AEPSHEP2022 Lecture
Higgs and BSM physics

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Plan:

Lecture 0+1: Overview

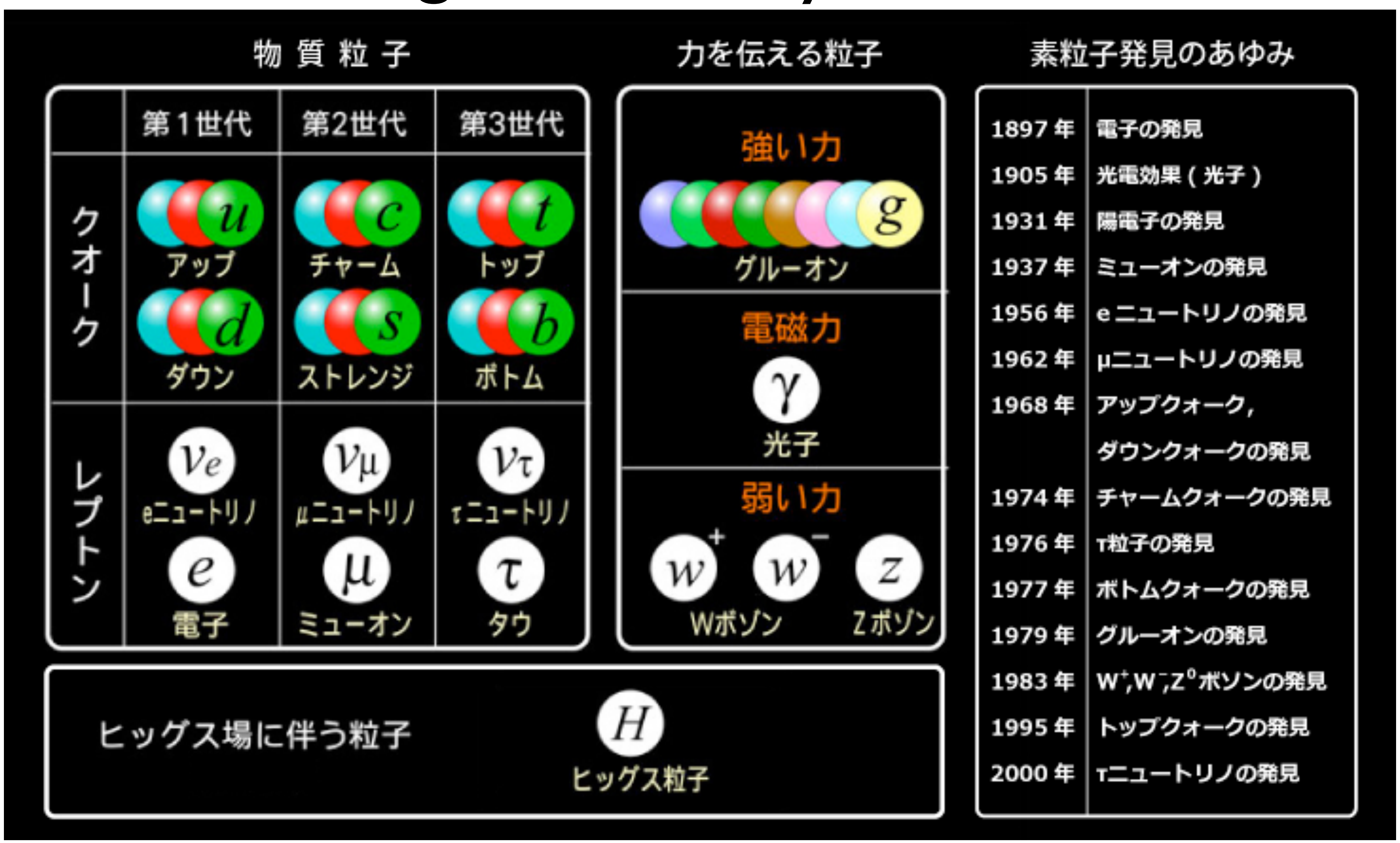
Lecture 2: renormalization and naturalness

Lecture 3: Beyond the Standard Model?

Lecture 0+1: Overview

What are elementary particles?

They are something elementary. We have a lot of them.



Field theory?

In our language, each “particles” are actually “fields.”

$\varphi(x,t)$

These are functions of space-time.

electron field, quark field, electromagnetic field, Higgs field..

Fields follow their equations of motion.
(classical theory)

$\varphi(x,t)$

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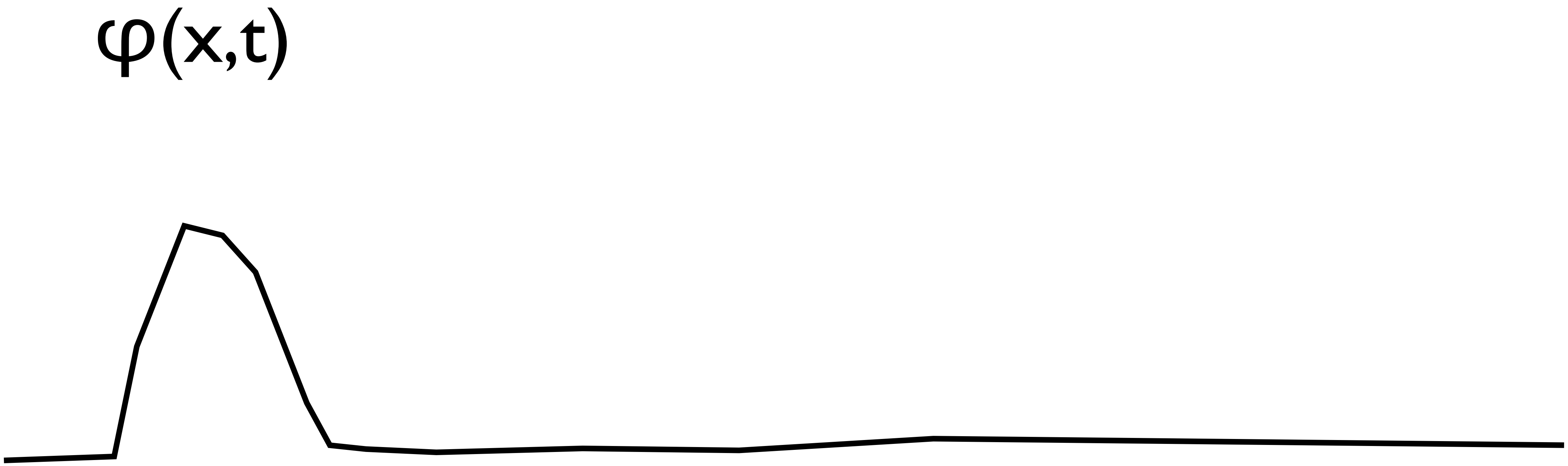
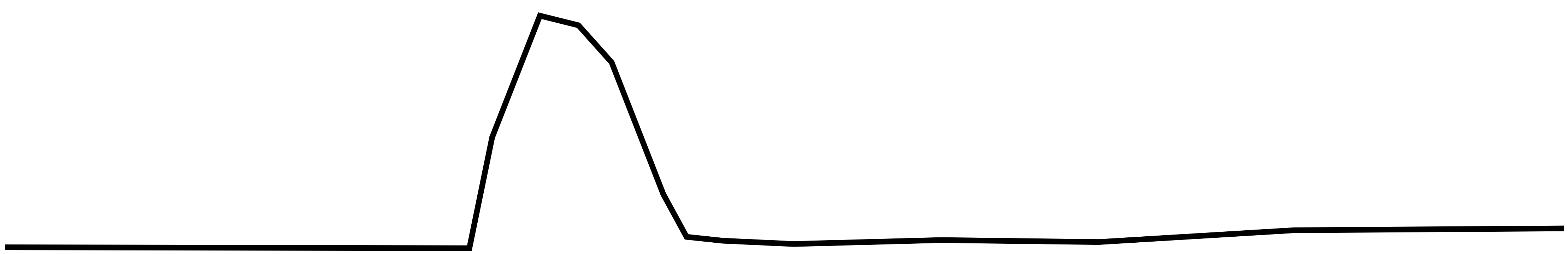
(classical theory)



Fields follow their equations of motion.
(classical theory)

φ(x,t)



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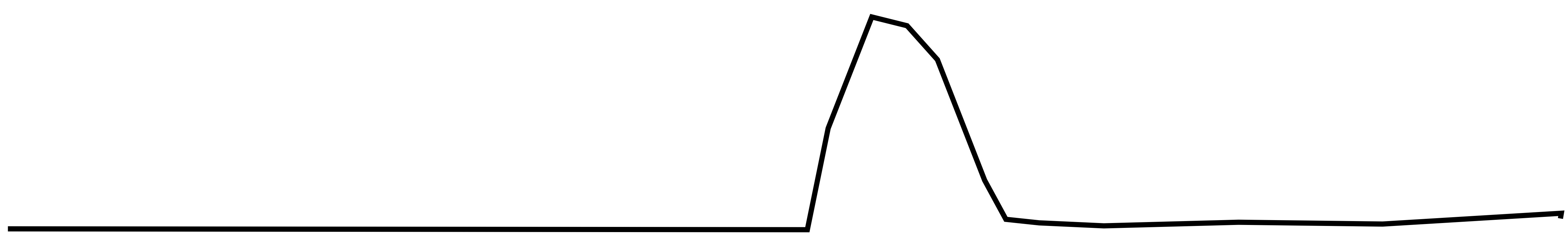


Fields follow their equations of motion.
(classical theory)

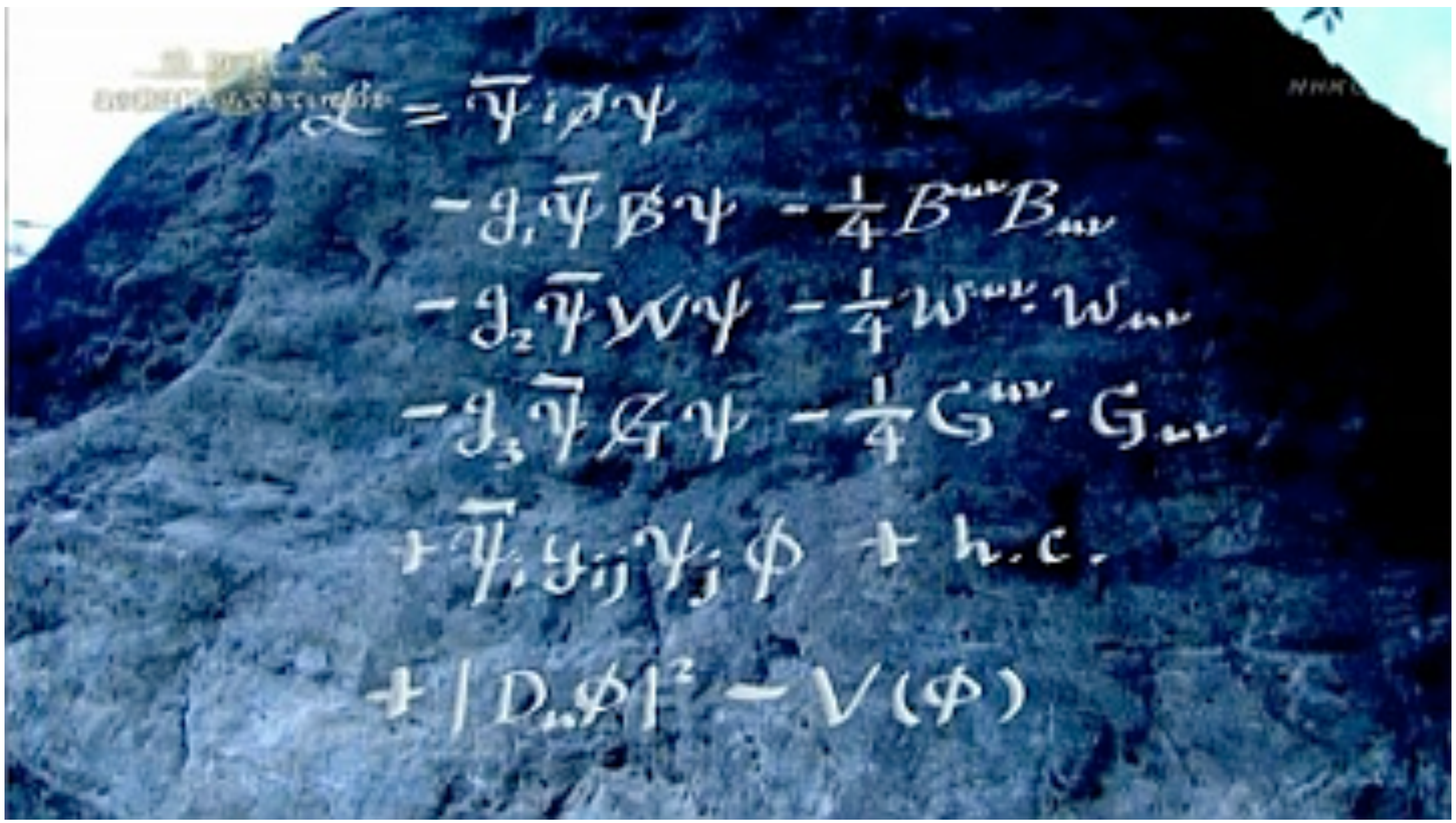
φ(x,t)

Fields follow their equations of motion. (classical theory)

$\varphi(x,t)$

This is the particle. This is one of the solution of the equations of motion. Nothing difficult.

We call the collection of equations of motion as "theory" or "model." The equations of motions are derived from a single quantity, the Lagrangian.



The Standard Model

(Taken from NHK)

Well, is that it?

Yes, but what's nice is that we can predict infinite numbers of physical quantity out of finite numbers of parameters.

(Renormalizable theory)

Table 10.5: Principal Z pole observable[illegible] and [illegible]r SM [illegible]rediction[illegible] ([illegible] Table 10.[illegible]). The first $\bar{s}_\ell^2$ is the effective weak mixing [illegible] from th[illegible] ha[illegible]ronic cha[illegible]ge asymmetry, the second is the combined v[illegible] Tevatron [illegible]63[illegible]64,165], [illegible] the third from the LHC [168,169]. The va[illegible] of [illegible] are [illegible] from A_L[illegible] for hadronic final states [154]; (ii) from A_{LR} for leptonic final states an[illegible] polariz[illegible] Bhabba scattering [156]; and (iii) from the angular distribution of the τ polarization at LEP 1. The A_τ values are from SLD and the total τ polarization, respectively.

Quantity	Value	Standard Model	Pull
M_Z [GeV]	91.1876 ± 0.0021	91.1880 ± 0.0020	-0.2
Γ_Z [GeV]	2.4952 ± 0.0023	2.4955 ± 0.0009	-0.1
$\Gamma(\text{had})$ [GeV]	1.7444 ± 0.0020	1.7420 ± 0.0008	—
$\Gamma(\text{inv})$ [MeV]	499.0 ± 1.5	501.66 ± 0.05	—
$\Gamma(\ell^+\ell^-)$ [MeV]	83.984 ± 0.086	83.995 ± 0.010	—
σ_{had} [nb]	41.541 ± 0.037	41.479 ± 0.008	1.7
[illegible]	[illegible]	[illegible]	[illegible]
R_μ	20.785 ± 0.033	20.740 ± 0.010	1.4
R_τ	20.764 ± 0.045	20.785 ± 0.010	-0.5
[illegible]	[illegible]	[illegible]	[illegible]
[illegible]	[illegible]	[illegible]	[illegible]
$A_{FB}^{(0,e)}$	0.0145 ± 0.0025	0.01616 ± 0.00008	-0.7
$A_{FB}^{(0,\mu)}$	0.0169 ± 0.0013		0.6
$A_{FB}^{(0,\tau)}$	0.0188 ± 0.0017		1.6
$A_{FB}^{(0,b)}$	0.0992 ± 0.0016	[illegible]	[illegible]
$A_{FB}^{(0,c)}$	0.0707 ± 0.0035	0.0735 ± 0.0002	-0.8
$A_{FB}^{(0,s)}$	0.0976 ± 0.0114	0.1030 ± 0.0003	-0.5
$\bar{s}_\ell^2$	0.2324 ± 0.0012	0.23155 ± 0.00005	0.7
	0.23176 ± 0.00060		0.3
	0.2297 ± 0.0010		-1.9
A_e	0.15138 ± 0.00216	0.1468 ± 0.0004	2.1
	0.1544 ± 0.0060		1.3
	0.1498 ± 0.0049		0.6
A_μ	0.142 ± 0.015		-0.3
A_τ	0.136 ± 0.015		-0.7
	0.1439 ± 0.0043		-0.7
A_b	0.923 ± 0.020	0.9347	-0.6
A_c	0.670 ± 0.027	0.6676 ± 0.0002	0.1
A_s	0.895 ± 0.091	0.9356	-0.4

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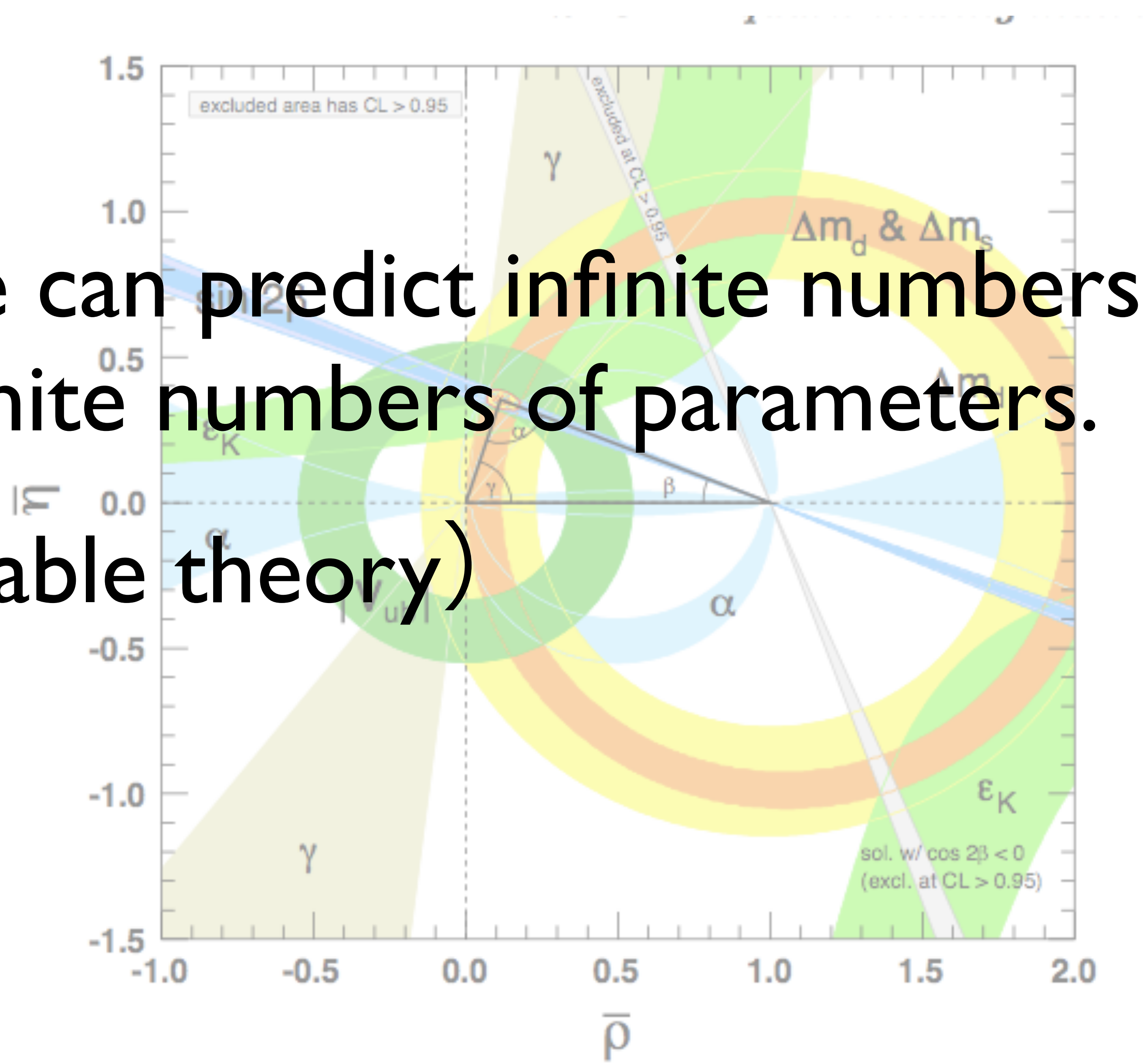

gure 12.2: Constraints on the $\bar{\rho}, \bar{\eta}$ plane. The shaded areas have 95% CL.

Table 10.5: Principal Z pole observables and their SM predictions (*cf.* Table 10.4). The first $\bar{s}^2_\ell$ is the effective weak mixing angle extracted from the hadronic charge asymmetry, the second is the combined value from the Tevatron [163,164,165], and the third from the LHC [168,169]. The values of A_e are (i) from A_{LR} for hadronic final states [154]; (ii) from A_{LR} for leptonic final states and from polarized Bhabba scattering [156]; and (iii) from the angular distribution of the τ polarization at LEP 1. The A_τ values are from SLD and the total τ polarization, respectively.

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R_e	[illegible]	[illegible]	[illegible]
R_μ	20.785 ± 0.033	20.740 ± 0.010	1.4
R_τ	20.764 ± 0.045	20.785 ± 0.010	-0.5
R_b	0.21629 ± 0.00066	0.21576 ± 0.00003	0.8
R_c	0.1721 ± 0.0030	0.17226 ± 0.00003	-0.1
$A_{FB}^{(0,e)}$	0.0145 ± 0.0025	0.01616 ± 0.00008	-0.7
$A_{FB}^{(0,\mu)}$	0.0169 ± 0.0013		0.6
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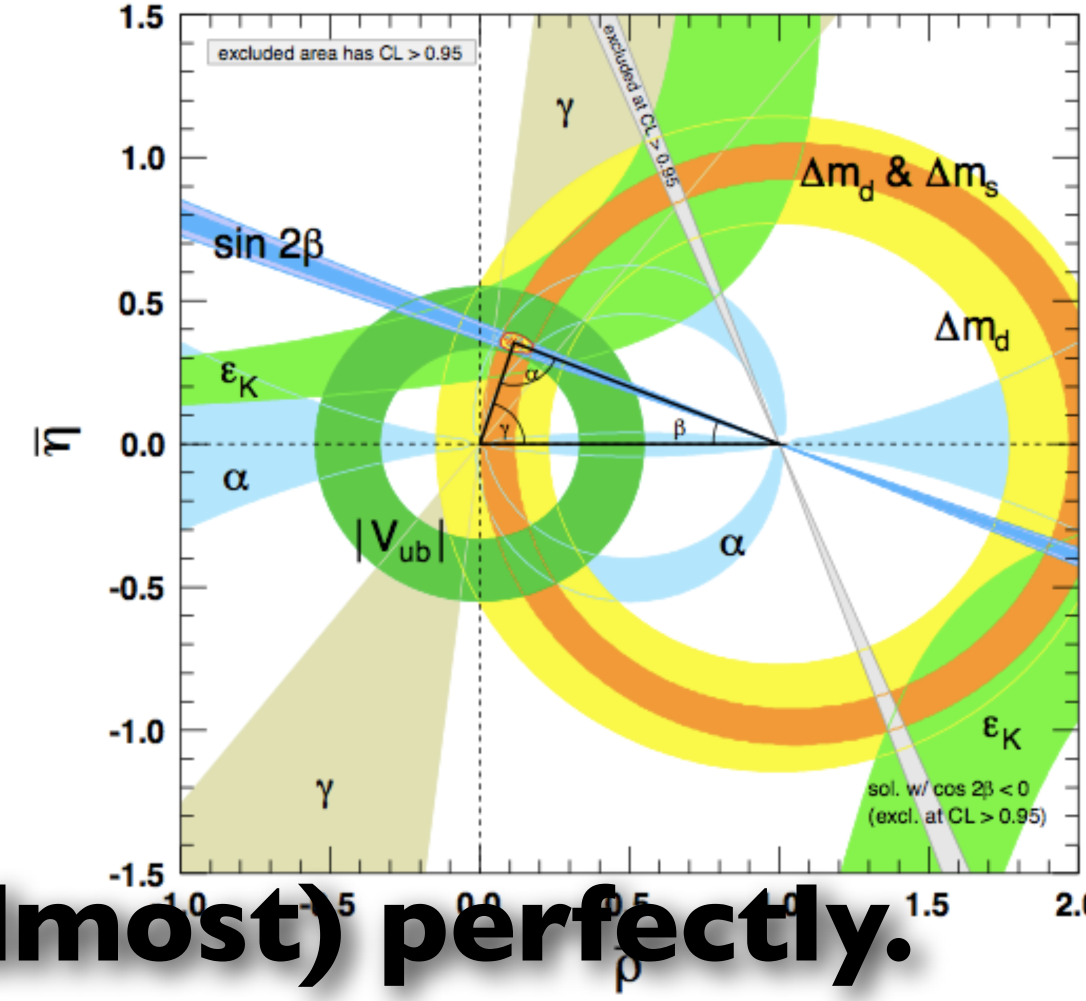
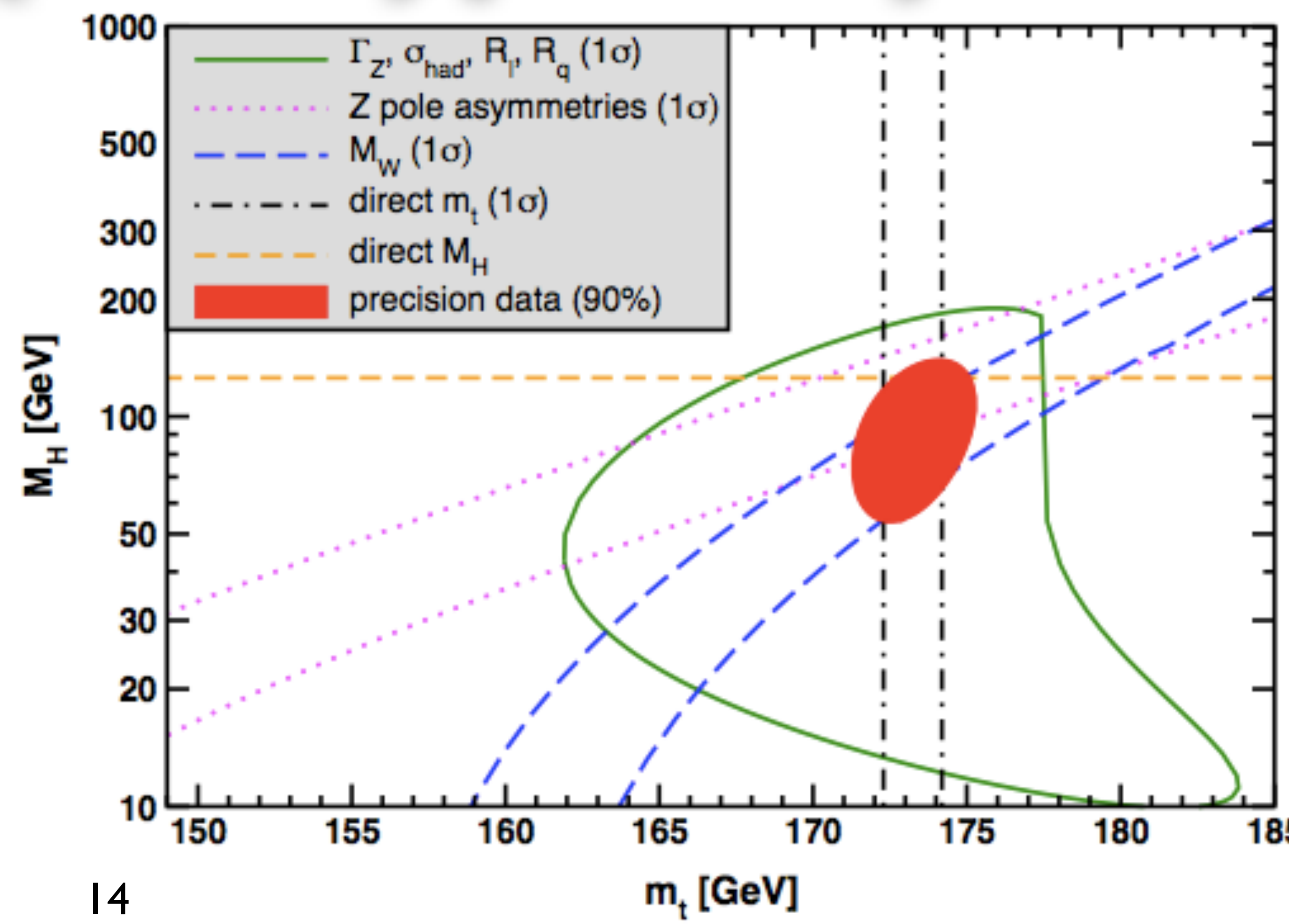
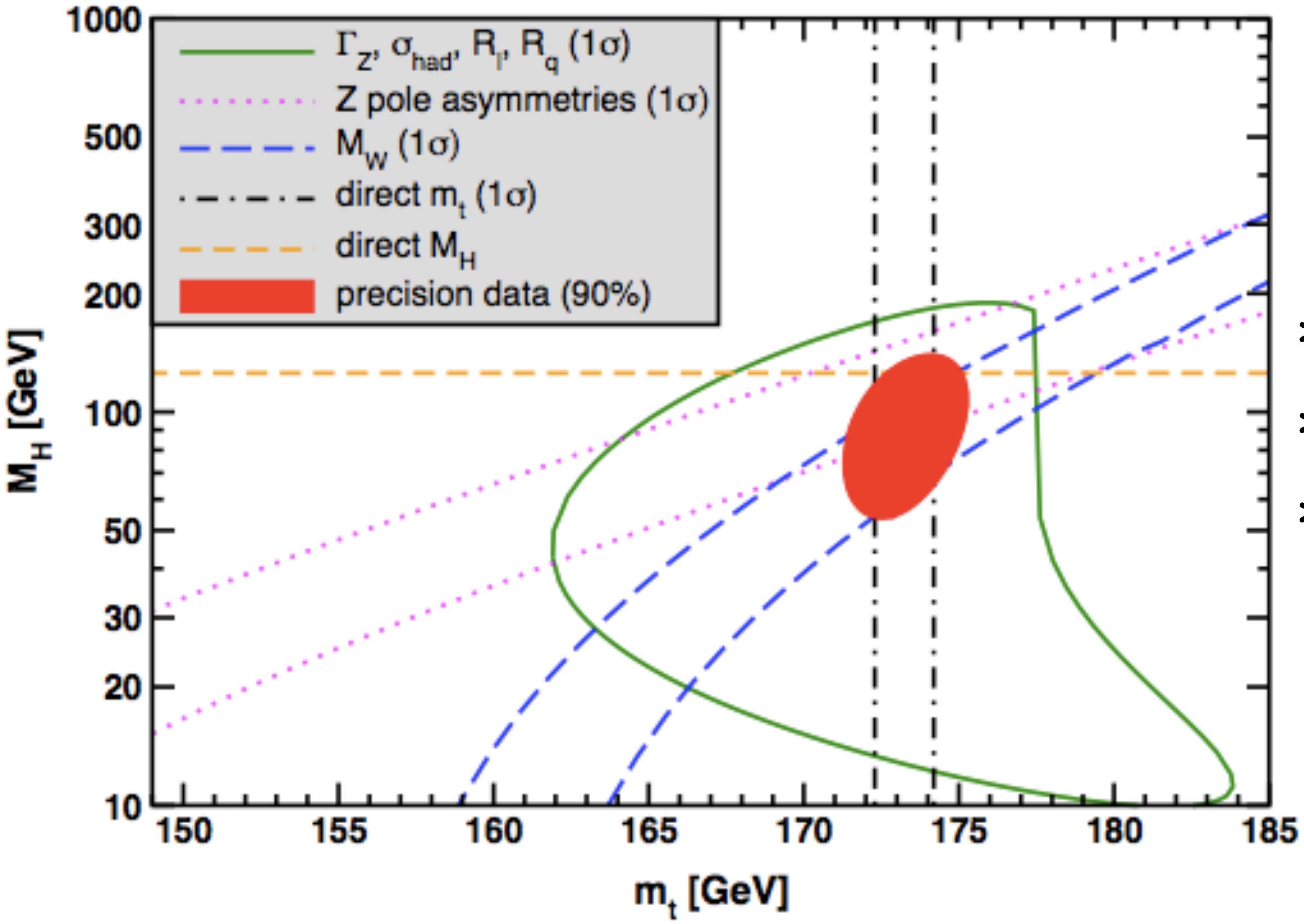


And, it works (almost) perfectly.



For example,



We can calculate the W boson mass by inputting

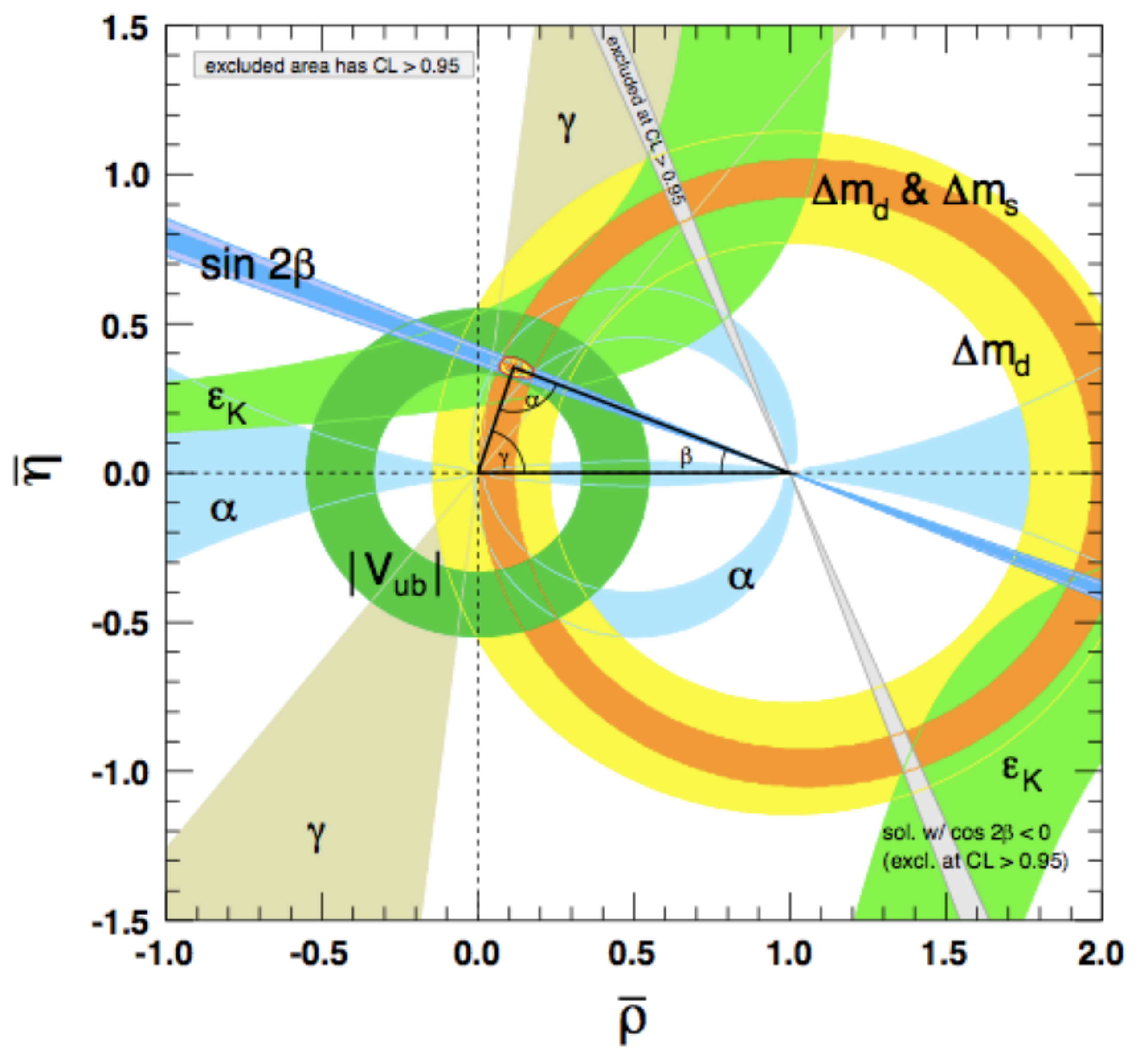
* the Z boson mass,

* strength of the Coulomb force α,

* and the muon lifetime.

very good agreement.

And,



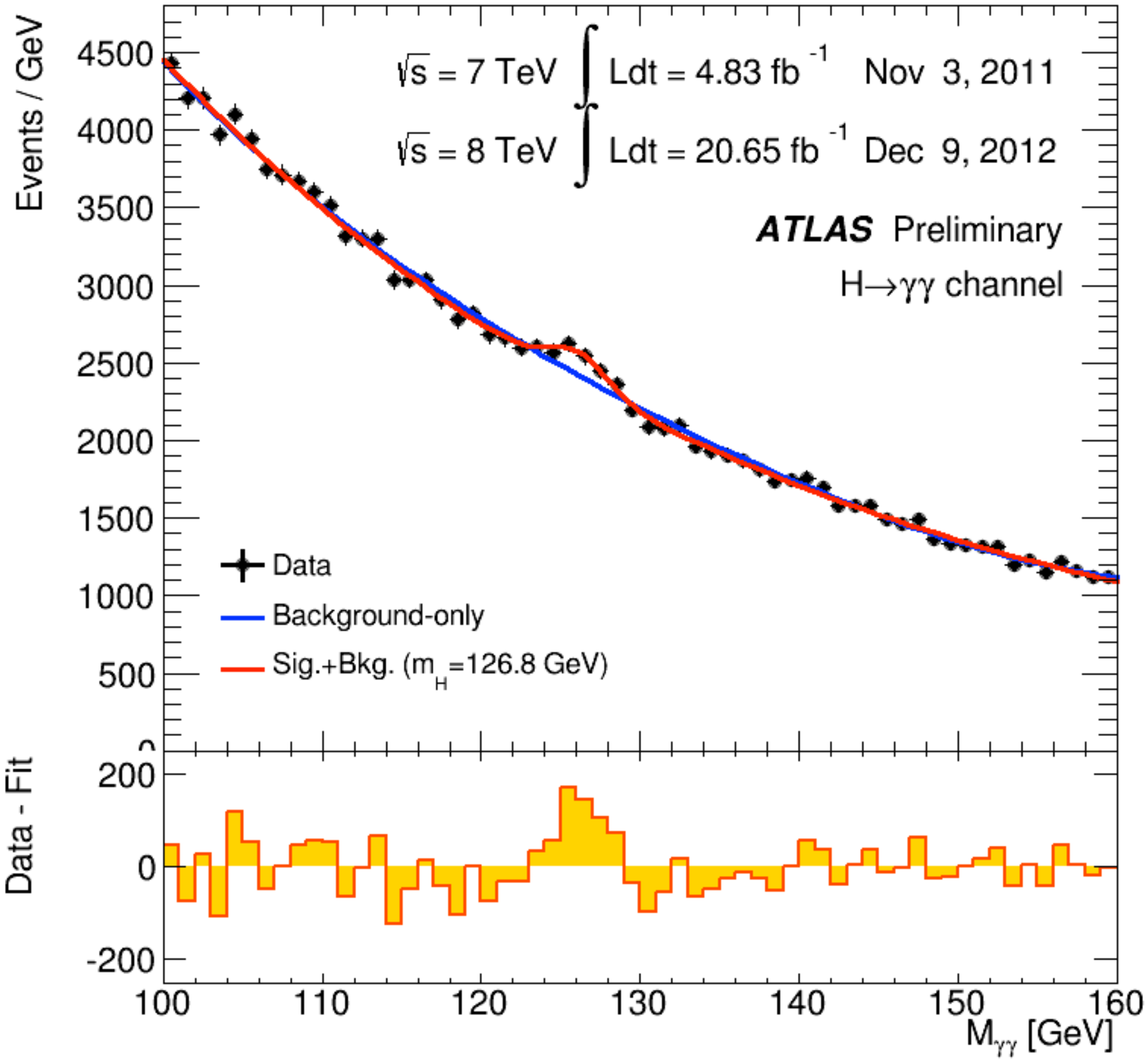
gure 12.2: Constraints on the $\bar{\rho}, \bar{\eta}$ plane. The shaded areas have 95% CL.

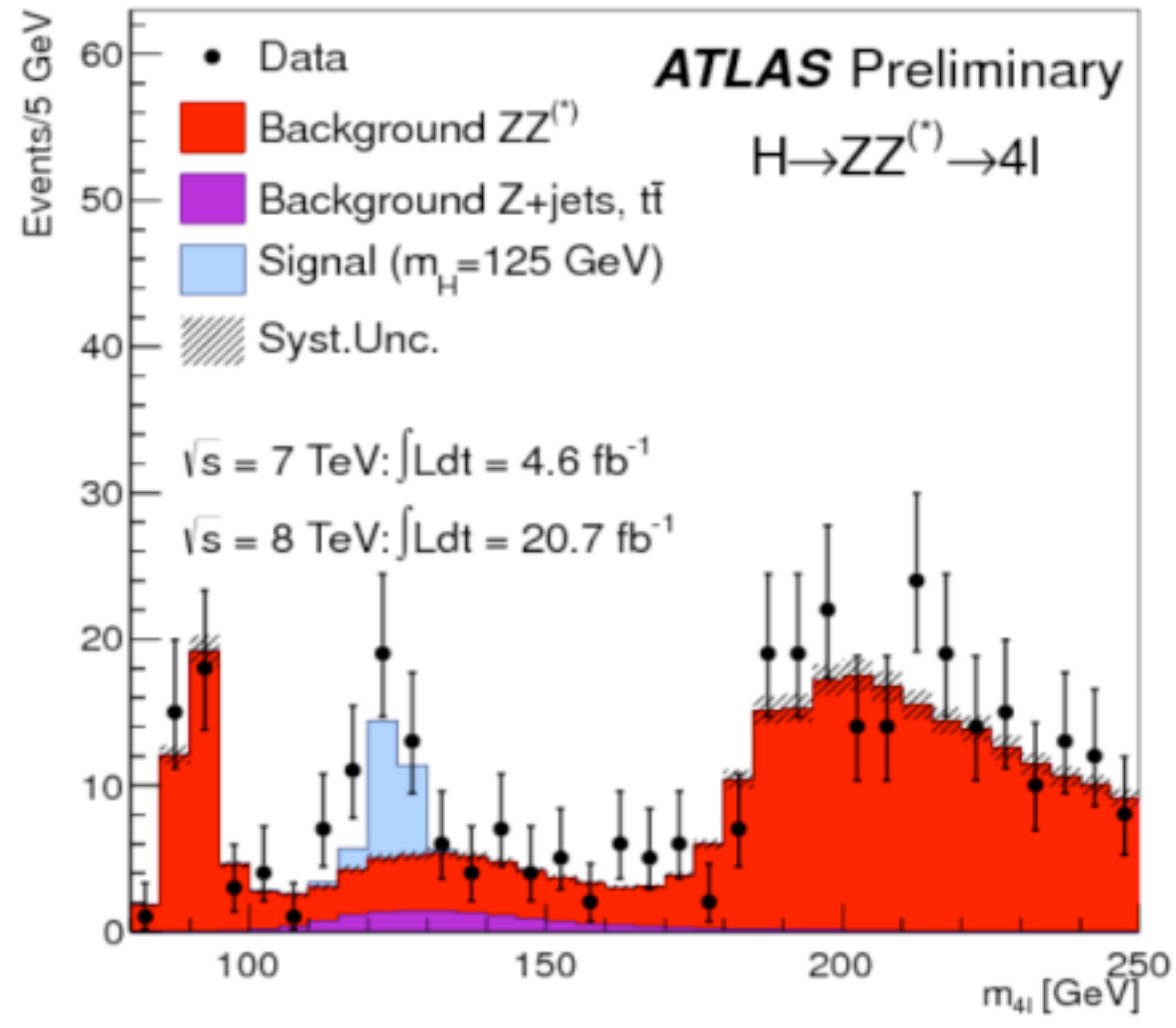
All the flavor changing processes are governed by a single 3x3 matrix, the CKM matrix.

(it has four parameters.)

Perfect.

And, the most non-trival one is this.





This is the most mysterious part of the Standard Model,
but it was there.

What's the Higgs field and the Higgs boson?

Let's follow a little bit of history.

In the strong interactions among hadrons, there are so called the isospin symmetry to mix the proton and the neutron:

$$\psi(x) = \begin{pmatrix} p \\ n \end{pmatrix} \to e^{i\sigma^a \theta^a} \begin{pmatrix} p \\ n \end{pmatrix}$$

They are indeed similar once we ignore the electric charge. They have similar masses and similar interactions.

1954: Yang-Mills theory (non-abelian gauge theory)

By the analogy of the electromagnetic interactions which are sourced by electric charges,
Yang and Mills constructed a theory in which the isospin sources the force.

As in the case of electromagnetism, there appeared a massless particle (gauge boson).

There aren’t such particles…

1961: Spontaneous symmetry breaking by Nambu and Jona-Lasinio

$$\psi(x) = \begin{pmatrix} p \\ n \end{pmatrix}$$

proton and neutron get masses from fermion condensation in the vacuum just as in superconductors.

This theory predicts massless spin 0 particle.

It is identified as the pions.

In general, massless particle appears when spontaneous symmetry breaking happens.
(Nambu-Goldstone theorem)

Simple model (and it is actually a part of the Standard Model)

Let's prepare a field: $\phi(x) = \begin{pmatrix} \phi_1(x) \\ \phi_2(x) \end{pmatrix}$

$$|\phi|^2 = |\phi_1|^2 + |\phi_2|^2$$

This quantity is invariant under SU(2) which mixes φ_1 and φ_2.

$$\begin{pmatrix} \phi_1 \\ \phi_2 \end{pmatrix} \to e^{i\sigma^a\theta^a} \begin{pmatrix} \phi_1 \\ \phi_2 \end{pmatrix} \Rightarrow |\phi|^2 \to |\phi|^2$$

Which means,

when the potential depends only on $|\phi|^2$

The theory has **SU(2) symmetry.**

For example, Potential

$$V(\phi) = -m^2|\phi|^2 + \frac{\lambda}{4}|\phi|^4$$
$$= \frac{\lambda}{4}\left(|\phi|^2 - \frac{2m^2}{\lambda}\right)^2 - \frac{m^4}{\lambda}$$

With this potential,
we have a smaller energy for nonvanishing $|\phi|^2$

At every point in the space

$$\phi(x) = \begin{pmatrix} v \\ 0 \end{pmatrix} \qquad v = \sqrt{\frac{2m^2}{\lambda}}$$

this configuration minimizes the potential. This means that that's a solution of the equation of motion.

Of course, this is also a solution:

$$\phi(x) = \begin{pmatrix} 0 \\ v \end{pmatrix}$$

But not distinguishable with $\phi(x) = \begin{pmatrix} v \\ 0 \end{pmatrix}$

What's important is

the solution picks up a **special direction in SU(2).**

Not SU(2) symmetric world anymore.

For example, in the Standard Model,

$$\phi(x) = \begin{pmatrix} v \\ 0 \end{pmatrix}$$

the Yukawa interactions between φ and leptons give a mass to the electron while the neutrinos remain massless.

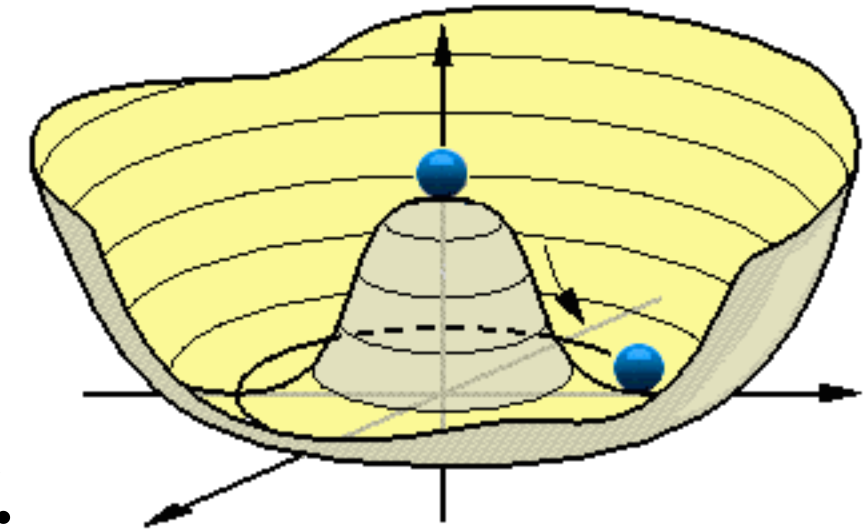
(actually we call the component "electron" when it gets a mass.)

Theory is symmetric, but the nature is not.

This is the spontaneous symmetry breaking.

Theory is SU(2) symmetric.

$$\phi(x) = \begin{pmatrix} v \\ 0 \end{pmatrix}$$



That means SU(2) transformed configuration has the same energy.

The potential has a flat direction.

This represents the Nambu-Goldstone boson.

In history, it seems that the theorists thought that the spontaneous symmetry breaking may be useful for approximate symmetry such as isospin, but the appearance of the Nambu-Goldstone boson doesn't quite match to real world…

1964: Higgs, and independently by Brout, Englert, Guralnik, Hagen and Kibble

In gauge theory, spontaneous symmetry breaking gives masses to the gauge bosons, while Nambu-Goldstone boson does not appear.

And, there appears the Higgs boson.

OK, now we can use this framework to describe the hadron world(!?)

BROKEN SYMMETRIES AND THE MASSES OF GAUGE BOSONS

Peter W. Higgs
Tait Institute of Mathematical Physics, University of Edinburgh, Edinburgh, Scotland
(Received 31 August 1964)

In a recent note[1] it was shown that the Goldstone theorem,[2] that Lorentz-covariant field theories in which spontaneous breakdown of symmetry under an internal Lie group occurs contain zero-mass particles, fails if and only if the conserved currents associated with the internal group are coupled to gauge fields. The

about the "vacuum" solution $\varphi_1(x)=0$, $\varphi_2(x)=\varphi_0$:

$$\partial^\mu\{\partial_\mu(\Delta\varphi_1)-e\varphi_0A_\mu\}=0, \tag{2a}$$

$$\{\partial^2-4\varphi_0^2V''(\varphi_0^2)\}(\Delta\varphi_2)=0, \tag{2b}$$

. . . .

It is worth noting that an essential feature of the type of theory which has been described in this note is the prediction of incomplete multiplets of scalar and vector bosons.[8] It is to be expected that this feature will appear also in theories in which the symmetry-breaking scalar fields are not elementary dynamic variables but bilinear combinations of Fermi fields.[9]

See S. Coleman and S. L. Glashow, Phys. Rev. 134, B671 (1964).

[8]Tentative proposals that incomplete SU(3) octets of scalar particles exist have been made by a number of people. Such a rôle, as an isolated $Y=\pm1$, $I=\frac{1}{2}$ state, was proposed for the κ meson (725 MeV) by Y. Nambu and J. J. Sakurai, Phys. Rev. Letters 11, 42 (1963). More recently the possibility that the σ meson (385 MeV) may be the $Y=I=0$ member of an incomplete octet has been considered by L. M. Brown, Phys. Rev. Letters 13, 42 (1964).

1967: Weinberg

It is actually the weak interaction that can be described by the Higgs mechanism.

Yang-Mills+Nambu+Higgs+Weinberg…

Now unified to the framework of the Standard Model.

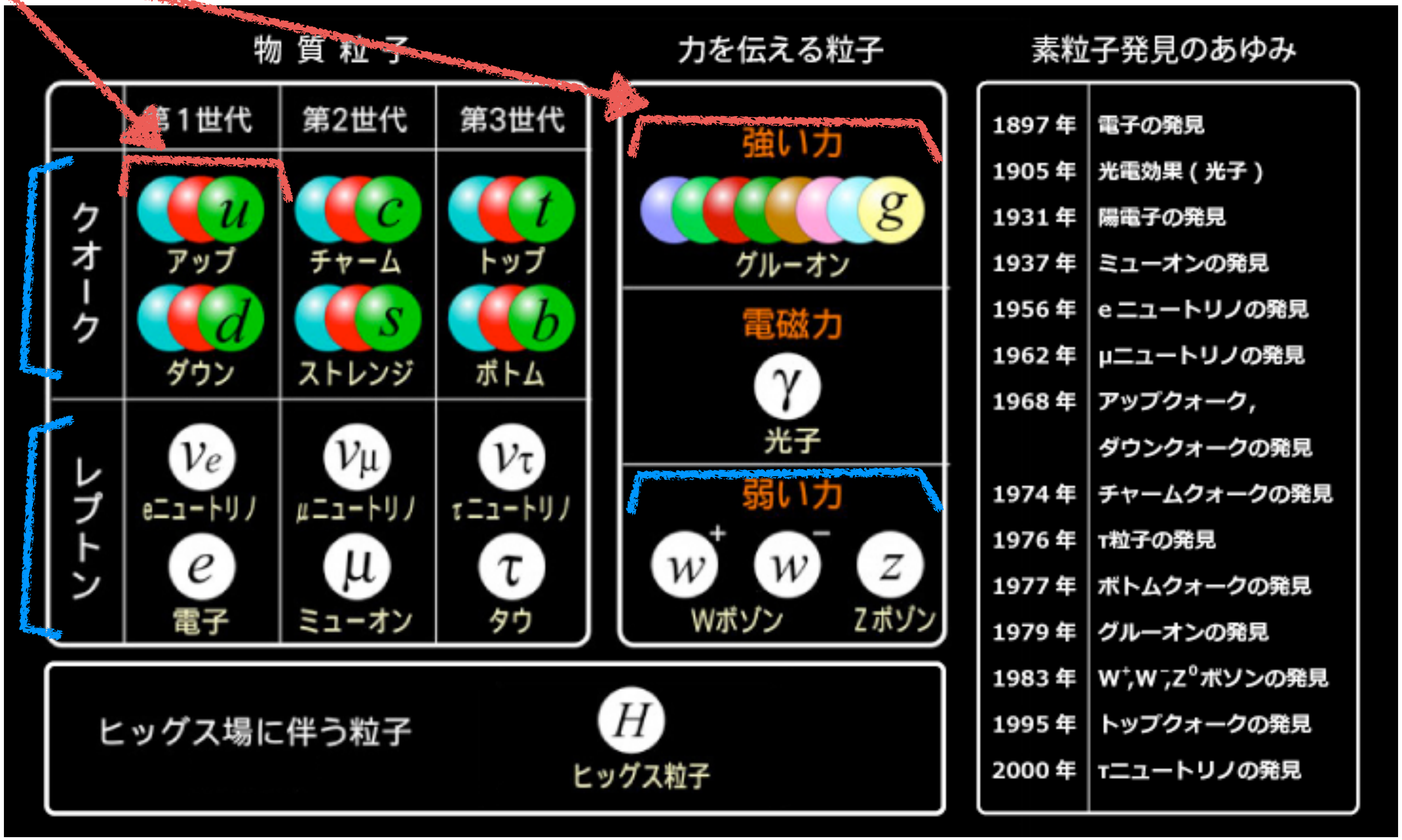
By the way, the hadron world also turns out to be the gauge theory, but yet another interesting realization called the confinement phase. Interesting.

SU(3)

Now look at the table again.

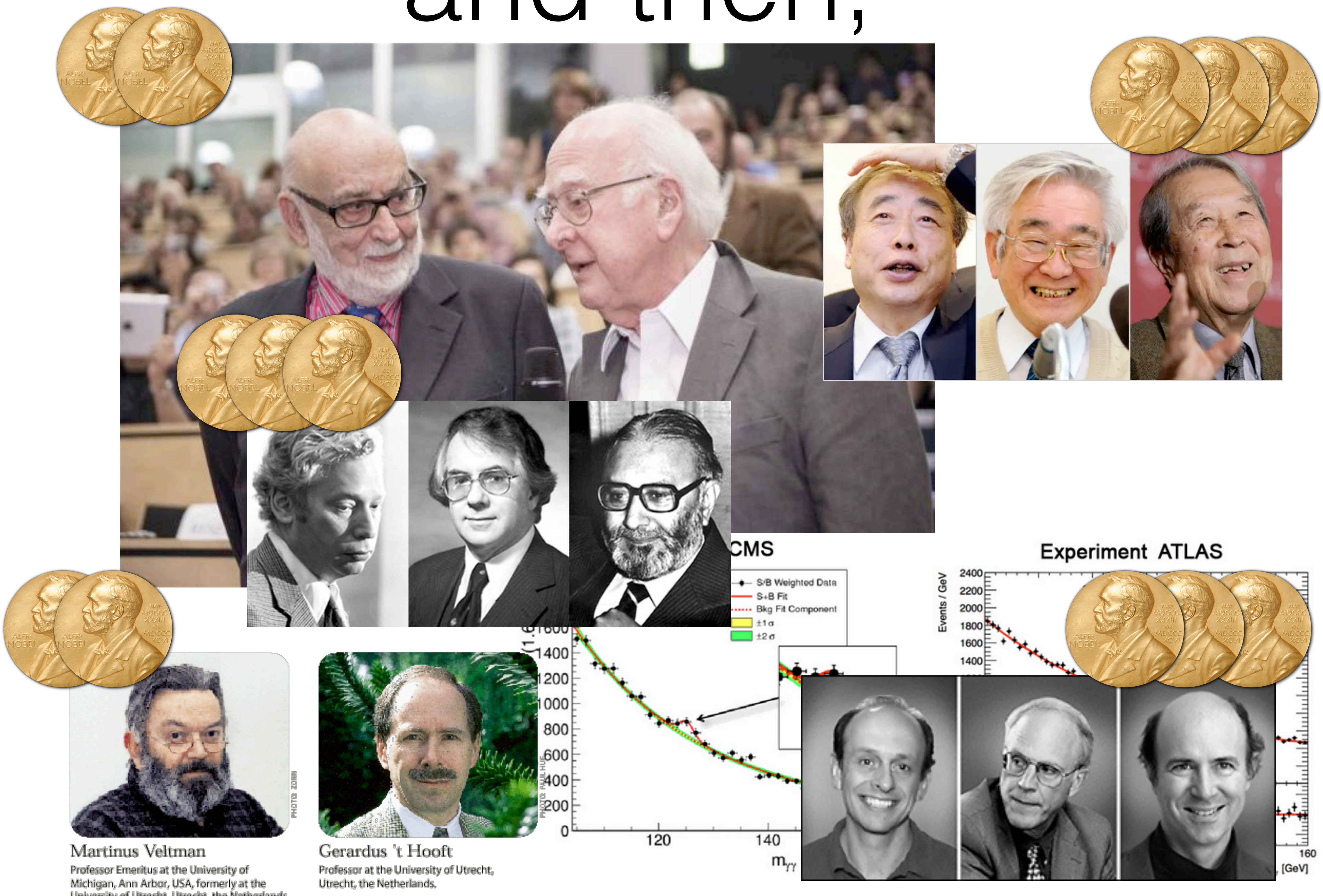
SU(2)

SU(2)



The Standard Model unifies particles which have different properties into common fields. For example, the electron and neutrino are indistinguishable originally but separated by the Higgs configuration!

and then,


CMS
S/B Weighted Data
S+B Fit
Bkg Fit Component
±1 σ
±2 σ
Experiment ATLAS
Events / GeV
Martinus Veltman
Professor Emeritus at the University of Michigan, Ann Arbor, USA, formerly at the University of Utrecht, Utrecht, the Netherlands.
Gerardus 't Hooft
Professor at the University of Utrecht, Utrecht, the Netherlands.

OK, nice history. Are we done?

Not quite. The Standard Model has a full of mystery.

Dark Matter?

Why three generation?

Inflation?

What's Higgs?

Dark energy?

Strong CP problem

Where are anti-particles?

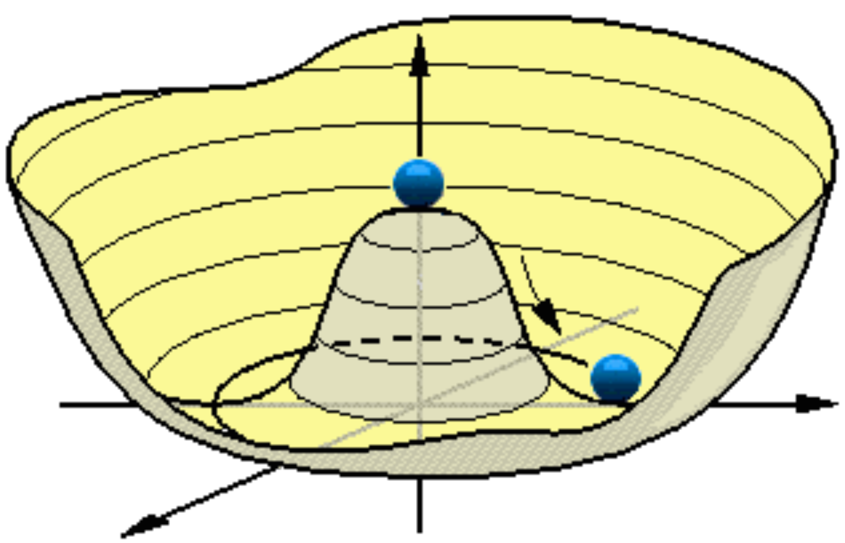
Neutrino mass?

Why three forces?

Why many kinds of particles?

Why charges quantized?

Higgs?



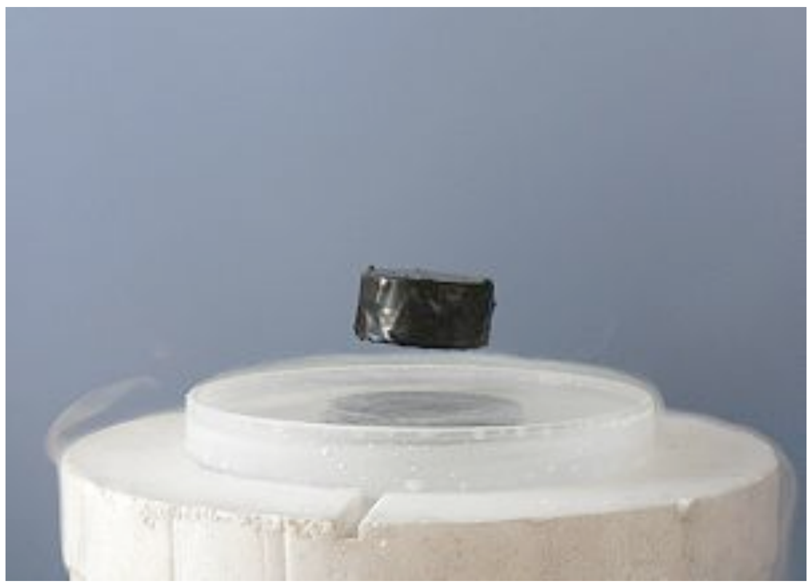
$$V(\phi) = -m^2|\phi|^2 + \frac{\lambda}{4}|\phi|^4$$
$$= \frac{\lambda}{4}\left(|\phi|^2 - \frac{2m^2}{\lambda}\right)^2 - \frac{m^4}{\lambda}$$

It does look like an artificial construction.

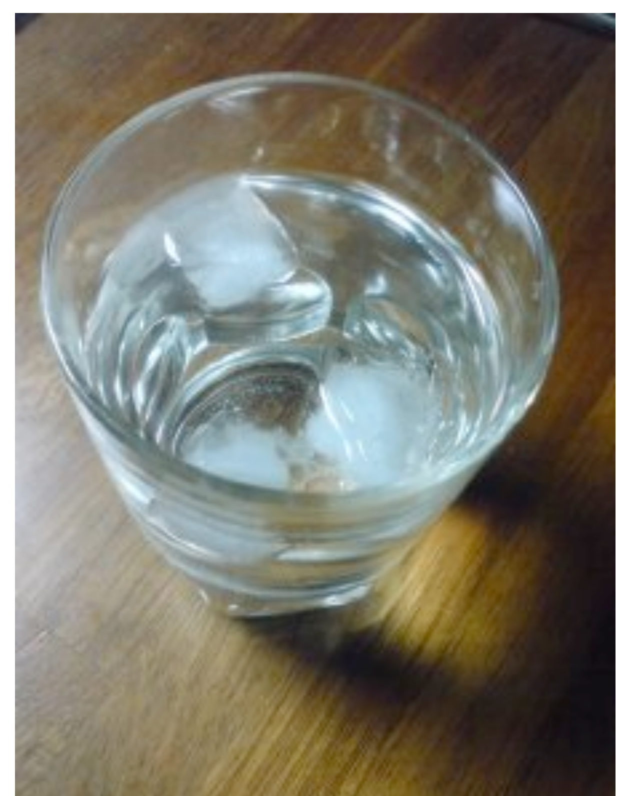
It is consistent, but why do we have such a field?

We know that there are always dynamics behind the spontaneous symmetry breaking.

chiral symmetry breaking



Superconductor

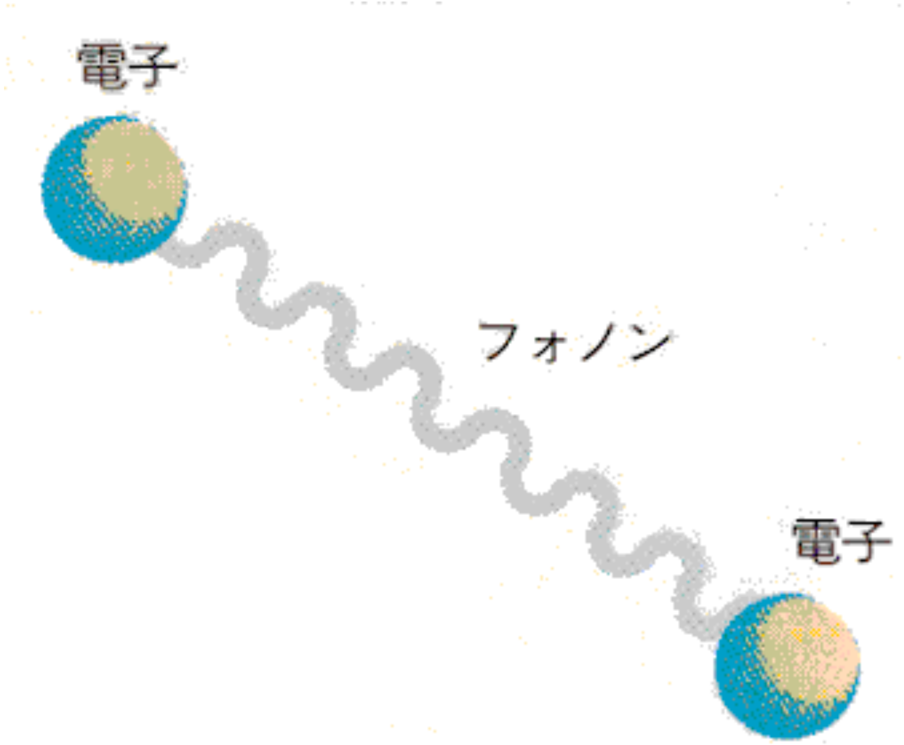


ice to water

Higgs is probably some effective degree of freedom.

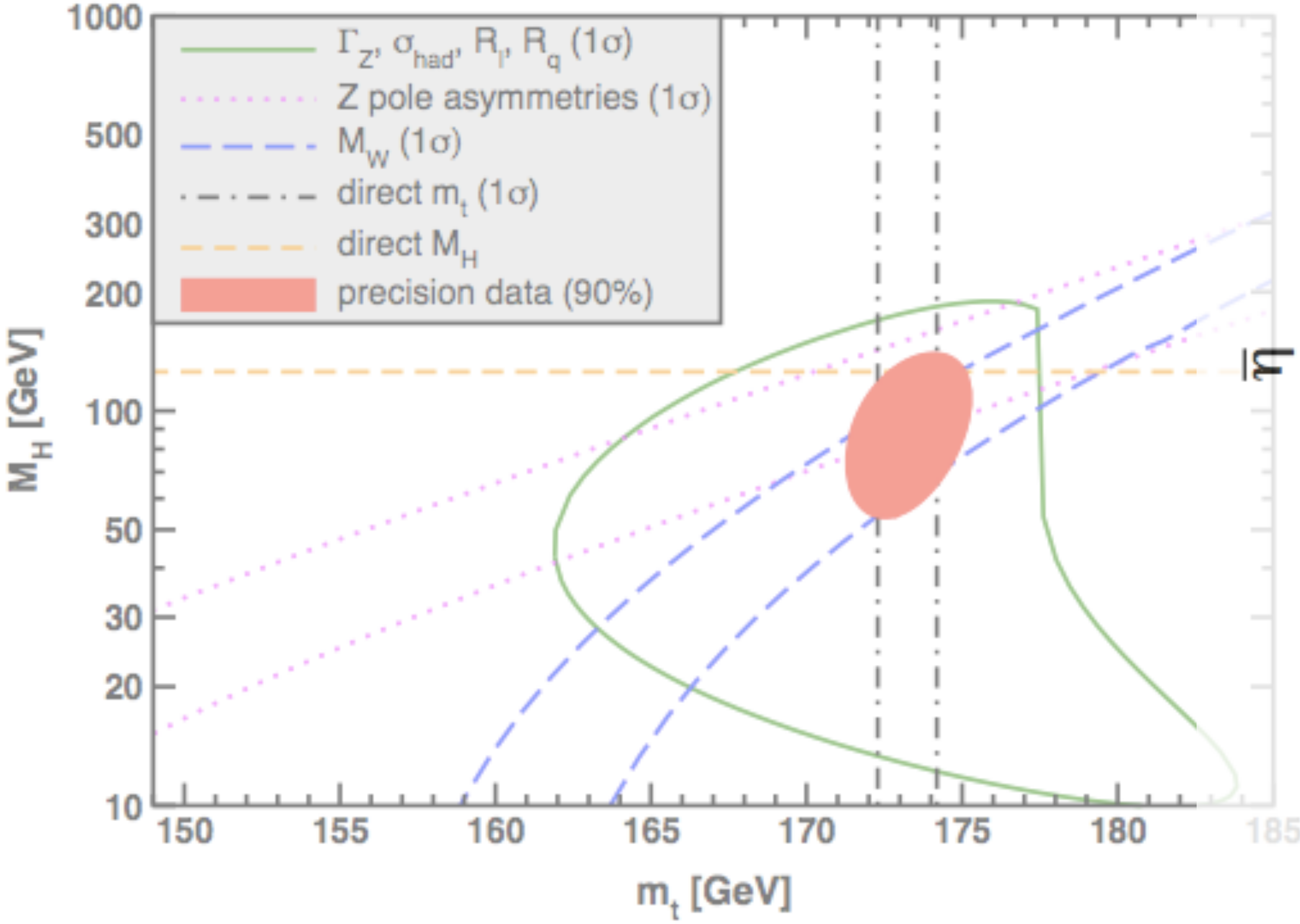
If so, the Standard Model should be replaced by a more fundamental theory at a microscopic scale.

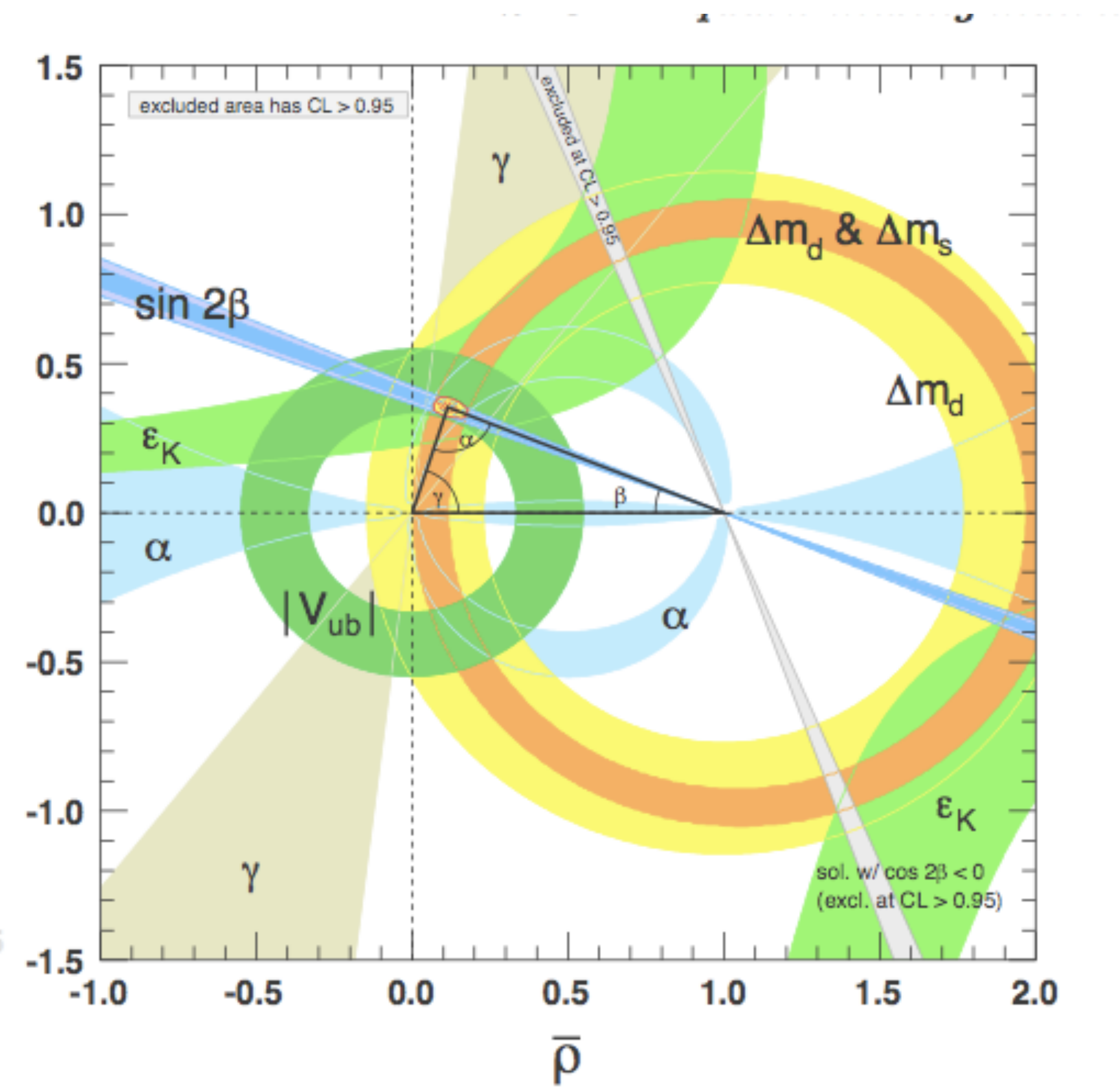
Which means, the Standard Model predictions should not be perfect.



If you look at carefully, they may have some structure (?)

But, it looks perfect..





gure 12.2: Constraints on the $\bar{\rho}, \bar{\eta}$ plane. The shaded areas have 95% CL.

Maybe the replacement happens at a very microscopic scale, i.e., the high energy scale such as 10-100TeV. (energy scale = 1/distance scale)

That’s strange.

We do not expect a hierarchy between the electroweak scale and the scale of the replacement.

$$\text{10-100TeV} \quad \xleftrightarrow{???} \quad \text{125GeV}$$

What is going on?

Maybe there is some reason for this scale separation.

Supersymmetry?

this predicts $m_h \sim m_Z = 91\mathrm{GeV}$
not so bad. But 125GeV is a bit too heavy?

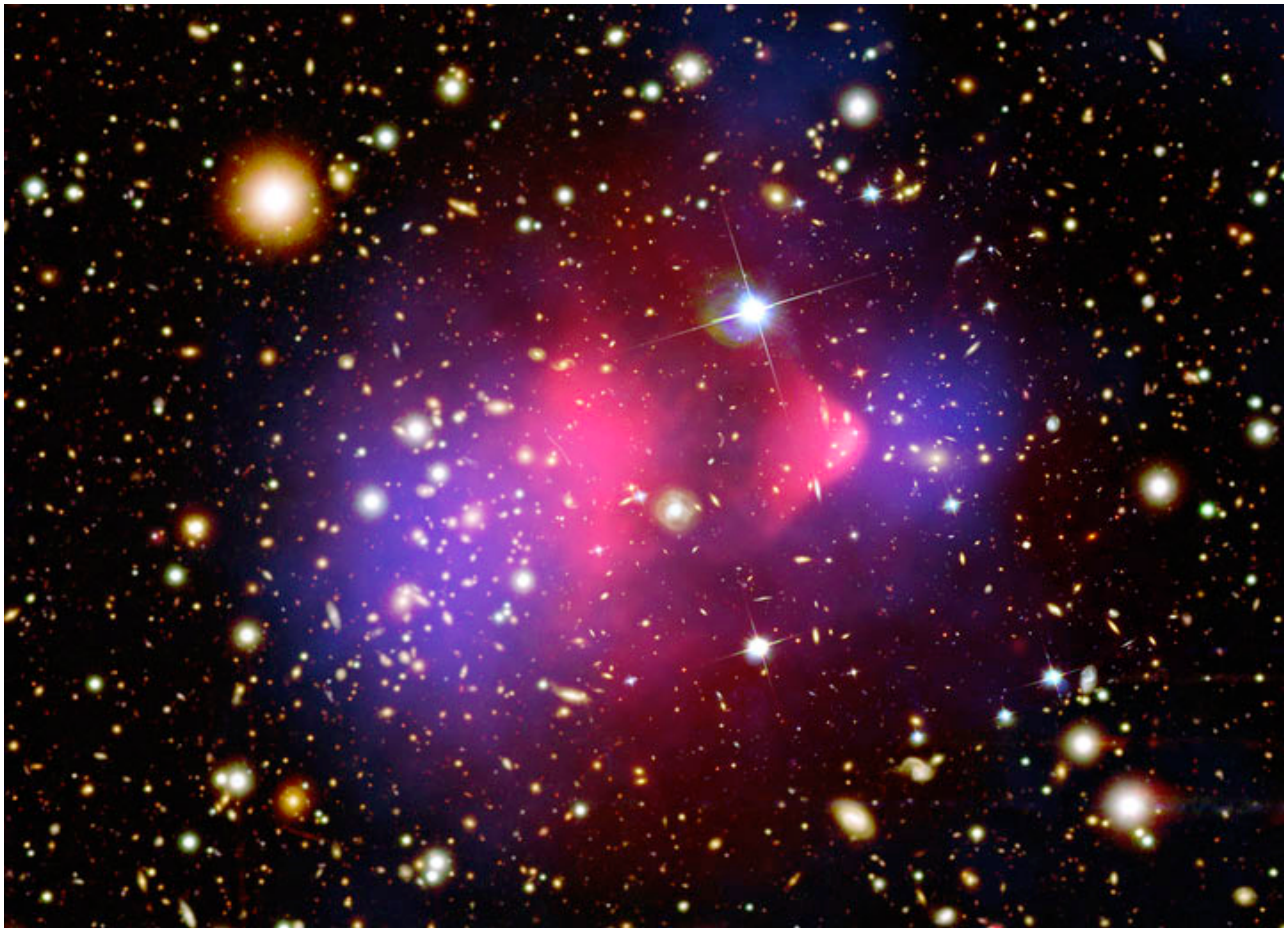
Nambu-Goldstone Higgs?

The Higgs boson maybe originated as the NG boson. Higgs as an approximately massless boson.

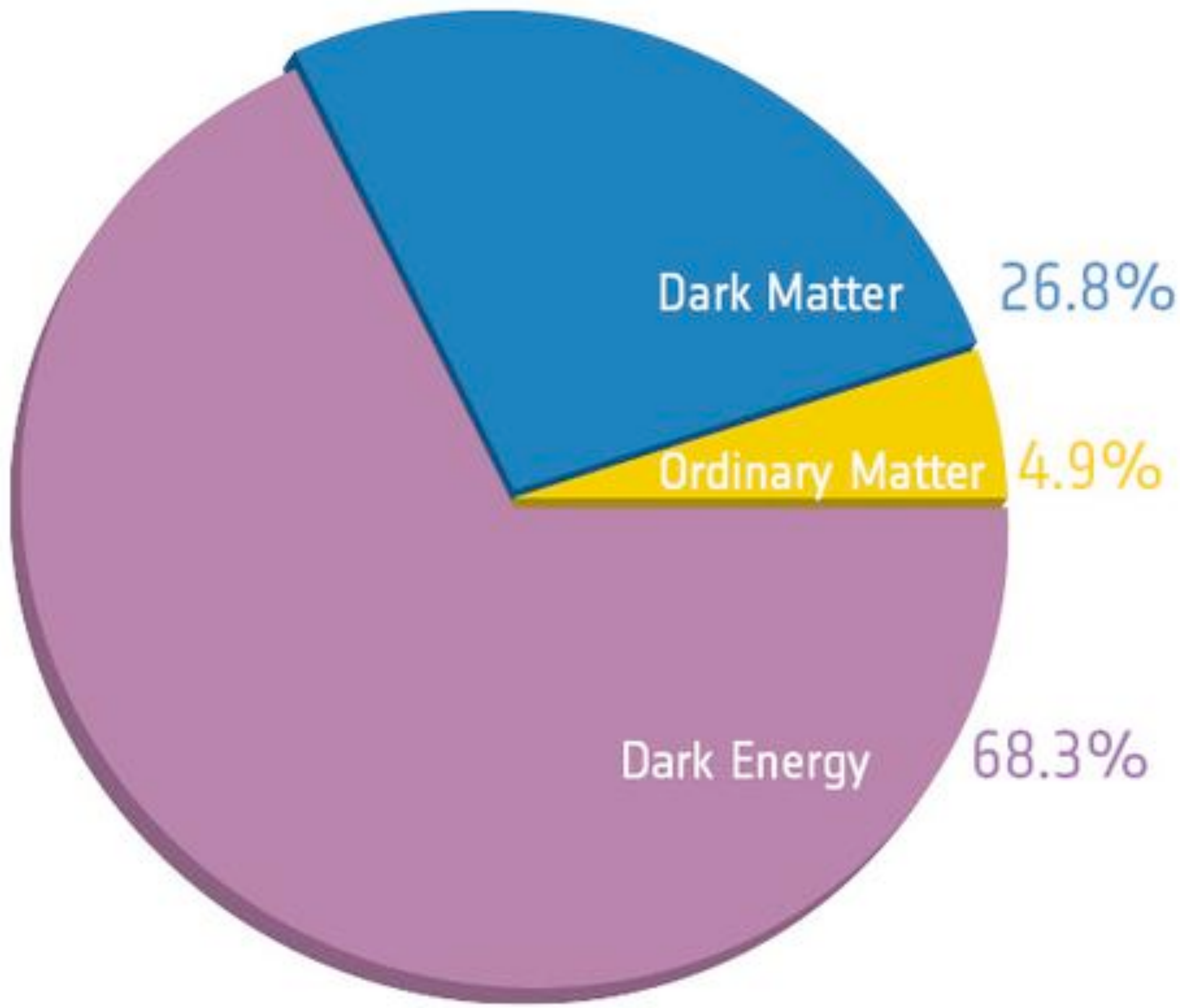
In either cases, there should be new particles at O(TeV) scale.

Where are they hiding?

Dark Matter?



The Standard Model does not explain this component of the Universe.



Actually, the SM of particle physics does not explain any of them.

A possible link to the Higgs physics?

A new stable TeV scale particle can naturally explain dark matter component of the Universe.

WIMP Scenario:

At a very high temperature, the new particle, X, populates.

$$\downarrow$$

At a low temperature, the number density is suppressed by the Boltzmann factor

$$e^{-m_X/T}$$

X can reduce their numbers by pair annihilation processes.

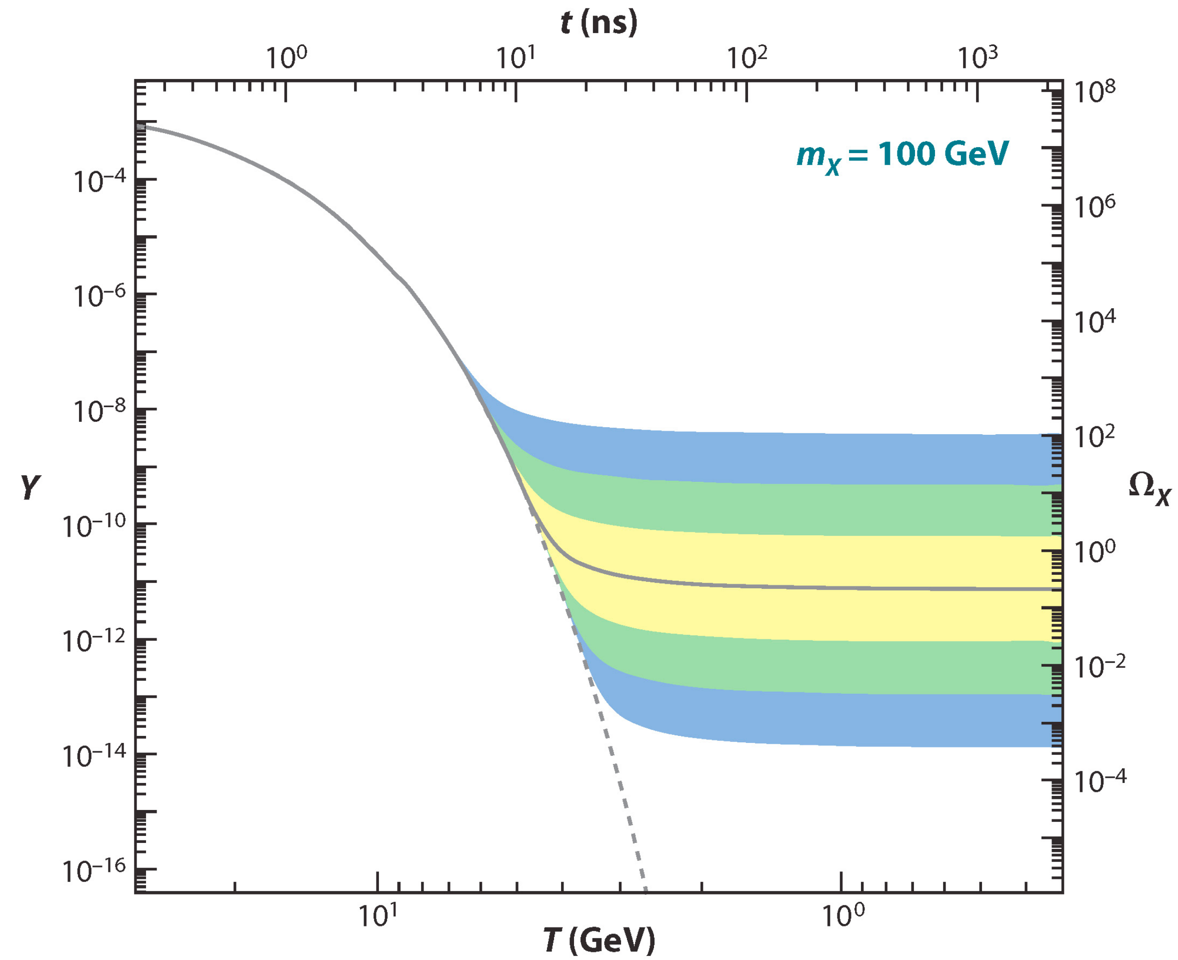
$X + X \to$ SM particles

$\downarrow$

At some point, the typical time scale to find X gets beyond the age of the Universe.

$\downarrow$

number of X doesn't change anymore.
Remain today as the dark matter.



Energy density is completely determined by the annihilation cross section.

It turns out, the required cross section is of order TeV^{-2} scale.

Interesting.

Looking for WIMP dark matter

WIMP must interact with us.

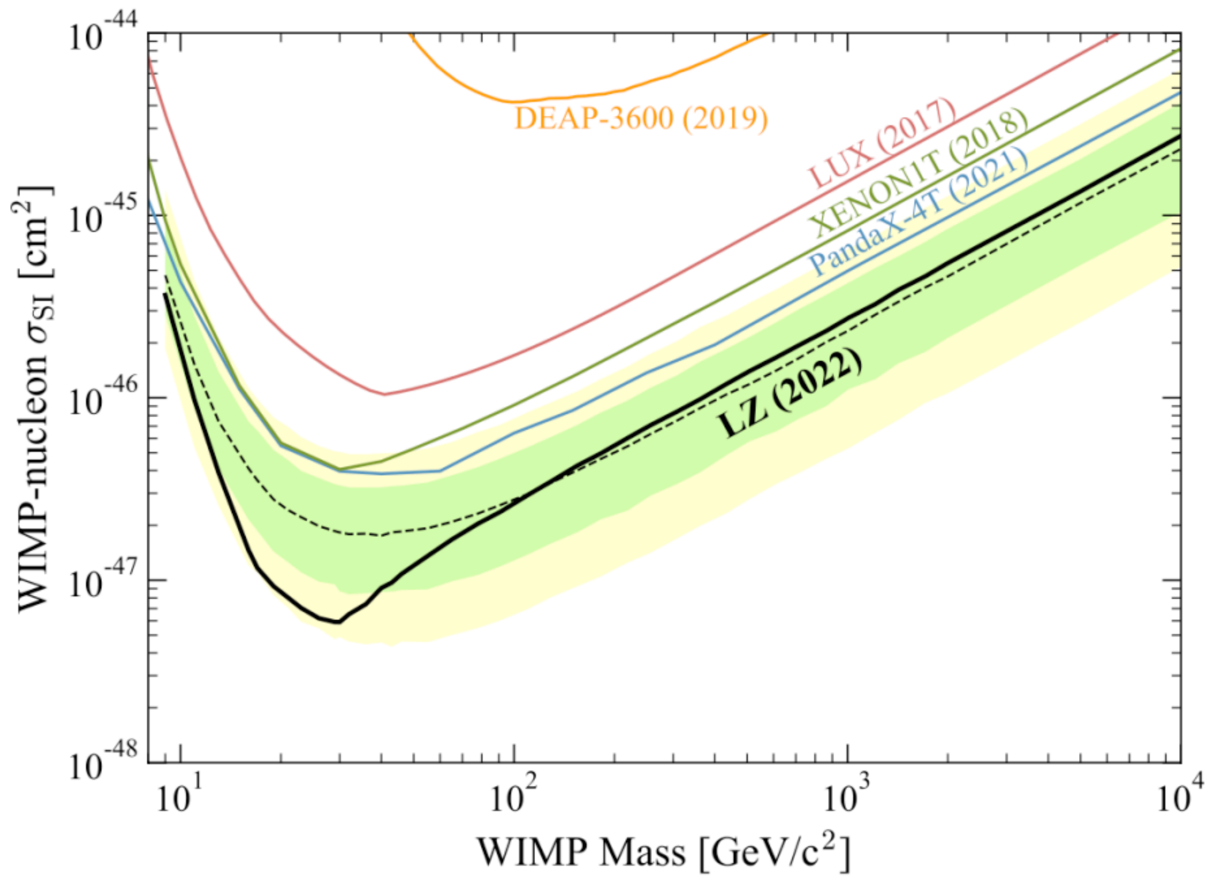
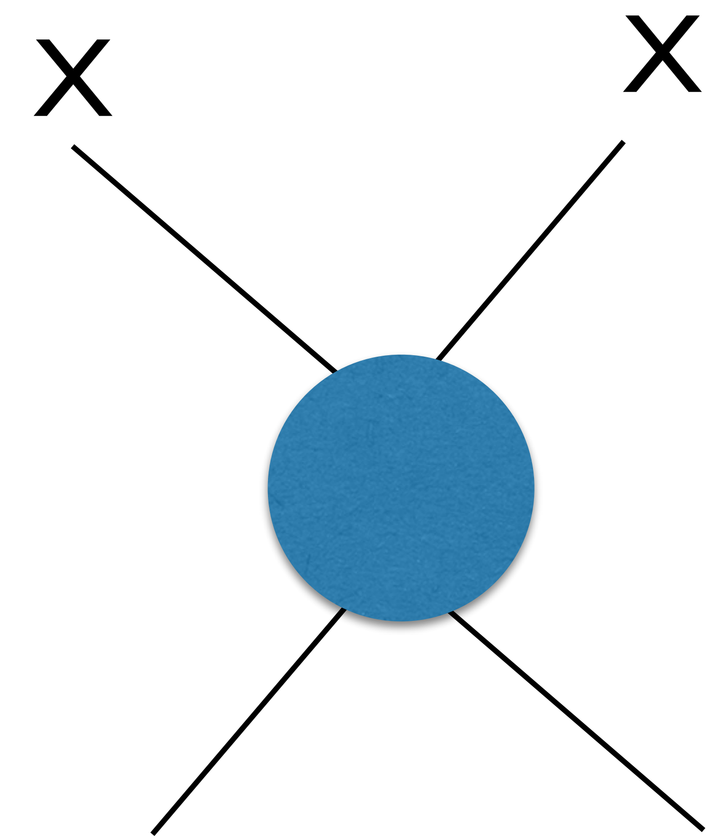
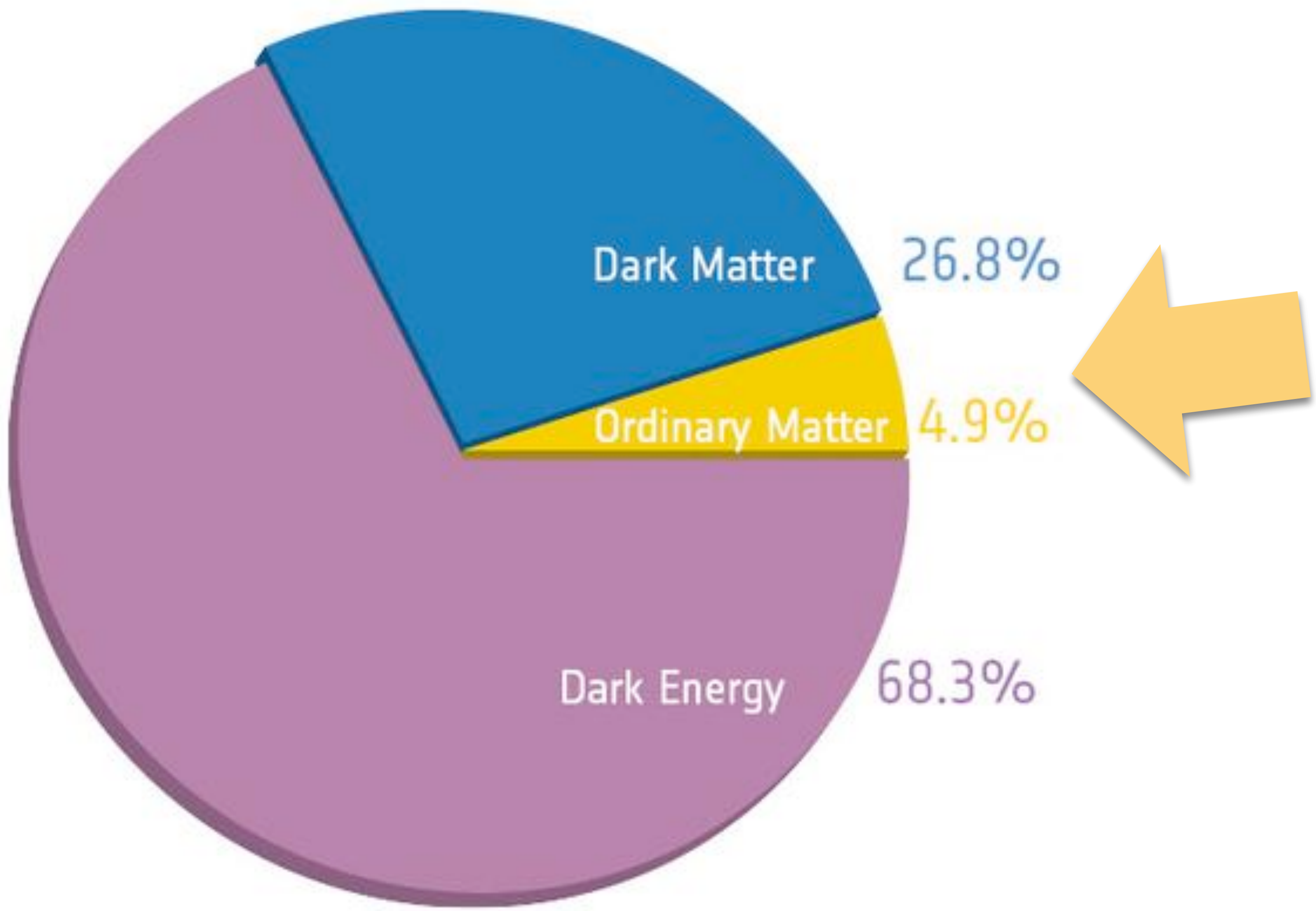


FIG. 5. The 90% confidence limit (black line) for the spin-independent WIMP cross section vs. WIMP mass. The green and yellow bands are the 1σ and 2σ sensitivity bands. The dotted line shows the median of the sensitivity projection. Also shown are the PandaX-4T [26], XENON1T [25], LUX [28], and DEAP-3600 [74] limits.



We may be able to see it soon.

Where are antiparticles?



SM does not have a mechanism for this.
We need CP violation in order for the particle and antiparticle to have different properties.
CP violation through CKM is too small for this purpose.

Speaking of CP…

We have seen that CKM theory explains all the flavor/CP violating processes.

But, actually, there is another source of CP violation in the Standard Model.

There is a parameter called θ which controls the way how the topologically non-trivial configurations contribute to physics.

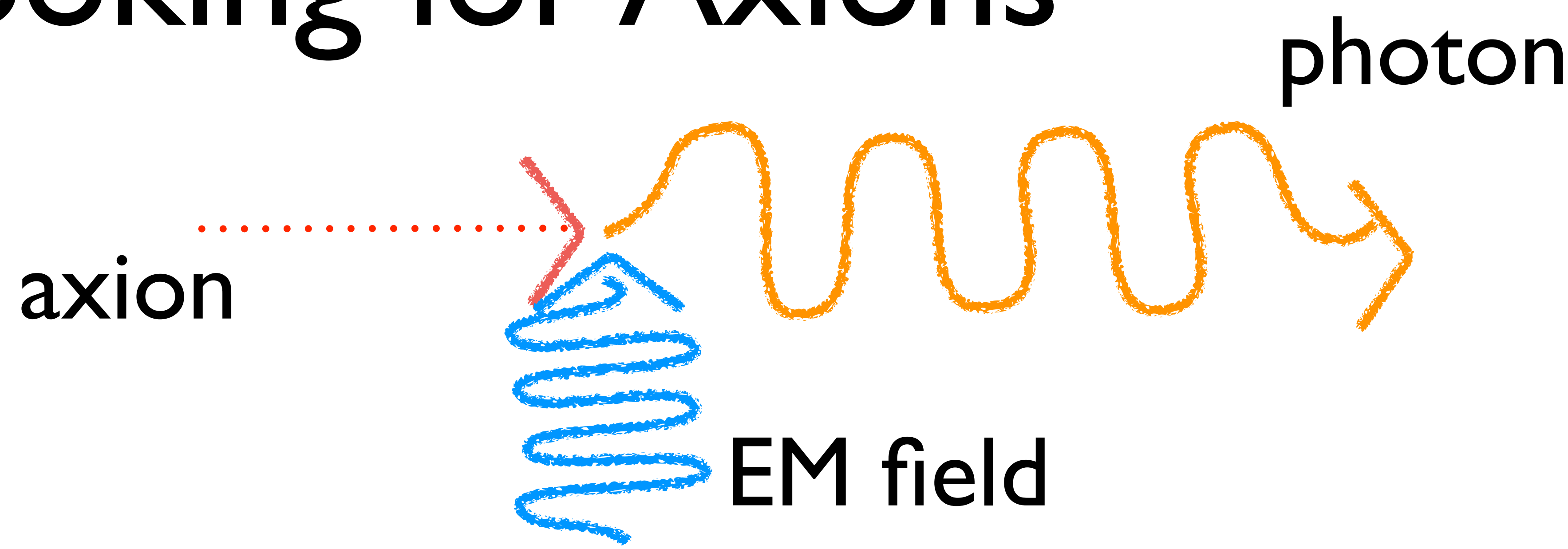
Strong CP problem

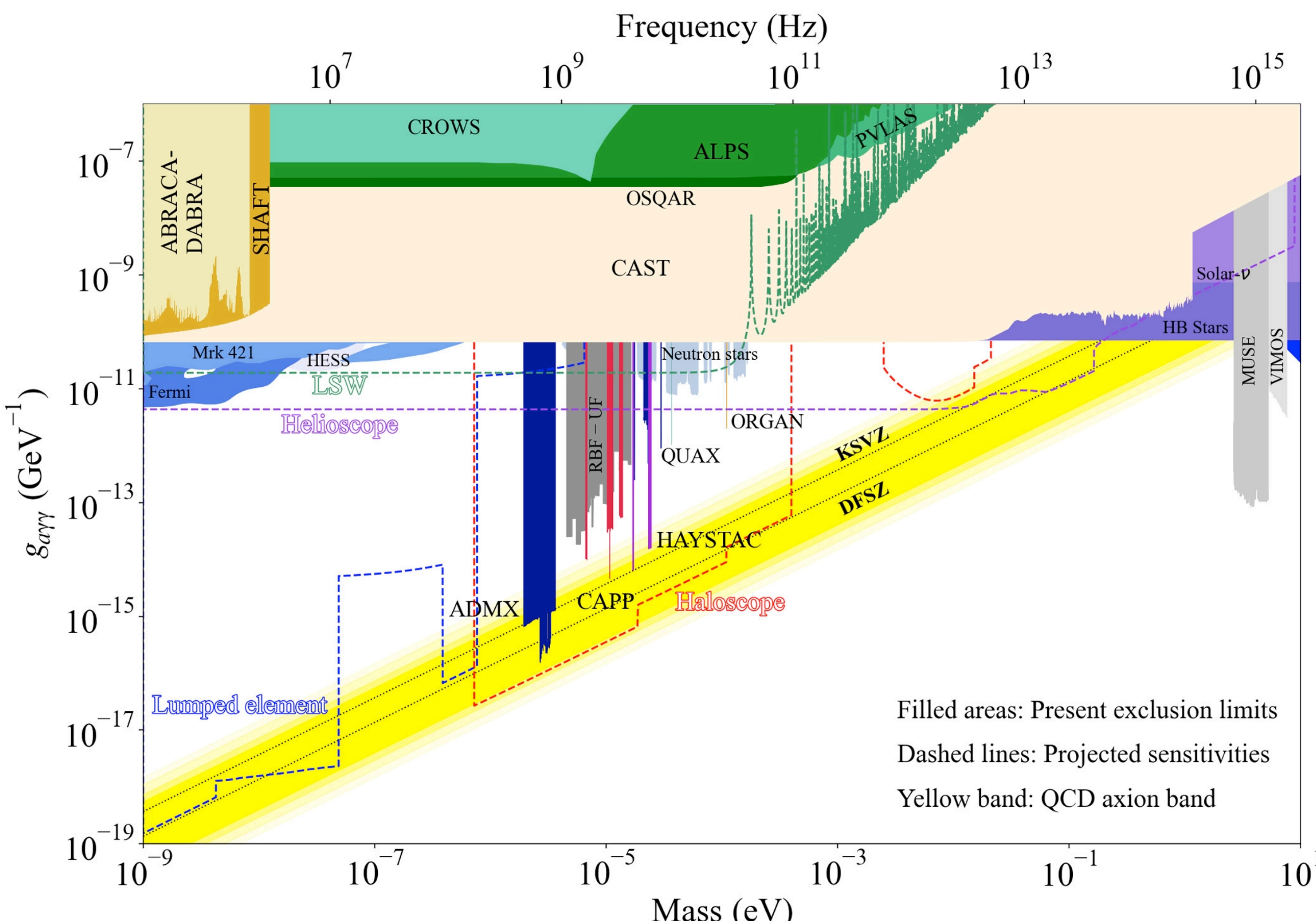
The θ parameter must be very small, such as $< 10^{-9}$.

This problem can be solved if there is a new particle called "Axion."

Interestingly, the axion with masses O(0.01meV) can be the dark matter of the Universe.

Looking for Axions



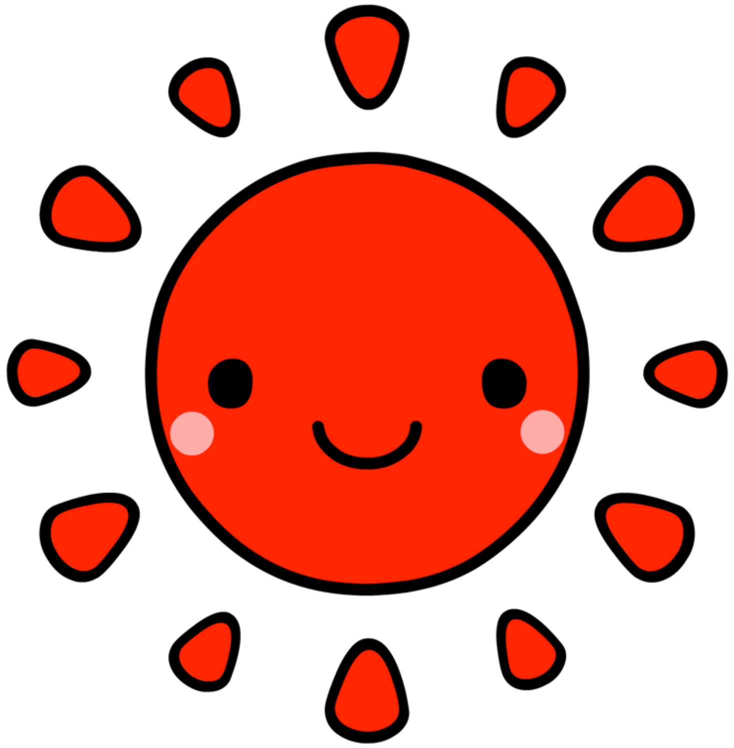


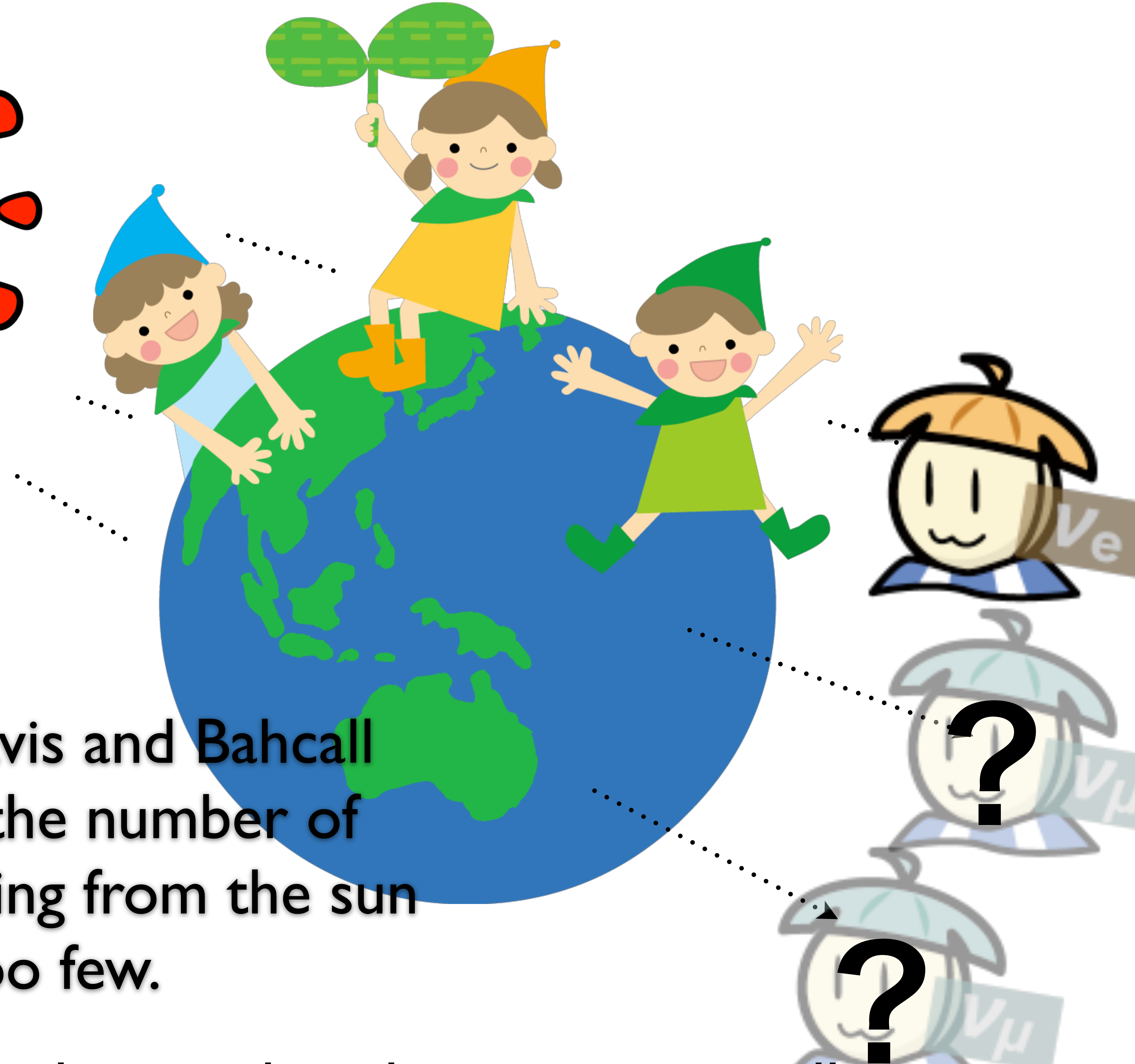
taken from [Y. Semertzidis and S. Youn '22]

Neutrino masses

We are now sure that the neutrino have finite masses.
Where do they come from?

It is pretty fun to learn how we found
neutrino masses.

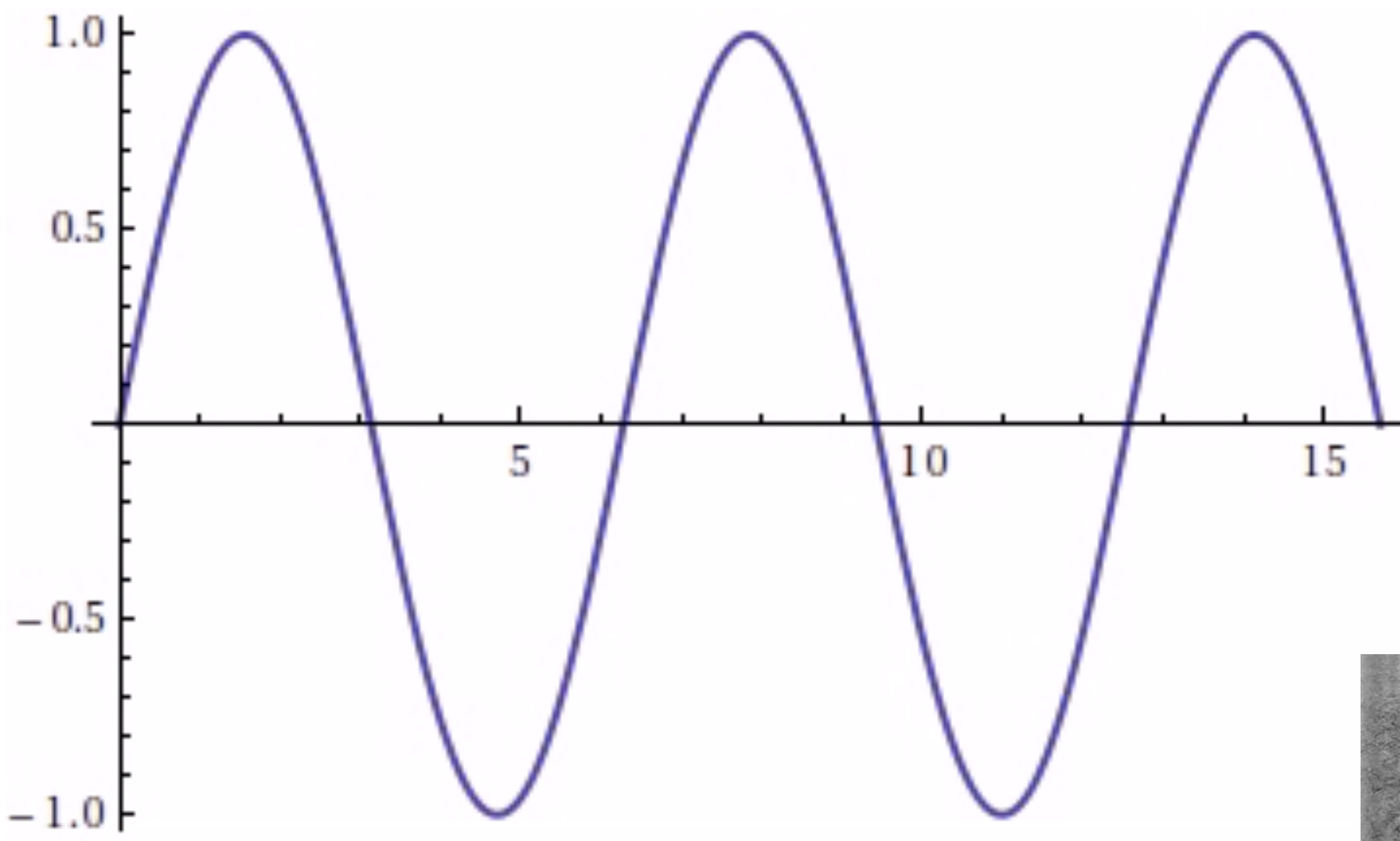




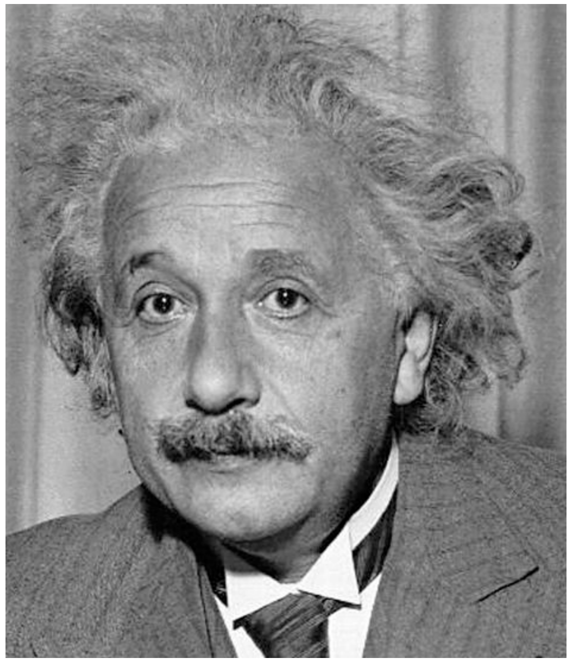
in 1960's Davis and Bahcall found that the number of neutrino coming from the sun is too few.

Now it is understood as the neutrino oscillation.

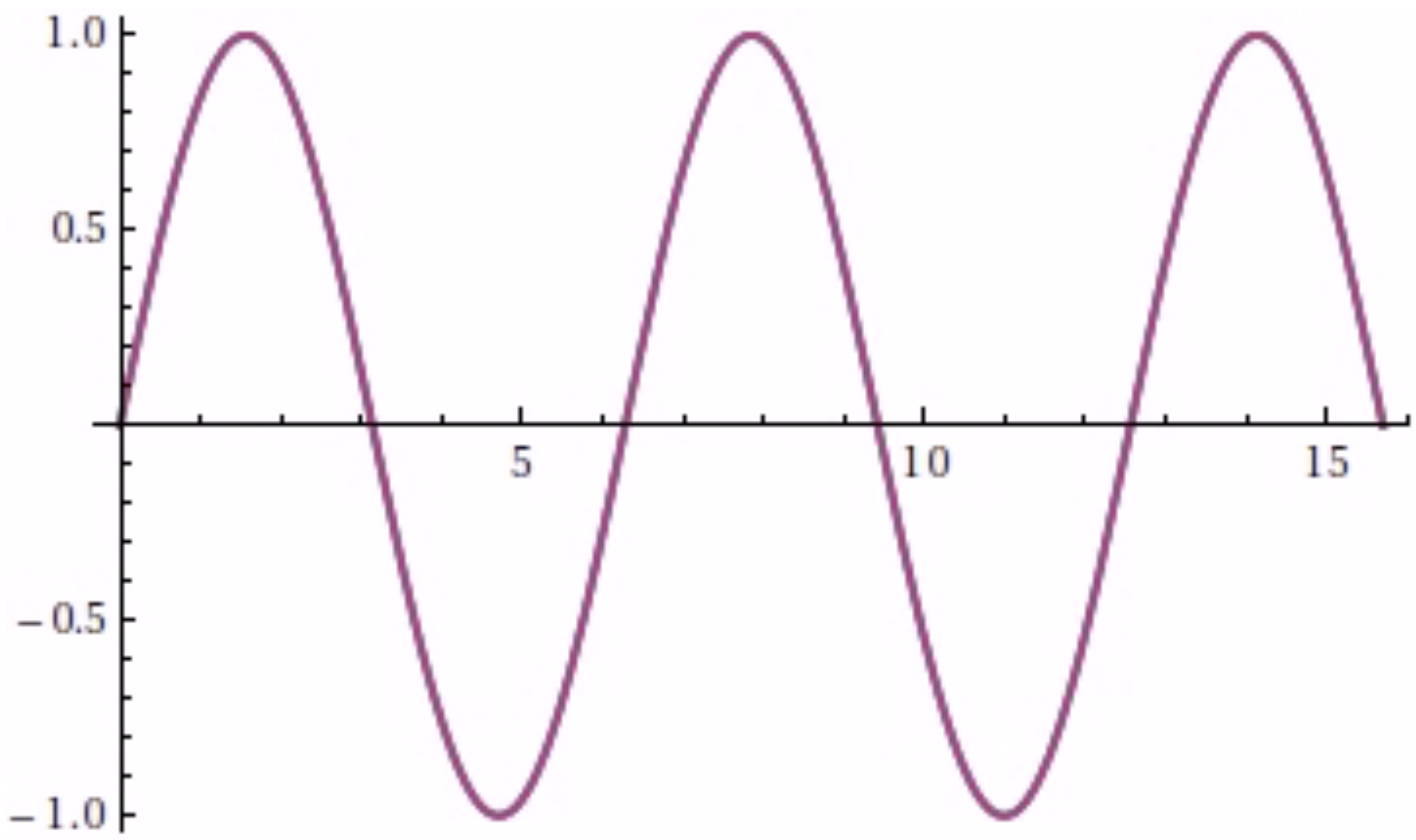
Neutrinos are waves.



$E=h\nu$



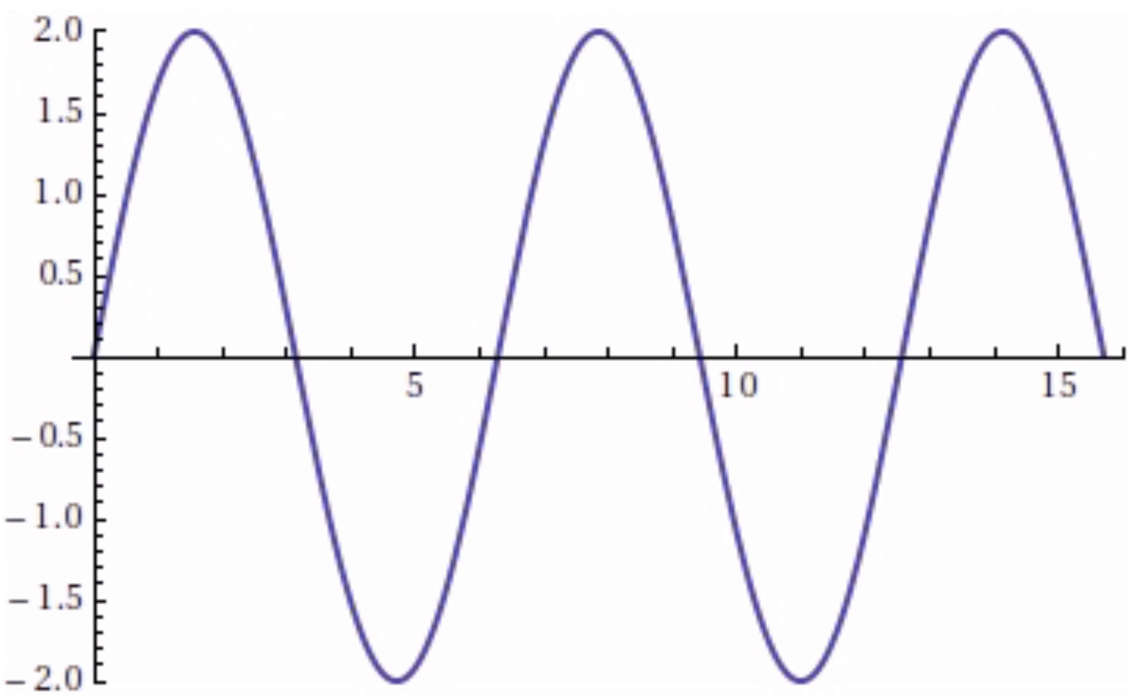
Now let's think of two waves with slightly different energies.



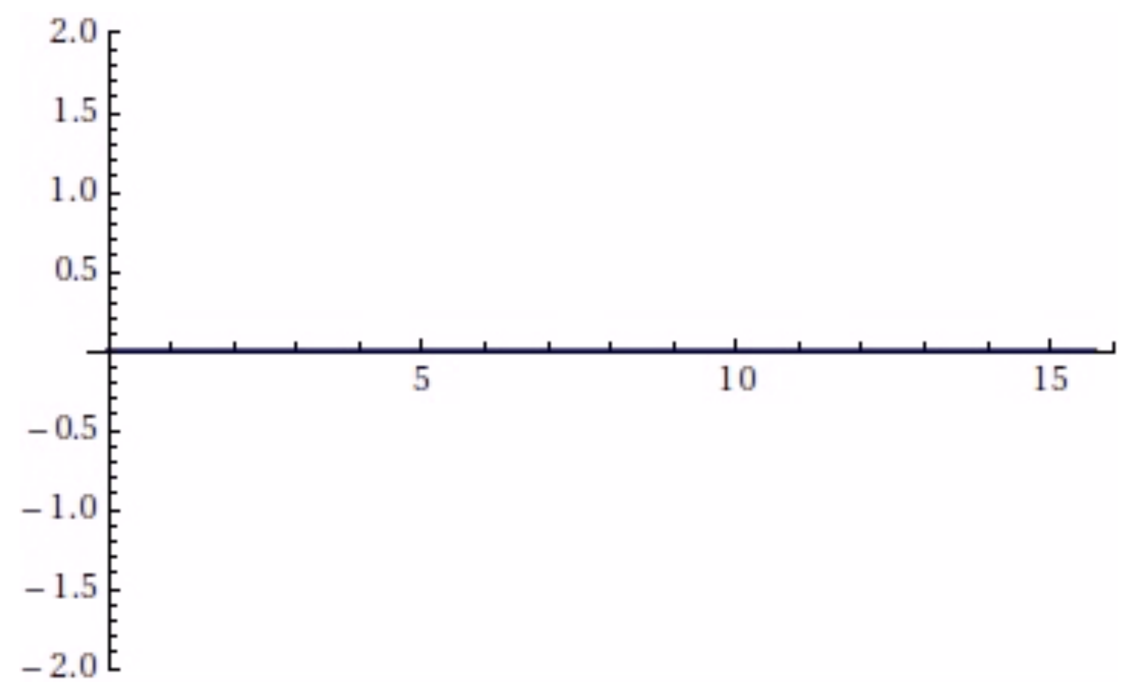
gradually waves separate away

Now look at the sum and the difference

<u>sum</u>



<u>diff</u>



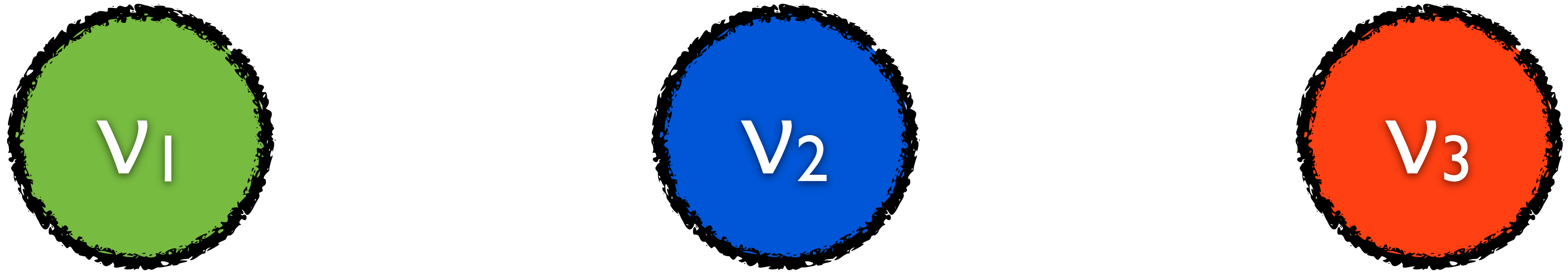
we see oscillation.

In this way, if the electron neutrino is a superposition of neutrinos with different masses, neutrinos can oscillate into different flavors, explaining the deficit.

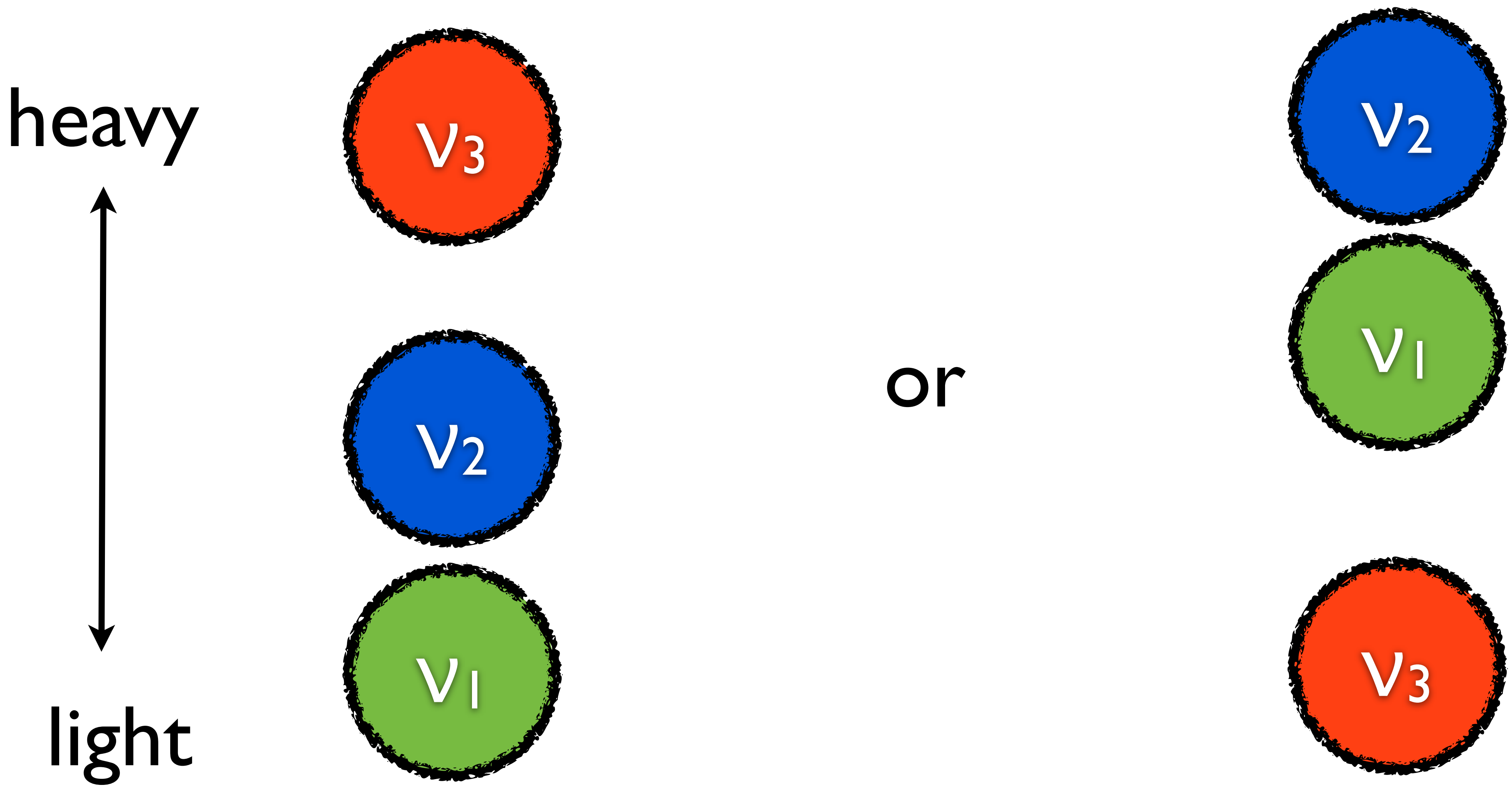
$= 0.8\ \nu_1 + 0.6\ \nu_2 + 0.15\ \nu_3$

$= -0.5\ \nu_1 + 0.5\ \nu_2 + 0.7\ \nu_3$

$= -0.3\ \nu_1 - 0.6\ \nu_2 + 0.7\ \nu_3$



three neutrinos with different masses



We don't know which is the case, yet.

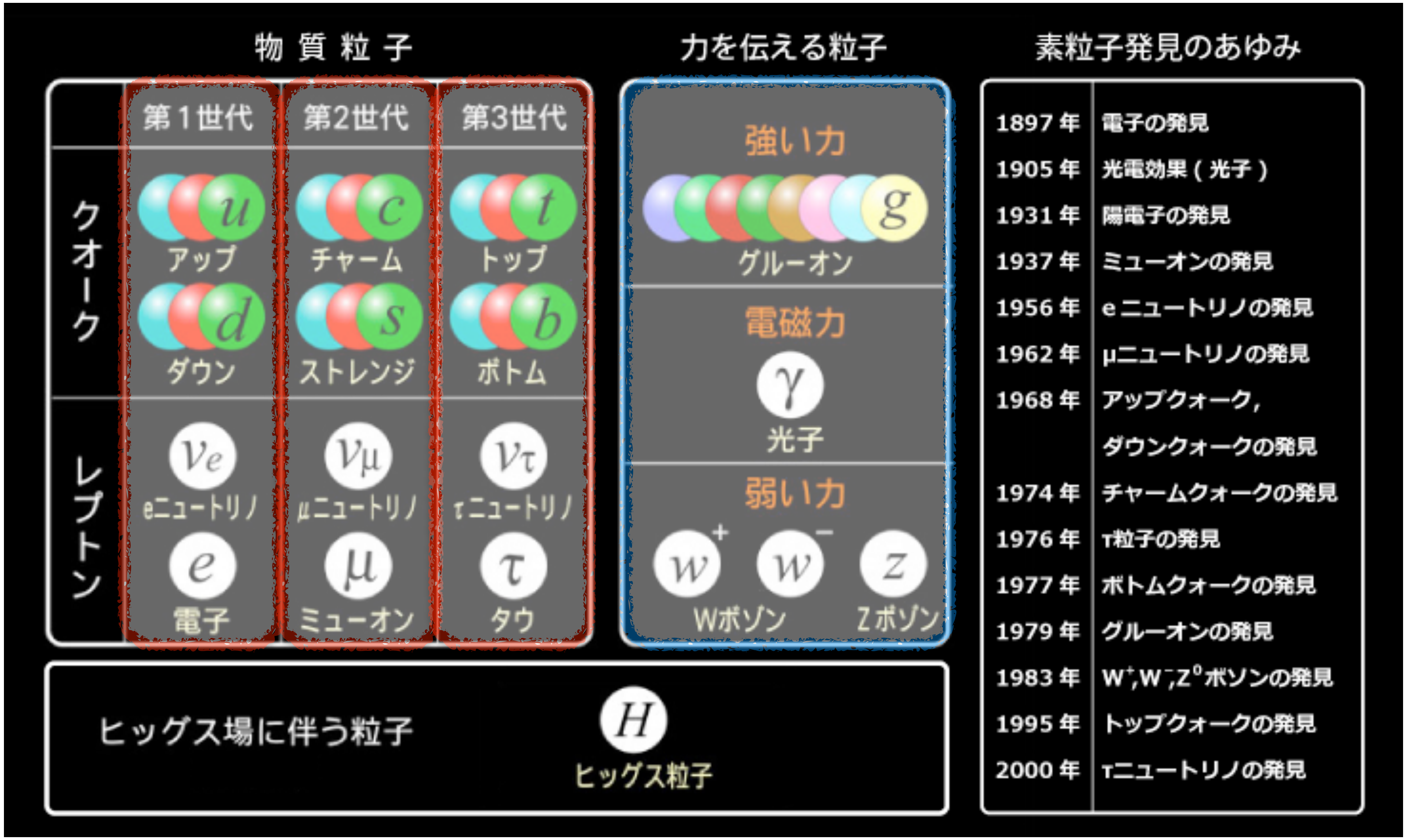
In any case, the masses are tiny,

at most O(0.1eV).

Probably, this came from a very high energy physics,

such as Grand Unified Theory.

Grand Unification?



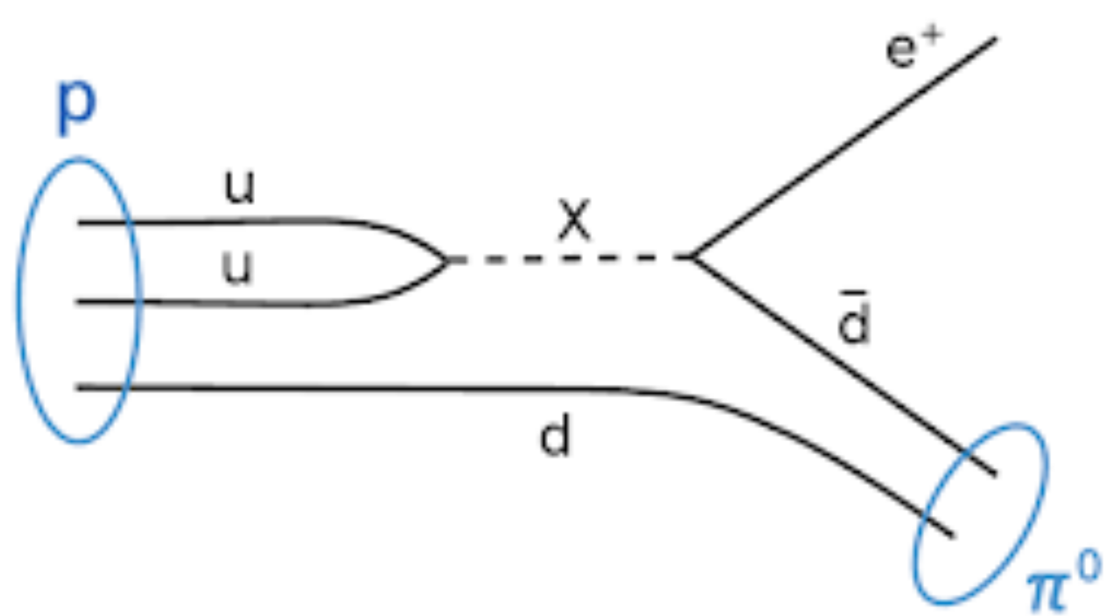
It is interesting that gauge theory based on SO(10) can unify all the forces and fermions.

It also contains right-handed neutrinos which account for the neutrino masses.

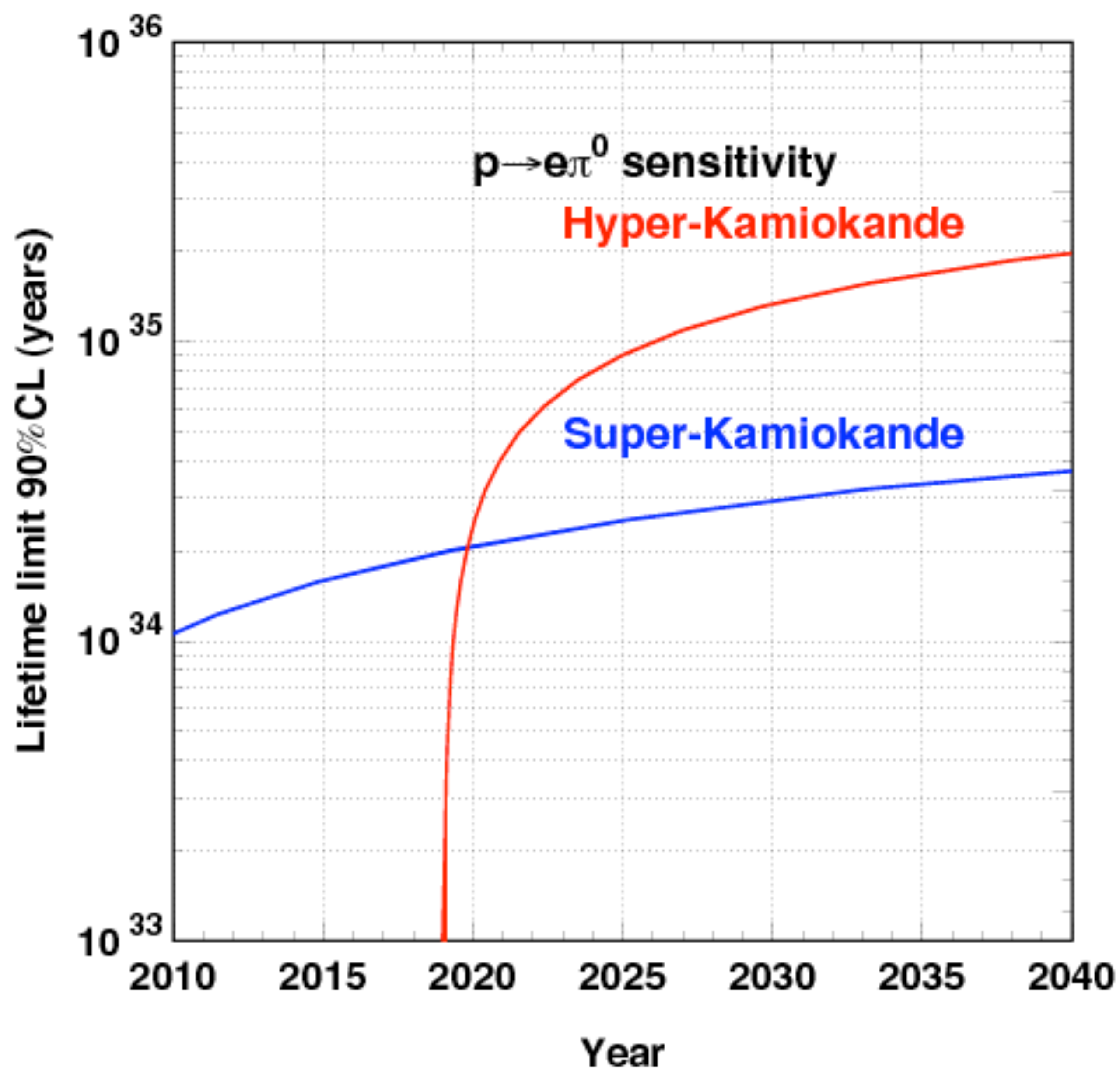
Higgs seems not be unified…

Could be related to the origin of the Higgs field?

One interesting consequence: proton decay



Looking for Proton decay



Theory predictions are around $10^{34\text{-}36}$ years.

we may see it!

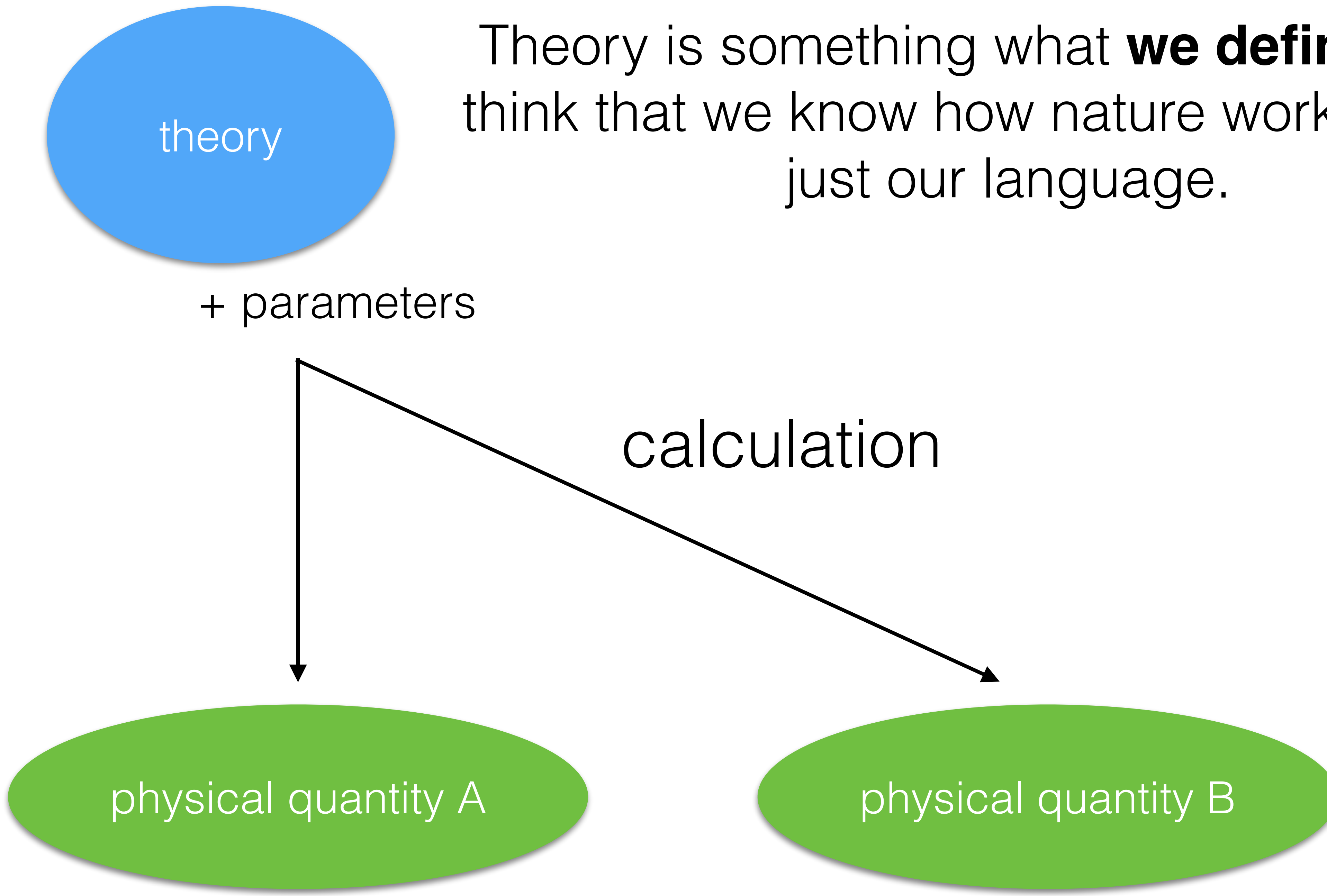
There are various mysteries in the SM…

Big discoveries may be waiting for us.

Lecture 2: Renormalization and naturalness

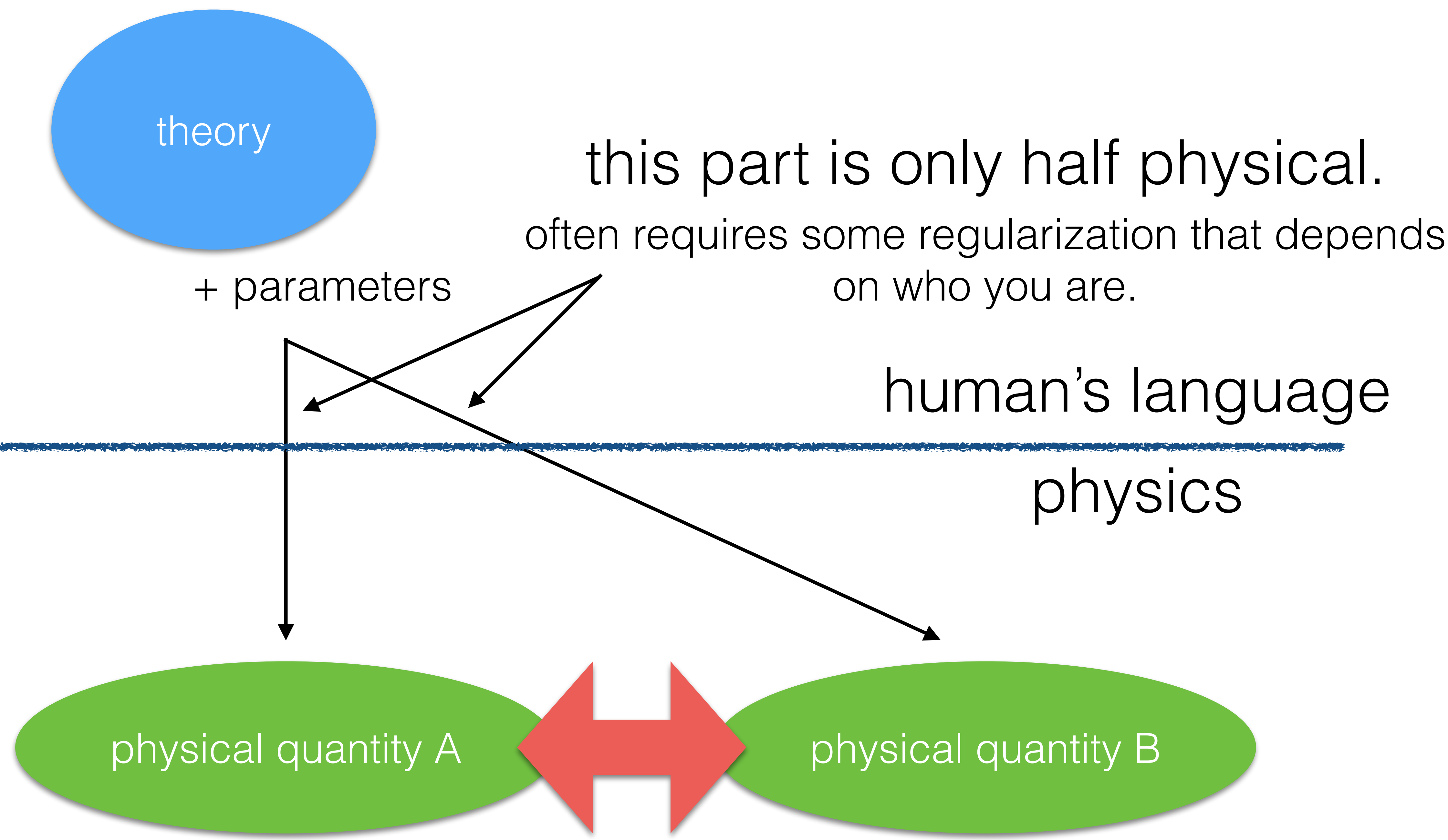
Let me explain how theory works in general.

Theory is something what **we define**. Don’t think that we know how nature works. This is just our language.



this is the usual way of thinking.

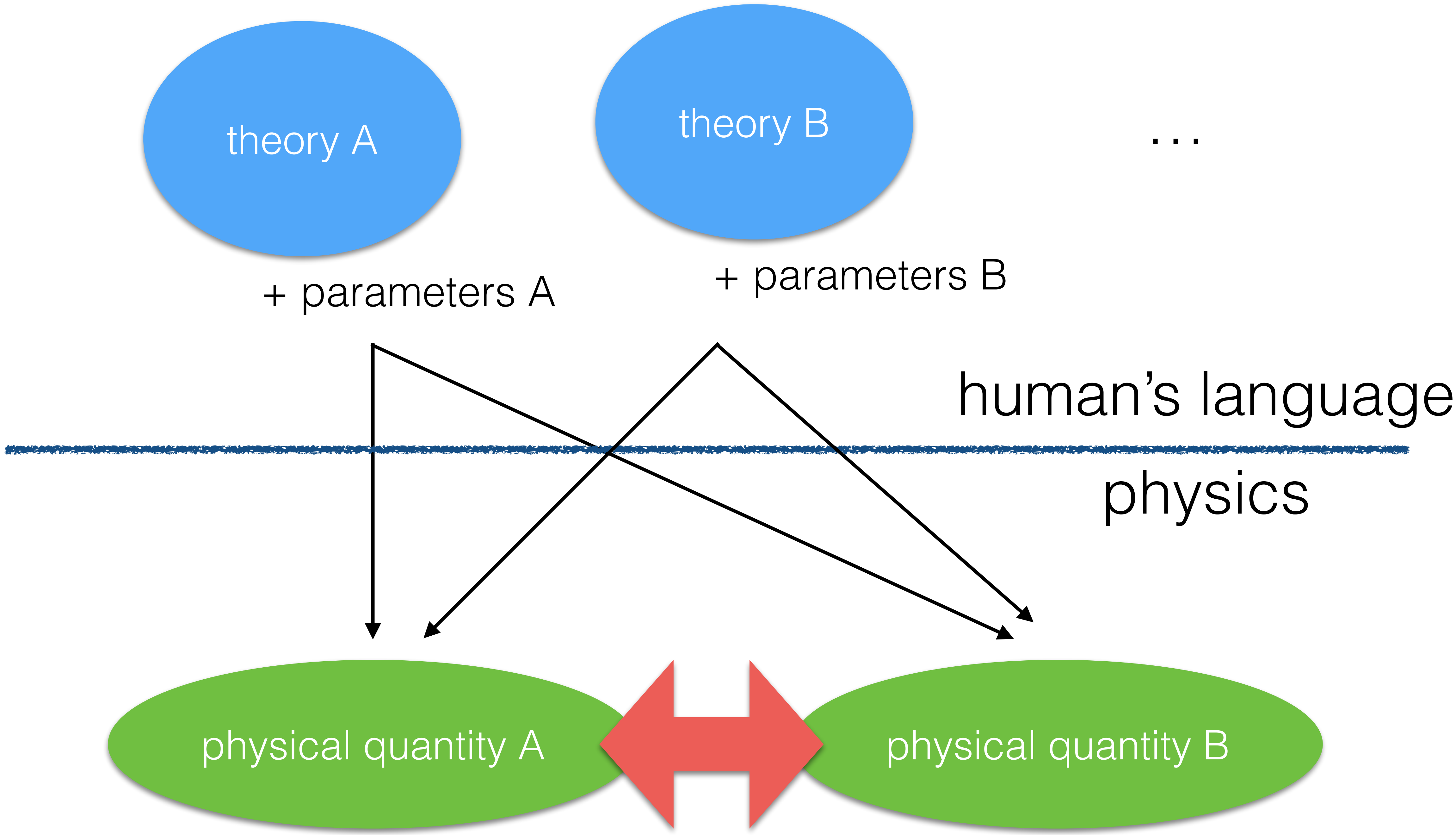
But, in reality



what we'd like to know is this relation!

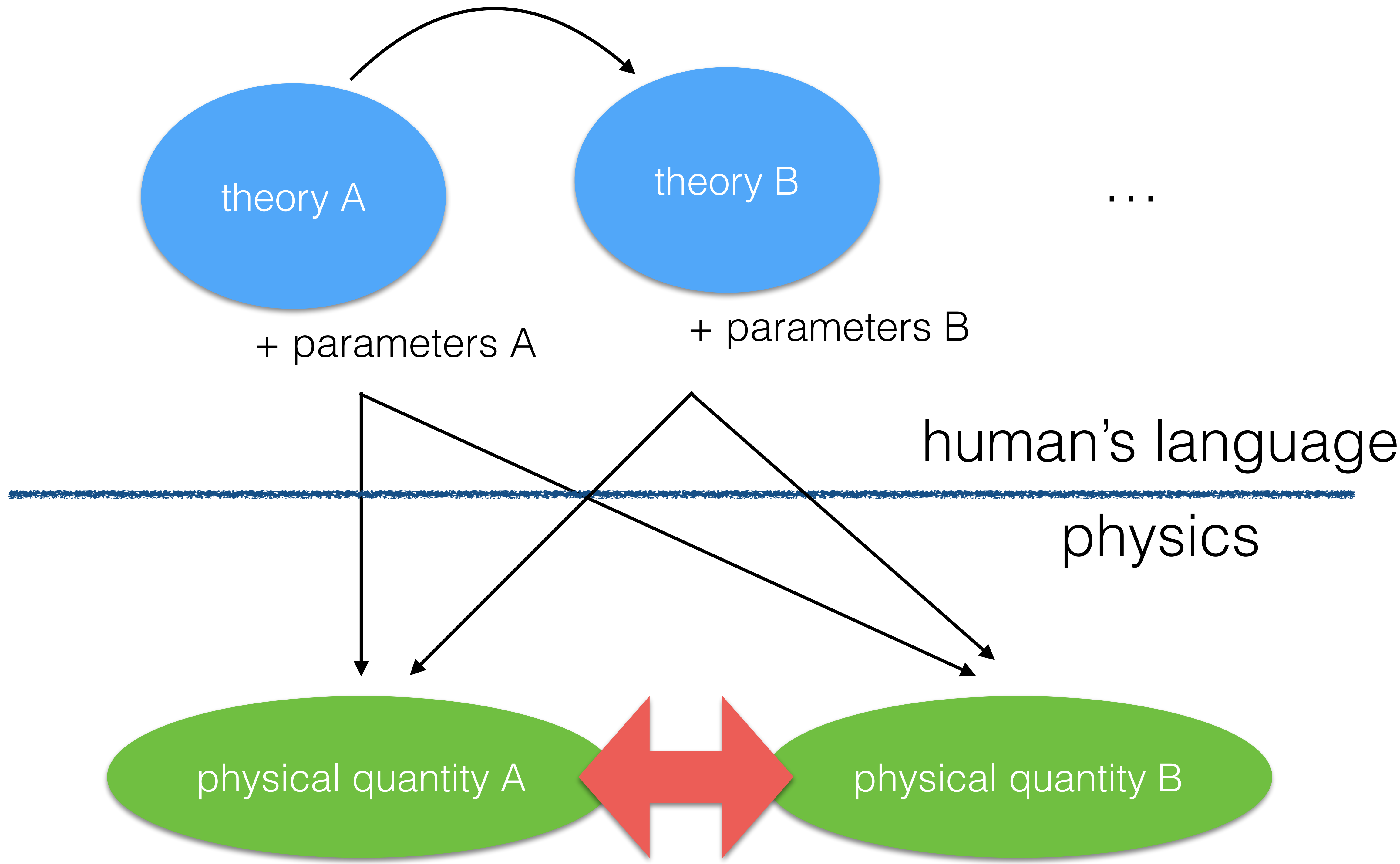
this is the physics question!

In fact, **there can be many different theories that give the same relation.**



what we'd like to know is this relation!

Renormalization is this type of transformation.



what we'd like to know is this relation!

[See textbook of A. Zee]

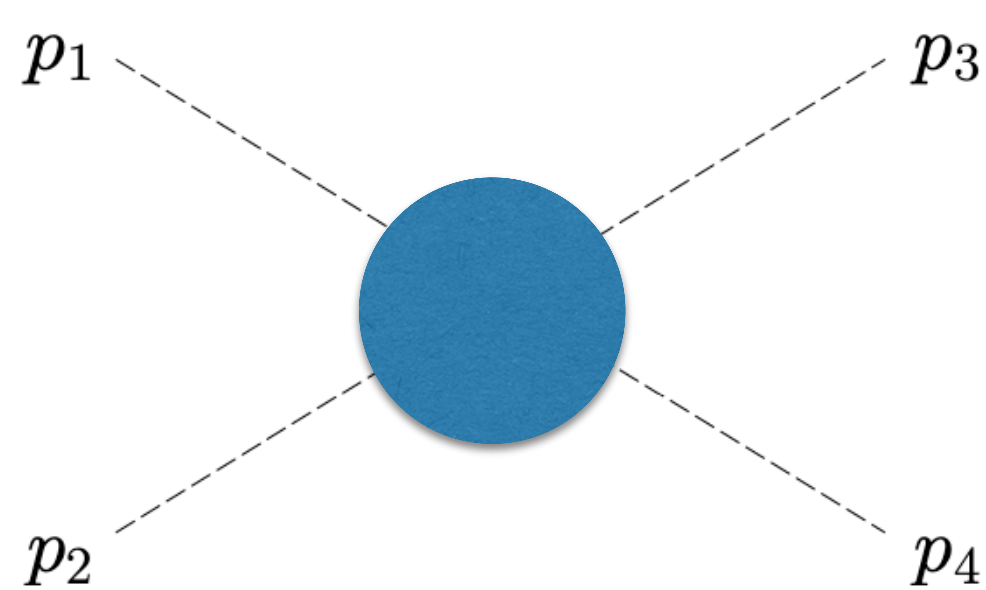
OK, Let's discuss in more detail.

Let's consider a simple model:

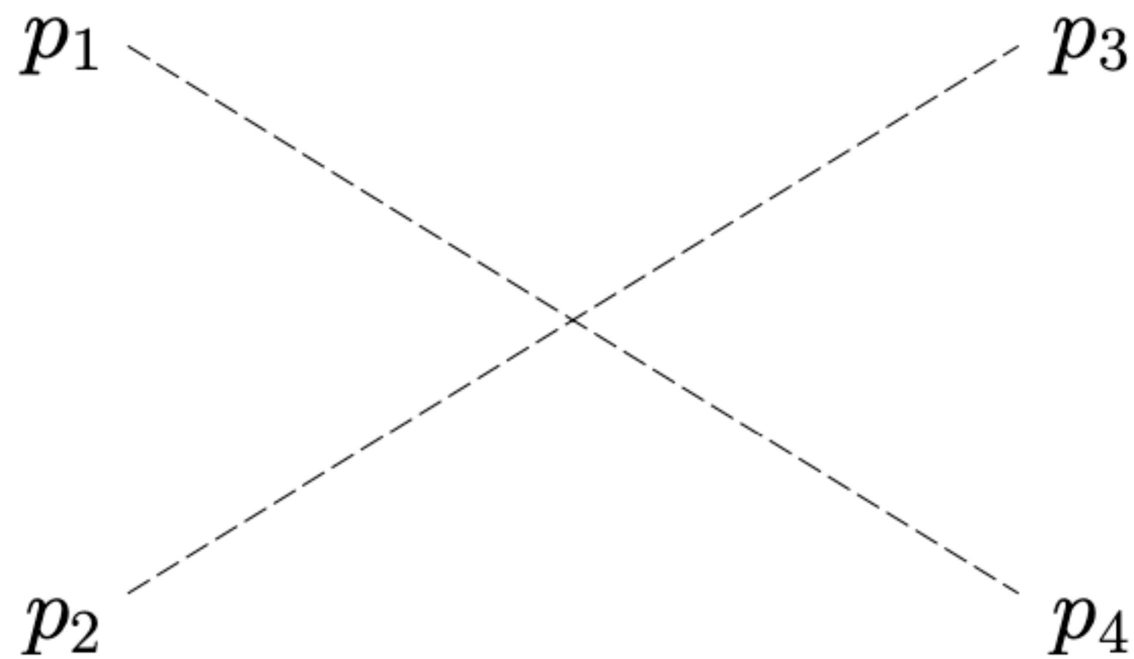
$$\mathcal{L}_{\rm int} = -\frac{\lambda}{4}|\phi|^4$$

and calculate a **physical quantity**.

scattering amplitude (or cross section with a fixed energy)



At the lowest order in perturbation theory,



$$M = -\lambda + O(\lambda^2)$$

Very simple. If we ignore $O(\lambda^2)$ terms,

we can determine the Lagrangian parameter λ, by an experiment with a fixed energy.

cross section: $M = -\lambda + O(\lambda^2)$

$$d\sigma = \frac{1}{16\pi}\frac{|M|^2}{s}\frac{d\Omega}{4\pi}, \qquad s = (p_1 + p_2)^2$$

energy

$$\frac{\sigma(s)}{\sigma(s_0)} = \frac{s_0}{s}$$ up to O(λ²)

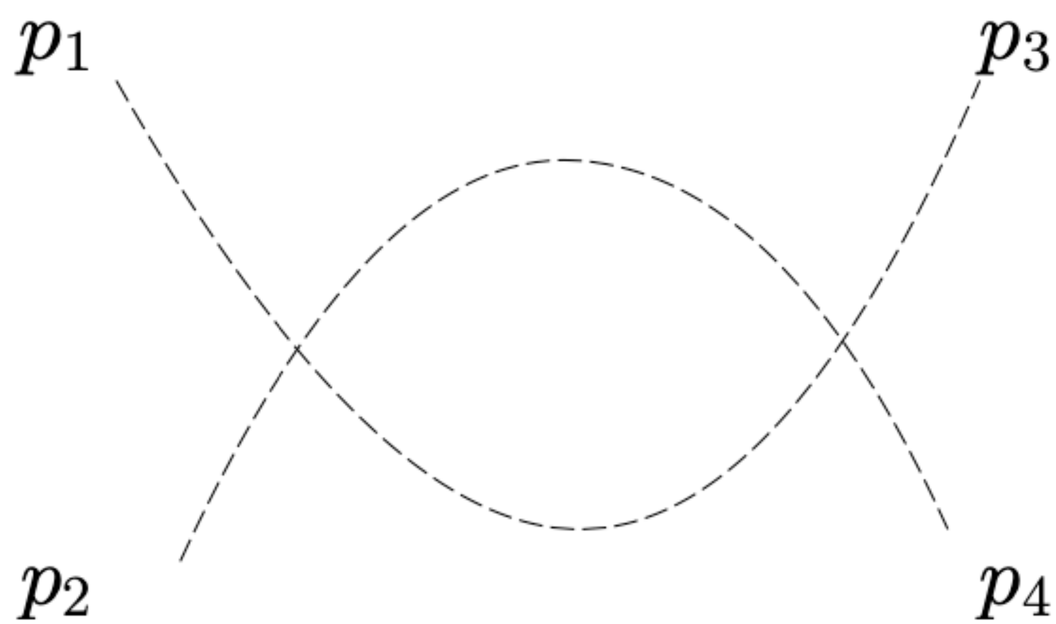
or more simply, $$\frac{M(s)}{M(s_0)} = 1$$ up to O(λ²)

This is the **prediction** of the theory.

This is a non-trivial relation between physical quantities.

It doesn't contain Lagrangian parameter, λ.

Now let's consider O(λ²) terms.



$+\ldots$

$$M = -\lambda + \lambda^2 \int \frac{d^4k}{i(2\pi)^4} \frac{1}{k^2 - m^2} \frac{1}{(k + p_1 + p_2)^2 - m^2} + O(\lambda^3)$$

This integral diverges….

Well, let's cut-off the integral.

(We are in the human side. Whatever you do is a part of the theory.)

Clearly artificial treatment, but let's just do that.

$$M_\Lambda = -\lambda + \lambda^2 \int_{|k|<\Lambda} \frac{d^4k}{i(2\pi)^4} \frac{1}{k^2 - m^2} \frac{1}{(k+p_1+p_2)^2 - m^2} + O(\lambda^3).$$

(let's ignore m^2 for simplicity)

$$M_\Lambda = -\lambda + C\lambda^2 \left[\log\left(\frac{\Lambda^2}{s}\right) + \log\left(\frac{\Lambda^2}{t}\right) + \log\left(\frac{\Lambda^2}{u}\right) \right] + O(\lambda^3).$$

$$s = (p_1+p_2)^2, \quad t = (p_1 - p_3)^2, \quad u = (p_1 - p_4)^2$$

(C is some numerical factor. Not important for this discussion.)

Then what?

Actually that's it. Remember that what we are doing is to obtain a correction to

$$\frac{M(s)}{M(s_0)} = 1 \quad \text{up to } O(\lambda^2)$$

We can actually do this already.

$$\begin{aligned}
M(s_1,t_1,u_1) &= -\lambda + C\lambda^2\left[\log\left(\frac{\Lambda^2}{s_1}\right) + \log\left(\frac{\Lambda^2}{t_1}\right) + \log\left(\frac{\Lambda^2}{u_1}\right)\right] + O(\lambda^3) \\
&= -\lambda + C\lambda^2\left[\log\left(\frac{\Lambda^2}{s_0}\right) + \log\left(\frac{\Lambda^2}{t_0}\right) + \log\left(\frac{\Lambda^2}{u_0}\right)\right] + O(\lambda^3) \\
&\quad + C\lambda^2\left[\log\left(\frac{s_0}{s_1}\right) + \log\left(\frac{t_0}{t_1}\right) + \log\left(\frac{u_0}{u_1}\right)\right] + O(\lambda^3) \\
&= M(s_0,t_0,u_0) + C\lambda^2\left[\log\left(\frac{s_0}{s_1}\right) + \log\left(\frac{t_0}{t_1}\right) + \log\left(\frac{u_0}{u_1}\right)\right] + O(\lambda^3) \\
&= M(s_0,t_0,u_0) + CM^2(s_0,t_0,u_0)\left[\log\left(\frac{s_0}{s_1}\right) + \log\left(\frac{t_0}{t_1}\right) + \log\left(\frac{u_0}{u_1}\right)\right] + O(\lambda^3).
\end{aligned}$$

The last line doesn't contain λ or Λ!
We now find that M depends on energies.

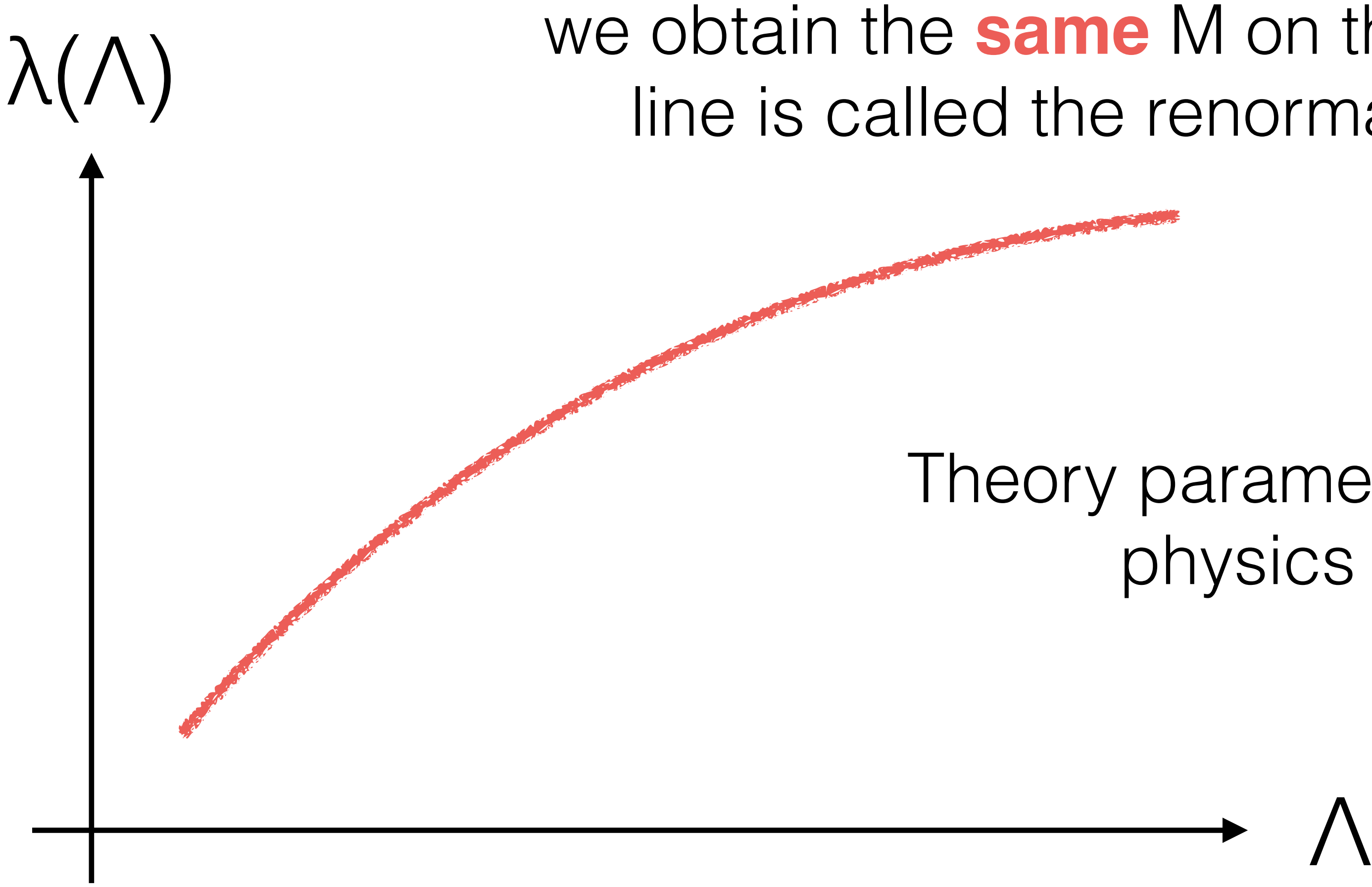
OK, we see that we can do it.

$$M_\Lambda = -\lambda + C\lambda^2 \left[\log\left(\frac{\Lambda^2}{s}\right) + \log\left(\frac{\Lambda^2}{t}\right) + \log\left(\frac{\Lambda^2}{u}\right)\right] + O(\lambda^3).$$

This is already a non-trivial prediction.

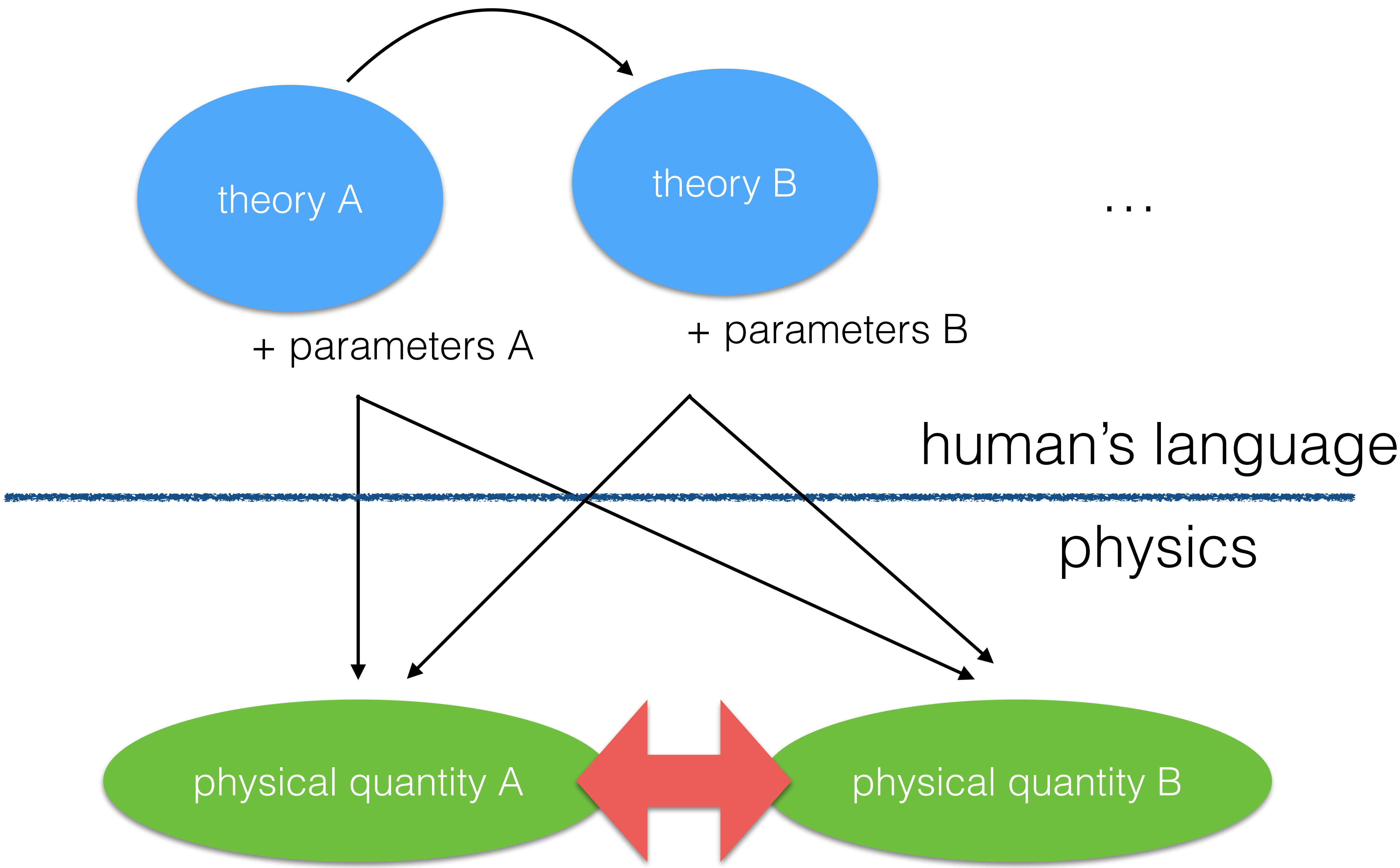
By looking at the formula, we realize that

we obtain the **same** M on this line. Moving on this line is called the renormalization group flow.



Theory parameters changed while physics unchanged.

Now we understand that the renormalization is not quite physics, just our language.

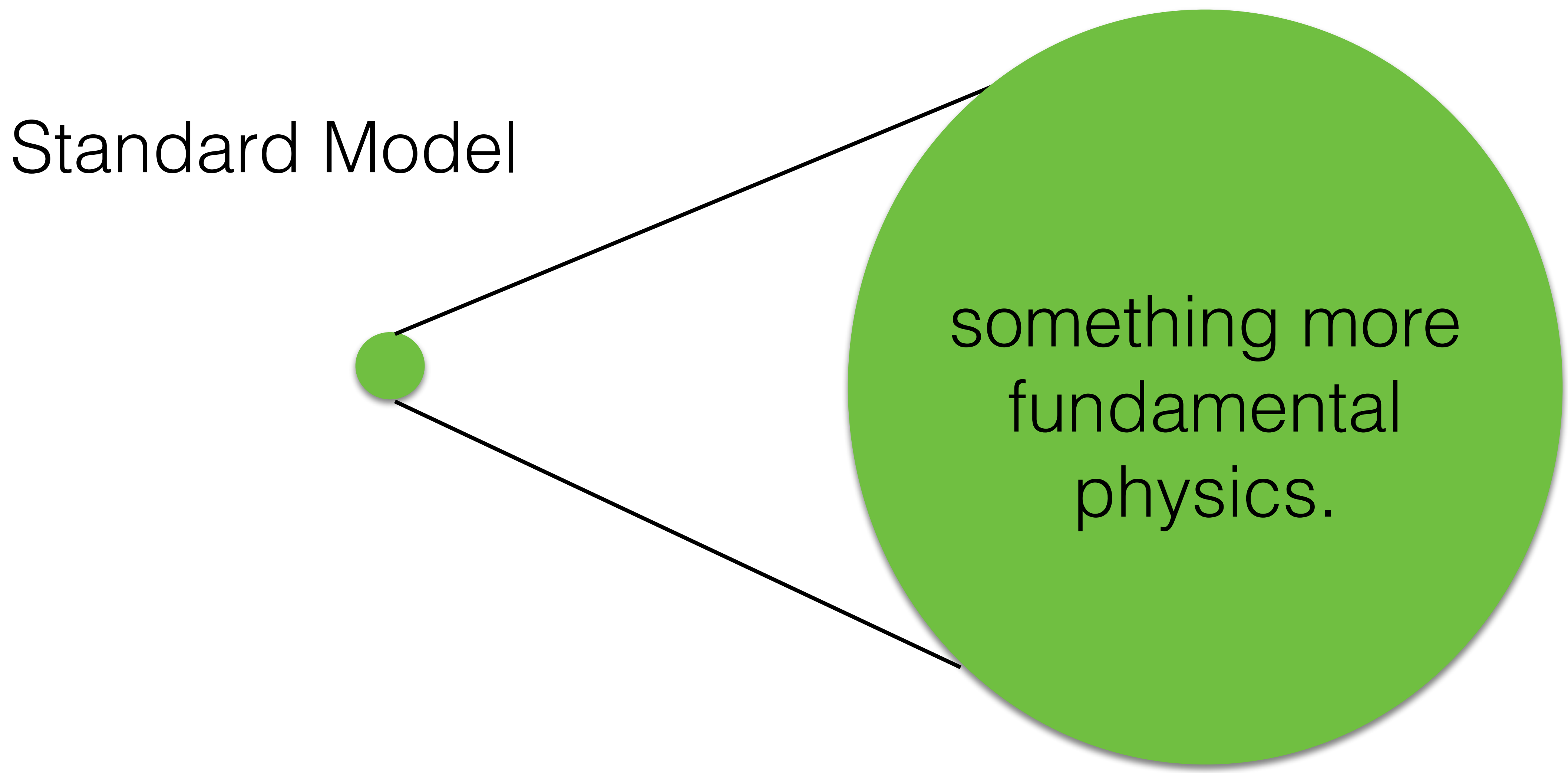


what we'd like to know is this relation!

Hierarchy problem is related to renormalization. But I'd like to stress that the it is a physics problem.

Let’s discuss the hierarchy problem.

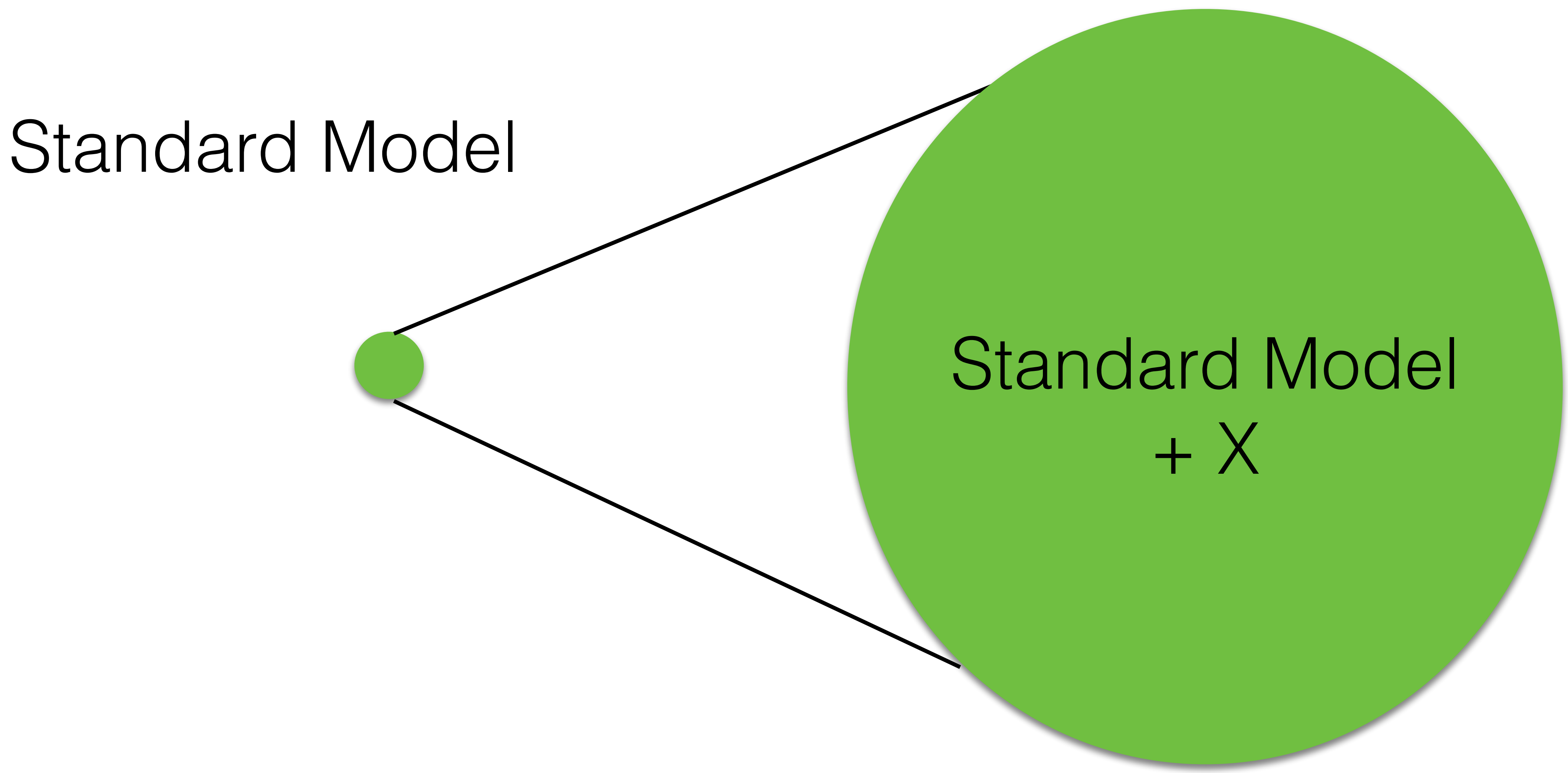
We start with the following belief.



As a toy example,

$$\mathcal{L} = \kappa|\phi|^2|X|^2 - M_X^2|X|^2$$

let's consider a model with new heavy particle X.



very simple example.

SM and SM+X both can give correct predictions to physical quantities as long as the scale of our interest is much lower than Mx.

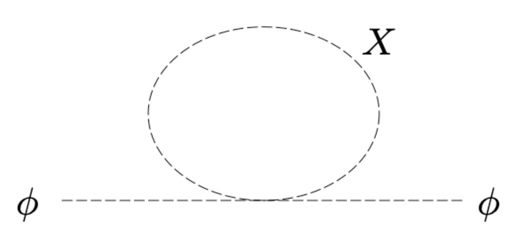
Let's calculate the Higgs boson mass m_ϕ in both theories at $O(\kappa)$.

$$m_\phi^2 = m_L^2 \qquad \text{(SM)}$$

Lagrangian parameter in SM.

$$m_\phi^2 = m_H^2 - \frac{\kappa}{(4\pi)^2} M_X^2 \log \frac{\Lambda^2}{M_X^2} + \cdots \qquad \text{(SM+X)}$$

this part even depends on how we calculate. But we learned already that we don't care.



Lagrangian parameter in SM+X.

We are calculating a physical quantity:

$$m_\phi^2 = m_L^2 = m_H^2 - \frac{\kappa}{(4\pi)^2} M_X^2 \log \frac{\Lambda^2}{M_X^2} + \cdots$$

$(125\mathrm{GeV})^2$

This just requires the Lagrangian parameters in each theory to satisfy these equations. (renormalization)

This just requires $m_H^2 \to m_H^2(\Lambda)$.

Similarly, $\kappa \to \kappa(\Lambda), M_X^2 \to M_X^2(\Lambda)$

fixed by physical scattering and masses.

We learned that this is not a problem. But..

If we take the belief that microscopic physics is more fundamental,

$$m_H^2(\Lambda), \kappa(\Lambda), M_X(\Lambda)$$

are more fundamental than m_L^2

If so, it is strange if $m_\phi^2 \ll M_X^2$

because, we need a special relation among

$$m_H^2(\Lambda), \kappa(\Lambda), M_X(\Lambda)$$

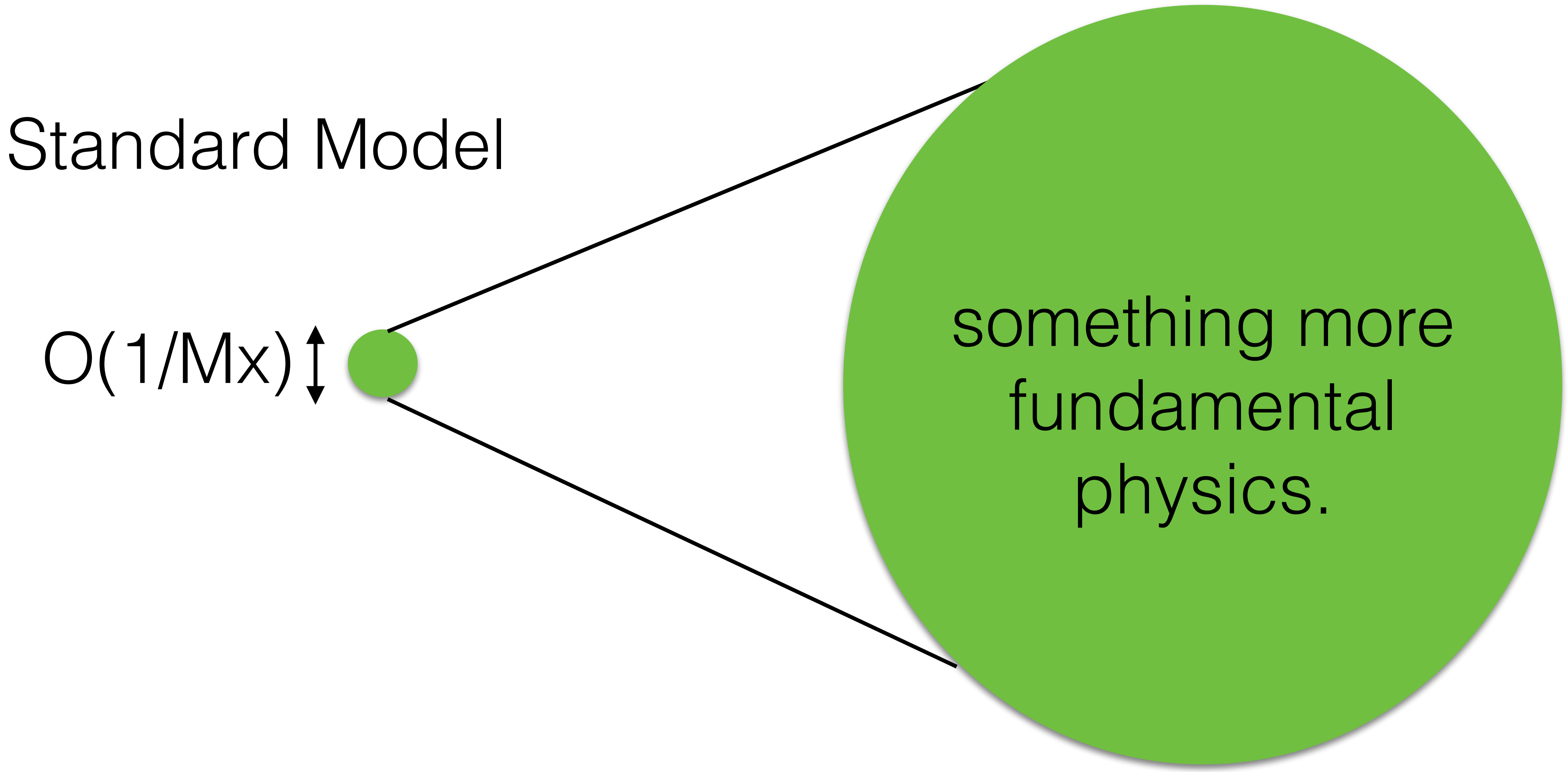
$$m_\phi^2 = m_L^2 = m_H^2 - \frac{\kappa}{(4\pi)^2} M_X^2 \log \frac{\Lambda^2}{M_X^2} + \cdots$$

for example,

$125^2 = 10000^2 - 9999.22^2$

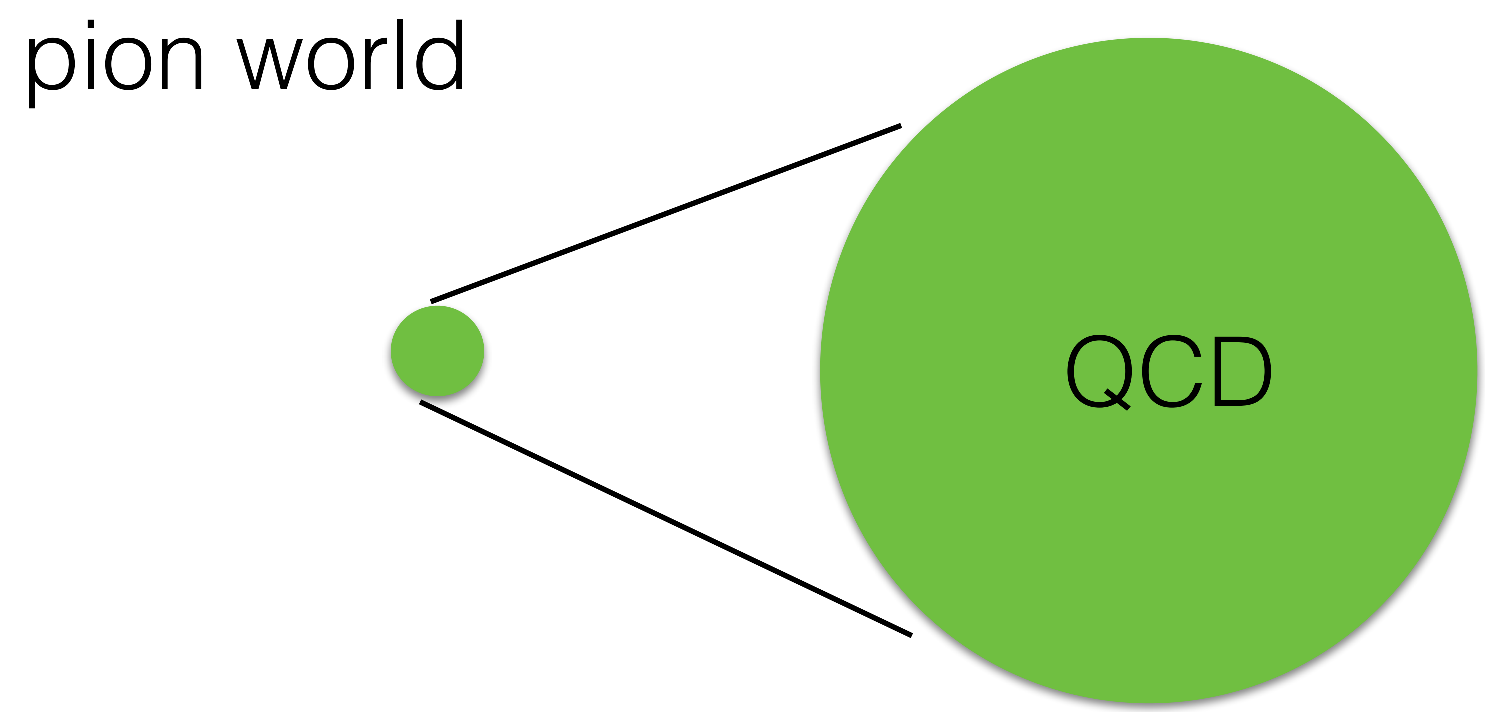
to realize the hierarchy.

In other words, a generic prediction of this framework is

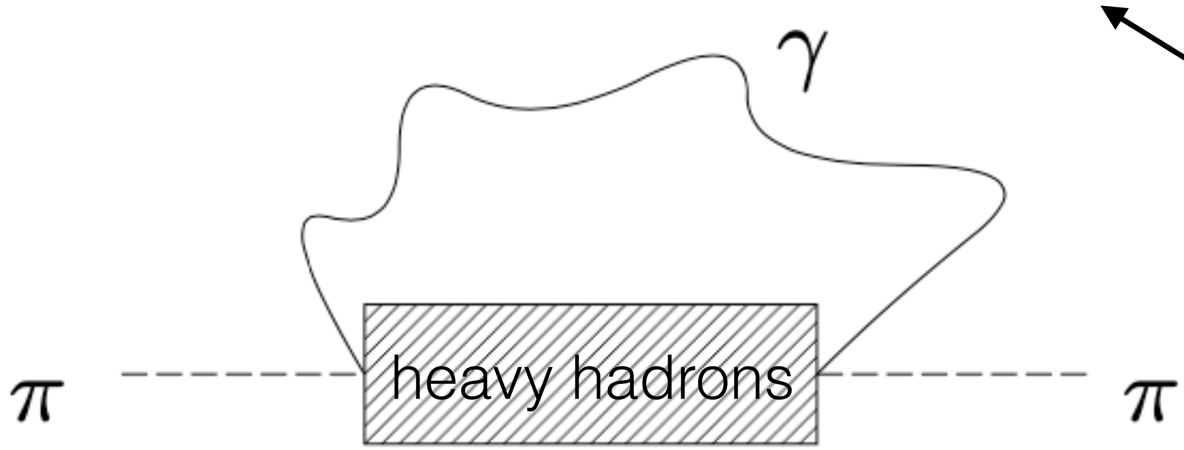


$$m_\phi^2 \sim m_X^2$$

example:



$m_{\pi^+}^2 - m_{\pi^0}^2$ ~ (35MeV)^2 (exp)



it works!

This would naively gives $\Delta m_\pi^2 \sim \frac{e^2}{(4\pi)^2}\Lambda_{\rm QCD}^2$ ~O((10MeV)^2)

(theory guess)

This discussion suggests that

the Standard Model should be replaced by some fundamental theory not so far from 125GeV.

Where are they hiding is really the question.

Lecture 3: Beyond the Standard Model?

Physics beyond the Standard Model

We've seen various mysteries
in the Standard Model.

Dark Matter?

three generations?

Grand Unification?

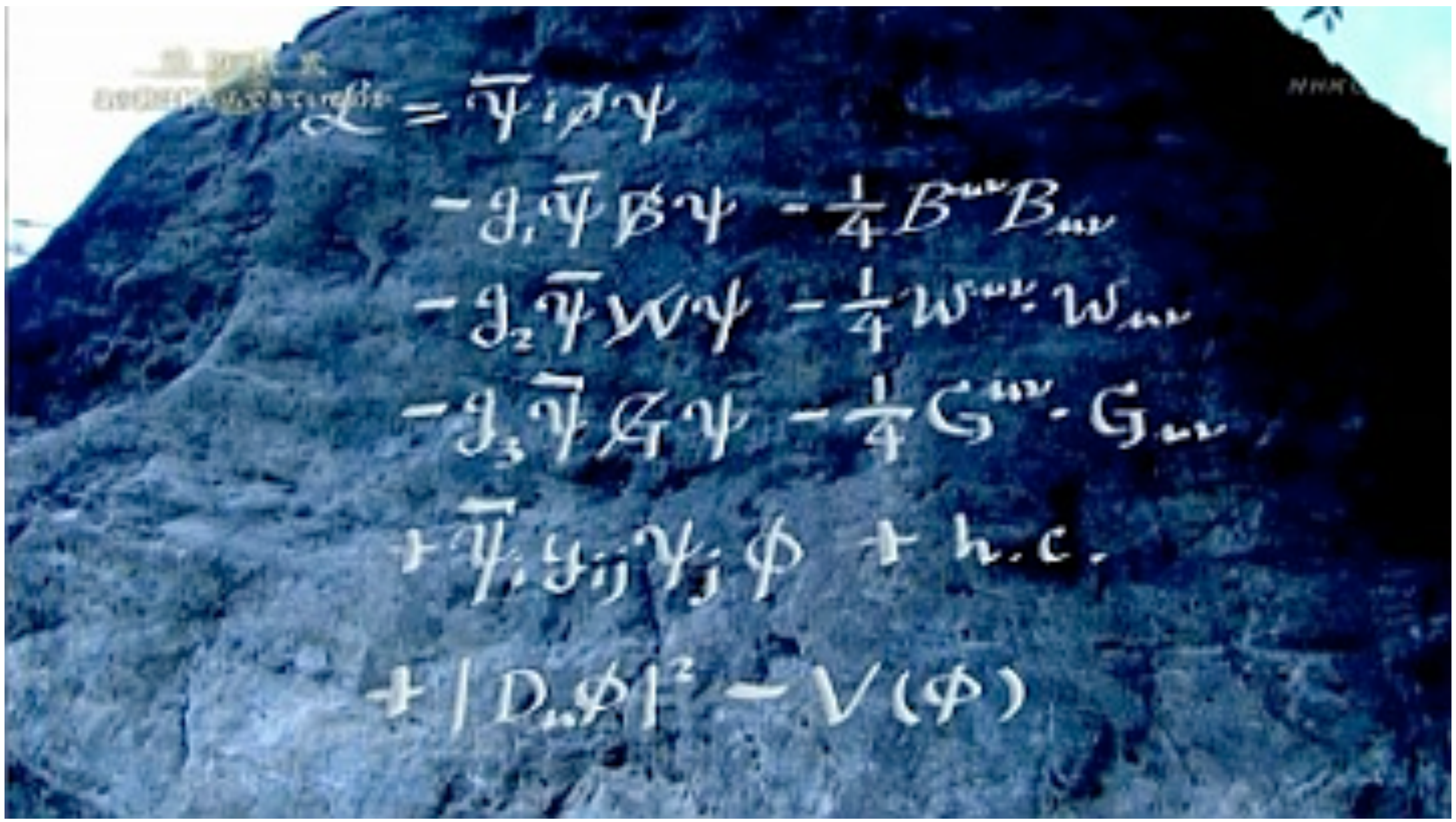
strong CP?

Higgs?

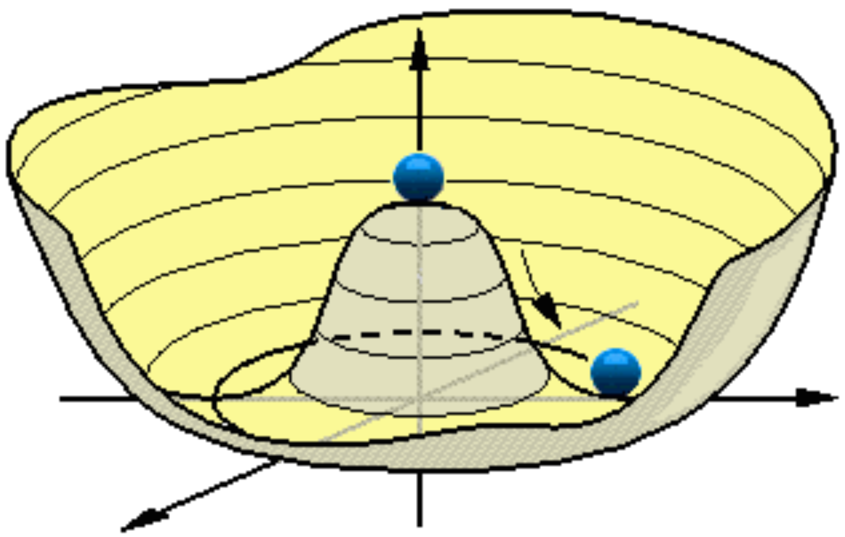
Baryon asymmetry?

Today, I'm going to tell you about trials toward the understanding of these mysteries.

Standard Model



Higgs mystery

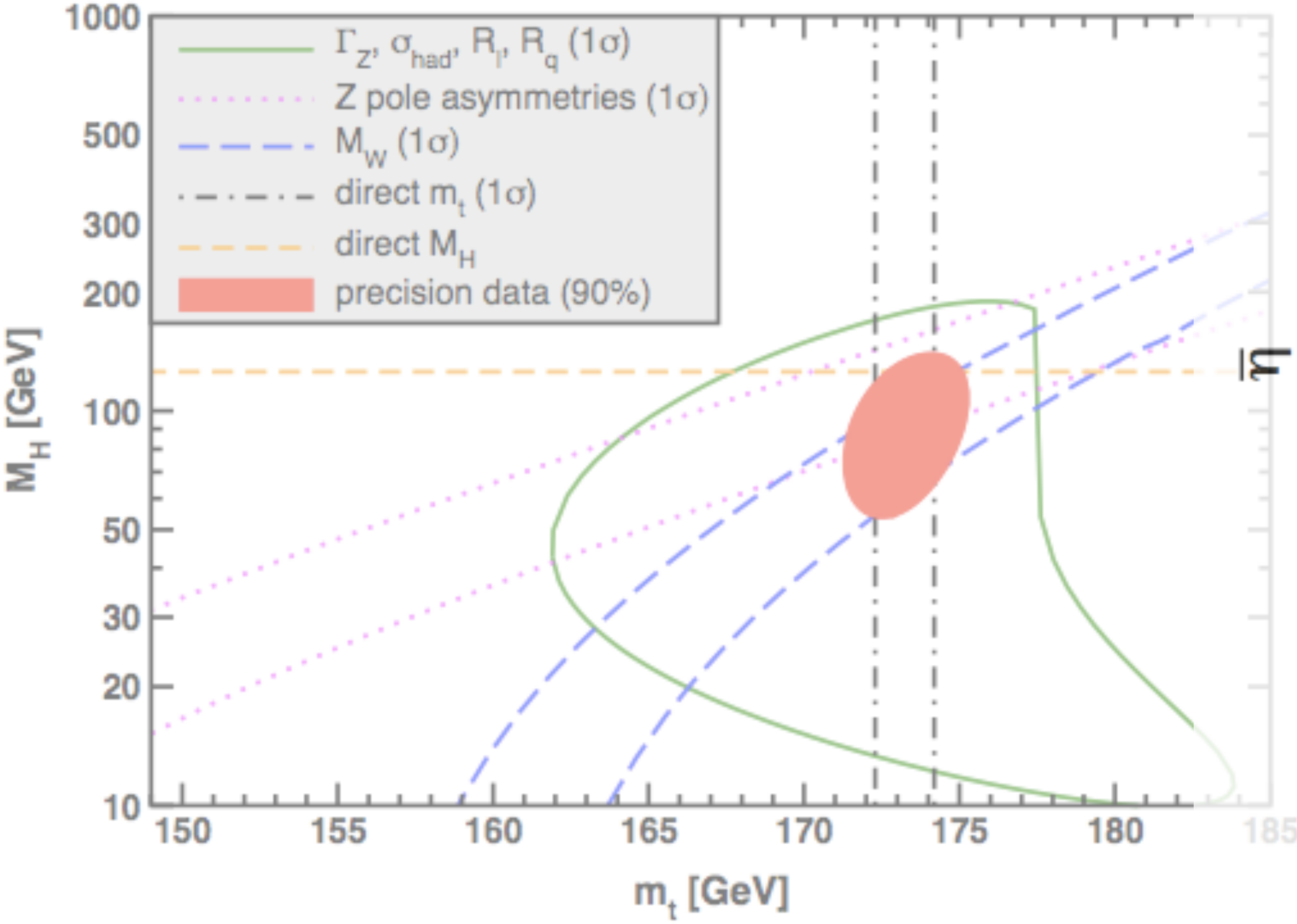


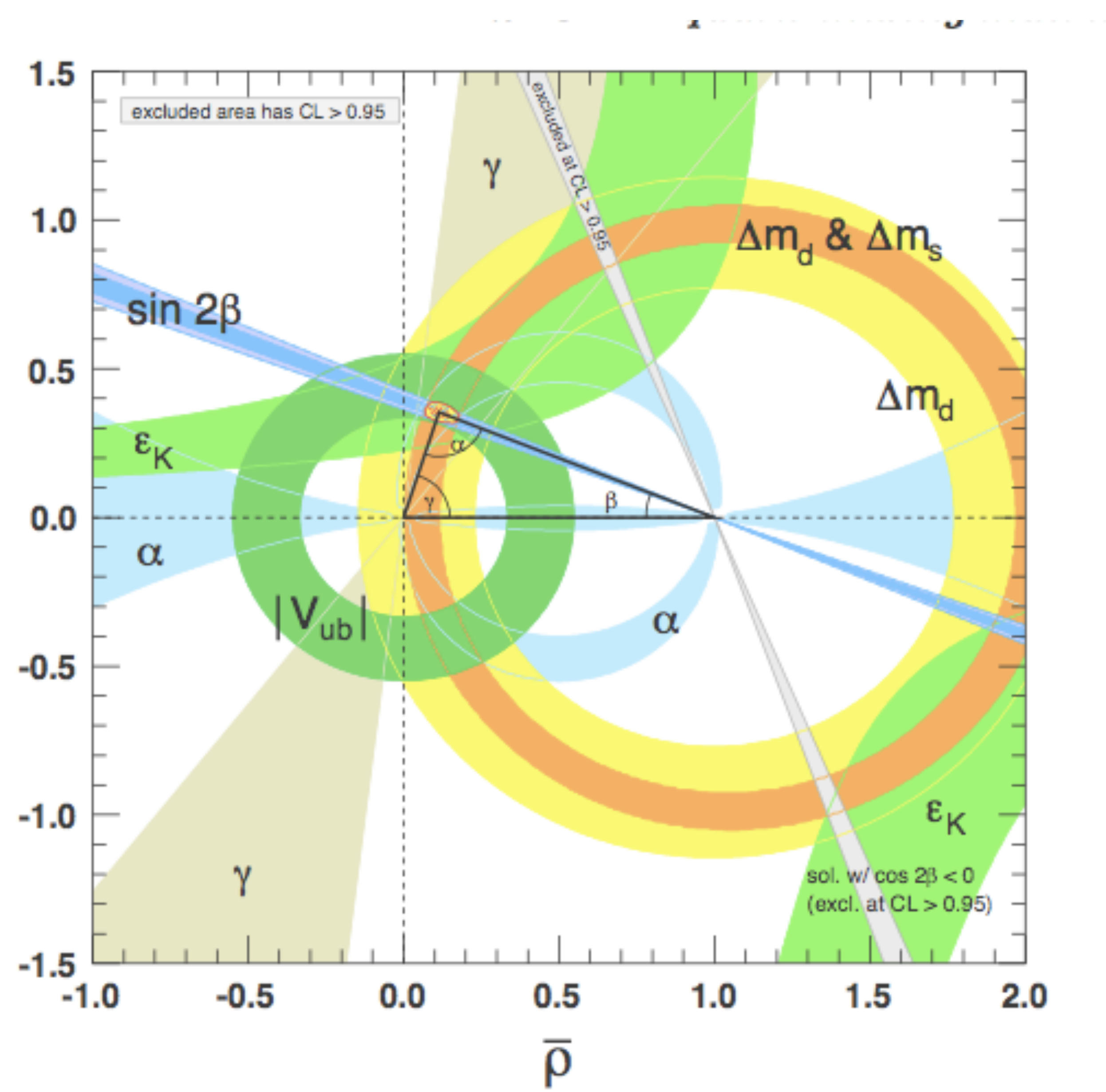
$$V(\phi) = -m^2|\phi|^2 + \frac{\lambda}{4}|\phi|^4$$
$$= \frac{\lambda}{4}\left(|\phi|^2 - \frac{2m^2}{\lambda}\right)^2 - \frac{m^4}{\lambda}$$

Looks pretty artificial.

This is the most important part of the SM,
and at the same time, the most unsatisfactory part of the SM.

Nevertheless, pretty good fit with data.





gure 12.2: Constraints on the $\bar{\rho}, \bar{\eta}$ plane. The shaded areas have 95% CL.

which means,

if we assume breakdown of the effective theory,
the scale of underlying theory to show up should be
much higher than the Higgs mass.

$$10\text{-}100\,\mathrm{TeV} \quad \xleftrightarrow{???} \quad 125\,\mathrm{GeV}$$

(Little) Hierarchy Problem

this should be an important hint
for physics beyond the Standard Model

Supersymmetry

fermionic dimension

space-time: x, y, z, t

field

$\phi(x,y,z,t)$

complex number

superspace: x, y, z, t, θ

superfield

$\Phi(x,y,z,t,\theta)$

complex number

The only possible extension of the special relativity.

θ ?

* θ is a spinor: (θ_1, θ_2)

* θ's are complex anti-commuting numbers..

$$\theta_1\theta_2 = -\theta_2\theta_1 \quad \Rightarrow \quad \theta_1^2 = \theta_2^2 = 0$$

* translation into θ-directions are a bit funny

$$\delta_1\delta_2 - \delta_2\delta_1 \neq 0$$

$$\left(\{Q_\alpha, \bar{Q}_{\dot{\beta}}\} = 2\sigma^a_{\alpha\dot{\beta}}P_a.\right)$$

superfield to field

$$\theta_1^2 = \theta_2^2 = 0$$

$\Rightarrow$ at most 2nd order

$$\Phi(y,\theta) = A(y) + \sqrt{2}\theta\psi(y) + \theta\theta F(y)$$

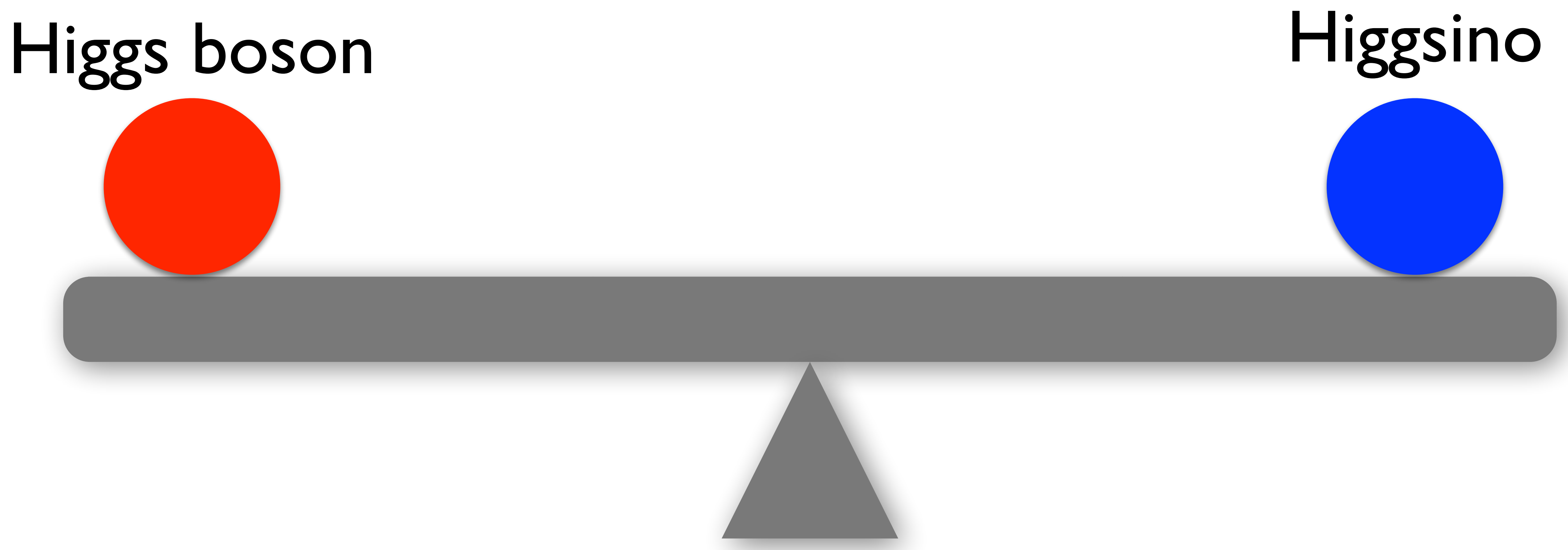
boson fermion boson

There are superpartners for each fields.

boson = fermion rule

particles and their superpartners
have the same properties.

especially,



The light boson is not mysterious anymore.

Light fermions such as the electron are common in nature.

Light bosons are also common with supersymmetry.

hypothesis

There is an interesting hypothesis.

there is no supersymmetry in nature.

→ supersymmetry is spontaneously broken.

↓

this phenomena triggers the Higgs mechanism?

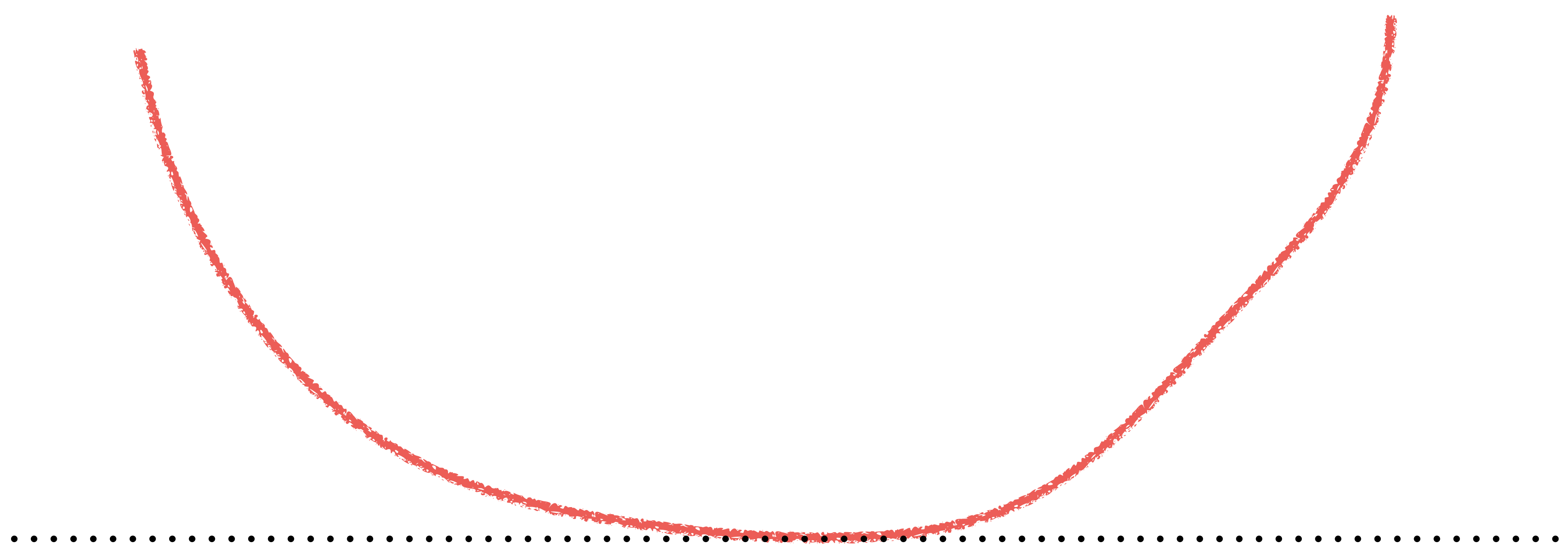
Supersymmetric world

Higgs Higgsino selectron electron

m=0

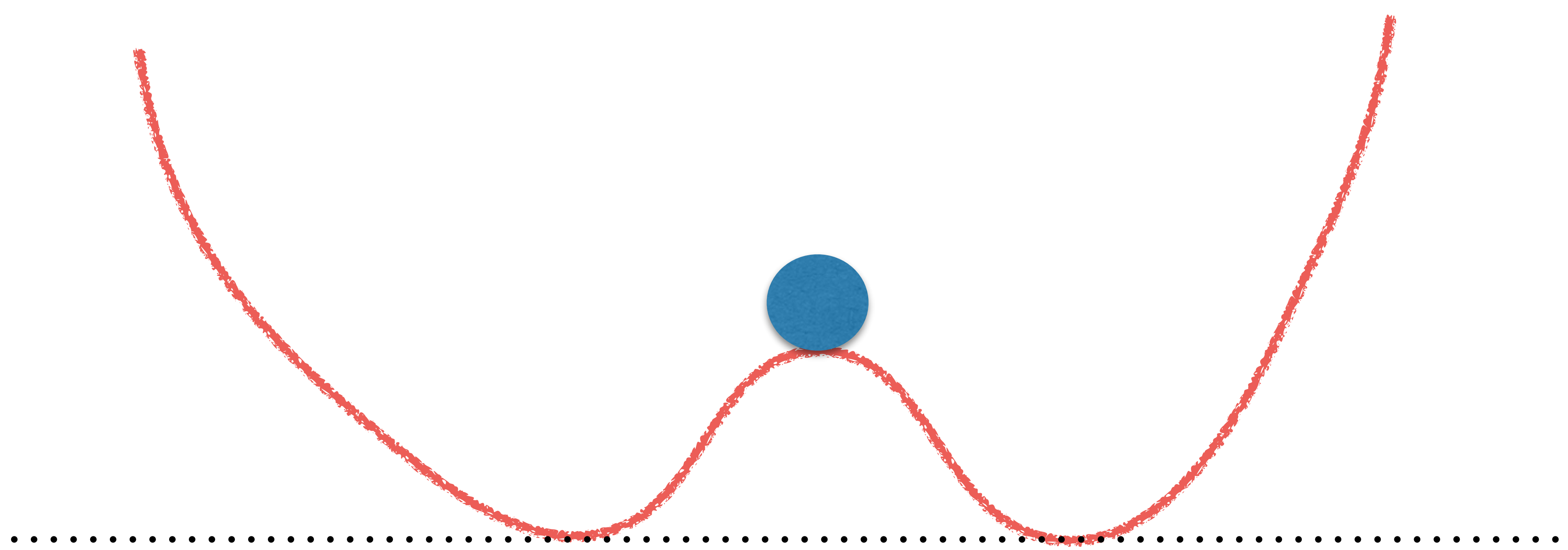
Higgs potential

ϕ^4 potential

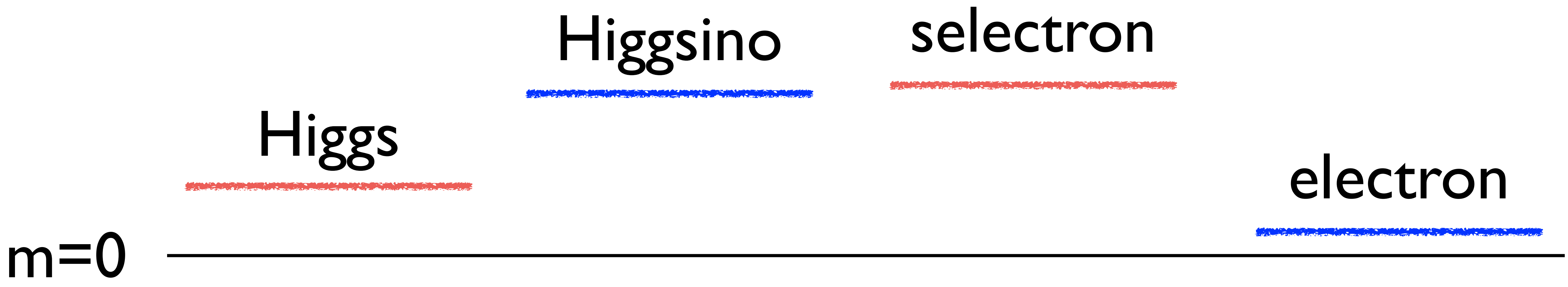


Supersymmety broken world

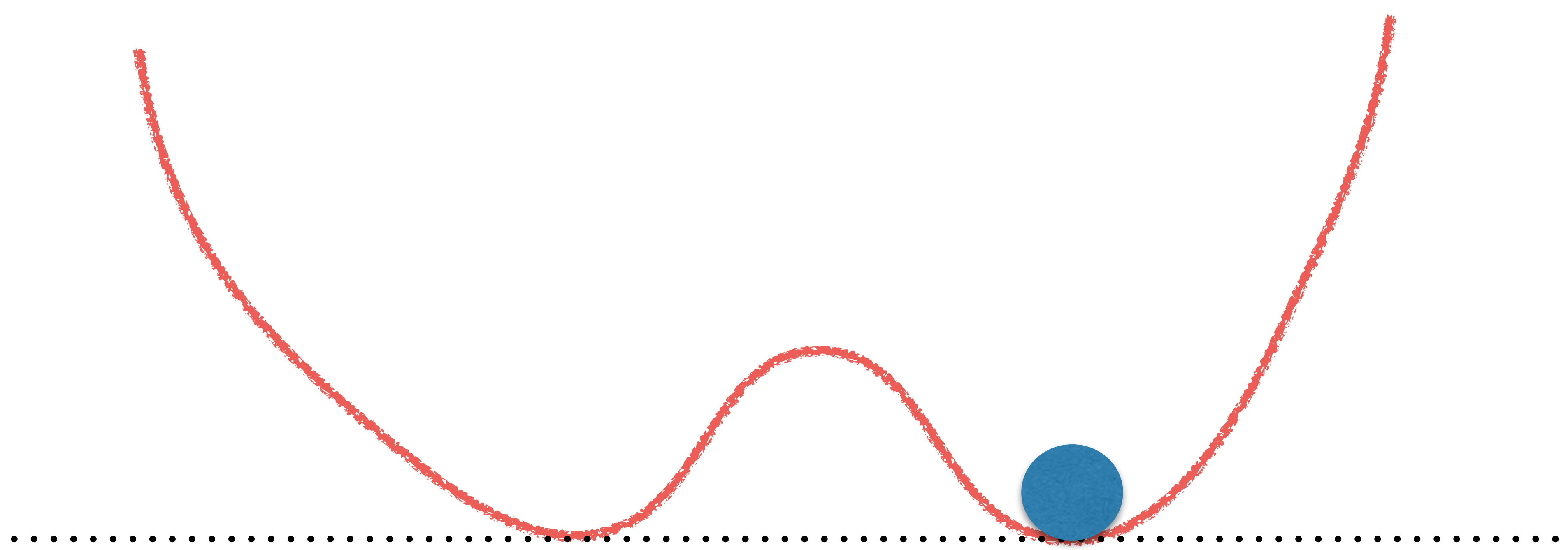
Higgs potential



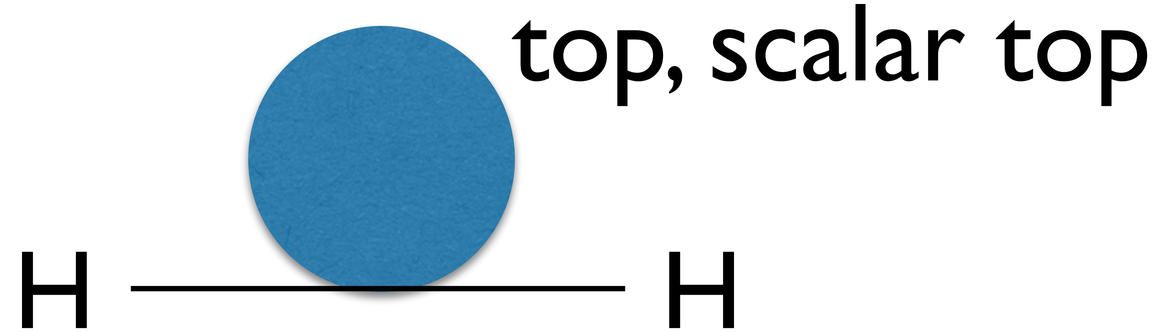
Supersymmety broken world



electroweak symmetry breaking!



Indeed, it is interesting to note that



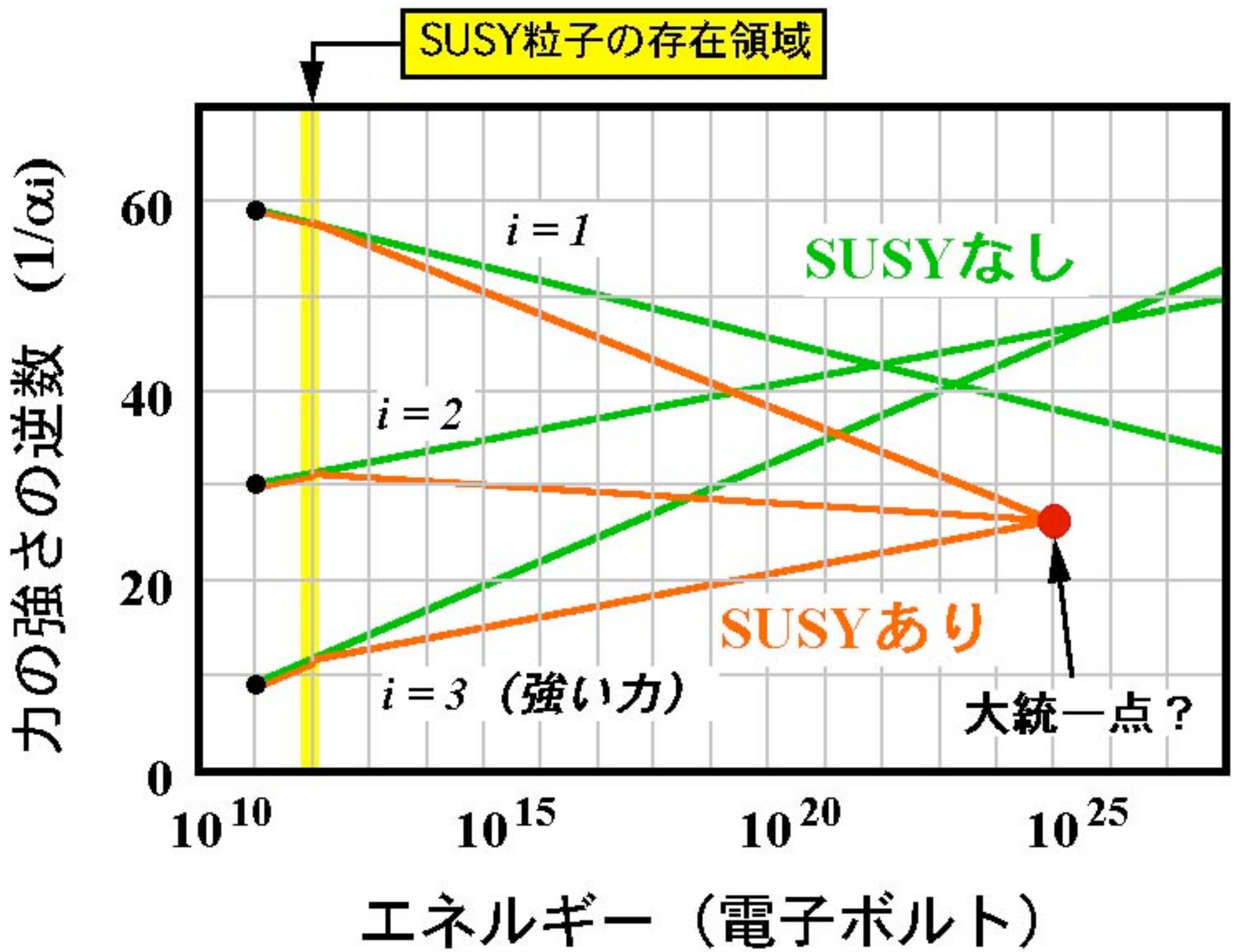
the quantum correction drives the Higgs mass parameter negative.

The minimal model predicts that

Higgs boson mass (125GeV) ~ Z boson mass (90GeV)

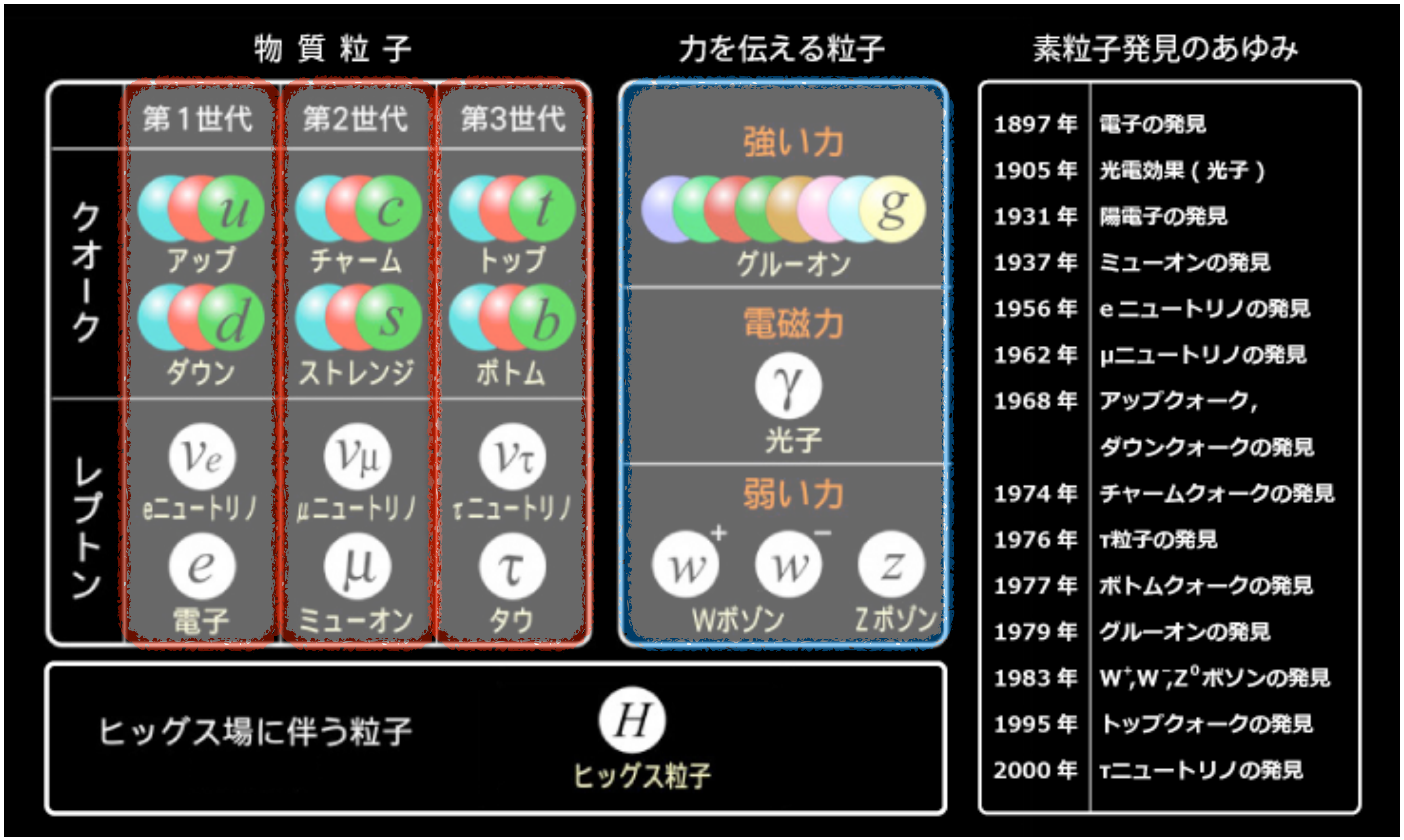
not too far!

Wow.



Interestingly, superparticles at TeV make all forces to be the same strength around 10^{16} GeV.

Supersymmetric Grand Unification?



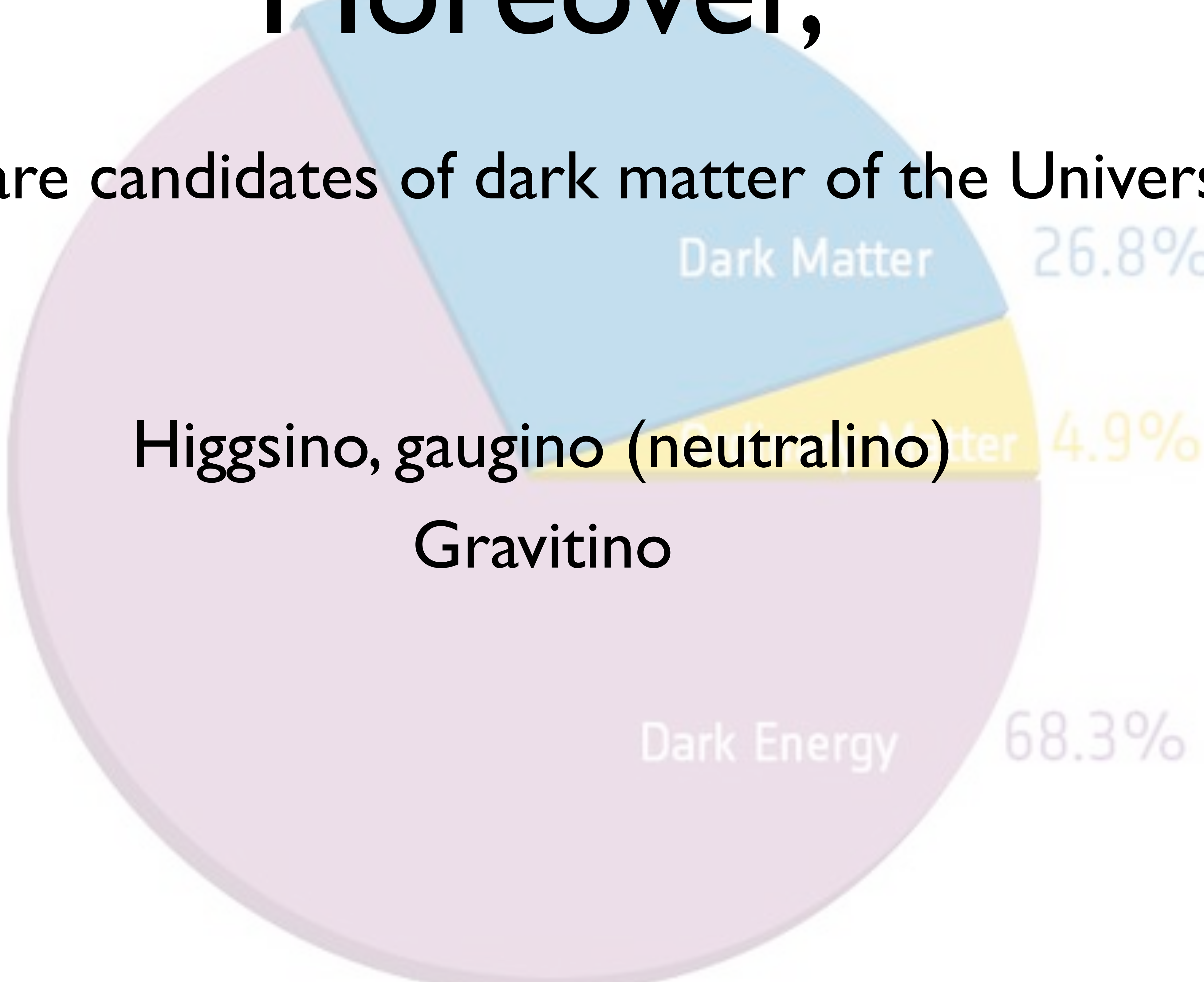
beautiful.

Moreover,

there are candidates of dark matter of the Universe.

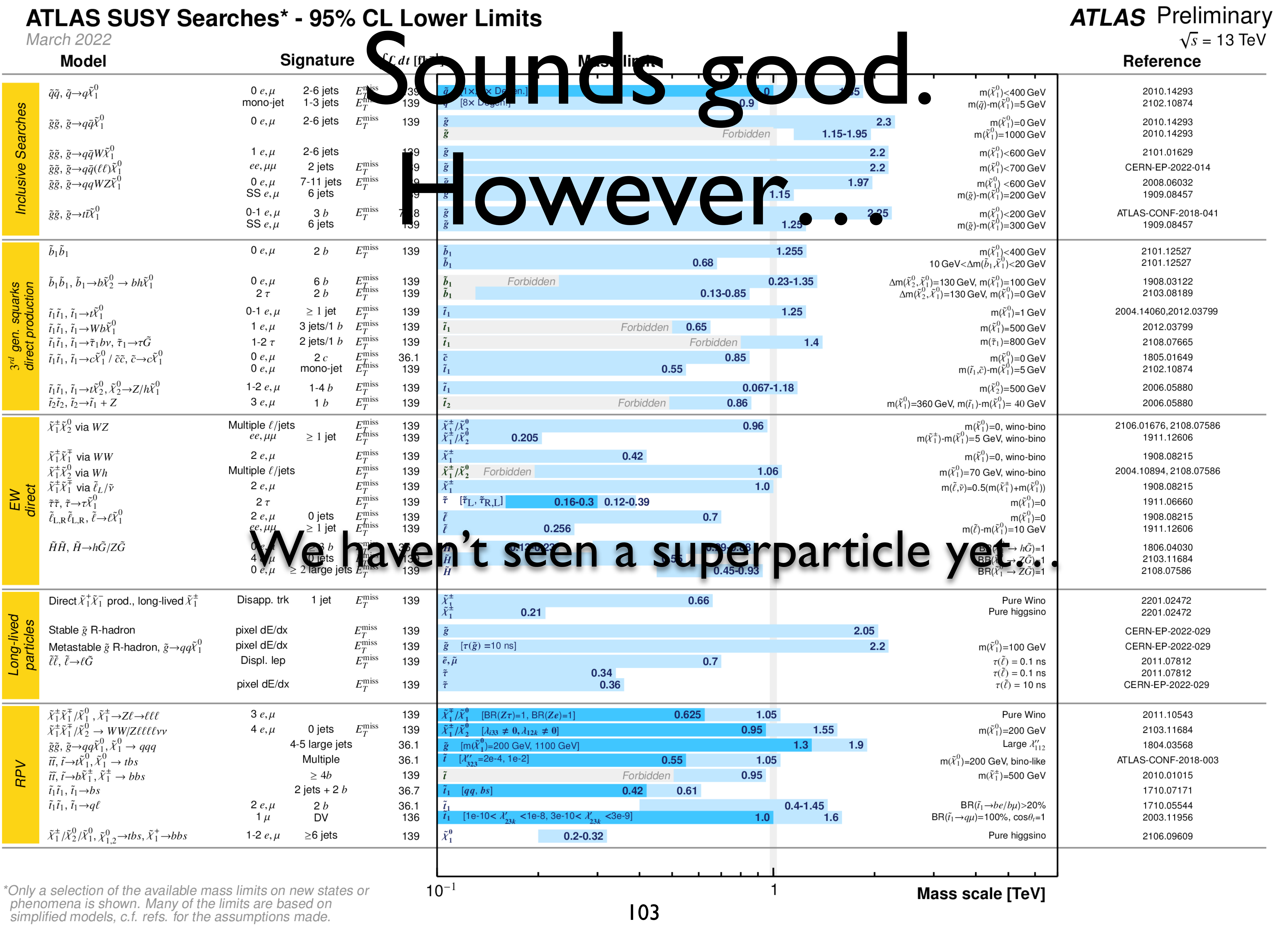
Higgsino, gaugino (neutralino)

Gravitino



Sounds good.

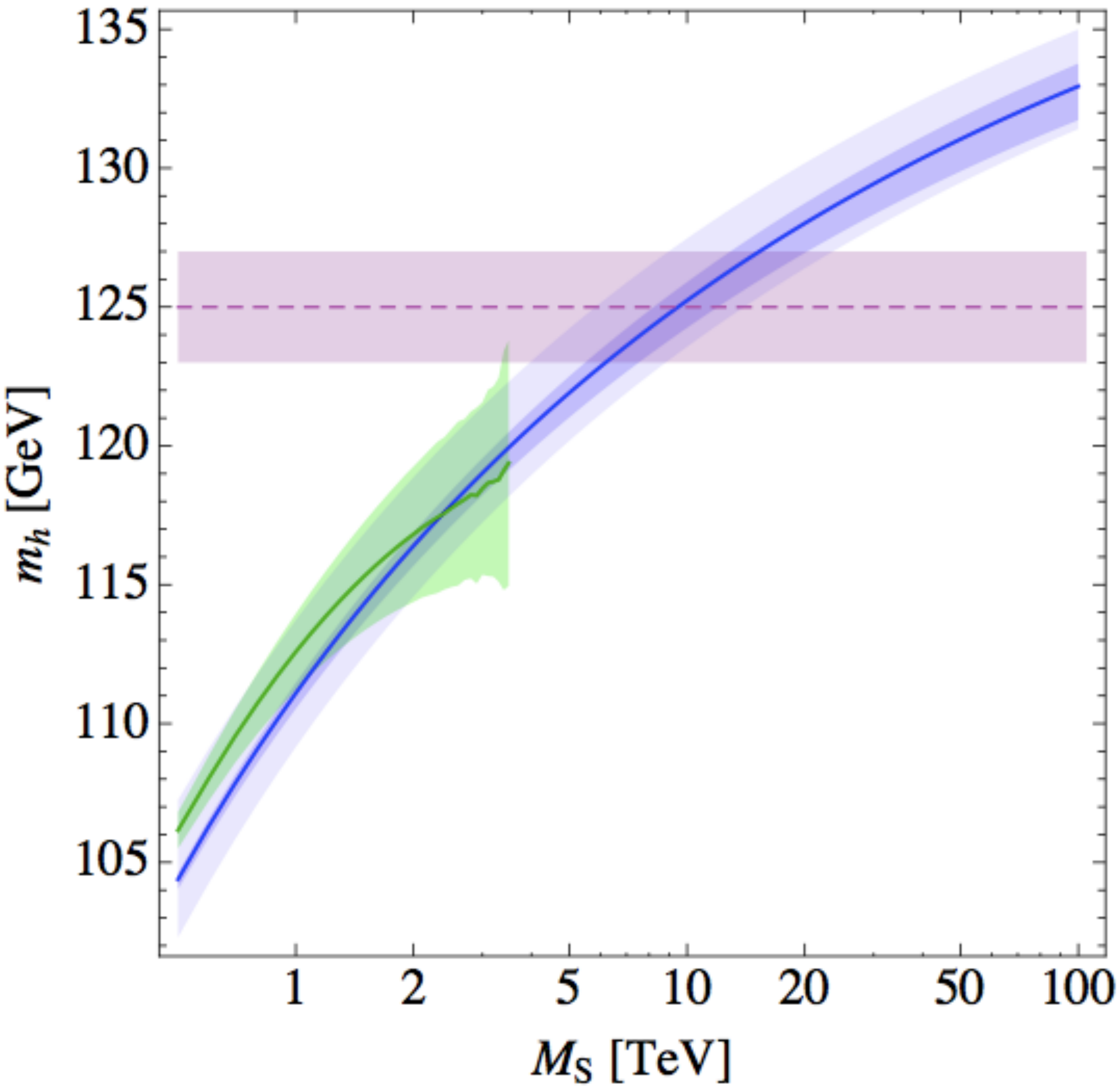
However. . .



We haven't seen a superparticle yet. . .

Also,

[Draper, Meade, Reece, Shih ’12]



125GeV Higgs is a bit heavy…

Maybe superparticles are heavy?

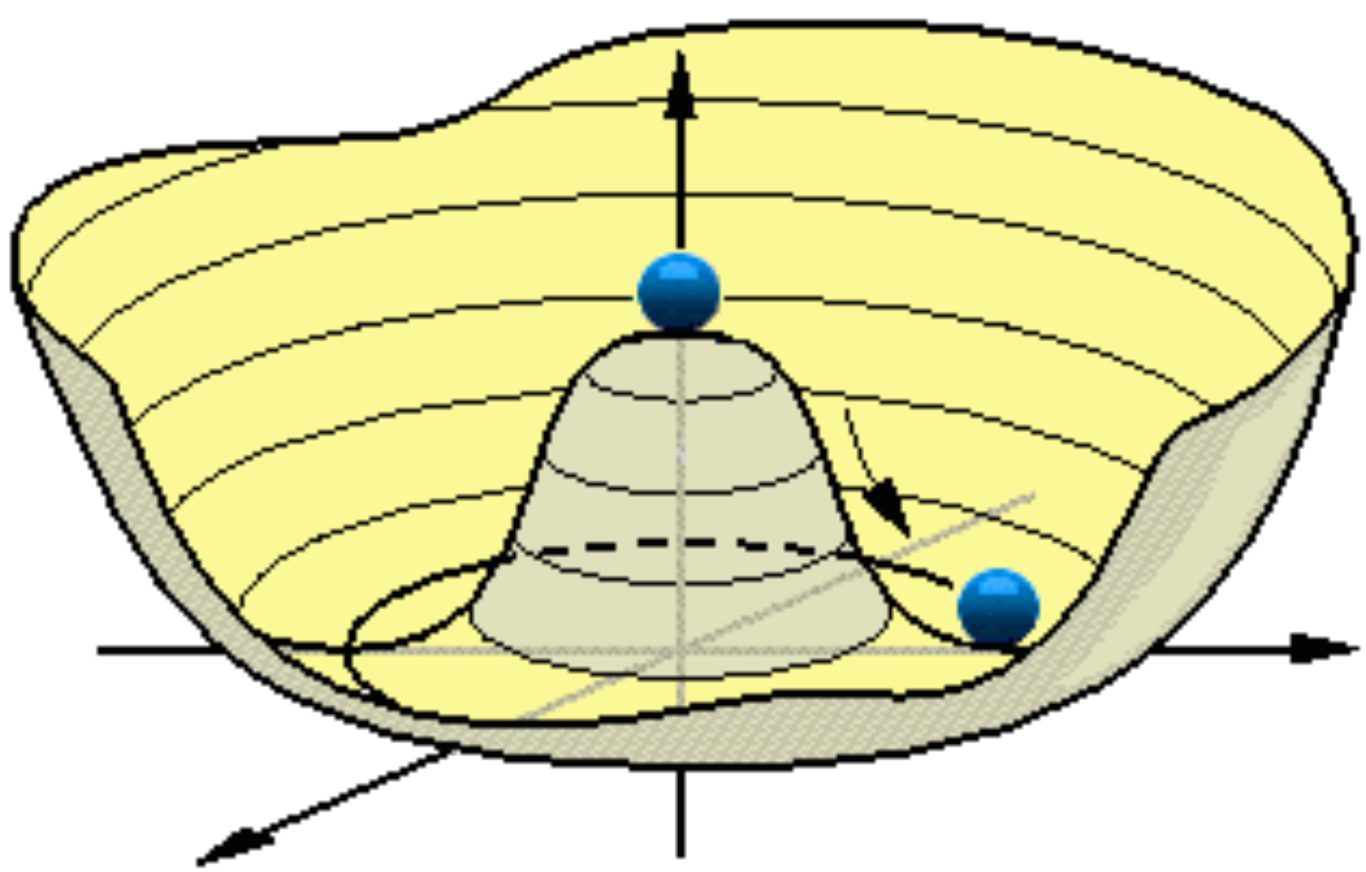
More complicated structure?

Stay tuned!

Another approach

Higgs is light

➡ Higgs as a Nambu-Goldstone boson?



[Kaplan, Georgi '84 …]

How come?

For example, let's consider a theory with global symmetry

SO(5)

and assume spontaneous break down to

SO(4)

The number of the Nambu-Goldstone bosons is

(5x4/2)-(4x3/2)=4=2x2

One can identify this four d.o.f. with the Higgs field.

If the Higgs boson is a Nambu-Goldstone boson,

1. the Higgs boson should be massless, and
2. the value of the Higgs field should not change physics.

But, in the real world, the Higgs boson has mass, 125GeV, and the gauge boson masses and the quark/lepton masses are proportional to the value of the Higgs field.

But these two problems provides us with an interesting hypothesis.

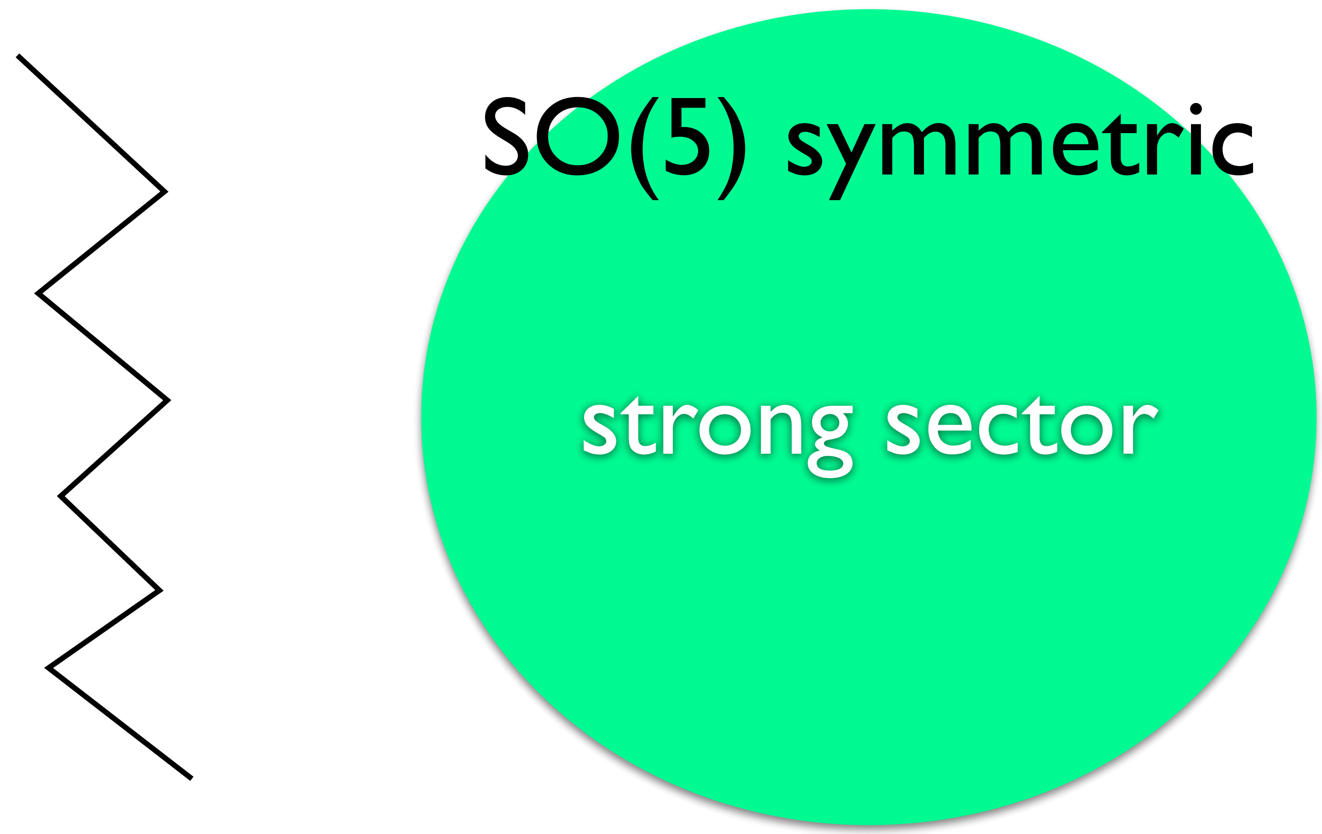
[Kaplan ’91 …]

Partially composite fermions

no SO(5) symmetry

top quark

elementary



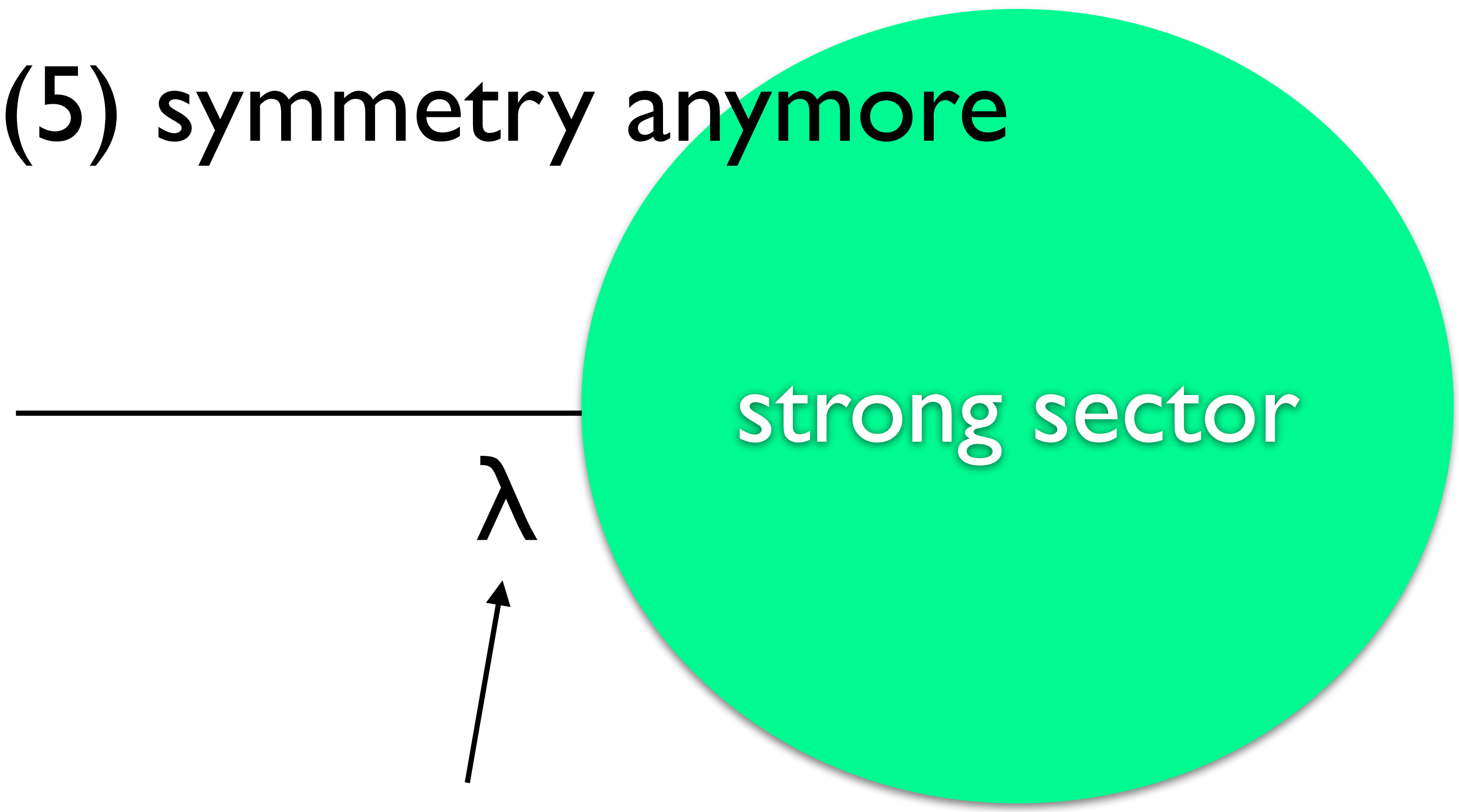
exactly massless

Higgs is a Nambu-Goldstone boson in the strong sector.

Partially composite fermions

no SO(5) symmetry anymore

top quark



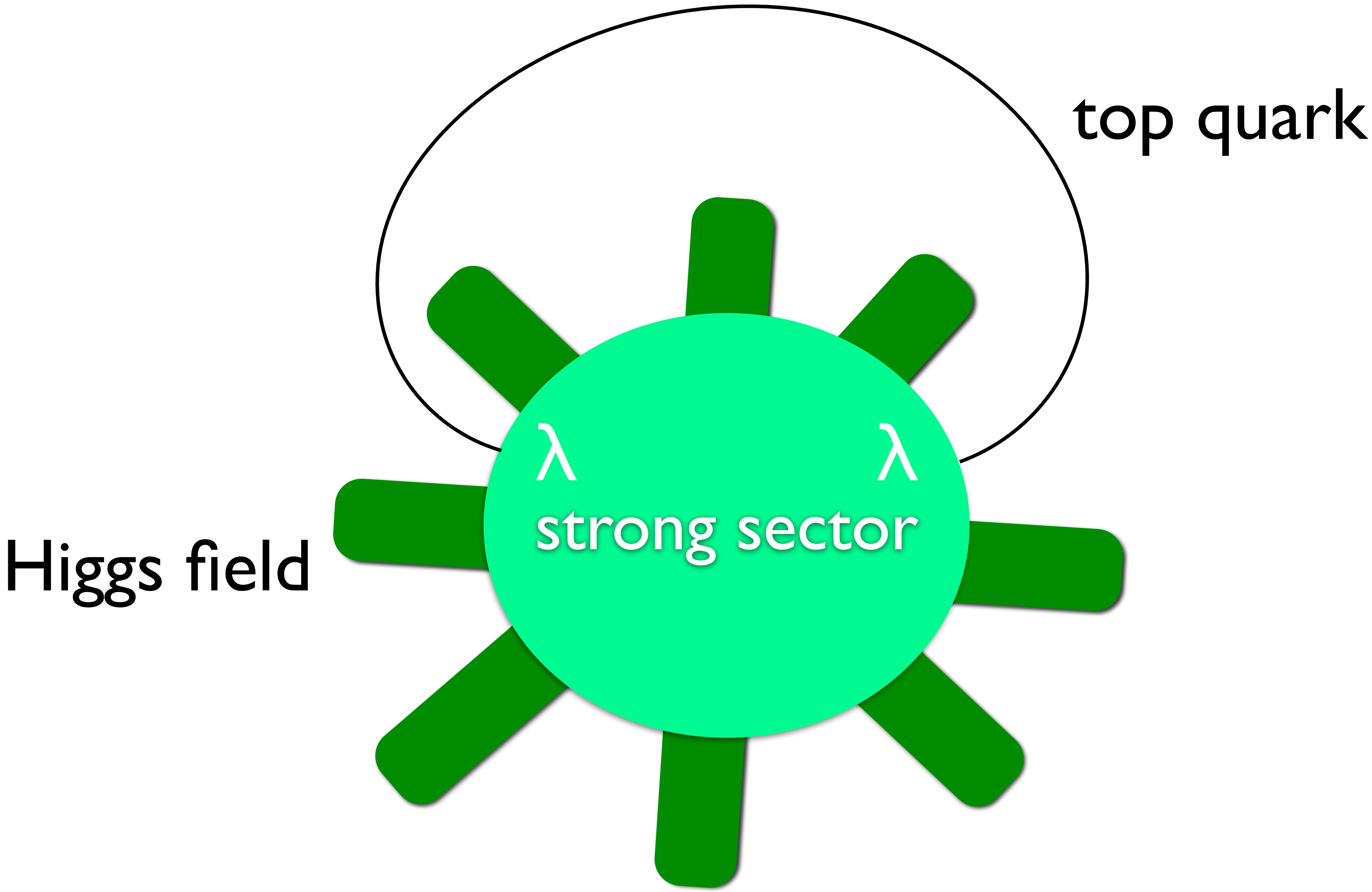
partially composite

small coupling $\mathcal{L}_{\rm int} = \lambda \bar{t} \cdot \mathcal{O}_t$

pseudo

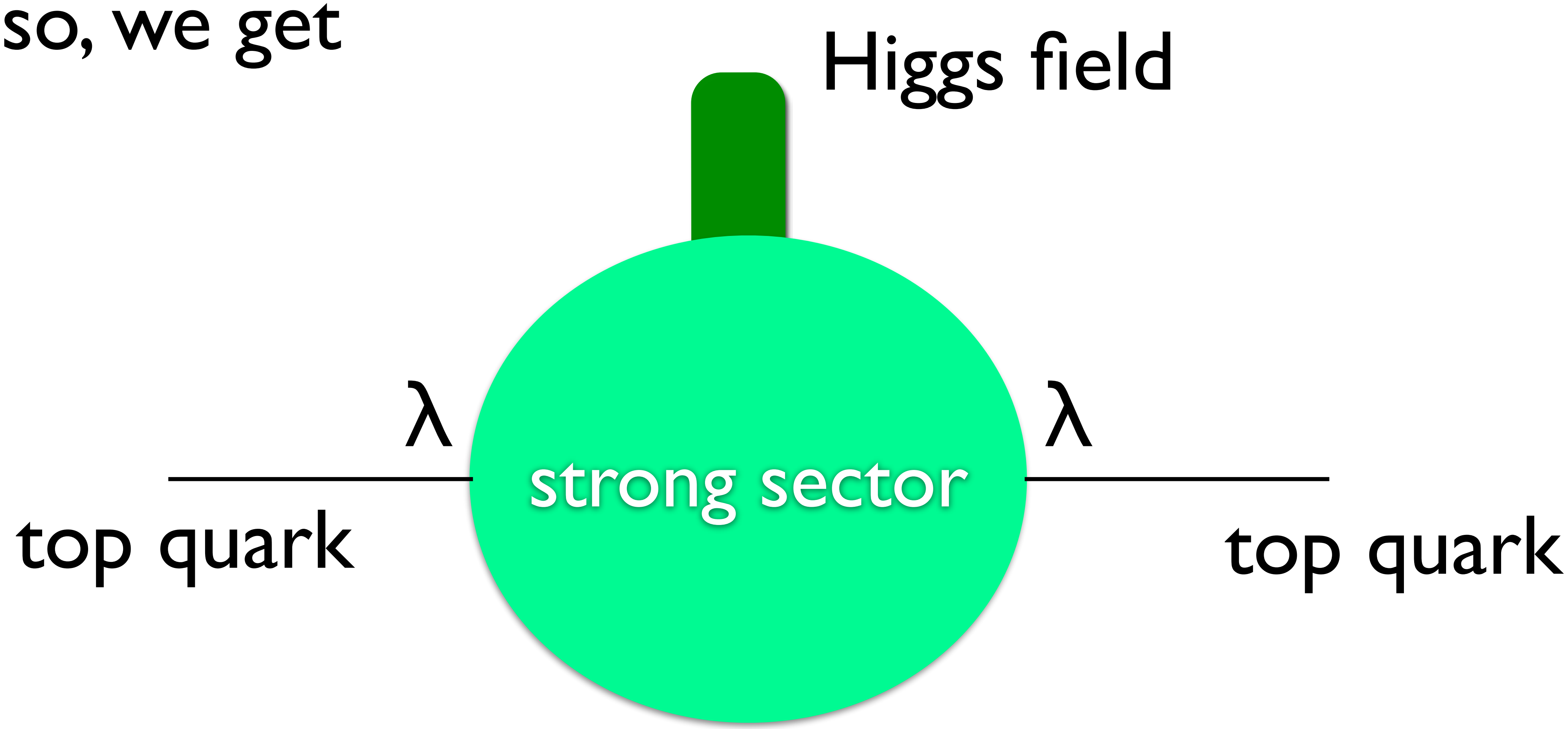
Higgs is a Nambu-Goldstone boson in the strong sector.

The explicit breaking provides



the Higgs potential at the λ^2 order.

Also, we get



the top quark mass at the λ^2 order.

small explicit breaking terms can generate both

top quark mass

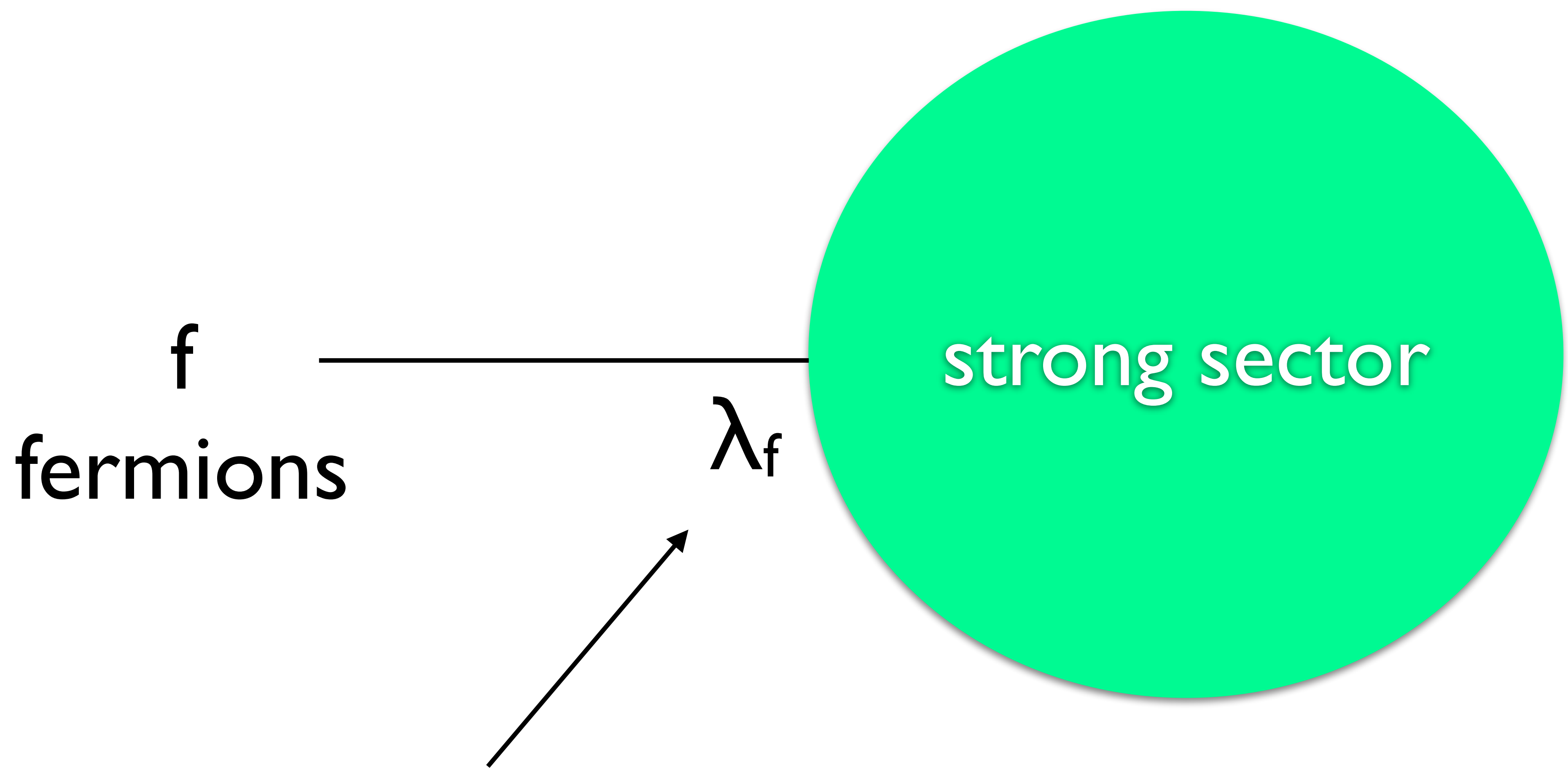
and

the Higgs potential

The gauge boson masses can also be generated in a similar way.

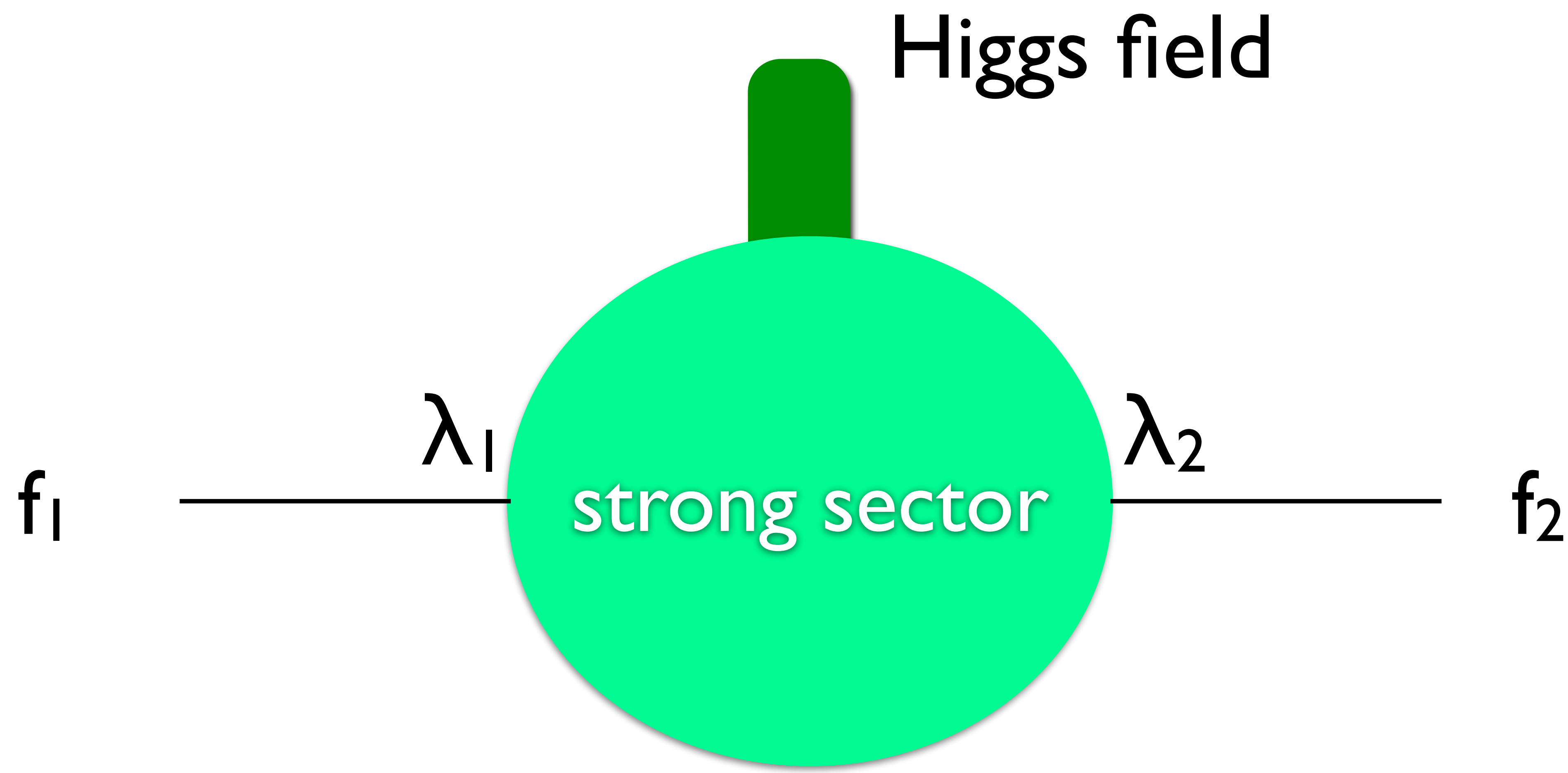
What about other fermions?

Actually, this scenario is friendly with the Yukawa structures.



This coupling can control the size of the Yukawa coupling.

fermion masses



The Yukawa coupling constant gets a structure of

$$y_{12} \sim \lambda_1 \lambda_2$$

$$y_{12} \sim \lambda_1 \lambda_2 \qquad y \sim \begin{pmatrix} \lambda_1\lambda_1 & \lambda_1\lambda_2 & \lambda_1\lambda_3 \\ \lambda_2\lambda_1 & \lambda_2\lambda_2 & \lambda_2\lambda_3 \\ \lambda_3\lambda_1 & \lambda_3\lambda_2 & \lambda_3\lambda_3 \end{pmatrix}$$

$$\Downarrow$$

This structure says

light and heavy fermions mix weakly.

fermions with similar masses have large mixing.

Looks consistent.

$$m_u = 1.5 - 3.3 \text{ MeV}, \quad m_c = 1.3 \text{ GeV}, \quad m_t = 1.1 \text{ GeV}$$

$$m_d = 3.5 - 6.0 \text{ MeV}, \quad m_s = 70 - 130 \text{ MeV}, \quad m_b = 4.2 \text{ GeV}$$

$$V_{\text{CKM}} \sim \begin{pmatrix} 0.97 & 0.23 & 0.004 \\ * & 0.96 & 0.04 \\ * & * & 1.0 \end{pmatrix}$$

Very nice, but…

The Higgs potential should be like,

$$V(H) \sim \frac{\lambda^2}{(4\pi)^2}\Lambda^2 f^2 \left[\alpha \cos(H/f) + \beta \cos(2H/f) + \cdots\right]$$

this means <H> ~ f with f being the SO(5) breaking scale.

- Such a scenario is severely constrained by the electroweak precision data.

- We need a few % level of fine-tuning to hide the dynamical sector…

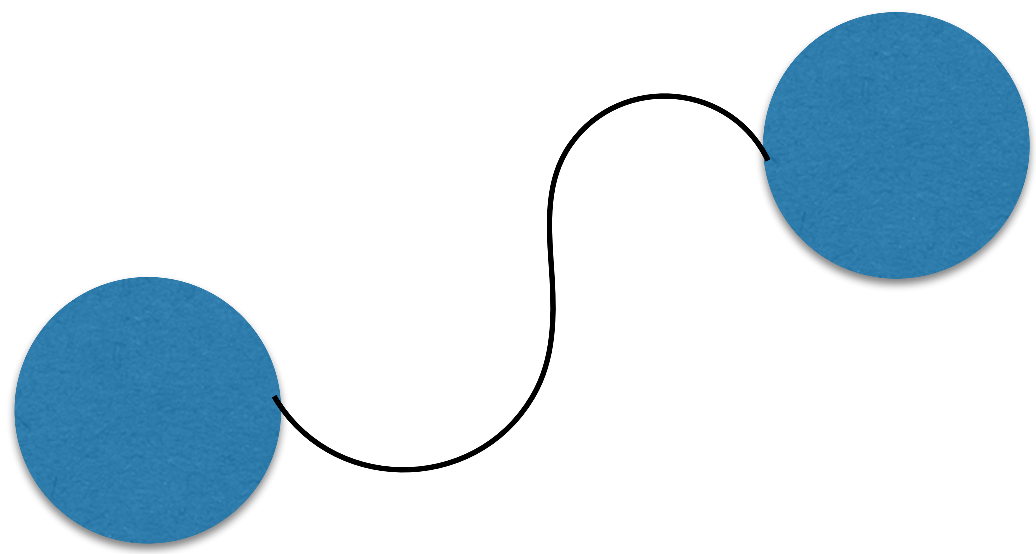
In any case, the properties of the Higgs boson will be quite different from the Standard Model.

Maybe we can see it at LHC, HL-LHC, ILC…

exciting!

Extra dimension?

It seems that we need more spacial dimension
to define quantum gravity(?)



The appearance of the light Higgs boson
may be indicating that breakdown of 4-dim field
theory is close?

5-dim gauge theory?

in 4-dim.	in 5-dim.
$V = -\dfrac{e^2}{4\pi r}$	$V = -\dfrac{e^2}{2\pi^2 r^2}$

dim[V]=M, dim[r]=1/M

"e" is dimensionless

dim[e]=$1/M^{1/2}$

From dimensional analysis

dimensionless physical quantity

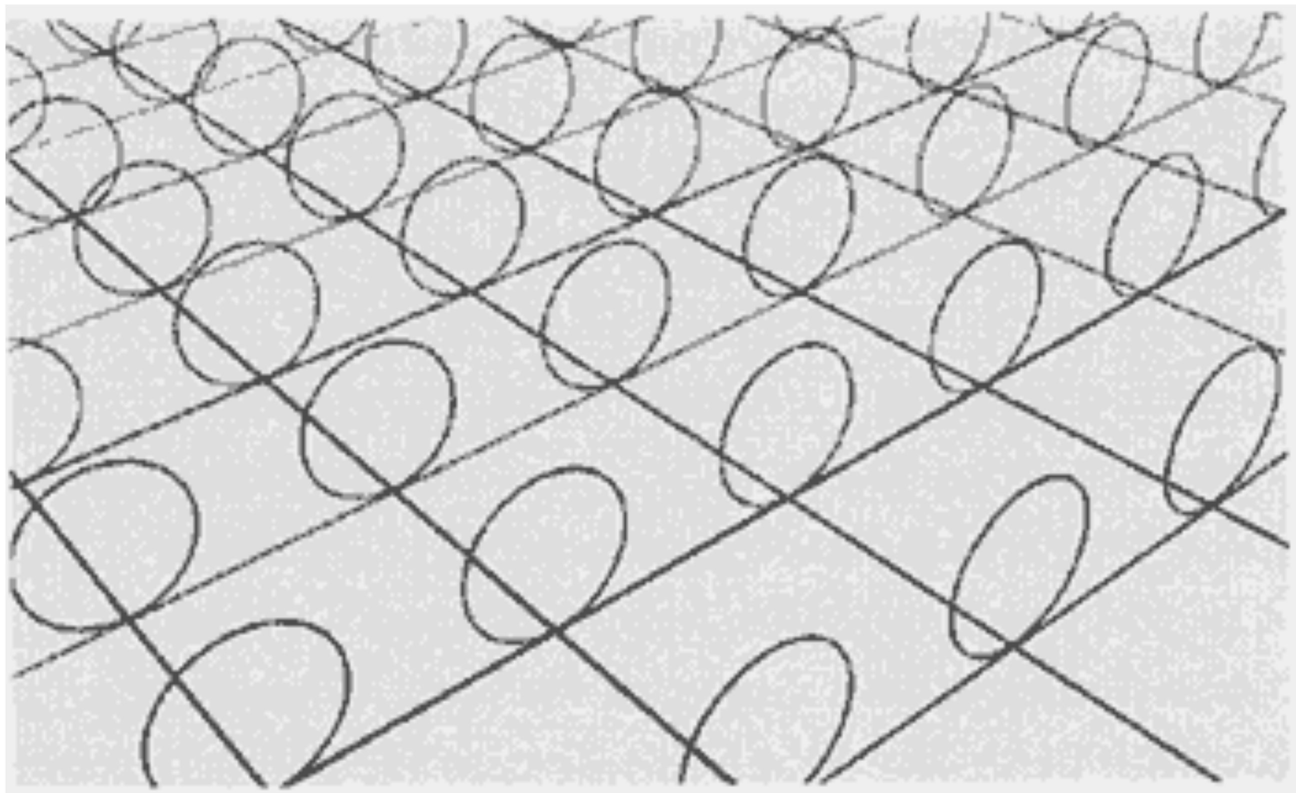
$$X = c_0 + c_1 e^2 E + c_2 e^4 E^2 + \cdots$$

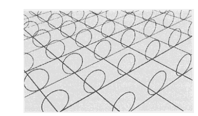
energy

perturbative expansion breaks down at high energies.

limited predictability, but makes sense as an effective theory.

Kaluza-Klein theory



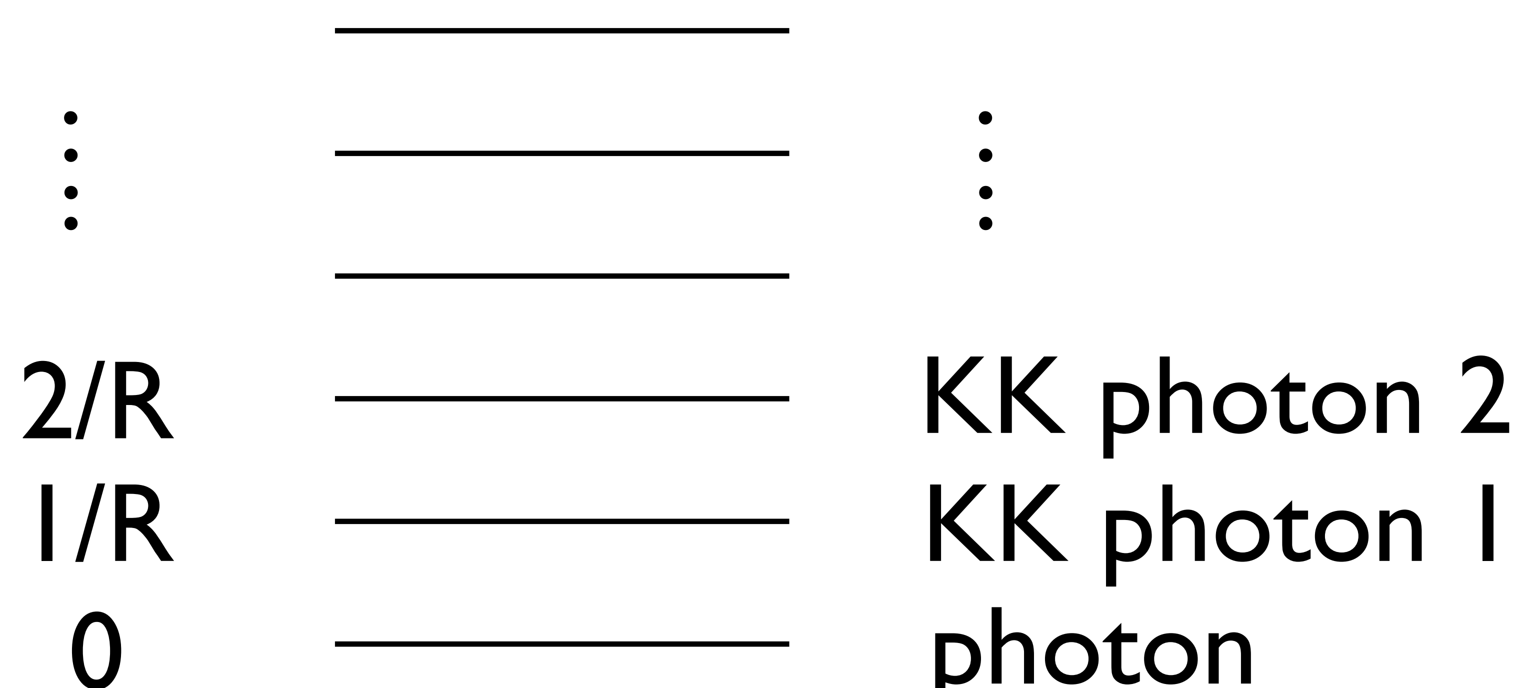


5th direction is compactified.

Looks like 4-dim for long-distance (low energy) physics.

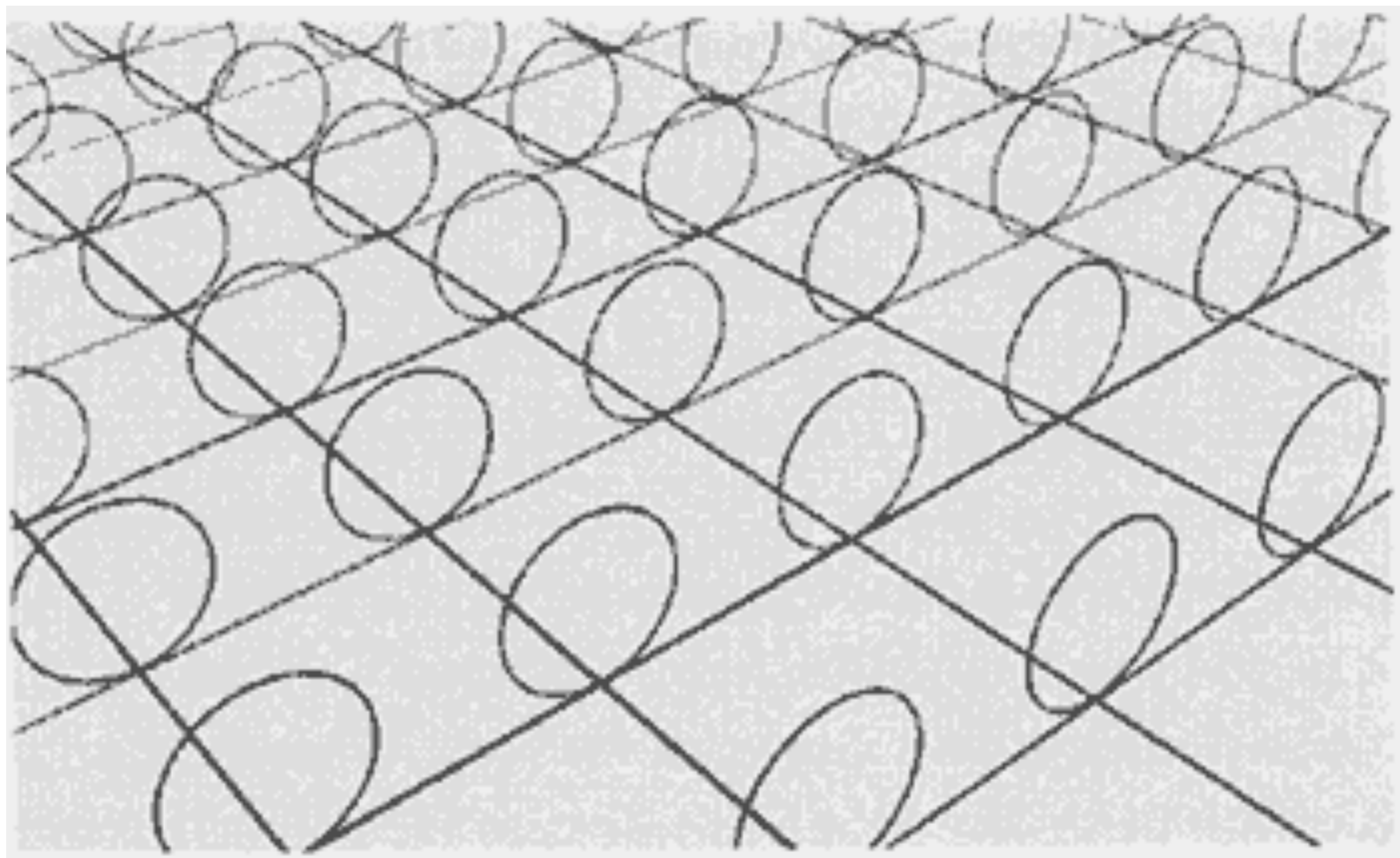
In this type of theories,
the momentum in the 5th dimension is quantized.

from 4-dim people, this looks like the appearance of the Kaluza-Klein modes.



Cute. It is wonderful if we see them!

Higgs from higher dimension?



In constructing models, it is a choice if we allow the Higgs to propagate into the extra dimension(s).

interesting possibilities

* Higgs from gauge field in 5dim: Hosotani mechanism.
(this model shares various features in the Nambu-Goldstone Higgs scenario.)

* Higgs from composite in 5dim: self breaking mechanism.

(Higgs as the condensation of the Standard Model fermions?)

* Higgs mass from supersymmetric higher dim. theory.
supersymmetry breaking via compactification

Higgs may be a window to physics of space-time.

Experimental hints of BSM?

anomalous magnetic moment of muon?

$$H = +g \frac{e}{2m} \mathbb{S} \cdot \mathbb{B}$$

$\uparrow$ spin

interaction Hamiltonian of muon.

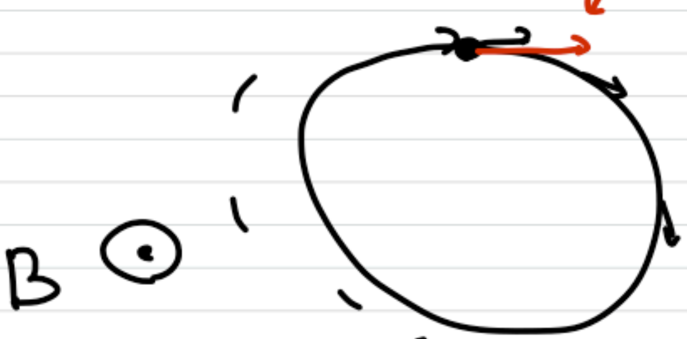
g=2 in at tree level.

quantum corr.

$\frac{\alpha}{\pi}$

$+$  $- \cdots$

calculated up to 5-loop level!



g-2 gives different frequency for spin rotation

compare

BNL + Fermilab $g = 2.00233184122(82)$

$g_{SM} = 2.00233183620(86)$

(4.2σ deviation).

very precise measurement!

theory calculation $\quad a = \frac{g-2}{2}$

$116\ldots \times 10^{-11}$

QED  + ... up to 5-loop!

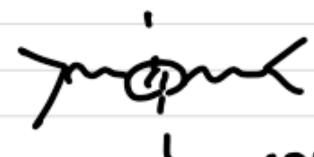
hadron  

LBL

EW

↑ hadron

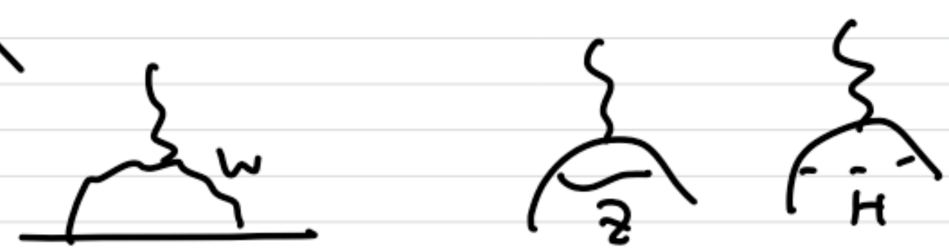
↑ e^+e^- exp



O(7000)

↑ Model lattice

O(100) $\times 10^{-11}$



up to 2-loop.

O(200)

deviation O(200)

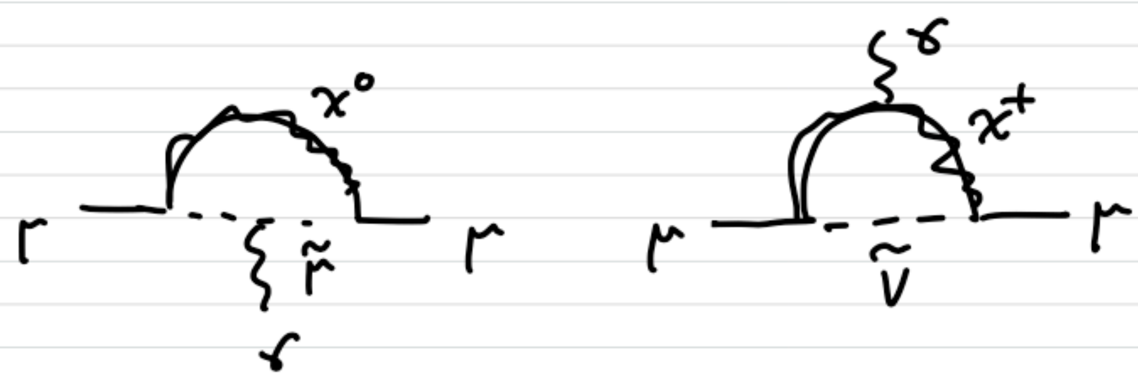
New physics at EW ?

New physic operator

$$\frac{m_\mu}{\Lambda^2}\,\bar{\mu}\,\sigma^{\mu\nu}\mu\, F_{\mu\nu}$$

$\Rightarrow$ $\Lambda \sim$ TeV is necessary.

Supersym. ?

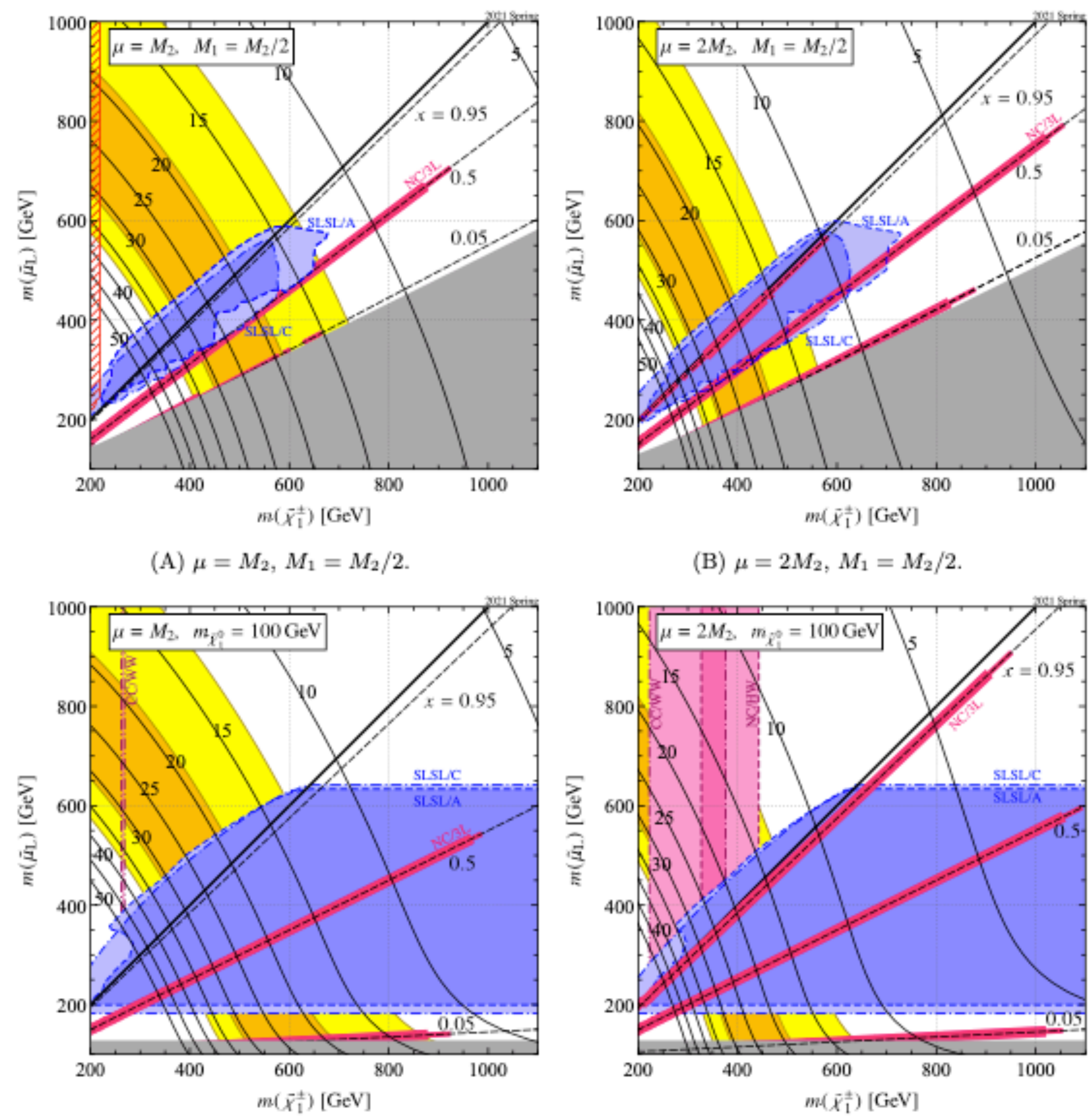


we need $\tilde{\mu}$ and $\tilde{\chi}$ below TeV.

Extra dim (many variations)

KK modes below TeV

[Endo, Hamaguchi, Iwamoto, Kitahara '21]



(A) $\mu = M_2,\ M_1 = M_2/2$.

(B) $\mu = 2M_2,\ M_1 = M_2/2$.

(C) $\mu = M_2,\ m_{\tilde{\chi}_1^0} = 100\,\mathrm{GeV}$.

(D) $\mu = 2M_2,\ m_{\tilde{\chi}_1^0} = 100\,\mathrm{GeV}$.

light particles?

dark photon?

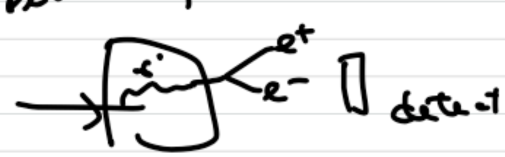
↑ small mixing κ

$$\Rightarrow \Delta(g-2) \sim \frac{\alpha\kappa^2}{2\pi} \quad \text{for } m_{\gamma'} \ll m_\mu$$

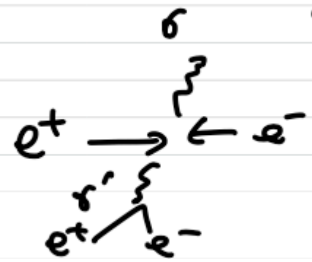
$\kappa \sim 10^{-3}$ can explain the deviation

$m_{\gamma'} \sim 100\,\mathrm{MeV}$ is allowed.
(if γ' does not couple to e)

beam dump



KLOE



$$\pi^0 \to \gamma\gamma', \quad \gamma' \to e^+e^-$$

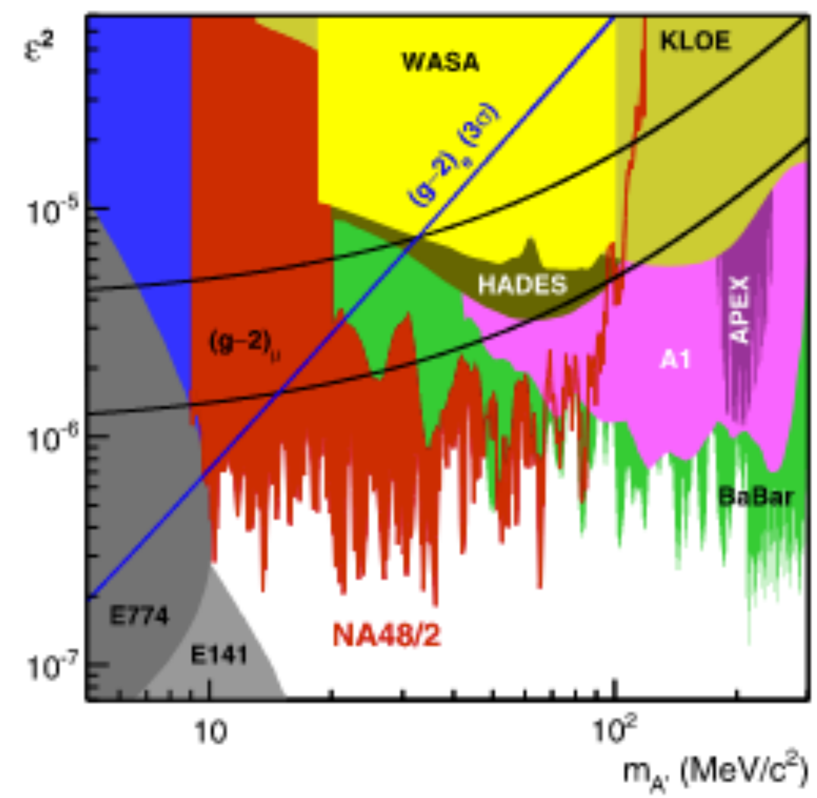
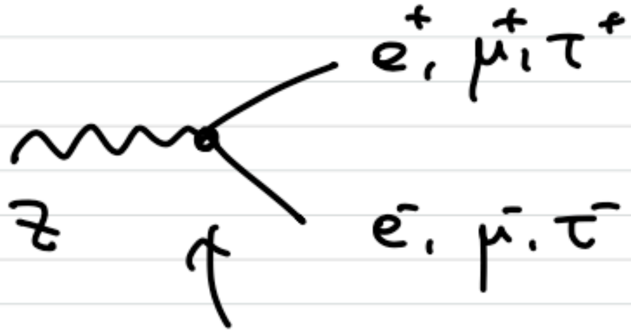


Fig. 4. Obtained upper limits at 90% CL on the mixing parameter ε^2 versus the DP mass $m_{A'}$, compared to other published exclusion limits from meson decay, beam dump and e^+e^- collider experiments [16–22]. Also shown is the band where the inconsistency of theoretical and experimental values of muon $(g-2)$ reduces to less than 2 standard deviations, as well as the region excluded by the electron $(g-2)$ measurement [2,23,24].

Wmm.. looks excluded,
but models of dark photon which only couples to mu and tau seem to evade the bounds.

Anomaly in B-physics?

flavor universality



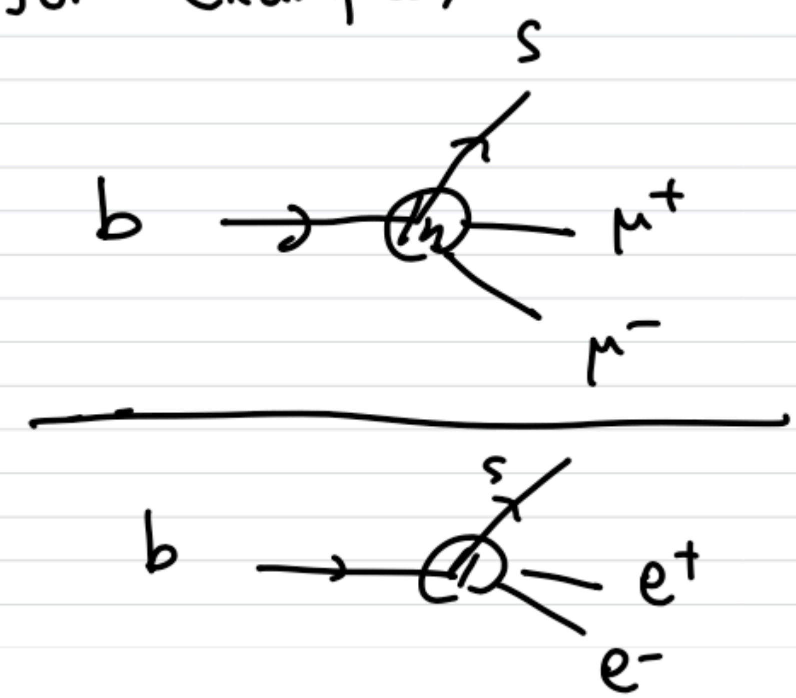
same coupling

in the SM

the differences btw e and μ

are $\propto m_\mu - m_e$

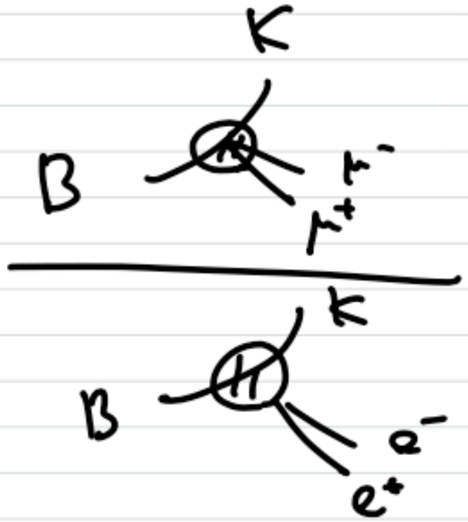
for example,

$R_X =$  $\simeq 1.0$

(theory prediction)

But…

LHCb

$R_K =$ 

$$R_K^{exp} = 0.846 \pm 0.05 \qquad (3\sigma)$$

$$R_{K^*}^{exp} = 0.69 \pm 0.1 \qquad (3\sigma)$$

This can be explained by adding new interaction terms to the SM:

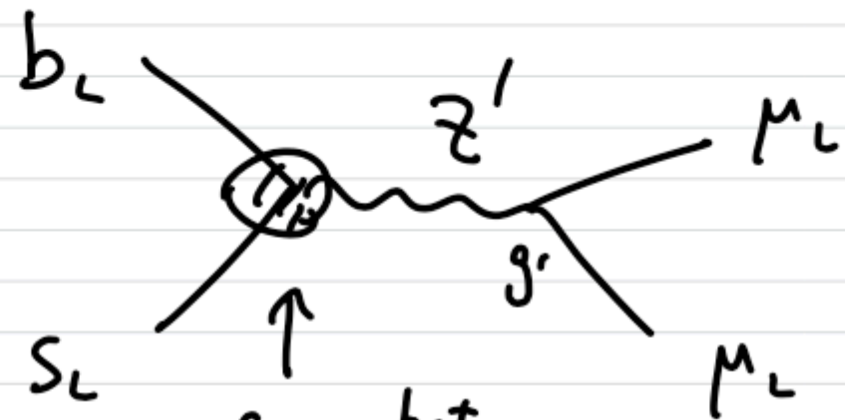
$$\frac{1}{\Lambda^2} \times \Big[(\bar{s}\gamma_\mu P_L b)(\bar{\mu}\gamma^\mu \mu) + (\bar{s}\gamma_\mu P_L b)(\bar{\mu}\gamma^\mu P_L \mu) + \text{h.c.} \Big]$$

with $\Lambda \sim 35\,\text{TeV}$

that's interesting.

Models?

20–30 TeV (?)

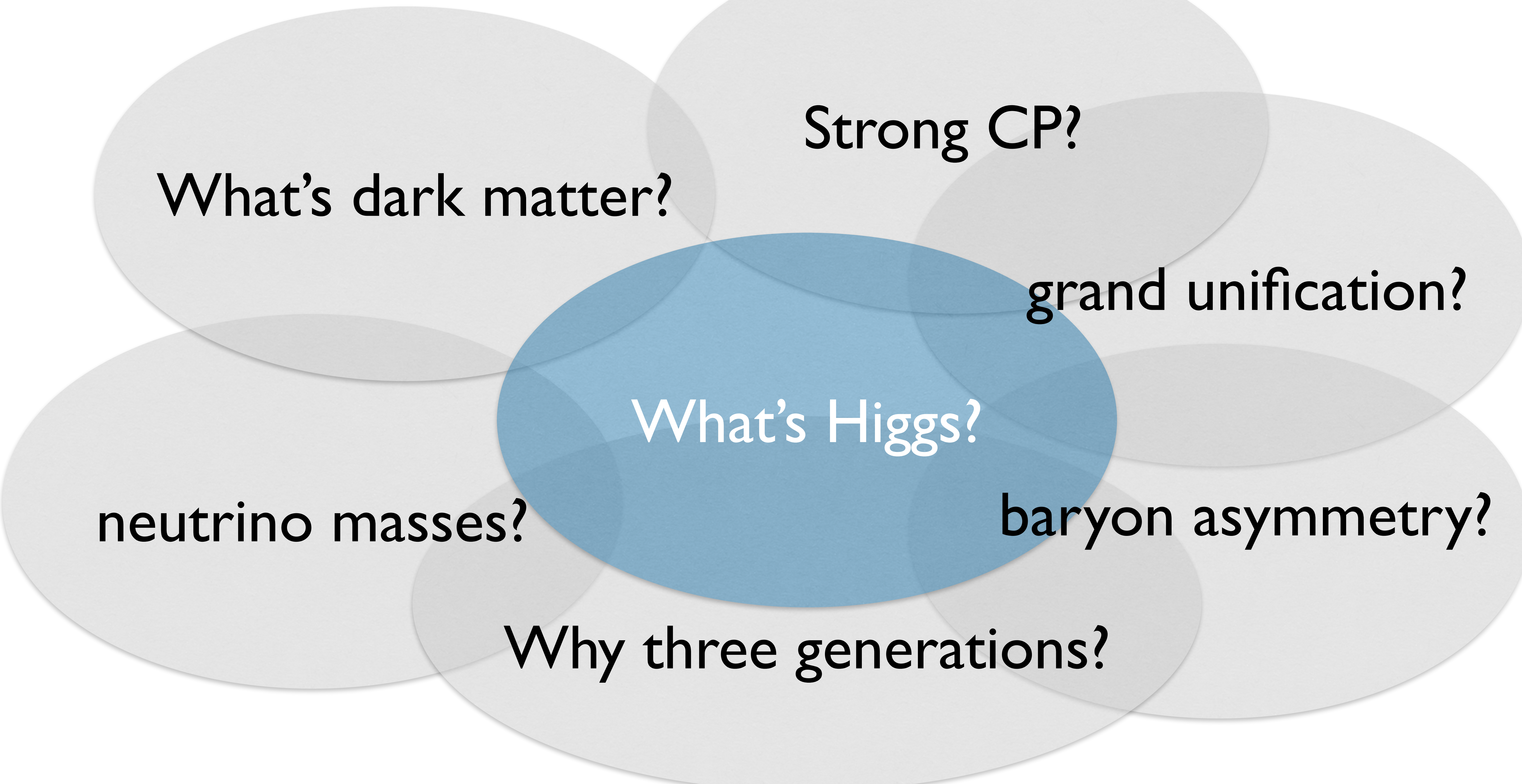


somewhat suppressed to avoid

$$\frac{g'}{m_{Z'}} \lesssim 1\,\mathrm{TeV}$$

exciting.

We've seen various mysteries:



These may be related. The central question is (I think) "what's Higgs?"