Molecular interpretation of the P_c states

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U.-G. Meißner, J. A. Oller, and Q. Wang

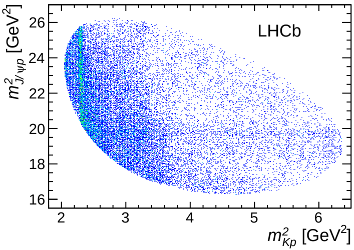
Based on PRL124(2020)072001 and JHEP08(2021)157

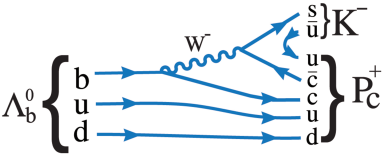
International Workshop on Partial Wave Analyses and Advanced Tools for
Hadron Spectroscopy (PWA 12 / ATHOS 7)

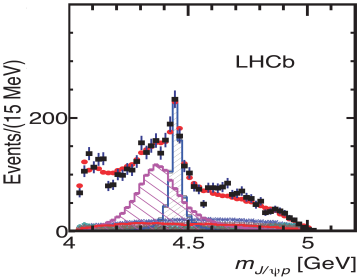
Charmonium-pentaquark states (I)

Observation of exotic structures (P_c) in $\Lambda_b^0 \to J/\psi p K^-$

LHCb, PRL 115, 072001 (2015)







$P_c(4380)^+$: $M = 4380 \pm 8 \pm 29$ MeV

$\Gamma = 205 \pm 18 \pm 86$ MeV

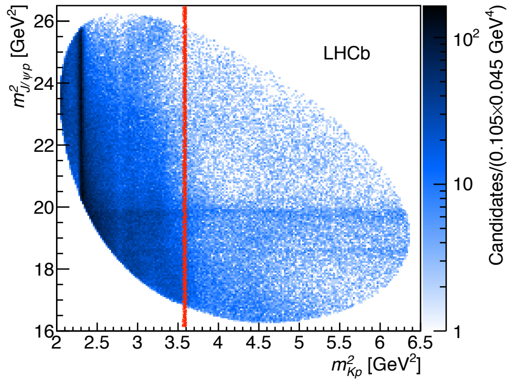
$P_c(4450)^+$: $M = 4449.8 \pm 1.7 \pm 2.5$ MeV

$\Gamma = 39 \pm 5 \pm 19$ MeV

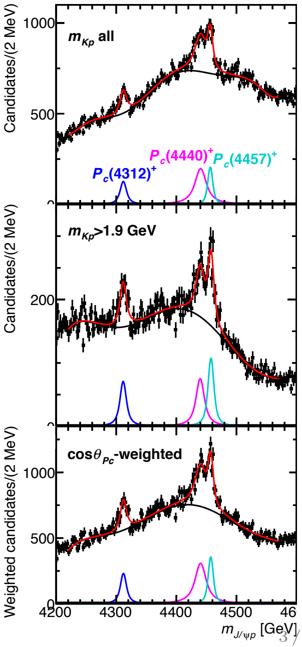
Preferred Parity: Opposite

Charmonium-pentaquark states (II)

LHCb, PRL 122, 222001 (2019)



State	M [MeV]	Γ [MeV]
$P_c(4312)^+$	$4311.9 \pm 0.7^{+6.8}_{-0.6}$	$9.8 \pm 2.7^{+\ 3.7}_{-\ 4.5}$
$P_c(4440)^+$	$4440.3 \pm 1.3^{+4.1}_{-4.7}$	$20.6 \pm 4.9^{+\ 8.7}_{-10.1}$
$P_c(4457)^+$	$4457.3 \pm 0.6^{+4.1}_{-1.7}$	$6.4 \pm 2.0^{+\ 5.7}_{-\ 1.9}$



Charmonium-pentaquark (theoretical)

- Compact pentaquark — Cheng et al., PRD100(2019)054002

  $P_c(4312)$, $P_c(4440)$, $P_c(4457)$: $J^P = 3/2^-, 1/2^-, 3/2^-$

- Compact diquark model — Ali et al., JHEP1910(2019)256

$3/2^-$	4240 ± 29
$3/2^+$	4440 ± 35
$5/2^+$	4457 ± 35

- P_cs as double triangle cusps — Nakamura, PRD103(2020)L111503
- $P_c(4312)$: virtual state — Fernández-Ramírez et al., PRL123(2019)092001
- K-matrix: $J/\psi p - \Sigma_c \bar{D} - \Sigma_c \bar{D}^*$ — Kuang et al., EPJC80(2020)433

  $\hookrightarrow$ P_cs have same J^P. $P_c(4312)$: $\Sigma_c \bar{D}$, $P_c(4457)$: ? cusp effect

- Molecule (HQSS) — Liu et al., PRL122,242001 (2019)

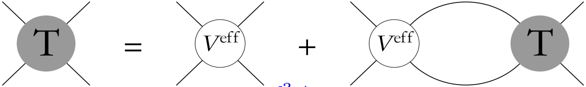
	Molecule	J^P	M (MeV)		Molecule	J^P	M (MeV)
A	$\bar{D}\Sigma_c$	$\frac{1}{2}^-$	4311.8 − 4313.0	B	$\bar{D}\Sigma_c$	$\frac{1}{2}^-$	4306.3 − 4307.7
A	$\bar{D}\Sigma_c^*$	$\frac{3}{2}^-$	4376.1 − 4377.0	B	$\bar{D}\Sigma_c^*$	$\frac{3}{2}^-$	4370.5 − 4371.7
A	$\bar{D}^*\Sigma_c$	$\frac{1}{2}^-$	4440.3*	B	$\bar{D}^*\Sigma_c$	$\frac{1}{2}^-$	4457.3*
A	$\bar{D}^*\Sigma_c$	$\frac{3}{2}^-$	4457.3*	B	$\bar{D}^*\Sigma_c$	$\frac{3}{2}^-$	4440.3*
A	$\bar{D}^*\Sigma_c^*$	$\frac{1}{2}^-$	4500.2 − 4501.0	B	$\bar{D}^*\Sigma_c^*$	$\frac{1}{2}^-$	4523.2 − 4523.6
A	$\bar{D}^*\Sigma_c^*$	$\frac{3}{2}^-$	4510.6 − 4510.8	B	$\bar{D}^*\Sigma_c^*$	$\frac{3}{2}^-$	4516.5 − 4516.6
A	$\bar{D}^*\Sigma_c^*$	$\frac{5}{2}^-$	4523.3 − 4523.6	B	$\bar{D}^*\Sigma_c^*$	$\frac{5}{2}^-$	4500.2 − 4501.0

and many more works...

- quantum numbers? line shape? the existence of $P_c(4380)$?

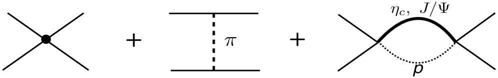
EFT approach at low energies: $\Sigma_c^{(*)}\bar{D}^{(*)}$ $(\Lambda_c \bar{D}^{(*)})$

Lippmann-Schwinger Equation:



$$T^J_{\alpha\gamma}(E,p,p') = V^J_{\alpha\gamma}(E,p,p') - \sum_\beta \int \frac{d^3\vec{q}}{(2\pi)^3} V^J_{\alpha\beta}(E,p,q) G_\beta(E,q) T^J_{\beta\gamma}(E,q,p')$$

☞ $V^{\rm eff}_{\rm LO}$ =



HQSS: 2 S-S wave LECs at O(Q^0)
1 S-D wave LEC at O(Q^2)

Long range: OPE

Imaginary part from inelastic channels

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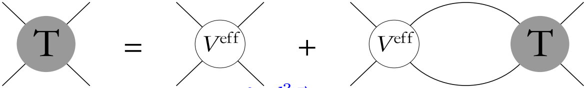

$$T^J_{\alpha\gamma}(E,p,p') = V^J_{\alpha\gamma}(E,p,p') - \sum_\beta \int \frac{d^3\vec{q}}{(2\pi)^3} V^J_{\alpha\beta}(E,p,q) G_\beta(E,q) T^J_{\beta\gamma}(E,q,p')$$

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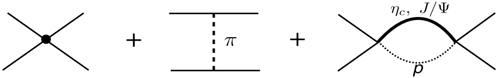

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Long range: OPE

Imaginary part
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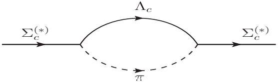
☞ Green function: $\Gamma(\Sigma_c^* \to \Lambda_c\pi) = 15.0$ MeV $\sim \Gamma(P_c)$

$$G_\beta(E,\mathbf{q}) = \frac{m_{\Sigma_c^{(*)}} m_{D^{(*)}}}{E_{\Sigma_c^{(*)}}(\mathbf{q}) E_{D^{(*)}}(\mathbf{q})} \frac{1}{E_{\Sigma_c^{(*)}}(\mathbf{q}) + E_{D^{(*)}}(\mathbf{q}) - E - \frac{\tilde{\Sigma}_R^{(*)}(s)}{2E_{\Sigma_c^{(*)}}(\mathbf{q})}},$$

$\hookrightarrow$ The self-energy: $\tilde{\Sigma}_R^{(*)}(s) \sim ig^2 \frac{p^3}{\sqrt{s}}$



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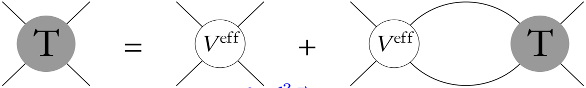

$$T^J_{\alpha\gamma}(E,p,p') = V^J_{\alpha\gamma}(E,p,p') - \sum_\beta \int \frac{d^3\vec{q}}{(2\pi)^3} V^J_{\alpha\beta}(E,p,q) G_\beta(E,q) T^J_{\beta\gamma}(E,q,p')$$

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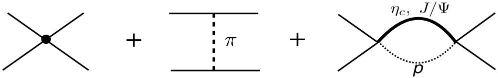

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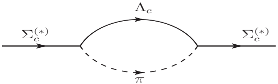
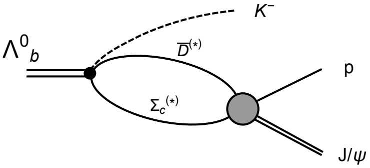


↪ Nonrelativistic limit:

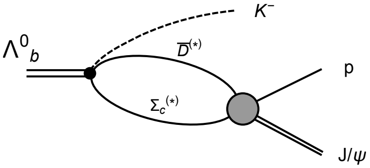
$$G_\beta(E,\mathbf{q}) = \frac{1}{\frac{\mathbf{q}^2}{2\mu} + m_{D^{(*)}} + m_{\Sigma_c^{(*)}} - E}.$$

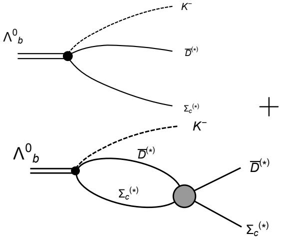
$\Lambda_b^0 \to K^- J/\psi p$



☞ $m_{J/\psi p} \sim 4440$ MeV
$\hookrightarrow$ $|\mathbf{p}| \sim 810$ MeV
$\hookrightarrow$ $J/\psi p(S)$, $J/\psi p(D)$

$\Lambda_b^0 \to K^- J/\psi p$





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$\hookrightarrow$ S-wave $\Sigma_c^{(*)}\bar{D}^{(*)}$
$\hookrightarrow$ 7 parameters: P_α^J

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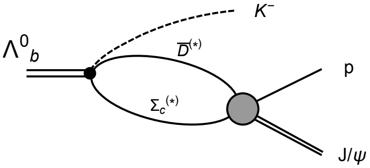

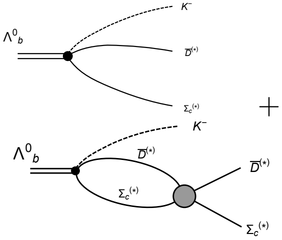

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☞

$$\text{channels} \quad \begin{cases} \underline{\Sigma_c^{(*)}\bar{D}^{(*)}(S/D)}, \Lambda_c\bar{D}^{(*)}(S/D) & \to \alpha,\beta,\gamma \\ J/\psi p(S/D), \eta_c p(S/D) & \to i,j,k \end{cases}$$

☞

$$U_\alpha^J(E,p) = P_\alpha^J(E,p) - \sum_\beta \int \frac{d\mathbf{q}^3}{(2\pi)^3} V_{\alpha\beta}^J(E,p,q) G_\beta(E,q) U_\beta^J(q),$$

$$U_i^J(E,p) = \sum_\beta \int \frac{d\mathbf{q}^3}{(2\pi)^3} \mathcal{V}_{\beta i} G_\beta(E,q) U_\beta^J(q).$$

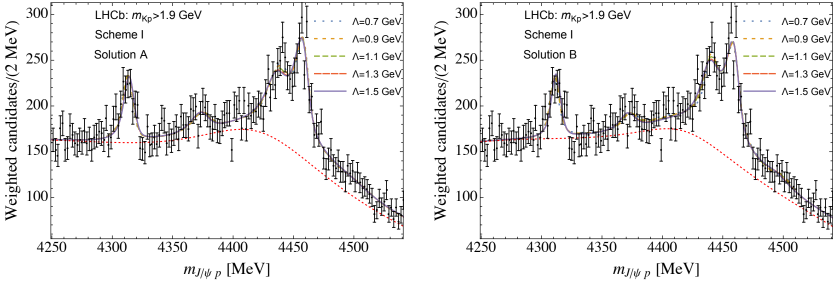
Fit Schemes

☞ Fit schemes:

- Scheme I: pure contact potential w/o $\Lambda_c\bar{D}^{(*)}$
- Scheme II: Scheme I + OPE + S-D counter term w/o $\Lambda_c\bar{D}^{(*)}$
  ↪ coupled channel
- Scheme III: contact + OPE + S-D counter terms w/ $\Lambda_c\bar{D}^{(*)}$

Scheme I: pure contact potential w/o $\Lambda_c\bar{D}^{(*)}$

Solution A | Solution B



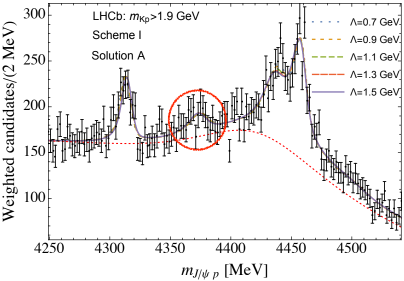
- ☞ $\Lambda > \Lambda_{\rm soft} \sim \sqrt{2\mu\delta} \sim 0.7$ GeV
- ☞ Cutoff-independent for both solution A and B

$\Sigma_c\bar{D}^*$	$P_c(4440)$	$P_c(4457)$
Fit A	$\frac{1}{2}^-$	$\frac{3}{2}^-$
Fit B	$\frac{3}{2}^-$	$\frac{1}{2}^-$

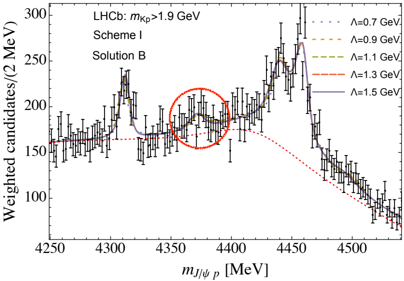
- ☞ No need for $\Lambda_c\bar{D}^{(*)}$

Scheme I: pure contact potential w/o $\Lambda_c \bar{D}^{(*)}$

Solution A



Solution B



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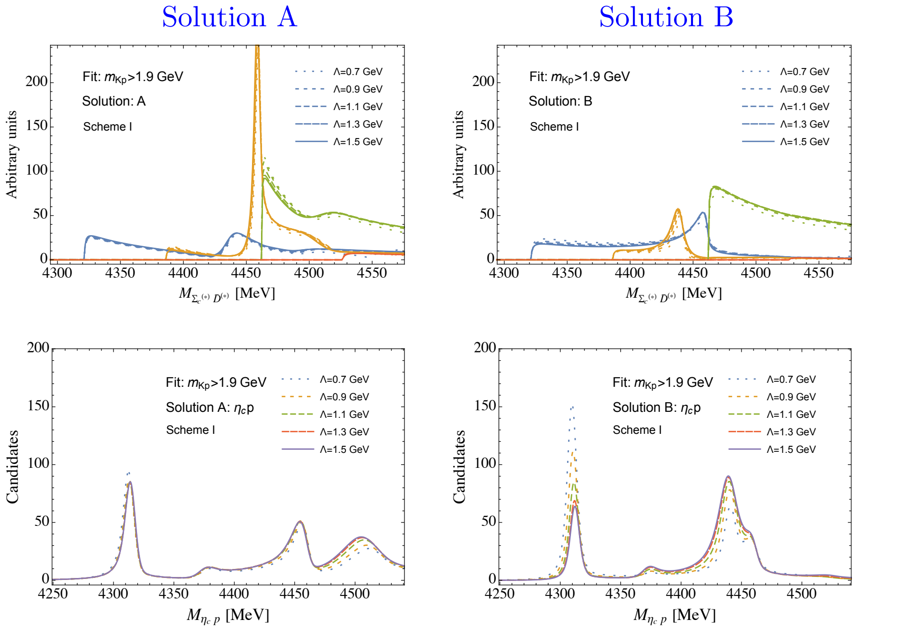
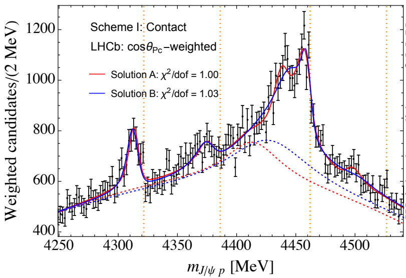
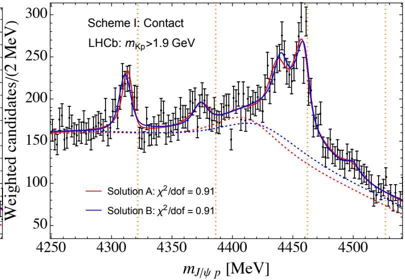
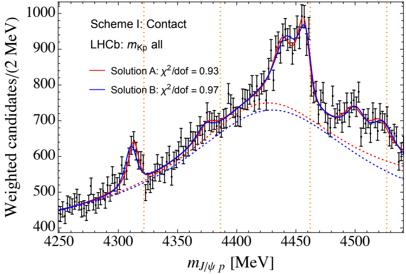


Scheme I: Contact fits to three sets of LHCb data







Scheme I: pole positions

		Solution A		Solution B	
	DC ([MeV])	J^P	Pole [MeV]	J^P	Pole [MeV]
$P_c(4312)$	$\Sigma_c\bar{D}$ (4321.6)	$\frac{1}{2}^-$	$4314(1)-4(1)i$	$\frac{1}{2}^-$	$4312(2)-4(2)i$
$\mathbf{P_c(4380)}^*$	$\Sigma_c^*\bar{D}$ (4386.2)	$\frac{3}{2}^-$	$4377(1)-7(1)i$	$\frac{3}{2}^-$	$4375(2)-6(1)i$
$P_c(4440)$	$\Sigma_c\bar{D}^*$ (4462.1)	$\frac{1}{2}^-$	$4440(1)-9(2)i$	$\frac{3}{2}^-$	$4441(3)-5(2)i$
$P_c(4457)$	$\Sigma_c\bar{D}^*$ (4462.1)	$\frac{3}{2}^-$	$4458(2)-3(1)i$	$\frac{1}{2}^-$	$4462(4)-5(3)i$
P_c	$\Sigma_c^*\bar{D}^*$ (4526.7)	$\frac{1}{2}^-$	$4498(2)-9(3)i$	$\frac{1}{2}^-$	$4526(3)-9(2)i$
P_c	$\Sigma_c^*\bar{D}^*$ (4526.7)	$\frac{3}{2}^-$	$4510(2)-14(3)i$	$\frac{3}{2}^-$	$4521(2)-12(3)i$
P_c	$\Sigma_c^*\bar{D}^*$ (4526.7)	$\frac{5}{2}^-$	$4525(2)-9(3)i$	$\frac{5}{2}^-$	$4501(3)-6(4)i$

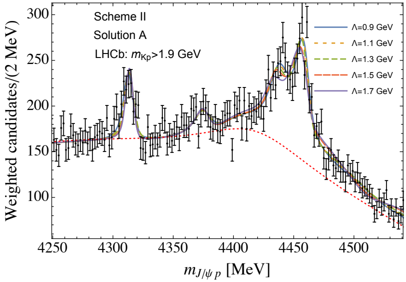
- ☞ * NOT the broad $P_c(4380)$ reported by LHCb in 2015
- ☞ Bound states with respect to the dominant channel (DC)
- ☞ $\Sigma_c^*\bar{D}^*$ states are not seen yet, production rate suppressed?

↪ prompt production in the pp collision in the LHC

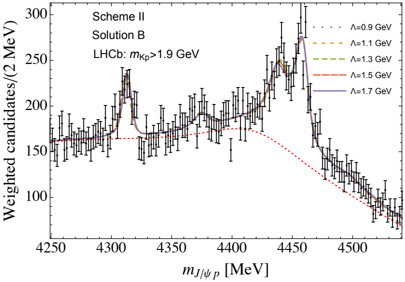
P. Ling, X.-H. Dai, MLD and Q. Wang, arXiv:2104.11133

Scheme II: contact + OPE + S-D w/o $\Lambda_c \bar{D}^{(*)}$

Solution A



Solution B

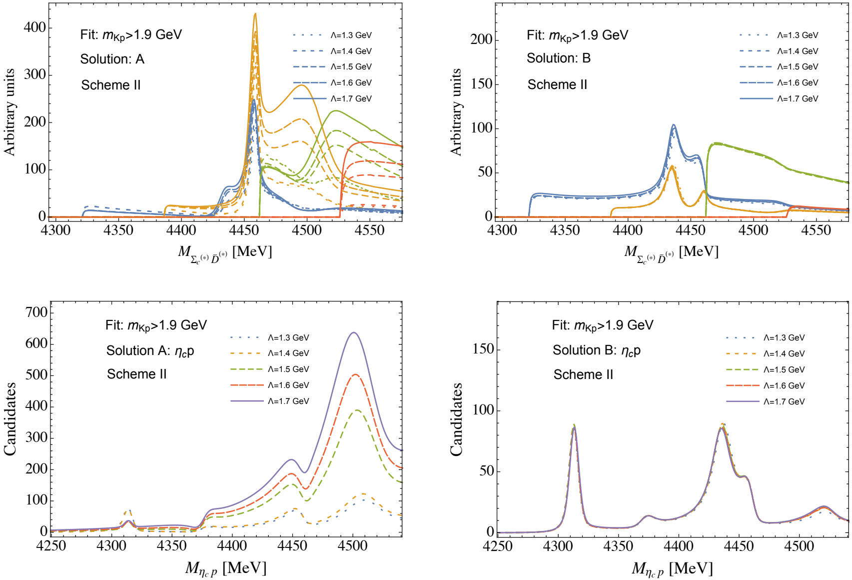


☞ $\Lambda_{\text{soft}} \sim 0.7$ GeV

☞ Cutoff-independent only for solution B

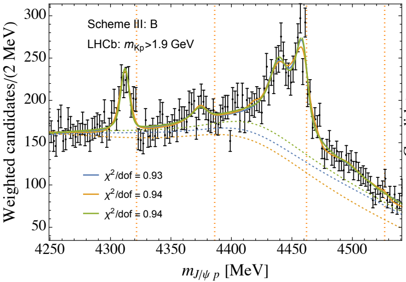
Scheme II: contact + OPE + S-D w/o $\Lambda_c \bar{D}^{(*)}$

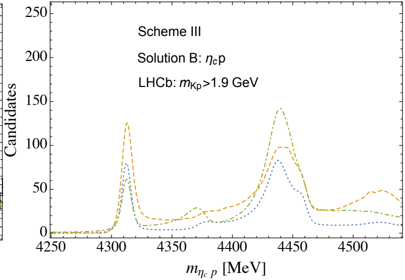
Solution A $\Lambda_{\rm soft} \sim 0.7$ GeV Solution B



Scheme III: CT + OPE + SD w/ $\Lambda_c\bar{D}^{(*)}$ $\Lambda_{\rm soft} \sim 0.9$ GeV

Solution B



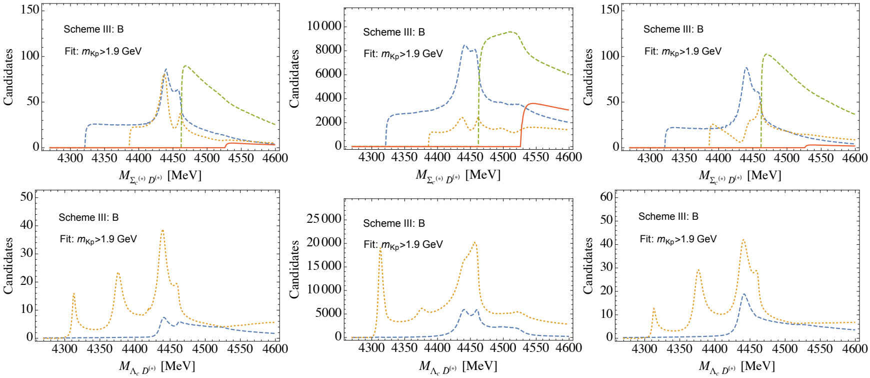


☞ $\Lambda = 1.3$ GeV.

☞ Overdetermined.

Scheme III: CT + OPE + SD w/ $\Lambda_c \bar{D}^{(*)}$ $\Lambda_{\rm soft} \sim 0.9$ GeV

Solution B



☞ $J/\psi p$ data alone are not enough to constrain $\Lambda_c \bar{D}^{(*)}$ interactions.

Summary & Outlook

☞ Solving Lippmann-Schwinger equation with respect to

- Unitarity, three-body cut
  $\hookrightarrow$ width of $\Sigma_c^{(*)}$
- Coupled-channels
  $\hookrightarrow$ cut-off independence: OPE $\to$ SD counter term
- Heavy quark spin symmetry
  $\hookrightarrow$ 7 $\Sigma_c^{(*)}\bar{D}^{(*)}$ molecular states

☞ Preferred spin assignment (Solution B)

$$\begin{array}{llll} P_c(4312) & : & 1/2^-, & (\bar{D}\Sigma_c) \\ P_c(4440) & : & 3/2^-, & (\bar{D}^*\Sigma_c) \\ P_c(4457) & : & 1/2^-, & (\bar{D}^*\Sigma_c) \end{array}$$

☞ We can not say much about $\Lambda_c\bar{D}^{(*)}$ interaction without data in this channel.

☞ A narrow $P_c(4380)$, different from the broad one reported by LHCb in 2015.

Thank you very much for your attention!

Heavy-quark spin symmetry (HQSS)

- In the limit $\Lambda_{\rm QCD}/m_Q \to 0$
  $\hookrightarrow$ strong interactions are independent on heavy-quark spin
- S-wave $\bar{D}^{(*)}\Sigma_c^{(*)}$ $\Lambda_c\bar{D}^{(*)}$ spin decomposition $|s_Q \otimes j_\ell\rangle$

$$\begin{pmatrix} |\Sigma_c\bar{D}\rangle \\ |\Sigma_c\bar{D}^*\rangle \\ |\Sigma_c^*\bar{D}^*\rangle \\ |\Lambda_c\bar{D}\rangle \\ |\Lambda_c\bar{D}^*\rangle \end{pmatrix}_{\frac{1}{2}} = \begin{pmatrix} \frac{1}{2} & \frac{1}{2\sqrt{3}} & \sqrt{\frac{2}{3}} & 0 & 0 \\ \frac{1}{2\sqrt{3}} & \frac{5}{6} & -\frac{\sqrt{2}}{3} & 0 & 0 \\ \sqrt{\frac{2}{3}} & -\frac{\sqrt{2}}{3} & -\frac{1}{3} & 0 & 0 \\ 0 & 0 & 0 & -\frac{1}{2} & \frac{\sqrt{3}}{2} \\ 0 & 0 & 0 & \frac{\sqrt{3}}{2} & \frac{1}{2} \end{pmatrix} \begin{pmatrix} |0\otimes\frac{1}{2}\rangle \\ |1\otimes\frac{1}{2}\rangle \\ |1\otimes\frac{3}{2}\rangle \\ |0\otimes\frac{1}{2}\rangle' \\ |1\otimes\frac{1}{2}\rangle' \end{pmatrix}, \tag{1}$$

$$\begin{pmatrix} |\Sigma_c\bar{D}^*\rangle \\ |\Sigma_c^*\bar{D}\rangle \\ |\Sigma_c^*\bar{D}^*\rangle \\ |\Lambda_c\bar{D}^*\rangle \end{pmatrix}_{\frac{3}{2}} = \begin{pmatrix} \frac{1}{\sqrt{3}} & -\frac{1}{3} & \frac{\sqrt{5}}{3} & 0 \\ -\frac{1}{2} & \frac{1}{\sqrt{3}} & \frac{1}{2}\sqrt{\frac{5}{3}} & 0 \\ \frac{1}{2}\sqrt{\frac{5}{3}} & \frac{\sqrt{5}}{3} & -\frac{1}{6} & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix} \begin{pmatrix} |0\otimes\frac{3}{2}\rangle \\ |1\otimes\frac{1}{2}\rangle \\ |1\otimes\frac{3}{2}\rangle \\ |1\otimes\frac{1}{2}\rangle' \end{pmatrix}, \tag{2}$$

$$|\Sigma_c^*\bar{D}^*\rangle_{\frac{5}{2}} = |1\otimes\frac{3}{2}\rangle. \tag{3}$$

Contact interactions

- Contact interaction: short-range interaction
- strong interaction: spin of light degrees of freedom

$\hookrightarrow$ $\Sigma_c^{(*)}\bar{D}^{(*)} \to \Sigma_c^{(*)}\bar{D}^{(*)}$:

$$C_{\frac{1}{2}} \equiv \langle s_Q \otimes \frac{1}{2}|\hat{\mathcal{H}}_I|s_Q \otimes \frac{1}{2}\rangle, \quad C_{\frac{3}{2}} \equiv \langle s_Q \otimes \frac{3}{2}|\hat{\mathcal{H}}_I|s_Q \otimes \frac{3}{2}\rangle,$$

$\hookrightarrow$ $\Sigma_c^{(*)}\bar{D}^{(*)} \to \Lambda_c\bar{D}^{(*)}$:

$$C'_{\frac{1}{2}} \equiv {}'\langle s_Q \otimes \frac{1}{2}|\hat{\mathcal{H}}_I|s_Q \otimes \frac{1}{2}\rangle = \langle s_Q \otimes \frac{1}{2}|\hat{\mathcal{H}}_I|s_Q \otimes \frac{1}{2}\rangle'.$$

$\hookrightarrow$ $\Lambda_c\bar{D}^{(*)} \to \Lambda_c\bar{D}^{(*)}$:

$$C''_{\frac{1}{2}} \equiv {}'\langle s_Q \otimes \frac{1}{2}|\hat{\mathcal{H}}_I|s_Q \otimes \frac{1}{2}\rangle',$$

- $J/\psi p$ $(\eta_c p)$ Heavy-Light spin decomposition:

$$|J/\psi p\rangle \begin{cases} S\text{-wave}: |1 \otimes \frac{1}{2}\rangle \\ D\text{-wave}: |1 \otimes \frac{3}{2}\rangle \end{cases}, \qquad |\eta_c p\rangle \begin{cases} S\text{-wave}: |0 \otimes \frac{1}{2}\rangle \\ D\text{-wave}: |0 \otimes \frac{3}{2}\rangle \end{cases}.$$

- $\Sigma_c^{(*)}\bar{D}^{(*)} \to J/\psi p$ $(\eta_c p)$:

$$g_S \equiv \langle 1 \otimes \frac{1}{2}|\hat{\mathcal{H}}_I|J/\psi p\rangle_S = \langle 0 \otimes \frac{1}{2}|\hat{\mathcal{H}}_I|\eta_c p\rangle_S,$$

$$g_D k^2 \equiv \langle 1 \otimes \frac{3}{2}|\hat{\mathcal{H}}_I|J/\psi p\rangle_D = \langle 0 \otimes \frac{3}{2}|\hat{\mathcal{H}}_I|\eta_c p\rangle_D$$

Contact potentials

$$V_C^{\frac{1}{2}^-} = \begin{pmatrix} \frac{1}{3}C_{\frac{1}{2}} + \frac{2}{3}C_{\frac{3}{2}} & \frac{2}{3\sqrt{3}}C_{\frac{1}{2}} - \frac{2}{3\sqrt{3}}C_{\frac{3}{2}} & \frac{1}{3}\sqrt{\frac{2}{3}}C_{\frac{1}{2}} - \frac{1}{3}\sqrt{\frac{2}{3}}C_{\frac{3}{2}} & 0 & \frac{1}{\sqrt{3}}C'_{\frac{1}{2}} \\ \frac{2}{3\sqrt{3}}C_{\frac{1}{2}} - \frac{2}{3\sqrt{3}}C_{\frac{3}{2}} & \frac{7}{9}C_{\frac{1}{2}} + \frac{2}{9}C_{\frac{3}{2}} & -\frac{\sqrt{2}}{9}C_{\frac{1}{2}} + \frac{\sqrt{2}}{9}C_{\frac{3}{2}} & \frac{1}{\sqrt{3}}C'_{\frac{1}{2}} & \frac{2}{3}C'_{\frac{1}{2}} \\ \frac{1}{3}\sqrt{\frac{2}{3}}C_{\frac{1}{2}} - \frac{1}{3}\sqrt{\frac{2}{3}}C_{\frac{3}{2}} & -\frac{\sqrt{2}}{9}C_{\frac{1}{2}} + \frac{\sqrt{2}}{9}C_{\frac{3}{2}} & \frac{8}{9}C_{\frac{1}{2}} + \frac{1}{9}C_{\frac{3}{2}} & -\sqrt{\frac{2}{3}}C'_{\frac{1}{2}} & \frac{\sqrt{2}}{3}C'_{\frac{1}{2}} \\ 0 & \frac{1}{\sqrt{3}}C'_{\frac{1}{2}} & -\sqrt{\frac{2}{3}}C'_{\frac{1}{2}} & C''_{\frac{1}{2}} & 0 \\ \frac{1}{\sqrt{3}}C'_{\frac{1}{2}} & \frac{2}{3}C'_{\frac{1}{2}} & \frac{\sqrt{2}}{3}C'_{\frac{1}{2}} & 0 & C''_{\frac{1}{2}} \end{pmatrix},$$

$$V_C^{\frac{3}{2}^-} = \begin{pmatrix} \frac{1}{9}C_{\frac{1}{2}} + \frac{8}{9}C_{\frac{3}{2}} & -\frac{1}{3\sqrt{3}}C_{\frac{1}{2}} + \frac{1}{3\sqrt{3}}C_{\frac{3}{2}} & -\frac{\sqrt{5}}{9}C_{\frac{1}{2}} + \frac{\sqrt{5}}{9}C_{\frac{3}{2}} & -\frac{1}{3}C'_{\frac{1}{2}} \\ -\frac{1}{3\sqrt{3}}C_{\frac{1}{2}} + \frac{1}{3\sqrt{3}}C_{\frac{3}{2}} & \frac{1}{3}C_{\frac{1}{2}} + \frac{2}{3}C_{\frac{3}{2}} & +\frac{1}{3}\sqrt{\frac{5}{3}}C_{\frac{1}{2}} - \frac{1}{3}\sqrt{\frac{5}{3}}C_{\frac{3}{2}} & \frac{1}{\sqrt{3}}C'_{\frac{1}{2}} \\ -\frac{\sqrt{5}}{9}C_{\frac{1}{2}} + \frac{\sqrt{5}}{9}C_{\frac{3}{2}} & \frac{1}{3}\sqrt{\frac{5}{3}}C_{\frac{1}{2}} - \frac{1}{3}\sqrt{\frac{5}{3}}C_{\frac{3}{2}} & \frac{5}{9}C_{\frac{1}{2}} + \frac{4}{9}C_{\frac{3}{2}} & \frac{\sqrt{5}}{3}C'_{\frac{1}{2}} \\ -\frac{1}{3}C'_{\frac{1}{2}} & \frac{1}{\sqrt{3}}C'_{\frac{1}{2}} & \frac{\sqrt{5}}{3}C'_{\frac{1}{2}} & C''_{\frac{1}{2}} \end{pmatrix},$$

$$V_C^{\frac{5}{2}^-} = C_{\frac{3}{2}}\,.$$

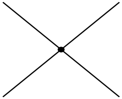
channels

J^P	S-wave
$\left(\frac{1}{2}\right)^-$	$\Sigma_c\bar{D}$, $\Sigma_c\bar{D}^*$,$\Sigma_c^*\bar{D}^*$, $\Lambda_c\bar{D}$, $\Lambda_c\bar{D}^*$
$\left(\frac{3}{2}\right)^-$	$\Sigma_c\bar{D}^*$, $\Sigma_c^*\bar{D}$, $\Sigma_c^*\bar{D}^*$, $\Lambda_c\bar{D}^*$
$\left(\frac{5}{2}\right)^-$	$\Sigma_c^*\bar{D}^*$

J^P	D-wave
$\left(\frac{1}{2}\right)^-$	$\Sigma_c\bar{D}^*$, $\Sigma_c^*\bar{D}$, $\Sigma_c^*\bar{D}^*_{\frac{3}{2}}$, $\Sigma_c^*\bar{D}^*_{\frac{5}{2}}$, $\Lambda_c\bar{D}^*$
$\left(\frac{3}{2}\right)^-$	$\Sigma_c\bar{D}$, $\Sigma_c\bar{D}^*_{\frac{1}{2}}$, $\Sigma_c\bar{D}^*_{\frac{3}{2}}$, $\Sigma_c^*\bar{D}$, $\Sigma_c^*\bar{D}^*_{\frac{1}{2}}$, $\Sigma_c^*\bar{D}^*_{\frac{3}{2}}$, $\Sigma_c^*\bar{D}^*_{\frac{5}{2}}$, $\Lambda_c\bar{D}$, $\Lambda_c\bar{D}^*_{\frac{1}{2}}$, $\Lambda_c\bar{D}^*_{\frac{3}{2}}$
$\left(\frac{5}{2}\right)^-$	$\Sigma_c\bar{D}$, $\Sigma_c\bar{D}^*_{\frac{1}{2}}$, $\Sigma_c\bar{D}^*_{\frac{3}{2}}$, $\Sigma_c^*\bar{D}$, $\Sigma_c^*\bar{D}^*_{\frac{1}{2}}$, $\Sigma_c^*\bar{D}^*_{\frac{3}{2}}$, $\Sigma_c^*\bar{D}^*_{\frac{5}{2}}$, $\Lambda_c\bar{D}$, $\Lambda_c\bar{D}^*_{\frac{1}{2}}$, $\Lambda_c\bar{D}^*_{\frac{3}{2}}$

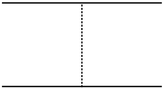
Effective Lagraingian $\Sigma_c^{(*)}\bar{D}^{(*)}$, $J/\psi p$, $\eta_c p$, $\Lambda_c\bar{D}^{(*)}$

- Contact Lagrangian



$$\begin{aligned}\mathcal{L} &= -C_a\vec{S}_c^\dagger\cdot\vec{S}_c\mathrm{Tr}[\bar{H}_c^\dagger\bar{H}_c] \\ &\quad -C_b i\epsilon_{ijk}(S_c^\dagger)_j(S_c)_k\mathrm{Tr}[\bar{H}_c^\dagger\sigma_i\bar{H}_c]. \\ &\quad +C_c\Big(S_{ab}^{i\dagger}T_{ca}\langle\bar{H}_c^\dagger\sigma^i\bar{H}_b\rangle - T_{ca}^\dagger S_{ab}^i\langle\bar{H}_b^\dagger\sigma^i\bar{H}_c\rangle\Big) \\ &\quad +C_d T_{ab}^\dagger T_{ba}\langle\bar{H}_c^\dagger\bar{H}_c\rangle\end{aligned}$$

- One-pion exchange (OPE)



$$\begin{aligned}\mathcal{L}_{DD\pi} &= \frac{g}{4}\langle\sigma\cdot u_{ab}\bar{H}_b\bar{H}_a^\dagger\rangle, \\ \mathcal{L}_{\Sigma_c\Sigma_c\pi} &= i\frac{3}{2}g_1\epsilon_{ijk}\langle\bar{S}_i u_j S_k\rangle. \\ \mathcal{L}_{\Sigma_c\Lambda_c\pi} &= -\frac{1}{\sqrt{2}}g_3(S_{ab}^{i\dagger}u_{bc}^i T_{ca} + T_{ab}^\dagger u_{bc}^i S_{ca}^i),\end{aligned}$$

☞ Effective Lagrangian for $\bar{D}^{(*)}\Sigma_c^{(*)}\to J/\psi p$ $(\eta_c p)$

$$\mathcal{L} = \frac{g_S}{\sqrt{3}}N^\dagger\sigma^i\bar{H}J^\dagger S^i - \sqrt{3}g_D N^\dagger\sigma^i\bar{H}(\partial^i\partial^j - \frac{1}{3}\delta^{ij}\partial^2)J^\dagger S^j,$$

$$\vec{S}_c = \frac{1}{\sqrt{3}}\vec{\sigma}\Sigma_c + \vec{\Sigma}_c^*, \qquad \bar{H}_c = \frac{1}{2}\Big(-\bar{D} + \vec{\sigma}\cdot\vec{\bar{D}}^*\Big), \qquad J = -\eta_c + \sigma\cdot\psi.$$

Effective potentials

☞ The effective potentials

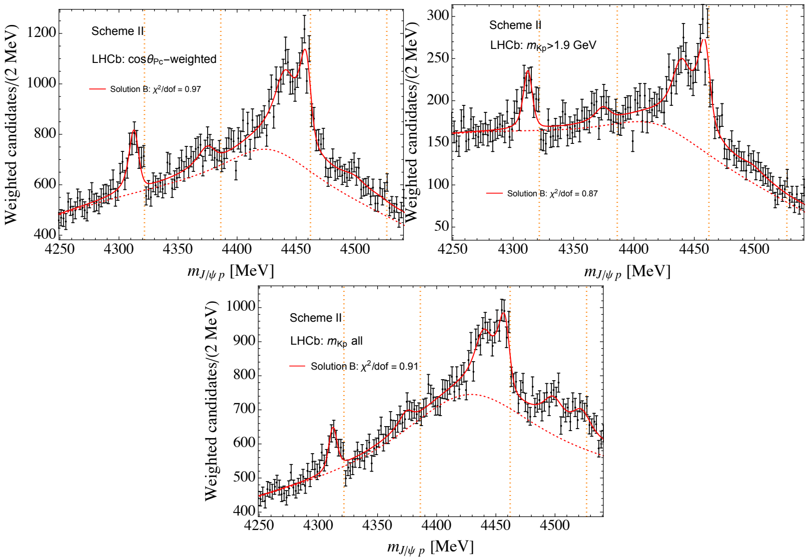
$$V^J_{\alpha\beta} = V^J_{\mathrm{CT},\alpha\beta} + V^J_{\mathrm{OPE},\alpha\beta} + \mathcal{G}^J_{\alpha\beta},$$

The effective contributions from the $J/\psi p$ and $\eta_c p$ bubble loop $(k \sim 0.9\ \mathrm{GeV})$

$$\mathcal{G}^J_{\alpha\beta} = \sum_i$$

$$= R_{\mathcal{G}} - \sum_i i \frac{1}{2\pi E} m_{\psi(\eta_c)} m_p g^{(\prime)J}_{\alpha i} g^{(\prime)J}_{\beta i} k^{2l_i+1}.$$

Fit result of Scheme II: LO + OPE + S-D w/o $\Lambda_c \bar{D}^{(*)}$



Some details on the OPE

- Pionic Lagrangian: $\mathcal{L} = \frac{g_1}{4}\underbrace{\langle \sigma \cdot u_{ab}\bar{H}_b\bar{H}_a^\dagger\rangle}_{\bar{D}^{(*)}\bar{D}^{(*)}\pi} + ig_2\underbrace{\epsilon_{ijk}S_{ab}^{i\dagger}u_{bc}^{j}S_{ca}^{k}}_{\Sigma_c^{(*)}\Sigma_c^{(*)}\pi} - \frac{1}{\sqrt{2}}g_3\underbrace{\left(S_{ab}^{i\dagger}u_{bc}^{i}T_{ca} + T_{ab}^{\dagger}u_{bc}^{i}S_{ca}^{i}\right)}_{\Sigma_c^{(*)}\Lambda_c\pi}$

  with $g_1 =$ 0.57 from D* →Dπ width, g_2 and g_3 are taken from lattice Detmold et al., PRD 85, 114508 (2012)

- Exemplary potential:

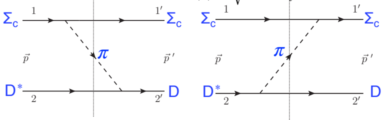
$$V_{\Sigma_c\bar{D}^*\to\Sigma_c\bar{D}}(\mathbf{p},\mathbf{p}') \propto \frac{g_1 g_2}{f_\pi^2}(\tau_1\cdot\tau_2^c)(\epsilon_{\bar{D}^*}\cdot\mathbf{q})(\sigma\cdot\mathbf{q})\left(\frac{1}{G_{\Sigma_c\bar{D}\pi}} + \frac{1}{G_{\Sigma_c\bar{D}^*\pi}}\right)$$

☛ TOPT propagators

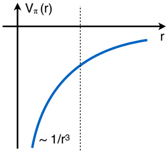
$$G_{\Sigma_c\bar{D}\pi} = 2E_\pi(q)\left(E_\pi(q) + E_{\Sigma_c}(p) + E_{\bar{D}}(p') - \sqrt{s}\right)$$

$$G_{\Sigma_c\bar{D}^*\pi} = 2E_\pi(q)\left(E_\pi(q) + E_{\Sigma_c}(p') + E_{\bar{D}^*}(p) - \sqrt{s}\right)$$

$E_i(p) = \sqrt{p^2 + m_i^2}$

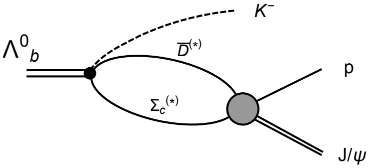


- Central part: S-wave to S-wave transitions
- Tensor part: S-wave to D-wave transitions



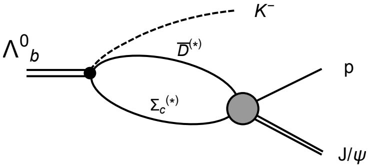
- At short distances OPE is not well defined w/o contact terms

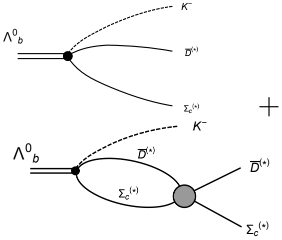
$\Lambda_b^0 \to K^- J/\psi p$



☞ $m_{J/\psi p} \sim 4440$ MeV
$\hookrightarrow |\mathbf{p}| \sim 810$ MeV
$\hookrightarrow J/\psi p(S),\ J/\psi p(D)$

$\Lambda_b^0 \to K^- J/\psi p$





☞ $m_{J/\psi p} \sim 4440$ MeV

$\hookrightarrow$ $|\mathbf{p}| \sim 810$ MeV

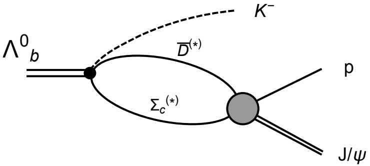
$\hookrightarrow$ $J/\psi p(S)$, $J/\psi p(D)$

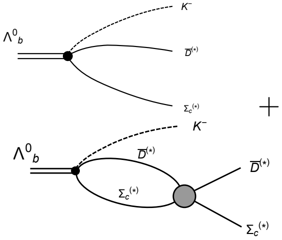
☞ Weak production:

$\hookrightarrow$ S-wave $\Sigma_c^{(*)} \bar{D}^{(*)}$

$\hookrightarrow$ 7 parameters: P_α^J

$\Lambda_b^0 \to K^- J/\psi p$





☞ $m_{J/\psi p} \sim 4440$ MeV
↪ $|\mathbf{p}| \sim 810$ MeV
↪ $J/\psi p(S)$, $J/\psi p(D)$

☞ Weak production:
↪ S-wave $\Sigma_c^{(*)} \bar{D}^{(*)}$
↪ 7 parameters: P_α^J

☞

$$\text{channels} \quad \begin{cases} \underline{\Sigma_c^{(*)} \bar{D}^{(*)}(S/D)}, \Lambda_c \bar{D}^{(*)}(S/D) & \to \ \alpha, \beta, \gamma \\ J/\psi p(S/D), \eta_c p(S/D) & \to \ i, j, k \end{cases}$$

☞

$$U_\alpha^J(E,p) \ = \ P_\alpha^J(E,p) - \sum_\beta \int \frac{d\mathbf{q}^3}{(2\pi)^3} V_{\alpha\beta}^J(E,p,q) G_\beta(E,q) U_\beta^J(q),$$

$$U_i^J(E,p) \ = \ \sum_\beta \int \frac{d\mathbf{q}^3}{(2\pi)^3} \mathcal{V}_{\beta i} G_\beta(E,q) U_\beta^J(q).$$

Scheme I + OPE w/o $\Lambda_c\bar{D}^{(*)}$

- ☞ No solution A
- ☞ Solution B:
  Cut-off dependent
- ☞ $\Lambda_{\rm soft} \sim 700$ MeV