Lectures on
Neutrino Physics (I)

AEPSHEP 2022

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The Neutrino Revolution (June, 1998)

Official Super-Kamiokande Press Release

MEDIA ADVISORY for June 5, 1998, Takayama, Japan
US EMBARGO EXPIRATION: 20:00
June 4, 1998, Pacific Daylight Time

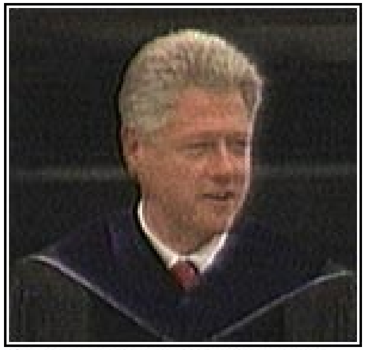
EVIDENCE FOR MASSIVE NEUTRINOS

The Neutrino Revolution (June, 1998)

Clinton on neutrino

President Clinton addresses the graduating class at MIT

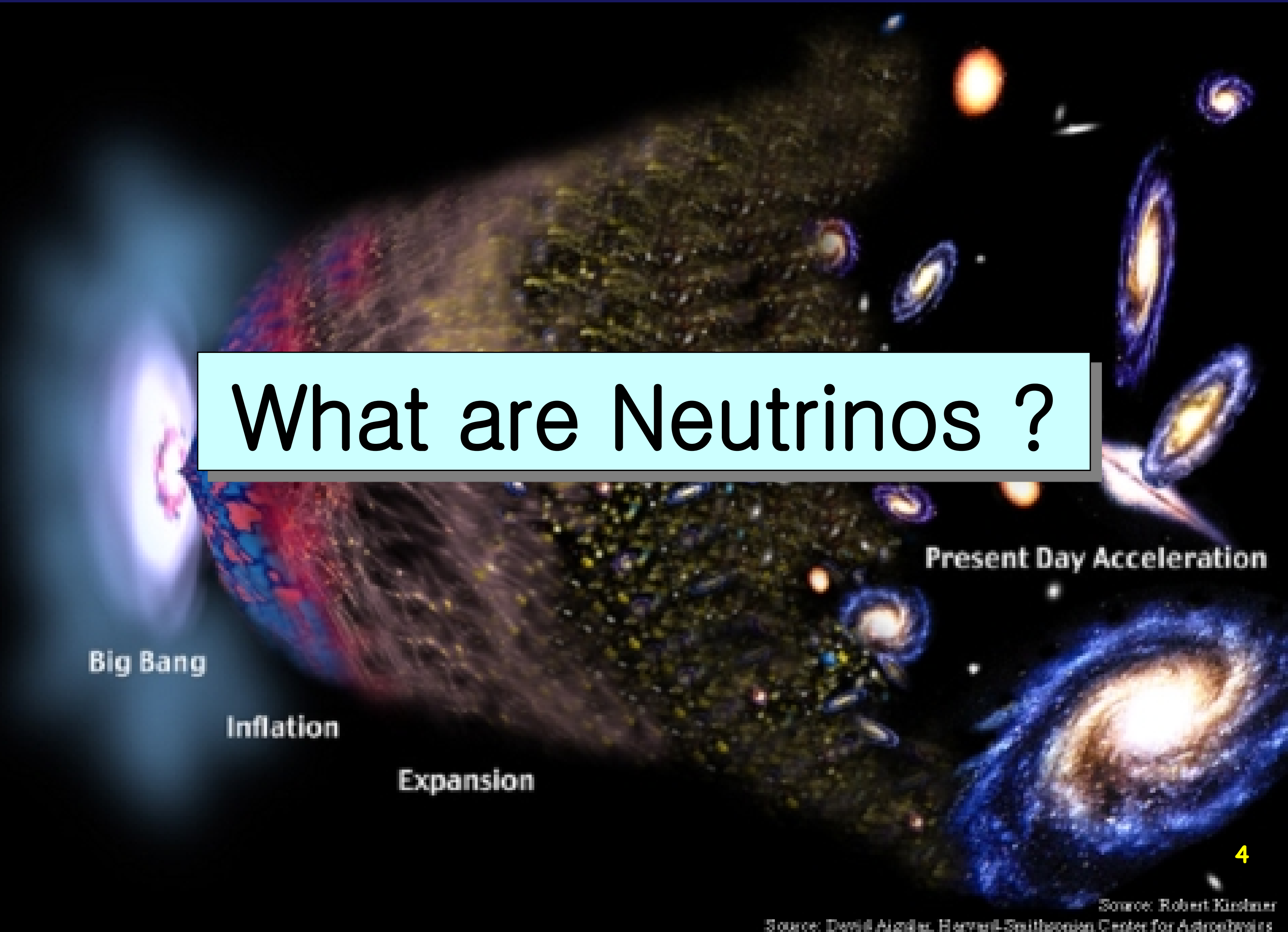
(excerpted from remarks at the MIT commencement, June 6, 1998)



President Clinton addresses the graduating class at MIT

[W]e must help you to ensure that America continues to lead the revolution in science and technology……..
Just yesterday in Japan, physicists announced a discovery that tiny neutrinos have mass. …..but it may change our most fundamental theories from the nature of the smallest subatomic particles to how the universe itself works, and indeed how it expands.

What are Neutrinos ?



Source: Robert Kirshner

Source: David Aguilar, Harvard-Smithsonian Center for Astrophysics

Elusive ghost particles

Source: Robert Kirshner

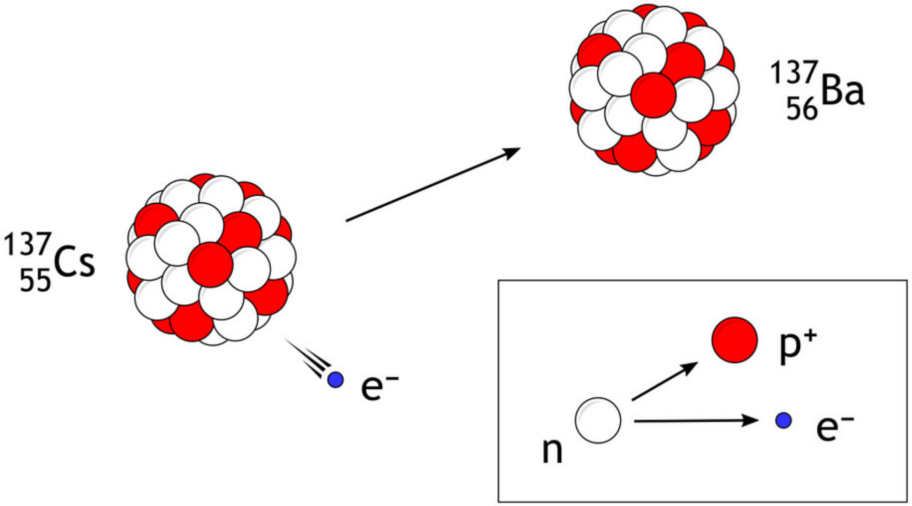
Source: David Aguilar, Harvard-Smithsonian Center for Astrophysics

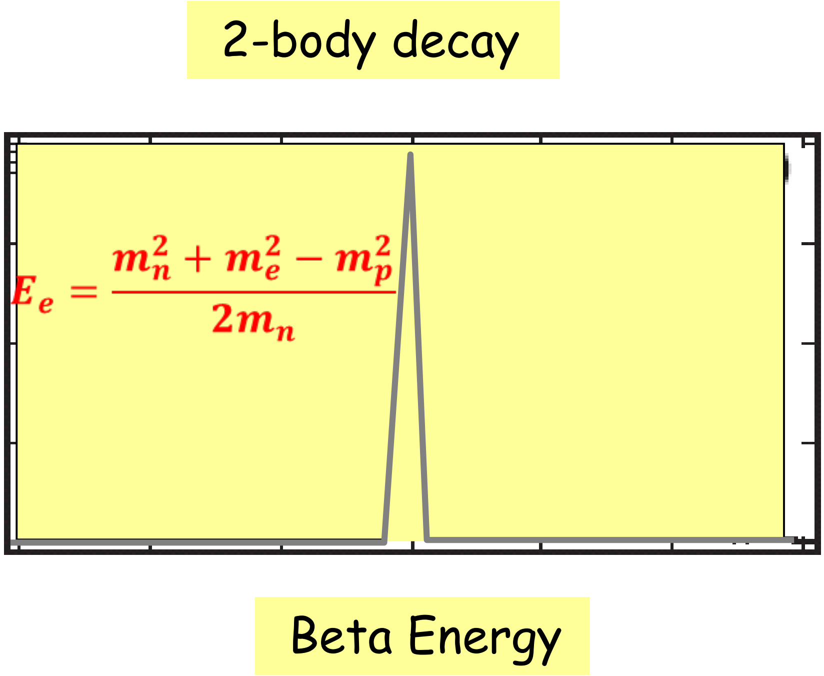
Invented to resolve a puzzle……

Invention of the Neutrino

Beta decay mystery (1920's puzzle)

- 2-body decay should give mono-energetic electron



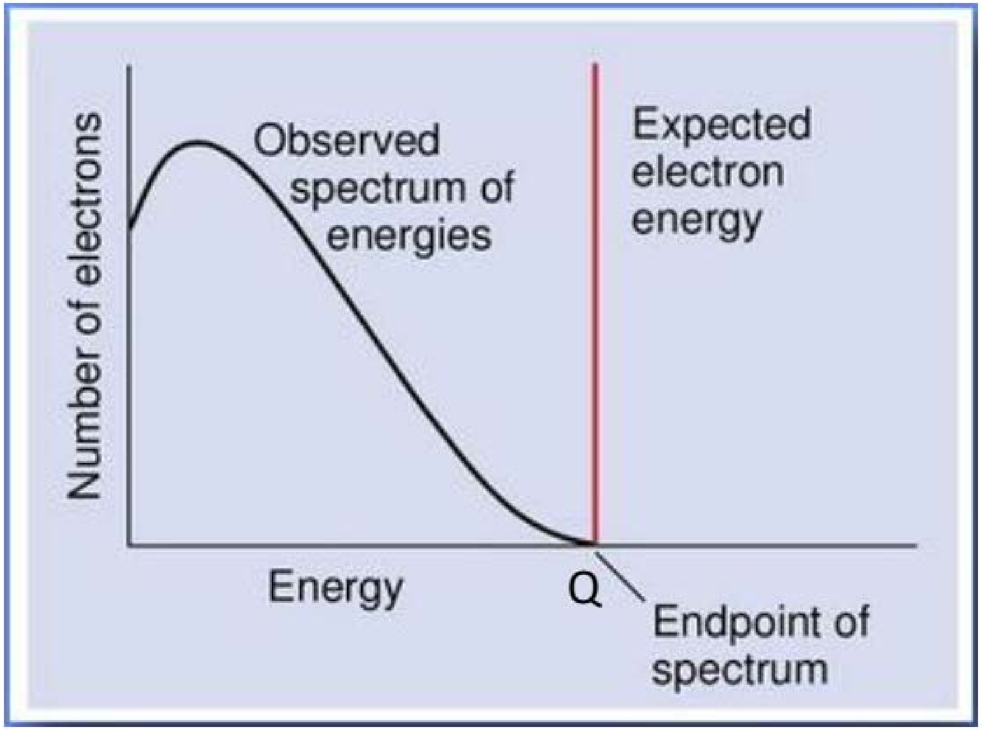


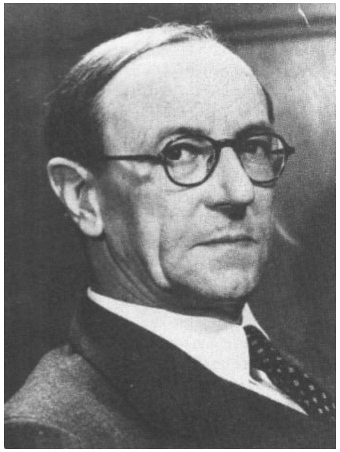
$$E_e = \frac{m_n^2 + m_e^2 - m_p^2}{2m_n}$$

Invention of the Neutrino

Observed spectrum was continuous

→ Breakdown of Energy Conservation ?





James Chadwick 1914

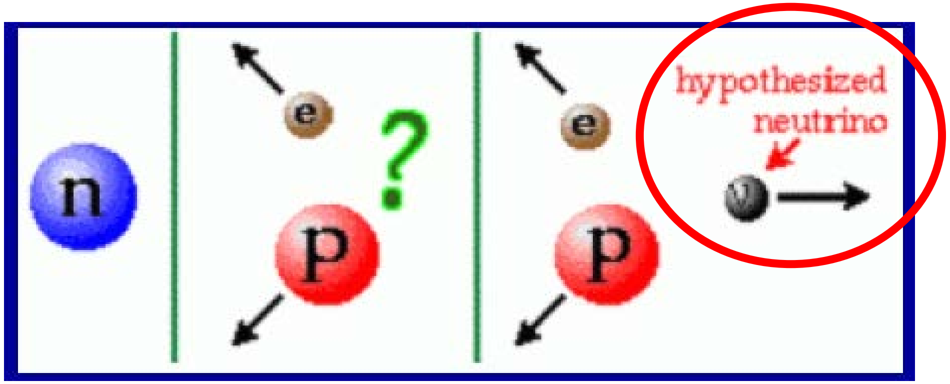
Also,conservation of angular momentum was broken

Invention of the Neutrino

W. Pauli's solution (1930)

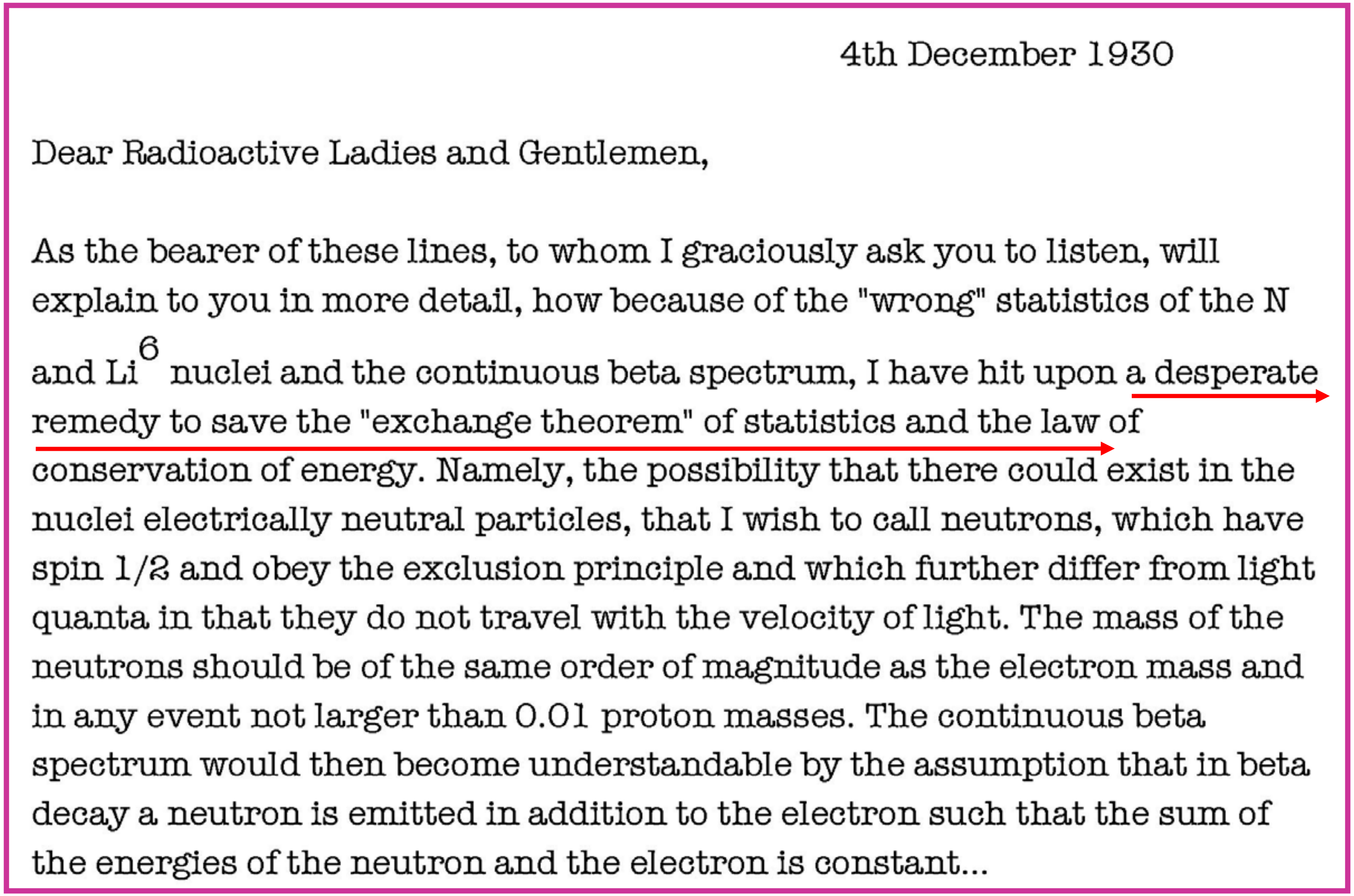

Photo: AIP, Emilio Segrè Visual Archives

In β−decay



Pauli proposed a hypothetical particle which is weakly interacting massless neutral fermion

Desperate remedy



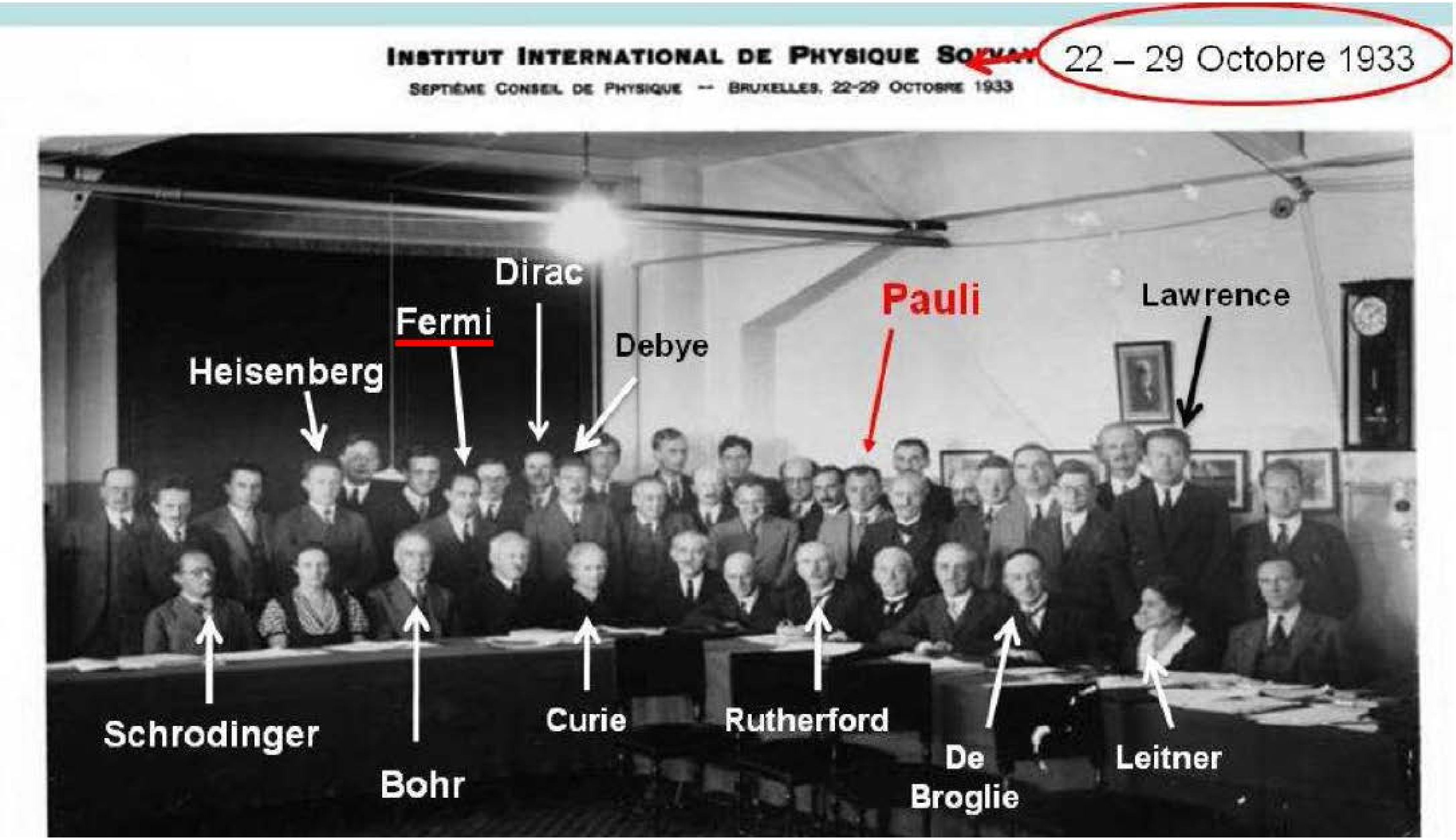
4th December 1930

Dear Radioactive Ladies and Gentlemen,

As the bearer of these lines, to whom I graciously ask you to listen, will explain to you in more detail, how because of the "wrong" statistics of the N and Li^6 nuclei and the continuous beta spectrum, I have hit upon a desperate remedy to save the "exchange theorem" of statistics and the law of conservation of energy. Namely, the possibility that there could exist in the nuclei electrically neutral particles, that I wish to call neutrons, which have spin 1/2 and obey the exclusion principle and which further differ from light quanta in that they do not travel with the velocity of light. The mass of the neutrons should be of the same order of magnitude as the electron mass and in any event not larger than 0.01 proton masses. The continuous beta spectrum would then become understandable by the assumption that in beta decay a neutron is emitted in addition to the electron such that the sum of the energies of the neutron and the electron is constant...

Solvay 1933

Pauli gave a talk on his idea of neutrino



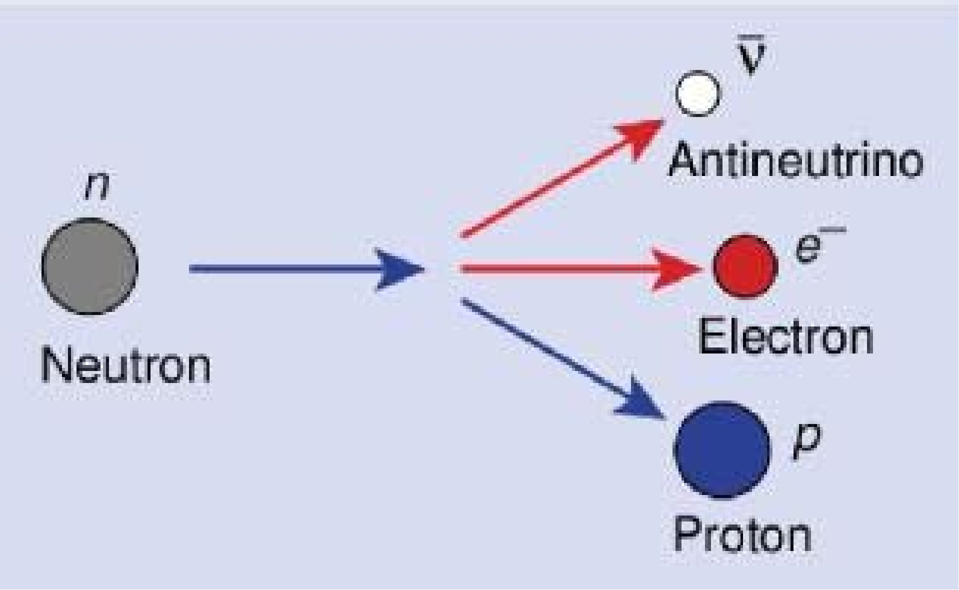
INSTITUT INTERNATIONAL DE PHYSIQUE SOLVAY

SEPTIÈME CONSEIL DE PHYSIQUE -- BRUXELLES, 22-29 OCTOBRE 1933

22 – 29 Octobre 1933

True Picture of Beta Decay

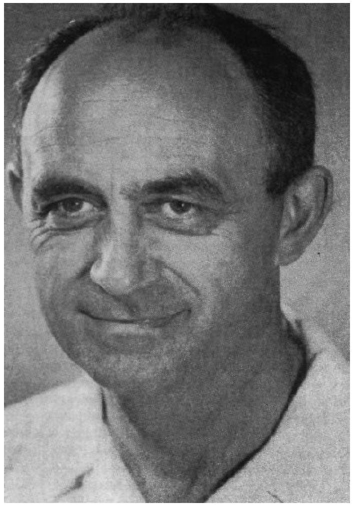
Three-body final state



Electron and antineutrino share the energy

Fermi Theory (1933)

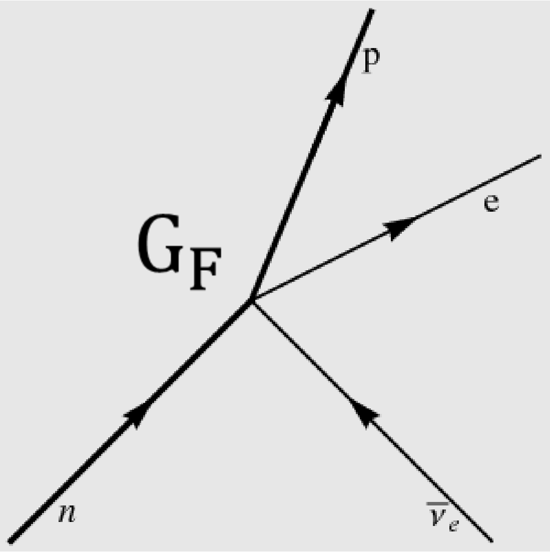
- Fermi formulated the theory of beta decay



$$\boxed{n \rightarrow p^{+} + e^{-} + \bar{\nu}}$$

$$\mathcal{L}_{\text{Fermi}} = -\frac{G_F}{\sqrt{2}}\, \bar{p}\gamma_\mu n\, \bar{e}\gamma^\mu \nu + \text{h.c.}$$

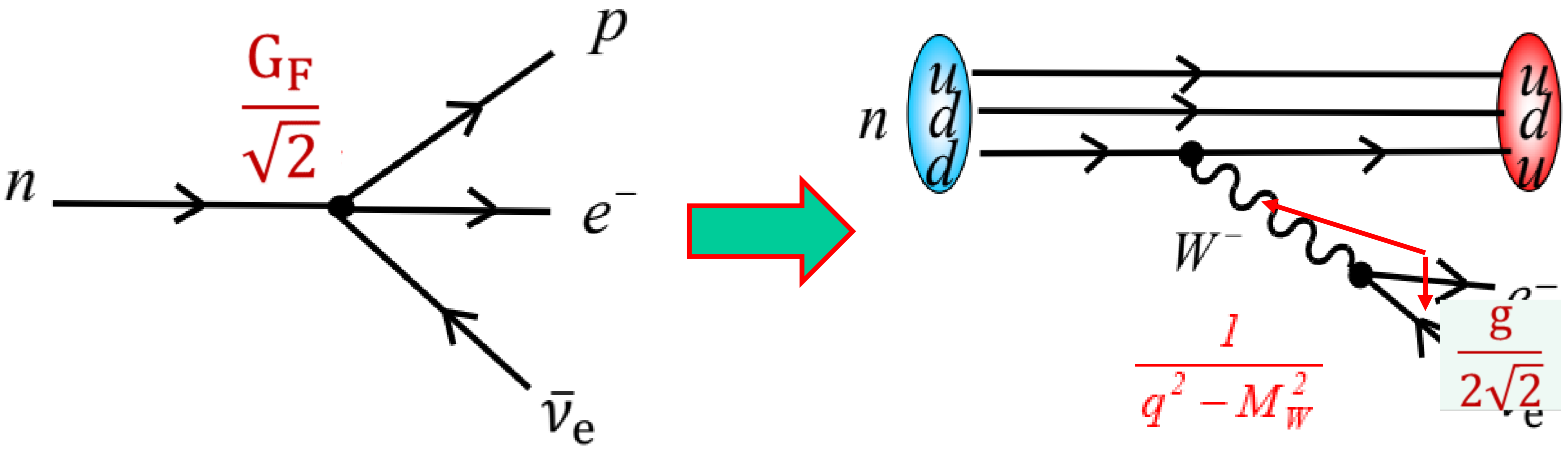
$G_F = 1.166 \cdot 10^{-5}\,\text{GeV}^{-2}$ Fermi constant



- point-like four fermion vertex
- works for muon decay and can be applied to any nucleu
- first successful theory of the creation of massive particle
- Nature didn't publish the article:" contained speculations too remote from reality to be of interest to the reader…"

Weak Theory

Interaction between (p^+, n) and $(e^-, \bar{\nu})$ is mediated by W^- of which the Fermi theory is the low-E effective field theory



$$\frac{G_F}{\sqrt{2}} = \frac{g^2}{8M_W^2} \qquad (q^2 \ll M_W^2) \qquad M_W \sim 80.4 \text{ GeV}$$

Discovery of neutrino

- Bethe-Peierls (1934) proposed a possible detection of ν from the phenomenon known as inverse β-decay

$$\bar{\nu}_e + p^+ \rightarrow n + e^+$$

- They computed ν cross section using Fermi theory

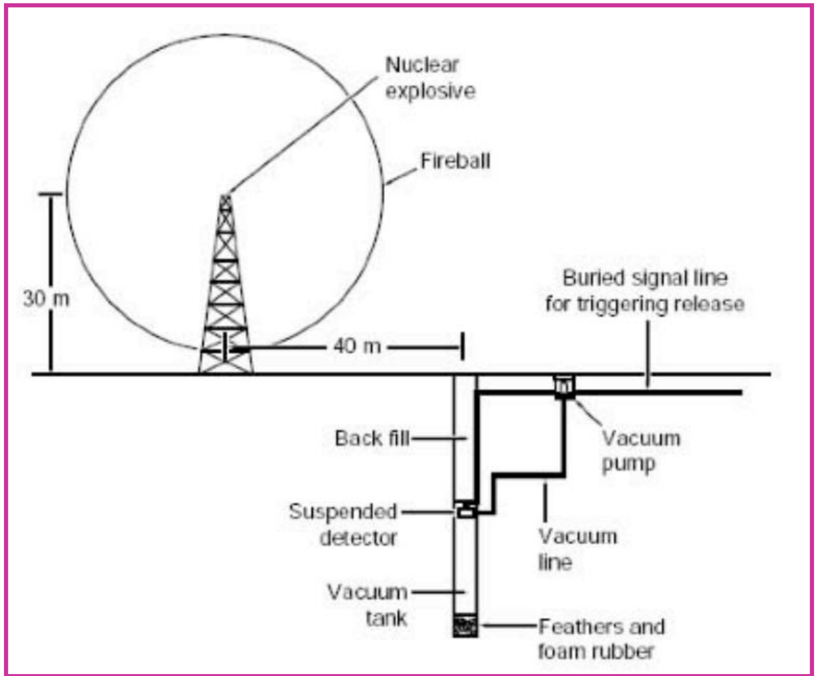
$$\sigma < \frac{h^3}{m^3 c^4 t} \sim 10^{-44}\,\mathrm{cm}^2 \quad (\text{for } E \sim 2\,\mathrm{MeV})$$

In fact, $\sigma \sim 10^{-43} (E/\mathrm{MeV})^2 \mathrm{cm}^2$

- The “mean free path” of such neutrinos in a block of lead is of the order of 1 light year !.
- Detecting neutrinos is next to impossible.

Discovery of neutrino

- In "atomic age", very intense sources of ν could be available ：nuclear bombs or nuclear reactors.
  → Reactors ~ isotropic flux of $10^{20}\ \nu/sec$
- *Poltergeist* project came up with : First idea is to put the detector close to a nuclear explosion

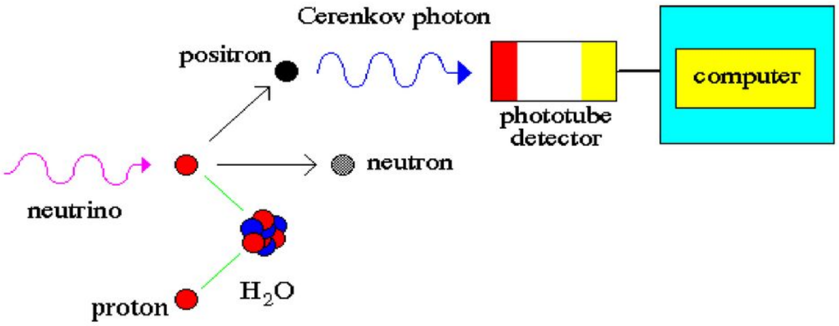


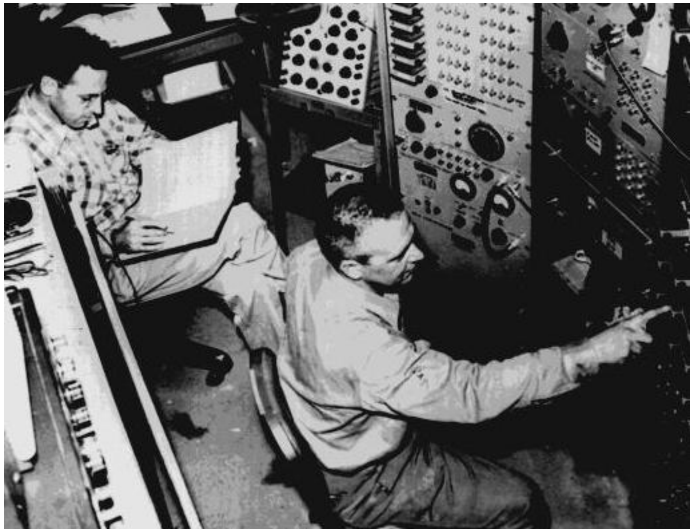
Discovery of Neutrino

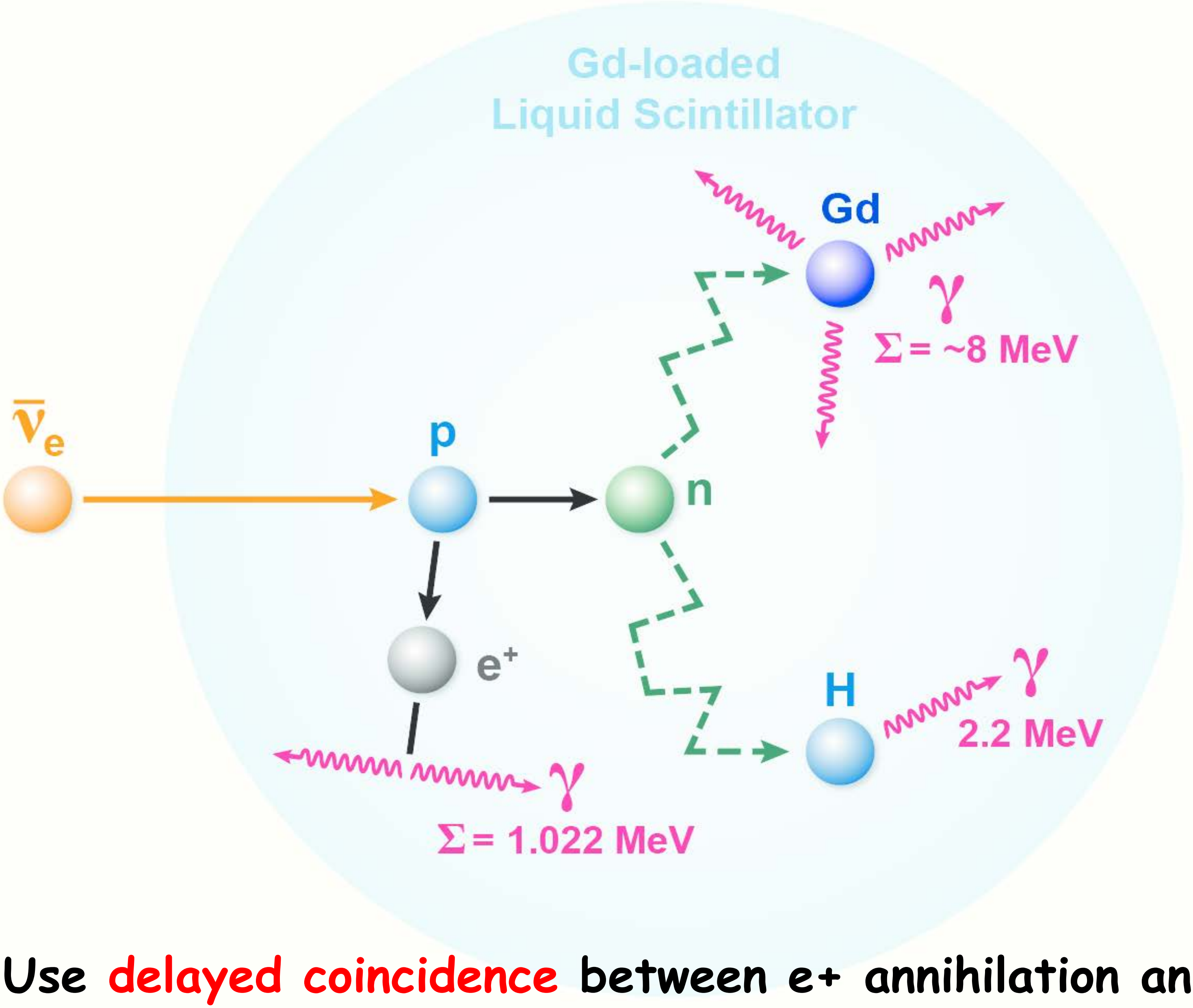
- Frederick Reines & Clyde Cowan’s Project (1951)
- “the first detection of anti-neutrinos using reactor``
- Project approved at Los Alamos (1952)

Anti-Neutrino Detector

$$\bar{\nu} + p \rightarrow e^{+} + n$$







Use delayed coincidence between e+ annihilation and neutron capture

Discovery of Neutrino

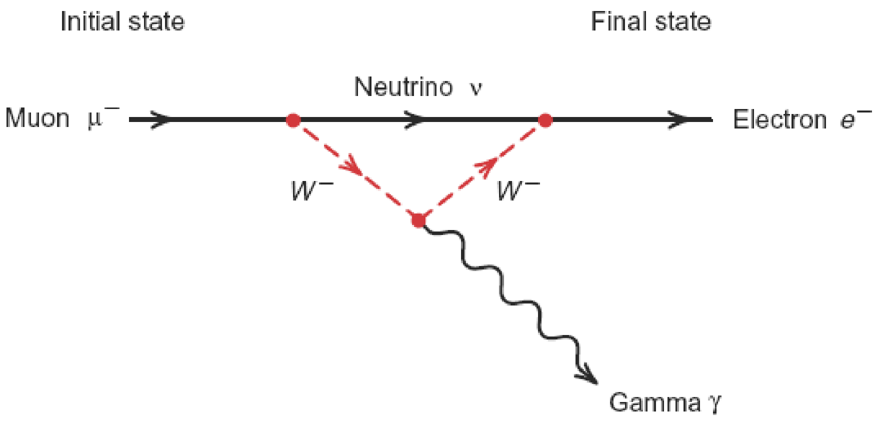
On June 14, 1956, Reines and Cowan sent a telegram to Pauli:

"We are happy to inform you that we have definitely detected neutrinos from fission fragments by observing inverse beta decay of protons. Observed cross section agrees well with expected…"

Reines: Nobel Prize in 1995

Discovery of Muon Neutrino

- **Muons were discovered by Anderson & Neddermeyer (1936) from cosmic source and confirmed by Street and Stevenson's cloud chamber experiment in 1937.**
- **It is like electron but heavier.**
- **It was thought to decay into an electron and a gamma**



But, it didn't happen

Discovery of Muon Neutrino

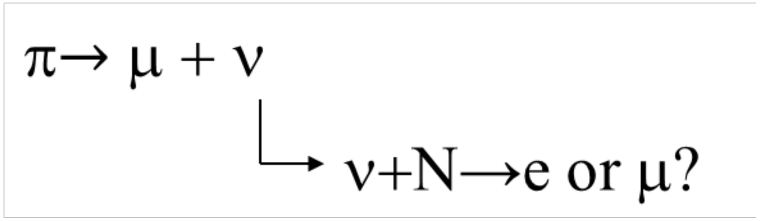
- **In the late 40's, it became clear that the μ^- decayed into more than one particle**
- **Presumably the unseen particles were neutrinos :**

$$\mu \rightarrow e + \nu + \bar{\nu}$$

- **There was a particle decaying into a μ^- and a $\bar{\nu}_x$:**

$$\pi^- \rightarrow \mu^- + \bar{\nu}_x$$

- **Are they all the same as neutrino emitting from β-decay ?**

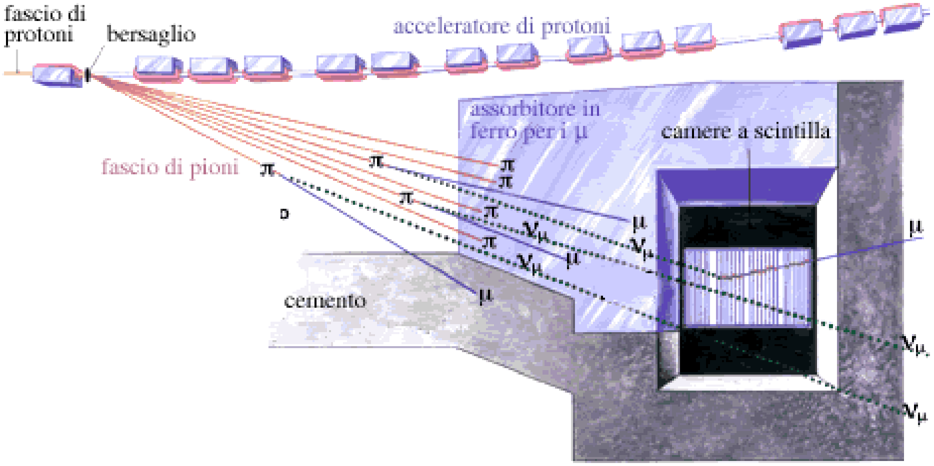


$\pi \rightarrow \mu + \nu$

$\nu + N \rightarrow e$ or μ?

Discovery of Muon Neutrino

- **Experiment was done at the Brookhaven 30 GeV accelerator in 1962 (Lederman, Schwartz, Steinberger).**



$$\pi^- \rightarrow \mu^- + \bar{\nu}_x$$

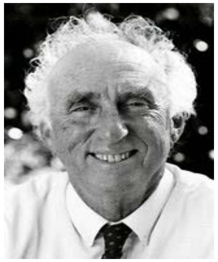
$$\bar{\nu}_x + p \rightarrow e^+ + n$$

L. Lederman, M. Schwartz and J. Steinberger

None was compatible with electron in the final state

Nobel Prize in 1988

Discovery of Tau Neutrino



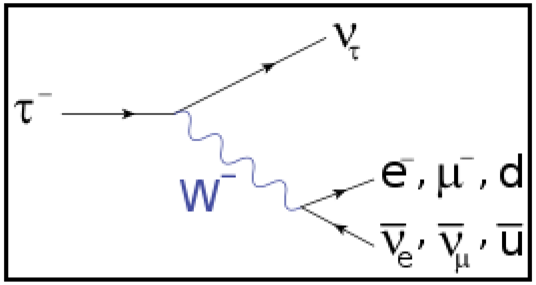
- **Tau was discovered by M. Perl in 1975 via**

$$e^+ + e^- \to e^\pm + \mu^\mp + X \text{ (undetected)}$$

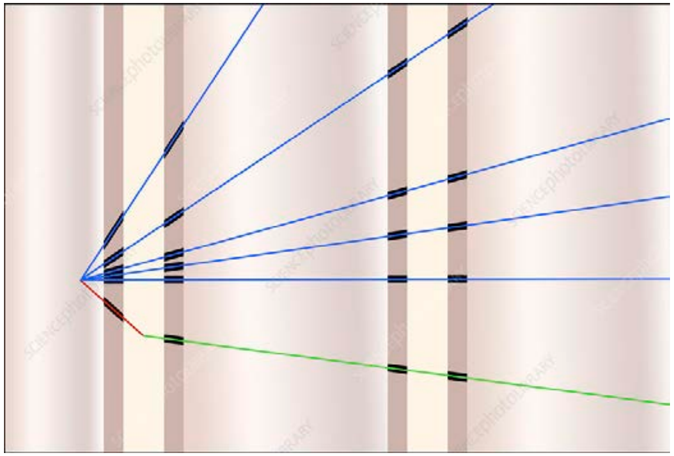
(at least two)

- **It was proposed that the above can be mediated by**

$$e^+ + e^- \to \tau^+ + \tau^- \to e^\pm + \mu^\mp + 4\nu$$



- **Tau neutrino was finally discovered by DONUT exp. at FERMILAB (2000)**

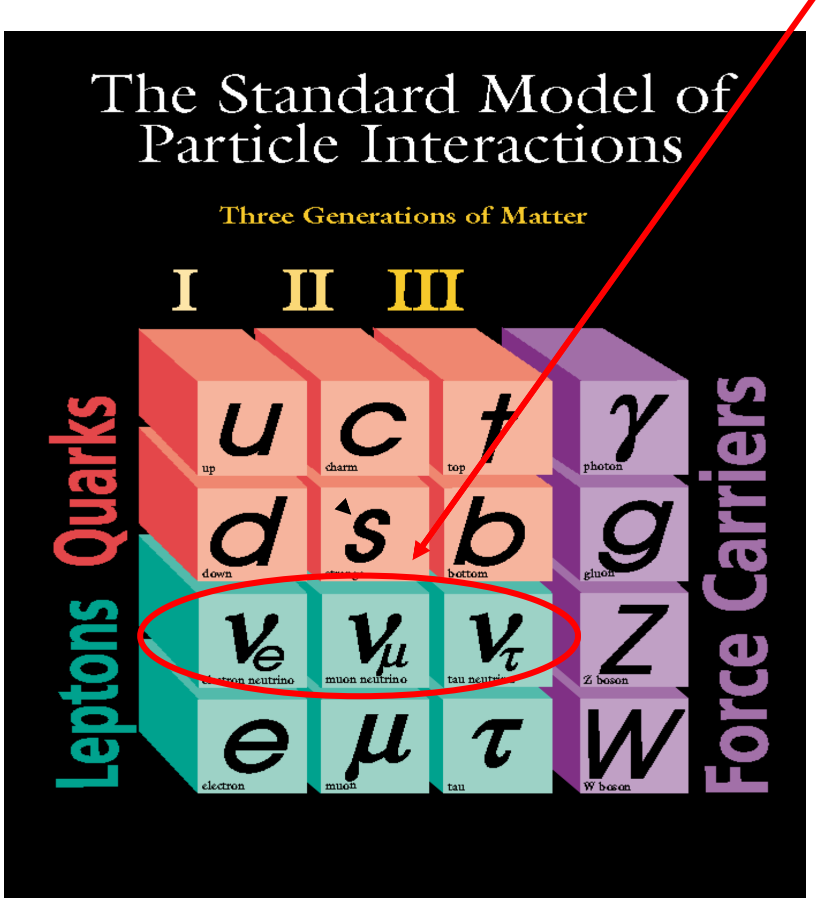


Lepton family of 3 generations is completed !

Prize in 1995
(discovery of τ^-)

Neutrinos in the SM

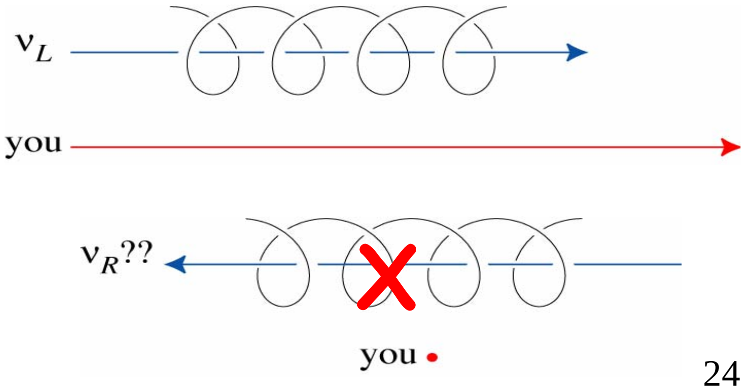
Our best model for the microscopic universe…



Neutrinos:

- Come in three `flavors'
- Are massless
- Interact weakly
- Cannot change flavor

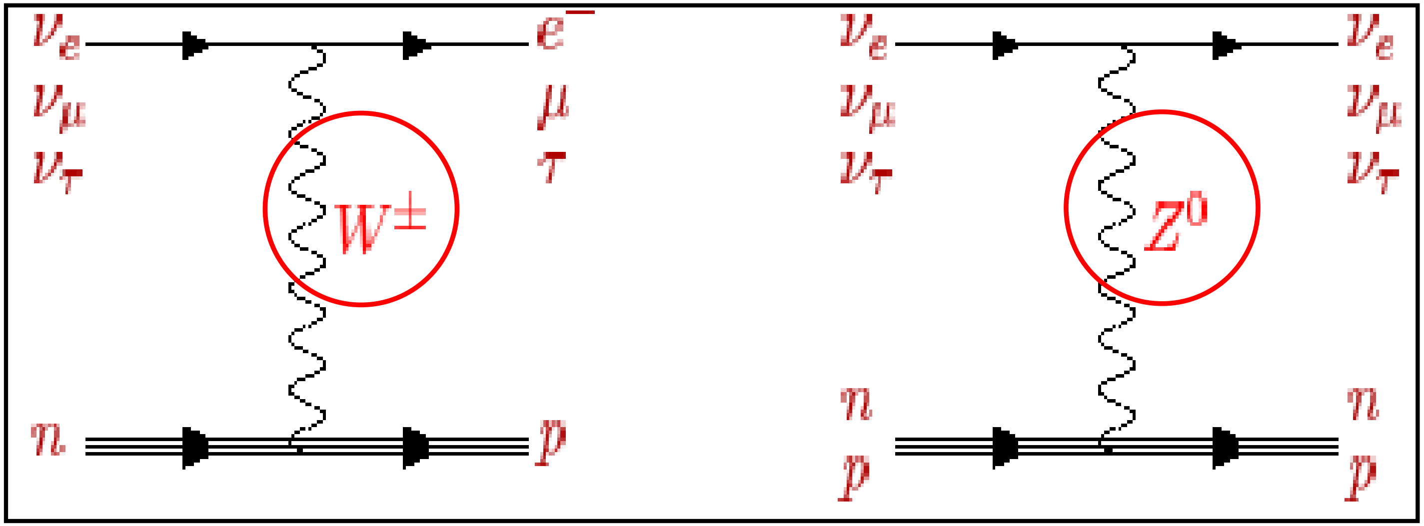
All neutrinos left-handed



Neutrinos in the SM

- **How neutrinos interact ?**

→ **weak interaction exchanging** $W^{\pm}, Z^0$



Charged Current

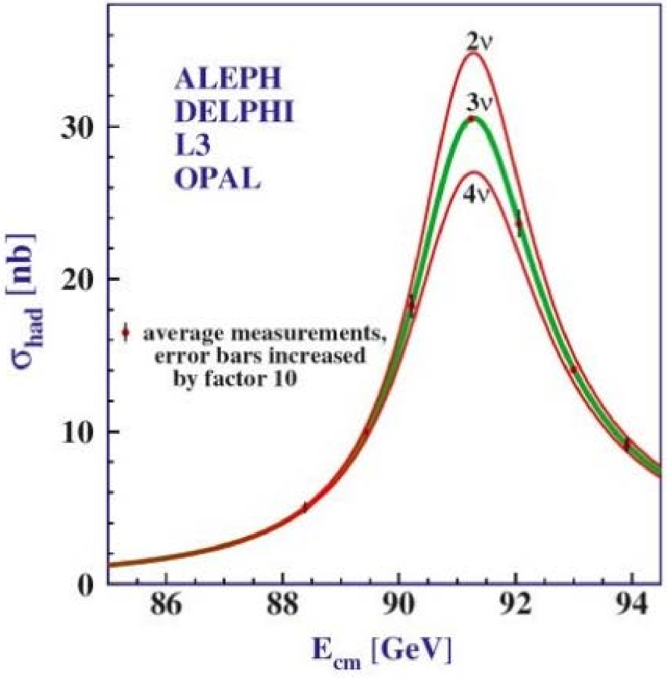
Neutral Current

$$\mathcal{L}_{SM} \supset -\frac{g}{\sqrt{2}}\sum_f \bar{\nu}_{Lf}\gamma_\mu l_{Lf} W^+_\mu - \frac{g}{2\cos\theta_W}\sum_f \bar{\nu}_{Lf}\gamma_\mu \nu_{Lf} Z_\mu + h.c.$$

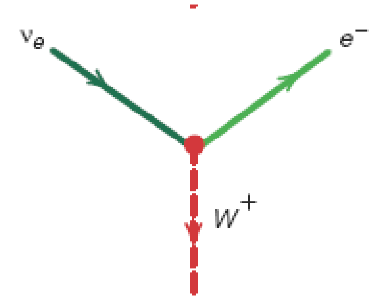
Neutrinos in the SM

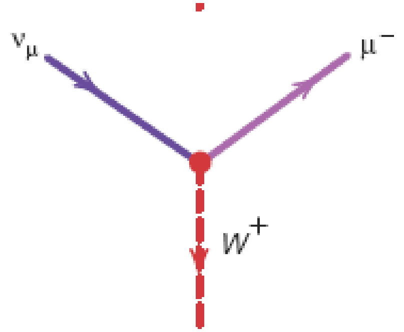
- **Only 3 neutrinos exist (below electroweak scale)**

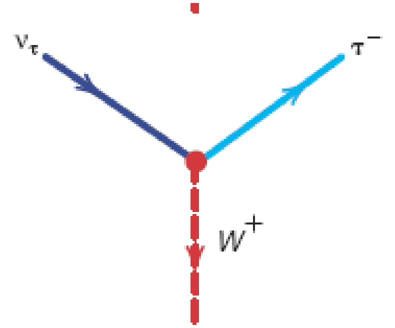
From $e^+ + e^- \rightarrow Z^0 \rightarrow f\bar{f}$



$$N_\nu = \frac{\Gamma_{\text{inv}}}{\Gamma_{\nu\bar{\nu}}} = 2.984 \pm 0.008$$





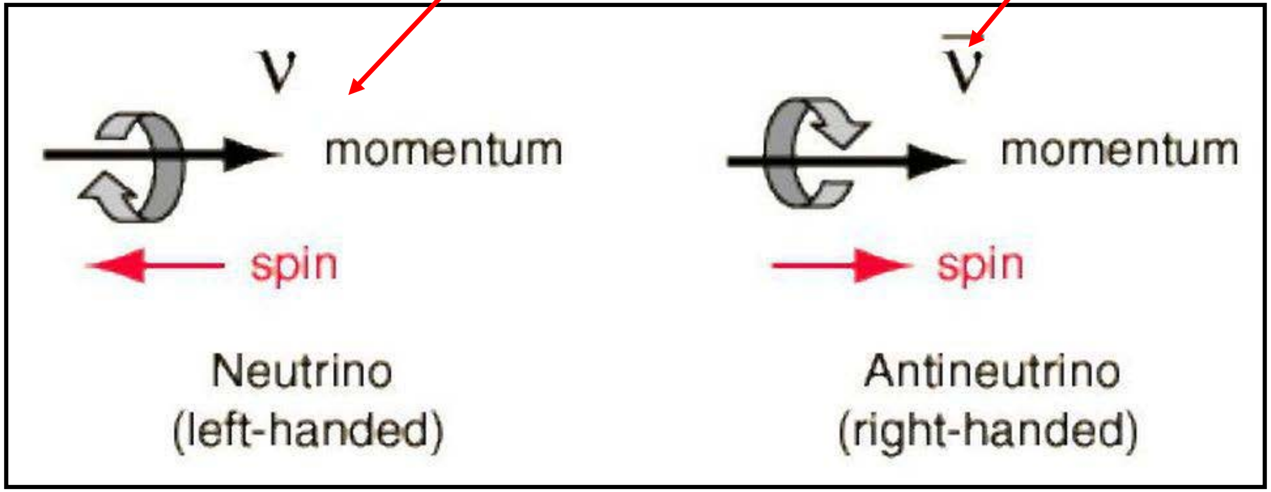


Neutrinos in the SM

- **Neutrinos in SM exist only in 2 states :**

  ν with (-) helicity & $\overline{\nu}$ with (+) helicity
- **Experiments have shown that ν $(\overline{\nu})$ are always left-handed (right-handed)**

$$W^+ \rightarrow l_\alpha^+ + \nu_\alpha \qquad W^- \rightarrow l_\alpha^- + \overline{\nu}_\alpha$$



Massless neutrinos: helicity=chirality

Neutrinos in the Universe

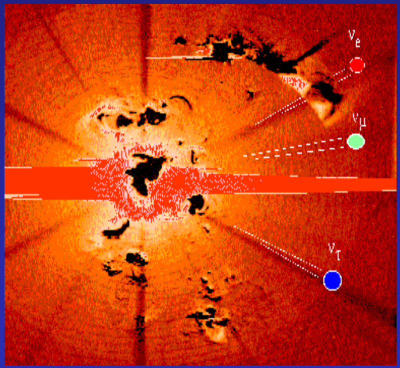
Neutrinos are everywhere:

Every second hundred trillion neutrinos from sun are passing through our body.

Neutrino Sources

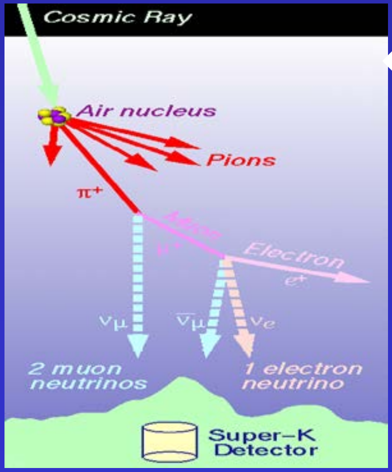
←Sun

~10^{38}/sec



←Cosmology

Big Bang ~300/cm^3



←Atmosphere

~1000/sec detected

Astronomy: →
Supernovae
GRBs
UHE ν's

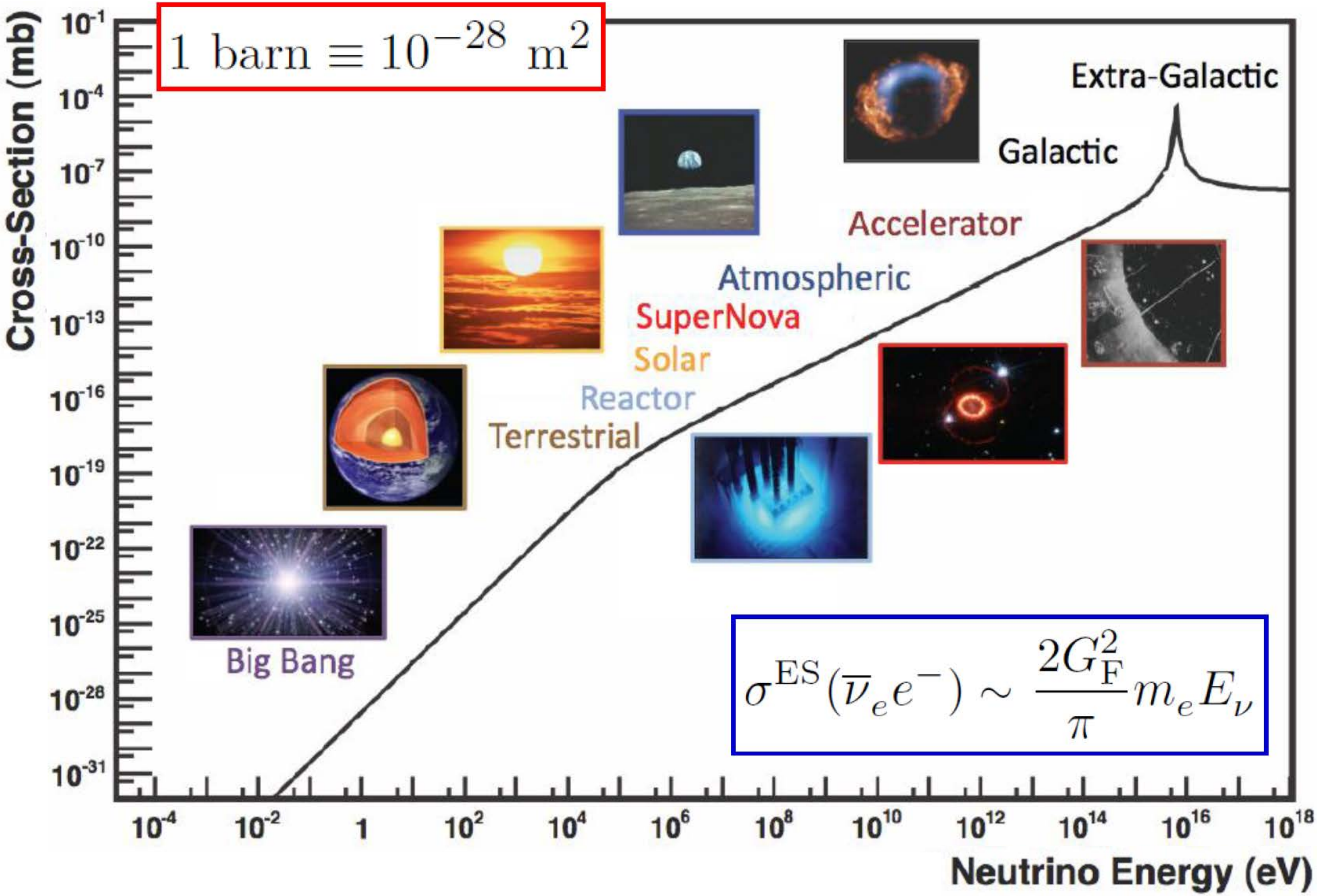
Reactors→

~10^{21}/sec

Accelerators→

←Earth

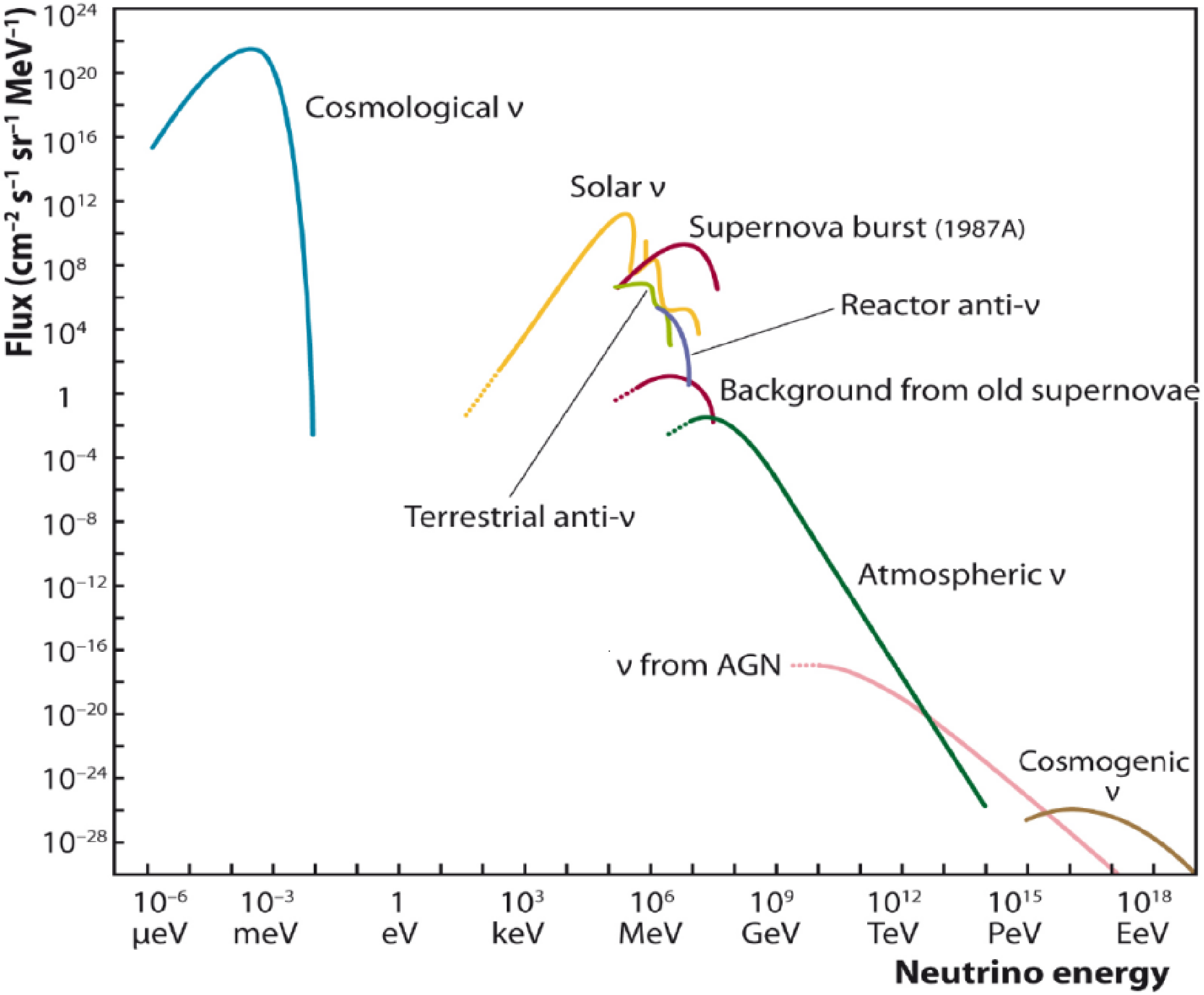
~10^9/second



$$1 \text{ barn} \equiv 10^{-28} \text{ m}^2$$

$$\sigma^{\mathrm{ES}}(\overline{\nu}_e e^-) \sim \frac{2G_{\mathrm{F}}^2}{\pi} m_e E_\nu$$

Prormaggio & Zeller, arXiv:1305.7513



Measured and expected fluxes of natural and reactor neutrinos (arXiv:1207.4952)

How did neutrino physics become so important ?

The Neutrino Revolution (1998-)

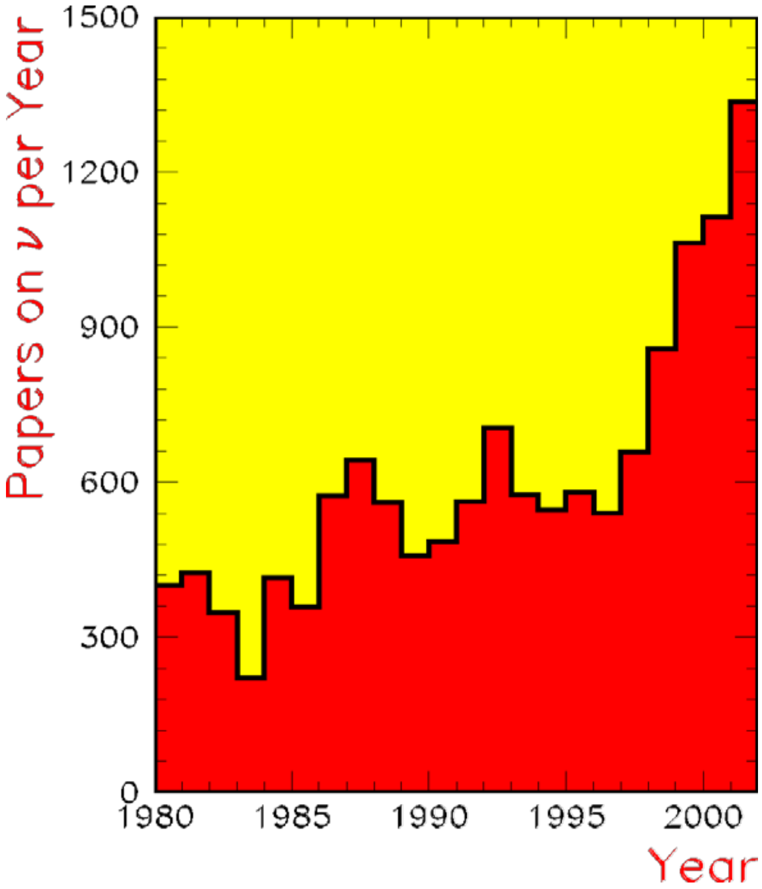
Discovery of Neutrino Oscillation

Neutrinos are massive.

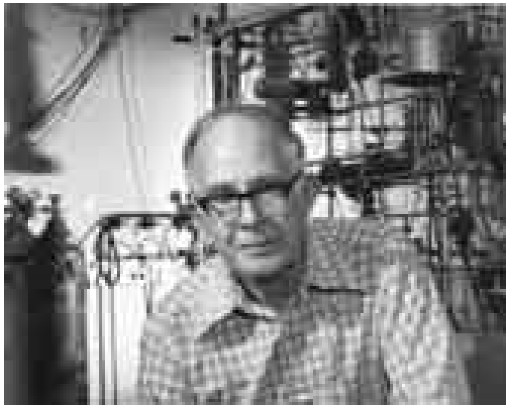
leptons mix.

- **Evidence for new physics beyond SM**
- **Playing a key role in understanding our universe**

Neutrino became hot !



2002-Nobel Prizes were awarded
to Davis & Koshiba
for detection of cosmic neutrinos

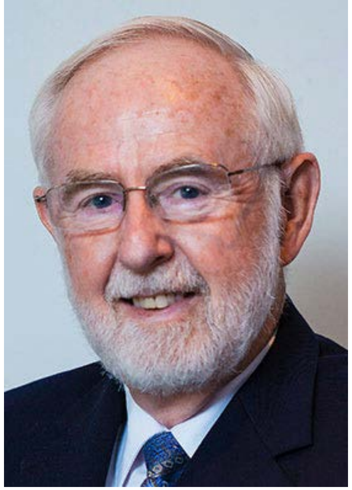


Neutrinos

from backstage to center stage

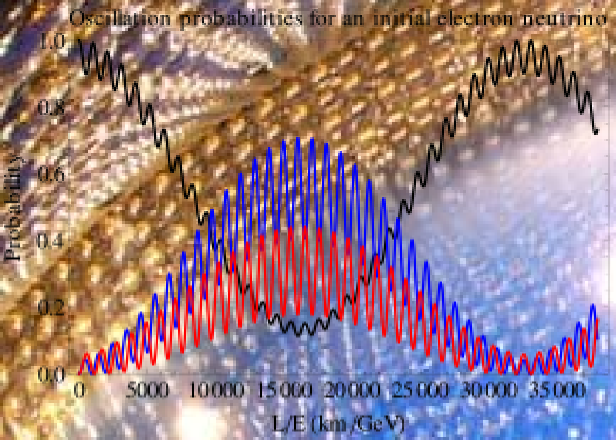
Neutrino is still hot!

2015-Nobel prizes were awarded to
“discovery of neutrino oscillation
Kajita & Mcdonald



Yoji Totsuka(March 6, 1942 – July 10, 2008)

***“The discovery of oscillation in neutrinos was largely due to the work done by Professor Totsuka,”* Kajita said.**



Physics of Neutrino Oscillations

History of Neutrino Oscillations

- After the experimental observation of $K - \bar{K}$ osc., Pontecorvo (1957) asked weather something similar could occur to other systems such as $\nu - \bar{\nu}, n - \bar{n}, ..$
- Maki, Nakagawa & Sakata (1962) mentioned the possible occurrence of virtual transmutation of neutrinos again without elaborating the details
- This was amended, once again, by Pontecorvo (1967).

  → developed the modern theory of NO in vacuum.

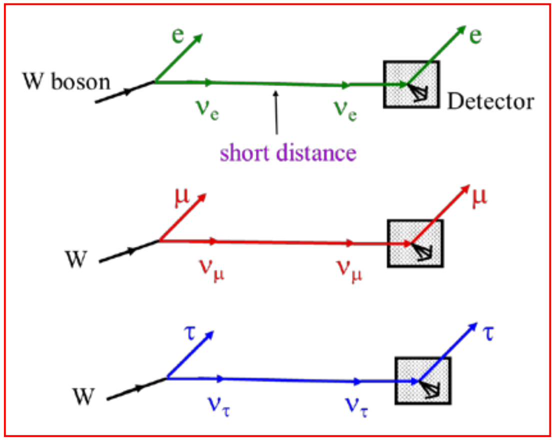
  → new ingredient is the mixing of different families of neutrinos introduced by MNS.

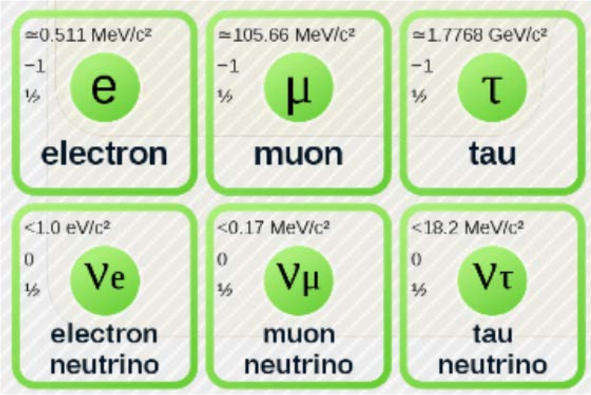
History of Neutrino Oscillations

- First indication on NO came from solar neutrino experiments: Homestake found solar neutrino flux deficit (1964,68)
- Atmospheric neutrino experiments (IMB, MACRO, Kamokande-II) found a deficit in the ratio of the flux of muon to electron neutrinos
- SuperK reported the first evidence of the atmospheric neutrino oscillations. (1998)
- SNO experiments provided clear evidence of the solar neutrino oscillations. (2001)

Lepton Mixing

- **Flavor eigenstates $(\nu_e, \nu_\mu, \nu_\tau)$: states produced or detected via weak interactions together with charged lepton with the same flavor (e, μ, τ)**





- **Mass eigenstates (ν_1, ν_2, ν_3) : states of definite masses that are created by the interactions with Higgs boson or other mechanisms**

- **Mismatch between flavor states and mass states of neutrinos gives rise to neutrino mixing**
- **A specific flavor state is a superposition of three mass eigenstates with definite masses.**

$$\nu_\ell = \sum_{i=1}^{N} U_{\ell i}\, \nu_i \quad \text{with} \quad \begin{cases} \ell = \mathrm{e}, \mu, \tau & \text{[flavor]} \\ i = 1, 2, 3 & \text{[mass]} \end{cases}$$

-Pontecorvo-Maki-Makagawa-Sakita mixing matrix (3x3 unitary)

$$\begin{pmatrix} \nu_e \\ \nu_\mu \\ \nu_\tau \end{pmatrix} = \left(\begin{matrix} \text{PMNS} \\ \text{matrix} \end{matrix} \right) \begin{pmatrix} \nu_1 \\ \nu_2 \\ \nu_3 \end{pmatrix}$$

Lepton Mixing

- **The unitary mixing matrix U occurs in C.C. weak interactions**

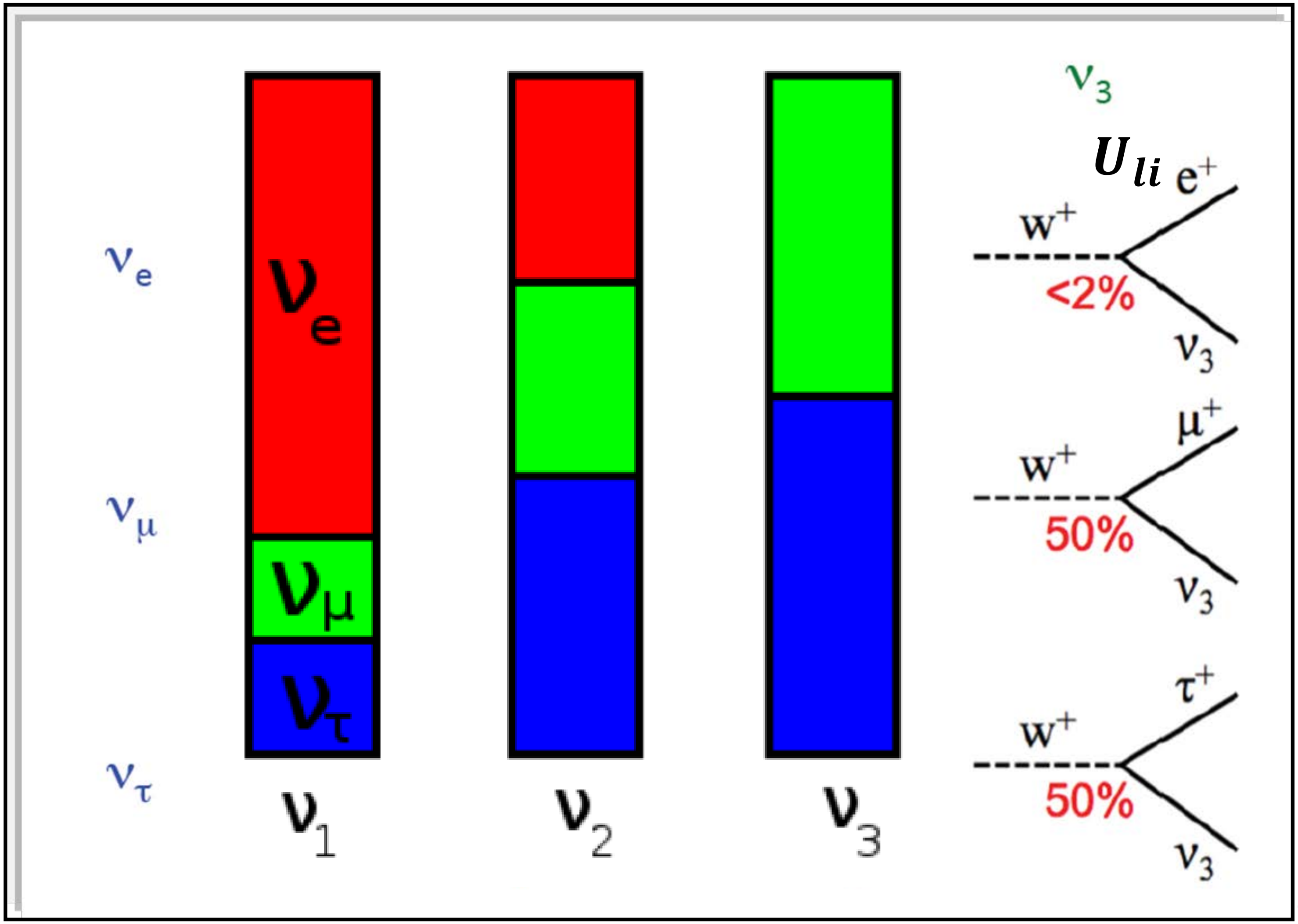
  (in the flavor basis) :

$$-\mathcal{L}_{\mathrm{cc}} = \frac{g}{\sqrt{2}}\, \overline{(\nu_e, \nu_\mu, \nu_\tau)_{\mathrm{L}}}\, \gamma^\mu \begin{pmatrix} e \\ \mu \\ \tau \end{pmatrix}_{\mathrm{L}} W_\mu^+ + \mathrm{h.c.}$$

  (in the mass basis) :

$$-\mathcal{L}_{\mathrm{cc}} = \frac{g}{\sqrt{2}}\, \overline{(\nu_1, \nu_2, \nu_3)_{\mathrm{L}}}\, \boxed{U^\dagger} \gamma^\mu \begin{pmatrix} e \\ \mu \\ \tau \end{pmatrix}_{\mathrm{L}} W_\mu^+ + \mathrm{h.c.}$$

What neutrino mixing elements mean



Parametrization of Mixing Matrix

$$U = V_L^{\ell\dagger} V_L^{\nu} = \begin{pmatrix} U_{e1} & U_{e2} & U_{e3} \\ U_{\mu 1} & U_{\mu 2} & U_{\mu 3} \\ U_{\tau 1} & U_{\tau 2} & U_{\tau 3} \end{pmatrix}$$

- **3x3 unitary mixing matrix depends on 9 independent parameters → 3 mixing angles and 6 phases**
- **Not all phases are physical observables.**
- **Let's see how many phases are physical by assuming neutrinos are Dirac particles.**

Parametrization of Mixing Matrix

- **Under global phase transformations:**

$$\nu_k \to e^{i\varphi_k}\,\nu_k \quad (k = 1, 2, 3)\,, \qquad \ell_\alpha \to e^{i\varphi_\alpha}\,\ell_\alpha \quad (\alpha = e, \mu, \tau)$$

$$\sum_{k=1}^{3} \sum_{\alpha=e,\mu,\tau} \overline{\nu_{kL}}\, e^{-i\varphi_k}\, U_{\alpha k}^{*}\, e^{i\varphi_\alpha}\, \gamma^\rho\, \ell_{\alpha L} \quad \Longrightarrow$$

$$\underbrace{e^{-i(\varphi_1-\varphi_e)}}_{1} \sum_{k=1} \sum_{\alpha=e,\mu,\tau} \overline{\nu_{kL}}\, \underbrace{e^{-i(\varphi_k-\varphi_1)}}_{2}\, U_{\alpha k}^{*}\, \underbrace{e^{i(\varphi_\alpha-\varphi_e)}}_{2}\, \gamma^\rho\, \ell_{\alpha L}$$

→ 5 phases can be eliminated by redefining fields

Parametrization of Mixing Matrix

- **Alternatively**

$$\begin{pmatrix} e^{i\gamma_2} & 0 & 0 \\ 0 & c_2 e^{i\alpha_2} & s_2 e^{-i\beta_2} \\ 0 & -s_2 e^{i\beta_2} & c_2 e^{-i\alpha_2} \end{pmatrix} \begin{pmatrix} c_3 e^{i\alpha_3} & 0 & s_3 e^{-i\beta_3} \\ 0 & e^{i\gamma_3} & 0 \\ -s_3 e^{i\beta_3} & 0 & c_3 e^{-i\alpha_3} \end{pmatrix} \begin{pmatrix} c_1 e^{i\alpha_1} & s_1 e^{-i\beta_1} & 0 \\ -s_1 e^{i\beta_1} & c_1 e^{-i\alpha_1} & 0 \\ 0 & 0 & e^{i\gamma_1} \end{pmatrix}$$

$$= \begin{pmatrix} c_1 c_3 e^{i(\alpha_1+\gamma_2+\alpha_3)} & s_1 c_3 e^{i(-\beta_1+\gamma_2+\alpha_3)} & s_3 e^{i(\gamma_1+\gamma_2-\beta_3)} \\ -s_1 c_2 e^{i(\beta_1+\alpha_2+\gamma_3)} - c_1 s_2 s_3 e^{i(\alpha_1-\beta_2+\beta_3)} & c_1 c_2 e^{i(-\alpha_1+\alpha_2+\gamma_3)} - s_1 s_2 s_3 e^{i(-\beta_1-\beta_2+\beta_3)} & s_2 c_3 e^{i(\gamma_1-\beta_2-\alpha_3)} \\ s_1 s_2 e^{i(\beta_1+\beta_2+\gamma_3)} - c_1 c_2 s_3 e^{i(\alpha_1-\alpha_2+\beta_3)} & -c_1 s_2 e^{i(-\alpha_1+\beta_2+\gamma_3)} - s_1 c_2 s_3 e^{i(-\beta_1-\alpha_2+\beta_3)} & c_2 c_3 e^{i(\gamma_1-\alpha_2-\alpha_3)} \end{pmatrix}$$

$$= \begin{pmatrix} e^{ia} & 0 & 0 \\ 0 & e^{ib} & 0 \\ 0 & 0 & e^{ic} \end{pmatrix} \begin{pmatrix} c_1 c_3 & s_1 c_3 & s_3 e^{-i\delta} \\ -s_1 c_2 - c_1 s_2 s_3 e^{i\delta} & c_1 c_2 - s_1 s_2 s_3 e^{i\delta} & s_2 c_3 \\ s_1 s_2 - c_1 c_2 s_3 e^{i\delta} & -c_1 s_2 - s_1 c_2 s_3 e^{i\delta} & c_2 c_3 \end{pmatrix} \begin{pmatrix} e^{ix} & 0 & 0 \\ 0 & e^{iy} & 0 \\ 0 & 0 & e^{iz} \end{pmatrix}$$

$$a = (\alpha_1 - \beta_1) - (\alpha_2 + \beta_2 - \gamma_2) - \gamma_3$$

Parametrization of Mixing Matrix

- **The mixing matrix contains 1 physical phase**
- **3x3 unitary mixing matrix can be expressed in terms of 3 mixing angles and 1 phase**

- **Standard parametrization :**

$$U = \begin{pmatrix} 1 & 0 & 0 \\ 0 & c_{23} & s_{23} \\ 0 & -s_{23} & c_{23} \end{pmatrix} \begin{pmatrix} c_{13} & 0 & s_{13}e^{-i\delta_{13}} \\ 0 & 1 & 0 \\ -s_{13}e^{i\delta_{13}} & 0 & c_{13} \end{pmatrix} \begin{pmatrix} c_{12} & s_{12} & 0 \\ -s_{12} & c_{12} & 0 \\ 0 & 0 & 1 \end{pmatrix}$$

$$= \begin{pmatrix} c_{12}c_{13} & s_{12}c_{13} & s_{13}e^{-i\delta_{13}} \\ -s_{12}c_{23}-c_{12}s_{23}s_{13}e^{i\delta_{13}} & c_{12}c_{23}-s_{12}s_{23}s_{13}e^{i\delta_{13}} & s_{23}c_{13} \\ s_{12}s_{23}-c_{12}c_{23}s_{13}e^{i\delta_{13}} & -c_{12}s_{23}-s_{12}c_{23}s_{13}e^{i\delta_{13}} & c_{23}c_{13} \end{pmatrix}$$

$$(\boldsymbol{s_{ij}} = \sin\theta_{ij}\,, \boldsymbol{c_{ij}} = \cos\theta_{ij})$$

Basics for Neutrino Oscillation

- **Non-relativisitic Schrodinger Eq. for a free particle**

$$E = \frac{p^2}{2m}.$$

Substituting $E \to i\hbar \frac{\partial}{\partial t}, \quad \mathbf{p} = -i\hbar\boldsymbol{\nabla}$

$$i\frac{\partial\psi}{\partial t} + \frac{1}{2m}\nabla^2\psi = \mathbf{0}$$

- **Relativistic quantum equation from energy-momentum relation:**

$$E^2 = \mathbf{p}^2 + m^2.$$

$$-\frac{\partial^2\phi}{\partial t^2} + \nabla^2\phi = m^2\phi$$ → **Klein-Gordon Eq.**

$$\phi = Ne^{i\mathbf{p}\cdot\mathbf{x} - iEt}$$ →**free particle solution**

Basics for Neutrino Oscillation

- **Choosing natural units** $\boxed{\hbar = c = 1}$

$$\hbar = \frac{h}{2\pi} = 1.055 \times 10^{-34}\ \mathrm{J\ sec}$$
$$c = 2.998 \times 10^{8}\ \mathrm{m\ sec^{-1}}$$

$\hbar$ (ML^2/T) and c (L/T)

- **In high *E* physics, quantities are measured in units of GeV (e.g. m_p~ 1 GeV)**

$$m\ \mathrm{kg} \equiv mc^2\ \text{Energy units}$$

$$\begin{aligned} 1\ \mathrm{kg} &\equiv 1 \times (2.998 \times 10^{8})^2\ \mathrm{J} \\ &= \frac{(2.998 \times 10^{8})^2\ \mathrm{J}}{1.6 \times 10^{-19}\ \frac{\mathrm{J}}{\mathrm{eV}}} \\ &= 5.618 \times 10^{-35}\ \mathrm{eV} \\ &= 5.618 \times 10^{-26}\ \mathrm{GeV}. \end{aligned}$$

Table 2.3: Conversion factors for MKS to Natural units

Quantity	**Conversion factor**	**Actual dimension**
Mass	$1\ \mathrm{kg}\ = 5.62 \times 10^{-26}\ \mathrm{GeV}$	$\frac{\mathrm{GeV}}{c^2}$
Length	$1\ \mathrm{m}\ = 5.07 \times 10^{15}\ \mathrm{GeV}^{-1}$	$\frac{\hbar c}{\mathrm{GeV}}$
Time	$1\ \mathrm{sec}\ = 1.52 \times 10^{24}\ \mathrm{GeV}^{-1}$	$\frac{\hbar}{\mathrm{GeV}}$

Basics for Neutrino Oscillation

Relativistic Schrodinger Eq. in a form linear in $\frac{\partial}{\partial t}$

$$H\psi = (\boldsymbol{\alpha}\cdot\mathbf{P} + \beta m)\psi \;= i\frac{\partial}{\partial t}\psi$$

$\alpha_i \beta$ are constants and satisfy $E^2 = \mathbf{p}^2 + m^2.$

$$H^2\psi = (\mathbf{P}^2 + m^2)\psi$$

- $\alpha_i^2 = \beta^2 = 0$
- $\alpha_i\alpha_j + \alpha_j\alpha_i = \alpha_i\beta + \beta\alpha_i = 0$. Hence α_i's and β anticommute with one another.

- **Due to the last relations, $\alpha_i \beta$ cannot be numbers but matrices**

$$\boldsymbol{\alpha} = \begin{pmatrix} 0 & \boldsymbol{\sigma} \\ \boldsymbol{\sigma} & 0 \end{pmatrix}, \quad \beta = \begin{pmatrix} I & 0 \\ 0 & -I \end{pmatrix}$$

$$\sigma_1 = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}, \quad \sigma_2 = \begin{pmatrix} 0 & -i \\ i & 0 \end{pmatrix}, \quad \sigma_3 = \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}$$

Basics for Neutrino Oscillation

Relativistic Schrodinger Eq. :

$$\left(i\gamma^{\mu}\partial_{\mu} - m\right)\psi = 0$$

where $\gamma^{\mu} \equiv (\beta, \beta\boldsymbol{\alpha})$ and $\partial_{\mu} = \left(\frac{\partial}{\partial \mathrm{t}}, \boldsymbol{\nabla}\right)$ (in four vector notation). γ^{μ}'s are known as the Dirac γ matrices

Neutrino Oscillation

- Quantum mechanical effects when

Flavor states 🔵 🔴 🟢

Mass states ν_1 ν_2 ν_3

$$\boxed{|\,\nu_\alpha\rangle = \sum_k U_{\alpha k}|\nu_k\rangle}$$

Evolution of ν_k (mass states):

$$|\,\nu_k(t,x)\rangle = e^{-iE_k+ip_kx}|\nu_k\rangle$$

Evolution of ν_α (flavor states):

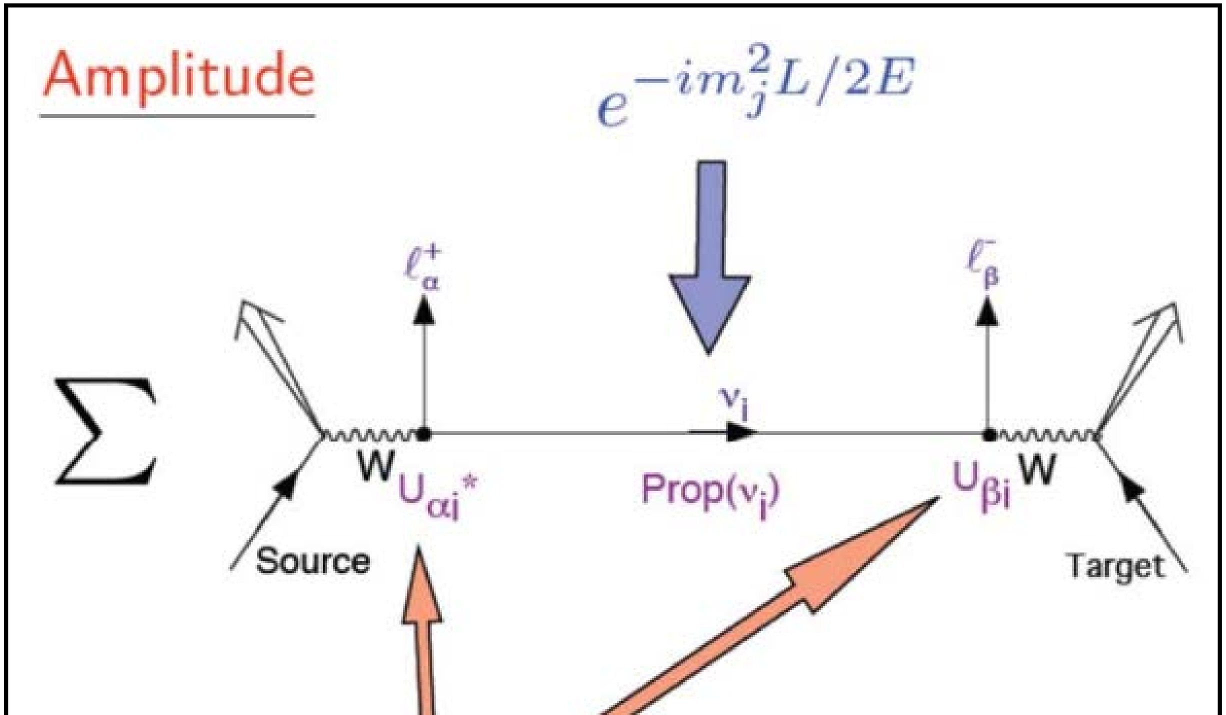
$$|\,\nu_\alpha(t,x)\rangle = \sum_k U_{\alpha k}e^{-iE_k+ip_kx}\underline{|\nu_k\rangle}$$

$$|\,\nu_k\rangle = \sum_\beta U^*_{\beta k}|\nu_\beta\rangle$$

Neutrino Oscillation

$$|\nu_\alpha(t,x)\rangle = \sum_\beta \left(\sum_k U_{\alpha k} e^{-iE_k + ip_k x} U^*_{\beta k} \right) |\nu_\beta\rangle$$

$$= \sum_\beta A_{\nu_\alpha \to \nu_\beta}(t,x) |\nu_\beta\rangle$$



Neutrino Oscillation

$$\boxed{P_{\nu_\alpha\to\nu_\beta}(t,x) = \left|A_{\nu_\alpha\to\nu_\beta}(t,x)\right|^2 = \left|\sum_k U_{\alpha k} e^{-iE_k t + i p_k x} U^*_{\beta k}\right|^2}$$

$$E_k t - p_k x \simeq (E_k - p_k)\, L = \frac{E_k^2 - p_k^2}{E_k + p_k} L = \frac{m_k^2}{E_k + p_k} L \simeq \frac{m_k^2}{2E} L$$

(In natural unit, $t = L$)

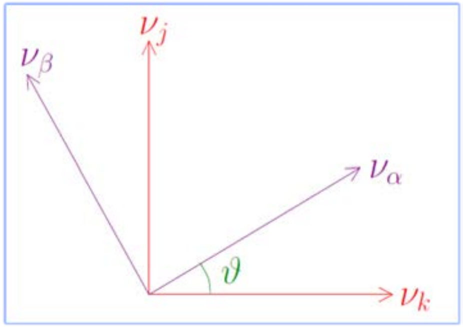
$$P_{\nu_\alpha\to\nu_\beta}(L,E) = \left|\sum_k U_{\alpha k} e^{-i m_k^2 L/2E} U^*_{\beta k}\right|^2$$

$$= \sum_{kj} U_{\alpha k} U^*_{\beta k} U_{\alpha j} U^*_{\beta j}\, exp\left(-i\frac{\Delta m_{kj}^2 L}{2E}\right)$$

$$\boxed{\Delta m_{kj}^2 \equiv m_k^2 - m_j^2}$$

2-Neutrino Oscillation

$$\begin{pmatrix} \nu_e \\ \nu_\mu \end{pmatrix} = \begin{pmatrix} \cos\theta & \sin\theta \\ -\sin\theta & \cos\theta \end{pmatrix} \begin{pmatrix} \nu_1 \\ \nu_2 \end{pmatrix}$$



Production
(Flavor state)

$$|\nu_\mu\rangle = -\sin\theta|\nu_1\rangle + \cos\theta\,|\nu_2\rangle$$

Propagation :
(mass states)

$$\begin{cases} \nu_1 :\ e^{-ip_1x} \\ \nu_2 :\ e^{-ip_2x} \end{cases}$$

$$|\nu_\mu(x)\rangle = -\sin\theta e^{-ip_1x}|\nu_1\rangle + \cos\theta e^{-ip_2x}|\nu_2\rangle$$

Detection :
(flavor states)

$$|\nu_e\rangle = \cos\theta|\nu_1\rangle + \sin\theta\,|\nu_2\rangle$$

$$P_{\nu_\mu\to\nu_e} = \left|\langle\nu_e(0)|\nu_\mu(x)\rangle\right|^2 = \left|-\sin\theta\cos\theta\, e^{-ip_1x} + \cos\theta\sin\theta e^{-ip_2x}\right|^2$$

$$p_j = \sqrt{E^2 - m_j^2} \approx E - \frac{m_j^2}{2E}$$

2-Neutrino Oscillation

$$P_{\nu_\mu \to \nu_e}(L,E) = 2\sin^2\theta\cos^2\theta\left(1-\cos\left(\frac{\Delta m_{21}^2 L}{2E}\right)\right)$$

$$= \sin^2 2\theta \sin^2\left(\frac{\Delta m_{21}^2 L}{4E}\right)$$ ⇨ transition probability

- **2 fundamental parameters** : $(\theta, \Delta m_{21}^2)$

oscillation length : $L^{osc} = \dfrac{4\pi E}{\Delta m^2}$

- **Converting natural unit to lab. unit :**

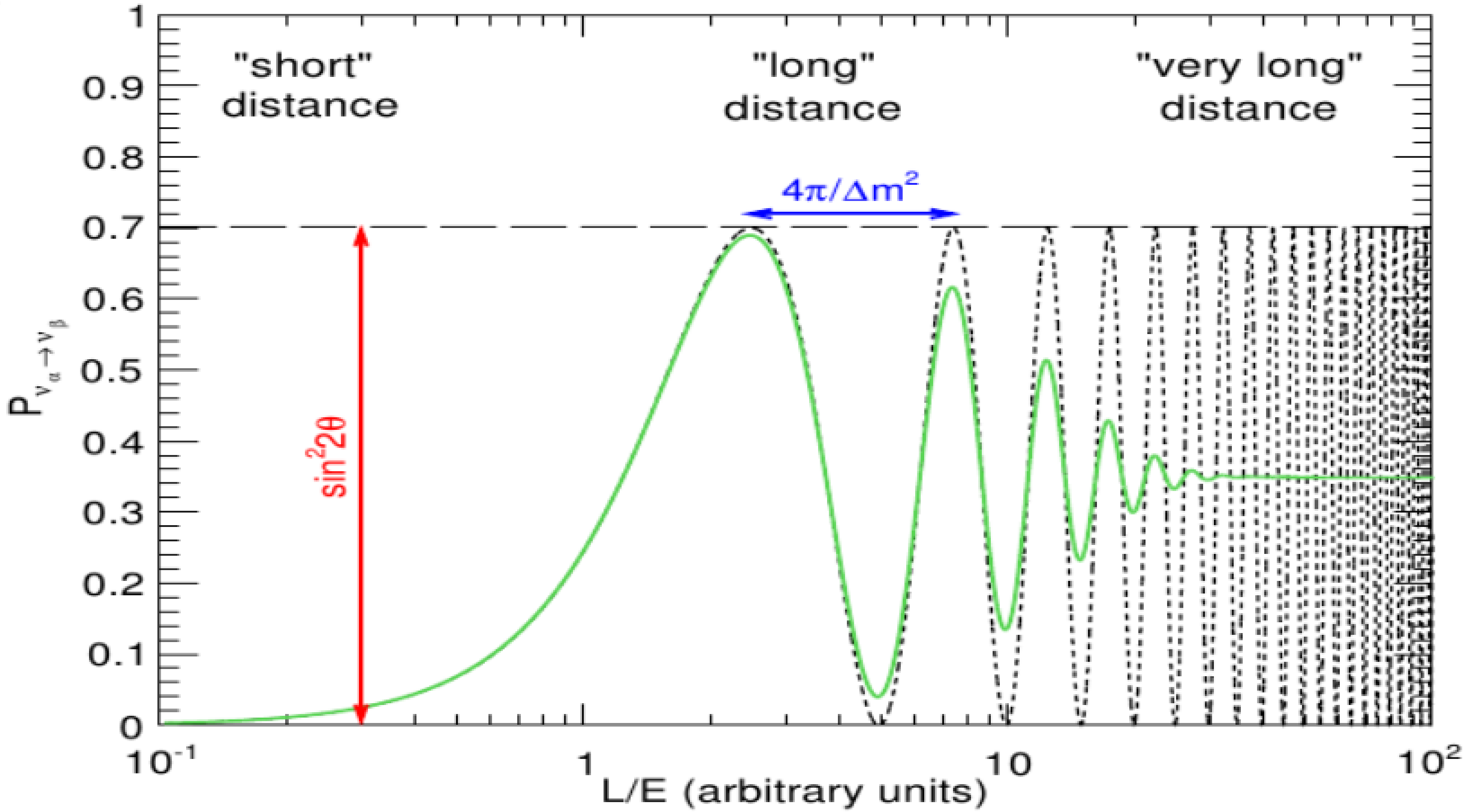
$$\frac{\Delta m^2 L}{4E} = 1.27\frac{\Delta m^2[\mathrm{eV}^2]\,L[\mathrm{m}]}{E[\mathrm{MeV}]} = 1.27\frac{\Delta m^2[\mathrm{eV}^2]\,L[\mathrm{km}]}{E[\mathrm{GeV}]}$$

- **Survival (disappearance) probability**

$$P_{\nu_\mu \to \nu_\mu}(L,E) = 1 - P_{\nu_\mu \to \nu_e}(L,E) = 1 - \sin^2 2\theta \sin^2\left(\frac{\Delta m_{21}^2 L}{4E}\right)$$

2-Neutrino Oscillation

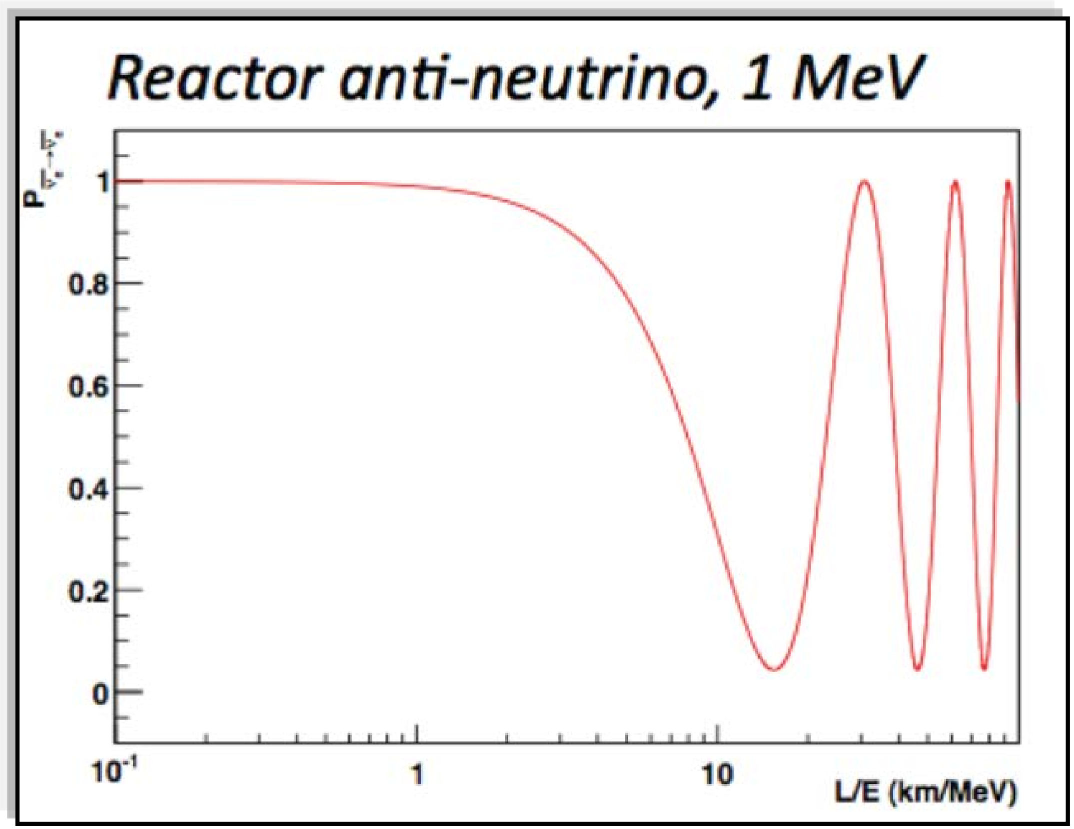
Example of $P_{\nu_\alpha \to \nu_\beta}$



θ

2-Neutrino Oscillation

$$P_{\nu_\alpha \to \nu_\alpha} = 1 - P_{\nu_\alpha \to \nu_\beta}$$



E=1 MeV,

θ=40 deg

$\Delta m^2 = 8 \times 10^{-5} (\mathrm{eV})^2$

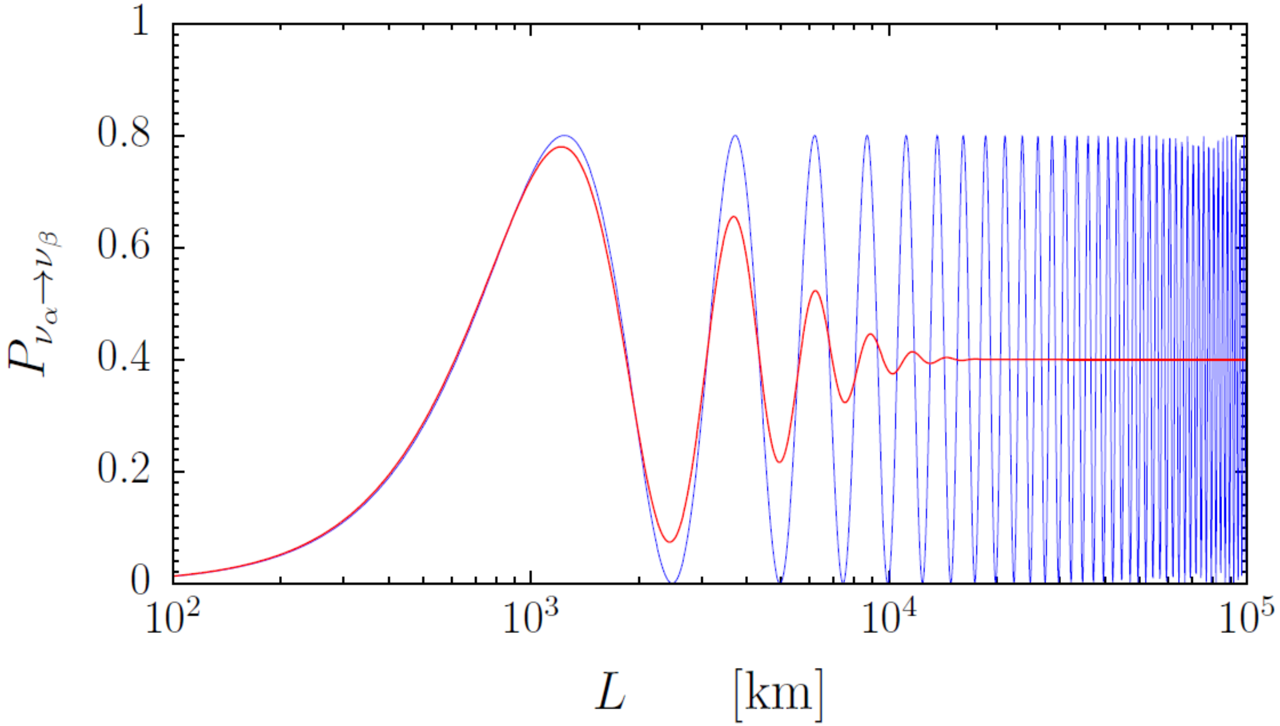
θ

Averaging over *E*

$$< P_{\nu_\alpha \to \nu_\beta}(L,E) > = \delta_{\alpha\beta} - \left(2\delta_{\alpha\beta} - 1\right)\sin^2 2\theta < \sin^2\left(\frac{\Delta m^2_{21} L}{4E}\right) >$$

Practically,

$$< P_{\nu_\alpha \to \nu_\beta} > = \frac{\int dE_\nu \frac{d\Phi}{dE_\nu} \sigma_{CC}(E_\nu)\varepsilon(E_\nu) P_{\nu_\alpha \to \nu_\beta}(L,E)}{\int dE_\nu \frac{d\Phi}{dE_\nu} \sigma_{CC}(E_\nu)\varepsilon(E_\nu)}$$



$\Delta m^2 = 10^{-3}\,\text{eV}$

$\sin^2 2\vartheta = 0.8$

$\langle E \rangle = 1\,\text{GeV}$

θ

Exclusion Curve

$$< P_{\nu_\alpha \to \nu_\beta}(L, E) > = \delta_{\alpha\beta} - (2\delta_{\alpha\beta} - 1)\sin^2 2\theta < \sin^2\left(\frac{\Delta m_{21}^2 L}{4E}\right) >$$

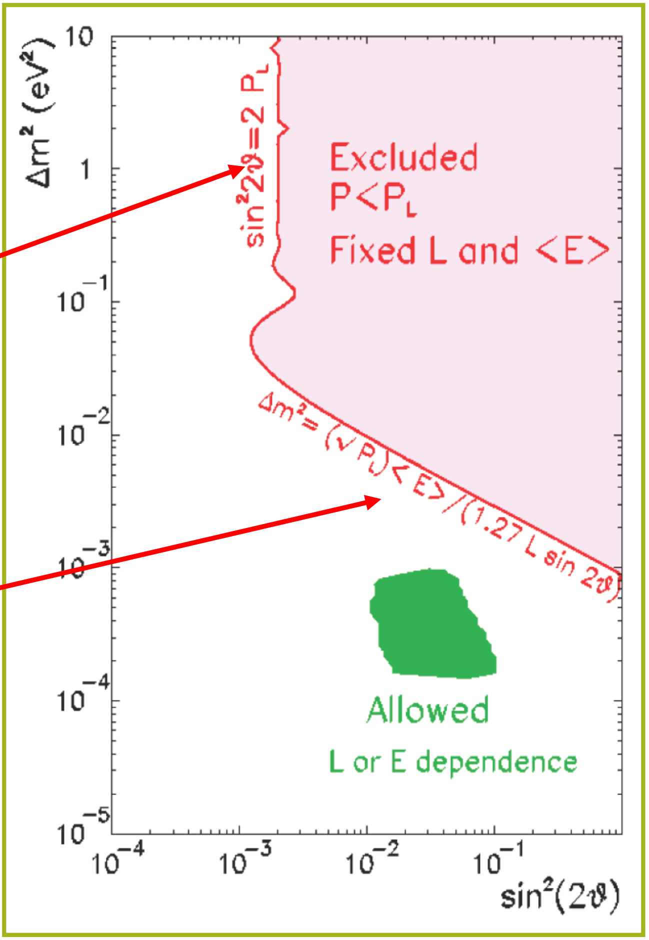
Taking data at fixed $< L >$ and $< E >$.

-For $\Delta m^2 \gg \frac{1}{<L/E>}$, $< \sin^2(x) > = \frac{1}{2}$,

a vertical line at $\boxed{\sin^2 2\theta = 2 < P_{\alpha\beta} >}$

-For $\Delta m^2 \ll \frac{1}{<L/E>}$, $< \sin^2(x) > \sim x^2$

$$\boxed{\Delta m^2 \sin 2\theta = 4\sqrt{< P_{\alpha\beta} >} / < L/E >}$$



Three-Neutrino Oscillation

- **Oscillation probability**

$$P_{\nu_\alpha\to\nu_\beta} = \sum_{i=1}^{3}\left|U^*_{\alpha i}U_{\beta i}\right|^2 + 2\sum_{i<j}^{3} Re\left(U^*_{\alpha i}U_{\beta i}U_{\alpha j}U^*_{\beta j}\right)\cos\frac{\Delta m^2_{ji}L}{2E} - 2\sum_{i<j}^{3} Im\left(U^*_{\alpha i}U_{\beta i}U_{\alpha j}U^*_{\beta j}\right)\sin\frac{\Delta m^2_{ji}L}{2E}$$

$$= \underbrace{\left|\underbrace{\sum_{i=1}^{3} U^*_{\alpha i}U_{\beta i}}_{\delta_{\alpha\beta}}\right|^2 - 4\sum_{i<j}^{3} Re\left(U_{\alpha i}U_{\beta i}U^*_{\alpha j}U^*_{\beta j}\right)\sin^2\frac{\Delta m^2_{ji}L}{2E}}_{\text{CP conserving part : } P^{CPC}_{\nu_\alpha\to\nu_\beta}} + 2\sum_{i<j}^{3} \underbrace{\boxed{Im\left(U_{\alpha i}U_{\beta i}U^*_{\alpha j}U^*_{\beta j}\right)}}_{= J\sum_{\gamma,k}\varepsilon_{\alpha\beta\gamma}\varepsilon_{ijk}}\sin\frac{\Delta m^2_{ji}L}{2E}$$

$$P^{CPV}_{\nu_\alpha\to\nu_\beta} = 8J\sum_{\gamma}\varepsilon_{\alpha\beta\gamma}\sin\frac{\Delta m^2_{21}L}{4E}\sin\frac{\Delta m^2_{31}L}{4E}\sin\frac{\Delta m^2_{32}L}{4E}$$

Three-Neutrino Oscillation

$$P_{\nu_\alpha \to \nu_\alpha} = 1 - 4|U_{\alpha 1}|^2 |U_{\alpha 2}|^2 \sin^2 \frac{\Delta m_{21}^2 L}{4E} - 4|U_{\alpha 1}|^2 |U_{\alpha 3}|^2 \sin^2 \frac{\Delta m_{31}^2 L}{4E}$$

$$-4|U_{\alpha 2}|^2 |U_{\alpha 3}|^2 \sin^2 \frac{\Delta m_{32}^2 L}{4E}$$

$$P_{\nu_\alpha \to \nu_\alpha} \cong 1 - 4|U_{\alpha 1}|^2 |U_{\alpha 2}|^2 \sin^2 \frac{\Delta m_{21}^2 L}{4E} - 4(1 - |U_{\alpha 3}|^2)|U_{\alpha 3}|^2 \sin^2 \frac{\Delta m_{31}^2 L}{4E}$$

$$P_{\nu_\alpha \to \nu_\beta} \cong -4\left(U_{\alpha 1} U_{\beta 1} U_{\alpha 2}^* U_{\beta 2}^*\right) \sin^2 \frac{\Delta m_{21}^2 L}{4E} + 4|U_{\alpha 3}|^2 \left|U_{\beta 3}\right|^2 \sin^2 \frac{\Delta m_{31}^2 L}{4E}$$

$$(\Delta m_{21}^2 = \Delta m_{sol}^2 \ll |\Delta m_{atm}^2| = |\Delta m_{31}^2| \cong \left|\Delta m_{32}^2\right|)$$

This hierarchy & small θ_{13} lead to two-neutrino oscillation approximation for many experiments.

Three-Neutrino Oscillation

- ν_e survival (appearance) probability

$$|U_{e3}|^2 \ll |U_{e1}|^2, |U_{e2}|^2 \quad \Longrightarrow \quad |U_{e1}|^2 \simeq \cos\vartheta_{12}\,,\; |U_{e2}|^2 \simeq \sin\vartheta_{12}$$

$$\boxed{P_{\nu_e \to \nu_e} = 1 - \sin^2 2\vartheta_{12} \sin^2\left(\frac{\Delta m^2_{21} L}{4E}\right)}$$

⇨ It is decoupled from atmospheric *v* osc.
so, good at probing solar neutrinos

Three-Neutrino Oscillation

In the case that $|\Delta m^2_{21}| \ll |\Delta m^2_{31}| \cong |\Delta m^2_{32}|$

$$P_{\nu_\alpha \to \nu_\beta} = \delta_{\alpha\beta} - 4|U_{\alpha 3}|^2 \left(\delta_{\alpha\beta} - |U_{\beta 3}|^2\right) \sin^2 \frac{\Delta m^2_{31} L}{4E}$$

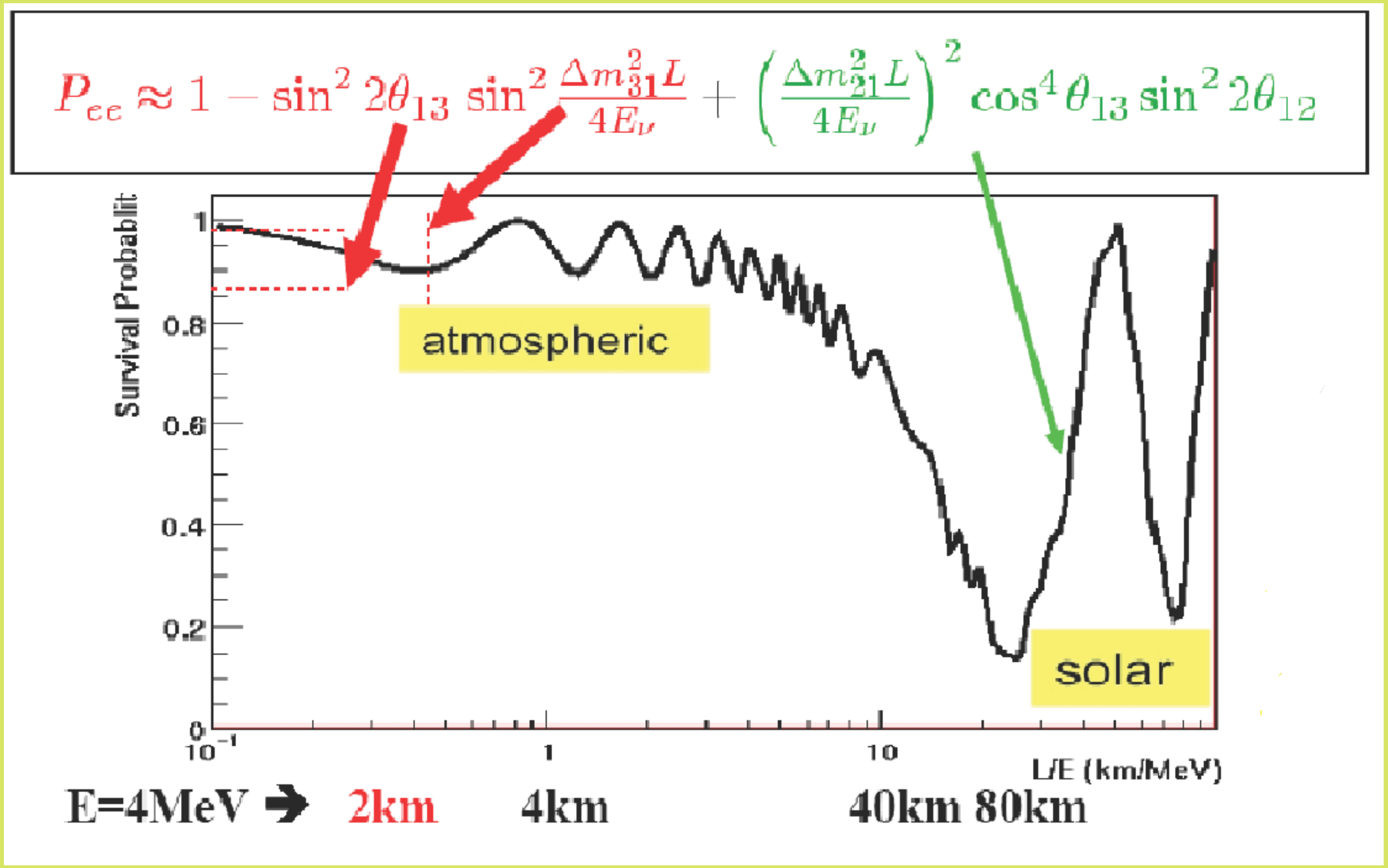
$$\alpha \neq \beta \quad \Longrightarrow \quad P_{\nu_\alpha \to \nu_\beta} = 4|U_{\alpha 3}|^2 |U_{\beta 3}|^2 \sin^2 \left(\frac{\Delta m^2_{31} L}{4E}\right)$$

$$\alpha = \beta \quad \Longrightarrow \quad P_{\nu_\alpha \to \nu_\alpha} = 1 - 4|U_{\alpha 3}|^2 \left(1 - |U_{\alpha 3}|^2\right) \sin^2 \left(\frac{\Delta m^2_{31} L}{4E}\right)$$

relevant to atmospheric & SBL reactor exp.

Three-Neutrino Oscillation

Oscillations for Short & Medium Baseline Reactor neutrinos

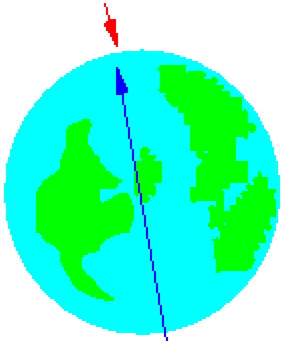


$$P_{ee} \approx 1 - \sin^2 2\theta_{13} \sin^2 \frac{\Delta m^2_{31} L}{4E_\nu} + \left(\frac{\Delta m^2_{21} L}{4E_\nu}\right)^2 \cos^4 \theta_{13} \sin^2 2\theta_{12}$$

Three-Neutrino Oscillation

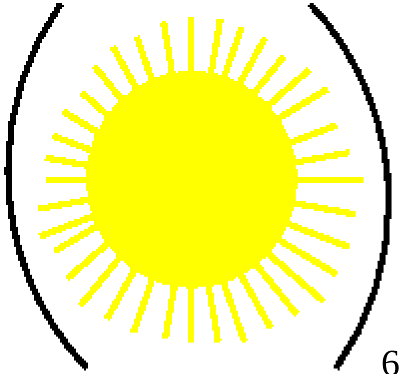
$$U = \underline{\begin{pmatrix}1&0&0\\0&c_{23}&s_{23}\\0&-s_{23}&c_{23}\end{pmatrix}}\;\underline{\begin{pmatrix}c_{13}&0&s_{13}e^{-i\delta_{13}}\\0&1&0\\-s_{13}e^{i\delta_{13}}&0&c_{13}\end{pmatrix}}\;\underline{\begin{pmatrix}c_{12}&s_{12}&0\\-s_{12}&c_{12}&0\\0&0&1\end{pmatrix}}$$

$$= \begin{pmatrix}\boxed{c_{12}c_{13}}&\boxed{s_{12}c_{13}}&\boxed{s_{13}}e^{-i\delta_{13}}\\-s_{12}c_{23}-c_{12}s_{23}s_{13}e^{i\delta_{13}}&c_{12}c_{23}-s_{12}s_{23}s_{13}e^{i\delta_{13}}&\boxed{s_{23}c_{13}}\\s_{12}s_{23}-c_{12}c_{23}s_{13}e^{i\delta_{13}}&-c_{12}s_{23}-s_{12}c_{23}s_{13}e^{i\delta_{13}}&\boxed{c_{23}c_{13}}\end{pmatrix}$$

U= ()(Reactor

and/or

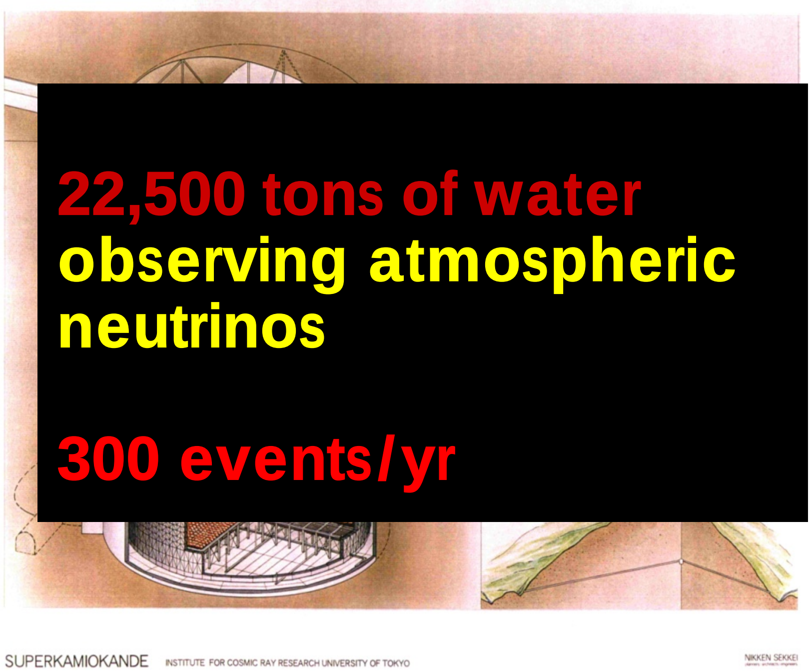
Accelerator

ν_e)()

How to Detect Neutrinos

- **Neutrinos don’t interact with light, so can’t directly be seen.**
- **But signatures are produced via weak interactions → large volume of detector is required to capture significant numbers of neutrino events**

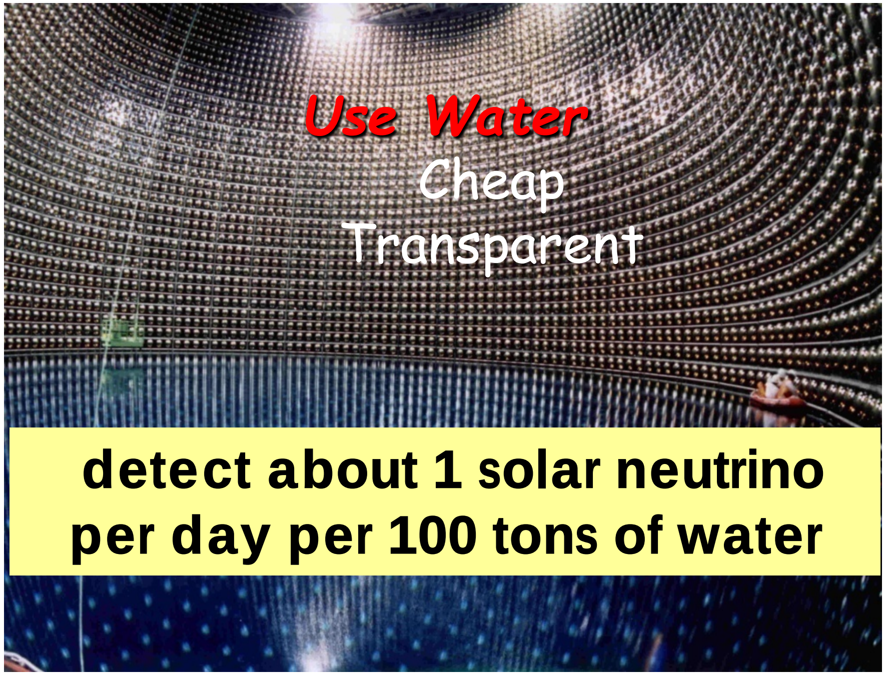
Neutrino detectors



22,500 tons of water
observing atmospheric neutrinos

300 events/yr

(ex) Super Kamiokande

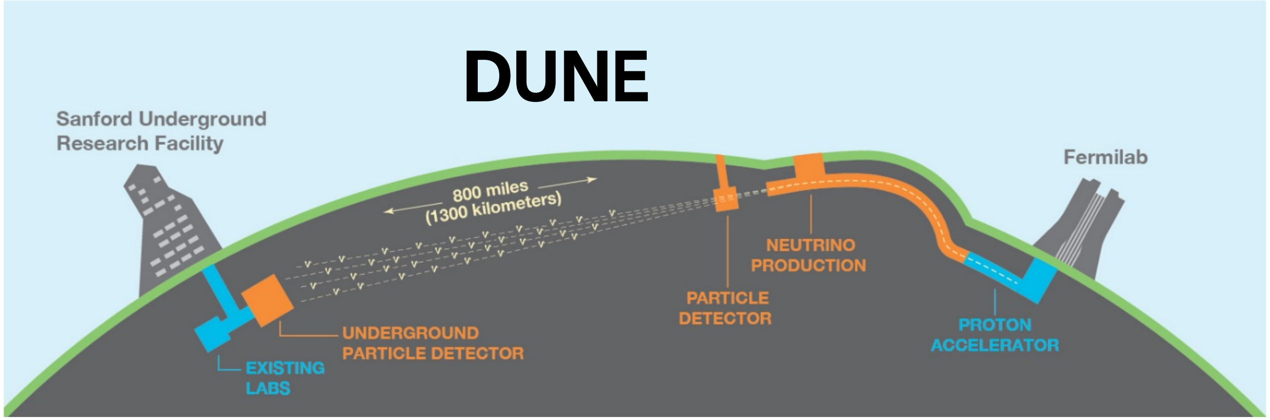


Use Water
Cheap
Transparent

detect about 1 solar neutrino per day per 100 tons of water

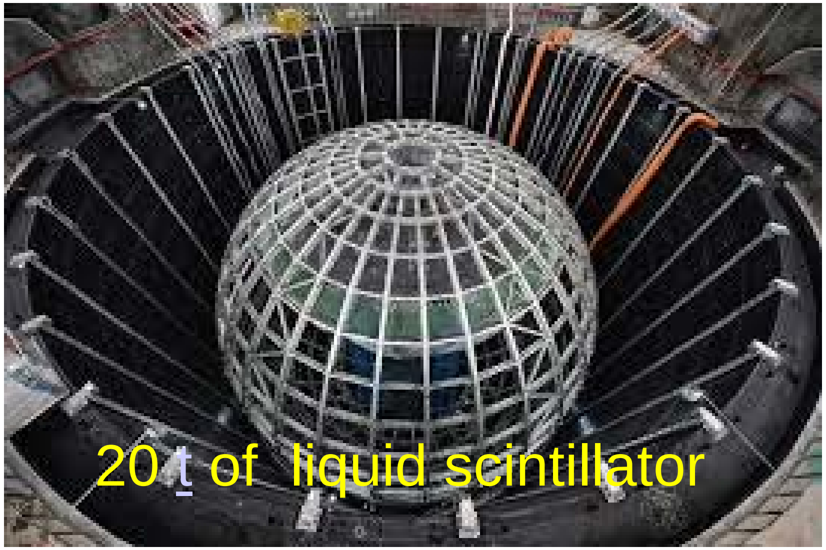
How to Detect Neutrinos

DUNE



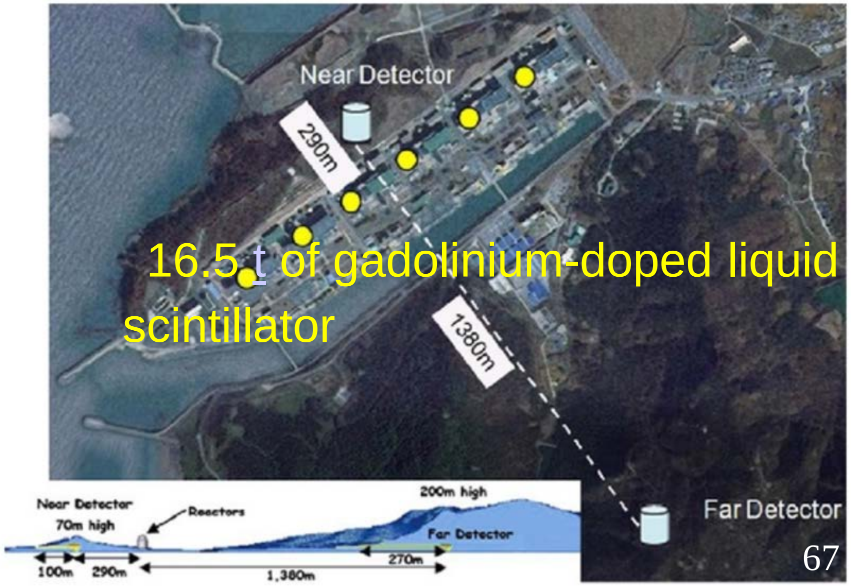
Use ~40 kt liquid Ar

JUNO



20 t of liquid scintillator

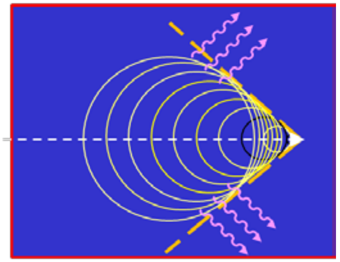
RENO

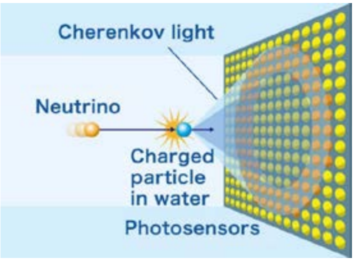


16.5 t of gadolinium-doped liquid scintillator

How to Detect Neutrinos

- **Liquid Scintillators** : first designed by Reines & Cowan to detect $\overline{\nu}_e$ from reactor by observing signals of radiation striking the scintillator (KamLAND, Borexino, No*v*A,SNO+)
- **Radiochemical detectors :** filled with radiochemical materials(^{37} Cl, ^{71} Ga) where neutrinos convert them into others (unstable isotopes) (Homestake, GALLEX/SAGE)
- **Cherenkov detectors :** filled with (heavy)water or oil to detect Cherenkov light produced whenever charged particles move through medium faster than the speed of light. (Super-K, SNO, IceCube,AMANDA etc.)





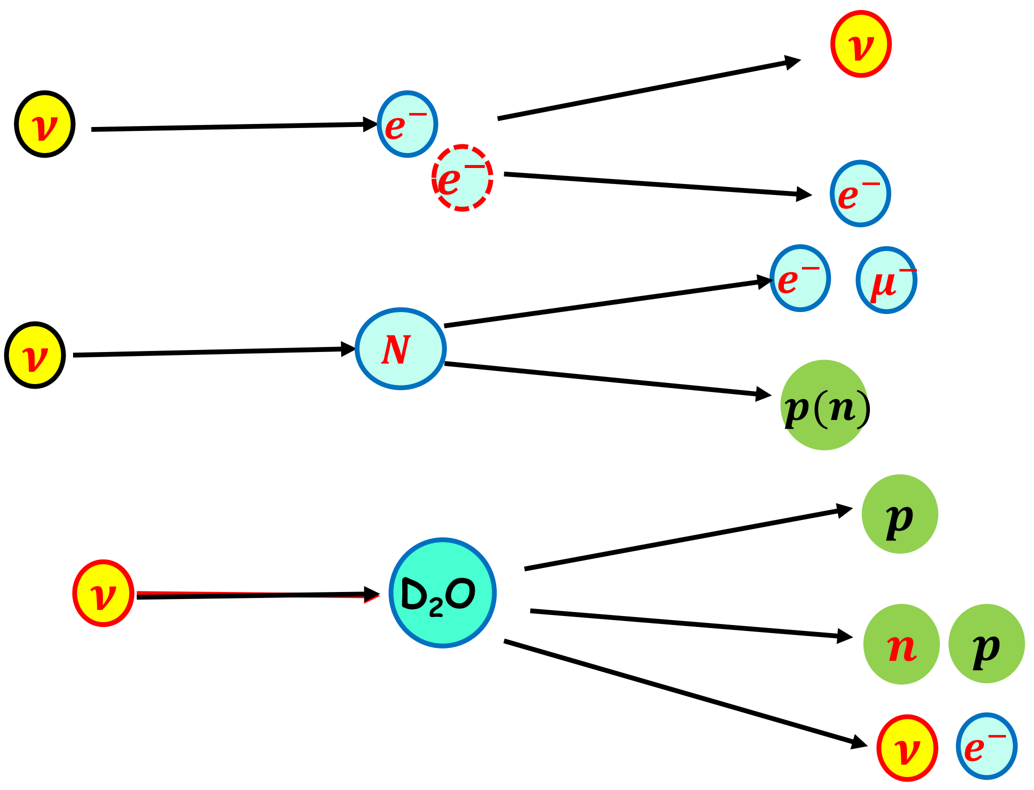
How to Detect Neutrinos

$$\nu + {}^{37}\mathrm{Cl} \longrightarrow e^- + {}^{37}\mathrm{Ar}$$

Homestake

$$\nu + {}^{71}\mathrm{Ga} \longrightarrow e^- + {}^{71}\mathrm{Ge}$$

GALLEX/SAGE

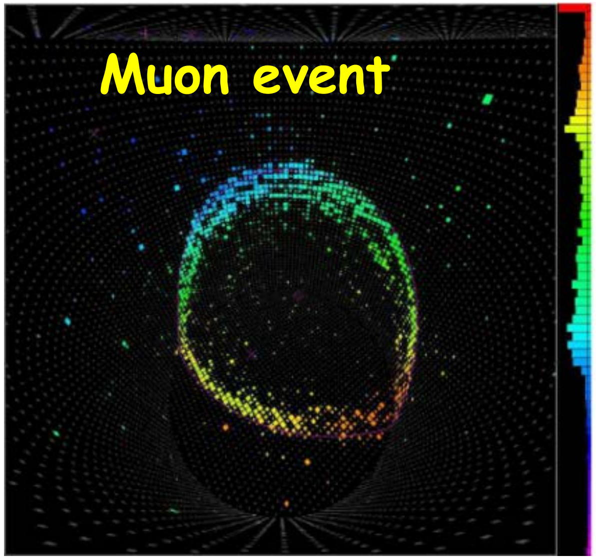


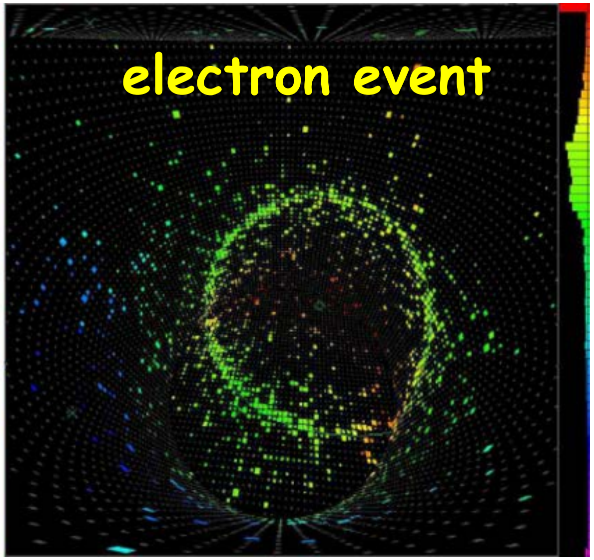
SuperKamiokande
SNO

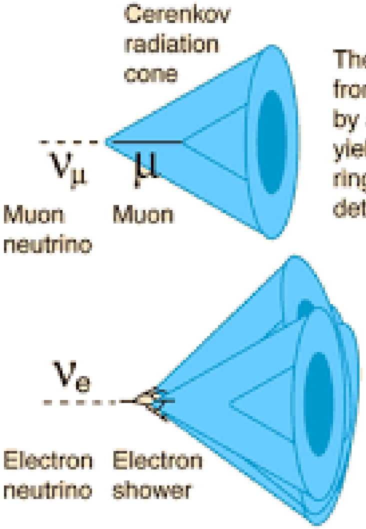
SuperKamiokande

SNO (NC: CC)

Cherenkov Radiations



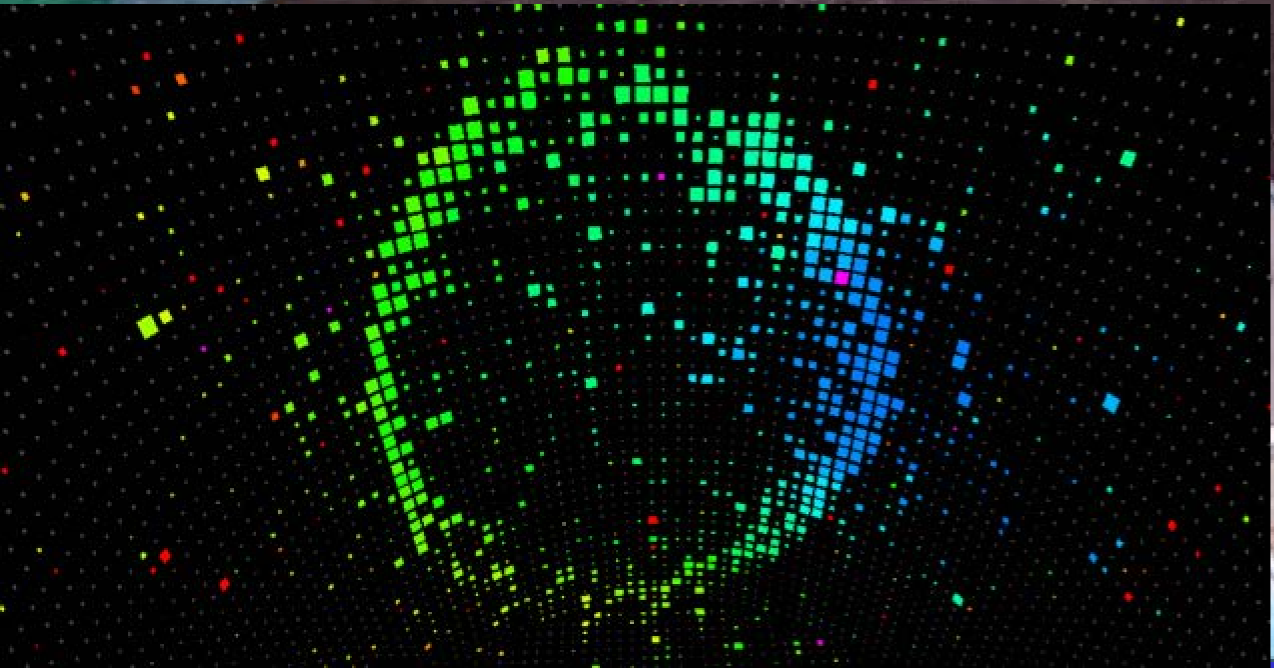




The Cerenkov radiation from a muon produced by a muon neutrino event yields a well defined circular ring in the photomultiplier detector bank.

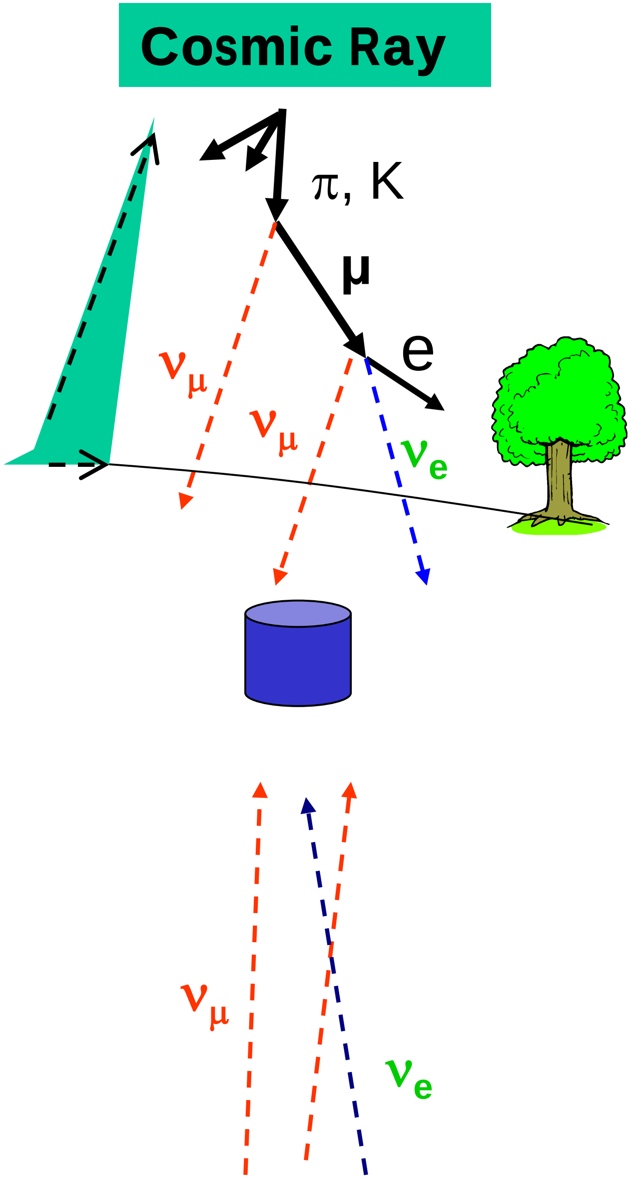
The Cerenkov radiation from the electron shower produced by an electron neutrino event produces multiple cones and therefore a diffuse ring in the detector array.

Neutrino Oscillation Experiments

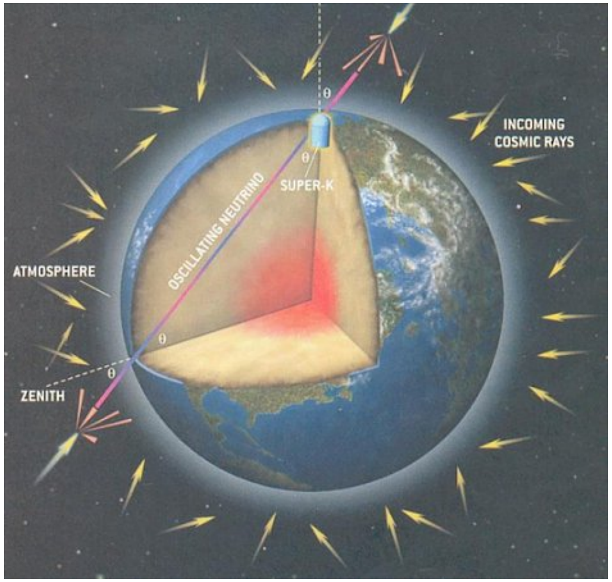


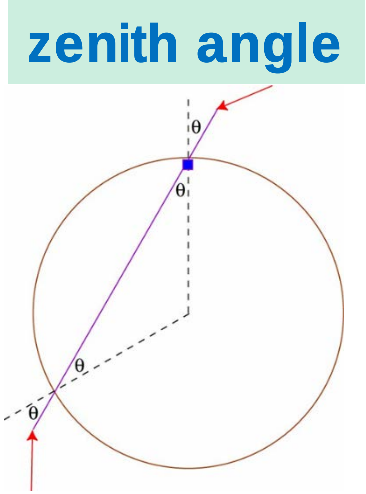
RENO : 서울대

Atmospheric Neutrinos



Neutrinos from the
other side of the Earth.



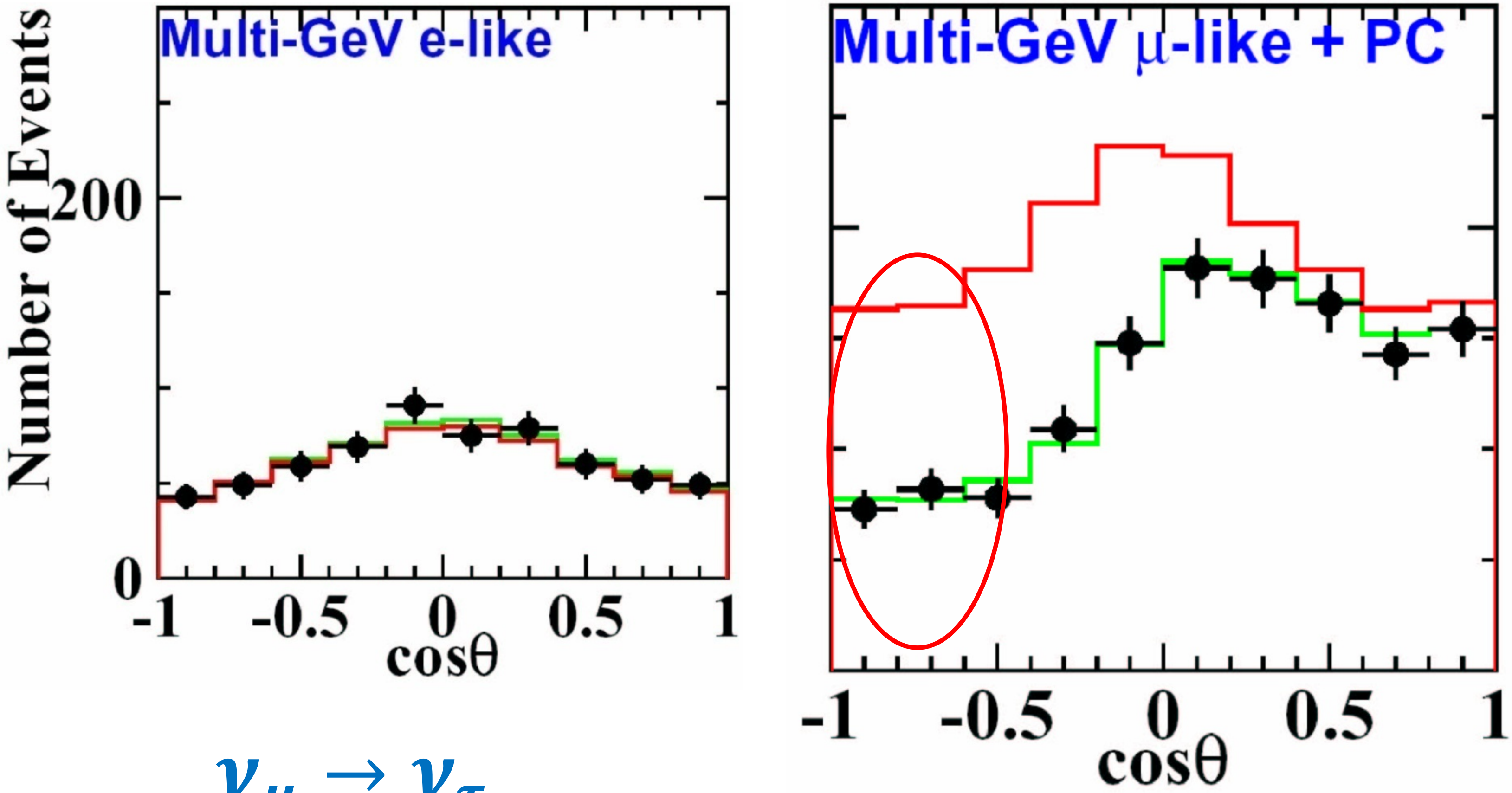


$$R_{\mu/e} = \frac{N_{\nu_\mu} + N_{\bar{\nu}_\mu}}{N_{\nu_e} + N_{\bar{\nu}_e}} \sim 2$$

A deficit was observed in the ratio
μ/e events : Saudan 2, IBM, Kamiokande

Atmospheric Neutrinos

A half of upward going ν_μ *lost !!*



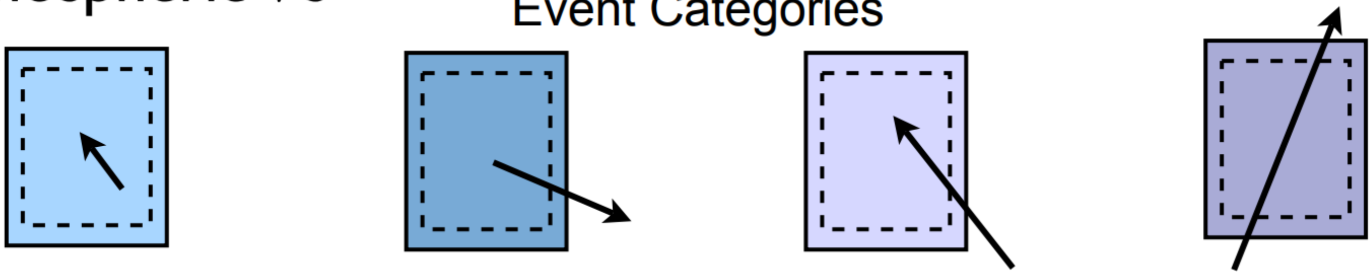
$\nu_\mu \to \nu_\tau$

$$\Phi(obs) = P_{surv}\Phi(prod)$$

Atmospheric Neutrinos

Atmospheric ν's

Event Categories



SK-III run period: July 29, 2006 - present

Event Category	**Event Rate (events/day)**		
	SK-I	SK-II	SK-III (Preliminary)
Fully Contained (FC)	8.18 ± 0.07	8.22 ± 0.10	8.31 ± 0.22
Partially Contained (PC)	0.61 ± 0.02	0.54 ± 0.03	0.57 ± 0.06
Upward-stopping μ (Upstop)	0.25 ± 0.01	0.28 ± 0.02	0.24 ± 0.03
Upward-thrugoing μ (Upthru)	1.12 ± 0.03	1.07 ± 0.04	1.11 ± 0.06

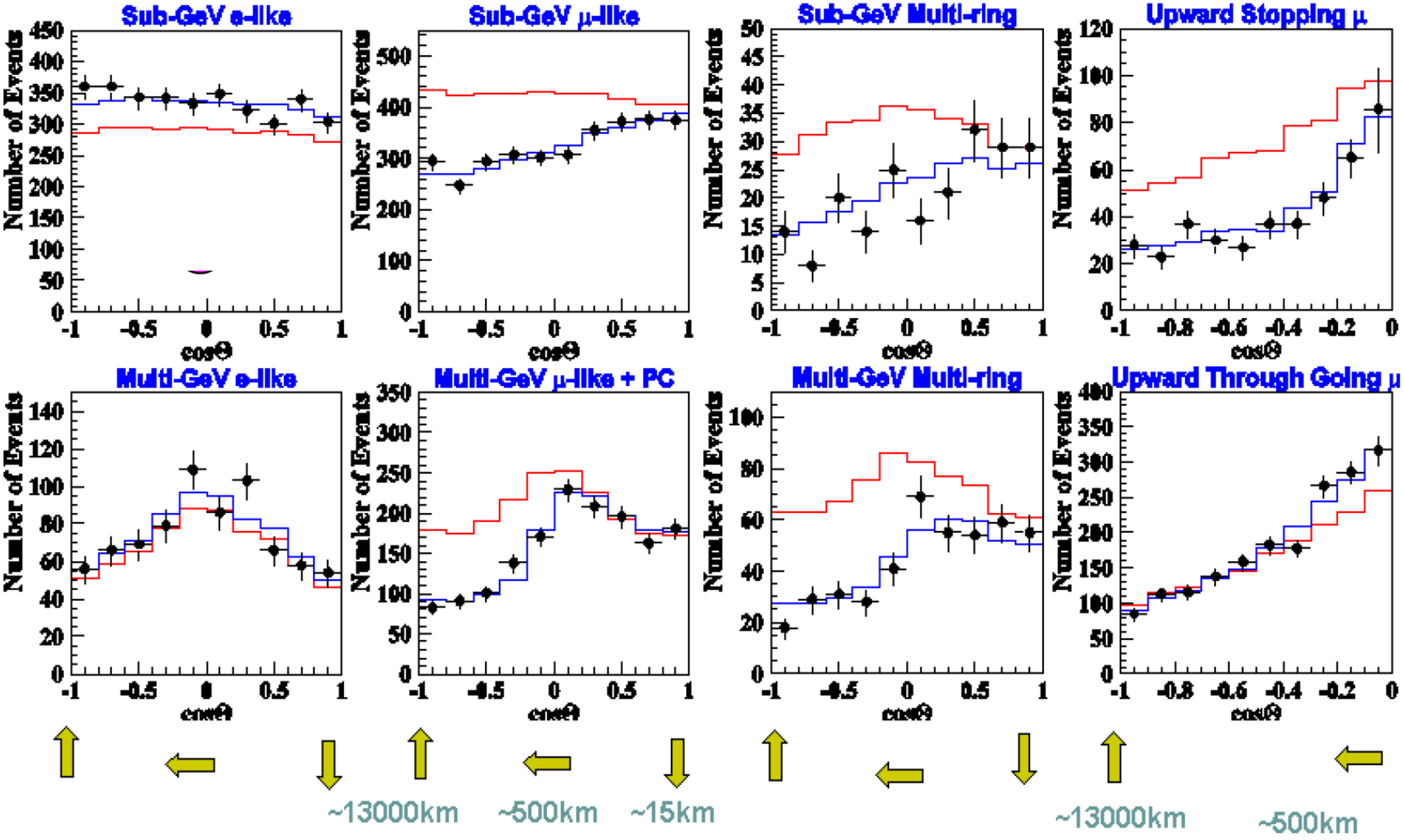
Event rates consistent across all phases of SK

Atmospheric Neutrinos

Zenith angle distributions (SuperKamiokande)

C. Sagi/ICHEP04

🟥 Null oscillation
🟦 **Best fit** $\sin^2 2\theta=1.0$, $\Delta m^2=2.1\times10^{-3}$ eV^2



Atmospheric Neutrinos

L/E analysis (SK I + SK II)

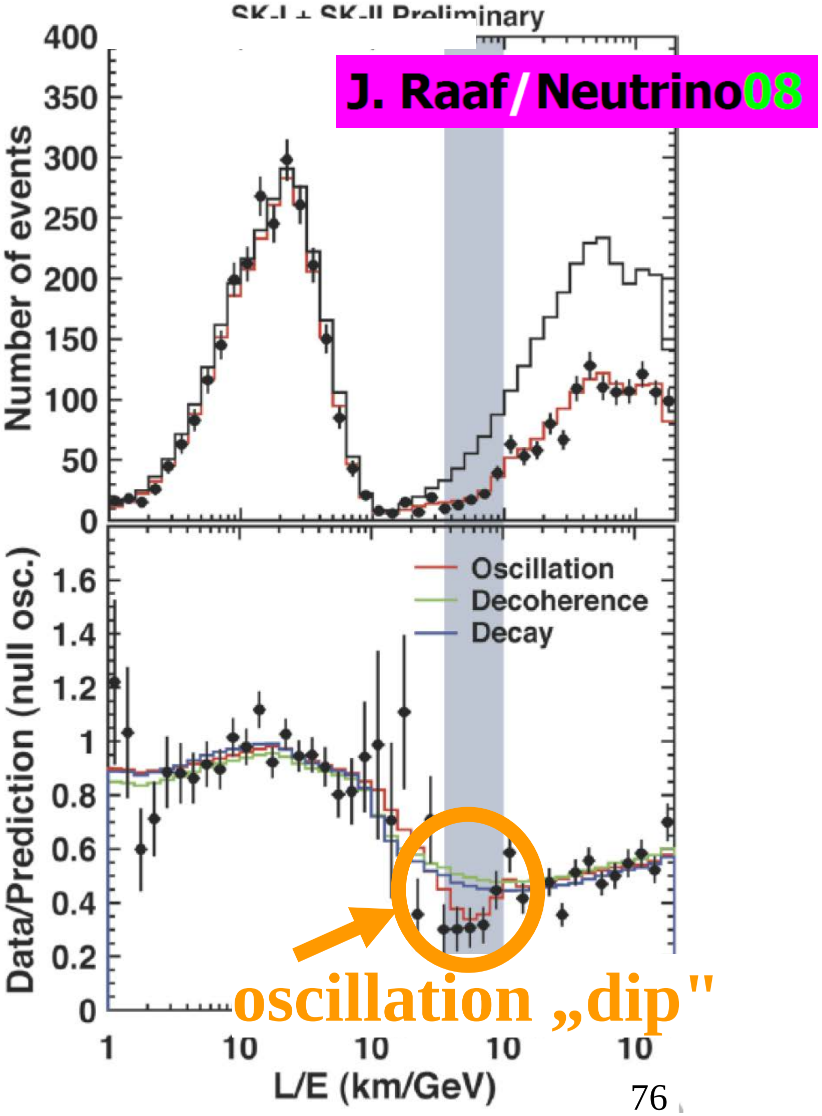
datasets

SK-I FC/PC μ-like:	1489 days
SK-II FC/PC μ-like:	799 days

- **Use only event categories with good L/E resolution:**
  - Partially-contained muons
  - Fully-contained muons

oscillation, decay and decoherence models tested

Rejecting alternatives to oscillation



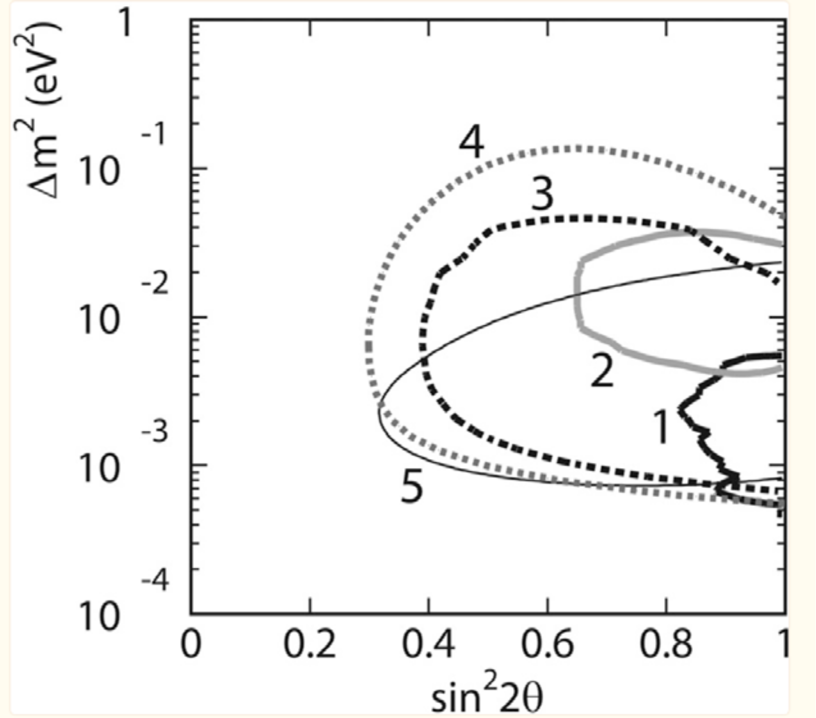
J. Raaf/Neutrino08

oscillation „dip"

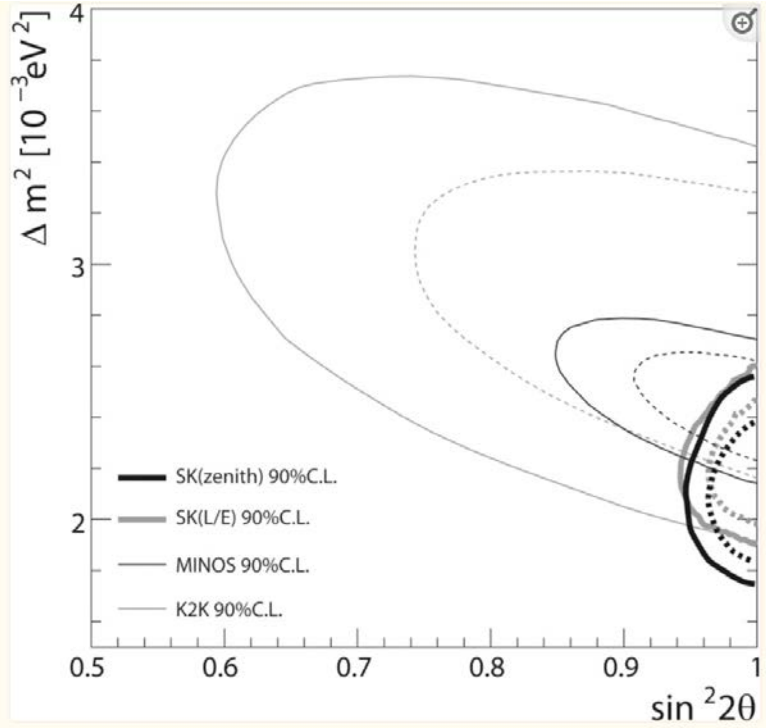
Atmospheric Neutrino

Experimental Results

$$P_{\nu_\mu\to\nu_\tau} \approx \sin^2 2\theta_{23} \sin^2\left(\frac{\Delta m^2_{32(1)}}{4E}\right.$$



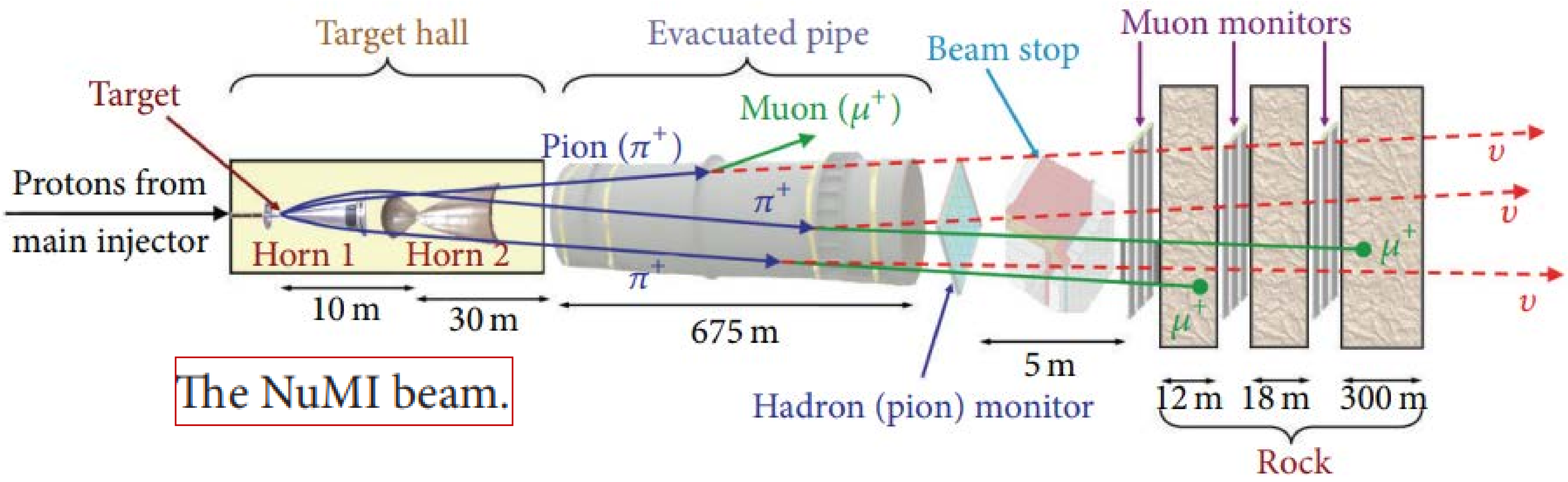
Kajita (Neutrino1998) : SK+Kamiokande



Kajita(2010) : SK I-III + MINOS+K2K

Evidence for the existence of neutrino masses and mixing

Accelerator Based Neutrino Experiments



The NuMI beam.

- **Making intense neutrino beams using particle accelerators.**
- **Neutrinos produced in accelerators are typically ν_μ**
- **Neutrino beams can be used for SBL or LBL ν detectors**

Accelerator Based Neutrino Experiments

SBL (short-based line) Experiments

- Sit close to the source of neutrinos, so the beam is very concentrated when it reaches the detector.
  (much higher number of neutrino interactions with
  a very pure neutrino beam.)
- Good for characterizing the beam and learning about the neutrinos before they oscillate
- Good place to hunt for sterile neutrinos and see how neutrinos interact with other particles.
- LSND, MiniBooNE, MicroBooNE etc.

Accelerator Based Neutrino Experiments

LBL (Long-based line) Experiments

- Focus on the oscillations while traveling great distances through Earth.
- Neutrinos have a lot of chances to interact with matter and sufficient distance to change flavors
- Good place to figure out mass ordering & CPV
- MINOS, T2K, NovA, DUNE, etc.

Accelerator Based Neutrino Experiments

LBL (Long-based line) Experiments

- For $\nu_\mu(\bar{\nu}_\mu)$ disappearance observations

$$P_{\nu_\mu\to\nu_\mu} \approx 1-(\cos^4\theta_{13}\sin^2 2\theta_{23}+\sin^2 2\theta_{13}\sin^2\theta_{23})\ \sin^2\left(\frac{\Delta m^2_{31}L}{4E}\right)$$

- For $\nu_e(\bar{\nu}_e)$ appearance observations

$$P_{\nu_\mu\to\nu_e} \approx \sin^2 2\theta_{23}\sin^2 2\theta_{13}\sin^2\left(\frac{\Delta m^2_{32}L}{4E}\right)$$

- **But , matter effects are not negligible in LBL exp.**

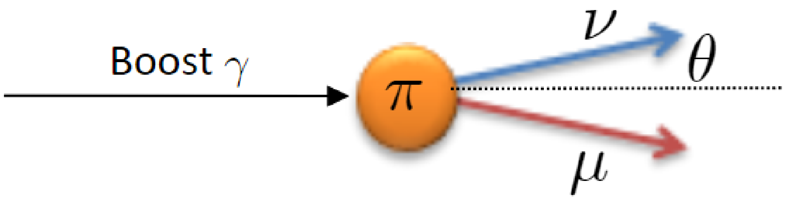
$$P_{matt\pm}[\nu_\mu(\bar{\nu}_\mu)\to\nu_e(\bar{\nu}_e)] = \pm\cos 2\theta_{13}\sin^2 2\theta_{13}s^2_{23}\left(\frac{2Ea(x)}{\Delta m^2_{13}}\right)\sin^2\left(\frac{\Delta m^2_{13}L}{4E}\right)$$
$$\mp\frac{a(x)L}{4}\sin^2 2\theta_{13}\cos 2\theta_{13}s^2_{23}\sin\left(\frac{\Delta m^2_{13}L}{2E}\right),$$

Accelerator Based Neutrino Experiments

On vs. Off-axis neutrino beam

In the pion rest frame the kinematics are all completely determined for the decay

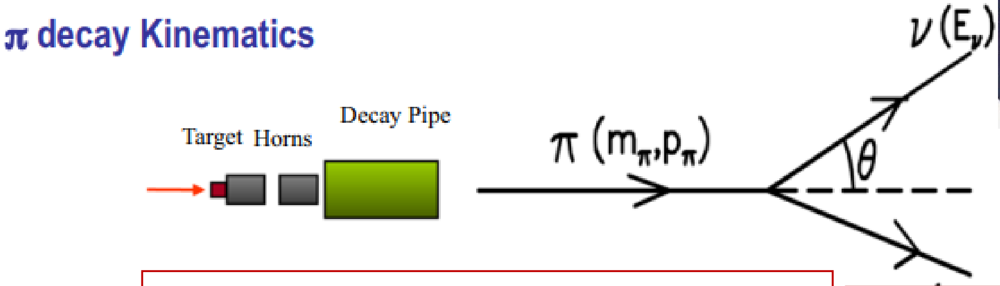
When we boost into the lab frame, neutrino energy depends on the angle relative to the boost direction

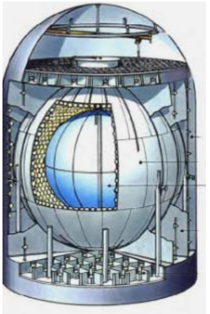


This ends up projecting neutrino *E* spectrum down till it's almost flat

Accelerator Based Neutrino Experiments

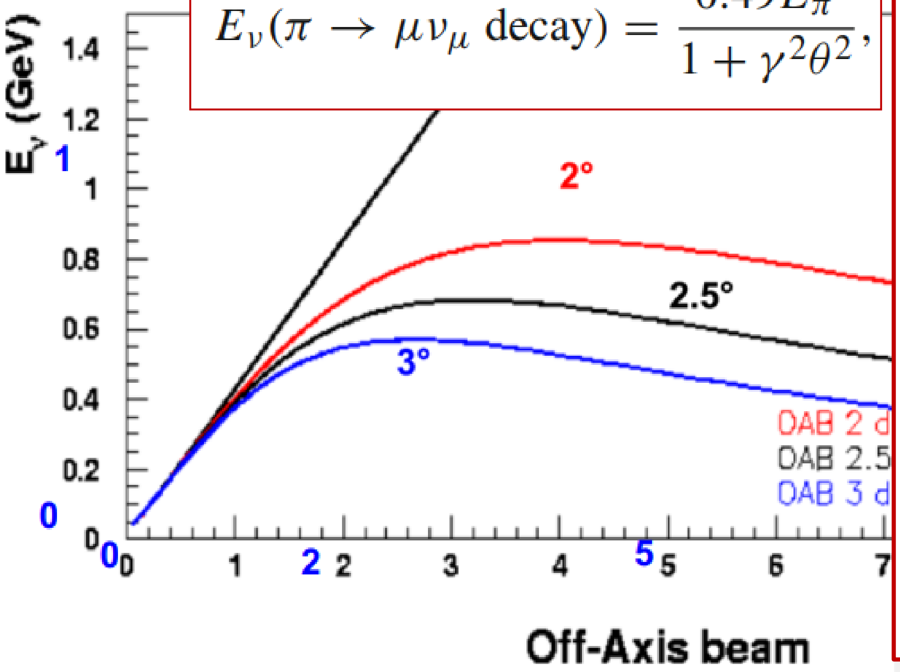
On vs. Off-axis neutrino beam

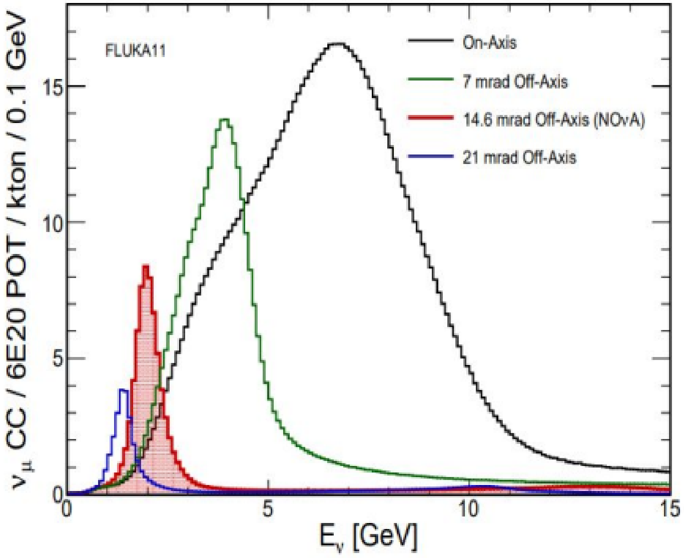




$$F = \left(\frac{2\gamma}{1+\gamma^2\theta^2}\right)^2 \frac{A}{4\pi z^2}$$

$$E_\nu(\pi \to \mu\nu_\mu \text{ decay}) = \frac{0.49 E_\pi}{1+\gamma^2\theta^2},$$



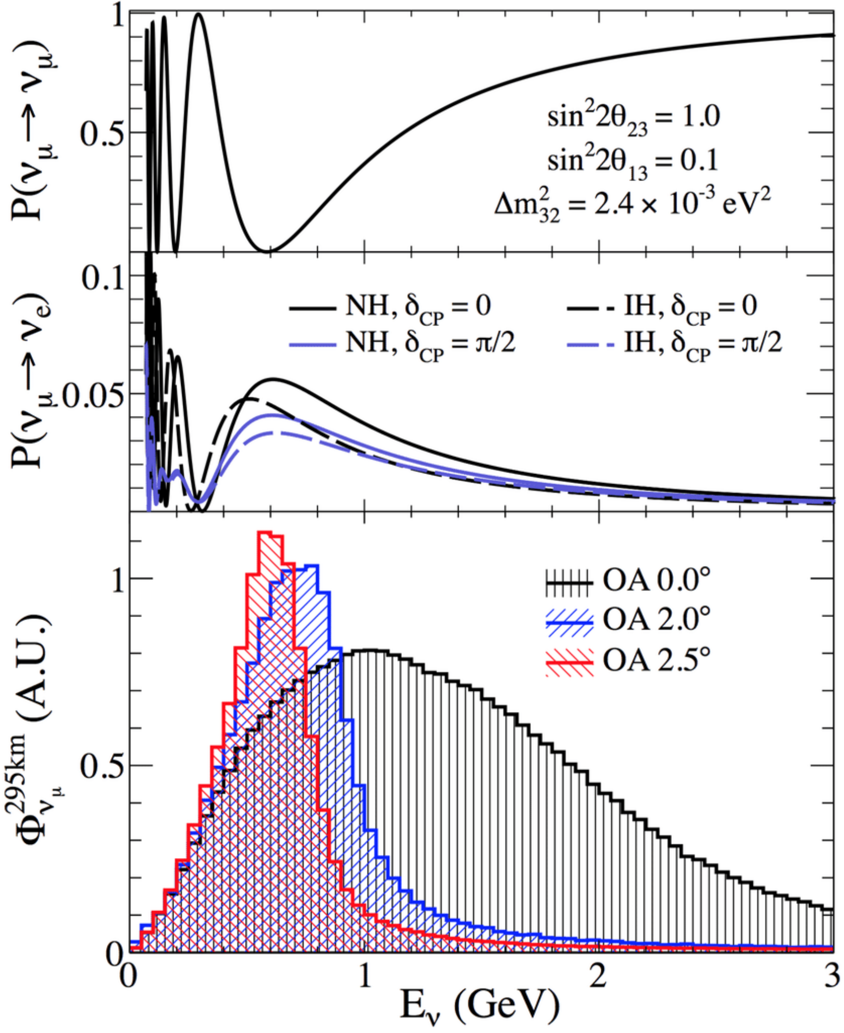


For NOvA

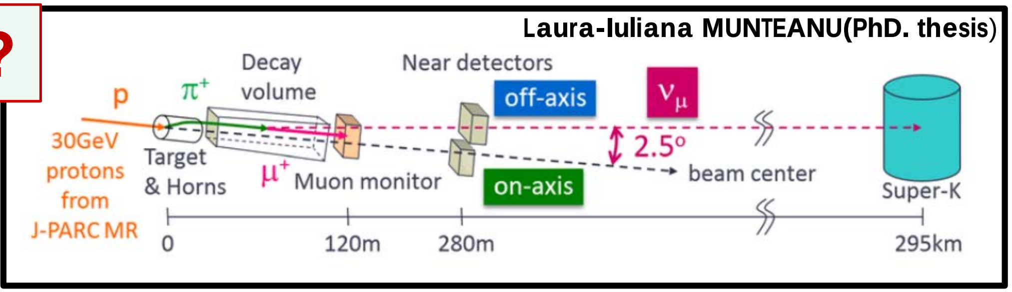
- Nakaya & Nishikawa , JHEP2020

Accelerator Based Neutrino Experiments

Why off-axis beam ?



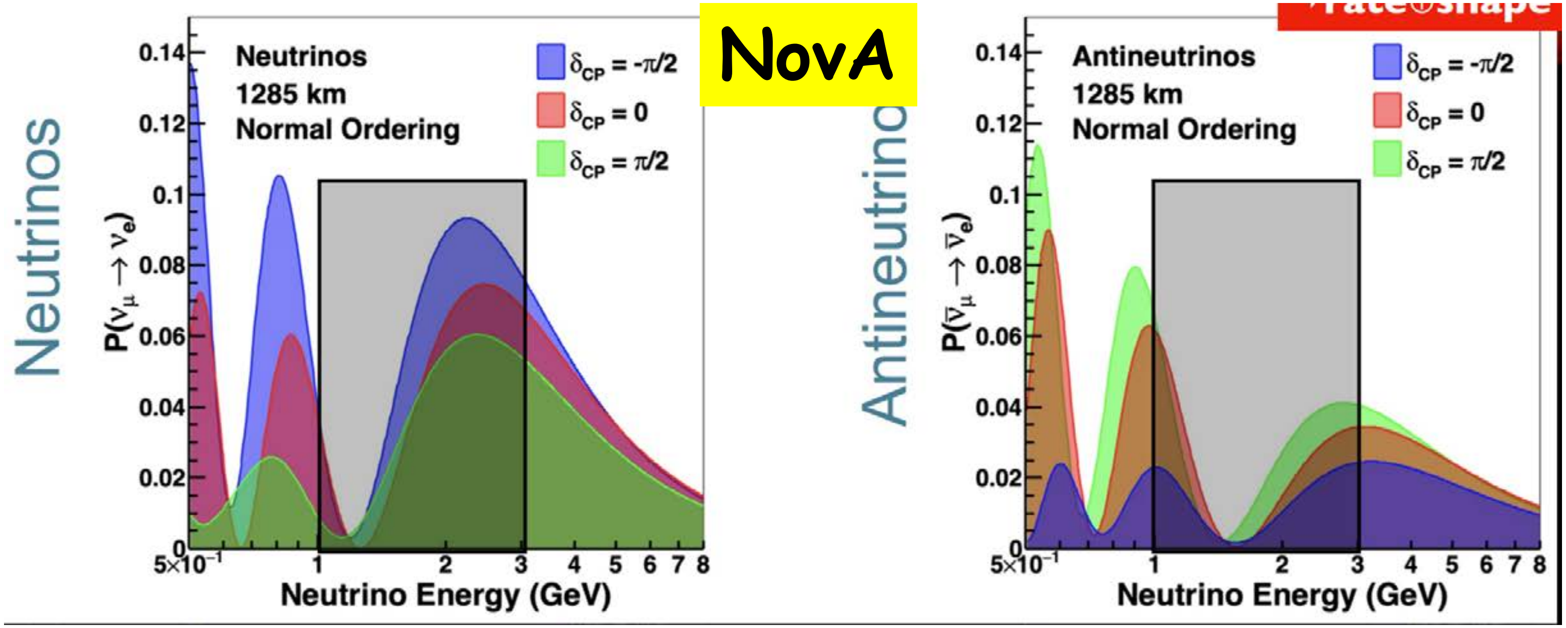
T2K

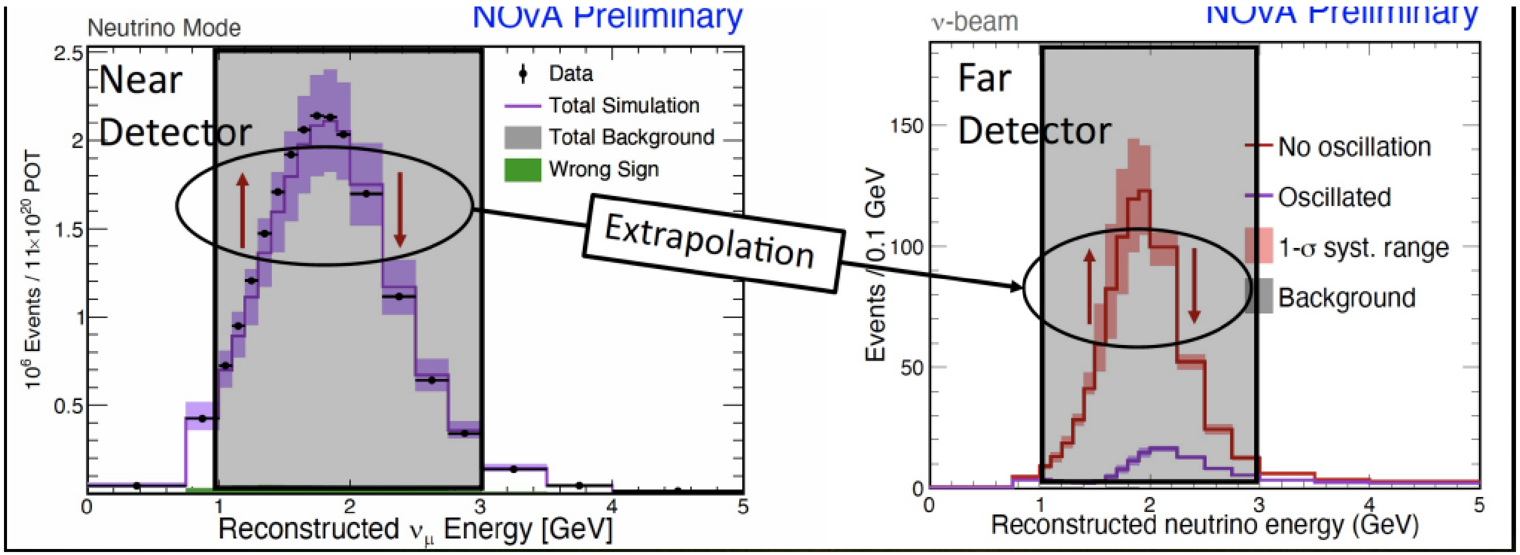


Laura-Iuliana MUNTEANU(PhD. thesis)

- Off-axis method provides narrow E_ν band peaking at a specific E.
- The off-axis angle is adjusted to maximize $P_{\alpha\beta}$ (oscillation maximum)
- Low background, few high E events
- Measure oscillations at a single L/E
- Essential for high precision of mixing angle

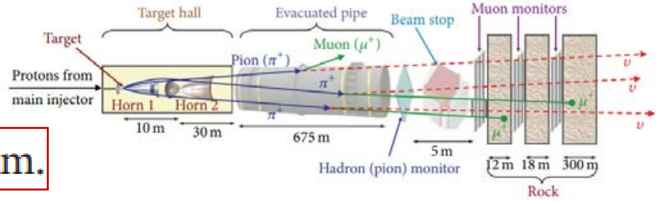
Accelerator Based Neutrino Experiments





Accelerator Based Neutrino Experiments

MINOS Experiment



The NuMI beam.

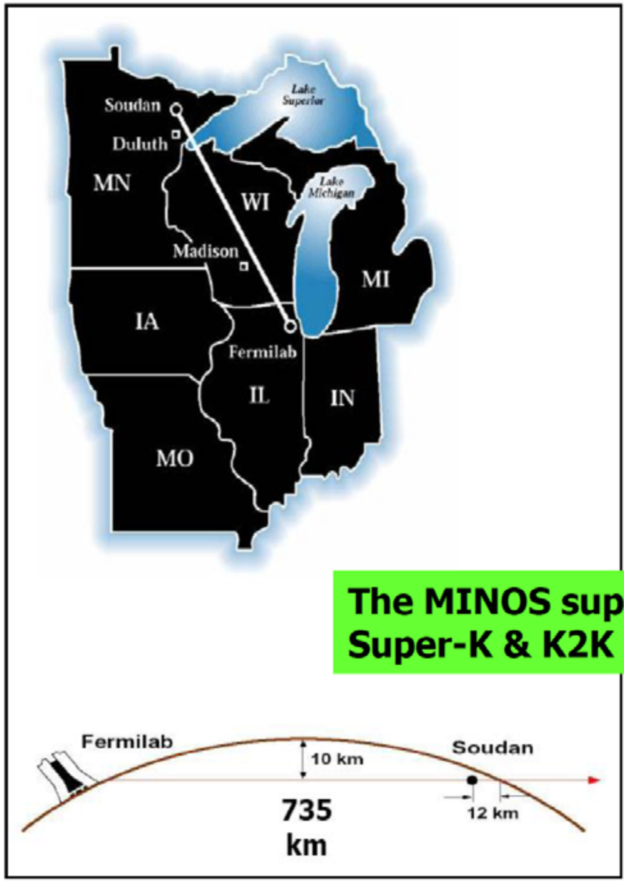
- ν_μ disappearance to probe atmospheric oscillation regime

-Neutrinos produced by NuMI beamline

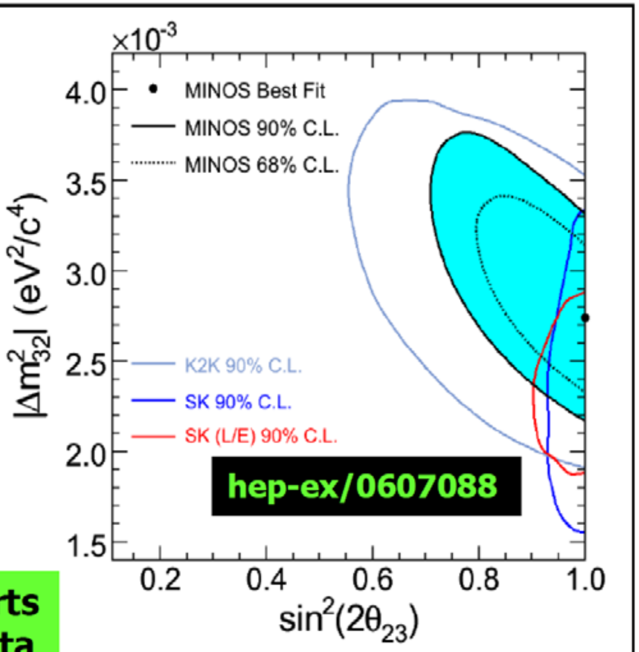
-2 detectors :

Near : close to beamline

Far (larger) : 735 km away



The MINOS supports Super-K & K2K data



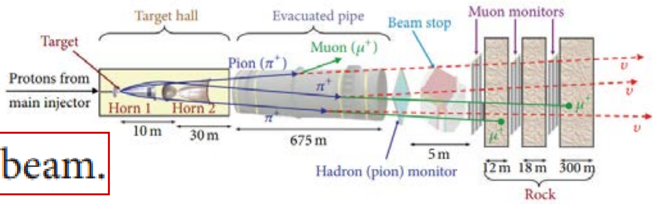
$$|\Delta m^2_{32}| = 2.74^{+0.44}_{-0.26}(\text{stat}+\text{syst})\times 10^{-3}\,\text{eV}^2$$

$$\sin^2 2\theta_{23} = 1.00_{-0.13}(\text{stat}+\text{syst})$$

Constrained to $\sin^2(2\theta_{23}) \leq 1$

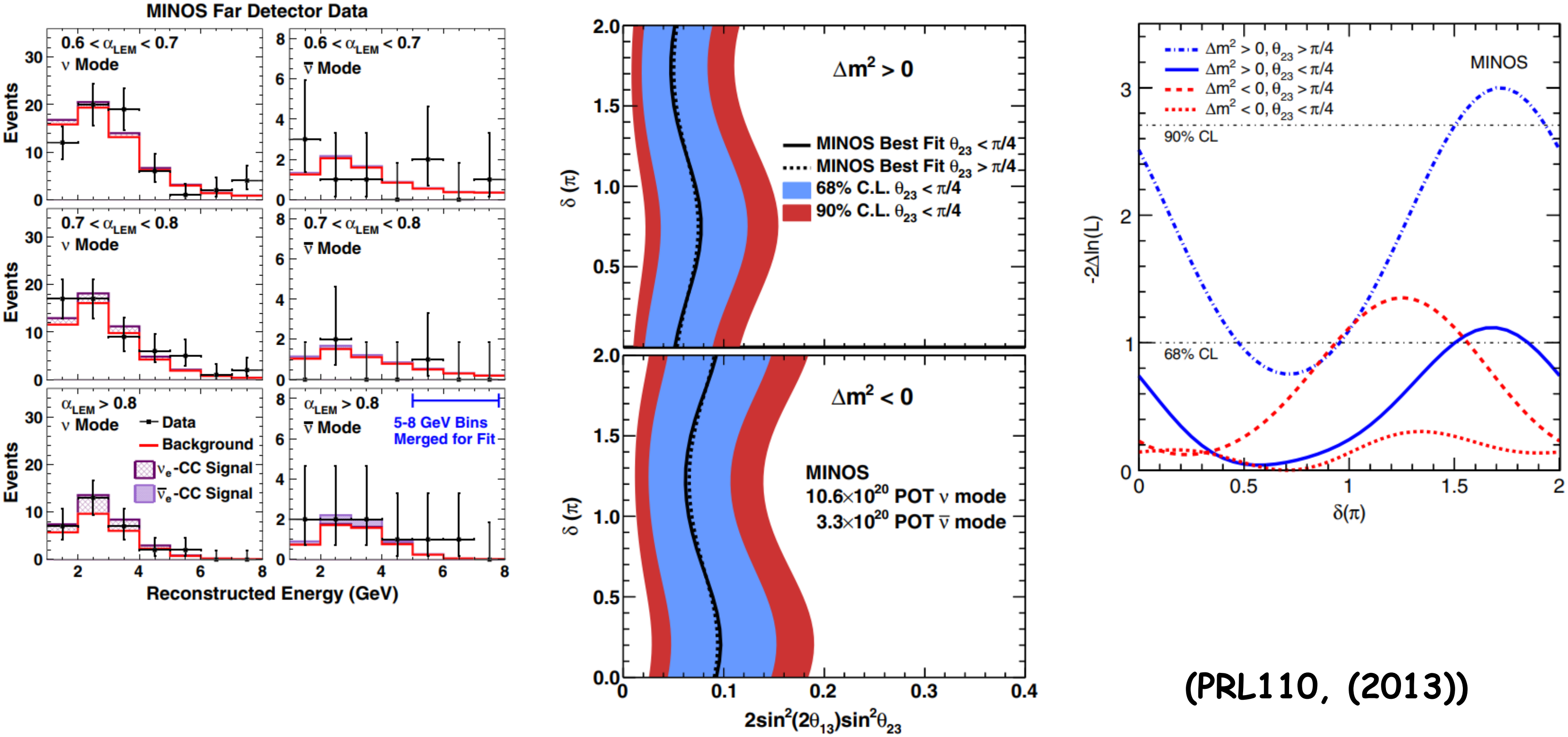
Accelerator Based Neutrino Experiments

MINOS Experiment



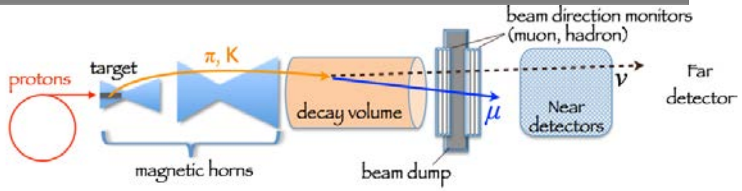
The NuMI beam.

- $\nu_e(\bar{\nu}_e)$ appearance to measure θ_{13}, δ_{CP} as well as MO



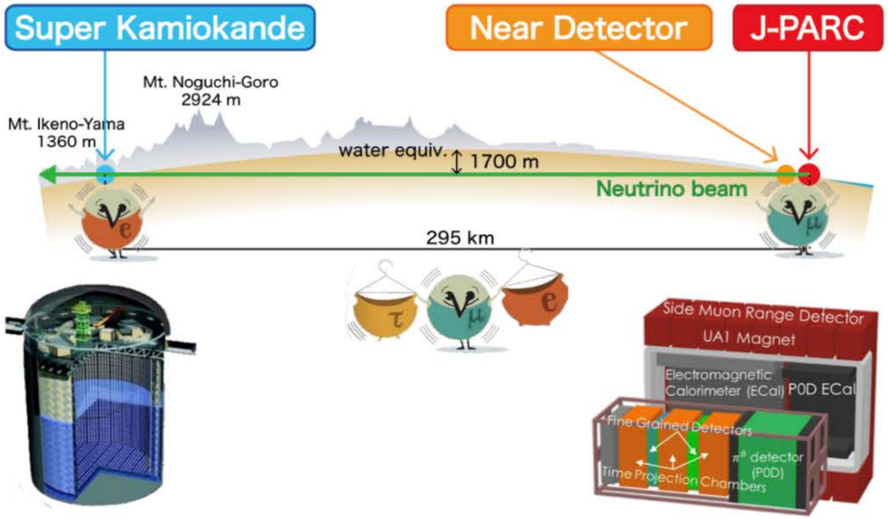
(PRL110, (2013))

Accelerator Based Neutrino Experiments



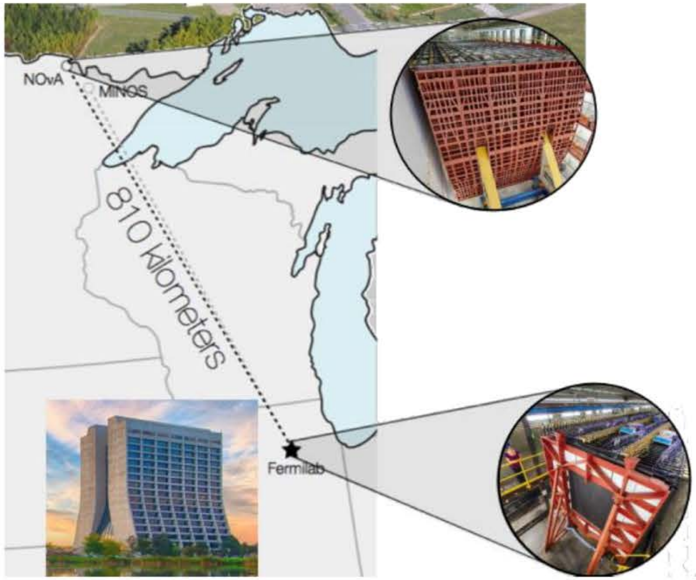
The Experiments

T2K



- Baseline: 295 km
- Peak E_ν: ~0.6 GeV (off-axis)
- Near detector: ND280 (~2 T C/O targets, TPC tracking, magnetised)
- Far detector: Super-K, 50 kT, Water-Cherenkov

NOvA



- Baseline: 810 km
- Peak E_ν: ~2 GeV (off-axis)
- Near detector: Scintillator tracker (300 T)
- Far detector: Scintillator tracker (14 kT)

Accelerator Based Neutrino Experiments

T2K (Tokai-to-Kamioka) experiment

arXiv:1106.2822 [hep-ex] 14 June 2011
Hint for unsuppressed theta(13) !

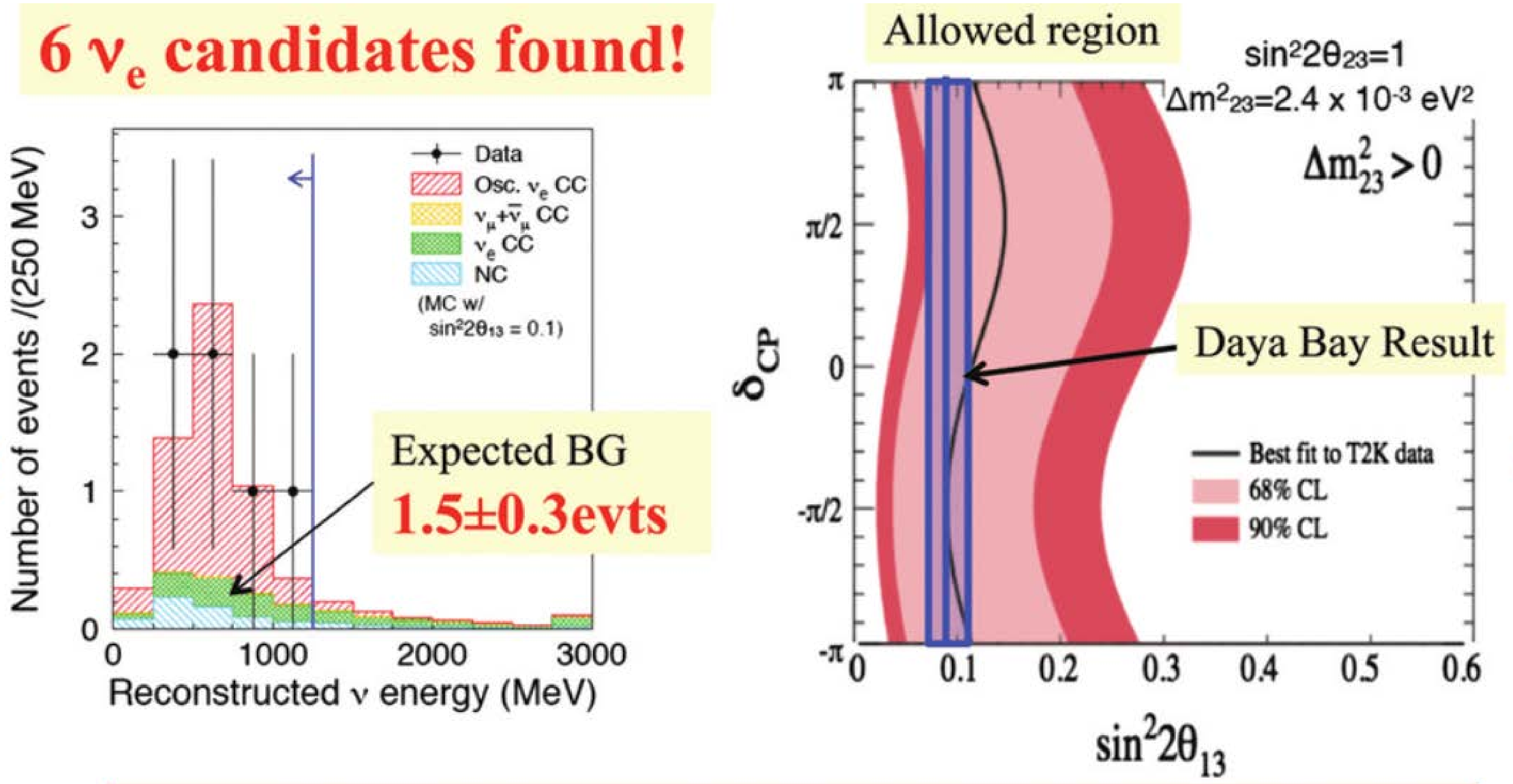
T2K Main Goals:

★ Discovery of $\nu_\mu \rightarrow \nu_e$ oscillation (ν_e appearance)

★ Precision measurement of ν_μ disappearance

Accelerator Based Neutrino Experiments

- **First observation of ν_e appearance**



6 ν_e candidates found!

Expected BG **1.5±0.3evts**

Allowed region

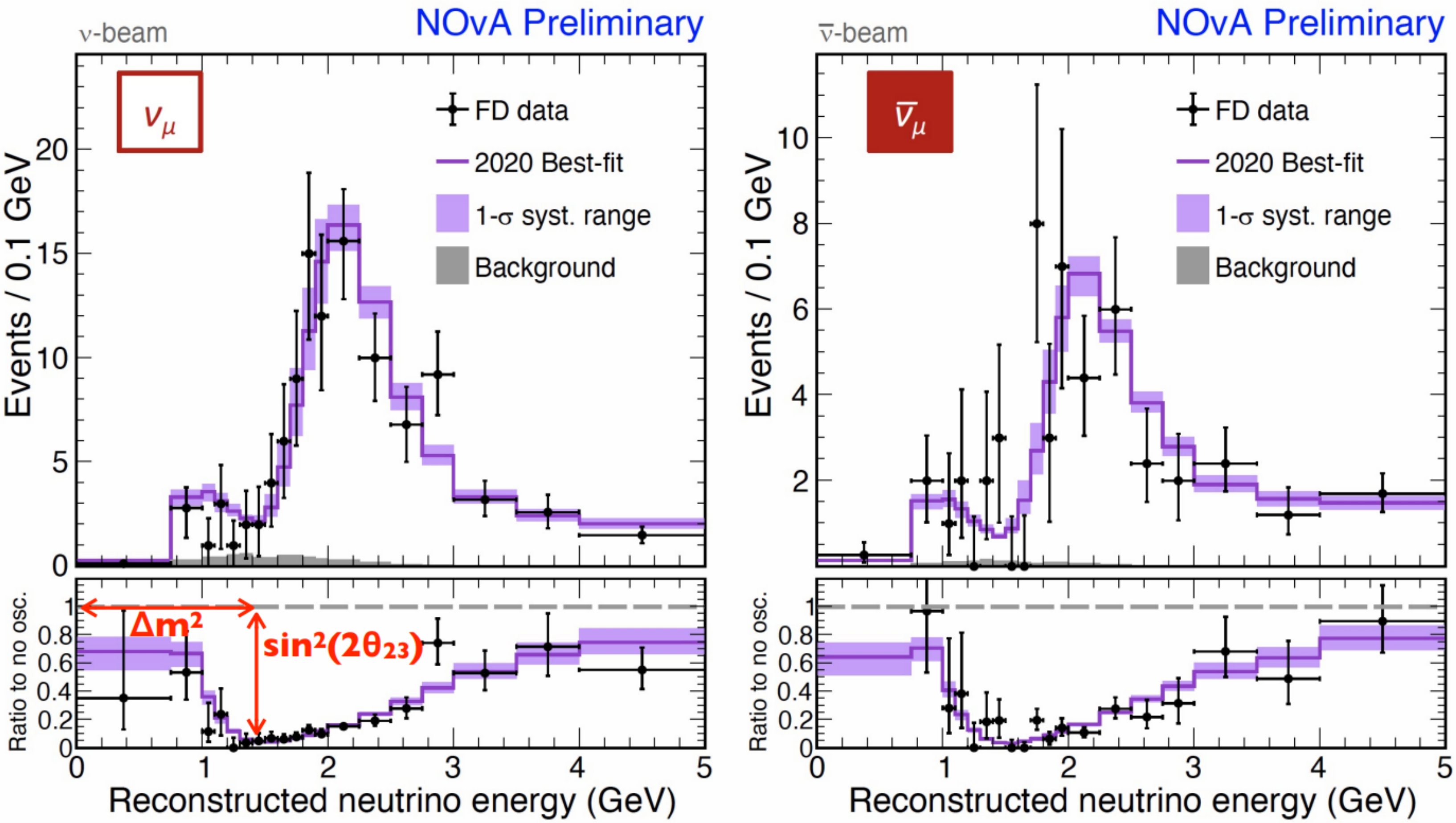
Daya Bay Result

Selected to Physics World Top 10 Breakthroughs (England) in 2011
http://physicsworld.com/cws/article/news/48126

T2K result for electron neutrino appearance.

Accelerator Based Neutrino Experiments

- **Search for ν_μ & $\bar{\nu}_\mu$ disappearance**

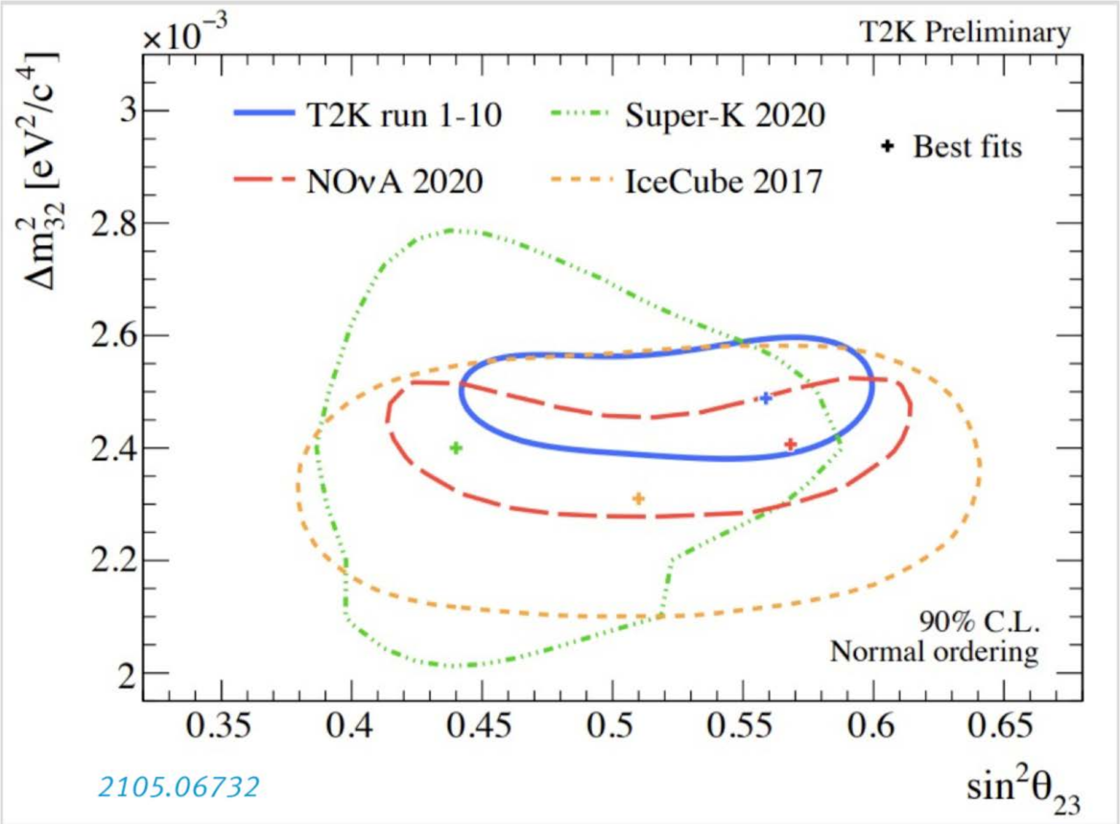


211 events, 8.2 background

105 events, 2.1 background

Accelerator Based Neutrino Experiments

Experimetal Results



2105.06732

- Δm^2_{32} -vs-$\sin^2\theta_{23}$: at 90% CL, θ_{23} contours overlap. T2K and NOνA favour upper octant while Super-K prefers lower