Flavor Physics and CP Violation

Xiao-Gang He

2022 ASIAN EYROPE PACIFIC SCHOOL
OF HIGH-ENERGY PHYSICS

Pyeongchang, SOUTH KOREA
5 – 18, 2022

Contents

Lecture II FPCP in Standard model

2.1 Flavor Physics tests for SM

2.2 CP violation tests for SM

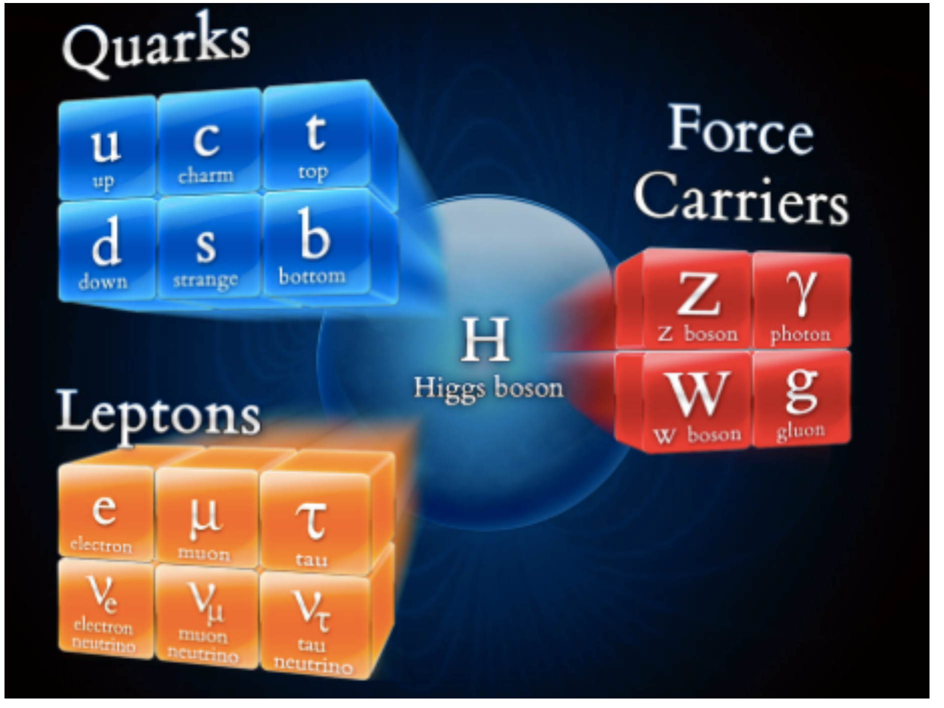
2.3 More CP violating experimental observables

2.1 Flavor Physics tests for SM

The standard model

Standard Model is based on $SU(3)_C x SU(2)_L x U(1)_Y$ gauge interaction.

In SM mis-match of weak and mass eigen-bases, leads to flavor mixing and CP violation.



Parameters in the standard model with 3 generations

Gauge boson couplings and masses: $g_1=g'$, $g_2=g$, $g_3=g_s$, m_W, m_Z

Fermion Masses: m_e, m_μ, m_τ, $m_{\nu e}$, $m_{\nu\mu}$, $m_{\nu\tau}$
m_u, m_d, m_c, m_s, m_t, m_b

Higgs boson mass and couplings: m_h or λ, m_i/v to ith fermion

(Weak mixing angle θ_W: $\tan\theta_W = g_2/g_1$, $e = g_2 \sin\theta_W$)

$\alpha_{em} = e^2/4\pi$, $\alpha_2=g_2^2/4\pi$, $\alpha_3=\alpha_{s=}=g_s^2/4\pi$; $G_F = g^2/(4\sqrt{2}\, m_W^2)$

Mixing: quark mixing (3 mixing angles + 1 Dirac-phase)
Neutrino mixing (3 mixing angles +1 Dirac-phase + 2 Majorana-phases)

1 possible strong CP violating parameter θ

Total independent model parameters: 18 +1 without neutrino masses.

Another 9 if include neutrino masses at low energies or more.

(3 gauge couplings + 1 W or Z mass + 1 Higgs coupling or Higgs mass + (6 quark + 3 charged lepton masses) + 3 quark mixing angle + 1 Dirac-phase, 1 strong phase, and 3+6 neutrino masses, mixing angles and phases)

In the SM flavor physics has a lot to do with these free parameters

What do we know about the SM parameters?

α_{em}=1/137.035999084(21) $\sin^2\theta_W$=0.23121(4) α_3=0.1179(9)
(G_F = 1.1663788(6)x10^{-5} GeV^{-2})

m_Z=91.1876(21) GeV m_h=125.25(0.17) GeV
(SM: m_W=80.357(6) GeV vs. Recent CDF II data: m_W=80.4335(94) GeV 7σ away!)

Charged lepton masses:
m_e=0.51099895000(15) MeV m_μ=105.6583755(23) MeV m_τ= 1776.86(12) MeV

Quark masses:
m_u =1.16(+0.49, -0.26) MeV m_d=4.67(+0.48, -0.17) MeV, m_s=93.4(+8.6, -3.4) MeV,
m_c=1.27(0.02) GeV, m_b=4.18(+0.03,-0.02) GeV, m_t= 172.69(0.30) GeV

Strong CP violating phase $\theta < 10^{-9}$

What about quark and neutrino mixing angles and CP violating phases, and neutrino masses?

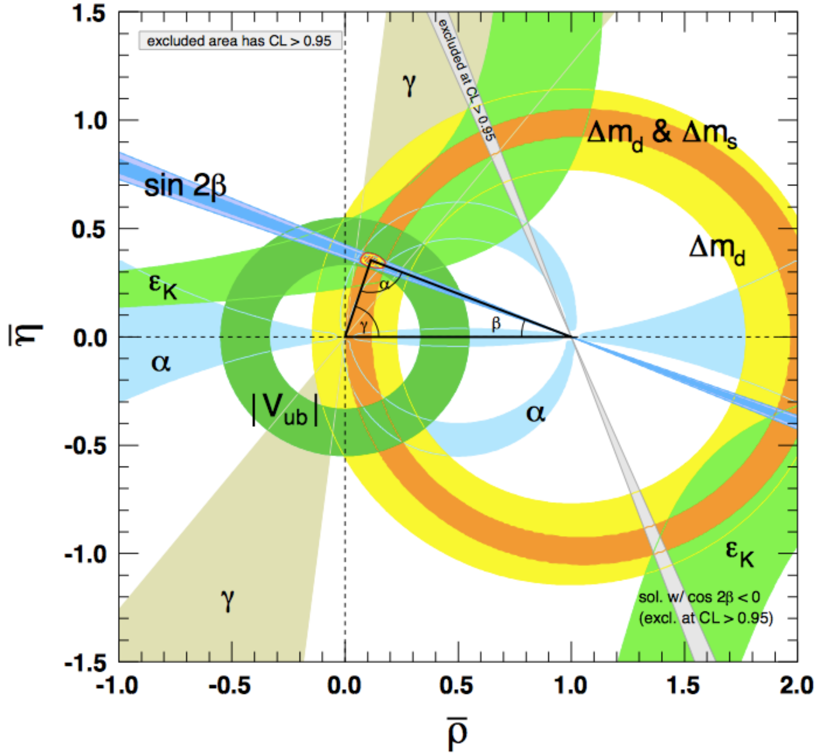
Status of Quark and Lepton Mixing

Quark Mixing

PDG

Neutrino Mixing

$$\begin{pmatrix} 1-\lambda^2/2 & \lambda & A\lambda^3(\rho - i\eta) \\ -\lambda & 1-\lambda^2/2 & A\lambda^2 \\ A\lambda^3(1-\rho-i\eta) & -A\lambda^2 & 1 \end{pmatrix} + \mathcal{O}(\lambda^4)$$

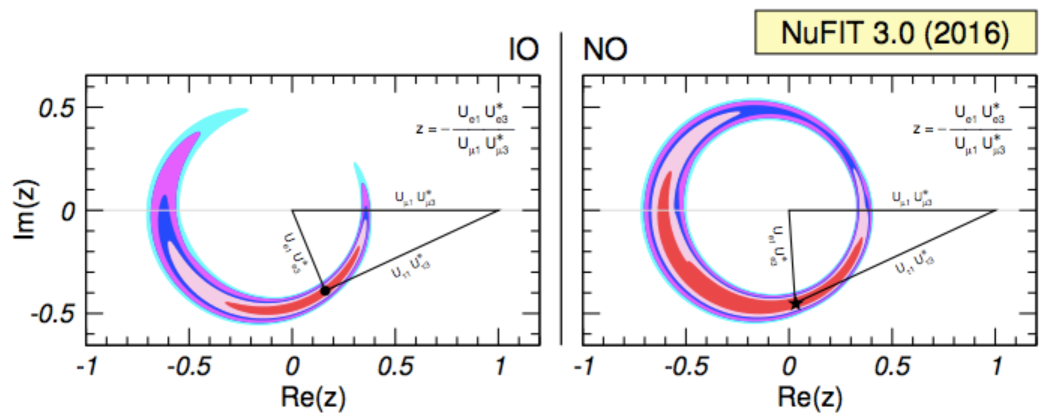


$$\lambda = 0.22500 \pm 0.00067\,, \qquad A = 0.826^{+0.018}_{-0.015}\,,$$
$$\bar\rho = 0.159 \pm 0.010\,, \qquad \bar\eta = 0.348 \pm 0.010\,.$$

$$\sin\theta_{12} = 0.22500 \pm 0.00067\,, \qquad \sin\theta_{13} = 0.00369 \pm 0.00011\,,$$
$$\sin\theta_{23} = 0.04182^{+0.00085}_{-0.00074}\,, \qquad \delta = 1.144 \pm 0.027\,.$$

$\Delta m^2 = m_3^2 - (m_2^2 + m_1^2)/2$. Thus, $\Delta m^2 = \Delta m_{31}^2 - \Delta m_{21}^2/2 > 0$, if $m_1 < m_2 < m_3$ and $\Delta m^2 = \Delta m_{32}^2 + \Delta m_{21}^2/2 < 0$ for $m_3 < m_1 < m_2$.

Parameter	best-fit	3σ
Δm_{21}^2 $[10^{-5}$ eV $^2]$	7.37	6.93 – 7.97
$\lvert\Delta m^2\rvert$ $[10^{-3}$ eV $^2]$	2.50 (2.46)	2.37 – 2.63 (2.33 – 2.60)
$\sin^2\theta_{12}$	0.297	0.250 – 0.354
$\sin^2\theta_{23}$, $\Delta m^2 > 0$	0.437	0.379 – 0.616
$\sin^2\theta_{23}$, $\Delta m^2 < 0$	0.569	0.383 – 0.637
$\sin^2\theta_{13}$, $\Delta m^2 > 0$	0.0214	0.0185 – 0.0246
$\sin^2\theta_{13}$, $\Delta m^2 < 0$	0.0218	0.0186 – 0.0248
δ/π	1.35 (1.32)	(0.92 – 1.99)
		((0.83 – 1.99))

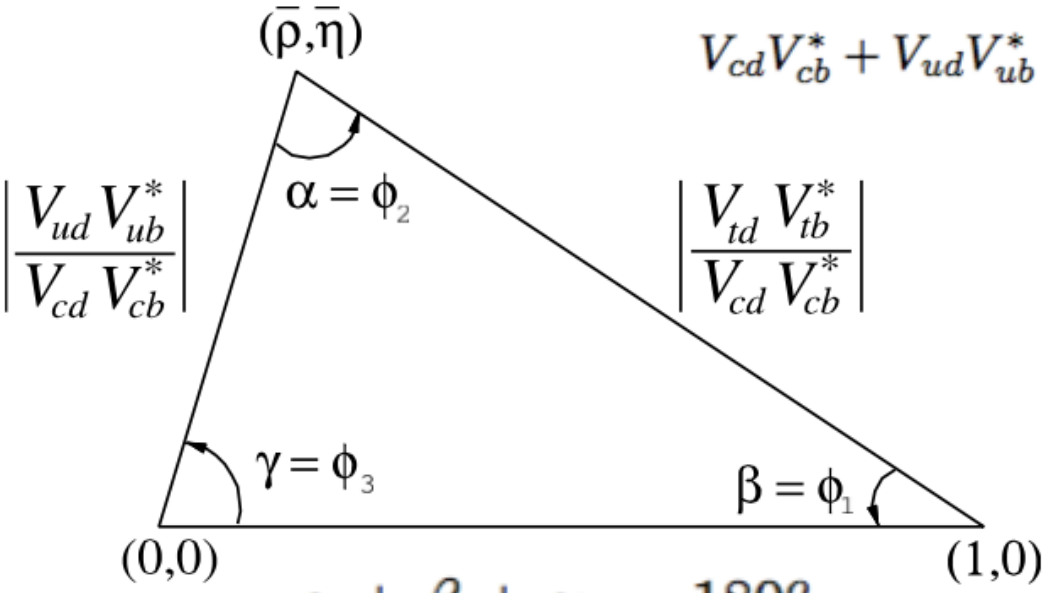


Why they mix the pattern shown above?

The Unitarity Triangle

$$\sum_i V_{ij} V^*_{ik} = \delta_{jk} \;, \quad \sum_i V_{ji} V^*_{ki} = \delta_{jk} \;,$$

$$V_{cd}V^*_{cb} + V_{ud}V^*_{ub} + V_{td}V^*_{tb} = 0 \;\rightarrow\; 1 + \frac{V_{ud}V^*_{ub}}{V_{cd}V^*_{cb}} + \frac{V_{td}V^*_{tb}}{V_{cd}V^*_{cb}} = 0 \,.$$



$$\alpha + \beta + \gamma = 180^o$$

The Jarlskog parameter J (1985)

$$\mathrm{Im}\left[V_{ij} V_{kl} V^*_{il} V^*_{kj}\right] = J \textstyle\sum_{m,n} \varepsilon_{ikm}\varepsilon_{jln}$$

$$J = s_{12}c_{12}s_{23}c_{23}s_{13}c_{13}^2 \sin\delta$$

$$= (3.08^{+0.15}_{-0.13}) \times 10^{-5}$$

PDG

$$\alpha + \beta + \gamma = (173 \pm 6)^\circ$$

The area of the triangle = J/2

CPV in SM is always proportional to J

$$\alpha = \mathrm{Arg}(-V_{td}V^*_{tb}/V^*_{ub}V_{ud}),\; \beta = \mathrm{Arg}(-V_{cd}V^*_{cb}/V^*_{tb}V_{td}),\; \text{and}\; \gamma = \mathrm{Arg}(-V_{ud}V^*_{ub}/V^*_{cb}V_{cd})$$

Flavor physics tests for SM

Discovering new phenomena, and testing various theoretical predictions

-> establishment of a theory (Determine the model parameters, looking for deviations -> modify the theory…)

Produce various particles and observe how they interact and decay

Production: e+e-, p anti-p, pp… colliders (γ, e-, ν, p..) hit on Nuclei target…

=> SM particles…

Observe various particle decays, quarks, leptons, gauge bosons, Higgs boson… t -> W + b -> l v + c light hadrons (for lighter quarks, one needs to study the hadrons containing the specific quark to see it decay properties…)

Interaction with probes: g-2 of muon (muon under know magnetic field)…

Cross sections, decay rates, production and decay asymmetries…. Obtain desired properties of a theory: coupling constants, mixing angles, parity and CP properties…

SM interaction features

interaction	fermions	force carrier	coupling	flavor
Electromagnetic	u, d, ℓ	A^0	eQ	universal
Strong	u, d	g	g_s	universal
NC weak	all	Z^0	$\frac{e(T_3 - s_W^2 Q)}{s_W c_W}$	universal
CC weak	$\bar{u}d/\bar{\ell}\nu$	$W^\pm$	gV/g	non-universal/universal
Yukawa	u, d, ℓ	h	y_q	diagonal

NC – neutral current; CC-charged current.

QED test: Experimental data test for photon interactions to multi-loop level precision. No flavor changing effects. Agree with SM.

QCD test: Gluon and quark jets observed, no conflict between data and theory. No flavor changing effects.

Charged Current Interaction

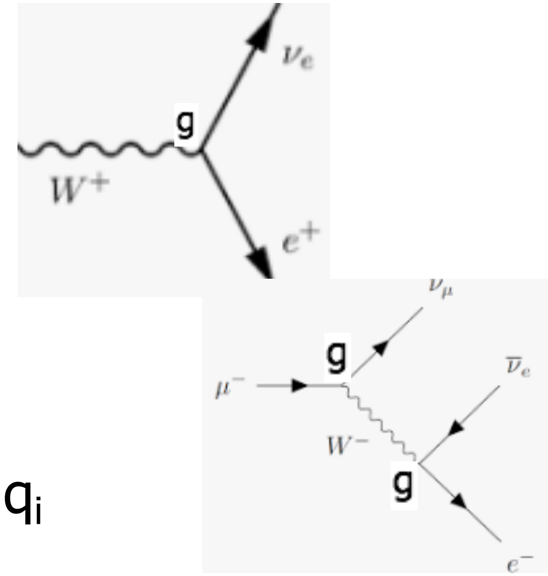
CN weak: W interactions,

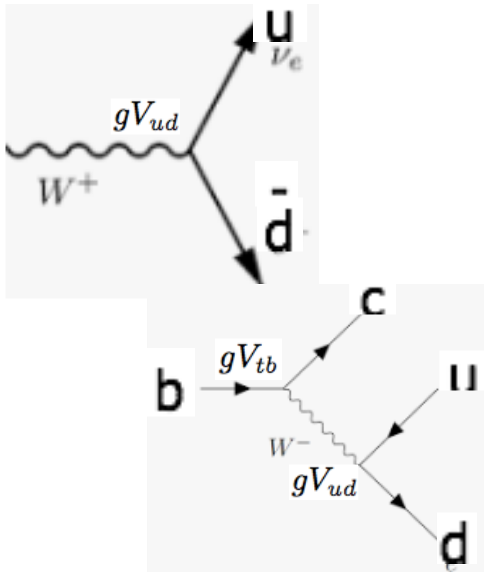
W -> l $\bar{\nu}$ ~ g

W -> $q_i \bar{q}_j$ ~ $g(V_{KM})_{ij}$

l_i -> W + ν_i -> $l_i \nu_j + \nu_i$;

q_i -> W + q_j -> $\bar{q}_k q_l + q_j q_i$





$$\Gamma(\mu \to e\nu_\mu\bar{\nu}_e) = \frac{G_F^2 m_\mu^5}{192\pi^3}\left(1 - 8\frac{m_e^2}{m_\mu^2} + 8(\frac{m_e^2}{m_m^2 u})^3 - (\frac{m_e^2}{m_\mu^2})^4 - 12(\frac{m_e^2}{m_\mu^2})^2 \ln(\frac{m_e^2}{m_\mu^2})\right)$$

providing information for G_F.

A quark can decay into another generation of quarks (flavor changing)
Providing information for V_{KM}.

$$Br(W^+ \to e^+\nu_e) = (10.71 \pm 0.16)\% \,,$$
$$Br(W^+ \to \mu^+\nu_\mu) = (10.63 \pm 0.15)\% \,,$$
$$Br(W^+ \to \tau^+\nu_\tau) = (11.38 \pm 0.21)\% \,.$$
$$Br(W \to hadrons) = (67.41 \pm 0.27)\%$$

Agree with SM predictions
$g^2_{\mu\nu}/g^2_{e\nu}$ =0.992 (0.020)
$g^2_{\tau\nu}/g^2_{e\nu}$ =1.063(0.025)
Universality better than 3σ

Normalizing $\Gamma(W \to l\,\nu) \sim 1$
$\to \Gamma(W \to q_i\, q_j) \sim 3\, |V_{ij}|^2$ (3-colors, 1st and 2nd generations)

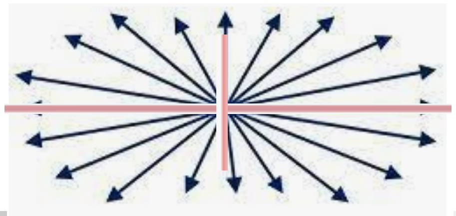
$$|V_{ud}|^2 + |V_{us}|^2 + |V_{ub}|^2 = |V_{cd}|^2 + |V_{cs}|^2 + |V_{cb}|^2 = 1$$

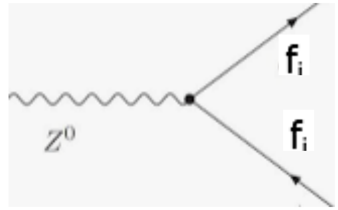
$$\Gamma(W \to \text{hadrons})/\Gamma(W \to \text{leptons}) \approx 2 \qquad \text{Data: } 2.06 \pm 0.1$$

Look into detailed W induce decays V_{ij} can be determined to good precisions

More later

Neutral current interaction





NC Neutral: Z interactions

forward-backward scattering asymmetry ~$g_V g_A$

e^+e^-	[h]	(3.3632±0.0042) %
$\mu^+\mu^-$	[h]	(3.3662±0.0066) %
$\tau^+\tau^-$	[h]	(3.3696±0.0083) %
$\ell^+\ell^-$	[b,h]	(3.3658±0.0023) %
invisible	[h]	(20.000 ±0.055) %
hadrons	[h]	(69.911 ±0.056) %
$(u\overline{u}+c\overline{c})/2$		(11.6 ±0.6) %
$(d\overline{d}+s\overline{s}+b\overline{b})/3$		(15.6 ±0.4) %
$c\overline{c}$		(12.03 ±0.21) %
$b\overline{b}$		(15.12 ±0.05) %

$\Gamma(\mu^+\mu^-)/\Gamma(e^+e^-) = 1.0001 \pm 0.0024$

$\Gamma(\tau^+\tau^-)/\Gamma(e^+e^-) = 1.0020 \pm 0.0032$

$g_V^\ell = -0.03783 \pm 0.00041$
$g_V^u = 0.266 \pm 0.034$
$g_V^d = -0.38^{+0.04}_{-0.05}$
$g_A^\ell = -0.50123 \pm 0.00026$
$g_A^u = 0.519^{+0.028}_{-0.033}$
$g_A^d = -0.527^{+0.040}_{-0.028}$
$g^{\nu_\ell} = 0.5008 \pm 0.0008$
$g^{\nu_e} = 0.53 \pm 0.09$
$g^{\nu_\mu} = 0.502 \pm 0.017$

Agree with SM predictions

Universality holds

In SM invisible width from Z -> $\nu\nu$ Data: number of light neutrinos N_ν =3.0026(0.0061)!

No FCNC

$e^\pm\mu^\mp$	LF	[j] < 7.5	$\times 10^{-7}$ CL=95%
$e^\pm\tau^\mp$	LF	[j] < 5.0	$\times 10^{-6}$ CL=95%
$\mu^\pm\tau^\mp$	LF	[j] < 6.5	$\times 10^{-6}$ CL=95%

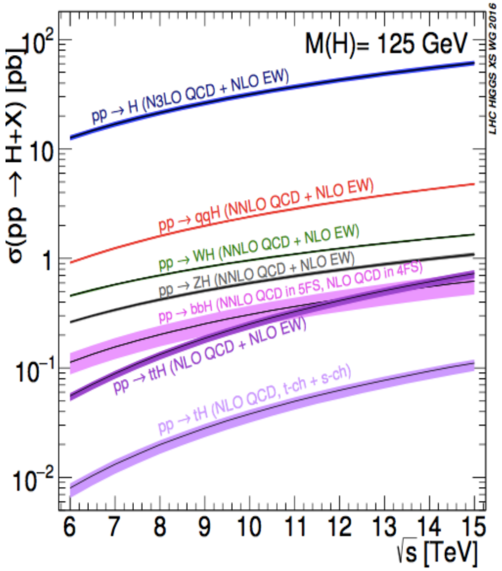
- no q_i q_j flavor changing quark decays neither!

How to explain FCNC observed in K-$\overline{K}$, D-$\overline{D}$ and B-$\overline{B}$ mixing? Loop. More later

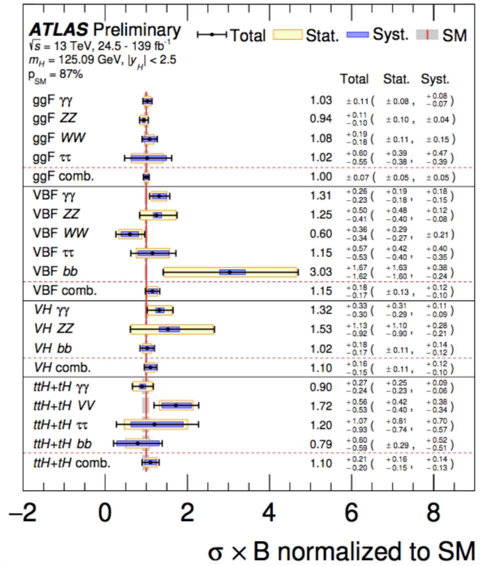
Higgs boson interactions

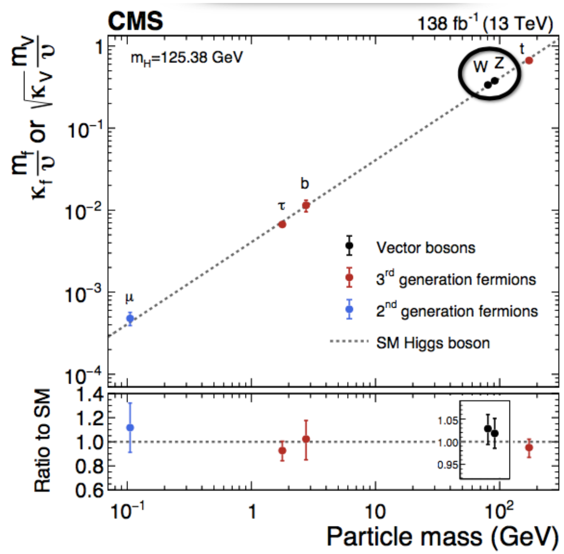
Couplings to fermion flavor proportional to particle masses.

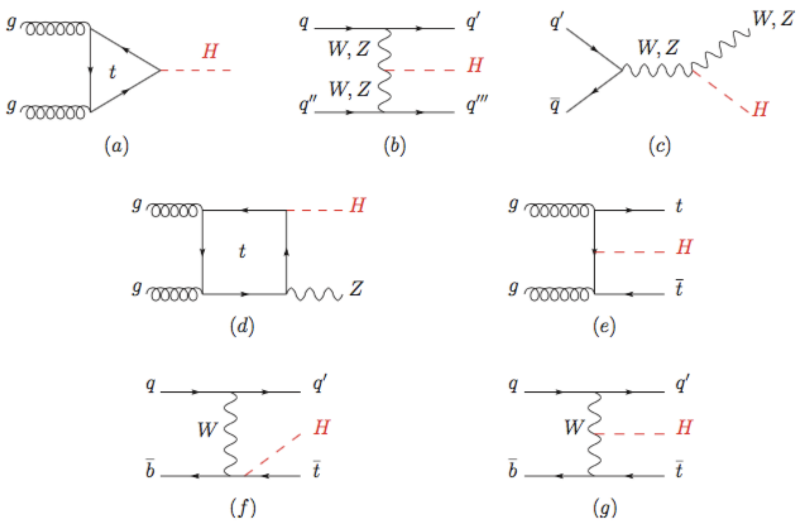
Production   Decay branching ratios   Compare data with SM prediction



Decay channel	Branching ratio	Rel. uncertainty
$H \to \gamma\gamma$	2.27×10^{-3}	2.1%
$H \to ZZ$	2.62×10^{-2}	±1.5%
$H \to W^+W^-$	2.14×10^{-1}	±1.5%
$H \to \tau^+\tau^-$	6.27×10^{-2}	±1.6%
$H \to b\bar{b}$	5.82×10^{-1}	$^{+1.2\%}_{-1.3\%}$
$H \to c\bar{c}$	2.89×10^{-2}	$^{+5.5\%}_{-2.0\%}$
$H \to Z\gamma$	1.53×10^{-3}	±5.8%
$H \to \mu^+\mu^-$	2.18×10^{-4}	±1.7%







Main production channel, pp -> gg -> H

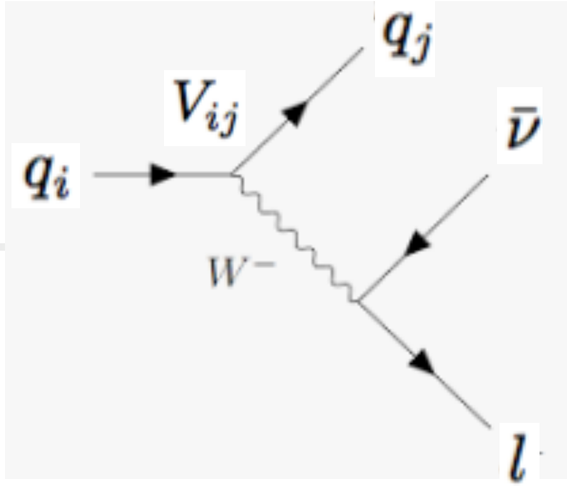
Cross section proportional to heavy quark number N in Figure (a). Cross section ~ N^2. If there are 4th generation quarks, the must be heavy, N=3 compared with 3 generation model, there is a factor of N^2 = 9 enhancement! Not in agreement with data.

No more than 3 generations!

Couplings consistent with standard model

Determination of KM matrix elements

Tree level determination of KM matrix



V_{ud}: n -> p l ν (supperallowed $0^+ \to 0^+$ nuclear β decays

π^+-> $\mu^+ \nu(\gamma)$... $|V_{ud}|$= 0.97435(0.00016)

V_{us}: K^+-> $\mu^+ \nu(\gamma)$, $K_L \to \pi e(\mu) \nu$, $K^+ \to \pi^0 e^+(\mu^+)\nu$,

$K_S \to \pi e \nu$, Hyperon β decays... $|V_{us}|$= 0.22500(0.00067)

V_{cd}: D^+ -> $\mu^+(\tau^+) \nu$, D_s^+ -> $\mu^+(\tau^+) \nu$,

$D(D_s)$->$\pi(K)$ l ν, ν d -> c X l... $|V_{us}|$ = 0.22486(0.00067)

V_{cs}: D_s^+ -> $\mu^+(\tau^+) \nu$, D_s^+ -> $\mu^+(\tau^+) \nu$,

$D_s(D)$->$\pi(K)$ l ν... $|V_{cs}|$ = 0.97349(0.00016)

V_{cb}: B -> D (D^*)l ν, B->D(D^*) X, B -> X_c l ν,

B_s ->$D^*\mu\nu$, Λ_b -> $\Lambda_c^+\mu^- \nu$... $|V_{cb}|$= 0.04182(+0.00085, – 0.00074)

V_{ub}: B -> X_u l ν, B -> π l ν, B $\to \tau \nu$,

B_s -> $K^- \mu^+ \nu$, Λ_b-> p $\mu^- \nu$... $|V_{ub}|$= 0.00369(0.00011)

V_{tb}: B(t $\to$ W b)/B(t->W q)... $|V_{tb}|$= 0.999118(+0.000031,-0.000036)

V_{td} and V_{ts} need to have loop effect to determine. More later

FCNC at one loop level in SM
Phenomenology of neutral Kaon mixing

The weak interaction mixes K^0 and $\bar{K}^0$

Work in the basis $\Phi(t) = (K^0(t), \bar{K}^0(t))^T$, $\; i\frac{d}{dt}\Phi = H\Phi(t)$,

H can written as the sum of two Hermitian 2×2 matrices M and Γ,

$$H = M - i\frac{\Gamma}{2} = \begin{pmatrix} M_{11} - i\Gamma_{11}/2 & M_{12} - i\Gamma_{12}/2 \\ M_{12}^* - i\Gamma_{12}^*/2 & M_{22} - i\Gamma_{22}/2 \end{pmatrix},$$

M is related to the masses of the particles, Γ is related to the life-times
Both separately must be Hermitian

The appearance i in front Γ, naive T violation because particles decay.

CPT symmetry is exact, $M_{11} = M_{22}$ and $\Gamma_{11} = \Gamma_{22}$.
The off diagonal ones M_{12} and Γ_{12} mix K^0 and $\bar{K}^0$.

CP violation requires M_{12} and/or Γ_{12} be complex!

$\frac{d}{dt}(\Phi^\dagger\Phi) = \frac{d\Phi^\dagger}{dt}\Phi + \Phi^\dagger\frac{d\Phi}{dt}\Phi = \Phi^\dagger(iM^\dagger - \Gamma^\dagger/2)\Phi - \Phi^\dagger(iM - \Gamma/2)\Phi = -\Phi^\dagger\Gamma\Phi$

Γ must be positively defined! $\Gamma_{11} = \Gamma_{22} > 0$ and $Det(\Gamma) > 0$.

Diagonalize the mixing Hamiltonian H
One obtains the mass and life-time eigenvalues for K_S and K_L

$$(m - i\frac{\Gamma}{2})_S = M_{11} - i\frac{\Gamma_{11}}{2} - E\,, \quad (m - i\frac{\Gamma}{2})_L = M_{11} - i\frac{\Gamma_{11}}{2} + E\,,$$
$$E = \sqrt{(M_{12} - i\Gamma_{12}/2)(M_{12}^* - i\Gamma_{12}^*/2)}\,.$$

One also finds, $\epsilon_1 = \epsilon_2$ which will be denoted by ϵ. One obtains:
$\Delta m_{L-S} = m_L - m_S$ and $\Delta\Gamma_{S-L} = \Gamma_S - \Gamma_L$.

$$K_L = \frac{K_2^0 + \epsilon K_1^0}{\sqrt{1+|\epsilon|^2}} = \frac{(1+\epsilon)K^0 + (1-\epsilon)\bar{K}^0}{\sqrt{1+|\epsilon|^2}}\,, \quad \left(\frac{1+\epsilon}{1-\epsilon}\right)^2 = \frac{M_{12}^* - i\Gamma_{12}^*/2}{M_{12} - i\Gamma_{12}/2}$$
$$K_S = \frac{K_1^0 + \epsilon K_2^0}{\sqrt{1+|\epsilon|^2}} = \frac{(1+\epsilon)K^0 - (1-\epsilon)\bar{K}^0}{\sqrt{1+|\epsilon|^2}}\,, \quad \epsilon \approx \frac{iIm(M_{12}) + Im(\Gamma_{12}/2)}{\Delta m_{L-S} + i\Delta\Gamma_{S-L}/2}$$

Data: $\Delta m_{L-S} \approx \Delta\Gamma_{S-L}/2 = (3.484 \pm 0.006) \times 10^{-12}$ MeV,

$\epsilon = (2.228 \pm 0.011) \times 10^{-3} exp(i\phi_\epsilon)$ with $(\phi_\epsilon = 43.52 \pm 0.05)^o$.

Assuming $Im(\Gamma_{12})$ is much smaller than $Im(M_{12})$
Theoretical estimate OK

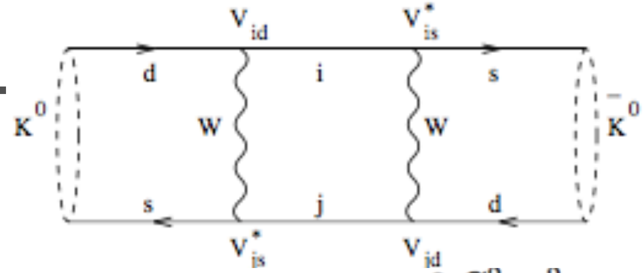
One finally obtains

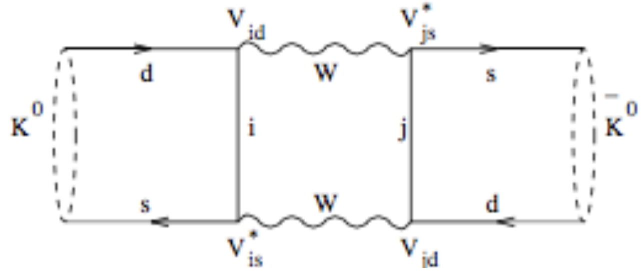
$$\epsilon \approx \frac{Im(M_{12})}{\sqrt{2}\Delta m_{L-S}} e^{i\phi_\epsilon} \ ,$$

To understand CP violation, one must understand

How $Im(M_{12})$ is generated and what is the origin of it.

Calculation of Im(M_{12})





$$H_{eff} = -\frac{2}{3}\frac{G_F^2 m_W^2}{\pi^2}\sum_{i,j}(V_{id}V_{is}^*)(V_{jd}V_{js}^*)B(\alpha_i,\alpha_j)\bar{s}\gamma_\mu L d\bar{s}\gamma^\mu L d\ ,$$

$$B(x,y) = (1+\frac{xy}{4})(\frac{1}{(1-x)(1-y)} + \frac{1}{x-y}[\frac{x^2 lnx}{(1-x)^2} - \frac{y^2 lny}{(1-y)^2}])$$
$$-2xy(\frac{1}{(1-x)(1-y)} + \frac{1}{x-y}[\frac{xlnx}{(1-x)^2} - \frac{ylny}{(1-y)^2}])\ ,$$

$$M_{12} \quad = \quad <\bar{K}^0|H_{eff}|K^0> = -\frac{1}{8}\frac{G_F^2 m_W^2}{\pi^2}\sum_{i,j}(V_{id}V_{is}^*)(V_{jd}V_{js}^*)B(\alpha_i,\alpha_j)C\ ,$$

$$C = <\bar{K}^0|\bar{s}\gamma_\mu L d\bar{s}\gamma^\mu L d|K^0>$$

Replacing (d, s) to (d,b) and (s,b), obtain $B_{d(s)}$-$B_{d(s)}$ mixing

Dominated by heavy top quark in the loop

For $B_{d(s)}$-$B_{d(s)}$ mixing, $V_{td}V_{tb}^*$($V_{td}V_{tb}^*$) term dominate! Determination of V_{td} and V_{ts} !

$2ReM_{12} = \Delta m_{b;}$ $\Delta m_B = 3.334(0.013)\mathrm{x}10^{-10}$ MeV; Δm_{Bs}=1.1693(0.0004)x10^{-8} MeV.

$\Delta m_D = 6.56(0.010)10^{-12}$ MeV. Need long distance contributions in SM.

Vacuum saturation approximation $C =< \bar{K}^0|\bar{s}\gamma_\mu L d \bar{s}\gamma^\mu L d|K^0 >$

$=< \bar{K}^0|(\bar{s}^\alpha\gamma_\mu L d_\alpha \bar{s}^\beta\gamma^\mu L d_\beta) + (\bar{s}^\alpha\gamma_\mu L d_\beta \bar{s}^\beta\gamma^\mu L d_\alpha)K^0 >$

$= 2 < \bar{K}^0|(\bar{s}^\alpha\gamma_\mu L d_\alpha|0 >< 0|\bar{s}^\beta\gamma^\mu L d_\beta) + (\bar{s}^\alpha\gamma_\mu L d_\beta|0 >< 0|\bar{s}^\beta\gamma^\mu L d_\alpha)K^0 >$

$= 2(1 + 1/3)(1/4) f_K^2 m_K^2/(2m_k) = -2/3 f_K^2 m_K.$

$\alpha_i = m_i^2/m_W^2$, $f_K = 160$ MeV is the kaon decay constant

$< 0|\bar{s}\gamma^\mu\gamma_5 d|K^0 >=< \bar{K}^0|\bar{s}\gamma^\mu\gamma_5 d|0 >= i f_K p_k^\mu$

None factorizeble effects introduce bag factor B_K, $C = -2/3 f_K^2 m_K B_K$

Vacuum saturation, $B_K = 1$. Lattice calculation gives $B_K = 0.766 \pm 0.010$

With QCD corrections, the matrix element M_{12} is given by

$$
\begin{aligned}
M_{12} &= \frac{f_K^2 m_K G_F^2 m_W^2}{12\pi^2} B_K[\eta_1 \tilde{B}_1 (V_{cd}V_{cs}^*)^2 + \eta_2 \tilde{B}_2 (V_{td}V_{ts}^*)^2 \\
&+ 2\eta_3 \tilde{B}_3 (V_{cd}V_{cs}^* V_{td}V_{ts}^*)] , \\
\tilde{B}_1 &= B(\alpha_c, \alpha_c) - B(\alpha_u, \alpha_c) - B(\alpha_c, \alpha_u) + B(\alpha_u, \alpha_u) , \\
\tilde{B}_2 &= B(\alpha_t, \alpha_t) - B(\alpha_u, \alpha_t) - B(\alpha_t, \alpha_u) + B(\alpha_u, \alpha_u) , \\
\tilde{B}_3 &= B(\alpha_u, \alpha_u) - B(\alpha_c, \alpha_u) - B(\alpha_t, \alpha_u) + B(\alpha_t, \alpha_c) ,
\end{aligned}
$$

η_i QCD correction factors η_i,next-to-leading order and are given by:

$\eta_1 = 1.38$, $\eta_2 = 0.574$, and $\eta_3 = 0.47$

The parameter ϵ is given by

$$
|\epsilon| = 4.39 A^2 B_K \eta [\eta_3 \tilde{B}_3 - \eta_1 \tilde{B}_1 + \eta_2 A^2 \lambda^4 (1-\rho) \tilde{B}_2] .
$$

Successful explain CP violation in Neutral Kaon Mixing!

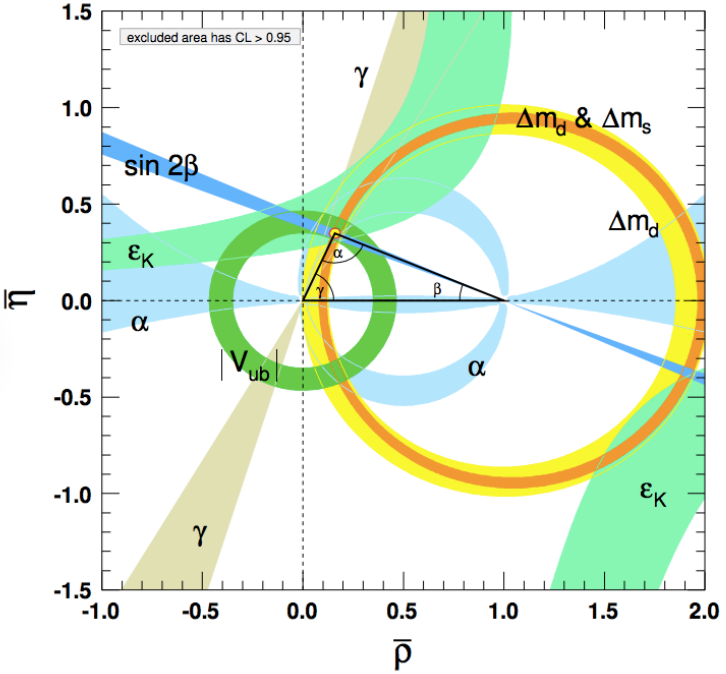
A consistent flavor physics picture in SM

For $B_{d(s)}$-$B_{d(s)}$ mixing, $V_{td}V_{tb}^*$($V_{td}V_{tb}^*$) term determination of V_{td} and V_{ts} !

M_{12}^K, M_{12}^{Bd}, M_{12}^{Bs} (include higher order corrections) explain data:

$\Delta m_K = 3.484(0.006)\times10^{-12}$ MeV, $|\varepsilon_K| = 2.228(0.011)\times10^{-3}$,
$\Delta m_{Bd} = 3.334(0.013)\times10^{-10}$ MeV, $\Delta m_{Bs} = 1.1693(0.0004)\times10^{-8}$ MeV

Combining tree level constraints, KM matrix elements are dertmined!



$$|V_{CKM}| = \begin{pmatrix} 0.97435 \pm 0.00016 & 0.22500 \pm 0.00067 & 0.00369 \pm 0.00011 \\ 0.22486 \pm 0.00067 & 0.97349 \pm 0.00016 & 0.04182^{+0.00085}_{-0.00074} \\ 0.00857^{+0.00020}_{-0.00018} & 0.04110^{+0.00083}_{-0.00072} & 0.999118^{+0.000031}_{-0.000036} \end{pmatrix}$$

$$\lambda = 0.22500 \pm 0.00067\,, \qquad A = 0.826^{+0.018}_{-0.015}\,,$$
$$\bar{\rho} = 0.159 \pm 0.010\,, \qquad \bar{\eta} = 0.348 \pm 0.010\,.$$

$$\sin\theta_{12} = 0.22500 \pm 0.00067\,, \qquad \sin\theta_{13} = 0.00369 \pm 0.00011\,,$$
$$\sin\theta_{23} = 0.04182^{+0.00085}_{-0.00074}\,, \qquad \delta = 1.144 \pm 0.027\,.$$

2.2 Tests for Standard Model of CV Violation

SM can explain CPV in neutral Kaon mixing. Only doing that job is not enough to become part of a SM and being awarded Nobel prize.

Predictions made and confirmed.

Many predictions been confirmed!

Observables: ε', time dependent A_{CP} and independent rate asymmetry S_f and C_f in K, D and B decays, and also to test unitarity triangle predicted by SM

ε'/ε: CP violation in K -> π π decay

The ϵ' in $K_{L,S} \to \pi\pi$, a measurement of direct CPV

$$\epsilon' = \frac{\eta_{+-} - \eta_{00}}{3}\ , \quad \eta_{+-} = \frac{A(K_L \to \pi^+\pi^-)}{A(K_S \to \pi^+\pi^-)}\ , \quad \eta_{00} = \frac{A(K_L \to \pi^0\pi^0)}{A(K_S \to \pi^0\pi^0)}\ .$$

What ϵ' is measuring?

$$A(K_L \to \pi\pi) = \frac{1+\epsilon}{\sqrt{1+|\epsilon|^2}} A(K^0 \to \pi\pi) + \frac{1-\epsilon}{\sqrt{1+|\epsilon|^2}} A(\bar{K}^0 \to \pi\pi)$$
$$A(K_S \to \pi\pi) = \frac{1+\epsilon}{\sqrt{1+|\epsilon|^2}} A(K^0 \to \pi\pi) - \frac{1-\epsilon}{\sqrt{1+|\epsilon|^2}} A(\bar{K}^0 \to \pi\pi)$$

Isospin decay decomposition for $K^0(\bar{K}^0) \to \pi\pi$ decay amplitudes

Isospins I of π is 1, isospin components $(\pi^+, \pi^0, \pi^-) \to (1, 0, -1)$,

Isospin I of K is 1/2, isospin components $(K^0, \bar{K}^0) \to (-1/2, 1/2)$

$$|\pi^+\pi^- >= \sqrt{1/3}|2,0> + \sqrt{2/3}|0,0>, \ |\pi^0\pi^0 >= \sqrt{2/3}|2,0> - \sqrt{1/3}|0,0>$$

$$< K^0| \ |\pi^+\pi^- >= |\frac{1}{2},\frac{1}{2}> |\pi^+\pi^- >= \sqrt{\frac{1}{5}}|\frac{5}{2},\frac{1}{2}> + \sqrt{\frac{2}{15}}|\frac{3}{2},\frac{1}{2}> + \sqrt{\frac{2}{9}}|\frac{1}{2},\frac{1}{2}>$$

$$< K^0| \ |\pi^0\pi^0 >= |\frac{1}{2},\frac{1}{2}> |\pi^0\pi^0 >= -\sqrt{\frac{2}{5}}|\frac{5}{2},\frac{1}{2}> + \sqrt{\frac{4}{15}}|\frac{3}{2},\frac{1}{2}> + \sqrt{\frac{2}{9}}|\frac{1}{2},\frac{1}{2}>$$

To induce the decay to happen, the Hamiltonian needs carry isospin I=5/2, I=3/2 and I=1/2 inducing $A_{5/2}$, $A_{3/2}$ and $A_{1/2}$ amplitudes

In the SM, 5/2 isospin amplitude is very small
(more than four quark operators to generate). Neglect them!

Renaming $-(2/\sqrt{5})A_{3/2} = A_2 e^{i\delta_2}$ and $A_{1/2} = -\sqrt{2}A_0 e^{i\delta_0}$

$$
\begin{aligned}
A(K^0 \to \pi^+\pi^-) &= \sqrt{\frac{2}{3}}A_0 e^{i\delta_0} + \sqrt{\frac{1}{3}}A_2 e^{i\delta_2}\ , \\
A(K^0 \to \pi^0\pi^0) &= \sqrt{\frac{1}{3}}A_0 e^{i\delta_0} - \sqrt{\frac{2}{3}}A_2 e^{i\delta_2}\ ,
\end{aligned}
$$

δ_i are the strong final state rescattering phases (strong phase)
A_0 and A_2 are complex in general due to weak CP violating phases.
The corresponding anti-particle decay amplitudes are

$$
\begin{aligned}
A(\bar{K}^0 \to \pi^+\pi^-) &= -\sqrt{\frac{2}{3}}A_0^* e^{i\delta_0} - \sqrt{\frac{1}{3}}A_2^* e^{i\delta_2}\ , \\
A(\bar{K}^0 \to \pi^0\pi^0) &= -\sqrt{\frac{1}{3}}A_0^* e^{i\delta_0} + \sqrt{\frac{2}{3}}A_2^* e^{i\delta_2}\ .
\end{aligned}
$$

$$\eta_{+-} = \epsilon + i\frac{ImA_0}{ReA_0} + e^{i(\pi/2+\delta_2-\delta_0)}\frac{ReA_2}{\sqrt{2}ReA_2}\left(\frac{ImA_2}{ReA_2} - \frac{ImA_0}{ReA_0}\right),$$

$$\eta_{00} = \epsilon + i\frac{ImA_0}{ReA_0} - 2e^{i(\pi/2+\delta_2-\delta_0)}\frac{ReA_2}{\sqrt{2}ReA_0}\left(\frac{ImA_2}{ReA_2} - \frac{ImA_0}{A_0}\right),$$

$$\epsilon' = \frac{\eta_{+-} - \eta_{00}}{3} = \frac{ReA_2}{\sqrt{2}ReA_0}\left(\frac{ImA_2}{ReA_2} - \frac{ImA_0}{ReA_0}\right)e^{i(\pi/2+\delta_2-\delta_0)}.$$

δ_i are determined from phase shift analyses in $\pi - \pi$ scattering, and $\pi/2 + \delta_2 - \delta_0$ is found to be close to $\pi/4$.

CPT symmetry implies that this phase is equal to the phase ϕ_ϵ for ϵ.

In the literature the quantity ϵ'/ϵ is usually used.

Experiment value from NA48 and KTeV: $\varepsilon'/\varepsilon = 16.6(2.3)\times 10^{-4}$

SM calculation for ε'/ε

Tree and penguin contributions

$$\mathcal{H}_{\rm eff}(\Delta S=1)=\frac{G_F}{\sqrt{2}}V_{us}^*V_{ud}\sum_{i=1}^{10}[z_i(\mu)+\tau\, y_i(\mu)]Q_i(\mu).$$

$$Q_1=(\bar{s}_i u_j)_{V-A}(\bar{u}_j d_i)_{V-A},$$

$$Q_2=(\bar{s}u)_{V-A}(\bar{u}d)_{V-A},$$

$$Q_3=(\bar{s}d)_{V-A}\sum_q(\bar{q}q)_{V-A},$$

$$Q_4=(\bar{s}_i d_j)_{V-A}\sum_q(\bar{q}_j q_i)_{V-A},$$

$$Q_5=(\bar{s}d)_{V-A}\sum_q(\bar{q}q)_{V+A},$$

$$Q_6=(\bar{s}_i d_j)_{V-A}\sum_q(\bar{q}_j q_i)_{V+A}.$$

$$Q_7=\frac{3}{2}(\bar{s}d)_{V-A}\sum_q e_q(\bar{q}q)_{V+A},$$

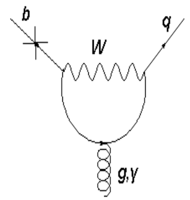
$$Q_8=\frac{3}{2}(\bar{s}_i d_j)_{V-A}\sum_q e_q(\bar{q}_j q_i)_{V+A},$$

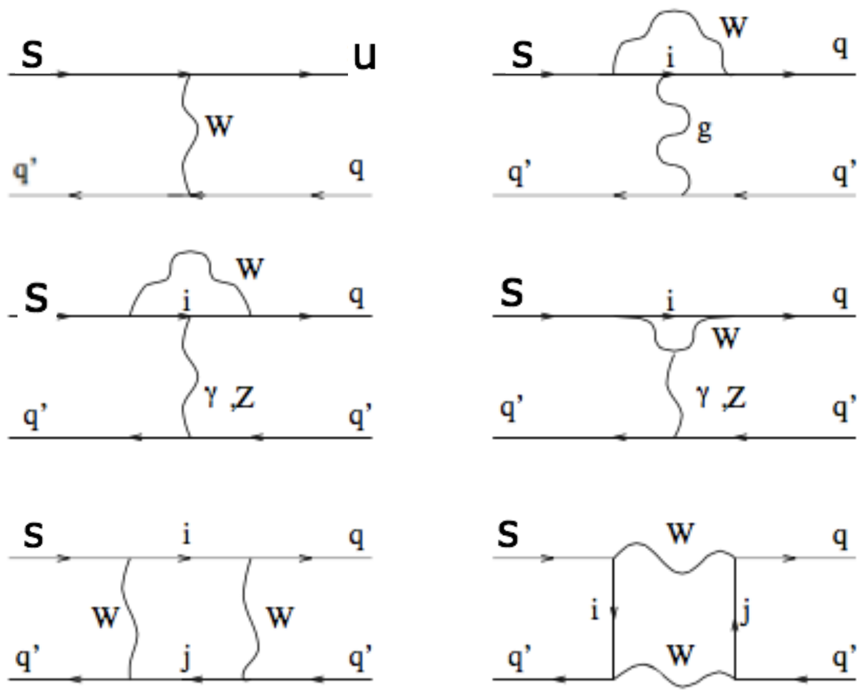
$$Q_9=\frac{3}{2}(\bar{s}d)_{V-A}\sum_q e_q(\bar{q}q)_{V-A},$$

$$Q_{10}=\frac{3}{2}(\bar{s}_i d_j)_{V-A}\sum_q e_q(\bar{q}_j q_i)_{V-A}.$$

$$\tau=-\frac{V_{ts}^*V_{td}}{V_{us}^*V_{ud}}.$$

s->u $\bar{q}'$ q, s -> d $\bar{q}'$q'





Replacing s to b, q to d, or s, apply to b -> u $\bar{q}''$ q, b -> q $\bar{q}''$ q' decays.

ΔS=1 Wilson coefficients at μ=1 GeV for m_t=170 GeV. $y_1 = y_2 \equiv 0$.

	$\Lambda_{\overline{MS}}^{(4)}$=215 MeV			$\Lambda_{\overline{MS}}^{(4)}$=325 MeV			$\Lambda_{\overline{MS}}^{(4)}$=435 MeV		
Scheme	LO	NDR	HV	LO	NDR	HV	LO	NDR	HV
z_1	−0.607	−0.409	−0.494	−0.748	−0.509	−0.640	−0.907	−0.625	−0.841
z_2	1.333	1.212	1.267	1.433	1.278	1.371	1.552	1.361	1.525
z_3	0.003	0.008	0.004	0.004	0.013	0.007	0.006	0.023	0.015
z_4	−0.008	−0.022	−0.010	−0.012	−0.035	−0.017	−0.017	−0.058	−0.029
z_5	0.003	0.006	0.003	0.004	0.008	0.004	0.005	0.009	0.005
z_6	−0.009	−0.022	−0.009	−0.013	−0.035	−0.014	−0.018	−0.059	−0.025
z_7/α	0.004	0.003	−0.003	0.008	0.011	−0.002	0.011	0.021	−0.001
z_8/α	0	0.008	0.006	0.001	0.014	0.010	0.001	0.027	0.017
z_9/α	0.005	0.007	0	0.008	0.018	0.005	0.012	0.034	0.011
z_{10}/α	0	−0.005	−0.006	−0.001	−0.008	−0.010	−0.001	−0.014	−0.017
y_3	0.030	0.025	0.028	0.038	0.032	0.037	0.047	0.042	0.050
y_4	−0.052	−0.048	−0.050	−0.061	−0.058	−0.061	−0.071	−0.068	−0.074
y_5	0.012	0.005	0.013	0.013	−0.001	0.016	0.014	−0.013	0.021
y_6	−0.085	−0.078	−0.071	−0.113	−0.111	−0.097	−0.148	−0.169	−0.139
y_7/α	0.027	−0.033	−0.032	0.036	−0.032	−0.030	0.043	−0.031	−0.027
y_8/α	0.114	0.121	0.133	0.158	0.173	0.188	0.216	0.254	0.275
y_9/α	−1.491	−1.479	−1.480	−1.585	−1.576	−1.577	−1.700	−1.718	−1.722
y_{10}/α	0.650	0.540	0.547	0.800	0.690	0.699	0.968	0.892	0.906

Buchalla et al., Rev. Mod. Phys. 68, 1125(1996)

Experimental measurement of ϵ'/ϵ

1993 NA31 at CERN, $\epsilon'/\epsilon = (2.3 \pm 0.7) \times 10^{-3}$

1993 E731 at Fermilab, $\epsilon'/\epsilon = (0.74 \pm 0.59) \times 10^{-3}$.

1999 KTeV at Fermlab, $\epsilon'/\epsilon = (2.8 \pm 0.41) \times 10^{-3}$

1999 NA48 at CERN, $Re(\epsilon'/\epsilon) = (1.85 \pm 0.45 \pm 0.58) \times 10^{-3}$

Experiment value from NA48 and KTeV: $\varepsilon'/\varepsilon = 16.6(2.3) \times 10^{-4}$

Lattice calculation: $21.7(8.4) \times 10^{-4}$ (PRD 102 (2020) 505459)

Chiral perturbation calculation: $14(5) \times 10^{-4}$ (Conf. Ser. 1562(2020) 012011)

SM is consistent with data

There are rooms for new physics beyond SM…

Time dependent and independent rate asymmetry

The mass eigenstates are

$$|M_L>= p|M> +q|\bar{M}> , \quad M_H>= p|M> -q|\bar{M}> ,$$
$$|M>= \frac{1}{2p}(|M_L> +|M_H>) , \; |\bar{M}>= \frac{1}{2q}(|M_L> -|M_H>) .$$

$$|M(t)>= \frac{1}{2p}\left(e^{-im_Lt-\Gamma_Lt/2}|M_L> +e^{-im_Ht-\Gamma_Ht/2}|M_H>\right) ,$$
$$= \frac{1}{2}e^{-im_Ht-\Gamma_Ht/2}\left((1+e^{i\Delta m+\Delta\Gamma t/2})|M> -\frac{q}{p}(1-e^{i\Delta mt+\Delta\Gamma t/2})|\bar{M}>\right) ,$$
$$|\bar{M}(t)>= \frac{1}{2q}\left(e^{-im_Lt-\Gamma_Lt/2}|M_L> -e^{-im_Ht-\Gamma_Ht/2}|M_H>\right) ,$$
$$= \frac{1}{2}e^{-im_Ht-\Gamma_Ht/2}\left(-\frac{p}{q}(1-e^{i\Delta m+\Delta\Gamma t/2})|M> +(1+e^{i\Delta mt+\Delta\Gamma t/2})|\bar{M}>\right) ,$$

$\Delta m = m_H - m_L$, $\Delta\Gamma = \Gamma_H - \Gamma_L$, $\Gamma = (\Gamma_H + \Gamma_L)/2$, $(\frac{p}{q})^2 = \frac{M^*_{12}-i\Gamma^*_{12}/2}{M_{12}-i\Gamma_{12}/2}$

Define decay amplitudes

$M \rightarrow f$: $A_f =< f|H|M >$, $M_{\rightarrow}\bar{f}$: $A_{\bar{f}} =< \bar{f}|H|M >$,

$\bar{M} \rightarrow \bar{f}$: $\bar{A}_{\bar{f}} =< \bar{f}|H|\bar{M} >$, $\bar{M} \rightarrow f$: $\bar{A}_f =< f|H\bar{M} >$

$$< f|H|M(t) >= \frac{1}{2}e^{-im_H t - \Gamma_H/2}(1 + e^{i\Delta m t + \Delta\Gamma t/2})A_f \ ,$$

$$< \bar{f}|H|M(t) >= -\frac{1}{2}e^{-im_H t - \Gamma_H/2}\frac{q}{p}(1 - e^{i\Delta m t + \Delta\Gamma t/2})A_{\bar{f}} \ ,$$

$$< \bar{f}|H|\bar{M}(t) >= \frac{1}{2}e^{-im_H t - \Gamma_H/2}(1 + e^{i\Delta m t + \Delta\Gamma t/2})\bar{A}_{\bar{f}} \ ,$$

$$< f|H|\bar{M}(t) >= -\frac{1}{2}e^{-im_H t - \Gamma_H/2}\frac{p}{q}(1 - e^{i\Delta m t + \Delta\Gamma t/2})\bar{A}_f \ ,$$

Uncorrelated M and $\bar{M}$ production

Flavor specific case, $f \neq \bar{f}$, $A_{\bar{f}} = \bar{A}_f = 0$

Time dependent CP asymmetry,

Such as $B_d^0 \to \pi^+ K^-$ and $B_s^0 \to \pi^- K^+$

$$A(t)_{CP} = \frac{\Gamma(\bar{M}(t) \to \bar{f}) - \Gamma(M(t) \to f)}{\Gamma(M(t) \to f) + \Gamma(\bar{M}(t) \to \bar{f})} = \frac{|\bar{A}_{\bar{f}}|^2 - |A_f|^2}{|A_f|^2 + |A_{\bar{f}}|^2} .$$

Actually no time dependence!

How to measure this experimentally?

Usually M and $\bar{M}$ are produced in pairs.

If produced uncorrelated, like production at hadron colliders

Make sure each decay is originated from M for $M(t) \to f$

by having good tracking measurement

One trace at origin, whether the particle is M or $\bar{M}$.

Time integrated CP asymmetry,

$$A_{CP} = \frac{\int_0^\infty \Gamma(\bar{M}(t) \to \bar{f}) - \int_0^\infty \Gamma(M(t) \to f)}{\int_0^\infty \Gamma(M(t) \to f) + \int_0^\infty \Gamma(\bar{M}(t) \to \bar{f})} = \frac{|\bar{A}_{\bar{f}}|^2 - |A_f|^2}{|A_f|^2 + |A_{\bar{f}}|^2} .$$

$A(t)_{CP} = A_{CP}$. Direct CP violation.

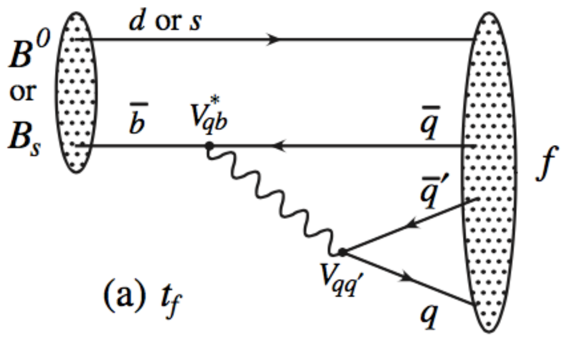
For charged M no need of tagging because there is no M and $\bar{M}$ oscillation.

There is no mixing between M and $\bar{M}$, such as $B^+ \to K^+\pi^0$...

Conditions for CP asymmetry: $|A_f| \neq |\bar{A}_{\bar{f}}|$

Parametrized

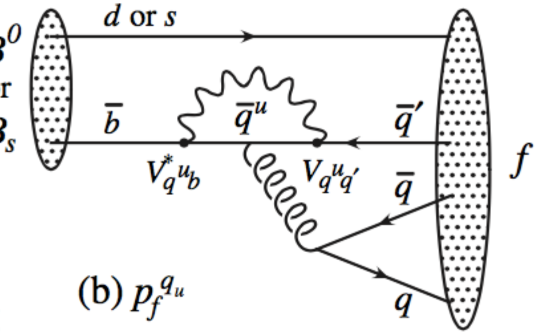
$$\begin{aligned} A_f &= A_1 e^{i(\delta_1^s+\delta_1^w)} + A_2 e^{i(\delta_2^s+\delta_2^w)} , \\ \bar{A}_{\bar{f}} &= \eta^{CP}(A_1 e^{i(\delta_1^s-\delta_1^w)} + A_2 e^{i(\delta_2^s-\delta_2^w)}) , \end{aligned}$$



δ_i^s are the strong phases and δ_i^w are the CP violating weak phases. $|\eta^{CP}| = 1$

$$A_{CP} = \frac{-2A_1A_2 sin(\delta_1^w - \delta_2^w) sin(\delta_1^s - \delta_2^s)}{A_1^2 + A_2^2 + 2A_1A_2 cos(\delta_1^w - \delta_2^w) cos(\delta_1^w - \delta_w^2)} .$$

There must more than one amplitudes with different strong and weak phases!



$A_{CP}(B_s^0 \to \pi^+ K$ 0.224 ± 0.012 4 and $A_{CP}(B^0 \to \pi^- K^+) =$ − 0.0834 ± 0.0032

These measurements are in consistent
With SM predictions!

$\bar{b} \to \bar{q}q\bar{q}'$	$B^0 \to f$	$B_s^0 \to f$	CKM dependence of A_f	Suppression
$\bar{b} \to \bar{c}c\bar{s}$	ψK_S	$\psi\phi$	$(V_{cb}^*V_{cs})T + (V_{ub}^*V_{us})P^u$	loop × λ^2
$\bar{b} \to \bar{s}s\bar{s}$	ϕK_S	$\phi\phi$	$(V_{cb}^*V_{cs})P^c + (V_{ub}^*V_{us})P^u$	λ^2
$\bar{b} \to \bar{u}u\bar{s}$	$\pi^0 K_S$	K^+K^-	$(V_{cb}^*V_{cs})P^c + (V_{ub}^*V_{us})T$	λ^2/loop
$\bar{b} \to \bar{c}c\bar{d}$	D^+D^-	ψK_S	$(V_{cb}^*V_{cd})T + (V_{tb}^*V_{td})P^t$	loop
$\bar{b} \to \bar{s}s\bar{d}$	K_SK_S	ϕK_S	$(V_{tb}^*V_{td})P^t + (V_{cb}^*V_{cd})P^c$	$\lesssim 1$
$\bar{b} \to \bar{u}u\bar{d}$	$\pi^+\pi^-$	$\rho^0 K_S$	$(V_{ub}^*V_{ud})T + (V_{tb}^*V_{td})P^t$	loop
$\bar{b} \to \bar{c}u\bar{d}$	$D_{CP}\pi^0$	$D_{CP}K_S$	$(V_{cb}^*V_{ud})T + (V_{ub}^*V_{cd})T'$	λ^2
$\bar{b} \to \bar{c}u\bar{s}$	$D_{CP}K_S$	$D_{CP}\phi$	$(V_{cb}^*V_{us})T + (V_{ub}^*V_{cs})T'$	$\lesssim 1$

Purely mixing induced CP

If measuring $M(t) \to \bar{f}$ and $\bar{M}(t) \to f$

$$A(t)_{CP} = \frac{\Gamma(M(t) \to \bar{f}) - \Gamma(\bar{M}(t) \to f)}{\Gamma(M(t) \to \bar{f}) + \Gamma(\bar{M}(t) \to f)} = \frac{|\frac{q}{p}|^2 |A_{\bar{f}}|^2 - |\frac{p}{q}|^2 |\bar{A}_f|^2}{|\frac{q}{p}|^2 |A_{\bar{f}}|^2 + |\frac{p}{q}|^2 |\bar{A}_f|^2}$$

Information about mixing can be extracted!

In the case $|A_f| = |\bar{A}_{\bar{f}}|$,

$$A^{mix}(t)_{CP} = A^{mix}_{CP} = \frac{\left|\frac{q}{p}\right|^2 - \left|\frac{p}{q}\right|^2}{\left|\frac{q}{p}\right|^2 + \left|\frac{p}{q}\right|^2} .$$

Example: $K^0 \to \mu^+\nu$ and $\bar{K}^0 \to \mu^-\bar{\nu}$. $A_f = \bar{A}_{\bar{f}}$

$p = (1+\epsilon)/\sqrt{1+|\epsilon|^2}$ and $q = (1-\epsilon)/\sqrt{1+|\epsilon|^2}$

$p/q = \sqrt{\frac{M_{12}^* - i\Gamma^*/2}{M_{12} - i\Gamma_{12}/1}}$

$A_{CP}^{mix} = (|1-\epsilon|^4 - |1+\epsilon|^4)/(|1+\epsilon|^4 + |1-\epsilon|^4) \approx -2Re(\epsilon) \approx 2\frac{Im(M_{12}\Gamma^*)}{|M_{12}|^2}$

If one can identify K_L first, then

$$\delta_L = \frac{\Gamma(K_L -> l^+\nu_l\pi^-) - \Gamma(K_L \to l^-\bar{\nu}_l\pi^+)}{\Gamma(K_L -> l^+\nu_l\pi^-) + \Gamma(K_L \to l^-\bar{\nu}_l\pi^+)}$$

$\delta_L \approx 2Re(\epsilon) = (3.32 \pm 0.06) \times 10^{-3}$. Agree with data!

For $B_s^0 \to l^- X$, $A_{SL}^s = A_{CP}^{mix} = (-7.5 \pm 4.1) \times 10^{-3}$.

Compared with SM $A_{SL}^s = (1.9 \pm 3.0) \times 10^{-5}$.

M **and** $\bar{M}$ **decay into CP eigenstate** $f = \bar{f} = f_{CP}$

Let $A_{CP} =< f_{CP}|H|M >$ and $\bar{A}_{CP} =< f_{CP}|H|\bar{M} >$

$$\Gamma(M(t) \to f_{CP}) \sim \frac{1}{2} e^{-\Gamma_H t} \Big(\frac{1+e^{\Delta\Gamma t}}{2} (|A_{CP}|^2 + |\frac{q}{p}\bar{A}_{CP}|^2) + e^{\Delta\Gamma t} cos(\Delta m t)(|A_{CP}|^2 - |\frac{q}{p}\bar{A}_{CP}|^2)$$

$$- \quad |A_{CP}|^2 (1-e^{\Delta\Gamma t}) Re(\frac{q}{p}\frac{\bar{A}_{CP}}{A_{CP}}) - |A_f|^2 sin(\Delta m t) e^{\Delta\Gamma t/2} Im(\frac{q}{p}\frac{\bar{A}_f}{A_f}) \Big) \ ,$$

$$\Gamma(\bar{M}(t) \to f_{CP}) \sim \frac{1}{2} e^{-\Gamma_H t} \Big(\frac{1+e^{\Delta\Gamma t}}{2} (|\bar{A}_{CP}|^2 + |\frac{p}{q}A_{CP}|^2) + e^{\Delta\Gamma t} cos(\Delta m t)(|\bar{A}_{CP}|^2 - |\frac{p}{q}A_{CP}|^2)$$

$$- \quad |\bar{A}_{CP}|^2 (1-e^{\Delta\Gamma t}) Re(\frac{p}{q}\frac{A_{CP}}{\bar{A}_{CP}}) - |\bar{A}_{CP}|^2 sin(\Delta m t) e^{\Delta\Gamma t} Im(\frac{p}{q}\frac{A_{CP}}{\bar{A}_{CP}}) \Big) \ ,$$

Time dependent asymmetry

$$A(t)_{CP} = \frac{\Gamma(\bar{M}(t) \to f_{CP}) - \Gamma(M(t) \to f_{CP})}{\Gamma(\bar{M}(t) \to f_{CP}) + \Gamma(M(t) \to f_{CP})} \ .$$

Time dependent asymmetry

$$A(t)_{CP} = \frac{\Gamma(\bar{M}(t) \to f_{CP}) - \Gamma(M(t) \to f_{CP})}{\Gamma(\bar{M}(t) \to f_{CP}) + \Gamma(M(t) \to f_{CP})} .$$

In the limit $|q/p| = 1$, one obtains

$$A(t)_{CP} = \frac{-C_f cos(\Delta m t) + S_f sin(\Delta m t)}{cosh(\Delta\Gamma t/2) + A_f^{\Delta\Gamma} sinh(\Delta\Gamma t/2)} ,$$

$$C_f = \frac{1-|\lambda_f|^2}{1+|\lambda_f|^2} , \quad S_f = \frac{2\Im(\lambda_f)}{1+|\lambda_f|^2} , \quad A_f^{\Delta\Gamma|} = \frac{2\Re(\lambda_f)}{1+|\lambda_f|^2} , \quad \lambda_f = \frac{q}{p}\frac{\bar{A}_{CP}}{A_{CP}} .$$

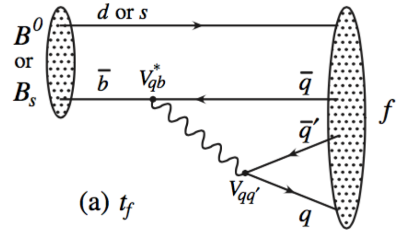
CPT sum rule: $|C_f|^2 + |S_f|^2 + |A_f^{\Delta\Gamma}|^2 = 1.$

In the SM, for $B_s^0 - \bar{B}_s^0$ system, good approximation $q/p = V_{tb}^* V_{ts} / V_{tb} V_{ts}^*$,
For $B_0 - \bar{B}^0$ system, $q/p = V_{tb}^* V_{td} / V_{tb} V_{td}^*$. $|q/p| = 1$.

Measurements of S_f and C_f in B decays played an important role in verifying the standard model for CP violation.

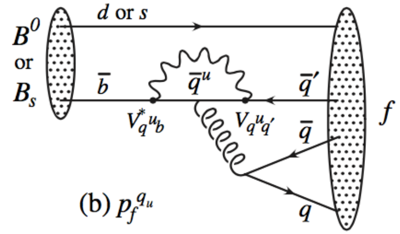
C_f type: D -> K^+K^-, $\pi^+\pi^-$; S_f type: B^0 -> $J/\psi\ K_S^0$, $\pi^+\pi^-$

S_f type: $B^0 \to J/\psi\, K_S^0$



$$Im\lambda_{\psi K_s} = Im\left(\left(\frac{q}{p}\right)_{B_d} \frac{V_{cb}V_{cs}^*}{V_{cb}^*V_{cs}}\left(\frac{q}{p}\right)_K\right) = -\sin(2\beta) = -0.699 \pm 0.017$$

Expected and consistent with SM



C_f type: $D \to K^+K^-,\ \pi^+\pi^-$

$$\Delta A_{CP} = A_{CP}(K^+K^-) - A_{CP}(\pi^+\pi^-) = (-0.154 \pm 0.029)\%$$

Unexpected! Short distance contributions are small

Long distance strong interaction effects important at Charm scale

Cannot be sure if SM is in conflict with data. Room for new physics.

See Appendices B and C for more time dependent Decay observables

Unitarity triangle test for SM

SM for CPV has many interesting predictions:

small EDM, Zero Aτ, CPV in Hyperon decay of order A ~ 10^{-4} ….

Anything bigger a sign of new physics…

It would nice to have some positive ones to veryfiy SM CPV!

One of the most prominent feature is that CP violation comes from the KM matrix. The unitary conditions: $\Sigma_i V_{ij} V_{ik}^* = \delta_{jk}$; $\Sigma_i V j_i V_{ki}^* = \delta_{jk}$.

can be represented by 6 unitarity triangles. The most experimentally accessible one is by the following

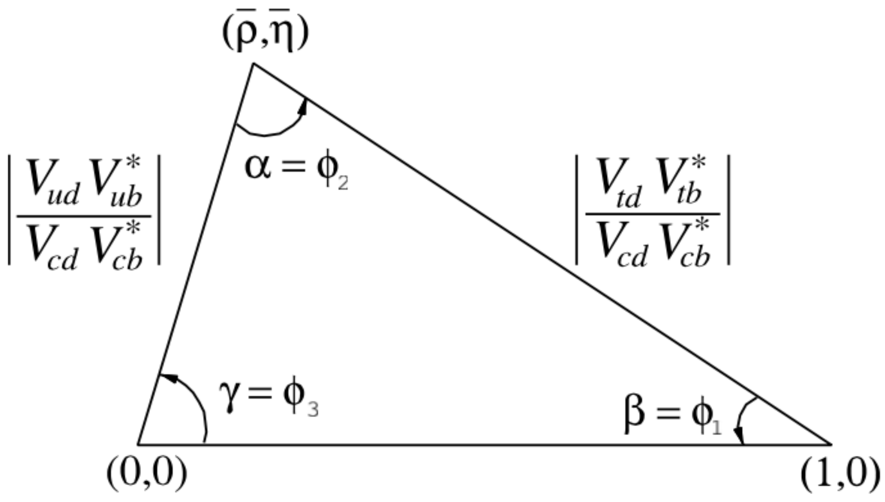
If the angles α, β and γ

can be independently measured,

whether $\alpha+\beta+\gamma$ =180^o can test the model.

This relation indeed holds!

Have been tested from B decays.



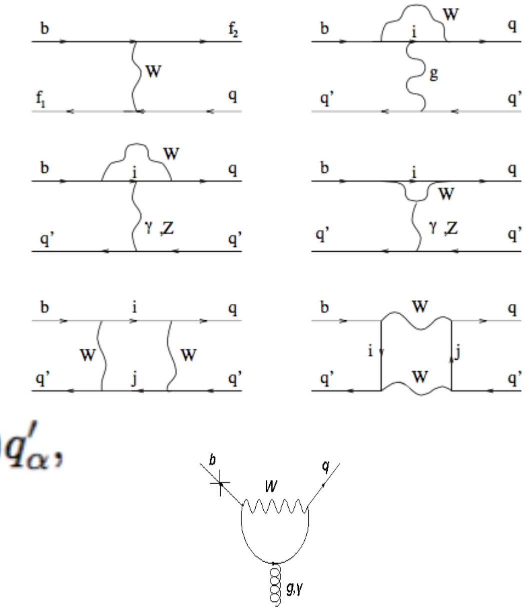
Hadronic B decays – The effective Hamiltonian

For hadronic B decays, the effective Hamiltonian is given by

$$
\begin{aligned}
H_{\Delta B=1}(q) &= \frac{4G_F}{\sqrt{2}}[V_{ub}V_{cq}^*(c_1O_1^{uc}(q)+c_2O_2^{uc}(q))+V_{cb}V_{uq}^*(c_1O_1^{cu}(q)+c_2O_2^{cu}(q))+V_{ub}V_{uq}^*(c_1O_1^{u}(q) \\
&+ c_2O_2^{u}(q))+V_{cb}V_{cq}^*(c_1O_1^{c}(q)+c_2O_2^{c}(q))-\sum_{j=u,c,t}V_{jb}V_{jq}^*\sum_{i=3}^{10}c_i^jO_i(q)]+H.C.,
\end{aligned}
$$

O_i's are defined as

$$
\begin{aligned}
&O_1^{f_1f_2}(q)=\bar{q}_\alpha\gamma_\mu Lf_{1\beta}\bar{f}_{2\beta}\gamma^\mu Lb_\alpha,\quad O_2^{f_1f_2}(q)=\bar{q}\gamma_\mu Lf_1\bar{f}_2\gamma^\mu Lb,\\
&O_1^{f}(q)=\bar{q}_\alpha\gamma_\mu Lf_\beta\bar{f}_\beta\gamma^\mu Lb_\alpha,\quad O_2^{f}(q)=\bar{q}\gamma_\mu Lf\bar{f}\gamma^\mu Lb,\\
&O_{3(5)}(q)=\bar{q}\gamma_\mu Lb\bar{q}'\gamma^\mu L(R)q',\quad O_{4(6)}(q)=\bar{q}_\alpha\gamma_\mu Lb_\beta\bar{q}'_\beta\gamma^\mu L(R)q'_\alpha,\\
&O_{7(9)}(q)=\frac{3}{2}\bar{q}\gamma_\mu Lbe_{q'}\bar{q}'\gamma^\mu R(L)q',\quad O_{8(10)}(q)=\frac{3}{2}\bar{q}_\alpha\gamma_\mu Lb_\beta e_{q'}\bar{q}'_\beta\gamma^\mu R(L)q'_\alpha,
\end{aligned}
$$



f can be u or c quark, q can be d or s quark,

q' is summed over u, d, s, and c quarks.

The leading QCD corrected Wilson Coefficients c_i at

$\alpha_s(m_Z) = 0.118$, $\alpha_{em}(m_Z) = 1/128$, $m_t = 176$ GeV and $\mu \approx m_b = 5$ GeV,

$$
\begin{aligned}
c_1 &= -0.3125, \quad c_2 = 1.1502, \quad c_3^t = 0.0174, \quad c_4^t = -0.0373, \\
c_5^t &= 0.0104, \quad c_6^t = -0.0459, \quad c_7^t = -1.050 \times 10^{-5}, \\
c_8^t &= 3.839 \times 10^{-4}, \quad c_9^t = -0.0101, \quad c_{10}^t = 1.959 \times 10^{-3}, \\
c_{3,5}^{u,c} &= -c_{4,6}^{u,c}/N_c = P_s^{u,c}/N_c \, , c_{7,9}^{u,c} = P_{em}^{u,c} \, , \quad c_{8,10}^{u,c} = 0 \, ,
\end{aligned}
$$

N_c is the number of color, $P_s^i = (\alpha_s/8\pi)c_2[10/9 + G(m_i, \mu, q^2)]$,

$P_{em}^i = \alpha_{em}/9\pi)(N_c c_1 + c_2)[10/9 + G(m_i, \mu, q^2)]$.

$G(m, \mu, q^2) = 4 \int_0^1 x(1-x) ln[(m^2 - x(1-x)q^2)/\mu^2] dx$.

Determination of α

The phase angle α can be determined from $B \to \pi\pi$ decays.

The decay amplitude can be parametrized as

$$\bar{A}_{\pi^+\pi^-} = V_{ub}V_{ud}^* T_{\pi^+\pi^-} + V_{tb}V_{td}^* P_{\pi^+\pi^-} \ ,$$

The decay $\bar{B}^0 \to \pi^+\pi^-$ is induced by $H_{\Delta B=1}(d)$, and can be written as

$$T_{\pi^+\pi^-} = \frac{4G_F}{\sqrt{2}} < \pi^+\pi^- | [c_1 O_1^u(d) + c_2 O_2^u(d)) + \sum_{i=3}^{10} (c_i^t - c_i^u) O_i(d)] | \bar{B}^0 >$$

$$P_{\pi^+\pi^-} = \frac{4G_F}{\sqrt{2}} \sum_{i=3}^{12} < \pi^+\pi^- | (c_i^t - c_i^c) O_i(d) | \bar{B}^0 > \ .$$

If the penguin amplitude $P_{\pi^+\pi^-}$ can be neglected,

$$Im\lambda_{\pi^+\pi^-} = Im\left(\frac{V_{tb}^* V_{td}}{V_{tb} V_{td}^*} \frac{V_{ub} V_{ud}^*}{V_{ub}^* V_{ud}}\right) = \sin(2\alpha) \ .$$

The angle α can therefore be determined.

However, if penguin effects are significant, the above method fails.

The KM factors for Tree $V_{ub}V_{ud}^*$ and penguin $V_{cb}V_{cd}^*$ is the same order

The error is of order $12°$.

It is necessary to find ways to isolate the penguin contributions.

When penguin effects are included,

$$Im\lambda_{\pi^+\pi^-} = \frac{|\bar{A}|}{|A|}\sin(2\alpha+\theta)\ .$$

To determine θ, Gronau and London[40] proposed to use isospin relation

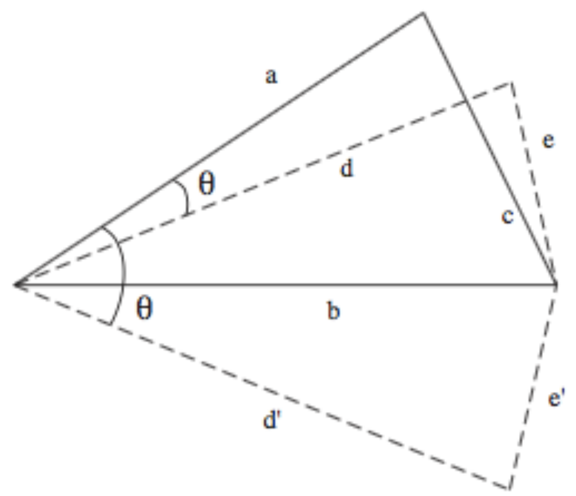
$$\sqrt{2}\bar{A}(\bar{B}^0 \to \pi^0\pi^0) + \sqrt{2}\bar{A}(B^- \to \pi^-\pi^0) = \bar{A}(\bar{B}^0 \to \pi^+\pi^-)\ ,$$

Similar relation for the corresponding the anti-particle decays.

If all amplitudes are measured, the angle θ can be determined.

Including $B \to \pi\rho, \rho\rho$, $\alpha = (85.2^{+4.8}_{-4.3})^\circ$

Gronau and London, PRL65, 3381(1990)
Snyder and Quinn, PRD48, 2139(1993)



Isospin triangles in the complex plane. Lines a, b, and c denote the amplitudes $\bar{A}(B^0 \to \pi^+\pi^-)$, $\sqrt{2}\bar{A}(B^- \to \pi^-\pi^0) = \sqrt{2}A(B^+ \to \pi^+\pi^0)$, and $\sqrt{2}\bar{A}(B^0 \to \pi^0\pi^0)$,respectively. The dashed lines d and e (or d' and e') denote the amplitudes $A(B^0 \to \pi^+\pi^-)$ and $\sqrt{2}A(B^0 \to \pi^0\pi^0)$, respectively.

Determination of β

The best way to determine β is to measure $Im\lambda_{\psi K_S}$ for $\bar{B}^0(B^0) \to J/\psi K_S$.

The decay amplitude can be parameterized as

$A(\bar{B}^0 \to J/\psi K_S) = \langle K_S J/\psi | H_{eff} | \bar{B}^0 \rangle = V_{cb}V_{cs}^* T_{\psi K} + V_{ub}V_{us}^* P_{\psi K}$.

The WC's involved indicate that $|T_{\psi K}|$ is much larger than $|P_{\psi K}|$,

Also $|V_{cb}V_{cs}^*|$ is about 50 times larger than $|V_{ub}V_{us}^*|$ from experimental data

The $P_{\psi K_S}$ term can be ignored, then $\frac{\bar{A}}{A} = \frac{V_{cs}V_{cd}^*}{V_{cs}^*V_{cd}^*}$.

To a very good approximation,

$$Im\lambda_{\psi K_s} = Im\left(\left(\frac{q}{p}\right)_{B_d} \frac{V_{cb}V_{cs}^*}{V_{cb}^*V_{cs}} \left(\frac{q}{p}\right)_K\right) = -\sin(2\beta) .$$

The Gold-plated place for CP violation (Cater, Sanda, and Bigi, 1980, 1981)

Data: $Im(\lambda_{B^0 \to J/\psi K_s}) = sin(2\beta) = 0.699 \pm 0.017$

Determination of γ

Gronau and Wyler, PLB256, 172(1991)
Atwood et al., PRL 78, 3257(1997)

Determination of the phase angle γ using: $B^- \to (D^0,\ \bar{D}^0,\ D_{CP})K^-$.

Here $D_{CP} = (D^0 - \bar{D}^0)/\sqrt{2}$ is the CP even state.

The decay amplitudes can be parameterised as

$$\bar{A}(\bar{D}^0 K^-) = V_{ub}V_{cs}^* T_{\bar{D}K}\ ,\quad \bar{A}(D^0 K^-) = V_{cb}V_{us}^* T_{D^0 K}\ ,$$
$$\bar{A}(D_{CP}K^-) = \frac{1}{\sqrt{2}}(\bar{A}(D^0 K^-) - \bar{A}(\bar{D}^0 K^-))\ .$$

The angle γ can be measured as shown in the figure

D_{CP} identified is through processes induced by

$c \to ud\bar{d}$ and $\bar{c} \to \bar{u}\bar{d}d$.

The angle γ' in the figure is given by the absolute value of

$\mathrm{Arg}[(V_{ub}V_{cs}^*/V_{cb}V_{us}^*)(V_{cd}V_{ud}^*/V_{cd}^*V_{ud})] = -2(\gamma - \sigma')$.

In the SM σ' is very small, so γ' is equal to 2γ to a very good approximation.

Including $B^- \to DK^{*-}, D^*K^{*-}$ and other similar decays: $\gamma = \left(66.2\,^{+3.4}_{-3.6}\right)^\circ$

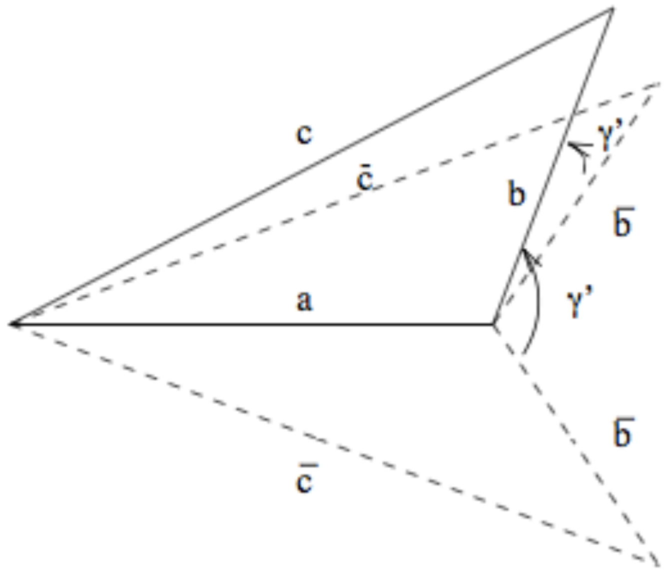


Figure 7: The measurement of γ through $B^-(B^+) \to DK^-(K^+)$ decays with $a = |\bar{A}(B^- \to D^0K^-)| = |A(B^+ \to \bar{D}^0K^+)|$, $b = \bar{A}(B^- \to \bar{D}^0)$, $\bar{b} = A(B^+ \to D^0K^+)$, $c = \sqrt{2}\bar{A}(B^- \to D_{CP}K^-)$, and $\bar{c} = \sqrt{2}A(B^+ \to D_{CP}K^+)$.

Unitarity triangle Consistent with SM prediction!

$$\alpha+\beta+\gamma = 173^{o}\ (6^{o})$$

Other tests? Any deviations from SM?

2.3 More CP violating experimental observables

CP violation with polarization measurement

a spin-1/2 -> spin-0 + spin-1/2

$$\mathcal{A} = \bar{\mathcal{F}}(A_v + iA_c\gamma_5)\mathcal{B} = \mathcal{S} + \mathcal{P}\sigma \cdot \vec{p}_c \qquad |\vec{p}_c| = \sqrt{E_{\mathcal{F}}^2 - m_{\mathcal{F}}^2}$$

$$\mathcal{S} = A_v\sqrt{\frac{(m_{\mathcal{B}} + m_{\mathcal{F}})^2 - m_{\mathcal{M}}^2}{16\pi m_{\mathcal{B}}^2}}\,, \quad \mathcal{P} = A_c\sqrt{\frac{(m_{\mathcal{B}} - m_{\mathcal{F}})^2 - m_{\mathcal{M}}^2}{16\pi m_{\mathcal{B}}^2}}\,.$$

$$\bar{\mathcal{A}} = -\bar{\mathcal{S}} + \bar{\mathcal{P}}\sigma \cdot \vec{p}_c\,.$$

$$\frac{4\pi}{\Gamma}\frac{d\Gamma}{d\Omega} = 1 + \alpha\vec{s}_{\mathcal{B}} \cdot \vec{n} + \vec{s}_{\mathcal{F}} \cdot [(\alpha + \vec{s}_{\mathcal{B}} \cdot \vec{n})\vec{n} + \beta\vec{s}_{\mathcal{B}} \times \vec{n} + \gamma(\vec{n} \times (\vec{s}_{\mathcal{B}} \times \vec{n}))]$$

$$\Delta = \frac{\Gamma - \bar{\Gamma}}{\Gamma + \bar{\Gamma}}\,, \quad A_\alpha = \frac{\Gamma\alpha + \bar{\Gamma}\bar{\alpha}}{\Gamma\alpha - \bar{\Gamma}\bar{\alpha}}\,, \quad B_\beta = \frac{\Gamma\beta + \bar{\Gamma}\bar{\beta}}{\Gamma\beta - \bar{\Gamma}\bar{\beta}}\,.$$

$$\vec{n} = \vec{p_c}/|p_c|$$

$$\beta = (1 - \alpha^2)^{1/2} sin\phi$$

CP violation in Hyperons

	Δ	A	B
$\Lambda^0 \to p\pi^-$	-5.4×10^{-7}	-0.5×10^{-4}	3.0×10^{-3}
$\Xi^- \to \Lambda^0\pi^-$	0	-0.7×10^{-4}	8.4×10^{-4}
$\Sigma^- \to n\pi^-$	0	1.6×10^{-4}	-1.2×10^{-2}
$\Sigma^+ \to p\pi^0$	-6.2×10^{-7}	-3.2×10^{-7}	-4.2×10^{-4}
$\Sigma^+ \to n\pi^+$	6.0×10^{-7}	-1.6×10^{-4}	-8.4×10^{-7}

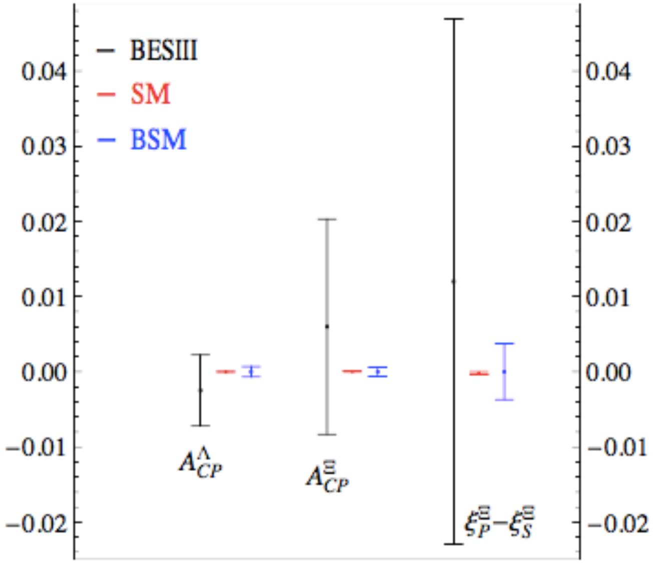
Signals of {CP} Nonconservation in Hyperon Decay
John F. Donoghue (Massachusetts U., Amherst), Sandip Pakvasa (Hawaii U.).
Published in **Phys.Rev.Lett. 55 (1985) 162**

Hyperon decays and *CP* nonconservation

John F. Donoghue
Department of Physics and Astronomy, University of Massachusetts, Amherst, Massachusetts 01003

Xiao-Gang He and Sandip Pakvasa
Department of Physics and Astronomy, University of Hawaii at Manoa, Honolulu, Hawaii 96822
(Received 7 March 1986)

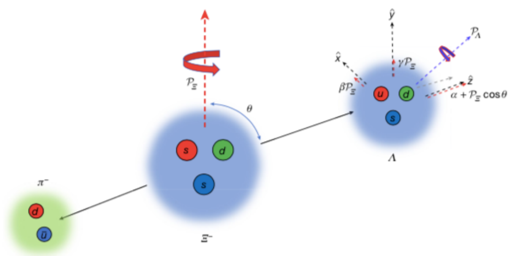
We study all modes of hyperon nonleptonic decay and consider the *CP*-odd observables which result. Explicit calculations are provided in the Kobayashi-Maskawa, Weinberg-Higgs, and left-right-symmetric models of *CP* nonconservation.



$A_{\Xi\Lambda} = A_\Xi + A_\Lambda$ HyperCP (Femilab E871): $A_{\Xi\Lambda} = [-6.0 \pm 2.1(\text{stat}) \pm 2.0(\text{syst})] \times 10^{-4}$,

Recent measurement from BESIII
(Nature 606(2022)64)

$$A_{CP} = \frac{\alpha + \bar{\alpha}}{\alpha - \bar{\alpha}}, \qquad B_{CP} = \frac{\beta + \bar{\beta}}{\alpha - \bar{\alpha}}$$



$$A^{\Xi}_{CP} = (6 \pm 13 \pm 6) \times 10^{-3},$$
$$B^{\Xi}_{CP} \simeq \xi^{\Xi}_P - \xi^{\Xi}_S = (1.2 \pm 3.4 \pm 0.8) \times 10^{-2},$$
$$A^{\Lambda}_{CP} = (-4 \pm 12 \pm 9) \times 10^{-3}$$

So far not CP violation effects have been established in baryon decay!.
Similar ideas can be used for c- and b-baryon decays.

CP violation in Higgs h decays into $\tau^+\tau^-$

(Hayreter, He, Valencia, arXiv:1603.06326, arXiv:1606.00951)

(He, Ma, McKellar, Mod. Phys Lett. A9, 205(1994);
Berge, Bereuther, Kirchner, PRD92,096012(2015))

General Higgs to fermion coupling: $L = -\bar{f}(r_f + i\tilde{r}_f\gamma_5)fh$

Define the density matrix R with polarization $\vec{n}_f(\vec{n}_{\bar{f}})$ for $f(\bar{f})$

$$R = N_f\beta_f[Im(r_f\tilde{r}_f^*)\hat{p}_f \cdot (\vec{n}_{\bar{f}} - \vec{n}_f) - Re(r_f\tilde{r}_f)\hat{p}_f \cdot (\vec{n}_f \times \vec{n}_{\bar{f}})]$$

N_f - normalization constant, $\hat{p}$ - three moment of f, $\beta_f = \sqrt{1 - 4m_f^2/m_h^2}$

Application to $h \to \tau^+\tau^-$

Using $\tau \to \pi^-\nu_\tau$ to measure $\vec{n}_f$, $\frac{4\pi}{\Gamma}\frac{d\Gamma}{d\Omega} = (1 + \alpha_\tau\vec{n}_\tau \cdot \hat{p}_\tau)$, $\alpha_\tau = 1$.

$$\hat{p}_\tau \cdot (\vec{n}_f \times \vec{n}_{\bar{f}}) \to \hat{p}_\tau \cdot (\hat{p}_{\pi^-} \times \hat{p}_{\pi^+})$$

One construct CP violating observable

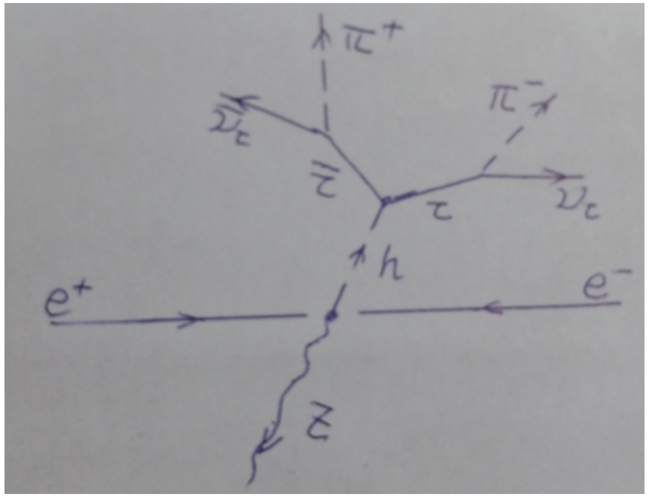
$$A_\tau = \frac{N(O_\pi > 0) - N(O_\pi < 0)}{N(O_\pi > 0) + N(O_\pi < 0)}, \quad O_\pi = \hat{p}_\tau \cdot (\hat{p}_{\pi^+} \times \hat{p}_{\pi^-}) .$$

Theoretically

$$A_\tau = \frac{N(O_\pi > 0) - N(O_\pi < 0)}{N(O_\pi > 0) + N(O_\pi < 0)} = \frac{\pi}{4}\beta_\tau \alpha_\tau \alpha_{\bar{\tau}} \frac{r_\tau \tilde{r}_\tau}{\beta_\tau^2 r_\tau^2 + \tilde{r}_\tau^2} .$$

Data still allow A to be as large as $\pi/8$. Experiments should look such CPV.

In the SM $A\tau = 0$



Br(h -> $\tau\tau$) ~ 5×10^{-2},

Br(τ –> π ν) ~ 0.1

10^6 Higgs bosons, sensitivity to A_τ can be 10% at CEPC.

The EDM of a fundamental particle

Classically a EDM $\vec{D} = \int d^3x\vec{x}\rho(\vec{x})$ interacts with an electric field $\vec{E}$

The interaction energy is given by $H = \vec{D}\cdot\vec{E}$, allowed by P and T symmetries.

Under P, $\vec{D} \to -\vec{D}$ and $\vec{E} \to -\vec{E}$, H conserves both P and T.

A fundamental particle, $\vec{D}$ is equal to $d\vec{S}$, $H_{edm} = d\vec{S}\cdot\vec{E}$.

Since under P, $\vec{S} \to \vec{S}$ and under T, $\vec{S} \to -\vec{S}$

H_{edm} violates both P and T, CPT is conserved, CP is also violated!

Quantum field theory, $H_{edm} = -i\frac{1}{2}d\bar{\psi}\sigma^{\mu\nu}\psi\tilde{F}_{mu\nu} = -i\frac{1}{2}d\bar{\psi}\sigma^{\mu\nu}\gamma_5\psi F_{\mu\nu}$

In non-relativestic limit H_{edm} reduce to $d\frac{\vec{\sigma}}{2}\cdot\vec{E} = d\vec{S}\cdot\vec{E}$.

One easily sees that H_{edm} violates P and T, violates CP, but conserve CPT.

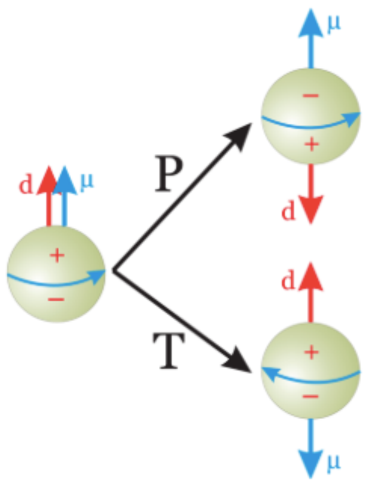
A non-zero fundamental particle EDM, violates P, T and CP!

Magnetic Dipole conserves P and T

$H_{mdm} = d_m\vec{S}\cdot\vec{B}$,

Under P: $\vec{B} \to \vec{B}$ and under T: $\vec{B} \to -\vec{B}$

Relativistic expression: $d_m\bar{\psi}\sigma^{\mu\nu}\psi F_{\mu\nu}$.



First fundamental particle EMD measurement: neutron EDM in 1950 by Purcell and Ramsey.

Landau first pointed out that EDM violates P and T symmetry.

No measurement of a fundamental particle EDM, yet!

Current 90% C.L. limits on EDM:

Neutron $|D_n| < 1.8 \times 10^{-26}$ ecm, electron $|D_e| < 1.1 \times 10^{-29}$ ecm

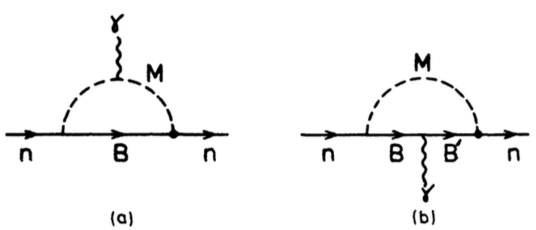
EDM of neutron and electron in KM model

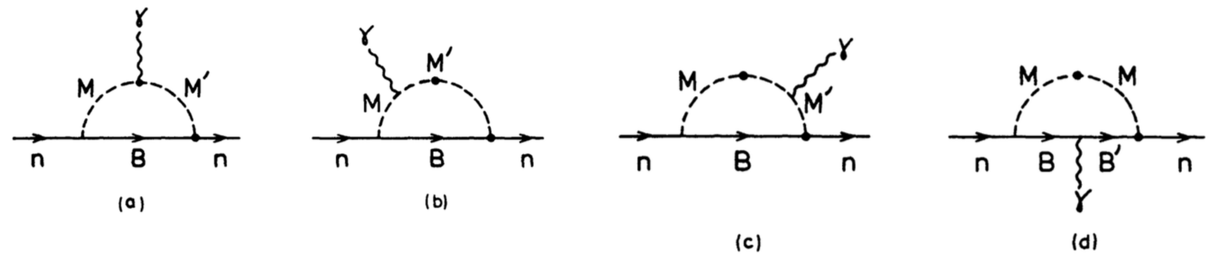
Quark EDM D_q and neutron EDM D_n, $D_n = (4D_d - D_u)/3$

In KM model, quark EDM only generated at two electroweak and one strong loop level (3 loop effects)l, very small ~ 10^{-33} e.cm. (Shabalin, 1978, 1980)

In fact with two weak and one strong interaction vertices, EDM can also be generated!
(He, McKellar and Pakvasa, PLB197, 556(1987), J. Mod. Phys. A4, 5011(1989)

$$1.6 \times 10^{-31}\ \text{e.cm} \geq |D_n| \geq 1.4 \times 10^{-33}\ \text{e.cm}$$





Electron EDM is even smaller, generated at fourth loop level, $D_e < 10^{-38}$ecm

Calculation of EDM from θ-term

Making on each of the light quarks (u, d, s).

With appropriate chiral transformation, assuming small θ

$$L = -m_u \bar{u}u + m_d \bar{d}d + m_s \bar{s}s - \theta \frac{g_3^2}{16\pi^2} Tr(\tilde{G}G)$$

$$\rightarrow \quad -m_u \bar{u}u + m_d \bar{d}d + m_s \bar{s}s + i\theta \frac{m_u m_d m_s}{m_u m_d + m_u m_s + m_d m_s} \bar{u}\gamma_5 u + \bar{d}\gamma_5 d + \bar{s}\gamma_5 s$$

Using current algebra, turn the above into nucleon-pion interactions

$$< P^a B_f | \bar{u}\gamma_5 u + \bar{d}\gamma_5 d + \bar{s}\gamma_5 s | n >= -i(\sqrt{2}/f_p i) < B_f | \bar{q}\lambda^a q | n >$$

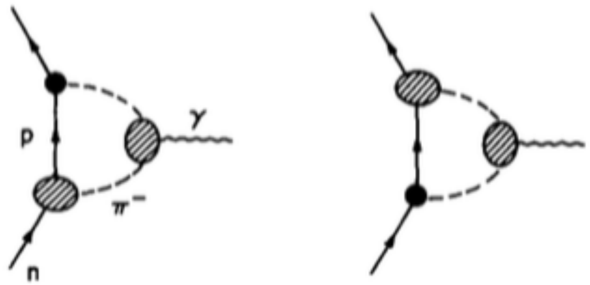
$$L_{\pi^i B_f B} = -\sqrt{2}\bar{N}_f \sigma^i (i\gamma_5 g_{\pi NN} + f_{\pi NN}|N >$$

$g_{\pi NN} \approx 14$ is CP conserving, and $f_{\pi NN}$ is CP violating coupling with

Crewther, Di Vecchia, Veneziano and Witten, PLB88, 13,(1979)

$$f_{\pi NN} = -2\frac{(m_\Xi - m_\Sigma)}{f_\pi}\frac{m_u m_d m_s}{m_u m_d + m_u m_s + m_d m_s},$$

$D_n \sim -3.8 \times 10^{-16}\theta$ ecm



Including all SU(3) octet contributions:

$2.5 \times 10^{-16}\theta ecm < |D_n| < 4.6 \times \theta ecm$ He, McKellar and Pakvasa, IJMP A4, 5011 (1989))

Using data $|D_n| < 3 \times 10^{-27} ecm$, $|\theta| < 10^{-11}$!

Why θ is small is the strong CP problem.

Homework

Problem 1 Diagonalize the Hamiltonian $H = M - i\,\Gamma/2$

Find the eigenvalues E_1, E_2, and the matrix V diagonalizing H

$V H V^{-1} = \mathrm{diag}(E_1, E_2)$

Note that H is not Hermitian, V is not Unitary.

Problem 2 Obtain expressions for r and a on page

For coherently produced M and $\bar{M}$, calculate

$A^{+-} =< f_1 \bar{f}_2 | H | \Psi(t_1; t_2 >$, $N^{+-}(+1, -2) \sim \int_0^\infty |A^{+-}|^2 dt_1 dt_2$
and $N^{+-}(+2, -1)$ (+i, -i indicate that t_i correspond to f and $\bar{f}$ respectively.)

obtain $N^{+-} = N^{+-}(+1, -2) + N^{+-}(+2, -1)$

$A^{++} =< f_1 f_2 | H | \Psi(t_1; t_2 >$, $N^{++}(+1, +2) \sim \int_0^\infty |A^{++}|^2 dt_1 dt_2$

$A^{--} =< \bar{f}_1 \bar{f}_2 | H | \Psi(t_1; t_2 >$, $N^{--}(-1, -2) \sim \int_0^\infty |A^{--}|^2 dt_1 dt_2$

Obtain r and a

Problem 3

SU3)/U-spin symmetry d <-> s channels, one has

Deshpande&X-G He(1995), X-G He(1999), Gronau&Rosner (2000)…He, Li, Ren and Yuan, arXiv:1704.05788

$$A(\bar{B}^0 \to K^-\pi^+) = V_{ub}V_{us}^* T + V_{tb}V_{ts}^* P\ , \quad A(B^0 \to K^+\pi^-) = V_{ub}^* V_{us} T + V_{tb}^* V_{ts} P$$
$$A(\bar{B}_s^0 \to K^+\pi^-) = V_{ub}V_{ud}^* T + V_{tb}V_{td}^* P\ , \quad A(B_s^0 \to K^-\pi^+) = V_{ub}^* V_{ud} T + V_{tb}^* V_{td} P$$

$$\Delta(B \to PP) = \Gamma(\bar{B} \to \bar{P}\,\bar{P}) - \Gamma(B \to P\,P) = \frac{\lambda_{ab}}{8\pi m_B}(|A(\bar{B} \to \bar{P}\,\bar{P})|^2 - |A(B \to P\,P)|^2)\ ,$$

Using the relation, $Im(V_{ub}V_{ud}^* V_{tb}^* V_{td}) = -Im(V_{ub}V_{us}^* V_{tb}^* V_{ts}),$ to show

$$\Delta(B^0 \to K^+\pi^-) = -\Delta(B_s^0 \to K^-\pi^+) \qquad \frac{A_{CP}(B^0 - K^+\pi^-)}{A_{CP}(B_s^0 \to K^-\pi^+)} = -r_c \frac{B(B_s^0 \to K^-\pi^+)\tau_{B_s^0}}{B(B^0 \to k^+\pi^-)\tau_{B^0}}$$

SU(3)/U symmetric, $r_c = 1$

Test for SU(3) flavor symmetry, and also SM with 3 generations!
Using current data from PDG, find the value for r_c

Lecture III FPCP beyond SM

3.1 The need of going beyond SM

3.2 Anomalies in flavor physics

3.2 Model buildings for FPCP beyond SM