

Simplified modelling for e-cloud instabilities

G. ladarola

Many thanks to:

L. Mether, E. Métral, N. Mounet, G. Rumolo L. Sabato



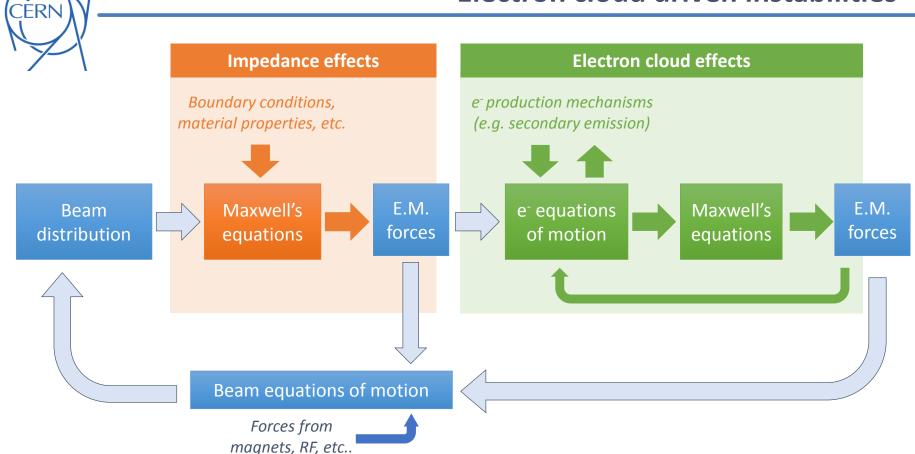
Synthetic description the e-cloud/beam interaction

- Introduction
- Harmonic response
- Superposition check

Instability modelling

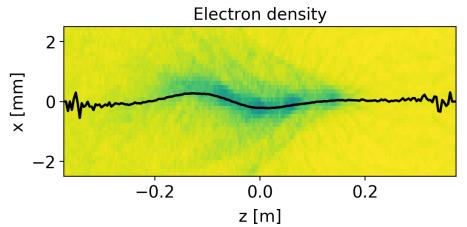
- Harmonic response alone
- Harmonic response + non-linearities
- RF voltage scan
- o e-cloud strength scan

Electron cloud driven instabilities



e-cloud instabilities are studied by simulating the coupled motion of beam and electron particles (very heavy)

→ can we find a simplified model for the effect of the electrons (avoiding the full simulation)?





We choose a set of **orthogonal functions**:

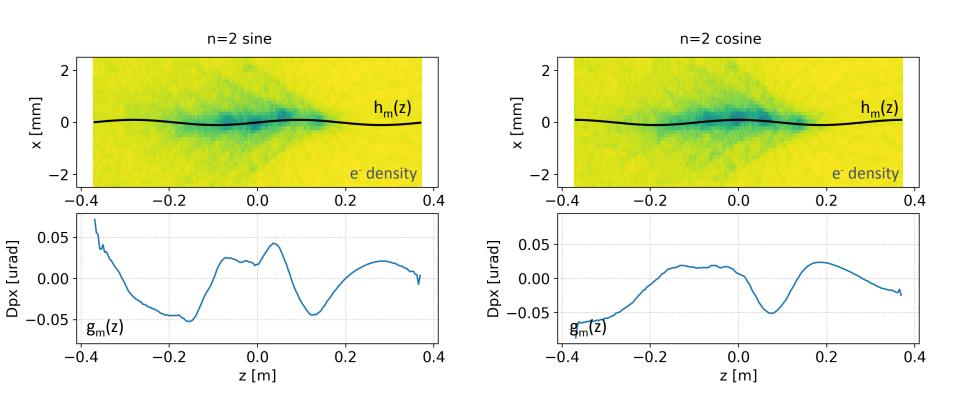
$$h_m\left(z\right) = egin{cases} A_m \cos\left(2\pi rac{m}{2} rac{z}{L_{
m bkt}}
ight), & ext{if } m ext{ is even} \ A_m \sin\left(2\pi rac{m-1}{2} rac{z}{L_{
m bkt}}
ight), & ext{if } m ext{ is odd} \end{cases}$$

We apply to the bunch a distortion equal to $h_m(z)$ to the horizontal position along a bunch and we call $g_m(z)$ the resulting dipolar kick along the bunch.

We can measure the $g_m(z)$ through short PyECLOUD simulations (single pass)

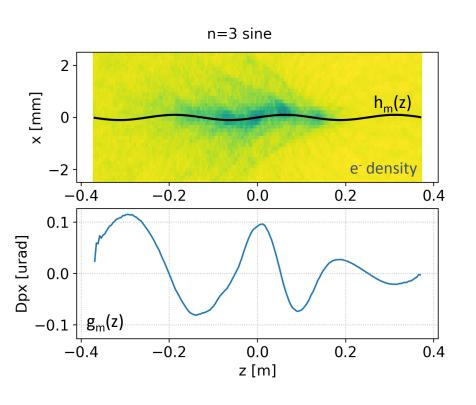


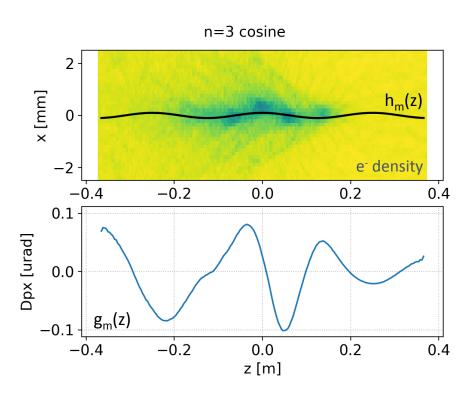






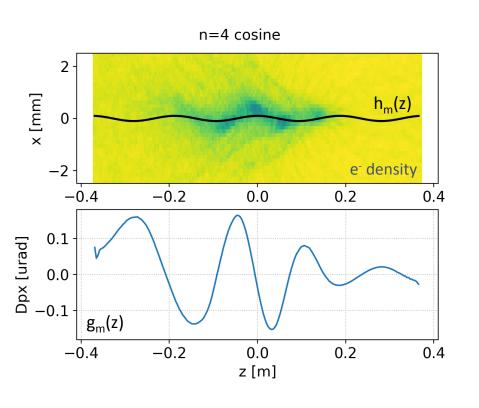


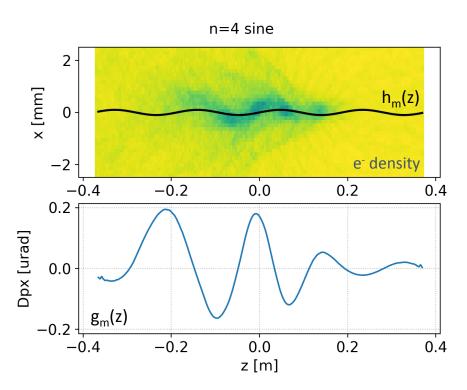






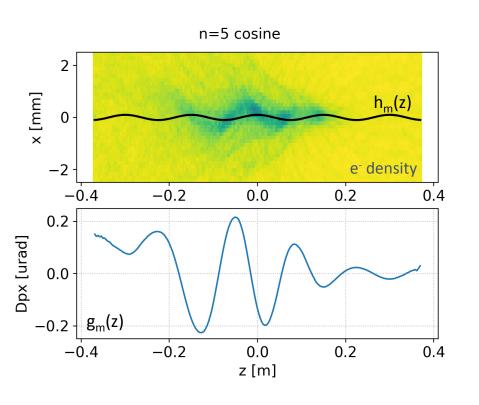


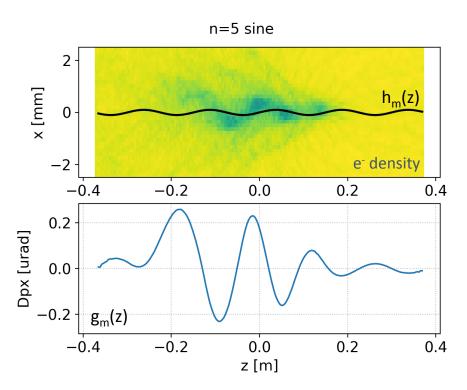






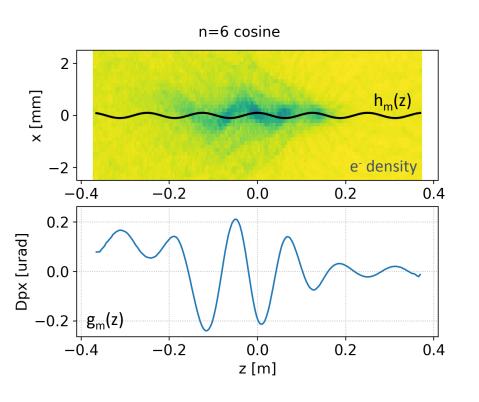


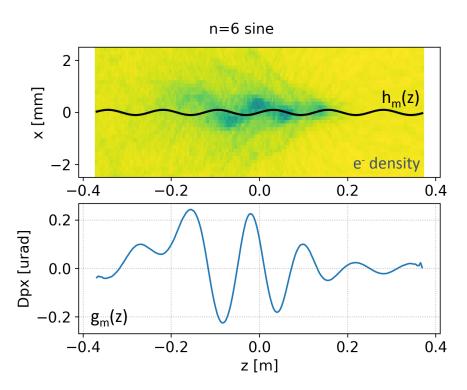






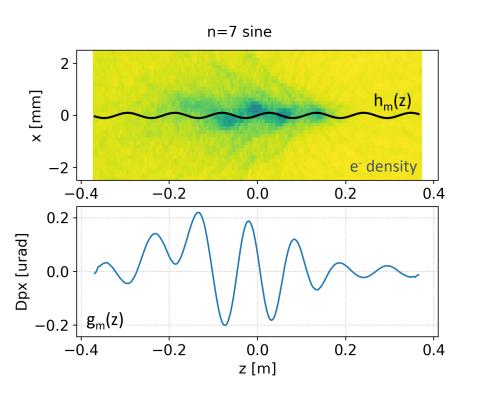


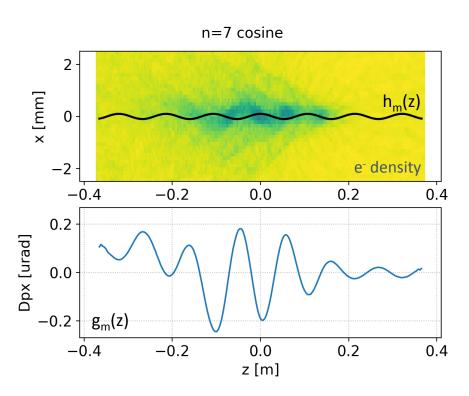






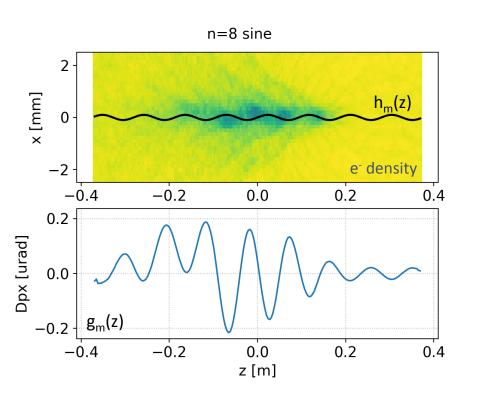


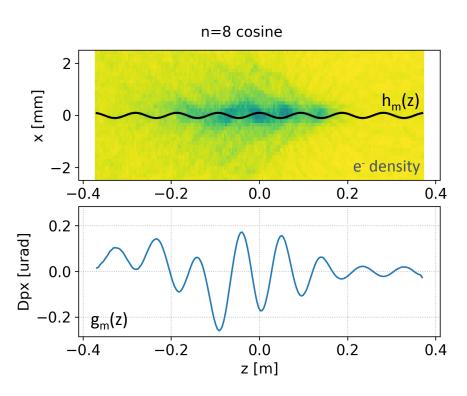






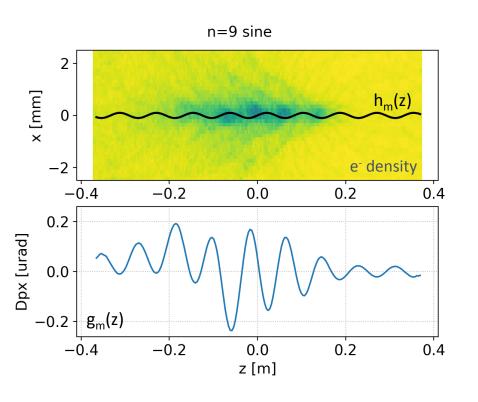


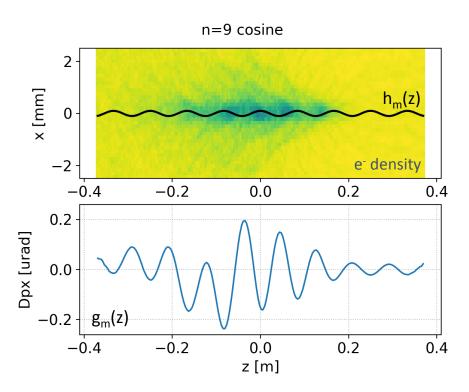






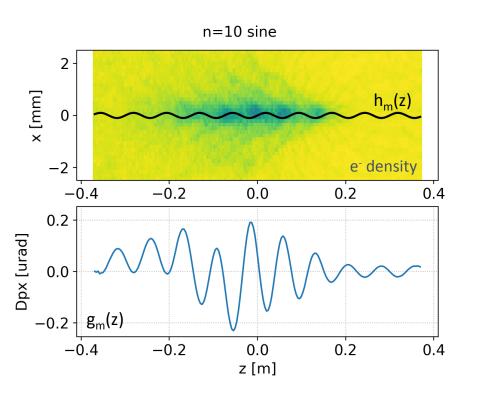


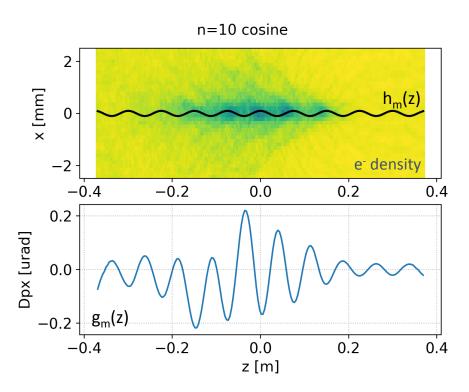






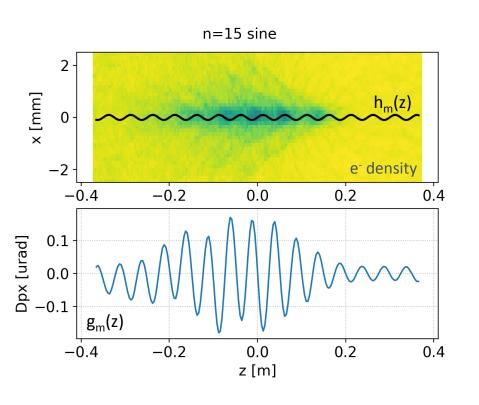


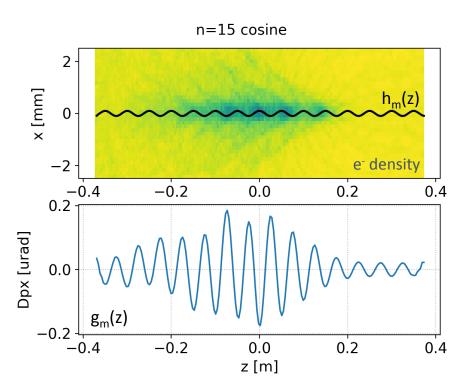






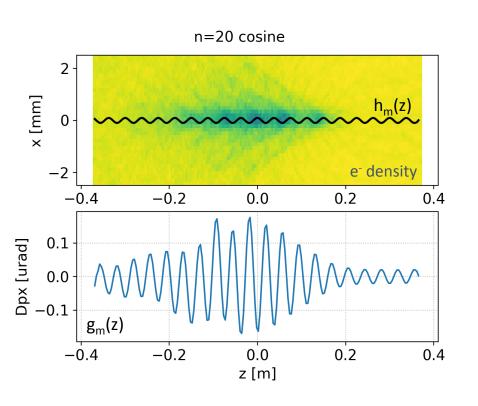


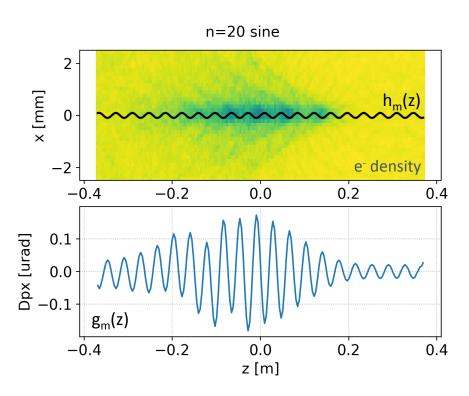






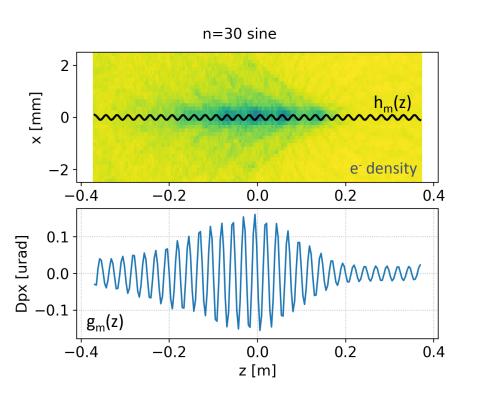


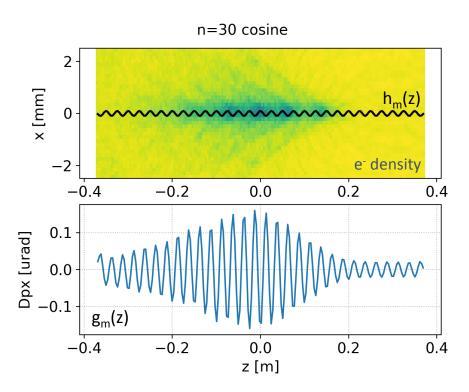






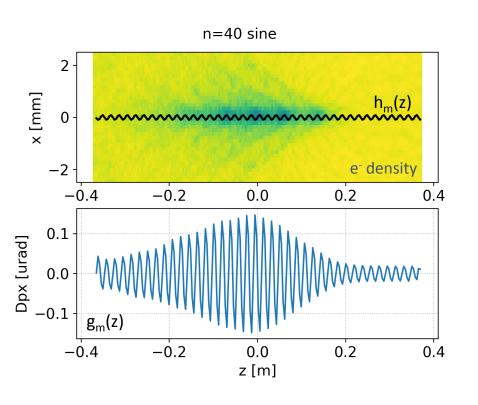


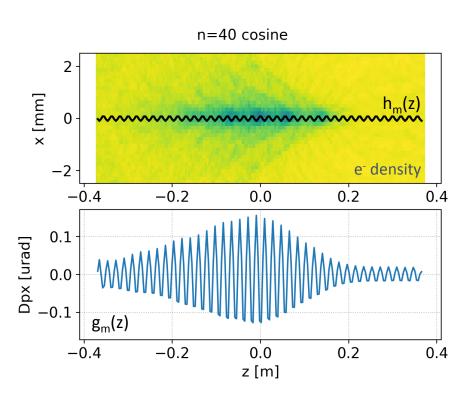














Superposition check





Can I use the calculated harmonic response to **predict the response to** an arbitrary distortion?

A generic distortion x(z) can be decomposed as a superposition of harmonics:

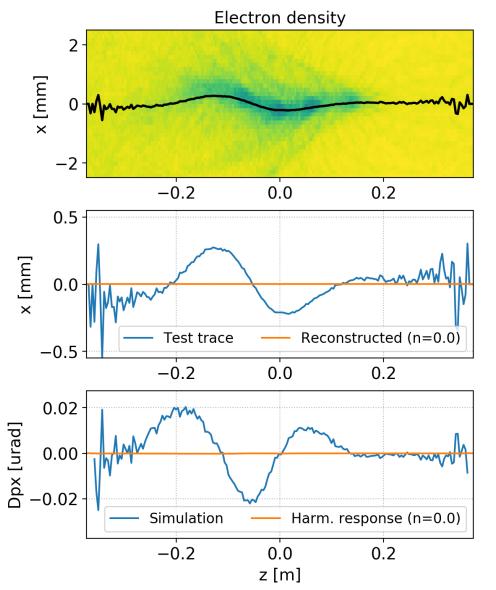
$$x\left(z
ight) = \sum_{m=0}^{M} a_m h_m\left(z
ight)$$
 with: $a_n = \int_{-L_{
m bkt}/2}^{L_{
m bkt}/2} x\left(z
ight) h_m\left(z
ight) dz$

Is the system sufficiently linear to use a linear superposition?

$$\Delta x'\left(z\right) = \sum_{m=0}^{M} a_{m} g_{m}\left(z\right) = \sum_{m=0}^{M} g_{m}\left(z\right) \int_{-L_{\text{bkt}}/2}^{L_{\text{bkt}}/2} x\left(z'\right) h_{m}\left(z'\right) dz'$$





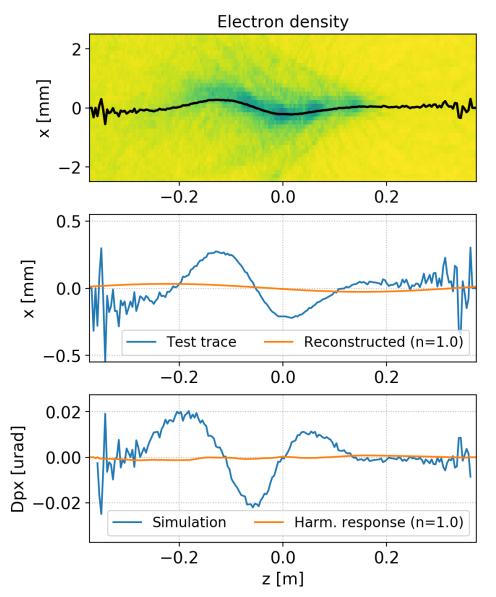


$$x\left(z\right) = \sum_{m=0}^{M} a_{m} h_{m}\left(z\right)$$

$$\Delta x'\left(z
ight) = \sum_{m=0}^{M} a_{m}g_{m}\left(z
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 :





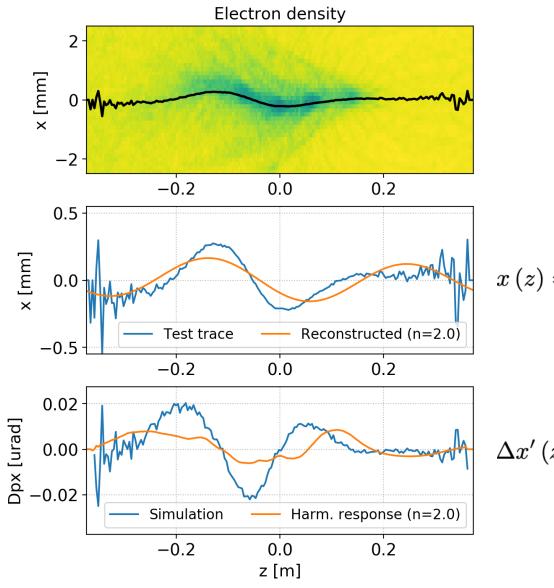


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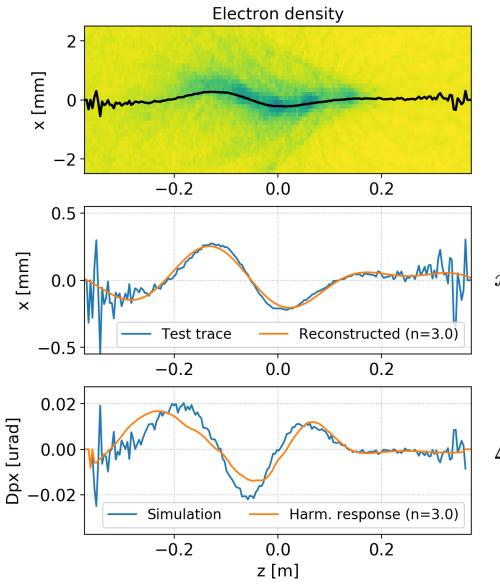


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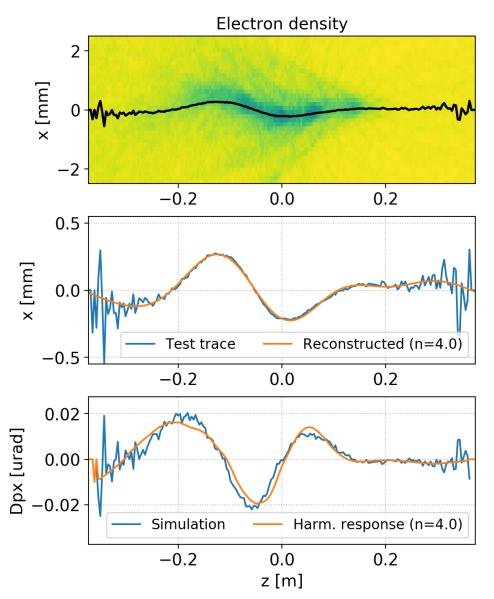


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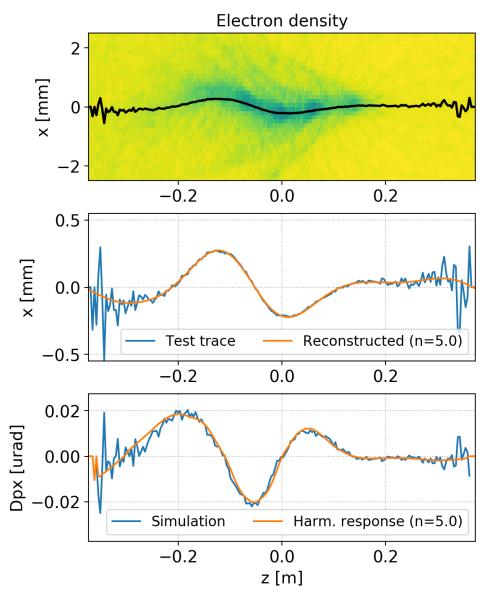


$$x\left(z\right) = \sum_{m=0}^{M} a_{m} h_{m}\left(z\right)$$

$$\Delta x'(z) = \sum_{m=0}^{M} a_m g_m(z)$$
:





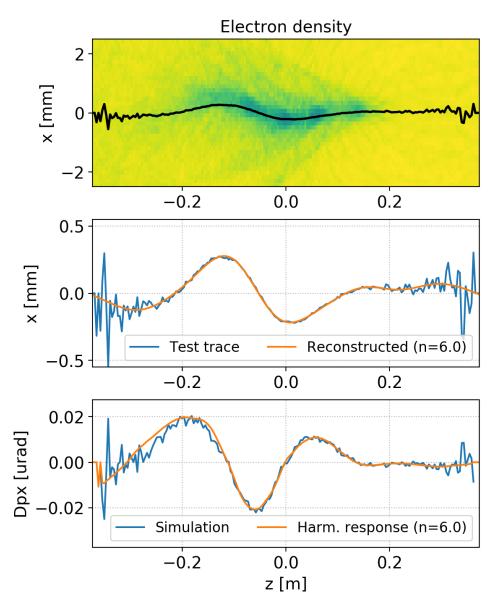


$$x\left(z\right) = \sum_{m=0}^{M} a_{m} h_{m}\left(z\right)$$

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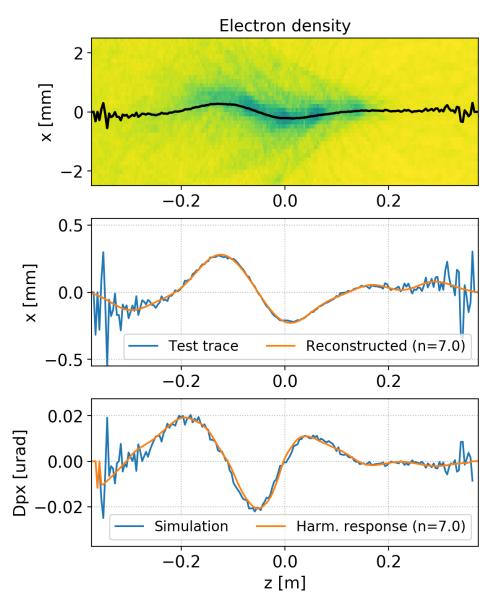


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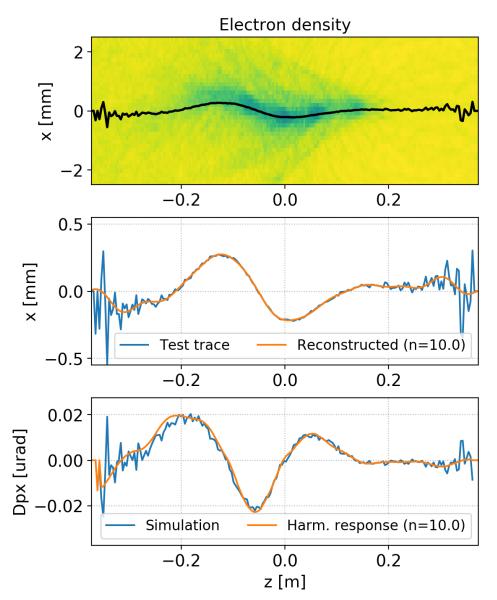


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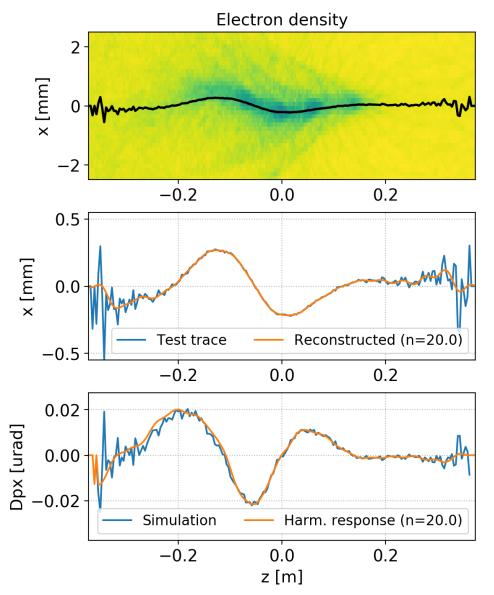


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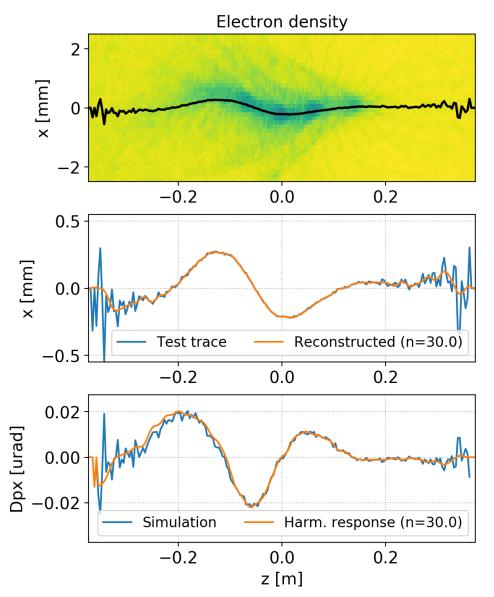


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:





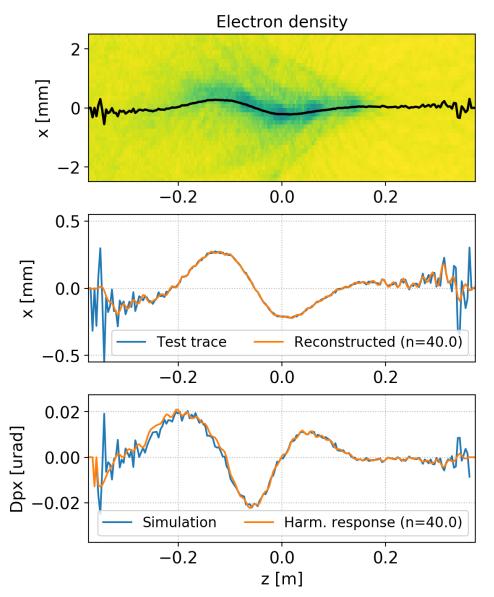


$$x\left(z\right) = \sum_{m=0}^{M} a_{m} h_{m}\left(z\right)$$

$$\Delta x'(z) = \sum_{m=0}^{M} a_m g_m(z)$$
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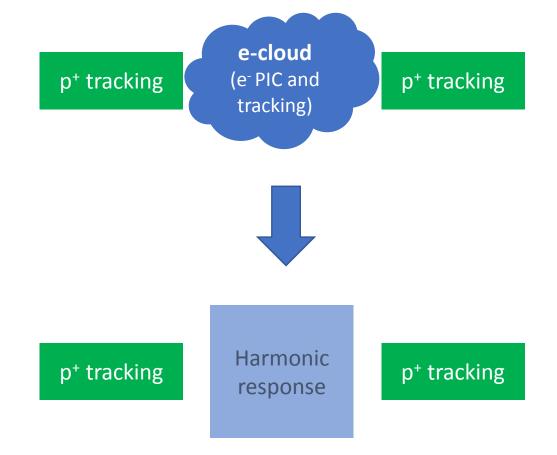
Instability modelling

- Harmonic response alone
- Harmonic response + non-linearities
- o RF voltage scan
- o e-cloud strength scan



Can we use the harmonic response to model instabilities?

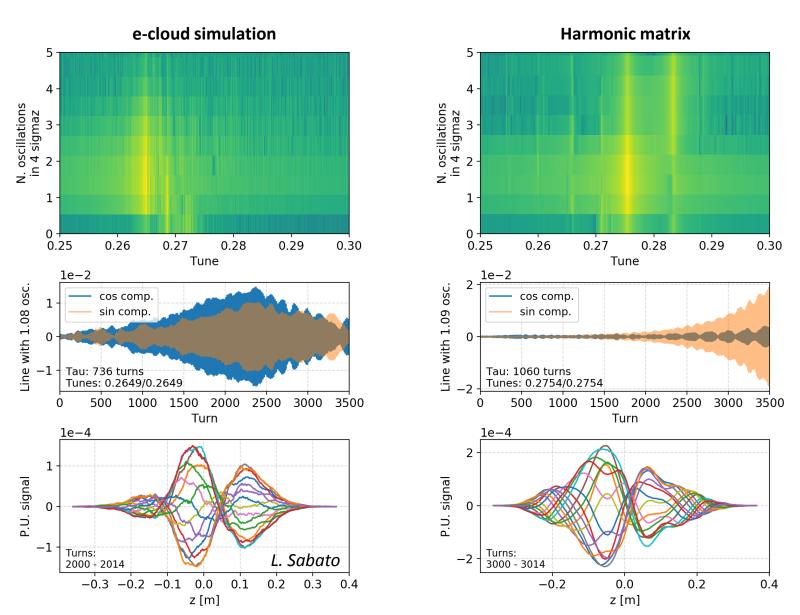
In the PyHEADTAIL model we replace the full e-cloud interactions with the computed harmonic response matrix





Comparison against simulations

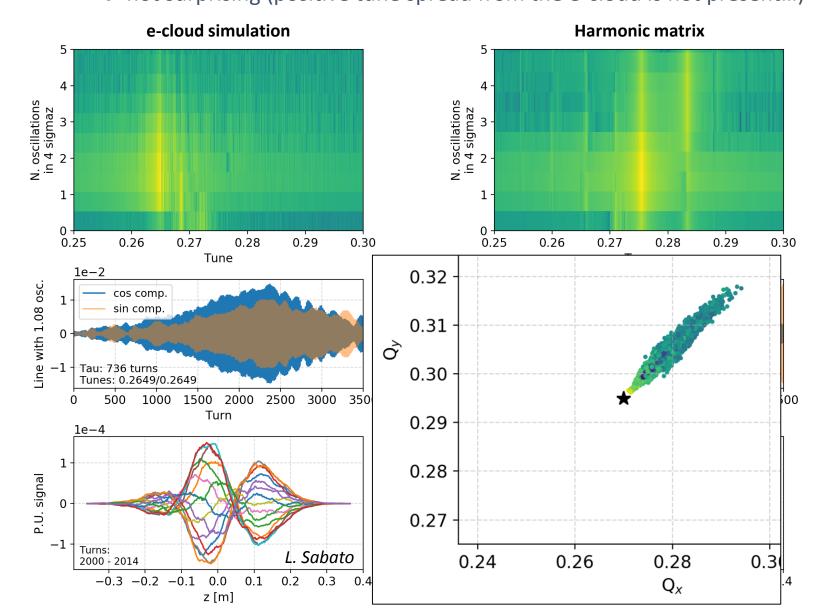
With the matrix alone the **instability is not reproduced** (frequency is different)





Comparison against simulations

With the matrix alone the **instability is not reproduced** (frequency is different) → not surprising (positive tune spread from the e-cloud is not present...)





Synthetic description the e-cloud/beam interaction

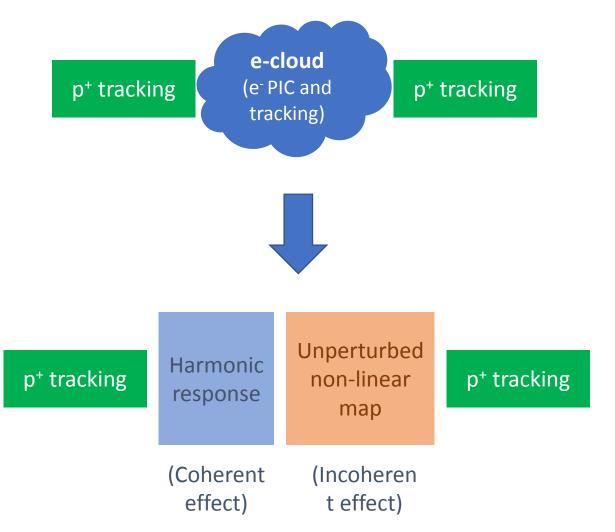
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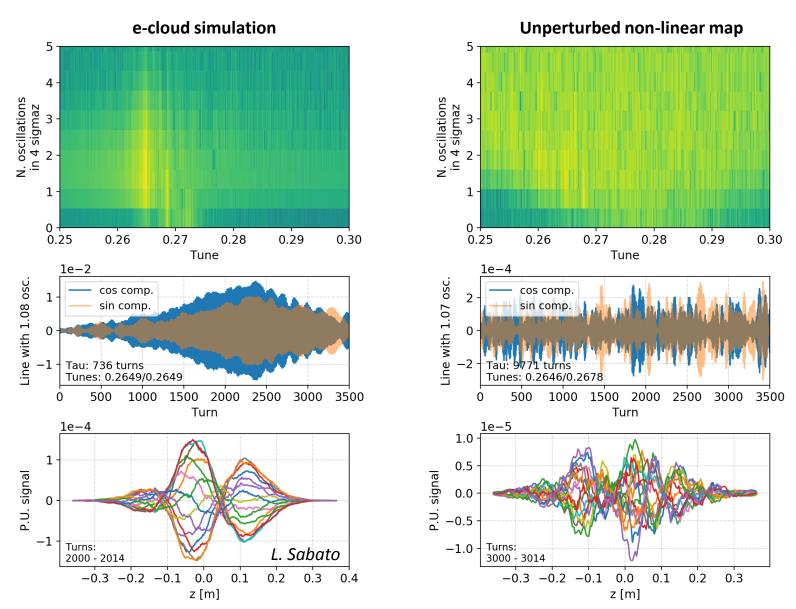


We model the single particle non-linear detuning of the e-cloud using the field map from the unperturbed distribution

(re-centered around each
slice → no additional dipolar
kick)



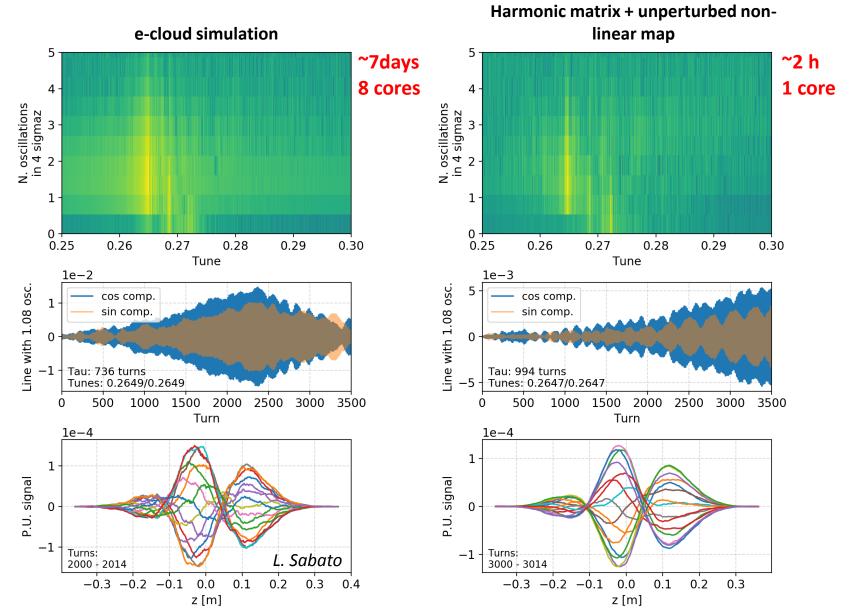
With the non-linear map alone no instability is observed





Comparison against simulations

This combined model reproduces very well the nature of the instability!





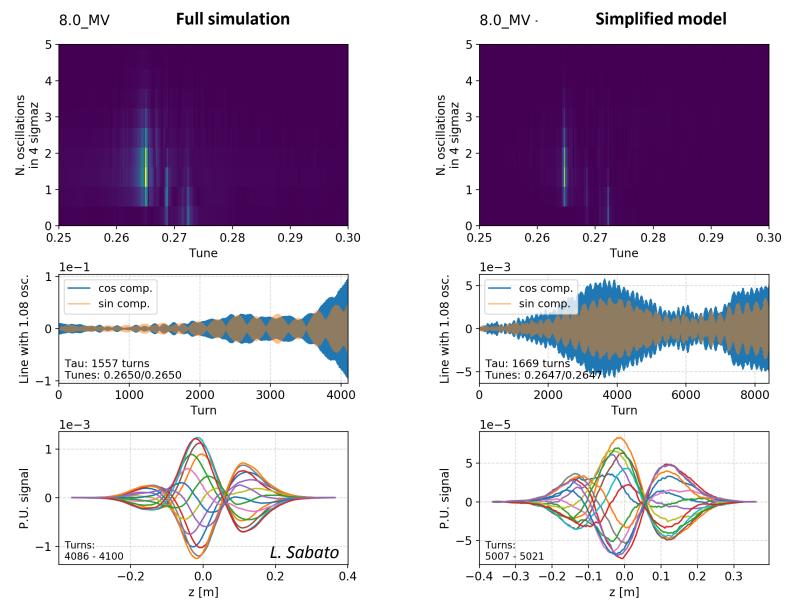
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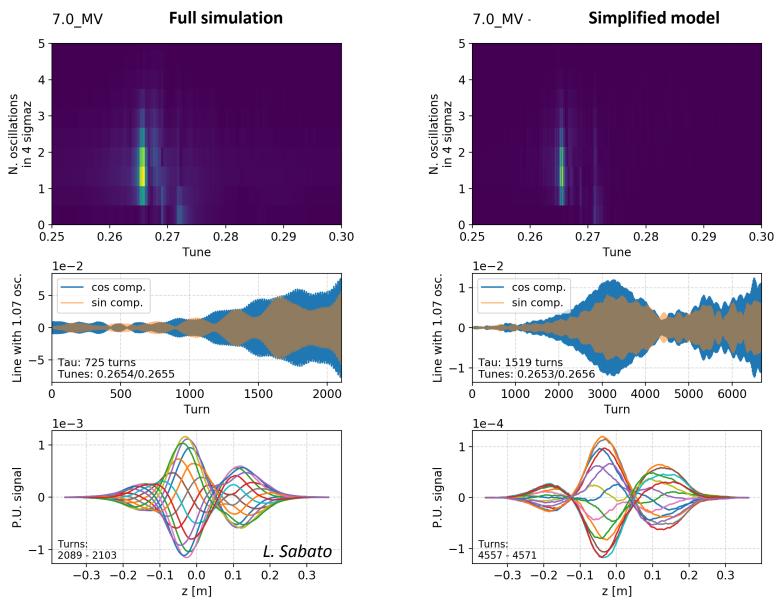
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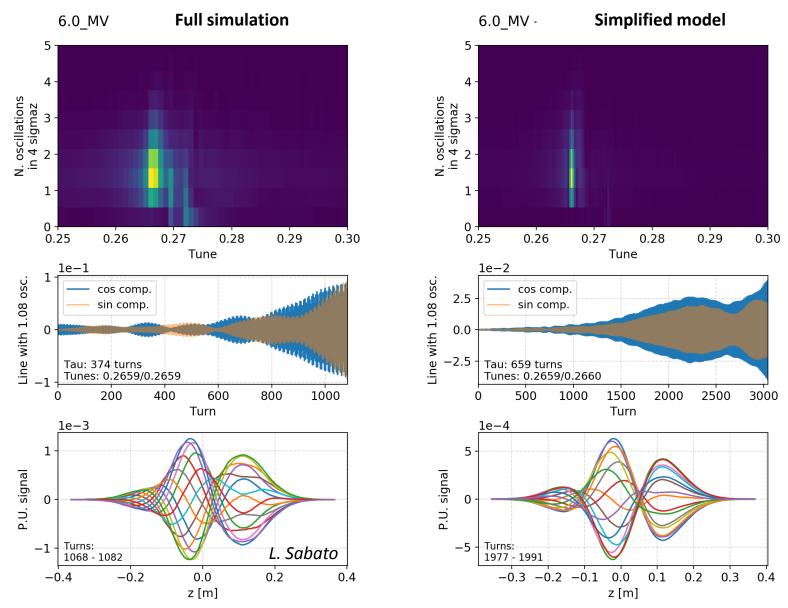




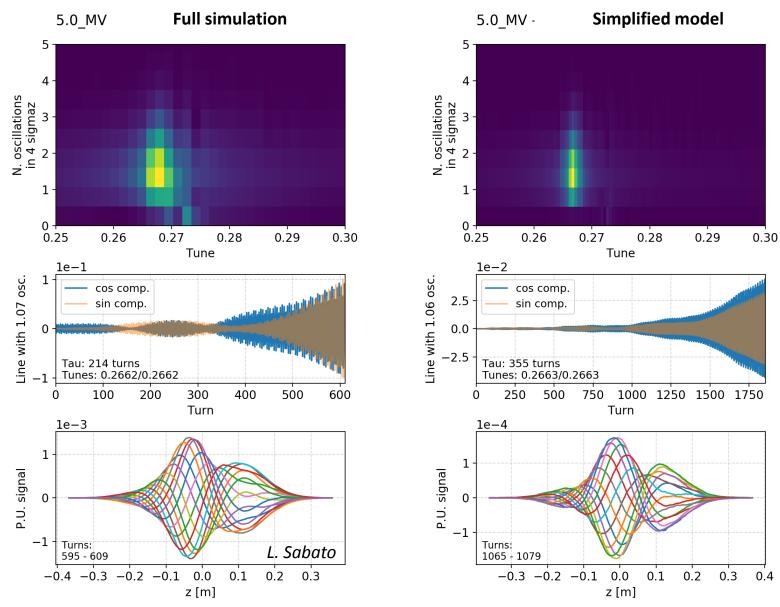




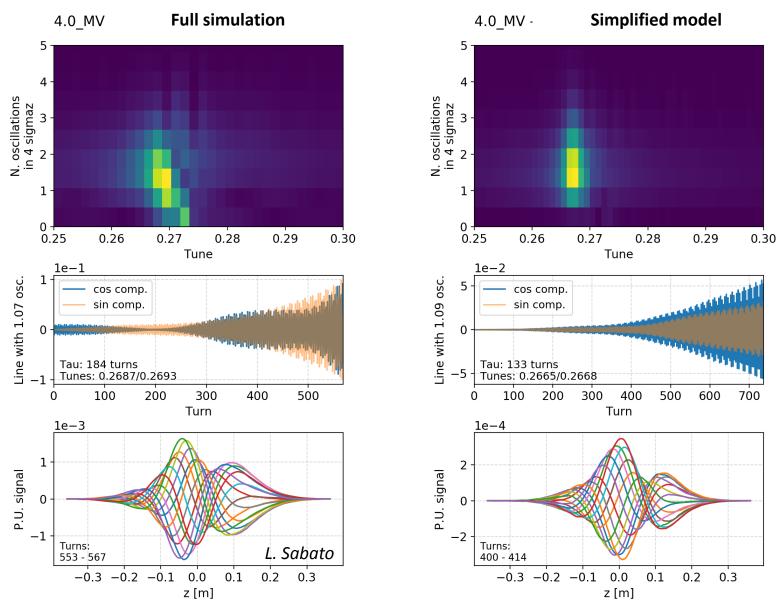




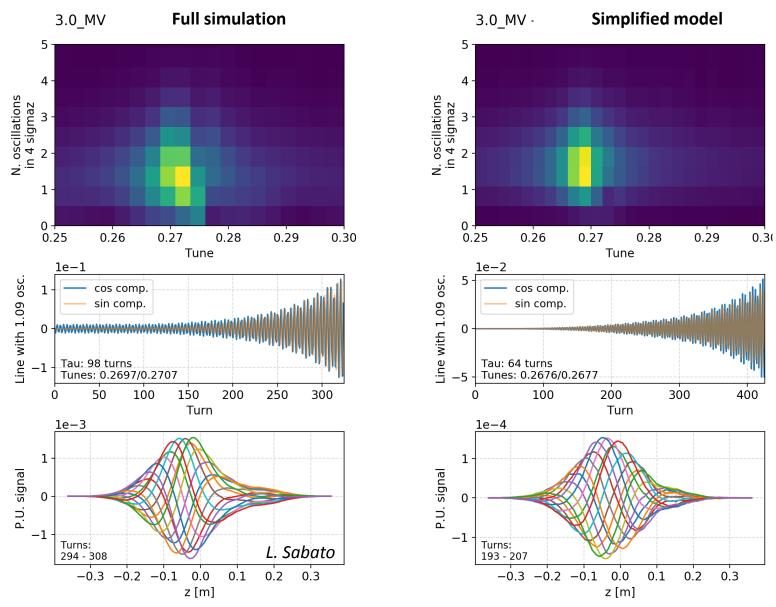














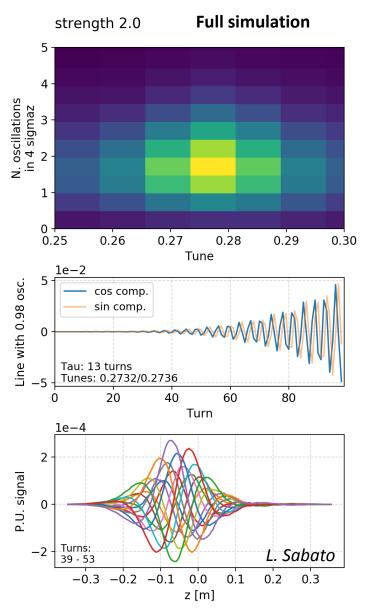
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strength 2.0 - h-matrix + unperturbed map 5 4 -N. oscillations in 4 sigmaz 1 -0 | 0.25 0.27 0.26 0.28 0.29 0.30 Tune 1e-1 Line with 1.09 osc. cos comp. sin comp. Tau: 19 turns Tunes: 0.2710/0.2712 40 60 80 20 100 120 Turn 1e-4 5 P.U. signal Turns: 56 - 70

z [m]

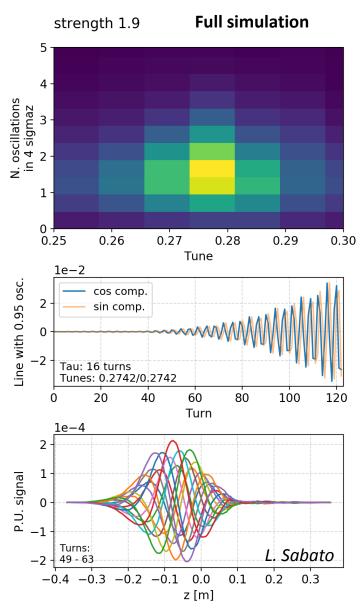
0.2

0.4

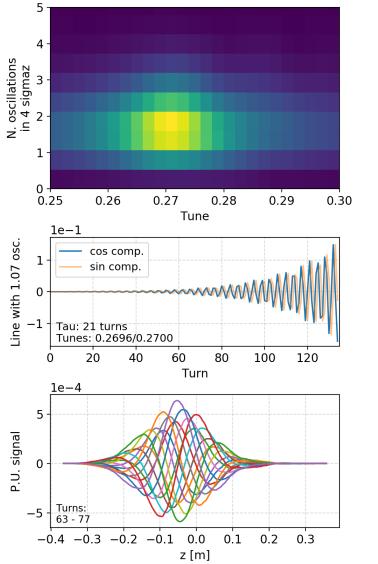
-0.2

-0.4

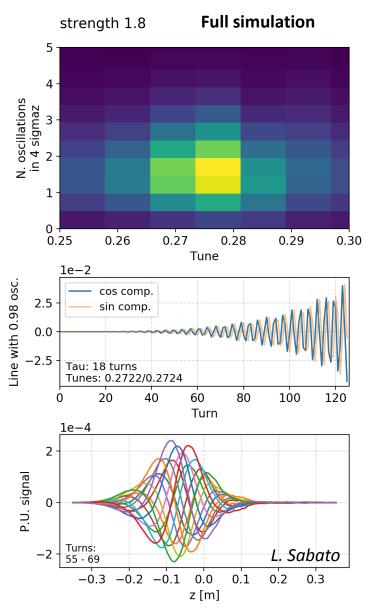




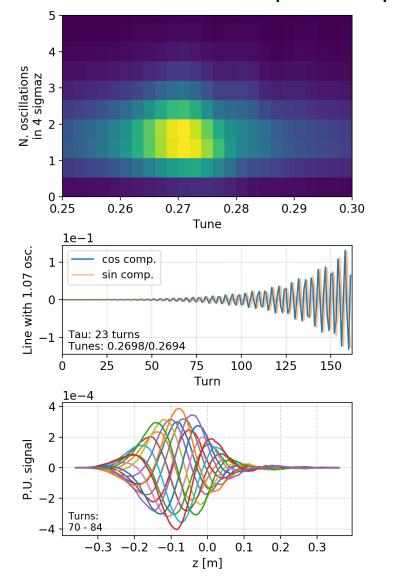
strength 1.9 - h-matrix + unperturbed map



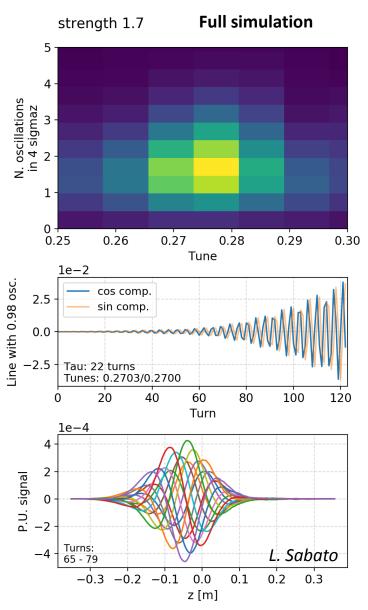




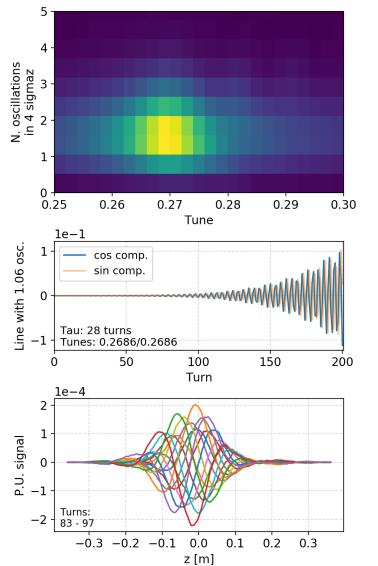
strength 1.8 - h-matrix + unperturbed map



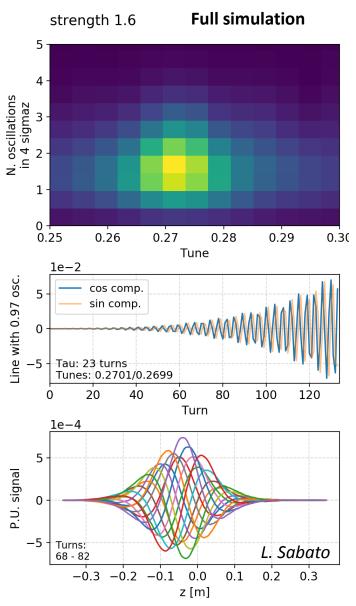




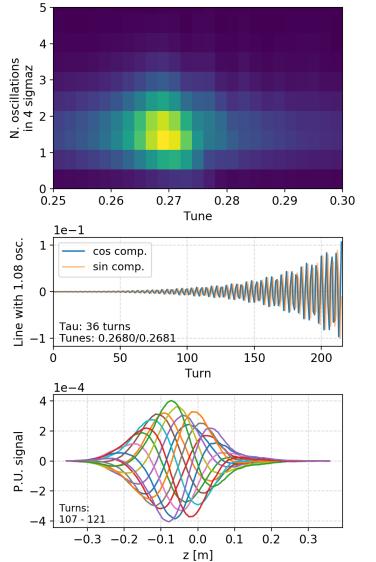
strength 1.7 - h-matrix + unperturbed map



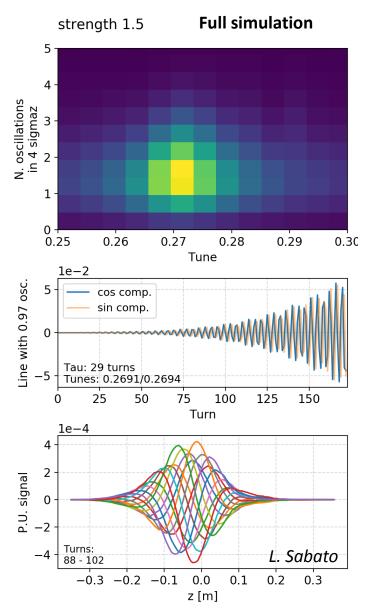




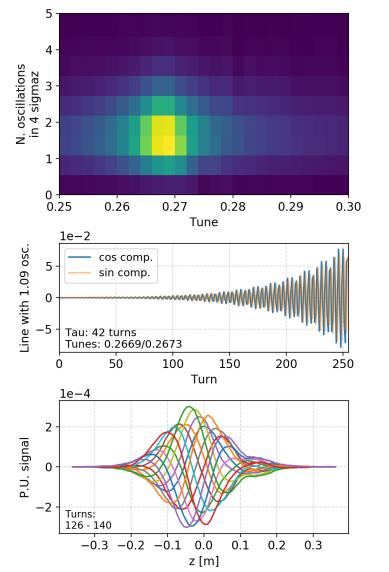
strength 1.6 - h-matrix + unperturbed map



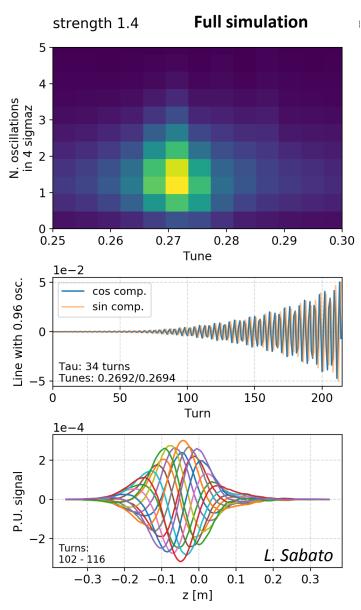




strength 1.5 - h-matrix + unperturbed map







5 4 -N. oscillations in 4 sigmaz 1 -0 | 0.25 0.27 0.26 0.28 0.29 0.30 Tune 1e-2 cos comp 5 sin comp. Tau: 52 turns Tunes: 0.2663/0.2663 200 50 100 150 250 Turn 1e-4

0.3

Line with 1.08 osc.

2

Turns: 155 - 169

 $-0.3 \quad -0.2 \quad -0.1$

0.0

z [m]

0.1

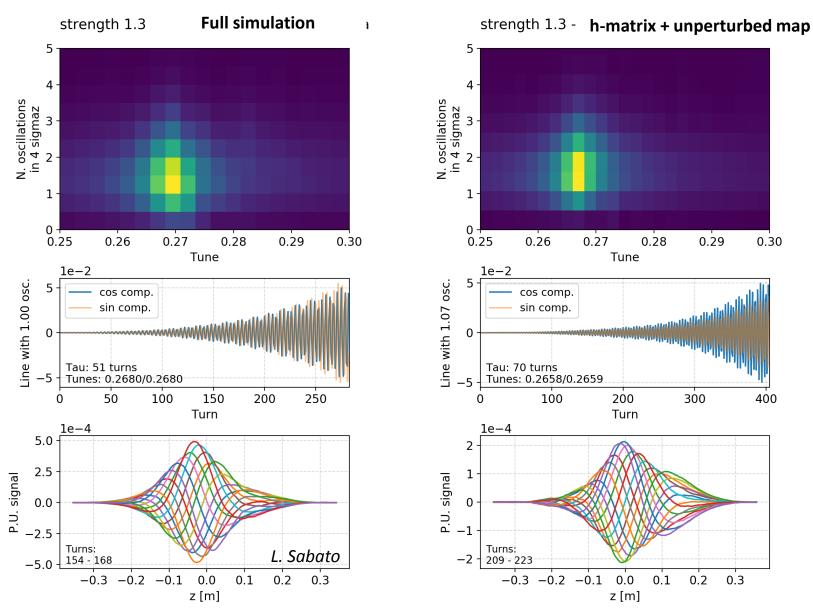
P.U. signal

strength 1.4 - h-matrix + unperturbed map

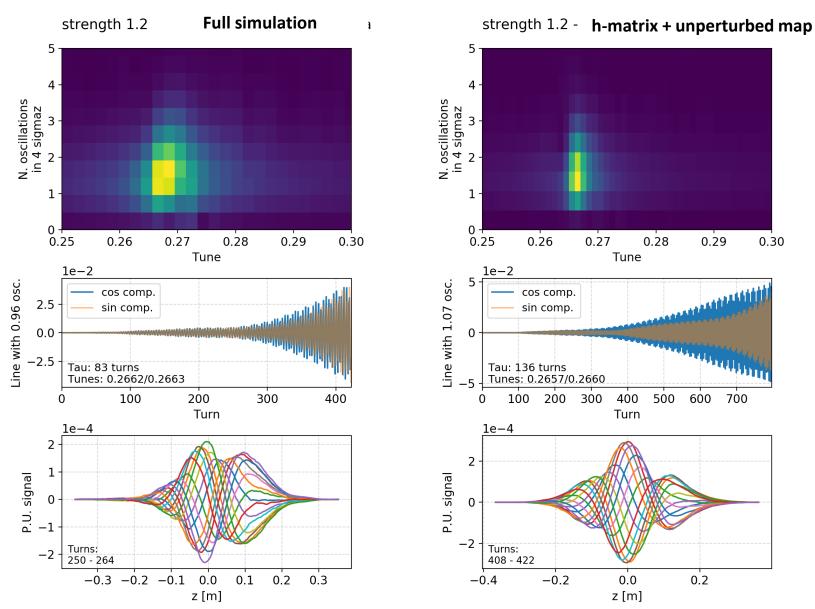
0.30

400

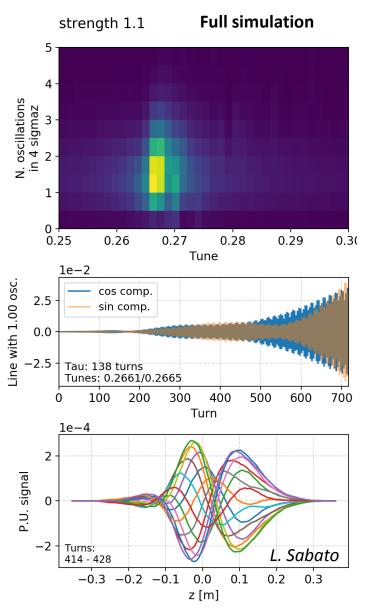




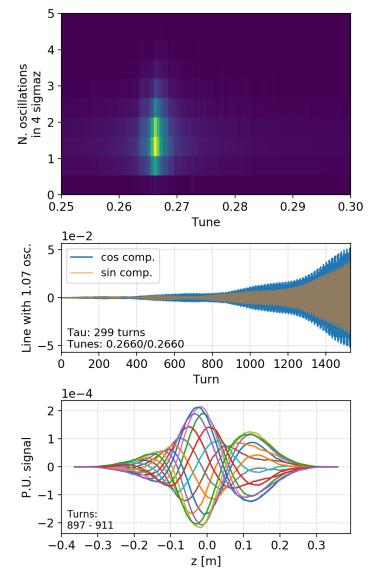




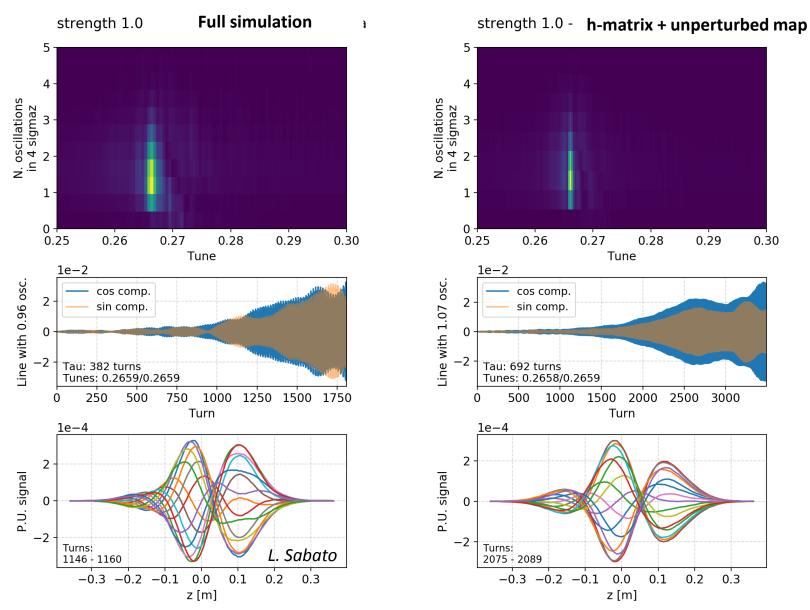




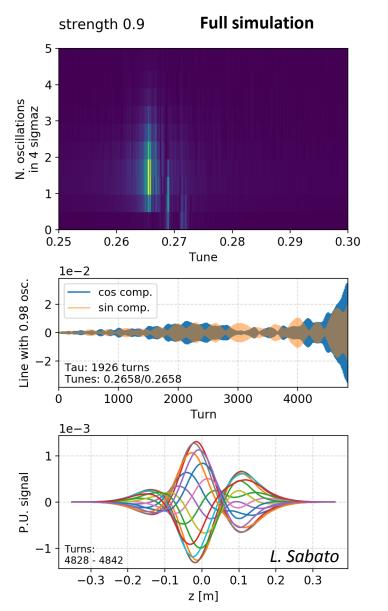
strength 1.1 - h-matrix + unperturbed map







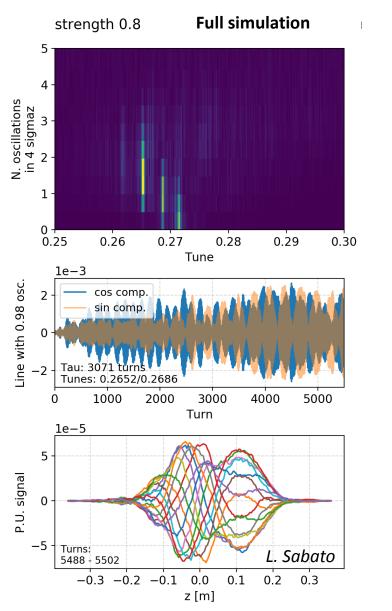




strength 0.9 - h-matrix + unperturbed map 5 4 -N. oscillations in 4 sigmaz 1 -0 | 0.25 0.27 0.26 0.28 0.29 0.30 Tune 1e-2 Line with 1.07 osc. cos comp sin comp. Tau: 2667 turns Tunes: 0.2656/0.2656 2000 4000 6000 8000 100001200014000 Turn 1e-4 2 P.U. signal Turns: 8002 - 8016 -0.4 -0.3 -0.2 -0.10.0 0.1 0.2

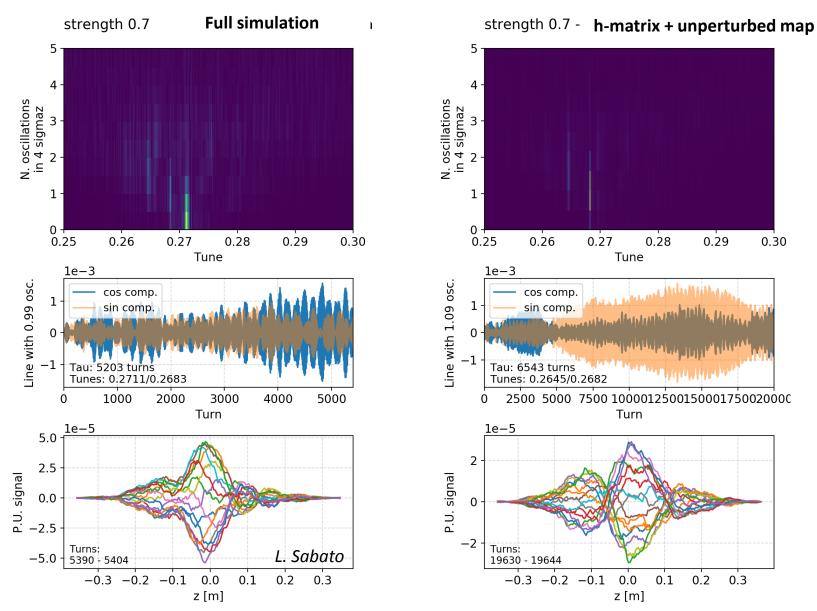
z [m]



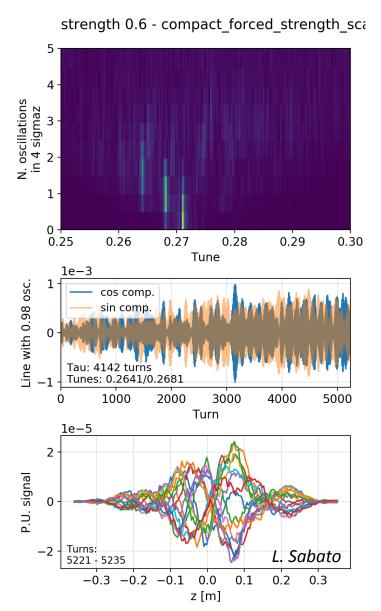


strength 0.8 - h-matrix + unperturbed map 5 4 · N. oscillations in 4 sigmaz 1 -0 | 0.25 0.27 0.26 0.28 0.29 0.30 Tune 1e-3 Line with 1.09 osc. cos comp. sin comp. Tau: 2123 turns Tunes: 0.2651/0.2651 6000 8000 10000 2000 4000 Turn 5.0 2.5 P.U. signal 0.0 -2.5 Turns: -5.0 - 6369 - 6383 -0.3 -0.2 -0.10.0 0.1 0.2 0.3 z [m]

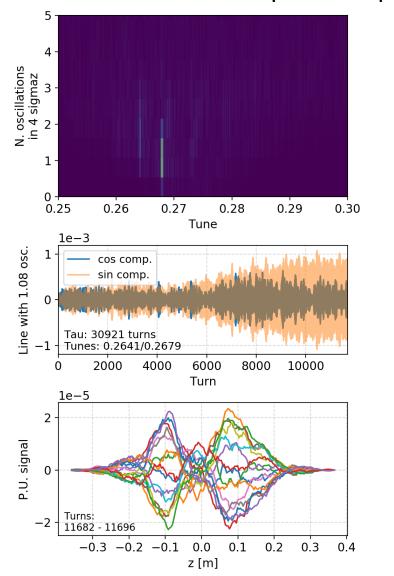




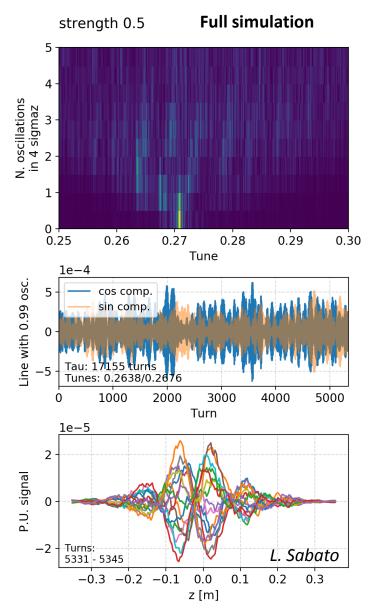




strength 0.6 - h-matrix + unperturbed map

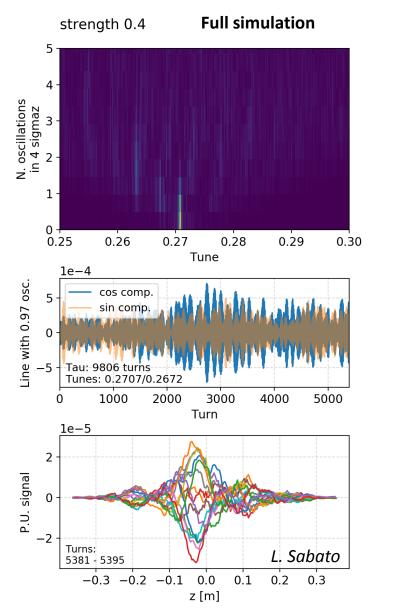






strength 0.5 - h-matrix + unperturbed map 5 4 · N. oscillations in 4 sigmaz 1 -0 | 0.25 0.27 0.26 0.28 0.29 0.30 Tune 1e-4 Line with 1.08 osc. cos comp sin comp Tau: 8371 turns Tunes: 0.2723/0.2676 2500 5000 7500 1000012500150001750020000 Turn 1e-5 1.0 P.U. signal 0.5 0.0 -0.5 Turns: -1.0 Turns: 19984 - 19998 -0.20.2 0.0 0.4 z [m]

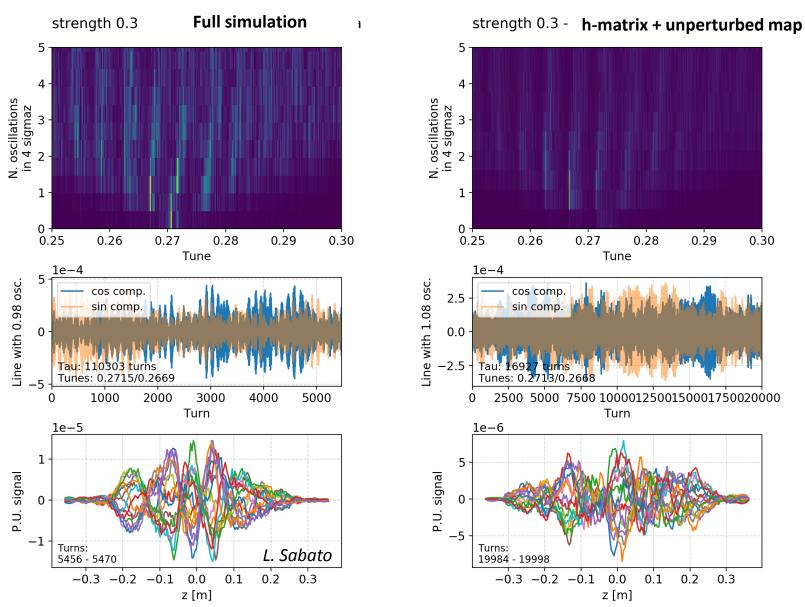




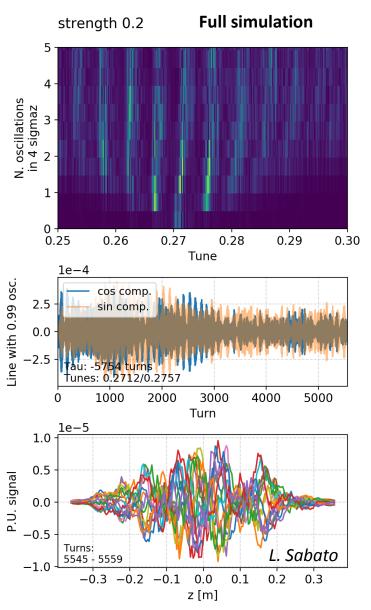
strength 0.4 - h-matrix + unperturbed map 5 4 · N. oscillations in 4 sigmaz 1 -0 | 0.25 0.26 0.27 0.28 0.29 0.30 Tune 1e-4 Line with 1.08 osc. cos comp. 2.5 sin comp. 0.0 -2.5 · Tau: 62274 turns Tunes: 0.2717/0.2672 2000 4000 6000 8000 10000 12000 14000 Turn 1e-6 5 P.U. signal Turns: 15005 - 15019 -0.20.2 0.0 0.4 -0.4

z [m]



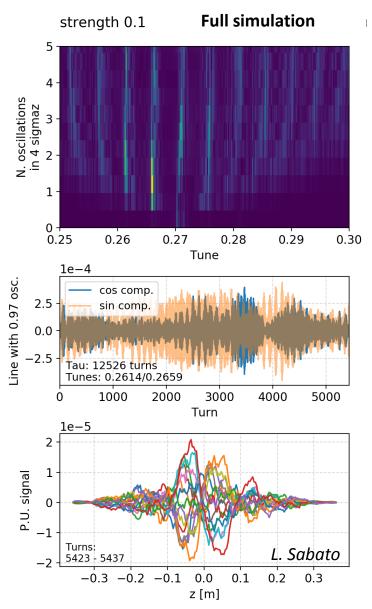






strength 0.2 - h-matrix + unperturbed map 5 4 N. oscillations in 4 sigmaz 1 -0 | 0.25 0.26 0.27 0.28 0.29 0.30 Tune 1e-4 Line with 1.07 osc. cos comp. sin comp Tau: 7126 turns Tunes: 0.2622/0.2663 2000 4000 6000 8000 10000 12000 14000 Turn 1e-5 1.0 P.U. signal 0.5 0.0 -0.5 Turns: 14901 - 14915 -1.0 -0.3 - 0.2 - 0.10.0 0.1 0.2 0.3 0.4 z [m]





strength 0.1 - h-matrix + unperturbed map 5 4 N. oscillations in 4 sigmaz 1 -0 | 0.25 0.26 0.27 0.28 0.29 0.30 Tune 1e-4 Line with 1.07 osc. Tau: 10059 turns Tunes: 0.2706/0.2756 2000 4000 6000 8000 10000 0 Turn 1e-6 5 P.U. signal Turns: 10452 - 10466 -0.3 -0.2 -0.10.0 0.1 0.2 0.3 0.4 z [m]



In the investigated case (LHC quad, 450 GeV), it is possible to model **instabilities driven by e-cloud** by including:

- The dipolar kicks along the bunch through an harmonic response
- The non-linearities as seen by a stable bunch

Potential applications:

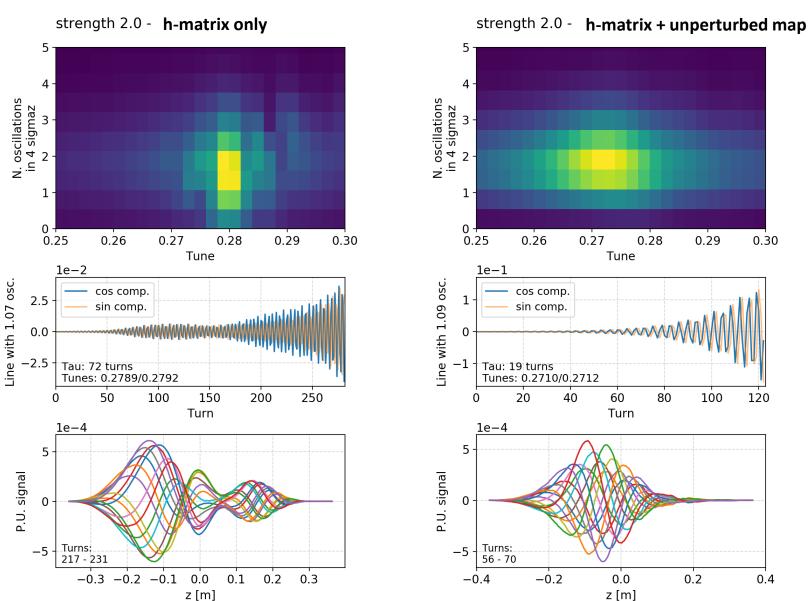
- Significant speedup for the simulations
- Modelling through Vlasov solver & dispersion integrals



Scanning the strength of the e-cloud (simplified model only – effect of non-linearities)



For high strength the instability is faster with non-linearity!





For high strength the instability is faster with non-linearity!

strength 1.9 - h-matrix only strength 1.9 - h-matrix + unperturbed map 5 5 4 4 N. oscillations in 4 sigmaz N. oscillations in 4 sigmaz N 3 -1 1 -0.25 0.27 0.28 0.29 0.27 0.28 0.29 0.25 0.26 0.30 0.26 0.30 Tune Tune 1e-21e-1 Line with 1.62 osc. Line with 1.07 osc. cos comp. cos comp. 2 sin comp. sin comp. Tau: 21 turns Tau: 83 turns Tunes: 0.2790/0.2790 Tunes: 0.2696/0.2700 200 250 20 80 100 50 100 150 300 60 120 0 40 Turn Turn 1e-41e-4 5.0 5 2.5 P.U. signal P.U. signal 0.0 -2.5 Turns: 248 - 262 Turns: 63 - 77 -5.0 -0.3 -0.2 -0.10.0 0.1 0.2 0.3 -0.3 -0.2 -0.10.0 0.1 0.2 z [m] z [m]

z [m]

0.30

150



For high strength the instability is faster with non-linearity!

strength 1.8 - h-matrix only strength 1.8 - h-matrix + unperturbed map 5 5 4 4 N. oscillations in 4 sigmaz N. oscillations in 4 sigmaz o 3 -1 1 -0.25 0.27 0.28 0.29 0.27 0.28 0.25 0.26 0.30 0.26 Tune Tune 1e-1 1e-2Line with 1.06 osc. Line with 1.07 osc. cos comp. cos comp. sin comp. sin comp. Tau: 23 turns Tunes: 0.2698/0.2694 Tau: 95 turns Tunes: 0.2788/0.2761 300 25 75 100 125 100 150 200 250 350 50 50 Turn Turn 1e-4 1e-4 2 2 P.U. signal P.U. signal Turns: 284 - 298 Turns: 70 - 84 -0.3 -0.2 -0.10.2 -0.2 -0.10.0 0.1 0.2 0.3 0.0 0.1

z [m]

150

0.2

z [m]

0.30

200



For high strength the instability is faster with non-linearity!

strength 1.7 - h-matrix only strength 1.7 - h-matrix + unperturbed map 5 5 4 4 N. oscillations in 4 sigmaz N. oscillations in 4 sigmaz o 3 -1 1 -0.25 0.27 0.29 0.26 0.27 0.28 0.25 0.26 0.28 0.30 Tune Tune 1e-21e-1 Line with 1.60 osc. Line with 1.06 osc. cos comp. cos comp. sin comp. sin comp. Tau: 28 turns Tunes: 0.2686/0.2686 Tau: 86 turns Tunes: 0.2789/0.2790 250 300 150 200 350 50 100 50 100 Turn Turn 2 2 1 1 P.U. signal P.U. signal Turns: 257 - 271 Turns: 83 - 97 -0.3 -0.2 -0.1-0.2 -0.10.0 0.1 0.2 0.3 0.0 0.1

200

0.3

z [m]



For high strength the instability is faster with non-linearity!

strength 1.6 - h-matrix only strength 1.6 - h-matrix + unperturbed map 5 5 4 4 -N. oscillations in 4 sigmaz N. oscillations in 4 sigmaz 3 -1 1 -0 | 0.25 0 | 0.25 0.27 0.28 0.29 0.26 0.27 0.28 0.29 0.26 0.30 Tune Tune 1e-2 1e-1 Line with 1.60 osc. Line with 1.08 osc. cos comp. cos comp. sin comp. sin comp. Tau: 113 turns Tunes: 0.2789/0.2784 Tau: 36 turns Tunes: 0.2680/0.2681 100 300 50 100 150 200 400 Turn Turn 1e-4 2 2 P.U. signal P.U. signal Turns: 338 - 352 Turns: 107 - 121 0.2 -0.20.0 0.4 -0.3 -0.2-0.10.0 0.1 0.2 -0.4

250

z [m]



For high strength the instability is faster with non-linearity!

strength 1.5 - h-matrix only strength 1.5 - h-matrix + unperturbed map 5 5 4 4 N. oscillations in 4 sigmaz N. oscillations in 4 sigmaz 3 -1 1 -0 | 0.25 0.27 0.28 0.29 0.26 0.27 0.28 0.29 0.25 0.26 0.30 Tune Tune 1e-21e-2 Line with 1.60 osc. Line with 1.09 osc. cos comp. cos comp. 2 sin comp. sin comp. Tau: 42 turns Tau: 111 turns Tunes: 0.2669/0.2673 Tunes: 0.2788/0.2789 300 500 50 100 150 100 200 400 600 200 0 Turn Turn 1e-4 1.0 2 0.5 P.U. signal P.U. signal -0.5 Turns: 332 - 346 Turns: 126 - 140 -1.0-0.3 -0.2 -0.10.2 -0.20.0 0.4 0.0 0.1 0.2

250

0.2

z [m]

0.30



For high strength the instability is faster with non-linearity!

strength 1.4 - h-matrix only strength 1.4 - h-matrix + unperturbed map 5 5 4 4 N. oscillations N. oscillations in 4 sigmaz in 4 sigmaz 3 1 1 -0 | 0.25 0 | 0.25 0.27 0.28 0.29 0.26 0.27 0.26 0.30 0.28 Tune Tune 1e-2 1e-2Line with 1.62 osc. Line with 1.08 osc. cos comp. cos comp. 2.5 sin comp. sin comp. 0.0 -2.5 · Tau: 110 turns Tau: 52 turns Tunes: 0.2785/0.2785 Tunes: 0.2663/0.2663 500 200 400 200 100 300 50 100 150 0 Turn Turn 1e-41e-4 1 2 P.U. signal P.U. signal Turns: 155 - 169 Turns: 331 - 345 -0.3 -0.2 -0.1-0.2 -0.10.1 0.2 0.3 0.0 0.1

0.30

400

0.3



For high strength the instability is faster with non-linearity!

strength 1.3 - h-matrix only strength 1.3 - h-matrix + unperturbed map 5 5 4 4 -N. oscillations in 4 sigmaz N. oscillations in 4 sigmaz 3 -1 1 -0 | 0.25 0.27 0.28 0.29 0.26 0.27 0.25 0.26 0.30 0.28 Tune Tune 1e-2Line with 1.61 osc. Line with 1.07 osc. cos comp. cos comp. 2 sin comp. sin comp. Tau: 124 turns Tau: 70 turns Tunes: 0.2784/0.2783 Tunes: 0.2658/0.2659 400 500 200 300 100 200 300 600 100 0 Turn Turn 1e-4 1e-4 2 1 P.U. signal P.U. signal Turns: 373 - 387 Turns: -2 + 209 - 223-0.3 -0.2 -0.10.1 0.2 0.3 -0.3 -0.2 -0.10.0 0.1 0.2 z [m] z [m]



For high strength the instability is faster with non-linearity!

strength 1.2 - h-matrix only strength 1.2 - h-matrix + unperturbed map 5 5 4 4 N. oscillations in 4 sigmaz N. oscillations in 4 sigmaz 3 -1 1 -0 | 0.25 0.27 0.28 0.29 0.26 0.27 0.29 0.25 0.26 0.30 0.28 0.30 Tune Tune 1e-2Line with 2.14 osc. Line with 1.07 osc. cos comp. cos comp. sin comp. sin comp. Tau: 165 turns Tunes: 0.2782/0.2782 Tau: 136 turns Tunes: 0.2657/0.2660 200 600 400 500 600 400 800 200 300 700 100 Turn Turn 1e-4 1e-5 2 5 P.U. signal P.U. signal Turns: 495 - 509 Turns: 408 - 422 0.2 -0.3 -0.2 -0.10.0 0.1 0.2 0.3 -0.20.0 -0.4

z [m]



strength 1.1 - h-matrix only 5 4 N. oscillations in 4 sigmaz 3 -1 0.27 0.28 0.29 0.25 0.26 0.30 Tune 1e-2 Line with 2.15 osc. cos comp. 2 sin comp. Tau: 194 turns Tunes: 0.2781/0.2781 400 600 800 200 1000 0 Turn 1.0 0.5 P.U. signal -0.5 Turns: 581 - 595 -1.0 +0.2 -0.4-0.20.0 0.4 z [m]

strength 1.1 - h-matrix + unperturbed map 5 4 -N. oscillations in 4 sigmaz 1 -0 | 0.25 0.26 0.27 0.28 0.29 0.30 Tune 1e-2 Line with 1.07 osc. cos comp. sin comp. Tau: 299 turns Tunes: 0.2660/0.2660 800 1000 1200 1400 400 600 200 Turn 1e-4 2 P.U. signal Turns: 897 - 911 -0.4 -0.3 -0.2 -0.10.0 0.1 0.2

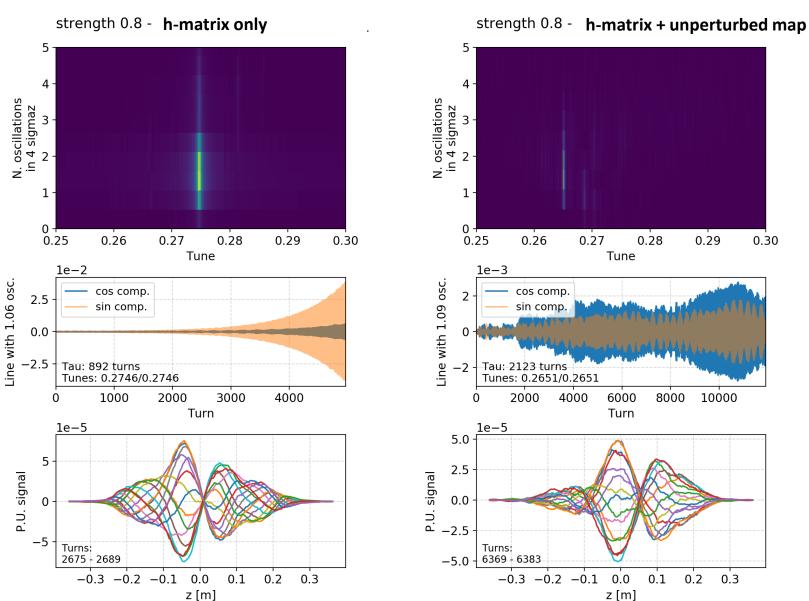


strength 1.0 - h-matrix only strength 1.0 - h-matrix + unperturbed map 5 5 4 · 4 -N. oscillations in 4 sigmaz N. oscillations in 4 sigmaz 3 -1 -1 -0 | 0.25 0.27 0.28 0.29 0.26 0.27 0.28 0.29 0.25 0.26 0.30 Tune Tune 1e-2 1e-2 Line with 2.16 osc. Line with 1.07 osc. cos comp. cos comp. sin comp. sin comp. Tau: 692 turns Tau: 341 turns Tunes: 0.2779/0.2779 Tunes: 0.2658/0.2659 750 1000 1250 1500 1750 1000 1500 2000 2500 3000 500 250 500 Turn Turn 1e-5 1e-4 5 2 P.U. signal P.U. signal Turns: 1023 - 1037 Turns: 2075 - 2089 0.2 -0.20.0 0.4 -0.3 -0.2 -0.10.0 0.1 0.2 -0.4z [m] z [m]



strength 0.9 - h-matrix only strength 0.9 - h-matrix + unperturbed map 5 5 4 4 · N. oscillations in 4 sigmaz N. oscillations in 4 sigmaz 3 -1 1 -0 | 0.25 0.27 0.28 0.29 0.27 0.28 0.29 0.25 0.26 0.30 0.26 0.30 Tune Tune 1e-2 Line with 1.08 osc. Line with 1.07 osc. cos comp. cos comp. sin comp. sin comp. Tau: 661 turns Tau: 2667 turns Tunes: 0.2746/0.2746 Tunes: 0.2656/0.2656 1000 1500 2000 2500 3000 3500 2000 4000 6000 8000 100001200014000 Turn Turn 1e-4 1e-4 2 1 P.U. signal P.U. signal Turns: 8002 - 8016 Turns: 1983 - 1997 0.2 -0.4 -0.3 -0.2 -0.1-0.20.0 0.4 0.0 0.1 0.2 -0.4z [m] z [m]



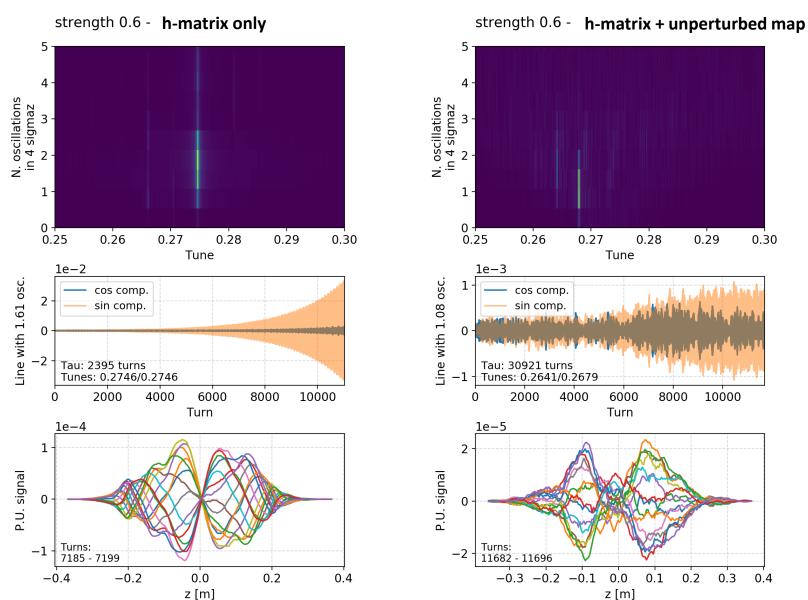


0.3



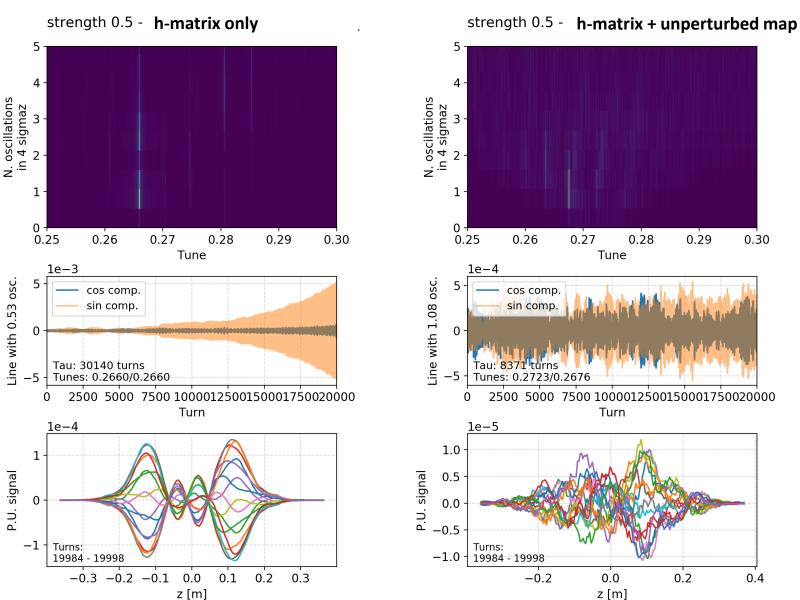
strength 0.7 - h-matrix only strength 0.7 - h-matrix + unperturbed map 5 5 4 4 N. oscillations in 4 sigmaz N. oscillations in 4 sigmaz N 3 -1 1 -0 | 0.25 0.27 0.28 0.29 0.26 0.27 0.29 0.25 0.26 0.30 0.28 Tune Tune 1e-21e-3 Line with 1.61 osc. Line with 1.09 osc. cos comp. cos comp. 2.5 sin comp. sin comp. 0.0 -2.5 -Tau: 6543 turns Tunes: 0.2645/0.2682 Tau: 1235 turns Tunes: 0.2746/0.2746 5000 2500 5000 7500 1000012500150001750020000 1000 2000 3000 4000 6000 Turn Turn 1e-5 1.0 2 0.5 P.U. signal P.U. signal -0.5 Turns: 3704 - 3718 Turns: 19630 - 19644 -0.3 -0.2 -0.10.0 0.1 0.2 0.3 -0.3 -0.2 -0.10.0 0.1 0.2 z [m] z [m]





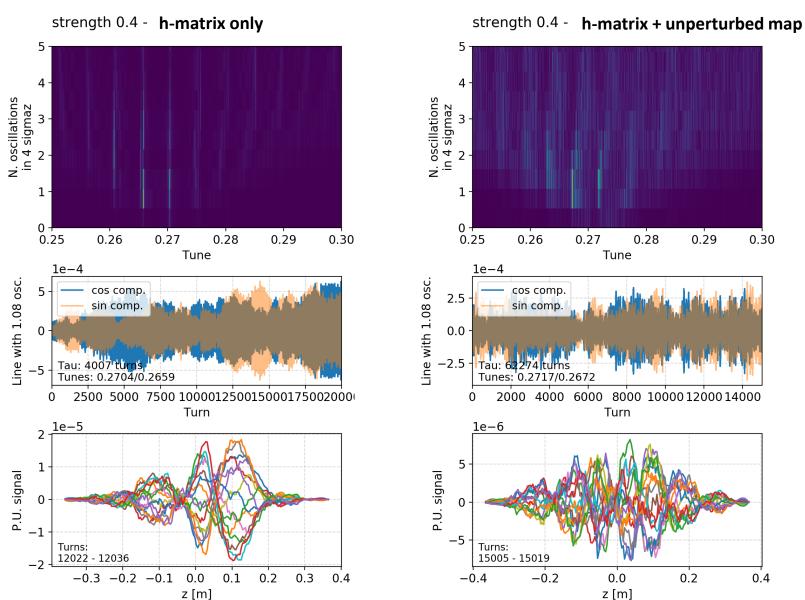


With non-linearities, the threshold in strength is higher

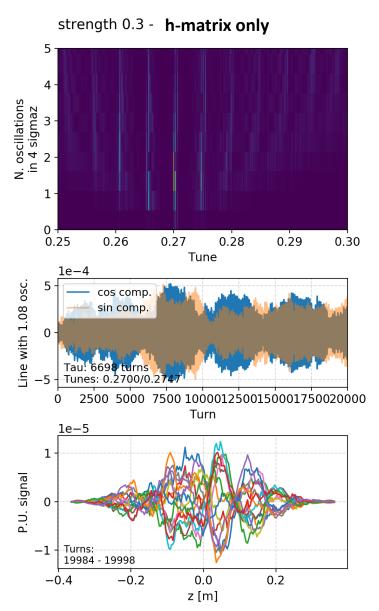




With non-linearities, the threshold in strength is higher







strength 0.3 - h-matrix + unperturbed map 5 4 N. oscillations in 4 sigmaz 1 -0 | 0.25 0.27 0.26 0.28 0.29 0.30 Tune 1e-4 Line with 1.08 osc. cos comp 2.5 sin comp 0.0 -2.5 Tunes: 0.2713/0.2668 2500 5000 7500 1000012500150001750020000 Turn 1e-6 5 P.U. signal Turns: 19984 - 19998 -0.3 - 0.2 - 0.10.0 0.1 0.2 0.3 z [m]

0.2

0.3

0.4

0.30



strength 0.2 - h-matrix only strength 0.2 - h-matrix + unperturbed map 5 5 4 · 4 N. oscillations in 4 sigmaz N. oscillations in 4 sigmaz 3 -1 -1 -0 | 0.25 0 | 0.25 0.27 0.28 0.29 0.26 0.27 0.28 0.26 0.30 Tune Tune 1e-4 1e-4 Line with 1.06 osc. Line with 1.07 osc. cos comp. cos comp. sin comp. sin comp Tau: 4124 turns Tunes: 0.2702/0.2653 Tau: 7126 turns Tunes: 0.2622/0.2663 2500 5000 7500 1000012500150001750020000 2000 4000 6000 8000 10000 12000 14000 Turn Turn 1e-5 1e-5 1.0 1 P.U. signal P.U. signal 0.5 0.0 -0.5 -1Turns: -1.0 - 14901 - 14915 Turns: 12372 - 12386 -0.3 -0.2 -0.1-0.3 - 0.2 - 0.10.2 0.3 0.0 0.1 0.0 0.1 z [m] z [m]

8000

0.2

0.3

0.4

0.30

10000



