Light-Meson spectroscopy at COMPASS

— Studies of the $\pi^-\pi^-\pi^+$ Background in $K^-\pi^-\pi^+$ Data —

Stefan Wallner
(stefan.wallner@tum.de)

Institute for Hadronic Structure and Fundamental Symmetries - Technical University of Munich

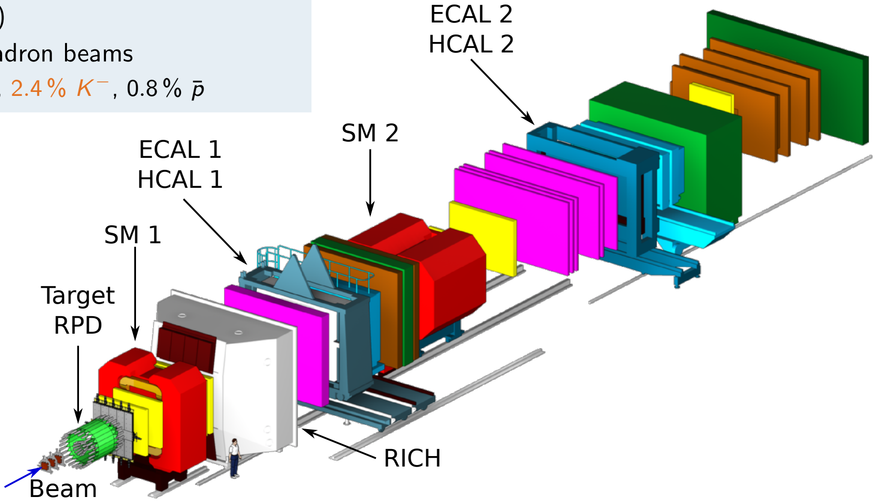
International Workshop on Partial Wave Analyses and Advanced Tools for Hadron Spectroscopy
(PWA12/ATHOS7)
September 9, 2021

COMPASS Setup for Hadron Beams

[COMPASS, Nucl. Instrum. Methods **779** (2015) 69]

M2 beam line

- Located at CERN (SPS)
- 190 GeV/c secondary hadron beams
  - h^- beams: 97 % π^-, 2.4 % K^-, 0.8 % $\bar{p}$

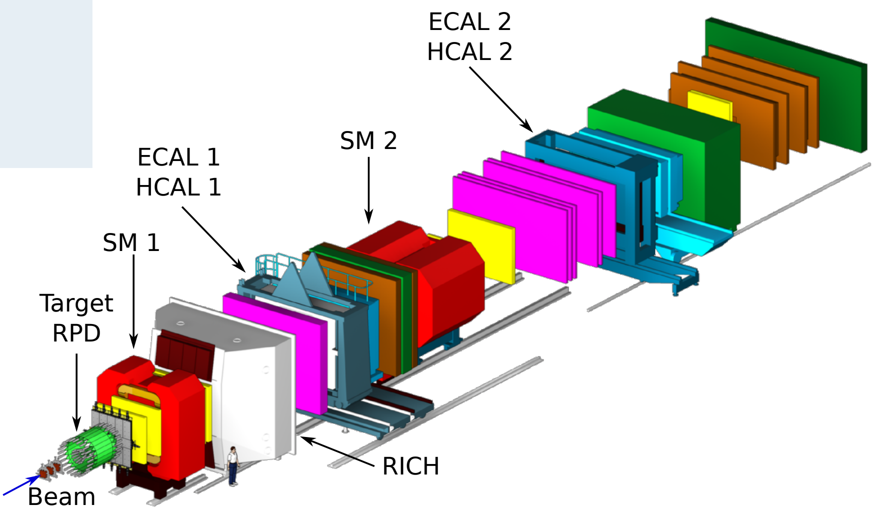


COMPASS Setup for Hadron Beams

[COMPASS, Nucl. Instrum. Methods **779** (2015) 69]

Target

- Various targets:
  - ℓH_2
  - Ni
  - Pb
  - W

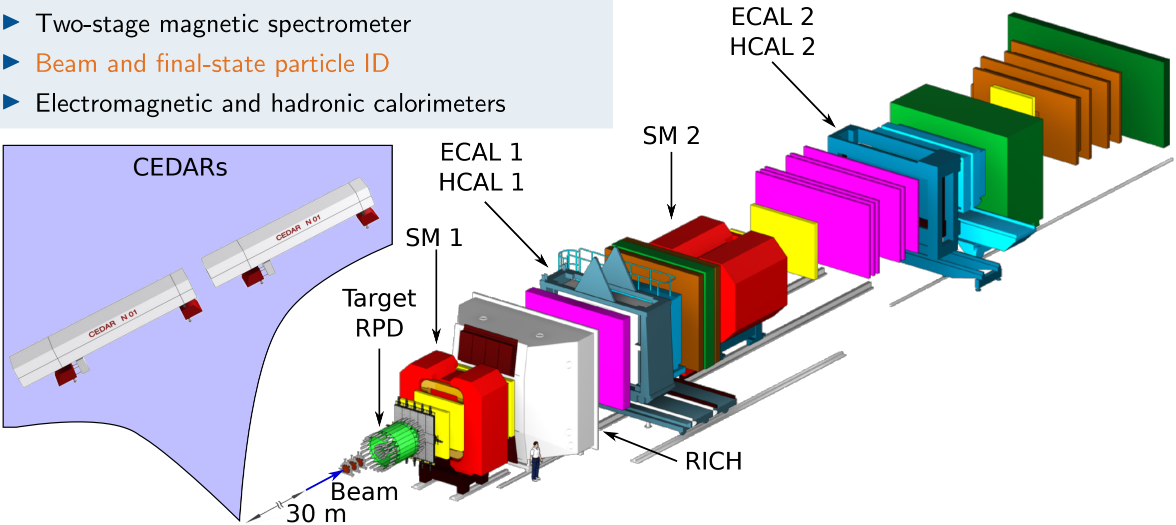


COMPASS Setup for Hadron Beams

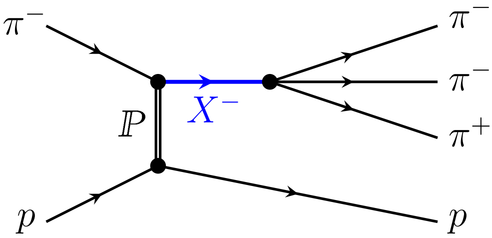
[COMPASS, Nucl. Instrum. Methods **779** (2015) 69]

COMPASS spectrometer

- Two-stage magnetic spectrometer
- Beam and final-state particle ID
- Electromagnetic and hadronic calorimeters

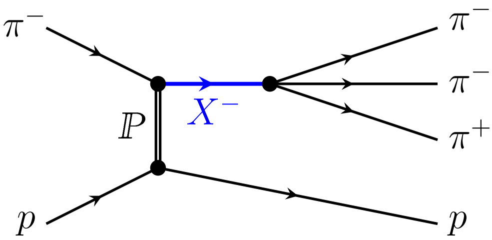


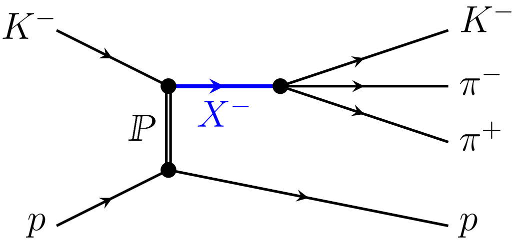
Diffractive Production



- Diffractive production in high-energy scattering
- Light mesons appear as intermediate states X^-
- Observed in decays into quasi-stable particles:
  - $\pi^-\pi^-\pi^+$ final state: Access to a_J and π_J states
  - $K^-\pi^-\pi^+$ final state: Access to K_J and K_J^* states

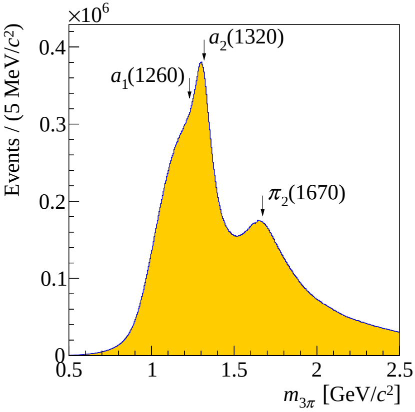
Diffractive Production

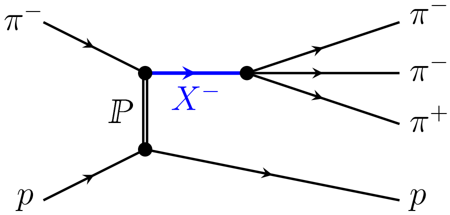




- Diffractive production in high-energy scattering
- Light mesons appear as intermediate states X^-
- Observed in decays into quasi-stable particles:
  - $\pi^-\pi^-\pi^+$ final state: Access to a_J and π_J states
  - $K^-\pi^-\pi^+$ final state: Access to K_J and K_J^* states

The $\pi^-\pi^-\pi^+$ Final State from COMPASS



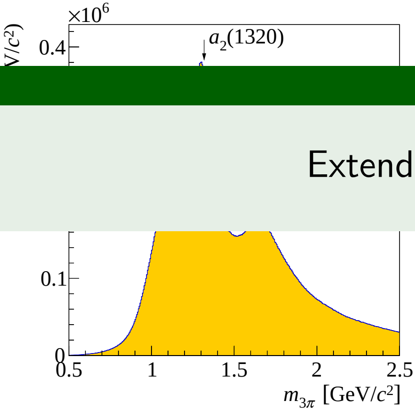


- Largest data set of about 50 M events
- Rich spectrum of overlapping and interfering X^-

Most comprehensive study of $\pi^-\pi^-\pi^+$

- Sophisticated partial-wave decomposition
  [COMPASS, Phys. Rev. D **95** (2015) 032004]
- Extensive resonance-model fit
  [COMPASS, Phys. Rev. D **98** (2018) 092003]
- Fit of triangle amplitude to COMPASS data
  [COMPASS, arXiv:2006.05342 (2021)]
- Study of spin-exotic $\pi_1(1600)$
  [COMPASS, arXiv:2108.01744 (2021)]

The $\pi^-\pi^-\pi^+$ Final State from COMPASS



See talk by F. Kaspar

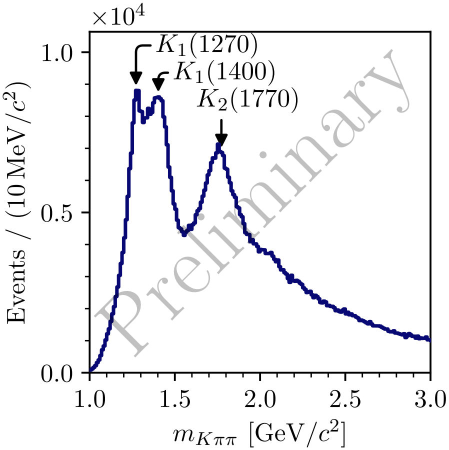
Extended analysis ongoing

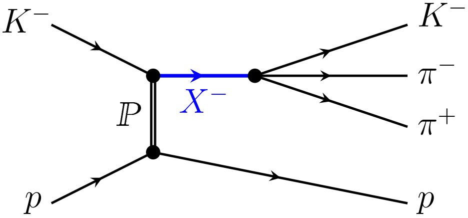
- Largest data set of about 50 M events
- Rich spectrum of overlapping and interfering X^-

Most comprehensive study of $\pi^-\pi^-\pi^+$

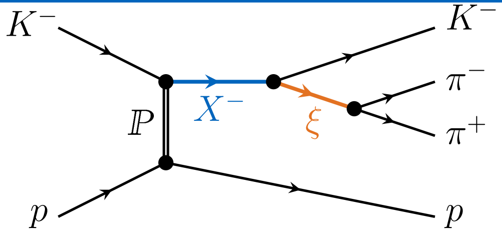
- Sophisticated partial-wave decomposition
  [COMPASS, Phys. Rev. D **95** (2015) 032004]
- Extensive resonance-model fit
  [COMPASS, Phys. Rev. D **98** (2018) 092003]
- Fit of triangle amplitude to COMPASS data
  [COMPASS, arXiv:2006.05342 (2021)]
- Study of spin-exotic $\pi_1(1600)$
  [COMPASS, arXiv:2108.01744 (2021)]

The $K^-\pi^-\pi^+$ Final State from COMPASS





- World's largest data set of about 720 k events
- Rich spectrum of overlapping and interfering X^-
  - Dominant well known states
  - States with lower intensity are "hidden"

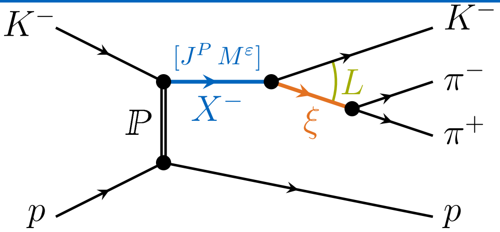


- Partial wave $a = J^{P(C)}\, M^\varepsilon\, \xi^0\, b^-\, L$ at fixed invariant mass of X^- system
  - ➥ Calculate 5D decay phase-space distribution $\psi_a(\tau)$ of final state
- Total intensity distribution: Coherent sum of partial-wave amplitudes

$$\mathcal{I}(\tau) = \left| \sum_a^{\text{waves}} \mathcal{T}_a \psi_a(\tau) \right|^2$$

- Perform maximum-likelihood fit in cells of $(m_{K\pi\pi}, t')$
  - ➥ Extract mass and t' dependence of transition amplitudes $\mathcal{T}_a$

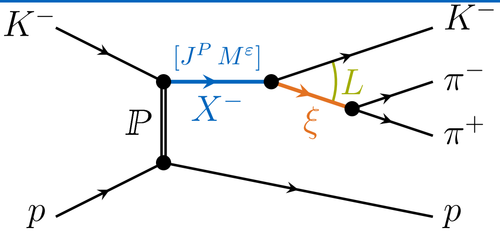
Partial-Wave Decomposition
Isobar Model



- Partial wave $a = J^{P(C)}\,M^\varepsilon\,\xi^0\,b^-\,L$ at fixed invariant mass of X^- system
  - ➥ Calculate 5D decay phase-space distribution $\Psi(\tau)$ of final state
- Total intensity distribution: Coherent sum of partial-wave amplitudes

$$\mathcal{I}(\tau) = \left|\sum_a^{\text{waves}} \mathcal{T}_a\,\Psi_a(\tau)\right|^2$$

- Perform maximum-likelihood fit in cells of $(m_{K\pi\pi}, t')$
  - ➥ Extract mass and t' dependence of transition amplitudes $\mathcal{T}_a$



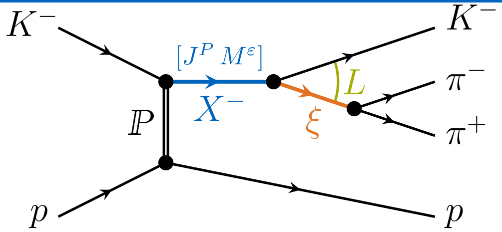
- Partial wave $a = J^{P(C)}\ M^\varepsilon\ \xi^0\ b^-\ L$ at fixed invariant mass of X^- system
  - ➥ Calculate 5D decay phase-space distribution $\Psi(\tau)$ of final state
- Total intensity distribution: Coherent sum of partial-wave amplitudes

$$\mathcal{I}(\tau) = \left| \sum_a^{\text{waves}} \mathcal{T}_a \Psi_a(\tau) \right|^2$$

- Perform maximum-likelihood fit in cells of $(m_{K\pi\pi}, t')$
  - ➥ Extract mass and t' dependence of transition amplitudes $\mathcal{T}_a$

Partial-Wave Decomposition

Isobar Model



- Partial wave $a = J^{P(C)}\ M^\varepsilon\ \xi^0\ b^-\ L$ at fixed invariant mass of X^- system
  - ➥ Calculate 5D decay phase-space distribution $\Psi(\tau)$ of final state
- Total intensity distribution: Coherent sum of partial-wave amplitudes

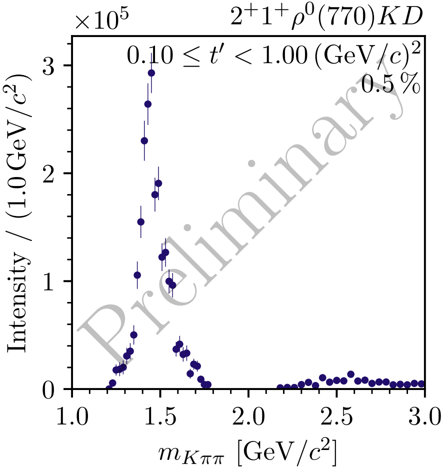
$$\mathcal{I}(\tau) = \left| \sum_a^{\text{waves}} \mathcal{T}_a \Psi_a(\tau) \right|^2$$

- Perform maximum-likelihood fit in cells of $(m_{K\pi\pi}, t')$
  - ➥ Extract mass and t' dependence of transition amplitudes $\mathcal{T}_a$

Selected Results from the $K^-\pi^-\pi^+$ Final state

Partial waves with $J^P = 2^+$

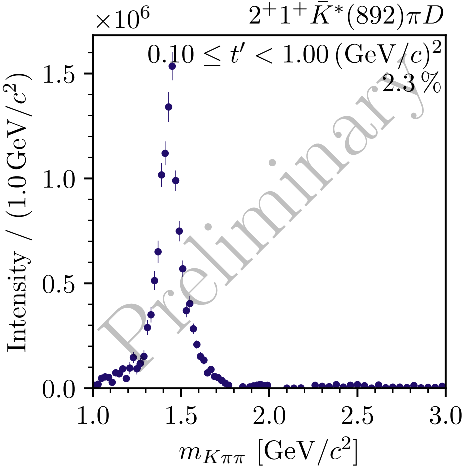
- Signal in $K_2^*(1430)$ mass region
- In Different decays
  - $\rho(770)\,K\,D$
  - $K^*(892)\,\pi\,D$
- In agreement with previous measurement



Selected Results from the $K^-\pi^-\pi^+$ Final state

Partial waves with $J^P = 2^+$

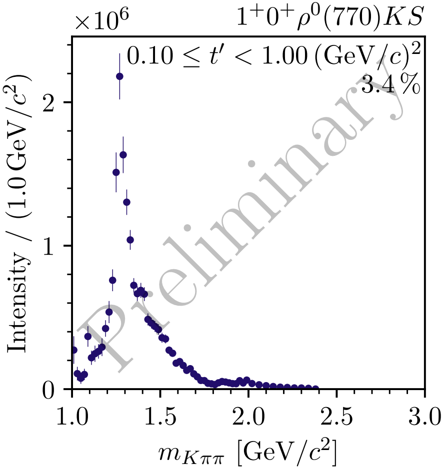
- Signal in $K_2^*(1430)$ mass region
- In Different decays
  - $\rho(770)\,K\,D$
  - $K^*(892)\,\pi\,D$
- In agreement with previous measurement



Selected Results from the $K^-\pi^-\pi^+$ Final state

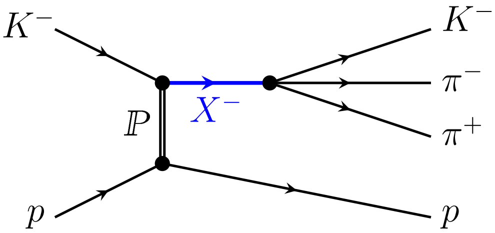
$1^+ 0^+ \rho(770) K S$ partial wave

- Dominated by $K_1(1270)$
- Small potential signal from $K_1(1650)$
- Consistent with previous observations



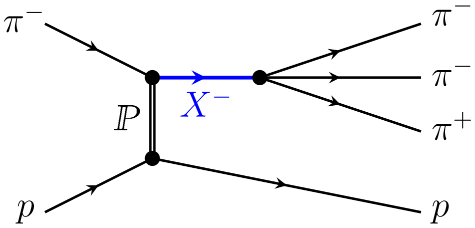
Studying the $\pi^-\pi^-\pi^+$ Background in $K^-\pi^-\pi^+$

- $K^-\pi^-\pi^+$ and $\pi^-\pi^-\pi^+$ similar experimental footprint
- Distinguishable only by
  - Beam particle identification
  - Final-state particle identification
- Excellent beam PID: Mis-ID $\pi^- \to K^-$ about 1 ‰
- But, about 35 times more π^- in beam
- Final-state PID does not suppress $\pi^-\pi^-\pi^+$ background
  - ➥ Non-negligible $\pi^-\pi^-\pi^+$ background in $K^-\pi^-\pi^+$ sample of about 7 %
  - ➥ Dominant background in $K^-\pi^-\pi^+$ sample



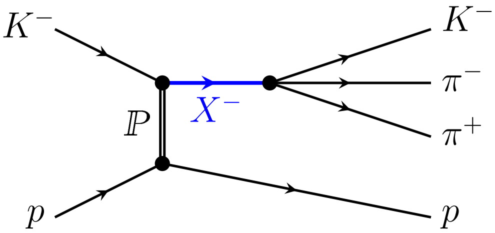
Studying the $\pi^-\pi^-\pi^+$ Background in $K^-\pi^-\pi^+$

- $K^-\pi^-\pi^+$ and $\pi^-\pi^-\pi^+$ similar experimental footprint
- Distinguishable only by
  - Beam particle identification
  - Final-state particle identification
- Excellent beam PID: Mis-ID $\pi^- \to K^-$ about 1 ‰
- But, about 35 times more π^- in beam
- Final-state PID does not suppress $\pi^-\pi^-\pi^+$ background
  - ➥ Non-negligible $\pi^-\pi^-\pi^+$ background in $K^-\pi^-\pi^+$ sample of about 7 %
  - ➥ Dominant background in $K^-\pi^-\pi^+$ sample



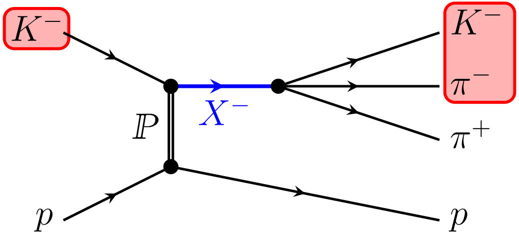
Studying the $\pi^-\pi^-\pi^+$ Background in $K^-\pi^-\pi^+$

- $K^-\pi^-\pi^+$ and $\pi^-\pi^-\pi^+$ similar experimental footprint
- Distinguishable only by
  - Beam particle identification
  - Final-state particle identification
- Excellent beam PID: Mis-ID $\pi^- \to K^-$ about 1 ‰
- But, about 35 times more π^- in beam
- Final-state PID does not suppress $\pi^-\pi^-\pi^+$ background
  - ➥ Non-negligible $\pi^-\pi^-\pi^+$ background in $K^-\pi^-\pi^+$ sample of about 7 %
  - ➥ Dominant background in $K^-\pi^-\pi^+$ sample



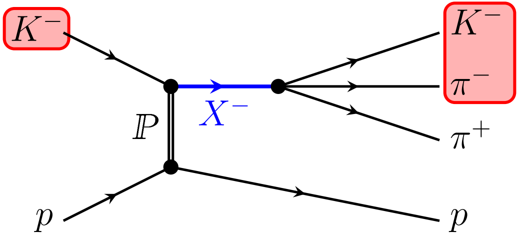
Studying the $\pi^-\pi^-\pi^+$ Background in $K^-\pi^-\pi^+$

- $K^-\pi^-\pi^+$ and $\pi^-\pi^-\pi^+$ similar experimental footprint
- Distinguishable only by
  - Beam particle identification
  - Final-state particle identification
- Excellent beam PID: Mis-ID $\pi^- \to K^-$ about 1 ‰
- But, about 35 times more π^- in beam
- Final-state PID does not suppress $\pi^-\pi^-\pi^+$ background
  - ➥ Non-negligible $\pi^-\pi^-\pi^+$ background in $K^-\pi^-\pi^+$ sample of about 7 %
  - ➥ Dominant background in $K^-\pi^-\pi^+$ sample



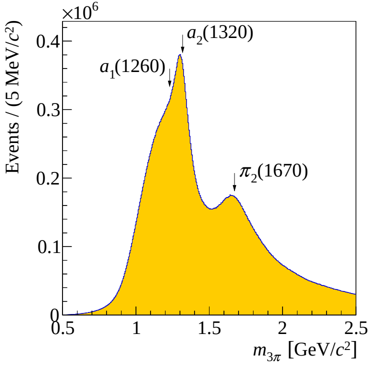
Studying the $\pi^-\pi^-\pi^+$ Background in $K^-\pi^-\pi^+$

- $K^-\pi^-\pi^+$ and $\pi^-\pi^-\pi^+$ similar experimental footprint
- Distinguishable only by
  - Beam particle identification
  - Final-state particle identification
- Excellent beam PID: Mis-ID $\pi^- \to K^-$ about 1 ‰
- But, about 35 times more π^- in beam
- Final-state PID does not suppress $\pi^-\pi^-\pi^+$ background
  - ➥ Non-negligible $\pi^-\pi^-\pi^+$ background in $K^-\pi^-\pi^+$ sample of about 7 %
  - ➥ Dominant background in $K^-\pi^-\pi^+$ sample



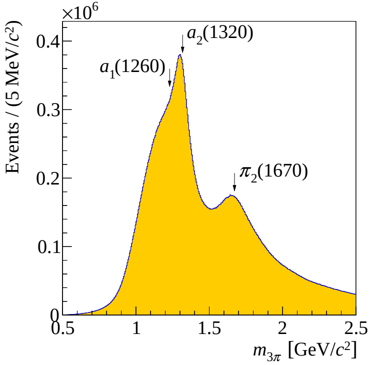
Studying the $\pi^-\pi^-\pi^+$ Background in $K^-\pi^-\pi^+$

- Well established model for $\pi^- + p \to \pi^-\pi^-\pi^+ + p$
  - From very same data set
  - Measured with high precision
  - Acceptance corrected
- Generate $\pi^-\pi^-\pi^+$ Monte Carlo sample
- Mis-interpret $\pi^-\pi^-\pi^+$ Monte Carlo events as $K^-\pi^-\pi^+$
  - Apply wrong mass assumption
  - Same event reconstruction
  - Same event selection
- Perform partial-wave decomposition of mis-interpreted $\pi^-\pi^-\pi^+$ Monte Carlo sample
  - Using the same PWA model as for measured $K^-\pi^-\pi^+$ sample



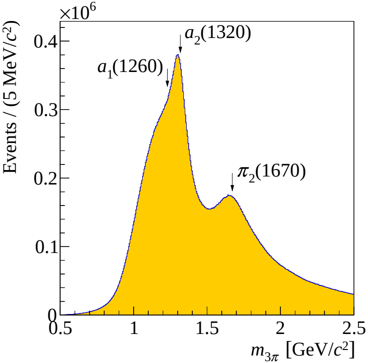
Studying the $\pi^-\pi^-\pi^+$ Background in $K^-\pi^-\pi^+$

- Well established model for $\pi^- + p \rightarrow \pi^-\pi^-\pi^+ + p$
  - From very same data set
  - Measured with high precision
  - Acceptance corrected
- Generate $\pi^-\pi^-\pi^+$ Monte Carlo sample
- Mis-interpret $\pi^-\pi^-\pi^+$ Monte Carlo events as $K^-\pi^-\pi^+$
  - Apply wrong mass assumption
  - Same event reconstruction
  - Same event selection
- Perform partial-wave decomposition of mis-interpreted $\pi^-\pi^-\pi^+$ Monte Carlo sample
  - Using the same PWA model as for measured $K^-\pi^-\pi^+$ sample



Studying the $\pi^-\pi^-\pi^+$ Background in $K^-\pi^-\pi^+$

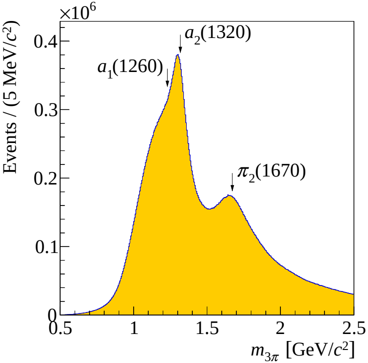
- Well established model for $\pi^- + p \to \pi^-\pi^-\pi^+ + p$
  - From very same data set
  - Measured with high precision
  - Acceptance corrected
- Generate $\pi^-\pi^-\pi^+$ Monte Carlo sample
- Mis-interpret $\pi^-\pi^-\pi^+$ Monte Carlo events as $K^-\pi^-\pi^+$
  - Apply wrong mass assumption
  - Same event reconstruction
  - Same event selection
- Perform partial-wave decomposition of mis-interpreted $\pi^-\pi^-\pi^+$ Monte Carlo sample
  - Using the same PWA model as for measured $K^-\pi^-\pi^+$ sample



Studying the $\pi^-\pi^-\pi^+$ Background in $K^-\pi^-\pi^+$

- Well established model for $\pi^- + p \to \pi^-\pi^-\pi^+ + p$
  - From very same data set
  - Measured with high precision
  - Acceptance corrected
- Generate $\pi^-\pi^-\pi^+$ Monte Carlo sample
- Mis-interpret $\pi^-\pi^-\pi^+$ Monte Carlo events as $K^-\pi^-\pi^+$
  - Apply wrong mass assumption
  - Same event reconstruction
  - Same event selection
- Perform partial-wave decomposition of mis-interpreted $\pi^-\pi^-\pi^+$ Monte Carlo sample
  - Using the same PWA model as for measured $K^-\pi^-\pi^+$ sample

➥ Study $\pi^-\pi^-\pi^+$ background in individual $K^-\pi^-\pi^+$ partial waves



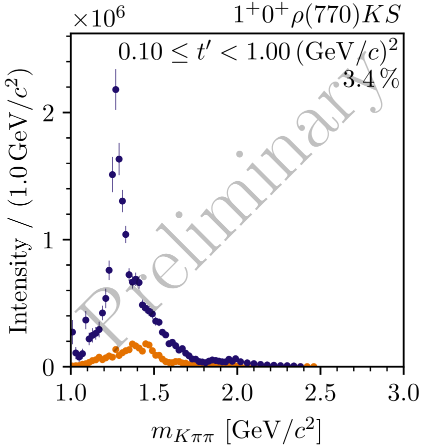
Studying the $\pi^-\pi^-\pi^+$ Background in $K^-\pi^-\pi^+$

Measured $K^-\pi^-\pi^+$ sample
$\pi^-\pi^-\pi^+$ Background

$\pi^-\pi^-\pi^+$ background in individual $K^-\pi^-\pi^+$ partial waves

- Broad bump in $1^+\,0^+\,\rho(770)\,K\,S$ wave
- Large peaking intensity in $2^+\,1^+\,\rho(770)\,K\,D$ wave
  - At similar mass as $K_2^*(1430)$ peak
- Small background in $2^+\,1^+\,K^*(892)\,\pi\,D$ wave

- Background very different in individual partial waves
- May produce peaking structures

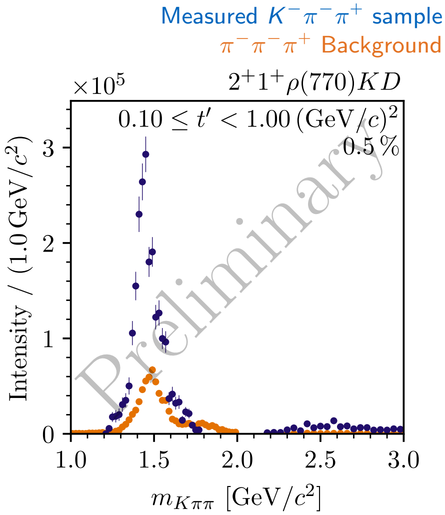


Studying the $\pi^-\pi^-\pi^+$ Background in $K^-\pi^-\pi^+$

$\pi^-\pi^-\pi^+$ background in individual $K^-\pi^-\pi^+$ partial waves

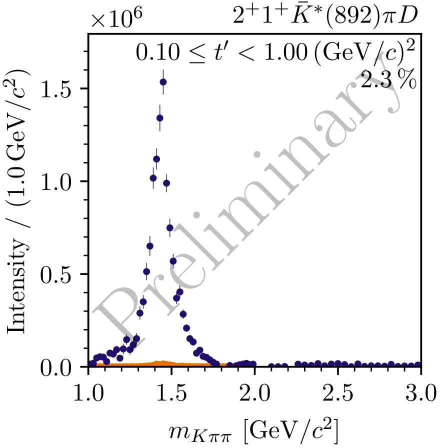
- Broad bump in $1^+ 0^+ \rho(770)\, K\, S$ wave
- Large peaking intensity in $2^+ 1^+ \rho(770)\, K\, D$ wave
  - At similar mass as $K_2^*(1430)$ peak
- Small background in $2^+ 1^+ K^*(892)\, \pi\, D$ wave

- Background very different in individual partial waves
- May produce peaking structures



Studying the $\pi^-\pi^-\pi^+$ Background in $K^-\pi^-\pi^+$

Measured $K^-\pi^-\pi^+$ sample
$\pi^-\pi^-\pi^+$ Background



$\pi^-\pi^-\pi^+$ background in individual $K^-\pi^-\pi^+$ partial waves

- Broad bump in $1^+ 0^+ \rho(770)\, K\, S$ wave
- Large peaking intensity in $2^+ 1^+ \rho(770)\, K\, D$ wave
  - At similar mass as $K_2^*(1430)$ peak
- Small background in $2^+ 1^+ K^*(892)\,\pi\, D$ wave

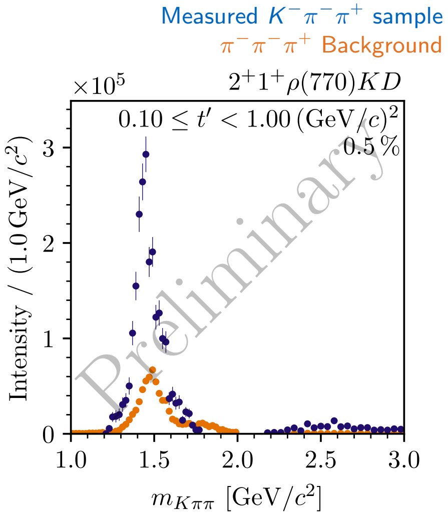
- Background very different in individual partial waves
- May produce peaking structures

Studying the $\pi^-\pi^-\pi^+$ Background in $K^-\pi^-\pi^+$

$\pi^-\pi^-\pi^+$ background in individual $K^-\pi^-\pi^+$ partial waves

- Broad bump in $1^+\,0^+\,\rho(770)\,K\,S$ wave
- Large peaking intensity in $2^+\,1^+\,\rho(770)\,K\,D$ wave
  - At similar mass as $K_2^*(1430)$ peak
- Small background in $2^+\,1^+\,K^*(892)\,\pi\,D$ wave

- Background very different in individual partial waves
- May produce peaking structures



True physics intensity distribution

$$\mathcal{I}(\tau) = \left| \sum_{a}^{\text{waves}} \mathcal{T}_a \Psi_a(\tau) \right|^2$$

Experimentally measured intensity distribution

$$\mathcal{I}_{\text{measured}}(\tau) = \eta(\tau)\mathcal{I}(\tau)$$

- Take into account different processes $\mathfrak{p}$
  - Different model intensities $\mathcal{I}^{\mathfrak{p}}$
  - Different experimental acceptance $\eta^{\mathfrak{p}}$
  - Formulated in terms of different phase-space variables $\tau^{\mathfrak{p}}$
    - Jacobian terms $J(\tau^{K\pi\pi} \to \tau^{\mathfrak{p}})$ from variable transformation

Treating the $\pi^-\pi^-\pi^+$ and Other Backgrounds

In the Partial-Wave Decomposition

True physics intensity distribution for process $\mathfrak{p}$

$$\mathcal{I}^{\mathfrak{p}}(\tau) = \left| \sum_{a}^{\text{waves}} \mathcal{T}_a^{\mathfrak{p}} \Psi_a^{\mathfrak{p}}(\tau) \right|^2$$

Experimentally measured intensity distribution

$$\mathcal{I}_{\text{measured}}(\tau) = \sum_{\mathfrak{p}} \eta^{\mathfrak{p}}(\tau)\, \mathcal{I}^{\mathfrak{p}}(\tau)$$

- Take into account different processes $\mathfrak{p}$
  - Different model intensities $\mathcal{I}^{\mathfrak{p}}$
  - Different experimental acceptance $\eta^{\mathfrak{p}}$
  - Formulated in terms of different phase-space variables $\tau^{\mathfrak{p}}$
    - Jacobian terms $J(\tau^{K\pi\pi} \to \tau^{\mathfrak{p}})$ from variable transformation

Treating the $\pi^-\pi^-\pi^+$ and Other Backgrounds

In the Partial-Wave Decomposition

True physics intensity distribution for process $\mathfrak{p}$

$$\mathcal{I}^{\mathfrak{p}}(\tau^{\mathfrak{p}}) = \left| \sum_{a}^{\text{waves}} \mathcal{T}_a^{\mathfrak{p}}\, \Psi_a^{\mathfrak{p}}(\tau^{\mathfrak{p}}) \right|^2$$

Experimentally measured intensity distribution

$$\mathcal{I}_{\text{measured}}(\tau^{K\pi\pi}) = \sum_{\mathfrak{p}} \eta^{\mathfrak{p}}(\tau^{\mathfrak{p}})\, \mathcal{I}^{\mathfrak{p}}(\tau^{\mathfrak{p}})\, J(\tau^{K\pi\pi} \to \tau^{\mathfrak{p}})$$

- Take into account different processes $\mathfrak{p}$
  - Different model intensities $\mathcal{I}^{\mathfrak{p}}(\tau^{\mathfrak{p}})$
  - Different experimental acceptance $\eta^{\mathfrak{p}}(\tau^{\mathfrak{p}})$
  - Formulated in terms of different phase-space variables $\tau^{\mathfrak{p}}$
    - Jacobian terms $J(\tau^{K\pi\pi} \to \tau^{\mathfrak{p}})$ from variable transformation

Treating the $\pi^-\pi^-\pi^+$ and Other Backgrounds

In the Partial-Wave Decomposition

True physics intensity distribution for process $\mathfrak{p}$

$$\mathcal{I}^{\mathfrak{p}}(\tau^{\mathfrak{p}}) = \left| \sum_{a}^{\text{waves}} \mathcal{T}_a^{\mathfrak{p}} \, \Psi_a^{\mathfrak{p}}(\tau^{\mathfrak{p}}) \right|^2$$

Experimentally measured intensity distribution

$$\mathcal{I}_{\text{measured}}(\tau^{K\pi\pi}) = \sum_{\mathfrak{p}} \eta^{\mathfrak{p}}(\tau^{\mathfrak{p}}) \, \mathcal{I}^{\mathfrak{p}}(\tau^{\mathfrak{p}}) \, J(\tau^{K\pi\pi} \to \tau^{\mathfrak{p}})$$

- $\mathcal{I}^{\pi\pi\pi}$ known by COMPASS analysis
- $\eta^{\pi\pi\pi}$ from detector simulation

- $\eta^{\pi\pi\pi}$ computationally expensive
- Different $m_{3\pi}$ bins enter one $m_{K\pi\pi}$ bin
- Other background channels: $K^-K^-K^+$, ...
  - $\mathcal{I}^{\mathfrak{p}}$ unknown
  - Unknown background channels

True physics intensity distribution for process $\mathfrak{p}$

$$\mathcal{I}^{\mathfrak{p}}(\tau^{\mathfrak{p}}) = \left| \sum_{a}^{\text{waves}} \mathcal{T}_a^{\mathfrak{p}} \, \Psi_a^{\mathfrak{p}}(\tau^{\mathfrak{p}}) \right|^2$$

Experimentally measured intensity distribution

$$\mathcal{I}_{\text{measured}}(\tau^{K\pi\pi}) = \sum_{\mathfrak{p}} \eta^{\mathfrak{p}}(\tau^{\mathfrak{p}}) \, \mathcal{I}^{\mathfrak{p}}(\tau^{\mathfrak{p}}) \, J(\tau^{K\pi\pi} \to \tau^{\mathfrak{p}})$$

- $\mathcal{I}^{\pi\pi\pi}$ known by COMPASS analysis
- $\eta^{\pi\pi\pi}$ from detector simulation

- $\eta^{\pi\pi\pi}$ computationally expensive
- Different $m_{3\pi}$ bins enter one $m_{K\pi\pi}$ bin
- Other background channels: $K^-K^-K^+$, ...
  - $\mathcal{I}^{\mathfrak{p}}$ unknown
  - Unknown background channels

Treating the $\pi^-\pi^-\pi^+$ and Other Backgrounds

In the Partial-Wave Decomposition

True physics intensity distribution for process $\mathfrak{p}$

$$\mathcal{I}^{\mathfrak{p}}(\tau^{\mathfrak{p}}) = \left| \sum_{a}^{\text{waves}} \mathcal{T}_a^{\mathfrak{p}} \, \Psi_a^{\mathfrak{p}}(\tau^{\mathfrak{p}}) \right|^2$$

Experimentally measured intensity distribution

$$\mathcal{I}_{\text{measured}}(\tau^{K\pi\pi}) = \sum_{\mathfrak{p}} \eta^{\mathfrak{p}}(\tau^{\mathfrak{p}}) \, \mathcal{I}^{\mathfrak{p}}(\tau^{\mathfrak{p}}) \, J(\tau^{K\pi\pi} \to \tau^{\mathfrak{p}})$$

- $\mathcal{I}^{\pi\pi\pi}$ known by COMPASS analysis
- $\eta^{\pi\pi\pi}$ from detector simulation

- $\eta^{\pi\pi\pi}$ computationally expensive
- Different $m_{3\pi}$ bins enter one $m_{K\pi\pi}$ bin
- Other background channels: $K^-K^-K^+$, ...
  - $\mathcal{I}^{\mathfrak{p}}$ unknown
  - Unknown background channels

Approximate model for process $\mathfrak{p}$ by $K^-\pi^-\pi^+$ partial waves

$$\eta^{\mathfrak{p}}(\tau^{\mathfrak{p}}) \left| \sum_a^{\text{waves}} \mathcal{T}_a^{\mathfrak{p}} \Psi_a^{\mathfrak{p}}(\tau^{\mathfrak{p}}) \right|^2 \approx \eta^{K\pi\pi}(\tau^{K\pi\pi}) \left| \sum_a^{\text{waves}} \tilde{\mathcal{T}}_a^{\mathfrak{p}} \Psi_a^{K\pi\pi}(\tau^{K\pi\pi}) \right|^2$$

Total true physics intensity distribution

$$\mathcal{I}(\tau^{K\pi\pi}) = \sum_{\mathfrak{p}} \left| \sum_a^{\text{waves}} \mathcal{T}_a^{\mathfrak{p}} \Psi_a^{K\pi\pi}(\tau^{K\pi\pi}) \right|^2$$

Experimentally measured intensity distribution

$$\mathcal{I}_{\text{measured}}(\tau^{K\pi\pi}) = \eta^{K\pi\pi}(\tau^{K\pi\pi}) \, \mathcal{I}(\tau^{K\pi\pi})$$

- How well can $K^-\pi^-\pi^+$ partial waves approximate the distribution of process $\mathfrak{p}$
  - Is the set of $K^-\pi^-\pi^+$ partial waves sufficient?
    - ➥ Automatic wave-set selection using model-selection techniques

Treating the $\pi^-\pi^-\pi^+$ and Other Backgrounds

In the Partial-Wave Decomposition

Approximate model for process $\mathfrak{p}$ by $K^-\pi^-\pi^+$ partial waves

$$\eta^{\mathfrak{p}}(\tau^{\mathfrak{p}}) \left| \sum_a^{\text{waves}} \mathcal{T}_a^{\mathfrak{p}} \, \Psi_a^{\mathfrak{p}}(\tau^{\mathfrak{p}}) \right|^2 \approx \eta^{K\pi\pi}(\tau^{K\pi\pi}) \left| \sum_a^{\text{waves}} \tilde{\mathcal{T}}_a^{\mathfrak{p}} \, \Psi_a^{K\pi\pi}(\tau^{K\pi\pi}) \right|^2$$

Total true physics intensity distribution

$$\mathcal{I}(\tau^{K\pi\pi}) = \sum_{\mathfrak{p}} \left| \sum_a^{\text{waves}} \mathcal{T}_a^{\mathfrak{p}} \, \Psi_a^{K\pi\pi}(\tau^{K\pi\pi}) \right|^2$$

Experimentally measured intensity distribution

$$\mathcal{I}_{\text{measured}}(\tau^{K\pi\pi}) = \eta^{K\pi\pi}(\tau^{K\pi\pi}) \, \mathcal{I}(\tau^{K\pi\pi})$$

- How well can $K^-\pi^-\pi^+$ partial waves approximate the distribution of process $\mathfrak{p}$
  - Is the set of $K^-\pi^-\pi^+$ partial waves sufficient?
    - ➥ Automatic wave-set selection using model-selection techniques

Treating the $\pi^-\pi^-\pi^+$ and Other Backgrounds

In the Partial-Wave Decomposition

Approximate model for process $\mathfrak{p}$ by $K^-\pi^-\pi^+$ partial waves

$$\eta^{\mathfrak{p}}(\tau^{\mathfrak{p}}) \left| \sum_a^{\text{waves}} \mathcal{T}_a^{\mathfrak{p}} \, \Psi_a^{\mathfrak{p}}(\tau^{\mathfrak{p}}) \right|^2 \approx \eta^{K\pi\pi}(\tau^{K\pi\pi}) \left| \sum_a^{\text{waves}} \tilde{\mathcal{T}}_a^{\mathfrak{p}} \, \Psi_a^{K\pi\pi}(\tau^{K\pi\pi}) \right|^2$$

Total true physics intensity distribution

$$\mathcal{I}(\tau^{K\pi\pi}) = \sum_{a,b}^{\text{waves}} \Psi_a^{K\pi\pi}(\tau^{K\pi\pi}) \, \rho_{a,b} \, [\Psi_b^{K\pi\pi}(\tau^{K\pi\pi})]^*$$

Spin-density matrix with rank $N_\mathrm{r} > 1$

$$\rho_{a,b} = \sum_{\mathfrak{p}}^{N_\mathrm{r}} \mathcal{T}_a^{\mathfrak{p}} [\mathcal{T}_b^{\mathfrak{p}}]^*$$

- How well can $K^-\pi^-\pi^+$ partial waves approximate the distribution of process $\mathfrak{p}$
  - Is the set of $K^-\pi^-\pi^+$ partial waves sufficient?
    - ➥ Automatic wave-set selection using model-selection techniques

Treating the $\pi^-\pi^-\pi^+$ and Other Backgrounds

In the Partial-Wave Decomposition

Approximate model for process $\mathfrak{p}$ by $K^-\pi^-\pi^+$ partial waves

$$\eta^{\mathfrak{p}}(\tau^{\mathfrak{p}}) \left| \sum_a^{\text{waves}} \mathcal{T}_a^{\mathfrak{p}} \, \Psi_a^{\mathfrak{p}}(\tau^{\mathfrak{p}}) \right|^2 \approx \eta^{K\pi\pi}(\tau^{K\pi\pi}) \left| \sum_a^{\text{waves}} \tilde{\mathcal{T}}_a^{\mathfrak{p}} \, \Psi_a^{K\pi\pi}(\tau^{K\pi\pi}) \right|^2$$

Total true physics intensity distribution

$$\mathcal{I}(\tau^{K\pi\pi}) = \sum_{a,b}^{\text{waves}} \Psi_a^{K\pi\pi}(\tau^{K\pi\pi}) \, \rho_{a,b} \, [\Psi_b^{K\pi\pi}(\tau^{K\pi\pi})]^*$$

Spin-density matrix with rank $N_\text{r} > 1$

$$\rho_{a,b} = \sum_{\mathfrak{p}}^{N_\text{r}} \mathcal{T}_a^{\mathfrak{p}} [\mathcal{T}_b^{\mathfrak{p}}]^*$$

- How well can $K^-\pi^-\pi^+$ partial waves approximate the distribution of process $\mathfrak{p}$
  - Is the set of $K^-\pi^-\pi^+$ partial waves sufficient?
    - ➡ Automatic wave-set selection using model-selection techniques

Treating the $\pi^-\pi^-\pi^+$ and Other Backgrounds

In the Partial-Wave Decomposition

Approximate model for process $\mathfrak{p}$ by $K^-\pi^-\pi^+$ partial waves

$$\eta^{\mathfrak{p}}(\tau^{\mathfrak{p}}) \left| \sum_{a}^{\text{waves}} \mathcal{T}_a^{\mathfrak{p}} \, \Psi_a^{\mathfrak{p}}(\tau^{\mathfrak{p}}) \right|^2 \approx \eta^{K\pi\pi}(\tau^{K\pi\pi}) \left| \sum_{a}^{\text{waves}} \tilde{\mathcal{T}}_a^{\mathfrak{p}} \, \Psi_a^{K\pi\pi}(\tau^{K\pi\pi}) \right|^2$$

Total true physics intensity distribution

$$\mathcal{I}(\tau^{K\pi\pi}) = \sum_{a,b}^{\text{waves}} \Psi_a^{K\pi\pi}(\tau^{K\pi\pi}) \, \rho_{a,b} \, [\Psi_b^{K\pi\pi}(\tau^{K\pi\pi})]^*$$

Spin-density matrix with rank $N_\mathrm{r} > 1$

$$\rho_{a,b} = \sum_{\mathfrak{r}}^{N_\mathrm{r}} \mathcal{T}_a^{\mathfrak{r}} [\mathcal{T}_b^{\mathfrak{r}}]^*$$

- Experimentally measurable quantities are spin-density matrix elements
  - ➡ Transition amplitudes $\mathcal{T}_a^{\mathfrak{p}}$ are only effective parameters
  - ➡ Cannot determine $\mathcal{T}_a^{\mathfrak{p}}$ of individual processes
  - ➡ Cannot separate different processes

Treating the $\pi^-\pi^-\pi^+$ and Other Backgrounds

In the Partial-Wave Decomposition

Approximate model for process $\mathfrak{p}$ by $K^-\pi^-\pi^+$ partial waves

$$\eta^{\mathfrak{p}}(\tau^{\mathfrak{p}}) \left| \sum_a^{\text{waves}} \mathcal{T}_a^{\mathfrak{p}} \, \Psi_a^{\mathfrak{p}}(\tau^{\mathfrak{p}}) \right|^2 \approx \eta^{K\pi\pi}(\tau^{K\pi\pi}) \left| \sum_a^{\text{waves}} \tilde{\mathcal{T}}_a^{\mathfrak{p}} \, \Psi_a^{K\pi\pi}(\tau^{K\pi\pi}) \right|^2$$

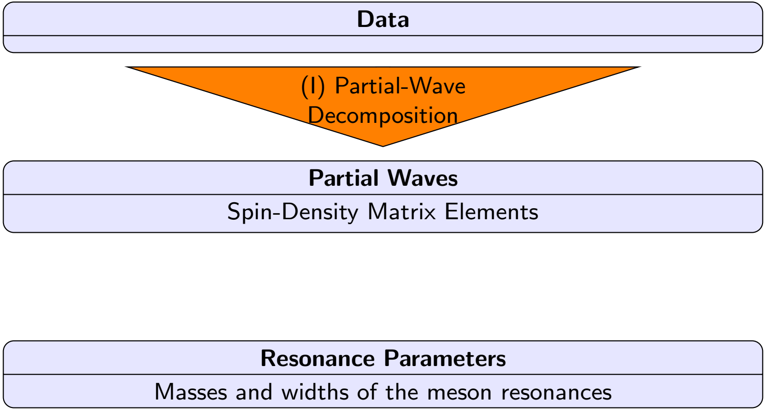
Total true physics intensity distribution

$$\mathcal{I}(\tau^{K\pi\pi}) = \sum_{a,b}^{\text{waves}} \Psi_a^{K\pi\pi}(\tau^{K\pi\pi}) \, \rho_{a,b} \, [\Psi_b^{K\pi\pi}(\tau^{K\pi\pi})]^*$$

Spin-density matrix with rank $N_\mathrm{r} > 1$

$$\rho_{a,b} = \sum_{\mathfrak{r}}^{N_\mathrm{r}} \mathcal{T}_a^{\mathfrak{r}} [\mathcal{T}_b^{\mathfrak{r}}]^*$$

- Large number of fit parameters: $N_\mathrm{para} = N_\mathrm{r}(2N_\mathrm{waves} - N_\mathrm{r})$
- Sufficient rank of spin-density matrix must be determined
  - Rank two needed to describe pure $\pi^-\pi^-\pi^+$ Monte Carlo sample using $K^-\pi^-\pi^+$ partial waves
  - Used rank three to model $K^-\pi^-\pi^+$ sample



Data
(I) Partial-Wave Decomposition
Partial Waves
Spin-Density Matrix Elements
Resonance Parameters
Masses and widths of the meson resonances

Data

(I) Partial-Wave Decomposition

Partial Waves

Spin-Density Matrix Elements

(II) Resonance-Model Fit

Resonance Parameters

Masses and widths of the meson resonances

Resonance-Model Fit

$$\hat{\rho}_{a,b}(m_{K\pi\pi}) = \hat{\rho}_{a,b}^{K\pi\pi}(m_{K\pi\pi}) + \hat{\rho}_{a,b}^{\pi\pi\pi}(m_{K\pi\pi}) + \hat{\rho}_{a,b}^{\text{eff}}(m_{K\pi\pi})$$

- Model the $m_{K\pi\pi}$ dependence of spin-density matrix elements
- Model for $K^- + p \to K^-\pi^-\pi^+ + p$
  - Sum of Breit-Wigner plus non-resonant processes
  - Coherent: $\hat{\rho}_{a,b}^{K\pi\pi}$ has rank one
- Model for $\pi^- + p \to \pi^-\pi^-\pi^+ + p$
  - Potentially largest background
  - Explicitly modeled using $\pi^-\pi^-\pi^+$ Monte Carlo sample
  - $\hat{\rho}_{a,b}^{\pi\pi\pi}$ has rank two
- Model for further background processes
  - Phenomenological model
  - $\hat{\rho}_{a,b}^{\text{eff}}$ has rank one

Resonance-Model Fit

$$\hat{\rho}_{a,b}(m_{K\pi\pi}) = \hat{\rho}^{K\pi\pi}_{a,b}(m_{K\pi\pi}) + \hat{\rho}^{\pi\pi\pi}_{a,b}(m_{K\pi\pi}) + \hat{\rho}^{\text{eff}}_{a,b}(m_{K\pi\pi})$$

- Model the $m_{K\pi\pi}$ dependence of spin-density matrix elements
- Model for $K^- + p \to K^-\pi^-\pi^+ + p$
  - Sum of Breit-Wigner plus non-resonant processes
  - Coherent: $\hat{\rho}^{K\pi\pi}_{a,b}$ has rank one
- Model for $\pi^- + p \to \pi^-\pi^-\pi^+ + p$
  - Potentially largest background
  - Explicitly modeled using $\pi^-\pi^-\pi^+$ Monte Carlo sample
  - $\hat{\rho}^{\pi\pi\pi}_{a,b}$ has rank two
- Model for further background processes
  - Phenomenological model
  - $\hat{\rho}^{\text{eff}}_{a,b}$ has rank one

Resonance-Model Fit

$$\hat{\rho}_{a,b}(m_{K\pi\pi}) = \hat{\rho}^{K\pi\pi}_{a,b}(m_{K\pi\pi}) + \hat{\rho}^{\pi\pi\pi}_{a,b}(m_{K\pi\pi}) + \hat{\rho}^{\text{eff}}_{a,b}(m_{K\pi\pi})$$

- Model the $m_{K\pi\pi}$ dependence of spin-density matrix elements
- Model for $K^- + p \to K^-\pi^-\pi^+ + p$
  - Sum of Breit-Wigner plus non-resonant processes
  - Coherent: $\hat{\rho}^{K\pi\pi}_{a,b}$ has rank one
- Model for $\pi^- + p \to \pi^-\pi^-\pi^+ + p$
  - Potentially largest background
  - Explicitly modeled using $\pi^-\pi^-\pi^+$ Monte Carlo sample
  - $\hat{\rho}^{\pi\pi\pi}_{a,b}$ has rank two
- Model for further background processes
  - Phenomenological model
  - $\hat{\rho}^{\text{eff}}_{a,b}$ has rank one

Treating the $\pi^-\pi^-\pi^+$ and Other Backgrounds

Modeling Background Components in the Resonance-Model Fit

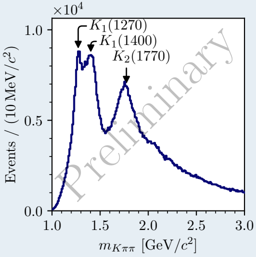
Resonance-Model Fit

$$\hat{\rho}_{a,b}(m_{K\pi\pi}) = \hat{\rho}^{K\pi\pi}_{a,b}(m_{K\pi\pi}) + \hat{\rho}^{\pi\pi\pi}_{a,b}(m_{K\pi\pi}) + \hat{\rho}^{\mathrm{eff}}_{a,b}(m_{K\pi\pi})$$

- Model the $m_{K\pi\pi}$ dependence of spin-density matrix elements
- Model for $K^- + p \to K^-\pi^-\pi^+ + p$
  - Sum of Breit-Wigner plus non-resonant processes
  - Coherent: $\hat{\rho}^{K\pi\pi}_{a,b}$ has rank one
- Model for $\pi^- + p \to \pi^-\pi^-\pi^+ + p$
  - Potentially largest background
  - Explicitly modeled using $\pi^-\pi^-\pi^+$ Monte Carlo sample
  - $\hat{\rho}^{\pi\pi\pi}_{a,b}$ has rank two
- Model for further background processes
  - Phenomenological model
  - $\hat{\rho}^{\mathrm{eff}}_{a,b}$ has rank one

Summary and Outlook

- World's largest sample on $K^-\pi^-\pi^+$
- Clear signals of well known states
- Interesting potential signals of excited states
- Non-negligible incoherent background
  - Treatment at the resonance-model fit

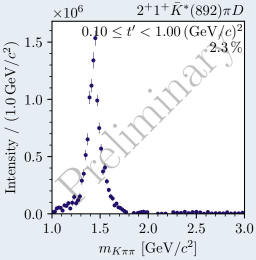


Further challanges and solutions

- Improved uncertainty estimates for spin-density matrix elements
  - Bootstrapping starting at the event sample level
- Extensive Monte Carlo studies (input-output studies) to verify our approach
- Automatic wave-set selection
- ..

Summary and Outlook

- World's largest sample on $K^- \pi^- \pi^+$
- Clear signals of well known states
- Interesting potential signals of excited states
- Non-negligible incoherent background
  - Treatment at the resonance-model fit

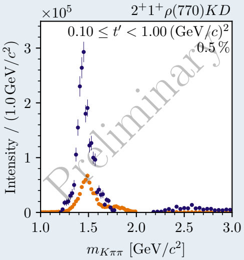


Further challanges and solutions

- Improved uncertainty estimates for spin-density matrix elements
  - Bootstrapping starting at the event sample level
- Extensive Monte Carlo studies (input-output studies) to verify our approach
- Automatic wave-set selection
- ..

Summary and Outlook

- World's largest sample on $K^-\pi^-\pi^+$
- Clear signals of well known states
- Interesting potential signals of excited states
- Non-negligible incoherent background
  - Treatment at the resonance-model fit

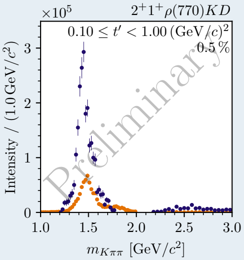


Further challanges and solutions

- Improved uncertainty estimates for spin-density matrix elements
  - Bootstrapping starting at the event sample level
- Extensive Monte Carlo studies (input-output studies) to verify our approach
- Automatic wave-set selection
- ..

Summary and Outlook

- World's largest sample on $K^-\pi^-\pi^+$
- Clear signals of well known states
- Interesting potential signals of excited states
- Non-negligible incoherent background
  - Treatment at the resonance-model fit



Further challanges and solutions

- Improved uncertainty estimates for spin-density matrix elements
  - Bootstrapping starting at the event sample level
- Extensive Monte Carlo studies (input-output studies) to verify our approach
- Automatic wave-set selection
- ..