The $(g-2)_\mu$ in the Standard Model: review of the calculation of hadronic contributions

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FOR FUNDAMENTAL PHYSICS

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Outline

Introduction: $(g-2)_\mu$ in the Standard Model

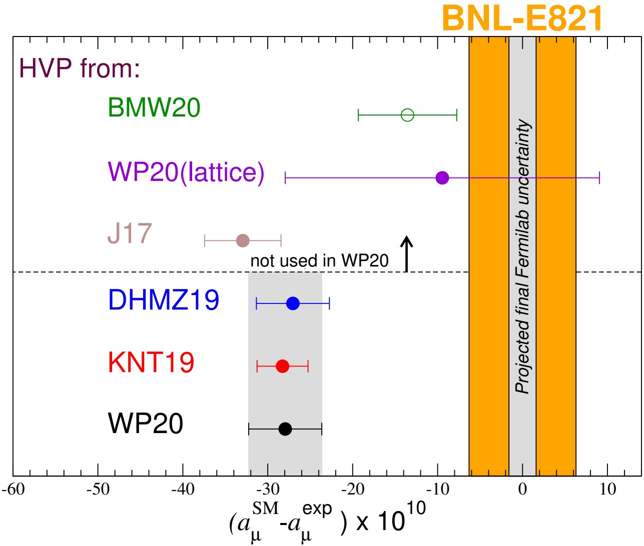
Hadronic Vacuum Polarization contribution to $(g-2)_\mu$

Hadronic light-by-light contribution to $(g-2)_\mu$

Conclusions and Outlook

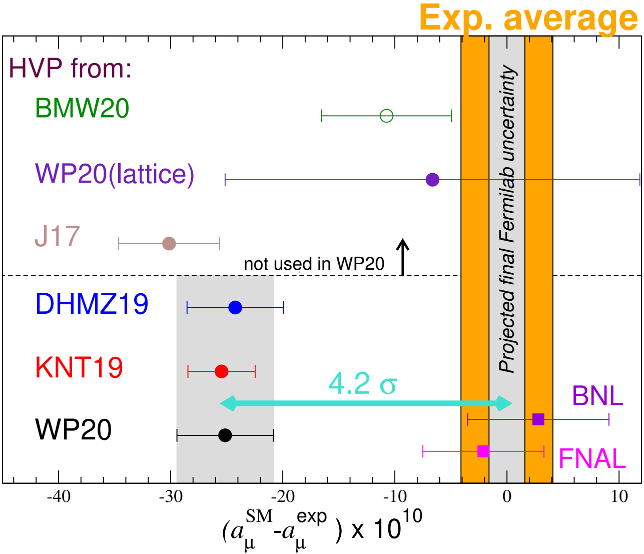
Present status of $(g-2)_\mu$, experiment vs SM

Before the Fermilab result



Present status of $(g-2)_\mu$, experiment vs SM

After the Fermilab result



Present status of $(g-2)_\mu$, experiment vs SM

$$a_\mu(BNL) = 116\,592\,089(63) \times 10^{-11}$$

$$a_\mu(FNAL) = 116\,592\,040(54) \times 10^{-11}$$

$$a_\mu(Exp) = 116\,592\,061(41) \times 10^{-11}$$

White Paper (2020): $(g-2)_\mu$, experiment vs SM

Contribution	Value $\times 10^{11}$
HVP LO (e^+e^-)	6931(40)
HVP NLO (e^+e^-)	−98.3(7)
HVP NNLO (e^+e^-)	12.4(1)
HVP LO (lattice , *udsc*)	7116(184)
HLbL (phenomenology)	92(19)
HLbL NLO (phenomenology)	2(1)
HLbL (lattice, *uds*)	79(35)
HLbL (phenomenology + lattice)	90(17)
QED	116 584 718.931(104)
Electroweak	153.6(1.0)
HVP (e^+e^-, LO + NLO + NNLO)	6845(40)
HLbL (phenomenology + lattice + NLO)	92(18)
Total SM Value	116 591 810(43)
Experiment	116 592 061(41)
Difference: $\Delta a_\mu := a_\mu^{\text{exp}} - a_\mu^{\text{SM}}$	251(59)

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HVP LO (lattice BMW(20), *udsc*)	7075(55)
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White Paper (2020): $(g-2)_\mu$, experiment vs SM

White Paper:
T. Aoyama et al. Phys. Rep. 887 (2020) = WP(20)

Muon $g-2$ Theory Initiative
Steering Committee:
GC
Michel Davier (vice-chair)
Simon Eidelman
Aida El-Khadra (chair)
Martin Hoferichter
Christoph Lehner (vice-chair)
Tsutomu Mibe (J-PARC E34 experiment)
(*Andreas Nyffeler* until summer 2020)
Lee Roberts (Fermilab E989 experiment)
Thomas Teubner
Hartmut Wittig

White Paper (2020): $(g-2)_\mu$, experiment vs SM

White Paper:
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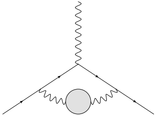
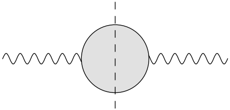
Muon $g-2$ Theory Initiative
Workshops:

- First plenary meeting, Q-Center (Fermilab), 3-6 June 2017
- HVP WG workshop, KEK (Japan), 12-14 February 2018
- HLbL WG workshop, U. of Connecticut, 12-14 March 2018
- Second plenary meeting, Mainz, 18-22 June 2018
- Third plenary meeting, Seattle, 9-13 September 2019
- Lattice HVP workshop, virtual, 16-20 November 2020
- Fourth plenary meeting, KEK (virtual), 28 June-02 July 2021

Theory uncertainty comes from hadronic physics

- Hadronic contributions responsible for most of the theory uncertainty
- Hadronic vacuum polarization (HVP) is $\mathcal{O}(\alpha^2)$, dominates the total uncertainty, despite being known to $< 1\%$



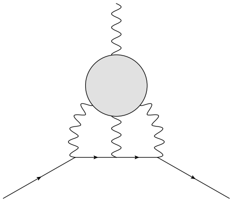


- unitarity and analyticity $\Rightarrow$ dispersive approach
- $\Rightarrow$ direct relation to experiment: $\sigma_{\text{tot}}(e^+e^- \to \text{hadrons})$
- e^+e^- Exps: BaBar, Belle, BESIII, CMD2/3, KLOE2, SND
- alternative approach: lattice, becoming competitive

(BMW, ETMC, Fermilab, HPQCD, Mainz, MILC, RBC/UKQCD)

Theory uncertainty comes from hadronic physics

- Hadronic contributions responsible for most of the theory uncertainty
- Hadronic vacuum polarization (HVP) is $\mathcal{O}(\alpha^2)$, dominates the total uncertainty, despite being known to $< 1\%$
- Hadronic light-by-light (HLbL) is $\mathcal{O}(\alpha^3)$, known to $\sim 20\%$, second largest uncertainty (now subdominant)

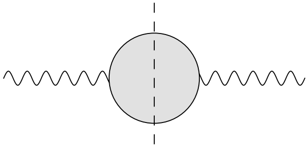


- 4-point fct. of em currents in QCD: more complicated than HVP
- recently: dispersive approach $\Rightarrow$ data-driven, systematic treatment
- lattice QCD is becoming competitive

(Mainz, RBC/UKQCD)

HVP contribution: Master Formula

Unitarity relation: simple, same for all intermediate states



$$\mathrm{Im}\Pi(q^2) \propto \sigma(e^+e^- \to \text{hadrons}) = \sigma(e^+e^- \to \mu^+\mu^-)R(q^2)$$

Analyticity $\Rightarrow$ Master formula for HVP

Bouchiat, Michel (61)

$$a_\mu^{\text{hvp}} = \frac{\alpha^2}{3\pi^2}\int_{s_{th}}^{\infty}\frac{ds}{s}K(s)R(s)$$

$K(s)$ known, depends on m_μ and $K(s) \sim \frac{1}{s}$ for large s

Comparison between DHMZ19 and KNT19

	DHMZ19	KNT19	Difference
$\pi^+\pi^-$	507.85(0.83)(3.23)(0.55)	504.23(1.90)	3.62
$\pi^+\pi^-\pi^0$	46.21(0.40)(1.10)(0.86)	46.63(94)	-0.42
$\pi^+\pi^-\pi^+\pi^-$	13.68(0.03)(0.27)(0.14)	13.99(19)	-0.31
$\pi^+\pi^-\pi^0\pi^0$	18.03(0.06)(0.48)(0.26)	18.15(74)	-0.12
K^+K^-	23.08(0.20)(0.33)(0.21)	23.00(22)	0.08
K_SK_L	12.82(0.06)(0.18)(0.15)	13.04(19)	-0.22
$\pi^0\gamma$	4.41(0.06)(0.04)(0.07)	4.58(10)	-0.17
Sum of the above	626.08(0.95)(3.48)(1.47)	623.62(2.27)	2.46
$[1.8, 3.7]$ GeV (without $c\bar{c}$)	33.45(71)	34.45(56)	-1.00
J/ψ, $\psi(2S)$	7.76(12)	7.84(19)	-0.08
$[3.7, \infty)$ GeV	17.15(31)	16.95(19)	0.20
Total $a_\mu^{\text{HVP, LO}}$	$694.0(1.0)(3.5)(1.6)(0.1)_\psi(0.7)_{\text{DV+QCD}}$	692.8(2.4)	1.2

DHMZ = Davier, Hoecker, Malaescu, Zhang, KNT = Keshavarzi, Nomura, Teubner

2π: comparison with the dispersive approach

The 2π channel can itself be described dispersively $\Rightarrow$ more constrained theoretically Ananthanarayan, Caprini, Das (19), GC, Hoferichter, Stoffer (18)

Energy range	ACD18	CHS18	DHMZ19	KNT19
≤ 0.6 GeV		110.1(9)	110.4(4)(5)	108.7(9)
≤ 0.7 GeV		214.8(1.7)	214.7(0.8)(1.1)	213.1(1.2)
≤ 0.8 GeV		413.2(2.3)	414.4(1.5)(2.3)	412.0(1.7)
≤ 0.9 GeV		479.8(2.6)	481.9(1.8)(2.9)	478.5(1.8)
≤ 1.0 GeV		495.0(2.6)	497.4(1.8)(3.1)	493.8(1.9)
[0.6, 0.7] GeV		104.7(7)	104.2(5)(5)	104.4(5)
[0.7, 0.8] GeV		198.3(9)	199.8(0.9)(1.2)	198.9(7)
[0.8, 0.9] GeV		66.6(4)	67.5(4)(6)	66.6(3)
[0.9, 1.0] GeV		15.3(1)	15.5(1)(2)	15.3(1)
≤ 0.63 GeV	132.9(8)	132.8(1.1)	132.9(5)(6)	131.2(1.0)
[0.6, 0.9] GeV		369.6(1.7)	371.5(1.5)(2.3)	369.8(1.3)
$[\sqrt{0.1}, \sqrt{0.95}]$ GeV		490.7(2.6)	493.1(1.8)(3.1)	489.5(1.9)

WP(20)

Combination method and final result

Complete analyses DHMZ19 and KNT19, as well as CHS19 (2π) and HHK19 (3π), have been so combined:

- central values are obtained by simple averages (for each channel and mass range)
- the largest experimental and systematic uncertainty of DHMZ and KNT is taken
- $1/2$ difference DHMZ$-$KNT (or BABAR$-$KLOE in the 2π channel, if larger) is added to the uncertainty

Final result:

$$a_\mu^{\text{HVP, LO}} = 693.1(2.8)_{\text{exp}}(2.8)_{\text{sys}}(0.7)_{\text{DV+QCD}} \times 10^{-10}$$
$$= 693.1(4.0) \times 10^{-10}$$

WP(20)

The BMW result

Borsanyi et al. Nature 2021

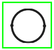
State-of-the-art lattice calculation of $a_\mu^{\text{HVP, LO}}$ based on

- current-current correlator, summed over all distances, integrated in time with appropriate kernel function
- using staggered fermions on an $L \sim 6$ fm lattice ($L \sim 11$fm used for finite volume corrections)
- at (and around) physical quark masses
- including isospin-breaking effects

The BMW result

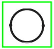
Borsanyi et al. Nature 2021

Isospin-symmetric



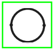
Connected light

$633.7(2.1)_{stat}(4.2)_{syst}$



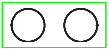
Connected strange

$53.393(89)_{stat}(68)_{syst}$



Connected charm

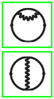
$14.6(0)_{stat}(1)_{syst}$



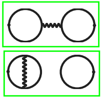
Disconnected

$-13.36(1.18)_{stat}(1.36)_{syst}$

QED isospin breaking: valence

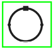


Connected $-1.23(40)_{stat}(31)_{syst}$



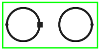
Disconnected $-0.55(15)_{stat}(10)_{syst}$

Strong–isospin breaking



Connected

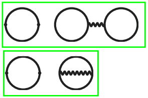
$6.60(63)_{stat}(53)_{syst}$



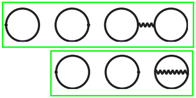
Disconnected

$-4.67(54)_{stat}(69)_{syst}$

QED isospin breaking: sea



Connected $0.37(21)_{stat}(24)_{syst}$



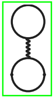
Disconnected $-0.040(33)_{stat}(21)_{syst}$

Other

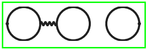
Bottom; higher-order; perturbative

$0.11(4)_{tot}$

QED isospin breaking: mixed



Connected $-0.0093(86)_{stat}(95)_{syst}$



Disconnected $0.011(24)_{stat}(14)_{syst}$

Finite-size effects

Isospin-symmetric

$18.7(2.5)_{tot}$

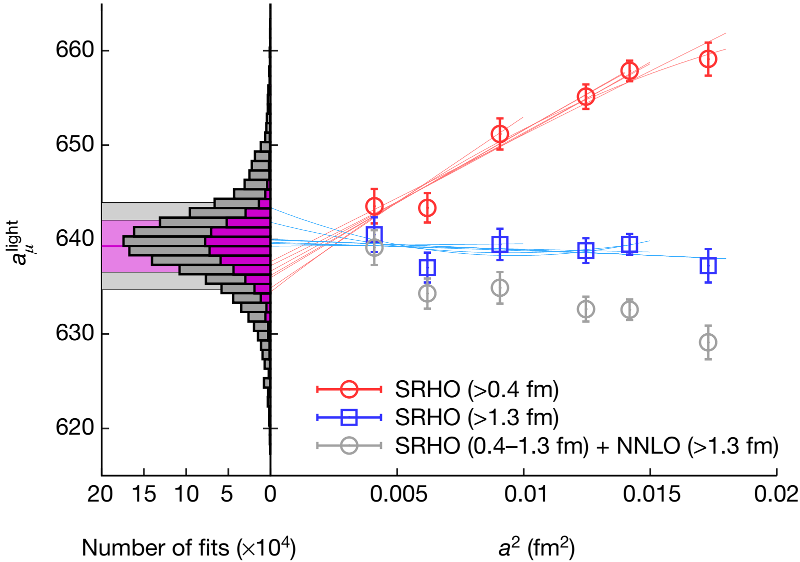
Isospin-breaking

$0.0(0.1)_{tot}$

$$a_\mu^{\text{LO-HVP}}\ (\times 10^{10}) = 707.5(2.3)_{stat}(5.0)_{syst}(5.5)_{tot}$$

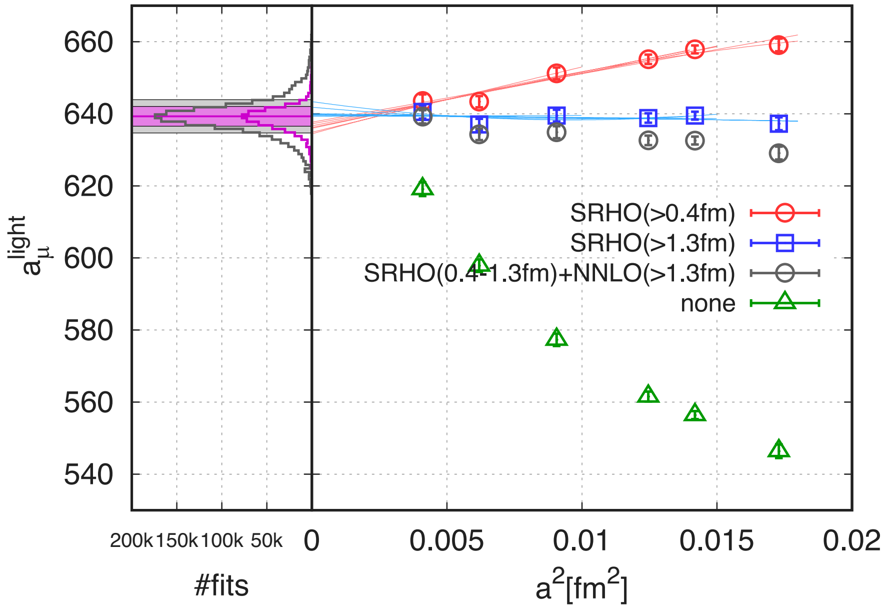
The BMW result

Borsanyi et al. Nature 2021



The BMW result

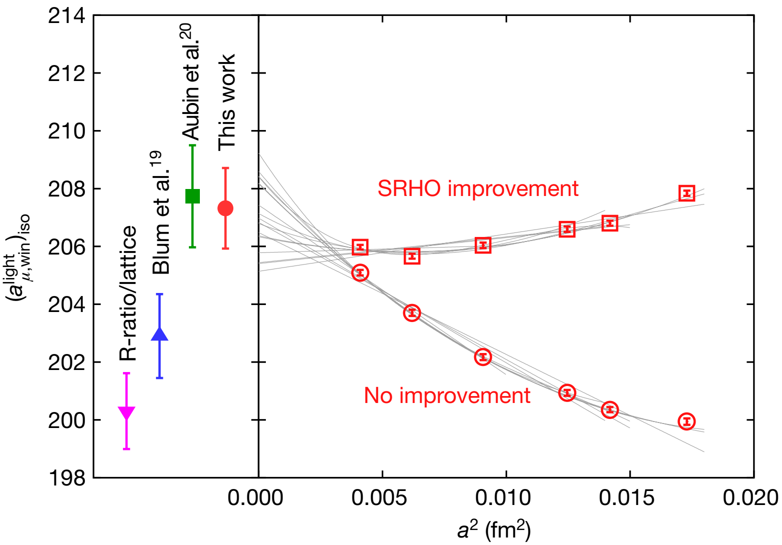
Borsanyi et al. Nature 2021



The BMW result

Borsanyi et al. Nature 2021

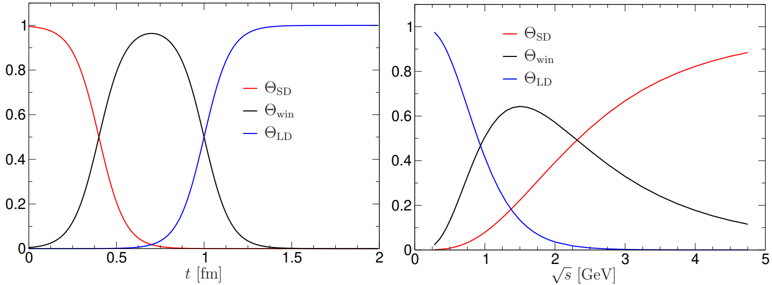
Article



The BMW result

Borsanyi et al. Nature 2021

Weight functions for window quantities

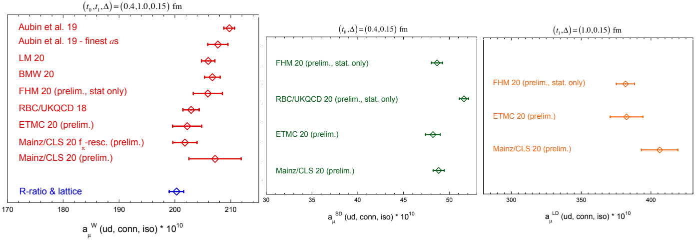


The BMW result

Borsanyi et al. Nature 2021

Summary: *ud* contribution

f	$a_\mu^{SD}(f) \cdot 10^{10}$	$a_\mu^{W}(f) \cdot 10^{10}$	$a_\mu^{LD}(f) \cdot 10^{10}$
ud	48.2 (0.8)	202.2 (2.6)	382.5 (11.7)



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D. Giusti, talk at Lattice 2021

Consequences of the BMW result

A shift in the value of $a_\mu^{\text{HVP, LO}}$ would have consequences:

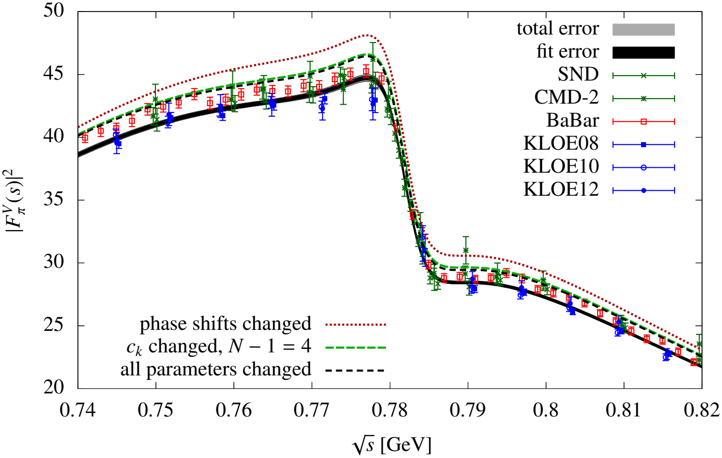
- $\Delta a_\mu^{\text{HVP, LO}} \quad \Leftrightarrow \quad \Delta\sigma(e^+e^- \to \text{hadrons})$
- $\Delta\alpha_{\text{had}}(M_Z^2)$ is determined by an integral of the same $\sigma(e^+e^- \to \text{hadrons})$ (more weight at high energy)
- changing $a_\mu^{\text{HVP, LO}}$ necessarily implies a shift in $\Delta\alpha_{\text{had}}(M_Z^2)$:
  $\Rightarrow$ impact on the EW-fit
- to save the EW-fit $\Delta\sigma(e^+e^- \to \text{hadrons})$ must occur below ~ 1 (max 2) GeV

  Crivellin, Hoferichter, Manzari, Montull (20)/Keshavarzi, Marciano, Passera, Sirlin (20)/Malaescu, Schott (20)
- or the need for BSM physics would be moved elsewhere

Changes in $\sigma(e^+e^- \to \text{hadrons})$ below 1 GeV?

- Below 1 – 2 GeV only one significant channel: $\pi^+\pi^-$
- Strongly constrained by analyticity and unitarity ($F_\pi^V(s)$)
- $F_\pi^V(s)$ parametrization which satisfies these
  $\Rightarrow$ small number of parameters — GC, Hoferichter, Stoffer (18)
- $\Delta a_\mu^{\text{HVP, LO}} \quad \Leftrightarrow \quad$ shifts in these parameters
  analysis of the corresponding scenarios — GC, Hoferichter, Stoffer (21)

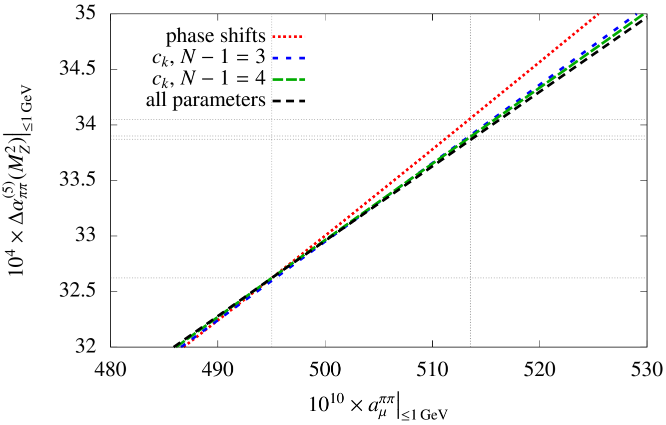
Changes in $\sigma(e^+e^- \to \text{hadrons})$ below 1 GeV?



GC, Hoferichter, Stoffer (21)

Tension [BMW20 vs e^+e^- data] stronger for KLOE than for BABAR

Changes in $\sigma(e^+e^- \to \text{hadrons})$ below 1 GeV?



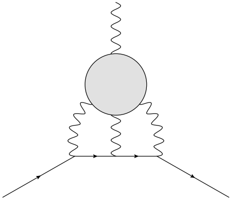
GC, Hoferichter, Stoffer (21)

$$10^4 \Delta\alpha^{(5)}_{\text{had}}(M_Z^2) = \begin{cases} 272.2(4.1) & \text{EW fit} \\ 276.1(1.1) & \sigma_{\text{had}}(s) \end{cases}$$

Calculating the HLbL contribution

Calculating the HLbL contribution is complicated

- 4-point function of em currents in QCD



- a data-driven approach like for HVP seemed hopeless but has been recently developed and used

  GC, Hoferichter, Procura, Stoffer=CHPS (14,15,17), Hoferichter, Hoid, Kubis, Leupold, Schneider (18)

- lattice QCD is an alternative and is making fast progress

  RBC/UKQCD (20), Mainz (19,20)

HLbL contribution: Master Formula

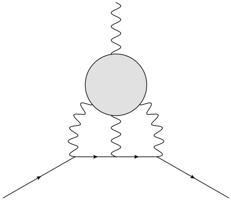
$$a_\mu^{\mathrm{HLbL}} = \frac{2\alpha^3}{48\pi^2}\int_0^\infty dQ_1 \int_0^\infty dQ_2 \int_{-1}^{1} d\tau \sqrt{1-\tau^2}\sum_{i=1}^{12} T_i(Q_1,Q_2,\tau)\bar{\Pi}_i(Q_1,Q_2,\tau)$$

Q_i^μ are the Wick-rotated four-momenta and τ the four-dimensional angle between Euclidean momenta:

$$Q_1 \cdot Q_2 = |Q_1||Q_2|\tau$$

The integration variables $Q_1 := |Q_1|$, $Q_2 := |Q_2|$. CHPS (15)

- T_i: known kernel functions
- $\bar{\Pi}_i$ are amenable to a dispersive treatment: their imaginary parts are related to measurable subprocesses



Improvements obtained with the dispersive approach

Contribution	PdRV(09) *Glasgow consensus*	N/JN(09)	J(17)	WP(20)
π^0, η, η'-poles	114(13)	99(16)	95.45(12.40)	93.8(4.0)
π, K-loops/boxes	−19(19)	−19(13)	−20(5)	−16.4(2)
S-wave $\pi\pi$ rescattering	−7(7)	−7(2)	−5.98(1.20)	−8(1)
subtotal	88(24)	73(21)	69.5(13.4)	69.4(4.1)
scalars	−	−	−	} − 1(3)
tensors	−	−	1.1(1)	
axial vectors	15(10)	22(5)	7.55(2.71)	6(6)
u, d, s-loops / short-distance	−	21(3)	20(4)	15(10)
c-loop	2.3	−	2.3(2)	3(1)
total	105(26)	116(39)	100.4(28.2)	92(19)

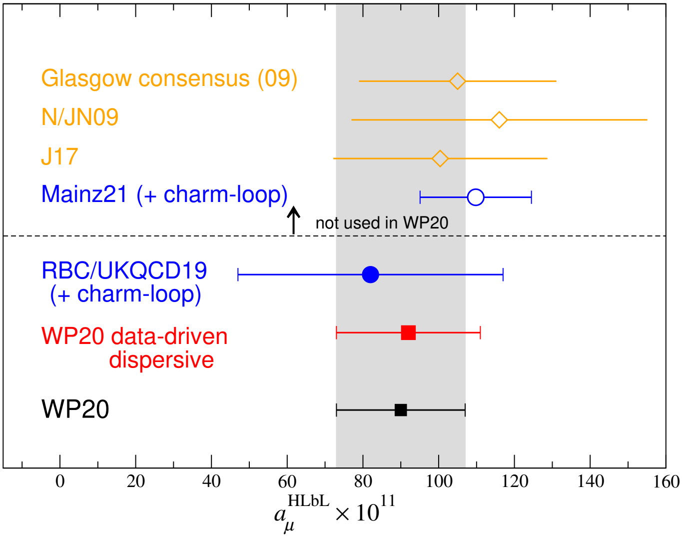
- significant reduction of uncertainties in the first three rows: low-energy region well constrained by a dispersive approach

  CHPS (17), Masjuan, Sánchez-Puertas (17) Hoferichter, Hoid et al. (18), Gerardin, Meyer, Nyffeler (19)

- 1 − 2 GeV and asymptotic region (short distance constraints) have been improved, but still work in progress (see WP(20))

  Melnikov, Vainshtein (04), (.......), Bijnens, Hermansson-Truedsson, Laub, Rodríguez-Sánchez (20,21)

Situation for HLbL



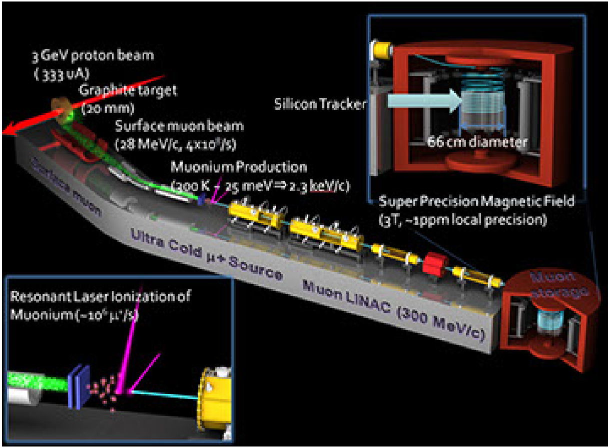
Conclusions

- The WP provides the current status of the SM evaluation of $(g-2)_\mu$: 4.2σ discrepancy with experiment (w/ FNAL)
- Evaluation of the HVP contribution based on the dispersive approach: 0.6% error $\Rightarrow$ dominates the theory uncertainty
- Recent lattice calculation [BMW(20)] has reached a similar precision but differs from the dispersive one (=from e^+e^- data).
  If confirmed $\Rightarrow$ discrepancy with experiment $\searrow$ below 2σ
- Evaluation of the HLbL contribution based on the dispersive approach: 20% accuracy. Two recent lattice calculations [RBC/UKQCD(20), Mainz(21)] agree with it

Outlook

- The Fermilab experiment aims to reduce the BNL uncertainty by a factor four $\Rightarrow$ potential 7σ discrepancy
- Improvements on the SM theory side:
  - HVP data-driven:
    Other e^+e^- experiments are available or forthcoming:
    SND, BaBar, Belle II, BESIII, CMD3 $\Rightarrow$ Error reduction
    MuonE will provide an alternative way to measure HVP
  - HVP lattice:
    More calculations w/ precision $\sim$ BMW are awaited
    Difference to data-driven evaluation must be understood
  - HLbL data-driven: goal of $\sim$ 10% uncertainty within reach
  - HLbL lattice: RBC/UKQCD $\Rightarrow$ similar precision as Mainz.
    Good agreement with data-driven evaluation.

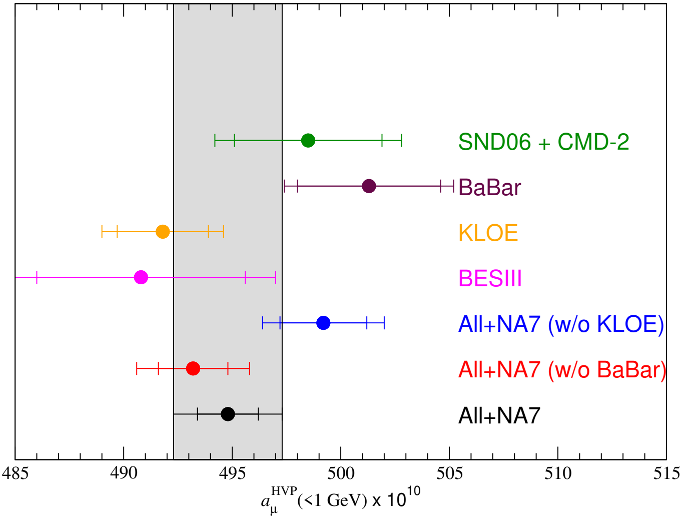
Future: Muon $g - 2$/EDM experiment @ J-PARC



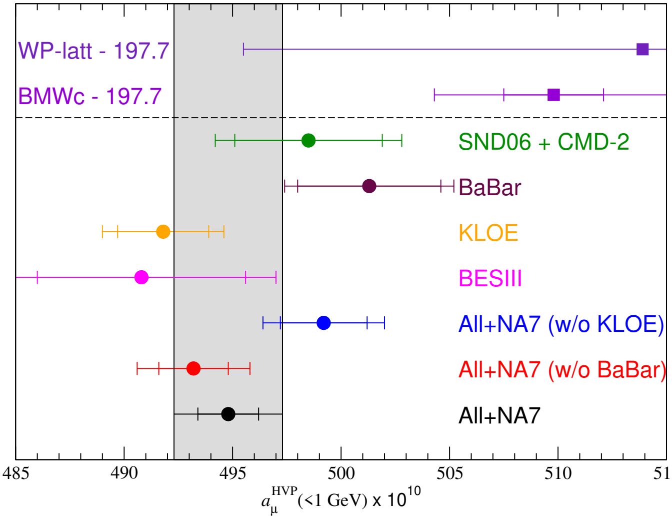
Credit: J-PARC

Backup Slides

BMW vs individual $\pi^+\pi^-$ experiments

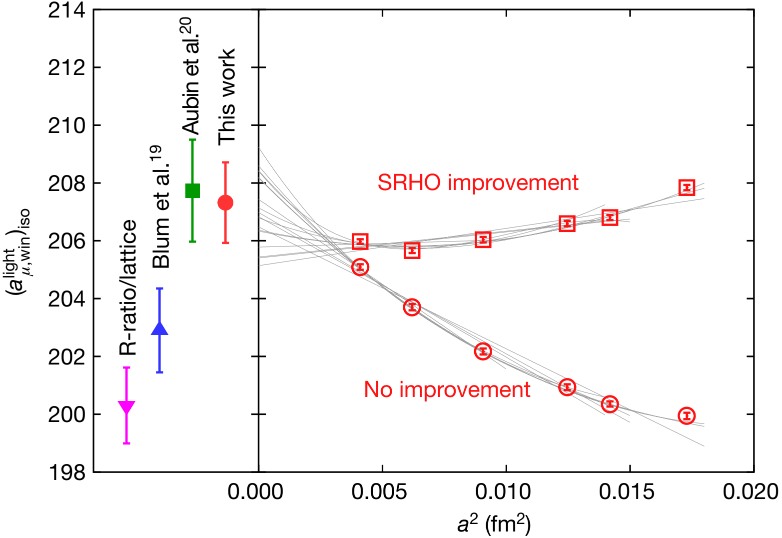


BMW vs individual $\pi^+\pi^-$ experiments



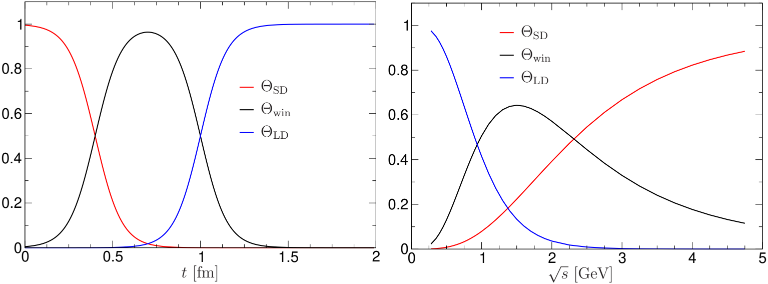
BMW vs individual $\pi^+\pi^-$ experiments

Article

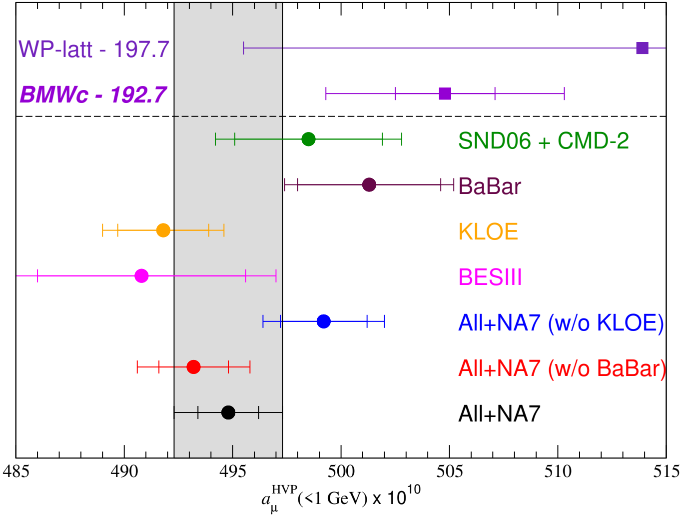


BMW vs individual $\pi^+\pi^-$ experiments

Weight functions for the window quantities



BMW vs individual $\pi^+\pi^-$ experiments



$a_\mu^{\rm win}$ suggests that $\sim 5 \times 10^{-10}$ must come from above 1 GeV

GC, Hoferichter, Stoffer