Lecture III FPCP beyond SM

3.1 The need of going beyond SM

3.2 Anomalies in flavor physics

3.2 Model buildings for FPCP beyond SM

3.1 The need of going beyond SM

The SM is a beautiful and successful model to describe strong and electroweak interactions. But how good is it and is there indications that is may not be the complete theory addressing all problems facing particle physics?

Yes, there are many hints. Some of the prominent phenomenological ones are:

The neutrino mass problem. Neutrino oscillations observed requires some of the neutrinos (at least two of them) to have non-zero masses. To give a mass to a fermion in the SM, one needs to pair up a left and right handed partners, example up, down quarks and charged leptons

$$- \quad \bar{Q}_L Y_u \tilde{H} u_R + \bar{Q}_L Y_d H d_R + \bar{L}_L Y_e H e_R + H.C.$$

In the minimal SM, there is not right handed neutrinos in model available, therefore need to introduce them.

Need to introduces ν_R in the model. Then one has $-\bar{L}_L Y_\nu \tilde{H} \nu_R \to \bar{\nu}_L (Y_\nu v/\sqrt{2}) \nu_R$

Then $m_\nu = Y^\nu$ v/sqrt[2]! Problem: $m_\nu/m_e = Y_\nu/Y_e < 10^{-6}$

Why such a small number?

Solutions

Seesaw models

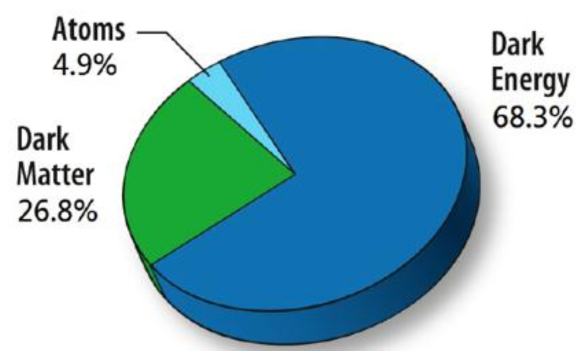
Type I seesaw model: ν_R (1, 1)(0) neutrinos, $-\bar{L}_L Y_\nu \tilde{H} \nu_R - (1/2) m_R \bar{\nu}_R^c \nu_R$, $\quad m_\nu = (Y_\nu v)^2/2m_R$

Type II seesaw model: $\chi(1,3)(-1)$ small vev v_χ, $-L_L Y_\nu \chi L_L^c \to -\nu_L (Y^\nu v_\chi/\sqrt{2}) \nu_L^c$

Type III seesaw model: N_R (1,3)(0),- $\bar{L}_L Y_\nu \tilde{H} N_R - (1/2) m_R \bar{N}_R^c N_R$, $\quad m_\nu = (Y_\nu v)^2/2m_R$

And models of generating neutrino masses at loop levels.

If only confined to leptons, flavor physics and CP violation will be affected in the lepton sector.



Cosmological evidences: Dark matter, Dark energy and matter-antimatter asymmetry

More theoretical ones

The hierarchy problem: Why electroweak scale ~ v~ 300 GeV is so much lower than the Planck scale ~ m_{Planck} ~ 10^{19} GeV.

Cosmological constant problem, very small but none zero.

Strong CP problem, why $\theta < 10^{-10}$ is so small?

Unification of all forces:

a) Grand unification of strong and electroweak interaction

b) Theory of Everything, unification of strong, electroweak and gravity

Too many free parameters in the theory? Possible to reduce them?

Why there are just 3 generations

…

The strong CP problem and neutron EDM

The CP violating term

$$\delta L_{QCD} = -\theta \frac{g^2}{16\pi^2} Tr(\tilde{G}^{\mu\nu} G_{\mu\nu}) = -\theta \frac{g^2}{32\pi^2} \tilde{G}^{\mu\nu}_a G^a_{\mu\nu} \,.$$

can be written as surface integral for the action

$$\delta S = -\theta \frac{g^2}{32\pi^2} \int d^4x \tilde{G}^{\mu\nu}_a G^a_{\mu\nu} = -\theta \frac{g^2}{32\pi^2} \int d^4x \partial_\mu K^\mu = -\theta \frac{g^2}{32\pi^2} \int d\sigma_\mu K^\mu$$

with $K^\mu = (\epsilon^{\mu\alpha\beta\gamma} G^a_\alpha (G^a_{\beta\gamma} - (g_3/3) f^{abc} G^b_\beta G^c_\gamma)$

The surface integral is usually dropped.

However, there are configuration the above integral is non-zero, θ-vacuum.

The integral is actually corresponding to the topological winding number ν

$$\nu = \frac{g^2}{32\pi^2} \int d\sigma_\mu K^\mu$$

One cannot simply disregard it!

Some useful relations

The chiral anomaly relation, $\partial^\mu(\bar{\psi}\gamma_\mu\gamma_5\psi) = \frac{g_3^2}{16\pi^2}Tr(\tilde{G}G)$,

leads to a chiral rotation $\psi \to e^{i\alpha\gamma_5/2}\psi$ generates in the Lagrangian

$$\delta L_\alpha = -\alpha\frac{g_3^2}{16\pi^2}Tr(\tilde{G}G)\ .$$

An imaginary matter term $\delta L = -\bar{\psi}m(cos\alpha + isin\alpha\gamma_5)\psi$

can be transformed away by define $\psi' = e^{-\alpha\gamma_5/2}\psi$ and to

$$\delta L = -\bar{\psi}'m\psi' + \alpha\frac{g_3^2}{16\pi^2}Tr(\tilde{G}G)\ .$$

If one write with more than one ψ the mass matrices as $\psi_R M \psi_L$

In general M is complex. Then

$$\begin{aligned}\delta L &= -(\bar{\psi}_R M\psi_L + H.C.) - \theta\frac{g_3^2}{16\pi^2}Tr(\tilde{G}G) \\ &= -\bar{\psi}\hat{M}\psi - (\theta - Arg(Det(M))\frac{g_3^2}{16\pi^2}Tr(\tilde{G}G)\ .\end{aligned}$$

$\hat{M} = diag(m_1, m_2, ...)$, with $m_i > 0$

Calculation of EDM from θ-term

Making on each of the light quarks (u, d, s).

With appropriate chiral transformation, assuming small θ

$$L = -m_u \bar{u}u + m_d \bar{d}d + m_s \bar{s}s - \theta \frac{g_3^2}{16\pi^2} Tr(\tilde{G}G)$$

$$\rightarrow \quad -m_u \bar{u}u + m_d \bar{d}d + m_s \bar{s}s + i\theta \frac{m_u m_d m_s}{m_u m_d + m_u m_s + m_d m_s} \bar{u}\gamma_5 u + \bar{d}\gamma_5 d + \bar{s}\gamma_5 s$$

Using current algerbra, turn the above into nucleon-pion interactions

$$< P^a B_f | \bar{u}\gamma_5 u + \bar{d}\gamma_5 d + \bar{s}\gamma_5 s | n >= -i(\sqrt{2}/f_p i) < B_f | \bar{q}\lambda^a q | n >$$

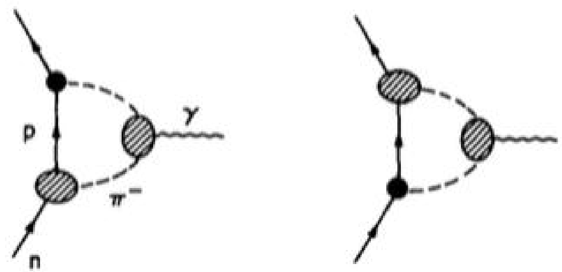
$$L_{\pi^i B_f B} = -\sqrt{2}\bar{N}_f \sigma^i (i\gamma_5 g_{\pi NN} + f_{\pi NN} | N >$$

$g_{\pi NN} \approx 14$ is CP conserving, and $f_{\pi NN}$ is CP violating coupling with

Crewther, Di Vecchia, Veneziano and Witten, PLB88, 13,(1979)

$$f_{\pi NN} = -2\frac{(m_\Xi - m_\Sigma)}{f_\pi}\frac{m_u m_d m_s}{m_u m_d + m_u m_s + m_d m_s},$$

$D_n \sim -3.8 \times 10^{-16}\theta$ ecm



Including all SU(3) octet contributions:

$2.5 \times 10^{-16}\theta ecm < |D_n| < 4.6 \times \theta ecm$ He, McKellar and Pakvasa, IJMP A4, 5011 (1989))

Using data $|D_n| < 3 \times 10^{-27} ecm$, $|\theta| < 10^{-11}$!

Why θ is small is the strong CP problem.

Solutions to the strong CP problems

1. One of the quark mass is zero, since D_n is proportional to $m_u m_d m_s$. But all quarks have non-zero masses!

2. Making the theory left-right symmetric (parity conservation, θ is zero to start with) and quark mass matrices Hermintian (Arg(M) = 0).

3. Spontaneous CP violation, making θ equal to zero first. Need to check whether after symmetry breaking, θ is not generated.

4. Dynamic solution driving θ small by imposing an additional chiral symmetry, the Peccei-Quinn symmetry. This solution leads to Axion which has not been discovered.

See Appendix D for PQ symmetry and Axion.

PQ symmetry

The most attractive solution is provided by Peccei-Quinn symmetry

Lagrangian symmetric under a $U(1)_A$ global chiral transformation

Chiral transformation for SM Q_L, u_R, d_R, L_L and e_R are independent

How f_L and f_R, if f transforms under $e^{i\alpha\gamma_5} f$?

$f_R = \frac{1}{2}(1+\gamma_5)f \to \frac{1}{2}(1+\gamma_5)e^{i\alpha\gamma_5} f$

$$\frac{1}{2}(1+\gamma_5)e^{i\alpha\gamma_5} f = \frac{1}{2}(1+\gamma_5)(cos\alpha + i\gamma_5 sin\alpha)f = \frac{1}{2}(cos\alpha + isin\alpha)(1+\gamma_5)f = e^{i\alpha} f_R \ ,$$

$$\frac{1}{2}(1-\gamma_5)e^{i\alpha\gamma_5} f = \frac{1}{2}(1-\gamma_5)(cos\alpha + i\gamma_5 sin\alpha)f = \frac{1}{2}(cos\alpha - isin\alpha)(1-\gamma_5)f = e^{-i\alpha} f_L \ .$$

$U(1)_A$ chiral model of QP symmetry for strong CP problem

$L = L_{SM} + \delta L_\theta,\ \delta L_\theta = -\theta(g_3^2/16\pi^2)Tr(\tilde{G}G)$

$u_R \to e^{i\alpha}u_R,\ d_R \to e^{i\alpha},\ Q_L \to Q_L,\ L_L \to L_l$ and $e_R \to e^{i\alpha}e_R$

$\bar{\theta} = \theta \to \theta - 2\alpha,$

If L_{SM} is symmetric under $U(1)_A$, $L \to L_{SM} + \delta L_{\bar{\theta}=\theta-2\alpha}$

For L_{SM}, α is arbitrary, choose one such that $\bar{\theta} = \theta - 2\alpha = 0.$

No strong CP term!

One then needs to show that the corresponding potentials are minimal to have a stable solution. More discussions in Appendix D.

3.2 Anomalies in flavor physics

There are also a few anomalies in flavor physics which show some deviations from SM predictions at some level, but not up to 5σ yet.

But they have generated a lot of concerns and people are trying to provide solutions to them. Here I discuss a few related to flavor physics.

The unitarity of CKM matrix; The g-2 muon anomaly;

B -> $\mu\mu$ $K^{(*)}$, $D^{(*)}$ τ ν…

CKM unitarity: $$|V_{ud}|^2 + |V_{us}|^2 + |V_{ub}|^2 = 1,$$

$|V_{ub}|^2 \sim 10^{-5}$ negligible, so usually study

$\Delta = |V_{ud}|^2 + |V_{us}|^2 - 1$

Zoom in superallowed 0+ -> 0+ nuclei transition and K -> π l ν show about 3σ level deviation

$\Delta^{(1)}_{\rm CKM}$	$-0.00176(56)$	$-0.00173(55)$	$-0.00162(56)$	$-0.00185(56)$	$-0.00171(55)$	$-0.00151(56)$	$-0.00195(56)$
	-3.1σ	-3.1σ	-2.9σ	-3.3σ	-3.1σ	-2.7σ	-3.5σ

aeXiv:2208.11707

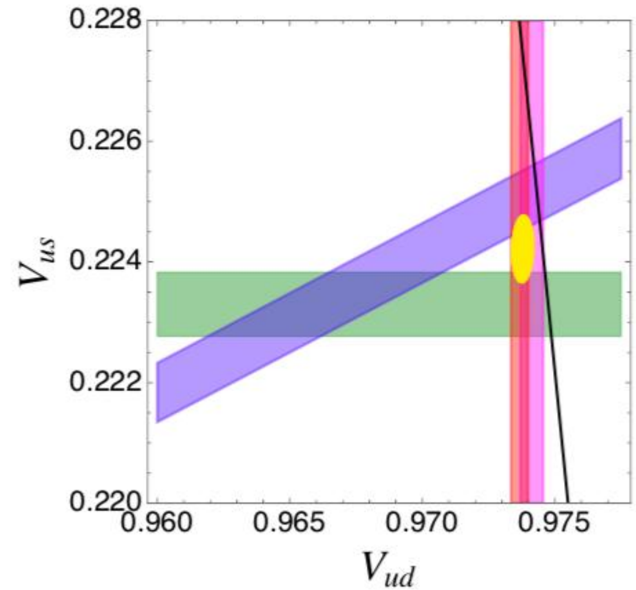
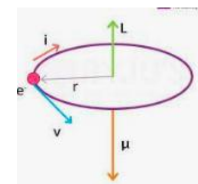


Figure 1: Constraints in the V_{ud}–V_{us} plane. The partially overlapping vertical bands correspond to $V_{ud}^{0^+\to 0^+}$ (leftmost, red) and $V_{ud}^{n,\,\rm best}$ (rightmost, violet). The horizontal band (green) corresponds to $V_{us}^{K_{\ell 3}}$. The diagonal band (blue) corresponds to $(V_{us}/V_{ud})_{K_{\ell 2}/\pi_{\ell 2}}$. The unitarity circle is denoted by the black solid line. The 68% C.L. ellipse from a fit to all four constraints is depicted in yellow ($V_{ud} = 0.97378(26)$, $V_{us} = 0.22422(36)$, $\chi^2/\rm dof = 6.4/2$, p-value 4.1%), it deviates from the unitarity line by 2.8σ. Note that the significance tends to increase in case τ decays are included.

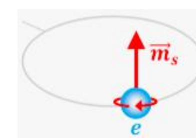
Muon g-2 anomaly

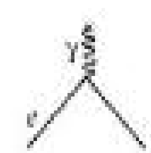
The energy of a particle with magnetic dipole μ interact with a magnetic field **B** is given by: $H = - \mu \cdot \mathbf{B}$

Classically, a particle of charge q moving in circle in magnetic field with angular momentum L, the magnetic dipole: $\mu_{\mathbf{L}} = q\mathbf{L}/2m$.

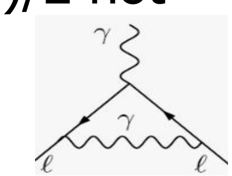


Quantum mechanically, a Dirac particle has an intrinsic magnetic dipole moment: $\mu = q\mathbf{S}/m$ which can be written as $\mu = g\, q\mathbf{S}/2m$, ------- g=2.
g is called the g-factor. (Dirac)

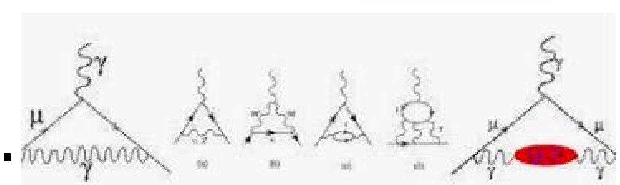




In quantum field theory, there is correction at loop level making $a = (g-2)/2$ not zero. This is the anomalous dipole moment of a particle. At one loop for charge leptons, $i = e, \mu, \tau$, $a_i = \alpha/2\pi$ (Schwinger)



In the SM, including QEC, Strong and electroweak contributions, a_μ has been calculated to very high precision.



Muon a_μ has also been measured to high precision.

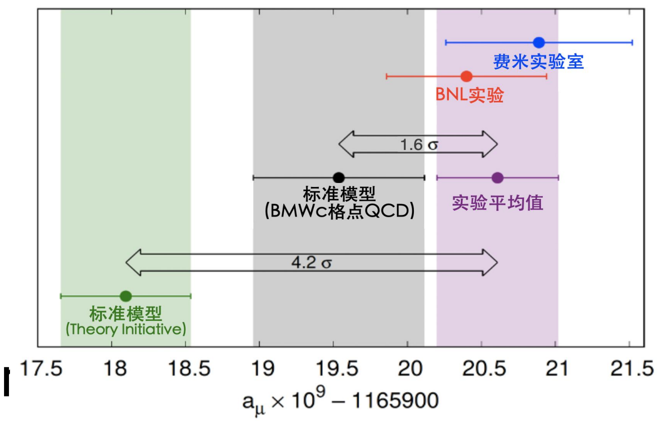
BNL experiment (1997 – 2001) final result

for $\Delta a_\mu = a_\mu(\text{exp}) - a_\mu(\text{SM})$ at 2.7σ larger than zero.

FNL experiment first result announce in April, 2021, conf
high confidence level at 3.3σ.

Combining BNL and FNL results, $\Delta a_\mu = 251(59)\times10^{-11}$.

The deviation away from SM is at 4.2σ level!



Recent Lattice calculation indicate the deviation is only at one σ level. More accurate theory calculations and Experimental measurement needed to confirm this anoı

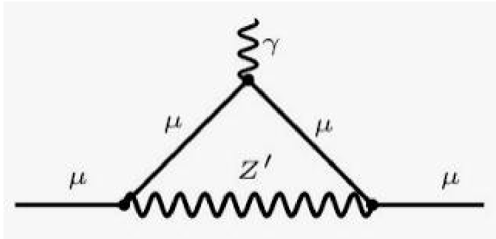
Even the anomaly itself still needs to be confirmed, but a lot of efforts have been made to explore the anomaly through beyond SM physics to match data.

Z', leptoquark, higgs….

Z′ contribution to muon g-2, a U(1)μ−τ example

arXiv: 2112.0992

The simplest version has Z′ coupling to μ and τ in diagonal form. An additional anomalous Δa_μ will be generated at one loop level



$$\Delta a_\mu^{Z'} = \frac{\tilde{g}^2}{8\pi^2}\frac{m_\mu^2}{m_{Z'}^2}\int_0^1 \frac{2x^2(1-x)dx}{1-x+(m_\mu^2/m_{Z'}^2)x^2}\,.$$

In the large $m_{Z'} \gg m_\mu$ limit, $\Delta a_\mu^{Z'} = (\tilde{g}^2/12\pi^2)(m_\mu^2/m_{Z'}^2)$.

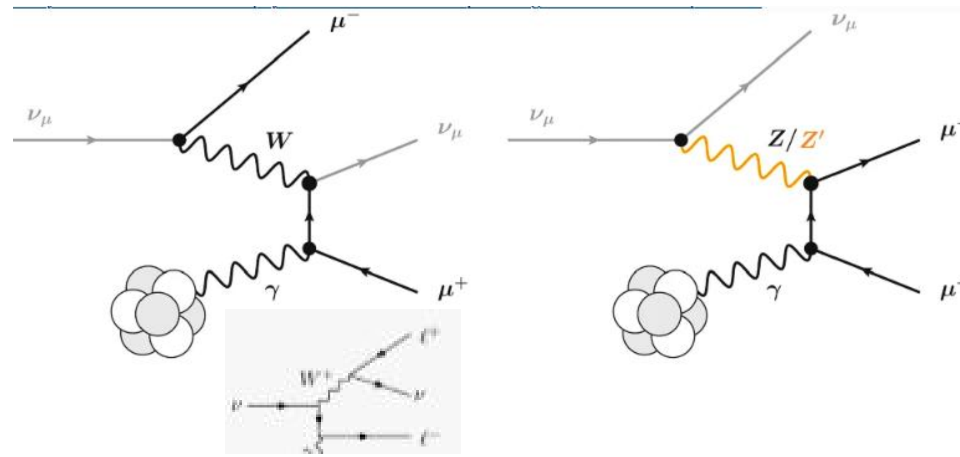
To explain the muon g-2 anomaly: $\tilde{g}^2/m_{Z'}^2 = (2.66 \pm 0.63)\times 10^{-5}\mathrm{GeV}^{-2}$.

One must check if the above region is ruled out by other processes.

The trident neutrino scattering data come in and rule out the above solution For large Z′ mass indicated above!

Small Z′ Mass Solution for Muon g-2

The trident neutrino constraint



Normalize SM contribution to experimental measurement, σ_{exp}/σ_{SM}.

If data agree with SM, $\sigma/\sigma(SM) = 1$.

Experimental data: $\sigma_{exp}/\sigma_{SM}|_{trident}$: 1.58 ± 0.57, 0.82 ± 0.28 and $0.72^{+1.73}_{-0.72}$ from CHARM-II, CCFR and NuTeV.

With Z′ contribution:

$$\frac{\sigma_{Z'}}{\sigma_{SM}}\Big|_{trident} = \frac{(1+4s_W^2+8\tilde{g}^2 m_W^2/g^2 m_{Z'}^2)^2+1}{1+(1+4s_W^2)^2}$$

Using central value for 2.66×10^{-5} GeV^{-2} for $\tilde{g}^2/m_{Z'}^2$ one obtains $\sigma_{Z'}/\sigma_{SM} = 5.86$

The model is ruled out as a solution for muon g-2 anomaly for large Z′ mass!

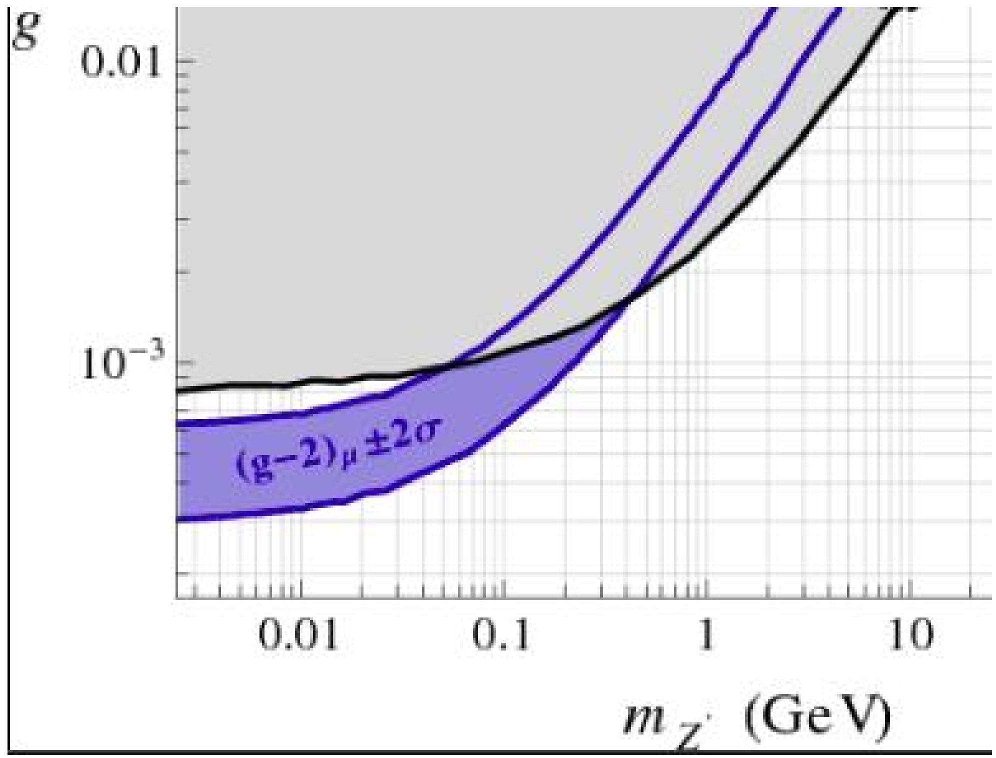
Small Z′ solution

What about small mass $m_{Z'} << m_T$?

In this region, the previous result for trident neutrino scattering is not valid because the q^2 exchange by Z′ is comparable, the heavy Z′ mass limit Cannot be applied!

More involved numerical calculations obtain the results shown in the figure On the right.

It is a folklore that in $U(1)_{L_\mu - L_\tau}$ in order to explain muon g-2 anomaly, $m_{Z'}$ must be less than 300 MeV!

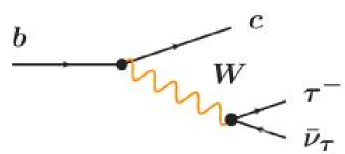


There are model which can invade this folklore, example arXiv:2112.09920

$R_{D^{(*)}}$ anomalies in B -> D$^{(*)}$ $\tau\nu$

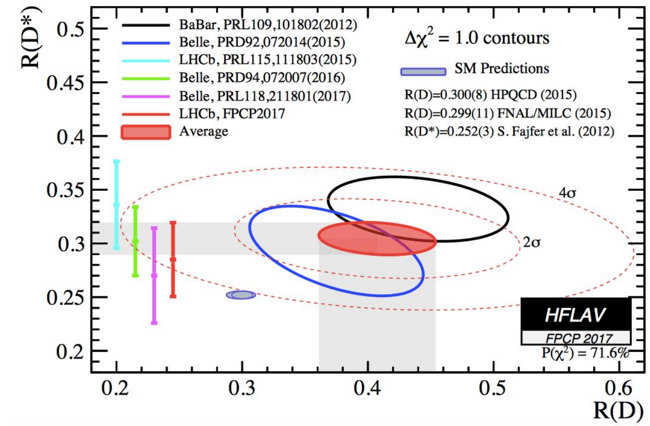
Before 2019

$$R(D^{(*)}) = \frac{Br(D^{(*)} \to \tau\bar{\nu}_\tau)}{Br(D^{(*)} \to l\bar{\nu})}$$

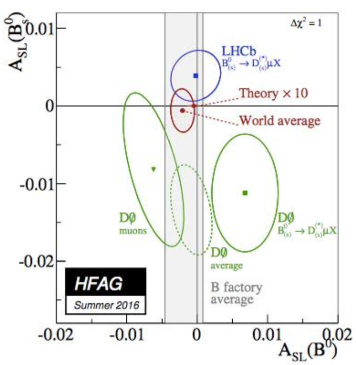


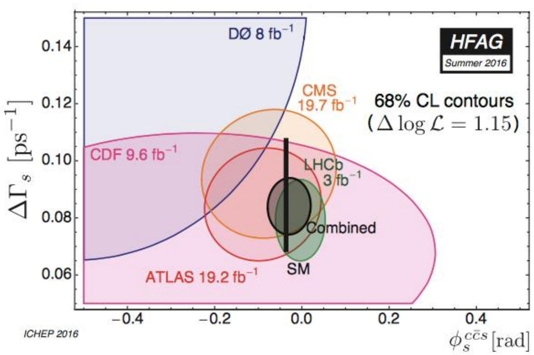
$$R(D) = \frac{B(\bar{B} \to D\tau^-\bar{\nu}_\tau)}{B(\bar{B} \to D\ell^-\bar{\nu}_\ell)} = 0.407 \pm 0.039 \pm 0.024$$

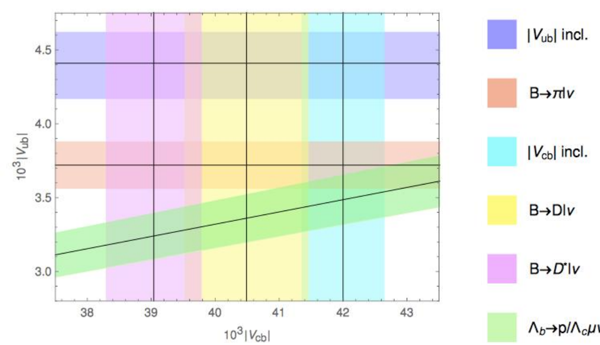
$$R(D^\star) = \frac{B(\bar{B} \to D^\star\tau^-\bar{\nu}_\tau)}{B(\bar{B} \to D^\star\ell^-\bar{\nu}_\ell)} = 0.304 \pm 0.013 \pm 0.007$$



4σ effects!







...Others....

$$R(J/\psi) = \frac{B(B_c^+ \to J/\psi\tau^+\nu_\tau)}{B(B_c^+ \to J/\psi\mu^+\nu_\mu)} = 0.71 \pm 0.17 \pm 0.18$$

R. Aaij *et al.* [LHCb Collaboration], arXiv:1711.05623 [hep-ex]

SM prediction

$$R(J/\psi) = 0.283 \pm 0.048$$

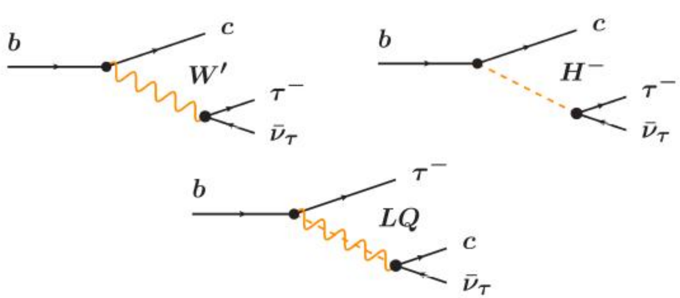
Measurement of $\mathcal{R}(D)$ and $\mathcal{R}(D^*)$ with a semileptonic tagging method

We report a measurement of the ratios of branching fractions $\mathcal{R}(D) = \mathcal{B}(\bar{B} \to D\tau^-\bar{\nu}_\tau)/\mathcal{B}(\bar{B} \to D\ell^-\bar{\nu}_\ell)$ and $\mathcal{R}(D^*) = \mathcal{B}(\bar{B} \to D^*\tau^-\bar{\nu}_\tau)/\mathcal{B}(\bar{B} \to D^*\ell^-\bar{\nu}_\ell)$, where ℓ denotes an electron or a muon. The results are based on a data sample containing 772×10^6 $B\bar{B}$ events recorded at the $\Upsilon(4S)$ resonance with the Belle detector at the KEKB e^+e^- collider. The analysis utilizes a method where the tag-side B meson is reconstructed in a semileptonic decay mode, and the signal-side τ is reconstructed in a purely leptonic decay. The measured values are $\mathcal{R}(D) = 0.307 \pm 0.037 \pm 0.016$ and $\mathcal{R}(D^*) = 0.283 \pm 0.018 \pm 0.014$, where the first uncertainties are statistical and the second are systematic. These results are in agreement with the Standard Model predictions within 0.2 and 1.1 standard deviations, respectively, while their combination agrees with the Standard Model predictions within 1.2 standard deviations.

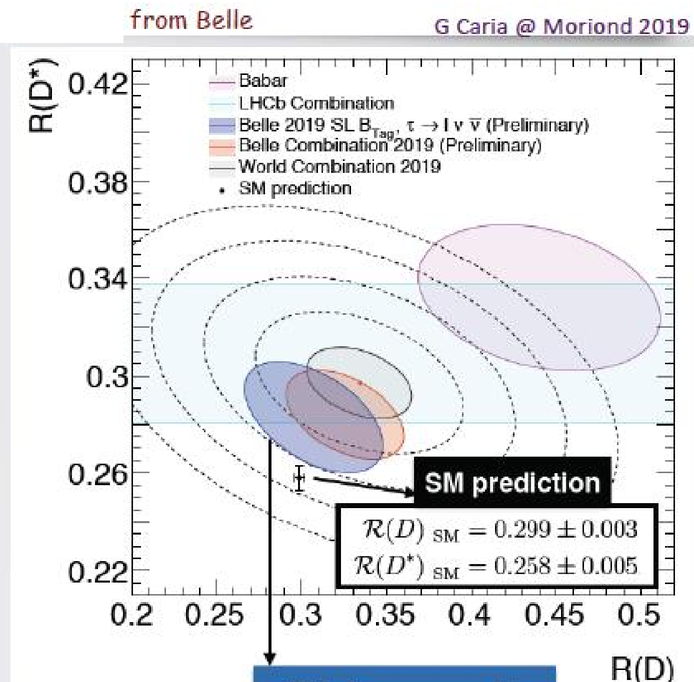
arXiv:1904.0879

Still 3.6 to 3.1 sigma effect for world average.

New physics?



- **Most precise measurement** of R(D) and R(D*) to date
- First **R(D)** measurement performed with a **semileptonic tag**
- Results **compatible with SM** expectation within **1.2σ**
- **R(D) - R(D*) Belle average** is now within **2σ** of the SM prediction
- **R(D) - R(D*) exp. world average** tension with SM expectation **decreases from 3.8σ to 3.1σ**



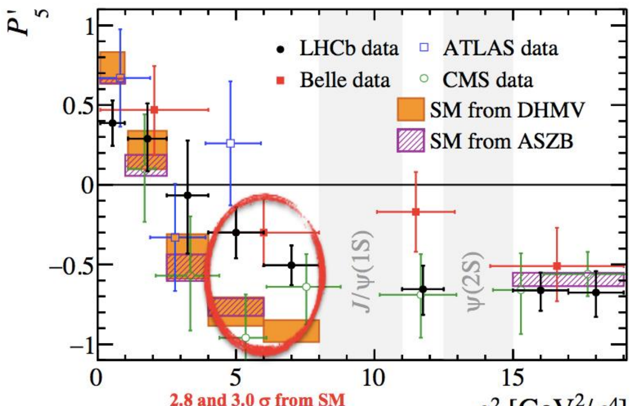
This result

$\mathcal{R}(D) = 0.307 \pm 0.037 \pm 0.016$

$\mathcal{R}(D^*) = 0.283 \pm 0.018 \pm 0.014$

B ->μμ K$^{(*)}$ anomalies

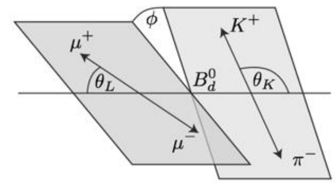
Before 2019

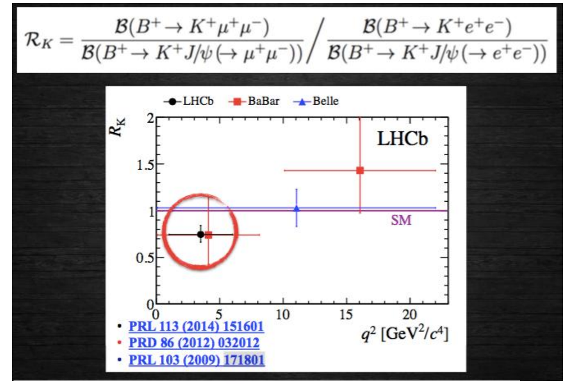


- JHEP 02 (2016) 104
- PRL 118 (2017)
- ATLAS-CONF-2017-023
- CMS-PAS-BPH-15-008

$$\frac{1}{\mathrm{d}\Gamma/\mathrm{d}q^2}\frac{\mathrm{d}^4\Gamma}{\mathrm{d}\cos\theta_L\,\mathrm{d}\cos\theta_K\,\mathrm{d}\phi\,\mathrm{d}q^2} = \frac{9}{32\pi}\Big[\frac{3(1-F_L)}{4}\sin^2\theta_K + F_L\cos^2\theta_K + \frac{1-F_L}{4}\sin^2\theta_K\cos 2\theta_L$$
$$-F_L\cos^2\theta_K\cos 2\theta_L + S_3\sin^2\theta_K\sin^2\theta_L\cos 2\phi$$
$$+S_4\sin 2\theta_K\sin 2\theta_L\cos\phi + S_5\sin 2\theta_K\sin\theta_L\cos\phi$$
$$+S_6\sin^2\theta_K\cos\theta_L + S_7\sin 2\theta_K\sin\theta_L\sin\phi$$
$$+S_8\sin 2\theta_K\sin 2\theta_L\sin\phi + S_9\sin^2\theta_K\sin^2\theta_L\sin 2\phi\Big].$$

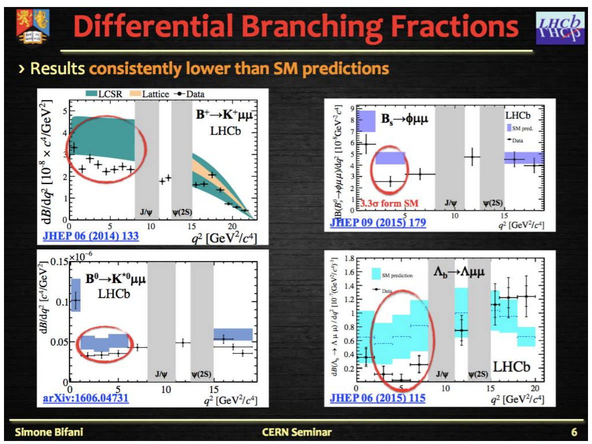
$$P_1 = \frac{2S_3}{1-F_L}$$
$$P_2 = \frac{2}{3}\frac{A_{FB}}{1-F_L}$$
$$P_3 = -\frac{S_9}{1-F_L}$$
$$P'_{i=4,5,6,8} = \frac{S_{j=4,5,7,8}}{\sqrt{F_L(1-F_L)}}.$$

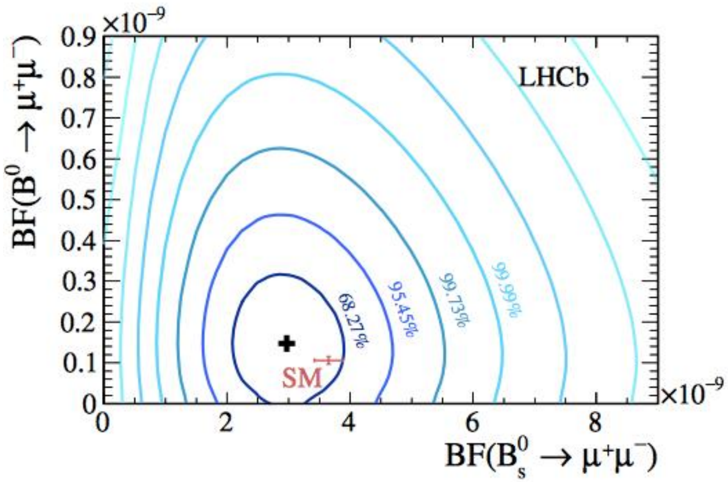




$R_K = 0.745^{+0.090}_{-0.074}\,(\mathrm{stat}) \pm 0.036\,(\mathrm{syst})$

$1\ \mathrm{GeV}^2 < q^2 < 6\ \mathrm{GeV}^2,$





LHCb arXiv:1705.03274

New data on B_s -> $\mu^+\mu^-$ from LHCb, lowered the differences

$\overline{\mathcal{B}}(B_s \to \mu^+\mu^-)_{\mathrm{LHCb}} = (3.0 \pm 0.6^{+0.3}_{-0.2}) \times 10^{-9}$,

$\overline{\mathcal{B}}(B_s \to \mu^+\mu^-)_{\mathrm{CMS}} = (3.0^{+1.0}_{-0.9} \times 10^{-9}$,

$\overline{\mathcal{B}}(B_s \to \mu^+\mu^-)_{\mathrm{avrage}} = (3.0 \pm 0.5) \times 10^{-9}$,

$B(B_s \to \mu^+\mu^-) = (3.65 \pm 0.23) \times 10^{-9}$

$B(B_d \to \mu^+\mu^-) = (1.06 \pm 0.09) \times 10^{-10}$

[Bobeth et al, PRL 112 (2014) 101801]

All these processes are induced by b -> s ll interaction.

Consistently lower than SM predictions. Combined effects are now about 4σ !

What's new since 2019?

New results: R_K from LHCb

R_K result with 2011 to 2016 data LHCb-Paper-2019-009

Imperial College London
T Humair @ Moriond 2019

Using 2011 and 2012 LHCb data, R_K was:

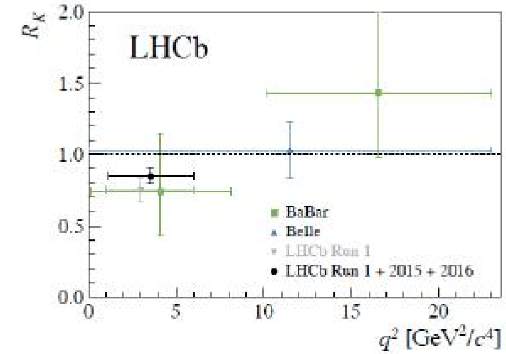
$$R_K = 0.745^{+0.090}_{-0.074}(\text{stat.}) \pm 0.036(\text{syst.}),$$

$\sim 2.6\,\sigma$ from SM (PRL113(2014)151601).

Adding 2015 and 2016 data, R_K becomes:

$$\boxed{\boldsymbol{R_K = 0.846\ {}^{+0.060}_{-0.054}(\text{stat.})\ {}^{+0.016}_{-0.014}(\text{syst.})}}$$

$\sim$ **2.5** σ from SM.



Search for lepton-universality violation in $B^+ \to K^+\ell^+\ell^-$ decays

LHCb collaboration[†]

Abstract

A measurement of the ratio of branching fractions of the decays $B^+ \to K^+\mu^+\mu^-$ and $B^+ \to K^+e^+e^-$ is presented. The proton-proton collision data used correspond to an integrated luminosity of 5.0 fb^{-1} recorded with the LHCb experiment at centre-of-mass energies of 7, 8 and 13 TeV. For the dilepton mass-squared range $1.1 < q^2 < 6.0\,\text{GeV}^2/c^4$ the ratio of branching fractions is measured to be $R_K = 0.846\,^{+0.060}_{-0.054}\,^{+0.016}_{-0.014}$, where the first uncertainty is statistical and the second systematic. This is the most precise measurement of R_K to date and is compatible with the Standard Model at the level of 2.5 standard deviations.

New results: R_K from LHCb

Branching fractions and other results LHCb-Paper-2019-009

Imperial College London
T Humair @ Moriond 2019

If instead the Run 1 and Run 2 were fitted separately:

$$R_{K\ \text{Run 1}}^{\text{new}} = 0.717^{+0.083+0.017}_{-0.071-0.016}, \quad R_{K\ \text{Run 2}} = 0.928^{+0.089+0.020}_{-0.076-0.017},$$

$$R_{K\ \text{Run 1}}^{\text{old}} = 0.745^{+0.090}_{-0.074} \pm 0.036 \quad \text{(PRL113(2014)151601)},$$

Compatibility taking correlations into account:

- Previous Run 1 result vs. this Run 1 result (new reconstruction selection): $< 1\,\sigma$;
- Run 1 result vs. Run 2 result: $1.9\,\sigma$.

$B^+ \to K^+\mu^+\mu^-$ **branching fraction**:

- Compatible with previous result (JHEP06(2014)133) at $< 1\,\sigma$;
- Run 1 and Run 2 results compatible at $< 1\,\sigma$.

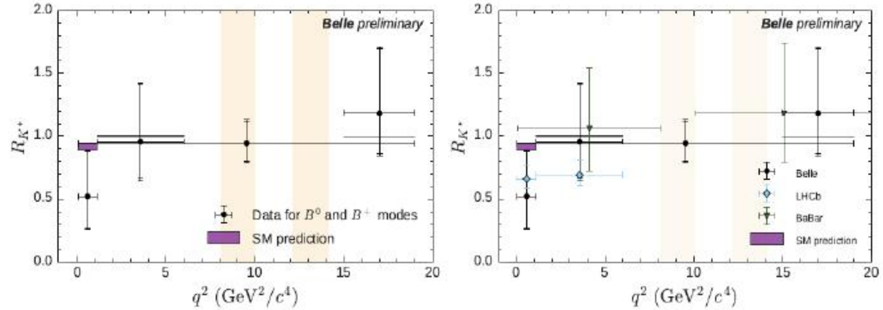
Test of lepton flavor universality in $B \to K^* \ell^+ \ell^-$ decays at Belle

arXiv:1904.002440

We present a measurement of R_{K^*}, the ratio of the branching fractions $\mathcal{B}(B \to K^* \mu^+ \mu^-)$ and $\mathcal{B}(B \to K^* e^+ e^-)$, for both charged and neutral B mesons. The ratio for charged B mesons, $R_{K^{*+}}$, is the first measurement ever performed. The analysis is based on a data sample of 711 fb^{-1}, containing 772×10^6 $B\bar{B}$ events, recorded at the $\Upsilon(4S)$ resonance with the Belle detector at the KEKB asymmetric-energy e^+e^- collider. The obtained results are compatible with standard model expectations.

$R(K^*)$: (Preliminary) Result

M Prim @ Moriond 2019



q^2 in GeV^2/c^4	All modes	B^0 modes	B^+ modes
[0.045, 1.1]	$0.52^{+0.36}_{-0.26} \pm 0.05$	$0.46^{+0.55}_{-0.27} \pm 0.07$	$0.62^{+0.60}_{-0.36} \pm 0.10$
[1.1, 6]	$0.96^{+0.45}_{-0.29} \pm 0.11$	$1.06^{+0.63}_{-0.38} \pm 0.13$	$0.72^{+0.99}_{-0.44} \pm 0.18$
[0.1, 8]	$0.90^{+0.27}_{-0.21} \pm 0.10$	$0.86^{+0.33}_{-0.24} \pm 0.08$	$0.96^{+0.56}_{-0.35} \pm 0.14$
[15, 19]	$1.18^{+0.52}_{-0.32} \pm 0.10$	$1.12^{+0.61}_{-0.36} \pm 0.10$	$1.40^{+1.99}_{-0.68} \pm 0.11$
[0.045,]	$0.94^{+0.17}_{-0.14} \pm 0.08$	$1.12^{+0.27}_{-0.21} \pm 0.09$	$0.70^{+0.24}_{-0.19} \pm 0.07$

- All measured values are in accordance with the SM and other recent measurements.
- First measurement of $R(K^{*+})$.

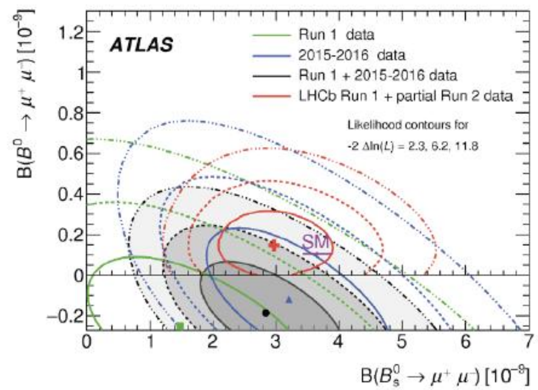
Bs -> $\mu^+\mu^-$ from ATLAS

Nikhef

Results

ATLAS, arXiv:1812.03017

O Igonkina @ Moriond 2019



- SM :
  - $Br(B_s \to \mu\mu) = (3.65 \pm 0.23) \times 10^{-9}$
  - $Br(B^0 \to \mu\mu) = (1.06 \pm 0.09) \times 10^{-10}$
- Best fit of Run 2 data :
  - $Br(B_s \to \mu\mu) = (3.2 \pm 0.9) \times 10^{-9}$
  - $Br(B^0 \to \mu\mu) = (-1.3 \pm 2.1) \times 10^{-10}$
- Run 1 + Run 2 result @ 95% CL
  - $Br(B_s \to \mu\mu) = (2.8 \pm 0.8) \times 10^{-9}$
  - $Br(B^0 \to \mu\mu) < 2.1 \times 10^{-10}$

PDG 2018: $\mathcal{B}(B_s \to \mu\mu) = (2.7^{+0.6}_{-0.5}) \times 10^{-9}$

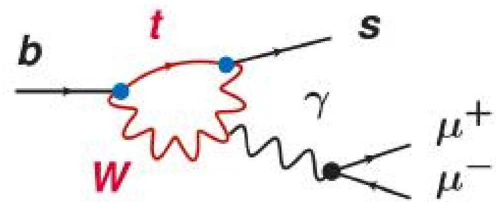
B^0 limit is most stringent at the moment

$$R^{\mathrm{exp}}_{K^+[1.1,6.0]} = 0.846^{+0.042}_{-0.039}{}^{+0.013}_{-0.012}\ [1] \qquad R^{\mathrm{th}}_{K^+} = 1.00 \pm 0.01\ [3,\ 4] \qquad 3.1\ \sigma$$

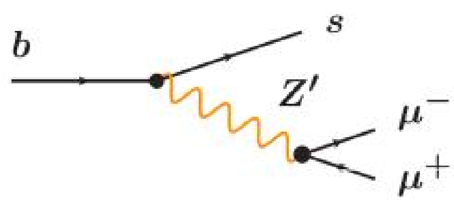
$$R^{\mathrm{exp}}_{K^{*0}[0.045,1.1]} = 0.66^{+0.11}_{-0.07} \pm 0.03\ [2] \qquad R^{\mathrm{th}}_{K^{*0}[0.045,1.1]} = 0.922 \pm 0.022\ [4] \qquad 2.3\ \sigma$$

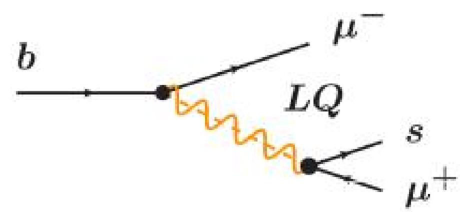
$$R^{\mathrm{exp}}_{K^{*0}[1.1,6.0]} = 0.69^{+0.11}_{-0.07} \pm 0.05\ [2] \qquad R^{\mathrm{th}}_{K^{*0}[1.1,6.0]} = 1.000 \pm 0.006\ [4] \qquad 3.4\ \sigma$$

LHCb data still show a 3.1 σ deviation from SM prediction



New physics?





All the anomalies need further experimental confirmation! But one can explore possible directions new physics may come in.

What Anomalies tell us?

anomaly – Cambridge Dictionary

noun [C or U] · **UK** /əˈnɒm.ə.li/ **US** /əˈnɑː.mə.li/ FORMAL

a person or thing that is different from what is usual, or not in agreement with something else and therefore not satisfactory:

Statistical anomalies can make it difficult to compare economic data from one year to the next.

The anomaly of the social security system is that you sometimes have more money without a job.

B decays and muon g-2 that are different from SM predictions and therefore not satisfactory.

These anomalies might be some hints of something more that just SM.

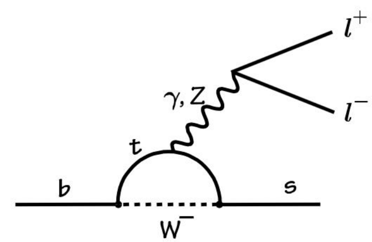
Will these anomalies stand with time??? More Data!!!

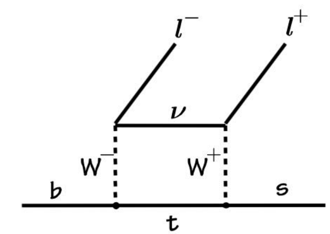
B -> s ll in the SM and beyond

$$\mathcal{H}^{bs\ell\ell}_{\text{eff, NP}} = -\mathcal{N}\left(C_7^{bs}O_7^{bs} + C_7'^{bs}O_7'^{bs} + \sum_{\ell=e,\mu}\sum_{i=9,10,S,P}\left(C_i^{bs\ell\ell}O_i^{bs\ell\ell} + C_i'^{bs\ell\ell}O_i'^{bs\ell\ell}\right)\right) + \text{h.c.}\,,$$

with the normalization factor

$$\mathcal{N} = \frac{4G_F}{\sqrt{2}}V_{tb}V_{ts}^*\frac{e^2}{16\pi^2}\,.$$





The dipole operators are given by[1]

$$O_7^{bs} = \frac{m_b}{e}(\bar{s}\sigma_{\mu\nu}P_R b)F^{\mu\nu}\,, \qquad O_7'^{bs} = \frac{m_b}{e}(\bar{s}\sigma_{\mu\nu}P_L b)F^{\mu\nu}\,,$$

where $\sigma^{\mu\nu} = \frac{i}{2}[\gamma^\mu, \gamma^\nu]$, and the semi-leptonic operators

$$\begin{aligned}
O_9^{bs\ell\ell} &= (\bar{s}\gamma_\mu P_L b)(\bar{\ell}\gamma^\mu\ell)\,, & O_9'^{bs\ell\ell} &= (\bar{s}\gamma_\mu P_R b)(\bar{\ell}\gamma^\mu\ell)\,,\\
O_{10}^{bs\ell\ell} &= (\bar{s}\gamma_\mu P_L b)(\bar{\ell}\gamma^\mu\gamma_5\ell)\,, & O_{10}'^{bs\ell\ell} &= (\bar{s}\gamma_\mu P_R b)(\bar{\ell}\gamma^\mu\gamma_5\ell)\,,\\
O_S^{bs\ell\ell} &= m_b(\bar{s}P_R b)(\bar{\ell}\ell)\,, & O_S'^{bs\ell\ell} &= m_b(\bar{s}P_L b)(\bar{\ell}\ell)\,,\\
O_P^{bs\ell\ell} &= m_b(\bar{s}P_R b)(\bar{\ell}\gamma_5\ell)\,, & O_P'^{bs\ell\ell} &= m_b(\bar{s}P_L b)(\bar{\ell}\gamma_5\ell)\,.
\end{aligned}$$

$$C^{\text{SM}}_{7\text{eff},9,10}(\mu_b) = (-0.29, 4.07, -4.31) \qquad \mu_b = 4.8\ \text{GeV}$$

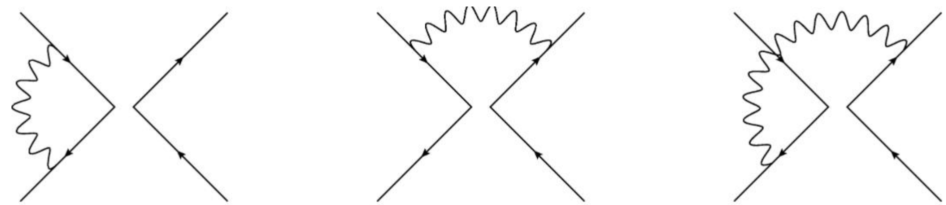
Coeff.	best fit	1σ	2σ	pull
$C_9^{bs\mu\mu}$	-0.95	$[-1.10, -0.79]$	$[-1.26, -0.63]$	5.8σ
$C_9^{\prime bs\mu\mu}$	$+0.09$	$[-0.07, +0.24]$	$[-0.23, +0.39]$	0.5σ
$C_{10}^{bs\mu\mu}$	$+0.73$	$[+0.59, +0.87]$	$[+0.46, +1.01]$	5.6σ
$C_{10}^{\prime bs\mu\mu}$	-0.19	$[-0.30, -0.07]$	$[-0.41, +0.04]$	1.6σ
$C_9^{bs\mu\mu} = C_{10}^{bs\mu\mu}$	$+0.20$	$[+0.05, +0.35]$	$[-0.09, +0.51]$	1.4σ
$C_9^{bs\mu\mu} = -C_{10}^{bs\mu\mu}$	-0.53	$[-0.62, -0.45]$	$[-0.70, -0.36]$	6.5σ
C_9^{bsee}	$+0.88$	$[+0.62, +1.15]$	$[+0.36, +1.44]$	3.4σ
$C_9^{\prime bsee}$	$+0.32$	$[+0.09, +0.61]$	$[-0.16, +0.91]$	1.3σ
C_{10}^{bsee}	-0.82	$[-1.06, -0.59]$	$[-1.31, -0.37]$	3.7σ
$C_{10}^{\prime bsee}$	-0.27	$[-0.52, -0.05]$	$[-0.78, +0.17]$	1.2σ
$C_9^{bsee} = C_{10}^{bsee}$	-1.65	$[-1.93, -1.36]$	$[-2.19, -1.02]$	4.0σ
$C_9^{bsee} = -C_{10}^{bsee}$	$+0.45$	$[+0.31, +0.59]$	$[+0.19, +0.74]$	3.6σ
$(C_S^{bs\mu\mu} = -C_P^{bs\mu\mu}) \times \mathrm{GeV}$	-0.005	$[-0.008, -0.003]$	$[-0.013, -0.001]$	2.6σ
$(C_S^{\prime bs\mu\mu} = C_P^{\prime bs\mu\mu}) \times \mathrm{GeV}$	-0.005	$[-0.008, -0.003]$	$[-0.013, -0.001]$	2.6σ

Latest fit: J. Aebischer et a;., arxiv:1903.1043
Older fits: arXiv:1307.5683, 1510.04239, 1703.09189

b -> c τ ν_τ

$$\mathcal{H}_{\rm eff} = 2\sqrt{2} G_F V_{cb} \left[(1 + C_V^L) O_V^L + C_S^R O_S^R + C_S^L O_S^L + C_T O_T \right]$$

M. Blanke et al., arXiv:1901811.09603



$$O_V^L = (\bar{c}\gamma^\mu P_L b)(\bar{\tau}\gamma_\mu P_L \nu_\tau) \, ,$$

$$O_S^R = (\bar{c} P_R b)(\bar{\tau} P_L \nu_\tau) \, ,$$

$$O_S^L = (\bar{c} P_L b)(\bar{\tau} P_L \nu_\tau) \, ,$$

$$O_T = (\bar{c}\sigma^{\mu\nu} P_L b)(\bar{\tau}\sigma_{\mu\nu} P_L \nu_\tau) \, ,$$

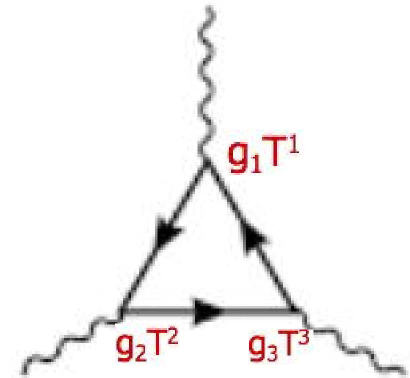
1D hyp.	best-fit	$1\,\sigma$ range	$2\,\sigma$ range	p-value (%)	pull$_{\rm SM}$
C_V^L	0.11	[0.09, 0.13]	[0.06, 0.15]	35	4.6
$C_S^R\vert_{10\%}$	0.15	[0.13, 0.15]	[0.08, 0.15]	1.7	3.8
$C_S^R\vert_{30\%,60\%}$	0.16	[0.13, 0.20]	[0.08, 0.23]	1.8	3.8
C_S^L	0.12	[0.07, 0.16]	[0.01, 0.20]	0.02	2.2
$C_S^L = 4C_T$	−0.07	[−0.12, −0.03]	[−0.15, 0.02]	0.01	1.6

Similar work: P. Asadi and D. Shih, arXiv: 1905.03311

3.3 Model buildings for FPCP beyond SM

Model building with new gauge bosons and new fermions

Quantum field theory with chiral fermion fields, have triangle anomalies generated as shown in the figure with three gauge fields as external ones and the chiral fermion in the loop. These anomalies if exist in a theory, ward identities will be destroyed and cannot be an consistent theory level. They must be cancelled – gauge anomaly cancellation.

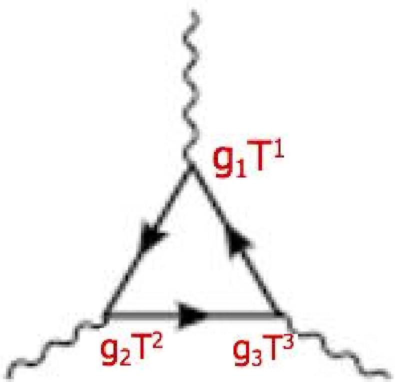


Normalizing the contributions by right handed $1+\gamma_5$ chiral fermion in the loop to be positive proportional to the couplings, then left handed $1-\gamma_5$ chiral fermion in the loop will be negative. The total sign also depends on the couplings $g_1T^1\, g_2T^2\, g_3T^3$.

The cancellation can happen by summing up left and right handed Fermion with appropriate couplings contributions. If vector fermion, no anomalies generated.

This is a powerful tool for model building with new fermions and gauge bosons.

Gauge anomaly cancellation in the SM



The standard model of strong and electroweak interaction has gauge group

$SU(3)_C \times SU(2)_L \times U(1)_Y$ with gauge bosons

8 $SU(3)_C$ Gluons : $G^\mu = \frac{\lambda^a}{2} G^\mu_a$, $Tr(\frac{\lambda^a}{2}\frac{\lambda^b}{2}) = \frac{\delta^{ab}}{2}$.

3 $SU(2)_L$ W-bosons : $W^\mu = \frac{\sigma^i}{2} W^\mu_i$, $Tr(\frac{\sigma^i}{2}\frac{\sigma^j}{2}) = \frac{\delta^{ij}}{2}$.

1 $U(1)_Y$ B boson : B^μ

The building blocks of fermions are chiral fields $f_{L,R} = \frac{1 \mp \gamma_5}{2} f$

The SM fermions are leptons L_L, E_R and quarks Q_L, U_R and D_R

$L_L = (\nu_L, e_L : (1,2)(-1/2)^T$, $e_R : (1,1)(-1)$,

$Q_L = (u_L, d_L)^T : (3,2)(1/6)$, $u_R : (3,1)(2/3)$, $d_R : (3,1)(-1/3)$.

Type of anomalies:

GGG (3 grkuons): automatically zero, because under $SU(3)_C$ all fermions are vector like.

WWW: also automatically zero, because $T_i = \sigma_i$ $Tr(\sigma_i\sigma_j+\sigma_j\sigma_i)\sigma_k)=0$

GGW, WWG, BBG, BBW, GWB all are zero due to trace of one single T_i is zero.

Nonzero ones: GGB, WWB, BBB, and ggB for individual fermion in the loop

Two gravitation gg and a B

One generation of SM fermion contributions to gauge anomalies

	u_R	d_R	u_L	d_L	e_R	ν_L	e_L	sum
GGB	2/3	-1/3	-(1/6)	-(1/6)	0	0	0	0
WWB	0	0	-3(1/6)	-3(1/6)	0	-(-1/2)	-(-1/2).	0
BBB	$3(2/3)^3$.	$3(-1/3)^3$.	$-3(1/6)^3$.	$-3(1/6)^3$.	$(-1)^3$	$-(-1/2)^3$	$-(-1/2)^3$	0
ggB	2/3	-1/3	-(1/6)	-(1/6)	-1	-(-1/2)	-(-1/2)	0

All anomalies are automatically cancelled!

One of the reasons for having two Higgs doublets H_1 (1,2)(-1/2) and H_2(1,2)(1/2)
Because Higgsino is a chiral fermion, it produce gauge anomalies
WWB -1/2 + 1/2= 0; BBB $(-1/2)^3 + (1/2)^3 = 0$; ggB. (-1/2) + (1/2) = 0!

Another example: $SU(3)_C \times SU(2)_L \times U(1)_Y \times U(1)_{\mu-\tau}$

(PRD 43 (1991) R22; PRD 44(1991) 2118)

$U(1)_{\mu-\tau}$ charges: 0 for u_R, d_R, u_L d_L and e_R e_L,

+1 for μ_R, $\nu_{\mu L}$, μ_L, -1 for τ_R $\nu_{\tau L}$, τ_L

New anomalies (indicate $U(1)_{\mu-\tau}$ gauge boson as Z')

	μ_R	$\nu_{\mu L}$	μ_L	τ_R	$\nu_{\tau L}$,	τ_L	sum
WWZ'	1	-1	-1	-1	-(-1)	-(-1)	0
BBZ'	$(-1)^2$x1	$-(-1/2)^2$x(1)	$-(-1/2)^2$x(1).	$(-1)^2$x(-1)	$-(-1/2)^2$x(-1)	$-(-1/2)^2$x(-1)	0
Z'Z'B	$(1)^2$x(-1)	$-1^2(-1/2)$	$-1^2(-1/2)$	$(-1)^2$x(-1)	$-(-1)^2(-1/2)$	$-(-1)^2(-1/2)$	0
Z'Z'Z'	1^3	-1^3	-1^3	$(-1)^3$.	$-(-1)^3$	$-(-1)^3$.	0
ggZ'	1	-1	-1	-1	-(-1)	-(-1)	0

Gauge anomaly free. The simplest model with a new Z' model!

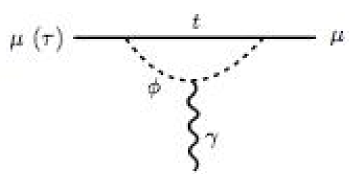
Models for muon g-2, $R_{D(*)}$ and $R_{K(*)}$ anomalies

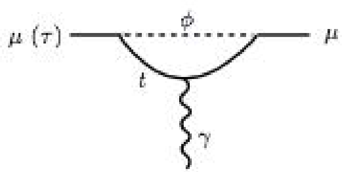
Many theoretical models have been proposed to solve the muon g-2, $R_{D(*)}$ and $R_{K(*)}$ anomalies discussed previously.

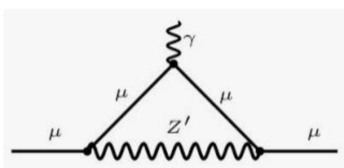
Multy Higgs, SUSY, Z', Leptoquarks…

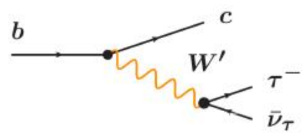
Some of them dealing with only for muon g-2, or $R(D_{(*)})$ or $R(K^{(*)})$

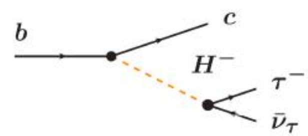
Relatively easy.

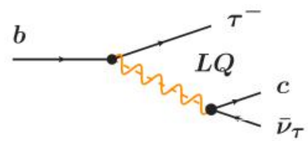


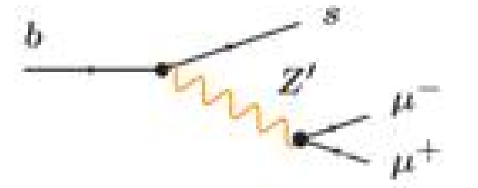












When want to solve two of them, the constrain become more stringent

When want to take on three of them together, the task becomes more difficult

But efforts have been made and possible!

I now discuss some examples.

A gauge model solving $R_{D(*)}$ and $R_{K(*)}$ anomalies

$$SU(3)\times SU(2)_l \times SU(2)_h \times U(1)_Y$$

S. Bouncenna et al., arxiv:1604.03088
C-W. Chiang, X-G He, G. Valencia,PRD93,074003.

$$Q_L^{1,2}:(3,2,1,1/3)\,,\quad Q_L^3:(3,1,2,1/3)\,,\quad U_R^{1,2,3}:(3,1,1,4/3)\,,\quad D_R^{1,2,3}:(3,1,1,-2/3)\,,$$

$$L_L^{1,2}:(1,2,1,-1)\,,\quad L_L^3:(1,1,2,-1)\,,\quad E_R^{1,2,3}:(1,1,1,-2)\,,$$

$SU(2)_l$ for 1st generation $SU(2)_h$ for 2nd and 3rd generations SU(2)L

The charged quark currents can be in the quark mass eigen-basis as

$$\begin{aligned}\mathcal{L}_{charged} &= \frac{g}{\sqrt{2}s_E c_E} W_h^{+\mu}\left(s_E^2 \bar{U}_L\gamma_\mu V_{KM} D_L - \bar{U}_L T_U N T_D^\dagger D_L\right)\\ &+ \frac{g}{\sqrt{2}} W_l^{+\mu}\left(\bar{U}_L \gamma_\mu V_{KM} D_L\right) + h.c.\ .\end{aligned}$$

The matrices $T_{\psi,U,D}$ diagonalize the left handed fermion weak eigen-states to obtain the mass eigen-states. The weak eigen-states are given by $T_\psi \psi$. In the limit $s_E^2 c_\beta^2 - c_E^2 s_\beta^2 = 0$, $Z_{l,h}$ and $W_{l,h}$ are mass eigen-states with

$$m^2_{Z_h,W_h} = \frac{u^2 g^2}{2c_E^2 s_E^2} + \frac{v^2 g^2}{4}\,,\quad m^2_{Z_l} = \frac{v^2(g^2+g'^2)}{4}\,,\quad m^2_{W_l} = \frac{v^2 g^2}{4}\,.$$

The quark neutral currents can be expressed in the physical basis as

$$\begin{aligned}\mathcal{L}_{NC} &= \bar{\psi}\gamma_\mu\left\{eA^\mu Q + gZ_h^\mu T_\psi\left[\frac{s_E}{c_E}T_3^l - \frac{c_E}{s_E}T_3^h\right]T_\psi^\dagger + \frac{gZ_l^\mu}{c_W}\left[-s_W^2 Q + T_3\right]\right\}\psi\\ &= \bar{\psi}\gamma_\mu\left\{eA^\mu Q + \frac{g}{s_E c_E}Z_h^\mu\left[s_E^2 T_3 - T_\psi T_3^h N T_\psi^\dagger\right] + \frac{gZ_l^\mu}{c_W}\left[-s_W^2 Q + T_3\right]\right\}\psi\end{aligned}$$

with $T_3 = T_3^l + T_3^h$, $Q = Y + T_3$ and $N = \mathrm{diag}(0,0,1)$.

Exchange W_h solve $R_{D(*)}$!

Exchange Z_H solve $R_{K(*)}$!

$$\mathcal{L}_{NC} = \frac{g}{s_E c_E} Z_h^\mu\left[\bar{D}_L\gamma_\mu(s_E^2 T_3 + \frac{1}{2}T_D N T_D^\dagger)D_L + \bar{U}_L\gamma_\mu(s_E^2 T_3 - \frac{1}{2}V_{KM}T_D N T_D^\dagger V_{KM}^\dagger)U_L\right]$$

$$\mathcal{L}_{charged} = \frac{g}{\sqrt{2}s_E c_E}W_h^{+\mu}\left(s_E^2 \bar{U}_L V_{KM}\gamma_\mu D_L - \bar{U}_L V_{KM}\gamma_\mu T_D N T_D^\dagger D_L\right)$$

Similar for Z_h and W_h interactions with leptons.

S_E small limit works well !

Triplet vector and $SU(3)_C x SU(2)_l x SU(2)_h x U(1)_Y$

D. Buttazzo et al., arXiv: 1706.07808,   Cheng-Wei Chiang, X-G He and G. Valencia

X_μ: (3,1)(0) This is the W_h in $SU(3)_C x SU(2)_l x SU(2)_h x U(1)_Y$

$$X_\mu = \frac{1}{\sqrt{2}} \begin{pmatrix} X^0_\mu & \sqrt{2} X^+_\mu \\ \sqrt{2} X^-_\mu & -X^0_\mu \end{pmatrix} \qquad L_{int} = \bar{Q}_L \gamma^\mu X_\mu \Delta^Q Q_L + \bar{L}_L \gamma^\mu X_\mu \Delta^L L_L$$

$$\begin{aligned} H_{eff} = &\frac{1}{m^2_{X^\pm}} [\bar{u}_L \gamma^\mu V_{KM} \Delta^Q d_L \bar{e}_L \gamma_\mu \Delta^L \nu_L + \bar{d}_L \gamma^\mu \Delta^Q V^\dagger_{KM} u_L \bar{\nu}_L \gamma_\mu \Delta^L e_L] \\ + &\frac{1}{2 m^2_{X^0}} [\bar{u}_L \gamma^\mu V_{KM} \Delta^Q V^\dagger_{KM} u_L \bar{\nu}_L \gamma_\mu \Delta^L \nu_L - \bar{d}_L \gamma^\mu \Delta^Q d_L \bar{\nu}_L \gamma_\mu \Delta^L \nu_L \\ &- \bar{u}_L \gamma^\mu V_{KM} \Delta^Q V^\dagger_{KM} u_L \bar{e}_L \gamma_\mu \Delta^L e_L + \bar{d}_L \gamma^\mu \Delta^Q d_L \bar{e}_L \gamma_\mu \Delta^L e_L] \, . \end{aligned}$$

Exchange Xμ obtains

$$H_{eff}(R_{D^{(*)}}) = \frac{(V_{KM}\Delta^Q)_{23}\Delta^L_{3l}}{m^2_{X^\pm}} \bar{c}_L\gamma^\mu b_L \bar{\tau}_L\gamma_\mu \nu^l_L \ ,$$

$$H_{eff}(R_{K^{(*)}}) = \frac{\Delta^Q_{23}\Delta^L_{22}}{2m^2_{X^0}} \bar{s}_L\gamma^\mu b_L \bar{\mu}_L\gamma_\mu \mu_L \ .$$

For $R_{D(*)}$

$$H_{eff} = \frac{4G_F}{\sqrt{2}} V_{cb}(\delta_{3,l} + \epsilon_{3,l})\bar{c}\gamma^\mu P_L b \bar{\tau}\gamma_\mu P_L \nu^l \ ,$$

$$\epsilon_{3,l} = \frac{\sqrt{2}}{4G_F V_{cb}} \frac{V_{2i}\Delta^Q_{i3}\Delta^L_{3l}}{m^2_{X^\pm}} \ .$$

$$H_{eff} = -\frac{4G_F}{\sqrt{2}} V_{tb}V^*_{ts} \sum C_i O_i$$

For $R_{K(*)}$

$$C_9^{NP} = -C_{10}^{NP} = -\frac{\sqrt{2}\pi}{\alpha G_F V_{tb} V^*_{ts}} \frac{\Delta^Q_{23}\Delta^L_{22}}{4m^2_{X^0}} \ .$$

Try the best fit values: $\epsilon_{3,\tau} = 0.11$ **and** $C_9^N = -0.53$. $V_{cb} \approx 0.04$, $V_{ts} \approx -0.04$

Allow $\frac{\Delta m_{B_s}^{NP}}{\Delta m_{B_s}^{SM}}$ **to be 0.1 (2σ bound),** $\Delta_{23}^Q \approx \pm 0.0068$ **with** $m_X = 1 TeV$.

Solve $R_{K^{(*)}}$ **anomaly,** $C_9^{NP} \approx -0.56 \rightarrow \Delta_{22}^L \approx \mp 0.26$.

Solve $R_{D^{(*)}}$ **anomaly,** $\sum_l \epsilon_{3,l} \approx 0.11$.

$\epsilon \sim (V_{cd}\Delta_{13}^Q + V_{cs}\Delta_{23}^Q + V_{cb}\Delta_{33}^Q)\Delta_{33}^L$

Prefer to have Δ_{23}^Q **positive.**

Then $\Delta_{33}^Q \Delta_{33}^L \approx 4$**, taking each about 2,**

large, but OK solution, although kind of large.

limit from $R_{B \rightarrow K\nu\bar{\nu}} < 4$**, OK**

$D \rightarrow \mu^+\mu^-$ **OK, since can set** $\Delta_{12}^Q = 0$.

The model can have reasonable solution for both $R_{D(*)}$ and $R_{K(*)}$ anomalies. However, difficult to get muon g-2 right.

Leptonquark solution to $RD_{(*)}$, $R_{K(*)}$ and muon g-2 anomalies

Scalar Leptoquarkls

$$\bar{Q}_L e_R R_1\ ,\quad \bar{U}_R L_L R_2\ ,\ ;\bar{D}_R L_L \tilde{R}_2\ \bar{L}^c_L Q_L S_{1,3}\ ,\quad \bar{e}^c_R U_R S_1\ ,\bar{e}^c_R D_R S_2\ ,$$

$$S_1 : (\bar{3},1)(1/3)\ ,\quad S_3 : (\bar{3},3)(1/3)\ ,\quad R_2 : (3,2)(7/6)\ ,\quad \tilde{R}_2 : (3,2)(1/6)$$

$R_2 : C_9 = C_{10}$ *out* ; $\tilde{R}_2$ cannot explain $R_{D^{(*)}}$ *out* ;

S_2 cannot explain $R_{D^{(*)}}$ *out* ; S_3 dose not allow $R^{exp}_{D^{(*)}} > R^{SM}_{D^{(*)}}$ *out*

S_3 and $S_1 = \phi$ interactions

$$L_1 = \bar{L}^c_L X Q_L \phi + \bar{U}_R Y e^c_R \phi^* + h.c.\ ,\quad X = (x_{ij})\ ,\quad X V^\dagger_{KM} = (z_{ij})\ ,\quad Y = (y_{ij})$$

$$L_3 = \bar{L}^c_L \tilde{X} Q_L S_3 + \bar{U}_R \tilde{Y} e^c_R S^*_3 + h.c.\ ,\quad \tilde{X} = (\tilde{x}_{ij})\ ,\quad \tilde{X} V^\dagger_{KM} = (\tilde{z}_{ij})\ ,\quad Y = (\tilde{y}_{ij})\ .$$

The S_3 case: problem with $R_{D(*)}$

$$
\begin{aligned}
H_3 = \quad & -\frac{\tilde{x}_{ij}\tilde{x}^*_{kl}}{m^2_{S_3}}\Big(\bar{d}^l_L\gamma_\mu d^j_L \bar{e}^k_L\gamma^\mu e^i_L + (\bar{u}_L V_{KM})^l\gamma_\mu(V^\dagger_{KM}u_L)^j\bar{\nu}^k_L\gamma^\mu\nu^i_L \\
& +\frac{1}{2}((\bar{u}_L V_{KM})^l\gamma_\mu d^j_L\bar{e}^k_L\gamma^\mu\nu^i_L + \bar{d}^l_L\gamma_\mu(V^\dagger_{KM}u_L)^j\bar{\nu}^k_L\gamma^\mu e^i_L) \\
& +\frac{1}{2}(\bar{d}^l_L\gamma_\mu d^j_L\bar{\nu}^l_L\gamma^\mu\nu^i_L + (\bar{u}_L V_{KM})^l\gamma_\mu(V^\dagger_{KM}u_L)^j\bar{e}^k_L\gamma^\mu e^i_L)\Big) \ ,
\end{aligned}
$$

The contribution to $R_{D^{(*)}}$ is proportional to

$$
-\tilde{x}_{33}(\tilde{x}^*_{33} + \frac{V_{cd}}{V_{cb}}\tilde{x}^*_{31} + \frac{V_{cs}}{V_{cb}}\tilde{x}^*_{32}) \ .
$$

The first term dominate.

This make $R^{exp}_{D^{(*)}} < R^{SM}_{D^{(*)}}$, and therefore is ruled out.

The S_1 case: tree and one loop level

$$L_{int} = \frac{1}{2m_\phi^2}[x_{ij}x_{i'j'}\bar{\nu}_L^{i'}\gamma^\mu \nu_L^i \bar{d}_L^{j'}\gamma_\mu d_L^j + z_{ij}z_{i'j'}\bar{e}_L^{i'}\gamma^\mu e_L^i \bar{u}_L^{j'}\gamma_\mu u_L^j]$$

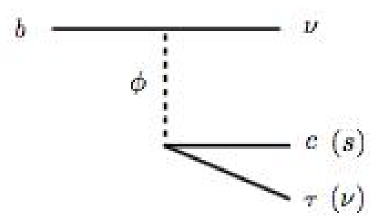
K-> $\pi\, \nu\nu$, B-> $K^{(*)}\, \nu\nu$ D->$\mu\mu$, $\pi\mu\mu$

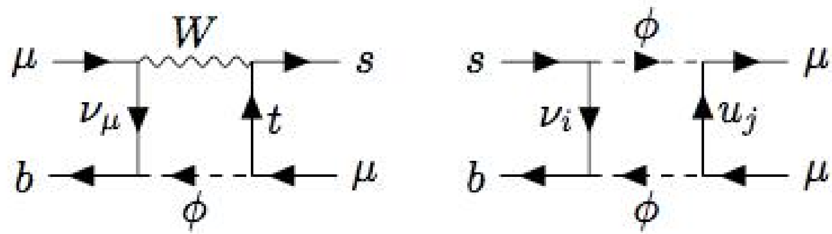
$$-\frac{1}{2m_\phi^2}[x_{ij}z_{i'j'}\bar{\nu}_L^{i'}\gamma^\mu e_L^i \bar{d}_L^{j'}\gamma_\mu u_L^j - x_{ij}z_{i'j'}\bar{e}_L^{i'}\gamma^\mu \nu_L^i \bar{u}_L^{j'}\gamma_\mu d_L^j]$$

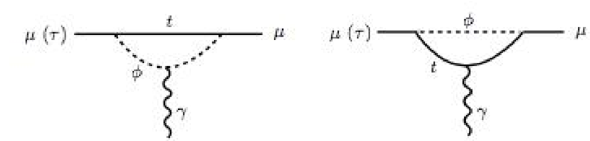
$R(D^{(*)})$, B->$D^{(*)}$ (ρ,π) $l\nu$, B_c -> $\tau\, \nu$

$$+\frac{1}{2m_\phi^2}[y_{ij}y_{i'j'}\bar{e}_R^{i'}\gamma^\mu e_R^i \bar{u}_R^{j'}\gamma_\mu u_R^j + x_{ij}y_{i'j'}(\bar{e}_R^{i'}\nu_L^i \bar{u}_R^{j'}\gamma_\mu d_L^j - \frac{1}{2}\bar{e}_R^{i}\sigma_{\mu\nu}\nu_L^{i'}\bar{u}_R^{j'}\sigma_{\mu\nu}d_L^j)]$$

D->$\mu\mu$, $\pi\, \mu\mu$ $R(D^{(*)})$, B->$D^{(*)}$ (ρ,π) $l\nu$, B_c -> $\tau\, \nu$







Solution to $R(D^{(*)})$ Solution to b-> s $\mu^+\mu^-$ induced anomalies Solution to $(g-2)_\mu$

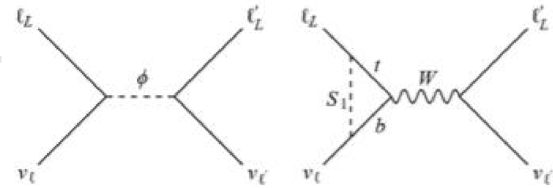
M. Bauer and M. Neubert, arXiv: 1511.01900; X-G. He and N. Deshpande, arXiv: 1608.04817

If R-parity violating interaction, exchange sd-quark, the last line is absent. That is the reason why R-parity cannot solve $R(_{D(*)})$ and b -> s $\mu^+\mu^-$ anomalies (Deshpande and He)

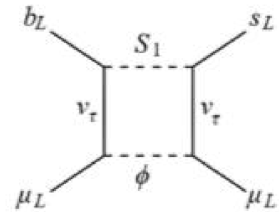
Also why Baur&Neubert, and Becrivic et al could not work, neglect last term contributions to R(D(*)) and lead to conflict for b -> s $\mu^+\mu^-$ when other constraints are included, important one B -> K(*) $\nu\nu$! (R = $B(B \to K^{(*)} \nu\nu)_{exp}$/ $B(B \to K(*)\, \nu\nu)_{SM}$ < 4.3! (Becirevic et al.)

Combining S_1 and a ϕ^+ to also solve KM unitarity anomaly

D. Marzocca and s. Trifinopoulos, arXiv: 2104.0573



S1: (3*, 1)(1/3); ϕ^+ (1,1)(1). New contributions



$$\mathcal{L}_{S1+\phi} = \frac{1}{2}\lambda_{\alpha\beta}\bar{\ell}^c_\alpha \epsilon \ell_\beta \phi^+ + \lambda^{1L}_{i\alpha}\bar{q}^c_i \epsilon \ell_\alpha S_1 + \lambda^{1R}_{i\alpha}\bar{u}^c_i e_\alpha S_1 + \text{h.c.}$$

$$\Delta a_\mu \approx \frac{m_\mu m_t \lambda^{1L}_{b\mu}\lambda^{1R}_{t\mu}}{4\pi^2 M_1^2}\left(\log M_1^2/m_t^2 - \frac{7}{4}\right)$$

$$\delta(\mu \to e\nu\nu) \approx \frac{v^2|\lambda_{12}|^2}{4M_\phi^2} + \frac{3m_t^2|\lambda^{1L}_{b\mu}|^2}{32\pi^2 M_1^2}\left(\frac{1}{2} - \log\frac{M_1^2}{m_t^2}\right),$$

$$R_D \approx 0.299 - 0.235\frac{\lambda^{1L}_{b\tau}\lambda^{1R}_{c\tau}}{m_1^2}\left(1 + 0.05\log m_1^2\right),$$

$$R_{D^*} \approx 0.258 - 0.088\frac{\lambda^{1L}_{b\tau}\lambda^{1R}_{c\tau}}{m_1^2}\left(1 + 0.02\log m_1^2\right),$$

$$V^\beta_{us} \equiv \sqrt{1 - (V^\beta_{ud})^2 - |V_{ub}|^2} \simeq V^{\rm CKM}_{us}\left[1 - \left(\frac{V^{\rm CKM}_{ud}}{V^{\rm CKM}_{us}}\right)^2 \delta(\mu \to e\nu\nu)\right]$$

$$C_{LL} \approx -\lambda^{1L}_{b\tau}\lambda^{1L*}_{s\tau}\left(\frac{|\lambda^{1L}_{b\mu}|^2}{64\pi^2 M_1^2} + \frac{|\lambda_{\mu\tau}|^2\log M_\phi^2/M_1^2}{64\pi^2(M_\phi^2 - M_1^2)}\right)$$

$$C_{LR} \approx -\frac{|\lambda^{1R}_{c\mu}|^2\lambda^{1L}_{b\tau}\lambda^{1L*}_{s\tau}}{64\pi^2 M_1^2}.$$

$$\lambda_{e\mu} = 1.35\,, \quad \lambda_{\mu\tau} = 3.17\,,$$
$$\lambda^{1L}_{b\tau} = 1.46\,, \quad \lambda^{1L}_{s\tau} = -0.54\,, \quad \lambda^{1L}_{b\mu} = 2.07\,,$$
$$\lambda^{1R}_{c\tau} = -3.28\,, \quad \lambda^{1R}_{t\mu} = 0.01\,, \quad \lambda^{1R}_{c\mu} = 2.35\,,$$

Muon g-2, $R_{D(*)}$, $R_{K(*)}$ and KM unitarity anomalies can all be addressed!

Beyond SM CP Violation

What is the origin of CPV? Is there other ways CPV can present in a model?

There are many ways CPV can show up when going beyond SM:

Superweak model, CP is only violated in $\Delta S=2$ current-current interactions. Too small ε'/ε. Ruled out by data.

In left-right $SU(3)_C xSU(2)_L xSU(2)_R xU(1)_{B-L}$ symmetric model, there are similar mixng matrix $V^R{}_{KM}$ charged current for right-handed fermions by exchanging $W_{R.}$ More phases, only two generations can have CPV.

Seesaw Model, there are new phases in Right-handed neutrino mass matrix.

…

CP violated by vacuum? Not explicitly violated as that in SM, T-D Lee, spontaneous CP violation.

…

Spontaneous CP violation in two Higg doublet model

In the SM it is not possible to have spontaneous CP violation.

It requires at least two Higgs doublets to have a realistic model,

H_1 and H_2 transforming as $(1, 2, 1/2)$

$$
\begin{aligned}
V(H_1, H_2) &= \mu_{ij}^2 H_i^\dagger H_j + \lambda_i (H_i^\dagger H_i)^2 + \lambda'_{ij} (H_i^\dagger H_j)(H_j^\dagger H_i) \\
&+ [\delta (H_1^\dagger H_2)(H_1^\dagger H_2) + \delta_1 (H_1^\dagger H_2)(H_1^\dagger H_1) \\
&+ \delta_2 (H_1^\dagger H_2)(H_2^\dagger H_2) + H.C.] \, .
\end{aligned}
$$

The vevs minimizes the potentais are: $\langle H_i \rangle \to v_i exp(i\theta_i)/\sqrt{2}$,

The Higgs fields can be written as: $H_i = (h_i^+, \frac{1}{\sqrt{2}}(v_i e^{i\theta_i} + h_i^0 + iI_i^0))$

The minmimaztion condition for the phase is

$$(\mu_{12}^2+\mu_{21}^2)v_1v_2\sin\theta+2\delta v_1^2v_2^2\sin(2\theta)+(\delta_1 v_1^3 v_2+\delta_2 v_1 v_2^3)\sin\theta=0\ ,$$

$$\theta=0\ ,\ \text{or}\ ,\ \theta=-arc\cos\left(\frac{(\mu_{12}^2+\mu_{21}^2)+\delta_1 v_1^2+\delta_2 v_2^2}{4\delta v_1 v_2}\right)$$

Here $\theta=\theta_1-\theta_2$.

$\sin\theta=0$ solution is no CP violating, $\cos\theta\neq\pm1$, violates CP.

The most general Yukawa interactions with quarks are given by

$$L_Y = -\bar{Q}_L(\lambda_1^U \tilde{H}_1 + \lambda_2^U \tilde{H}_2)U_R - \bar{Q}_L(\lambda_1^D H_1 + \lambda_2^D H_2)D_R + H.C.$$

All $\lambda_i^{U,D}$ are real (spontaneous CPV)

$$\begin{aligned}
L_Y &= -\bar{D}_L[(\lambda_1^U)(-(h_1^+)^*) + (\lambda_2^U)(-(h_2^+)^*))D_L - \bar{U}_L(\lambda_1^D h_1^+ + \lambda_2^D h_2^+]D_R \\
&- \bar{U}_L[\lambda_1^U \frac{1}{\sqrt{2}}(v_1 e^{-i\theta_1} + (h_1^0 + iI_1^0)^*) + \lambda_2^U \frac{1}{\sqrt{2}}(v_2 e^{-i\theta_2} + (h_2^0 + iI_2^0)^*)]U_R \\
&- \bar{D}_L[\lambda_1^D \frac{1}{\sqrt{2}}(v_1 e^{i\theta_1} + h_1^0 + iI_1^0) + \lambda_2^D \frac{1}{\sqrt{2}}(v_2 e^{i\theta_2} + h_2^0 + iI_2^0)]D_R \\
&+ H.C.
\end{aligned}$$

Remove Goldstone bosons Z_I and h_W^+ "eaten" by Z and W^+

$$Z_I = \frac{v_1 e^{-i\theta_1} I_1^0 + v_2 e^{-i\theta_2} I_2^0}{\sqrt{v_1^2 + v_2^2}}, \quad h_W^+ = \frac{v_1 e^{-i\theta_1} h_1^+ + v_2 e^{-i\theta_2} h_2^+}{\sqrt{v_1^2 + v_2^2}},$$

The orthogonal components are

$$A = \frac{-v_2 e^{i\theta_2} I_1^0 + v_1 e^{i\theta_1} I_2^0}{\sqrt{v_1^2 + v_2^2}}, \quad H^+ = \frac{-v_2 e^{i\theta_2} h_1^+ + v_1 e^{i\theta_1} h_2^+}{\sqrt{v_1^2 + v_2^2}},$$

Normalize in a same way for real scalars

$$h = \frac{v_1 e^{-i\theta_1} h_1^0 + v_2 e^{-i\theta_2} h_2^0}{\sqrt{v_1^2 + v_2^2}}, \quad H = \frac{-v_2 e^{i\theta_2} h_1^0 + v_1 e^{i\theta_1} h_2^0}{\sqrt{v_1^2 + v_2^2}},$$

Quark mass matrices

$$\bar{U}_L M^u U_R = \bar{U}_L \frac{1}{\sqrt{2}}(\lambda_1^U v_1 e^{-i\theta_1} + \lambda_2^U v_2 e^{-i\theta_2}) U_R$$
$$\bar{D}_L M^d D_R = \bar{D}_L \frac{1}{\sqrt{2}}(\lambda_1^D v_1 e^{i\theta_1} + \lambda_2^D v_2 e^{i\theta_2}) D_R$$

$$\begin{aligned} L_Y &= -\bar{D}_L(\lambda_1^U v_2 e^{-i\theta_2} - \lambda_2^U v_1 e^{-i\theta_1}) U_R \frac{h^-}{v} + \bar{U}_L(\lambda_1^D v_2 e^{i\theta_2} - \lambda_2^D v_1 e^{i\theta_1}) D_R \frac{h^+}{v} \\ &- \bar{U}_L[M^u(1+\frac{h}{v}) - (\lambda_1^U v_2 e^{-i\theta_2} - \lambda_2^U v_1 e^{-i\theta_1})\frac{H - iA}{\sqrt{2}v}] U_R \\ &- \bar{D}_L[M^d(1+\frac{h}{v}) - (\lambda_1^D v_2 e^{i\theta_2} - \lambda_2^D v_1 e^{i\theta_1})\frac{H + iA}{\sqrt{2}v}] D_R \\ &+ H.C. \end{aligned}$$

Making making chiral rotations: $U_R \to e^{-i\theta_1\gamma_5} U_R$ and $D_R \to e^{i\theta_1\gamma_5} D_R$

Note that the net contribution to strong θ is zero: $\theta_u + \theta_d = \theta_1 - \theta_1 = 0$

Finally, we have

$$
\begin{aligned}
L_Y &= -\bar{D}_L(\lambda_1^U v_2 e^{-i\theta} - \lambda_2^U v_1) U_R \frac{h^-}{v} + \bar{U}_L(\lambda_1^D v_2 e^{i\theta} - \lambda_2^D v_1) D_R \frac{h^+}{v} \\
&- \bar{U}_L[M^u(1 + \frac{h}{v}) - (\lambda_1^U v_2 e^{-i\theta} - \lambda_2^U v_1)\frac{H - iA}{v}]U_R \\
&- \bar{D}_L[M^d(1 + \frac{h}{v}) - (\lambda_1^D v_2 e^{i\theta} - \lambda_2^D v_1)\frac{H + iA}{v}]D_R \\
&+ H.C.
\end{aligned}
$$

In this basis,

$$
\bar{U}_L M^u U_R = \bar{U}_L \frac{1}{\sqrt{2}}(\lambda_1^U v_1 + \lambda_2^U v_2 e^{-i\theta}) U_R
$$

$$
\bar{D}_L M^d D_R = \bar{D}_L \frac{1}{\sqrt{2}}(\lambda_1^D v_1 + \lambda_2^D v_2 e^{i\theta}) D_R
$$

Where are CP violation?

1. The $M^{u,d}$ are complex, rotating into real quark eigen-mass basis,

generate a $\theta - term = Arg(Det(M^u M^d))$.

2. Diagonalize quark mass matrices, generating complex V_{KM}.

3. h^+, H and A have complex couplings, new source of CP violation!

4. h, H and A are not mass eigen-state yet.

Mixing of h and H with A, new source of CP violation.

Similar analysis can be carried out.

5. Where new FCNC come in?

Neutral Higgs H and A coupling to both λ_1 and λ_2

Can avoid FCNC, then need more Higgs doublets, Weinberg Model, 3 Higgs doublets.

Forbidden FCNC by symmetry principle

The previous Two has Higgs doublet model has FCNC. If want does not want such FCNC to mediated by the new neutral Higgs bosons H and A, what can one do? Usual practice by symmetry principle.

Example: Z_2 symmetry: H_1 -> H_1, H_2 -> - H_2, U_R -> U_R, D_R -> -D_R, Q_L -> Q_L
others no changes, require Lagrangian L does not change under this transformation, the Yukawa and Higgs potential term on L are given by

$$L_Y = -\bar{Q}_L \lambda_1^U \tilde{H}_1 U_R - \bar{Q}_L \lambda_2^D H_2 D_R + H.C.$$

$$\begin{aligned} V(H_1, H_2) = & \mu_1^2 H_1^\dagger H_1 + \mu_2^2 H_2^\dagger H_2 + \lambda_1 (H_1^\dagger H_1)^2 + \lambda_2 (H_2^\dagger H_2)^2 + \lambda_{12} (H_1^\dagger H_1)(H_2^\dagger H_2) \\ & + \lambda_{12}' (H_1^\dagger H_2)(H_2^\dagger H_1) + (\delta (H_1^\dagger H_2)(H_1^\dagger H_1) + H.C.) \, . \end{aligned}$$

h, H and A have no FCNC Interaction!

Removing Goldstone bosons 'eaten' by W and Z,

$$\begin{aligned} L_Y = & -\bar{U}_L \hat{M}^u (1 + \frac{h}{v} - \sin\beta \frac{H - iA}{v}) U_R - \bar{D}_L \hat{M}^d (1 + \frac{h}{v} - \cos\beta \frac{H + iA}{v}) D_R \\ - & \sqrt{2} \bar{D}_L V_{KM} \hat{M}^u U_R \tan\beta \frac{h^-}{v} - \sqrt{2} \bar{D}_R \hat{M}^d V_{KM}^\dagger U_L \frac{1}{\tan\beta} \frac{h^-}{v} + H.C. \end{aligned}$$

But because $\mu^2_{12,21}$, δ_1, and δ_2 are zero, no spontaneous CP violation!

$\sin\beta = v_2/v$ and $\cos\beta = v_1/v$

Type II THDM

A practice model solving all the problems

S-L. Chen, Deshpand, X-G He, J. Jiang and L-H Tsai, Eur. Phys. J. C53, 607(2008)

Solve strong CP problem, implement PQ symmetry,

Identify spontaneous CP breaking phase as KM phase,

Making Axion invisible,

Three doublets and a singlet.

Is there any solution exist! Yes!

Work with M_d is diagonal example.

$$M_u = M_{u1} + e^{i\delta} M_{u2} \qquad \hat{M}_u = V_{CKM} M_u V_R^\dagger$$

$V_R = I$ $\qquad M_u = V_{CKM}^\dagger \hat{M}_u \qquad V_{CKM}^\dagger = (M_{u1} + e^{i\delta} M_{u2}) \hat{M}_u^{-1}$

$$V_{CKM} = \begin{pmatrix} e^{-i\delta_{13}} & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix} \begin{pmatrix} c_{12}c_{13}e^{i\delta_{13}} & s_{12}c_{13}e^{i\delta_{13}} & s_{13} \\ -s_{12}c_{23} - c_{12}s_{23}s_{13}e^{i\delta_{13}} & c_{12}c_{23} - s_{12}s_{23}s_{13}e^{i\delta_{13}} & s_{23}c_{13} \\ s_{12}s_{23} - c_{12}c_{23}s_{13}e^{i\delta_{13}} & -c_{12}s_{23} - s_{12}c_{23}s_{13}e^{i\delta_{13}} & c_{23}c_{13} \end{pmatrix},$$

$$M_{u1} = \begin{pmatrix} 0 & -s_{12}c_{23} & s_{12}s_{23} \\ 0 & c_{12}c_{23} & -c_{12}s_{23} \\ s_{13} & s_{23}c_{13} & c_{23}c_{13} \end{pmatrix} \hat{M}_u\,, \quad M_{u2} = \begin{pmatrix} c_{12}c_{13} & -c_{12}s_{23}s_{13} & -c_{12}c_{23}s_{13} \\ s_{12}c_{13} & -s_{12}s_{23}s_{13} & -s_{12}c_{23}s_{13} \\ 0 & 0 & 0 \end{pmatrix} \hat{M}_u$$

There is solution to identify KM phase with spontaneous CP violating phase!

Consequence? large fermion EDMs

One loop

Review, He et al.

IJMPA, A4, 5011(1989)

Weinberg operator

PRL63, 2333, (1989)

Braaten, C-S Li, T-C Yuan

PRL 64, 1709(1990)

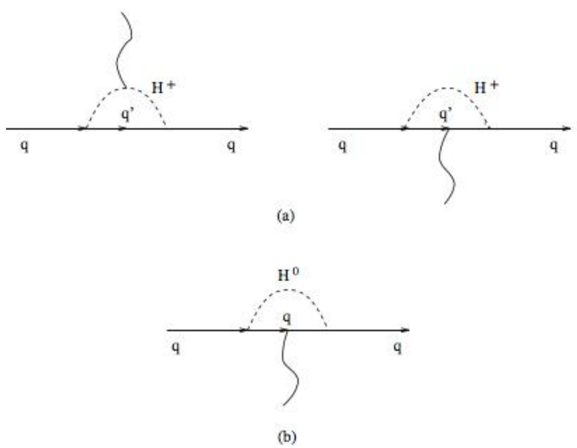
Correct CD running

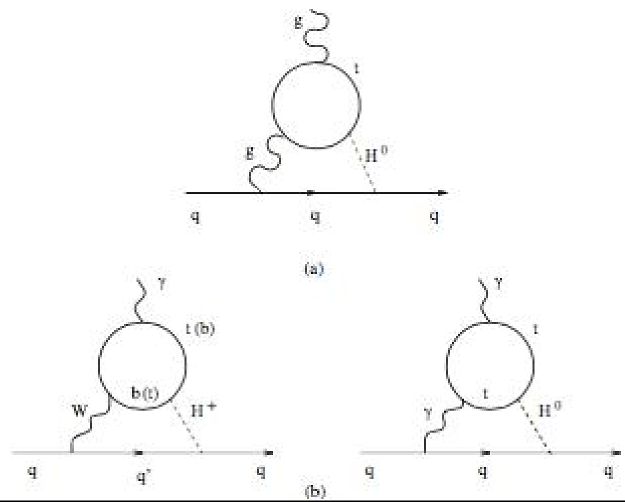
Barr-Zee, Gunion-Wyler

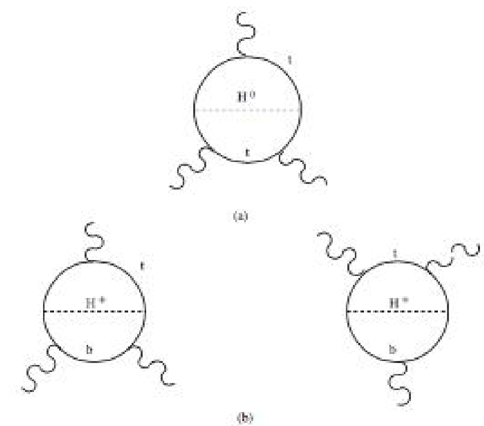
BZ, PRL 65, 21(1990)

GW, PLB 248, 170(1990)

Neutron and electron EDMs can be as large as experimental bounds.







Reducing model parameters and Grand Unified theory

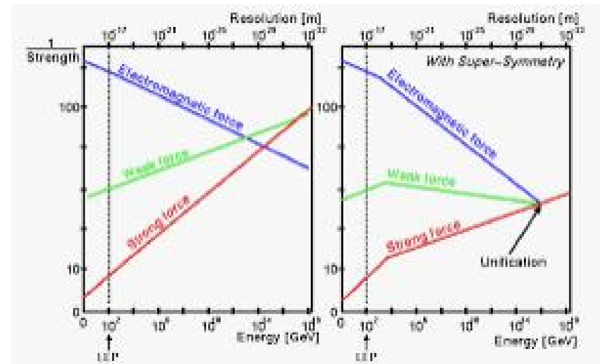
Grand Unification: Unify the 3 gauge groups into a single

Looking for a gauge group which contains SU(3)xSU(2)xU(1) as subgroups

The minimal one SU(5). Georgi and Glashaw Reduce 3 gauge couplings into 1.

Unification scale is about 10^{16} GeV!

$$V_\mu = \begin{pmatrix} G^1_1 - \frac{2}{\sqrt{30}}B & G^1_2 & G^1_3 & \bar{X}^1 & \bar{Y}^1 \\ G^2_1 & G^2_2 - \frac{2}{\sqrt{30}}B & G^2_3 & \bar{X}^2 & \bar{Y}^2 \\ G^3_1 & G^3_2 & G^3_3 - \frac{2}{\sqrt{30}}B & \bar{X}^3 & \bar{Y}^3 \\ X^1 & X^2 & X^3 & \frac{1}{\sqrt{2}}W^3 + \frac{3}{\sqrt{30}}B & W^+ \\ Y^1 & Y^2 & Y^3 & W^- & -\frac{1}{\sqrt{2}}W^3 + \frac{3}{\sqrt{30}}B \end{pmatrix}$$



non-susy. susy

The Fermions

$$\bar{\mathbf{5}}_F = \begin{pmatrix} d^c_1 \\ d^c_2 \\ d^c_3 \\ e \\ -\nu \end{pmatrix} \qquad \mathbf{10}_F = \begin{pmatrix} 0 & u^c_3 & -u^c_2 & u_1 & d_1 \\ -u^c_3 & 0 & u^c_1 & u_2 & d_2 \\ u^c_2 & -u^c_1 & 0 & u_3 & d_3 \\ -u_1 & -u_2 & -u_3 & 0 & e_R \\ -d_1 & -d_2 & -d_3 & -e_R & 0 \end{pmatrix}$$

Higgs:

$$\mathbf{24} \to (8,1)_0 \oplus (1,3)_0 \oplus (1,1)_0 \oplus (3,2)_{-\frac{5}{6}} \oplus (\bar{3},2)_{\frac{5}{6}}$$

vev breaks SU(5) -> SU(3)xSU(2)xU(1)

$$\mathbf{5}_H = (T_1, T_2, T_3, H^+, H^0)^T$$

vev give masses to fermions and breaks SM symmetry.

Relate quark and lepton masses... $m_e = m_d$, $m_\mu = m_s$, $m_\tau = m_b$ at gut scale,

at low energy $m_b \sim 3\, m_\tau$ (good), but not good for the other two!

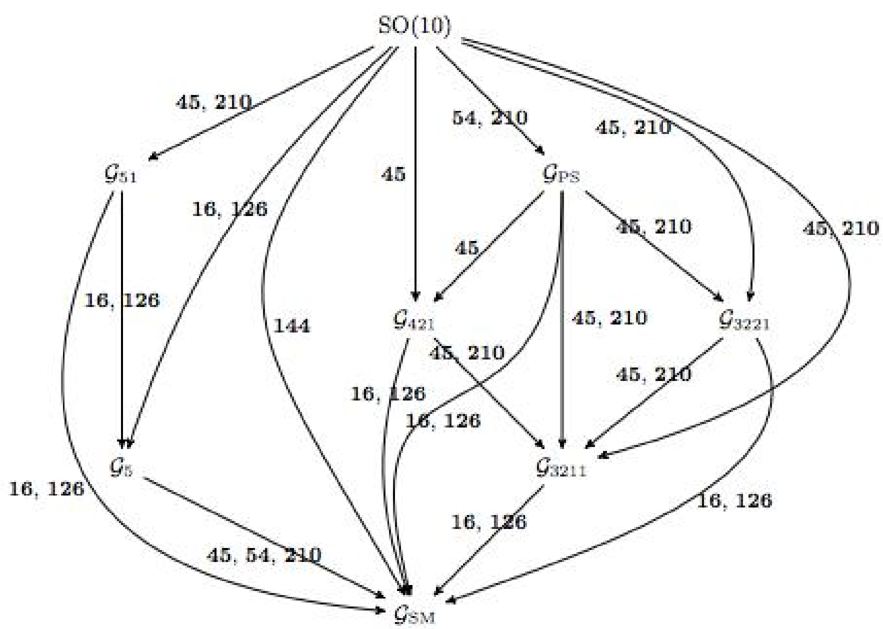
Smoking gun prediction: Proton decay. Not yet discovered!

SO(10) gut model

Gauge boson in 45 representation

Fermions in 16

Higgs fields 10 and 120, anti-126, 210…



$$45 \to 24_0 \oplus 10_{-4} \oplus \overline{10}_4 \oplus 1_0$$

$$16 \to 10_1 \oplus \bar{5}_{-3} \oplus 1_5$$

$$\mathbf{10}_H \supset (\mathbf{1},\mathbf{2})_{1/2} \oplus (\mathbf{1},\mathbf{2})_{-1/2} \equiv \Phi^d_{10} \oplus \Phi^u_{10}$$

$$\mathbf{120}_H \supset (\mathbf{1},\mathbf{2})_{1/2} \oplus (\mathbf{1},\mathbf{2})_{-1/2} \oplus (\mathbf{1},\mathbf{2})_{1/2} \oplus (\mathbf{1},\mathbf{2})_{-1/2}$$
$$\equiv \Phi^d_{120} \oplus \Phi^u_{120} \oplus \Sigma^d_{120} \oplus \Sigma^u_{120}$$

$$\overline{\mathbf{126}}_H \supset (\mathbf{1},\mathbf{2})_{1/2} \oplus (\mathbf{1},\mathbf{2})_{-1/2} \oplus (\mathbf{1},\mathbf{1})_0 \oplus (\mathbf{1},\mathbf{3})_1$$
$$\equiv \Sigma^d_{126} \oplus \Sigma^u_{126} \oplus \Delta_R \oplus \Delta_L.$$

16 contains the SM one generation of fermions plus a right handed neutrino!

Naturally have neutrino mass and also natural Seesaw mechanism.

SO(10) Predictions

$$16_F(Y_{10}10_H + Y_{\overline{126}}\overline{126}_H + Y_{120}120_H)16_F$$

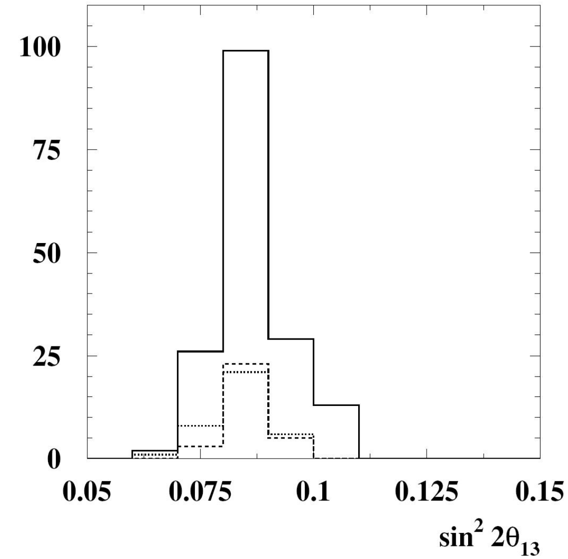
Minimal SO(10) Model without 120

$$\mathcal{L}_{\text{Yukawa}} = Y_{10}\,\mathbf{16}\,\mathbf{16}\,\mathbf{10}_H + Y_{126}\,\mathbf{16}\,\mathbf{16}\,\overline{\mathbf{126}}_H$$

Two Yukawa matrices determine all fermion masses and mixings, including the neutrinos

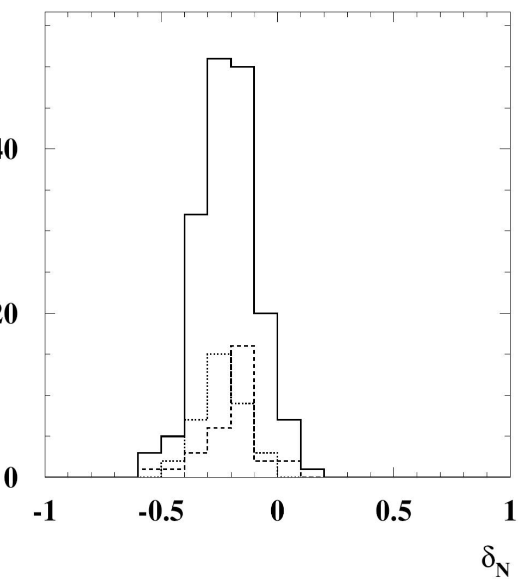
$$\begin{aligned} M_u &= \kappa_u Y_{10} + \kappa'_u Y_{126} \\ M_d &= \kappa_d Y_{10} + \kappa'_d Y_{126} \\ M_\nu^D &= \kappa_u Y_{10} - 3\kappa'_u Y_{126} \\ M_l &= \kappa_d Y_{10} - 3\kappa'_d Y_{126} \end{aligned}$$

$$\begin{aligned} M_{\nu R} &= \langle \Delta_R \rangle Y_{126} \\ M_{\nu L} &= \langle \Delta_L \rangle Y_{126} \end{aligned}$$



Good prediction for θ_{13}

δ Away from $-\pi/2$!!! Tobe tested!!



Model has only 11 real parameters plus 7 phases

Babu, Mohapatra (1993)
Fukuyama, Okada (2002)
Bajc, Melfo, Senjanovic, Vissani (2004)
Fukuyama, Ilakovac, Kikuchi, Meljanac,
Okada (2004)
Aulakh et al (2004)
Bertolini, Frigerio, Malinsky (2004)
Babu, Macesanu (2005)
Bertolini, Malinsky, Schwetz (2006
Dutta, Mimura, Mohapatra (2007)
Bajc, Dorsner, Nemevsek (2009)
Jushipura, Patel (2011).

Homework

Problem 1

Work out the masses (mass matrices) for h^+, h, H and A for the two Higgs doublet potential given in the lecture.

Problem 2

Show that $SU(3)_C xSU(2)_L xU(1)_Y xU(1)_{B-L}$ is gauge anomaly free.

In this model all quarks have B = 1/3 and all leptons have L = 1 (with also 3 generation of right handed neutrinos).

Problem 3

Work out the Z′ gauge boson of the $U(1)_{B-L}$ couplings to quarks and Leptons.

Appendix A: Lagrangian under P, C, T transformation (examples)

How fields and Lagrangian transform under C, P and T?

Take QED with fermion ψ and scalar ϕ fields as example.

$$L = -\frac{1}{4}F_{\mu\nu}F^{\mu\nu} + \bar{\psi}(i\gamma_\mu D^\mu - m_\psi)\psi + (D^\mu\phi)^\dagger(D_\mu\phi) - m_\phi^2\phi^\dagger\phi - V(\phi^\dagger\phi) ,$$
$$F_{\mu\nu} = \partial_\mu A_\nu - \partial_\nu A_\mu , \quad D_\mu = \partial_\mu + ieQA_\mu ,$$

ϕ spin-0, A^μ spin-1 (communting), ψ spin-1/2 (anti-communting)
$V(\phi^\dagger\phi)$ - potential of ϕ and is invariant under Lorentz Transformation.

The theory is invariant under the following gauge transformation,
$A_\mu \to A_\mu - \partial_\mu\alpha(x)$, $\psi(x) \to e^{ieQ\alpha(x)}\psi(x)$ and $\phi(x) \to e^{ieQ\alpha}\phi(x)$.

The Dirac γ-matrices are

$$\{\gamma^\mu, \gamma^\nu\} = 2g^{\mu\nu} , \quad g^{\mu\nu} = \mathrm{Diag}(1, -1, -1, -1) ,$$

$$\gamma^0 = \begin{pmatrix} I & 0 \\ 0 & -I \end{pmatrix} , \quad \gamma^i = \begin{pmatrix} 0 & \sigma_i \\ -\sigma_i & 0 \end{pmatrix} .$$

P transformed Lagrangian $L^P(x) = L^P(x, \partial^\mu, \psi_P(x), \phi_P(x), A^\mu_P(x))$

Using transformation table $L^P(x) = L(x')$ $(x = (x^0, x^i)$ and $x' = (x^0, -x^i))$

Example:
$F^{\mu\nu}_P(x) F_{P,\mu\nu}(x) = (\partial^\mu A^\nu_P(x) - \partial^\nu A^\mu_P(x))(\partial_\mu A_{P,\nu}(x) - \partial_\nu A_{P,\mu}(x))$

$= (\partial^\mu A_\nu(x') - \partial^\nu A_\mu(x'))(\partial_\mu A^\nu(x') - \partial_\nu A^\mu(x'))$

$= (\partial'_\mu A_\nu(x') - \partial'_\nu A_\mu(x'))(\partial'^\mu A^\nu(x') - \partial'^\nu A^\mu(x')) = F^{\mu\nu}(x') F_{\mu\nu}(x')$

$\bar{\psi}_P(x) \psi_P(x) = \bar{\psi}(x') \psi(x')$, ...

$S = \int_{-\infty}^{\infty} dx^4 L^P(x) = \int_{-\infty}^{\infty} dx^4 L(x' = (x^0, -x^i))$

$= \int_{-\infty}^{\infty} dx^4 L(x = (x^0, x^i)) = \int_{-\infty}^{\infty} dx^4 L(x)$

The action does not change!

Similarly $S = \int_{-\infty}^{\infty} L^C d^4x = \int_{-\infty}^{\infty} L^T d^4x = \int_{-\infty}^{\infty} L^{CP} d^4x = \int_{-\infty}^{\infty} L^{CPT} d^4x$

Similarly $S = \int_{-\infty}^{\infty} L^C d^4x = \int_{-\infty}^{\infty} L^T d^4x = \int_{-\infty}^{\infty} L^{CP} d^4x = \int_{-\infty}^{\infty} L^{CPT} d^4x$

Be careful when when making T transformed L^T,
a complex conjugate action should be taken

$L^T = L^*(\phi_T, \psi_T, A_T^\mu)$

Any constant c in L is transformed to c^*.

Example: $\bar{\psi}(x) i\gamma^\mu \partial_\mu \psi(x) \;\rightarrow\; \bar{\psi}_T(x)(-i\partial^\mu)\gamma_\mu^* \psi_T(x)$

$= \bar{\psi}(x')(-i\gamma^{3\dagger}\gamma^{1\dagger})\gamma^0(-i\partial^\mu)\gamma_\mu^*)(i\gamma^1\gamma^3)\psi(x') = \bar{\psi}(x')(-i\partial^\mu)\gamma^\mu\psi(x') = \bar{\psi}(x')i\partial'_\mu\gamma^\mu\psi(x')$.

An outline for CTP theorem proof

For a more general L, there may be tensors with arbitrary numbers of Lorentz

indices $\Gamma^{\mu_1\mu_2\cdots\mu_N}$ results from combinations of ∂^μ, $\phi(S,P)$, $T^{\mu\nu}$, A^μ and a^μ

The $\epsilon^{\mu\nu\alpha\beta}$ tensor

Under P and T it changes to $-\epsilon_{\mu\nu\alpha\beta}$. It does not change under C.

Using transformation properties of S, P, $T^{\mu\nu}$, A^μ, a^μ and $\epsilon^{\mu\nu\alpha\beta}$
and under CPT a constant c is transformed into c^*

One obtains: $(\Gamma^{\mu_1\cdots\mu_N})^{CPT} = (-1)^N(\Gamma^{\mu_1\cdots\mu_N})^\dagger$

Since L is Lorentz scalar, the Lorentz indices must be contracted
Then $L^{CPT}(x) = (-1)^{2N}L^\dagger(-x)$

If the theory is Hermmitian $L^\dagger = L \rightarrow L^{CPT}(x) = L(-x)$!

Where spin-statistics matters?

When prove the transformation table for C transformation.

A sample calculation: $\bar{\psi}'_C(x)\psi_C(x)$

$$\psi_C = C\gamma^0\psi^* \ , \quad \bar{\psi}_C = -\psi^T C^\dagger \ ,$$
$$\bar{\psi}'_C\psi_C = -\psi'^T C^\dagger C\gamma^0\psi^* = -\psi'^T\gamma^{0T}\psi^* \ .$$

Switching ψ and ψ' a minus sign is generated because they are fermion fields

$$\bar{\psi}'_C\psi_C = -\psi'^T\gamma^{0T}\psi^* = \psi^\dagger\gamma^0\psi' = \bar{\psi}\psi'$$

Particle and anti-Particle masses and lifetimes

a) The existence of an anti-particle for each particle ($|P>_m$, $m - spin$)

$\Theta|P>_m = \eta^{CPT}|\bar{P}>_{-m}$. If $\Theta = CPT$ is good, $|\bar{P}>_{-m}$ exists.

b) $m_P = m_{\bar{P}}$

$$m_P =< P|H|P>_m =< P|\Theta^{-1}\Theta H\Theta^{-1}\Theta|P>_m$$

$$=< \bar{P}|H^\dagger|\bar{P}>_{-m} =< \bar{P}|H|\bar{P}>_{-m} = m_{\bar{P}}.$$

c) $\tau_P = \tau_{\bar{P}}$

$$\tau_P^{-1} = 2\pi \sum_i \delta(E_i - E_P)| < i(\infty)|U(\infty,0)H_{int}|P >_m |^2$$
$$\tau_{\bar{P}}^{-1} = 2\pi \sum_{\bar{i}} \delta(E_{\bar{i}} - E_{\bar{P}})| < \bar{i}(\infty)|U(\infty,0)H_{int}|\bar{P} >_m |^2$$

$$\tau_P^{-1} = 2\pi \sum_i \delta(E_i - E_P)| < i(\infty)|\Theta^{-1}\Theta U(\infty,0)H_{int}\Theta\Theta^{-1}|P >_m |^2$$
$$= 2\pi \sum_i \delta(E_i - E_P)| < \bar{i}(-\infty)|U(-\infty,0)H_{int}|\bar{P} >_{-m} |^2$$

$E_P = E_{\bar{P}}$ and $E_i = E_{\bar{i}}$

$$\rightarrow 2\pi \sum_{\bar{i}} \delta(E_{\bar{i}} - E_{\bar{P}})| \sum_{\bar{j}} < \bar{i}(-\infty)|S\dagger|\bar{j}(\infty) >< \bar{j}(\infty)|U(\infty,0)H_{int}|\bar{P} >_{-m} |^2$$
$$= 2\pi \sum_{\bar{i},\bar{j},\bar{j}'} < \bar{i}(-\infty)|S\dagger|\bar{j}(\infty) >< \bar{j}(\infty)|U(\infty,0)H_{int}|\bar{P} >_{-m}$$
$$(< \bar{i}(-\infty)|S\dagger|\bar{j}'(\infty) >)^*(< \bar{j}'(\infty)|U(\infty,0)H_{int}|\bar{P} >_{-m})^*$$

Summ over $\bar{i}$,

$$\rightarrow 2\pi \sum_{\bar{j},\bar{j}'} \delta_{jj'} < \bar{j}(\infty)|U(\infty,0)H_{int}|\bar{P} >_{-m} (< \bar{j}'(\infty)|U(\infty,0)H_{int}|\bar{P} >_{-m})^*$$

$$= 2\pi \sum_{\bar{j}} \delta(E_{\bar{j}} - E_{\bar{P}})| < \bar{j}(\infty)|U(\infty,0)H_{int}|\bar{P} >_{-m} |^2$$

There is no difference in lifetime with different third spin component: $\tau_P = \tau_{\bar{P}}$!

Appendix B: Measurement of DM in meson oscillations

Without Tagging, one can also obtain important information

Let f to be a positively charged particle and $\bar{f}$ then has negative charge.

The time integrated event number N^+ is proportional to

$$N^+(M) \sim \int_0^\infty |<f|H|M(t)>|^2 dt = \frac{|A_f|^2}{2}\frac{\Gamma}{\Gamma_H\Gamma_L}(1+\frac{\Gamma_H\Gamma_L}{\Delta m^2+\Gamma^2})$$

$$N^+(\bar{M}) \sim \int_0^\infty |<f|H|M(t)>|^2 dt = \frac{|A_f|^2}{2}\frac{\Gamma}{\Gamma_H\Gamma_L}(1-\frac{\Gamma_H\Gamma_L}{\Delta m^2+\Gamma^2})|p/q|^2$$

$$N^-(\bar{M}) \sim \int_0^\infty |<\bar{f}|H|\bar{M}(t)>|^2 dt = \frac{|\bar{A}_{\bar{f}}|^2}{2}\frac{\Gamma}{\Gamma_H\Gamma_L}(1+\frac{\Gamma_H\Gamma_L}{\Delta m^2+\Gamma^2})$$

$$N^-(M) \sim \int_0^\infty |<\bar{f}|H|M(t)>|^2 dt = \frac{|\bar{A}_{\bar{f}}|^2}{2}\frac{\Gamma}{\Gamma_H\Gamma_L}(1-\frac{\Gamma_H\Gamma_L}{\Delta m^2+\Gamma^2})|q/p|^2$$

The event numbers N^{+-}, N^{++} and N^{--} for observing $f\bar{f}$, ff and $\bar{f}\bar{f}$ are

$$N^{+-} = N^+(M)N^-(\bar{M}) + N^+(\bar{M})N^-(M)$$
$$\sim (\tfrac{\Gamma}{2\Gamma_H\Gamma_L})^2|A_f|^2|\bar{A}_{\bar{f}}|^2((1+\tfrac{\Gamma_H\Gamma_L}{\Delta m^2+\Gamma^2})^2 + (1-\tfrac{\Gamma_H\Gamma_L}{\Delta m^2+\Gamma^2})^2)$$

$$N^{++} = N^+(M)N^+(\bar{M}) \sim (\tfrac{\Gamma}{2\Gamma_H\Gamma_L})^2|A_f|^2|A_f|^2(1-(\tfrac{\Gamma_H\Gamma_L}{\Delta m^2+\Gamma^2})^2)|p/q|^2$$

$$N^{--} = N^-(M)N^-(\bar{M}) \sim (\tfrac{\Gamma}{2\Gamma_H\Gamma_L})^2|\bar{A}_{\bar{f}}|^2|\bar{A}_{\bar{f}}|^2(1-(\tfrac{\Gamma_H\Gamma_L}{\Delta m^2+\Gamma^2})^2)|p/q|^2$$

Defining $\Delta = \frac{\Gamma_H\Gamma_L}{\Delta m^2+\Gamma^2} = \frac{\Gamma^2-\Delta\Gamma^2/4}{\Delta m^2+\Gamma^2}$

$$r = \frac{N^{++}+N^{--}}{N^{+-}} = \frac{|q/p|^2|\bar{A}_{\bar{f}}|^4 + |p/q|^2|A_f|^4}{2|A_f|^2|\bar{A}_{\bar{f}}|^2}\frac{1-\Delta^2}{1+\Delta^2},$$
$$a = \frac{N^{--}-N^{++}}{N^{--}+N^{++}} = \frac{|q/p|^2|\bar{A}_{\bar{f}}|^4 - |p/q|^2|A_f|^4}{|q/p|^2|\bar{A}_{\bar{f}}|^4 + |p/q|^2|A_f|^4}$$

In the limit $|A_f| = \bar{A}_{\bar{f}}|$, $\Delta\Gamma = 0$ and $p/q| = 1$
such as $B^0 \to l^+X$ and $\bar{B}^0 \to l^-\bar{X}$

$$r = \frac{(1+\Delta m^2/\Gamma^2)^2 - 1}{(1+\Delta m^2/\Gamma^2)^2+1}, \quad \Delta m_{B^0}/\Gamma_{B^2} = 0.77 \pm 0.004 .$$

Appendix C: The need of asymmetric e^+e^- collider for B factories

The need of asymmetric e^+e^- collider for B factories

Produce $B^0\bar{B}^0$ pair at $\Upsilon(4S)$
B are almost at rest and decay at production point and $\Delta\Gamma = 0$.

Aim to measure $Im(\lambda_f)$

Coherent production $M\bar{M}$ at resonance in $e + e^-$ collider at $t = 0$,

Wave function $\Psi(t_1, r_1; t_2, r_2)$ system at for M or $\bar{M}$ at t_1, r_1 and t_2, r_2 is

$$\Psi(t_1, r_1; t_2, r_2) = \frac{1}{\sqrt{2}}(|M(t_1, r_1)\bar{M}(t_2, r_2) > +(-1)^l |M(t_2, r_2)\bar{M}(t_1, r_1) >)$$

For $M(t_1)\bar{M}(t_2)$ decay to $f_{CP}f$ and $f_{CP}\bar{f}$, the decay amplitudes are

$$< f_{CP}(t_1)f(t_2)|\Psi(t_1;t_2> = \frac{\bar{A}_{CP}A_f}{4}e^{-im_H(t_1+t_2)-\Gamma_H(t_1+t_2)/2}$$
$$\times\left\{[1-\bar{\lambda}_f-(1+\bar{\lambda}_f)e^{i\Delta m t_1+\Delta\Gamma t_1/2}](1-e^{i\Delta m t_2+\Delta\Gamma t_2/2})\right.$$
$$\left.+(-1)^l[1-\bar{\lambda}_f+(1+\bar{\lambda}_f)e^{i\Delta m t_1+\Delta\Gamma t_1/2}](1+e^{i\Delta m t_2+\Delta\Gamma t_2/2})\right\}$$

$$< f_{CP}(t_1)\bar{f}(t_2)|\Psi(t_1;t_2> = \frac{A_{CP}\bar{A}_{\bar{f}}}{4}e^{-im_H(t_1+t_2)-\Gamma_H(t_1+t_2)/2}$$
$$\times(-1)^l\left\{(-1)^l[1-\lambda_f+(1+\lambda_f)e^{i\Delta m t_1+\Delta\Gamma t_1/2}](1+e^{i\Delta m t_2+\Delta\Gamma t_2/2})\right.$$
$$\left.+[1-\lambda_f-(1+\lambda_f)e^{i\Delta m t_1+\Delta\Gamma t_1/2}](1+e^{i\Delta m t_2-\Delta\Gamma t_2/2})\right\}$$

Consider $B^0\bar{B}^0$ system, $\Delta\Gamma = 0$

$$\bar{\Gamma}(t_1,t_2) \sim \bar{R}(t_1,t_2) = |< f_{CP}(t_1)\bar{f}(t_2)|\Psi(t_1;t_2) > |^2 \frac{|A_{CP}\bar{A}_{\bar{f}}|^2}{2} e^{-\Gamma(t_1+t_2)}$$

$$\times \{1 + |\lambda_f|^2 + [(1-|\lambda_f|^2)cos(\Delta m t_1) - 2Im\lambda_f sin(\Delta m t_1)]cos(\Delta m t_2)$$

$$-(-1)^l\ [(1-|\lambda_f|^2 sin(\Delta m t_1) + 2Im\lambda_f cos(\Delta m t_1)]sin(\Delta m t_2)\}$$

$$\Gamma(t_1,t_2) \sim \bar{R}(t_1,t_2) = |< f_{CP}(t_1)f(t_2)|\Psi(t_1;t_2) > |^2 \frac{|\bar{A}_{CP}A_{\bar{f}}|^2}{2} e^{-\Gamma(t_1+t_2)}$$

$$\times \{1 + |\bar{\lambda}_f|^2 + [(1-|\bar{\lambda}_f|^2)cos(\Delta m t_1) - 2Im\bar{\lambda}_f sin(\Delta m t_1)]cos(\Delta m t_2)$$

$$-(-1)^l\ [(1-|\bar{\lambda}_f|^2 sin(\Delta m t_1) + 2Im\bar{\lambda}_f cos(\Delta m t_1)]sin(\Delta m t_2)\}$$

If one does not need to know when f or $\bar{f}$, integrate t_2,

$$\bar{R}(t_1) = \int_0^\infty \bar{R}(t_1, t_2) dt_2 = \frac{|A_{CP}\bar{A}_{\bar{f}}|^2}{2} e^{-\Gamma t_1}$$

$$\times [\frac{1+|\bar{\lambda}_f|^2}{\Gamma} + ((1-|\lambda_f|^2)cos(\Delta m t_1) - 2Im\bar{\lambda}_f sin(\Delta m t_1))\frac{\Gamma}{\Delta m^2 + \Gamma^2}$$

$$-(-1)^l((1-|\lambda_f|^2)sin(\Delta m t_1) + 2Im\bar{\lambda}_f cos(\Delta m t_1))\frac{\Delta m}{\Delta m^2 + \Gamma^2}] \, ,$$

Further if one does not needs to know where f_{CP} was borne, integrate t_1

$$\bar{R} = \int_0^\infty \bar{R}(t_1) dt_1 = \frac{|A_{CP}\bar{A}_{\bar{f}}}{2} \left(\frac{1+|\lambda_f|^2}{\Gamma^2} + (1-|\lambda_f|^2)\frac{\Gamma^2 - (-1)^l \Delta m^2}{(\Delta m^2 + \Gamma^2)^2} - 2Im\lambda_f \frac{\Delta m \Gamma}{(\Gamma^2 + \Delta m^2)^2}(1+(-1)^l) \right)$$

Similarly one obtains

$$R = \int_0^\infty R(t_1) dt_1 = \frac{|\bar{A}_{CP}A_f}{2} \left(\frac{1+|\bar{\lambda}_f|^2}{\Gamma^2} + (1-|\bar{\lambda}_f|^2)\frac{\Gamma^2 - (-1)^l \Delta m^2}{(\Delta m^2 + \Gamma^2)^2} - 2Im\bar{\lambda}_f \frac{\Delta m \Gamma}{(\Gamma^2 + \Delta m^2)^2}(1+(-1)^l) \right)$$

For B factories, resonant production of $e^+e^- \to \Upsilon(4S) \to B^0\bar{B}^0$, $l = 1$

$$\bar{R} = \frac{|A_{CP}\bar{A}_{\bar{f}}|}{2}\left(\frac{1+|\lambda_f|^2}{\Gamma^2} + \frac{1-|\lambda_f|^2}{\Delta m^2 + \Gamma^2}\right)$$
$$R = \frac{|\bar{A}_{CP}A_f|}{2}\left(\frac{1+|\bar{\lambda}_f|^2}{\Gamma^2} + \frac{1-|\bar{\lambda}_f|^2}{\Delta m^2 + \Gamma^2}\right)$$

Contains no information about $Im\lambda_f$ and $\bar{\lambda}_f$ at all.

One should keep time dependent information without integrating

Not only that, one should boost B^0 and $\bar{B}^0$ to move after been produced

Asymmetric e^+ energy E_+ and e^- energy E_- are needed

For example, with $E_+ > E_-$, B^0 and $\bar{B}^0$ would be boosted in the e^+ direction

With $l = 1$

$$\bar{R}(t_1, t_2) = \frac{|A_{CP}\bar{A}_{\bar{f}}|}{2} e^{-\Gamma(t_1+t_2)}$$

$$\times \left(2(1 + |\lambda_f|^2) + (1 - |\lambda_f|^2) cos(\Delta m(t_1 - t_2) + 2Im\lambda_f sin(\Delta(t_1 - t_2))\right) \ ,$$

$$R(t_1, t_2) = \frac{|\bar{A}_{CP}A_f|}{2} e^{-\Gamma(t_1+t_2)}$$

$$\times \left(2(1 + |\bar{\lambda}_f|^2) + (1 - |\bar{\lambda}_f|^2) cos(\Delta m(t_1 - t_2) + 2Im\bar{\lambda}_f sin(\Delta(t_1 - t_2))\right) \ .$$

By measuring where f_{CP}, f and $\bar{f}$ are produced

Information on $Im\lambda_f$ and $\bar{\lambda}_f$ can be extracted!

This is the principle for Belle and Babar to measure CP violation

$$Im(\lambda_{B^0 \to J/\psi K_s}) = sin(2\beta) = 0.699 \pm 0.017$$

Appendix D: PQ symmetry and Axion

$U(1)_A$ chiral model of QP symmetry for strong CP problem

$L = L_{SM} + \delta L_\theta$, $\delta L_\theta = -\theta(g_3^2/16\pi^2)Tr(\tilde{G}G)$

$u_R \to e^{i\alpha}u_R$, $d_R \to e^{i\alpha}$, $Q_L \to Q_L$, $L_L \to L_l$ and $e_R \to e^{i\alpha}e_R$

$\bar{\theta} = \theta \to \theta - 2\alpha$,

If L_{SM} is symmetric under $U(1)_A$, $L \to L_{SM} + \delta L_{\bar{\theta}=\theta-2\alpha}$

For L_{SM}, α is arbitrary, choose one such that $\bar{\theta} = \theta - 2\alpha = 0$.

No strong CP term!

One then needs to show that the corresponding potentials are minimal to have a stable solution.

How to make L_{SM} symmetric under $U(1)_A$?

With just one Higgs, H, $L_Y = -\bar{Q}_L Y_u \tilde{H} u_R - \bar{Q}_L Y_d H d_R$

If require the first term be PQ invariant, $\tilde{H} \to e^{-i\alpha} \tilde{H}$

Since $\tilde{H} = i\sigma_2 H^*$, then $H \to e^{i\alpha} H$,

Second term not allowed, d-quarks do not get masses for vev of H

Not possible to make the second term PQ invariant.

Minimal SM does not work!

Extend the Higgs secotor to have two Higgs doublets H_1 and H_2

$H_1 \to e^{i\alpha} H_1$ and $H_2 \to e^{-i\alpha} H_2$,

Then $L_Y = -\bar{Q}_L Y_u \tilde{H}_1 u_R - \bar{Q}_L Y_d H_2 d_R$

Should make the potential $V(H_1, H_2)$ invariant.

Both H_1 and H_2 should have non-zero vev, v_1 and v_2

The PQ symmetry in $V(H_1, H_2)$ is spontaneously broken by v_i,

There is a massless GOLDSTONE boson, Axion.

QCD global anomaly make Axion mass non-zero.

Finding the Goldstone boson

Two Higgs doublet model: (v_1, v_2) breaks $U(1)_Y$ symmetry

The imaginary fields (I_1, I_2) carries Y charges is proportional to $(1, 1)$

The Z needs to "eats" a neutral pesudo-Goldstone boson to become massive

The one "eaten" is just proportional to $\sum_i v_i Y_i I_i$.

If there is another $U(1)_A$ is broken by the same vev,

If no Z to "eats" anything, the Goldstone boson is proportional to: $\sum_i v_i A_i I_i$.

If there is a combination Z_I "eaten" by a gauge boson Z, find it first

The one orthogonal to Z_I is the other Goldstone boson!

In PQ model, there are two neutral imaginary fields I_1 and I_2

$Z_I = \frac{v_1 I_1 + v_2 I_2}{\sqrt{v_1^2+v_2^2}}$ "eaten" by Z

The orthoganol combination is massless Axion: $a = \frac{-v_2 I_1 + v_1 I_2}{\sqrt{v_1^2+v_2^2}}$.

The PQ symmetry is anomalous, there is a mass of order hundrads of KeV.

Couplings to quarks: $\sim i[-\bar{u}\frac{\hat{M}_u}{v}\frac{v_2}{v}u + \bar{d}\frac{\hat{M}}{v}\frac{v_1}{v}d]a$

Too large couplings. Ruled out!

Invisible Axion. Introduce an additional singlet $S:(1,1)(1)_{PQ}$ with vev v_s

Axion; $a = \frac{-2v_1 v_2^2 I_1 + 2v_2 V_1^2 I_2 + v^2 v_s I_s}{\sqrt{4v_1^2 v_2^2 (v_1^2 + v_2^2) + v^4 v_s^2}}$

Couplings to quarks: $\sim i \frac{-\bar{u}(\hat{M}_u/v)(2v_1 v_2^2)u + \bar{d}(\hat{M}/v)(2v_1^2 v_2)d}{\sqrt{4v_1^2 v_2^2 (v_1^2 + v_2^2) + v^4 v_s^2}} a$

If $v_s >> v_1$, the couplings are: $\sim i[-\bar{u}\frac{\hat{M}_u}{v}\frac{2v_1 v_2^2}{v^2 v_s} u + \bar{d}\frac{\hat{M}}{v}\frac{2v_1^2 v_2}{v^2 v_s} d]a$

Axion mass of order $m_\pi^2 \frac{v^2}{v_s^2}$.

Invisible Axion. Still alive!

A lot of interestgin physics related to Axion: find the Axion, applicatin to astrophysics, cosmology and etc.!!!

Thank you all for listen to my lectures