Amplitude- and truncated partial-wave analyses combined: A single-channel method for extracting photoproduction multipoles directly from measured data

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***Main idea:** We all speak about partial waves as something self-understood and unique, so we need*
MODEL INDEPENDENT PWA

***Focus of my interest:** SINGLE ENERGY, SINGLE CHANNEL PWA*

***Main effort:** MINIMIZE (ELIMINATE?) MODEL DEPENDENCE*

After years of research the present state is (for me) disappointing:

PHYSICAL REVIEW C **104**, 014605 (2021)

Each single-energy, single-channel partial-wave analysis is inherently model-dependent

A. Švarc

WHY ?

1. CONTINUUM AMBIGUITY :

Complete set of observables of any single channel scattering reaction in inelastic region where unitarity is lost is invariant with respect to the simultaneous phase rotation of all reaction amplitudes with arbitrary energy and angle dependent function $\Phi(W, \vartheta)$ (Bowcock-Burkhard Rep. Prog. Phys. 1975, 38, 1099 – 1141, et al)

Consequence: overall phase of a SINGLE CHANNEL process is undetermined

2. Angular dependent part of phase rotation $\Phi(W, \vartheta)$ (CONTINUUM AMBIGUITY) MIXES MULTIPOLES (Omalaenko Sov. Jour. Nucl. Phys. 34(3) 1981, A. Švarc et al. Few-Body Systems (2018) 59:96, Phys. Rev. C 97, 054611 (2018) Phys. Rev. C 98, 056206 (2018))

In other words: For one set of observables there exist a number of EQUIVALENT sets of multipoles (chi**2 is identical for all of them) WITH DIFFERENT analytic structure (pole content)!

Consequence: As resonance quantum numbers are defined by the partial wave decomposition which is non-unique in single-channel measurements, they CONNOT BE UNIQUELY DEFINED in a single channel process.

3. Fixing the phase $\Phi(W, \vartheta)$ (eliminating continuum ambiguity) is achieved by RESTORING UNITARITY! This can ONLY be done if all open channels are analyzed SIMULTANEOUSLY in a coupled channel formalism!

Consequence: Resonance quantum numbers CANNOT be assigned WITHOUT coupled channel formalism, so each single-channel PWA must be model dependent!

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So, ANY SINGLE-CHANNEL PWA IS MODEL DEPENDENT!

Question: How do we see it?

In SE PWA: It produces discontinuities!

In ED PWA: The missing phase is implicitly defined by choosing the model!

AA/PWA Method

For unique, sensible SC-SE PWA we need to introduce some MODEL DEPENDENCE.

I introduce MINIMAL MODEL DEPENDENCE (existing because of first principles) by fixing the only quantity which is undetermined, and this is THE REACTION AMPLITUDE PHASE.

I choose to FIX all reaction amplitude phases TO THE VALUE GIVEN BY SOME THEORETICAL MODEL. In my choice this is Bonn-Gatchina coupled-channel model.

A reminder: the phase has two parts:

1. *The part determined by certain polarization observables*
2. *The overall phase*

In the proposed method we take over both parts from the theoretical model.

Remark:
This is only the first step. Only four observables in, for example, η-photoproduction are enough to uniquely determine reaction amplitude absolute values at a single energy, all other observables determine all reaction amplitude phases. So, if one has sufficient number of observables (a complete set) one could, at least in principle, determine all 4 phases too. However, the overall, continuum ambiguity phase remains undermined. We can get an ideal fit to the experiment, but analytic structure (pole content) of a solution is still undefined. So, we still need the overall phase which can be given only by a coupled-channel model. However, this is the next step. I have to stress that I hope to get the reliable results as used theoretical ED model by Bonn-Gatchina group fits all polarization observables fairly well.

Warning!

The method relies ENTIRELY on the data, so quality of the data is essential !

DATA INCONSISTENCY WILL BE QUITE A PROBLEM!

Discontinuities are possible!

If the data are consistent, multipoles are smooth. It is checked for pseudo or numeric data, and shown in previous publications.

So, all discontinuities which might occur in AA/PWA are entirely DUE TO DATA INCONSISTENCY!

If we want to get smooth multipoles we need more constraints! An example: FIXED-t ANALYTICITY

It Is introduced by Karlsruhe-Helsinki group in 1980-es.

It is revived recently, and used for eta and pion photoproduction!

Phys. Rev. C 97, 015207 (2018), Phys. Rev. C 100, 055203 (2019), Phys. Rev. C (2021) in press

Amplitude analysis/partial wave analysis (AA/PWA) method is fully explained and tested in η-photoproduction in ref:

PHYSICAL REVIEW C **102**, 064609 (2020)

Amplitude- and truncated partial-wave analyses combined: A single-channel method for extracting photoproduction multipoles directly from measured data

A. Švarc, Y. Wunderlich, and L. Tiator

It is important to stress that the method uses MEASURED DATA ONLY, and the only model dependence is the ED phase. Analyticity is enforced via partial wave decomposition.

The method is a two step process:

Step 1 – amplitude analysis of all observables with FIXING THE PHASE to the Bonn-Gatchina ED phase

Step 2 – constrained PWA using the results of Step 1

$$\chi^2(W) = \chi^2_{\text{data}}(W) + \chi^2_{\text{pen}}(W),$$

$$\chi^2_{\text{data}}(W) = \sum_{i=1}^{N_{\text{data}}} w^i \left[\mathcal{O}_i^{\text{expt.}}(W, \Theta_i) - \mathcal{O}_i^{\text{theor.}}(\mathcal{M}^{\text{fit}}(W, \Theta_i))\right]^2,$$

$$\chi^2_{\text{pen}}(W) = \lambda_{\text{pen.}} \sum_{i=1}^{N_{\text{data}}} \sum_{k=1}^{N_{\text{amp}}} \left|\mathcal{A}_k(\mathcal{M}^{\text{fit}}(W, \Theta_i)) - \mathcal{A}_k^{\text{pen.}}(W, \Theta_i)\right|^2,$$

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We used the world collection of data

TABLE I. Experimental data from A2@MAMI, GRAAL, and CBELSA/TAPS used in our PWA. Data from CBELSA/TAPS are taken at the center of the energy bin.

Obs.	N	E_{lab} [MeV]	N_E	$\theta_{\text{c.m.}}$ [deg.]	N_θ	Reference
σ_0	2400	710–1395	120	18–162	20	A2@MAMI(2010) [15]
Σ	150	724–1472	15	40–160	10	GRAAL(2007) [16]
T	144	725–1350	12	24–156	12	A2@MAMI(2016) [17]
F	144	725–1350	12	24–156	12	A2@MAMI(2016) [17]
E	64	750–1450	8	29–151	8	CBELSA/TAPS(2020) [18]
P	66	725–908	6	41–156	11	CBELSA/TAPS(2020) [18]
G	48	750–1250	6	48–153	8	CBELSA/TAPS(2020) [18]
H	66	725–908	6	41–156	11	CBELSA/TAPS(2020) [18]

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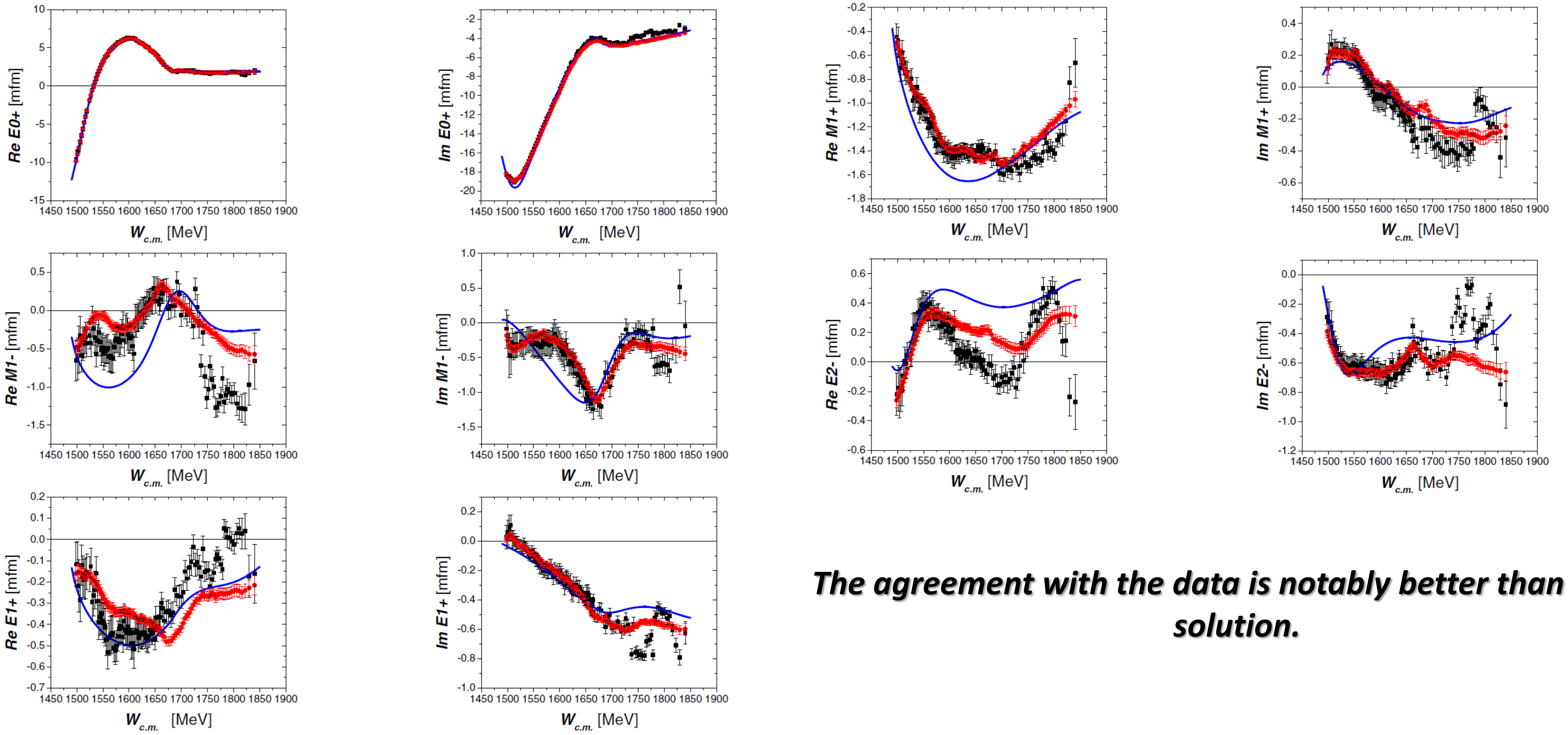
Results are stable and confident.



The agreement with the data is notably better than for ED solution.

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The analytic structure of the obtained solution is given in PHYSICAL REVIEW C **104**, 014605 (2021) ***and is completely consistent with PDG.***

TABLE I. Pole parameters for BG 2014-2, Sol 1, and Sol 1/21 extracted using L+P expansion. M_i, Γ_i, r_i, and Θ_i, $i = 1, 2$ are pole masses, widths, absolute values of the residue, and its phase, while χ^2 and χ^2_{red} are total and reduced χ-squared values (reduced χ-squared is defined as total χ-squared divided by the difference of total number of points and free fitting parameters). Particle Data Group values from Ref. [13] are for reference given in bold text.

	Model	M_1	Γ_1	$\|a_1\|$	Θ_1	M_2	Γ_2	$\|a_2\|$	Θ_2	χ^2	χ^2_{red}
S_{11} $1/2^-$	PDG	**1510(19)**	**130(20)**	-	-	**1655(15)**	**135(35)**	-	-		
	BG 2014-2	1498(107)	158(157)	1780(5300)	164(345)	1661(5)	85(12)	126(47)	24(19)	18	0.13
E_0^+	Sol 1	1489(66)	158(78)	2043(5054)	148(146)	1664(5)	92(9)	140(36)	37(15)	140	0.6
	Sol 1/21	1484(37)	196(189)	2926(7330)	166(172)	1662(3)	101(7)	158(26)	34(9)	97	0.43
P_{11} $1/2^+$	PDG	**1379(10)**	**175(15)**	-	-	**1700(20)**	**120(40)**	-	-		
	BG 2014-2	-	-	-	-	1698(1)	123(1)	105(2)	-90(1)	102	0.75
M_1^-	Sol 1	-	-	-	-	1730(6)	80(10)	48(12)	-22(18)	140	0.7
	Sol 1/21	-	-	-	-	1660(6)	112(13)	49(15))	-168(16)	106	0.47
	Sol 1/21	1526(25)	73(37)	19(32)	-123(110)	1681(7)	103(12)	39(10))	-124(17)	25	0.11
P_{13} $3/2^+$	PDG	-	-	-	-	**1675(15)**	**250(150)**	-	-		
	BG 2014-2	-	-	-	-	1705(7)	195(21)	$\begin{pmatrix}38(10)\\38(13)\end{pmatrix}$	$\begin{pmatrix}-133(15)\\-107(16)\end{pmatrix}$	58	0.23
$\begin{pmatrix}E_1^+\\M_1^+\end{pmatrix}$	Sol 1	-	-	-	-	1879(46)	200(68)	$\begin{pmatrix}328(359)\\260(280)\end{pmatrix}$	$\begin{pmatrix}-8(44)\\67(50)\end{pmatrix}$	310	0.6
	Sol 1/21	-	-	-	-	1714(7)	102(13)	$\begin{pmatrix}10(3)\\1(1)\end{pmatrix}$	$\begin{pmatrix}-167(16)\\20(47)\end{pmatrix}$	297	0.6
D_{13} $3/2^-$	PDG	**1510(5)**	**110(10)**	-	-	**1700(50)**	**200(100)**	-	-		
	BG 2014-2	1508(3)	106(7)	$\begin{pmatrix}52(11)\\25(6)\end{pmatrix}$	$\begin{pmatrix}122(12)\\118(13)\end{pmatrix}$	1664(76)	399(159)	$\begin{pmatrix}119(155)\\72(86)\end{pmatrix}$	$\begin{pmatrix}73(71)\\103(77)\end{pmatrix}$	1.7	0.06
$\begin{pmatrix}E_2^-\\M_2^-\end{pmatrix}$	Sol 1	1528(23)	63(37)	$\begin{pmatrix}11(22)\\2(3)\end{pmatrix}$	$\begin{pmatrix}-160(82)\\148(98)\end{pmatrix}$	1721(6)	64(13)	$\begin{pmatrix}10(3)\\4(1)\end{pmatrix}$	$\begin{pmatrix}149(19)\\-168(18)\end{pmatrix}$	368	0.8
	Sol 1/21	1525(23)	121(60)	$\begin{pmatrix}37(52)\\24(39)\end{pmatrix}$	$\begin{pmatrix}-156(91)\\158(94)\end{pmatrix}$	1664(12)	121(24)	$\begin{pmatrix}11(6)\\13(7)\end{pmatrix}$	$\begin{pmatrix}-31(33)\\46(33)\end{pmatrix}$	50	0.1
D_{15} $5/2^-$	PDG	-	-	-	-	**1660(5)**	**135(15)**	-	-		
	BG 2014-2	-	-	-	-	1673(4)	225(6)	$\begin{pmatrix}1(0.3)\\23(1)\end{pmatrix}$	$\begin{pmatrix}54(17)\\-17(6)\end{pmatrix}$	45	0.16
$\begin{pmatrix}E_2^+\\M_2^+\end{pmatrix}$	Sol 1	-	-	-	-	1784(1)	11(1)	$\begin{pmatrix}0.5(0.01)\\0.8(0.1)\end{pmatrix}$	$\begin{pmatrix}23(9)\\95(9)\end{pmatrix}$	310	0.6
	Sol 1/21	-	-	-	-	1659(10)	145(23)	$\begin{pmatrix}6(2)\\9(4)\end{pmatrix}$	$\begin{pmatrix}17(20)\\-40(22)\end{pmatrix}$	90	0.2
F_{15} $5/2^+$	PDG	-	-	-	-	**1675(10)**	**120(15)**	-	-		
	BG 2014-2	-	-	-	-	1677(1)	117(1)	$\begin{pmatrix}13(1)\\7(0.5)\end{pmatrix}$	$\begin{pmatrix}147(1)\\145(11)\end{pmatrix}$	12	0.05
$\begin{pmatrix}E_3^-\\M_3^-\end{pmatrix}$	Sol 1	-	-	-	-	1767(2)	34(4)	$\begin{pmatrix}3(0.5)\\2(0.5)\end{pmatrix}$	$\begin{pmatrix}-91(9)\\33(10)\end{pmatrix}$	690	1.5
	Sol 1/21	-	-	-	-	1690(4)	166(11)	$\begin{pmatrix}11(2)\\23(4)\end{pmatrix}$	$\begin{pmatrix}172(8)\\164(77)\end{pmatrix}$	156	0.34

New results
$\gamma p \rightarrow K^+ \Lambda$

Preliminary

Advantage: there exist another SE-SC PWA to compare with

PRL **119,** 062004 (2017)

Strong Evidence for Nucleon Resonances near 1900 MeV

A. V. Anisovich, V. Burkert, M. Hadžimehmedović, D. G. Ireland, E. Klempt, V. A. Nikonov, R. Omerović, H. Osmanović, A. V. Sarantsev, J. Stahov, A. Švarc, and U. Thoma

Eur. Phys. J. A (2017) **53**: 242

N* resonances from KΛ amplitudes in sliced bins in energy

A.V. Anisovich, V. Burkert, M. Hadžimehmedović, D.G. Ireland, E. Klempt, V.A. Nikonov, R. Omerović, A.V. Sarantsev, J. Stahov, A. Švarc, and U. Thoma

Method: classical constrained SE-SC PWA

1. ***Four lowest multipoles E_{0+}, E_{1+}, M_{1+}, M_{1-} FREE***
2. ***Next three multipoles E_{2-}, M_{2-}, E_{2+} CONSTRAINED TO BG MODEL***
3. ***All higher multipoles FIXED TO BG MODEL***

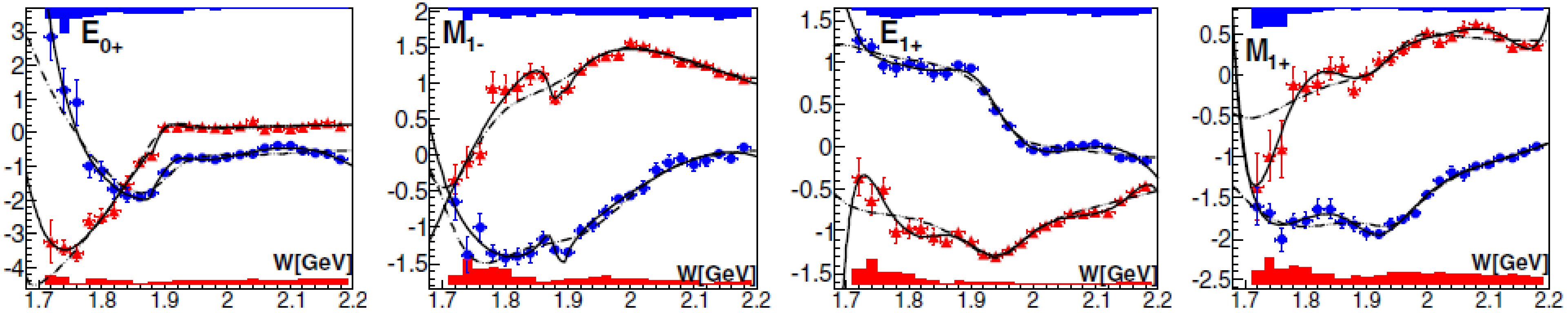

FIG. 2. Real (red triangles) and imaginary (blue dots) part of the E_{0+}, M_{1-}, E_{1+}, and M_{1+} multipoles for the reaction $\gamma p \to K^+\Lambda$. The systematic errors are given at the top (real part) and bottom (imaginary part) of the subfigures. E_{0+} excites the partial wave $J^P = 1/2^-$, M_{1-}: $J^P = 1/2^+$, E_{1+} and M_{1+}, $J^P = 3/2^+$. The solid curve shows the $L+P$ fit, the dashed curve the energy-dependent BNGA fit.

COMPARISON BETWEEN SE/SC PWA BG AND AA/PWA POSSIBLE!

Data base

Data base is identical to Bonn-Gatchina publications!

TABLE I: Experimental data from CLAS, and GRAAL used in our PWA.

Obs.	N	$E_{c.m.}$ [MeV]	N_E	θ_{cm} [deg]	N_θ	Reference
$d\sigma/d\Omega \equiv \sigma_0$	3615	1625 – 2295	268	28 – 152	5 – 19	CLAS(2007) [18], CLAS(2010) [19]
Σ	252	1649 – 2179	34	35 – 143	6 – 16	GRAAL(2007) [20], CLAS(2016) [22]
T	247	1645 – 2179	34	31 – 142	6 – 16	GRAAL(2007) [20],CLAS(2016) [22]
P	1259	1625 – 2295	78	28 – 143	6 – 18	CLAS(2010) [19], GRAAL(2007) [20]
$O_{x'}$	252	1645 – 2179	34	31 – 143	6 – 16	GRAAL(2007) [20], CLAS(2016) [22]
$O_{z'}$	252	1645 – 2179	34	31 – 143	6 – 16	GRAAL(2007) [20], CLAS(2016) [22]
C_x	138	1678 – 2296	14	31 – 139	9	CLAS(2007) [18]
C_z	138	1678 – 2296	14	31 – 139	9	CLAS(2007) [18]

Problem: CLAS(2016) data

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Problem with BG2019 solution

Baryon Spectroscopy

Transition amplitudes for pion induced reactions, multipoles and polarization observables.
The polarization observables are defined as in Phys.Rev.C46:2430-2455,1992.

Solutions BG2011: Eur.Phys.J. A47 (2011) 153; Eur.Phys.J. A48 (2012) 15.
Solutions BG2014: Eur.Phys.J. A51 (2015) 95; Eur.Phys.J. A52 (2016) 284.

List of the data sets	gamma-proton reactions	gamma-neutron reactions	pion induced reactions
	○ BG2011-01	○ BG2014-01	◉ BG2019
	○ BG2011-02	○ BG2014-02	

pi N Amplitudes	Multipoles	Single meson photoproduction off proton	Single meson photoproduction off neutron (BG2011-02) (BG2014-02)	Double meson photoproduction observables

Publicly available BG2019 solution fits CLAS(2016) data rather poorly, while the solution used in both Bonn-Gatchina 2017 publications, and which I call BG2017 is much better!

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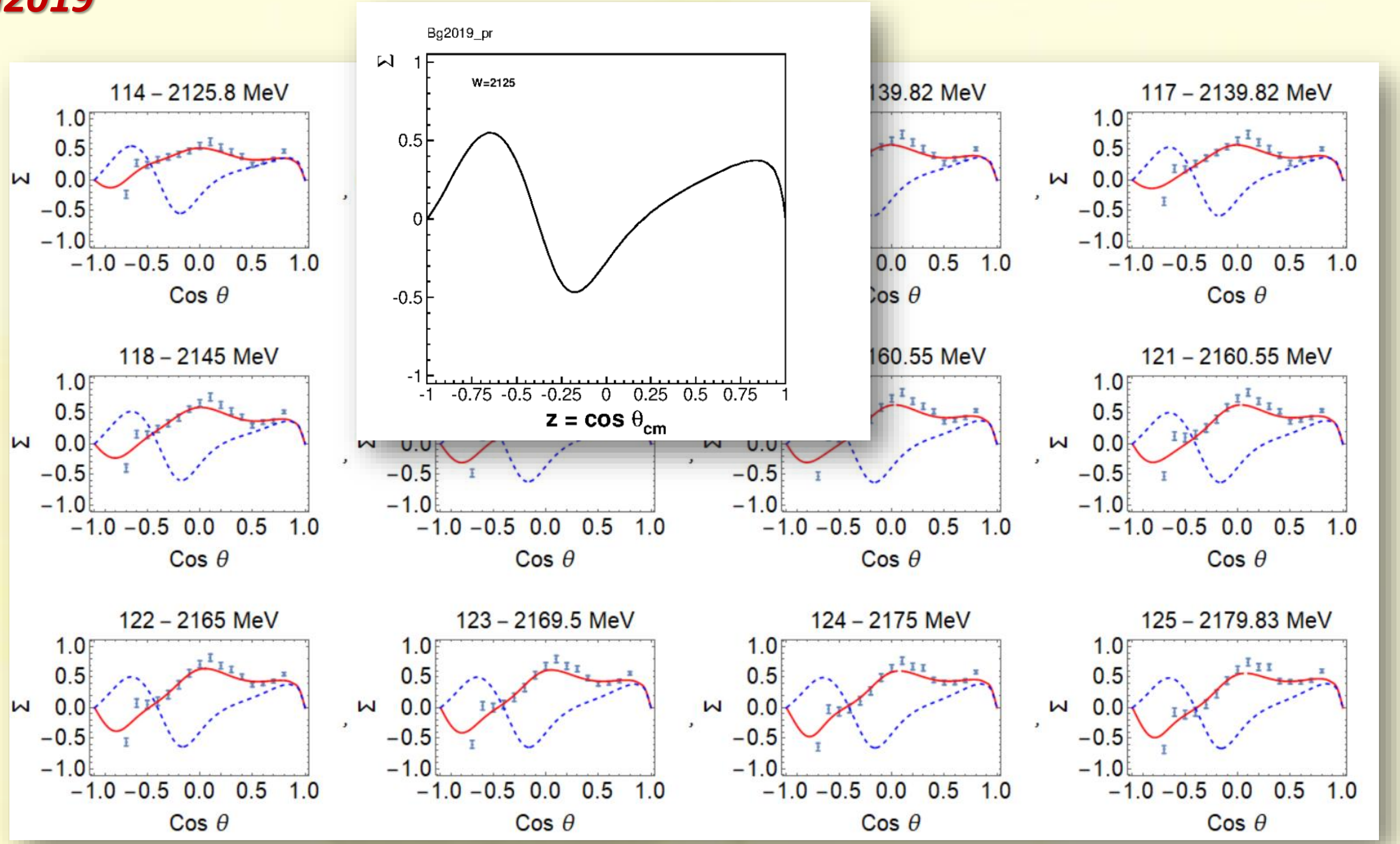
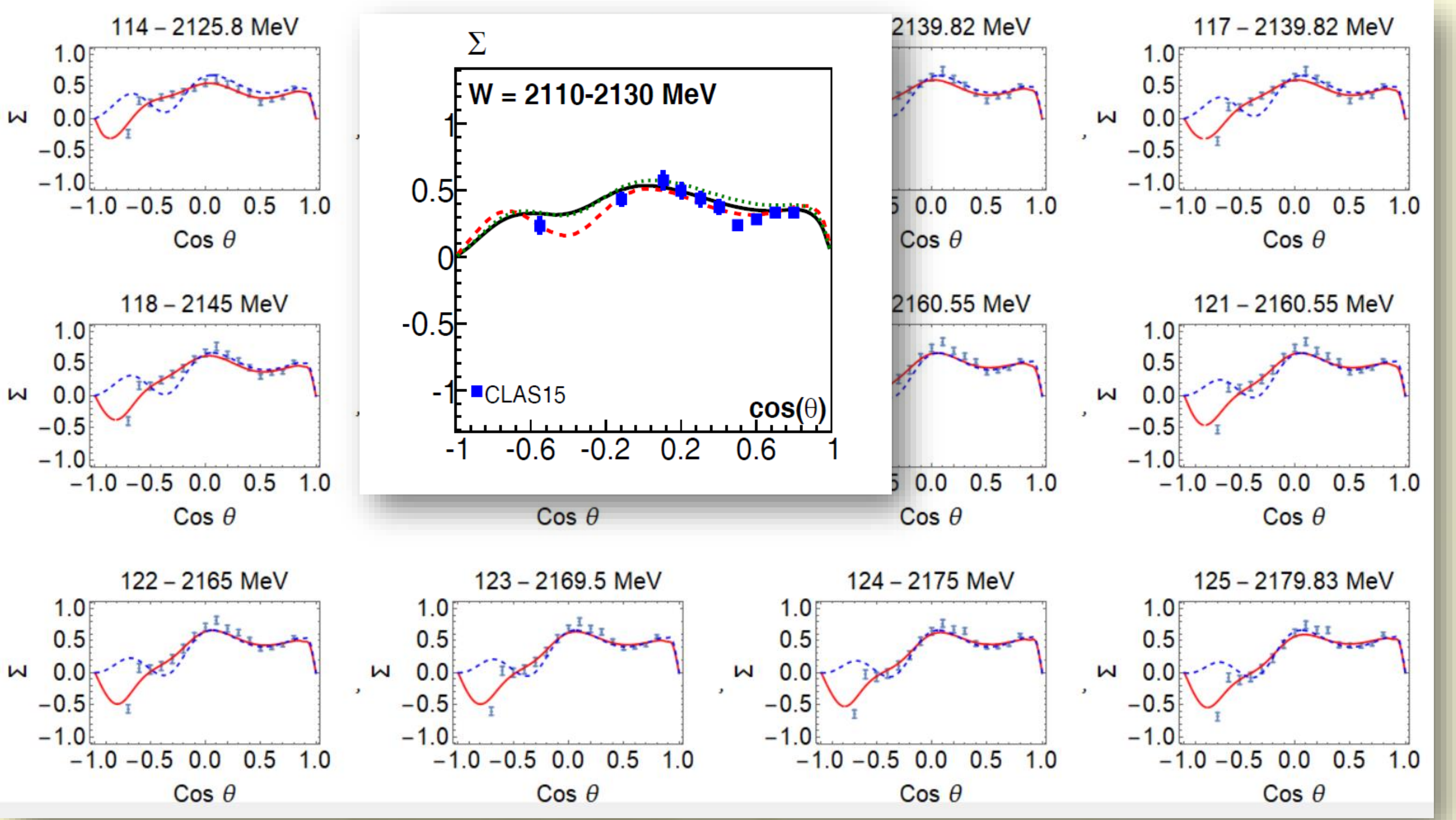
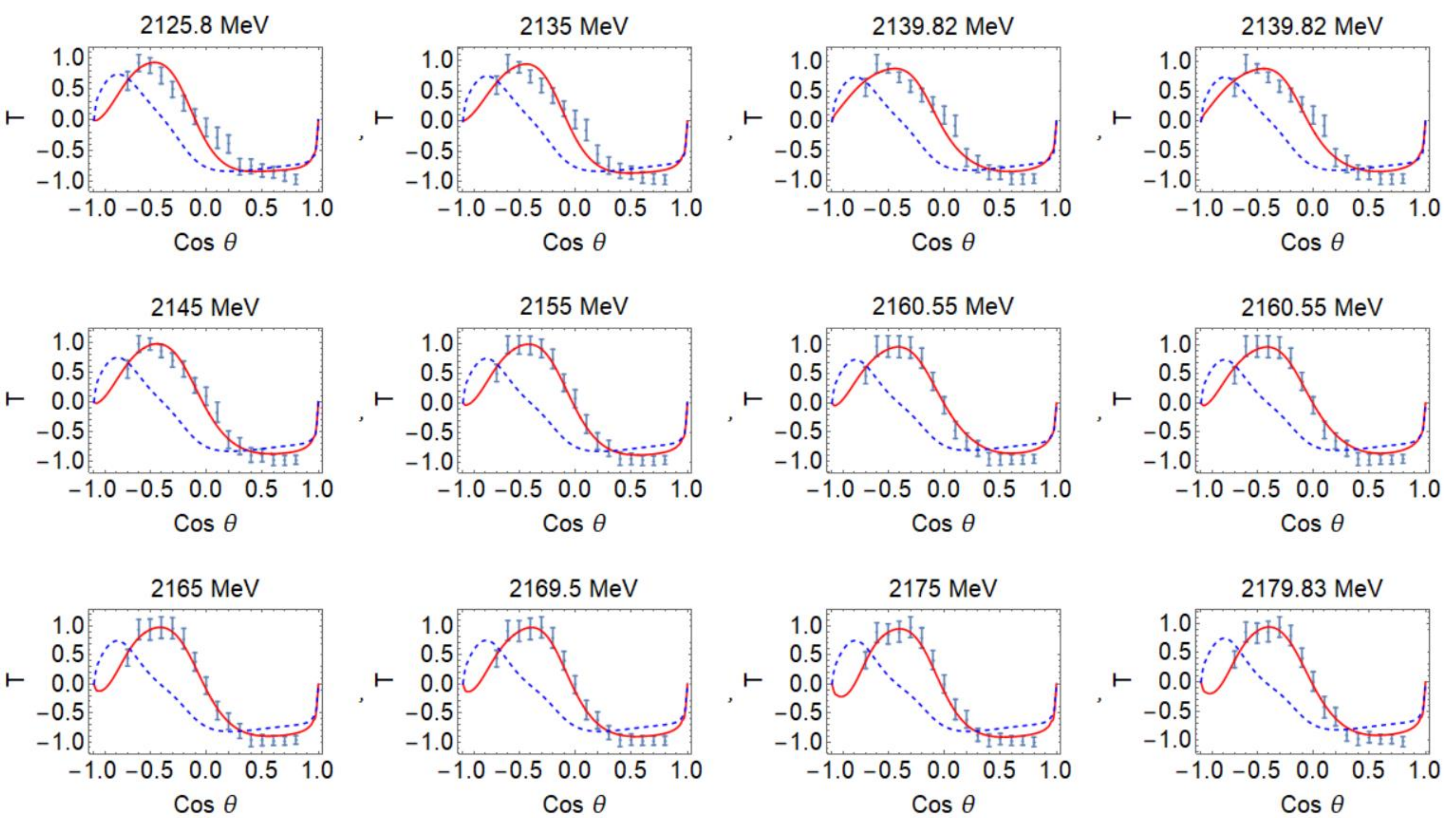
BG2019



BG2017


114 – 2125.8 MeV
2139.82 MeV
117 – 2139.82 MeV
118 – 2145 MeV
2160.55 MeV
121 – 2160.55 MeV
122 – 2165 MeV
123 – 2169.5 MeV
124 – 2175 MeV
125 – 2179.83 MeV
Σ
Cos θ
W = 2110-2130 MeV
CLAS15
cos(θ)

BG2019


2125.8 MeV
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T
Cos θ
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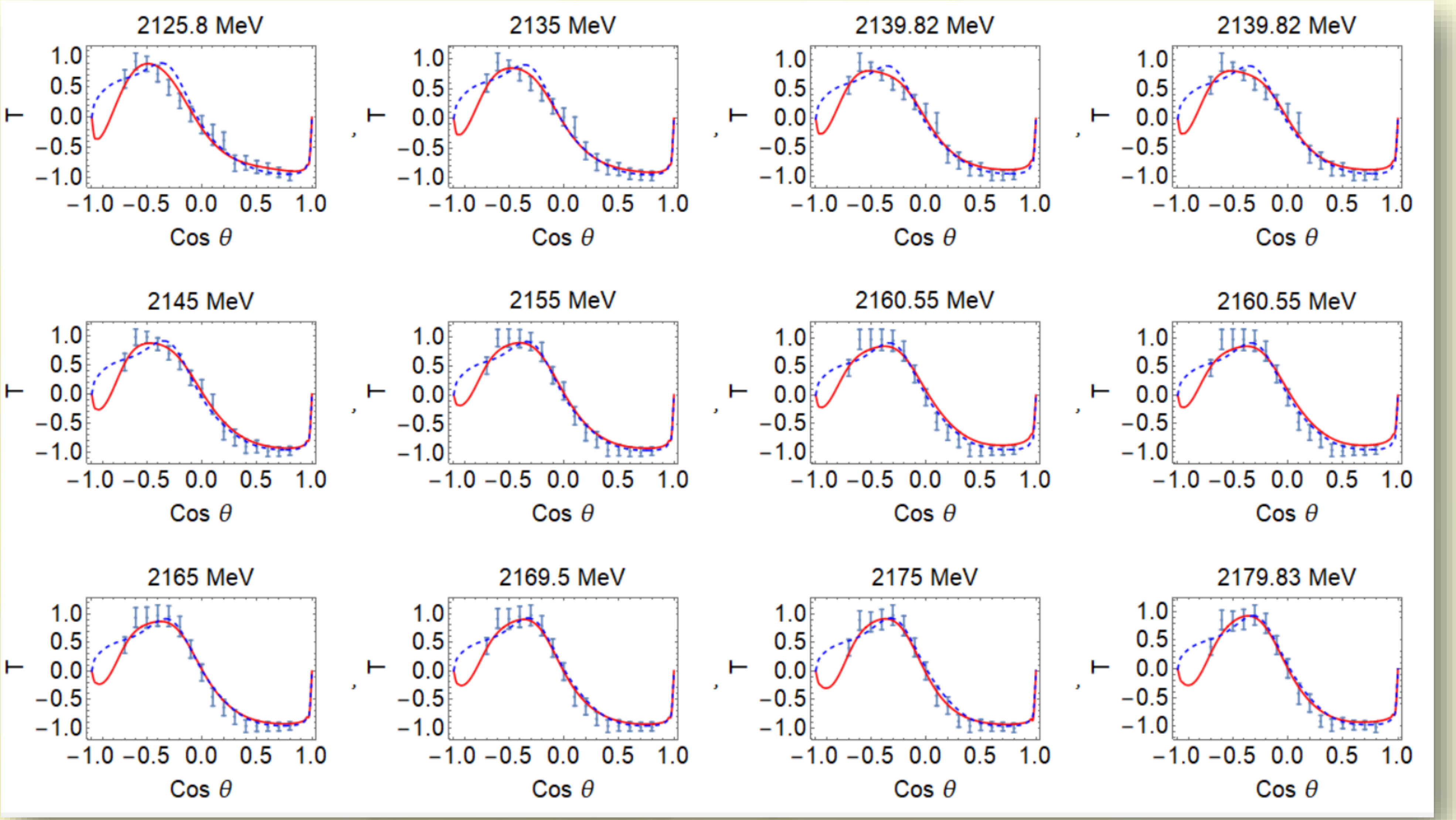
BG2017



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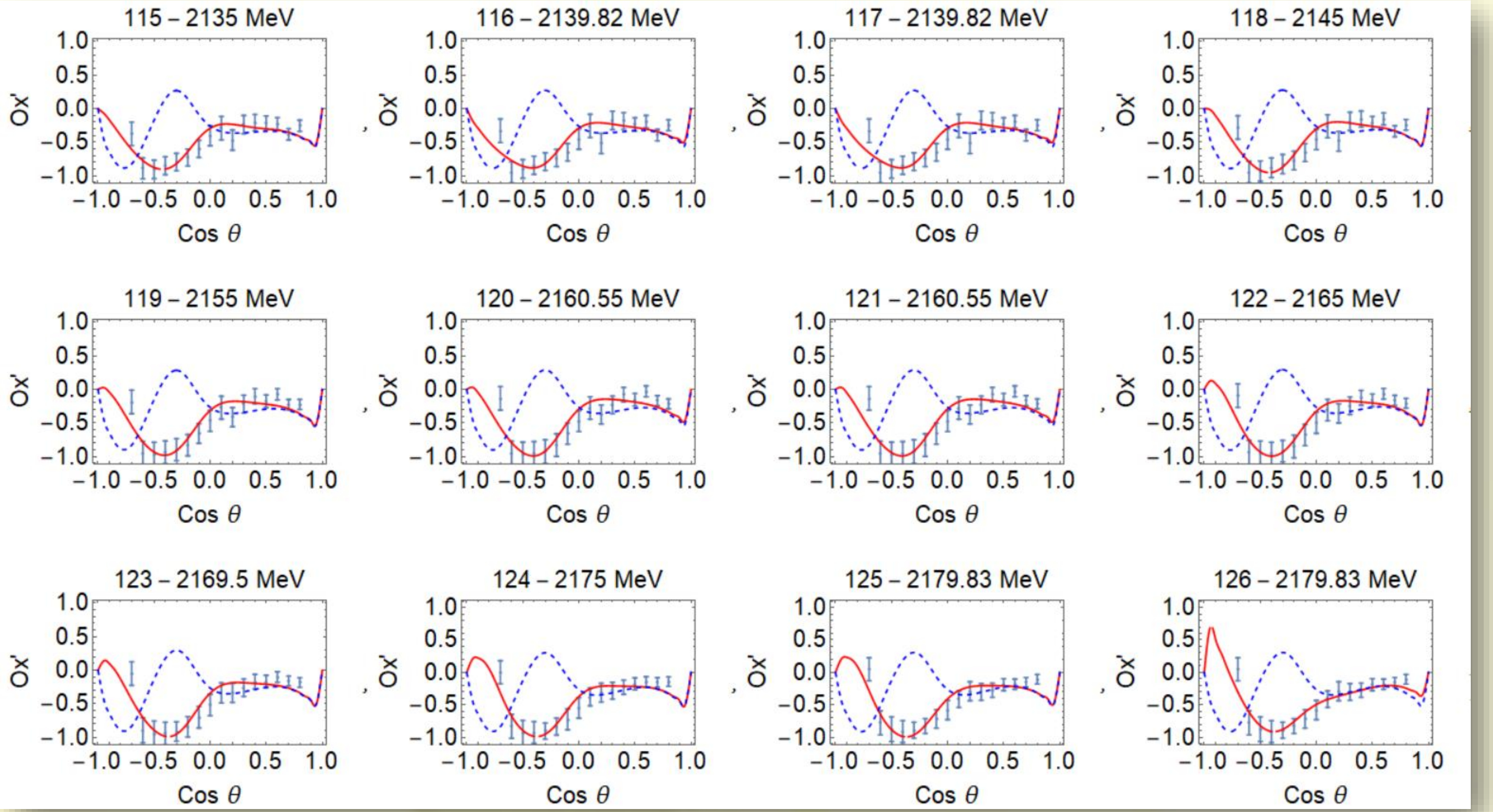
2175 MeV
2179.83 MeV
T
Cos θ

BG2019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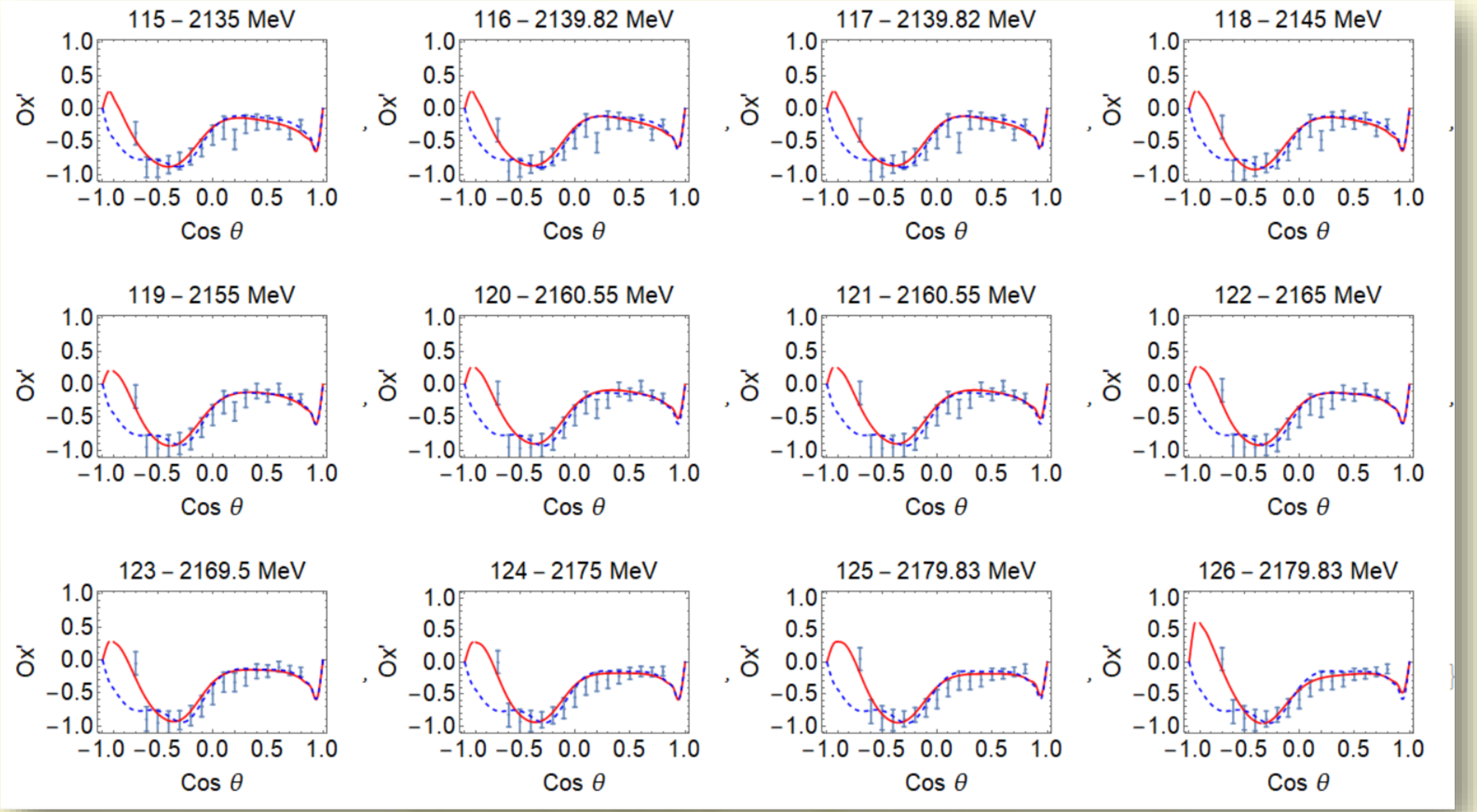
124 – 2175 MeV
125 – 2179.83 MeV
126 – 2179.83 MeV
Ox'
Cos θ

BG2017



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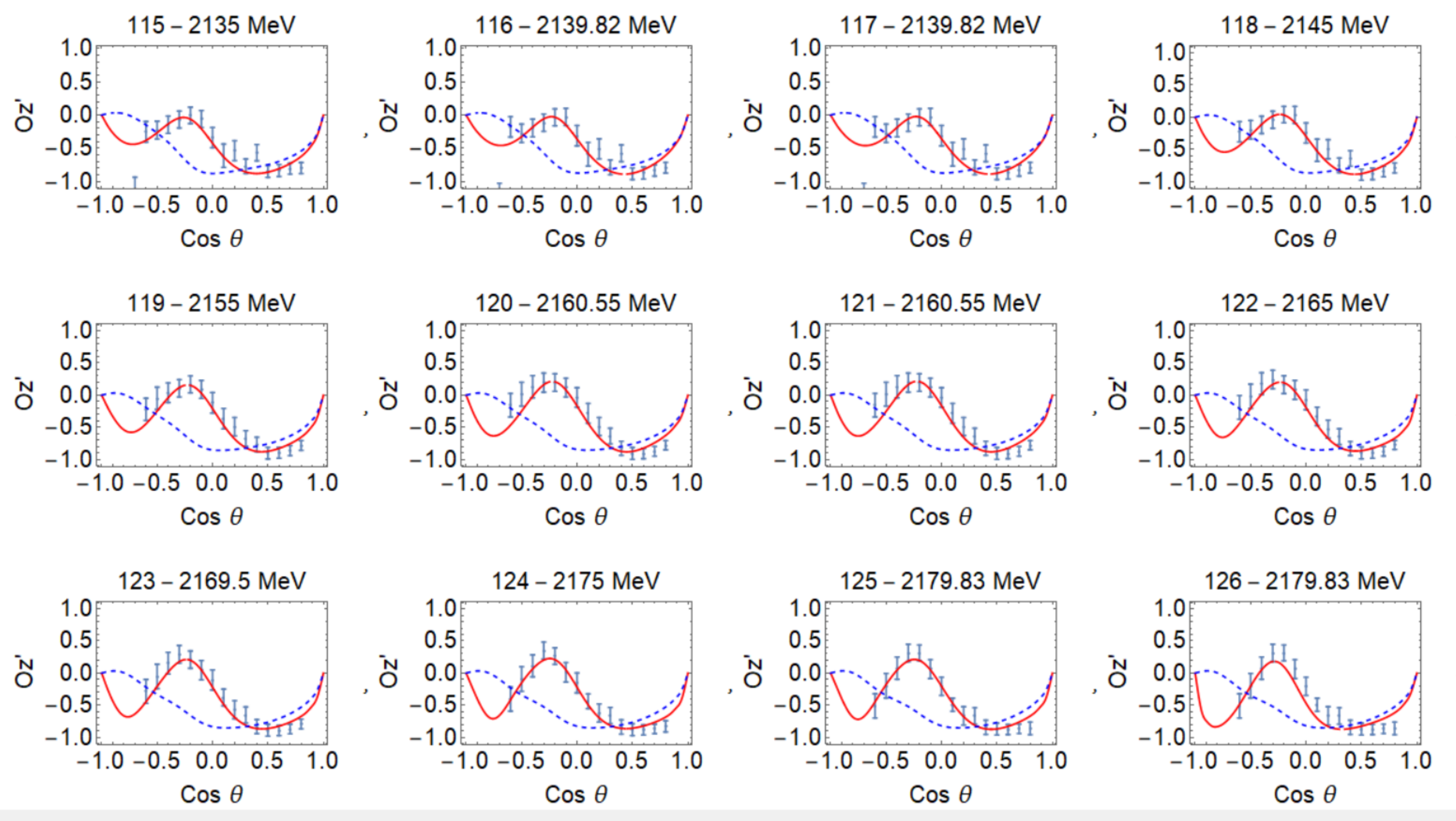
126 – 2179.83 MeV
Ox'
Cos θ

BG2019



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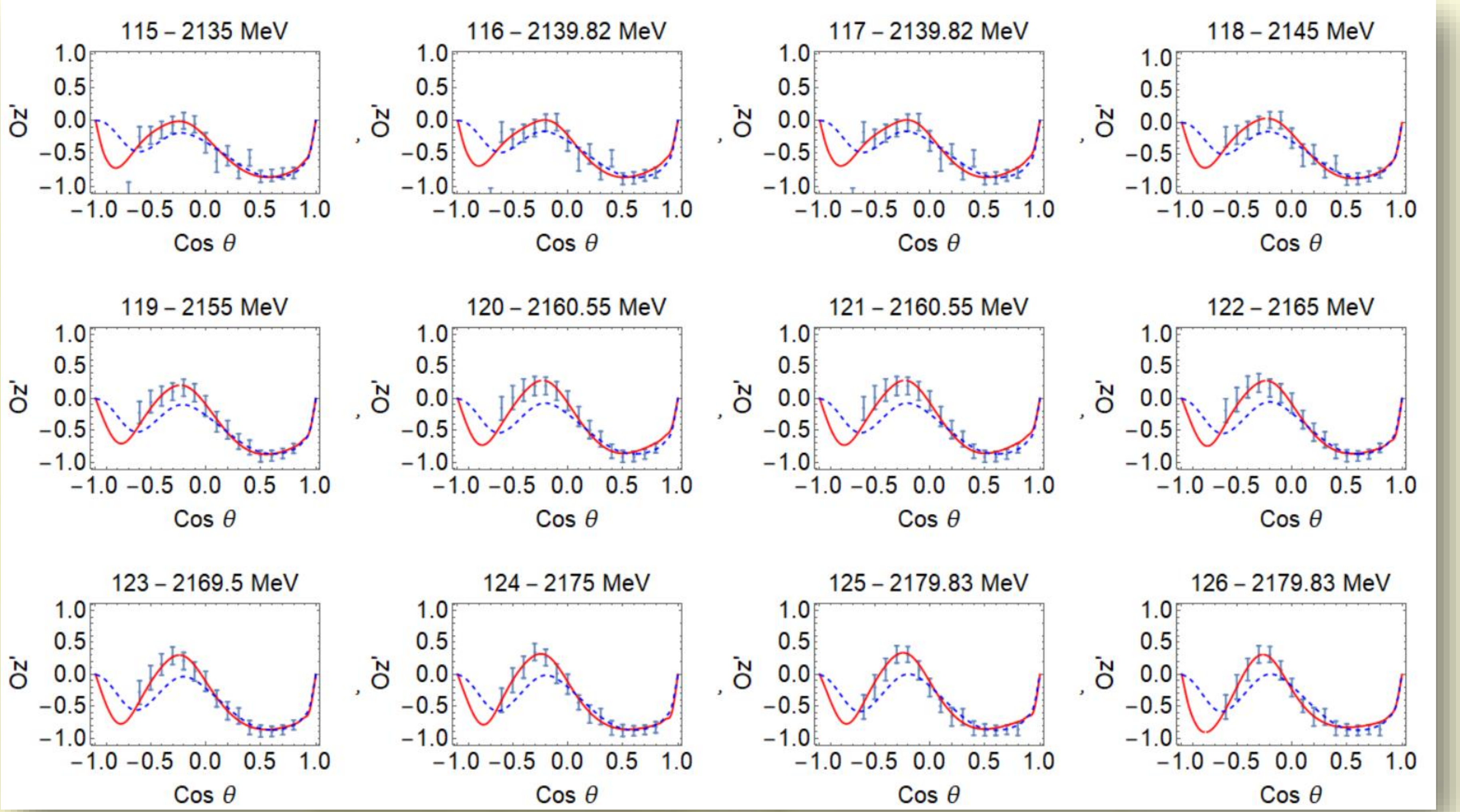
126 – 2179.83 MeV
Oz'
Cos θ

BG2017



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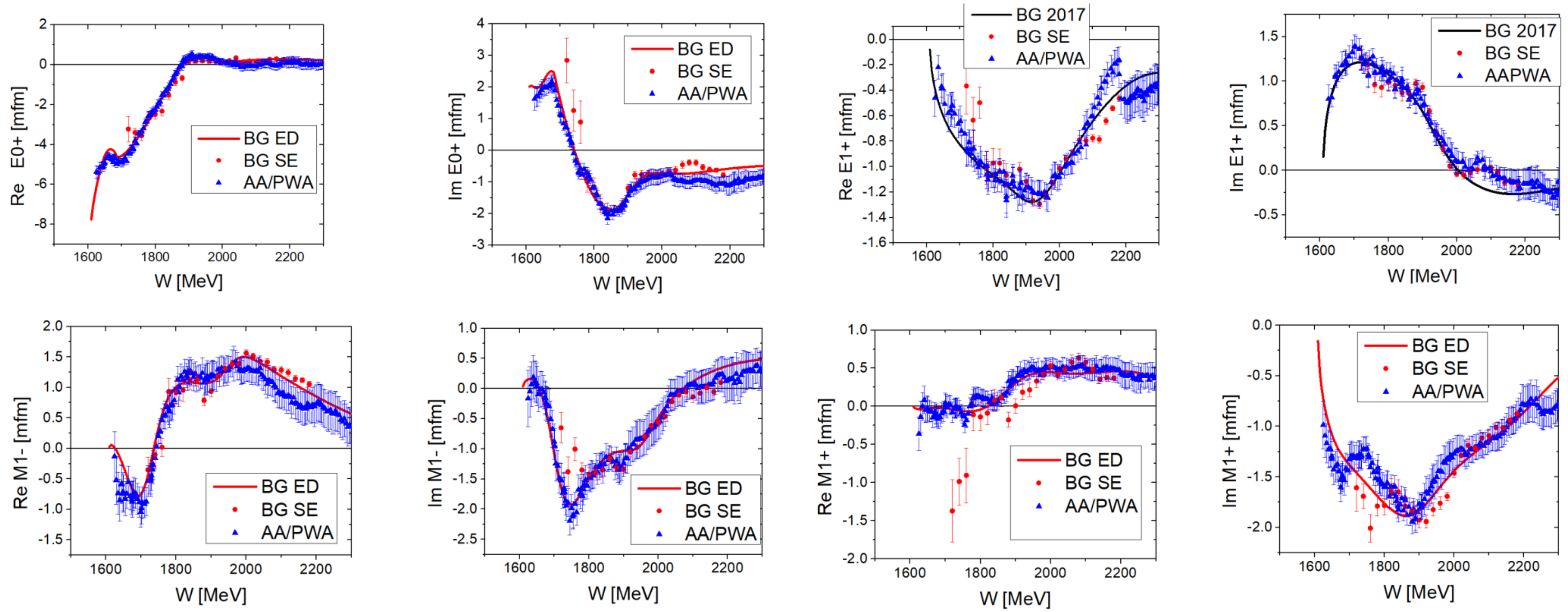
122 – 2165 MeV
123 – 2169.5 MeV
124 – 2175 MeV
125 – 2179.83 MeV
126 – 2179.83 MeV
Oz'
Cos θ

The choice of input: **BG2017**

Results

Comparison with BG SE result



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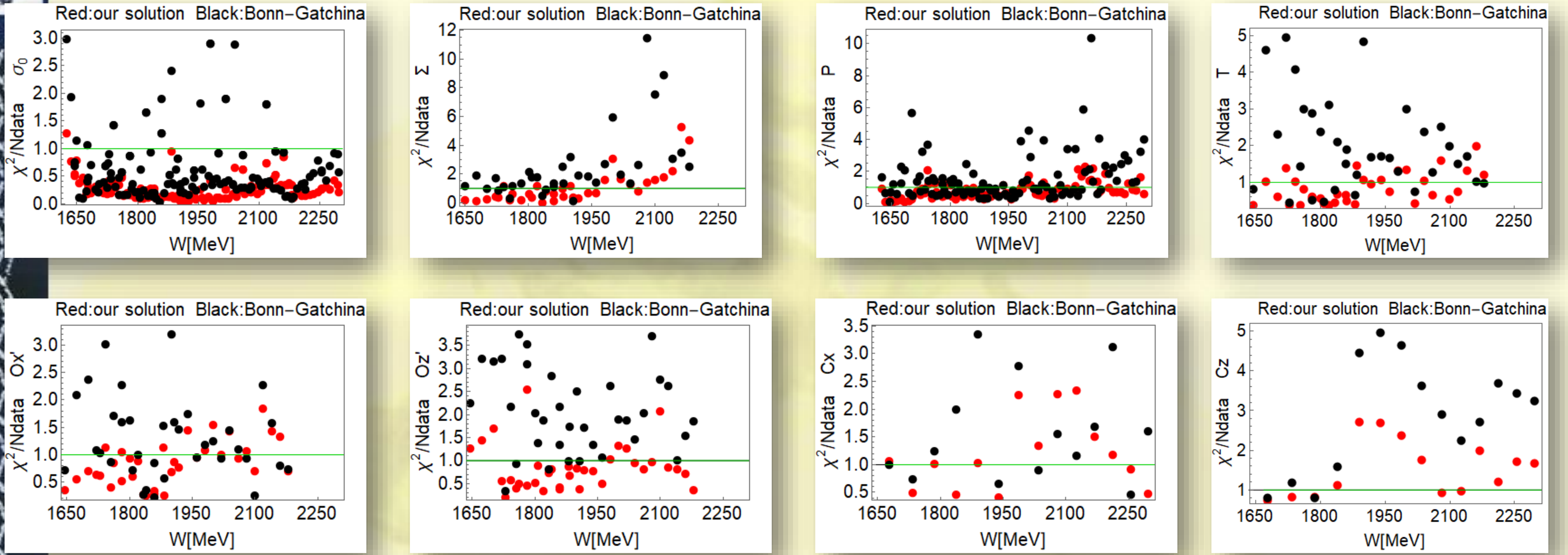
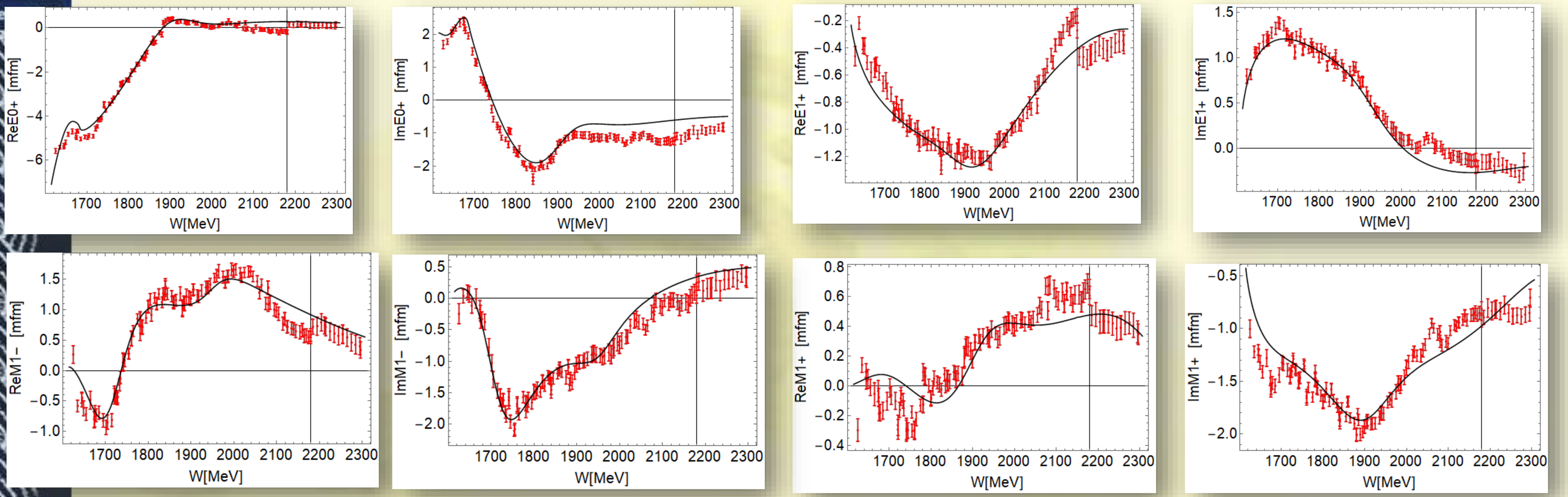
Comparison of χ^2 between AA/PWA and BG ED



All multipoles



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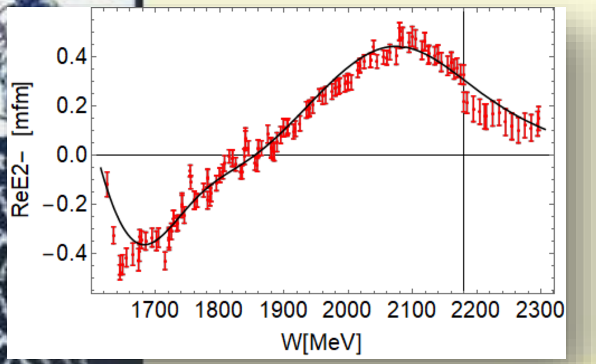
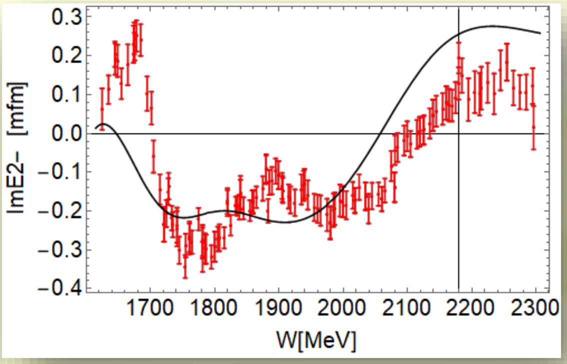
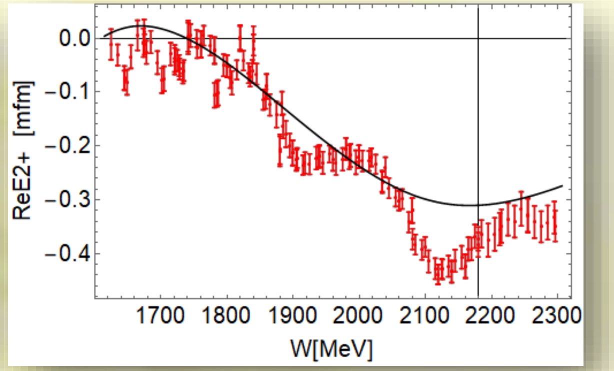
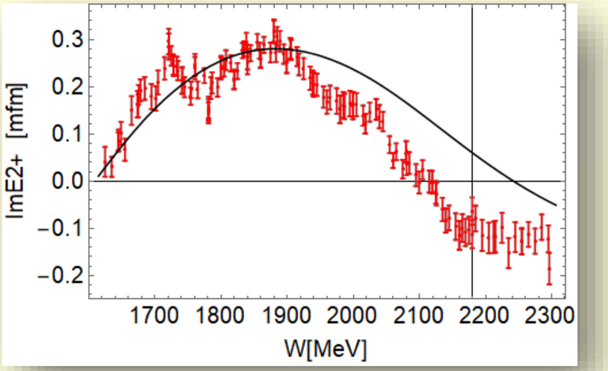
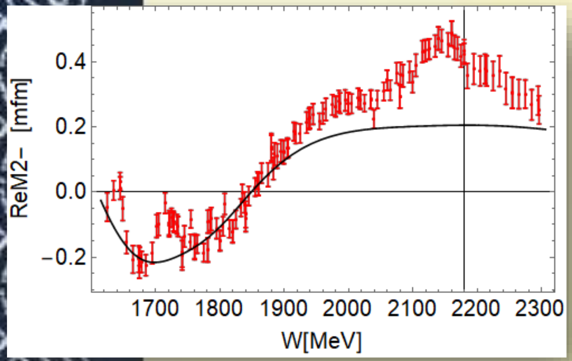
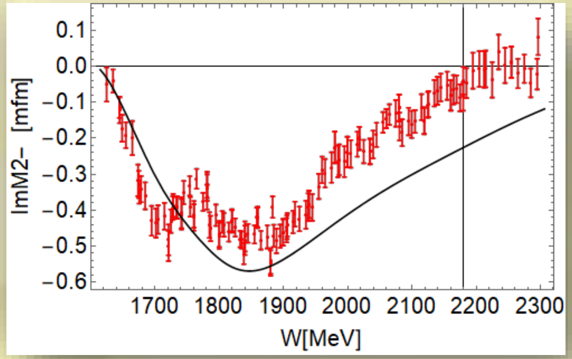


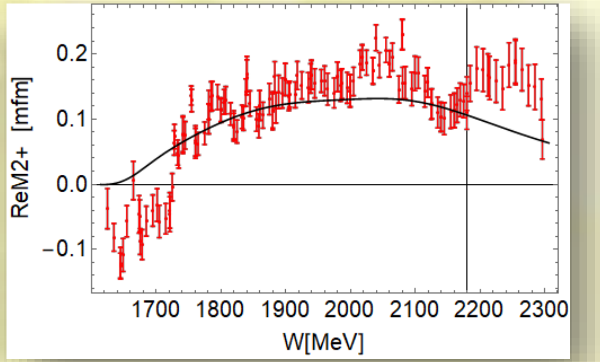


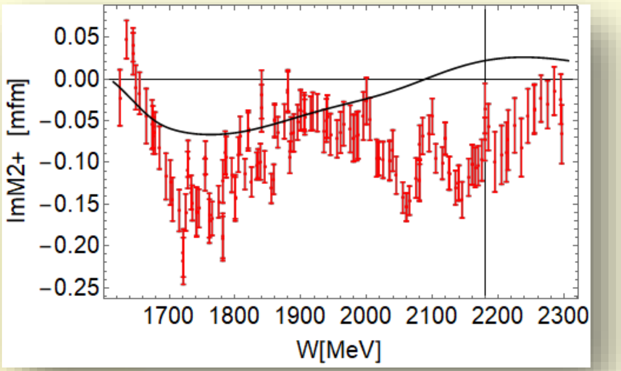






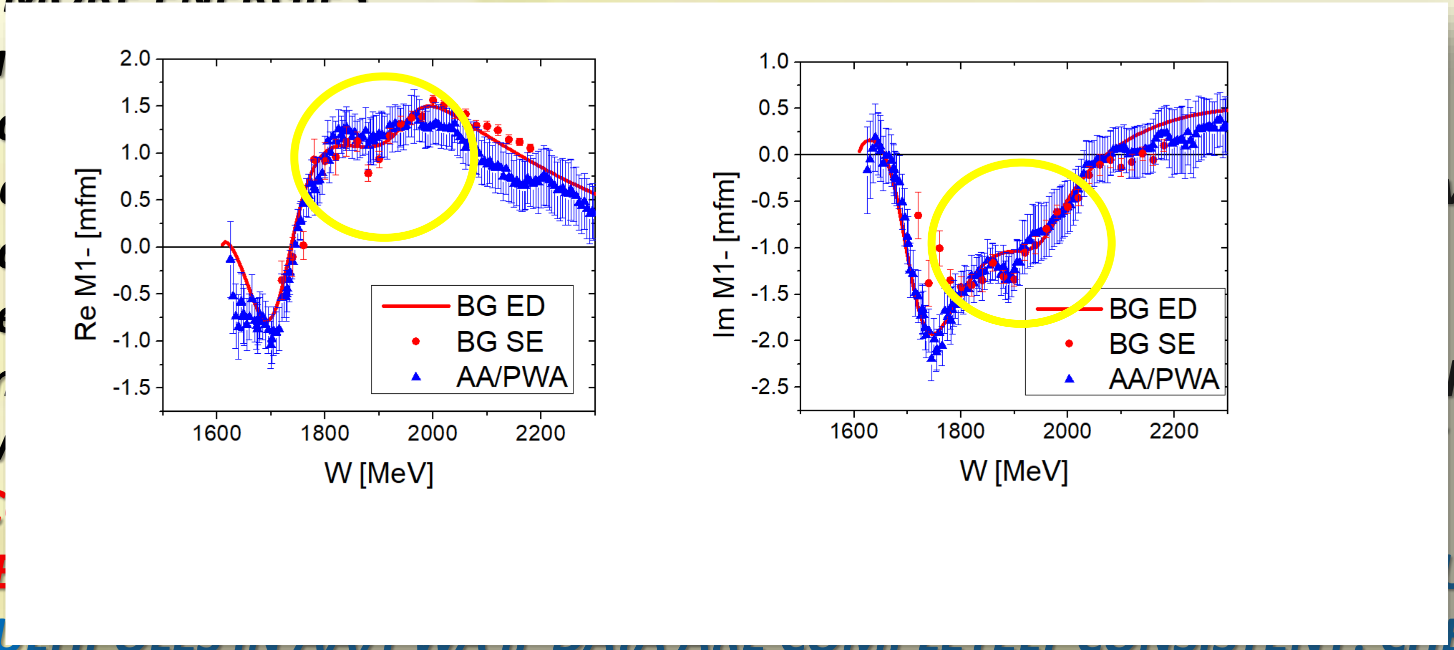






Results and conclusions

1. AA/PWA reliably reproduces the complete data base for $K^+\Lambda$ too
2. The solution is consistent with Bonn-Gatchina SE PWA, but CAN BE DONE AT MUCH MORE ENERGIES
3. Structu
4. Multip
   - Th vables (fr
   - eve
5. If minin hases, AA/PW ON THE DATA C
   (RE UM MU ECKED FOR PSEUDO AND NUMERIC DATA IN η-PHOTOPRODUCTION.)



Working with real data is complicated!

Future

If we want to avoid discontinuities, and work with real data, we have TO INCREASE MODEL dependence by introducing fixed-t analyticity!

Alternative to fixed-t analyticity is FIXED AMPLITUDE ANALYTICITY as a constrained, but it yet has to be developed!