

Charm meson decays: spectroscopy and strong phase distribution

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BESIII



Introduction

Amplitude analysis of charmed meson decays at BESIII

$$D_S^+ \rightarrow \pi^+ \pi^- \pi^+ \quad [\text{arXiv: 2108.10050}]$$

$$D_S^+ \rightarrow K_S^0 \pi^+ \pi^0 \quad [\text{JHEP 06 (2021) 181}]$$

$$D^+ \rightarrow K^+ K_S^0 \pi^0 \quad [\text{PRD 104 (2021) 012006}]$$

$$D_S^+ \rightarrow \pi^+ \pi^+ \pi^- \eta \quad [\text{arXiv:2106.13536}]$$

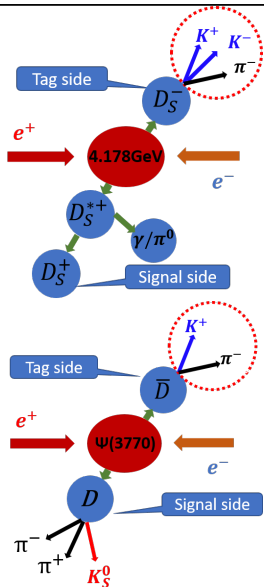
$$D_S^+ \rightarrow K^- K^+ \pi^+ \pi^0 \quad [\text{PRD 104 (2021) 032011}]$$

Strong phase parameters in neutral $D\bar{D}$ meson decays at BESIII

Phase variation across $D \rightarrow K_S^0 \pi^+ \pi^-$ Dalitz plot [PRD 101 (2020) 112002]

Phase variation in $D \rightarrow K3\pi$ phase space [JHEP 05 (2021) 164]

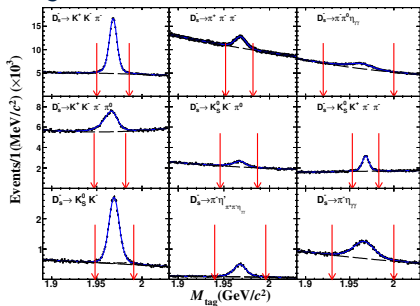
Summary



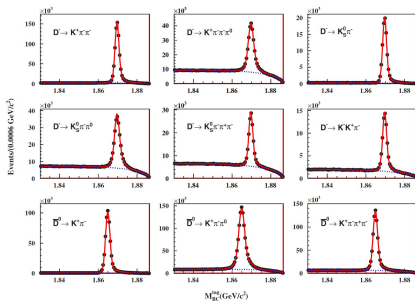
- $D_S^{*+} D_S^{*-}$: 6.32 fb^{-1} at energy points $4.178\text{--}4.226\text{ GeV}$
- $D\bar{D}$: 2.93 fb^{-1} at $\psi(3770)$ peak
- Single tag (ST): reconstruct one $D_{(S)}$
- Double tag (DT): $D_{(S)}$ in the other side
- Absolute branching fraction measurements
- Systematics in the tag side almost cancel out

Single tag $D_{(S)}$ samples at BESIII

D_S^- [JHEP 06 (2021) 181]



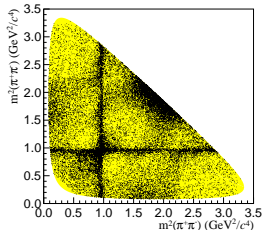
D^-/D^0 [PRD 97 (2018) 072015]



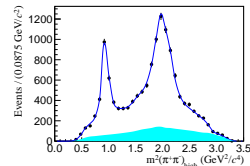
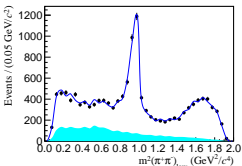
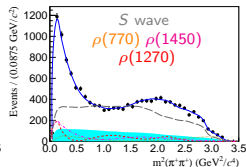
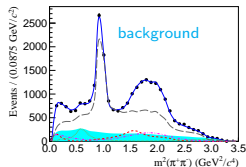
- Large ST samples by reconstructing hadronic decays with large branching fractions in the tag side
- Clean environment for amplitude analyses as continuum background is highly suppressed

Amplitude analysis of $D_S^+ \rightarrow \pi^+ \pi^- \pi^+$

[arXiv: 2108.10050]



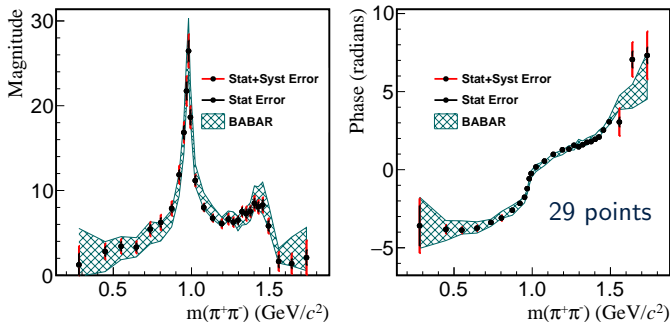
13797 signal events, $\sim 80\%$ purity



Decay mode	Fit fraction (%)	Magnitude	Phase (rad.)
$f_2(1270)\pi^+$	$10.5 \pm 0.8 \pm 1.2$	1. (Fixed)	0. (Fixed)
$\rho(770)\pi^+$	$0.9 \pm 0.4 \pm 0.5$	$0.13 \pm 0.03 \pm 0.04$	$5.44 \pm 0.25 \pm 0.62$
$\rho(1450)\pi^+$	$1.3 \pm 0.4 \pm 0.5$	$0.91 \pm 0.16 \pm 0.22$	$1.03 \pm 0.32 \pm 0.51$
S wave	$84.2 \pm 0.8 \pm 1.3$		
Total	$96.8 \pm 2.4 \pm 3.5$		

- Dominant by $\pi^+\pi^-$ S-wave contribution

Amplitude analysis of $D_S^+ \rightarrow \pi^+ \pi^- \pi^+$

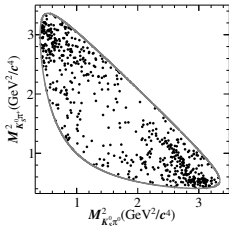
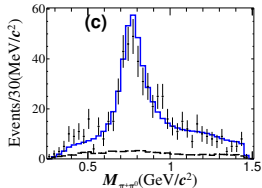
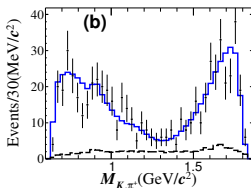
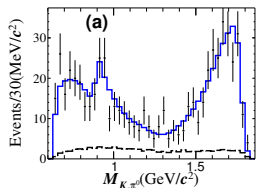


- Extract dominant $\pi^+ \pi^-$ S -wave contribution with model-independent PWA [[PRD 73 \(2006\) 032004](#)]
- Consistent with the BABAR measurement with an improved precision [[PRD 79 \(2009\) 032003](#)]

Amplitude analysis of $D_S^+ \rightarrow K_S^0 \pi^+ \pi^0$

607 signal events with full dataset, background $\sim 20\%$

[JHEP 06 (2021) 181]



Amplitude	Magnitude (ρ_n)	Phase (ϕ_n)	FF (%)	(σ)
$K_S^0 \rho^+$	1.0(fixed)	0.0(fixed)	$50.2 \pm 7.2 \pm 3.9$	>10
$K_S^0 \rho(1450)^+$	2.7 ± 0.5	$2.2 \pm 0.2 \pm 0.1$	$20.4 \pm 4.3 \pm 4.4$	>10
$K^*(892)^0 \pi^+$	0.4 ± 0.1	$3.2 \pm 0.2 \pm 0.1$	$8.4 \pm 2.2 \pm 0.9$	5.0
$K^*(892)^+ \pi^0$	0.3 ± 0.1	$0.2 \pm 0.2 \pm 0.2$	$4.6 \pm 1.4 \pm 0.4$	4.0
$K^*(1410)^0 \pi^+$	0.8 ± 0.2	$0.2 \pm 0.3 \pm 0.1$	$3.3 \pm 1.6 \pm 0.5$	3.7

- Singly Cabibbo suppressed, precision measurement help to study the SU(3) breaking effects and test the theoretical predictions (see next slide)

Amplitude analysis of $D_S^+ \rightarrow K_S^0 \pi^+ \pi^0$

Channel ($\times 10^{-3}$)	PDG	Y.L. Wu <i>et al.</i> ¹	H.Y. Cheng <i>et al.</i> ²	F.S. Yu <i>et al.</i> ³
$K^0 \rho^+$	–	9.1 ± 7.7	11.47 ± 0.48	7.5 ± 2.1
$K^*(892)^0 \pi^+$	2.13 ± 0.36	3.3 ± 3.5	3.65 ± 0.24	1.5 ± 0.7
$K^*(892)^+ \pi^0$	–	1.3 ± 1.3	1.02 ± 0.07	0.1 ± 0.1

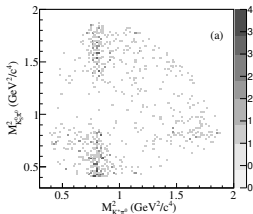
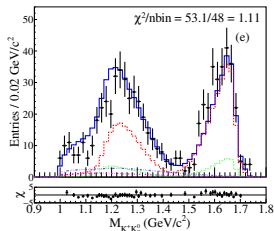
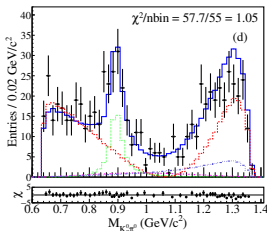
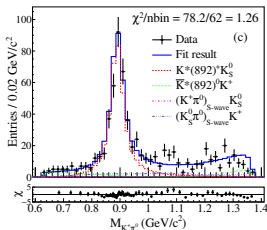
Table: ¹[EPJC 42 (2005) 391]; ²[PRD 100 (2019) 093002]; ³[PRD 84 (2011) 074019]

- Measured \mathcal{B} are marginally consistent with Ref.[3]
 - $\mathcal{B}(D_S^+ \rightarrow K_S^0 \pi^+ \pi^0) = (5.43 \pm 0.30 \pm 0.15) \times 10^{-3}$
 - $\mathcal{B}(D_S^+ \rightarrow K^0 \rho^+) = (5.46 \pm 0.84 \pm 0.44) \times 10^{-3}$
 - $\mathcal{B}(K^*(892)^0 \pi^+) = (2.71 \pm 0.72 \pm 0.30) \times 10^{-3}$
 - $\mathcal{B}(K^*(892)^+ \pi^0) = (0.75 \pm 0.24 \pm 0.06) \times 10^{-3}$
- $A_{CP} = \frac{\mathcal{B}(D_S^+ \rightarrow K_S^0 \pi^+ \pi^0) - \mathcal{B}(D_S^- \rightarrow K_S^0 \pi^- \pi^0)}{\mathcal{B}(D_S^+ \rightarrow K_S^0 \pi^+ \pi^0) + \mathcal{B}(D_S^- \rightarrow K_S^0 \pi^- \pi^0)} = (2.7 \pm 5.5 \pm 0.9)\%$

Amplitude analysis of $D^+ \rightarrow K^+ K_S^0 \pi^0$

692 signal events with background less than 2%

[PRD 104 (2021) 012006]

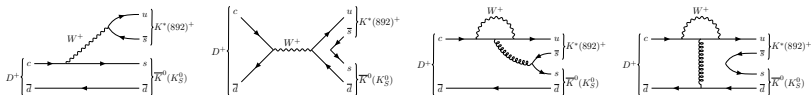


Amplitude	Magnitude	Phase ϕ ($^\circ$)	FF (%)	Significance
$K^*(892)^+ K_S^0$	1.0 (fixed)	0.0 (fixed)	57.1 ± 2.6	29.6σ
$K^*(892)^0 K^+$	0.41 ± 0.04	162 ± 10	10.2 ± 1.5	11.6σ
$(K^+ \pi^0)_S K_S^0$	2.02 ± 0.37	140 ± 14	3.9 ± 1.5	5.2σ
$(K_S^0 \pi^0)_S K^+$	3.14 ± 0.46	-173.7 ± 9.7	9.7 ± 2.6	7.4σ

- Dominant by $K^*(892)^+ K_S^0$, provides important data to test theoretical QCD predictions and associated CP violating effects in the charmed SCS 2-body decays (see next slide)

Amplitude analysis of $D^+ \rightarrow K^+ K_S^0 \pi^0$

- $\mathcal{B}(D^+ \rightarrow K^*(892)^+ K_S^0)$ contributed by color-favoured tree diagram, W -annihilation diagram and penguin diagrams
 - Pole model: $(6.2 \pm 1.2) \times 10^{-3}$ [PRD 84 (2011) 074019]
 - Factorization: 5.5×10^{-3} [PRD 89 (2014) 054006]
 - Topological diagram (tree-level): $(5.02 \pm 1.31) \times 10^{-3}$ [PRD 93 (2016) 114010]
 - Topological diagram (penguin): $(4.90 \pm 0.21) \times 10^{-3}$ [PRD 100 (2019) 093002]

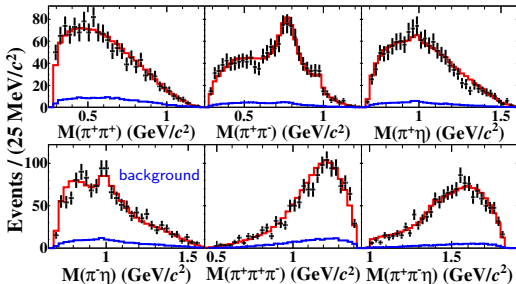


BF	This work	PDG
$\mathcal{B}(D^+ \rightarrow K^*(892)^+ K_S^0)$	$(8.69 \pm 0.40_{\text{stat.}} \pm 0.64_{\text{syst.}} \pm 0.51_{\text{Br.}}) \times 10^{-3}$	$(17 \pm 8) \times 10^{-3}$
$\mathcal{B}(D^+ \rightarrow K^*(892)^0 K^+)$	$(3.10 \pm 0.46_{\text{stat.}} \pm 0.68_{\text{syst.}} \pm 0.18_{\text{Br.}}) \times 10^{-3}$	$(3.74^{+0.12}_{-0.20}) \times 10^{-3}$

- $\mathcal{B}(D^+ \rightarrow K^*(892)^+ K_S^0)$ consistent with pole model prediction
- $\mathcal{B}(D^+ \rightarrow K^*(892)^0 K^+)$ agrees well with the predictions in above works

Amplitude analysis of $D_S^+ \rightarrow \pi^+ \pi^+ \pi^- \eta$

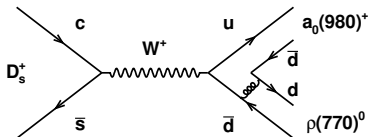
[arXiv:2106.13536]



Amplitude	Phase	FF(%)
$a_1(1260)^+(\rho(770)^0 \pi^+) \eta$	0.0(fixed)	$55.4 \pm 3.9 \pm 2.0$
$a_1(1260)^+(f_0(500) \pi^+) \eta$	$5.0 \pm 0.1 \pm 0.1$	$8.1 \pm 1.9 \pm 2.1$
$a_0(980)^+ \rho(770)^0$	$2.5 \pm 0.1 \pm 0.1$	$6.7 \pm 2.5 \pm 1.5$
$\eta(1405)(a_0(980)^- \pi^+) \pi^+$	$0.2 \pm 0.2 \pm 0.1$	$0.7 \pm 0.2 \pm 0.1$
$\eta(1405)(a_0(980)^+ \pi^-) \pi^+$	$0.2 \pm 0.2 \pm 0.1$	$0.7 \pm 0.2 \pm 0.1$
$f_1(1420)(a_0(980)^- \pi^+) \pi^+$	$4.3 \pm 0.2 \pm 0.4$	$1.9 \pm 0.5 \pm 0.3$
$f_1(1420)(a_0(980)^+ \pi^-) \pi^+$	$4.3 \pm 0.2 \pm 0.4$	$1.7 \pm 0.5 \pm 0.3$
$[a_0(980)^- \pi^+]_S \pi^+$	$0.1 \pm 0.2 \pm 0.2$	$5.1 \pm 1.2 \pm 0.9$
$[a_0(980)^+ \pi^-]_S \pi^+$	$0.1 \pm 0.2 \pm 0.2$	$3.4 \pm 0.8 \pm 0.6$
$[f_0(980) \eta]_S \pi^+$	$1.4 \pm 0.2 \pm 0.3$	$6.2 \pm 1.7 \pm 0.9$
$[f_0(500) \eta]_S \pi^+$	$2.5 \pm 0.2 \pm 0.3$	$12.7 \pm 2.6 \pm 2.0$

- First measurement
- 2139 signal events, $\sim 85\%$ purity
- $\mathcal{B}(D_S^+ \rightarrow \pi^+ \pi^+ \pi^- \eta) = (3.12 \pm 0.13 \pm 0.09)\%$
- Dominant by $\mathcal{B}(D_S^+ \rightarrow a_1(1260)^+(\rho(770)^0 \pi^+) \eta) = (1.73 \pm 0.14 \pm 0.08)\%$
- Help background estimation in semileptonic B decays for R_{D^*} measurement [PRL 120 (2018) 171802]
- Observation of $a_0(980)^+ \rho(770)^0$, $a_0(980)^+ \rightarrow \pi^+ \eta$ with $> 7\sigma$ significance

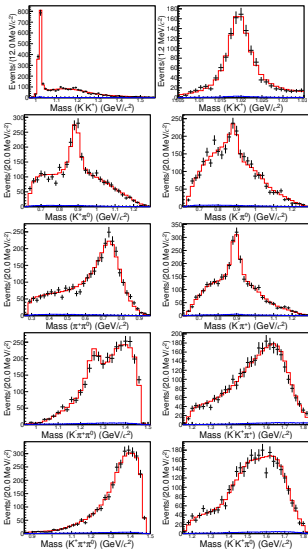
Amplitude analysis of $D_S^+ \rightarrow \pi^+ \pi^+ \pi^- \eta$



- $D_S^+ \rightarrow a_0(980)^+ \rho(770)^0$ (VS state) is a W -annihilation transition
- $\mathcal{B} = (0.21 \pm 0.08 \pm 0.05)\%$, similar as $D_S^+ \rightarrow a_0(980)^+ \pi^0$ (SP), is much larger than the pure WA processes $\mathcal{B}_{D_S^+ \rightarrow \pi^0 \pi^+} = 0.037\%$ (PP) and $\mathcal{B}_{D_S^+ \rightarrow \rho^0 \pi^+} = 0.019\%$ (VP)
- Final-state rescattering may play an important role [EPJC 80 (2020) 895]
- Helpful to interpret the $a_0(980)^+$ state [Prog. Theor. Exp. Phys. 2020 (2020) 083C01]

Amplitude analysis of $D_S^+ \rightarrow K^- K^+ \pi^+ \pi^0$ [PRD 104 (2021) 032011]

~ 3000 signal events, background less than 3%



Process	$\mathcal{B}_{\text{BESIII}}(\%)$	$\mathcal{B}_{\text{PDG}}(\%)$
$[\phi\rho^+]_S$	$2.10 \pm 0.09 \pm 0.13$	
$[\phi\rho^+]_P$	$0.52 \pm 0.05 \pm 0.02$	
$[\phi\rho^+]_D$	$0.18 \pm 0.04 \pm 0.02$	
$\phi\rho^+$	$2.75 \pm 0.07 \pm 0.15$	
Resonance Corrected	$5.59 \pm 0.15 \pm 0.30$	$8.4^{+1.9}_{-2.3}$
$[\bar{K}^{*0}K^{*+}]_S$	$0.88 \pm 0.05 \pm 0.03$	
$[\bar{K}^{*0}K^{*+}]_P$	$0.37 \pm 0.03 \pm 0.02$	
$[\bar{K}^{*0}K^{*+}]_D$	$0.18 \pm 0.03 \pm 0.01$	
$\bar{K}^{*0}K^{*+}$	$1.25 \pm 0.05 \pm 0.06$	
Resonance Corrected	$5.64 \pm 0.23 \pm 0.27$	7.2 ± 2.6
$\bar{K}_1^0(1270)K^+$,		
$\bar{K}_1^0(1270) \rightarrow K^- \rho^+$	$0.57 \pm 0.05 \pm 0.04$	
$\bar{K}_1^0(1270)[S] \rightarrow K^* \pi$	$0.21 \pm 0.04 \pm 0.03$	
$\bar{K}_1^0(1270)[D] \rightarrow K^* \pi$	$0.07 \pm 0.02 \pm 0.01$	
$\bar{K}_1^0(1270) \rightarrow K^* \pi$	$0.29 \pm 0.04 \pm 0.04$	
$\bar{K}_1^0(1400)K^+$,		
$\bar{K}_1^0(1400) \rightarrow K^* \pi$	$0.44 \pm 0.06 \pm 0.07$	
$a_0^0(980)\rho^+$	$0.19 \pm 0.03 \pm 0.03$	
$f_1(1420)\pi^+$,		
$f_1(1420) \rightarrow K^* \mp K^\pm$	$0.13 \pm 0.02 \pm 0.01$	
$f_1(1420) \rightarrow a_0^0(980)\pi^0$	$0.04 \pm 0.01 \pm 0.01$	
$\eta(1475)\pi^+$,		
$\eta(1475) \rightarrow a_0^0(980)\pi^0$	$0.07 \pm 0.02 \pm 0.02$	
$K^- K^+ \pi^+ \pi^0$	$5.42 \pm 0.10 \pm 0.17$	6.3 ± 0.6

Amplitude analysis of $D_S^+ \rightarrow K^- K^+ \pi^+ \pi^0$

- Dominant by the $D_S^+ \rightarrow VV$ processes $\phi\rho^+$ and $\bar{K}^{*0}K^{*+}$
- $R_{K_1(1270)} = \frac{K_1^0(1270) \rightarrow K^* \pi}{K_1^0(1270) \rightarrow K \rho}$ is determined to be $0.99 \pm 0.15 \pm 0.18$, consistent with [LHCb, JHEP 02 (2019) 126] and [CLEO, PRD 85 (2012) 12202]
- Favor the prediction of $R_{K_1(1270)} = 1$ under narrow width approximation for $K_1^0(1270)$ and CP conservation [NPR 36(2) (2019) 125]

$R_{K_1(1270)}$	Process	Ref.
0.81 ± 0.10	$D^0 \rightarrow K^+ K^- \pi^+ \pi^-$	[LHCb, JHEP 02 (2019) 126]
1.18 ± 0.43	$D^0 \rightarrow K^- K_1^+(1270)$	[CLEO, PRD 85 (2012) 12202]
0.11 ± 0.06	$D^0 \rightarrow K^+ K_1^-(1270)$	[CLEO, PRD 85 (2012) 12202]
0.19 ± 0.10	$D^0 \rightarrow K^- \pi^+ \pi^+ \pi^-$	[BESIII, PRD 95 (2017) 072010]
0.24 ± 0.04	$D^0 \rightarrow K^- \pi^+ \pi^+ \pi^-$	[LHCb, EPJC 78 (2018) 443]
0.45 ± 0.05	$B^+ \rightarrow J/\psi K^+ \pi^+ \pi^-$	[Belle, PRD 83 (2011) 032005]
0.30 ± 0.04	$B^+ \rightarrow J/\psi K^+ \pi^+ \pi^-$	[Belle, PRD 83 (2011) 032005]
0.38 ± 0.13	$K^- p \rightarrow K^- \pi^- \pi^+ p$	[ACCMOR, NPB 187 (1981) 1]
0.45 ± 0.14	$D^0 \rightarrow K^- K_1^+(1270)$	[CLEO, JHEP 05 (2017) 143]

Introduction

Amplitude analysis of charmed meson decays at BESIII

$$D_S^+ \rightarrow \pi^+ \pi^- \pi^+ \quad [\text{arXiv: 2108.10050}]$$

$$D_S^+ \rightarrow K_S^0 \pi^+ \pi^0 \quad [\text{JHEP 06 (2021) 181}]$$

$$D^+ \rightarrow K^+ K_S^0 \pi^0 \quad [\text{PRD 104 (2021) 012006}]$$

$$D_S^+ \rightarrow \pi^+ \pi^+ \pi^- \eta \quad [\text{arXiv:2106.13536}]$$

$$D_S^+ \rightarrow K^- K^+ \pi^+ \pi^0 \quad [\text{PRD 104 (2021) 032011}]$$

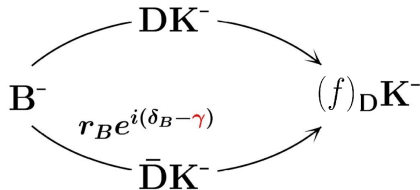
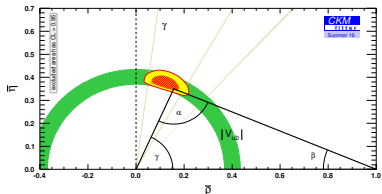
Strong phase parameters in neutral $D\bar{D}$ meson decays at BESIII

Phase variation across $D \rightarrow K_S^0 \pi^+ \pi^-$ Dalitz plot [PRD 101 (2020) 112002]

Phase variation in $D \rightarrow K3\pi$ phase space [JHEP 05 (2021) 164]

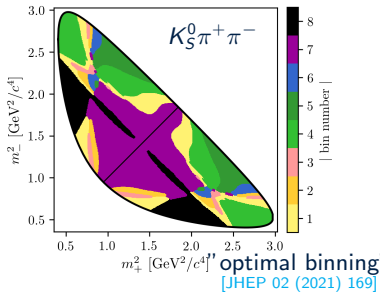
Summary

Inputs to the determination of CKM angle γ BESIII



- $\gamma = (72.1_{-5.7}^{+5.4})^\circ$, $\gamma_{\text{ind.}} = (65.66_{-2.65}^{+0.90})^\circ$ (CKM fitter)
- Interference between the two paths into same final states, depends on the B and D decay parameters, i.e. strong phase parameters
- Extracting γ with **multi-body D decays** have advantage with exploiting strong-phase variations in phase space:
 - Self-conjugate decays (eg. $K_S^0 h^+ h^-$): "BPGGSZ" [PRD 68 (2003) 054018; PRD 67 (2003) 071301; A. Bondar] $\Rightarrow C_i, S_i$
 - DCS decays (eg. $K\pi\pi\pi$): "ADS" [PRL 78 (1997) 3257; PRD 63 (2001) 036005] $\Rightarrow \delta_D R_D$
 - Other methods...

Phase variation in $D \rightarrow K_S^0 \pi^+ \pi^-$



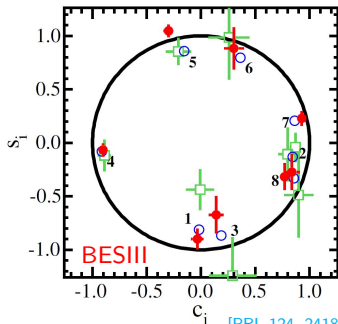
- The phase difference of the amplitude $\mathcal{A}_D(m_+^2, m_-^2)$ and $\mathcal{A}_D(m_-^2, m_+^2)$
 $\Rightarrow \Delta\delta_D(m_+^2, m_-^2)$
- $\Delta\delta_D$ varies across the Dalitz plane
- Idea: divide the Dalitz plot into bins
 \Rightarrow bin averaged $\Delta\delta_D$
 - $c_i = \int dp \mathcal{A}_D(m_+^2, m_-^2) \mathcal{A}_D(m_-^2, m_+^2) \cos \Delta\delta_D(m_+^2, m_-^2)$
 - $s_i = \int dp \mathcal{A}_D(m_+^2, m_-^2) \mathcal{A}_D(m_-^2, m_+^2) \sin \Delta\delta_D(m_+^2, m_-^2)$
- Model-independent

- Quantum correlated C -odd $D^0 \bar{D}^0$ produced at BESIII

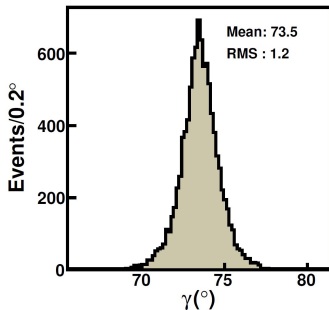
$$e^+ e^- \rightarrow \psi(3770) \rightarrow D^0 \bar{D}^0$$

- CP tags (eg. $\pi\pi\pi^0$, $K_S^0\pi^0$): information of c_i
- Self tags (tagged by itself): information of c_i , s_i

Measurement of c_i and s_i in $D \rightarrow K_S^0 \pi^+ \pi^-$



[PRL 124, 241802 (2020)]
[PRD 101, 112002 (2020)]



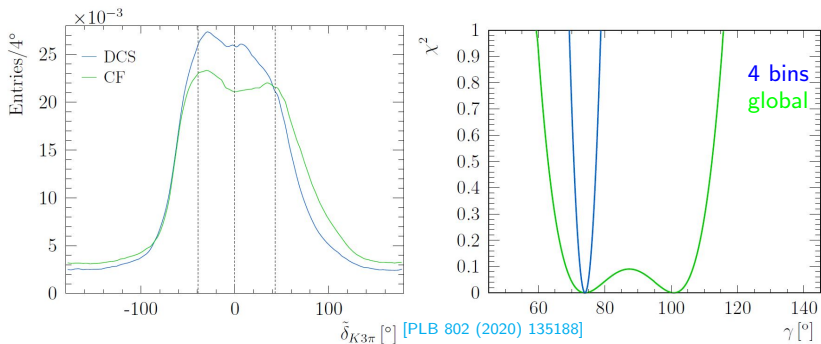
Contribution to δ_γ

- Improvement of the γ sensitivity compare to that with only CLEO-c inputs [PRD 82, 112006 (2010)]
- BESIII uncertainties of $K_S^0 \pi^+ \pi^-$ contribute around 1° in the latest LHCb measurement, leading to the best single measurement of γ (5°) [JHEP 02 (2021) 169]
- $K_S^0 K^+ K^-$ is measured with a similar method, see [PRD 102, 052008 (2020)]

Phase variation in $D \rightarrow K3\pi$ phase space

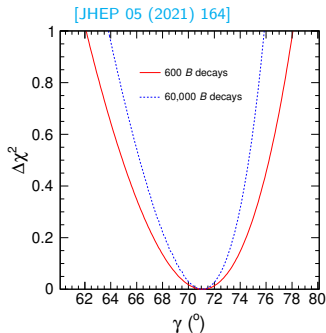
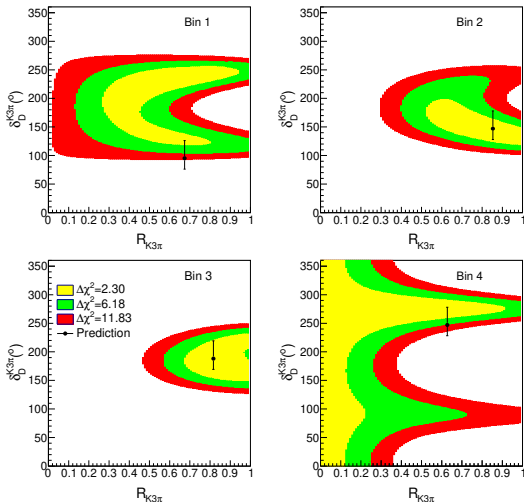
- $D \rightarrow K3\pi$ is rich of resonances, $\delta_D^{K3\pi}$ varies in phase space regions
- Divide the phase space according to LHCb model of the DCS and CF decay

[EPJC 78, 443 (2018)]



- Model-independent measurement, only binning method depends on the DCS and CF model of $D \rightarrow K3\pi$ [EPJC 78 (2018) 443]

Results of binned $\delta_D^{K3\pi}$ and $R_{K3\pi}$ at BESIII



- $\delta_{\text{LHCb}}^{\text{Run1\&2}} = \begin{pmatrix} +7 \\ -9 \end{pmatrix}^\circ, \begin{pmatrix} +5 \\ -7 \end{pmatrix}^\circ$ comes from BESIII
- $\delta_{\text{LHCb}}^{K_S^0\pi\pi} \sim 5^\circ$ [JHEP 02 (2021) 169]

- Significant improvement compare to CLEO-c results [PLB 802 (2020) 135188]

- Amplitude analysis of $D_{(S)}$ decays with "double-tag" method at BESIII
 - Contain rich physics, eg. SU(3) breaking effect, final state interaction
 - Test the theoretical calculation approaches
 - Provide inputs for the LFU tests with B semileptonic decays
 - More results not in this talk:
 - $D_S^+ \rightarrow K^+ K^- \pi^+$ [PRD 104 (2021) 012016]
 - $D_S^+ \rightarrow K_S^0 K^- \pi^+ \pi^+$ [PRD 103 (2021) 092006]
- Quantum correlated measurement at BESIII with neutral $D\bar{D}$
 - Give access to the strong phase variations in multi-body D decays
 - Inputs for CKM angle γ measurement
 - Inputs for charm mixing and CP violation studies [arXiv: 2106.03744]
- 20 fb^{-1} $\psi(3770)$ data at BESIII in the near future [CPC 44, 040001 (2020)]

Thanks!

Dalitz plot analysis

- **Amplitude analysis:**

- **Unbinned maximum likelihood Fit:**

- PDF = $f_s S + (1 - f_s) B = \epsilon(p_j) R_4(p_j) \left[f_s \frac{|A_{D_s}(a_i, p_j)|^2}{\int \epsilon(p_j) |A_{D_s}(a_i, p_j)|^2 R_4(p_j) dp_j} + (1 - f_s) \frac{B_\epsilon(p_j)}{\int B(p_j) R_4(p_j) dp_j} \right]$

Bkg function
([RoosNDKeysPdf](#))

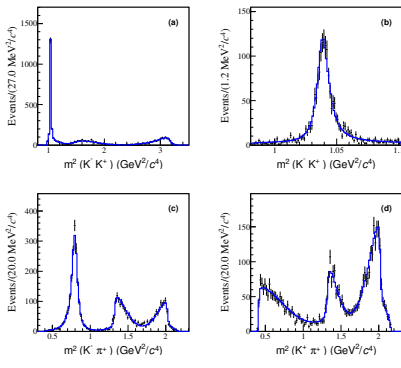
Acceptance function

MC integration

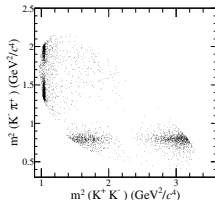
$$B_\epsilon = B/\epsilon$$

- $A_{D_s \rightarrow \text{sig}} = \sum_n c_n A_n$, $c_n = \rho_n e^{i\phi_n}$
- **Covariant tensor formalism**
- $A_n = P_n^1 P_n^2 S_n F_n^1 F_n^2 F_n^{D_s}$, P_n^i propagator, F_n^i barrier, S_n angular distribution.
- **Log-likelihood:**
 - $\ln L_{\text{total}} = \sum_k^{N_{\text{data}}} \log(f_s S(p_k) + (1 - f_s) B(p_k))$

Amplitude analysis of $D_S^+ \rightarrow K^+ K^- \pi^+$



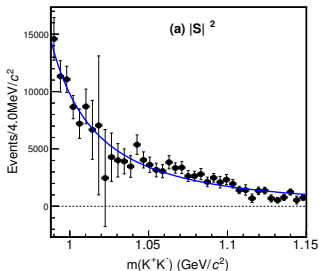
- The "golden" normalization mode for other D_S^+ decays
- Free of background



	FF%(BESIII)	FF%(CLEO)	FF%(BaBar)
$\bar{K}^*(892)^0 K^+$	$48.3 \pm 0.9 \pm 0.6$	$47.4 \pm 1.5 \pm 0.4$	$47.9 \pm 0.5 \pm 0.5$
$\phi(1020)\pi^+$	$40.5 \pm 0.7 \pm 0.9$	$42.2 \pm 1.6 \pm 0.3$	$41.4 \pm 0.8 \pm 0.5$
$S_0(980)\pi^+$	$19.3 \pm 1.7 \pm 2.0$	$28.2 \pm 1.9 \pm 1.8$	$16.4 \pm 0.7 \pm 2.0$
$\bar{K}_0^*(1430)^0 K^+$	$3.0 \pm 0.6 \pm 0.5$	$3.9 \pm 0.5 \pm 0.5$	$2.4 \pm 0.3 \pm 1.0$
$f_0(1710)\pi^+$	$1.9 \pm 0.4 \pm 0.6$	$3.4 \pm 0.5 \pm 0.3$	$1.1 \pm 0.1 \pm 0.1$
$f_0(1370)\pi^+$	$1.2 \pm 0.4 \pm 0.2$	$4.3 \pm 0.6 \pm 0.5$	$1.1 \pm 0.1 \pm 0.2$
\sum FF(%)	$110.2 \pm 0.6 \pm 2.0$	$129.5 \pm 4.4 \pm 2.0$	$110.2 \pm 0.6 \pm 2.0$
Events	4399	12226 ± 122	96307 ± 369

[PRD 104 (2021) 012016]

Amplitude analysis of $D_S^+ \rightarrow K^+ K^- \pi^+$

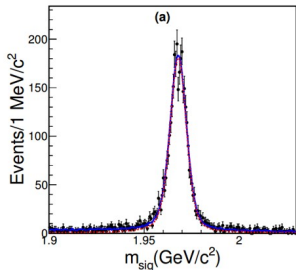


- Extract $K^+ K^-$ S -wave ($S(980)$) from model-independent PWA

$$A_{S(980)} = \frac{1}{m_0^2 - m^2 - im_0 \Gamma_0 \rho_{KK}}$$

$$m_0 = (0.919 \pm 0.006_{\text{stat}} \pm 0.030_{\text{stat}}) \text{ GeV}/c^2,$$

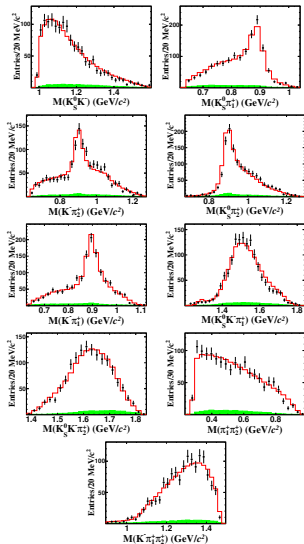
$$\Gamma_0 = (0.272 \pm 0.040_{\text{stat}} \pm 0.024_{\text{stat}}) \text{ GeV}.$$



- $\mathcal{B}_{K^+ K^- \pi^+} = (5.47 \pm 0.08 \pm 0.13)\%$, best measurement up to date
- $\mathcal{B}_{\bar{K}^{*0}(892) K^+} = (3.94 \pm 0.12)\%$,
 $\mathcal{B}_{\phi(1020) \pi^+} = (4.60 \pm 0.17)\%$, consistent with theoretical prediction
[\[PRD 93, 114010 \(2016\)\]](#)

Amplitude analysis of $D_S^+ \rightarrow K_S^0 K^- \pi^+ \pi^+$

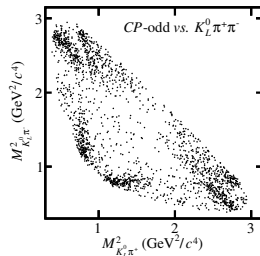
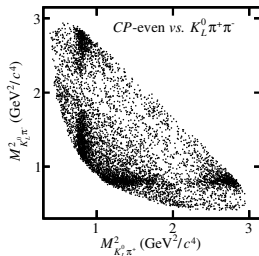
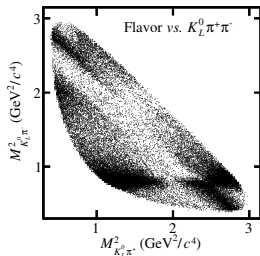
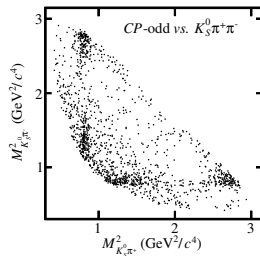
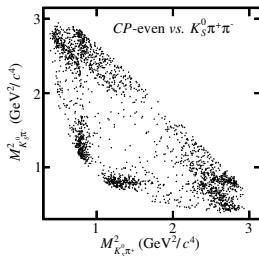
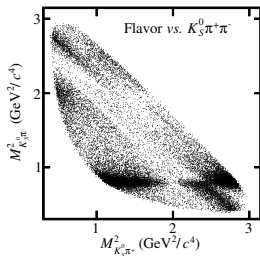
[PRD 103 (2021) 092006]



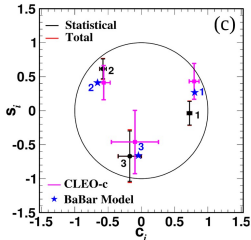
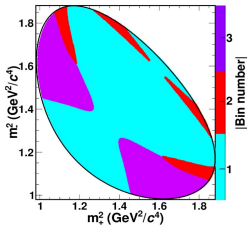
Process	BF(10^{-3})	
	This analysis	PDG
$[K^*(892)^+ \bar{K}^*(892)^0]_S$	$5.01 \pm 0.49 \pm 0.78$	
$[K^*(892)^+ \bar{K}^*(892)^0]_P$	$1.10 \pm 0.16 \pm 0.10$	
$[K^*(892)^+ \bar{K}^*(892)^0]_D$	$0.65 \pm 0.12 \pm 0.10$	
$K^*(892)^+ \bar{K}^*(892)^0$	$5.93 \pm 0.47 \pm 0.74$	7.98 ± 2.88
$K^*(892)^+(K^-\pi^+)_S$	$0.73 \pm 0.17 \pm 0.15$	
$\bar{K}^*(892)^0(K_S^0\pi^+)_S$	$1.06 \pm 0.16 \pm 0.13$	
$\eta(1475)\pi^+$,	$1.57 \pm 0.39 \pm 0.76$	
$\eta(1475) \rightarrow a_0(980)^-\pi^+$		
$\eta(1475)\pi^+$,	$0.32 \pm 0.10 \pm 0.10$	
$\eta(1475) \rightarrow \bar{K}^*(892)^0 K_S^0$		
$\eta(1475)\pi^+$,	$0.32 \pm 0.10 \pm 0.10$	
$\eta(1475) \rightarrow K^*(892)^+ K^-$		
$\eta(1475)\pi^+$,	$0.72 \pm 0.21 \pm 0.14$	
$\eta(1475) \rightarrow K^*(892)K$		
$\eta(1475)\pi^+$,	$3.44 \pm 0.54 \pm 1.10$	
$\eta(1475) \rightarrow (K_S^0\pi^+)_S K^-$		
$f_1(1285)\pi^+$,	$0.33 \pm 0.08 \pm 0.10$	
$f_1(1285) \rightarrow a_0(980)^-\pi^+$		
$K_S^0 K^- \pi^+ \pi^+$	$14.60 \pm 0.46 \pm 0.48$	16.50 ± 1.00

- Improved determination of $\mathcal{B}(K_S^0 K^- \pi^+ \pi^+)$ and $\mathcal{B}(K^*(892)^+ \bar{K}^*(892)^0)$

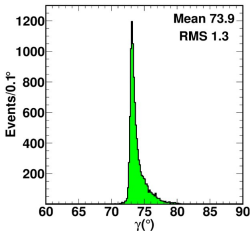
DT events of $D \rightarrow K_S^0 \pi^+ \pi^-$



Measurement of c_i and s_i in $D \rightarrow K_S^0 K^+ K^-$

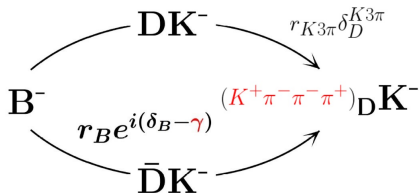


[PRD 102, 052008 (2020)]



- Improvement of the γ sensitivity with CLEO-c inputs (3.2-3.9) $^\circ$ ($K_S^0 K^+ K^-$) [PRD 82, 112006 (2010)]

ADS method with multi-body D decays

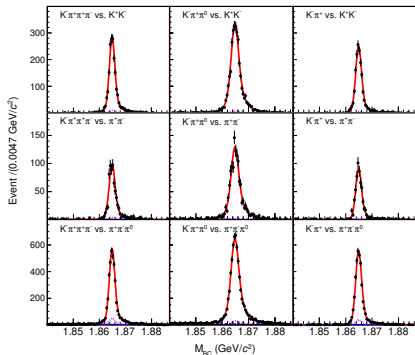


- Both DCS and CF processes contribute to $B^\mp \rightarrow (K^\pm 3\pi)_D K^\mp$
- Phase-space integrated decay rate

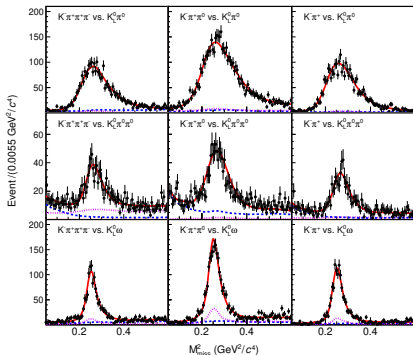
$$\Gamma(B^\mp \rightarrow (K^\pm 3\pi)_D K^\mp) \propto r_{K3\pi}^2 + r_B^2 + 2r_B R_{K3\pi} r_{K3\pi} \cos(\delta_B \pm \gamma - \delta_D^{K3\pi})$$

- $R_S e^{-i\delta_D^S} = \frac{\int \mathcal{A}_S^*(\mathbf{x}) \mathcal{A}_{\bar{S}}(\mathbf{x}) d\mathbf{x}}{A_S A_{\bar{S}}}$, coherence factor and strong phase
- R indicates the intermediate resonances, $0 < R < 1$ for $K3\pi$ and $K\pi\pi^0$, $R = 1$ for $K\pi$

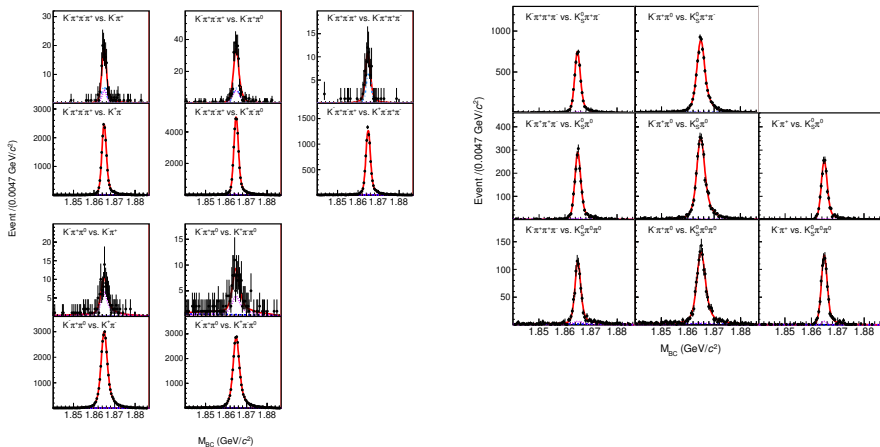
Back-up: $K3\pi/K\pi\pi^0$ DT modes



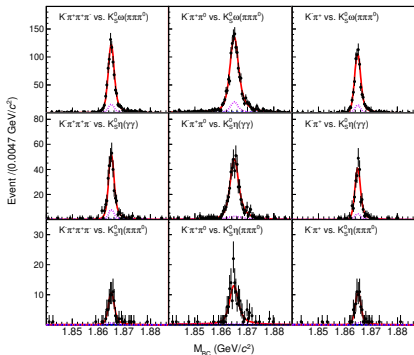
[JHEP 05 (2021) 164]



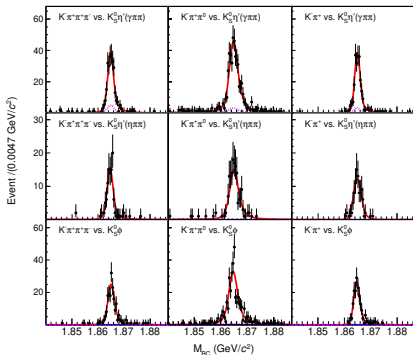
Back-up: $K3\pi/K\pi\pi^0$ DT modes



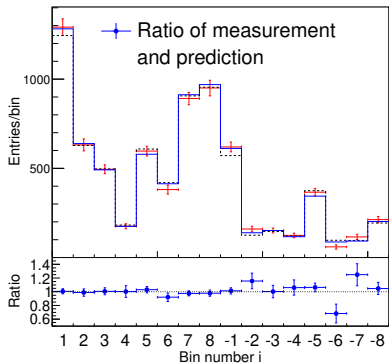
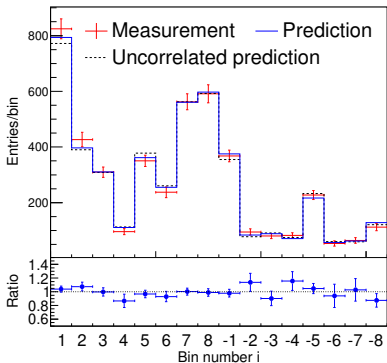
Back-up: $K3\pi/K\pi\pi^0$ DT modes



[JHEP 05 (2021) 164]

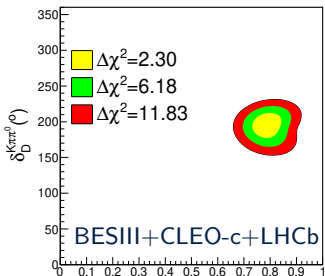
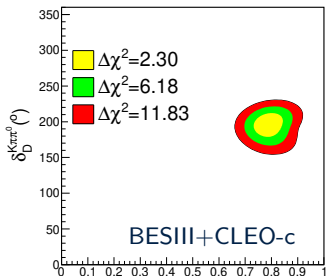
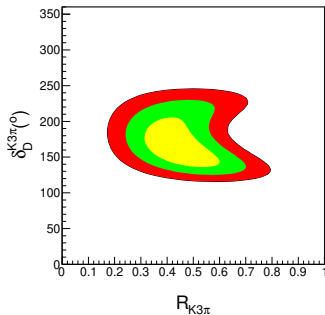
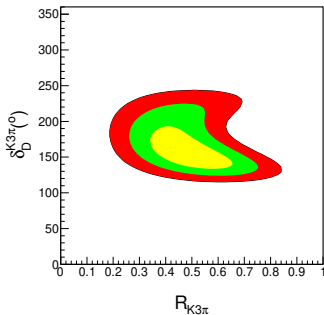


Back-up: $K_S^0 \pi^+ \pi^-$ tag for $K3\pi / K\pi\pi^0$



[JHEP 05 (2021) 164]

Back-up: Combination of $\delta_D^{K3\pi}$ and $R_{K3\pi}$



Back-up: Combination of binned $\delta_D^{K3\pi}$ and $R_{K3\pi}$

