Lectures on
Neutrino Physics (II)

AEPSHEP 2022

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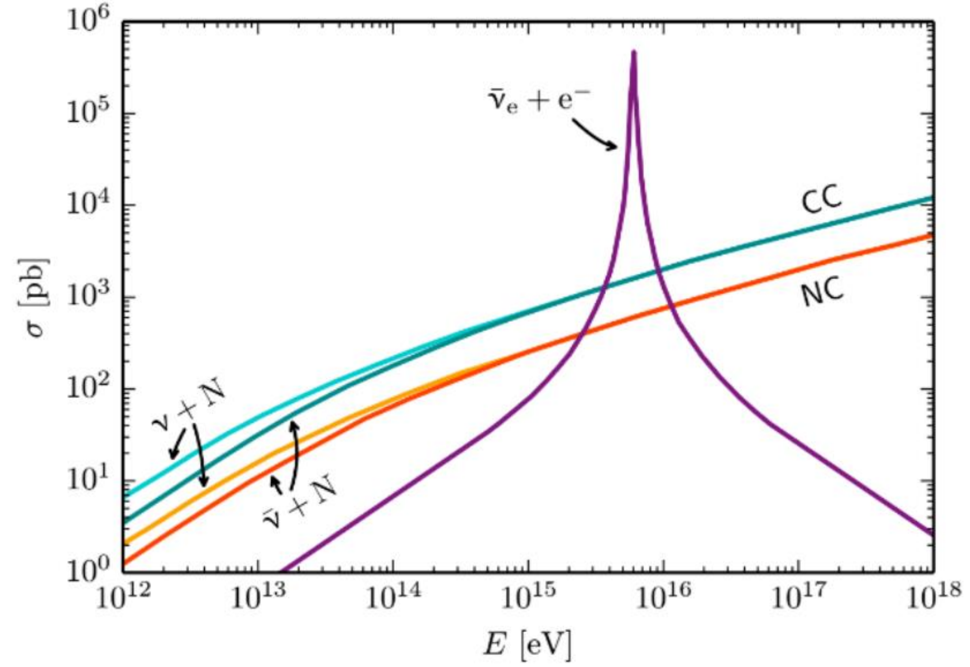
Glashow Resonance

$\bar{\nu}_e + e^- \to \underline{W^-} \to anything$

on-shell

- The threshold $E_{\bar{\nu}_e}$ for this process : $E_{\bar{\nu}_e} = \frac{M_W^2 - (m_e^2 + m_\nu^2)}{2m_e} \cong \frac{M_W^2}{2m_e} \sim 6.3 \text{ PeV}$
- This process is considered for detection of high E cosmic neutrinos at IceCube



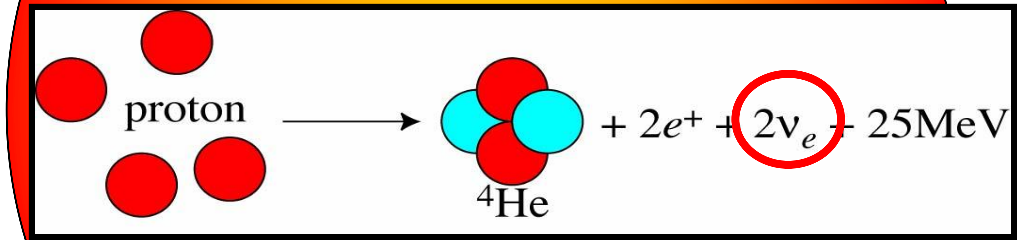
$$\sigma_{GR}(E_\nu) = \frac{4}{3}\frac{G_F^2 m E_\nu}{2\pi}\frac{1}{\left(1 - \frac{2mE_\nu}{M_W^2}\right)^2 + \frac{\Gamma_W^2}{M_W^2}}$$

Halzen

Solar Neutrinos

Nuclear Fusion

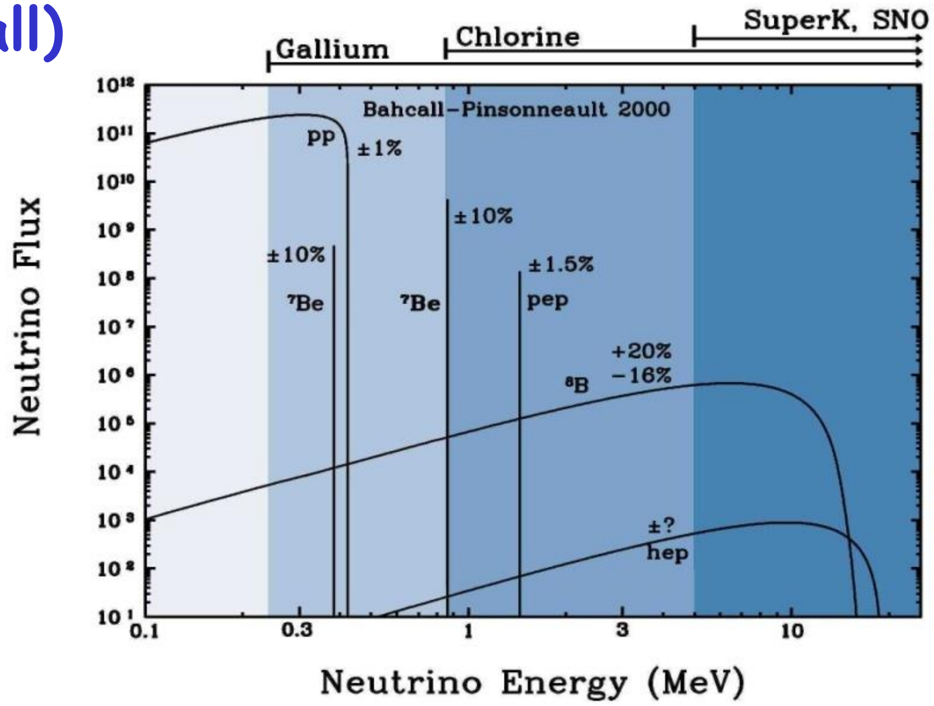
Sun burns!!!



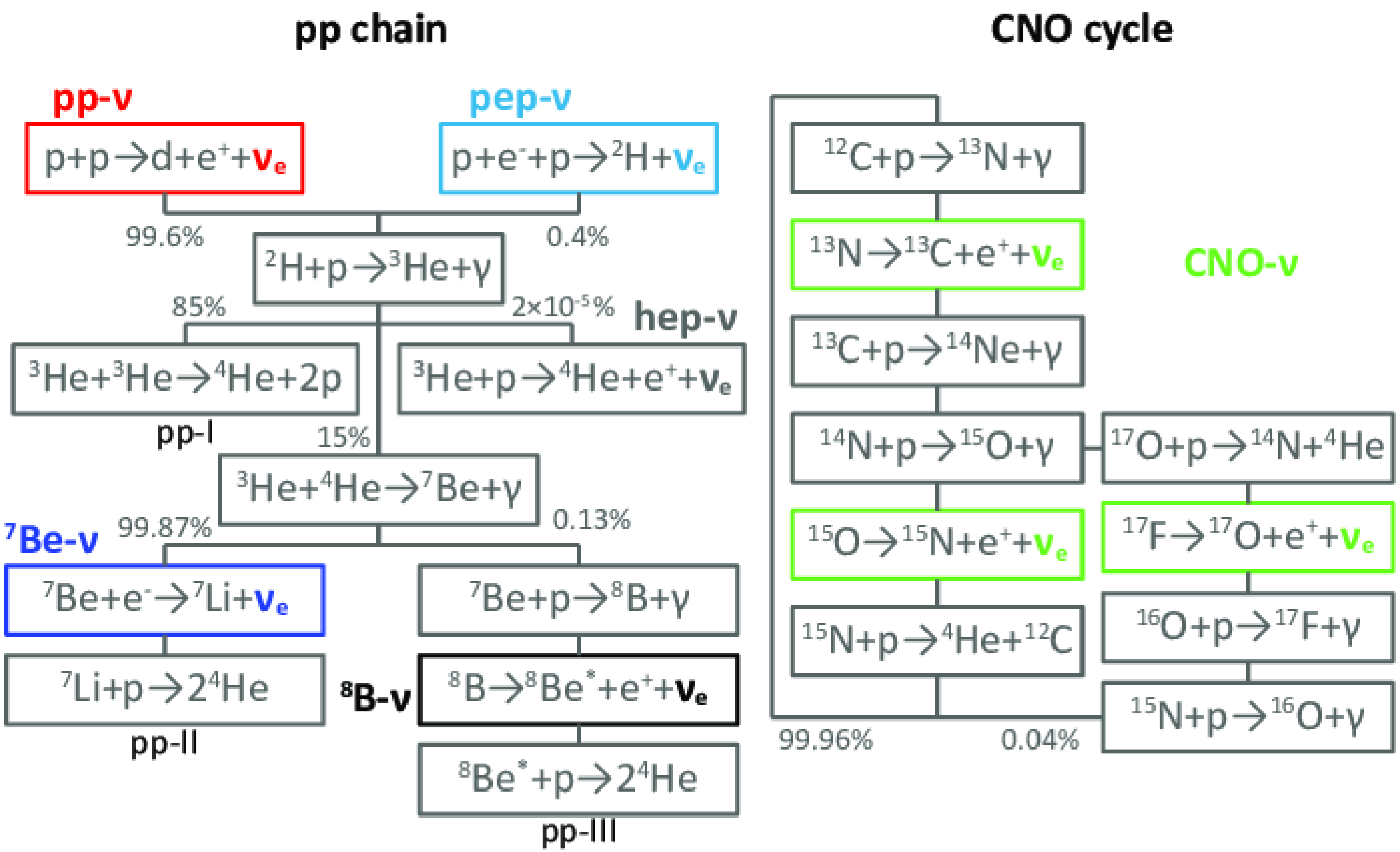
Production of ν_e

Standard Solar Model (Bahcall)

J. N. Bahcall, S. Basu and M. Pinsonneault (1998)



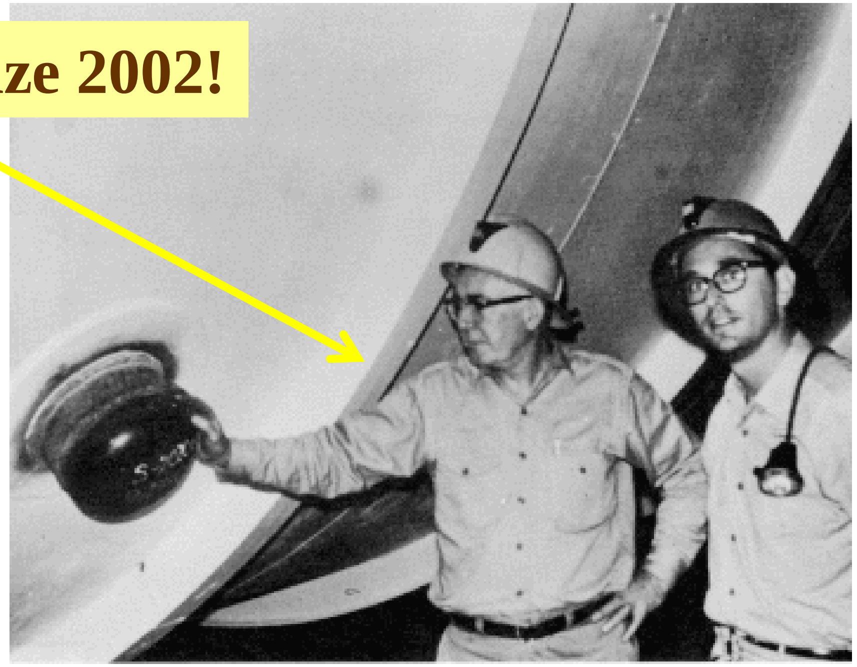
Production of solar neutrinos



Solar Neutrino Experiment

Homestake

Won Nobel Prize 2002!

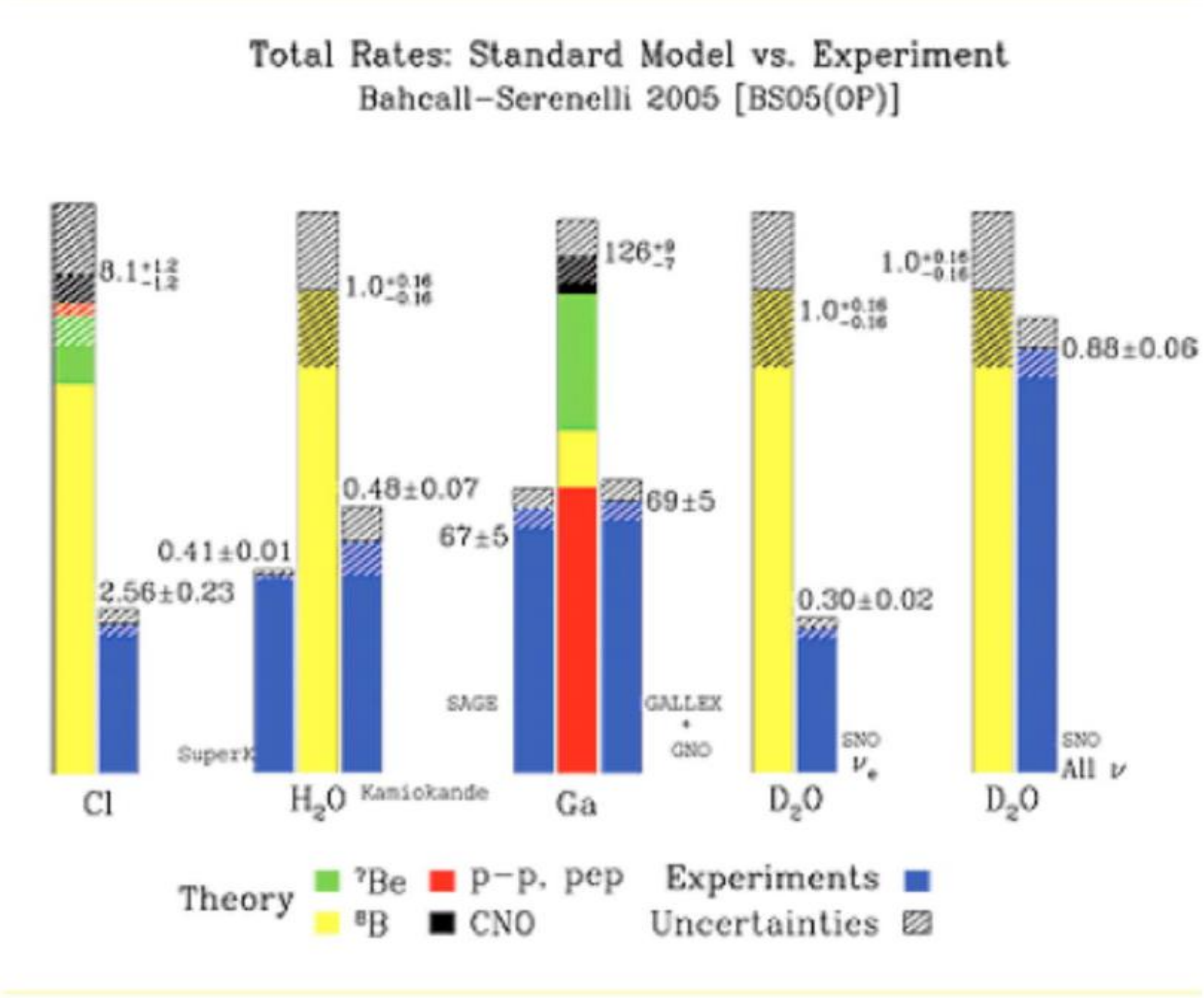


First experiment by Davis *et al.* in 1960's

Radiochemical Method (Chlorine): $\nu_e + {}^{37}Cl \rightarrow {}^{37}Ar + e^-$

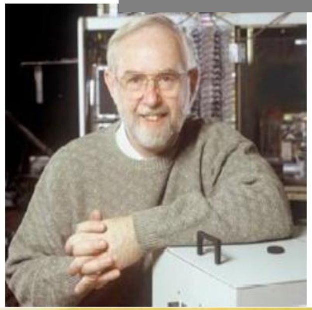
→ **found ~ 1/3 of expected rate ! (1968)**

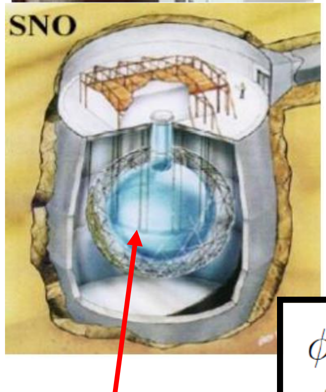
Solar Neutrino Experiment



Theory v Exp.

Solar Neutrino Experiment





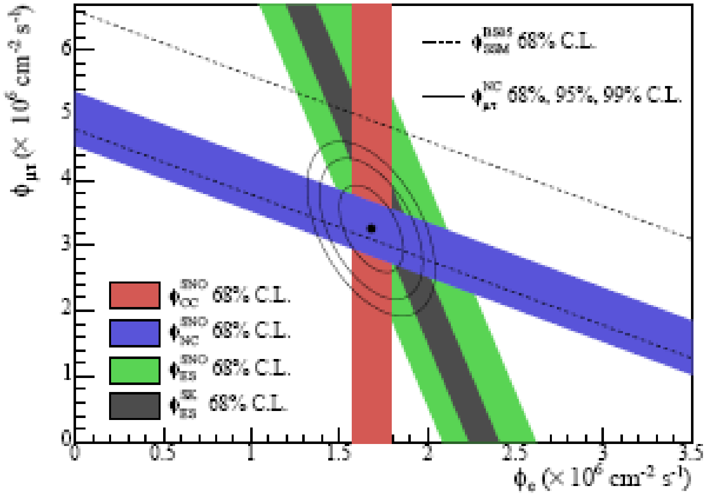
Heavy water

Using neutrinos from ^{8}B

CC: $\nu_e + d \rightarrow e^- + p + p$

NC : $\nu_x + d \rightarrow \nu_x + p + n$

ES: $\nu_\alpha + e^- \rightarrow \nu_\alpha + e^-$



$$\phi_{CC} = 1.76^{+0.06}_{-0.05}(\text{stat.})^{+0.09}_{-0.09}\ (\text{syst.}) \times 10^6\ \text{cm}^{-2}\ \text{s}^{-1}$$
$$\phi_{ES} = 2.39^{+0.24}_{-0.23}(\text{stat.})^{+0.12}_{-0.12}\ (\text{syst.}) \times 10^6\ \text{cm}^{-2}\ \text{s}^{-1}$$
$$\phi_{NC} = 5.09^{+0.44}_{-0.43}(\text{stat.})^{+0.46}_{-0.43}\ (\text{syst.}) \times 10^6\ \text{cm}^{-2}\ \text{s}^{-1}$$

$$\phi(\nu_e) = 1.76^{+0.05}_{-0.05}(\text{stat.})^{+0.09}_{-0.09}\ (\text{syst.}) \times 10^6\ \text{cm}^{-2}\ \text{s}^{-1}$$
$$\phi(\nu_{\mu\tau}) = 3.41^{+0.45}_{-0.45}(\text{stat.})^{+0.48}_{-0.45}\ (\text{syst.}) \times 10^6\ \text{cm}^{-2}\ \text{s}^{-1}$$

Nucl-ex/0610020

$$\begin{aligned}\phi_{CC} &= \phi(\nu_e)\\ \phi_{ES} &= \phi(\nu_e) + 0.1559\phi(\nu_{\mu\tau})\\ \phi_{NC} &= \phi(\nu_e) + \phi(\nu_{\mu\tau})\end{aligned}$$

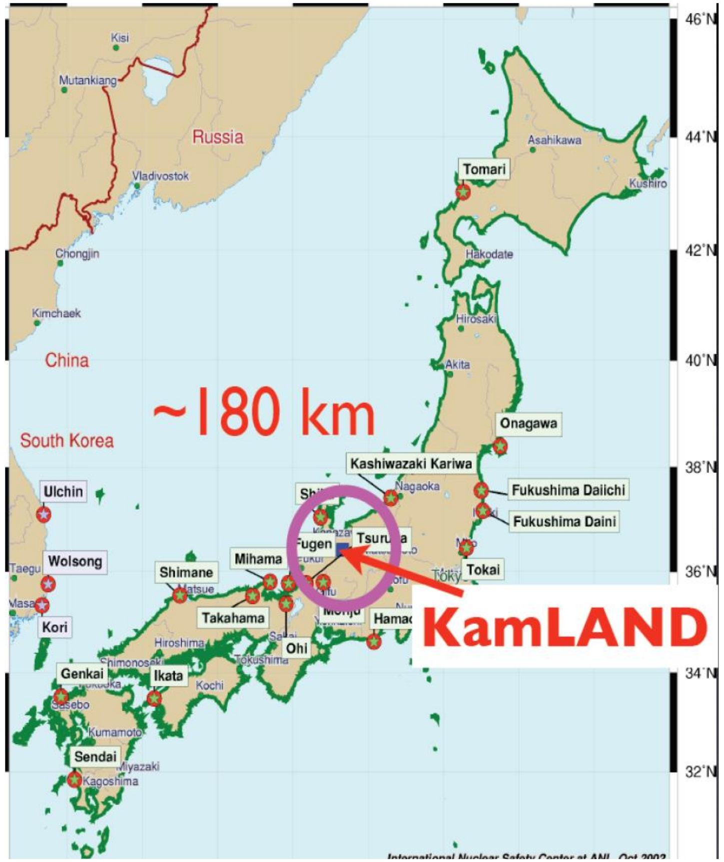
$SSM(2004){:}\ \Phi(^8B) = 5.26(1 \pm 0.23) \times 10^6 cm^{-2} s^{-1}$ **Bahcall & Pinsonneault**

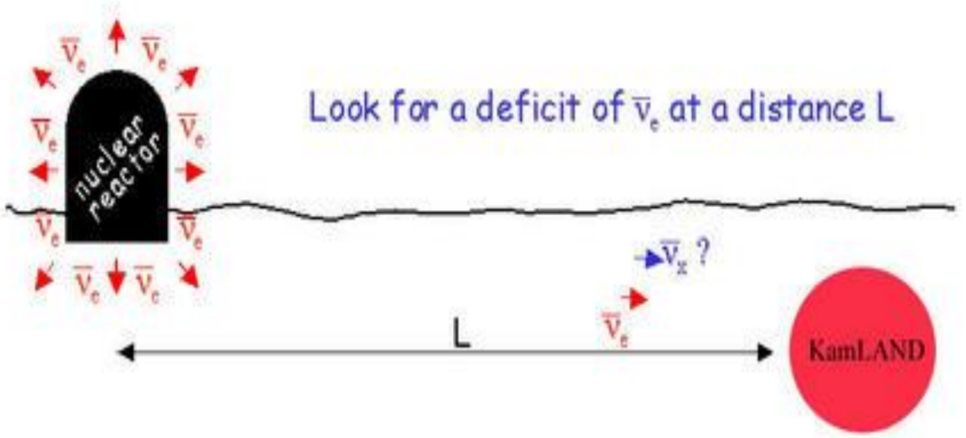
Supporting neutrino transition as well as verifying SSM

KamLAND results

Reactor long baseline experiment

~180 km **(E_{pr} > 2.6 MeV)**





$$p^+ + \bar{\nu}_e \rightarrow e^+ + n^0$$

- **in consistent with the results of solar neutrino oscillation**

KamLAND results

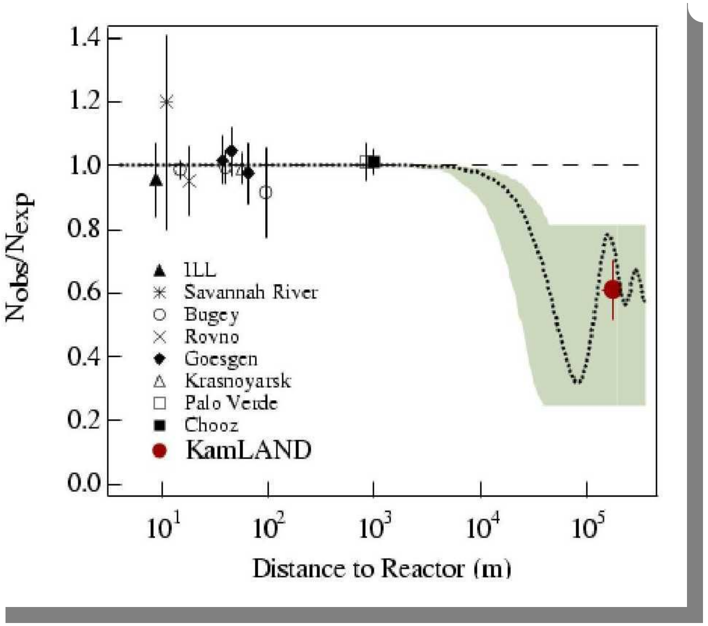
Testing solar neutrino osc. with reactor experiment

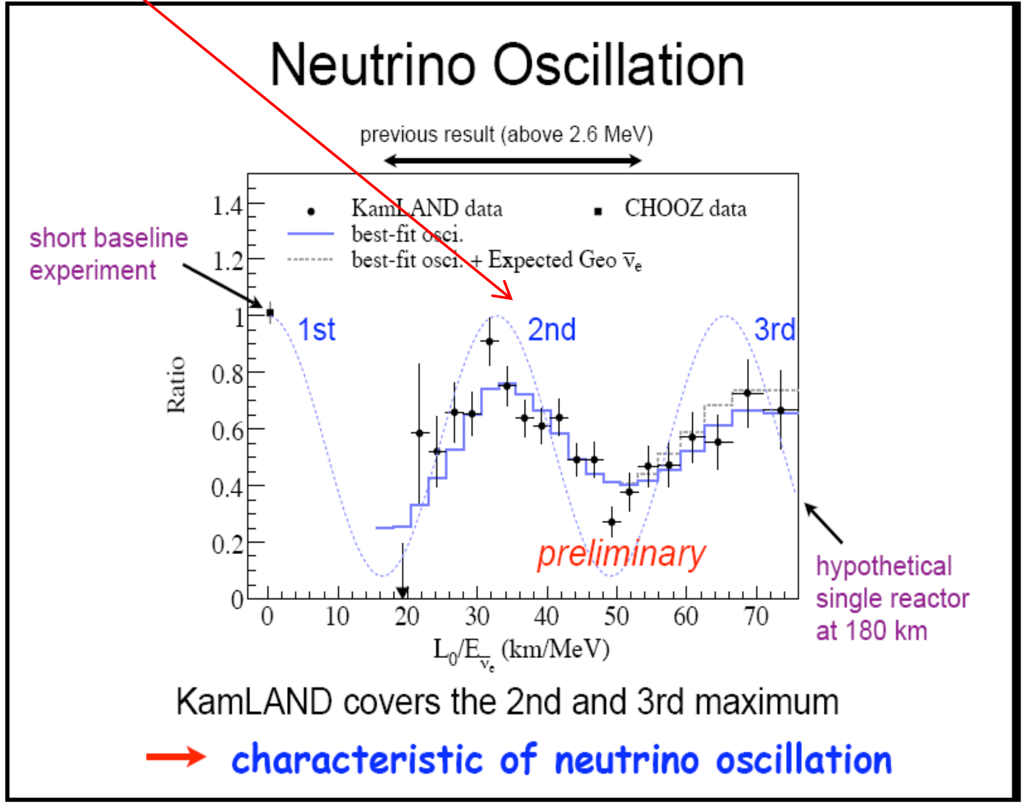
[K. Eguchi et al., Phys. Rev. Lett., 90, 021802 (2003)]

L/E analysis(2008)

A period oscillation

Itaru Shimizu





N_{obs}/N_{exp} = 0.611 + 0.094

$$\Delta m^2 \sim 7.5 \times 10^{-5}\ \mathrm{eV}^2,\ \sin^2\theta \sim 0.32$$

Solar Neutrino Experiment

With $\Delta m^2 \sim 7.5\times10^{-5}\ \mathrm{eV}^2$, $\sin^2\theta \sim 0.32$

For solar neutrinos,

$$1.27\frac{\Delta m^2 L}{E} \sim 1.27\frac{(7.5\times10^{-5}\ \mathrm{eV}^2)(1.5\times10^{11} m)}{0.1-10\ MeB} \sim 10^{7\pm1}$$

$$\Rightarrow \quad < \sin^2\left(1.27\frac{\Delta m^2 L}{E}\right) > \sim \frac{1}{2}$$

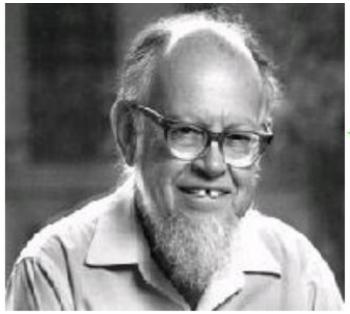
$$< P_{\nu_e\to\nu_e} > = 1-\sin^2 2\theta < \sin^2\left(1.27\frac{\Delta m_{21}^2 L}{E}\right) >$$

$$\approx 1-\frac{1}{2}\sin^2 2\theta = \cos^2\theta\cos^2\theta + \sin^2\theta\sin^2\theta$$

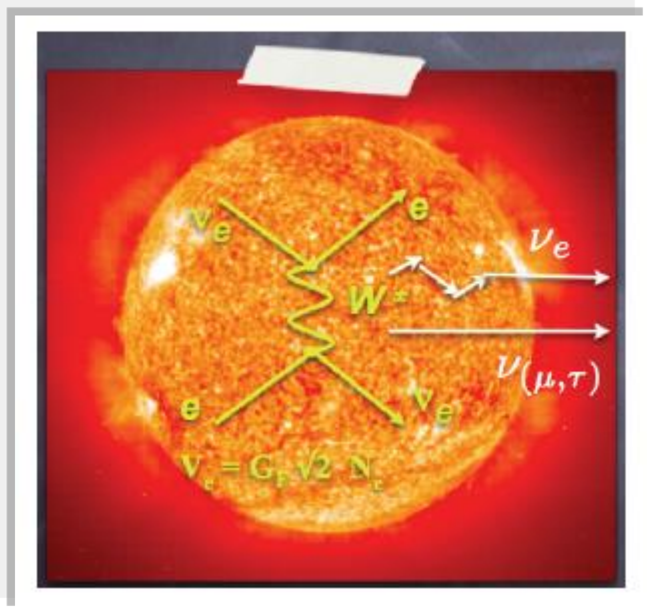
Tension : for ^8B, $< P_{\nu_e\to\nu_e} > \approx 0.32$, for pp, ^7Be: $< P_{\nu_e\to\nu_e} > \approx 0.6$

Matter Effect

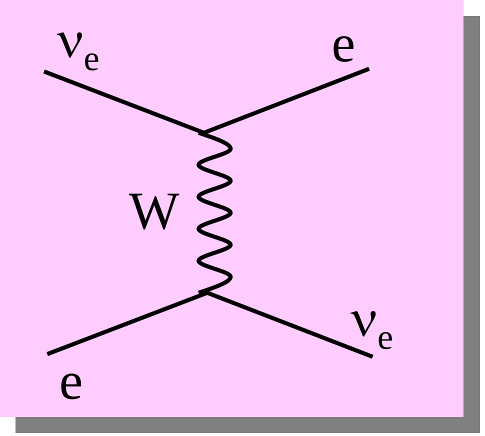
(Wolfestein '79)



- When neutrinos travel through a medium, they interact with the background of electron, proton and neutron, and then acquire effective mass.



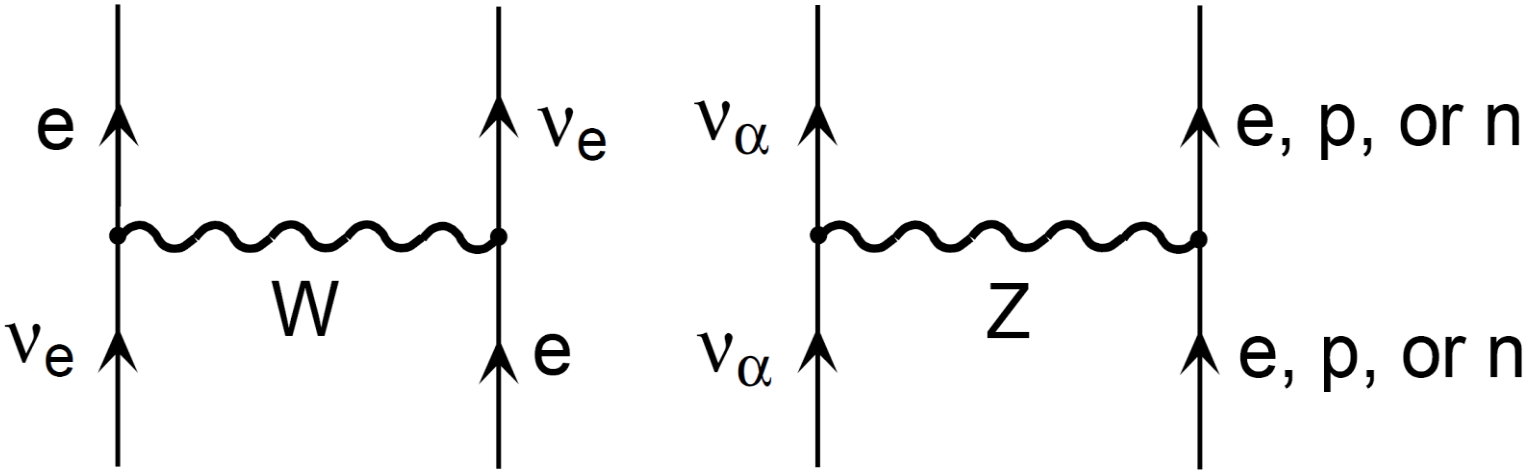
Elastic forward scattering



Matter Effect

- **This modifies mixing between flavor states and mass states, and eigenvalues of Hamiltonian, leading to different oscillation probability**

- ν_e has C.C. and N.C. while ν_μ, ν_τ have only N.C.



Matter Effect

- **The Hamiltonian in matter can be obtained by adding the potential terms**

$$\mathcal{H}_{\mathrm{mat}} = \begin{bmatrix} V_e & 0 \\ 0 & V_\mu \end{bmatrix} = \begin{bmatrix} V & 0 \\ 0 & 0 \end{bmatrix} + \lambda'' \mathbf{I}$$

irrelevant for flavor evolution

- **Difference of *V* plays a crucial role**

$$V \equiv V_{\nu_e} - V_{\nu_\mu} = V_{\nu_e} - V_{\nu_\tau} = +\sqrt{2}\, G_F\, N_e$$

Matter Effect

- **In vacuum, time evolution of neutrino states**

$$i\frac{d}{dt}\begin{pmatrix}|\nu_\alpha\rangle\\|\nu_\beta\rangle\end{pmatrix} = U\begin{pmatrix}E_1 & 0\\0 & E_2\end{pmatrix}U^\dagger\begin{pmatrix}|\nu_1\rangle\\|\nu_2\rangle\end{pmatrix}$$

$$= \begin{pmatrix}-\frac{\Delta m^2}{4E}\cos 2\theta & \frac{\Delta m^2}{4E}\sin 2\theta\\ \frac{\Delta m^2}{4E}\sin 2\theta & \frac{\Delta m^2}{4E}\cos 2\theta\end{pmatrix}\begin{pmatrix}|\nu_\alpha\rangle\\|\nu_\beta\rangle\end{pmatrix}$$

- **In matter,**

$$i\frac{d}{dt}\begin{pmatrix}|\nu_e\rangle\\|\nu_\mu\rangle\end{pmatrix} = \begin{pmatrix}-\frac{\Delta m^2}{4E}\cos 2\theta + \sqrt{2}G_F N_e & \frac{\Delta m^2}{4E}\sin 2\theta\\ \frac{\Delta m^2}{4E}\sin 2\theta & \frac{\Delta m^2}{4E}\cos 2\theta\end{pmatrix}\begin{pmatrix}|\nu_e\rangle\\|\nu_\mu\rangle\end{pmatrix}$$

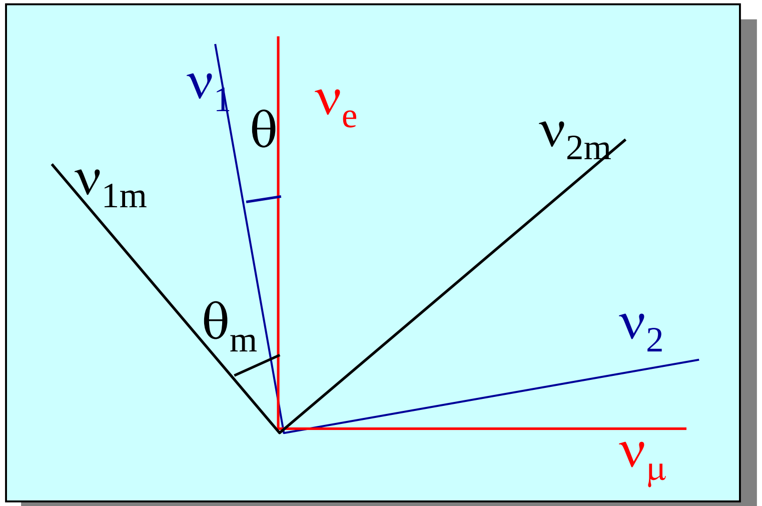
- **For antineutrinos, new term has opposite sign**

Matter Effect

- **If N_e is constant, diagonalizing the Hamiltonian**

$$\tan 2\theta_M = \frac{\tan 2\theta}{1 - \frac{A_{CC}}{\Delta m^2 \cos 2\theta}}, \qquad A_{CC} = 2\sqrt{2} G_F N_e E$$

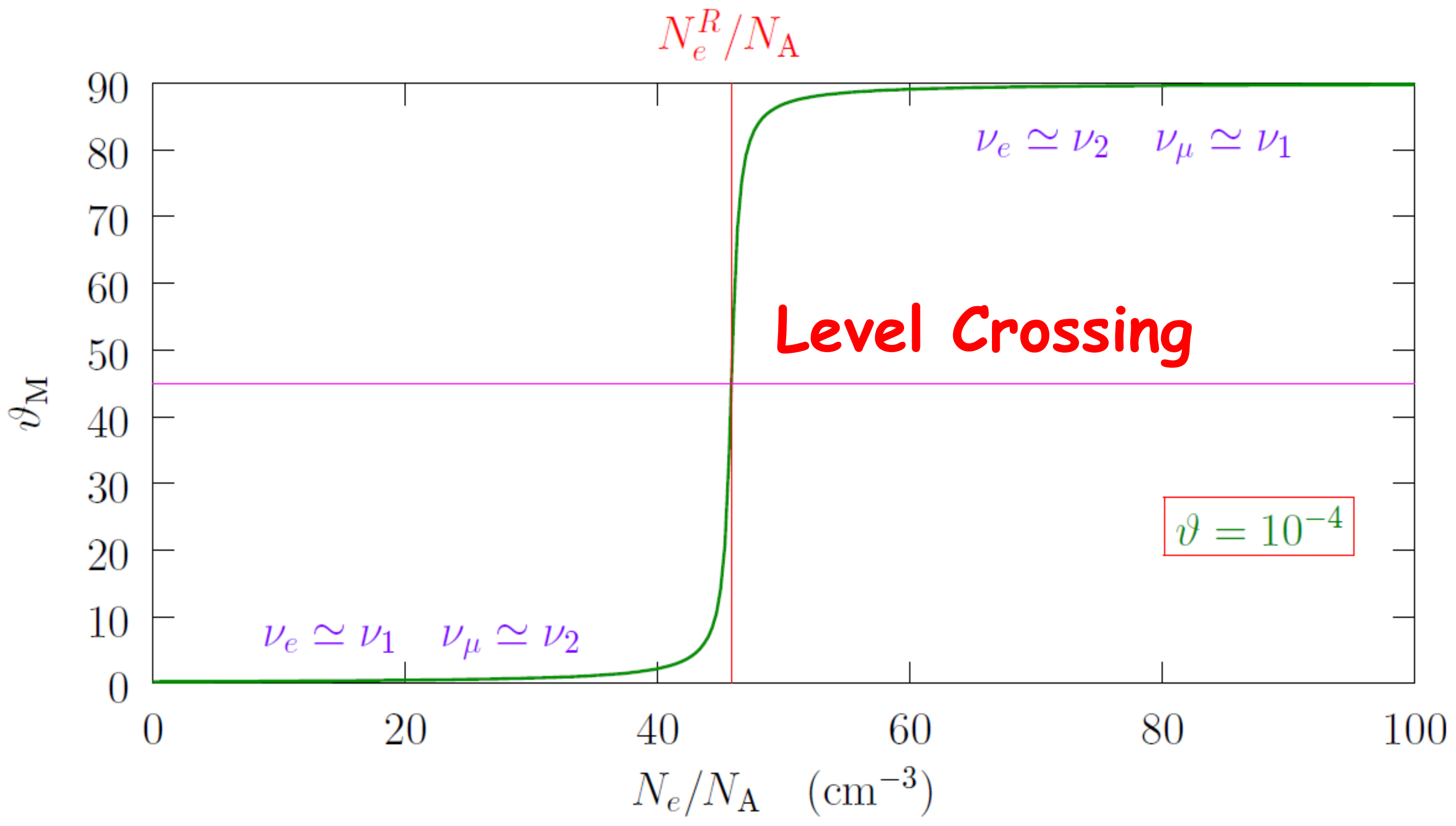
$$\nu_e = \cos\theta_M \nu_{1m} + \sin\theta_M \nu_{2m}$$
$$\nu_\mu = -\sin\theta_M \nu_{1m} + \cos\theta_M \nu_{2m}$$



$$\Delta m_M^2 = \sqrt{(\Delta m^2 \cos 2\vartheta - A_{CC})^2 + (\Delta m^2 \sin 2\vartheta)^2}$$

Matter Effect

$$\nu_e = \cos\theta_M \nu_{1m} + \sin\theta_M \nu_{2m}$$
$$\nu_\mu = -\sin\theta_M \nu_{1m} + \cos\theta_M \nu_{2m}$$



Matter Effect

(Mikheyev, Smirnov '85)

- If $A_{CC} = \Delta m^2 \cos 2\theta$, resonance occurs and mixing becomes maximal $\theta_M = \pi/4$

Oscillations in matter

- In medium with constant density
  - There is no $\nu_{1m} \leftrightarrow \nu_{2m}$ transitions, ν_{1m}, ν_{2m} are the eigenstates of propagation
  - Oscillation probability in matter looks similar to in vacuum

$$\bar{P}_{\nu_e \to \nu_\mu} = \sin^2(2\theta_M)\sin^2\frac{(E_A - E_B)L}{2}$$

Matter Effect

- In case that matter density varies with time, it is hard to solve analytically.
- ν_{1m}, ν_{2m} are not propagation eigenstates and transition between them occurs.

- Adiabatic limit : evolution is sufficiently slow.
  → each component evolves independently
  → $\nu_{1m} \leftrightarrow \nu_{2m}$ transitions are neglected

In-Matter Survival Probability

$$\boldsymbol{\nu_e} = \cos\theta_M \nu_{1m} + \sin\theta_M\, \nu_{2m}$$: **production**

$$\boldsymbol{\nu_e(x)} = \cos\theta\, \nu_1(x) + \sin\theta\, \nu_2(x)$$: **detection**

Neglecting interference term (averaged over *E* spectrum)

$$\overline{P}_{\nu_e\to\nu_e}(x) = |\langle\psi_e(x)\rangle|^2 = \cos^2\vartheta\,\cos^2\vartheta_M^0\,|\mathcal{A}_{11}^R|^2 + \cos^2\vartheta\,\sin^2\vartheta_M^0\,|\mathcal{A}_{21}^R|^2 + \sin^2\vartheta\,\cos^2\vartheta_M^0\,|\mathcal{A}_{12}^R|^2 + \sin^2\vartheta\,\sin^2\vartheta_M^0\,|\mathcal{A}_{22}^R|^2$$

$$|\mathcal{A}_{12}^R|^2 = |\mathcal{A}_{21}^R|^2 = P_c \qquad |\mathcal{A}_{11}^R|^2 = |\mathcal{A}_{22}^R|^2 = 1 - P_c$$

$P_c \equiv$ crossing probability **for non-adiabatic case**

$$\overline{P}_{\nu_e\to\nu_e}(x) = \frac{1}{2} + \left(\frac{1}{2} - P_c\right)\cos 2\vartheta_M^0\,\cos 2\vartheta$$

(S. Parke, PRL57, '86)

In-Matter Survival Probability

Assuming adiabatic limit

$$\bar{P}_{\nu_e \to \nu_e} = \cos^2\theta \cos^2\theta_M + \sin^2\theta \sin^2\theta_M$$

- **two interesting limits**

  ❖ **Matter dominates :** $\frac{\Delta m^2}{2E} \ll \sqrt{2} G_F N_e \Rightarrow \theta_M \sim \pi/2$

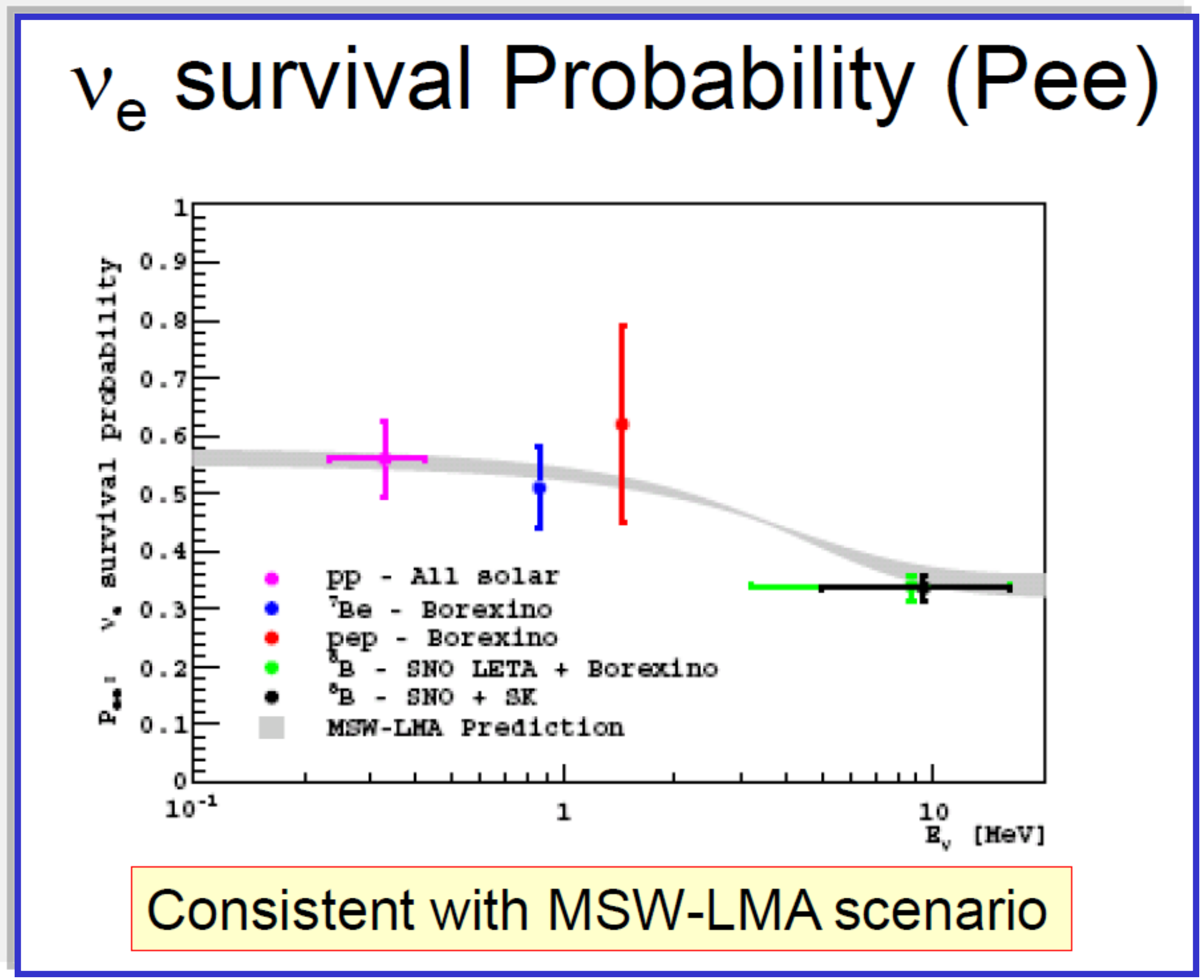
  $$\boxed{\bar{P}_{\nu_e \to \nu_e} \approx \sin^2\theta} \quad \left({}^8B : \bar{P}_{\nu_e \to \nu_e} \approx 0.32 \right)$$

  (${}^8B$ solar neutrino is pure ν_2 due to matter effect)

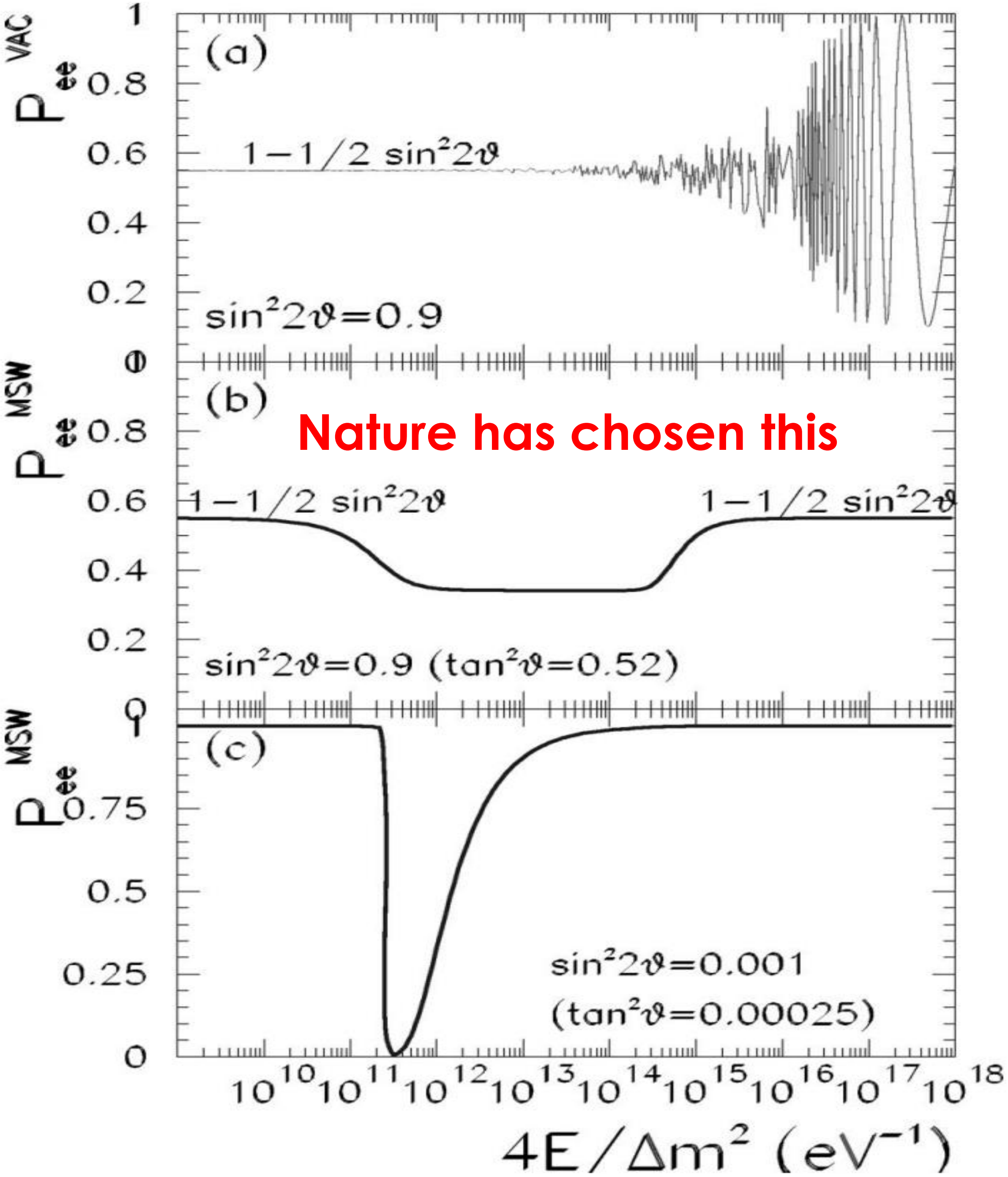
  ❖ **Vacuum dominates :** $\frac{\Delta m^2}{2E} \gg G_F N_e \Rightarrow \theta_M \sim \theta$

  $$\boxed{\bar{P}_{\nu_e \to \nu_e} \approx 1 - \frac{1}{2}\sin^2 2\theta} \quad \left(pp, {}^7Be : \bar{P}_{\nu_e \to \nu_e} \approx 0.6 \right)$$

Confirming Matter Effect

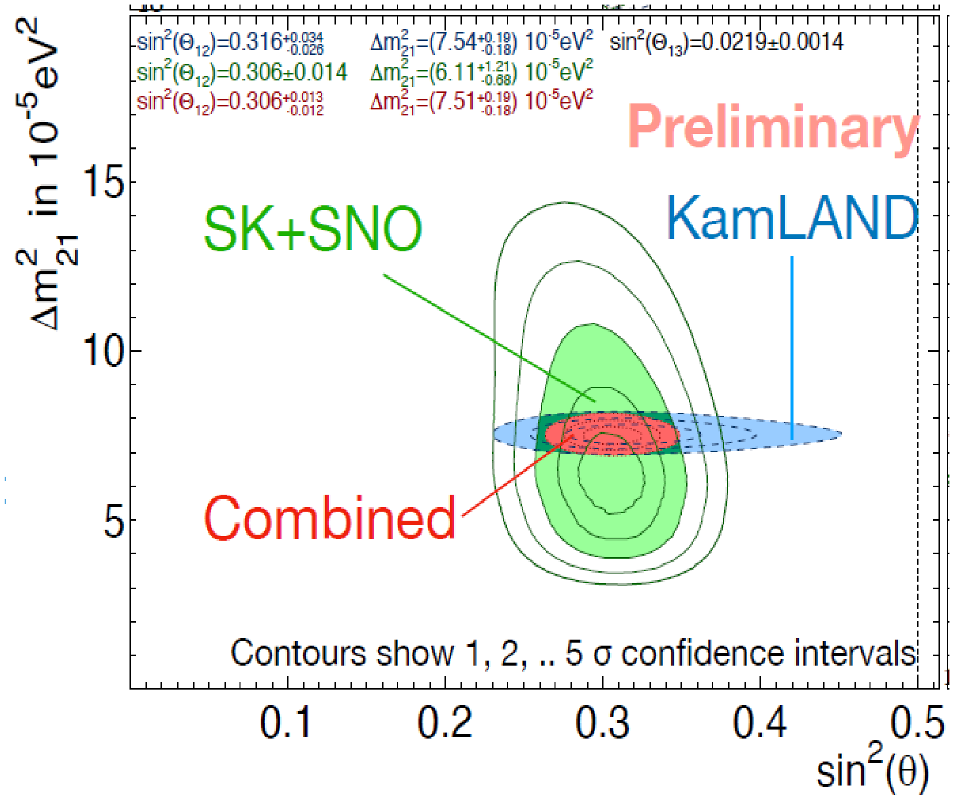


Borexino(2011)


(a)
1−1/2 sin²2ϑ
sin²2ϑ=0.9
(b)
Nature has chosen this
1−1/2 sin²2ϑ
1−1/2 sin²2ϑ
sin²2ϑ=0.9 (tan²ϑ=0.52)
(c)
sin²2ϑ=0.001
(tan²ϑ=0.00025)
4E/Δm² (eV⁻¹)

Experimental Results

- **Solar neutrino experiments and KamLAND**

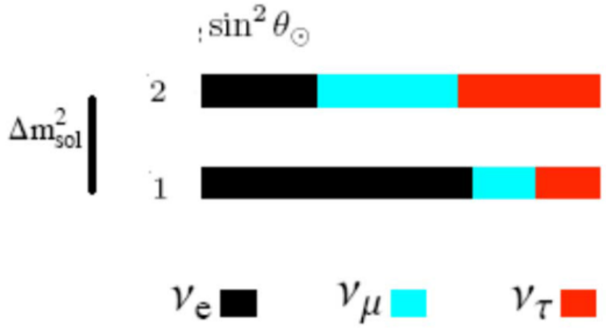


P. Salas, D. Forero, S. Gariazzo, O. Martinez, O. Mena, C. Ternes, M. Tortola, J. Valle, JHEP (2021)

- New solar neutrino data from Super-K (IV) lead to an upward shift of the allowed region for Δm^2_{21} which significantly reduced the tension between solar global & KamLAND data.
- They are now compatible at 1.1σ

Day/Night asymmetry:

$A_{DN}^{Fit} = (-3.6 \pm 1.6(stat) \pm 0.6(syst))\ \% \ \rightarrow A_{DN}^{Fit} = (-2.1 \pm 1.1)\ \%$



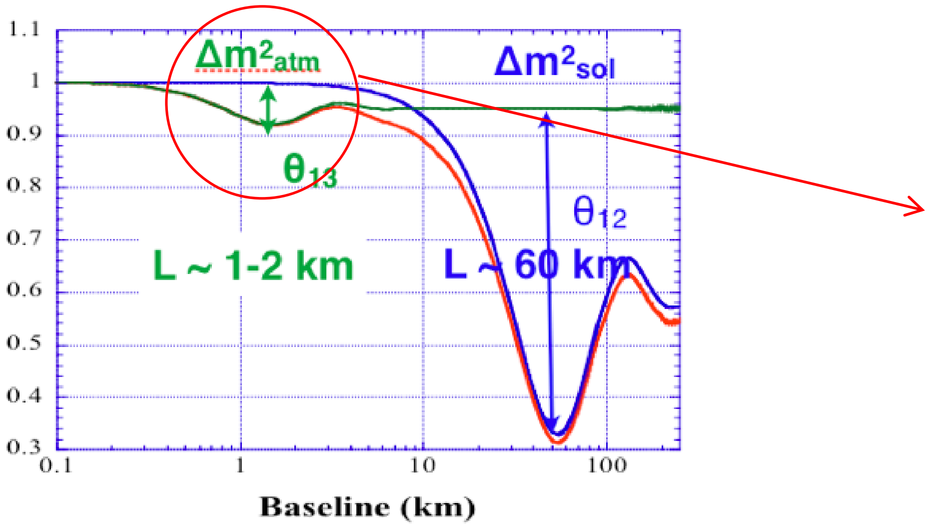
θ_{13}

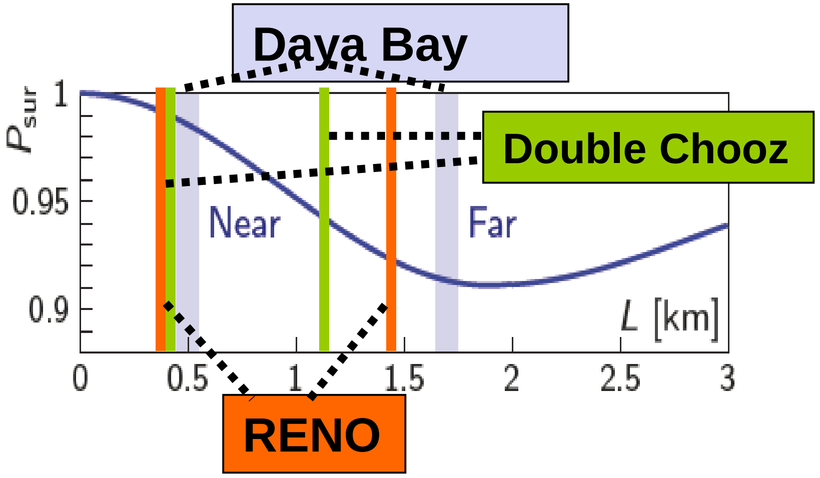
- **Measuring θ_{13} : important role in determining CPV & mass ordering**

$$J_{CP} = \mathrm{Im}(U_{\mu 3}U^*_{e3}U_{e2}U^*_{\mu 2}) = \frac{1}{8}\sin 2\theta_{12}\sin 2\theta_{23}\sin 2\theta_{13}\cos\theta_{13}\sin\delta$$

- **Measured from SBL reactor experiments**

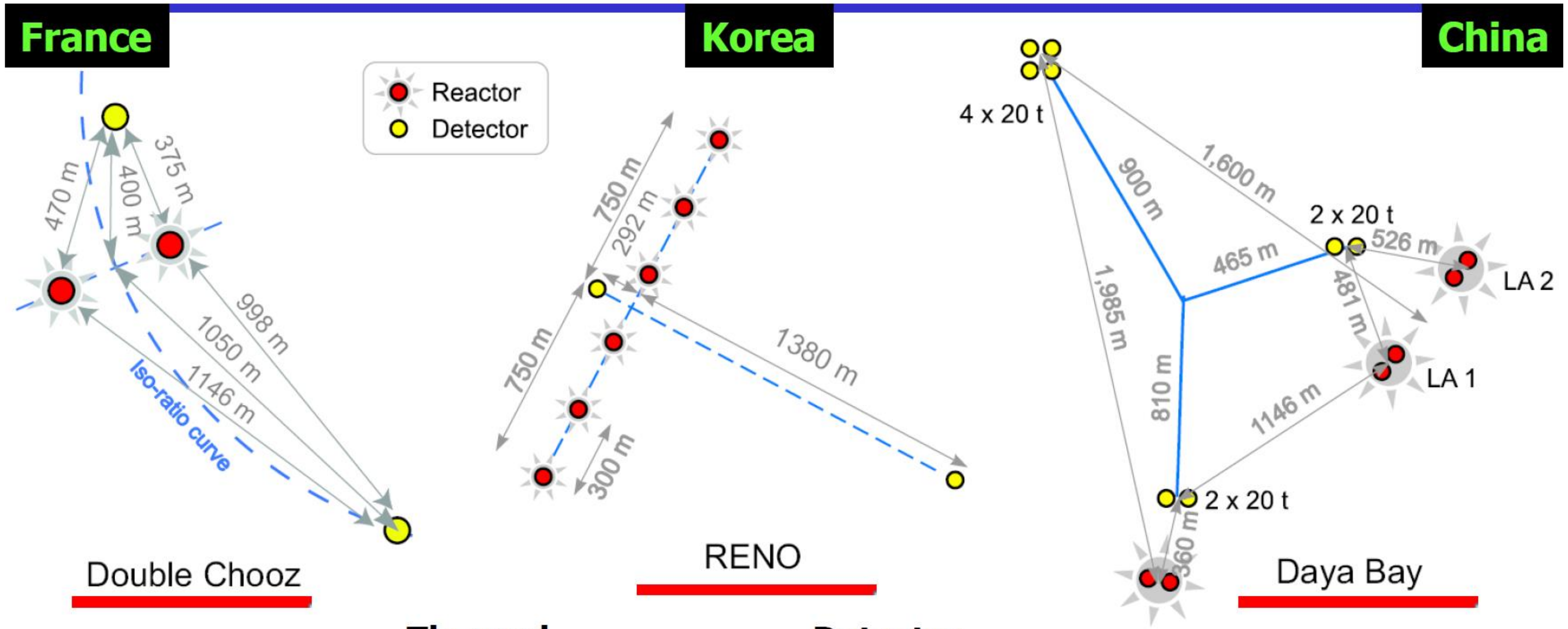
$$P_{\nu_e\to\nu_e} \approx 1 - \sin^2 2\theta_{13}\sin^2\frac{\Delta m^2_{31}L}{4E} - \cos^4\theta_{13}\sin^2 2\theta_{12}\sin^2\frac{\Delta m^2_{21}L}{4E}$$





θ_{13}

3 Reactor Experiments



Setup	**Thermal power** P_{Th} (GW)	**Baseline** L (m)	**Detector mass** m_{Det} (t)	Events/year	Backgrounds/day
Daya Bay [20]	17.4	1700	80	10×10^4	0.4
Double CHOOZ [21]	8.6	1050	8.3	1.5×10^4	3.6
RENO [22]	16.4	1400	15.4	3×10^4	2.6

θ_{13}

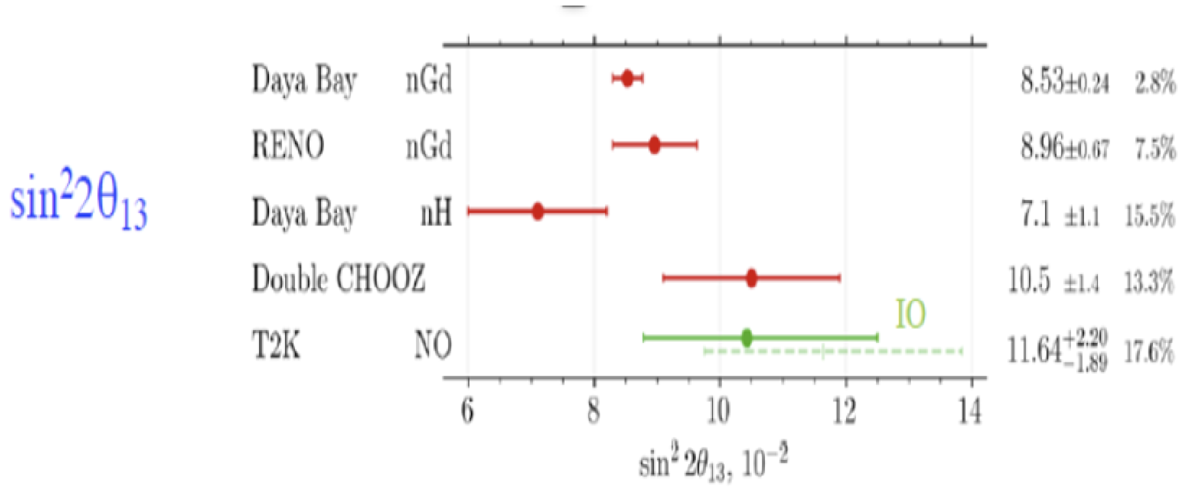
Experimental Results

- New results from Daya Bay nGd capture:

$$\sin^2 2\theta_{13} = 0.0853^{+0.0024}_{-0.0024} \qquad (2.8\% \text{ precision})$$

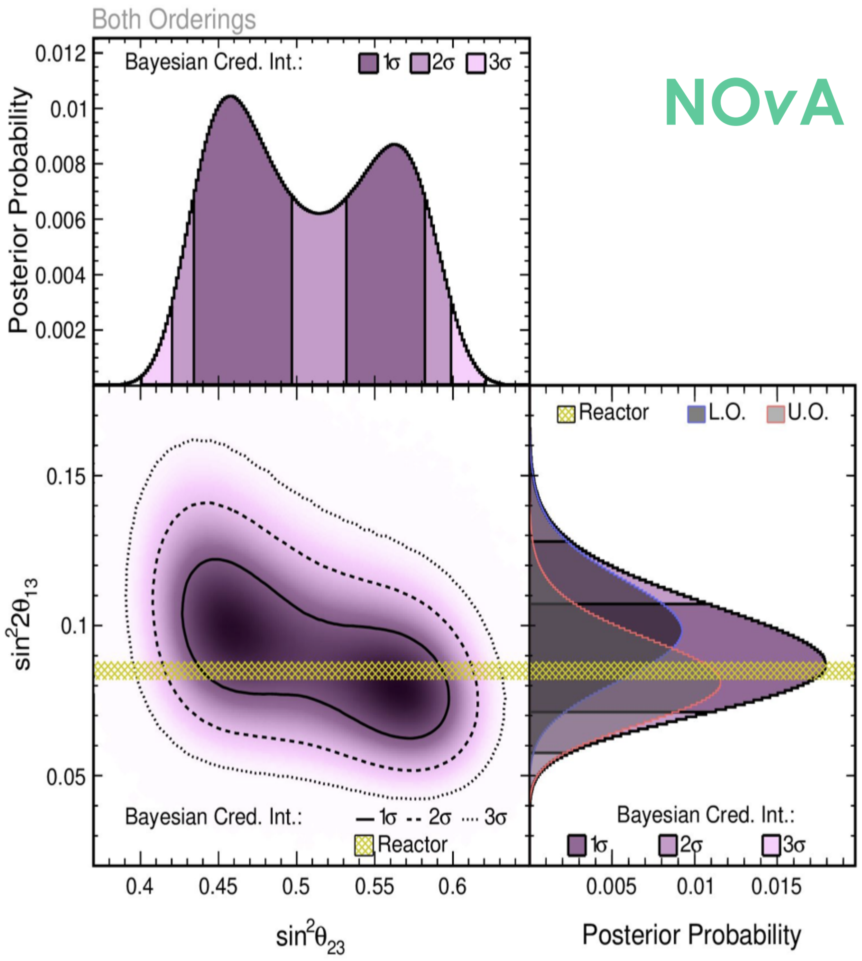
- Expect final results from Daya Bay on combined nGd+nH analysis: 2.6% for $\sin^2 2\theta_{13}$?
- RENO reported new results(up to 2019)

$$\sin^2 2\theta_{13} = 0.0892 \pm 0.0044(\text{stat.}) \pm 0.0045(\text{sys.}) \qquad (\pm 7.0\ \%)$$



θ_{13}

Measurement of θ_{13} from accelerator experiment



NOvA

- The results so far all use a constraint on θ_{13} from reactor experiments.
- The Bayesian interpretation of our data allows us to drop this constraint and make a NOvA measurement of θ_{13}.

$$\sin^2(2\theta_{13}) = 0.085^{+0.020}_{-0.016}$$

- Consistent with the measurements from reactor experiments.
- Good test of PMNS consistency → NOvA measurement uses a very different strategy to reactor experiments.

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Global Fit

Recent 3-neutrino global analysis

Gonzalez-Garcia, Maltoni, Schwetz (NuFIT), 2111.03086

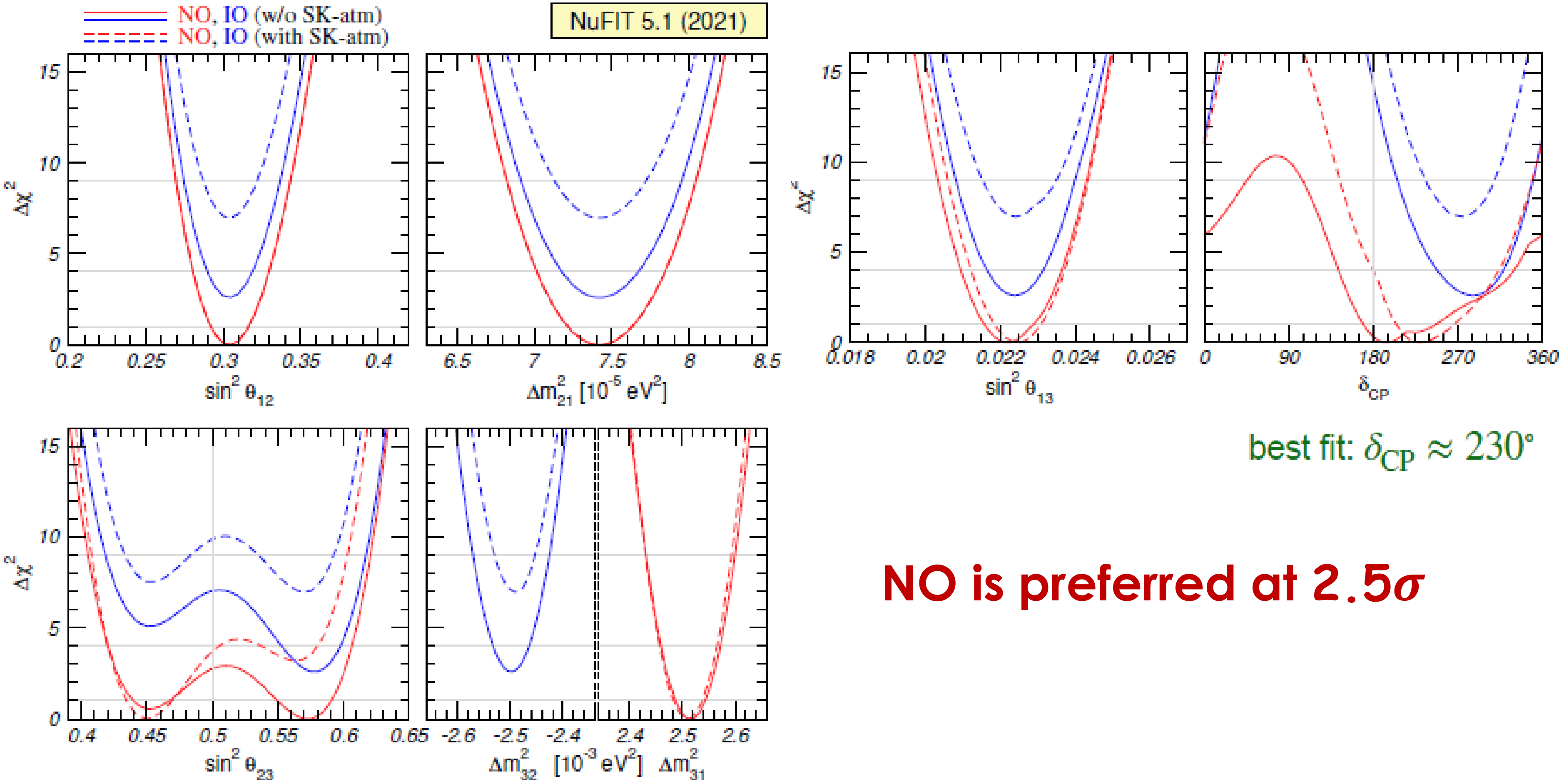
NuFIT5.1

with SK atmospheric data

	Normal Ordering (Best Fit)		Inverted Ordering ($\Delta\chi^2 = 7.0$)	
	bfp $\pm 1\sigma$	3σ range	bfp $\pm 1\sigma$	3σ range
$\sin^2\theta_{12}$	$0.304^{+0.012}_{-0.012}$	$0.269 \to 0.343$	$0.304^{+0.013}_{-0.012}$	$0.269 \to 0.343$
$\theta_{12}/^\circ$	$33.45^{+0.77}_{-0.75}$	$31.27 \to 35.87$	$33.45^{+0.78}_{-0.75}$	$31.27 \to 35.87$
$\sin^2\theta_{23}$	$0.450^{+0.019}_{-0.016}$	$0.408 \to 0.603$	$0.570^{+0.016}_{-0.022}$	$0.410 \to 0.613$
$\theta_{23}/^\circ$	$42.1^{+1.1}_{-0.9}$	$39.7 \to 50.9$	$49.0^{+0.9}_{-1.3}$	$39.8 \to 51.6$
$\sin^2\theta_{13}$	$0.02246^{+0.00062}_{-0.00062}$	$0.02060 \to 0.02435$	$0.02241^{+0.00074}_{-0.00062}$	$0.02055 \to 0.02457$
$\theta_{13}/^\circ$	$8.62^{+0.12}_{-0.12}$	$8.25 \to 8.98$	$8.61^{+0.14}_{-0.12}$	$8.24 \to 9.02$
$\delta_{CP}/^\circ$	230^{+36}_{-25}	$144 \to 350$	278^{+22}_{-30}	$194 \to 345$
$\frac{\Delta m^2_{21}}{10^{-5}\ \mathrm{eV}^2}$	$7.42^{+0.21}_{-0.20}$	$6.82 \to 8.04$	$7.42^{+0.21}_{-0.20}$	$6.82 \to 8.04$
$\frac{\Delta m^2_{3\ell}}{10^{-3}\ \mathrm{eV}^2}$	$+2.510^{+0.027}_{-0.027}$	$+2.430 \to +2.593$	$-2.490^{+0.026}_{-0.028}$	$-2.574 \to -2.410$

- Hints for deviation of θ_{23} from $\pi/4$
- Mild hints for a Dirac *CP* phase δ
- Mild hint in favor of Normal Ordering

Global Fit

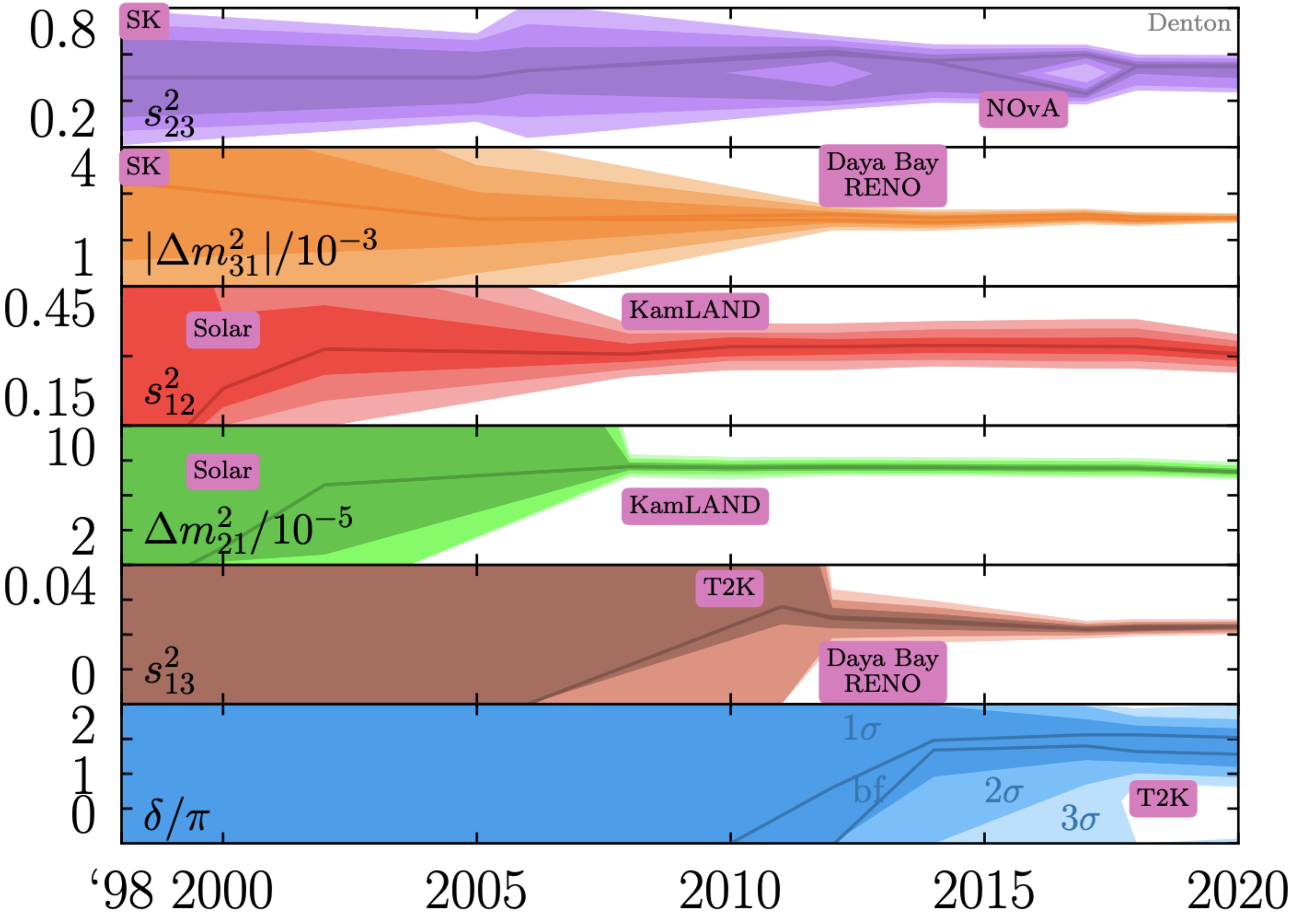


best fit: $\delta_{CP} \approx 230°$

NO is preferred at 2.5σ

Parameter Precision Evolution

Towards precision physics



See P. Denton's talk

The past 20 years have seen a remarkable progress in determining neutrino properties!

Parameter Precision Evolution

(2111.03086)

	2012 NuFIT 1.0	2014 NuFIT 2.0	2016 NuFIT 3.0	2018 NuFIT 4.0	2021 NuFIT 5.1
θ_{12}	15%	14%	14%	14%	14%
θ_{13}	30%	15%	11%	8.9%	9.0%
θ_{23}	43%	32%	32%	27%	27%
Δm^2_{21}	14%	14%	14%	16%	16%
$\lvert\Delta m^2_{3\ell}\rvert$	17%	11%	9%	7.8%	6.7% [6.5%]
δ_{CP}	100%	100%	100%	100% [92%]	100% [83%]
$\Delta\chi^2_{\text{IO-NO}}$	± 0.5	-0.97	$+0.83$	$+4.7$ [$+9.3$]	$+2.6$ [$+7.0$]

w/o [w] SK atm data

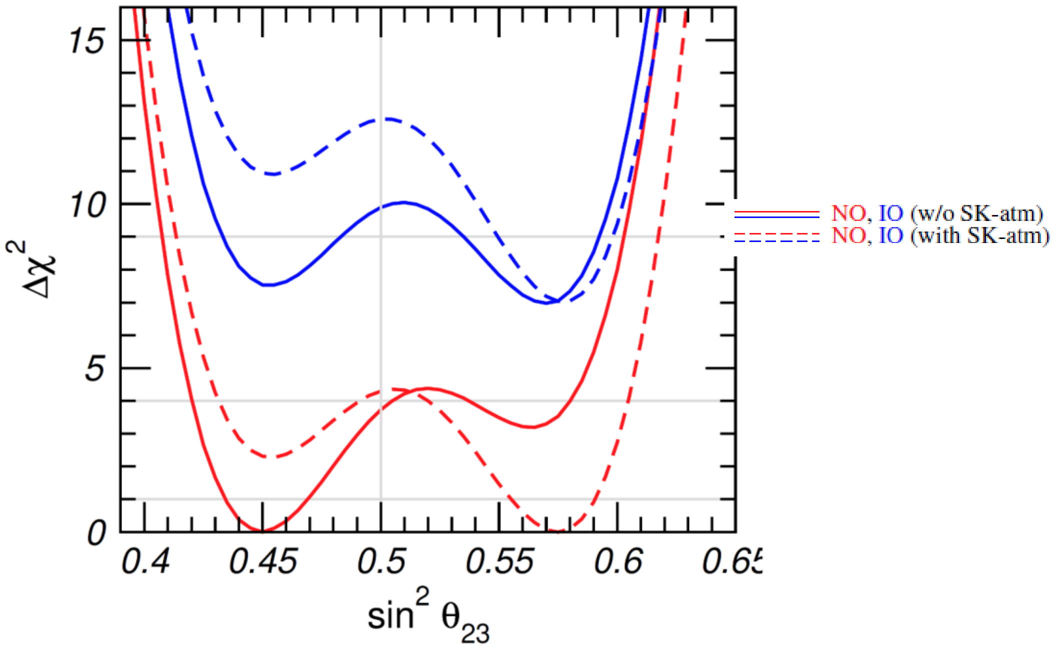
- 4 well-measured parameters : $\theta_{13}, \theta_{12}, \Delta m^2_{21}, |\Delta m^2_{31}|$
- Future exps. such as JUNO,DUNE,Hyper-K will achieve a few percent precision.

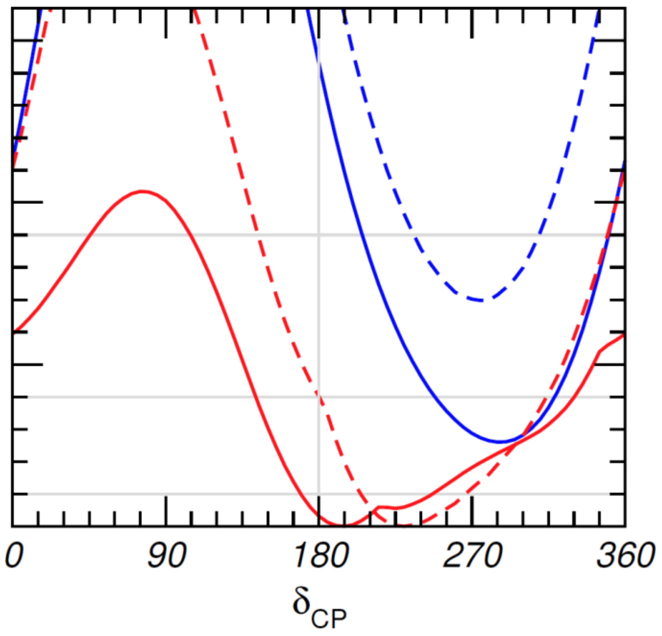
The Unknowns

- Neutrino mass ordering : (red vs. blue) NO is preferred at 2.5σ

- Octant of θ_{23}

- Leptonic CP violation



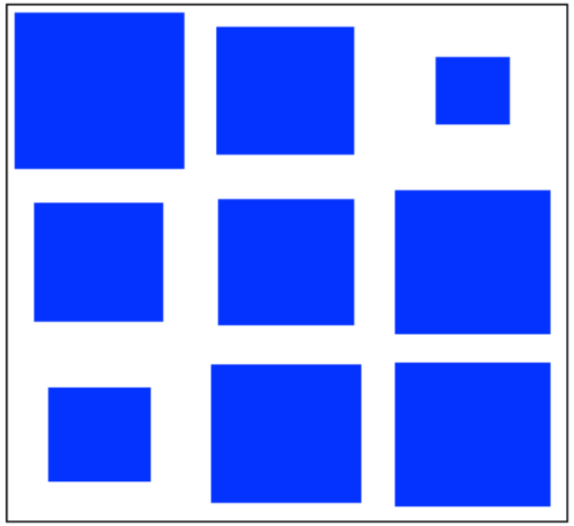


best fit: $\delta_{CP} \approx 230°$

PMNS *vs.* CKM

- **From fit to neutrino data in 3-neutrino paradigm**

$$|U_{PMNS}| = \begin{pmatrix} 0.800-0.844 & 0.515-0.581 & 0.139-0.155 \\ 0.229-0.516 & 0.438-0.699 & 0.614-0.790 \\ 0.249-0.528 & 0.462-0.715 & 0.595-0.776 \end{pmatrix}$$



Looks different from quark mixing matrix !!

$$|V_{CKM}| = \begin{pmatrix} 0.97434 & 0.22506 & 0.00357 \\ 0.22492 & 0.97351 & 0.0414 \\ 0.00875 & 00403 & 0.99915 \end{pmatrix}$$

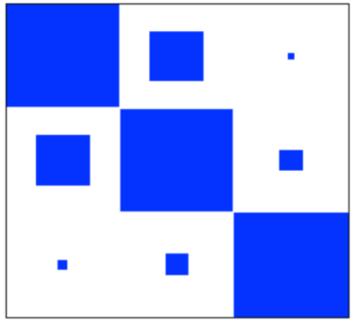
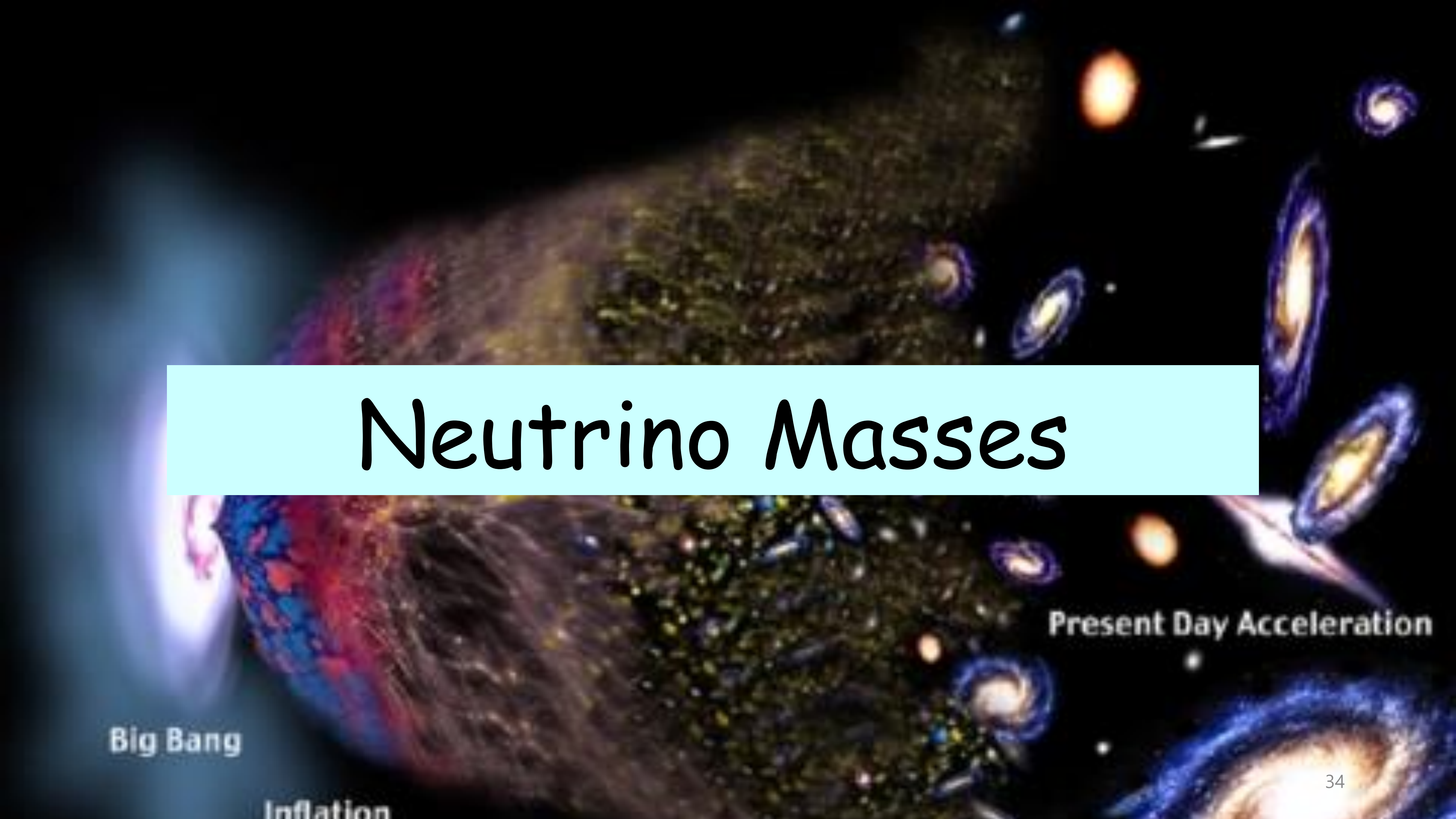


Image from Mark Messier

Neutrino Masses



Neutrino Mass Scale

- Solar neutrino; $\Delta m^2_{sol} \approx \Delta m^2_{21} \sim 7.5 \times 10^{-5} \mathrm{eV}^2$
- Atmospheric neutrino; $\Delta m^2_{atm} \approx |\Delta m^2_{31}| \approx |\Delta m^2_{32}| \sim 2.5 \times 10^{-3} \mathrm{eV}^2$
- Sum of 3 Δm^2 should be 0; $\Delta m^2_{12} + \Delta m^2_{23} + \Delta m^2_{31} = 0$

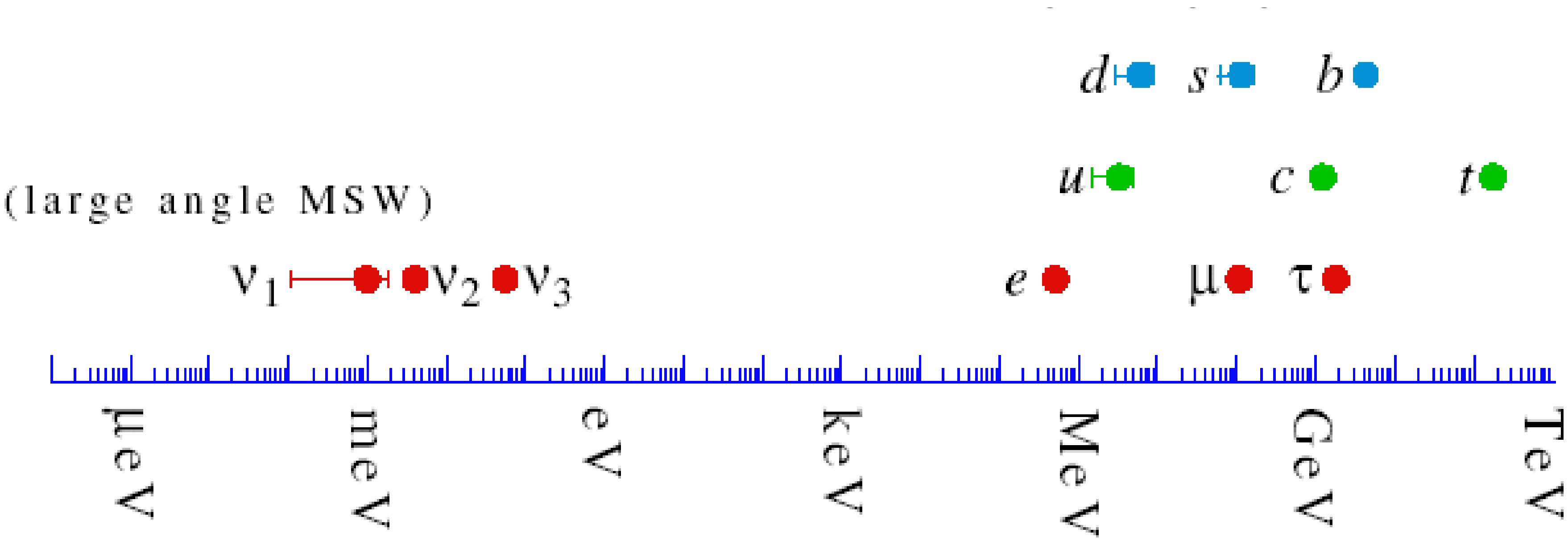
While we could determine the mass squared difference, each mass has not been measured .

- Assuming hierarchical mass spectrum, $m_{\nu_i} \leq \sqrt{\Delta m^2_{atm}} \sim 0.05$ eV

How small are neutrino masses?

- **Cosmological mass limit :**
  - Under the assumption of ΛCDM
  - from Planck CMB + BAO +Planck high-l polar. + optical depth to reion.

⇨ $m_\nu \leq 0.12\ (\mathrm{eV})$ (Sunny Vagnozzi, et al. arXiv:1701.08172)



2003 snowmass report

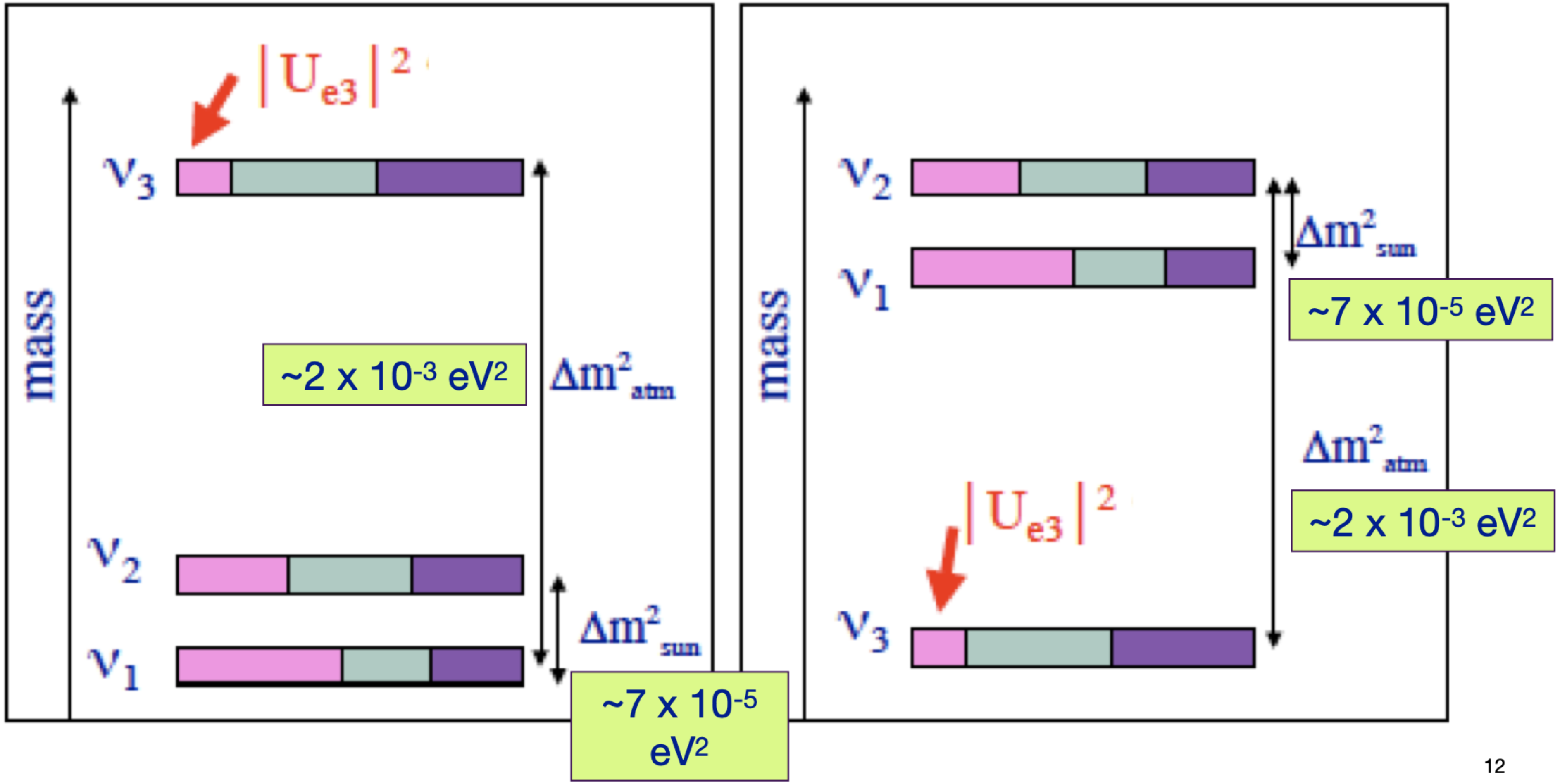
Mass Ordering

- Oscillation probability in vacuum is invariant under $\Delta m^2 \rightarrow -\Delta m^2$
- Matter effect depends on sign of Δm^2
- Solar neutrino experiments fix $\Delta m^2_{21} > 0$
- But, we do not determine the sign of $\Delta m^2_{31(2)}$

→ we have 2 options for mass ordering

Mass Ordering

Normal Ordering **Inverted Ordering** (NuFIT group)



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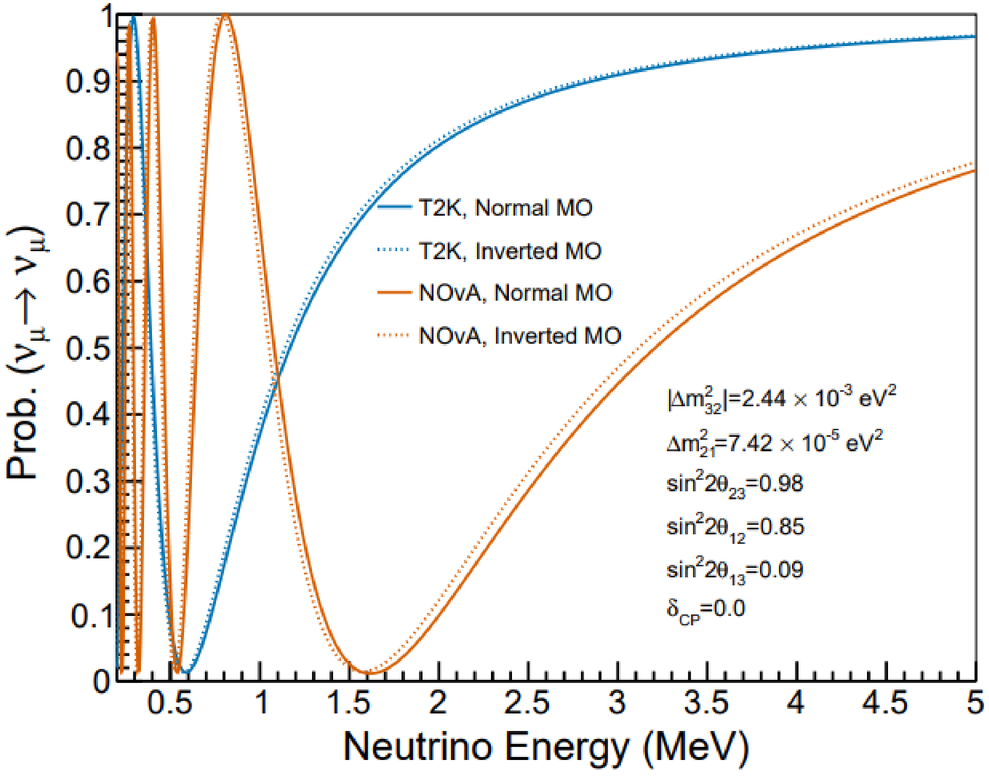
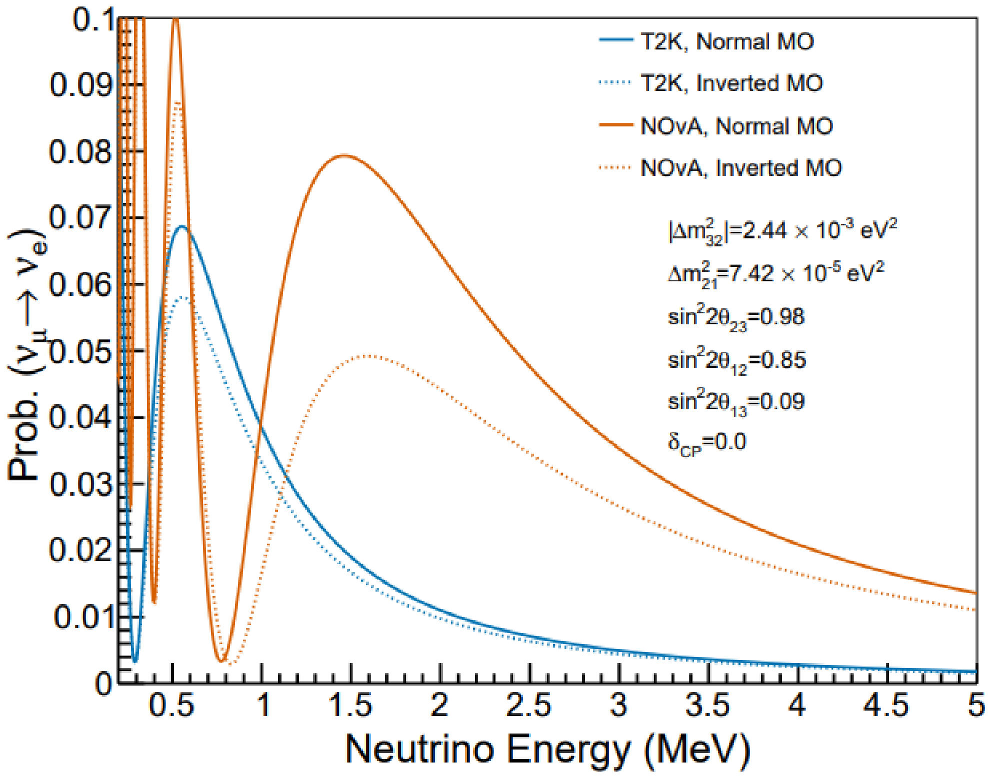
Mass Ordering

How to discriminate between two ?

→ **accelerator-based LBL experiments.**



Disappearance



Appearance

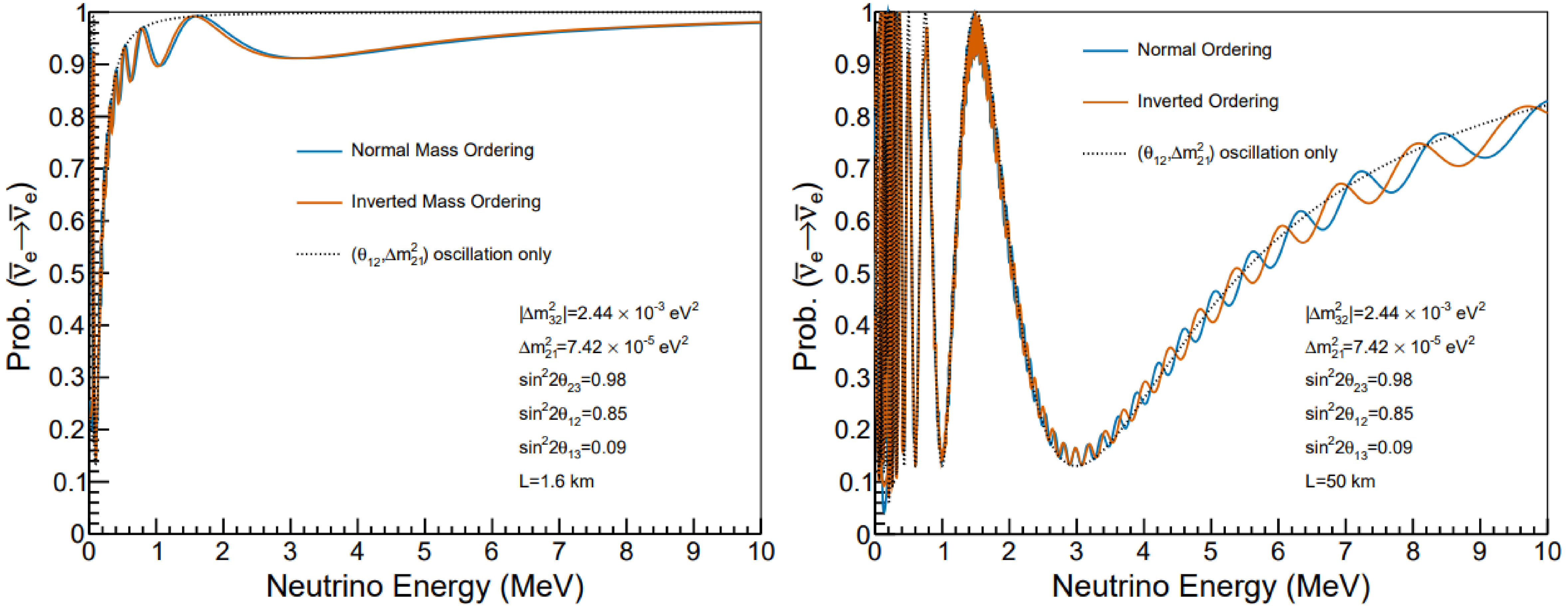
Son Cao *et al.*, symmetry2022

Mass Ordering

- Senstivity to MO is marginal in the $\nu_\mu \to \nu_\mu$ (or $\bar{\nu}_\mu \to \bar{\nu}_\mu$) modes
- The effect of MO is much stronger in the $\nu_\mu \to \nu_e$ (or $\bar{\nu}_\mu \to \bar{\nu}_e$) modes
- The relatively large modification of the oscillation probabilities in the appearance is due to the coherent scattering of ν_e ($\bar{\nu}_e$) on the electron present in the matter.
- But, appearance probability is just a few %, limiting the statistics of the collected data sample.
- Moreover, extracting MO effect from the appearance probabilities is non-trivial since the sign of Δm^2_{31} is tangled severely with δ_{CP} & θ_{23} which have been measured with relatively large uncertainty.

Mass Ordering

Short & medium base line experiments

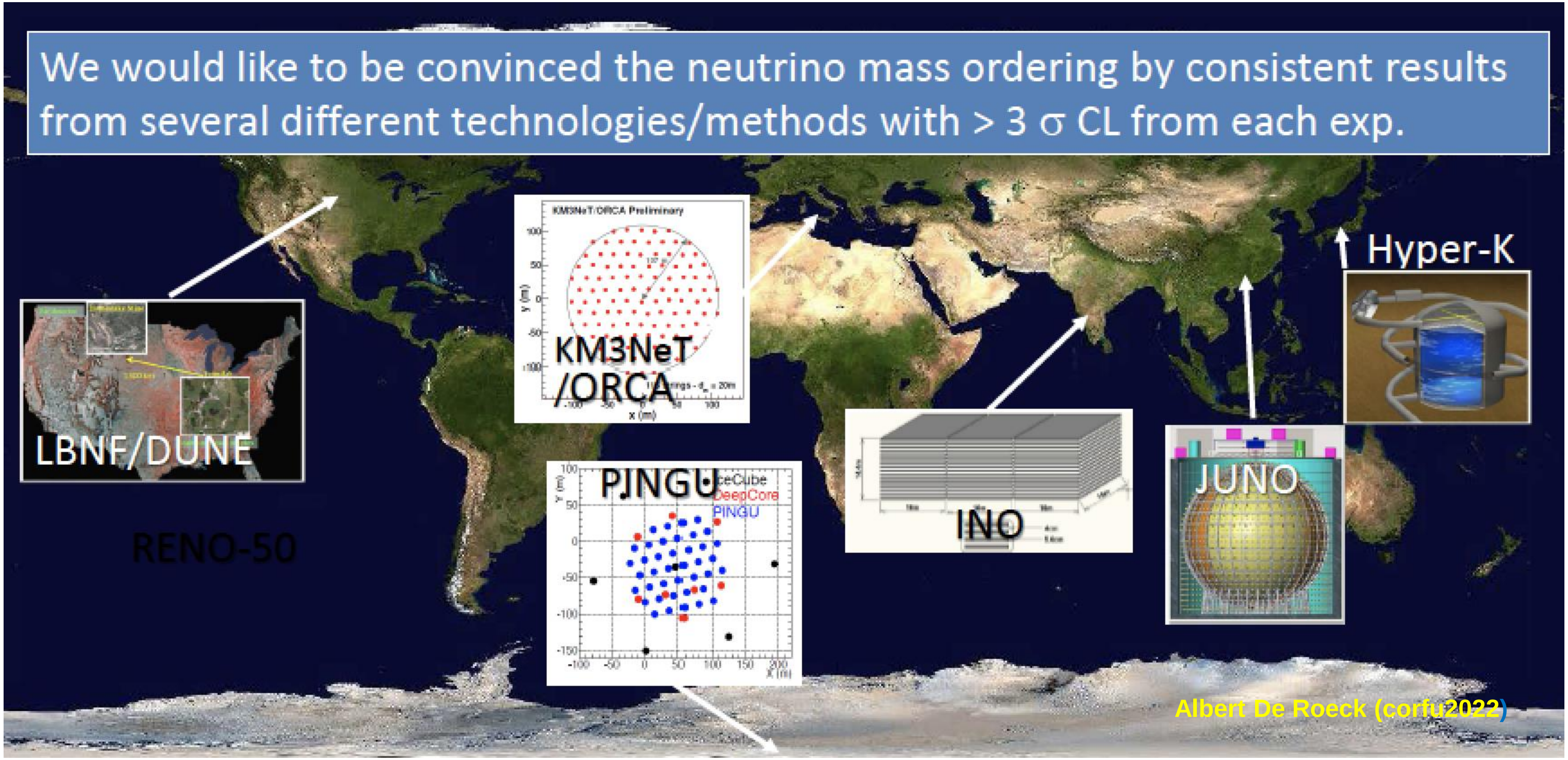


Son Cao etal, symmetry2022

Mass Ordering

- For SBL reactor exp., the sensitivity to MO is marginal .
- But, for JUNO with a medium-baseline of 50 km, can improve the sensitivity to MO thanks to the interference between two oscillation terms driven by Δm_{21}^2 & Δm_{31}^2, respectively.
- The most challenging thing is to achieve an excellent resolution of reconstructed neutrino energy for unravelling MO from the detector response effect (more detail : Abusleme, eta., JHEP2021.2)

Future Experiments for MO



Neutrino Mass

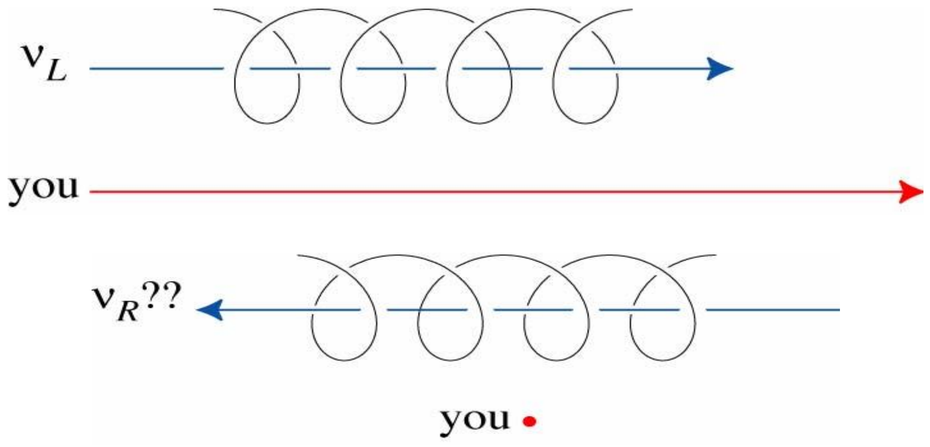
In SM, neutrinos are massless

- SM particle table

	I	I_3	Y	$Q = I_3 + \frac{Y}{2}$
lepton doublet $L_L = \begin{pmatrix} \nu_L \\ \ell_L \end{pmatrix}$	1/2	1/2 −1/2	−1	0 −1
lepton singlet ℓ_R	0	0	−2	−1
Higgs doublet $\Phi(x) = \begin{pmatrix} \phi_+(x) \\ \phi_0(x) \end{pmatrix}$	1/2	1/2 −1/2	+1	1 0

All neutrinos left-handed

$\Rightarrow$ massless



Right-handed Neutrinos ?

No observation

Massive neutrinos imply the standard model is incomplete !

Neutrino Mass

(1) Dirac Mass

- **Some basics for understanding neutrino masses**

$$\gamma^0 = \begin{pmatrix} 0 & I_2 \\ I_2 & 0 \end{pmatrix}, \gamma^i = \begin{pmatrix} 0 & \sigma_i \\ -\sigma_i & 0 \end{pmatrix}$$

$$\gamma^\alpha \gamma^\beta + \gamma^\beta \gamma^\alpha = 2g^{\alpha\beta}$$

$$g^{\alpha\beta} = \text{diag}(1, -1, -1, -1)$$

$$\gamma^5 = -i\gamma^0\gamma^1\gamma^2\gamma^3$$

$$\gamma^5 = \begin{pmatrix} -I_2 & 0 \\ 0 & I_2 \end{pmatrix}$$

chiral projection operators

$$P_L = \frac{1}{2}(I - \gamma^5) = \begin{pmatrix} I_2 & 0 \\ 0 & 0 \end{pmatrix}$$

$$P_R = \frac{1}{2}(I + \gamma^5) = \begin{pmatrix} 0 & 0 \\ 0 & I_2 \end{pmatrix}$$

$$\gamma^{0(5)+} = \gamma^{0(5)}, \quad \gamma^{i+} = -\gamma^i$$

$$P_{L(R)}^2 = P_{L(R)}, P_L P_R = 0$$

$$\sigma_1 = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}, \quad \sigma_2 = \begin{pmatrix} 0 & -i \\ i & 0 \end{pmatrix}, \quad \sigma_3 = \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}$$

Neutrino Mass

(1) Dirac Mass

For a Dirac Field $\psi = \begin{pmatrix} \varphi \\ \chi \end{pmatrix}$

$$\psi_L = P_L\psi = \frac{1}{2}(1-\gamma^5)\psi = \begin{pmatrix} \varphi \\ 0 \end{pmatrix}, \quad \psi_R = P_R\psi = \frac{1}{2}(1+\gamma^5) = \begin{pmatrix} 0 \\ \chi \end{pmatrix}$$

$$\bar{\psi}_L = (\psi_L)^+\gamma^0 = (P_L\psi)^+\gamma^0$$
$$= \psi^+P_L\gamma^0 = \psi^+\gamma^0P_R$$
$$= \bar{\psi}P_R$$

$$\bar{\psi}_R = \bar{\psi}P_L$$

$$\psi = \psi_L + \psi_R$$

Neutrino Mass

(1) Dirac Mass

we can easily check that $\bar{\psi}_R \psi_R = \bar{\psi}_L \psi_L = 0.$

$$
\begin{aligned}
\bar{\psi}\psi &= \overline{(\psi_R + \psi_L)}(\psi_R + \psi_L) \\
&= \bar{\psi}_R \psi_R + \bar{\psi}_L \psi_L + \bar{\psi}_R \psi_L + \bar{\psi}_L \psi_R \\
&= \bar{\psi}_R \psi_L + \bar{\psi}_L \psi_R
\end{aligned}
$$

We can write down Dirac mass term :

$$\bar{\psi}_R M^\dagger \psi_L + \bar{\psi}_L M \psi_R$$

Neutrino Mass

(1) Dirac Mass

- A fermion mass can be thought of as a $L \leftrightarrow R$ transition
- For electron & positron

$$\begin{array}{lccc} I_3 = \pm\frac{1}{2} & : & e_L & | & \bar{e}_R \\ I_3 = 0 & : & e_R & | & \bar{e}_L \end{array}$$

Dirac mass

- Mass terms (Dirac mass) : $e_L \leftrightarrow e_R\ ,\ \bar{e}_R \leftrightarrow \bar{e}_L$
- Mass terms $e_L \leftrightarrow \bar{e}_R\ ,\ e_R \leftrightarrow \bar{e}_L$ not allowed due to violation of conservation of electric charge
- For neutrino Dirac mass, we need RH neutrino ($I_3 = 0$)

Neutrino Mass

(1) Dirac Mass

- **Dirac type mass matrix M is in general $N \times N$ complex matrix**
- **M is diagonalized by bi-unitary matrices**

$$M = U m_D V^\dagger \qquad M M^\dagger = U m_D^2 U^\dagger$$

$$m_D^2 = \begin{pmatrix} m_1^2 & & & \\ & m_2^2 & & \\ & & m_3^2 & \\ & & & \ddots \end{pmatrix}$$

U : *a unitary matrix*

Neutrino Mass

(1) Dirac Mass

Origin : Yukawa interactions $Y\bar{\psi}_R H^\dagger \psi_L \to \text{SSB}$

$$\bar{\psi}_L M \psi_R = \bar{\psi}_L U m_D V^\dagger \psi_R = \overline{(U^\dagger \psi_L)} m_D V^\dagger \psi_R$$

$$\psi_L = U\psi_L^M$$
$$\psi_R = V\psi_R^M$$

This shows that weak eigenstates are different from mass eigenstates.

Neutrino Mass

(1) Dirac Mass

- Dirac type mass matrix M is invariant under

$$\psi \rightarrow e^{i\alpha}\psi$$

which implies lepton number $L\ (= L_e + L_\mu + L_\tau)$ conservation.

$$\begin{pmatrix} \overline{\nu_{eL}} & \overline{\nu_{\mu L}} & \overline{\nu_{\tau L}} \end{pmatrix} \begin{pmatrix} m^{\mathrm{D}}_{ee} & m^{\mathrm{D}}_{e\mu} & m^{\mathrm{D}}_{e\tau} \\ m^{\mathrm{D}}_{\mu e} & m^{\mathrm{D}}_{\mu\mu} & m^{\mathrm{D}}_{\mu\tau} \\ m^{\mathrm{D}}_{\tau e} & m^{\mathrm{D}}_{\tau\mu} & m^{\mathrm{D}}_{\tau\tau} \end{pmatrix} \begin{pmatrix} \nu_{eR} \\ \nu_{\mu R} \\ \nu_{\tau R} \end{pmatrix} + \mathrm{H.c.}$$

$L_e,\ L_\mu,\ L_\tau$ are not conserved

Neutrino Mass

(2) Majorana Mass

- **Some basics for understanding Majorana mass**

Charge conjugate :

$$C = i\gamma^2\gamma^0 = \begin{pmatrix} i\sigma_2 & 0 \\ 0 & -i\sigma_2 \end{pmatrix}$$

$$(C^+C = 1, \quad C^T = -C)$$

$$C(\gamma^\mu)^T C^{-1} = -\gamma^\mu, \quad C(\gamma^5)^T C^{-1} = \gamma^5$$

Neutrino Mass

(2) Majorana Mass

- **Some basics for understanding Majorana mass**

$$\psi^c = C\bar{\psi}^T, \quad (C = i\gamma^2\gamma^0)$$

$$\overline{\psi^c} = \overline{C\bar{\psi}^T} = (C\bar{\psi}^T)^\dagger\gamma^0 = -\psi^T C^{-1}$$

- **Exercise : Prove that**

$$(\psi_R)^c = (\psi^c)_L$$

$$(\psi_L)^c = (\psi^c)_R$$

Neutrino Mass

(2) Majorana Mass

- Condition for Majorana neutrino : $\boldsymbol{\psi = \psi^c}$

- Let $\psi_L = \begin{pmatrix} \varphi \\ 0 \end{pmatrix}$

$$(\psi_L)^C = C\gamma^0(\psi_L^*) = i\gamma^2\psi_L^* = \begin{pmatrix} 0 \\ -i\sigma^2\varphi^* \end{pmatrix}$$

- We need only φ to describe a Majorana neutrino.

$$\psi_M = \psi_L + (\psi_L)^C = \begin{pmatrix} \varphi \\ -i\sigma^2\varphi^* \end{pmatrix} = \psi_M^C$$

Neutrino Mass

(2) Majorana Mass

- Note that in the same representation, a Dirac fermion can be written as

$$\psi_D = \begin{pmatrix} \varphi \\ -i\sigma^2\chi^* \end{pmatrix}, \quad \varphi \neq \chi,$$

- If $\varphi = \chi$, it is a Majorana fermion

- For spinors φ, χ $\quad (\bar{\chi}\varphi)^\dagger = \varphi^\dagger\gamma^{0\dagger}\chi = \bar{\varphi}\chi$

- For Majorana spinors $\quad \bar{\chi}\varphi = \bar{\varphi}\chi$

Neutrino Mass

(2) Majorana Mass

- **Using**

$$(\psi_R)^c = (\psi^c)_L$$

$$(\psi_L)^c = (\psi^c)_R$$

- **Then, a Majorana fermion can be written**

$$\psi_M = \begin{cases} \psi_L + (\psi_L)^c = \psi_L + (\psi^c)_R \\ \psi_R + (\psi_R)^c = \psi_R + (\psi^c)_L \end{cases}$$

Neutrino Mass

(2) Majorana Mass

- **Majorana mass term :**

$$
\begin{aligned}
L^M_{\text{mass}} &= -\frac{1}{2}\overline{\psi_M} M \psi_M + h.c. \\
&= -\frac{1}{2}\left(\bar{\psi}_L + \overline{(\psi^c)}_R\right) M \left(\psi_L + (\psi^c)_R\right) + h.c. \\
&= -\frac{1}{2}\left(\bar{\psi}_L M (\psi^c)_R + \overline{(\psi^c)}_R M \psi_L\right) + h.c. \\
&= -\frac{1}{2}\left(\bar{\psi}_L M (\psi_L)^c + \overline{(\psi^c)}_L M \psi_L\right) + h.c.
\end{aligned}
$$

- **Prove :** $\bar{\psi}_L M (\psi_L)^c = \overline{(\psi^c)}_L M^T \psi_L$ ⇨ $M = M^T$

Neutrino Mass

(2) Majorana Mass

- Majorana mass term is not invariant under

$$\psi \ \rightarrow \ e^{i\alpha}\psi$$

- So, lepton number is not conserved.

$$\psi_L \rightarrow e^{+i\alpha}\psi_L,$$
$$\psi_R \rightarrow e^{(\cdots)}\psi_R$$

$$\bar{\psi}_L M \left(\psi_L\right)^c \rightarrow e^{-2i\alpha}\bar{\psi}_L M \left(\psi_L\right)^c$$

Neutrino Mass

(2) Majorana Mass

- Diagonalization of Majorana mass matrix :
- In general, $$X^\dagger M Y = M_D, \rightarrow X^\dagger M M^\dagger X = M_D^2$$
- Since $M = M^T$ $$Y^T M X^* = M_D, \rightarrow Y^T M M^\dagger Y^* = M_D^2$$
- Therefore $$X M_D^2 X^\dagger = Y^* M_D^2 Y^T$$ $$(Y^T X) M_D^2 = M_D^2 (Y^T X)$$
- Then, $(Y^T X)$ must be digonal
- Define $(Y^T X) \equiv P^2$ being diagonal, and $XP^* \equiv U$

$$U^+ M U^* = M_D$$

Adding Right-Handed Neutrino

- For ν & $\bar{\nu}$

forbidden by weak isospin

$I_3 = \pm\frac{1}{2}$	:	ν_L	|	$\bar{\nu}_R$
$I_3 = 0$	:	ν_R	|	$\bar{\nu}_L$

Dirac mass

allowed but unprotected : Majorana mass
(i.e. can be large : *L* violation)

- Mass term : $\nu_L \leftrightarrow \bar{\nu}_R$ implies $I_3 = 1, \ \ Y = -2$,
  so we need a new scalar with $I_3 = 1, \ \ Y = 2$
  → SU(2) triplet scalar

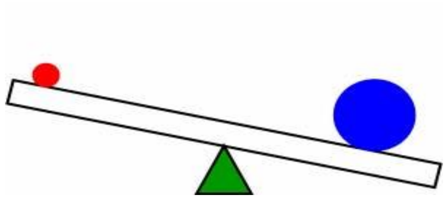
Adding Right-Handed Neutrino

- Putting possible mass terms together

$$\overline{(\nu_L, \bar{\nu}_R)} \begin{bmatrix} 0 & m_D \\ m_D^T & M \end{bmatrix} \begin{pmatrix} \nu_R \\ \bar{\nu}_L \end{pmatrix}$$

- Assuming $M \gg m_D$, diagonalization of the mass matrix →

$$-\frac{m_D^2}{M}$$



M

- Type-I Seesaw Mechanism

tiny neutrino mass : $-m_D M^{-1} m_D^T$

Type –I Seesaw Mechanism

- Scales : no guide, but
  - m_D : electroweak scale
  - M : L violation scale $\leftrightarrow$ embedding into GUT

- To obtain $m_\nu \sim \sqrt{\Delta m^2_{atm}}$, $m_D \sim 100$ GeV, we need
  $M \sim 10^{15}$ GeV → GUT scale !

- This seesaw idea was originally mentioned in a paper's footnote : PLB59 (1975) 256 by Fritzsch, Gell-Mann & Minkowski

Type –I Seesaw Mechanism

- This idea was clearly elaborated by Minkowski in his paper, PLB67 (1977) 421

- But, today, the following papers are mostly cited :

  Minkowski (1977), Yanagida (1979),

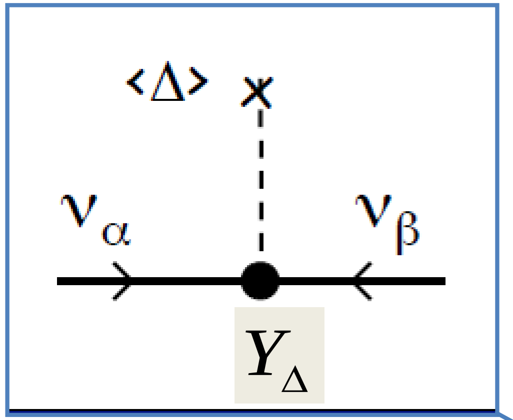
  Gell-Mann, Ramond & Slanski (1979)

  Glashow (1979), Mohapatra & Senjanovic (1980), ….

Type –II Seesaw Mechanism

(Higgs Triplet Mechanism)

Konetschny, Kummer PLB70(1977), Magg PLB94(1980), Schechter, Valle PRD22(1980), Mohapatra, Senjanovic PRL44(1981), Lazzarides,Shafi, Wetterich NPB181(1981)



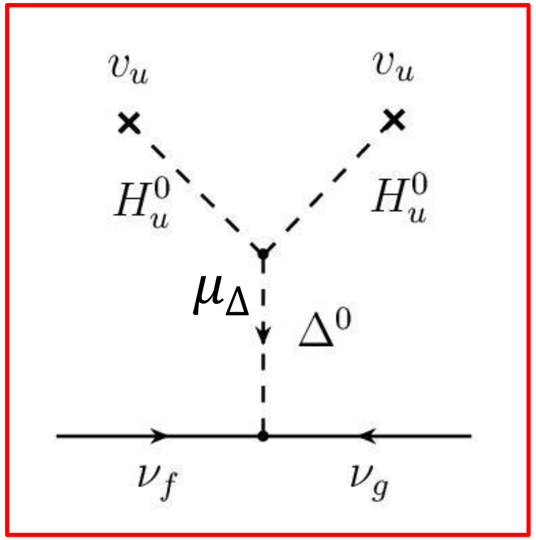
No RH neutrinos

Higgs triplet: $(\Delta^{++}, \Delta^{+}, \Delta^{0})$

EW precision measurements:

$< \Delta^0 > / < H > \; < 0.03$

$$m_\nu = Y_\Delta < \Delta >$$



$$\langle\Delta\rangle = \mu_\Delta \frac{v_{H_u}^2}{M_\Delta^2}$$

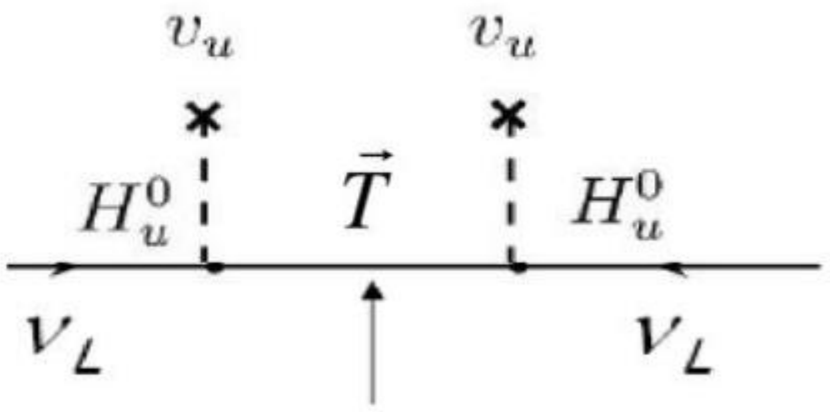
Breaking of *B*-*L*

Type –III Seesaw Mechanism

(Fermion Triplet Mechanism)

LR. Foot, H. Lew, X.-G. He and G.C. Joshi, Z. Phys. C44 (1989)

SU(2) fermion triplet with Y=0



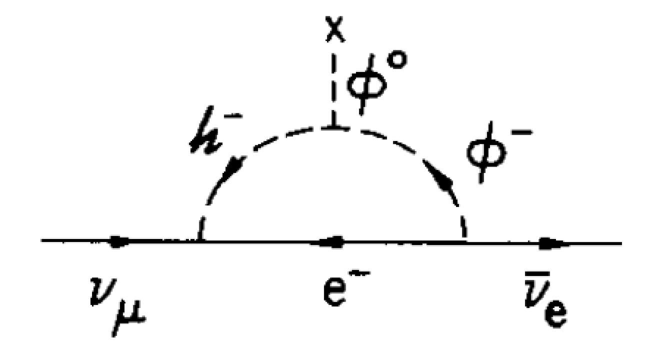
$$L = Y_\nu \overline{L}\vec{\sigma} H_u \vec{T} + M\vec{T}\cdot\vec{T} + h.c.$$

$$m_\nu = -Y_\nu^2 \frac{v_{H_u}^2}{M}$$

same formular as in type-I seesaw

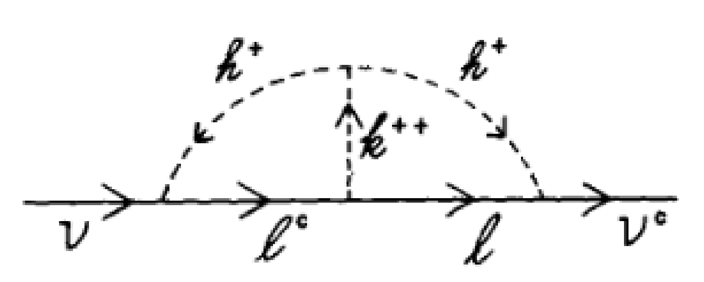
Radiative Generation

- 1-loop generation of light neutrino masses (Zee, 1980)



- SM be extended to include h^-(SU(2) singlet) & 2nd scalar doublet $(\phi^0,\ \phi^-)$

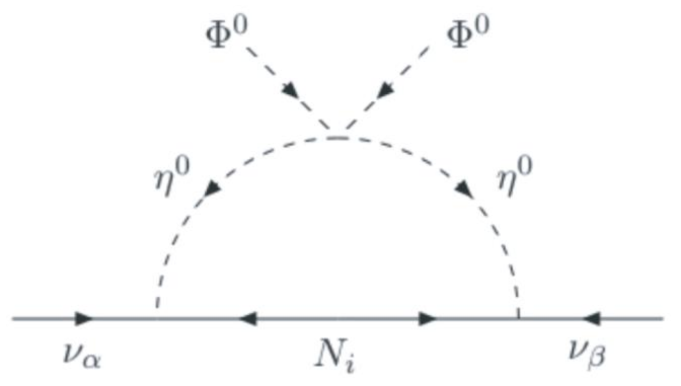
- 2-loop generation of light neutrino masses (Zee, 1986; Babu, 1988)



- SM be extended to include h^+(SU(2) singlet)& k^{++}(SU(2) singlet)

Radiative Generation

- 1-loop generation of light neutrino masses ("scotogenic model")



- SM be extended to include $3\nu_R$ & 2nd scalar doublet (η^+, η^0) : → odd under Z_2 (E.Ma,2006)

- In the flavor basis where the charged lepton mass matrix is real and diagonal, neutrino mass matrix becomes

$$(m_\nu)_{\alpha\beta} = \sum_i \frac{\Delta m^2_{\eta_i}}{16\pi^2} \frac{(\tilde{Y}_\nu)_{\alpha i}(\tilde{Y}_\nu)_{\beta i}}{M_i} f\left(\frac{M_i^2}{\bar{m}^2_{\eta_i}}\right),$$

$$z_i = M_i^2/\bar{m}^2_{\eta_i}.$$

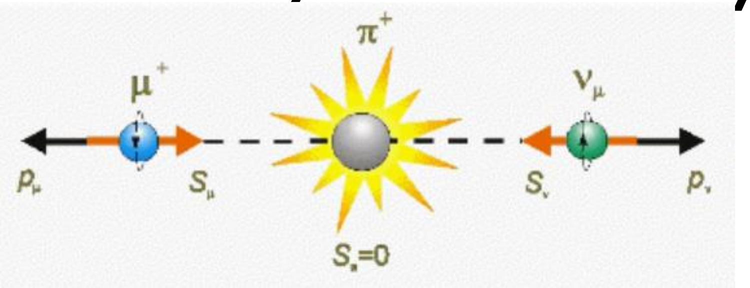
$$f(z_i) = \frac{z_i}{1-z_i}\left[1 + \frac{z_i \ln z_i}{1-z_i}\right], \quad \Delta m^2_{\eta_i} \equiv |m^2_{R_i} - m^2_{I_i}| = 4v^2\lambda_3^{\Phi\eta},$$

$$m^2_{R_i(I_i)} = \bar{m}^2_{\eta_i} \pm \Delta m^2_{\eta_i}/2$$

Determination of Neutrino Mass

Direct measurement

- Neutrino mass can be measured from decay kinematics.
- The simplest case is 2-body at-rest-decay kinematics of $\pi \rightarrow \mu\nu_\mu$



- Using $m_\nu^2 = m_\pi^2 + m_\mu^2 - \sqrt{4m_\pi^2(|\vec{p}_\mu|^2 + m_\mu^2)}$, we can obtain m_ν

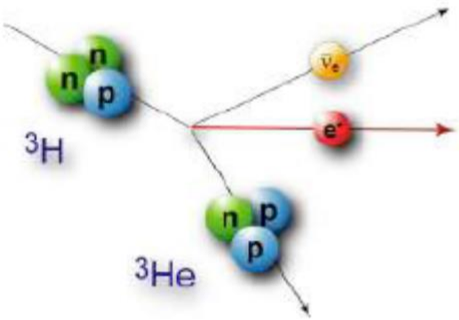
- But, it would hard to extract it from this method due to uncertainties in measuring m_π, m_μ and measurement of $|\vec{p}_\mu|$.

Determination of Neutrino Mass

Direct measurement

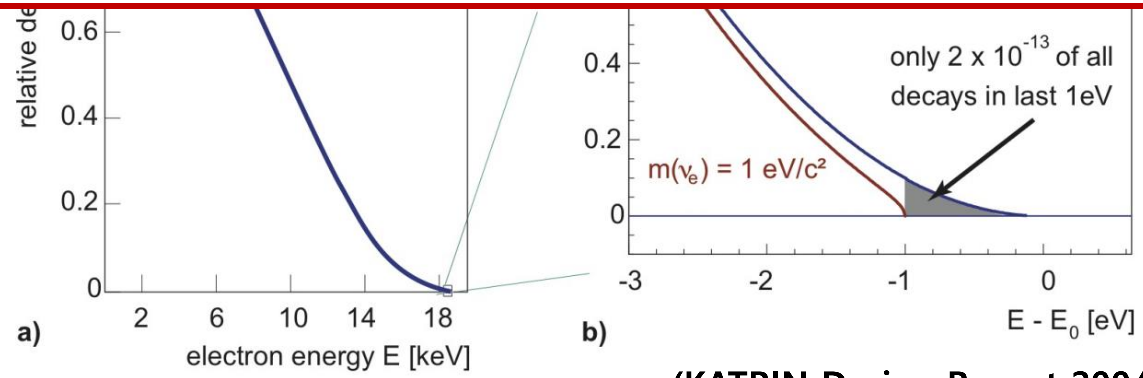
Using $E^2 = p^2c^2 + m^2c^4$, $m^2(\nu)$ can be extracted by endpoint spectrum of β-decay

$$m^2(\nu_e) = \sum |U_{ei}^2| m^2(\nu_i)$$



Tritium β-decay : $^3H \rightarrow {}^3He^+ + e^- + \bar{\nu}_e$ (E_0 = 18.6 keV)

$$\frac{dN}{dE} \propto p(E + m_e c^2)(E_0 - E)\sqrt{(E_0 - E)^2 - m_{\bar{\nu}_e}^{2(\mathrm{eff})} c^4}$$



(KATRIN Design Report 2004)

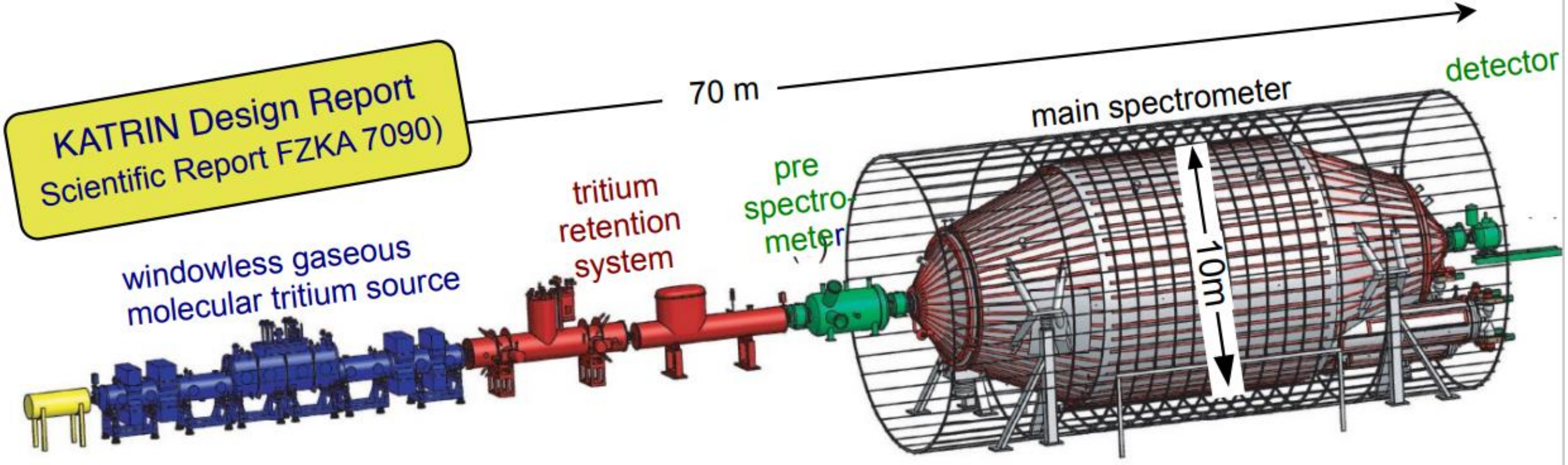
To observe modification of endpoint spectrum, we need
- **very high E resolution**
- **very high luminosity**
- **Very low background**

Determination of Neutrino Mass

KATRIN experiment

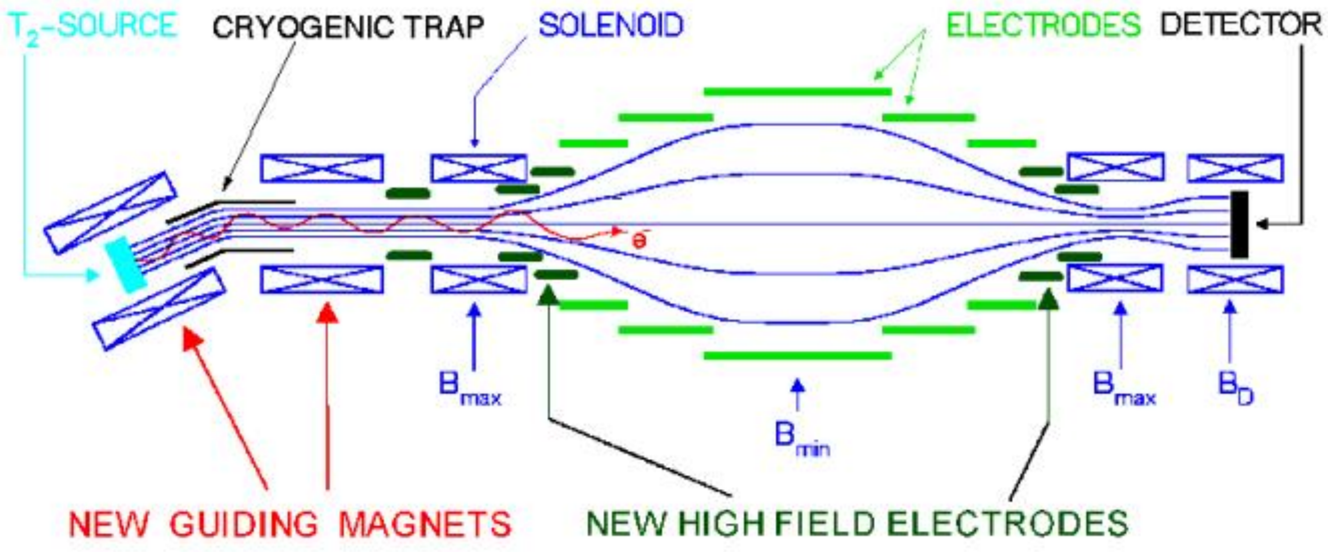
Aim : achieving $m(\nu_e)$ sensitivity of 0.2 eV
(current upper limit : 0.7 eV (2021))

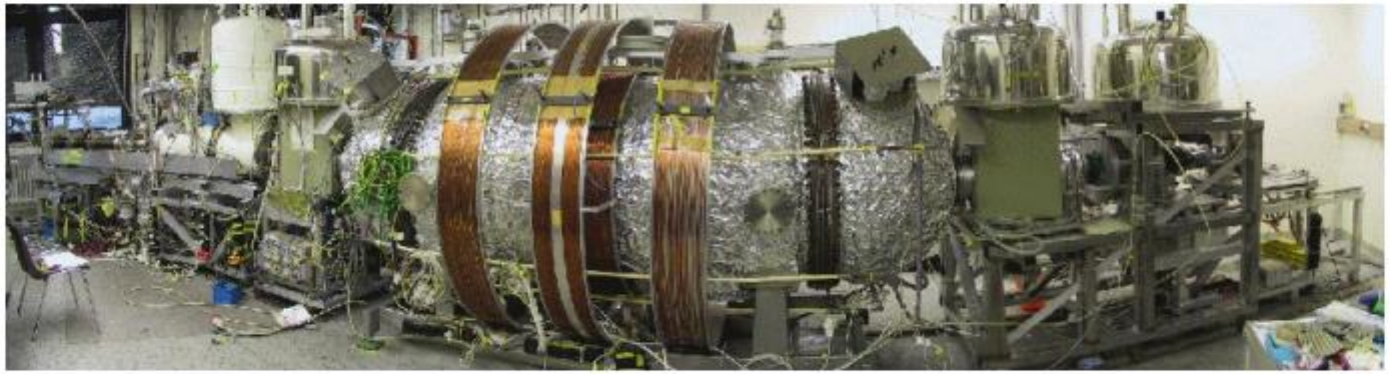
discovery potential:
m_ν = 0.3eV (3σ)
m_ν = 0.35eV (5σ)



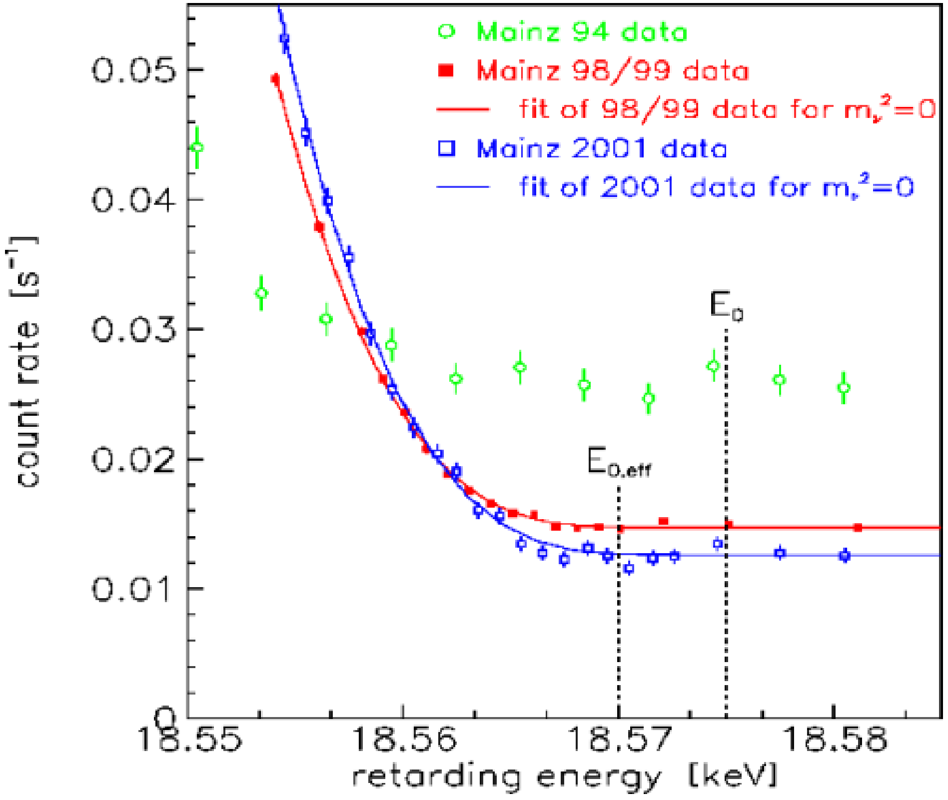
Determination of Neutrino Mass

Mainz Neutrino Mass Experiment





Final Results from phase II



$m^2(\nu) = -0.6 \pm 2.2 \pm 2.1\ eV^2 \Rightarrow m(\nu) < 2.3\ eV$ (95% C.L.)

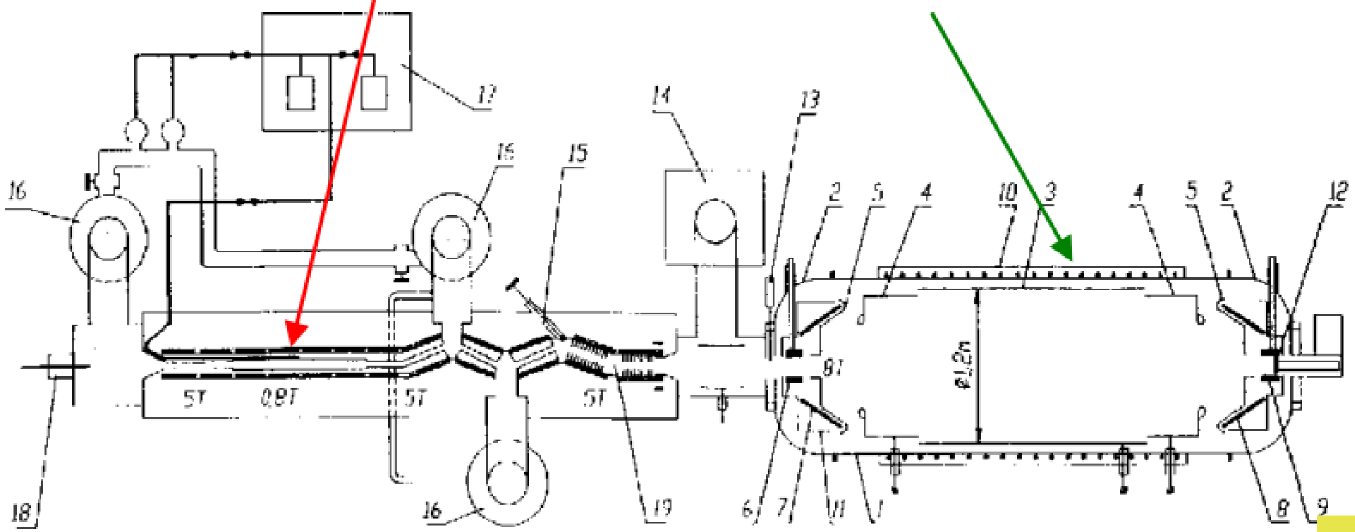
C. Kraus et al., Eur. Phys. J. C 40 (2005) 447

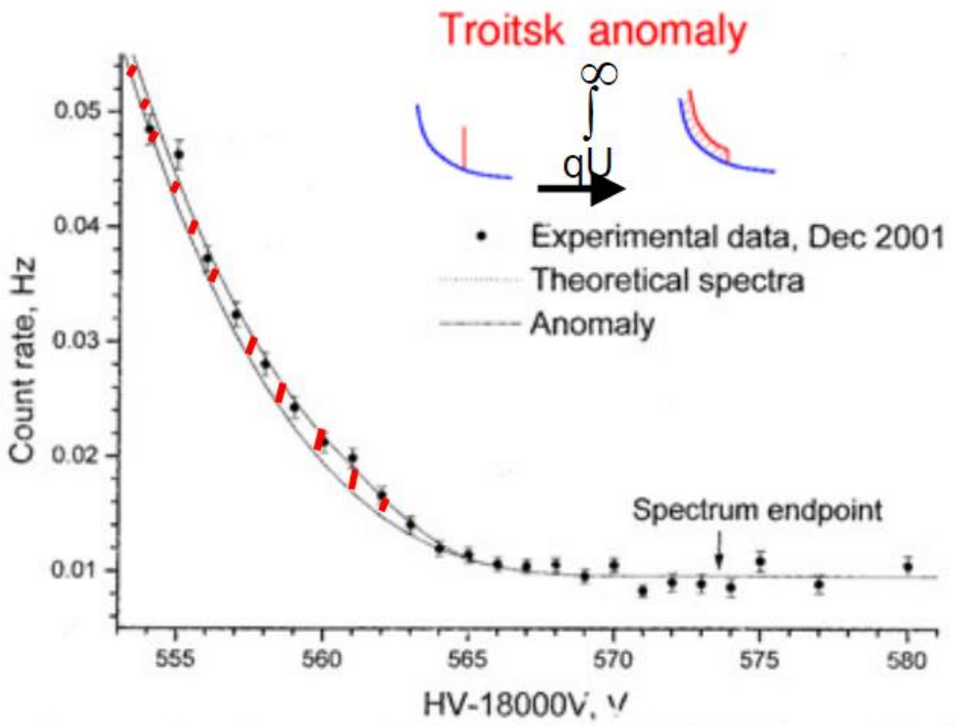
Determination of Neutrino Mass

Troitsk Neutrino Mass Experiment

windowless gaseous T_2 source, similar to LANL

MAC-E-Filter, similar to Mainz



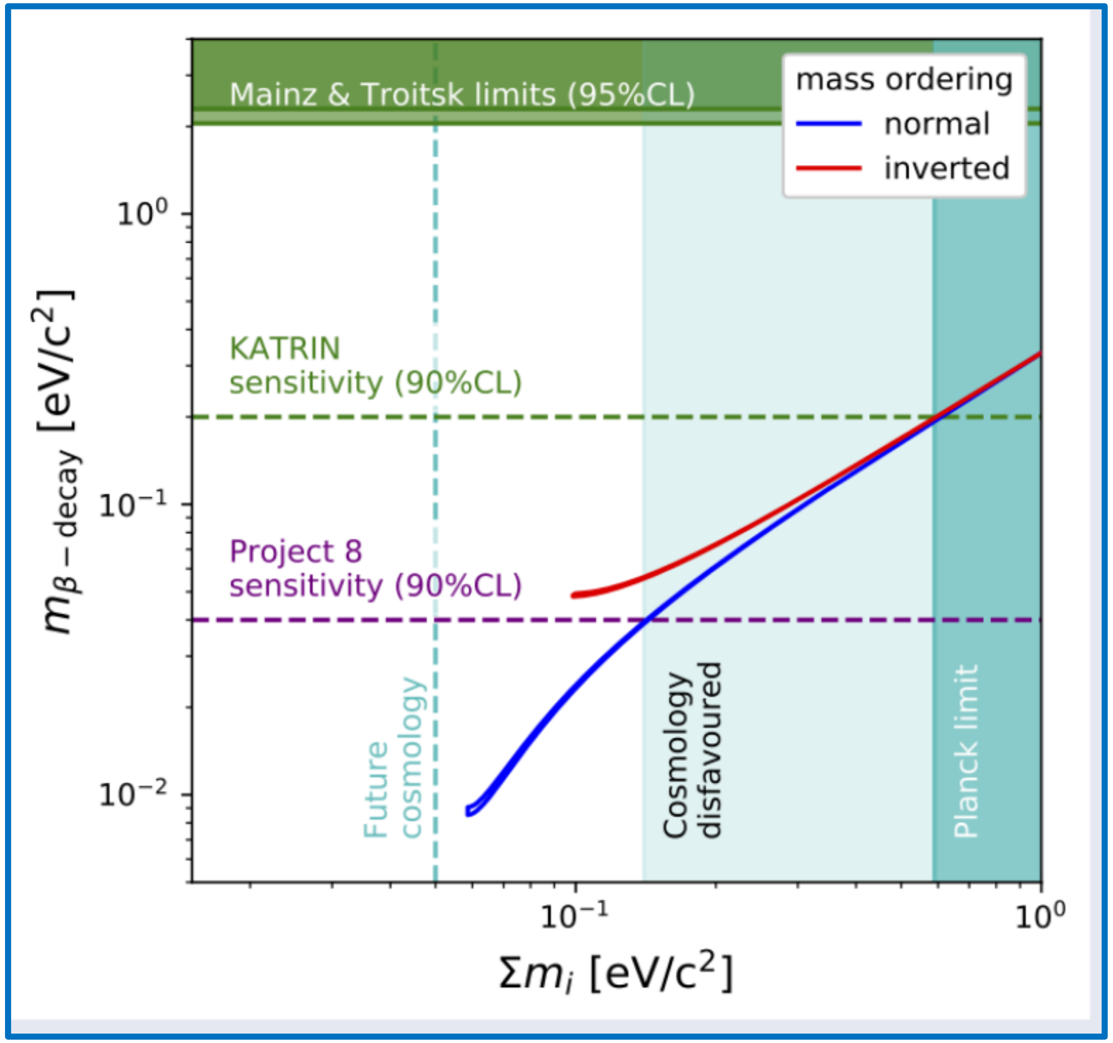


Luminosity: $L = 0.6\text{cm}^2$
($L = \Delta\Omega/2\pi * A_{source}$)

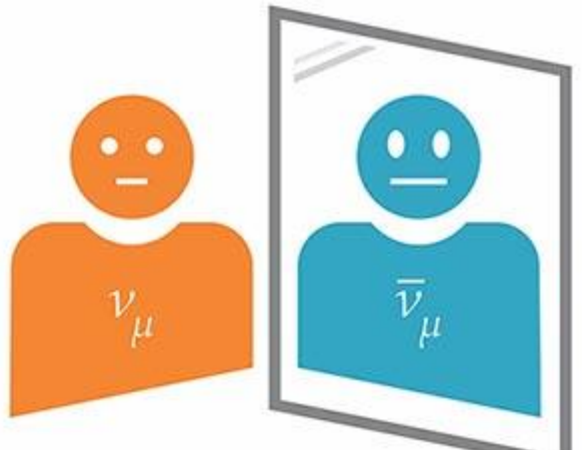
Energy resolution: $\Delta E = 3.5\text{eV}$
3 electrode system in 1.5m diameter UHV vessel ($p<10^{-9}$ mbar)

Re-analysis of Troitsk data
(better source thickness, better run selection)
Aseev et al, Phys. Rev. D 84, 112003 (2011)
$m_\beta < 2.2$ eV, 95% CL

Determination of Neutrino Mass

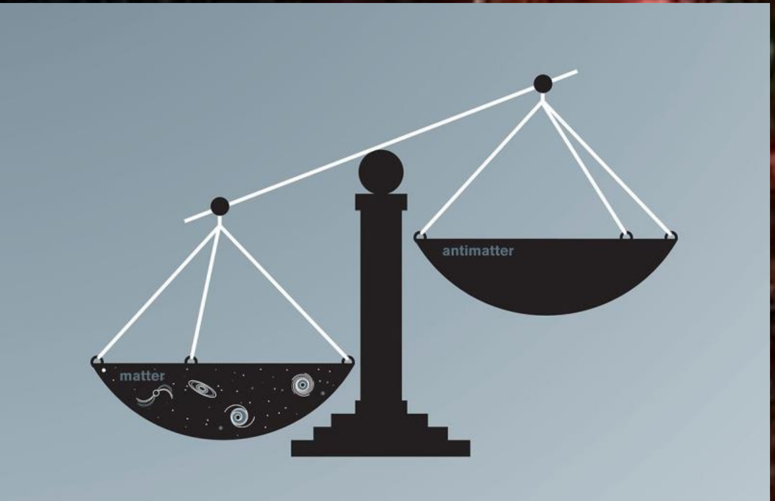


Project -8 experiment : Sensitivity of 0.04 eV



Donna Padian

CP Violation



SPITZER
SPACE TELESCOPE

CP violation in neutrino Oscillation

- C : charge conjugate

$$\psi^c = C\bar{\psi}^T, \quad (C = i\gamma^2\gamma^0)$$

$$\overline{\psi^c} = \overline{C\bar{\psi}^T} = (C\bar{\psi}^T)^\dagger\gamma^0 = -\psi^T C^{-1}$$

- P: parity

$$\psi \to \psi'(x') = S_P\psi(x) = \gamma^0\psi(x)$$

$$\bar{\psi} \to \bar{\psi}' = \bar{\psi}S_P^{-1}$$

$$a_P = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & -1 & 0 & 0 \\ 0 & 0 & -1 & 0 \\ 0 & 0 & 0 & -1 \end{bmatrix}$$

$$P^{-1}\gamma^\mu P = a^\mu{}_\nu\gamma^\nu$$

$$P^{-1}\gamma^0 P = \gamma^0$$

$$P^{-1}\gamma^i P = -\gamma^i$$

Note: $\gamma^0\gamma^0\gamma^0 = \gamma^0$

$\gamma^0\gamma^i\gamma^0 = -\gamma^i$

$$P : \psi_L \to \gamma^0\psi_L = \gamma^0\tfrac{1}{2}(1-\gamma_5)\psi = \tfrac{1}{2}(1+\gamma_5)\gamma^0\psi = (\psi')_R$$

C	$\Longrightarrow$	Particle $\leftrightarrows$ Antiparticle
P	$\Longrightarrow$	Left-Handed $\leftrightarrows$ Right-Handed

CP violation in neutrino Oscillation

- **Under CP transformation**

$$\psi_L \mapsto (\psi_L)^{\text{CP}} = i\sigma^2\psi_L^*, \ \psi_R \mapsto (\psi_R)^{\text{CP}} = -i\sigma^2\psi_R^*,$$

Leptonic Jarskog invariant: $\text{Im}\left[U_{\alpha k}^* U_{\beta k} U_{\alpha j} U_{\beta j}^*\right] = \pm J$

$$J = c_{12}s_{12}c_{23}s_{23}c_{13}^2 s_{13}\sin\delta_{13}$$

Under CP transformation :
$$U \rightarrow U^*$$
$$J \rightarrow -J$$

CP violation in neutrino Oscillation

- **Oscillation probability**

$$P_{\nu_\alpha\to\nu_\beta} = \sum_{i=1}^{3}\left|U_{\alpha i}^{*}U_{\beta i}\right|^2 + 2\sum_{i<j}^{3} Re\left(U_{\alpha i}^{*}U_{\beta i}U_{\alpha j}U_{\beta j}^{*}\right)\cos\frac{\Delta m_{ji}^2 L}{2E} - 2\sum_{i<j}^{3} Im\left(U_{\alpha i}^{*}U_{\beta i}U_{\alpha j}U_{\beta j}^{*}\right)\sin\frac{\Delta m_{ji}^2 L}{2E}$$

$$= \left|\sum_{i=1}^{3} U_{\alpha i}^{*}U_{\beta i}\right|^2 - 4\sum_{i<j}^{3} Re\left(U_{\alpha i}U_{\beta i}U_{\alpha j}^{*}U_{\beta j}^{*}\right)\sin^2\frac{\Delta m_{ji}^2 L}{2E} + 2\sum_{i<j}^{3} Im\left(U_{\alpha i}U_{\beta i}U_{\alpha j}^{*}U_{\beta j}^{*}\right)\sin\frac{\Delta m_{ji}^2 L}{2E}$$

$\left|\sum_{i=1}^{3} U_{\alpha i}^{*}U_{\beta i}\right|^2 \to \delta_{\alpha\beta}$

$Im\left(U_{\alpha i}U_{\beta i}U_{\alpha j}^{*}U_{\beta j}^{*}\right) \to = J\sum_{\gamma,k}\varepsilon_{\alpha\beta\gamma}\varepsilon_{ijk}$:

CP conserving part $: P_{\nu_\alpha\to\nu_\beta}^{CPC}$:

$$P_{\nu_\alpha\to\nu_\beta}^{CPV} = 8J\sum_{\gamma}\varepsilon_{\alpha\beta\gamma}\sin\frac{\Delta m_{21}^2 L}{4E}\sin\frac{\Delta m_{31}^2 L}{4E}\sin\frac{\Delta m_{32}^2 L}{4E}$$

$$P_{\nu_\alpha\to\nu_\beta} = P_{\nu_\alpha\to\nu_\beta}^{CPC} + P_{\nu_\alpha\to\nu_\beta}^{CPV}$$

→ **vanishing for α=β**

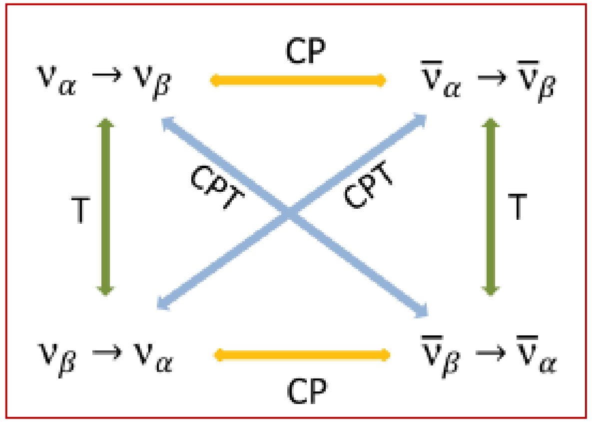
CP violation in neutrino Oscillation

- **CP transformation of oscillation probability:**

$$P_{\nu_\alpha \to \nu_\beta} \to P_{\bar{\nu}_\alpha \to \bar{\nu}_\beta} \qquad P_{\bar{\nu}_\alpha \to \bar{\nu}_\beta} = P^{CPC}_{\nu_\alpha \to \nu_\beta} - P^{CPV}_{\nu_\alpha \to \nu_\beta}$$

- **CPT invariance :** $P_{\nu_\alpha \to \nu_\beta} = P_{\bar{\nu}_\beta \to \bar{\nu}_\alpha}$

- **CP violation shows up a difference between $P_{\nu_\alpha \to \nu_\beta}$ and $P_{\bar{\nu}_\alpha \to \bar{\nu}_\beta}$**



$$A^{CP}_{\alpha\beta} = \frac{P_{\nu_\alpha \to \nu_\beta} - P_{\bar{\nu}_\alpha \to \bar{\nu}_\beta}}{P_{\nu_\alpha \to \nu_\beta} + P_{\bar{\nu}_\alpha \to \bar{\nu}_\beta}} = \frac{P^{CPV}_{\nu_\alpha \to \nu_\beta}}{P^{CPC}_{\nu_\alpha \to \nu_\beta}} \qquad (\alpha \neq \beta)$$

CP violation in neutrino Oscillation

- **CP asymmetry**

$$A_{\alpha\beta}^{CP} = \frac{2\sin 2\theta_{12}\sin\frac{\Delta m_{21}^2 L}{4E}\sin 2\theta_{23}\sin^2\frac{\Delta m_{31}^2 L}{4E}\sin 2\theta_{13}c_{13}\sin\delta}{P_{\nu_\alpha\to\nu_\beta}^{CPC}}$$

$$A_{\mu\tau}^{CP} \sim \sin 2\theta_{13}\frac{\Delta m_{21}^2 L}{E}$$: suppressed $\left(P_{\nu_\mu\to\nu_\tau}^{CPC} \sim c_{13}^4\sin^2 2\theta_{23}\sin^2\frac{\Delta m_{31}^2 L}{4E} \right)$

- **Detections of ν_e and ν_μ are far easier than ν_τ, so the golden channel for CP asymmetry is $A_{\mu e}^{CP}$**

CP violation in neutrino Oscillation

- Golden Channel for CP asymmetry

$$P_{\nu_\mu \to \nu_e} = \sin^2\theta_{23}\,\sin^2 2\theta_{13}\sin^2\frac{\Delta_{31}}{2}$$
$$+c_{13}^2\left(c_{23}^2\sin^2 2\theta_{12} + 4s_{13}^2 s_{23}^2 s_{12}^4 - 2s_{13}s_{12}^2\sin 2\theta_{12}\sin 2\theta_{23}\cos\delta\right)\sin^2\frac{\Delta_{21}}{2}$$
$$+c_{13}^2\left(s_{13}\sin 2\theta_{12}\sin 2\theta_{23}\cos\delta - 4s_{13}^2 s_{12}^2 s_{23}^2\right)\sin 2\frac{\Delta_{31}}{2}\sin\frac{\Delta_{21}}{2}$$
$$+8\hat{J}\sin\frac{\Delta_{31}}{2}\sin\frac{\Delta_{21}}{2}\sin\frac{\Delta_{32}}{2}\sin\delta$$

$(\hat{J}=\cos\theta_{13}\sin 2\theta_{13}\sin 2\theta_{23}\sin 2\theta_{12}\sim 0.039,\ \Delta_{ij}\equiv \Delta m_{ij}^2 L/2E\ ,\ c_{ij}=\cos\theta_{ij})$

CP violation in neutrino Oscillation

- ## Golden Channel for CP asymmetry

- To leading order in Δ_{21}

$$A_{\mu e}^{CP} \sim \frac{4\hat{J}\sin\Delta_{21}\sin\delta}{\sin^2\theta_{23}\sin^2 2\theta_{13}} \cong \frac{c_{23}\sin 2\theta_{12}}{s_{12}s_{13}}\left(\frac{\Delta m_{21}^2}{\Delta m_{31}^2}\right)\frac{\Delta m_{31}^2 L}{4E} + O\left(\Delta_{21}^2\right) \sim 0.26\left(\frac{\Delta m_{31}^2 L}{4E}\right)$$

- The asymmetry grows linearly with *L*, but for fixed detector size and neutrino energy, the flux of neutrinos decreases as ~ $1/L^2$.

- First oscillation maximum :

$$L_0 = \frac{2\pi E}{\Delta m_{31}^2} \approx 495\left(\frac{E}{\mathrm{GeV}}\right)\left(\frac{2.5\times 10^{-3}}{\Delta m_{31}^2}\right)\mathrm{km}$$

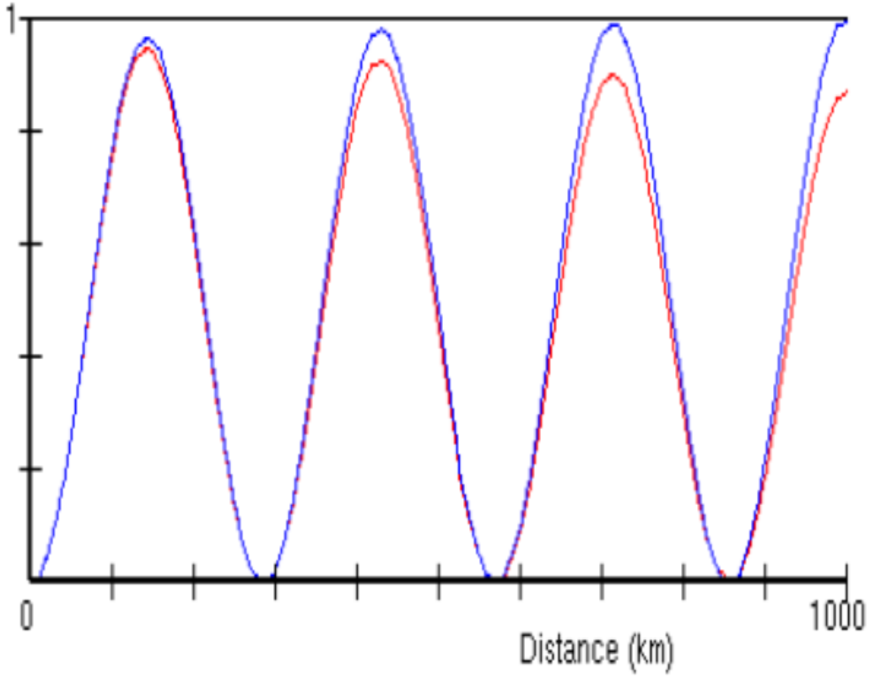
e.g.) T2K: 295 km → 0.6 GeV,

CP violation in neutrino Oscillation

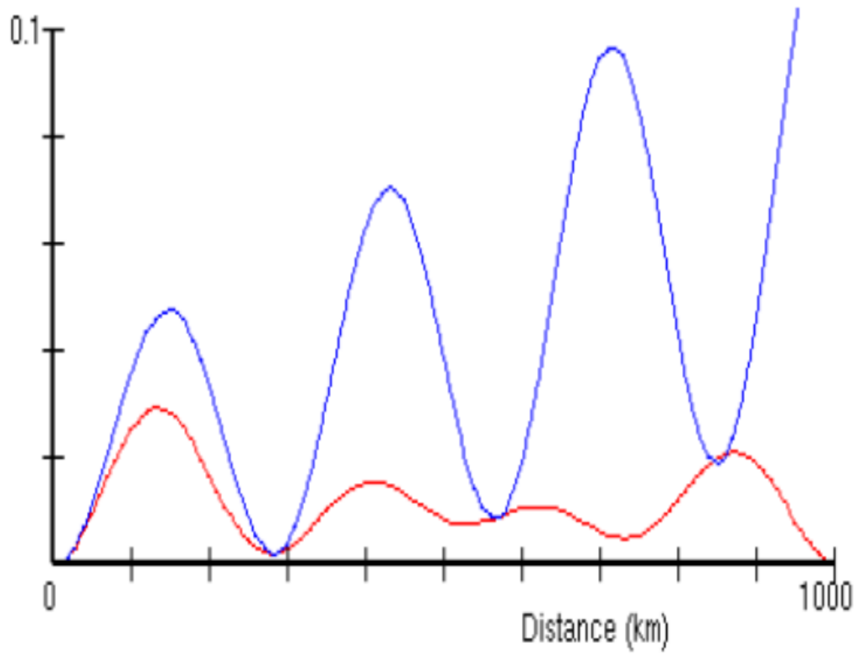
- Golden Channel for CP asymmetry

For $E = 500$ MeV, $\theta_3 = 8^o$, $\delta = 90^o$

$P_{\nu_\mu \to \nu_\tau}$ vs. $P_{\bar{\nu}_\mu \to \bar{\nu}_\tau}$



$P_{\nu_\mu \to \nu_e}$ vs. $P_{\bar{\nu}_\mu \to \bar{\nu}_e}$



CP violation in neutrino Oscillation

- **Matter Effects**

$$P_{\nu_\mu \to \nu_e} \sim \sin^2\theta_{23}\ \sin^2 2\theta_{13} \sin^2\frac{\Delta_{31}}{2}\left(1 - \frac{8a}{\Delta m_{31}^2}\cos 2\theta_{13}\right)$$

$$+ c_{13}^2\left(c_{23}^2 \sin^2 2\theta_{12} + 4 s_{13}^2 s_{23}^2 s_{12}^4 - 2 s_{13} s_{12}^2 \sin 2\theta_{12} \sin 2\theta_{23} \cos\delta\right) \sin^2\frac{\Delta_{21}}{2}$$

$$+ c_{13}^2\left(s_{13} \sin 2\theta_{12} \sin 2\theta_{23} \cos\delta - 4 s_{13}^2 s_{12}^2 s_{23}^2\right) \sin 2\frac{\Delta_{31}}{2} \sin\frac{\Delta_{21}}{2}$$

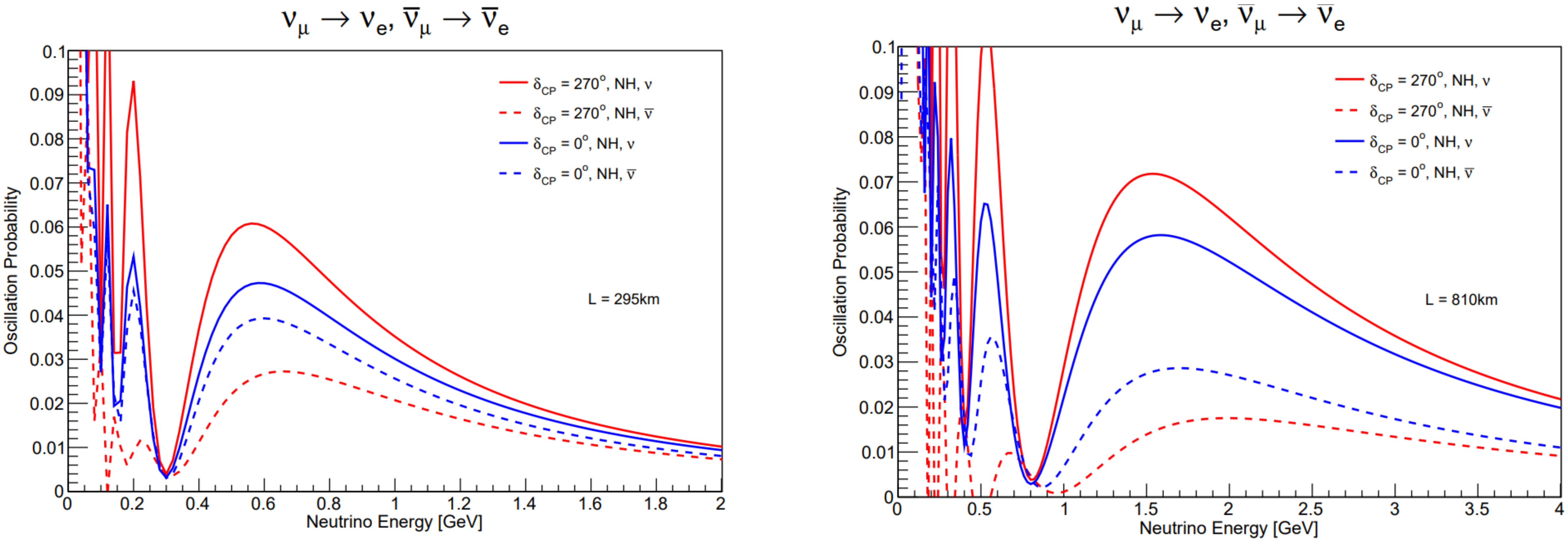
$$+ 8\hat{J} \sin\frac{\Delta_{31}}{2} \sin\frac{\Delta_{21}}{2} \sin\frac{\Delta_{32}}{2}\ \sin\delta$$

$$+ 2\cos 2\theta_{13} \sin^2 2\theta_{13}\ s_{23}^2 \left(\frac{aL}{4E}\right) \sin\frac{\Delta_{31}}{2} \cos\frac{\Delta_{32}}{2}$$

$(a[\mathrm{eV}^2] = 2\sqrt{2} G_F n_e E = 7.6 \times 10^{-3} \rho[\mathrm{g/cm3}] E[\mathrm{GeV}]$ (earth crust: ρ=2.76)

$P_{\bar{\nu}_\mu \to \bar{\nu}_e}$ ➔ $a \to -a$ (fake CPV), $\delta \to -\delta$

CP violation in neutrino Oscillation

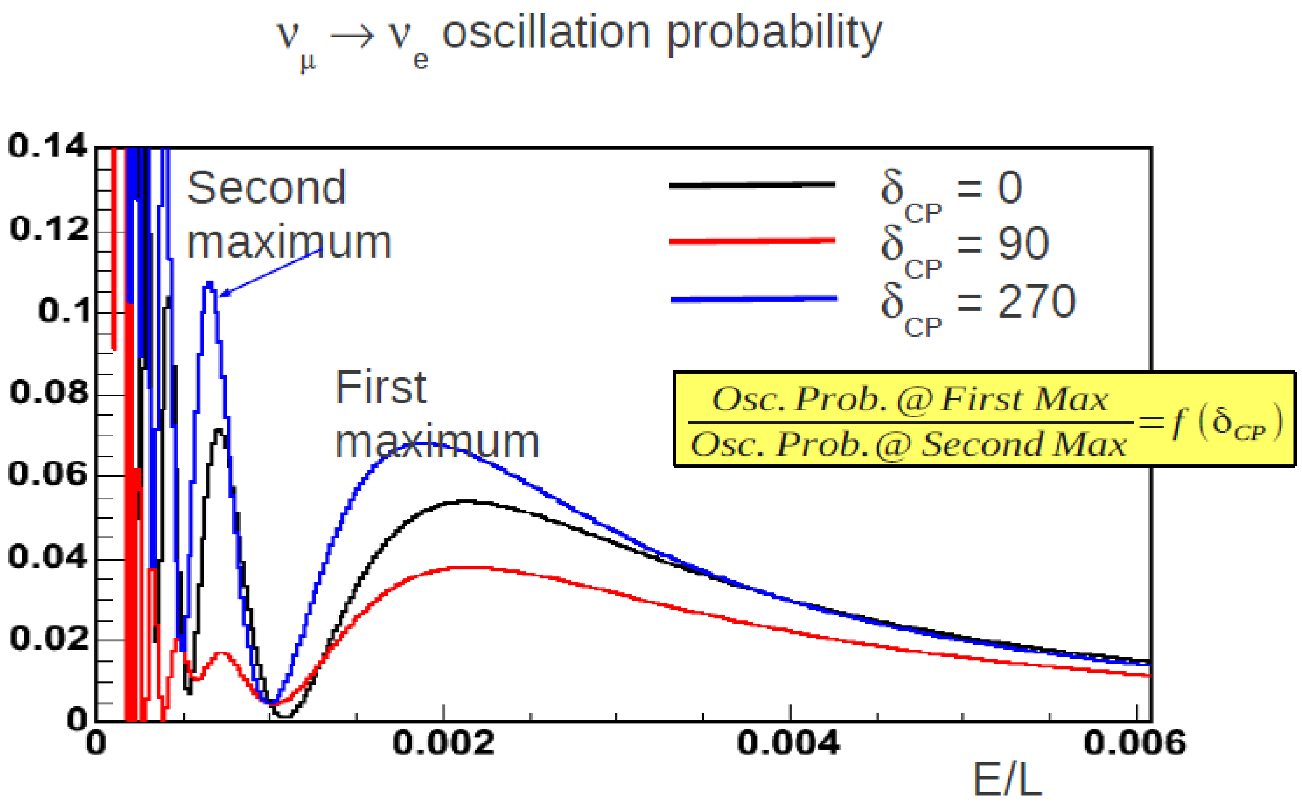


T2K

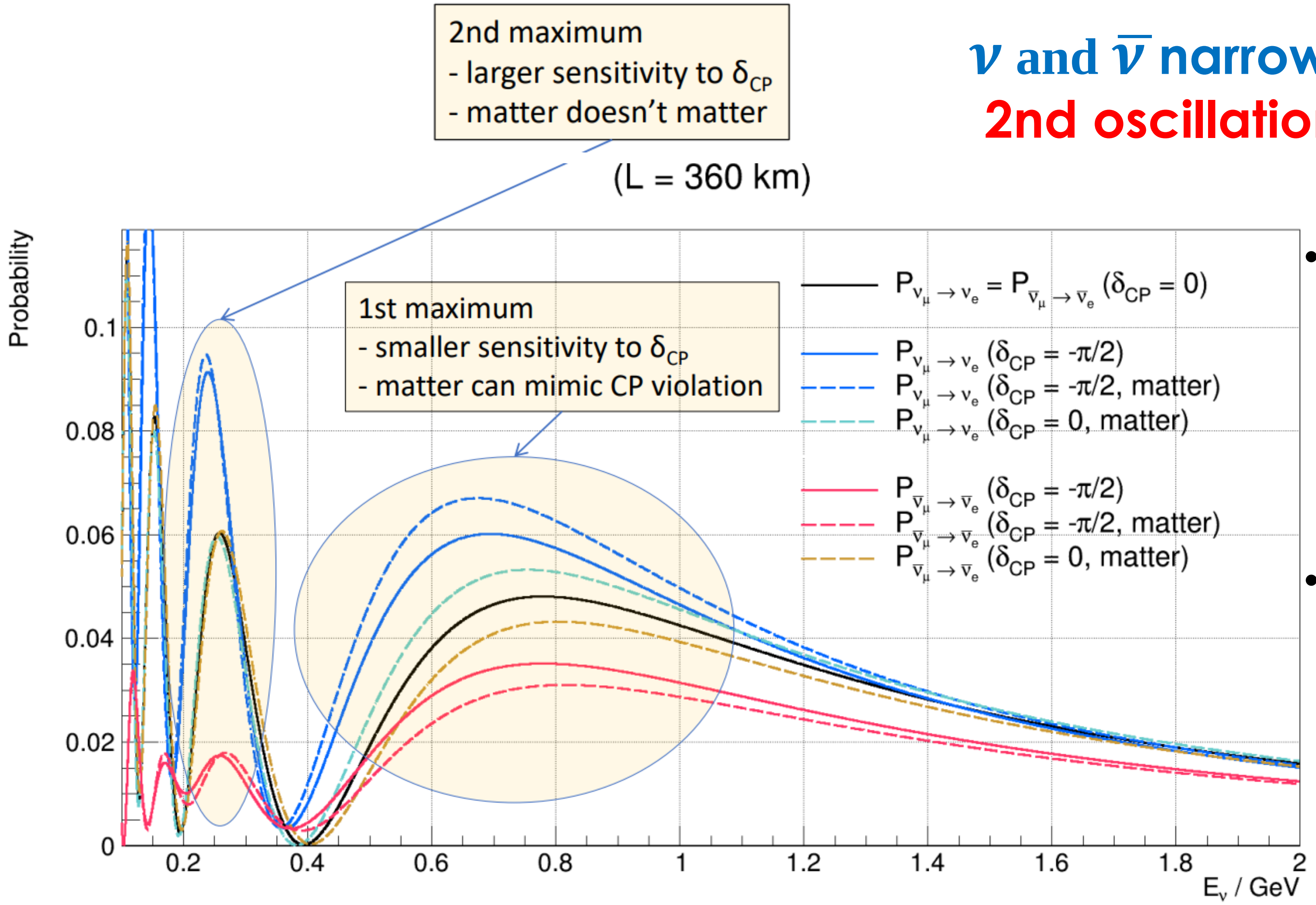
NOνA

Wide Beam

- CPV can be observed by measuring $P_{\nu_\mu \to \nu_e}$ & $P_{\bar{\nu}_\mu \to \bar{\nu}_e}$ at 1st and 2nd oscillation maxima, which are covered by wide ν ($\bar{\nu}$) beam



Narrow Beam



ν and $\bar{\nu}$ narrow beams tuned to 2nd oscillation max.

- **significantly less affected by syst. Uncertainties compared to the 1st osc. max.**
- **For this observation next generation very high intensive ν beams are needed. (ESSnuSB)**

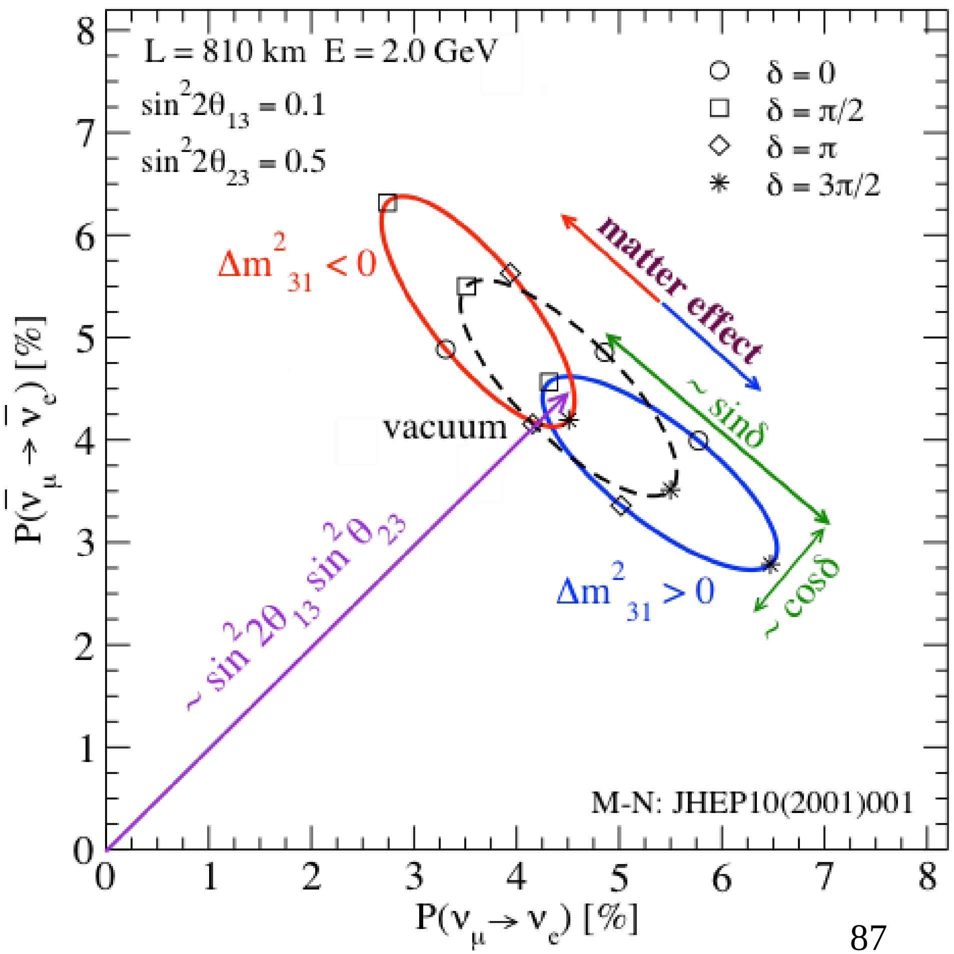
CP violation in neutrino Oscillation

Separating fake matter effects

- **Genuine CPV & the matter effect both lead to a difference between ν and $\bar{\nu}$ oscillation.**

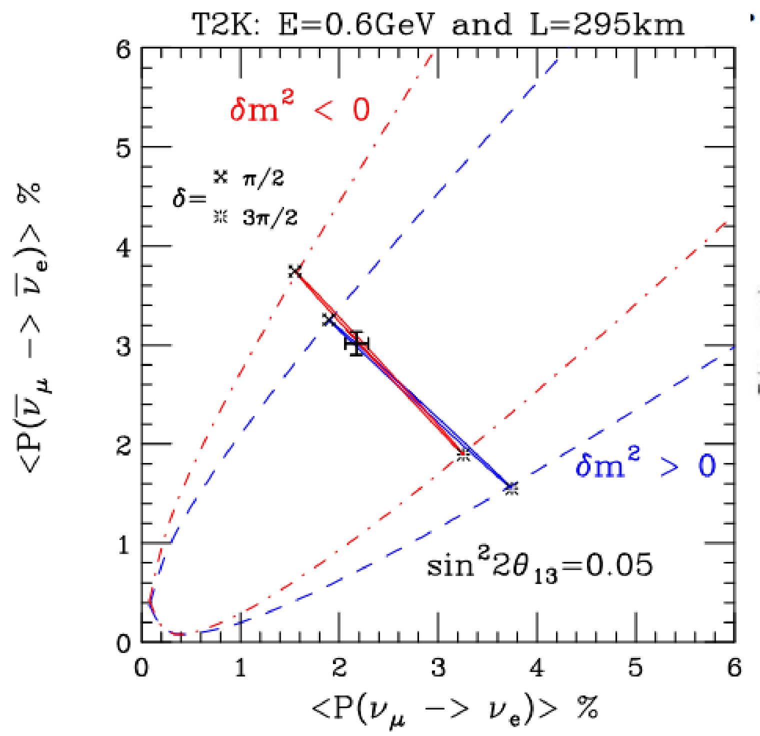
(Minakata & Nunokawa 2001)

- **trajectory in matter is shifted to 2 different directions, according to sign(Δm^2_{31})**
- **Octan of θ_{23} can be distinguishable**
- **To disentangle them, one may make oscillation measurements at different L and/or E.**



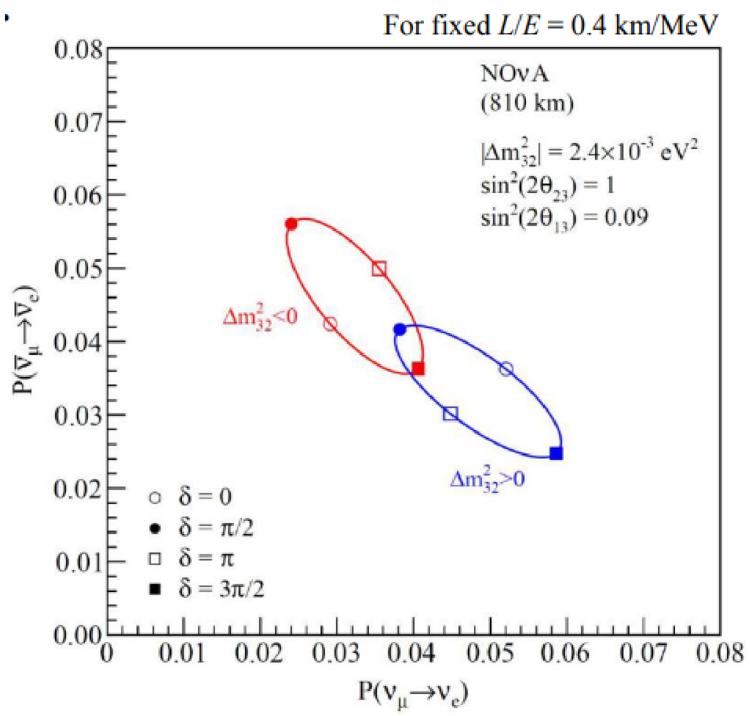
CP violation in neutrino Oscillation

T2K



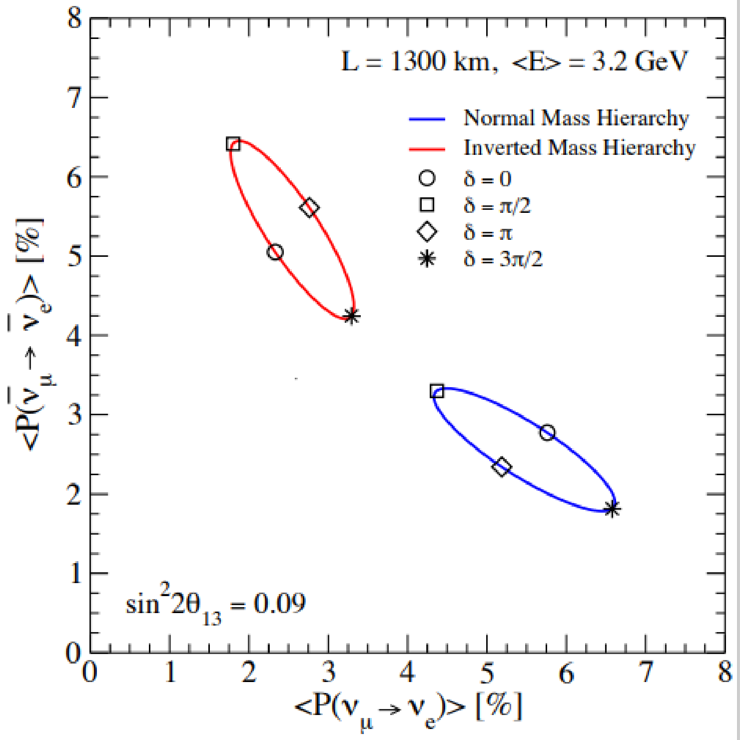
0.6 GeV

NOvA



2 GeV

DUNE



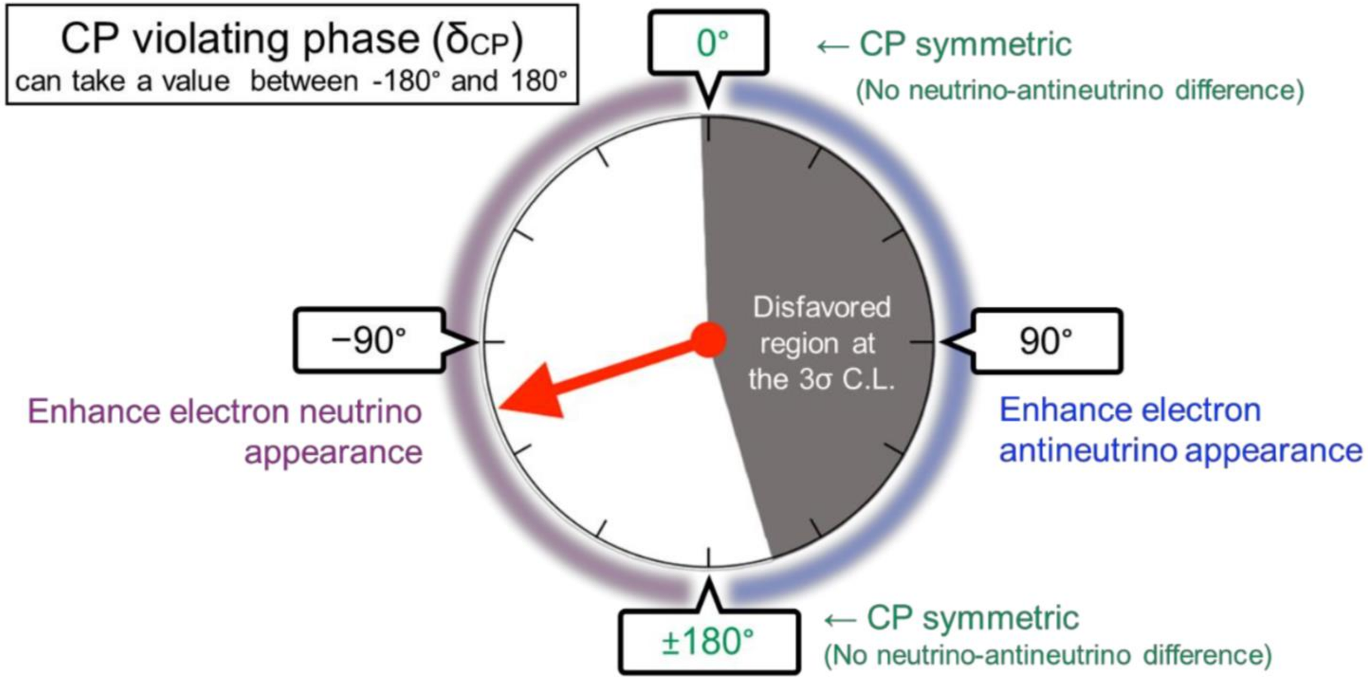
3 GeV

Increasing Energy

CP Violation : T2K result

Nature **:** April 16/4/2020
and arXiv:: 1910.03887

Determination of δ_{CP} from appearance of ν_e events



- The gray region is disfavored by 99.7% (3σ) CL
- The values 0 and 180 degrees are disfavoured at 95% CL

CP Violation : NOvA result

Determination of δ_{CP} from appearance of ν_e events

- Observed **82** events on a background prediction of **26.8**
  - Integral of total best-fit prediction is 85.8 events.

Selected ν_eCC candidates

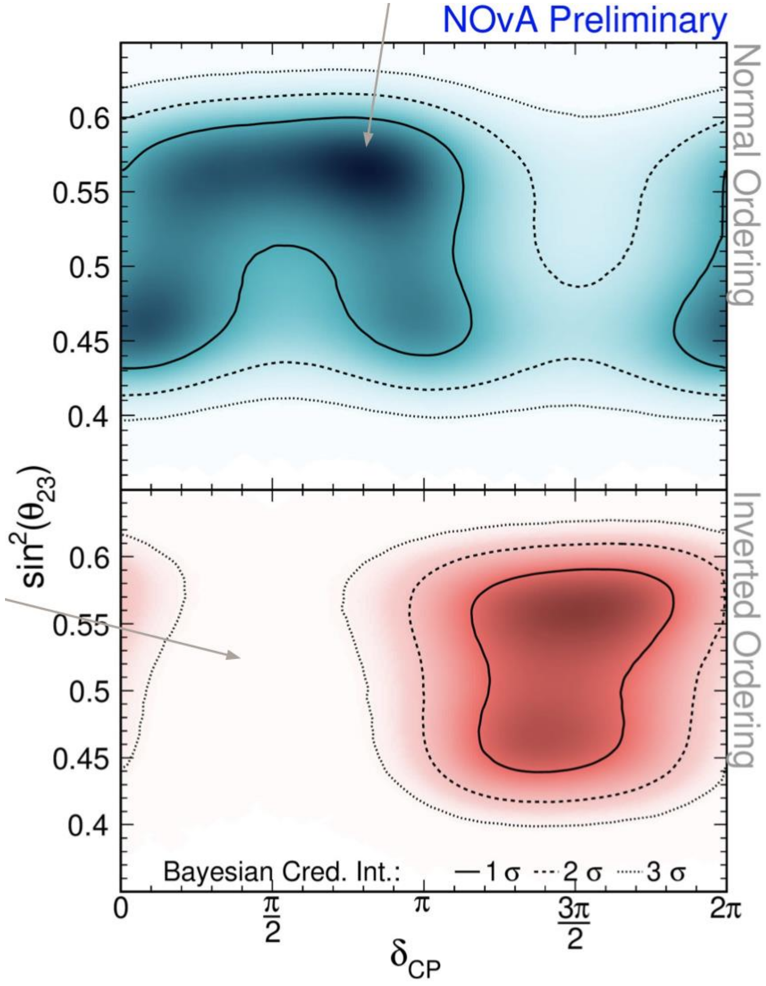
- Observed **33** events on a background prediction of **14.0**
  - Integral of total best-fit prediction is 33.2 events.

>4σ evidence of electron antineutrino appearance

- Rule out IO, $\delta = \pi/2$ region at >3σ.
- Weak preference for Normal Ordering, Upper Octant of θ_{23}.

Most Probable

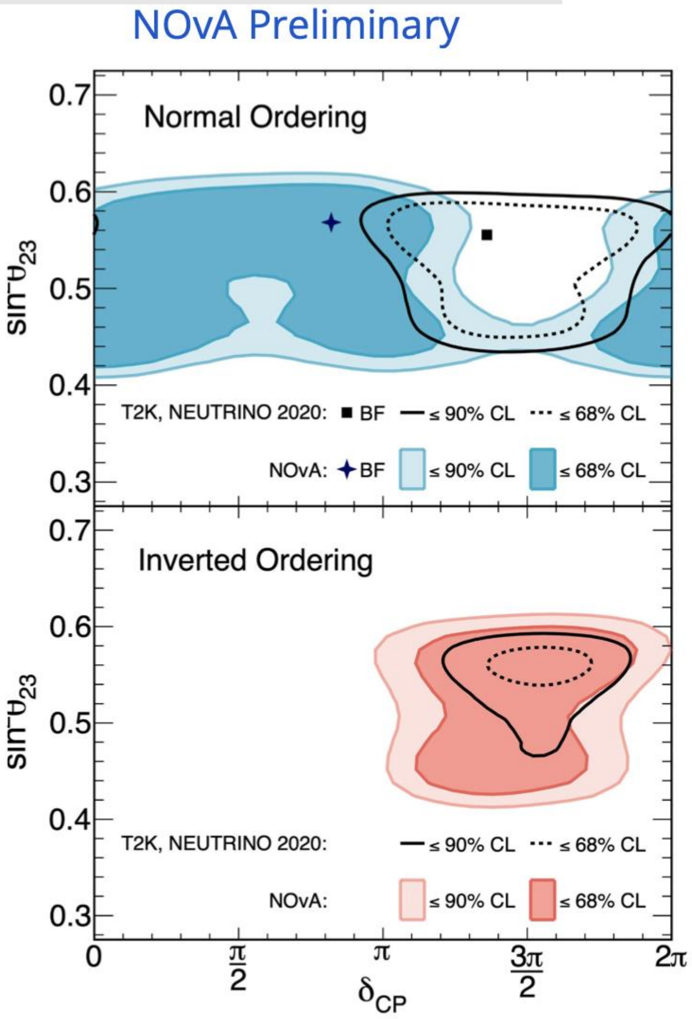
Excluded



CP Violation : NOvA result

Comparison with T2K

- Frequestist contours.
- Some tension between preferred regions for the Normal Ordering.
  - Agree on the preferred region in the Inverted Ordering.
- A joint fit of the data from the two experiments is needed to properly quantify consistency.
  - Significant progress made on a joint-fit → coming this year!

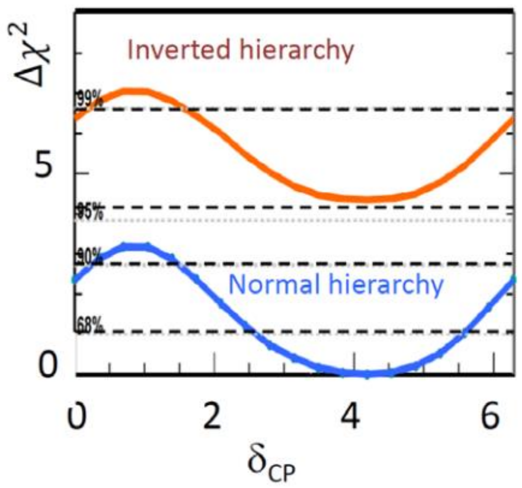


NOvA/T2K will continue to take data till 2026/2027
-> double the statistics of present analyses, reduce systematics

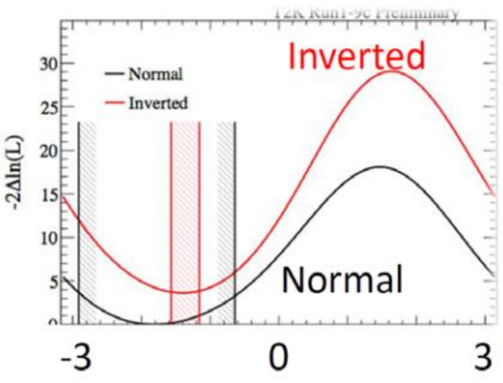
CP Violation :Future Experiments

	DUNE	Hyper-K
Baseline	1300km ➔ Large matter effect (Good for MO)	295km ➔ Small matter effect
Beam energy	~ Multi-GeV	~ Sub-GeV
Detector technology	Liq. Ar TPC	Water Cherenkov

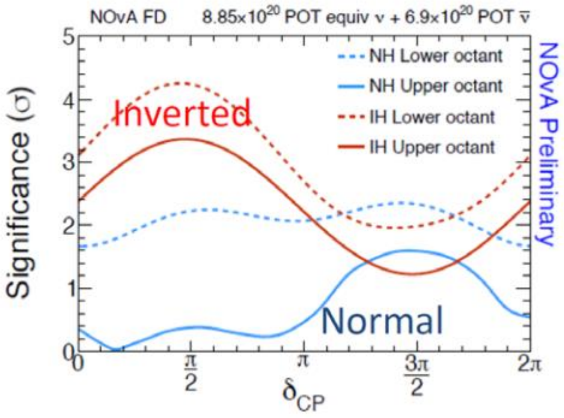
***Super-K** atmospheric (Y. Hayato)*



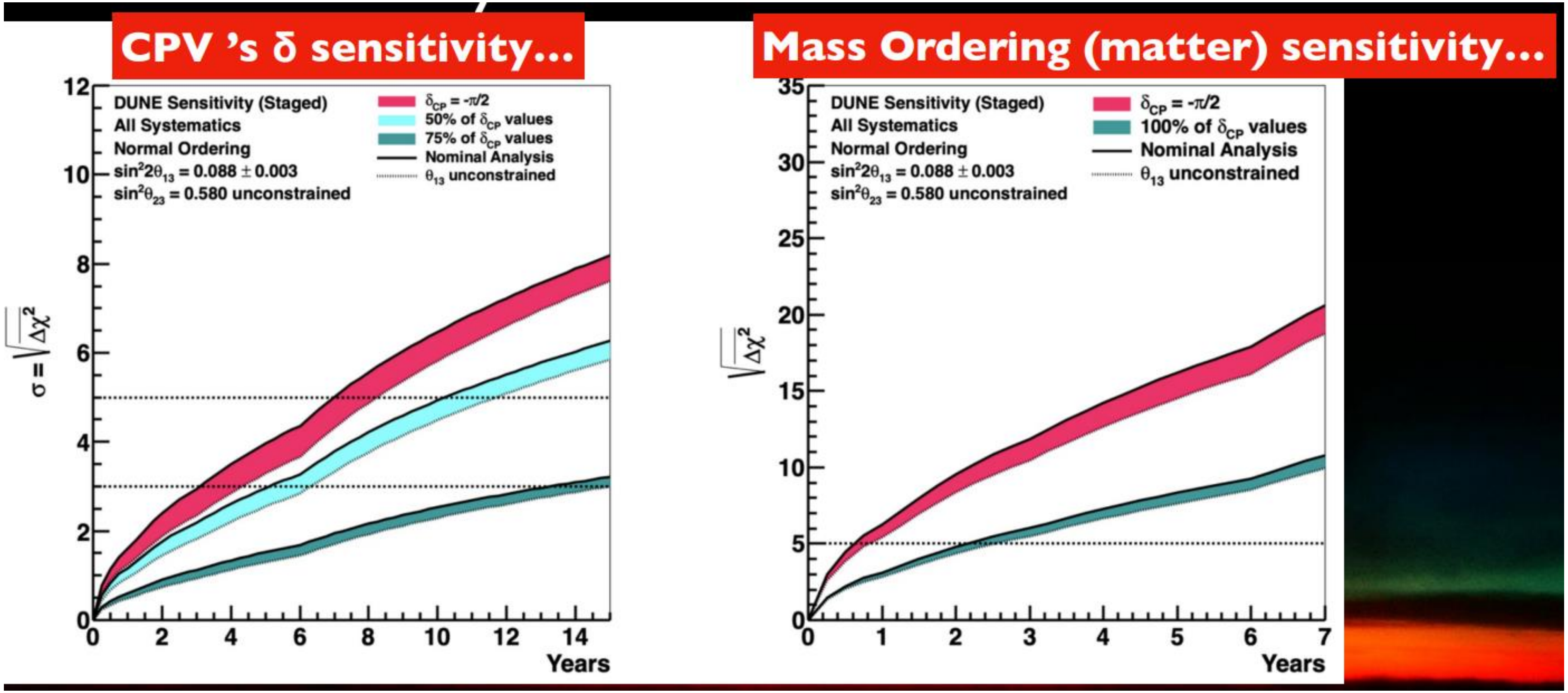
***T2K** (M. Wascko)*



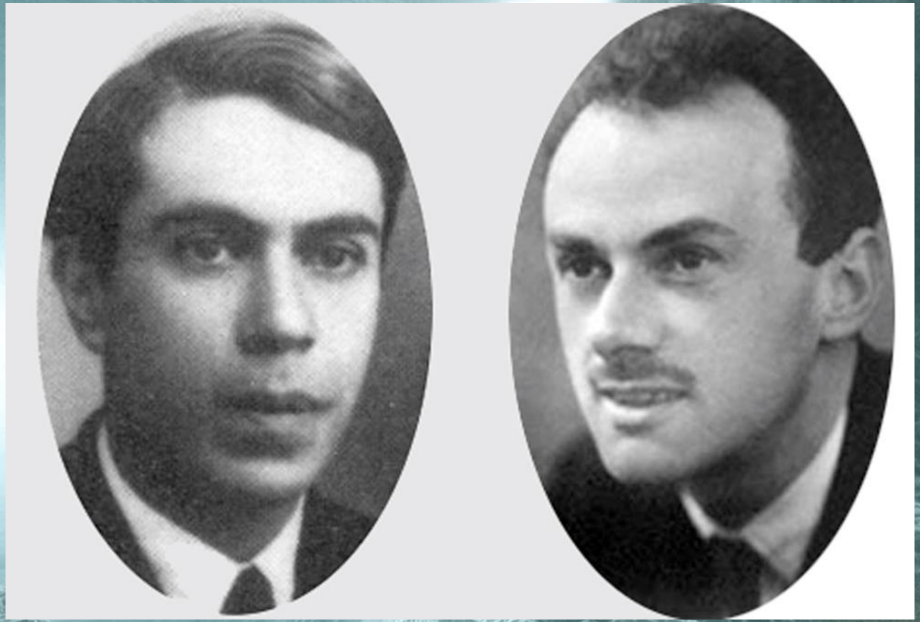
***NOvA** (M. Sanchez)*



DUNE Sensitivity



Majorana vs. Dirac



DESY, Science Communication Lab

Dirac vs. Majorana

❖ If massive neutrinos are *Majorana*, it implies *L* number is not conserved.

- L number violating phenomena :

Neutrinoless $\beta\beta$ decay : $(Z,A) \rightarrow (Z \pm 2, A) + 2e^{\pm}$ $\left(T_{1/2}^{^{136}\text{Xe}} / T_{1/2}^{total\,^{136}\text{Xe}} \sim 10^{-5}\right)$

Muon conversion : $\mu^{-} + (Z,A) \rightarrow e^{+} + (Z-2,A)$: $(\text{Br} \sim 10^{-12})$

Rare Kaon decays : $K^{+} \rightarrow \pi^{-}\mu^{+}\mu^{+}$: $(\text{Br} \sim 10^{-9})$

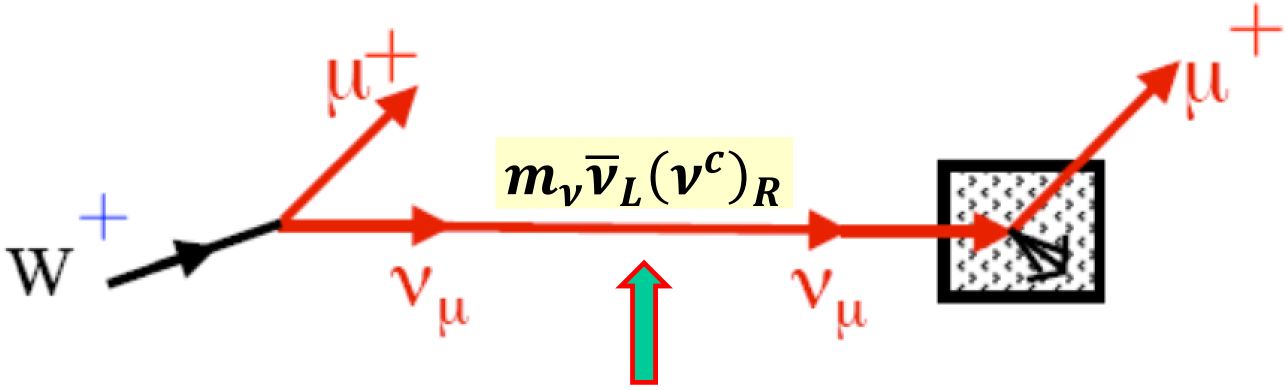
⇨ $0\nu\beta\beta$ decay dominates by a huge margin. That is so because many mols of the target can be studied for a long time, and the Avogadro number is much larger than typical beam flux.

(Vogel, 2017)

Dirac vs. Majorana

❖ **How difficult to test Majorana property**

If massive neutrinos are Majorana, the following process is allowed

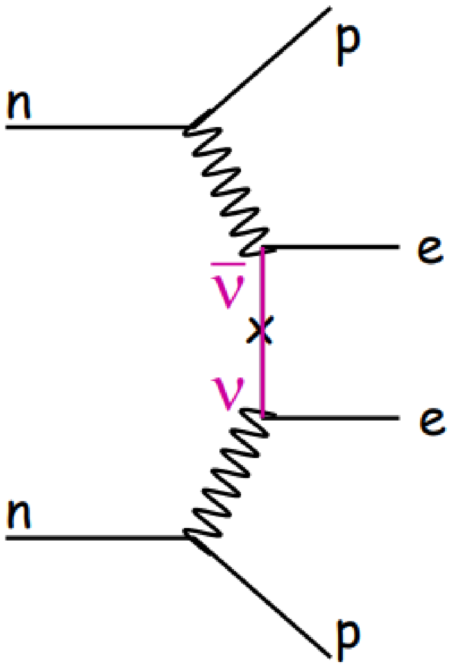


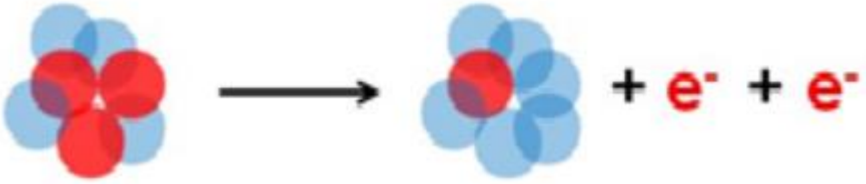
But, even rate $\propto \left(\frac{m_\nu}{E}\right)^2 \sim 10^{-20}$ suppressed.

Dirac vs. Majorana

- **Neutrinoless double beta decay (0νββ)**

- **If neutrinos are Majorana, two outgoing neutrino lines in the double β-decay diagram can be connected.**





Amplitude of 0νββ $\propto \langle m_{\beta\beta} \rangle \equiv \left| \sum m_{\nu_i} U_{ei}^2 \right|$

$$= \left| m_{\nu_1} c_{12}^2 c_{13}^2 + m_{\nu_2} s_{12}^2 c_{13}^2 e^{2i\alpha} + m_{\nu_3} s_{13}^2 e^{2i(\beta-\delta)} \right|$$

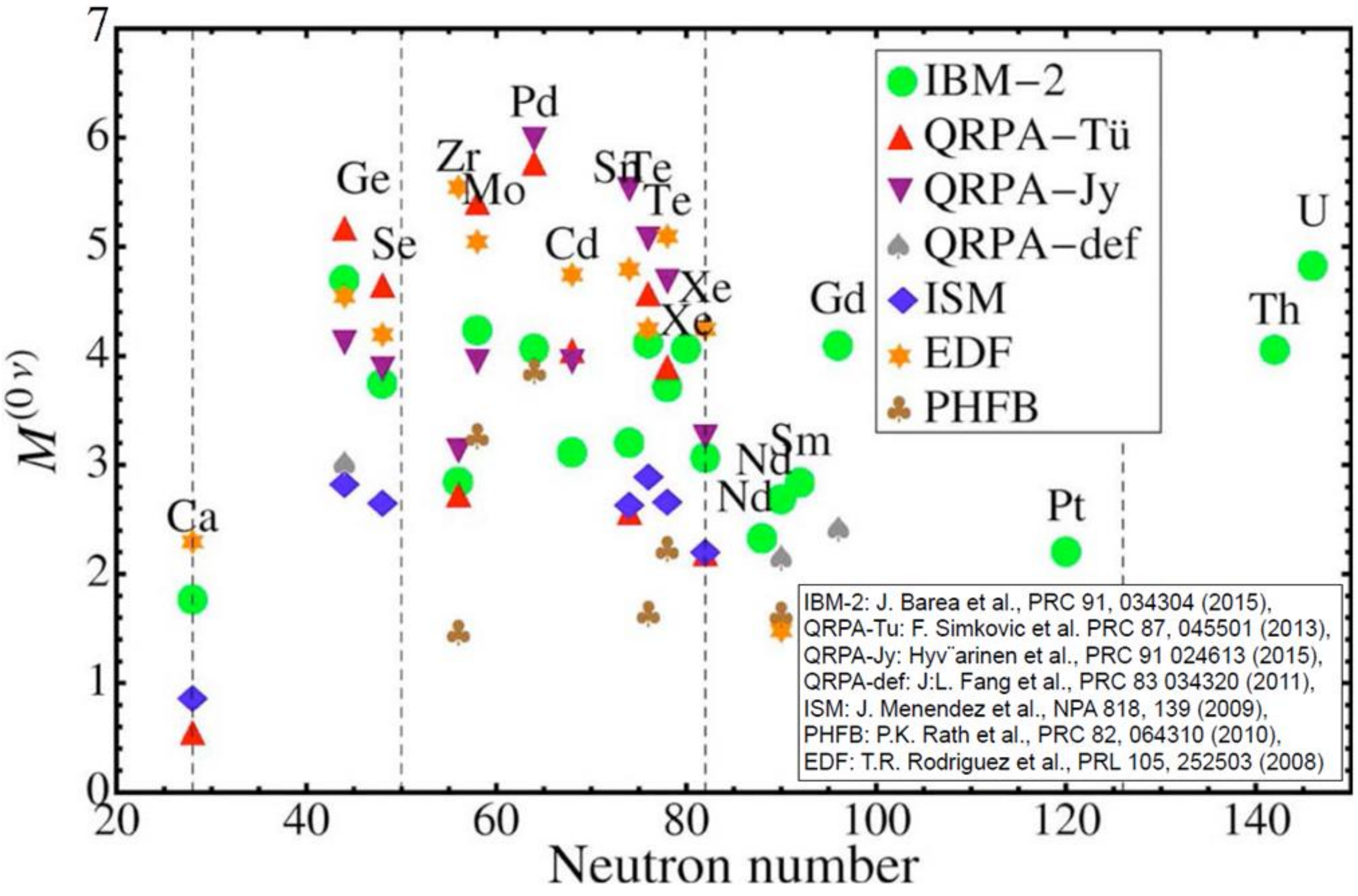
$$\Gamma_{0\nu} = 1/T_{1/2}^{0\nu} = G_{0\nu} \left| M_{0\nu} \right|^2 m_{\beta\beta}^2$$

Phase space factor ($G_{0\nu}$)

Nuclear Matrix Elements ($M_{0\nu}$)

Dirac *vs.* Majorana

- There is large uncertainty in the calculation of nuclear matrix elements



Dirac vs. Majorana

- **Neutrinoless double beta decay (0νββ)**
- **Why observation of 0νββ may indicate that neutrinos are Majorana ?**

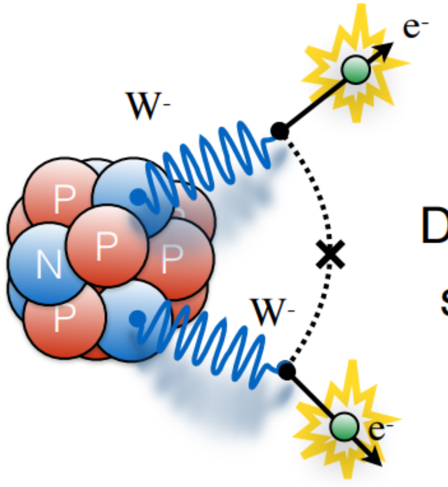
0νββ is the process $dd \rightarrow uu + e^{-}e^{-}$

Implies the amplitude for $e^{+}\bar{u}d \rightarrow e^{-}u\bar{d}$ **is not vanishing**

- **Amplitude for the chain** $\bar{\nu}_R \rightarrow e^{+}W^{-} \rightarrow e^{+}\bar{u}d \rightarrow e^{-}u\bar{d} \rightarrow e^{-}W^{+} \rightarrow \nu_L$ **is not vanishing**
- **This chain results in** $\bar{\nu}_R \rightarrow \nu_L$ **which is the effect of Majorana mass**

Dirac vs. Majorana

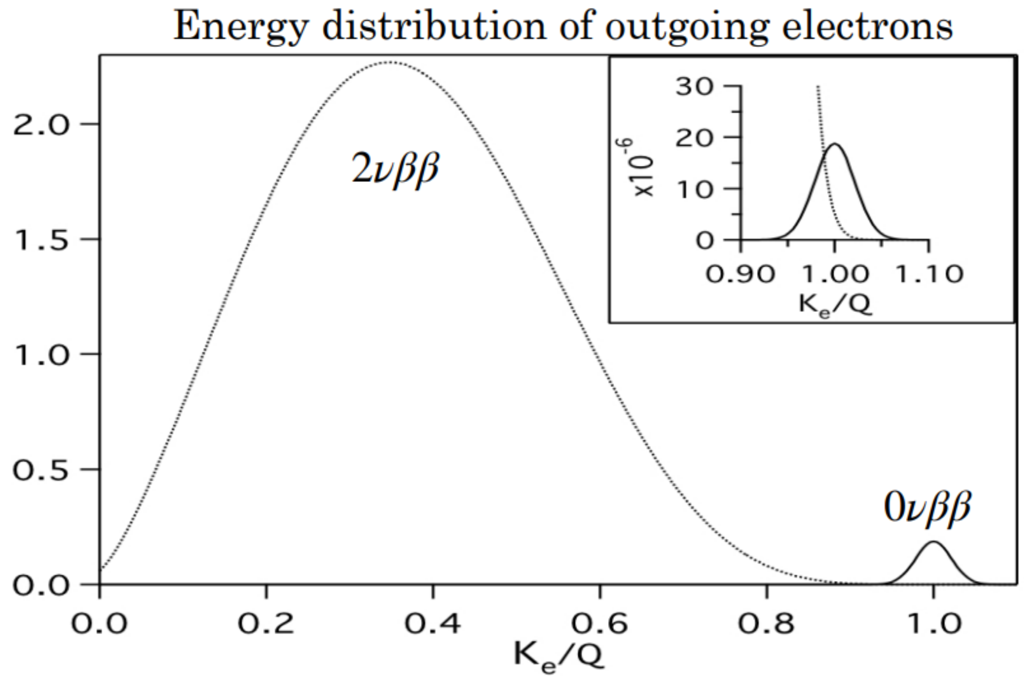
What is Observable signature of $0\nu\beta\beta$?



Decay energy ($Q_{\beta\beta}$) is shared by electrons.

Neutrinoless Double Beta Decay ($0\nu\beta\beta$)

$$(A, Z) \rightarrow (A, Z+2) + 2e^-$$



Observable is the $0\nu\beta\beta$ event rate (equivalently a half lifetime $T_{1/2}$)

Experiments

- **Several candidates of nuclei are being considered in experiments**

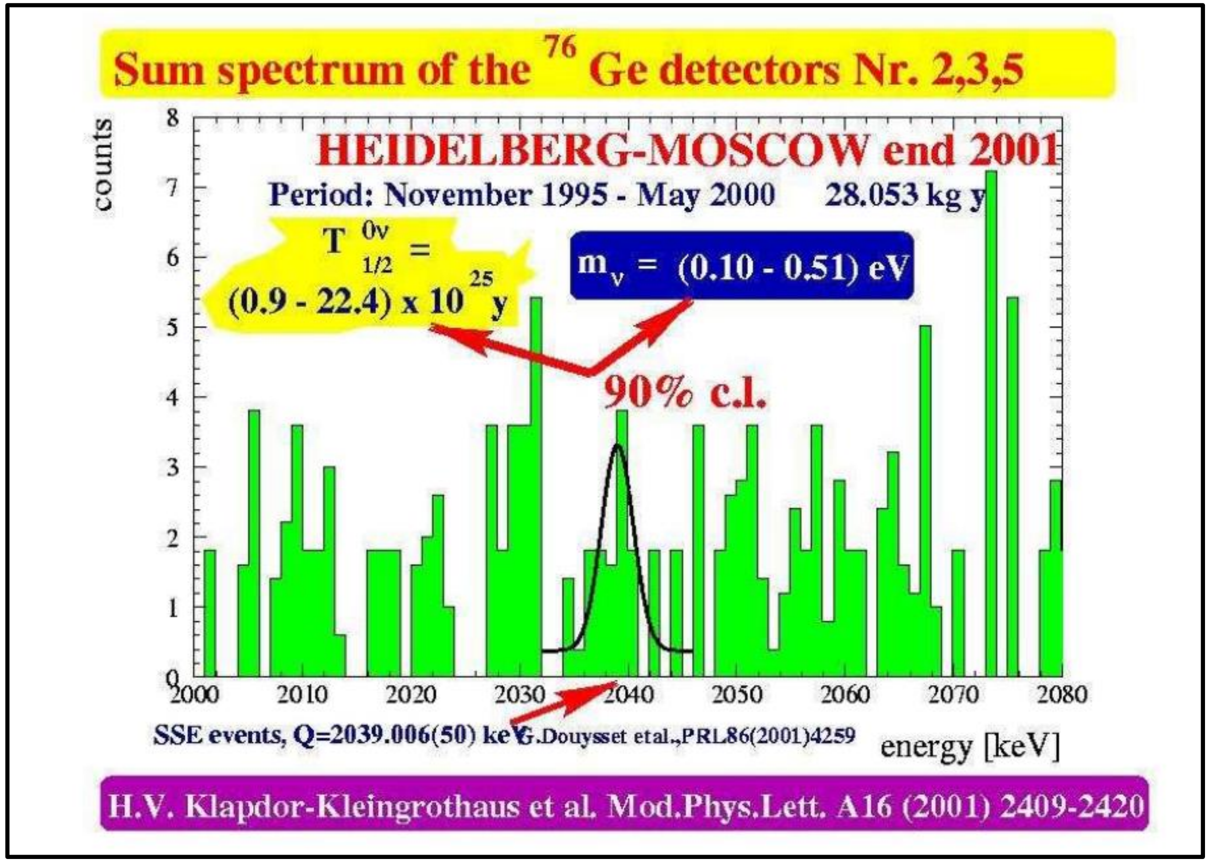
Experiment	Isotope	Technique	Mass $\beta\beta(0\nu)$ isotope	Status
CUORICINO	130Te	TeO2 Bolometer	10 kg	Complete
NEMO3	100Mo/82Se	Foils with tracking	6.9/0.9 kg	Complete
GERDA I	76Ge	Ge diodes in LAr	15 kg	Complete
EXO200	136Xe	Xe liquid TPC	160 kg	Operating
KamLAND-ZEN	136Xe	2.7% in liquid scint.	380 kg	Operating
CUORE-0	130Te	TeO2 Bolometer	11 kg	Operating
GERDA II	76Ge	Point contact Ge in LAr	30+35 kg	Commissioning
Majorana D	76Ge	Point contact Ge	30 kg	Commissioning
CUORE	130Te	TeO2 Bolometer	206 kg	Construction
SNO+	130Te	0.3% natTe suspended in Scint	55 kg	Construction
NEXT-100	136Xe	High pressure Xe TPC	80 kg	Construction
SuperNEMO D	82Se	Foils with tracking	7 kg	Construction
CANDLES	48Ca	305 kg of CaF2 crystals - liq. scint	0.3 kg	Construction
LUCIFER	82Se	ZnSe scint. bolometer	18 kg	Construction
1TGe (GERDA+MJ)	76Ge	Best of GERDA and MAJORANA	~ tonne	R&D
CUPID	-	Hybrid Bolometers	~ tonne	R&D
nEXO	136Xe	Xe liquid TPC	~ tonne	R&D
SuperNEMO	82Se	Foils with tracking	100 kg	R&D
AMoRE	100Mo	CaMoO4 scint. bolometer	50 kg	R&D
MOON	100Mo	Mo sheets	200 kg	R&D
COBRA	116Cd	CdZnTe detectors	10 kg/183 kg	R&D
CARVEL	48Ca	48CaWO4 crystal scint.	~ tonne	R&D
DCBA	150Nd	Nd foils & tracking chambers	20 kg	R&D

Jonathan Link, (TAU2016)

Experiments

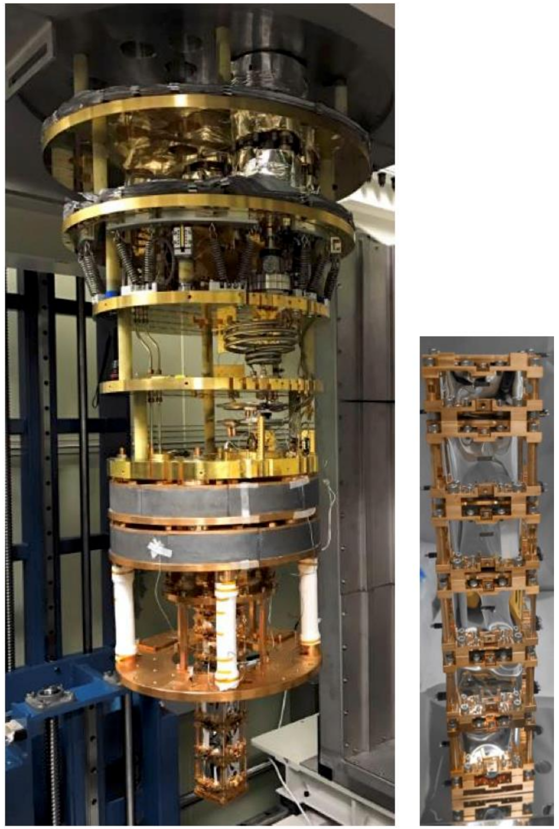
- Heidelberg-Moscow collab. (^{76} Ge in Gran Sasso)
  - 2001, they claimed to have found an evidence for $0\nu\beta\beta$

    (Klapdor-Kleingrothaus *et al*, MPLA16)

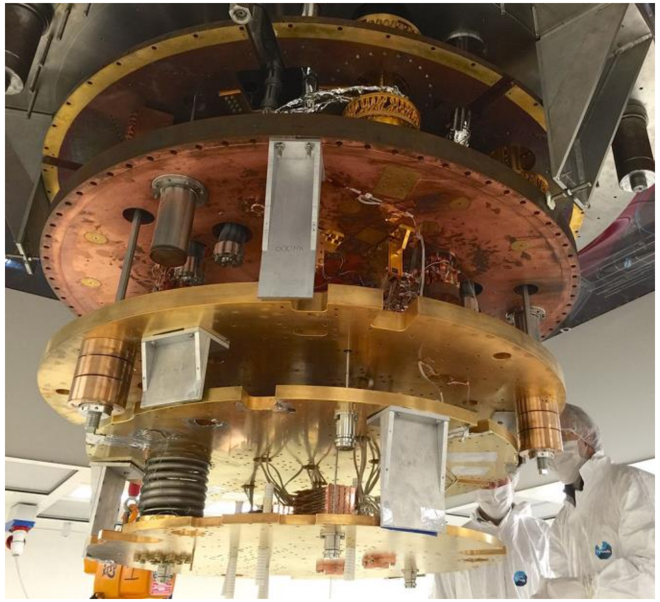


- This was ruled out by the results from GERDA exp. (arXiv:1411.4791)

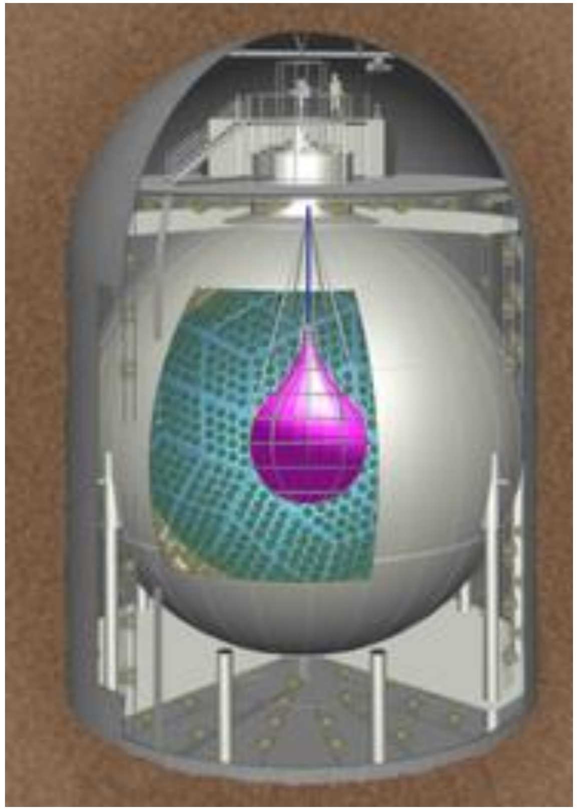
Experiments



AMORE (^{100}Mo)



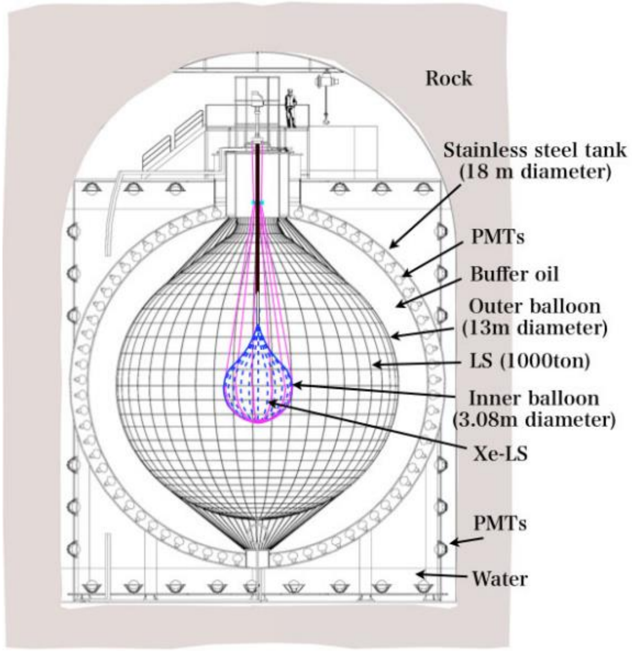
CUORE (^{130}Te)



KamLAND-Zen (^{136}Xe)

Experiments

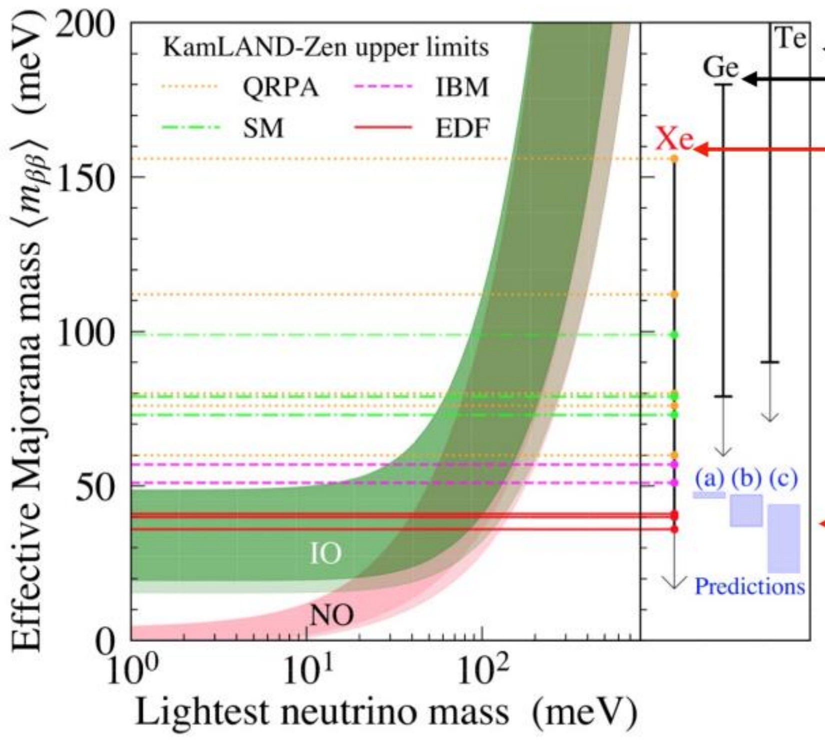
KamLAND-Zen



$$T^{0\nu}_{1/2} > 2.3 \times 10^{26}\ \text{yr at } 90\%\ \text{C.L.}$$

$$m_{\beta\beta} < \ 36 - 156\ \text{meV}$$

- ^{136}Xe(91% enriched) loaded LS
- **Reached IO region for the first time.**
- Improvement of KamLAND-Zen800 over KamLAND-Zen400



This Xe $0\nu\beta\beta$ search represents the **worlds most stringent limit** on the effective Majorana mass.

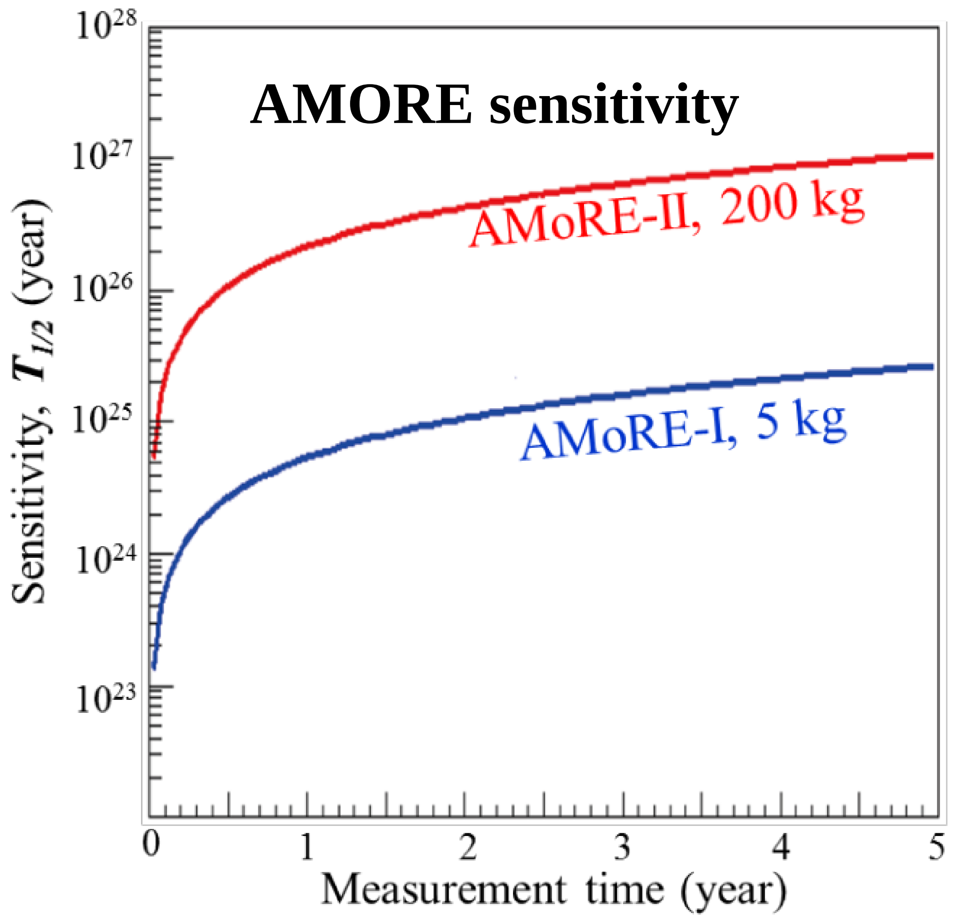
(Ge) GERDA: Phys.Lett. **125** 252502
(Te) CUORE: Nature 604, 53 (2022)

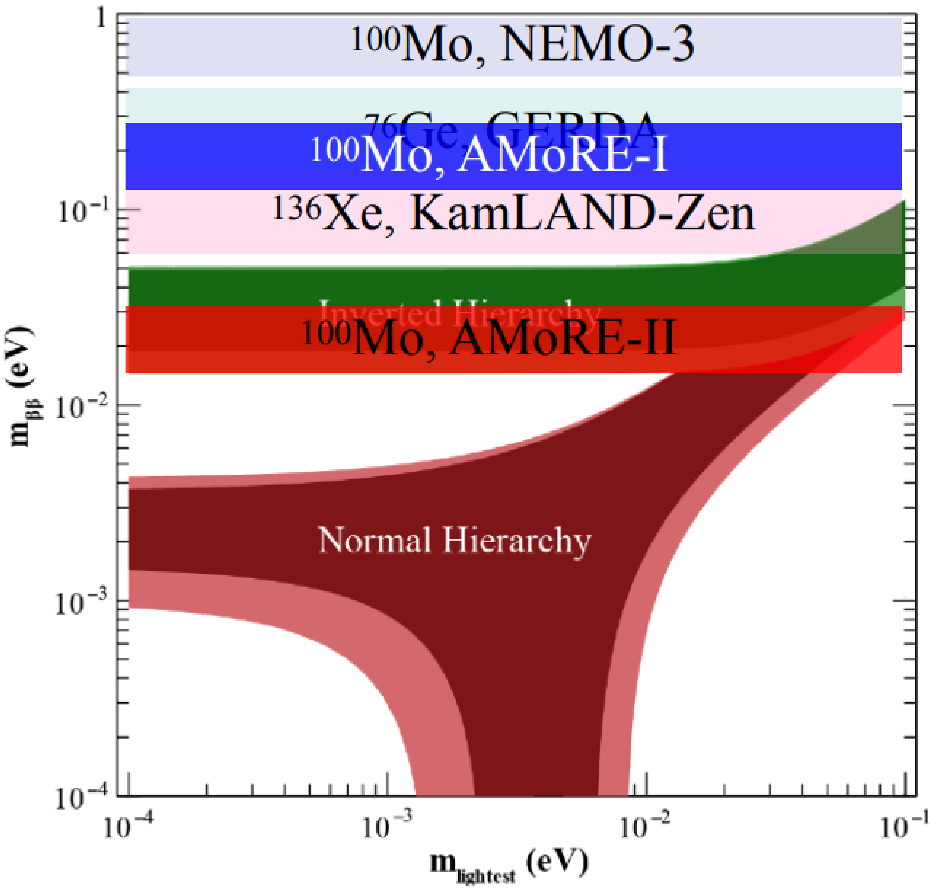
First tests of theoretical predictions.

(a) Phys. Rev. D 86, 013002
(b) Phys. Lett.B 811, 135956
(c) Euro.Phys.J.C 80, 76

Sensitivity

Neutrinoless double beta decay (0νββ)

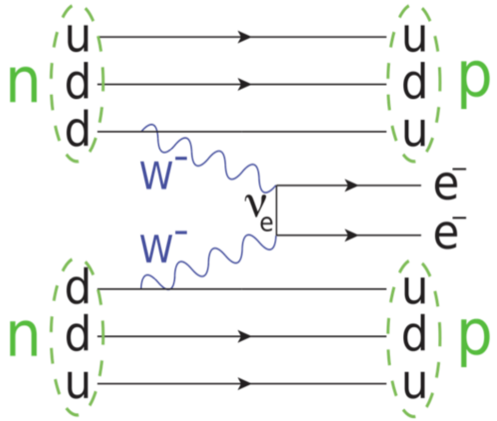


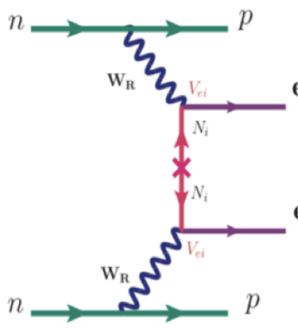


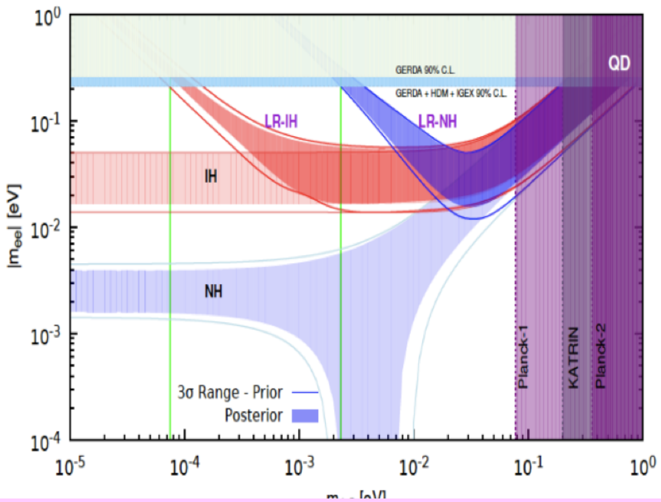
Dirac vs. Majorana

The question is still unanswered:

Are neutrinos their own antiparticles?







Ton scale 0nuBB experiments will cover the inverted hierarchy by 2035

Many experiments operating, planned or in R&D:
LEGEND, SNO+, NEXT, CUPID, THEIA…GERDAPhaseII, Majorana, SuperNEMO, CUORE, and nEXO

Conclusion & Outlook

- **Firmly established that neutrinos are massive particles and leptons mix.**
- **Determined three mixing angles and two mass-squared differences from various experiments.**
- **Made great effort on understanding neutrino properties.**
- **What we don't know yet**
  - Leptonic CP Violation
  - Origin of Neutrino Mass (ordering)
  - Octant of θ_{23}
  - Majorana *vs.* Dirac
  - Sterile Neutrinos ?
  - Non-unitarity of U ?
- **New opportunity through neutrinos.**