Quantum Chromodynamics

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AEPSHEP 2022
05-18 OCTOBER 2022,
Pyeongchang, SOUTH KOREA

QCD-3

Precision QCD

Plan

- Why Precision Calculation (PC)
- Impact of PCs on discoveries
- Part-1

  Methods for

  Multi-leg processes
- Part-2

  Methods for

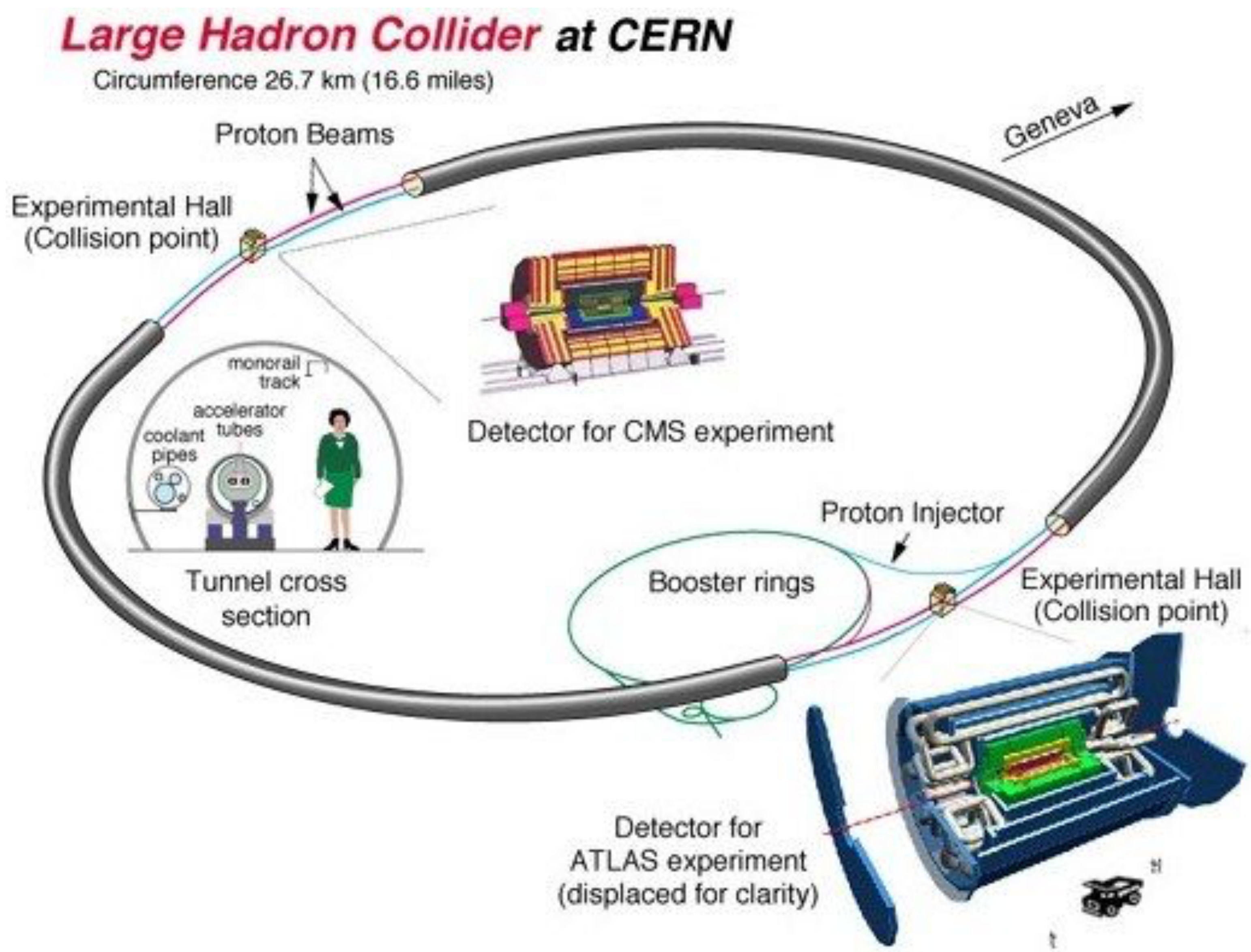
  Multi-loops processes
- Part-3

  Infrared physics

Large Hadron Collider

- Excellent Discovery Reach

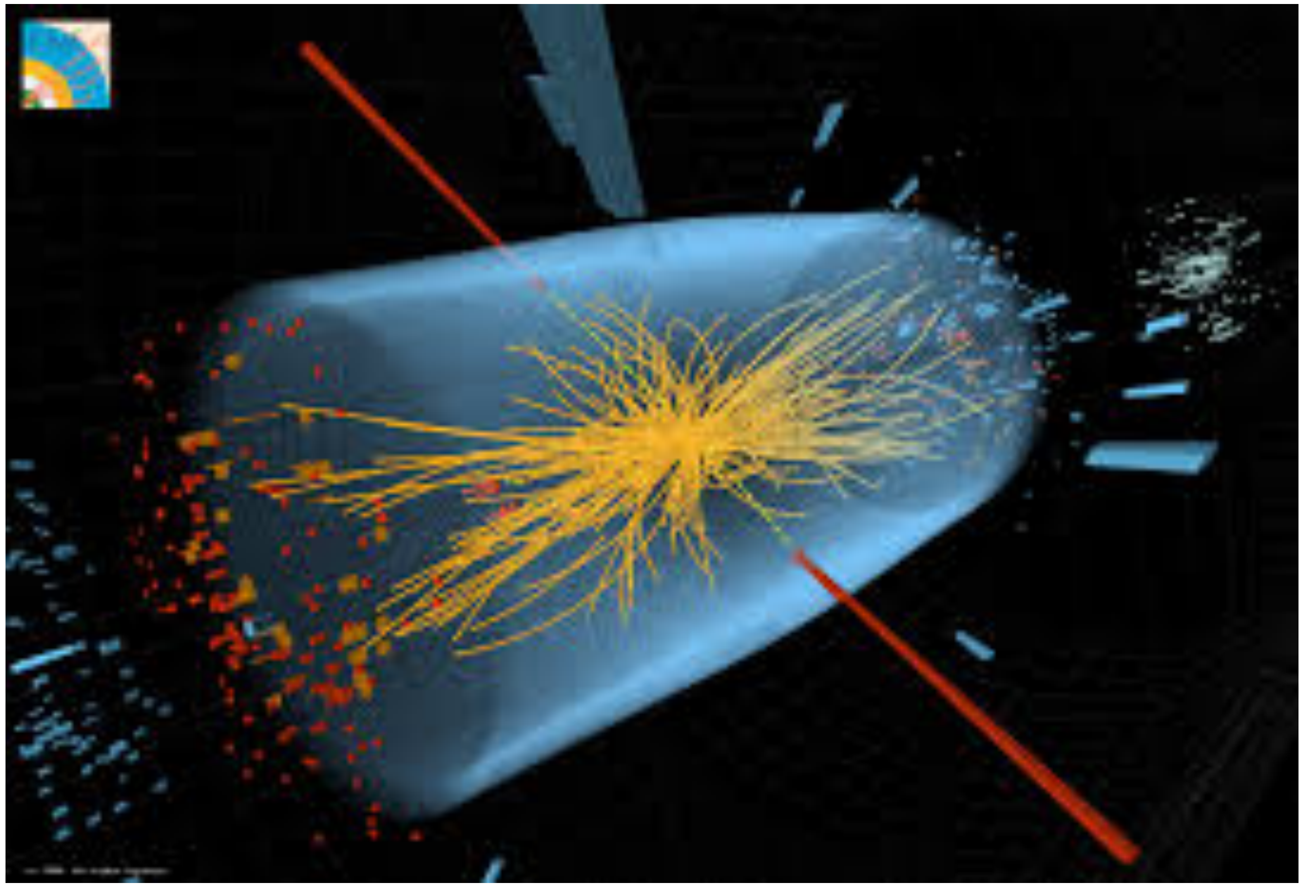
- Higgs
- Supersymmetry
- Extra-Dimensions
- Anything else



Large Hadron Collider

- Large amount of events

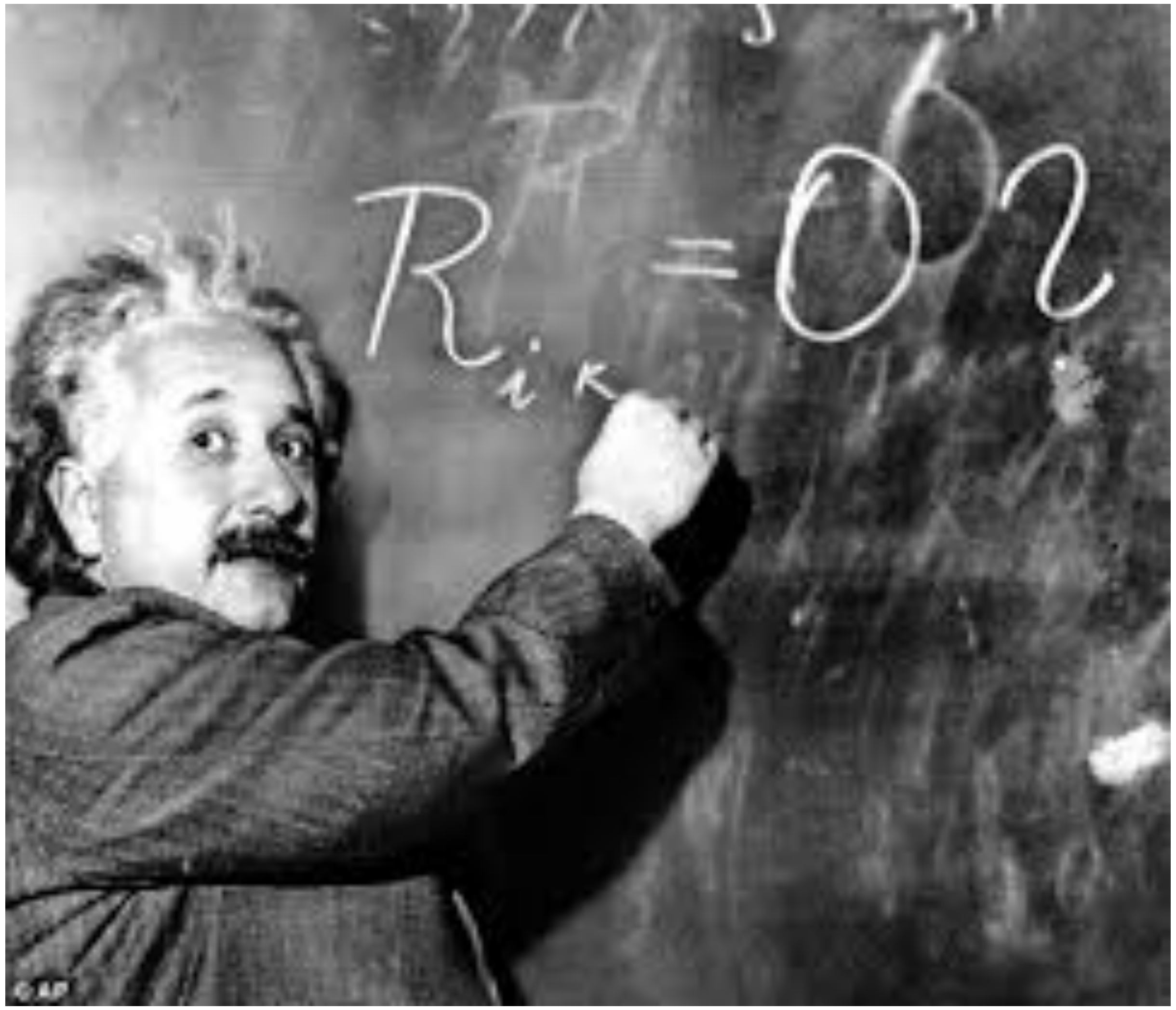
- $W \to e\nu$: 10^8 events
- $Z \to e^+e^-$: 10^7 events
- $t\bar{t}$ production 10^7 events
- Higgs production 10^5 events



Standard Model

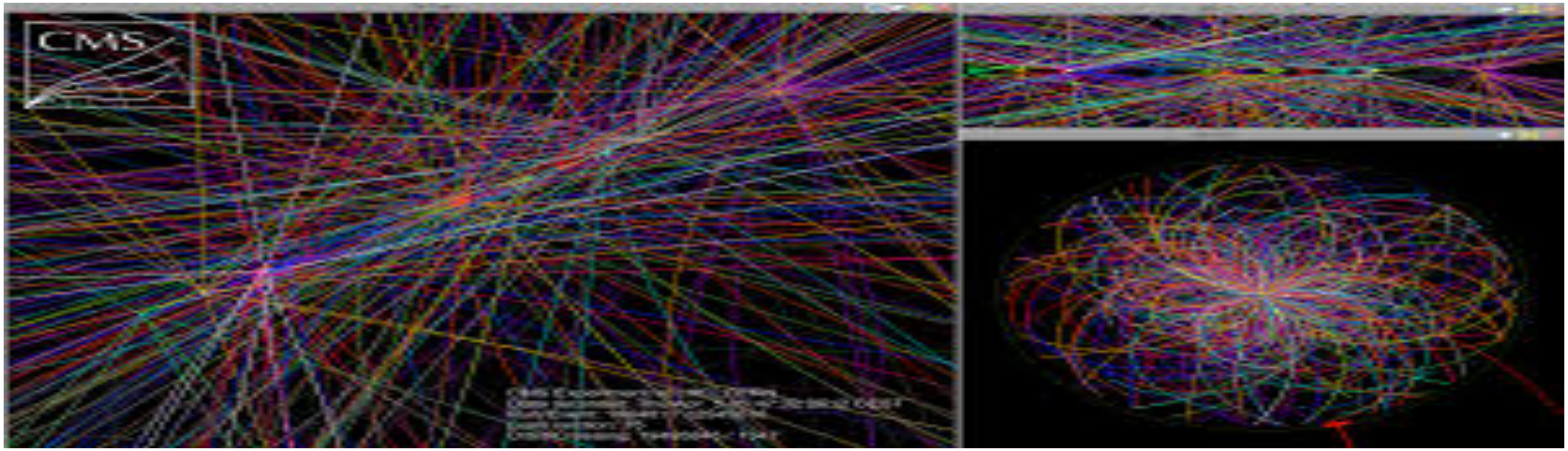
- Theories

- Quantum Chromodynamics
- Electroweak Theory (SM)
- Theory of Gravity



- Large background

- Large number of $\gamma, l^{\pm}, Z, W^{\pm}$
- Jets
- Large number of $t\bar{t}, b\bar{b}$



Theoretical Issues

- Issues to be tackled

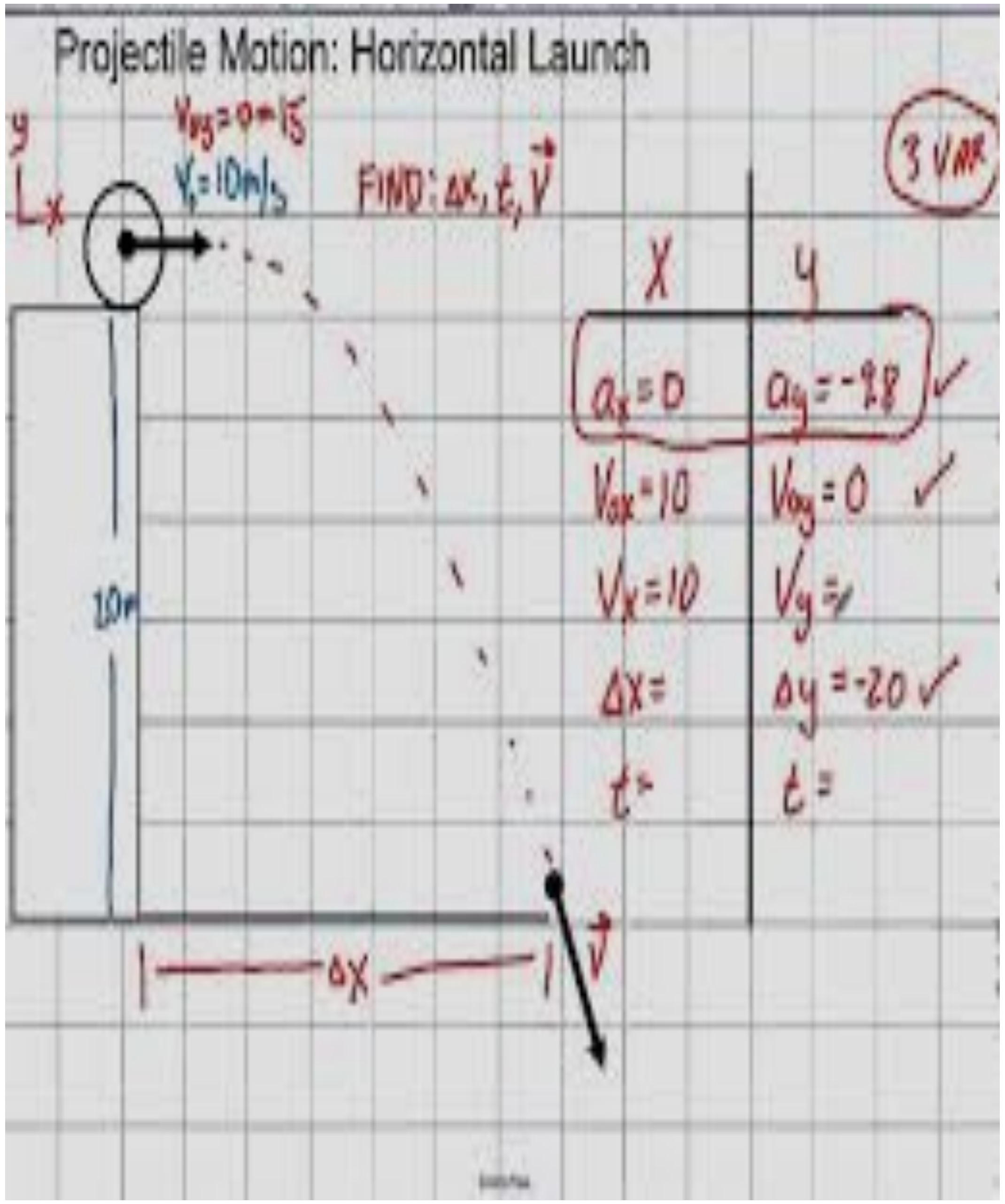
Kinematics

Normalisation

Renormalisation and Factorisation Scales

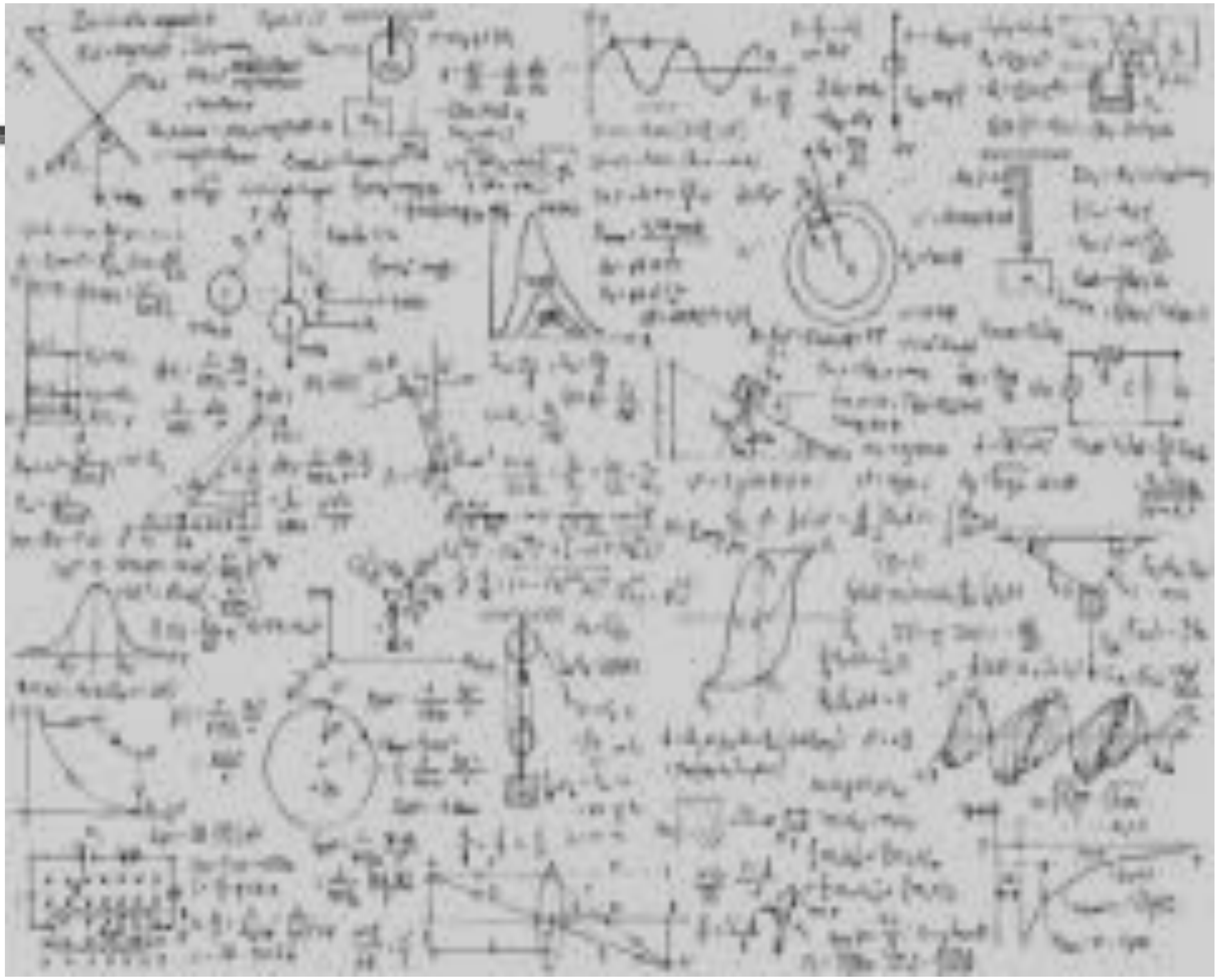
Parton distribution functions

Phase space boundary effects, resummation



Why Precision ?

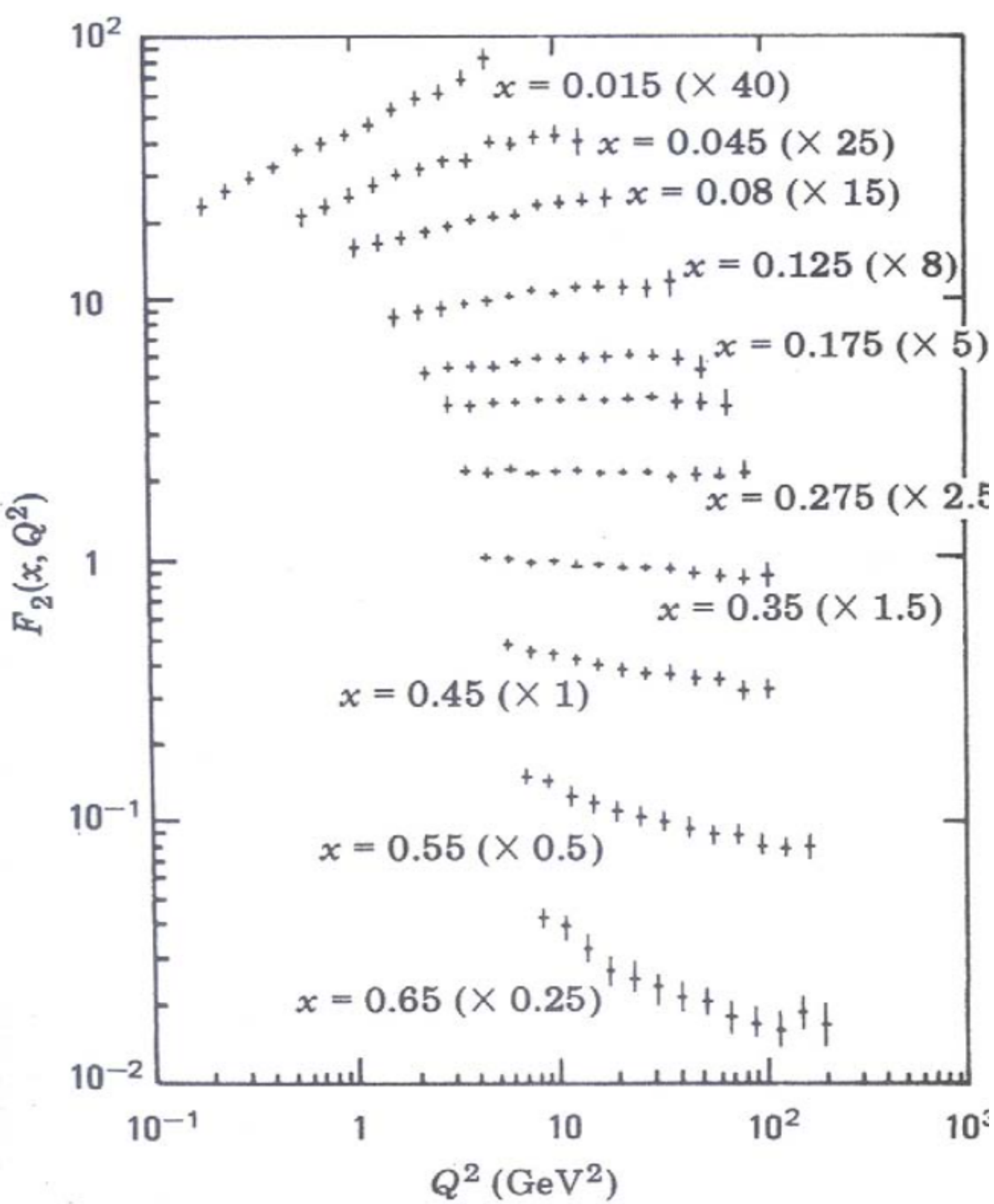
Precision
Units Ltd



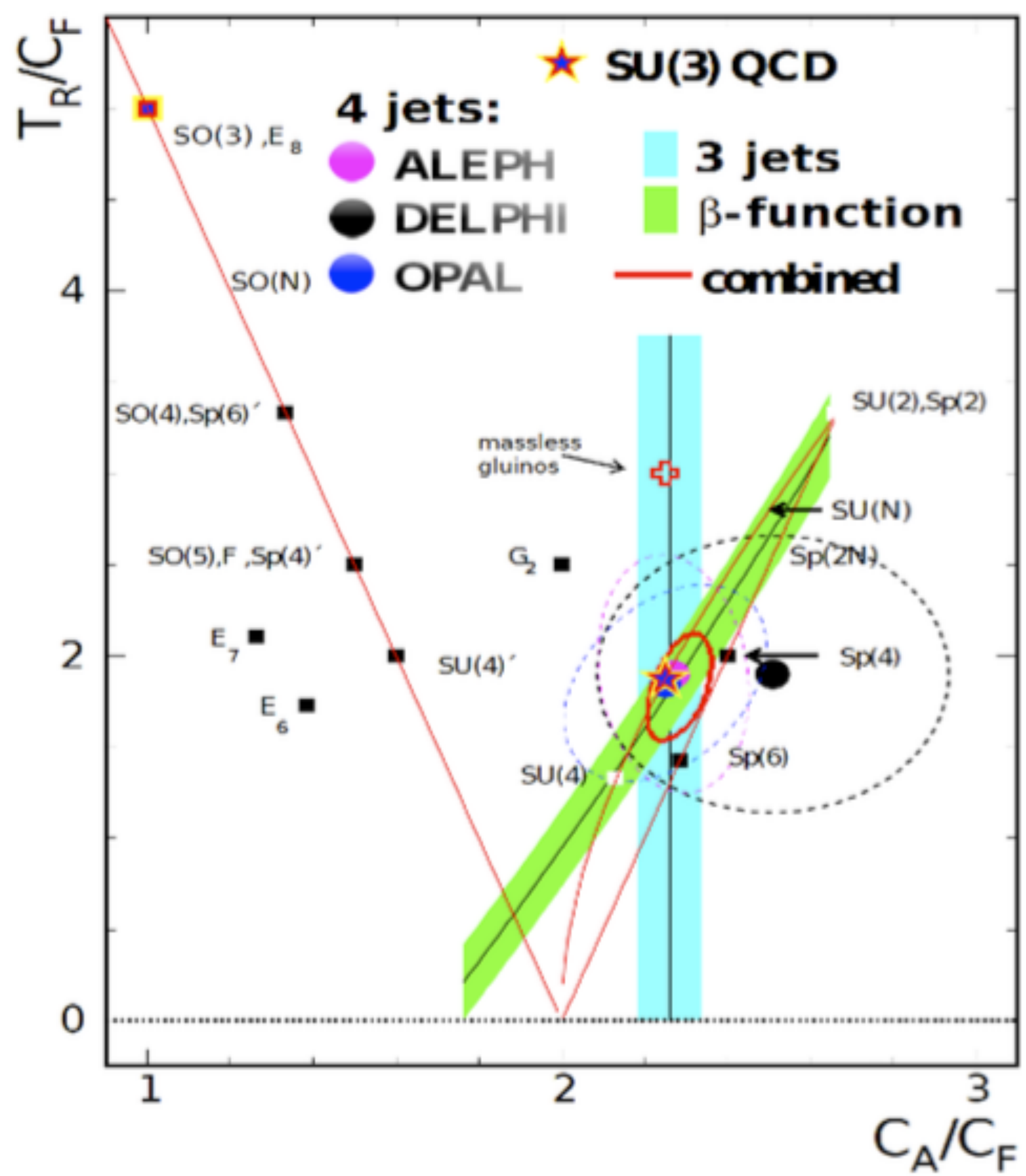
The truth is in the Details

Tests of Quantum Chromodynamics

QCD RGE prediction for DIS



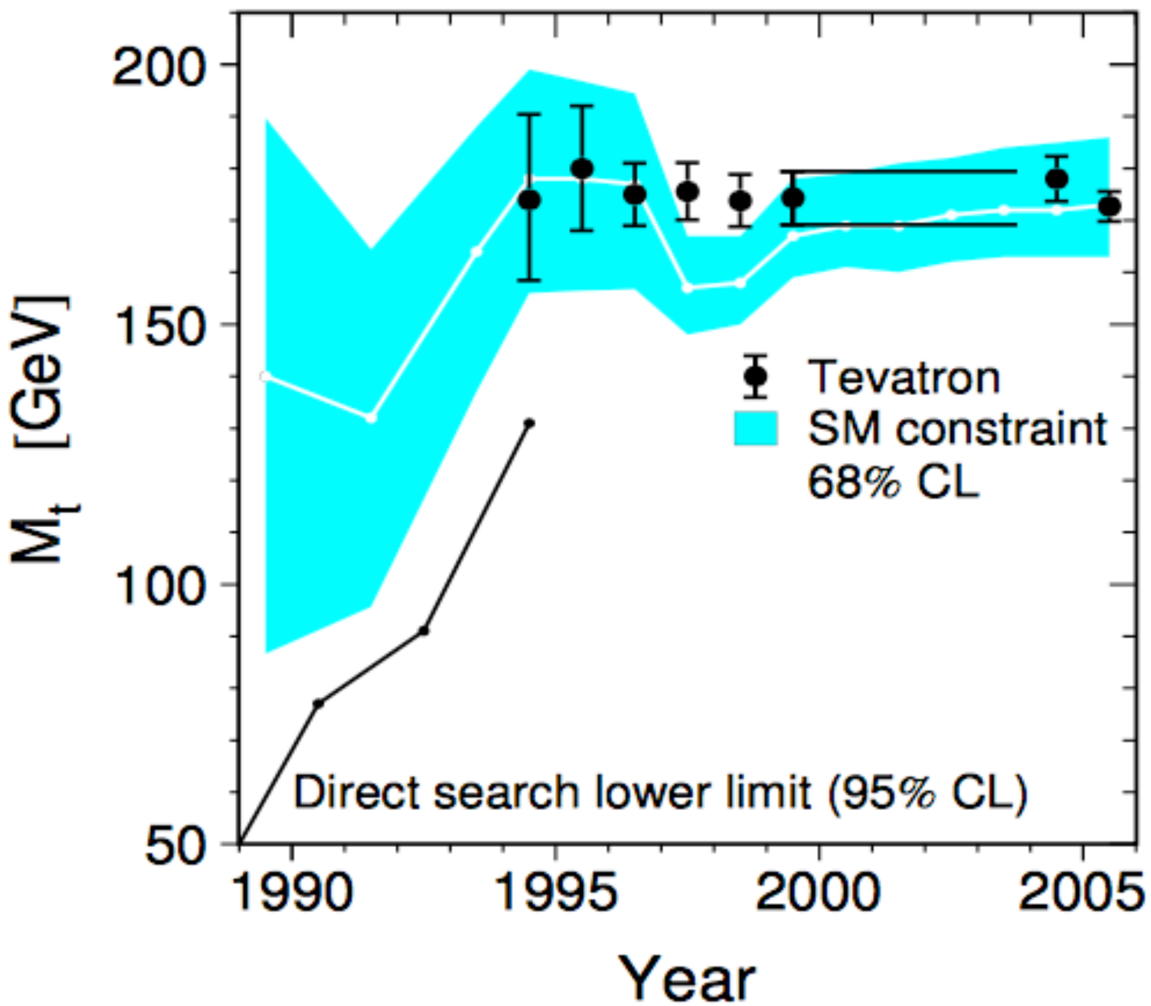
QCD Jets at LEP

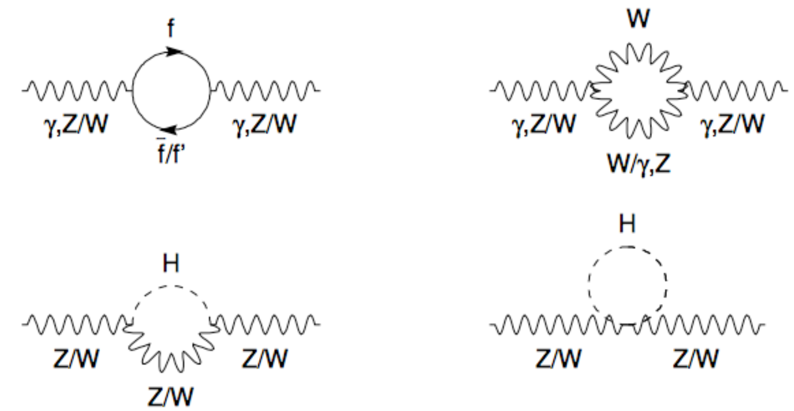


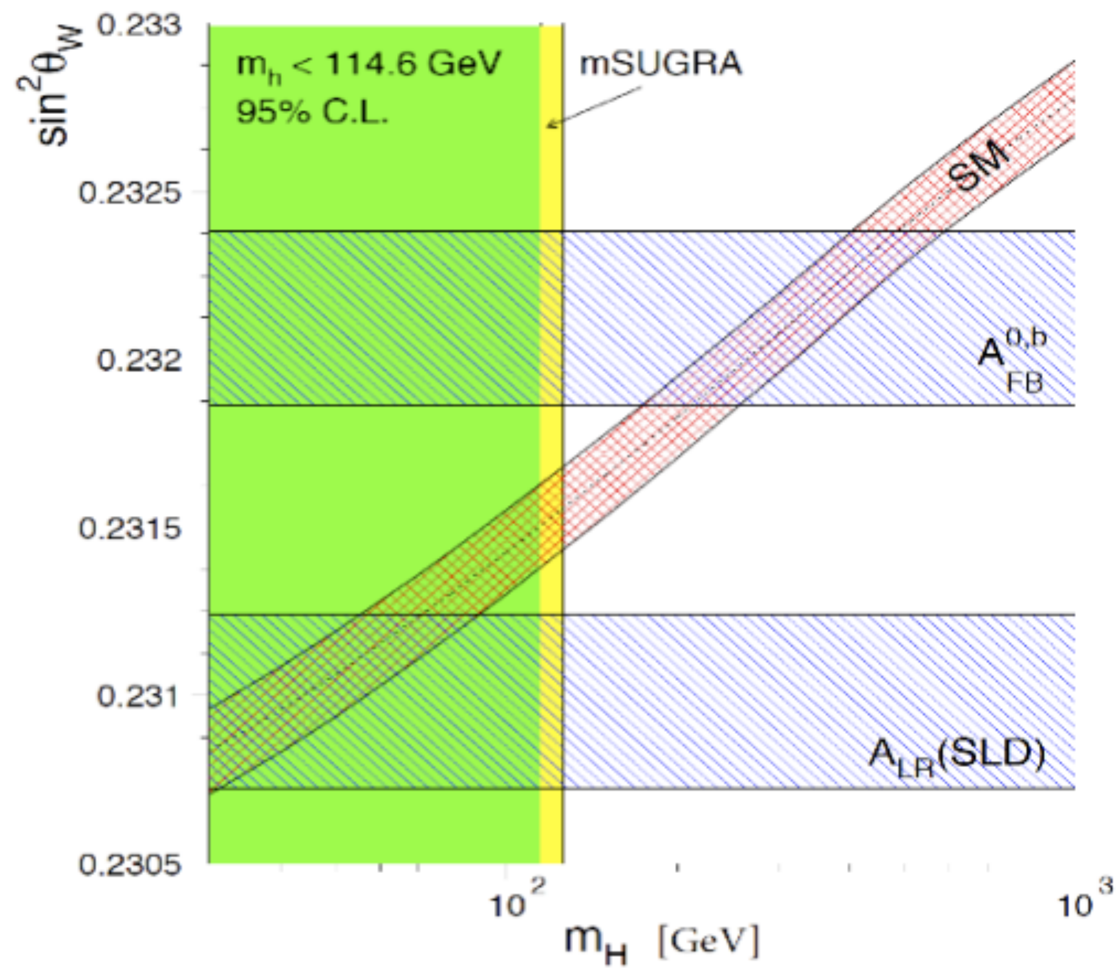
Hits from LEP for Top and Higgs

EW Radiative Corrections

$M_Z, M_H, m_t, \alpha_s(M_Z), \alpha(M_Z)$







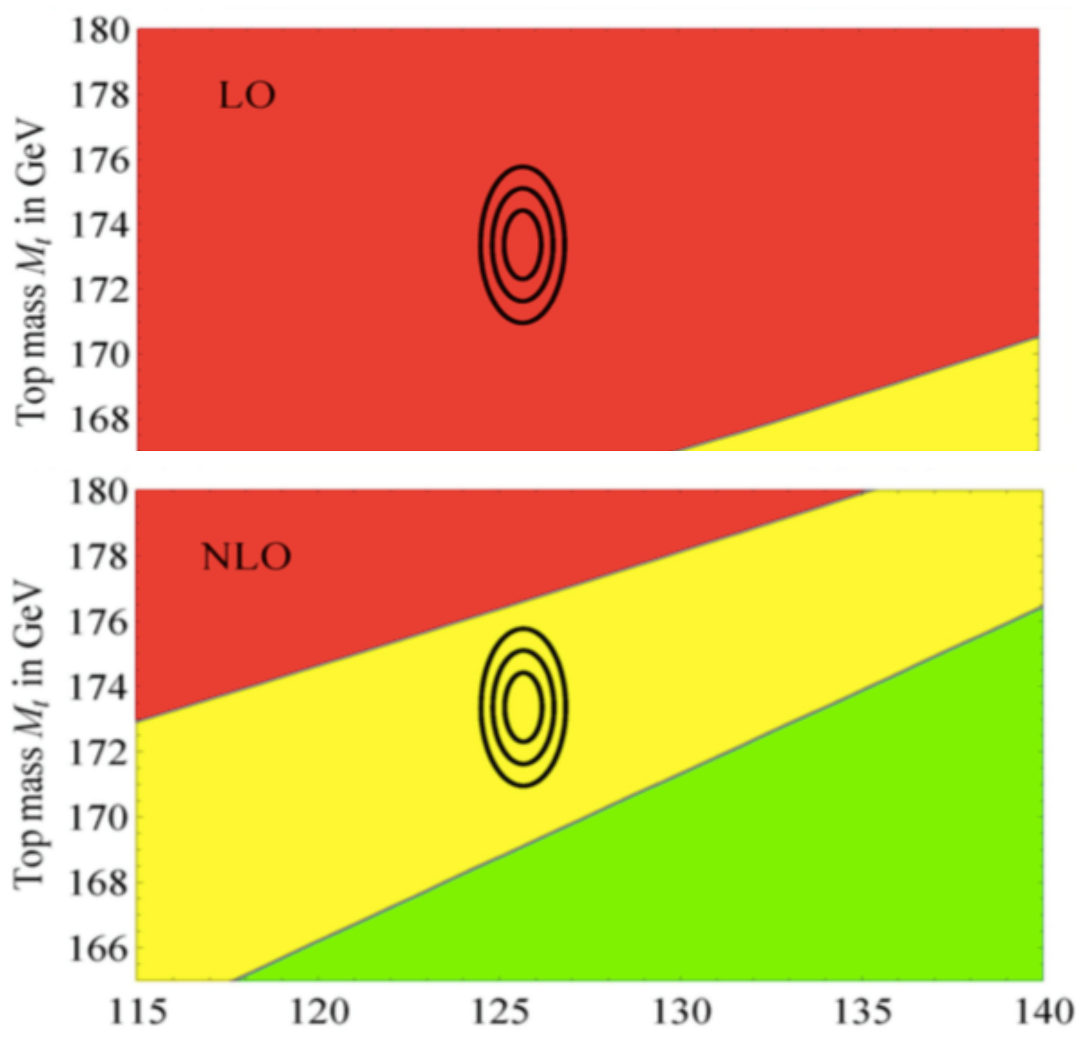
$$m_t = 178.5 \pm 3.9\ GeV$$

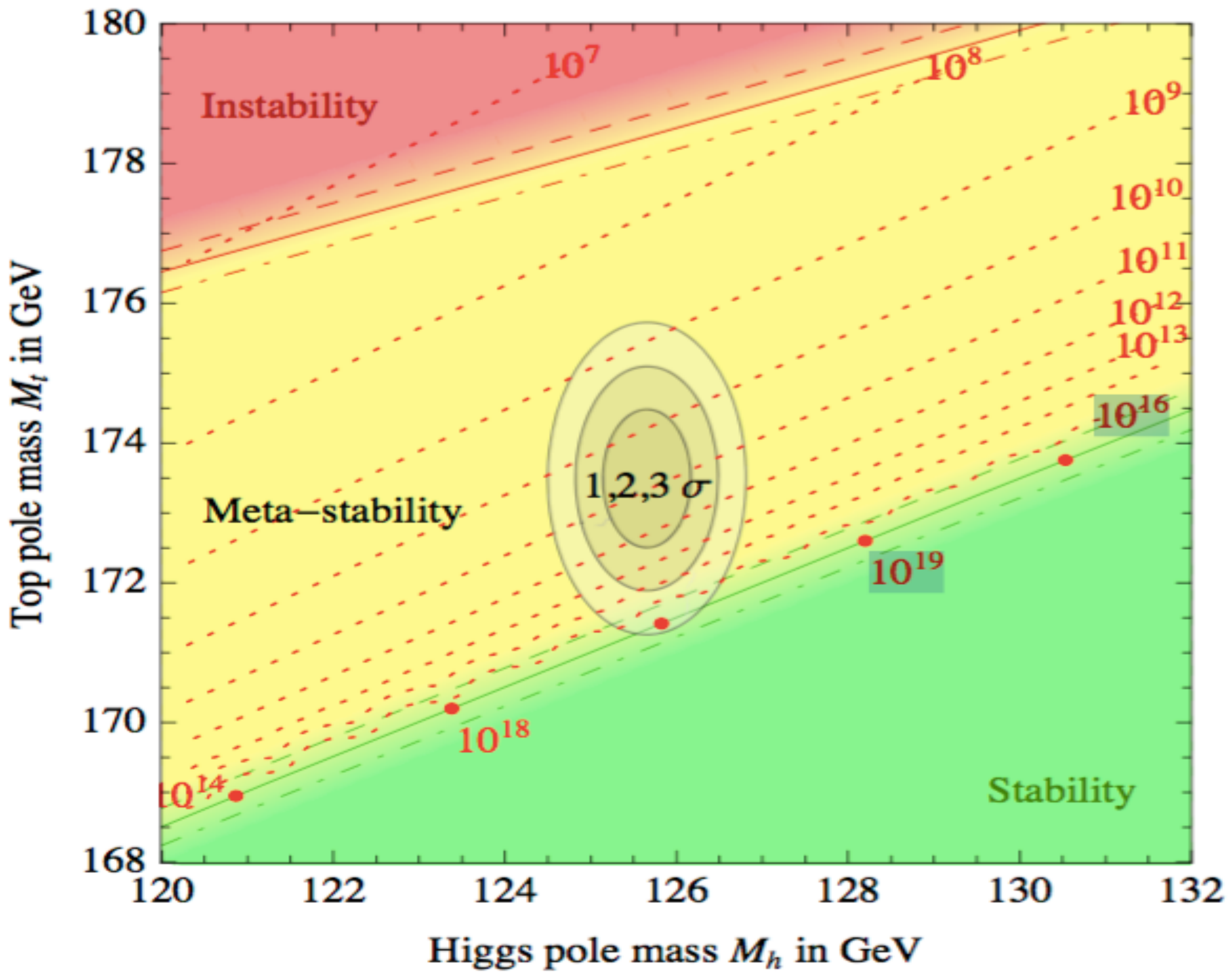
$$m_H = 129^{+74}_{-49}\ GeV$$

Stability of our Vacuum

NNLO Electroweak Correction :

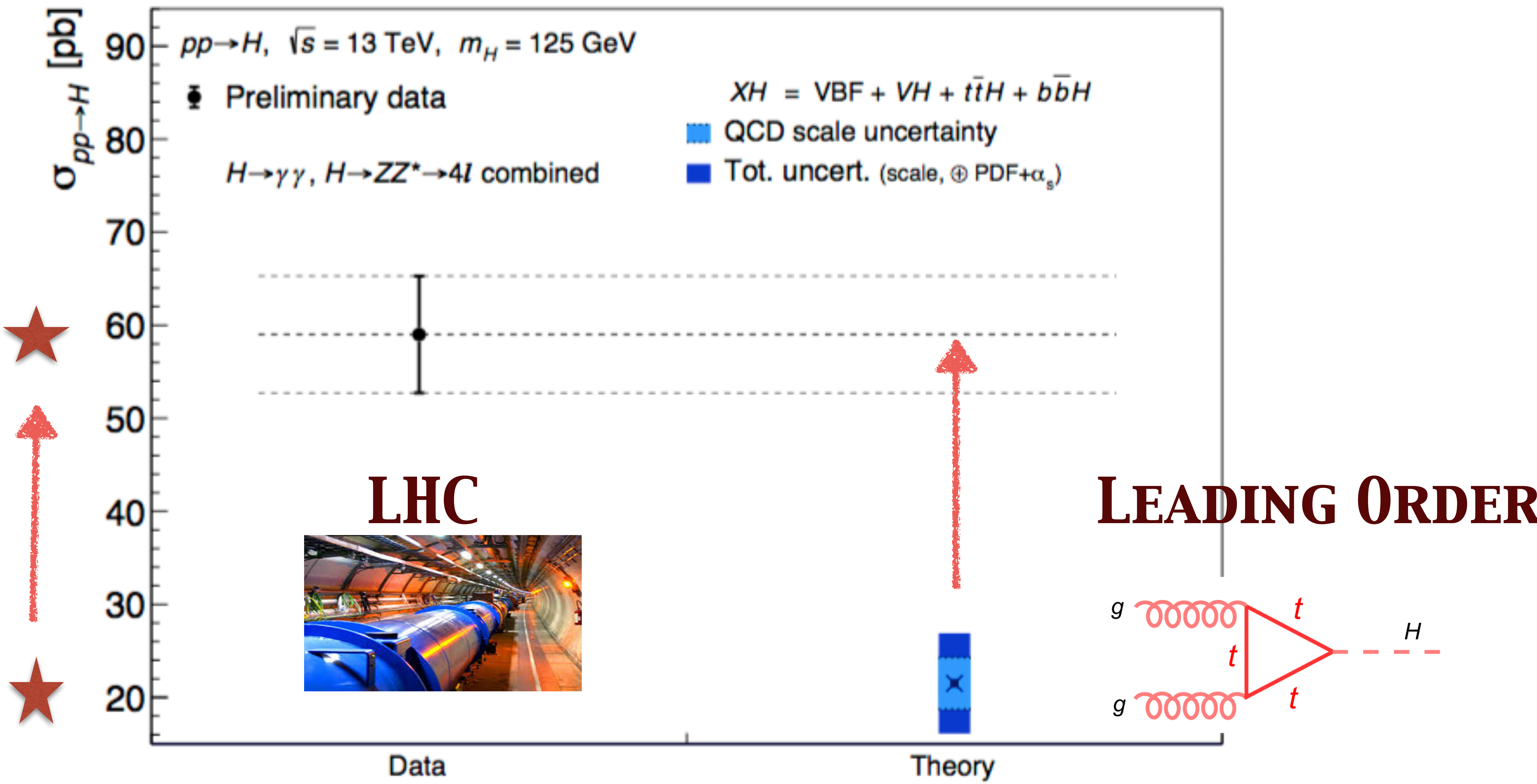
$$M_h[GeV] > 129.6 + 2.0\,[M_t(GeV) - 173.35] - 0.5\left[\frac{\alpha_s(M_z) - 0.1184}{0.0007}\right] \pm 0.3\,.$$



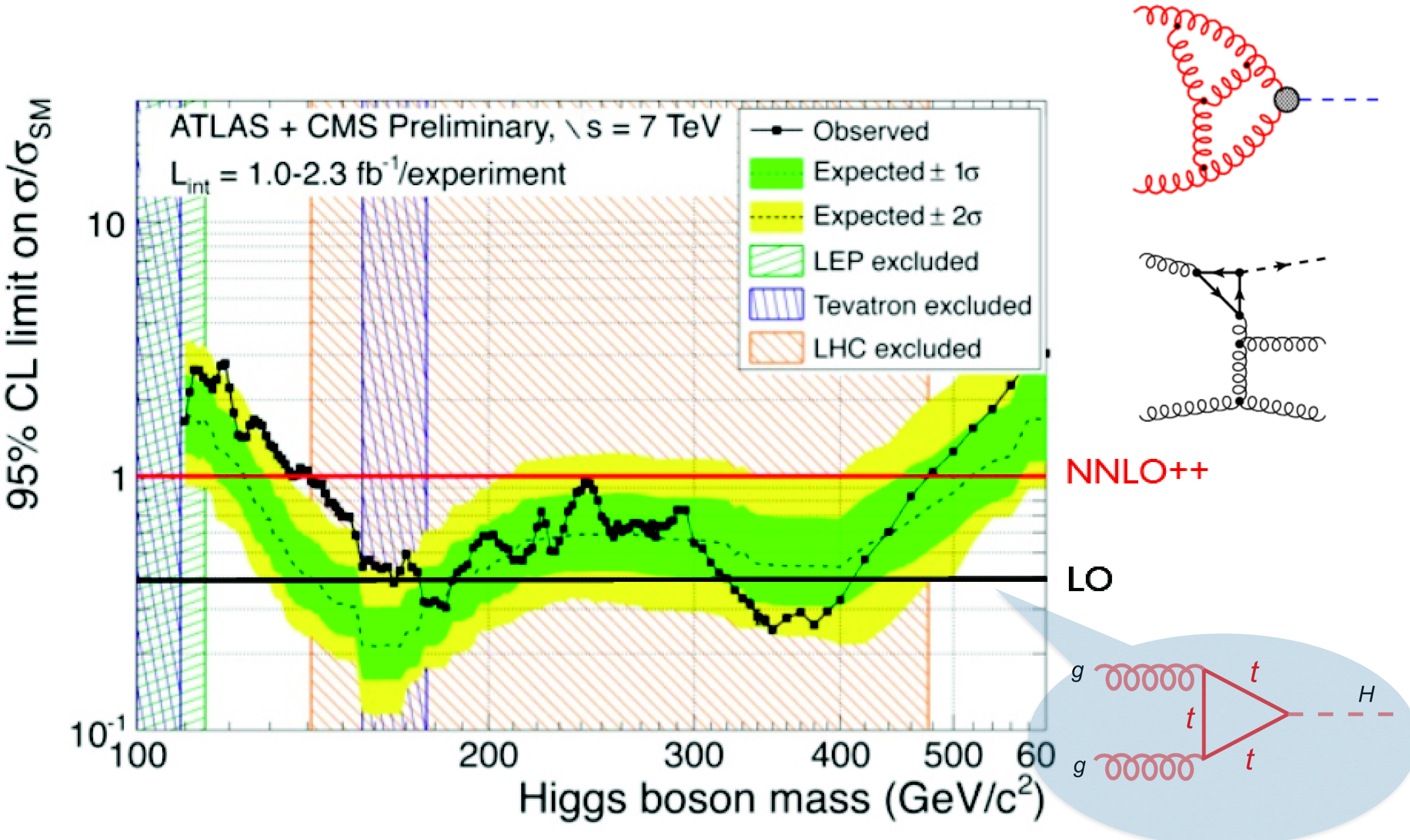


Fate of the universe depends on the mass of top

Leading order is often Crude in QCD

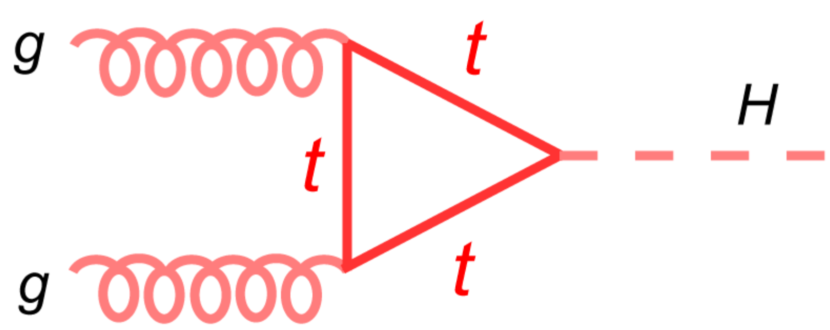


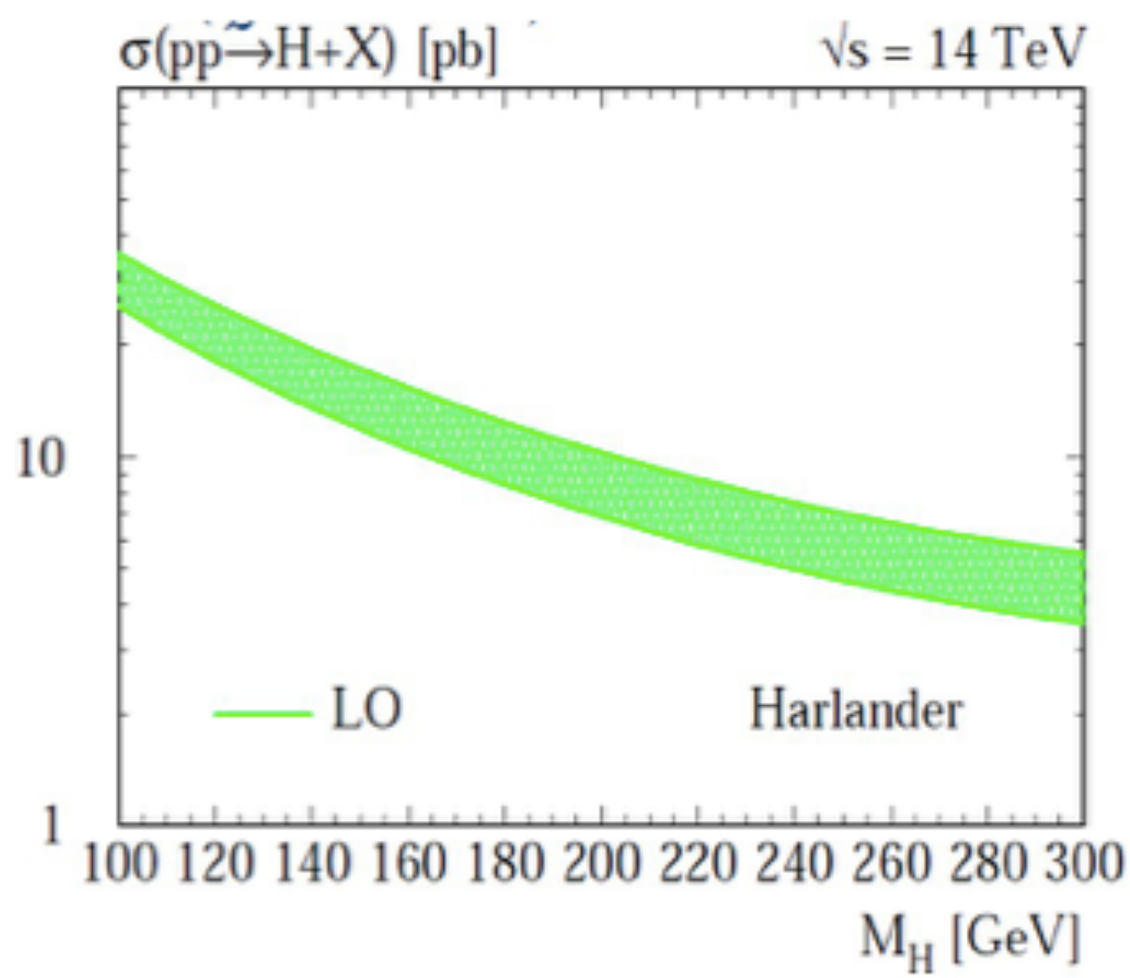
Exclusion Plot for Higgs mass



LO is often a crude estimate

$$2S\sigma^H(x, m_H) = \int_x^1 \frac{dz}{z} \Phi_{gg}^{(0)}(z, \mu_F) 2\hat{s}\hat{\sigma}_{gg}^{(0)}\left(\frac{x}{z}, m_H^2, \mu_R\right) + \cdots$$





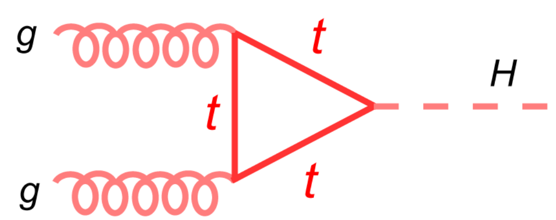
$$2\hat{s}\hat{\sigma}_{gg}^{(0)}\left(\frac{x}{z}, m_H^2, \mu_R\right) = \alpha_s^2(\mu_R) G_F F(m_t, m_H)$$

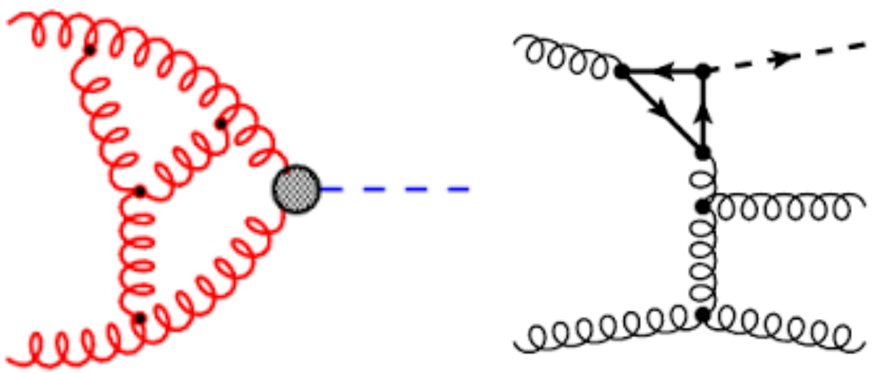
LO prediction is unreliable due to $100 - 200\%$ scale uncertainty

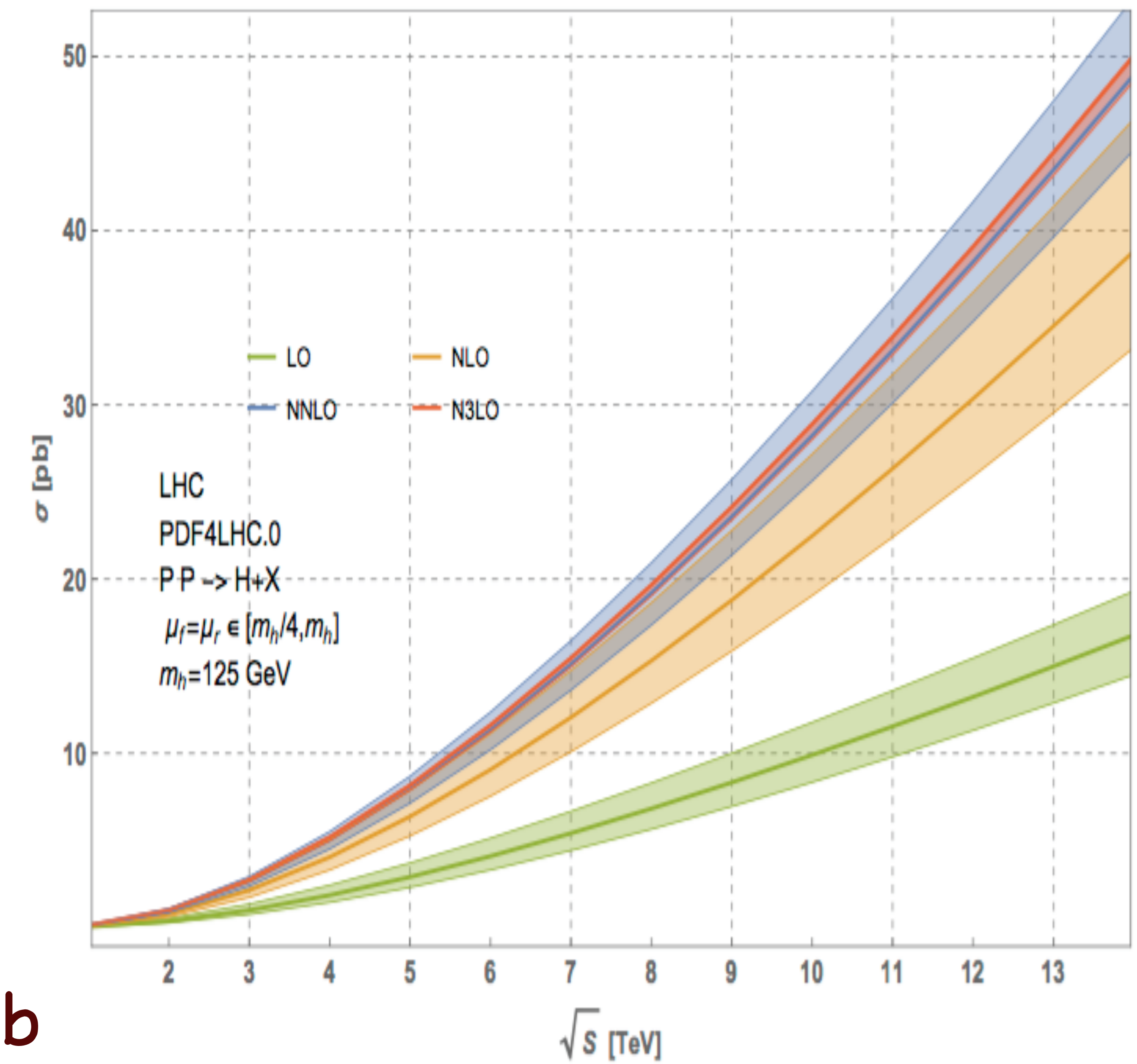
True Result for Higgs

Anastasiou et al

$$\hat{\sigma}(z) = \hat{\sigma}^{LO}(z) + \alpha_S \hat{\sigma}^{NLO}(z) + \alpha_S^2 \hat{\sigma}^{NNLO}(z) + \alpha_S^3 \hat{\sigma}^{N3LO}(z) + \mathcal{O}(\alpha_S^4)$$







LO	$15.05 \pm 14.8\%$
NLO	$38.2 \pm 16.6\%$
NNLO	$45.1 \pm 8.8\%$
N3LO	$45.2 \pm 1.9\%$

pb

QCD + EW for Higgs

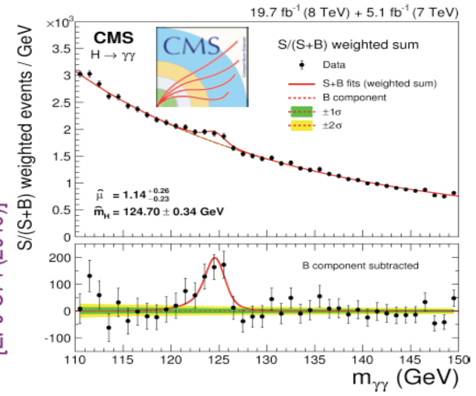
Anastasiou et al

13 Tev LHC

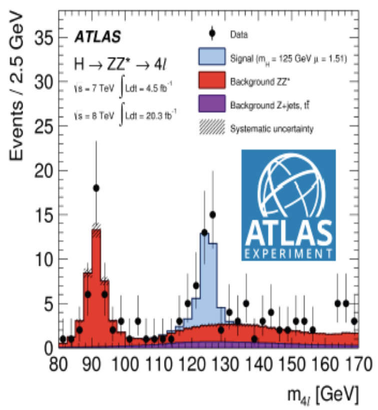
$$\sigma = 48.58\,\text{pb}^{+2.22\,\text{pb}\,(+4.56\%)}_{-3.27\,\text{pb}\,(-6.72\%)}\ (\text{theory}) \pm 1.56\,\text{pb}\,(3.20\%)\ (\text{PDF}+\alpha_s)\,.$$

$$\begin{aligned}
48.58\,\text{pb} = &\quad 16.00\,\text{pb} && (+32.9\%) && (\text{LO, rEFT}) \\
&+ 20.84\,\text{pb} && (+42.9\%) && (\text{NLO, rEFT}) \\
&- \quad 2.05\,\text{pb} && (-4.2\%) && ((t, b, c),\ \text{exact NLO}) \\
&+ \quad 9.56\,\text{pb} && (+19.7\%) && (\text{NNLO, rEFT}) \\
&+ \quad 0.34\,\text{pb} && (+0.7\%) && (\text{NNLO}, 1/m_t) \\
&+ \quad 2.40\,\text{pb} && (+4.9\%) && (\text{EW, QCD-EW}) \\
&+ \quad 1.49\,\text{pb} && (+3.1\%) && (\text{N}^3\text{LO, rEFT})
\end{aligned}$$

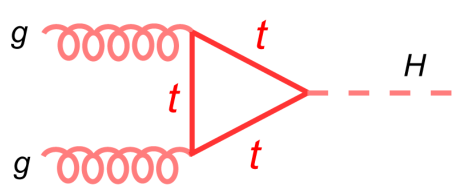
Theory Vs Experiment

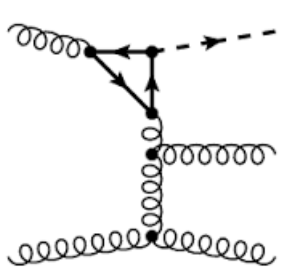


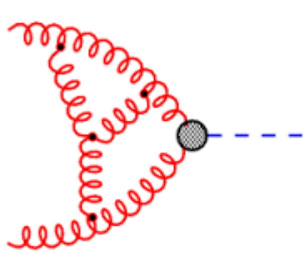
[EPJ C74 (2015)]



[PRD 91 (2015)]







Significance of excess:

$\gamma\gamma$: 5.6 σ (5.1 exp.)

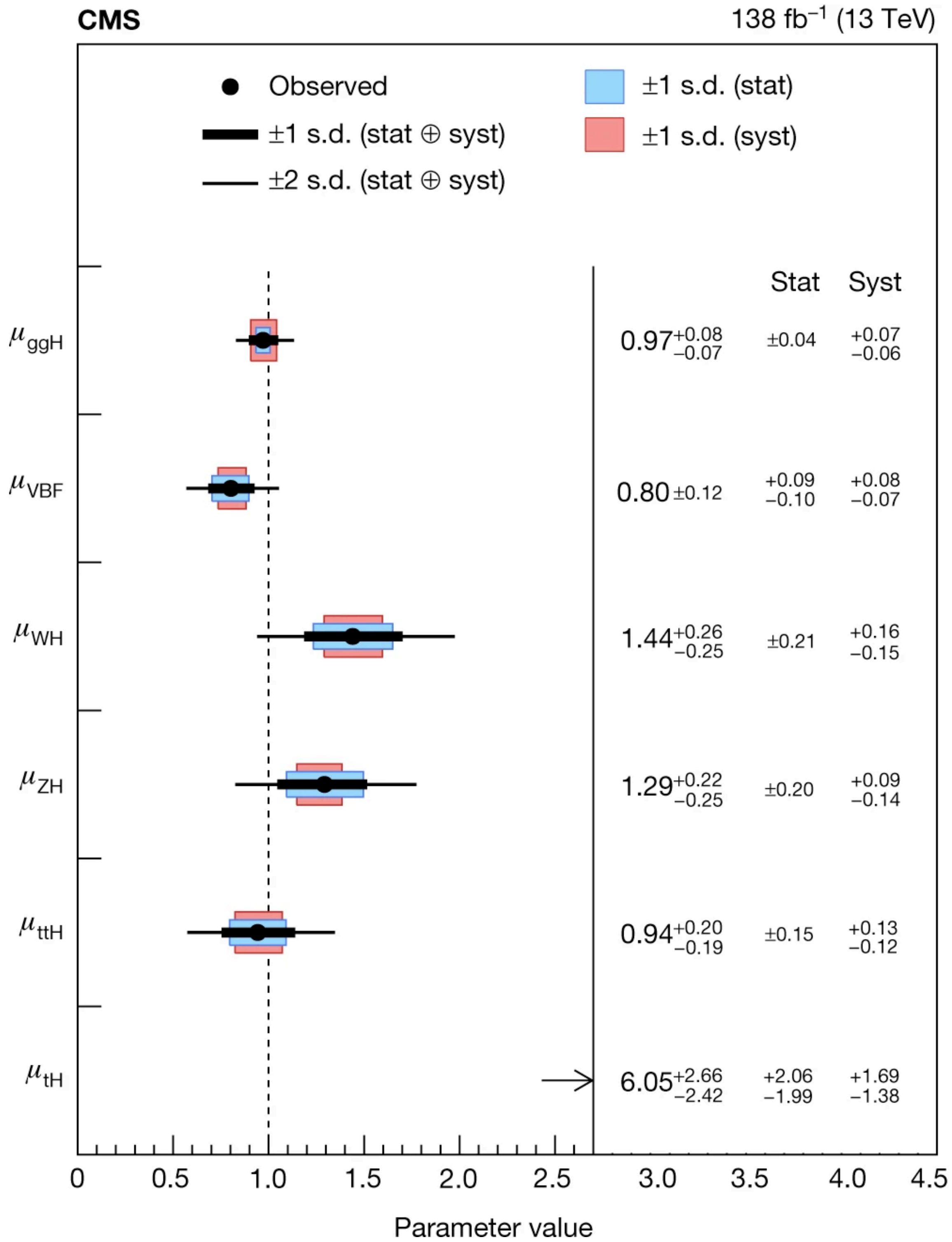
ZZ: 6.6 σ (5.5 exp.)

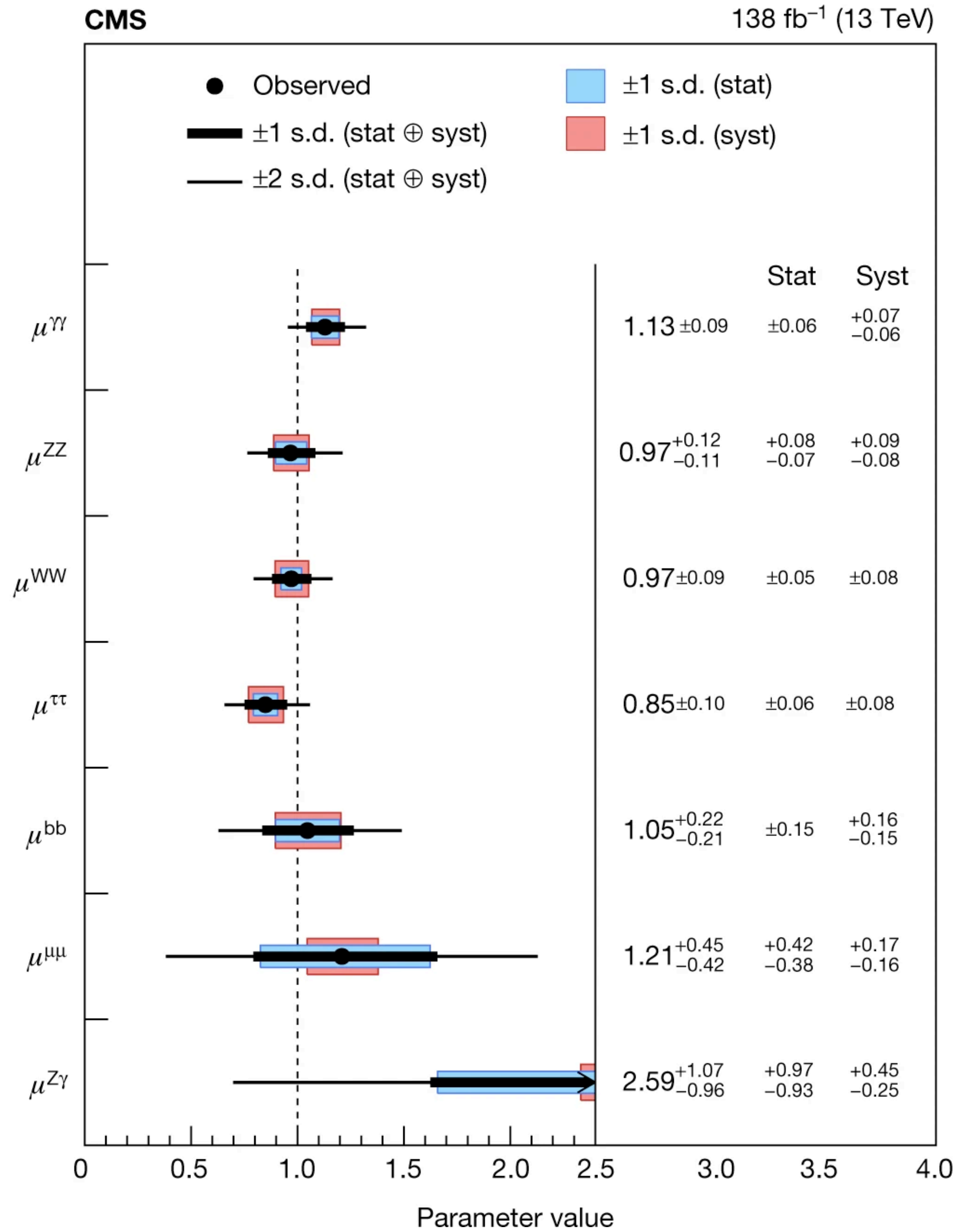
Signal strength $\mu = \sigma_{obs} / \sigma_{SM}$

$$\mu = 1.12^{+0.25}_{-0.23} \qquad \mu = 1.51^{+0.39}_{-0.34}$$

Agreement with SM Higgs Boson

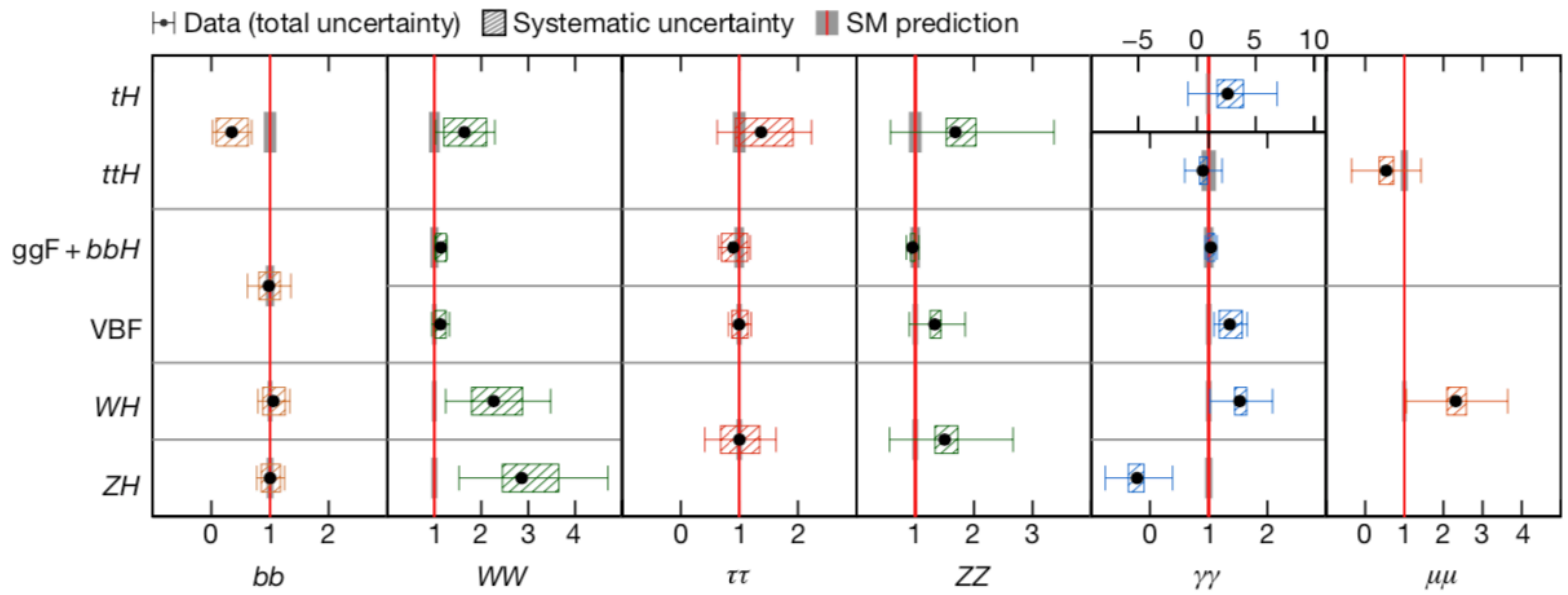
CMS: The agreement with the SM predictions for production modes and decay channels





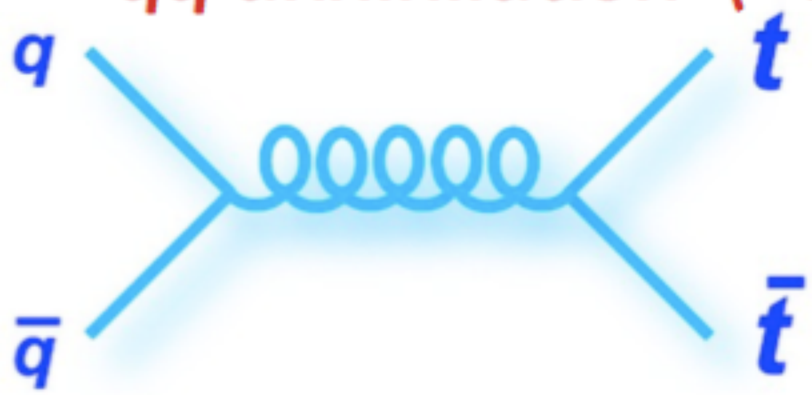
ATLAS:

Ratio of observed rate to predicted standard model event rate for different combinations of Higgs boson production and decay processes.

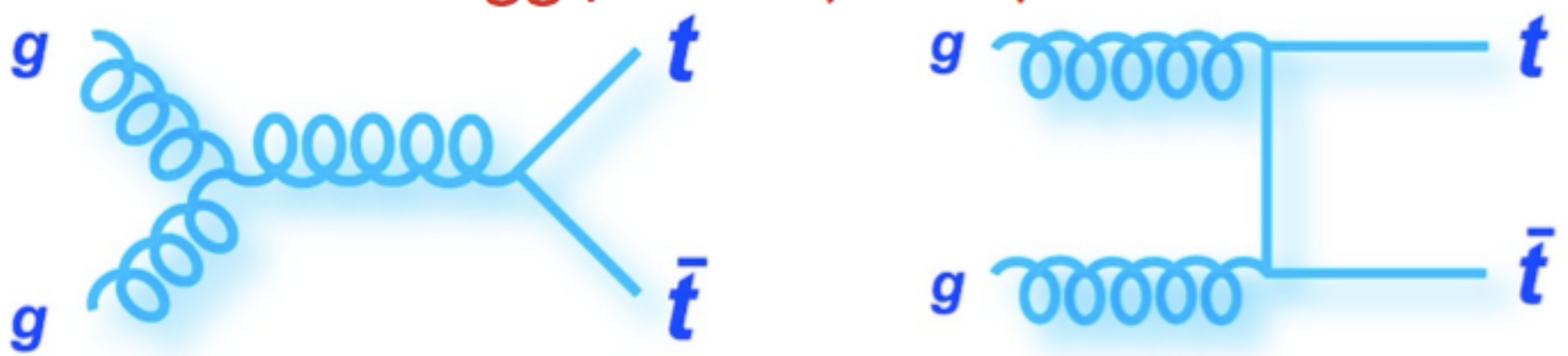


Top production

qq annihilation (~10%)



gg fusion (~90%)

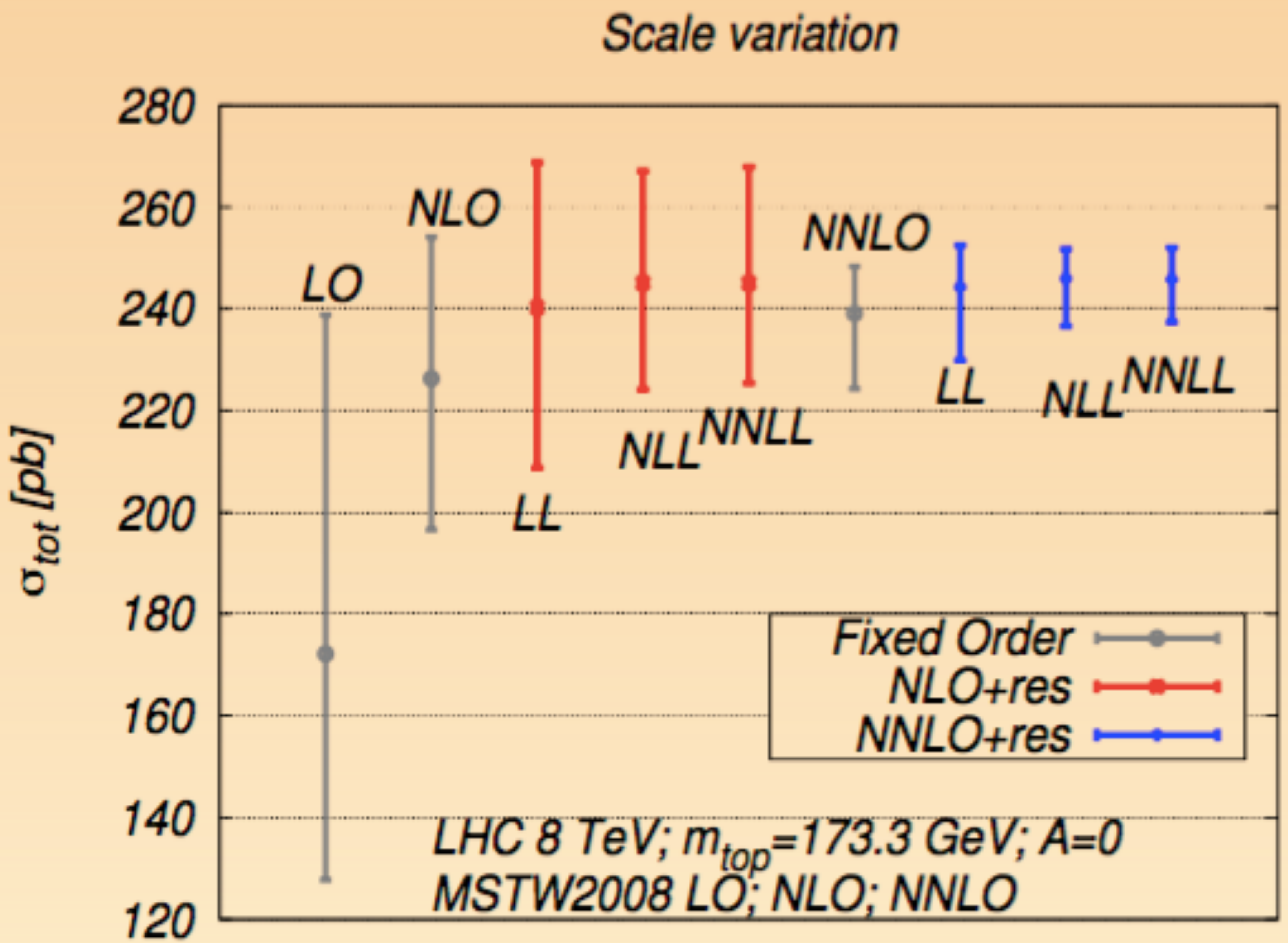


Large theory uncertainty

$$\alpha_s(\mu_R^2) \qquad f_g(x, \mu_F^2)$$

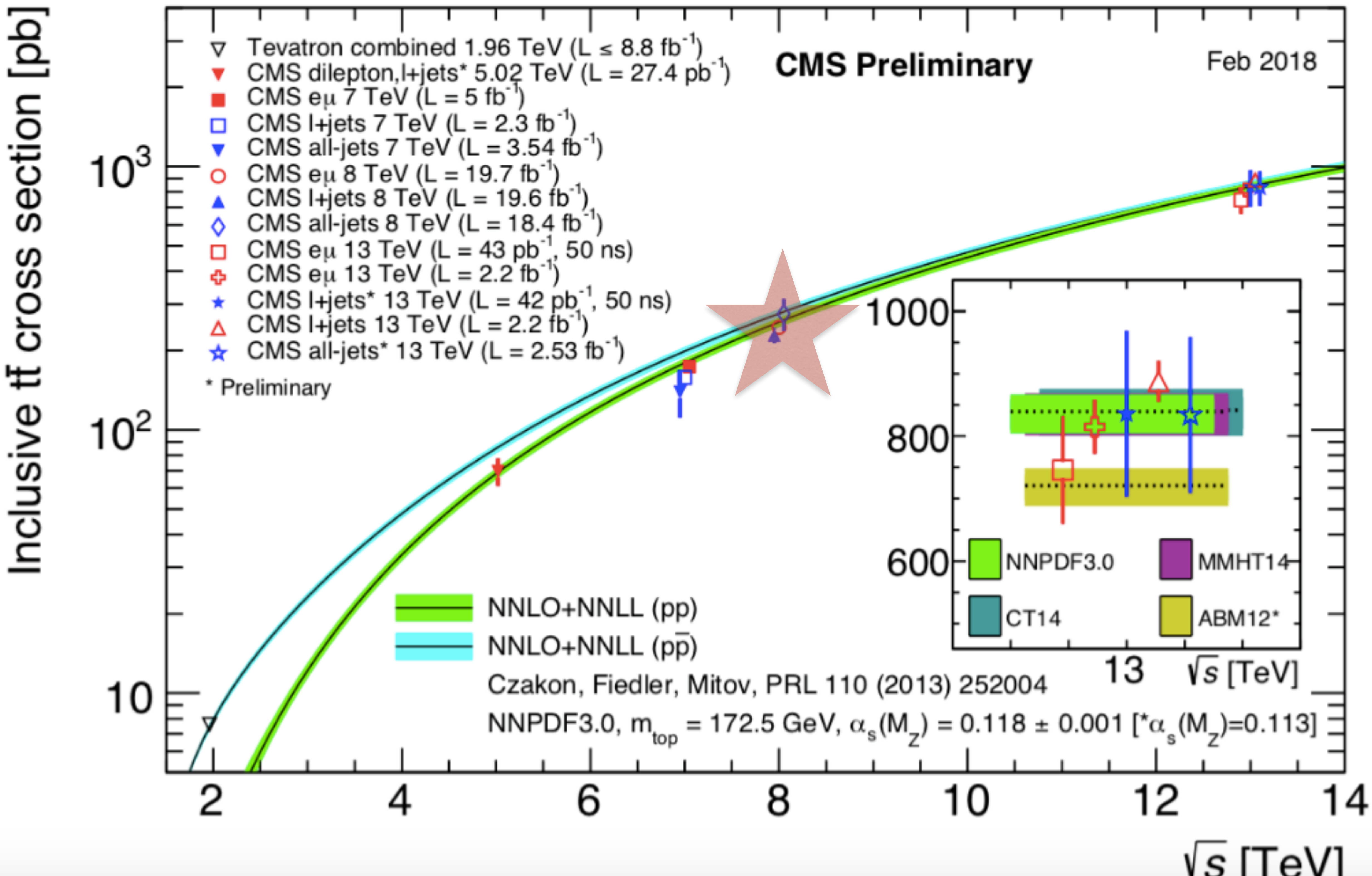
Theory Prediction

Czakon et al



Theory Vs Experiment

Czakon et al



Parton Model in QCD

Hadronic Cross section:

$$\sigma^A(\tau, m_A^2) = \sigma^{A,(0)}(\mu_R^2) \sum_{a,b=q,\bar{q},g} \int_\tau^1 dy\, \Phi_{ab}(y, \mu_F^2) \Delta_{ab}^A \left(\frac{\tau}{y}, m_A^2, \mu_R^2, \mu_F^2\right)$$

Partonic cross section:

Partonic Flux:

$$\Phi_{ab}(y, \mu_F^2) = \int_y^1 \frac{dx}{x} f_a(x, \mu_F^2) f_b\left(\frac{y}{x}, \mu_F^2\right),$$

Precision Measurements

Precise theory

Discover/Test Physics

Inputs that can affect

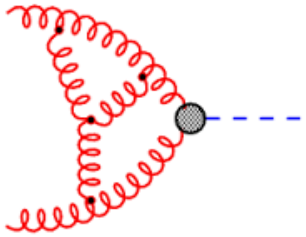
- UV Renormalisation Scale, Strong coupling

$$\alpha_s(\mu_R)$$

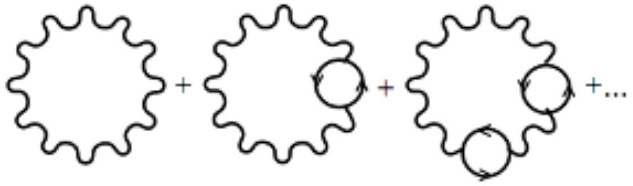
- Factorisation Scale and Parton Distribution Functions

$$f_a(x, \mu_F)$$

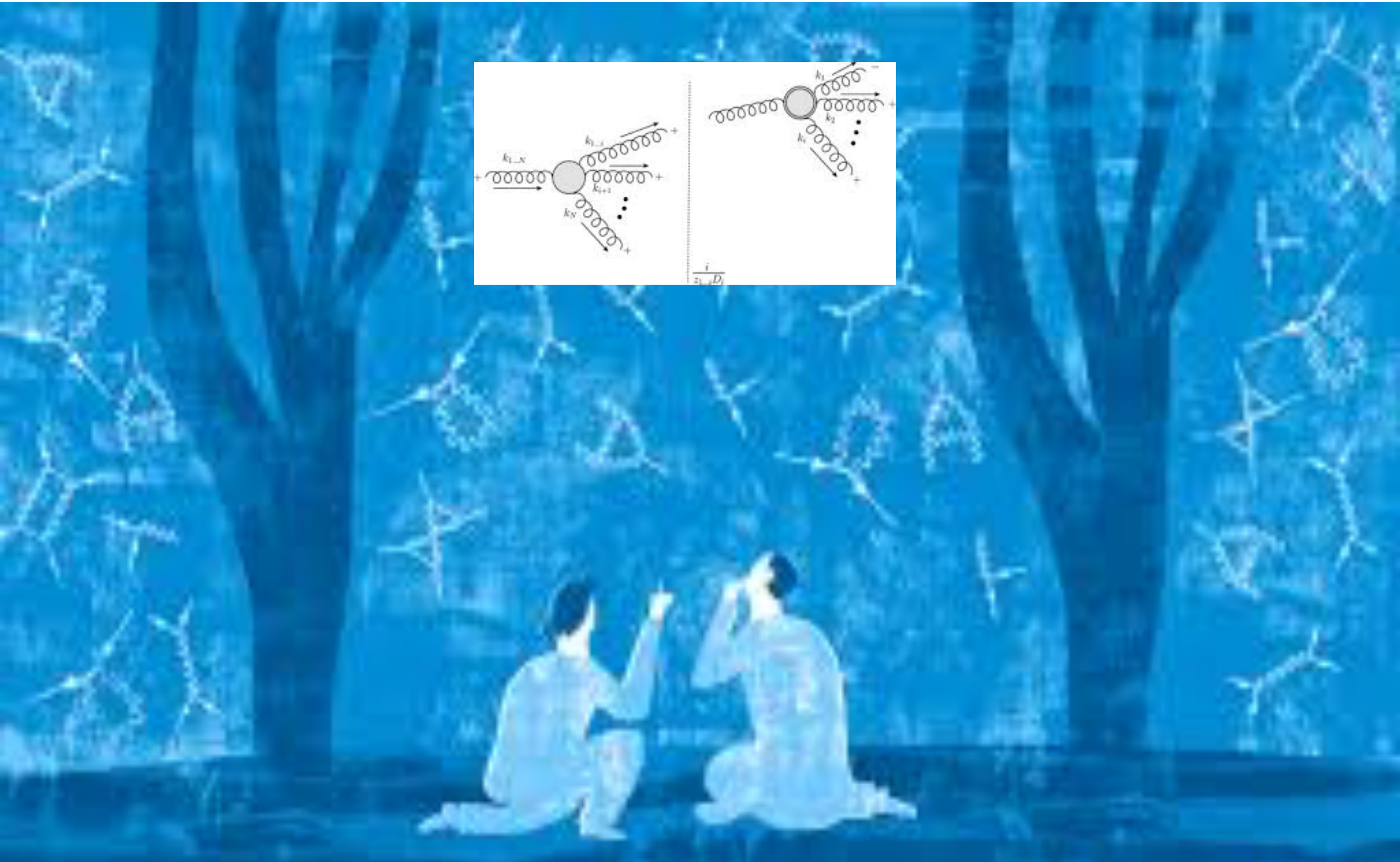
- Missing Higher Order corrections
- Stability of the perturbation theory



- Resummation Methods
- Hadronisation models



LO (Tree level)



No. of diagrams

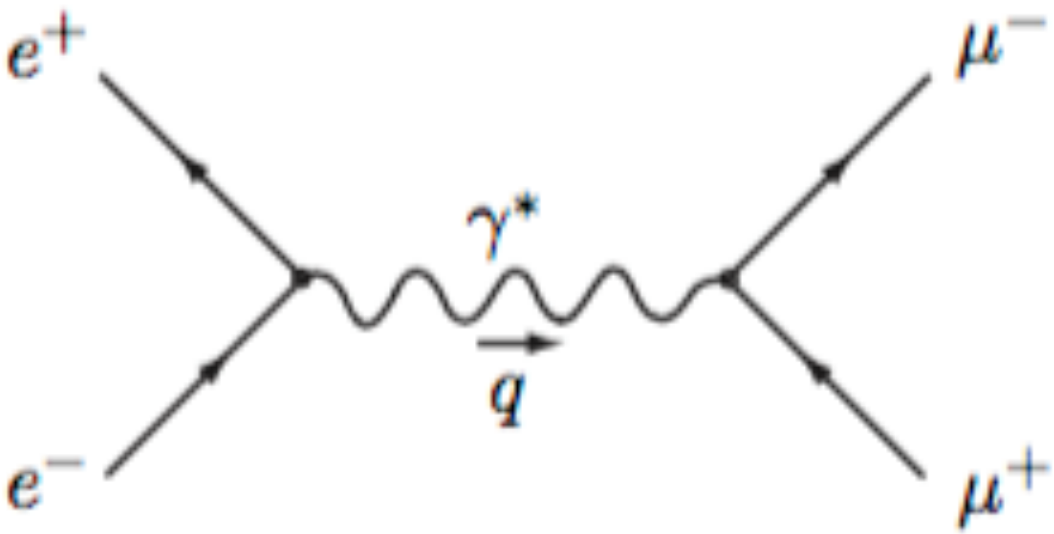
$$g + g \to n\ g$$

For Jet / background to BSM

$$g + g \to g + g$$
$$g + g \to g + g + g$$
$$g + g \to g + g + g + g$$

n	no. of diagrams
2	4
3	25
4	220
5	2485
6	34300
7	559405
8	10525900

Conventional Method



$$\frac{(4\pi\alpha)}{s^2}\,\mathrm{Tr}\left[\left(\not{p}_{e^+}-m_e\right)\gamma^\mu\left(\not{p}_{e^-}+m_e\right)\gamma^\nu\right]$$
$$\times\,\mathrm{Tr}\left[\left(\not{p}_{\mu^-}+m_\mu\right)\gamma_\mu\left(\not{p}_{\mu^+}-m_\mu\right)\gamma_\nu\right]$$

$$\sum_{\mathrm{spin}}|\mathcal{M}_2|^2 = 8\,(4\pi\alpha)^2\left[\frac{u^2+t^2}{s^2}+2\frac{m_\mu^2-m_e^2}{s^2}-8\frac{m_\mu^2 m_e^2}{s^2}\right]$$

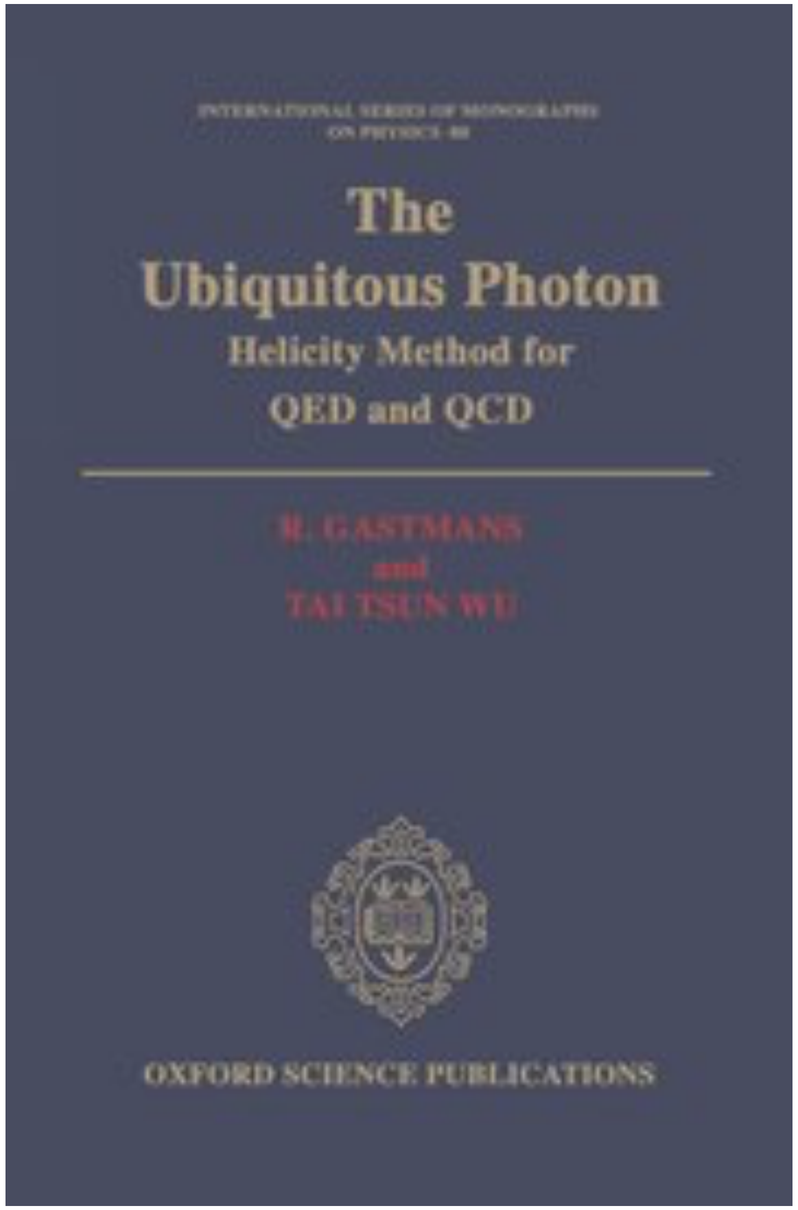
$$s = (p_{e^-}+p_{e^+})^2 = (p_{\mu^-}+p_{\mu^+})^2\,,$$
$$t = (p_{e^-}-p_{\mu^-})^2 = (p_{e^+}-p_{\mu^+})^2\,,$$
$$u = (p_{e^-}-p_{\mu^+})^2 = (p_{e^+}-p_{\mu^-})^2\,.$$

Helicity Amplitudes

- Helicity Amplitude - Convenient way

- Along fermion line Helicity is conserved

- Clever choice of photon/gluon polarization - Gauge invariance



- Different Helicity amplitudes do not interfere

Notations

Weyl Spinors

$$u_\pm(k) = \tfrac{1}{2}(1 \pm \gamma_5)u(k)$$

Particle

$$v_\mp(k) = \tfrac{1}{2}(1 \pm \gamma_5)v(k)$$

Anti-Particle

$$|i^\pm\rangle \equiv |k_i^\pm\rangle \equiv u_\pm(k_i) = v_\mp(k_i)$$

$$\langle i^\pm| \equiv \langle k_i^\pm| \equiv \overline{u_\pm(k_i)} = \overline{v_\mp(k_i)}.$$

$$|i\rangle = |i^+\rangle$$

$$|i] = |i^-\rangle$$

Dot Products

$$\overline{u_-(k_i)}u_+(k_j) = \langle i^-|j^+\rangle = \langle i\, j\rangle$$

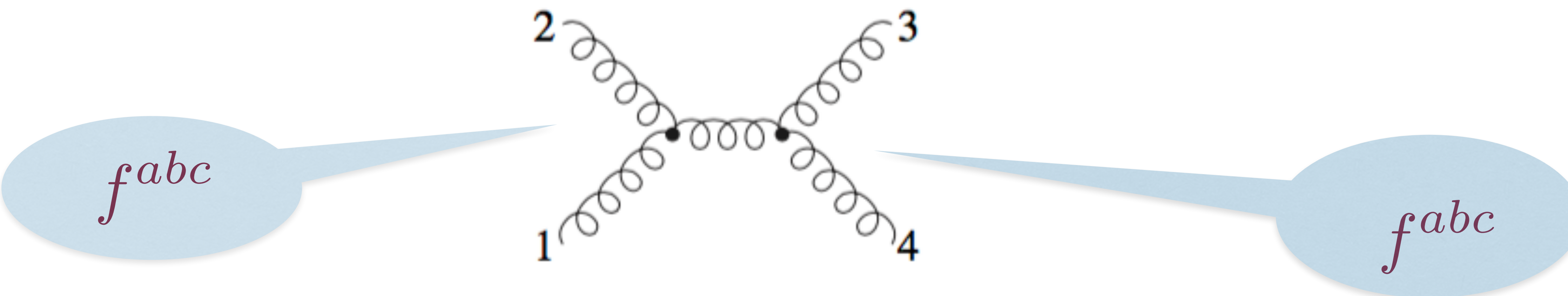
$$\overline{u_+(k_i)}u_-(k_j) = \langle i^+|j^-\rangle = [i\, j]$$

Useful Identities

- $\langle pq\rangle = -\langle qp\rangle, \quad [pq] = -[qp]$
- $\langle pp\rangle = 0 = [pp], \quad \langle pq] = 0 = [pq\rangle$
- $\langle pq\rangle = [qp]^*$
- $\langle pq\rangle[qp] = |\langle pq\rangle|^2 = |[qp]|^2 = 2\, p\cdot q = (p+q)^2 \equiv s_{pq}$
- $\langle p\gamma^\mu q] = [q\gamma^\mu p\rangle$
- $\gamma_\mu [p\gamma^\mu q\rangle = 2|p]\langle q| + |q\rangle[p|$
- Gordon Identity $[p\gamma^\mu p\rangle = 2p^\mu$
- Fierz identity $\langle p\gamma^\mu q][r\gamma_\mu s\rangle = 2\langle ps\rangle[rq]$
- Schouten identity $\langle pq\rangle\langle rs\rangle + \langle pr\rangle\langle sq\rangle + \langle ps\rangle\langle qr\rangle = 0$

SU(N) color algebra

Bern, Dixon, Kosower et al



$$[T^a, T^b] = i f^{abc} T^c \quad \longrightarrow \quad i f^{abc} = 2\left[\mathrm{Tr}\left(T^a T^b T^c\right) - \mathrm{Tr}\left(T^b T^a T^c\right)\right]$$

$$\begin{aligned} i f^{a_1 a_2 b} i f^{b a_3 a_4} &= 4\left[\mathrm{Tr}\left(T^{a_1} T^{a_2} T^b\right) - \mathrm{Tr}\left(T^{a_2} T^{a_1} T^b\right)\right]\left[\mathrm{Tr}\left(T^{a_3} T^{a_4} T^b\right) - \mathrm{Tr}\left(T^{a_4} T^{a_3} T^b\right)\right] \\ &= 2\,\mathrm{Tr}(T^{a_1} T^{a_2} T^{a_3} T^{a_4}) - 2\,\mathrm{Tr}(T^{a_1} T^{a_2} T^{a_4} T^{a_3}) - 2\,\mathrm{Tr}(T^{a_2} T^{a_1} T^{a_3} T^{a_4}) \\ &\quad + 2\,\mathrm{Tr}(T^{a_2} T^{a_1} T^{a_4} T^{a_3}) \end{aligned}$$

Color striped amplitudes:

Bern, Dixon, Kosower et al

$$\mathcal{A}_n^{(0)}(g_1, g_2, \ldots, g_n) = g^{n-2} \sum_{\sigma \in S_n/Z_n} 2\,\mathrm{Tr}\left(T^{a_{\sigma(1)}} \ldots T^{a_{\sigma(n)}}\right) A_n^{(0)}\left(g_{\sigma(1)}, \ldots, g_{\sigma(n)}\right)$$

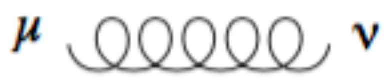
Partial Amplitude

Partial Amplitudes - No color information
- Gauge Invariant

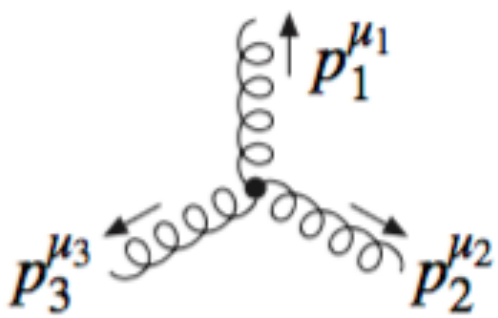
n	4	5	6	7	8	9	10
unordered	4	25	220	2485	34300	559405	10525900
cyclic ordered	3	10	38	154	654	2871	12925

Cyclic Ordered Feynman Rules

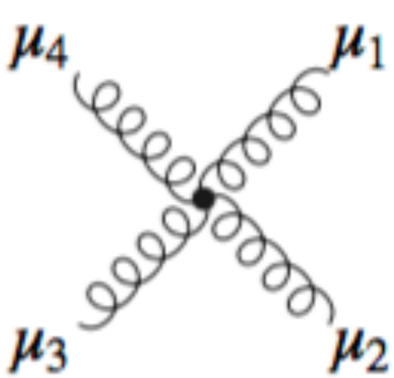
No Color Factors !!!



$$= \frac{-ig^{\mu\nu}}{p^2},$$



$$= i\left[g^{\mu_1\mu_2}\left(p_1^{\mu_3}-p_2^{\mu_3}\right)+g^{\mu_2\mu_3}\left(p_2^{\mu_1}-p_3^{\mu_1}\right)+g^{\mu_3\mu_1}\left(p_3^{\mu_2}-p_1^{\mu_2}\right)\right]$$

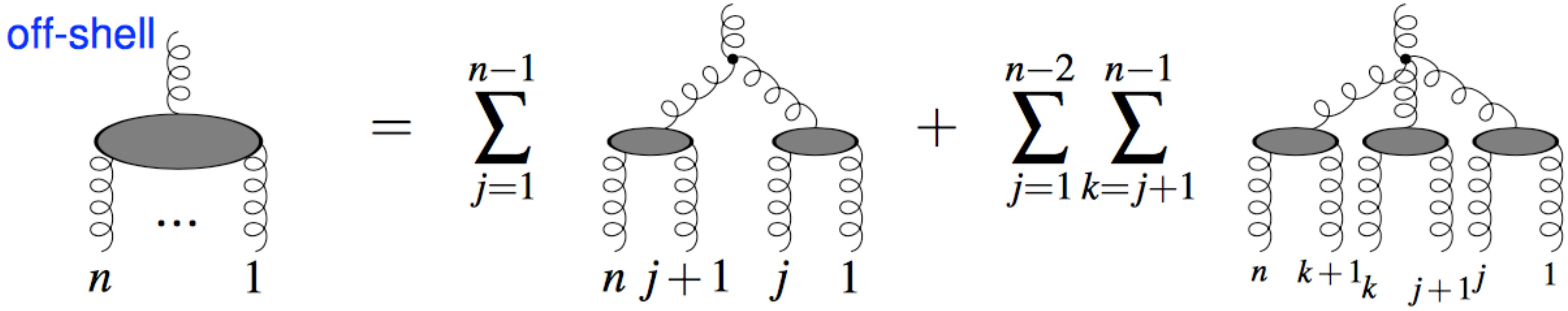


$$= i\left[2g^{\mu_1\mu_3}g^{\mu_2\mu_4}-g^{\mu_1\mu_2}g^{\mu_3\mu_4}-g^{\mu_1\mu_4}g^{\mu_2\mu_3}\right].$$

Berends-Giele Recursion

- Off-Shell currents

- No Feynman diagrams



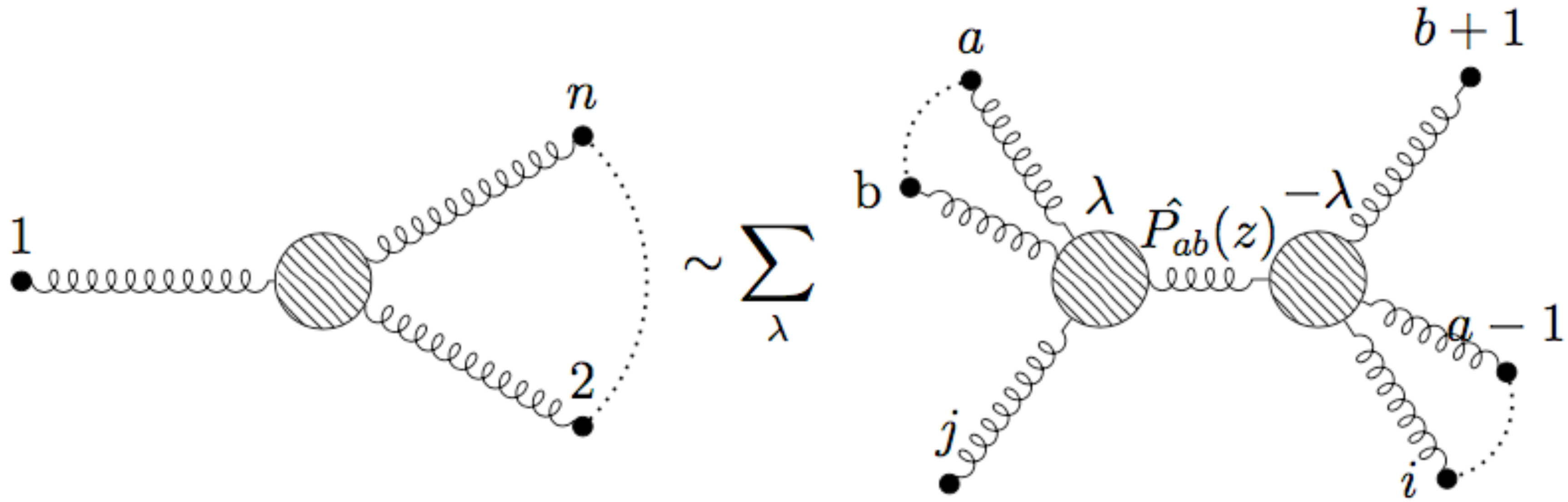
BCFW relation

Britto-Cachazo-Feng-Witten

$$0 = \frac{1}{2\pi i}\oint_C \mathrm{d}z \frac{A(z)}{z} = A(0) + \sum_{poles(z_\alpha \neq 0)} \mathrm{Res}\Big(\frac{A(z)}{z}, z_\alpha\Big)$$

$$\hat{P}_{ab}(z) = \sum_{k=a}^{b} |k\rangle\,[k| - z\,|i\rangle\,[j|$$

$$A(1,\cdots,n) \sim \sum_\lambda A_L(a,\cdots,b,-\hat{P}_{ab}^{\lambda})\frac{1}{\hat{P}_{ab}^2} A_R(\hat{P}_{ab}^{-\lambda}, b+1,\cdots,a-1)$$

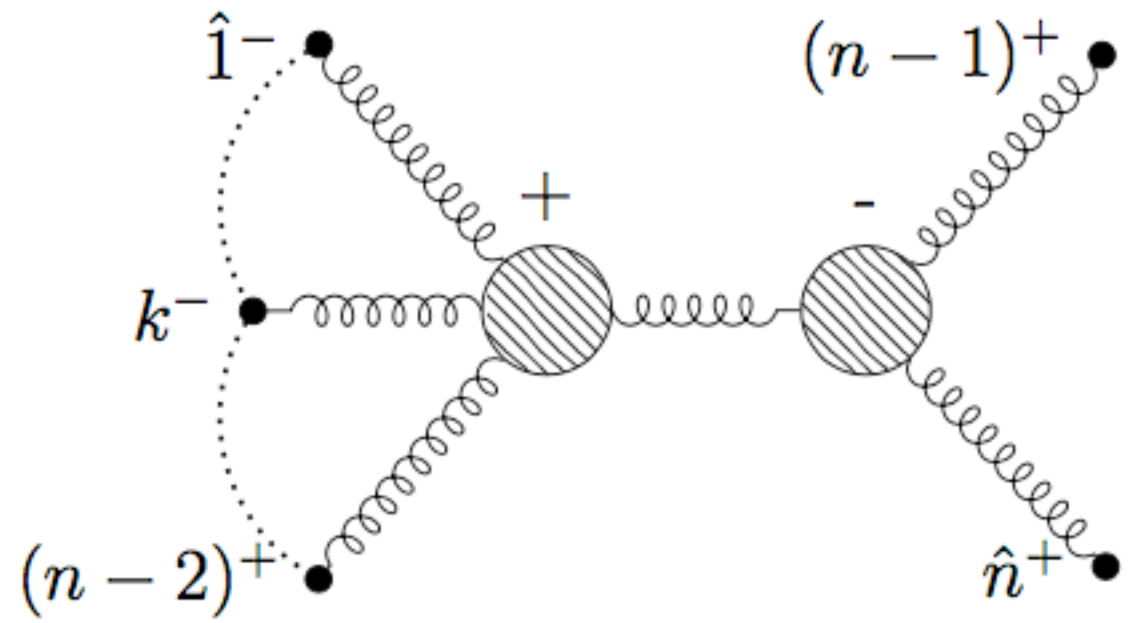


$, \hat{P}_{ab}^2 \to 0.$

Park-Taylor Amplitude

MHV n - gluon Amplitudes

$$A_n(1^-, 2^+, \cdots, k^-, \cdots, n^+) =$$



$$A_n(1^-, 2^+, \cdots, k^-, \cdots, n^+)$$

$$= \frac{\langle 1k \rangle^4}{\langle 12 \rangle \cdots \langle n-3|n-2 \rangle \langle n-2|n-1 \rangle \langle n-1|n \rangle \langle n1 \rangle}.$$

Twister space

Momenta in bi-spinor $p^\mu \to \lambda_a \lambda_{\dot{a}}$

Scaling

$$\lambda_a \to z\lambda_a$$
$$\lambda_{\dot{a}} \to \frac{1}{z}\lambda_{\dot{a}}$$

Transform

$$\lambda_{\dot{a}} \to i\frac{\partial}{\partial \lambda^{\dot{a}}}$$
$$-i\frac{\partial}{\partial \lambda^{\dot{a}}} \to \lambda_{\dot{a}}$$

Fourier Transform

$$f(\overline{\lambda}^{\dot{a}}) = \int \frac{d^2p}{(2\pi)^2}\; exp(i\overline{\lambda}^{\dot{a}}\lambda_{\dot{a}}) \;\; f(\lambda_{\dot{a}})$$

Weinzierl's comparison

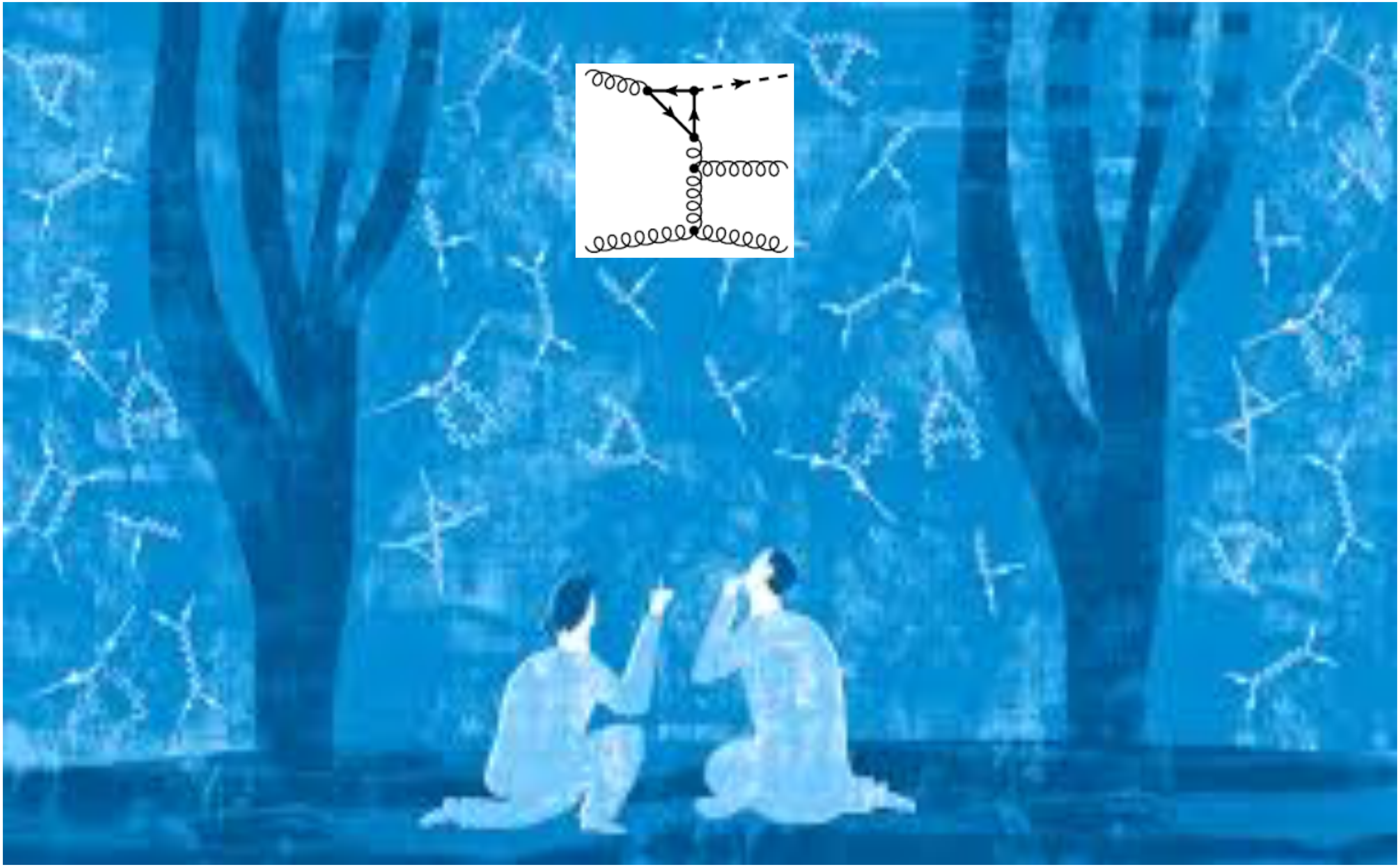
Compare algorithms based on different methods for the numerical computation of the Born gluon amplitude:

n	4	5	6	7	8	9	10	11	12
Berends-Giele	0.00011	0.00043	0.0015	0.005	0.016	0.047	0.13	0.37	1
Scalar	0.00014	0.00083	0.0033	0.011	0.033	0.097	0.26	0.7	1.8
MHV	0.00001	0.00053	0.0056	0.073	0.62	3.67	29	217	—
BCF	0.00002	0.00007	0.0004	0.003	0.017	0.083	0.47	2.5	14.5

CPU time in seconds for the computation of the n gluon amplitude on a standard PC (Pentium IV with 2 GHz), summed over all helicities.

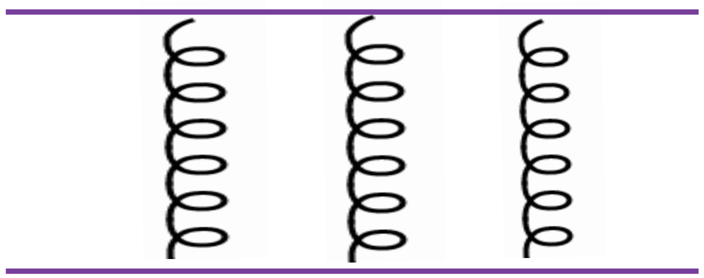
All methods give identical results within an accuracy of 10^{-12}.

NLO



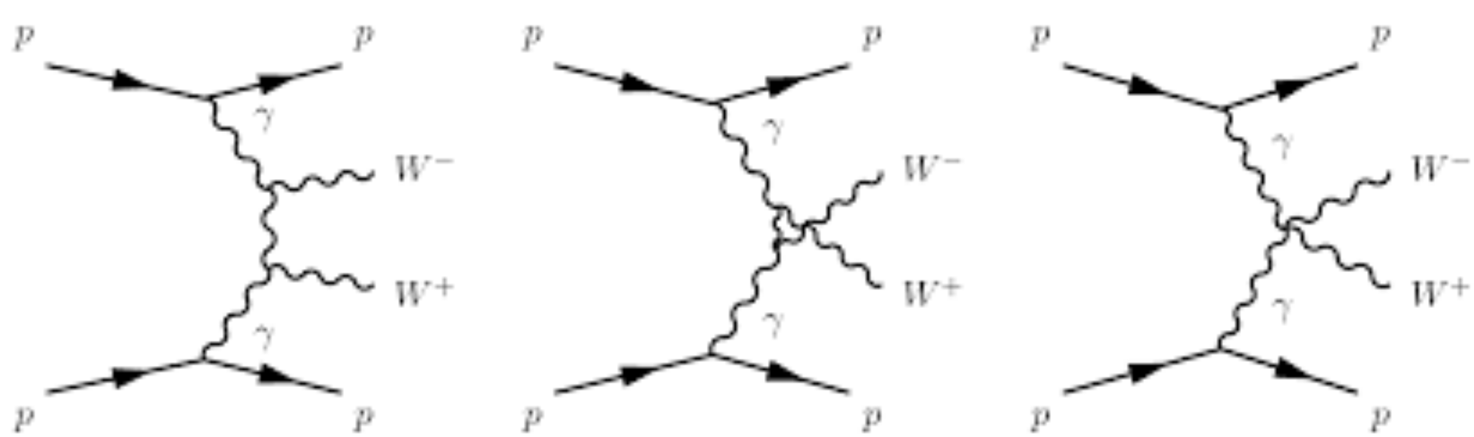
Beyond LO

Loop integral



$$\mathbf{I} = \prod_{i=1}^{N_k} \int \frac{d^n k_i}{(2\pi)^n} \frac{N(\{k_l\}, \{p_m\})}{D_1^{a_1} D_2^{a_2} \cdots D_{N_d}^{a_{N_d}}}$$

Phase space integrals



$$d\Phi_N = \prod_{i=1}^{N} \left(\int \frac{d^n p_i}{(2\pi)^n} \delta(p_i^2 - m_i^2) \right) (2\pi)^n \delta^n(q - \sum_i p_i)$$

Loop Integrals

Numerator: Reducible
Irreducible

Reducible if

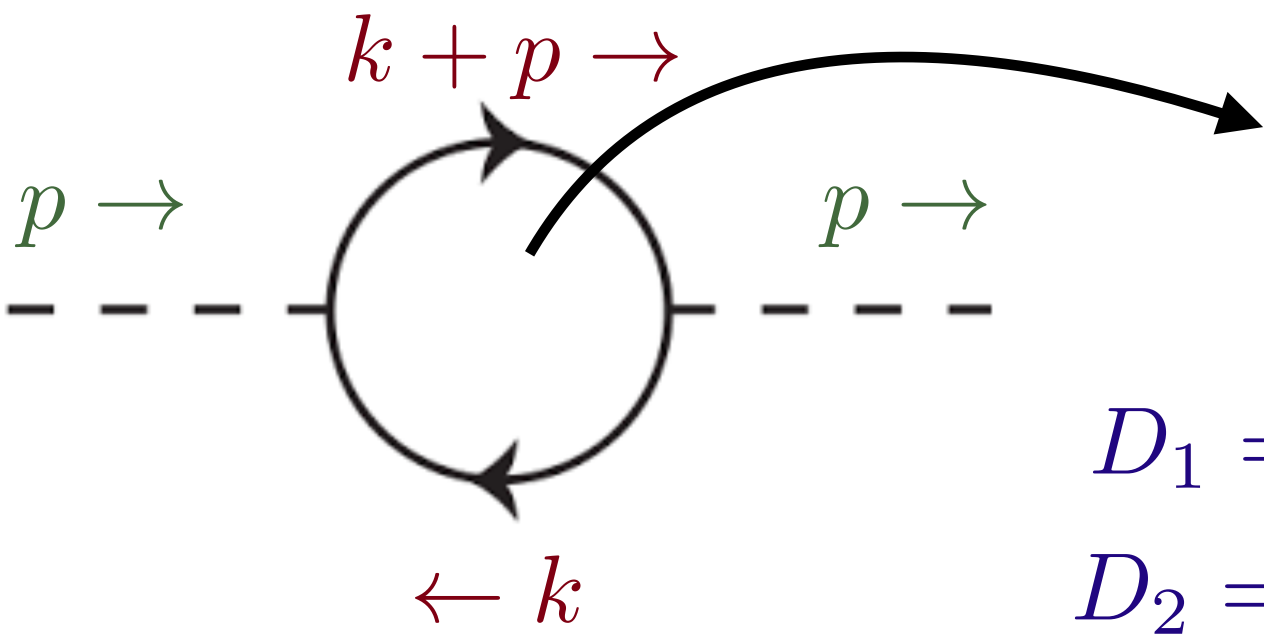
$$(p_i \cdot k_j)^{a_k}$$

$$(k_i \cdot k_j)^{b_k}$$

} are expressible in terms of Denominators

$$\frac{(p \cdot k)_j}{D_j} = \frac{1}{C_j}\left(1 - \frac{D_j - C_j\,(p \cdot k)_j}{D_j}\right)\,, \qquad j = 1, \ldots, N_d\,,$$

Loop Integrals



$$\int \frac{d^n k}{(2\pi)^n} \frac{(k \cdot p)^a}{D_1 D_2}$$

$$D_1 = k^2 + i\epsilon$$

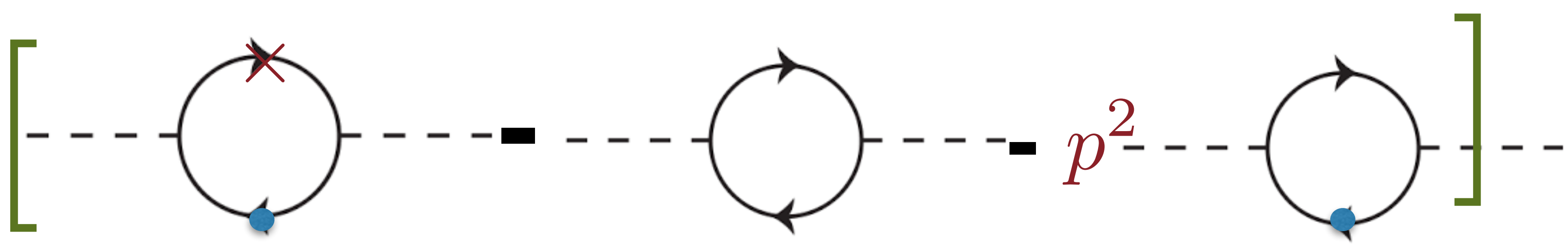
$$D_2 = (k+p)^2 + i\epsilon, \qquad p^2 < 0$$

$$\frac{k \cdot p}{D_1} = \frac{1}{2}\left(\frac{D_2}{D_1} - 1 - \frac{p^2}{D_1}\right)$$

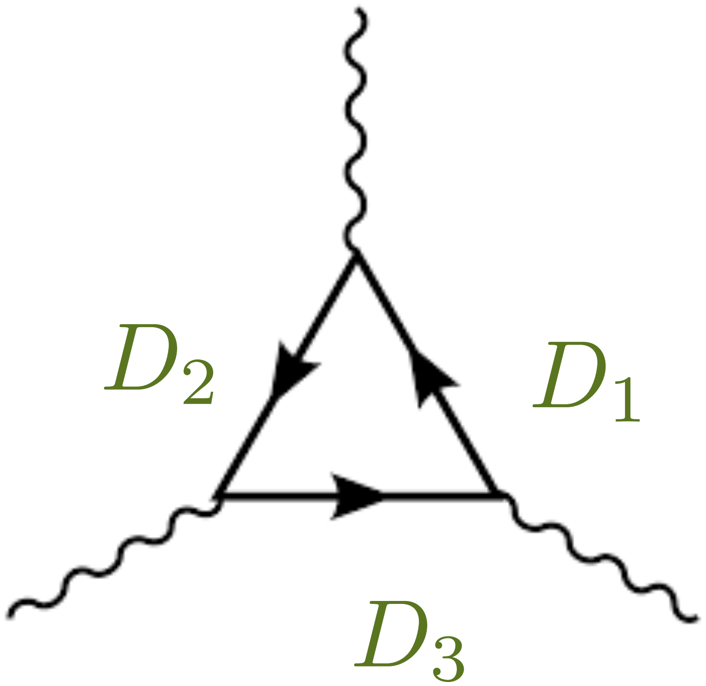
$$\frac{k \cdot p}{D_2} = \frac{1}{2}\left(1 - \frac{D_1}{D_2} - \frac{p^2}{D_2}\right)$$

Reducible

$$\int \frac{d^n k}{(2\pi)^n} \frac{k \cdot p}{D_1 D_2} = \frac{1}{2}$$



Tensorial Reduction



$$I_\mu = \int \frac{d^n k}{(2\pi)^n} \frac{k_\mu}{D_1 D_2 D_3}$$

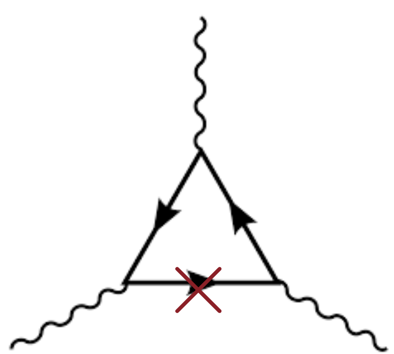
$$I_\mu = A_1\ p_{1\mu} + A_2\ p_{2\mu}$$

$$I_1 = p_1 \cdot I = A_2\ p_1 \cdot p_2$$

$$I_2 = p_2 \cdot I = A_1\ p_2 \cdot p_1$$

$$A_1 = \frac{1}{2 p_1 \cdot p_2}$$

[



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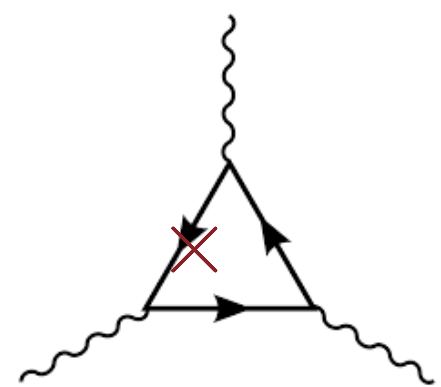

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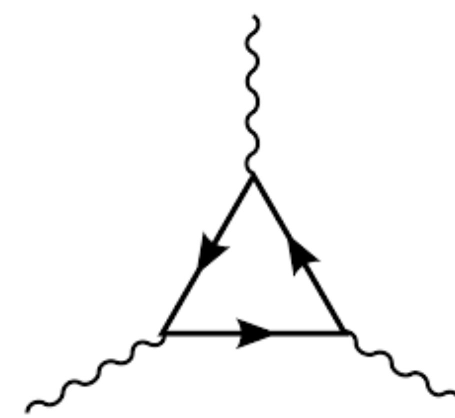


]

Ossola-Papadopoulos-Pittau (OPP method)

Integrand : $\dfrac{N(k,p_i)}{D_1 D_2 D_3 D_4}$ $\qquad D_i = (k + \sum_j c_j p_j)^2 - m_i^2$

$$
\begin{aligned}
N(k,p_i) = & \sum_{i_1<i_2<i_3<i_4}^{m} \left[d(i_1 i_2 i_3 i_4) + \tilde{d}(k, i_1 i_2 i_3 i_4)\right] \prod_{i \neq i_1 i_2 i_3 i_4}^{m} D_i \\
+ & \sum_{i_1<i_2<i_3}^{m} \left[c(i_1 i_2 i_3) + \tilde{c}(k, i_1 i_2 i_3)\right] \prod_{i \neq i_1 i_2 i_3}^{m} D_i \\
+ & \sum_{i_1<i_2}^{m} \left[b(i_1 i_2) + \tilde{b}(k, i_1 i_2)\right] \prod_{i \neq i_1 i_2}^{m} D_i \\
+ & \sum_{i_1}^{m} \left[b(i_1 i_2) + \tilde{b}(k, i_1)\right] \prod_{i \neq i_1}^{m} D_i + P(q) \prod_{i=1}^{m} D_i
\end{aligned}
$$

Best suited for Numerical Methods

NLO QCD - Tool Kits

Analytical Tools

Faster generation of Feynman diagram
QGRAF
Symbolic Manupulation:
FORM,Mathematica
On-Shell Methods
BCFW
Recursion techniques
BG

Merging NLO with Showers

MC@NLO
POWEG
SHERPA
VINCIA
GENeVa
aMC@NLO
KRKMC

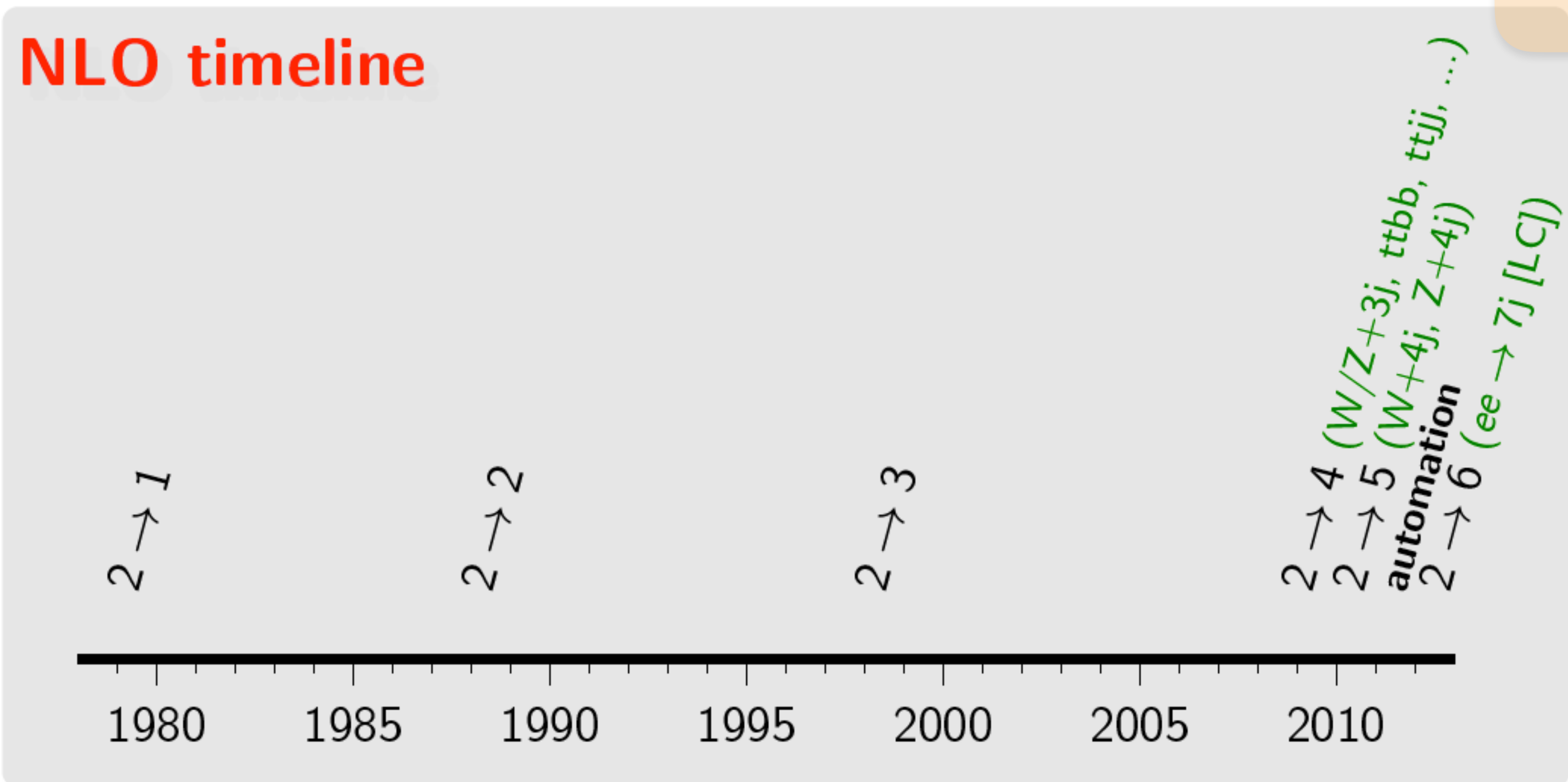
Semi-Numerical Methods

Madgraph
Helac-NLO
CutTools
BlackHat
Rocket
SAMURAI
MADLoop
GoSam

NLO revolution

1979: NLO Drell-Yan [Altarelli, Ellis & Martinelli]
1991: NLO $gg \to$ Higgs [Dawson; Djouadi, Spira & Zerwas]

1987: NLO high-p_t photoproduction [Aurenche et al]
1988: NLO $b\bar{b}$, $t\bar{t}$ [Nason et al]
1988: NLO dijets [Aversa et al]
1993: Vj [JETRAD, Giele, Glover & Kosower]

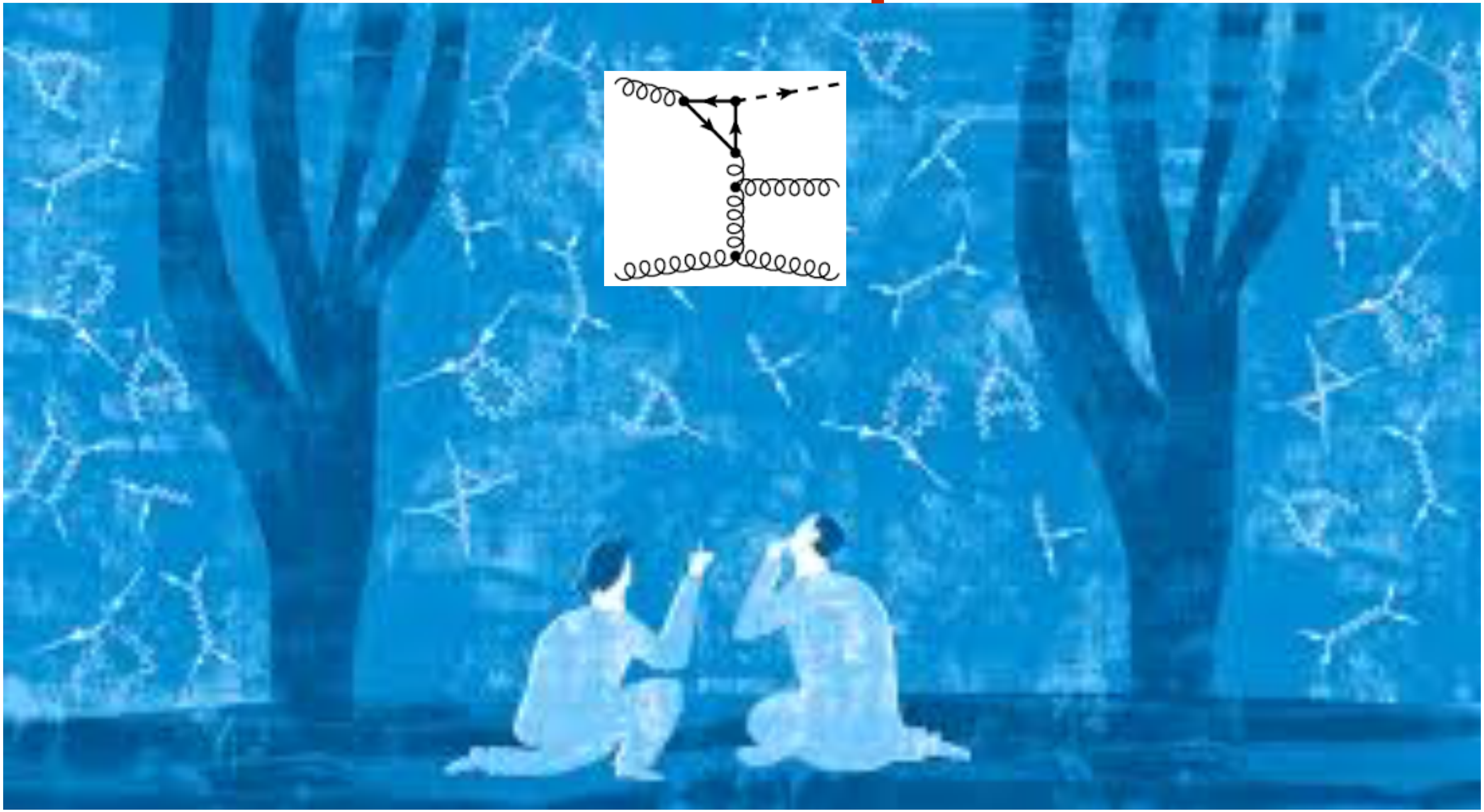


Golem, HELAC
BlackHat

1998: NLO $Wb\bar{b}$ [MCFM: Ellis & Veseli]
2000: NLO $Zb\bar{b}$ [MCFM: Campbell & Ellis]
2001: NLO 3j [NLOJet++: Nagy]
...
2007: NLO $t\bar{t}j$ [Dittmaier, Uwer & Weinzierl '07]
...

2009: NLO W+3j [Rocket: Ellis, Melnikov & Zanderighi]
2009: NLO W+3j [BlackHat+Sherpa: Berger et al]
2009: NLO $t\bar{t}b\bar{b}$ [Bredenstein et al]
2009: NLO $t\bar{t}b\bar{b}$ [HELAC-NLO: Bevilacqua et al]
2009: NLO $q\bar{q} \to b\bar{b}b\bar{b}$ [Golem: Binoth et al]
2010: NLO $t\bar{t}jj$ [HELAC-NLO: Bevilacqua et al]
2010: NLO Z+3j [BlackHat+Sherpa: Berger et al]
...

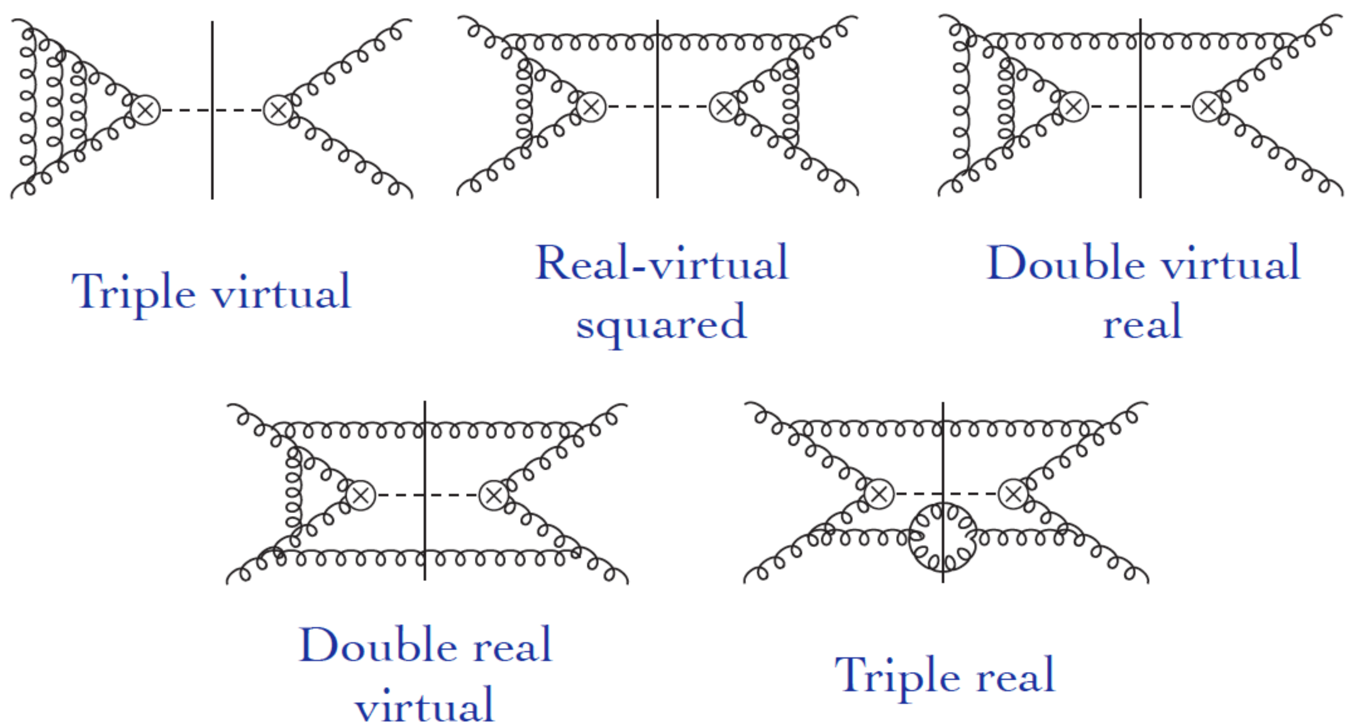
2- loop



Higgs production to N^3LO in QCD at the LHC

Anastasiou, Duhr, Dulat, Furlan, Gehrmann, Herzog, Mistlberger

100 000 diagrams



$$\prod_{i=1}^{N_k} \int \frac{d^n k_i}{(2\pi)^n} \frac{1}{D_1^{a_1} D_2^{a_2} \cdots D_{N_d}^{a_{N_d}}}$$

$$D_i = q_i^2 + i\epsilon \qquad q_i = \sum_j k_l + \sum_l p_j$$

	Integrals	Master Integrals
NNLO	50 000	
N3LO	517 531 178	

Integration By Parts (IBP)

K. Chetyrkin and F. Tkachev,

$$D_i = q_i^2 + i\epsilon \qquad q_i = \sum_j k_l + \sum_l p_j$$

$$\prod_{i=1}^{N_k} \int \frac{d^n k_i}{(2\pi)^n} \frac{1}{D_1^{a_1} D_2^{a_2} \cdots D_{N_d}^{a_{N_d}}}$$

n-Dimensional Gauss theorem

$$\int \prod_{i=1}^{N_k} \frac{d^n k_i}{(2\pi)^n} \frac{\partial}{\partial k_{j,\mu}} \left(\frac{k_{l,\mu}}{D_1^{a_1} D_2^{a_2} \cdots D_{N_d}^{a_{N_e}}} \right) = 0 \qquad j, l = 1, \cdots N_k$$

$$\int \prod_{i=1}^{N_k} \frac{d^n k_i}{(2\pi)^n} \frac{\partial}{\partial k_{j,\mu}} \left(\frac{p_{l,\mu}}{D_1^{a_1} D_2^{a_2} \cdots D_{N_d}^{a_{N_e}}} \right) = 0 \qquad l = 1, \cdots N_e - 1$$

Integration By Parts (IBP) identities

Integration By Parts (IBP)

Number of IBP identities = $N_k(N_k + N_e - 1)$

IBP identities:

$$\sum_i C_i(s_{ij}, n)\; I_i(\{p_j\}, n, a_1, \cdots a_{N_e}) = 0$$

Integrals:

$$i = 1, \cdots N_I$$

$$s_{ij} = (p_i + p_j)^2$$

Solving IBP identities -> **Master Integrals:**

$$I_i(\{p_j\}, a_1, \cdots a_{N_e})$$

$$i = 1, \cdots N_{MI}$$

Good News $N_{MI} << N_I$

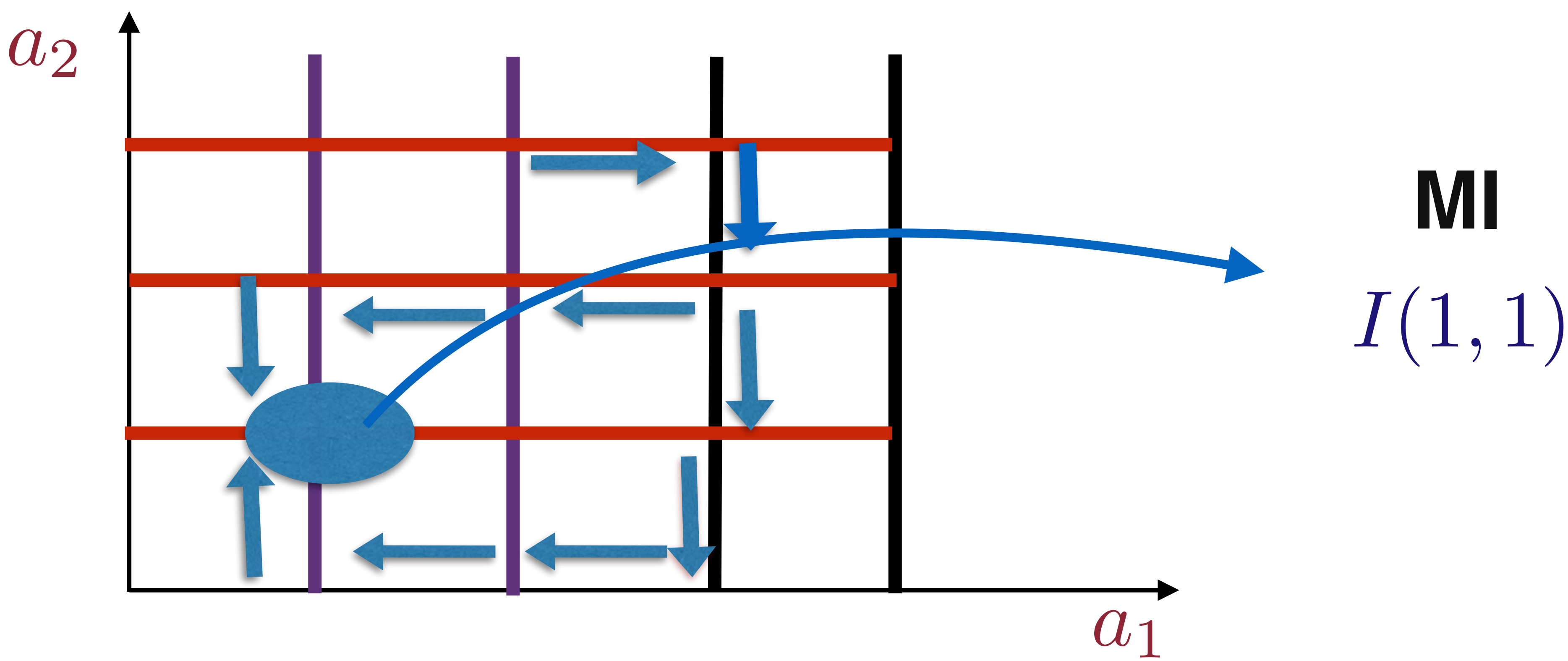
Integration By Parts (IBP)

$$I(a_1, a_2) = \int \frac{d^n k}{(2\pi)^n} \frac{1}{(k^2)^{a_1}((k+p)^2)^{a_2}}$$

$$\int \frac{d^n k}{(2\pi)^n} \frac{\partial}{\partial k_\mu} \left(\frac{v_\mu}{(k^2)^{a_1}((k+p)^2)^{a_2}} \right) = 0$$

$$v = k, p$$

$$I(a_1, a_2) = \frac{a_1 + a_2 - n - 1}{p^2(a_2 - 1)} I(a_1, a_2 - 1) + \frac{1}{p^2} I(a_1 - 1, a_2)$$



Lorentz Invariant Identities

Integrals are Lorentz scalars

$$\omega^{\mu\nu} = -\omega^{\nu\mu}$$

$$p_i^\mu \to p_i^\mu + \delta p_i^\mu = p_i^\mu + \omega^{\mu\nu} p_\nu,$$

$$I(p_i + \delta p_i) = I(p_i) + \omega^{\mu\nu} \sum_j p_{j,\nu} \frac{\partial}{\partial p_j^\mu} I(p_i) = I(p_i)$$

Anti-symmetry

$$\omega^{\mu\nu} = -\omega^{\nu\mu}$$

$$\sum_i p_{i,[\mu} \frac{\partial}{\partial p_i^{\nu]}} I(p_i) = 0$$

Anti-symmetry of $p_j^{[\mu} p_k^{\nu]}$

$$p_j^{[\mu} p_k^{\nu]} \sum_i p_{i,[\mu} \frac{\partial}{\partial p_i^{\nu]}} I(p_i) = 0$$

Solving Master Integrals

Kotikov, Remiddi, Gehrmann

Consider a Master Integral

$$I(s_{ij},n) = \prod_{i=1}^{N_k} \int \frac{d^n k_i}{(2\pi)^n} \frac{1}{D_1 D_2 \cdots D_{N_f}}$$

Define $\quad s_{12} = s$

$$D_i = q_i^2 + i\epsilon \qquad q_i = \sum_j k_l + \sum_l p_j$$

$$s = (p_1 + p_2)^2$$

Differential w.r.t s

$$s\frac{\partial}{\partial s} I(s_{ij},n) = \prod_{i=1}^{N_k} \int \frac{d^n k_i}{(2\pi)^n} s\frac{\partial}{\partial s}\left(\frac{1}{D_1 D_2 \cdots D_{N_f}}\right)$$

Master Integrals

Generalization with set of MIs

$$\vec{I} = \left(I_1, I_2, \cdot, \cdot, \cdot, I_N\right)$$

$\{I_i(\vec{x})\}$ depend on Scaling variables

$$x_i = f_i\left(\frac{s_{ij}}{Q^2}\right)$$

$$\vec{x} = \left(x_1, x_2, \cdot, \cdot, \cdot, x_M\right)$$

Differential equation:

$$d\vec{I} = \sum_{i=1}^{M} \mathbf{A}_i dx_i \ \vec{I}$$

$$\frac{\partial}{\partial x_i}\begin{bmatrix} I_1 \\ \cdot \\ I_N \end{bmatrix} = \begin{bmatrix} \mathbf{A_{11}} & \cdots & \mathbf{A_{1N}} \\ \cdot & \cdots & \cdot \\ \mathbf{A_{N1}} & \cdots & \mathbf{A_{NN}} \end{bmatrix} \begin{bmatrix} I_1 \\ \cdot \\ I_N \end{bmatrix}$$

$$i = 1, 2, \cdot, \cdot, \cdot M$$

Canonical/Henn's Basis

Consider Diff equation:

$$d\vec{I}(\vec{x},n) = \sum_i \mathbf{A}_i(\vec{x},n)dx_i \; \vec{I}(\vec{x},n)$$

Choose U Transformation such that

$$U^{-1}\mathbf{A}(\vec{x},n)U - U^{-1}dU = (n-4)\;\overline{\mathbf{A}}(\vec{x})$$

Diff equation contains ‚n' independent A

$$d\vec{\overline{I}}(\vec{x},n) = (n-4)\sum_i \overline{\mathbf{A}}_i(\vec{x})dx_i \; \vec{\overline{I}}(\vec{x},n)$$

Solution
$$\vec{\overline{I}}(\vec{x},n) = \vec{\overline{I}}(\vec{x}_0,n)\; \mathbf{P}\exp\left((n-4)\int \frac{d\lambda}{\lambda}\overline{\mathbf{A}}(\lambda)\right)$$

$\mathbf{P}$ - Path Ordered exponential

Canonical/Henn's Basis

Start with Henn's Diff equation:

$$s\frac{\partial}{\partial s}\overline{I}(s,n) = (n-4)\overline{A}(s)\ \overline{I}(s,n)$$

If $\overline{A}$ contains poses at s_i

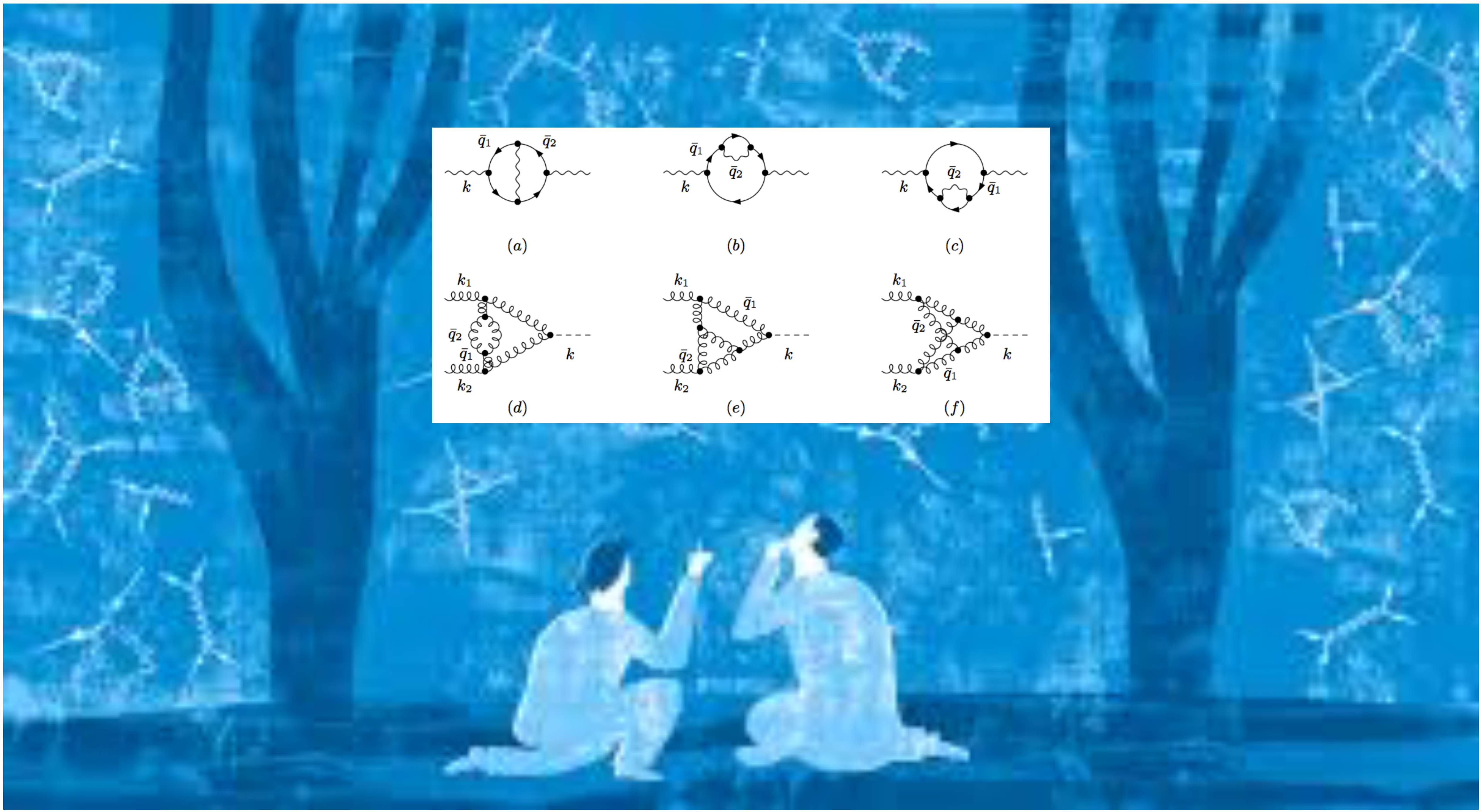
$$\overline{A}(s) = \sum_i \frac{\tilde{A}_i(s_i)}{s-s_i}$$

$$\overline{I}(s,n) = \overline{I}^{(0)}(s_0) + (n-4)\sum_i \tilde{A}_i(s_i)\log\left(\frac{s-s_i}{s_0-s_i}\right)$$

$$+(n-4)^2\sum_i \tilde{\tilde{A}}(s_i)\mathcal{L}_i(s_0,s_i) + \cdots$$

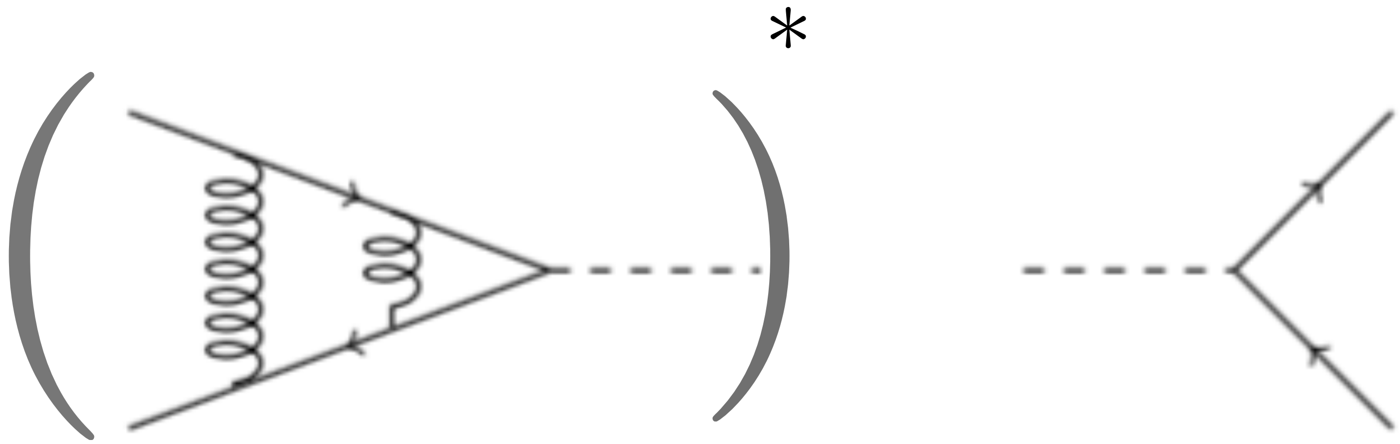
Polylogarithms - Uniform transcendental terms

NNLO


$\bar{q}_1$ $\bar{q}_2$ k
(a)
$\bar{q}_1$ $\bar{q}_2$ k
(b)
k $\bar{q}_2$ $\bar{q}_1$
(c)
k_1 $\bar{q}_2$ $\bar{q}_1$ k_2 k
(d)
k_1 $\bar{q}_1$ $\bar{q}_2$ k_2 k
(e)
k_1 $\bar{q}_2$ $\bar{q}_1$ k_2 k
(f)

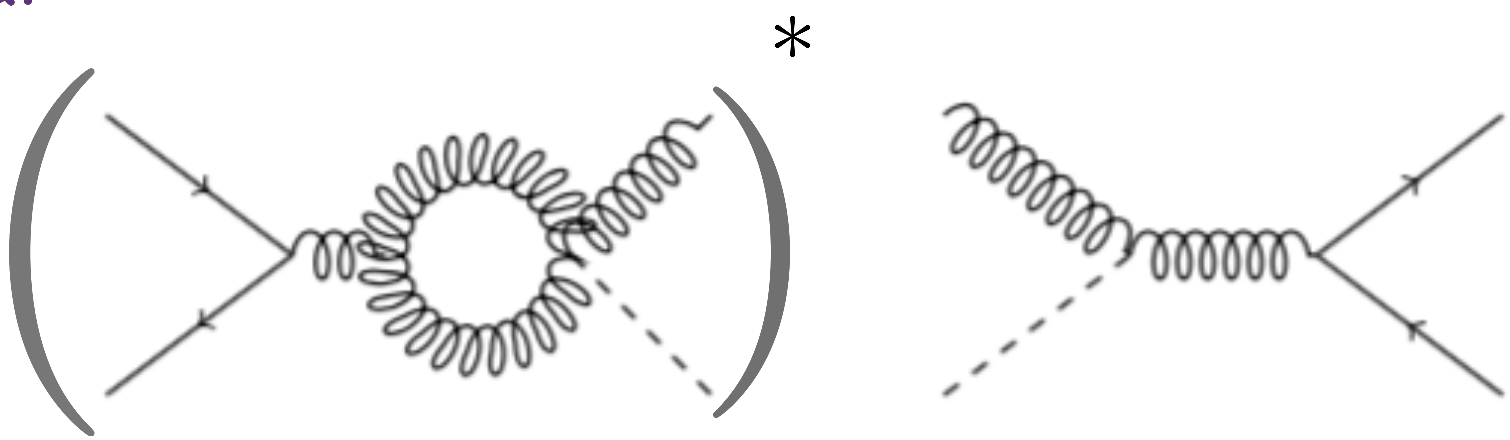
NNLO corrections:

Pure Virtual



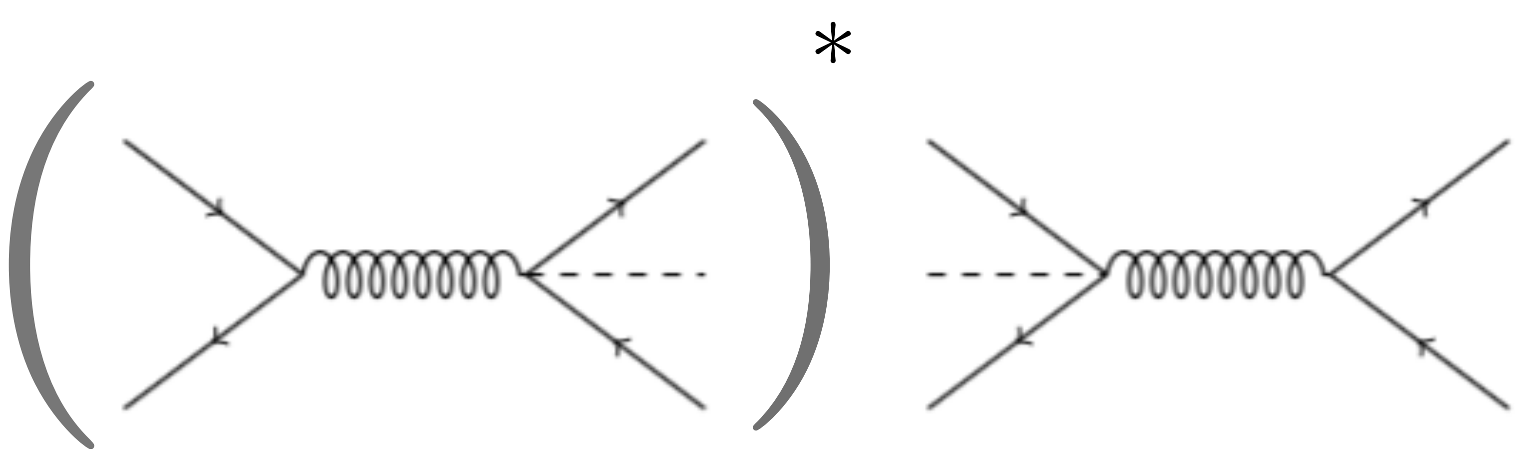
$+ 53$ terms.

Virtual - Real



$+ 171$ terms.

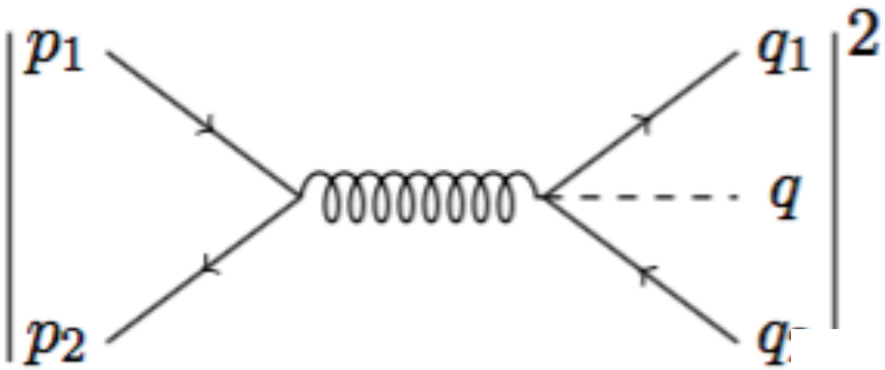
Real-Real



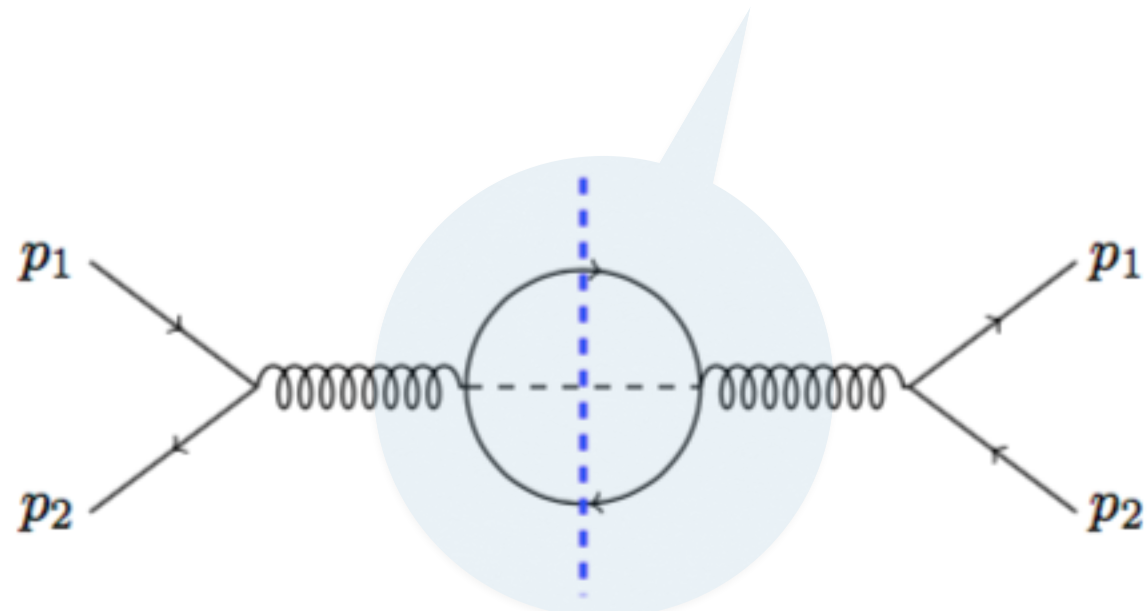
$+ 293$ terms.

Reverse Unitarity

Real-Real



$$\propto \int \frac{d^n q_1}{(2\pi)^{n-1}} \frac{d^n q_2}{(2\pi)^{n-1}} \delta_+(q_1^2)\delta_+(q_2^2)\delta_+(q^2 - m_h^2)[\cdots]$$



Reverse Unitarity

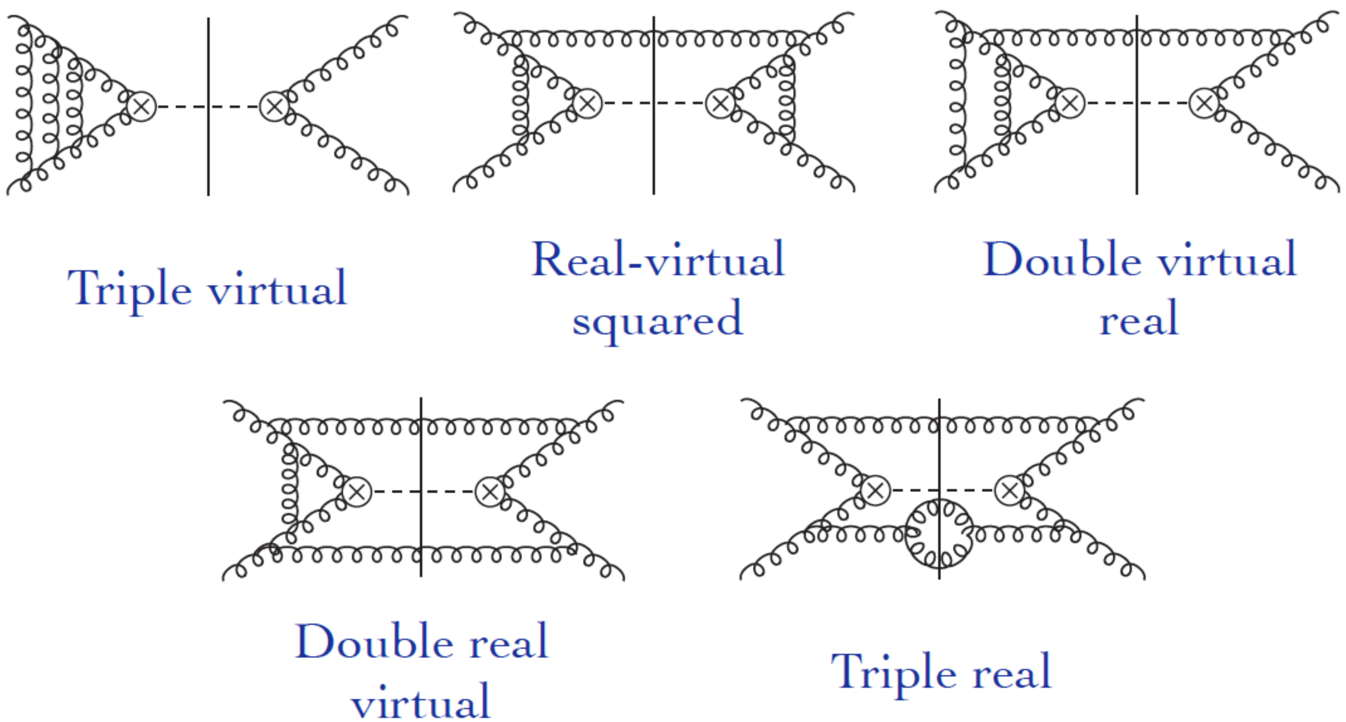
$$\delta_+(q^2 - m^2) \sim \frac{1}{q^2 - m^2 + i\varepsilon} - \frac{1}{q^2 - m^2 - i\varepsilon}$$

→ Loop Integrals

Higgs production to N^3LO in QCD at the LHC

Anastasiou, Duhr, Dulat, Furlan, Gehrmann, Herzog, Mistlberger

100 000 diagrams



Integration By Parts

$$\int \frac{d^dk_1}{(2\pi)^d}\cdots\int\frac{d^dk_3}{(2\pi)^d}\frac{\partial}{\partial k_i}\cdot\left(v_j\frac{1}{\prod_l D_l^{n_l}}\right)=0$$

Lorentz Invariance

$$p_i^\mu p_j^\nu\left(\sum_k p_{k[\nu}\frac{\partial}{\partial p_k^{\mu]}}\right)J(\vec{n})=0.$$

↓

Master Integrals

	Integrals	Master Integrals
NNLO	50 000	27
N3LO	517 531 178	1028

NNLO n-Jettiness:

$$\mathcal{T}_N = \sum_k \min_i \left\{ \frac{2p_i \cdot q_k}{Q_i} \right\}.$$

$$\mathcal{T}_N < \mathcal{T}_N^{cut}$$

$$\sigma(\mathcal{T}_N < \mathcal{T}_N^{cut}) = \int H \otimes B \otimes B \otimes S \otimes \left[\prod_n^N J_n \right] + \cdots .$$

NNLO - analytical

- H- pure virtual
- B - Initial state beam fns.
- S - Soft distribution fns.
- J - Final state jet fns.

$$\mathcal{T}_N \geq \mathcal{T}_N^{cut}$$

$$\sigma\left(\mathcal{T}_N \geq \mathcal{T}_N^{cut}\right)$$

NLO with two jets

finite- numerically

NNLO n-Jettiness:

$\mathcal{T}_N < \mathcal{T}_N^{cut}$ and $\mathcal{T}_N \geq \mathcal{T}_N^{cut}$

$$\mathcal{T}_N^{\text{cut}} \to \mathcal{T}_\delta = \delta_{\text{IR}} Q,$$

$$\sigma(X) = \int_0^{\mathcal{T}_\delta} \mathrm{d}\mathcal{T}_N \frac{\mathrm{d}\sigma(X)}{\mathrm{d}\mathcal{T}_N} + \int_{\mathcal{T}_\delta} \mathrm{d}\mathcal{T}_N \frac{\mathrm{d}\sigma(X)}{\mathrm{d}\mathcal{T}_N}$$
$$= \sigma^{\text{sing}}(X, \mathcal{T}_\delta) + \int_{\mathcal{T}_\delta} \mathrm{d}\mathcal{T}_N \frac{\mathrm{d}\sigma(X)}{\mathrm{d}\mathcal{T}_N} + \mathcal{O}(\delta_{\text{IR}}) .$$

$\sigma^{\text{nons}}(X, \mathcal{T}_\delta)$ is of $\mathcal{O}(\mathcal{T}_\delta / Q) = \mathcal{O}(\delta_{\text{IR}})$.

qT subtraction at N3LO:

$$d\sigma^F_{(N)NLO} = \mathcal{H}^F_{(N)NLO} \otimes d\sigma^F_{LO} + \left[d\sigma^{F+\text{jets}}_{(N)LO} - d\sigma^{CT}_{(N)LO} \right]$$

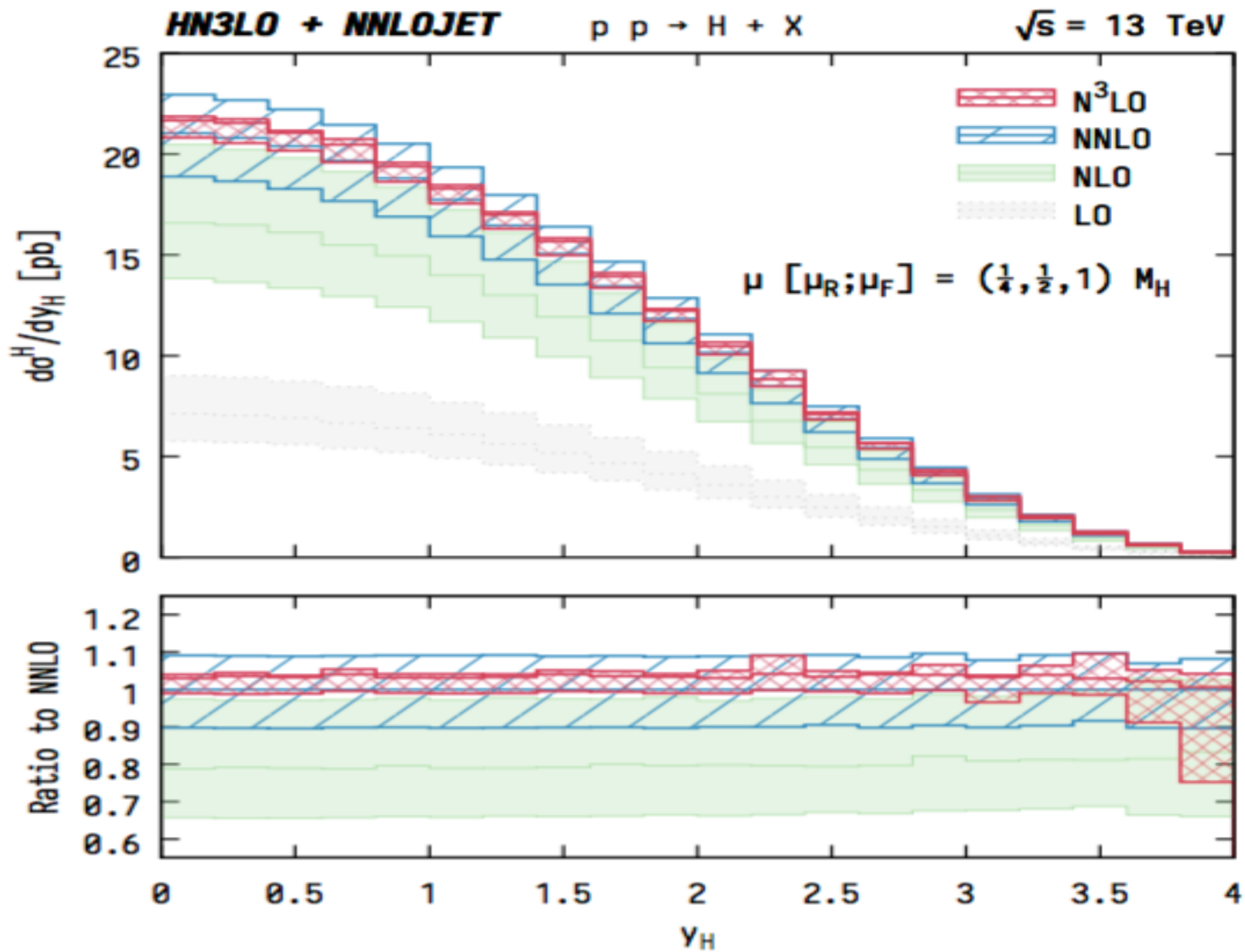
$$d\sigma^{CT} = d\sigma^F_{LO} \otimes \Sigma^F(q_T/Q)\, d^2\mathbf{q}_T .$$

Note that $$d\sigma^F_{(N)NLO}\big|_{q_T \neq 0} = d\sigma^{F+\text{jets}}_{(N)LO}$$

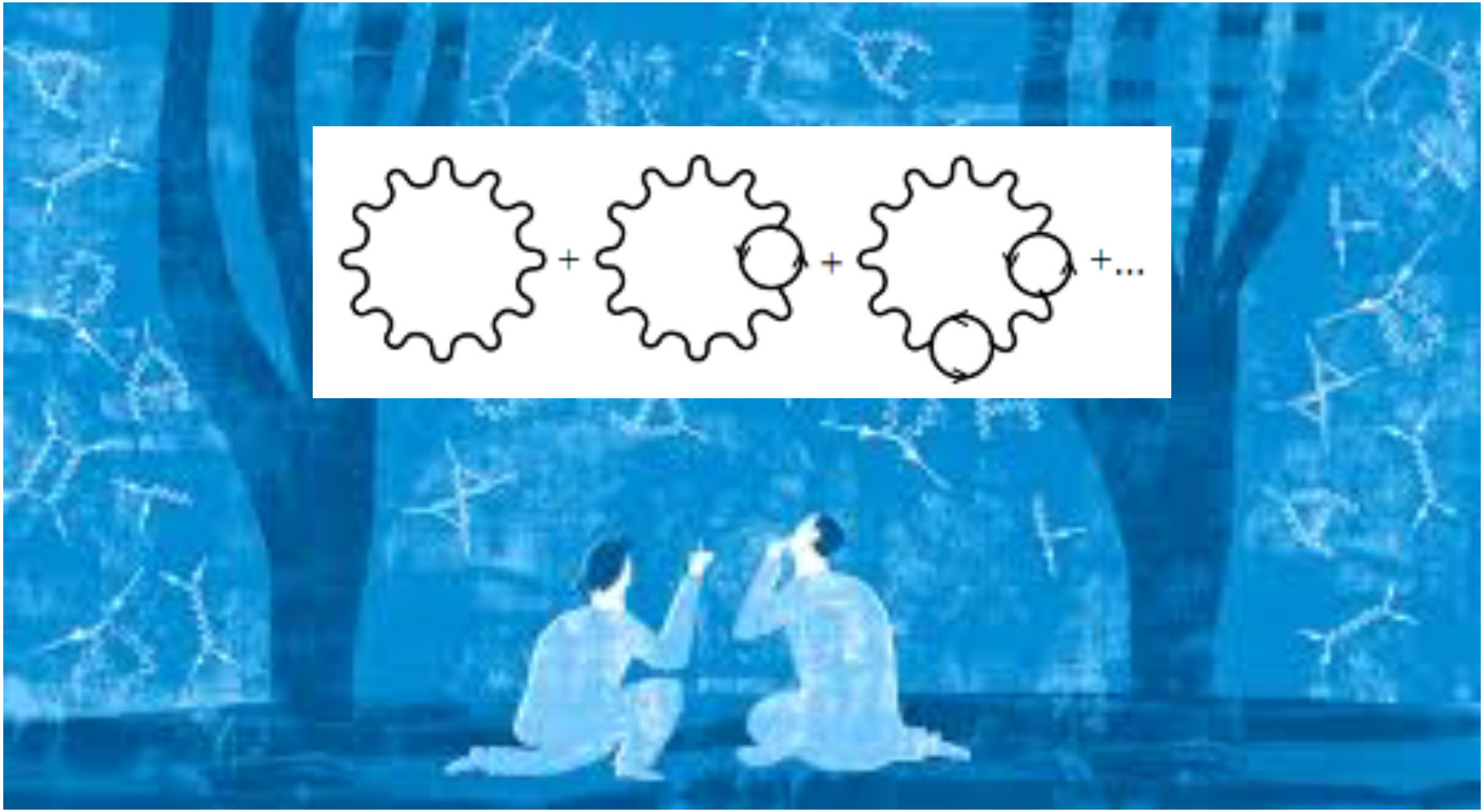
qT resummation gives

$$\Sigma^F(q_T/Q) \xrightarrow[q_T \to 0]{} \sum_{n=1}^{\infty} \left(\frac{\alpha_S}{\pi}\right)^n \sum_{k=1}^{2n} \Sigma^{F(n;k)} \frac{Q^2}{q_T^2} \ln^{k-1} \frac{Q^2}{q_T^2} \quad .$$

qT subtraction at N3LO:



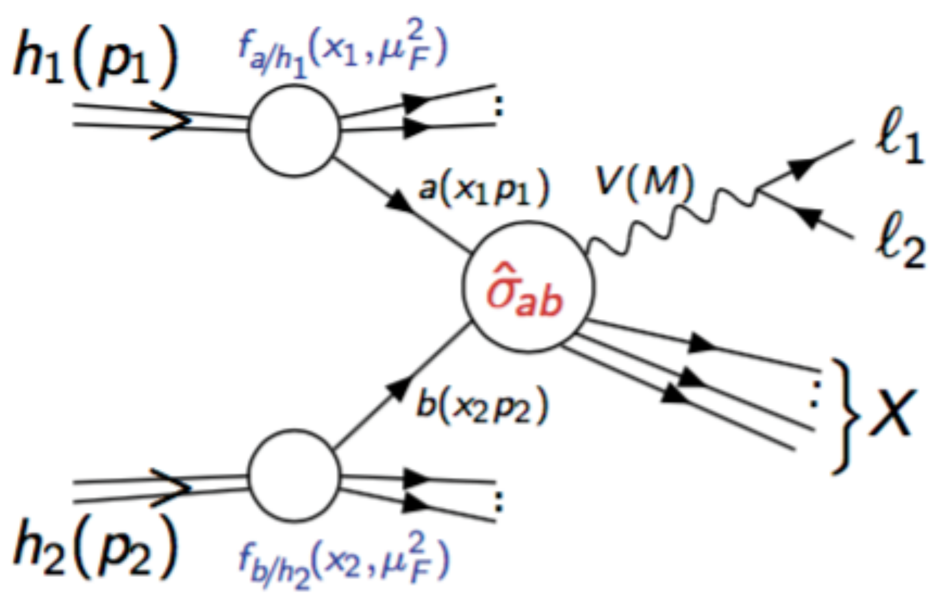
Resummation



Small qT Resummation for DY

Collins, Sterman, Soper, Canati et al

$$\frac{d\sigma}{dq_T^2}(q_T,M,s)=\sum_{a,b}\int_0^1 dx_1\int_0^1 dx_2\, f_{a/h_1}(x_1,\mu_F^2)\, f_{b/h_2}(x_2,\mu_F^2)\,\frac{d\hat{\sigma}_{ab}}{dq_T^2}(q_T,M,\hat{s};\alpha_S,\mu_R^2,\mu_F^2)$$



For $q_T \to 0$, $\alpha_S^n \log^m(M^2/q_T^2) \gg 1$

$$\frac{d\hat{\sigma}}{dq_T^2}=\frac{d\hat{\sigma}^{(res)}}{dq_T^2}+\frac{d\hat{\sigma}^{(fin)}}{dq_T^2}$$

$$\frac{d\hat{\sigma}^{(res)}}{dq_T^2}=\frac{M^2}{\hat{s}}\int_0^\infty db\,\frac{b}{2}J_0(bq_T)\,\mathcal{W}(b,M)$$

$$\mathcal{W}_N(b,M)=\mathcal{H}_N(\alpha_S)\times\exp\{\mathcal{G}_N(\alpha_S,L)\}$$

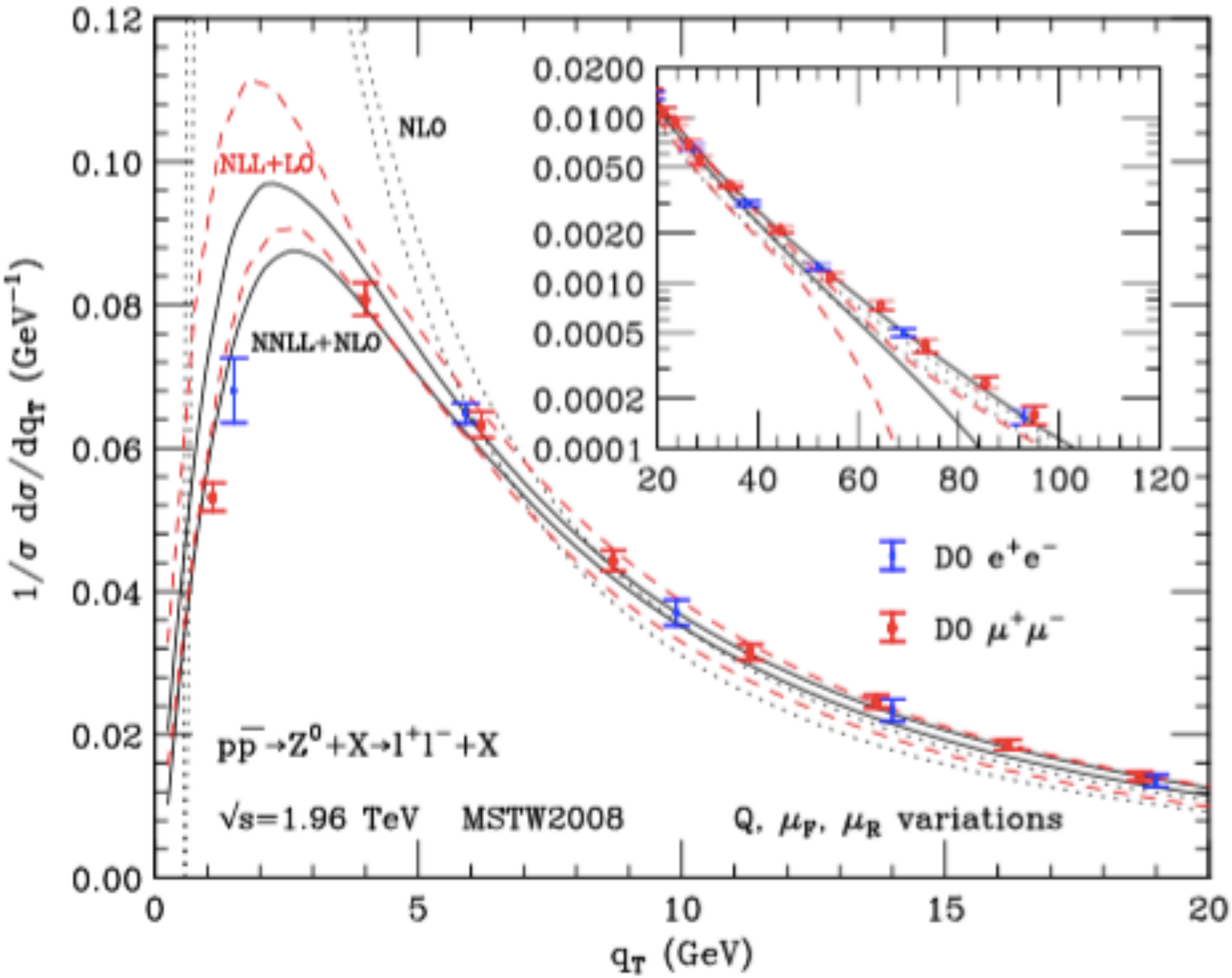
$$L\equiv\log(M^2b^2)$$

$$\mathcal{G}_N(\alpha_S,L)=L\,g^{(1)}(\alpha_S L)+g_N^{(2)}(\alpha_S L)+\frac{\alpha_S}{\pi}g_N^{(3)}(\alpha_S L)+\cdots$$

Small qT Resummation for DY

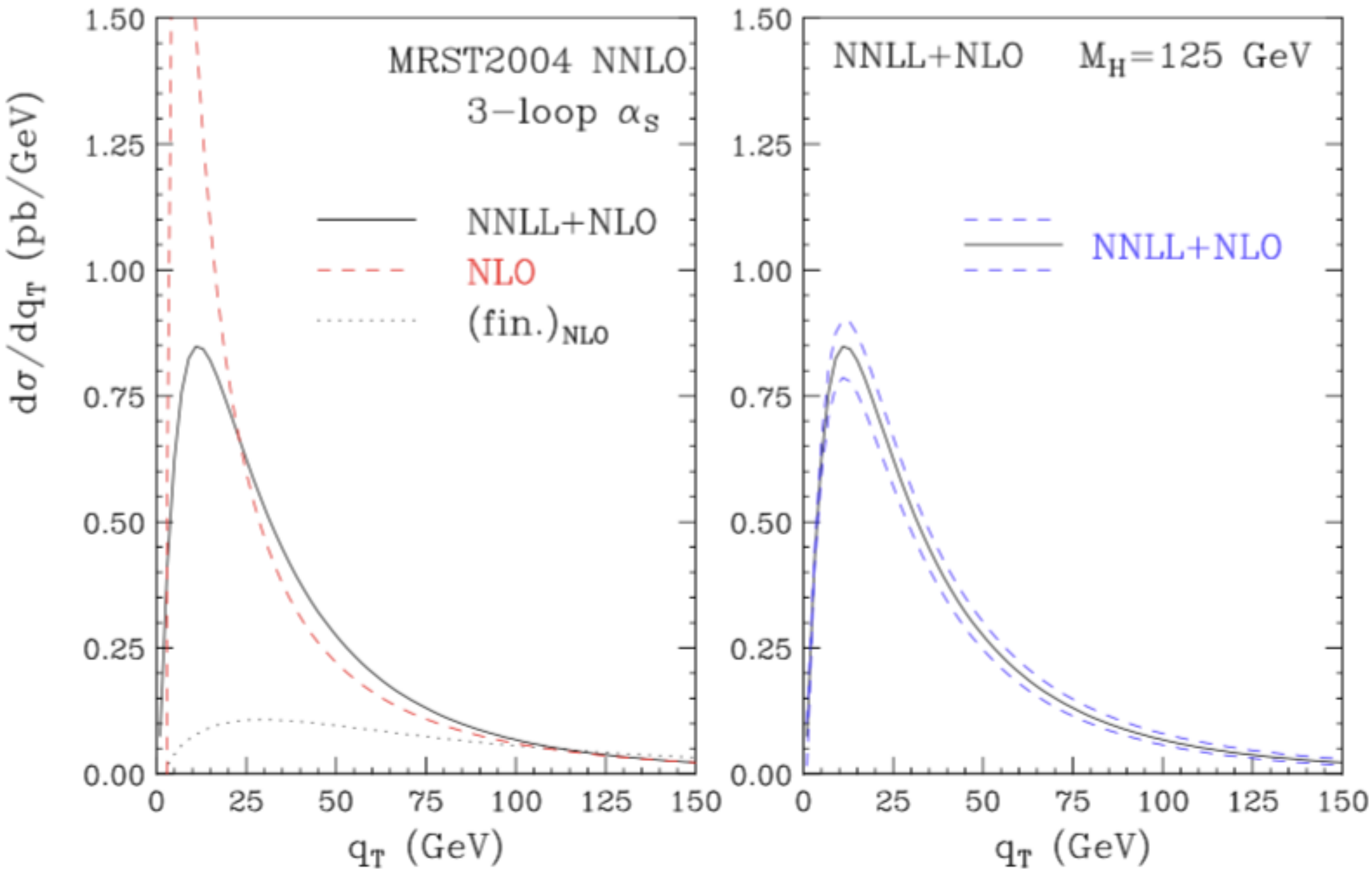
Canati et al

D0 data for the Z q_T spectrum compared with perturbative results.



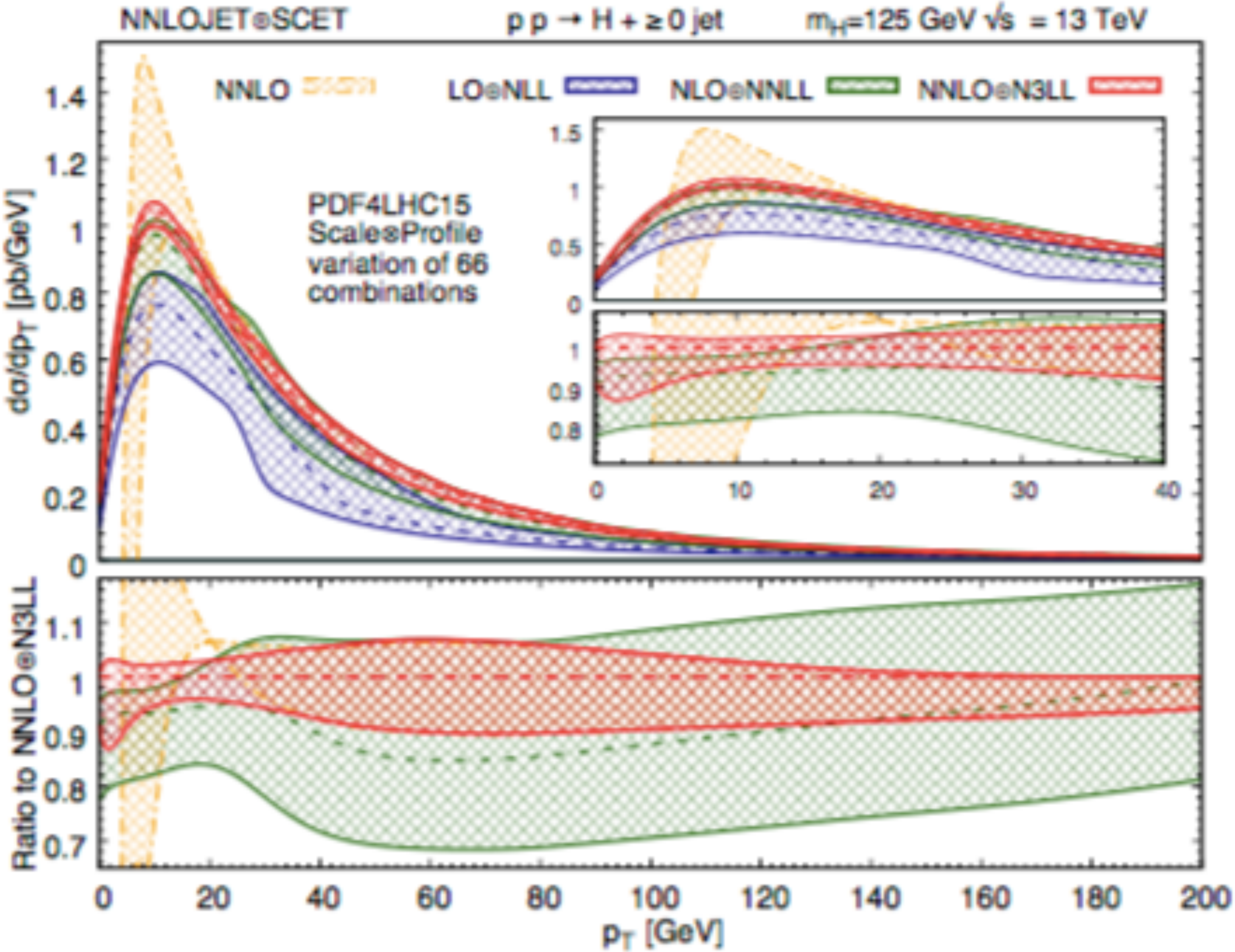
Small qT Resummation for Higgs

Canati et al



Small qT Resummation for Higgs

Canati et al



Rapidity Distribution

Rapidity Distribution of any colorless particle:

$$\frac{d\sigma^I}{dy} = \hat{\sigma}^I_B \sum_{ab=q,\bar{q},g} \int_{x_1^0}^1 \frac{dz_1}{z_1} \int_{x_2^0}^1 \frac{dz_2}{z_2} \hat{\mathcal{H}}^I_{ab}\left(\frac{x_1^0}{z_1}, \frac{x_2^0}{z_2}, \mu^2\right) \hat{\Delta}^I_{d,ab}\left(z_1, z_2, q^2, \mu^2\right)$$

DY production of lepton pairs $\sigma^I = \frac{d\sigma^q\left(\tau, q^2, y\right)}{dq^2}$.

Higgs through gluon (bottom anti-bottom), $\sigma^I = \sigma^{g(b)}\left(\tau, q^2, y\right)$.

Rapidity: $y = \frac{1}{2}\ln\left(\frac{p_2 \cdot q}{p_1 \cdot q}\right) = \ln\left(\frac{x_1^0}{x_2^0}\right)$, $\tau = x_1^0 x_2^0$

Partonic Scaling variables:

$$z_1 = \frac{x_1^0}{x_1}, \qquad z_1 = \frac{x_2^0}{x_2}$$

Soft and Virtual terms

$$\Delta_d^I = \delta(1-z_1)\delta(1-z_2) + a_s\Bigg\{ c_1^{(1)}\delta(1-z_1)\delta(1-z_2)$$

$$+c_2^{(1)}\left(\frac{\ln(1-z_1)}{1-z_1}\right)_+ + R^{(1)}(z_1,z_2) + z_1 \leftrightarrow z_2 \Bigg\} + \mathcal{O}(a_s^2)$$

$$\Delta_d^I(z_1,z_2) = \Delta_d^{I,SV}(z_1.z_2) + \Delta_d^{I,hard}(z_1,z_2)$$

Virtual , Soft $\quad \delta(1-z_i) \quad \left(\frac{\ln(1-z_i)}{(1-z_i)}\right)_+$

$$\tilde{\Delta}_d^{I,\mathrm{SV}}(\omega) = \int_0^1 dz_1 z_1^{N_1-1} \int_0^1 dz_2 z_2^{N_2-1} \Delta_d^{I,\mathrm{SV}}(z_1,z_2)$$

Resummation to NNLL

Logarithms that are resummed in g_d^I

$\mathcal{O}(a_s)$	$\ln^2(\bar{N}_1\bar{N}_2)$	$\ln(\bar{N}_1\bar{N}_2)$	
$\mathcal{O}(a_s^2)$	$\ln^3(\bar{N}_1\bar{N}_2)$	$\ln^2(\bar{N}_1\bar{N}_2)$	$\ln(\bar{N}_1\bar{N}_2)$
$\mathcal{O}(a_s^3)$	$\ln^4(\bar{N}_1\bar{N}_2)$	$\ln^3(\bar{N}_1\bar{N}_2)$	$\ln^2(\bar{N}_1\bar{N}_2)$
	LL	NLL	NNLL
Resummed terms:	$a_s^m \ln^{m+1}(\bar{N}_1\bar{N}_2)$	$a_s^m \ln^m(\bar{N}_1\bar{N}_2)$	$a_s^{m+1} \ln^m(\bar{N}_1\bar{N}_2)$
Function that resums :	$g_{d,1}^I \ln(\bar{N}_1\bar{N}_2)$	$g_{d,2}^I$	$a_s g_{d,3}^I$

Soft Gluon Resummation

Double Mellin Transformation:

$$\tilde{\Delta}_d^{I,\mathrm{SV}}(\omega) = \int_0^1 dz_1 z_1^{N_1-1} \int_0^1 dz_2 z_2^{N_2-1} \Delta_d^{I,\mathrm{SV}}(z_1, z_2)$$

Resumed Rapidity distribution:

Ni dependent

$$\tilde{\Delta}_d^{\mathrm{SV},I}(\omega) = \tilde{g}_{d,0}^I(a_s) \exp\left(g_d^I(a_s,\omega)\right)$$

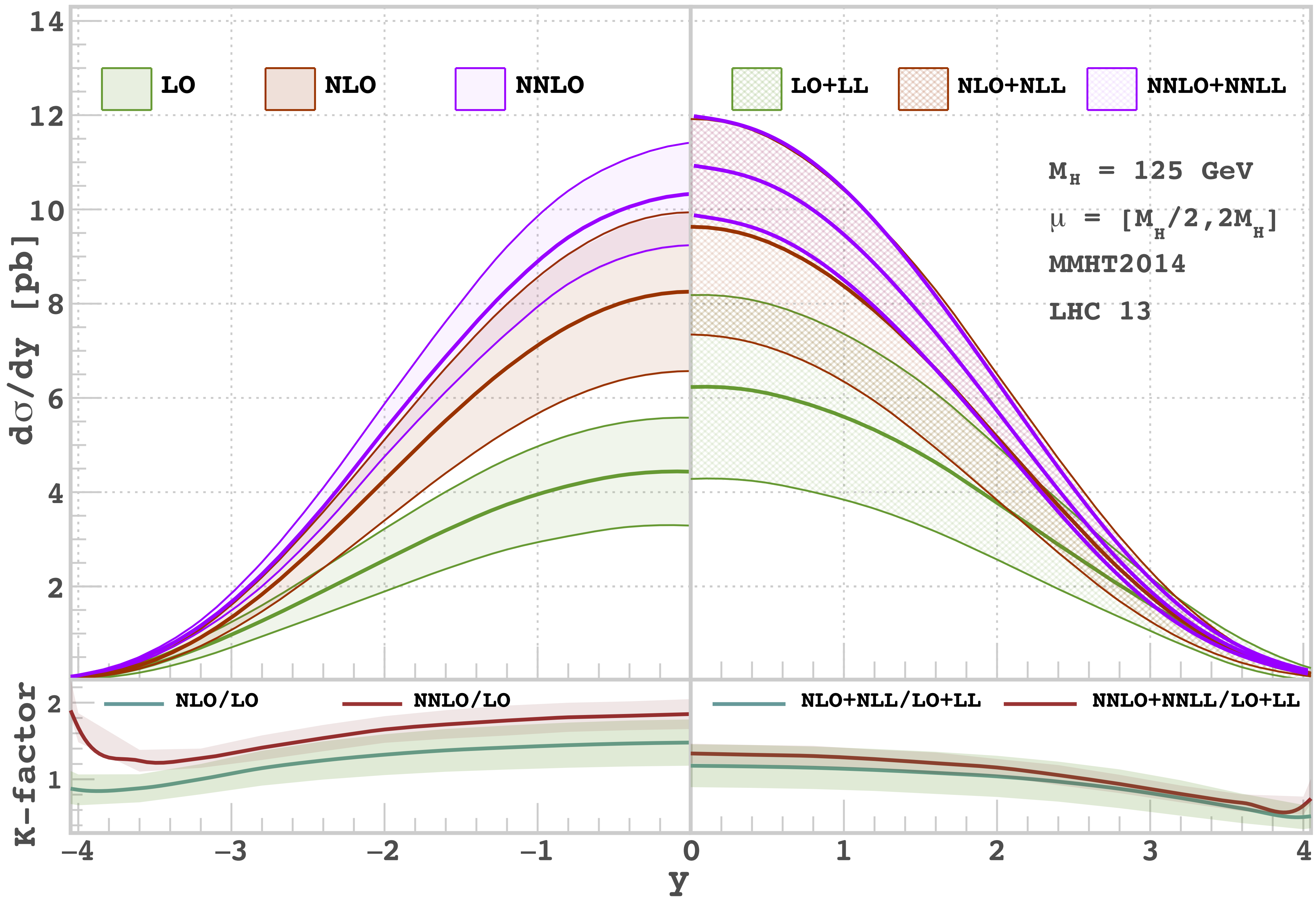
Ni independent

$$\omega = a_s \beta_0 \ln\left(\overline{N}_1 \overline{N}_2\right)$$

$$\overline{N}_i = e^{\gamma_E} N_i$$

Rapidity of Higgs at NNLO + NNLL

Banerjee, Das,Dhani and VR



Fixed order CS

$$M_H/2 \leq \mu_{R,F} \leq 2M_H$$

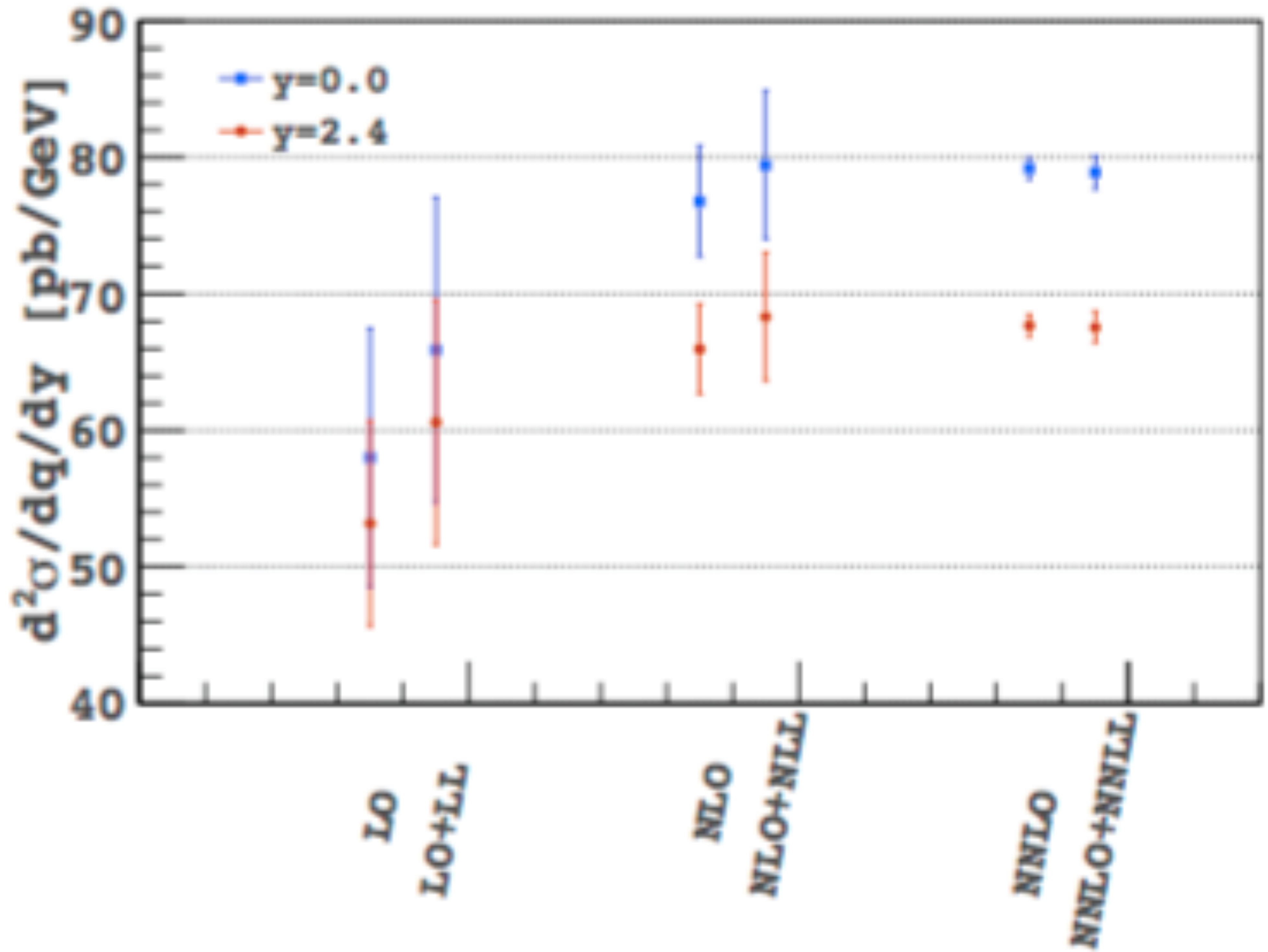
Resummed CS

$$\Longrightarrow$$

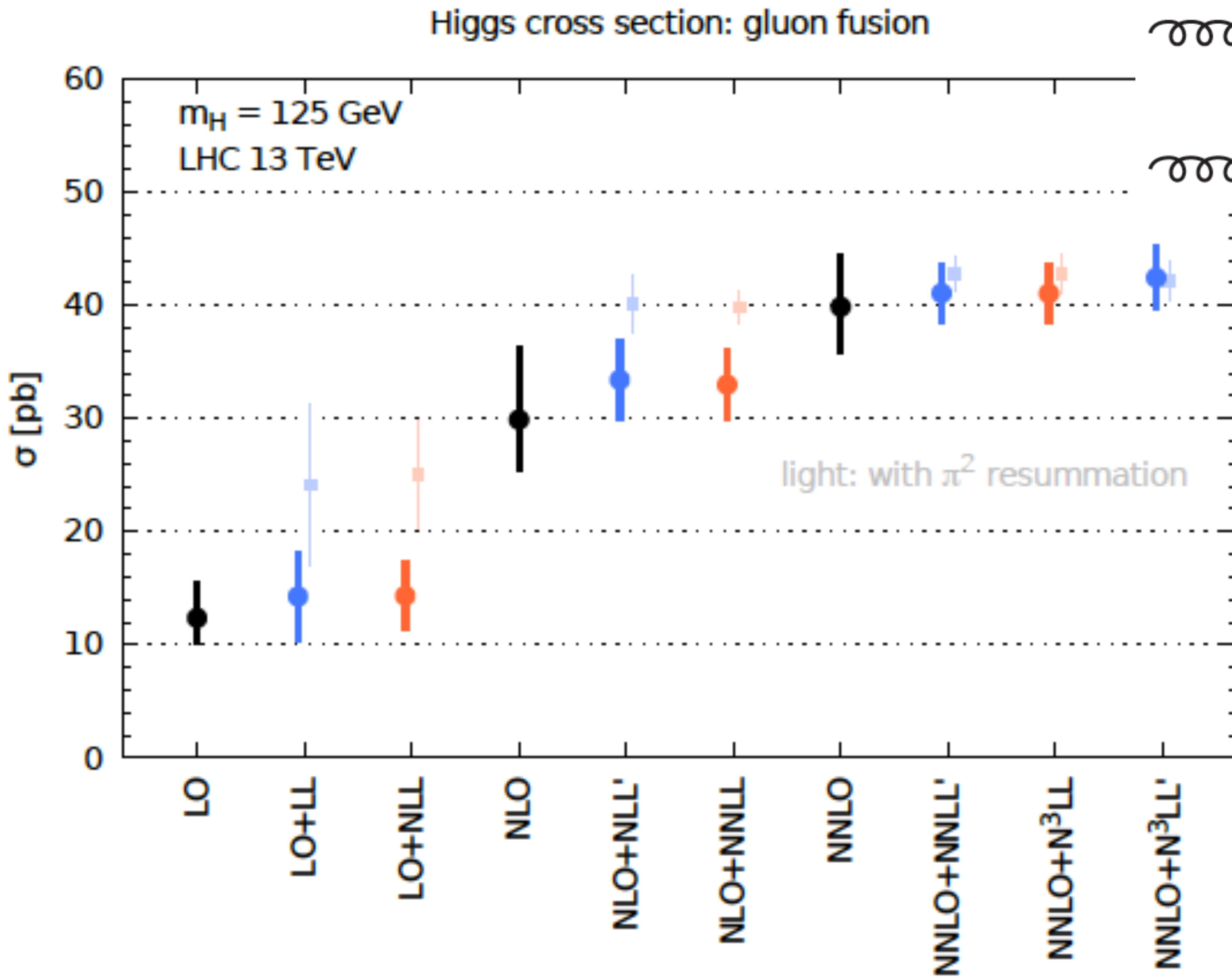
At NNLO+NNLL result stabilises convergence of perturbation series !

Rapidity of DY at NNLO + NNLL

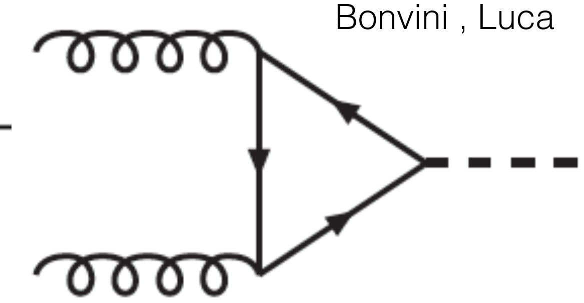
Ajjath, P. Mukherjee, Aparna, Surabhi, VR



Resummation at N3LL for Higgs

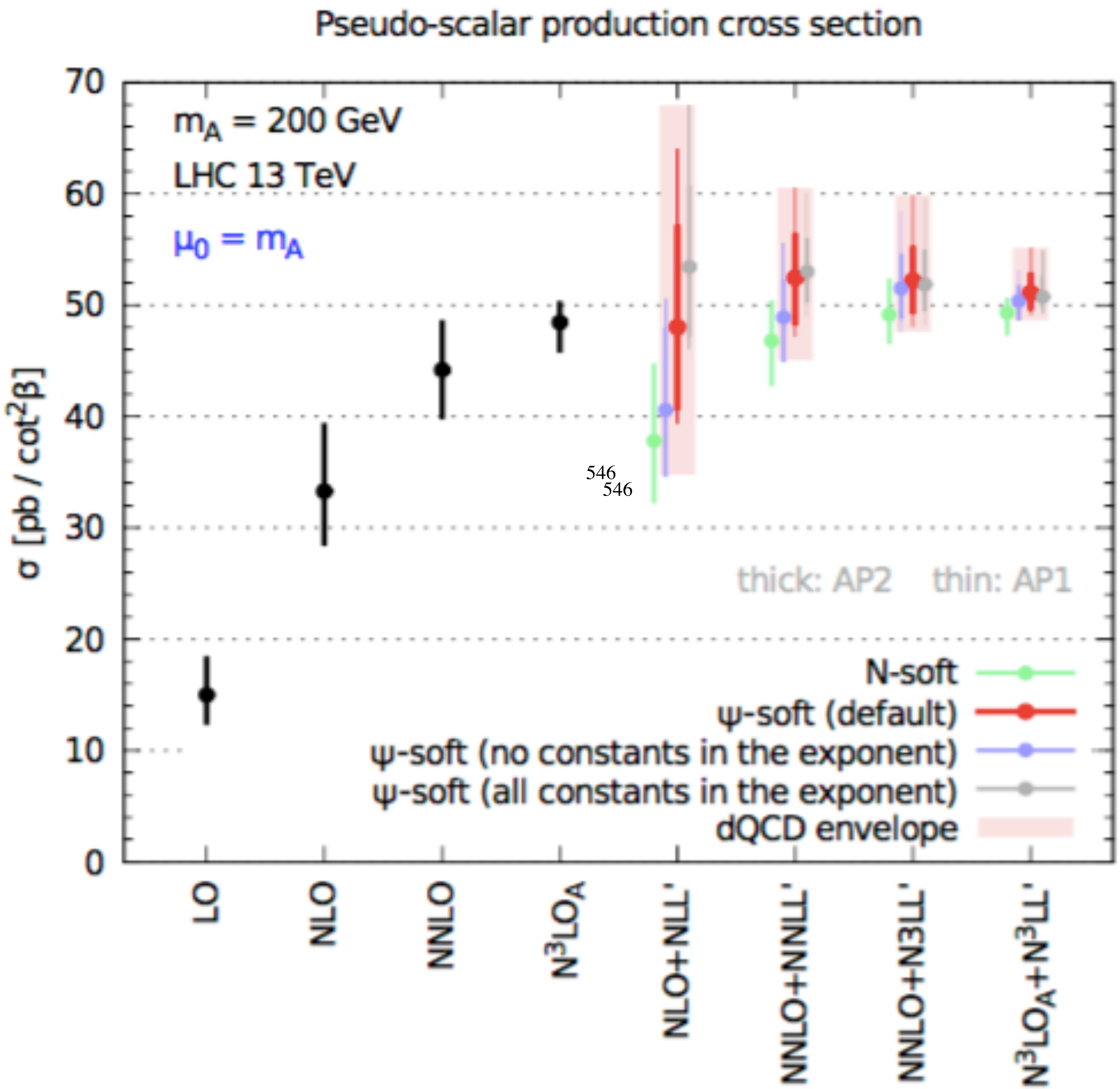


Bonvini , Luca



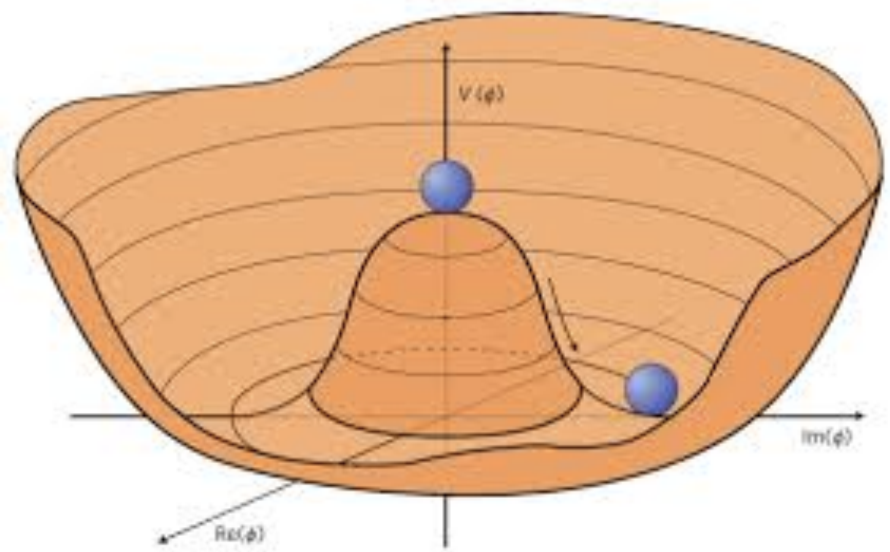
Pseudoscalar Higgs at N3LO(A) + N3LL

Kumar, P.Mathews, VR

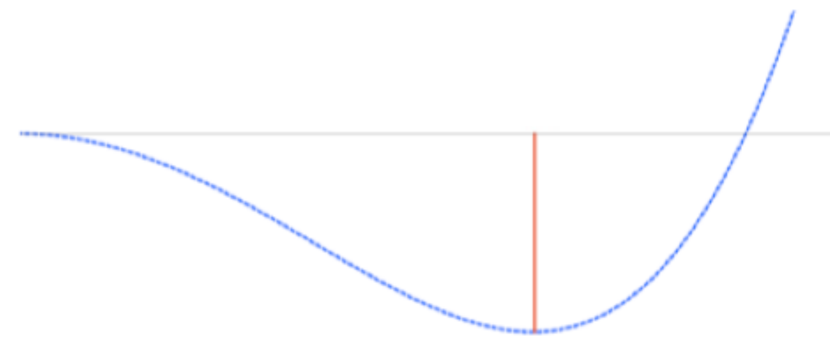


Di-Higgs Production

Higgs Potential in SM



$$V(\Phi) = -\mu^2 \Phi^\dagger \Phi + \lambda(\Phi^\dagger \Phi)^2$$



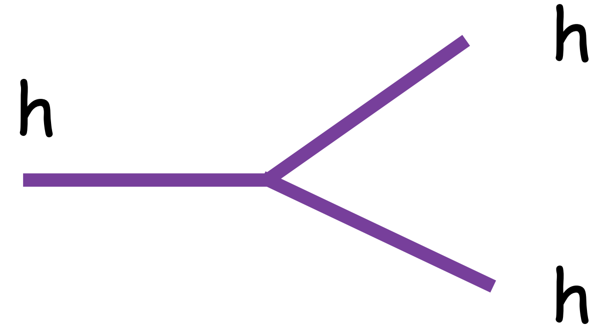
$$\frac{\mu}{\sqrt{\lambda}} \equiv v \quad 246 \text{ GeV}$$

- *Shape of the Potential*

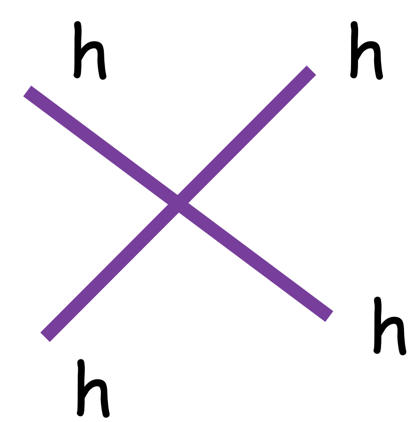
$$\mathcal{L} \supset -\frac{m_h^2}{2}\phi^2(x) - \lambda_3^{\rm SM} v \phi^3(x) - \lambda_4^{\rm SM}\phi^4(x),$$

- Test the *Predictions:*

$$\lambda_3^{\rm SM} = \frac{m_h^2}{2v^2}, \quad \lambda_4^{\rm SM} = \frac{m_h^2}{8v^2}$$

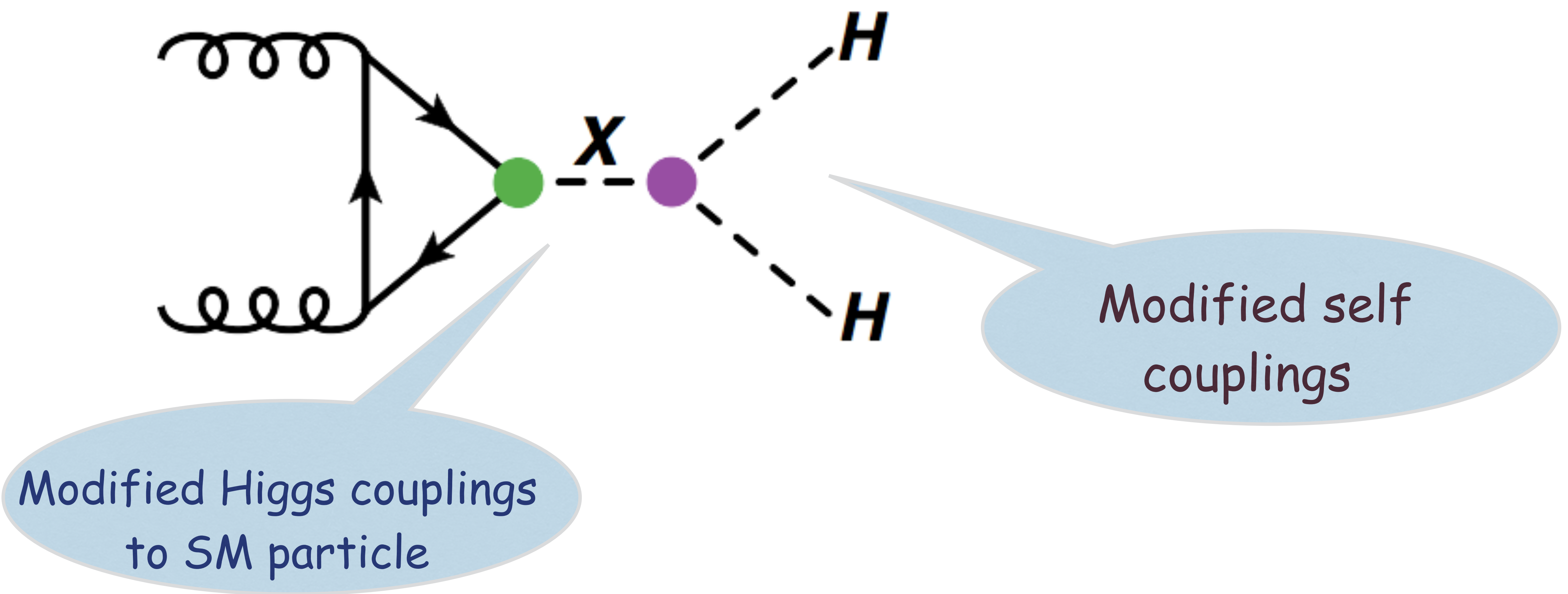


$\lambda_3^{SM} = 0.13$



$\lambda_4^{SM} = 0.03$

In BSM scenarios



- *Non-Resonant production*

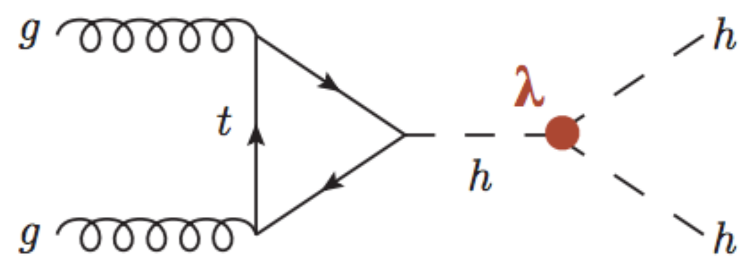
- *Resonant production:*

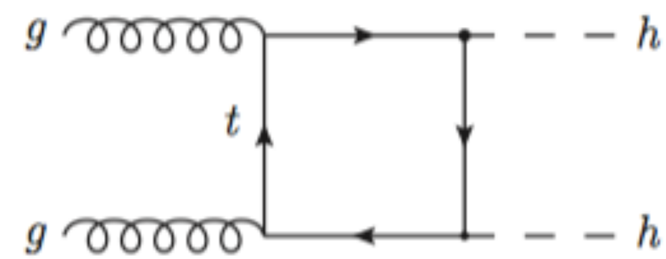
Heavy scalars in Two Higgs doublet models,
Spin-2 resonances from Randal Sundrum Model

Production Cross section

- Dominant ones:

Glover,van der Bij





- Relative Contributions

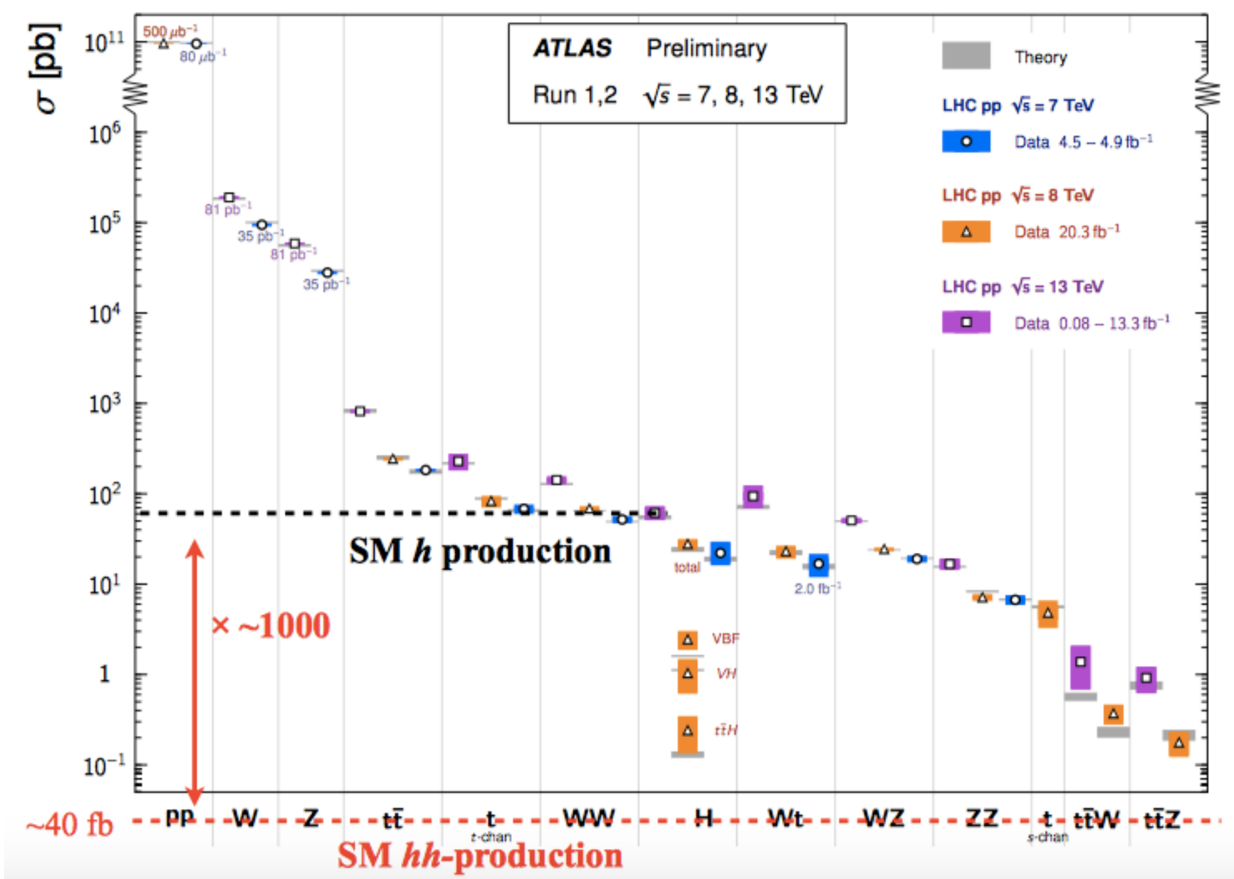
destructively interfere!

$\lambda_3^{SM} = 0.3$

ggF-hh	$\sim$ 40 fb
VBF-hh	~2 fb
V-hh	$\sim$ 1 fb
tt-hh	$\sim$ 1 fb

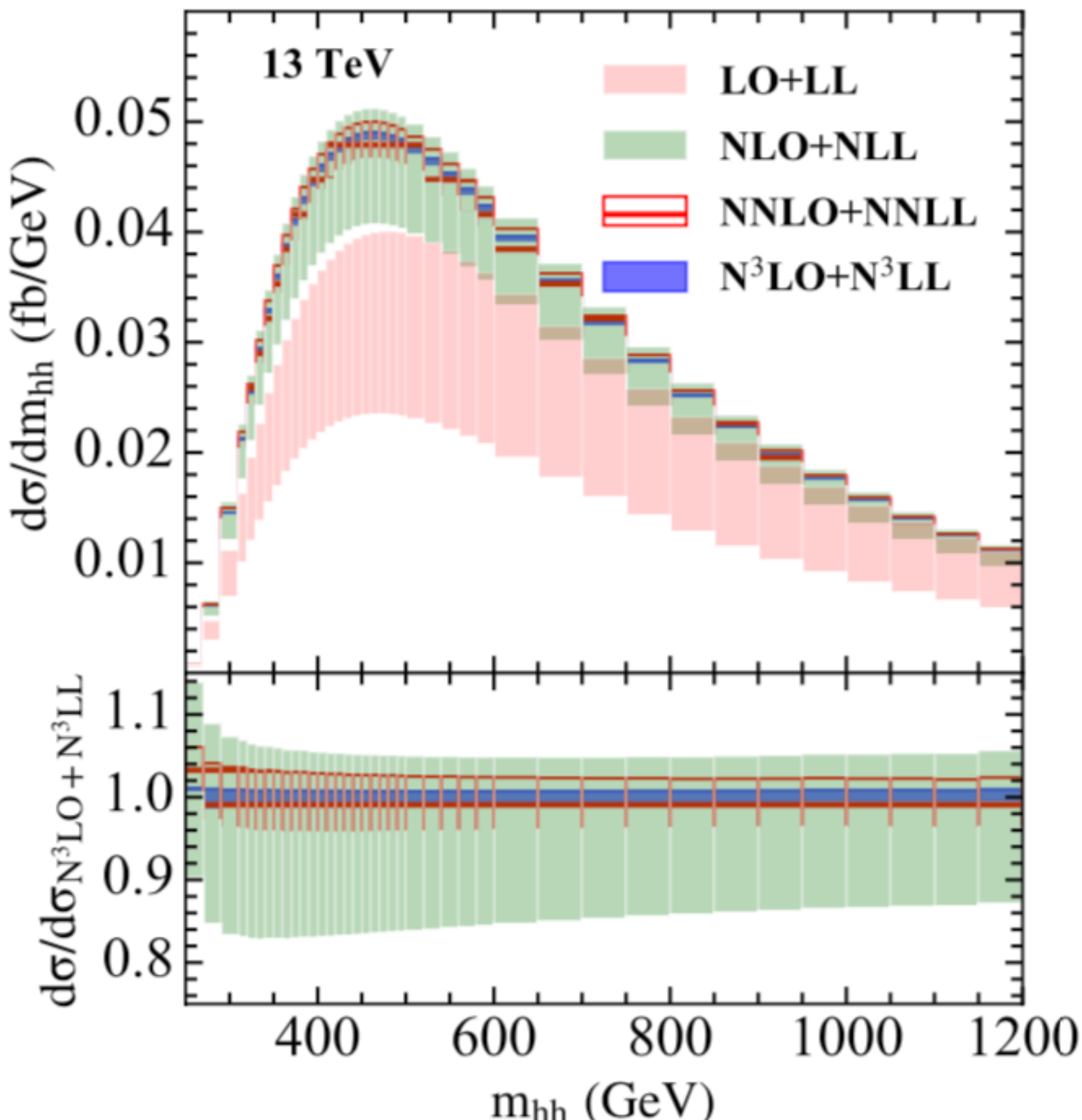
$$b + \bar{b} \rightarrow hh \quad \approx 0.1 fb$$

Tough Task



Production Cross section

A.H. Ajjath and Hua-Sheng Shao



Conclusion

The truth is in the Details