Flavor Physics and CP Violation

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1.1 Flavor physics and CP violation in particle physics

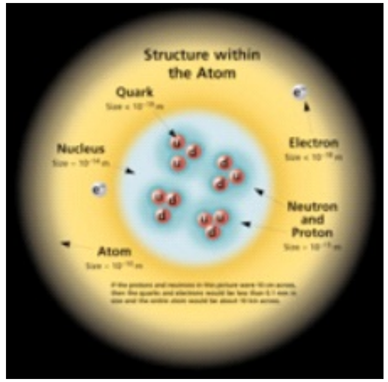
Particle Physics: The study of fundamental building blocks of matter and their interactions

Modern time: Matter → Molecular → Atom →

Nuclei(Hadron) + electrons + neutrinos (lepton) → ?

Periodic Table (Mendeleev, 1869):

Regularity hinted smaller building blocks



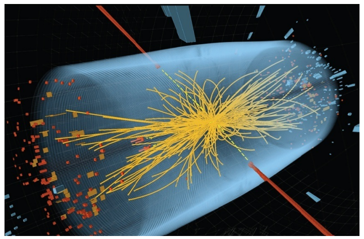
Hadrons have structures? Quarks ···

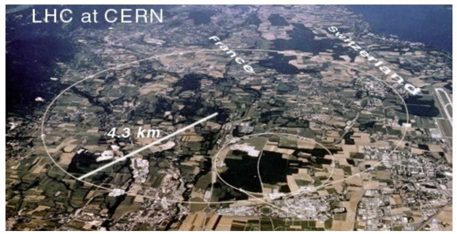
Leptons have structures? Not so far···

How do we know?

Experiments: New discoveries and verification of theories

Theories: Predictions and explain data





Fundamental building blocks for Hadrons

Nuclei → proton (p) (Rutherford, 1911) , neutron(n) (Chadwich, 1932)
More hadrons: π (Lattes, 1947), K (Rocheste, 1947)
Even more: Λ, Σ, Ξ ⋯ (1950).

Regularities? SU(3) flavor symmetry(Gell-Mann, Ne′eman, 1961) Octet: p, n, Λ, $\Sigma^{-,0,+}$, $\Xi^{0,-}$
Smaller building blocks?

Hypothesis of Quarks: u, d, s (Gell-Mann, Zweig, 1964)
up: u (2/3 e), down: d(-1/3 e), strange: s(-1/3 e)
p = (uud), n = (ddu), Λ = (uds) ⋯. Direct evidences: 1970' s
More quarks:
Charm c (2/3 e) (Ting, Richeter, 1974) (GIM prediction, 1970);
Beauty b(-1/3 e) (Lederman, 1980) (KM prediction, 1973); Top t (2/3 e) (Fermilab, 1994)

u c t (electric charge +2/3 e) Quarks are elementary particles
d s b (electric charge -1/3 e) Three generations/families

Cabibbo introduced mixing between d and s in 1963, KM: 3 generation mixing 1973

Fundamental Leptons

Heavy nuclei β decays(Becquerel, 1896)

(A,Z) → (A,Z+1) + e + ν

e (electron) discovered in 1897 by Thomas

ν (neutrino) before 1930, no such a particle is known to human being

Continuous electron energy spectrum

(Chadwick, Ellis,Wooster, 1914 - 1929)

Without neutrino, non-conservation of energy?

Pauli, 1930, existence of neutrino saved energy conservation.

Fermi, 1934, proposed four fermion interaction for nuclei β decays.

ν_e discovered in 1953 (Reines, … 1953)

More leptons: μ (Anderson, Sereet, 1937), ν_μ (Lederman et al 1962)

τ (Pearl, 1975), ν_τ (Fermilab, 2001)

ν_e ν_μ ν_τ (electric charge 0 e) Leptons are elementary particles

e μ τ (electric charge -1 e) Three generations/families

PMNS introduced the concept of neutrino oscillation, 1950’ s

Fundamental Interactions

Light → Wave nature(Young, 1803) → particle nature (Einstein, 1905)

Mediating electromagnetic interaction (Maxwell Equations)

W, Z (CERN 1984) → Mediating weak interactions

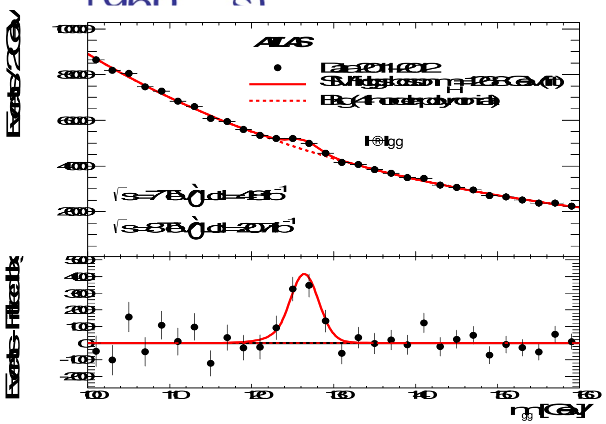
Unified with EM, electroweak theory (1960’ s)

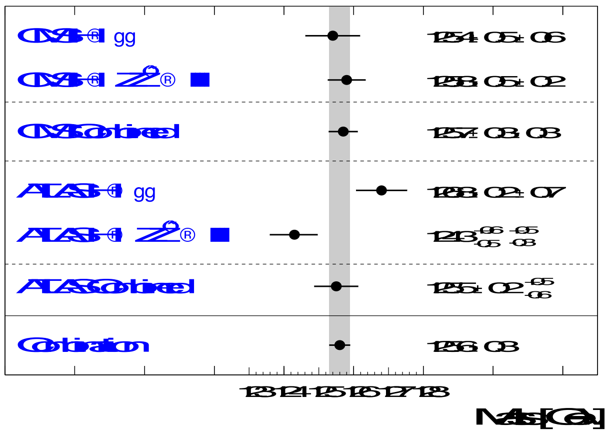
Gluon (1970’ s) → Mediating strong interaction at short

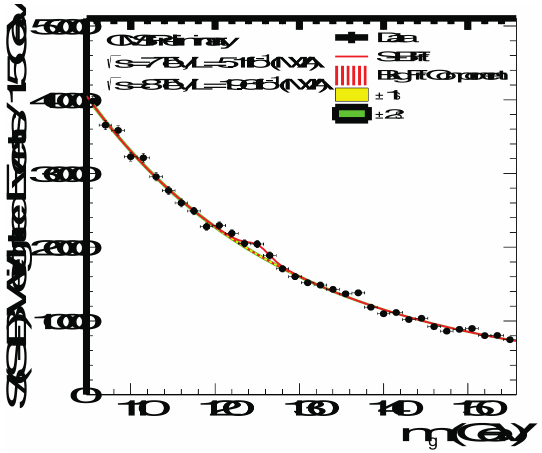
At long distance mediate π···, Yukawa 1935

Graviton→ Mediating gravitational attraction (gravitational wave, 2016)

The god particle---Higgs boson (CERN 2012) → Generating masses (early 1960’ s)

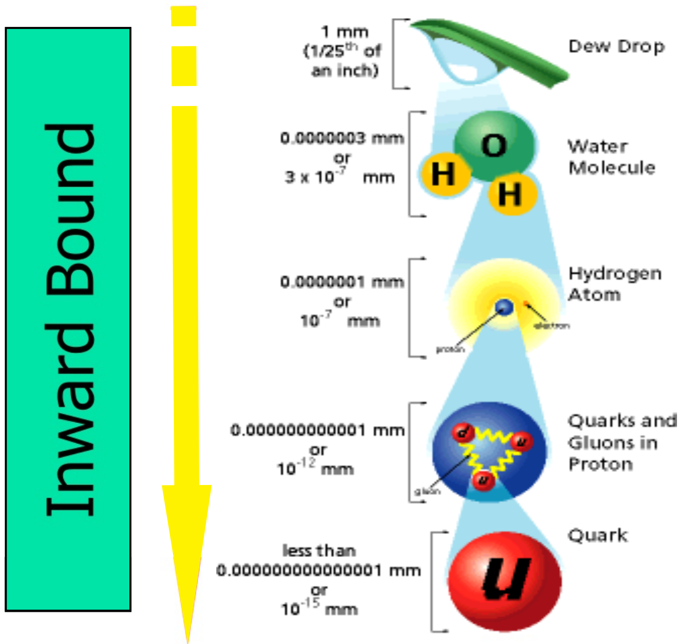


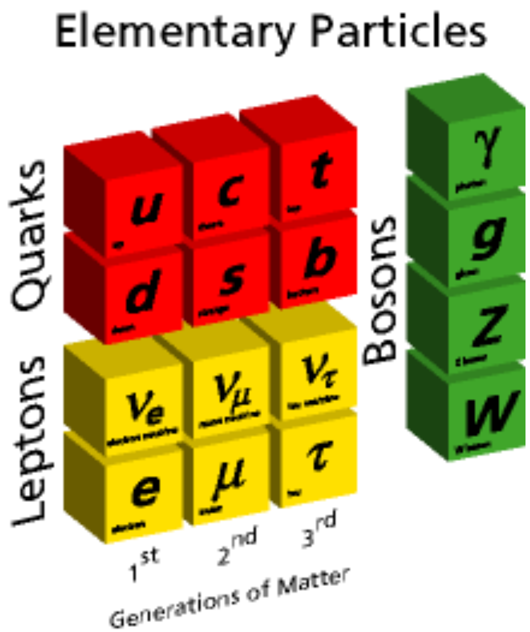


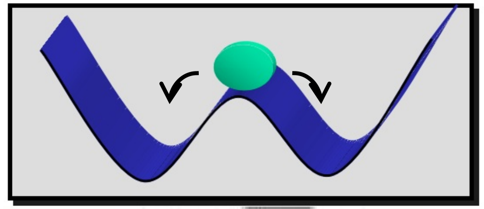


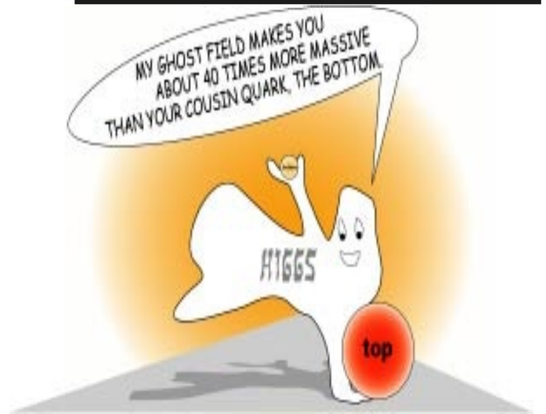
The standard model of strong and electroweak interactions

SU(3) x SU(2) x U(1) gauge theory for strong and electroweak interaction









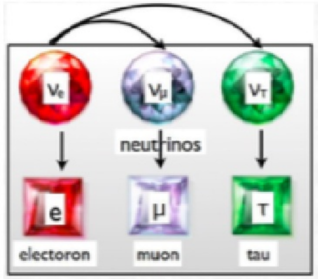
Can one negeclects gravitation interaction when studying particle interactions?
The coulomb force between two protons: $F_c = e^2/r^2$,
And Gravitational force: $F_g = -Gm^2/r^2$ $\quad\quad |F_g|/|F_c| = 7\times10^{-38}$

Gravitational force is much weaker than electromagnetism!

But when study cosmology › gravitational force always add up › but electromagnetism can cancel between positively and negatively charged particles!

Flavor physics and CP symmetry

Flavors: describe several copies of the same gauge representation, namely several fields that are assigned the same quantum charges:



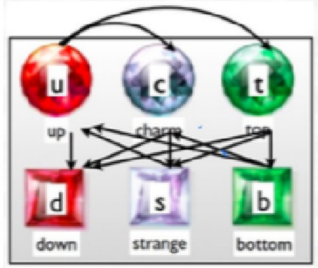
u, c, t; d, s, b; e, μ, τ; ν_ε, ν_μ, ν_τ; …

Flavor physics: the study of interactions that govern flavors.

Weak interaction one type of flavor change to another type

neutral current t → c, u; b → s, d; $\nu_\tau \rightarrow \nu_\mu$, ν_ε; ... ,

charged current b, s, d → t, c, u; τ, μ, ε → ν_τ, ν_μ, ν_ε…



CP symmetry: Combined symmetry of C-charge conjugation (particle and anti-particle symmetry) and P-space parity (inversion of space directions).

Strong and electromagnetism interactions respect these symmetries.

Weak interaction violates (breaks) these symmetries.

This lecture is about Flavor physics and CP violation with in and beyond the SM.

P, C, T and CP Violation, and CPT Theorem
P symmetry and violation

The parity operation P, a spatial inversion through the origin: $P\vec{x} = -\vec{x}$

The Schrödinger equation

$$i\hbar\frac{d\psi}{dt} = [-\frac{\hbar^2}{2m}\nabla^2 + V(\vec{x})]\psi \; ,$$

P operates on the wave function $\psi(\vec{x})$ of a state $|N, \vec{p}, \vec{s}>$,
N internal quantum numbers: electric charge, baryon number and etc.
$\vec{p}$ and $\vec{s}$ are the momentum and spin,

$$P\psi(\vec{x}) \to \psi(-\vec{x}) \; , \quad P|N, \; \vec{p}, \; \vec{s}> = \eta_P|N, \; -\vec{p}, \; \vec{s}> \; ,$$

η_P the intrinsic parity of the particle (system).

Parity symmetry (invariance) of the interactions, implies: $V(\vec{x}) = V(-\vec{x})$
Then: $\psi(-\vec{x})\psi^*(-\vec{x})$ is equal to $\psi(\vec{x})\psi^*(\vec{x})$,

The probability of the transition $i \to f$ is the same as that for $Pi \to Pf$.

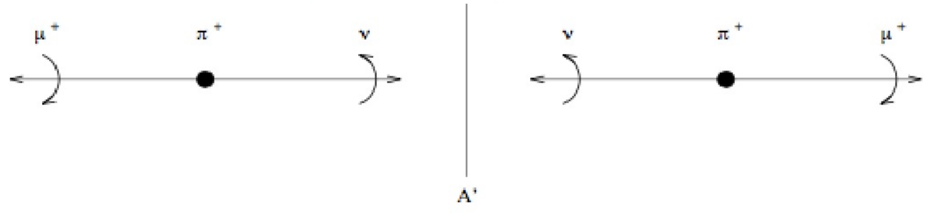


Figure 1: Mirror processes.

Observe left process, but not right one,

P is violated!

T symmetry and violation

In classic mechanics

Classically time reversal operation T: $t \to -t$.
Reverse momenta $\vec{p} \to = \vec{p}$, spins $\vec{s} \to -\vec{s}$
interchanging the initial state $|i>$ with final $|f>$ state.

A physics example: a damped pendulum
The equation of motion for the damped pendulum

$$m\frac{d^2x}{d^2t} + r\frac{dx}{dt} + kx = 0$$

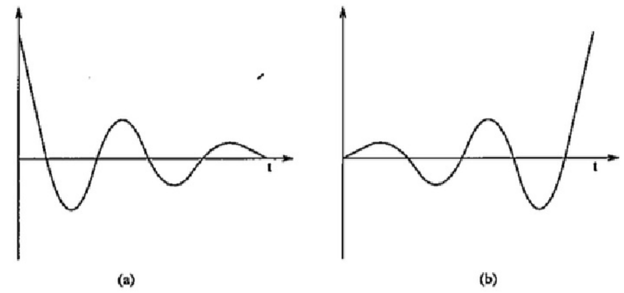


Figure 2: (a) The pendulum swing amplitude vs. time. (b) The time reversed processe.

gives the T transformed equation

$$m\frac{d^2x}{d^2(-t)} + r\frac{dx}{d(-t)} + kx = 0 \to m\frac{d^2x}{d^2t} - r\frac{dx}{dt} + kx = 0$$

Clearly the first order derivative in t is the reason why it is not T invariant.
$r = 0$, no damping, T invariant!

In quantum mechanics, the situation is more complicated.
The Schrödinger equation has a first order derivative in t

$$i\hbar \frac{d\psi}{dt} = [-\frac{\hbar^2}{2m}\nabla^2 + V(t)]\psi$$

T reversal operation : $t \rightarrow -t$

$$i\hbar \frac{d\psi(-t)}{d(-t)} = [-\frac{\hbar^2}{2m}\nabla^2 + V(-t)]\psi(-t)$$

$$\rightarrow \; -i\hbar \frac{d\psi(-t)}{dt} = [-\frac{\hbar^2}{2m}\nabla^2 + V(-t)]\psi(-t) \; .$$

Even $V(t) = V(-t)$, not possible to go back as before T transformation!

Contradiction with the observation for the damped pendulum
$V(t) = kx = V(-t)$, $r = 0$ is possible to define T invariance!

How to make T transformation still possible to be invariant?
This puzzle was solved by Wigner in 1932

T transformation: *Change t to $-t$ and take the complex conjugate.*
The T transformed version is

$$(-i)^* \hbar \frac{d\psi(-t)^*}{dt} = [-\frac{\hbar^2}{2m}\nabla^2 + V^*(-t)]\psi(-t)^*$$
$$\rightarrow \; i\hbar \frac{d\psi(-t)^*}{dt} = [-\frac{\hbar^2}{2m}\nabla^2 + V^*(-t)]\psi(-t)^*$$

As quantum observables are expectation values involving only $\psi^*\psi$,
If the interaction V is real: $V(-t)^* = V(t)$),
then $\psi(t)^*\psi(t) = \psi(-t)^*\psi(-t)$. Same physics!

Time reversal invariance imposes reality conditions on the interaction.
To break T symmetry, one needs to introduce complex valued interactions.

C symmetry and violation

Particle and anti-particle have opposite additive quantum numbers,
C parity changes the signs of all additive quantum numbers,

$$C|N,\ \vec{p},\ \vec{s}>=\eta_C|-N,\ \vec{p},\ \vec{s}>$$

η_C is a phase factor.
Only if $N=0$, a particle or a particle system can be eigen-state of C.

Example: π^0 which satisfies:

$$C|\pi^0>=+|\pi^0>\ \ \text{self-conjugate}$$

Charge conjugation symmetry also plays an important role in particle physics.
Apply C transformation on (a) the reactio becomes to (b)

$$(a)\ \pi^+\rightarrow\mu^+\nu\ ,\ \ (b)\ \pi^-\rightarrow\mu^-\bar{\nu}$$

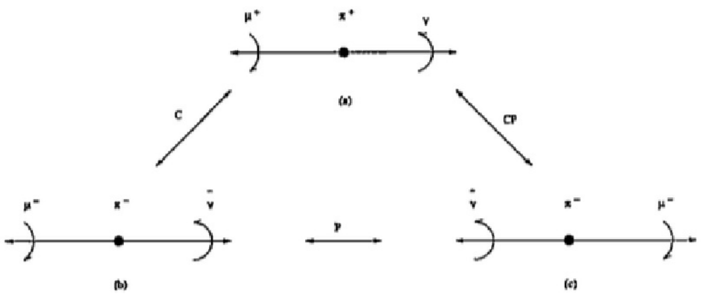


Figure 3: C, P and CP transformed processes.

(a) Process observed
(b) Process not observed.
C symmetry violated.
(c) process observed.
CP symmetry is respected.
Is CP symmetry always true?

CP symmetry and violation

CP was still considered to be exact untill experiment led by Fitch and Cronin found mixing between K_1 (K_S) and $K_2(K_L)$ in 1964!

Experimental data show: K_L mainly decays into $\pi\pi\pi$, but about a few per thousand times decays into $\pi\pi$.

Why the above fact implies violation of CP symmetry?

Kaons and pions are pseudoscalar, under P transformation

$$\pi \to -\pi \,, \;\; K \to -K \,,$$

Under a C transformation one has

$$\pi^+(u\bar{d}) \to \pi^-(d\bar{u}) \,, \;\; \pi^0((u\bar{u} - d\bar{d})/\sqrt{2}) \to \pi^0 \,, \;\; K^0(d\bar{s}) \to \bar{K}^0(s\bar{d}) \,.$$

Two neutral kaon CP eigenstates be constructed from K^0 and $\bar{K}^0$,

$$K_1^0 = \frac{1}{\sqrt{2}}(K^0 - \bar{K}^0)\ ,\ \ CP\ \text{even}\ ;\ \ K_2^0 = \frac{1}{\sqrt{2}}(K^0 + \bar{K}^0)\ ,\ \ CP\ \text{odd}\ .$$

The pion systems $\pi\pi$ and $\pi\pi\pi$ are all in S-wave states.
$(\pi^+\pi^-\ ,\pi^0\pi^0)$ in CP even states. $(\pi^+\pi^-\pi^0\ ,\pi^0\pi^0\pi^0)$ in CP odd states.

$$K_1^0 \to \pi^+\pi^-\ ,\ \pi^0\pi^0\ ,\ \ K_2^0 \to \pi^+\pi^-\pi^0\ ,\ \pi^0\pi^0\pi^0\ .$$

Phase space considerations, K_2^0 decays slower than the K_1^0 decay.
Experiment show the lifetimes are 10^{-7}s and 10^{-10}s, respectively.
K_L state mainly decay into $\pi\pi\pi$, it should be identified as K_2^0

At far enough distance (all original K_1^0 should have all decayed),
Still see $\pi\pi$ final state, K_L and K_S are admixture of K_1^0 and K_2^0.

$$K_L = \frac{K_2^0 + \epsilon_1 K_1^0}{\sqrt{1+|\epsilon_1|^2}}\ ,\ \ K_S = \frac{K_1^0 + \epsilon_2 K_2^0}{\sqrt{1+|\epsilon_2|^2}}\ .$$

$K^0 - \bar{K}^0$ mixing leads to CP violation!

CP symmetry is violated!

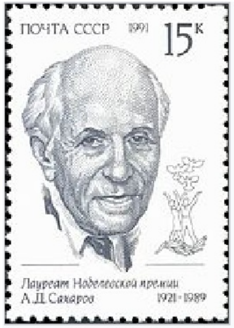
Importance of CP violation

Property of Fundamental laws of Nature

Played important roles in understanding fundamental laws of Nature!

One of the most crucial elements why we exist in the Universe.

Sakharov (1967): Violation of CP invariance, C asymmetry and baryon asymmetry of the universe.



Andrei Sakharov on *Soviet Nobel Peace Prize winners*, the USSR stamp issued on 14 May 1991

A matter dominating anti-Matter universe resulted from a symmetric one in the Big-Bang cosmology

- Baryon number B violation.
- C-symmetry and CP-symmetry violation.
- Interactions out of thermal equilibrium.

What about CPT symmetry?

No experimental evidence which shows violation of CPT symmetry.
There are more fundamental reasons for CPT symmetry to be exact.

In the 1950s, it was shown that CPT symmetry holds
if the Lagrangian of a local quantum field theory is

Lorentz invariance, Hermitician and the fields obey usual Spin-Statistics
(Bose-Einstein statistics for bosons, and Fermi-Dirac statistics for fermions).

Schwinger (1951); Lüders, (1954); Pauli(1955); Streater and Wightman(1964).

This is the so called CPT theorem.

See the outline of prove of CPT theorem in quantum field theory in Appendix A

There are many implications of the CPT theorem.

The masses, and life-times for particles and their anti-particles are all equal.

If CP is violated, T is violated in a way that CPT is still conserved!

These properties provide practical ways to test the CPT theorem.
The best limit on CPT symmetry is from

the mass difference between the masses of K^0 and $\bar{K}^0$,

one has $|m_{K^0} - m_{\bar{K}^0}|/m_{K^0} < 6 \times 10^{-19}$.

CPLEAR, observed direct T violation in $K^0 - \bar{K}^0$ system in 1998
as expected from CP violation in this system (PLB444, 43(1998)).

In most of the discussion later, CPT symmetry is assumed to hold.

1.2 Some Basics of QFT for CP Violation

How fields and Lagrangian transform under C, P and T?

Take QED with fermion ψ and scalar ϕ fields as example.

$$L = -\frac{1}{4}F_{\mu\nu}F^{\mu\nu} + \bar{\psi}(i\gamma_\mu D^\mu - m_\psi)\psi + (D^\mu\phi)^\dagger(D_\mu\phi) - m_\phi^2\phi^\dagger\phi - V(\phi^\dagger\phi) ,$$
$$F_{\mu\nu} = \partial_\mu A_\nu - \partial_\nu A_\mu , \quad D_\mu = \partial_\mu + ieQA_\mu ,$$

ϕ spin-0, A^μ spin-1 (communting), ψ spin-1/2 (anti-communting)
$V(\phi^\dagger\phi)$ - potential of ϕ and is invariant under Lorentz Transformation.

The theory is invariant under the following gauge transformation,
$A_\mu \to A_\mu - \partial_\mu\alpha(x)$, $\psi(x) \to e^{ieQ\alpha(x)}\psi(x)$ and $\phi(x) \to e^{ieQ\alpha}\phi(x)$.

The Dirac γ-matrices are

$$\{\gamma^\mu, \gamma^\nu\} = 2g^{\mu\nu} , \quad g^{\mu\nu} = \mathrm{Diag}(1, -1, -1, -1) ,$$

$$\gamma^0 = \begin{pmatrix} I & 0 \\ 0 & -I \end{pmatrix} , \quad \gamma^i = \begin{pmatrix} 0 & \sigma_i \\ -\sigma_i & 0 \end{pmatrix} . \quad \gamma_5 = \begin{pmatrix} 0 & I \\ I & 0 \end{pmatrix}$$

Mass term needs to pair up left and right handed fields

$$\psi_L = \frac{1-\gamma_5}{2}\psi , \quad \psi_R = \frac{1+\gamma_5}{2}\psi , \quad \psi = \psi_L + \psi_R \qquad \bar{\psi}\psi = \bar{\psi}_L\psi_R + \bar{\psi}_R\psi_L .$$

Equation of Motion

$$\delta \int L(\phi, \partial_\mu \phi) d^4 x = 0 \,, \qquad \frac{\partial L}{\partial \phi} - \partial_\mu \frac{\partial L}{\partial(\partial_\mu \phi)} = 0 \,.$$

$$\partial^\mu F_{\mu\nu} = e\bar{\psi}\gamma_\nu \psi - ie(\phi^* \partial_\nu \phi - \partial_\nu \phi^* \phi) \,,$$

$$(i\gamma^\mu \partial_\mu - eQ\gamma^\mu A_\mu - m_\psi)\psi = 0 \,,$$

$$(\partial^\mu \partial_\mu + m^2)\phi = e^2 A^\mu A_\mu - 2ieA^\mu \partial_\mu \phi - ie\partial^\mu A_\mu \phi + \frac{\partial V(\phi^* \phi)}{\partial \phi^*} \,.$$

Under C transformation, x^μ and ∂^μ do not transform,

$\phi \to \phi_C = \eta_C \phi^\dagger \; (\eta_C^2 = 1), \quad A^\mu \to A_C^\mu = -A^\mu \;, \quad \psi \to \phi_C = \psi\bar{\psi}^T = i\gamma^2\psi^*$

$C = i\gamma^2\gamma^0 \;, \quad C^{-1} = C^\dagger = -C \;, \quad C\gamma_\mu^T C^\dagger = -\gamma_\mu .$

Note that for $((1-\gamma_5)/2)\,\psi$, $\psi_L{}^C = ((1+\gamma_5)/2)\psi_C$

$\bar{\psi}\psi_C$ is not zero-> the Majorana mass term.

Under P transformation, $x^\mu = (x^0, x^i) \to x_\mu = (x^0, -x_i)$ and $\partial^\mu \to \partial_\mu$,

$\phi(\vec{x}) \to \phi_P = \pm\phi^\dagger(-\vec{x})$: $+scalar, -pesudoscalar$, $A^\mu(\vec{x}) \to A_P^\mu = A_\mu(-\vec{x})$,

$\psi(\vec{x}) \to \psi_P = \eta_P \gamma^0 \psi(-\vec{x})$ (η_P *phase factor*), $\gamma^0\gamma^\mu\gamma^0 = \gamma_\mu$

Under T transformation, $x^\mu \to -x_\mu$ and $\partial^\mu \to -\partial_\mu$,

$\phi(t) \to \phi_T = \eta_T^\phi \phi(-t) \;, \quad A^\mu(t) \to A_T^\mu = A_\mu(-t) \;,$

$\psi(t) \to \psi_T = \eta_T^\psi T\psi(-t)$ (η_T^ψ *phase factor*),

$T = i\gamma^1\gamma^3$, $T^\dagger = T^{-1}$, $T^\dagger\gamma_\mu^* T = \gamma^\mu$

axial vector $a^\mu \to a_C^\mu = a^\mu$, $a_P^\mu = -a_\mu(-\vec{x})$, $a_T^\mu = a_\mu(-t)$

Define the following bi-spiner products

$$S(x) = \bar{\psi}(x)\psi(x)\ , \quad A^\mu(x) = \bar{\psi}(x)\gamma^\mu\psi(x)\ , \quad T^{\mu\nu}(x) = \bar{\psi}(x)\sigma^{\mu\nu}\psi(x)\ ,$$

$$a^\mu(x) = \bar{\psi}(x)\gamma_5\gamma^\mu\psi(x)\ , \quad P(x) = i\bar{\psi}(x)\gamma_5\psi(x)\ , \quad \sigma^{\mu\nu} = \frac{i}{2}[\gamma^\mu, \gamma^\nu]\ .$$

Transformation properties under C, P, T and CPT

$S(x)$ as a scalar, $P(x)$ as a pesudoscalar, $A^\mu(x)$ as a vector,

$a^\mu(x)$ as an axial vector, and $T^{\mu\nu}(x)$ as tensor.

	$S(x)$	$A^\mu(x)$	$T^{\mu\nu}(x)$	$a^\mu(x)$	$P(x)$
C	$S(x)$	$-A^\mu(x)$	$-T^{\mu\nu}(x)$	$a^\mu(x)$	$P(x)$
P	$S(x')$	$A_\mu(x')$	$T_{\mu\nu}(x')$	$-a_\mu(x')$	$-P(x')$
T	$S(-x')$	$A_\mu(-x')$	$-T_{\mu\nu}(-x')$	$a_\mu(-x')$	$-P(-x')$
CPT	$S(-x)$	$-A^\mu(-x)$	$T^{\mu\nu}(-x)$	$-a^\mu(-x)$	$P(-x)$

Violation of P and C symmetries

How to have P, CP violation, but CTP conservation?

A toy model interaction violating P, C, CP, but conserving CTP

$$L = \bar{\psi}'\kappa\gamma^\mu(1-\gamma_5)\psi A_\mu + \bar{\psi}\kappa^\dagger\gamma^\mu(1-\gamma_5)\psi' A_\mu^\dagger$$

The P transformed Lagrangian, $x \to x^\mu$ goes to $x' = x_\mu$.

$$L^P(x) = \bar{\psi}'_P(x)\kappa\gamma^\mu(1-\gamma_5)\psi_P(x)A_{P,\mu}(x) + \bar{\psi}_P(x)\kappa^\dagger\gamma^\mu(1-\gamma_5)\psi'(x)A^\dagger_{P,\mu}(x)$$

Using the fact: $\psi_P(x) = \gamma^0\psi(x')$, $A_{P,\mu}(x) = -A^\mu(x')$

$$\bar{\psi}'_P(x)\gamma^\mu\psi_P(x) = \bar{\psi}'(x')\gamma_\mu\psi(x'),\ \bar{\psi}'_P(x)\gamma^\mu\gamma_5\psi_P(x) = -\bar{\psi}'(x')\gamma_\mu\gamma_5\psi(x')$$

$$L^P(x) = \bar{\psi}'(x')\kappa\gamma^\mu(1+\gamma_5)\psi(x')A_\mu(x') + \bar{\psi}(x')\kappa^\dagger\gamma^\mu(1+\gamma_5)\psi'(x')A^\dagger_\mu(x')$$

$L^P(x)$ does not goes to $L(x')$, mixture of $\psi'\gamma^\mu\psi$ and $\psi'\gamma^\mu\gamma_5\psi$!

Under C transformation, $\psi_C(x) = C\bar{\psi}^*$ and $A_C(x) = -A^\dagger(x)$

$$L^C(x) = \bar{\psi}'_C(x)\kappa\gamma^\mu(1-\gamma_5)\psi_C(x)A_{C,\mu}(x) + \bar{\psi}_C(x)\kappa^\dagger\gamma^\mu(1-\gamma_5)\psi'(x)A^\dagger_{C,\mu}(x)$$

$$L^C(x) = \bar{\psi}(x)\kappa^T\gamma^\mu(1+\gamma_5)\psi'(x)A^\dagger_\mu(x) + \bar{\psi}'(x)\kappa^*\gamma^\mu(1+\gamma_5)\psi(x)A_\mu(x)$$

$L^C(x)$ **does not goes to** $L(x)$**,**

mixture of $\psi'\gamma^\mu\psi$ **and** $\psi'\gamma^\mu\gamma_5\psi$**,** κ **is complex!**

Under CP transformation

$$L^{CP}(x) = \bar{\psi}(x')\kappa^T\gamma^\mu(1-\gamma_5)\psi'(x')A^\dagger_\mu(x') + \bar{\psi}'(x')\kappa * \gamma^\mu(1-\gamma_5)\psi(x')A_\mu(x')$$

$L^{CP}(x)$ **will goes to** $L(x')$ **is** κ **is real! CP is then conserved!!**

Under CPT transformation, because the T will change constant κ to κ^*,

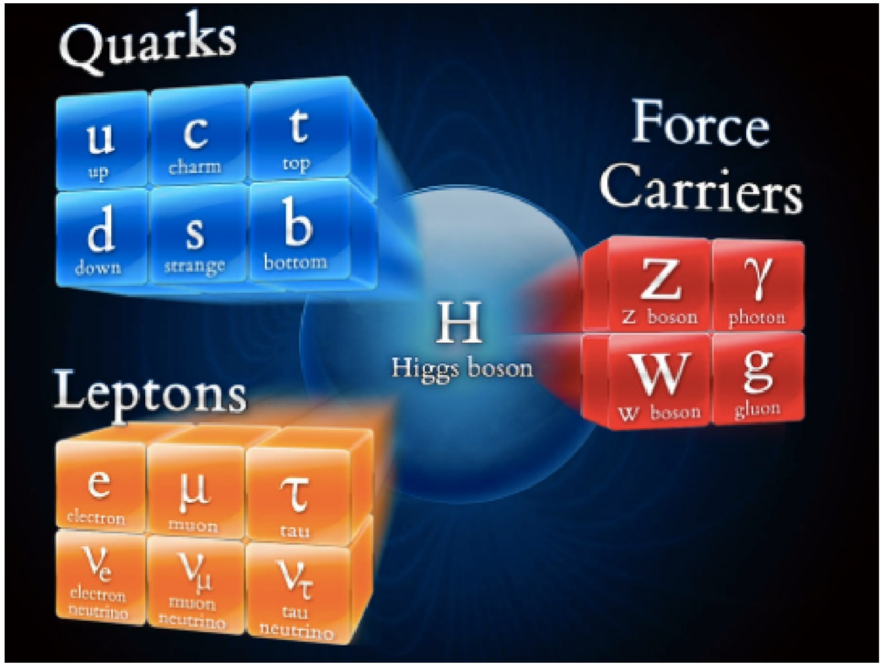
$$L^{CPT}(x) = \bar{\psi}(x')\kappa^\dagger\gamma^\mu(1-\gamma_5)\psi'(x')A^\dagger_\mu(x') + \bar{\psi}'(x')\kappa\gamma^\mu(1-\gamma_5)\psi(x')A_\mu(x')$$

$L^{CPT}(x) = L(-x)$**, CPT is conserved!!**

1.3 Flavor physics and CP violation in the SM

Standard Model is based on $SU(3)_C x SU(2)_L x U(1)_Y$ gauge interaction.

In SM mis-match of weak and mass eigen-bases, leads to flavor mixing and CP violation.



When going beyond SM, more possibilities!

Standard Model: CKM and Strong CP Violation

The standard model of strong and electroweak interaction has gauge group

$SU(3)_C \times SU(2)_L \times U(1)_Y$ with gauge bosons

8 $SU(3)_C$ Gluons : $G^\mu = \frac{\lambda^a}{2} G^\mu_a$, $Tr(\frac{\lambda^a}{2}\frac{\lambda^b}{2}) = \frac{\delta^{ab}}{2}$.

3 $SU(2)_L$ W-bosons : $W^\mu = \frac{\sigma^i}{2} W^\mu_i$, $Tr(\frac{\sigma^i}{2}\frac{\sigma^j}{2}) = \frac{\delta^{ij}}{2}$.

1 $U(1)_Y$ B boson : B^μ

$$\lambda_1 = \begin{pmatrix} 0 & 1 & 0 \\ 1 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix} \quad \lambda_2 = \begin{pmatrix} 0 & -i & 0 \\ i & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix} \quad \lambda_3 = \begin{pmatrix} 1 & 0 & 0 \\ 0 & -1 & 0 \\ 0 & 0 & 0 \end{pmatrix}$$

$$\lambda_4 = \begin{pmatrix} 0 & 0 & 1 \\ 0 & 0 & 0 \\ 1 & 0 & 0 \end{pmatrix} \quad \lambda_5 = \begin{pmatrix} 0 & 0 & -i \\ 0 & 0 & 0 \\ i & 0 & 0 \end{pmatrix}$$

$$\lambda_6 = \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & 1 \\ 0 & 1 & 0 \end{pmatrix} \quad \lambda_7 = \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & -i \\ 0 & i & 0 \end{pmatrix} \quad \lambda_8 = \frac{1}{\sqrt{3}} \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & -2 \end{pmatrix}$$

The building blocks of fermions are chiral fields $f_{L,R} = \frac{1 \mp \gamma_5}{2} f$

The SM fermions are leptons L_L, E_R and quarks Q_L, U_R and D_R

$$\sigma_1 = \sigma_x = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix} \quad \sigma_2 = \sigma_y = \begin{pmatrix} 0 & -i \\ i & 0 \end{pmatrix} \quad \sigma_3 = \sigma_z = \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}$$

$L_L = (\nu_L, e_L : (1,2)(-1/2)^T$, $e_R : (1,1)(-1)$,

$Q_L = (u_L, d_L)^T{}_{)^T:}(3,2)(1/6)$, $u_R : (3,1)(2/3)$, $d_R : (3,1)(-1/3)$.

This is the minimal SM. Neutrinos are massless. Non-zero neutrino mass need extension, at lead something to provide neutrino masses. More later

Also a Higgs boublet $H = (h^+, (v + h + iI)/\sqrt{2})^T : (1,2)(1/2)$

The non-zero vev v to break electroweak symmetries

and give masses to all particles.

$h^\pm$ and I are Goldstone bosons "eaten" by $W^\pm$ and Z bosons.

h is a neutral boson, the famous $Higgs$ boson discovered in 2012 at LHC.

Renormalizable SM Lagrangian

$$
\begin{aligned}
L &= -\frac{1}{2}Tr(G_{\mu\nu}G^{\mu\nu}) - \frac{1}{2}Tr(W_{\mu\nu}W^{\mu\nu}) - \frac{1}{4}B_{\mu\nu}B^{\mu\nu} - \theta\frac{g_3^2}{16\pi}Tr(\tilde{G}^{\mu\nu}G_{\mu\nu}) \\
&+ \bar{Q}_L i\gamma^\mu D_\mu Q_L + \bar{u}_R i\gamma^\mu D_\mu u_R + \bar{d}_R i\gamma^\mu D_\mu d_R + \bar{L}_L i\gamma^\mu D_\mu L_L + \bar{e}_R i\gamma^\mu D_\mu e_R \\
&- \bar{Q}_L Y_u \tilde{H} u_R + \bar{Q}_L Y_d H d_R + \bar{L}_L Y_e H e_R + H.C. + (D_\mu H)^\dagger (D^\mu H) - V(H)
\end{aligned}
$$

Fermion masses need to have left and right handed fermions to pair up, quarks and charged leptons do. But no right handed neutrinos in the minimal SM. Neutrinos are massless!

$\tilde{G}^{\mu\nu} = \frac{1}{2}\epsilon^{\mu\nu\alpha\beta}G_{\alpha\beta}$

$V(H) = \mu^2 H^\dagger H + \lambda(H^\dagger H)^2$ is the Higgs potential.
Positivity of potential for large h, $\lambda > 0$.

$D_\mu = \partial_\mu - ig_3 G_\mu - ig_2 W_\mu - ig_1 Y B_\mu$

If $D_f^\mu = \partial^\mu - igf^\mu$, then $f^{\mu\nu} = \partial^\mu f^\nu - \partial^\nu f^\mu - ig[f^\mu, f^\nu]$

At the vacuum $< H >= v/\sqrt{2}$, minimize $V(v)$ results in $\mu^2 = -\lambda v^2$

$V(h) = -\lambda \frac{v^4}{4} + \lambda v^2 h^2 + \lambda v h^3 + \lambda \frac{h^4}{4}, \;\; m_h^2 = 2\lambda v^2$

$(D_\mu H)^\dagger (D^\mu H) \to m_W^2 = g_2^2 v^2/4 \;\text{ and }\; m_Z^2 = (g_2^2 + g_1^2) v^2/4 = \frac{g_2^2}{4c_W^2} v^2$

$A_\mu = c_w B_\mu + s_W W_\mu^3,\; Z_\mu = -s_W B_\mu + c_W W_\mu^3,\; W_\mu^\pm = \frac{1}{\sqrt{2}}(W_\mu^1 \mp i W_\mu^2)$

$J_g^{a,\mu} G_\mu^a = -g_3 \bar{f} \frac{\lambda^a}{2} \gamma^\mu f G_\mu^a$. P, C, T symmetric.

$J_{em}^\mu A_\mu = -eQ_f \bar{f} \gamma^\mu f A_\mu$. P, C, T symmetric.

$J_Z^\mu Z_\mu = -\frac{g_2}{2c_W} \bar{f} \gamma^\mu (g_V^f - g_A^f \gamma_5) f Z_\mu$, $g_V^f = T_f^3 - 2Q_f s_W^2$ and $g_A^f = T_f^3$

T_f^3 weak isospin, up type (u, ν) $T_f^3 = 1/2$, down type (d, e) $T_f^3 = -1/2$

Violates P, C symmetry, but CPT, CP, T symmetric.

$J_W^\mu W_\mu^+ = -\frac{g_2}{\sqrt{2}} \bar{u}_L \gamma^\mu d_L W_\mu^+ = -\frac{g_2}{2\sqrt{2}} \bar{u} \gamma^\mu (1 - \gamma_5) d W_\mu^+$.

Violates P, C symmetry. CPT symmetric. What about CP, T?

Sources of flavor changing and CP violation in the SM

Needs to work with the charged current and also the Yukawa interactions.

$L_m = -(\bar{Q}_L Y_u \tilde{H} u_R + \bar{Q}_L Y_d H d_R + \bar{L}_L Y_e H e_R + H.C.)$,

$\bar{f} M_f (1 + \frac{h}{v}) f$ with $M_f = Y_f v/\sqrt{2}$ which is usually not diagonalized.

Bi-unitary diagonalization:

$M_f = V^\dagger_{fL} \hat{M}_f V_{fR}$, $\hat{M}_f = diag(m^1_f, m^2_f ... m^N_f)$, $V_{L,R}$ unitary matrices.

Mass eigenstate basis: $f^m_L = V_{fL} f_L$ and $f^m_R = V_{fR} f_R$

$L_m = -\bar{f}^m \hat{M}_f (1 + \frac{h}{v}) f^m$ is very simple. **P, C, T symmetric!**

$J^\mu_{g,em,Z}$ just replace f by f^m, form no change. **CP properties no change!**

But the charged current will be modified to

$$J_W^\mu W_\mu^+ = -\frac{g_2}{\sqrt{2}} \bar{u}_L^m \gamma^\mu V_{uL} V_{dL}^\dagger d_L^m W_\mu^+, \; V_{CKM} = V_{uL} V_{dL}^\dagger$$

$$L = -\frac{g_2}{\sqrt{2}} [\bar{u}_L^m \gamma^\mu V_{KM} d_L^m W_\mu^+ + \bar{d}_L^m \gamma^\mu V_{KM}^\dagger u_L^m W_\mu^-]$$

$$= -\frac{g_2}{2\sqrt{2}} [\bar{u}_i^m \gamma^\mu (1-\gamma_5) V_{KM}^{ij} d_j^m W_\mu^+ + \bar{d}_j^m \gamma^\mu (1-\gamma_5) (V_{KM}^{ij})^* u_i^m W_\mu^-]$$

$$L^{CP}(x) = -\frac{g_2}{2\sqrt{2}} [\bar{d}_j^m \gamma^\mu (1-\gamma_5) V_{KM}^{ij} u_i^m W_\mu^- + \bar{u}_i^m \gamma^\mu (1-\gamma_5) (V_{KM}^{ij})^* u_j^m W_\mu^+](x')$$

If V_{KM} is real, then $L^{CP}(x) = L(x')$, CP is conserved!

Is V_{KM} real? Or can V_{KM} be complex?

If V_{KM} is diagonal, no flavor changing interactions. But in general it has off diagonal entries.

Conditions for CP violation in the SM

Conditions for complex V_{KM}

$V_{KM} = (V^{ij})$, $N \times N$ unitary matrix. Naively $2N^2$ parameters, V^{ij} complex.

N^2 constraining equations: $\sum_i V^{ij} V^{ik*} = \delta^{jk}$ and $\sum_i V^{ji} V^{ki*} = \delta^{jk}$

So a unitary matrix contains N^2 parameters for N generations.

$2N - 1$ parameters, absorbed into quarks $q_i \to e^{i\alpha_i} q_i$, not physical.
Why not 2N but 2N-1?

Needs $N(N-1)/2$ parameters describe rotation angles (Euler angles)

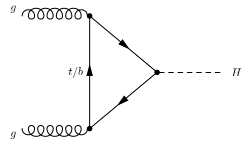
Finally $(N-1)(N-2)/2$ non-removable phases, physical, $\to V_{KM}$ complex.

The physical phases are the sources for KM model of CP violation!

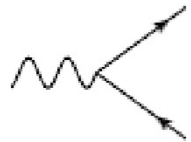
Number of SM generations

In the SM, only 3 generations of quarks and leptons are allowed.

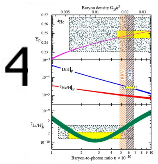
gg -> Higgs ~ (number of heavy quarks)^2, if fourth generation exist, their mass should be large, 9 times bigger production of Higgs. LHC data ruled out more than 3 generations of quarks.

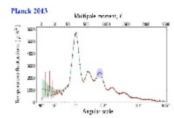


LEP already ruled out more than 3 neutrinos with mass less than $m_Z/2$.

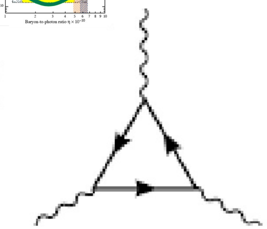


Cosmology and astrophysics, number of light neutrinos also less than 4





SM, triangle anomaly cancellation: equal number of quarks and leptons!



There are only three generations of sequential quarks and leptons!

Why 3 generations? How do they mix with each other?

Beyond SM, conclusions may change, X-G He and G. Valencia, PPLB707 (2012)

Idea of quark mixing and source for CP violation

UNITARY SYMMETRY AND LEPTONIC DECAYS

The Cabbibo angle

Nicola Cabibbo
CERN, Geneva, Switzerland
(Received 29 April 1963)

To determine θ, let us compare the rates for $K^+ \to \mu^+ + \nu$ and $\pi^+ \to \mu^+ + \nu$; we find

$$\Gamma(K^+ \to \mu\nu)/\Gamma(\pi^+ \to \mu\nu) = \tan^2\theta M_K(1 - M_\mu{}^2/M_K{}^2)^2/M_\pi(1 - M_\mu{}^2/M_\pi{}^2)^2. \quad (3)$$

From the experimental data, we then get[5,6]

$$\theta = 0.257. \quad (4)$$

The GIM mechanism

The Kobayashi-Maskawa Model in 1973!

Weak Interactions with Lepton-Hadron Symmetry*

S. L. Glashow, J. Iliopoulos, and L. Maiani†
Lyman Laboratory of Physics, Harvard University, Cambridge, Massachusetts 02139
(Received 5 March 1970)

We propose a model of weak interactions in which the currents are constructed out of four basic quark fields and interact with a charged massive vector boson. We show, to all orders in perturbation theory, that the leading divergences do not violate any strong-interaction symmetry and the next to the leading divergences respect all observed weak-interaction selection rules. The model features a remarkable symmetry between leptons and quarks. The extension of our model to a complete Yang-Milis theory is discussed.

Application of KM model for ε

CP violation in the six-quark model*

Sandip Pakvasa and Hirotaka Sugawara†
Department of Physics and Astronomy, University of Hawaii at Manoa, Honolulu, Hawaii 96822
(Received 29 September 1975)

We construct a Weinberg-Salam-type gauge theory of a weak interaction with CP violation based on the six-quark model. Under the assumption of the validity of the Zweig-Iizuka rule and (quark mass/W-meson mass)$^2 \ll 1$ this leads to the superweak theory of CP violation for both uncharmed and charmed hadrons. We also propose a new assignment for the J and other ψ particles, which predicts the existence of a 3.5-GeV 0^- meson using the 2.85-GeV 0^- state as input.

CP VIOLATION IN PURELY LEFTHANDED WEAK INTERACTIONS

L. MAIANI
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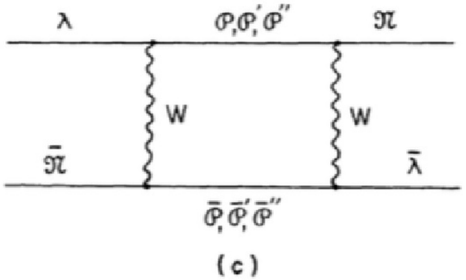
and Istituto Nazionale di Fisica Nucleare - Sezione Sanità, Roma, Italy

Received 3 November 1975
Revised manuscript received 18 February 1976

In a model with six quarks and pure V-A weak interactions CP violation can be introduced in weak currents by spontaneous breaking. The resulting milliweak model is shown to lead uniquely, with good approximation, to the results of the superweak theory both for K decays and for the neutron electric dipole moment.



(c) Second-order diagram for ϵ.

Birth of the SM of CP violation

Progress of Theoretical Physics, Vol. 49, No. 2, February 1973

CP-Violation in the Renormalizable Theory of Weak Interaction

Makoto KOBAYASHI and Toshihide MASKAWA

Department of Physics, Kyoto University, Kyoto

(Received September 1, 1972)

In a framework of the renormalizable theory of weak interaction, problems of CP-violation are studied. It is concluded that no realistic models of CP-violation exist in the quartet scheme without introducing any other new fields. Some possible models of CP-violation are also discussed.

$$\begin{pmatrix} \cos\theta_1 & -\sin\theta_1\cos\theta_3 & -\sin\theta_1\sin\theta_3 \\ \sin\theta_1\cos\theta_2 & \cos\theta_1\cos\theta_2\cos\theta_3-\sin\theta_2\sin\theta_3 e^{i\delta} & \cos\theta_1\cos\theta_2\sin\theta_3+\sin\theta_2\cos\theta_3 e^{i\delta} \\ \sin\theta_1\sin\theta_2 & \cos\theta_1\sin\theta_2\cos\theta_3+\cos\theta_2\sin\theta_3 e^{i\delta} & \cos\theta_1\sin\theta_2\sin\theta_3-\cos\theta_2\sin\theta_3 e^{i\delta} \end{pmatrix}.$$

a typo ! - $\cos\theta_3$ should be just $\cos\theta_3$

Mechanism for CP violation in SM. Predicted the existence of the third generation!

Yet, another CP violation source possible: under P and CP, $\varepsilon^{\mu\nu\alpha\beta}$ -> - $\varepsilon_{\mu\nu\alpha\beta}$,

$$-\theta\frac{g_3^2}{16\pi}Tr(\tilde{G}^{\mu\nu}G_{\mu\nu})$$ – term violates P and CP

This term θ-term gives too large neutron EDM and cause problem, Strong CP problem. Later

KM matrix parametrizations

More commonly used PDG parametrization of V_{KM}

$$V_{KM} = \begin{pmatrix} V_{ud} & V_{us} & V_{ub} \\ V_{cd} & V_{cs} & V_{cb} \\ V_{td} & V_{ts} & V_{tb} \end{pmatrix}$$

L.Maiani, 1976; L.L. Chau and W. Y. Keung, 1984

$$= \begin{pmatrix} c_{12}c_{13} & s_{12}c_{13} & s_{13}e^{-i\delta} \\ -s_{12}c_{23}-c_{12}s_{23}s_{13}e^{i\delta} & c_{12}c_{23}-s_{12}s_{23}s_{13}e^{i\delta} & s_{23}c_{13} \\ s_{12}s_{23}-c_{12}c_{23}s_{13}e^{i\delta} & -c_{12}s_{23}-s_{12}c_{23}s_{13}e^{i\delta} & c_{23}c_{13} \end{pmatrix}$$

$s_i = sin\theta_i$ and $c_i = cos\theta_i$ with θ_i

A non-zero value for δ violates CP.
The Wolfenstein parameterization

$$V_{KM} \approx \begin{pmatrix} 1-\lambda^2/2 & \lambda & A\lambda^3(\rho - i\eta) \\ -\lambda & 1-\lambda^2/2 & A\lambda^2 \\ A\lambda^3(1-\rho-i\eta) & -A\lambda^2 & 1 \end{pmatrix} .$$

When discussing CP violation

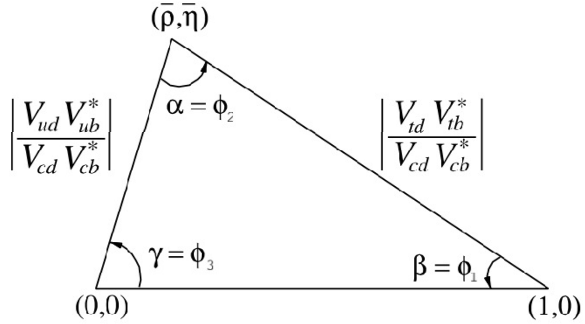
should add $-A^2\lambda^5(\rho + i\eta)$ and $-A\lambda^4(\rho + i\eta)$ to V_{cd} and V_{ts}, respectively.

Quark and Lepton mixing patterns

The mis-match of weak and mass eigen-state bases lead quark and lepton mix within generations.

Quark mixing the Cabibbo -Kobayashi-Maskawa (CKM) matrix $V_{\rm CKM}$,

lepton mixing the Pontecorvo -Maki-Nakawaga-Sakata (PMNS) matrix $U_{\rm PMNS}$



$$L = -\frac{g}{\sqrt{2}}\overline{U}_L\gamma^\mu V_{\rm CKM} D_L W_\mu^+ - \frac{g}{\sqrt{2}}\overline{E}_L\gamma^\mu U_{\rm PMNS} N_L W_\mu^- + H.C.\,,$$

$U_L = (u_L, c_L, t_L, ...)^T$, $D_L = (d_L, s_L, b_L, ...)^T$, $E_L = (e_L, \mu_L, \tau_L, ...)^T$, and $N_L = (\nu_1, \nu_2, \nu_3, ...)^T$

For n-generations, $V = V_{\rm CKM}$ or $U_{\rm PMNS}$ is an $n \times n$ unitary matrix.

A commonly used form of mixing matrix for three generations of fermions is given by

$$V = \begin{pmatrix} c_{12}c_{13} & s_{12}c_{13} & s_{13}e^{-i\delta} \\ -s_{12}c_{23} - c_{12}s_{23}s_{13}e^{i\delta} & c_{12}c_{23} - s_{12}s_{23}s_{13}e^{i\delta} & s_{23}c_{13} \\ s_{12}s_{23} - c_{12}c_{23}s_{13}e^{i\delta} & -c_{12}s_{23} - s_{12}c_{23}s_{13}e^{i\delta} & c_{23}c_{13} \end{pmatrix},$$

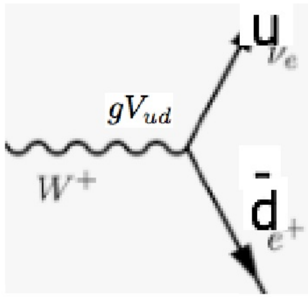
where $s_{ij} = \sin\theta_{ij}$ and $c_{ij} = \cos\theta_{ij}$ are the mixing angles and δ is the CP violating phase.

If neutrinos are of Majorana type, for the PMNS matrix one should include an additional diagonal matrix with two Majorana phases $\mathrm{diag}(e^{i\alpha_1/2}, e^{i\alpha_2/2}, 1)$ multiplied to the matrix from right in the above.

Homework

Problem 1

Using the C, P and T transformation properties for spiners (when exchange two spiners, be careful about the sign changes) and γ^μ matrices obtain S, A^μ, $T^{\mu\nu}$, a^μ, and P transformation table.



Problem 2

Obtain tree level $W^+ \to u\,\bar{d}$ decay width

$$\Gamma(W^+ \to u\bar{d}) = \frac{g^2}{48\pi} m_W N_c |V_{ud}|^2 \sqrt{(1 - \frac{(m_u + m_d)^2}{m_W^2})(1 - \frac{(m_u - m_d)^2}{m_W^2})}(1 - \frac{m_u^2 + m_d^2}{2m_W^2} - \frac{(m_u^2 - m_d^2)^4}{2m_W^4})$$

N_c = number of colors.

Problem 3

Show that for V_{KM}, the identify is true

$$\mathrm{Im}\left[V_{ij} V_{kl} V_{il}^* V_{kj}^*\right] = J \sum_{m,n} \varepsilon_{ikm} \varepsilon_{jln}.$$

Lecture II FPCP in Standard model

2.1 Flavor Physics tests for SM

2.2 CP violation tests for SM

2.3 More CP violating experimental observables