

Tool kit for baryon amplitude analyses and polarization measurements & the $\Lambda_c^+ \rightarrow pK^-\pi^+$ case

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Motivation: baryon amplitude analyses

- Spectroscopy
 - Baryon decays often feature complicated phase-space structures
 - Excellent place to search for new baryonic and exotic pentaquark states
- Parity-violation
 - Measurement of decay asymmetry parameters
 - Determines sensitivity to polarization (later in the talk)
- CP-violation
 - Still unobserved for baryons
 - Comparison of baryon-antibaryon amplitude models or basis to tailor model-independent searches
 - Resonance interference patterns can enhance sensitivity

Motivation: Polarization

- Baryon amplitude analyses can be used to measure baryon polarization vector \mathbf{P}
- Polarization is excellent probe for baryon production physics (with no meson counterpart)
 - Strong production: anchor point for low-energy QCD
 - Weak production: additional observable for New Physics tests
- For heavy baryons, multibody decays provide best statistics
 - e.g. $\Lambda_c^+ \rightarrow pK^-\pi^+$ is the main decay channel of Λ_c^+ (BF = 6.28%) with no neutral hyperons (problematic for fixed-target experiments)
- Essential for heavy baryon electromagnetic dipole moments measurement via spin precession in bent crystals, see [PRD 103 \(2021\) 072003](#) and Refs therein

The Tool Kit

Main tools for baryon amplitude analysis:

- Baryon polarization system
- Amplitude model & Decay rate
- Amplitude fit framework
- Will also present tests and sensitivity studies
- Illustrated taking as example the $\Lambda_c^+ \rightarrow pK^-\pi^+$ decay

Baryon polarization system

- Polarization components defined w.r.t. an external system
- Tailored according to production mechanism
 - Strong production: natural system is production plane formed by proton and baryon momenta: only orthogonal polarization allowed by parity conservation
 - Weak production: best system is helicity system reached from mother particle: z axis defined by baryon momentum in mother rest frame
- For Λ_c^+ produced in semileptonic decays:
 - $\hat{\mathbf{z}}_{\Lambda_c^+} = \hat{\mathbf{p}}(\Lambda_c^+)$
 - $\hat{\mathbf{x}}_{\Lambda_c^+}$ defined such that $x - z$ plane coincides with $\Lambda_c^+ - \mu$ one

Helicity formalism

- Considering amplitude models written in the helicity formalism
- Multi-body decays decomposed via single two-body $A \rightarrow BC$ amplitudes defined in terms of final-state helicities
- Structure:
 - Complex coupling: encodes the decay dynamics, to be determined from fit
 - Angular dependence: fixed from angular momentum conservation, expressed in terms of Wigner D-matrices
 - Invariant mass dependence: parametrization of the A particle width

$$\mathcal{A}_{\lambda_B, \lambda_C}^{A \rightarrow BC} = \mathcal{H}_{\lambda_B, \lambda_C}^{A \rightarrow BC} \times D_{m_A, \lambda_B - \lambda_C}^{J_A}(\phi_B, \theta_B, 0)^* \times \mathcal{R}(m_{BC}^2)$$

Writing the amplitude model

Given the three-body decay $\Lambda_c^+ \rightarrow pK^-\pi^+$

- Amplitudes built for each intermediate resonance $\Lambda_c^+ \rightarrow Rh$
 - $\Lambda^* \rightarrow pK^-$, $\Delta^* \rightarrow p\pi^+$, $K^* \rightarrow K^-\pi^+$
 - multiplying two-body helicity amplitudes, e.g.

$$\mathcal{A}_{m_{\Lambda_c^+}, \lambda_R, \lambda_p}^{[R]} = \mathcal{A}_{\lambda_R, 0}^{\Lambda_c^+ \rightarrow R\pi^+} \mathcal{A}_{\lambda_p, 0}^{R \rightarrow pK^-}$$

- Total helicity amplitudes for definite initial and final particles helicities to be obtained summing over all intermediate resonance helicity states

$$\mathcal{A}_{m_{\Lambda_c^+}, \lambda_p} = \sum_{i=1}^{N_R} \sum_{\lambda_{R_i} = -J_{R_i}}^{J_{R_i}} \mathcal{A}_{m_{\Lambda_c^+}, \lambda_{R_i}, \lambda_p}^{[R_i]}$$

Final particle spin matching issue

- But, for multibody decays featuring different decay chains, the definition of the final state helicity is different
 - For $\Lambda_c^+ \rightarrow pK^-\pi^+$ the proton helicity differs for Λ^* , Δ^* , K^*
- Different definitions must be matched to a reference one before summing amplitudes
- We addressed recently this problem for generic multi-body particle decays, [AHEP \(2020\) 6674595](#)
 - Obtained a set of final particle spin rotations transforming the different helicity definitions into canonical spin states

Decay amplitude

- Decay amplitude for a $A \rightarrow R(\rightarrow 1, 2)3$ topology
- (below) decay amplitude for final particle helicities
- (right) decay amplitude rotated to final particle canonical spin states

$$\mathcal{A}_{m_A, \lambda_1^R, \bar{\lambda}_2^R, \bar{\lambda}_3^R}^{A \rightarrow R, 3 \rightarrow 1, 2, 3}(\Omega) = \sum_{\lambda_R} \mathcal{H}_{\lambda_1^R, \bar{\lambda}_2^R}^{R \rightarrow 1, 2} D_{\lambda_R, \lambda_1^R + \bar{\lambda}_2^R}^{*S_R}(\phi_1^R, \theta_1^R, 0) \\ \times \mathcal{H}_{\lambda_R, \bar{\lambda}_3^R}^{A \rightarrow R, 3} D_{m_A, \lambda_R + \bar{\lambda}_3^R}^{*S_A}(\phi_R, \theta_R, 0).$$

$$\mathcal{A}_{m_A, m_1, m_2, m_3}^{A \rightarrow R, 3 \rightarrow 1, 2, 3}(\Omega) = \sum_{\lambda_1^R, \mu_1^R, \nu_1^R} D_{m_1, \nu_1^R}^{S_1}(\alpha_1^{W, R}, \beta_1^{W, R}, \gamma_1^{W, R}) \\ \times D_{\nu_1^R, \mu_1^R}^{S_1}(\phi_R, \theta_R, 0) \\ \times D_{\mu_1^R, \lambda_1^R}^{S_1}(\phi_1^R, \theta_1^R, 0) \\ \times \sum_{\lambda_2^R, \mu_2^R, \nu_2^R} D_{m_2, \nu_2^R}^{S_2}(\alpha_2^{W, R}, \beta_2^{W, R}, \gamma_2^{W, R}) \\ \times D_{\nu_2^R, \mu_2^R}^{S_2}(\phi_R, \theta_R, 0) \\ \times D_{\mu_2^R, \lambda_2^R}^{S_2}(\phi_1^R, \theta_1^R, 0) \\ \times \sum_{\lambda_3^R} D_{m_3, \lambda_3^R}^{S_3}(\phi_R, \theta_R, 0) \\ \times \mathcal{A}_{m_A, \lambda_1^R, \bar{\lambda}_2^R, \bar{\lambda}_3^R}^{A \rightarrow R, 3 \rightarrow 1, 2, 3}(\Omega)$$

Polarized decay rate

- Generic baryon polarization described by density matrix
- For the $\Lambda_c^+ \rightarrow pK^-\pi^+$ case: Λ_c^+ spin 1/2 density matrix

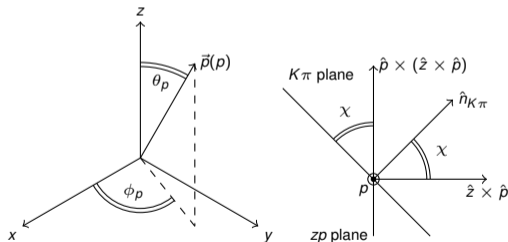
$$\rho_{\Lambda_c^+} = \frac{1}{2} \left(\mathcal{I} + \vec{P} \cdot \vec{\sigma} \right) = \frac{1}{2} \begin{pmatrix} 1 + P_z & P_x - iP_y \\ P_x + iP_y & 1 - P_z \end{pmatrix}$$

- Decay rate takes the form

$$\begin{aligned} \rho(\Omega, \vec{P}) \propto & \sum_{m_p=\pm 1/2} \left[(1 + P_z) |\mathcal{A}_{1/2, m_p}(\Omega)|^2 + (1 - P_z) |\mathcal{A}_{-1/2, m_p}(\Omega)|^2 \right. \\ & + (P_x - iP_y) \mathcal{A}_{1/2, m_p}^*(\Omega) \mathcal{A}_{-1/2, m_p}(\Omega) \\ & \left. + (P_x + iP_y) \mathcal{A}_{1/2, m_p}(\Omega) \mathcal{A}_{-1/2, m_p}^*(\Omega) \right] \end{aligned}$$

Baryon 3-body phase space

- Described by 5 variables
 - 2 two-body invariant mass “Dalitz” variables, describing decay dynamics
 - 3 decay orientation angles, describing decay plane orientation w.r.t. polarization system
- For $\Lambda_c^+ \rightarrow pK^-\pi^+$ decay, 5 uniform phase space variables can be chosen as
 - $\Omega = (m_{pK^-}^2, m_{K^-\pi^+}^2, \cos \theta_p, \phi_p, \chi)$
 - Angles being the polar and azimuthal proton angles, and the signed angle between $p, \hat{z}_{\Lambda_c^+}$ and K, π planes

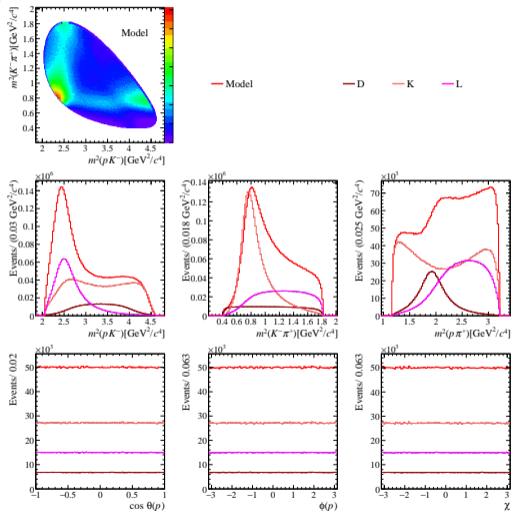


Amplitude fit

- We developed the fitting code for $\Lambda_c^+ \rightarrow pK^-\pi^+$ amplitude analysis basing on [TensorFlowAnalysis](#) package, by A. Poluektov and LHCb collaborators
- Exploits the machine-learning framework TensorFlow to build amplitude models
 - Based on computer algebra paradigm: users just specify the computational graph, with calculation, optimization and compilation for different architectures run automatically
 - Easier building of complicated amplitude models, otherwise prohibitive to build from basic, hard-coded functions
 - Caching of decay amplitude parts to speed up minimization process
- Minimization exploits the good-old MINUIT package
 - TF methods not suitable for physics purposes, not providing uncertainty estimates

Amplitude model tests

- We performed different tests for amplitude model implementation
- An important one was checking properties of the decay rate following from **rotational invariance**, valid **irrespective of the decay model** considered
 - Decay rate is **isotropic** in decay orientation angles for **zero polarisation** (right)
 - **Invariant mass distributions are independent** of the **polarisation**
- These tests showed inadequacy of previous spin matching method

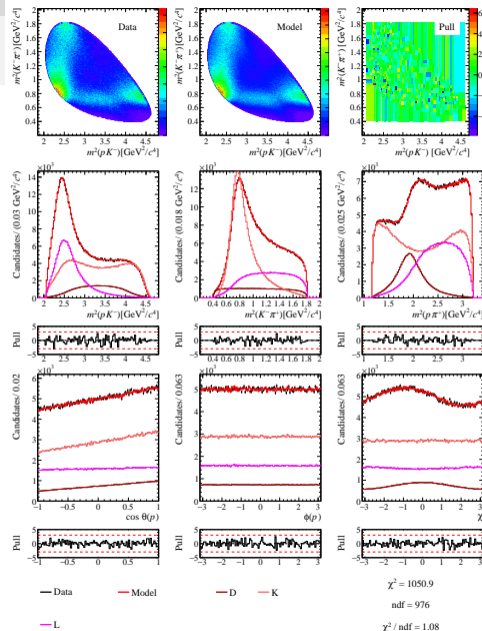


Sensitivity study for $\Lambda_c^+ \rightarrow pK^-\pi^+$

- We demonstrated the possibility to simultaneously measure $\Lambda_c^+ \rightarrow pK^-\pi^+$ amplitude model and Λ_c^+ polarization in [AHEP \(2020\) 7463073](#) via:
 - Analytical study of the constraints posed by the amplitude fit to the decay rate
 - Amplitude fit to pseudodata generated with toy $\Lambda_c^+ \rightarrow pK^-\pi^+$ description
- Results:
 - Interference effects among different decay chain contributions are crucial: they give sensitivity to single complex helicity coupling values and to the polarization magnitude
 - A non-zero polarization is needed to determine entirely the amplitude model: it gives sensitivity to the parity-violating part of the decay amplitude

Pseudodata toy fit

- Set toy amplitude model with interfering K^* , Λ^* , Δ^* resonances
- Generated pseudo-data for $P_z = 0.5$ and zero polarisation
- Complex couplings, polarisation and resonance parameters retrieved from amplitude fit
- $P_z = 0.5$ (left): all parameters correctly measured within uncertainties
- $P = 0$: measured resonance parameters and polarisation, complex couplings not fully determined
- Fit fractions correctly measured



Sensitivity to polarization

- Given the decay rate written as $p(\Omega) = f(\Omega) + Pg(\Omega)$
- Sensitivity to polarization can be measured as

$$S^2 = \int \frac{g^2}{f + P_0 g} d\Omega$$

- Cast into an effective decay asymmetry parameter $\alpha = \sqrt{3}S$, $0 \leq \alpha \leq 1$
 - Measure of parity-violation
 - Generalising definition of two-body α parameters to multi-body decays
- Preliminary studies show $\Lambda_c^+ \rightarrow pK^-\pi^+$ decays have a large sensitivity to polarization, $\alpha \approx 0.65$, [PRD 103 \(2021\) 072003](#)
 - Considering the large BF, $\Lambda_c^+ \rightarrow pK^-\pi^+$ is the best channel to access Λ_c^+ polarization

Summary

- Presented the main tools needed for baryon amplitude analyses
 - Polarization system: defined according to production process
 - Amplitude model: outlined method to write generic multi-body decays in helicity formalism
 - Amplitude fit framework: described our strategy based on TensorFlowAnalysis package
- Presented example tests for amplitude model implementation
- Presented sensitivity study for $\Lambda_c^+ \rightarrow pK^-\pi^+$ case
 - Demonstrated simultaneous extraction of amplitude model and baryon polarization
 - Large sensitivity to polarization estimated, making $\Lambda_c^+ \rightarrow pK^-\pi^+$ best probe for Λ_c^+ polarization

Thanks for your attention!

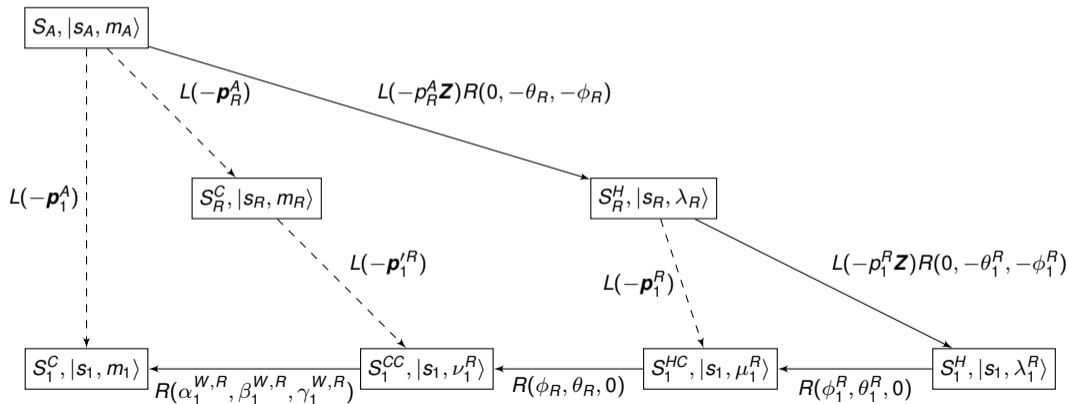
References:

- D. Marangotto, *Amplitude analysis and polarisation measurement of the Λ_c^+ baryon in the $pK^-\pi^+$ final state for the electromagnetic dipole moment determination*, PhD Thesis, <https://cds.cern.ch/record/2713231>
- D. Marangotto, *Helicity Amplitudes for Generic Multibody Particle Decays Featuring Multiple Decay Chains*, Advances in High Energy Physics (2020) 6674595, doi:10.1155/2020/6674595, arXiv:1911.10025.
- D. Marangotto, *Extracting maximum information from polarised baryon decays via amplitude analysis: the $\Lambda_c^+ \rightarrow pK^-\pi^+$ case*, Advances in High Energy Physics (2020) 7463073, doi:10.1155/2020/7463073, arXiv:2004.12318.
- S. Aiola et. al., *Progress towards the first measurement of charm baryon dipole moments*, Phys. Rev. D. **103** (2021) 072003, doi:10.1103/PhysRevD.103.072003, arXiv:2010.11902.

Backup Slides

Matching of final particle spin states, $A \rightarrow R \rightarrow 1$

- Apply Wigner rotation $R(\alpha_1^{W,R}, \beta_1^{W,R}, \gamma_1^{W,R}) = L(-\mathbf{p}_1^A)L(\mathbf{p}_R^A)L(\mathbf{p}_1^R)$: obtain 1 canonical states reached from $S_A, |s_A, m_A\rangle$ states, S_1^C system



Λ_c^+ polarization and time-reversal

- Polarization components defined as

$$\begin{aligned} P_z &\propto \mathbf{P} \cdot \hat{\mathbf{p}}(\Lambda_c^+) \\ P_x &\propto \mathbf{P} \cdot [(\mathbf{p}(\Lambda_c^+) \times \mathbf{p}(\mu^-)) \times \hat{\mathbf{p}}(\Lambda_c^+)] \\ P_y &\propto \mathbf{P} \cdot [\mathbf{p}(\Lambda_c^+) \times \mathbf{p}(\mu^-)], \end{aligned} \tag{1}$$

- Longitudinal (P_z) and transverse (P_x) polarization are T-even, while normal (P_y) polarization is T-odd
- P_y can be produced only by
 - T-violation: $\mathcal{O}(CPV)$, tiny in $b \rightarrow c$ transitions
 - Final state interactions: only EM btw Λ_c^+ and μ^- , should be $\mathcal{O}(1\%)$ at max
- Reference: Sozzi, Discrete symmetries and CPV

Maximum-likelihood fit

- Fit parameters determined minimizing

$$-\log \mathcal{L}(\omega) = -\sum_{i=1}^N \log \left[p(\Omega_i|\omega) + \frac{p_{bkg}(\Omega_i)I(\omega)}{\epsilon(\Omega_i)} \frac{n_{bkg}}{n_{sig}} \right] + N \log I(\omega) + \text{constant}$$

- Efficiency-corrected normalization computable using flat phase-space events simulated through full detector reconstruction

$$I(\omega) = \int p(\Omega|\omega)\epsilon(\Omega)d\Omega = \int p(\Omega|\omega)d\Omega' = \sum_{i=1}^{N_{MC}} p(\Omega_i|\omega)$$

- Background and efficiency parametrisation over phase space affect background term only
- For clean decays like $\Lambda_c^+ \rightarrow pK^-\pi^+$, $\frac{n_{bkg}}{n_{sig}} \ll 1$