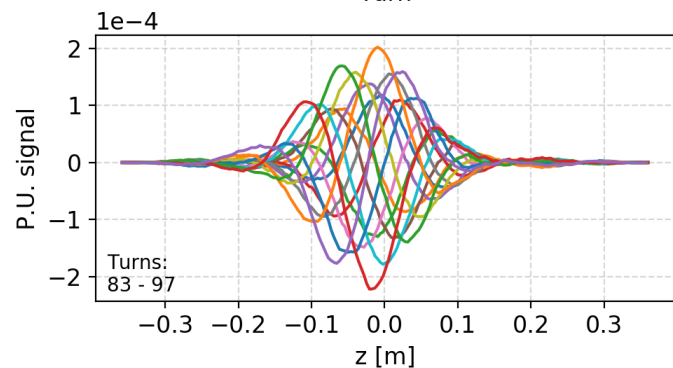
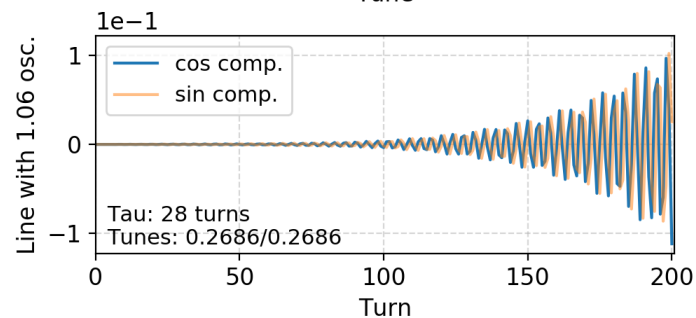
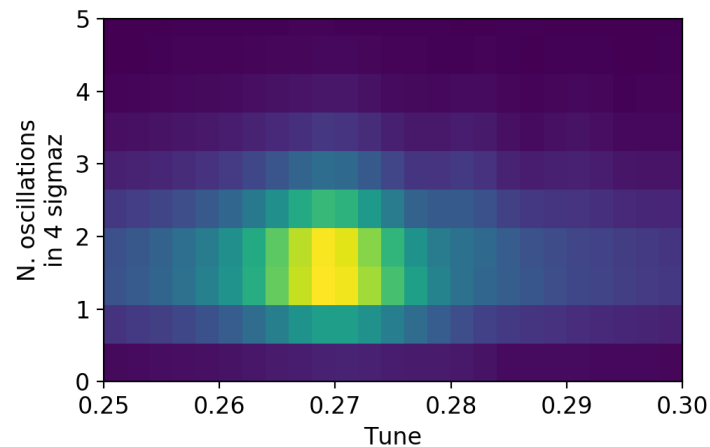


For high strength the instability is faster with non-linearity!

strength 1.7 - **h-matrix only**

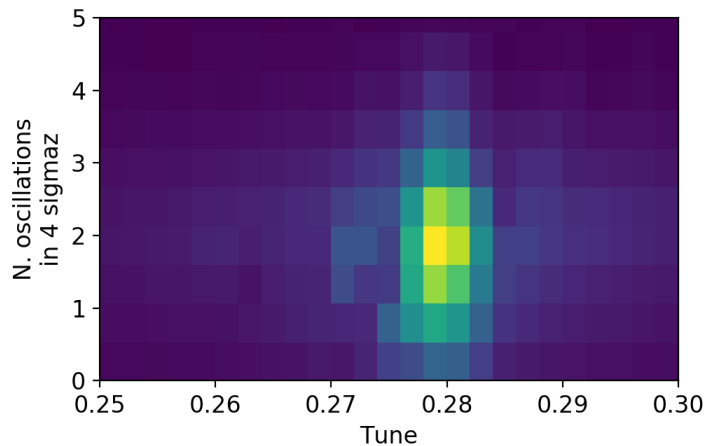


strength 1.7 - **h-matrix + unperturbed map**

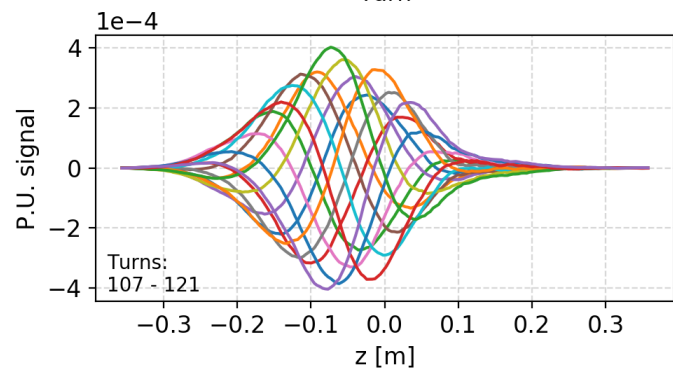
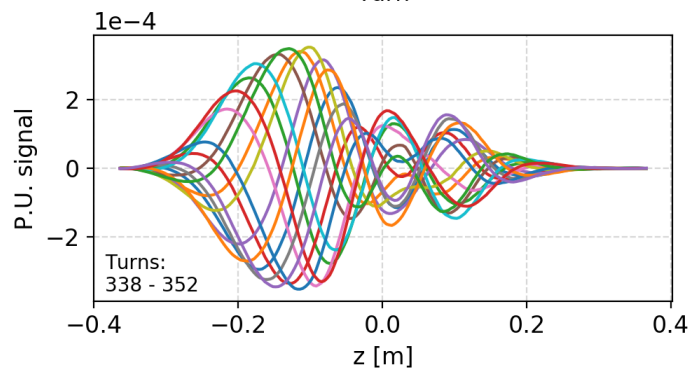
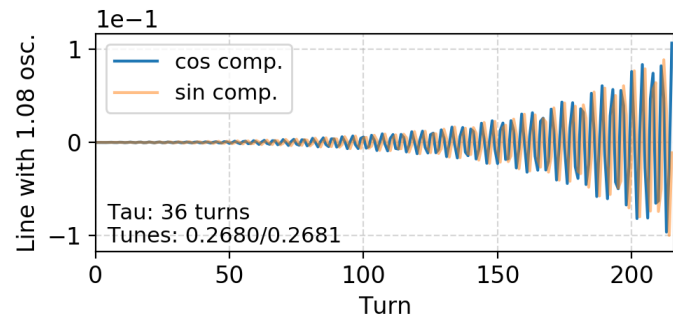
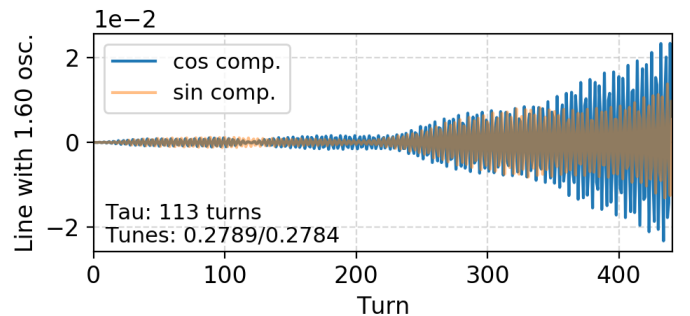
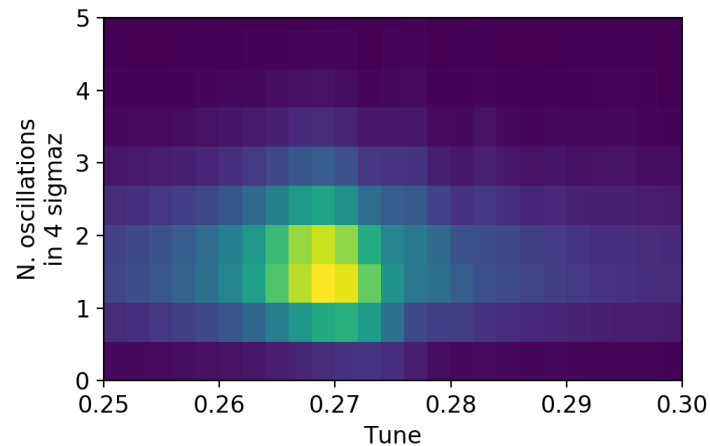


For high strength the instability is faster with non-linearity!

strength 1.6 - **h-matrix only**

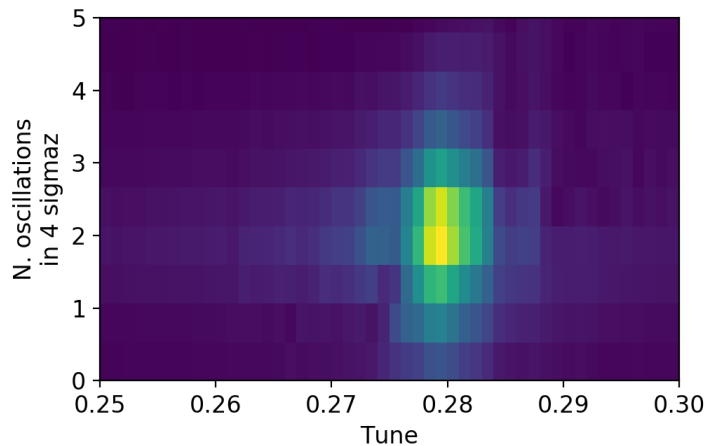


strength 1.6 - **h-matrix + unperturbed map**

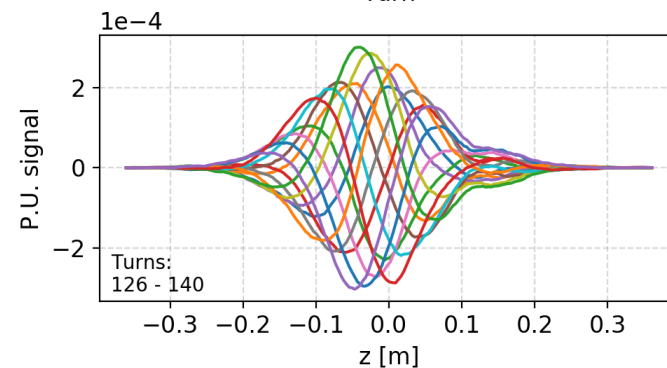
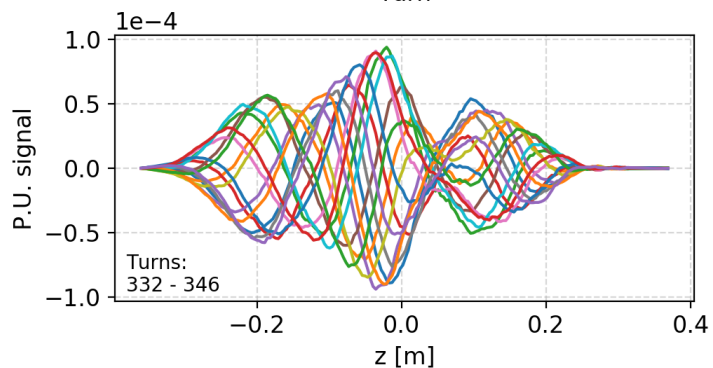
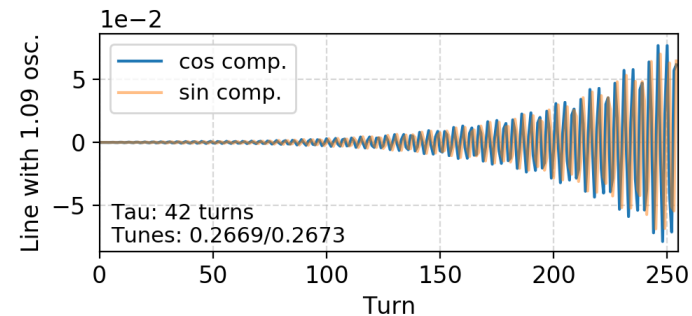
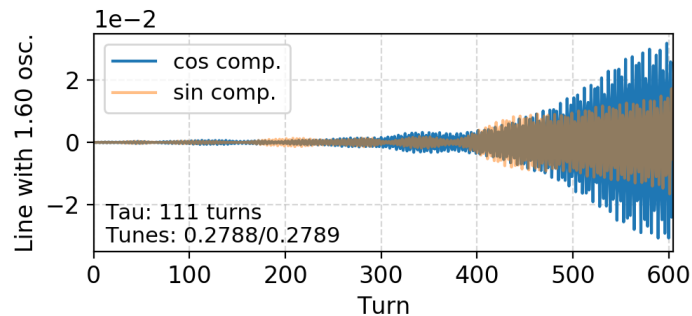
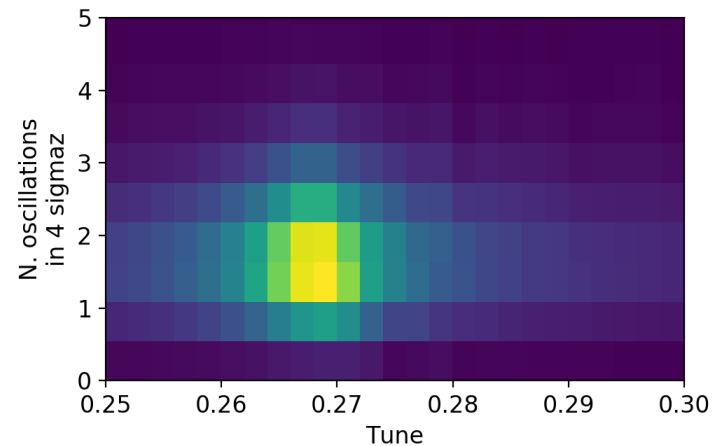


For high strength the instability is faster with non-linearity!

strength 1.5 - **h-matrix only**

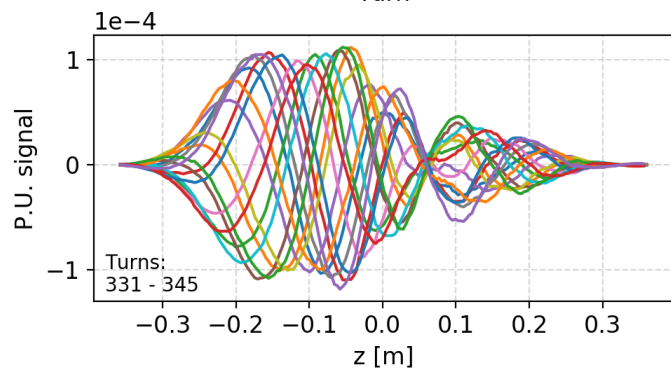
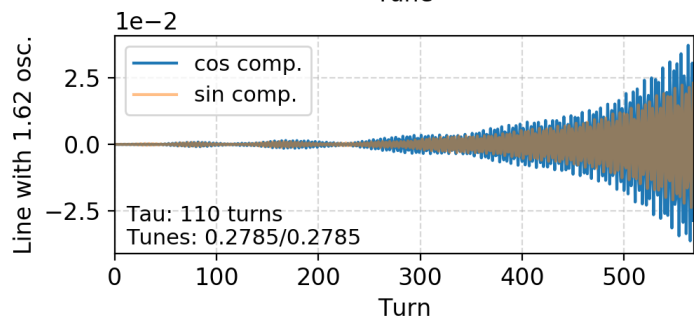
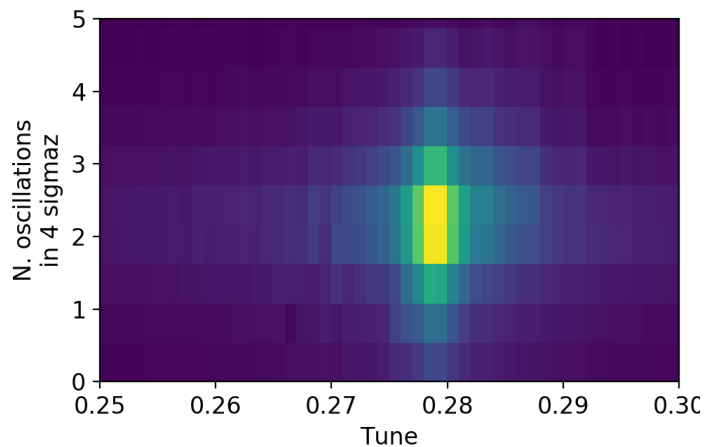


strength 1.5 - **h-matrix + unperturbed map**

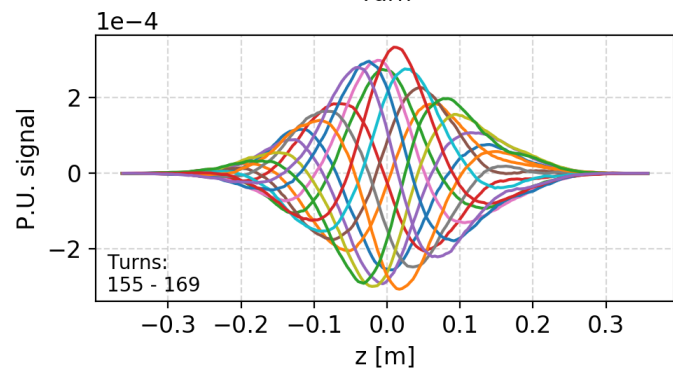
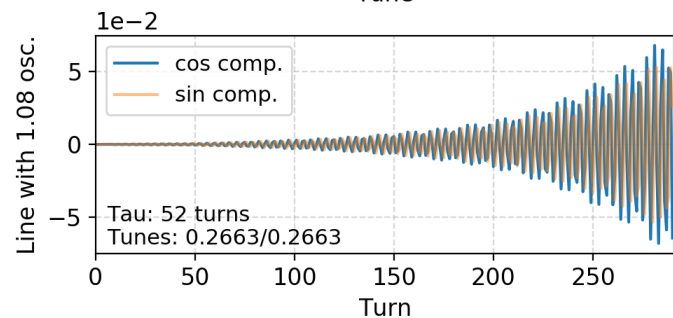
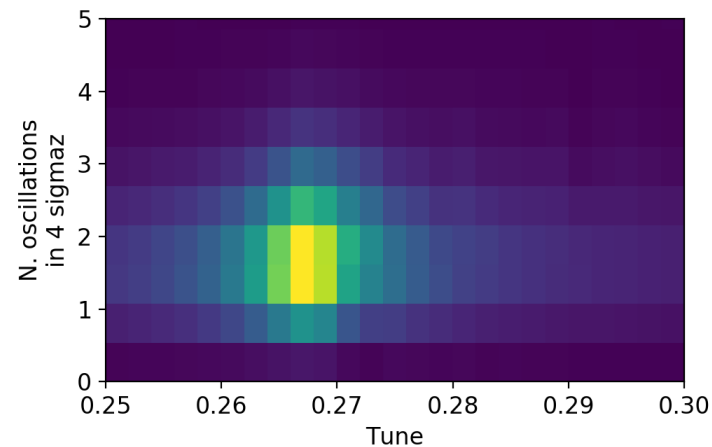


For high strength the instability is faster with non-linearity!

strength 1.4 - **h-matrix only**

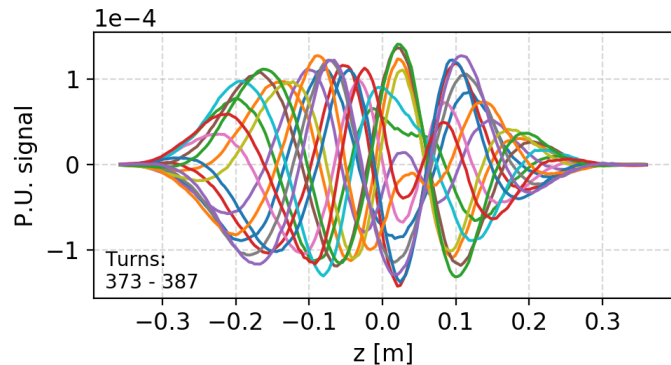
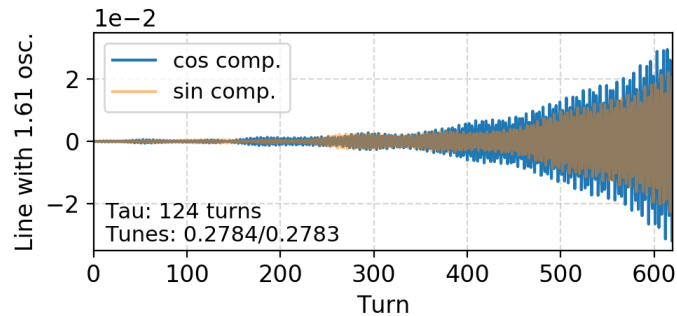
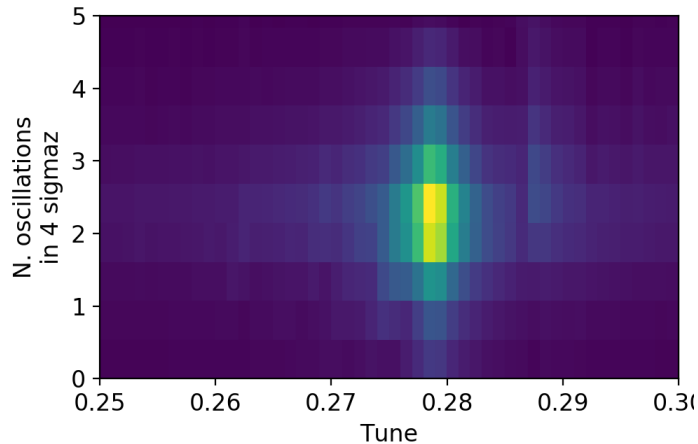


strength 1.4 - **h-matrix + unperturbed map**

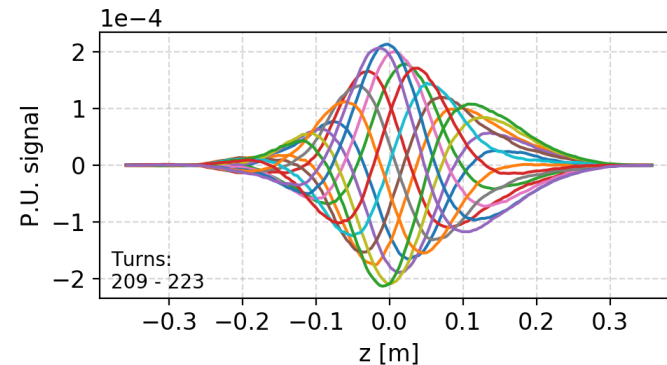
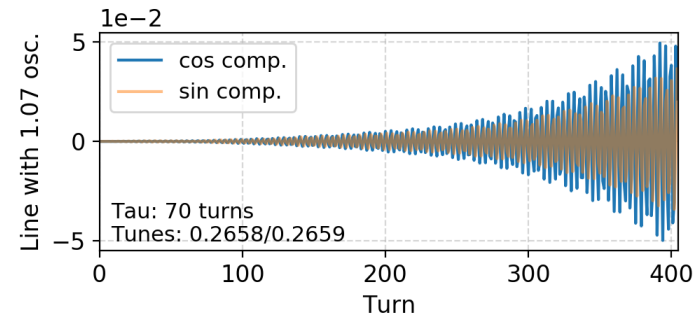
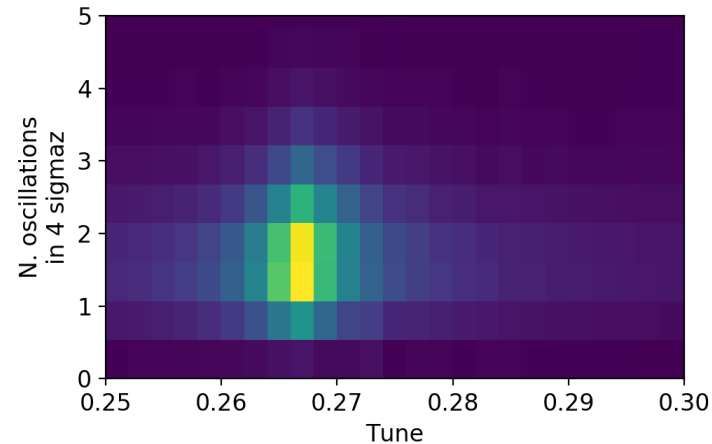


For high strength the instability is faster with non-linearity!

strength 1.3 - **h-matrix only**

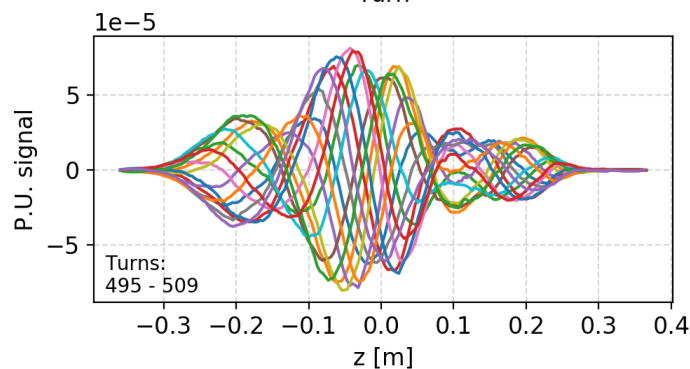
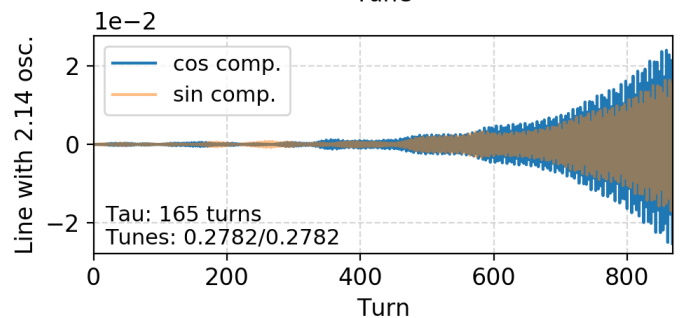
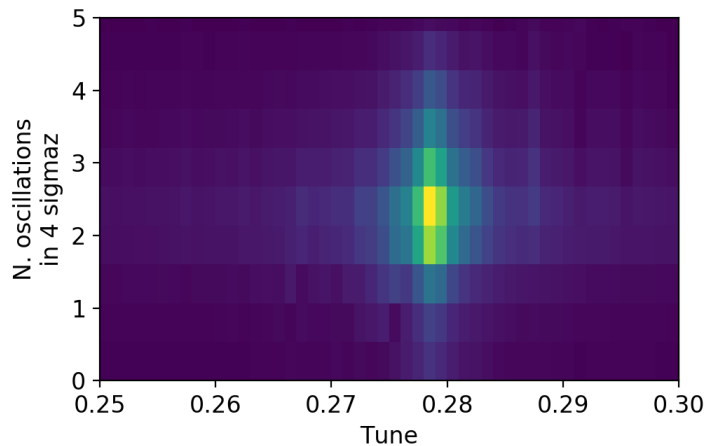


strength 1.3 - **h-matrix + unperturbed map**

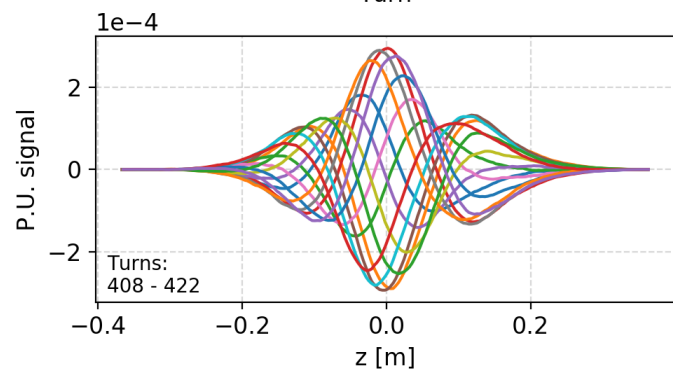
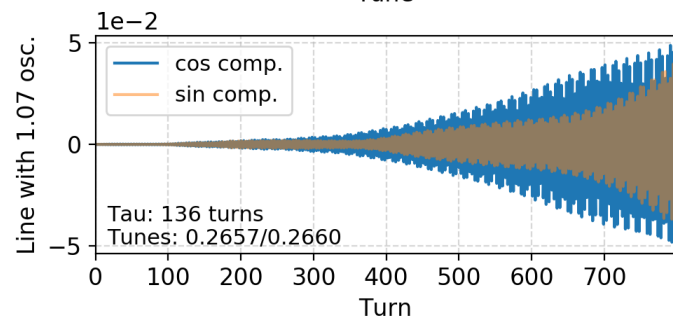
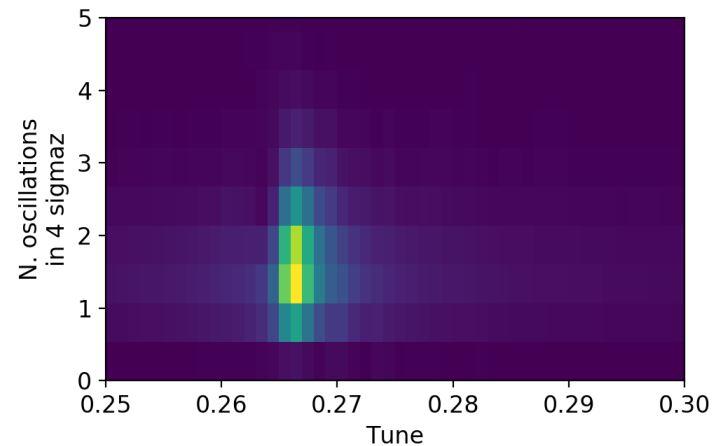


For high strength the instability is faster with non-linearity!

strength 1.2 - **h-matrix only**

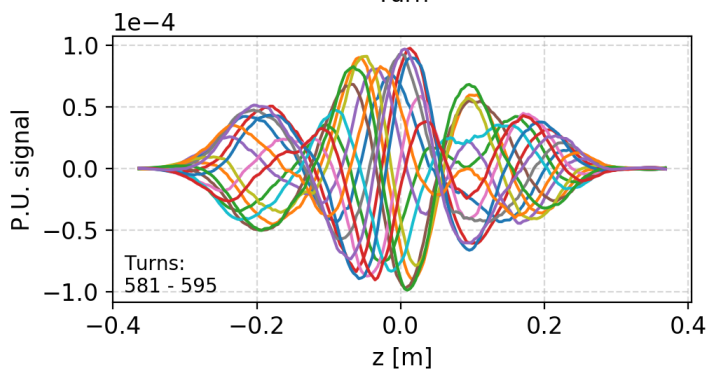
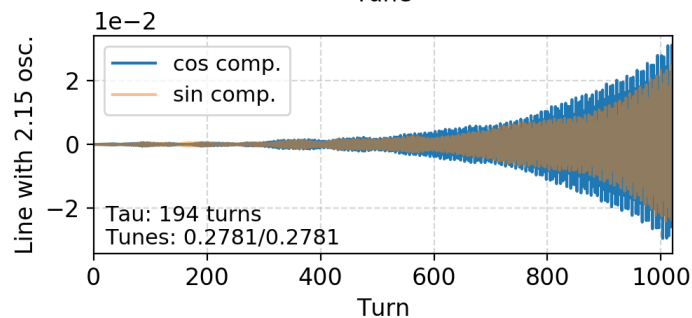
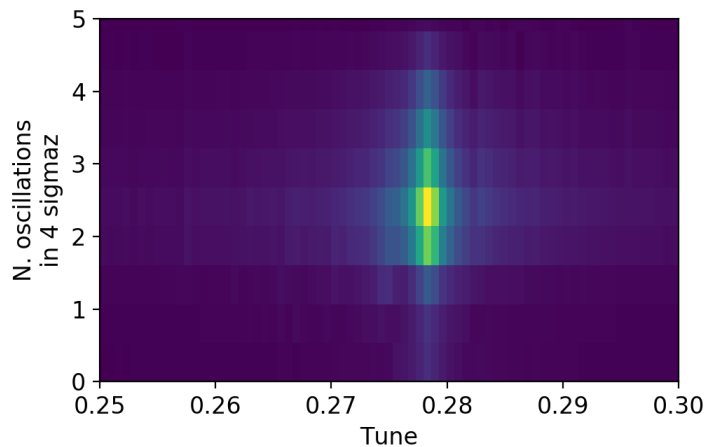


strength 1.2 - **h-matrix + unperturbed map**

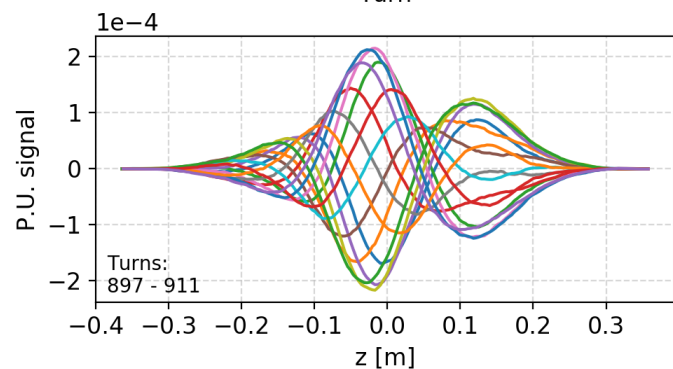
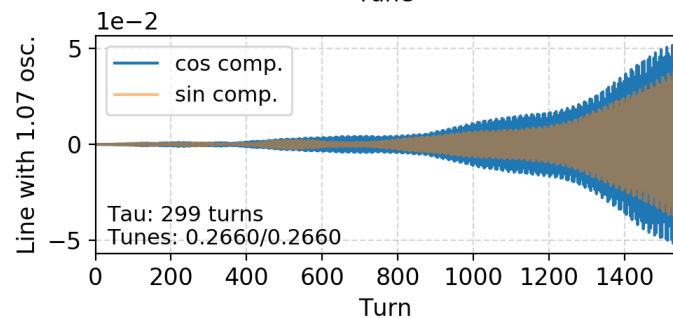
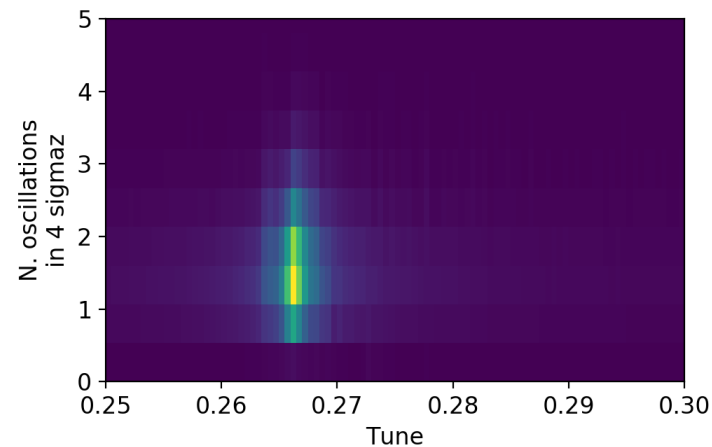


Around realistic strength the behavior changes

strength 1.1 - **h-matrix only**

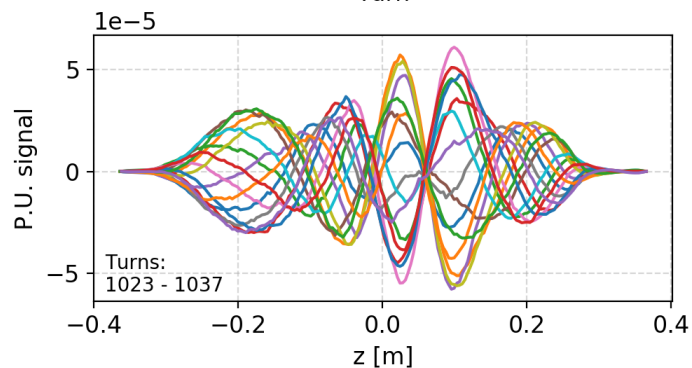
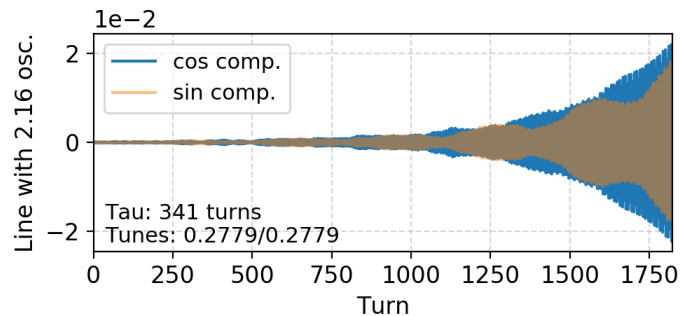
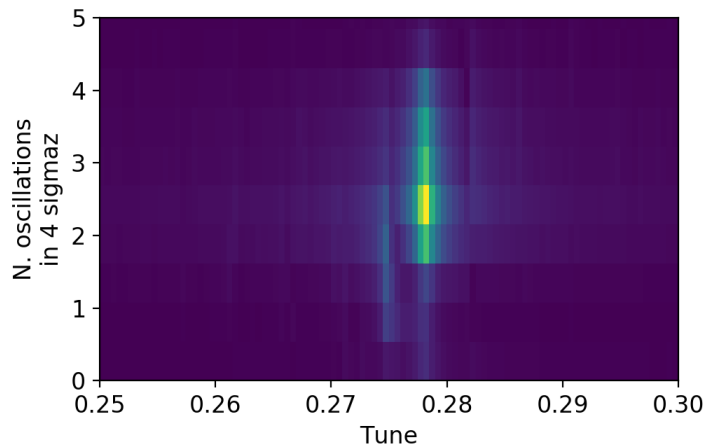


strength 1.1 - **h-matrix + unperturbed map**

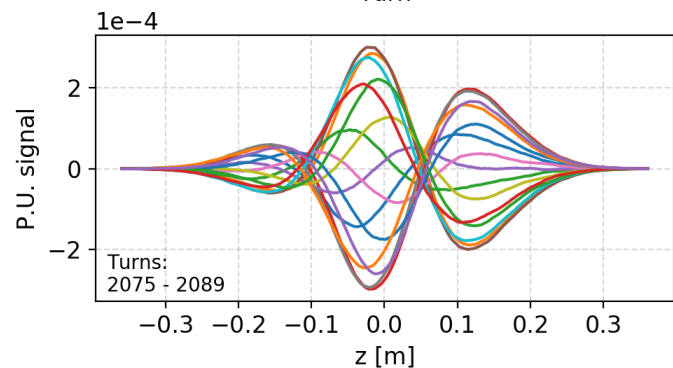
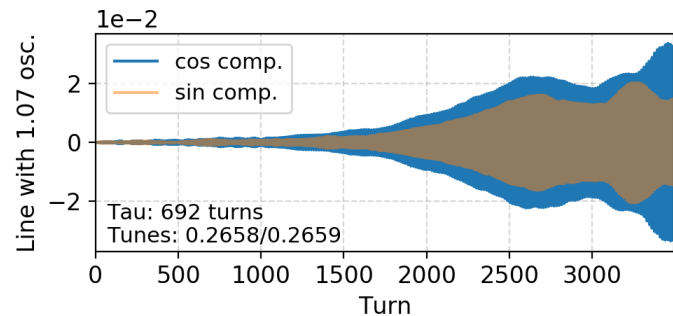
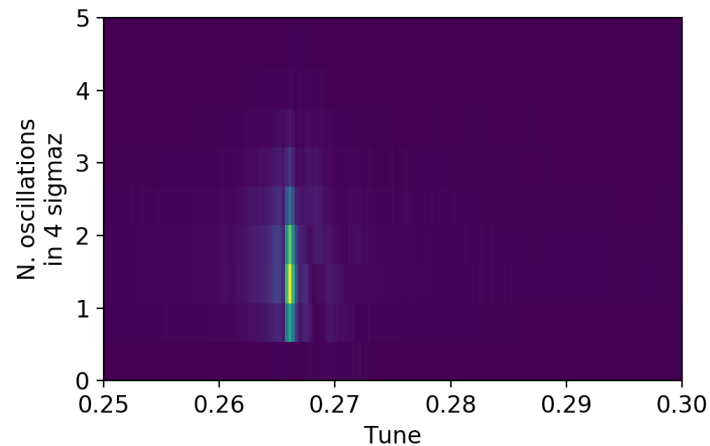


Around realistic strength the behavior changes

strength 1.0 - **h-matrix only**

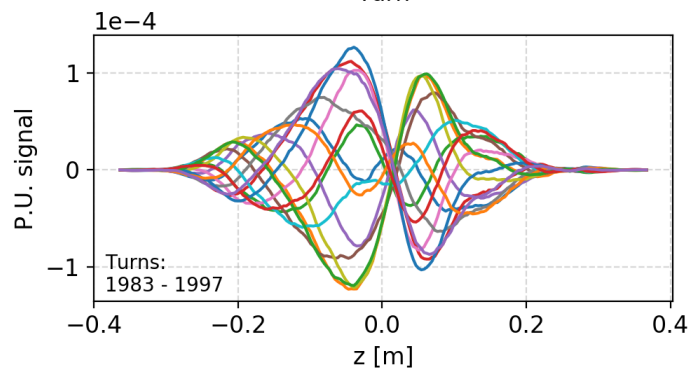
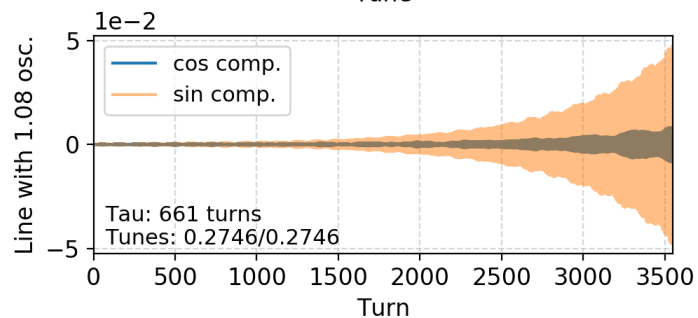
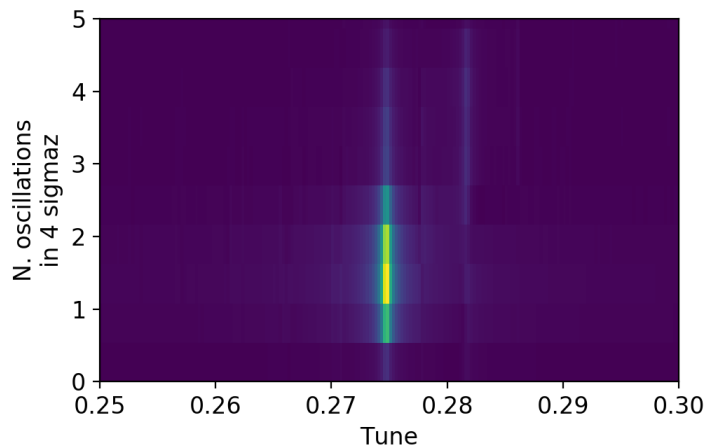


strength 1.0 - **h-matrix + unperturbed map**

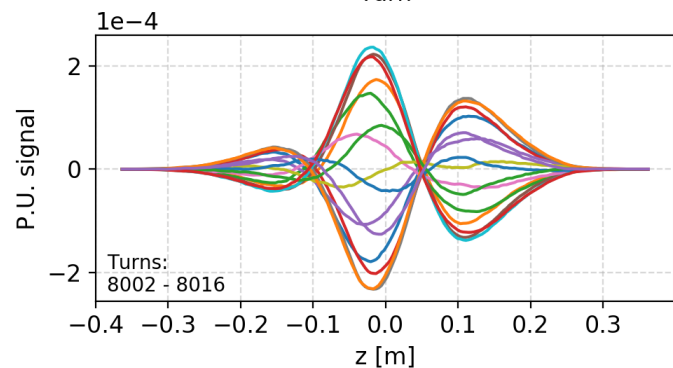
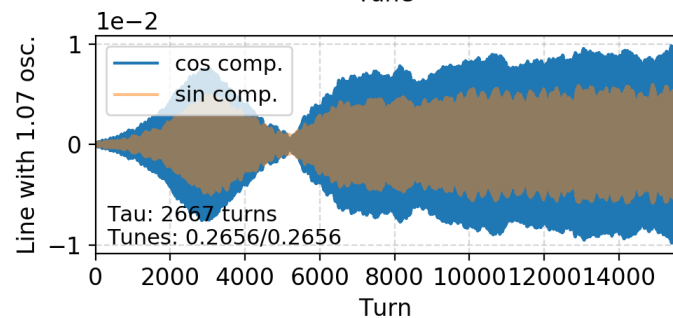
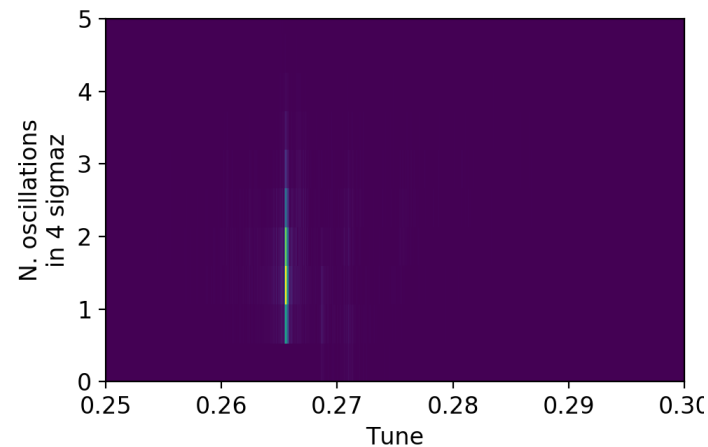


Around realistic strength the behavior changes

strength 0.9 - **h-matrix only**

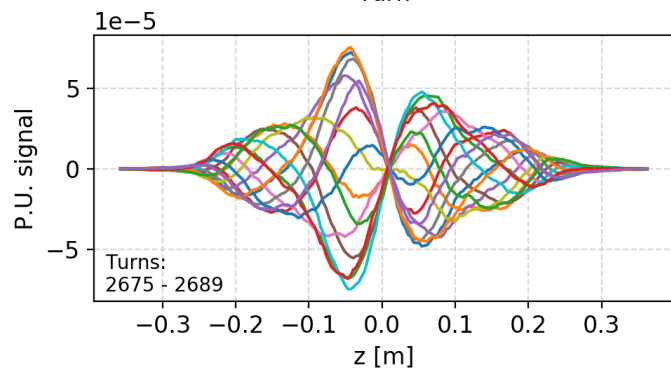
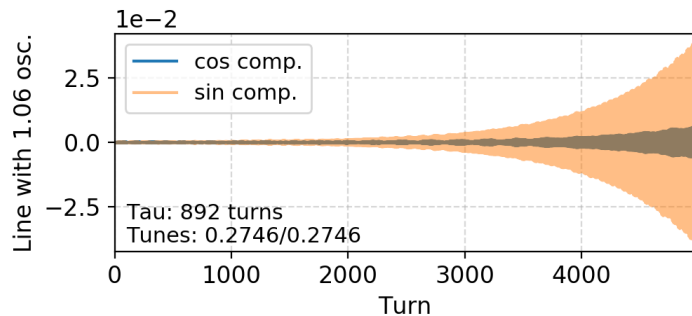
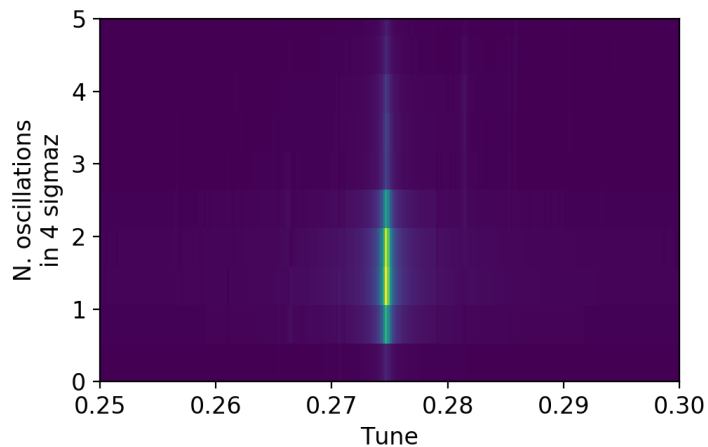


strength 0.9 - **h-matrix + unperturbed map**

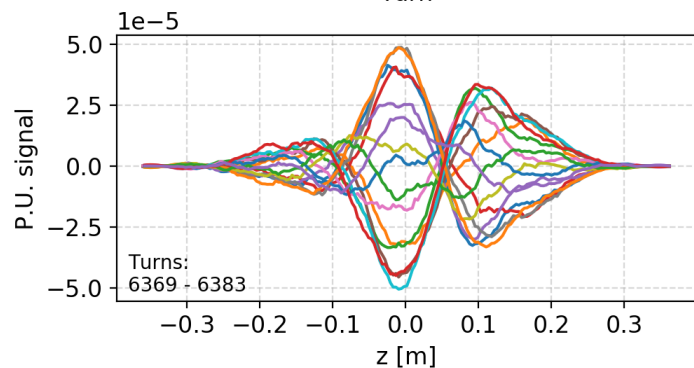
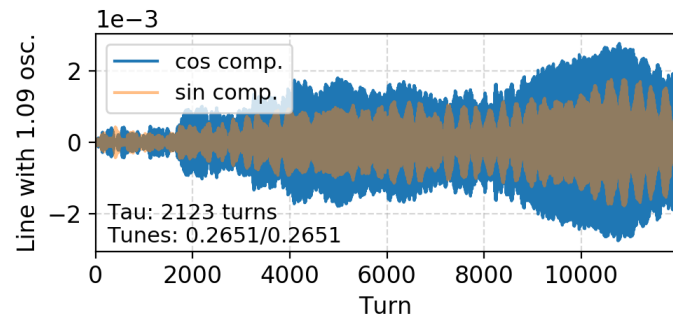
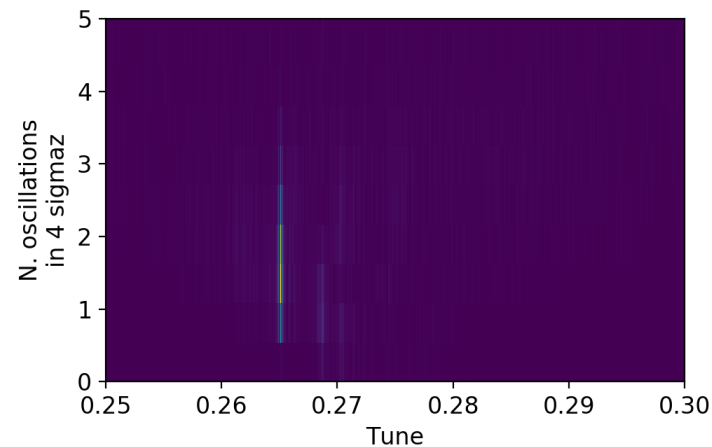


Around realistic strength the behavior changes

strength 0.8 - **h-matrix only**

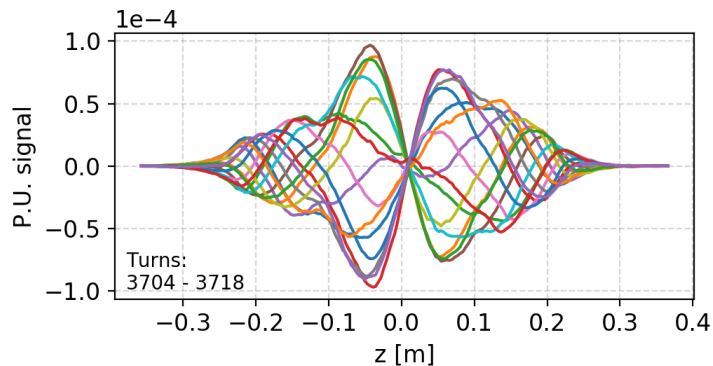
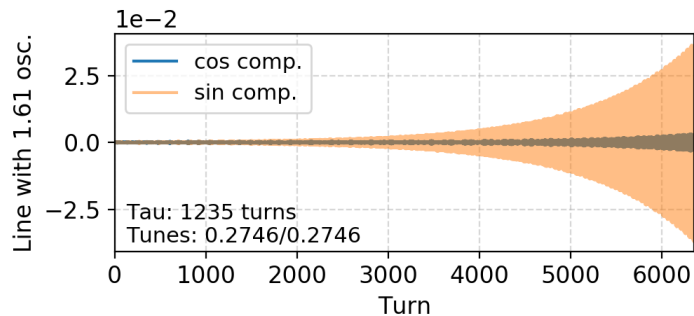
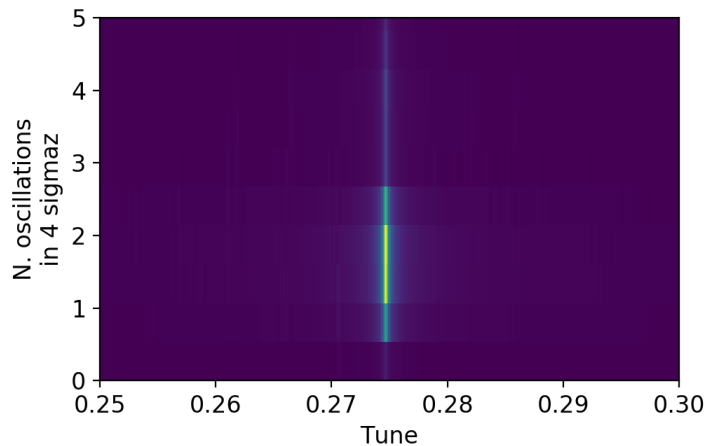


strength 0.8 - **h-matrix + unperturbed map**

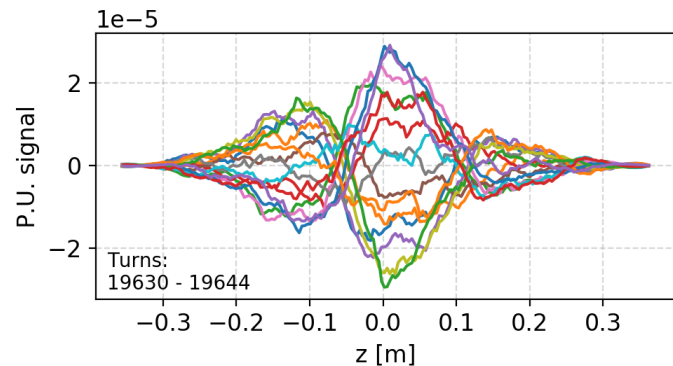
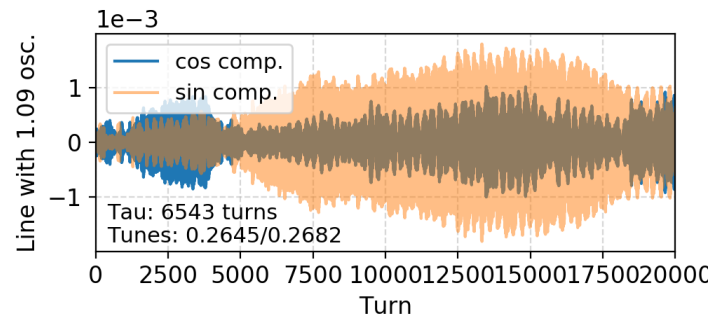
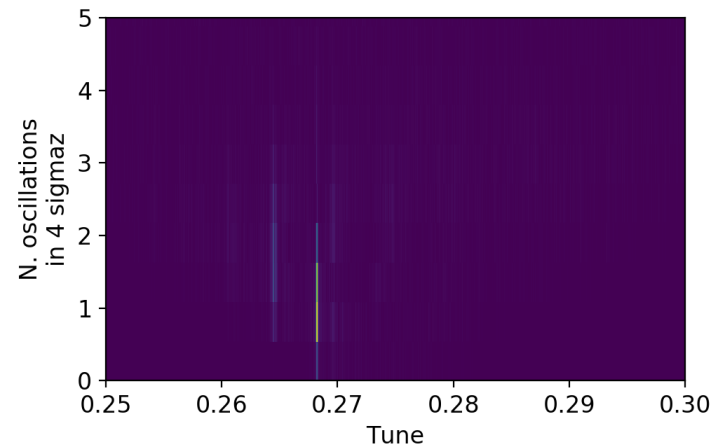


Around realistic strength the behavior changes

strength 0.7 - **h-matrix only**

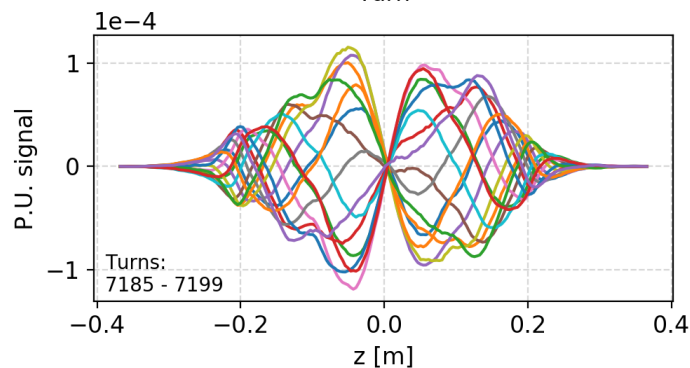
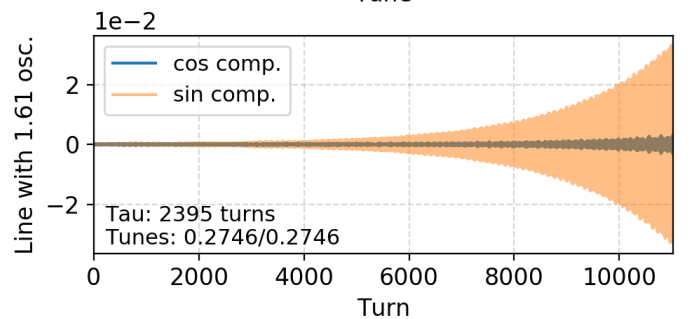
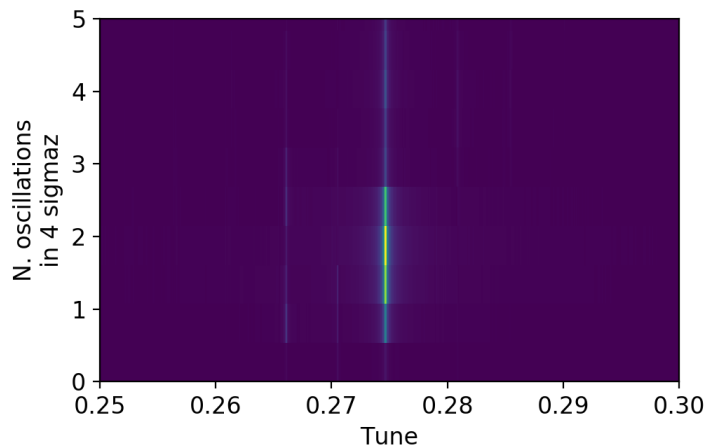


strength 0.7 - **h-matrix + unperturbed map**

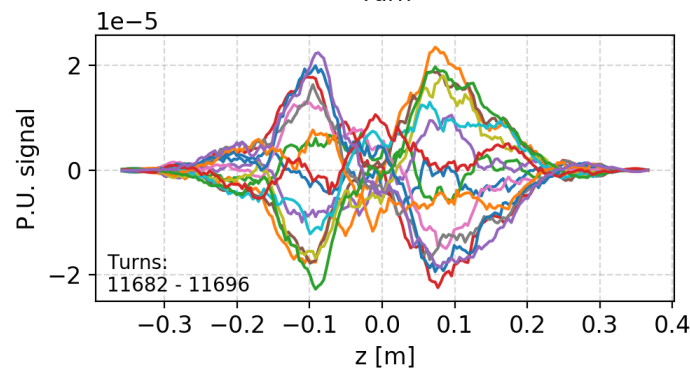
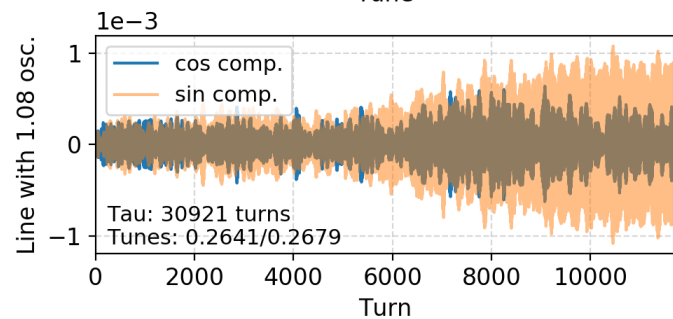
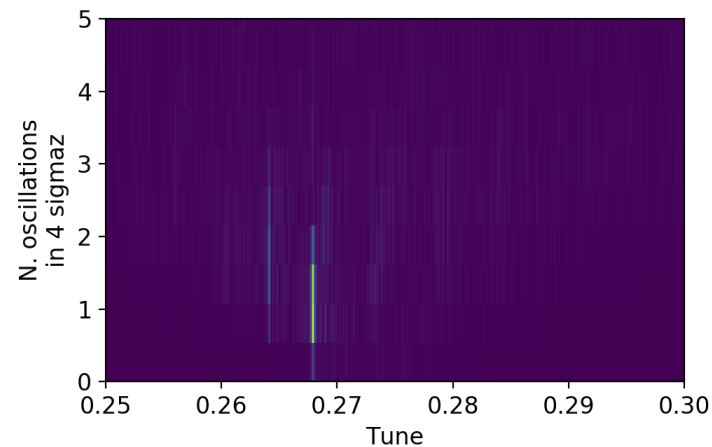


Around realistic strength the behavior changes

strength 0.6 - **h-matrix only**

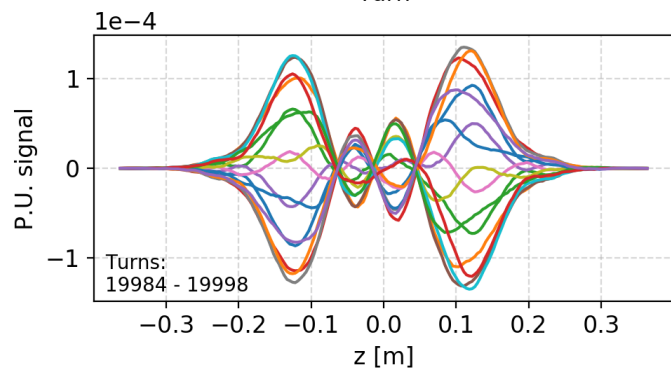
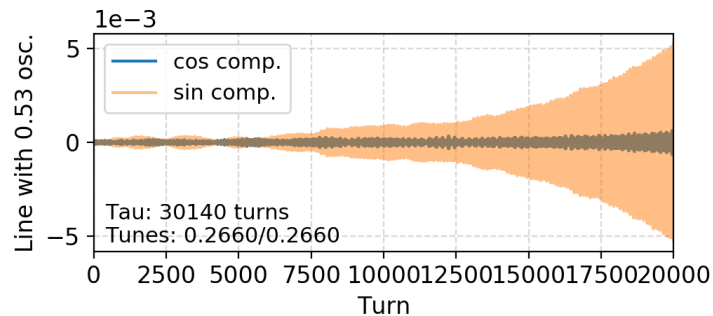
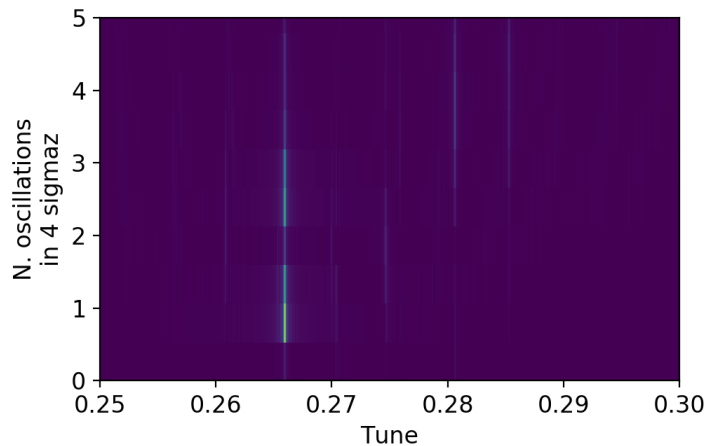


strength 0.6 - **h-matrix + unperturbed map**

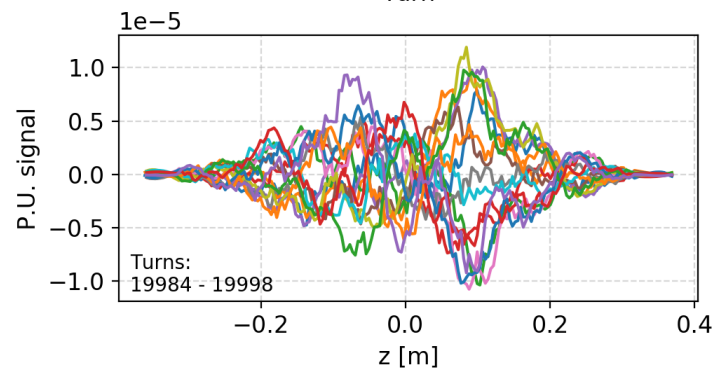
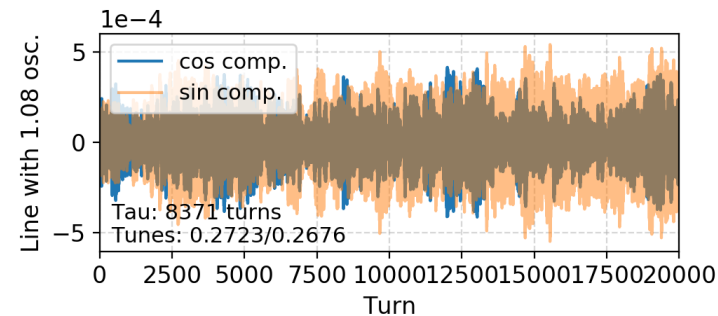
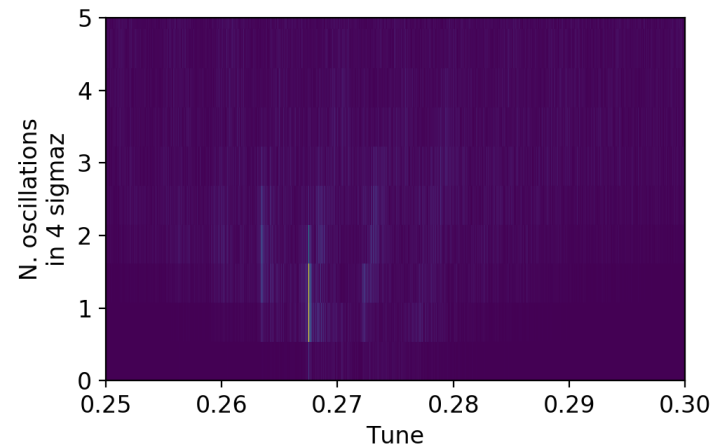


With non-linearities, the threshold in strength is higher

strength 0.5 - **h-matrix only**

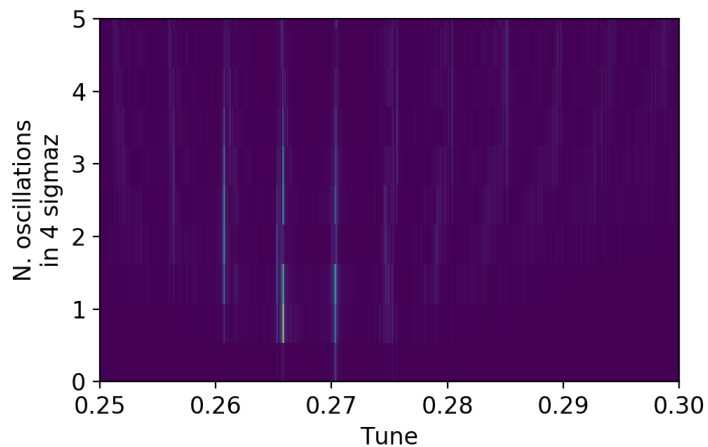


strength 0.5 - **h-matrix + unperturbed map**

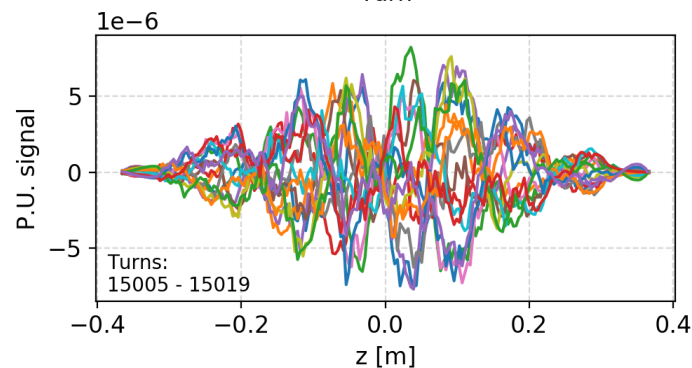
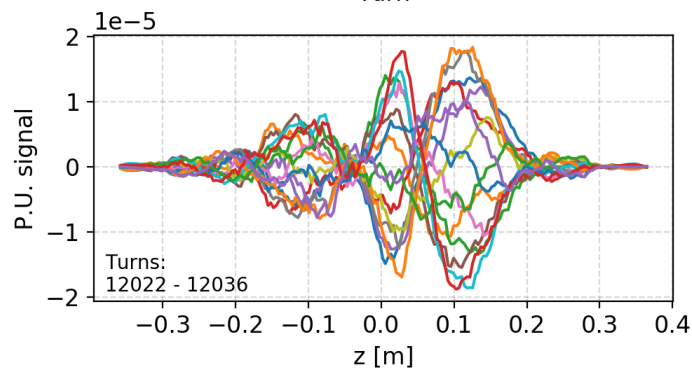
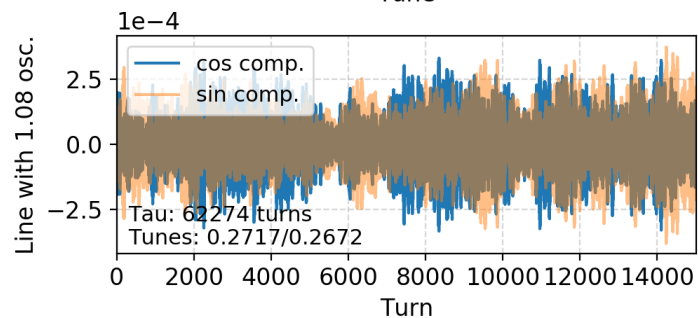
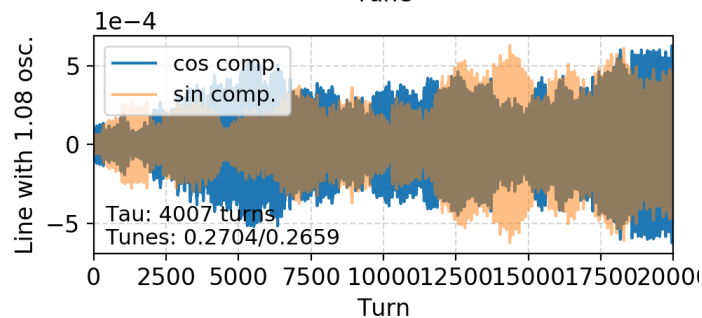
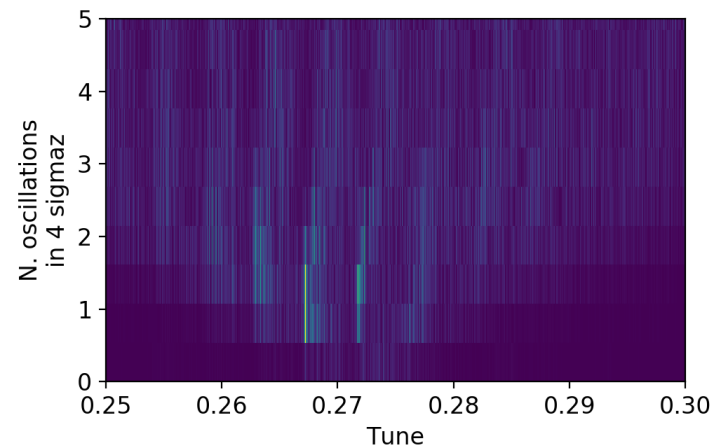


With non-linearities, the threshold in strength is higher

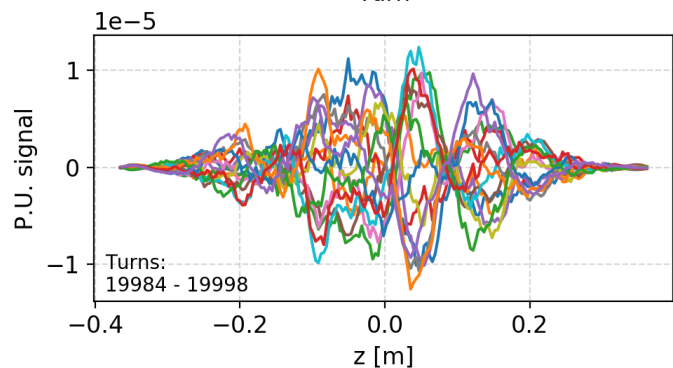
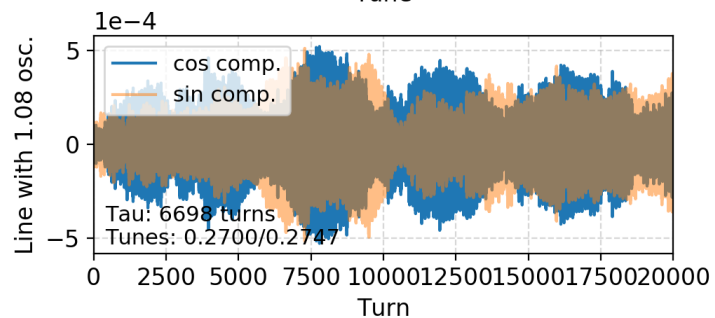
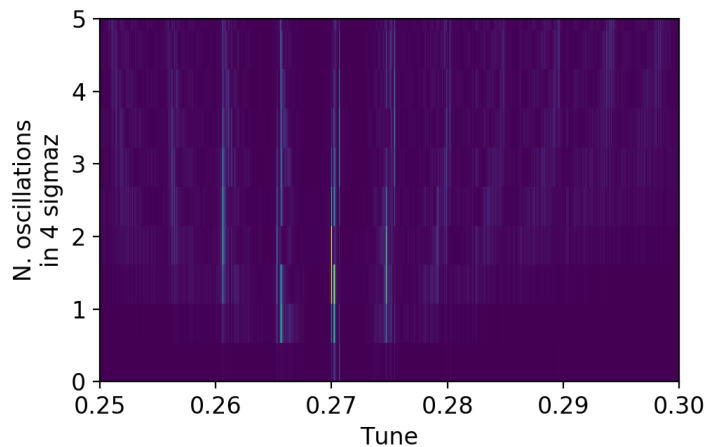
strength 0.4 - **h-matrix only**



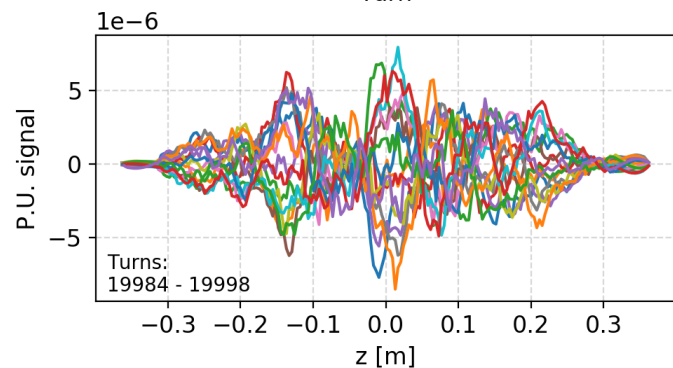
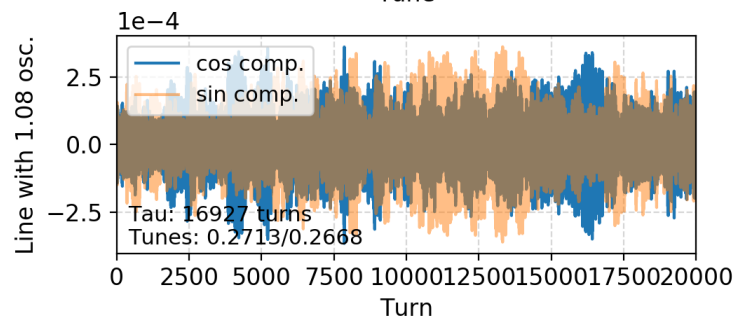
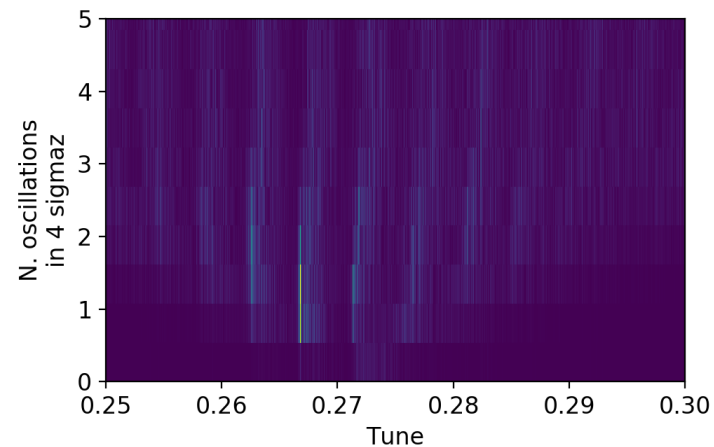
strength 0.4 - **h-matrix + unperturbed map**



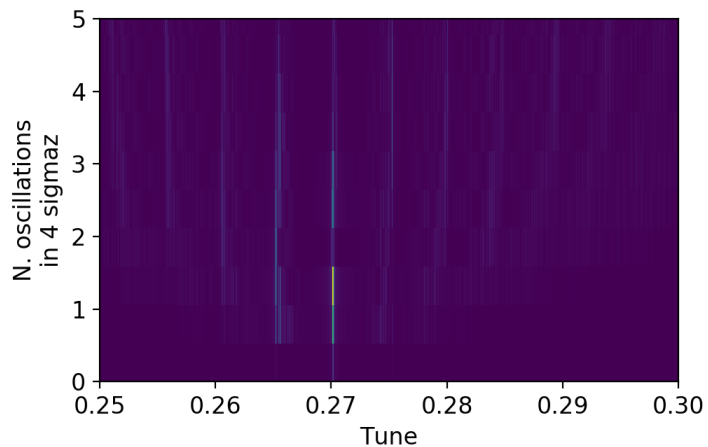
strength 0.3 - **h-matrix only**



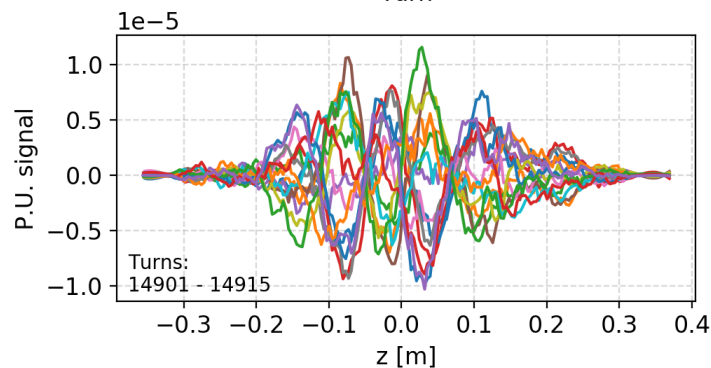
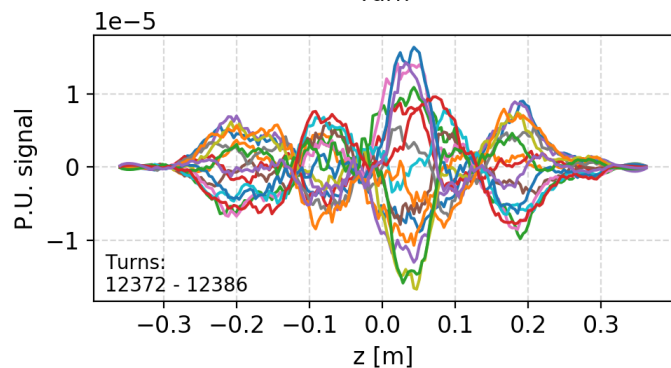
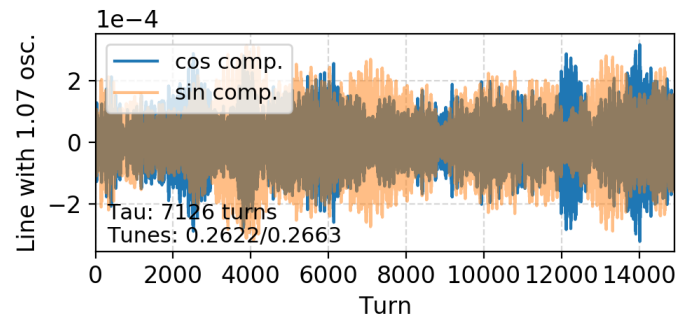
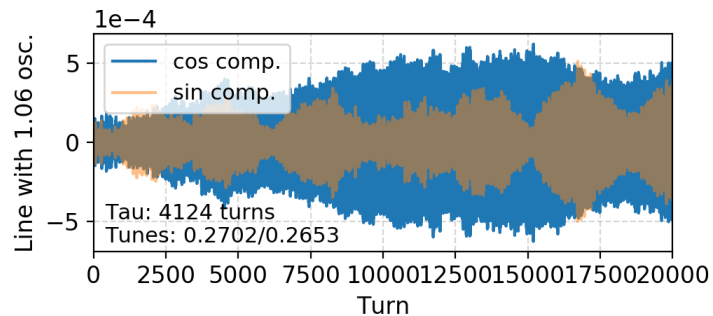
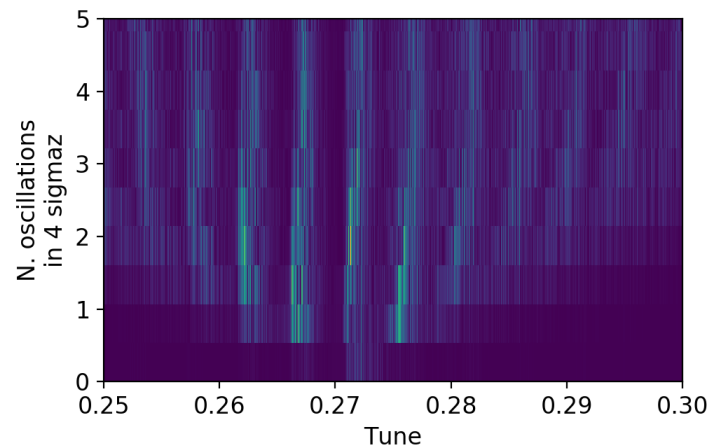
strength 0.3 - **h-matrix + unperturbed map**



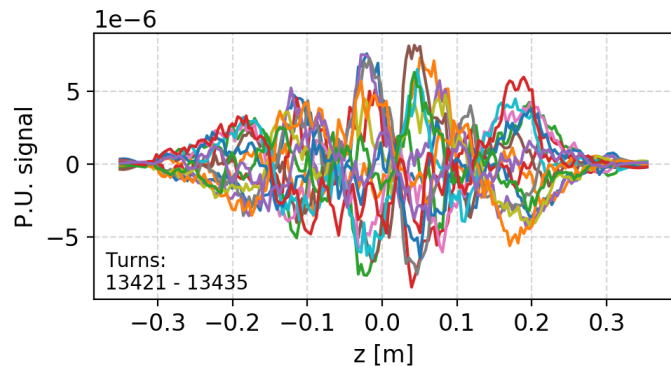
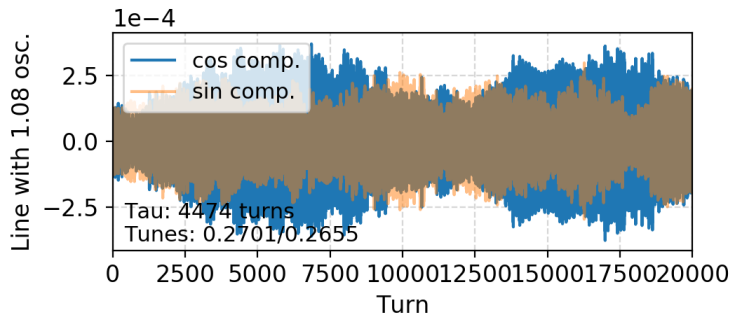
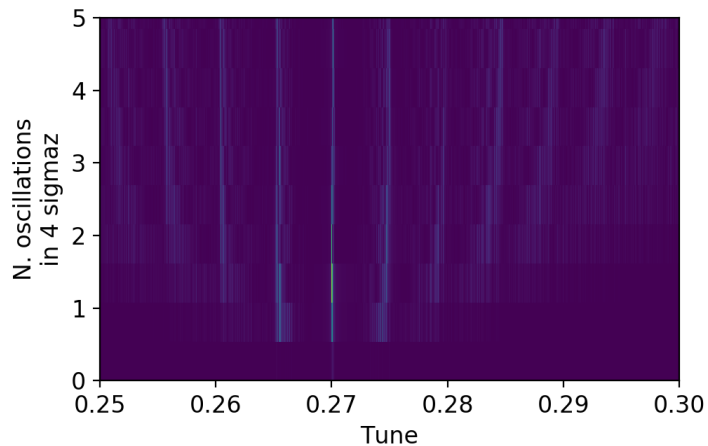
strength 0.2 - **h-matrix only**



strength 0.2 - **h-matrix + unperturbed map**



strength 0.1 - **h-matrix only**



strength 0.1 - **h-matrix + unperturbed map**

