$\eta' \to \eta\pi\pi$ DECAYS

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Based on:

S. Gonzàlez-Solís, E. Passemar; Eur.Phys.J.C 78 (2018) 9, 758; arXiv:1807.04313

Motivation

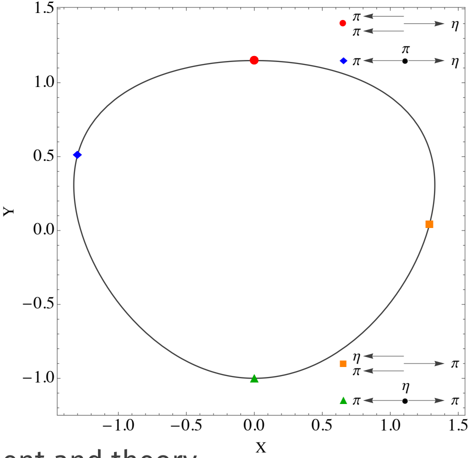
- The η and η' mesons are **special**:
  - Chiral Perturbation Theory:
    - Eight pGB associated to $SU(3)_L \otimes SU(3)_R \xrightarrow{\text{SCSB}} SU(3)_V$ exhibited by QCD (π, K and η_8)
    - η_1 not included due to the axial anomaly
  - Large-N_C $U(3)$ ChPT:
    - The axial anomaly is absent
    - η_1 as the ninth Goldstone boson
    - η-η' do mix
  - All their strong and EM decays are forbidden at lowest order
- **Important** experimental activities: A2, Belle-II, BESIII, KLOE-II, GlueX, WASA-at-COSY
- **Forthcoming** experiments: JLab Eta Factory (JEF) and REDTOP

Motivation

- Main decay channel of the η': $\mathrm{BR}(\eta' \to \eta\pi^0\pi^0) = 22.8(8)\%$, $\mathrm{BR}(\eta' \to \eta\pi^+\pi^-) = 42.6(7)\%$ [PDG]
- Cannot be described within $SU(3)$ ChPT
- Advantageous laboratory to test any of its extensions Large-N_C $U(3)$ ChPT and Resonance Chiral Theory
- G-parity conservation prevents vectors to contribute: analysis of the properties of scalar resonances
- Study of the η-η' mixing
- Access $\pi\eta$ scattering and phase-shift
- New data from the A2 and BESIII collaborations

- $s = (p_{\eta'} - p_\eta)^2, t = (p_{\eta'} - p_{\pi^+})^2, u = (p_{\eta'} - p_{\pi^-})^2,$
  $s + t + u = m_{\eta'}^2 + m_\eta^2 + 2m_\pi^2$

- Dalitz-plot variables:
  - $X = \frac{\sqrt{3}}{Q}(T_{\pi_1} - T_{\pi_2}) = \frac{\sqrt{3}}{Q}(u - t)$
  - $Y = \frac{m_\eta + 2m_\pi}{m_\pi}\frac{T_\eta}{Q} - 1 = \frac{m_\eta + 2m_\pi}{m_\pi}\frac{(m_{\eta'} - m_\eta)^2 - s}{2m_{\eta'}Q} - 1$
  - $Q = m_{\eta'} - m_\eta - 2m_\pi$



- Dalitz plot to compare experiment and theory

$$|M(X,Y)|^2 = |N|^2 \left(1 + aY + bY^2 + cX + dX^2 + \dots\right)$$

- Dalitz plot to compare experiment and theory

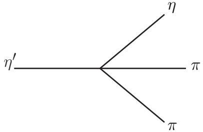
$$|M(X,Y)|^2 = |N|^2 \left(1 + aY + bY^2 + cX + dX^2 + \dots\right)$$

- a, b, c, d are the Dalitz plot parameters

$\eta' \to \eta\pi^0\pi^0$	$a[Y]$	$b[Y^2]$	$c[X]$	$d[X^2]$
GAMS-4 (2009)	−0.067(16)(4)	−0.064(29)(5)	= 0	−0.067(20)(3)
GAMS-4 (2009)	−0.066(16)(4)	−0.063(28)(4)	−0.107(96)(3)	0.018(78)(6)
A2 (2017)	−0.074(8)(6)	−0.063(14)(5)	—	−0.050(9)(5)
BESIII (2017)	−0.087(9)(6)	−0.073(14)(5)	0	−0.074(9)(4)

$\eta' \to \eta\pi^+\pi^-$	$a[Y]$	$b[Y^2]$	$c[X]$	$d[X^2]$
VES (2007)	−0.127(16)(8)	−0.106(28)(14)	0.015(11)(14)	−0.082(17)(8)
BESIII (2011)	−0.047(11)(3)	−0.069(19)(9)	0.019(11)(3)	−0.073(12)(3)
BESIII (2017)	−0.056(4)(3)	−0.049(6)(6)	$2.7(2.4)(1.8)\cdot 10^{-3}$	−0.063(4)(4)

- Lowest order calculation:



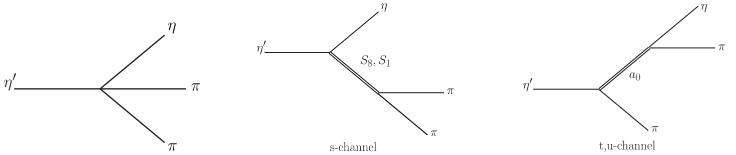
$$\mathcal{M}^{\text{LO}}_{\eta'\to\eta\pi\pi} = \frac{M_\pi^2}{6F_\pi^2}\left(2\sqrt{2}\cos 2\theta - \sin 2\theta\right) ,$$

- Branching ratio (for $\theta = -13.3(5)^\circ$):

Order	$\eta' \to \eta\pi^+\pi^-$	$\eta' \to \eta\pi^0\pi^0$
LO	1.1%	0.6%
PDG	42.6(7)%	22.8(8)%

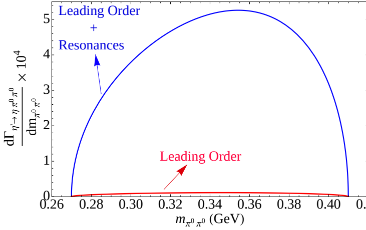
- Reason for this difference: amplitude is chirally suppressed (vanishes when $M_\pi^2 \to 0$)
- Higher order effects? resonances, $\pi\pi$ and $\pi\eta$ final state interactions

- LO+resonances:

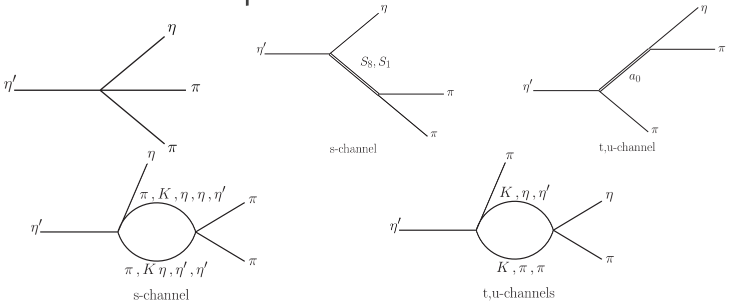


- Warm-up: using $c_m = 43$ and $c_d = 30$ [Jamin et.al. 0006045] and $M_S = 980$ MeV
- Branching ratio:

Order	$\eta' \to \eta\pi^+\pi^-$	$\eta' \to \eta\pi^0\pi^0$
LO	1.1%	0.6%
LO+resonances	$\sim$ 54%	$\sim$ 27%
PDG	42.6(7)%	22.8(8)%



- LO+resonances+loops:



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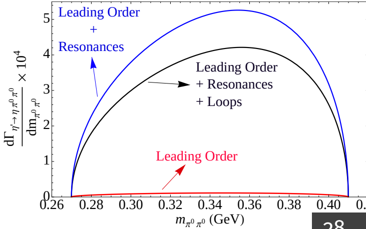
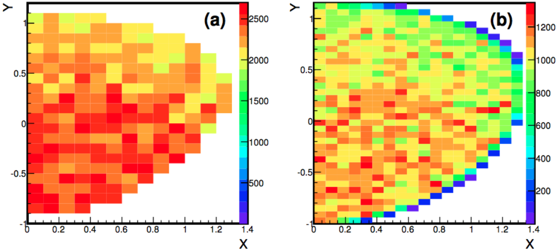
- Branching ratio:

Order	$\eta' \to \eta\pi^+\pi^-$	$\eta' \to \eta\pi^0\pi^0$
LO	1.1%	0.6%
LO+resonances	$\sim$ 54%	$\sim$ 27%
LO+res+loops	$\sim$ 50%	$\sim$ 25%
PDG	42.6(7)%	22.8(8)%



Fits to experimental data

A2 Coll. 1709.04230



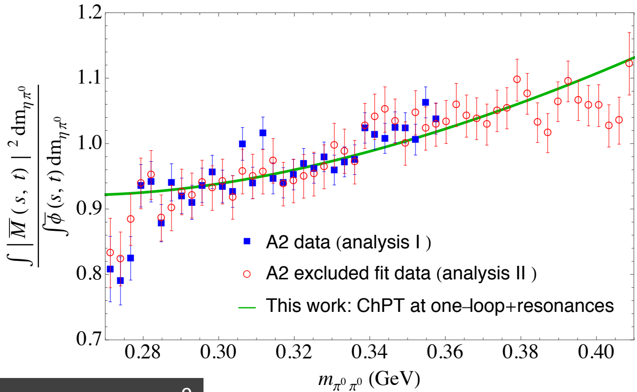
- We relate the experimental Dalitz plot data with the differential decay distribution from theory through

$$\frac{d^2 N_{\text{events}}}{dX dY} = \frac{d\Gamma(\eta' \to \eta\pi^0\pi^0)}{dX dY} \frac{N_{\text{events}}}{\Gamma_{\eta'} \bar{B}(\eta' \to \eta\pi^0\pi^0)} \Delta X \Delta Y ,$$

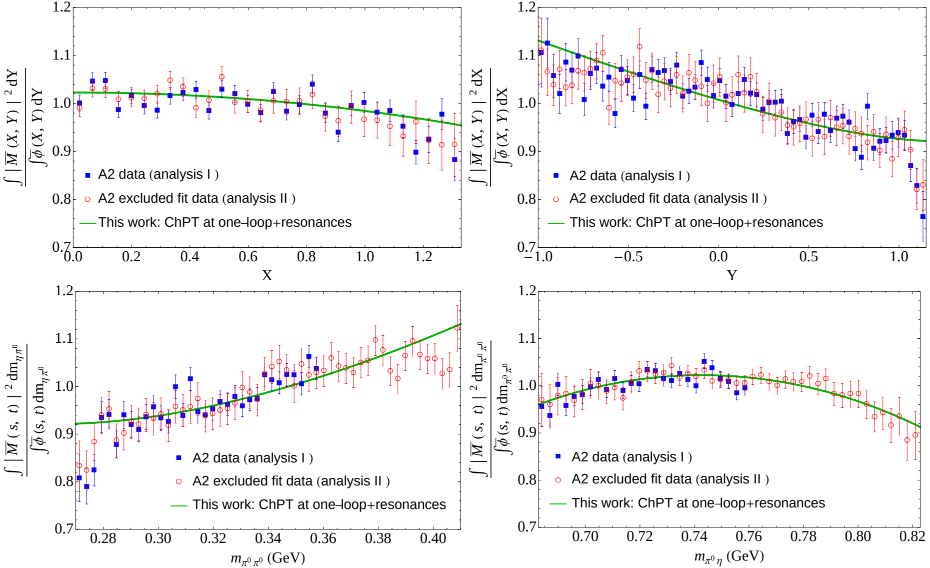
- $N_{\text{events}} = 463066$ (analysis I) and 473044 (analysis II)
- $\Delta X = \Delta Y = 0.10$

- Fit 1: $M_S = 973(5)$ MeV, $c_d = c_m = 30.1(4)$ MeV, $\tilde{c}_{d,m} = c_{d,m}/\sqrt{3}$, $\chi^2_{\rm dof} = 1.22$

Dalitz parameters	ChPT	A2 coll.
$a[Y]$	$-0.095(6)$	$-0.074(8)(6)$
$b[Y^2]$	$0.005(1)$	$-0.063(14)(5)$
$d[X^2]$	$-0.037(5)$	$-0.050(9)(5)$

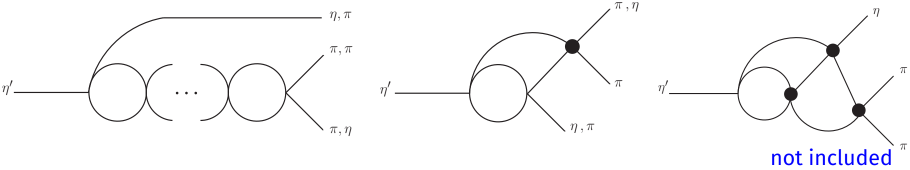


Fits to data: ChPT at one loop with resonances



- Unitarity relation

$$\mathrm{Im}\mathcal{M}_{\eta'\to\eta\pi\pi} = \frac{1}{2}\sum_n (2\pi)^4\,\delta^4\left(p_\eta + p_1 + p_2 - p_n\right)\mathcal{T}^*_{n\to\eta\pi\pi}\mathcal{M}_{\eta'\to n}$$



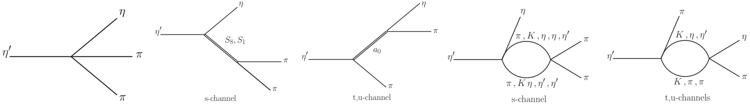
- Two-particle unitarity for the partial-wave amplitude
    - s-channel:

$$\mathrm{Im}\left(m^{IJ}_{\eta'\to\eta\pi\pi}(s)\right) = \sigma_\pi(s)\left(t^{IJ}_{\pi\pi\to\pi\pi}(s)\right)^* m^{IJ}_{\eta'\to\eta\pi\pi}(s)\times\theta(s-4m_\pi^2)\,,$$

    - similar for the t-and-u channels

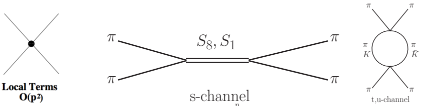
N/D unitarisation method applied to $\eta' \to \eta\pi\pi$

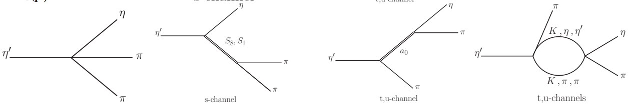
- Amplitude at one-loop in $U(3)$ ChPT with resonances

$\mathcal{M}^{\eta' \to \eta\pi\pi}(s) =$ 

- N/D representation of $\mathcal{M}^{\eta' \to \eta\pi\pi}(s)$

$$m^{IJ}_{\eta' \to \eta\pi\pi}(s) = [1 + N^{IJ}_{\pi\pi}(s) g_{\pi\pi}(s)]^{-1} R^{IJ}_{\eta' \to \eta\pi\pi}(s)\,,$$

$N^{IJ}_{\pi\pi \to \pi\pi}(s) =$ 

$R^{IJ}_{\eta' \to \eta\pi\pi}(s) =$ 

$$g_{\pi\pi}(s) = \frac{1}{16\pi^2}\left(a_{\pi\pi}(\mu) + \log\frac{m_\pi^2}{\mu^2} - \sigma_\pi(s)\log\frac{\sigma_\pi(s)-1}{\sigma_\pi(s)+1}\right) \qquad (4)$$

N/D applied to $\eta' \to \eta\pi\pi$

- Amplitude at one-loop in $U(3)$ ChPT with resonances

$$\mathcal{M}^{\eta'\to\eta\pi\pi}(s) = \mathcal{M}(s)^{(2)} + \mathcal{M}(s)^{\mathrm{Res(s,t,u)}} + \mathcal{M}(s)^{\mathrm{Loop(s,t,u)}}\,, \qquad (5)$$

- N/D representation of Eq. (5):

$$\begin{aligned} m^{IJ}_{\eta'\to\eta\pi\pi}(s) &= [1 + N^{IJ}_{\pi\pi}(s) g_{\pi\pi}(s)]^{-1} R^{IJ}_{\eta'\to\eta\pi\pi}(s)\,, \\ N^{IJ}_{\pi\pi}(s) &= t^{IJ}_{\pi\pi}(s)^{(2)+\mathrm{Res}+\mathrm{Loop}}\,, \\ R^{IJ}_{\eta'\to\eta\pi\pi}(s) &= m^{IJ}_{\eta'\to\eta\pi\pi}(s)^{(2)+\mathrm{Res}+\mathrm{Loop}}\,, \end{aligned} \qquad (6)$$

- Chiral expansion of Eq. (6) leads:

$$m^{IJ}_{\eta'\to\eta\pi\pi}(s) = m^{IJ}_{\eta'\to\eta\pi\pi}(s)^{(2)+\mathrm{Res}+\mathrm{Loop}} - t^{IJ}_{\pi\pi}(s)^{(2)} g_{\pi\pi}(s) m^{IJ}_{\eta'\to\eta\pi\pi}(s)^{(2)} + \cdots$$

$$\boxed{\mathrm{Im}\left(m^{IJ}_{\eta'\to\eta\pi\pi}(s)\right) = t^{IJ}_{\pi\pi}(s)\sigma_\pi(s) m^{IJ}_{\eta'\to\eta\pi\pi}(s)}$$

- Unitarized amplitude in terms of the *S*-and-*D*-waves

$$\mathcal{M}^{I=0}_{\eta'\to\eta\pi\pi}(s,\cos\theta_s) = \sum_J 32\pi(2J+1)P_J(\cos\theta_s)m^{IJ}(s)$$

$$= 32\pi P_0(\cos\theta_s)\frac{m^{00}(s)}{1+g_{\pi\pi}(s)t^{00}_{\pi\pi}(s)} + 160\pi P_2(\cos\theta_s)\frac{m^{02}(s)}{1+g_{\pi\pi}(s)t^{02}_{\pi\pi}(s)}$$

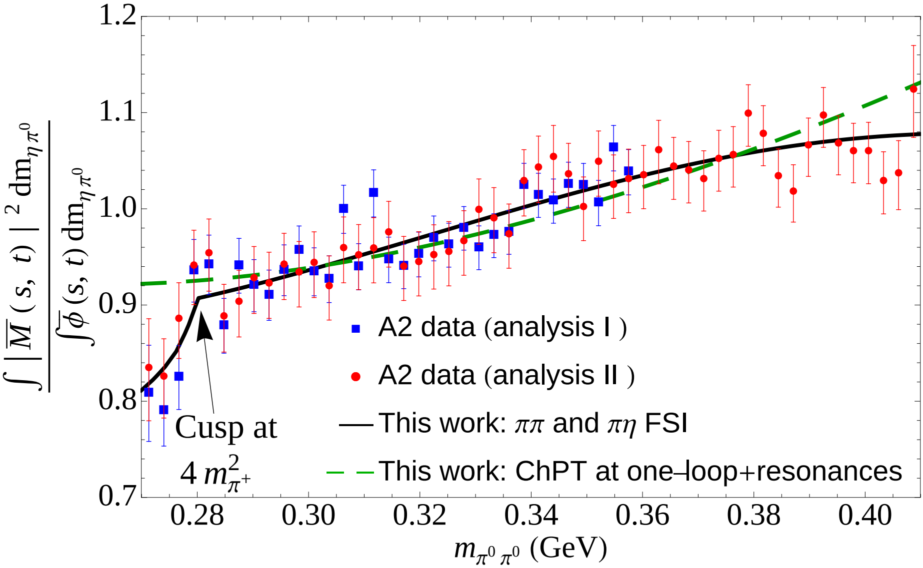
$$m^{IJ}(s) = \frac{1}{32\pi}\frac{s}{\lambda(s,m^2_{\eta'},m^2_\eta)^{1/2}\lambda(s,m^2_\pi,m^2_\pi)^{1/2}}\int_{t_{\min}}^{t_{\max}} dtP_J(\cos\theta_s)\mathcal{M}^I(s,t,u)$$

$$\cos\theta_s = -\frac{s\left(m^2_{\eta'}+m^2_\eta+2m^2_\pi-s-2t\right)}{\lambda(s,m^2_{\eta'},m^2_\eta)^{1/2}\lambda(s,m^2_\pi,m^2_\pi)^{1/2}},$$

$$P_0(\cos\theta_s) = 1\,,\quad P_2(\cos\theta_s) = \frac{1}{2}\left[-1+3(\cos\theta_s)^2\right] \propto X^2$$

Fit results

S. Gonzàlez-Solís, E. Passemar; Eur. Phys. J. C78 (2018) no.9, 758; arXiv:1807.04313



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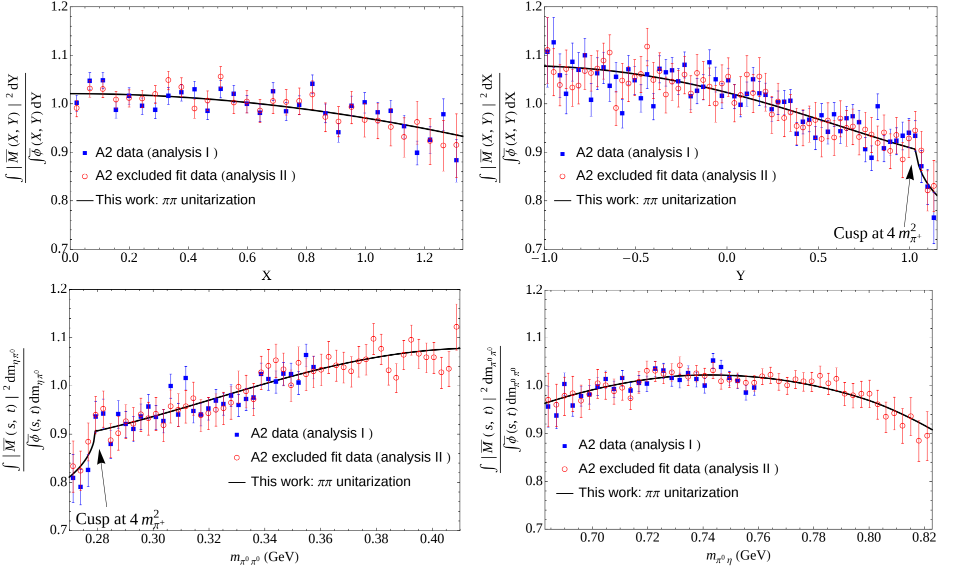
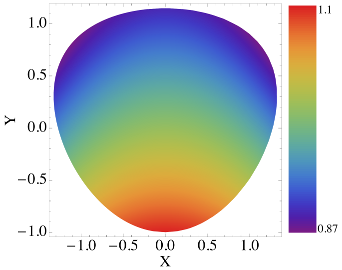
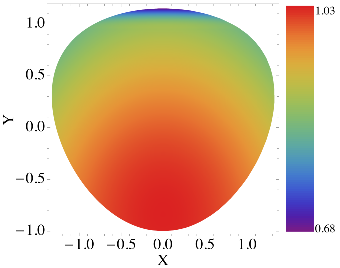


Fit results

- Fit 1: $M_S = 1001(24)$ MeV, $c_d = c_m = 29.5(1.8)$ MeV, $\tilde{c}_d = 17.0(1.0)$ MeV, $\tilde{c}_m = \tilde{c}_d$, $\chi^2_{\rm dof} = 1.12$
- Dalitz plot parameters:

Dalitz parameters	Our fit [$\pi\pi$ FSI]	Our Fit [ChPT]	A2 coll.
$a[Y]$	$-0.075(9)$	$-0.095(6)$	$-0.074(8)(6)$
$b[Y^2]$	$-0.051(1)$	$0.005(1)$	$-0.063(14)(5)$
$d[X^2]$	$-0.049(14)$	$-0.037(5)$	$-0.050(9)(5)$





- Perturbative expansion supplemented by S-wave $\pi\eta$ FSI

$$\mathcal{M}^{I=1}_{\eta'\to\eta\pi\pi}(s,t,u,\cos\theta_t,\cos\theta_u) = \mathcal{M}(s,t,u)^{(2)+\mathrm{Res}+\mathrm{Loop}}$$

$$+32\pi P_0(\cos\theta_t)\frac{m^{10}_{\eta'\to\eta\pi\pi}(t)^{(2)+\mathrm{Res}+\mathrm{Loop}}}{1+g_{\pi\eta}(t)t^{10}_{\pi\eta}(t)^{(2)+\mathrm{Res}+\mathrm{Loop}}}$$

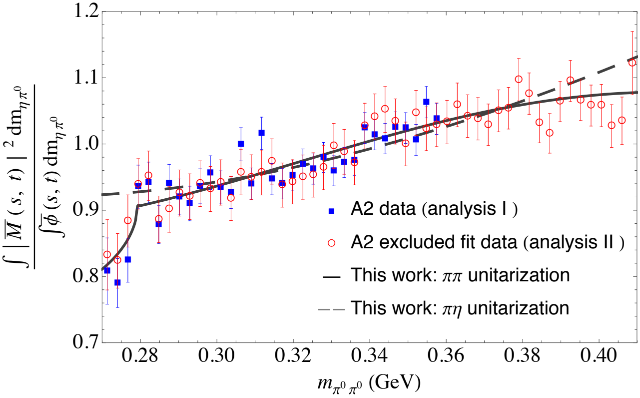
$$+32\pi P_0(\cos\theta_u)\frac{m^{10}_{\eta'\to\eta\pi\pi}(u)^{(2)+\mathrm{Res}+\mathrm{Loop}}}{1+g_{\pi\eta}(u)t^{10}_{\pi\eta}(u)^{(2)+\mathrm{Res}+\mathrm{Loop}}}$$

$$-32\pi P_0(\cos\theta_t)m^{10}_{\eta'\to\eta\pi\pi}(t)^{(2)+\mathrm{Res}+\mathrm{Loop}} - 32\pi P_0(\cos\theta_u)m^{10}_{\eta'\to\eta\pi\pi}(u)^{(2)+\mathrm{Res}+\mathrm{Lo}}$$

Fit results including $\pi\eta$ FSI

- Dalitz plot parameters:

Dalitz parameters	$\pi\eta$ FSI	$\pi\pi$ FSI	ChPT	A2 coll.
$a[Y]$	−0.094(6)(9)	−0.075(9)	−0.095(6)	−0.074(8)(6)
$b[Y^2]$	0.005(1)(1)	−0.051(1)	0.005(1)	−0.063(14)(5)
$d[X^2]$	−0.031(5)(4)	−0.049(14)	−0.037(5)	−0.050(9)(5)



INCLUSION OF $\pi\pi$ AND $\pi\eta$ FINAL-STATE INTERACTIONS

- Perturbative expansion + S-and D-wave $\pi\pi$ and S-wave $\pi\eta$ FSI

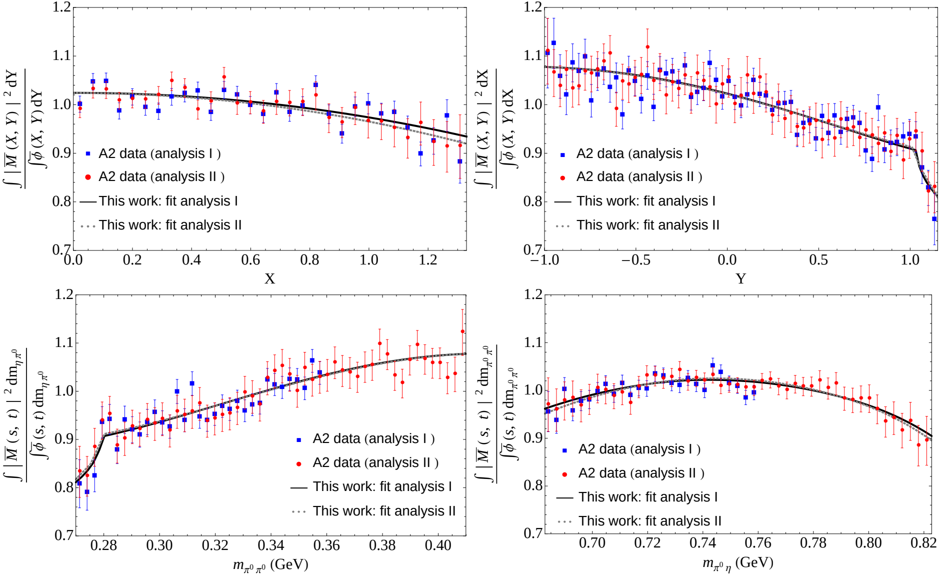
$$
\begin{aligned}
\mathcal{M}(s,t,u,\cos\theta_{s,t,u}) = \;& \mathcal{M}(s,t,u)^{(2)+\text{Res}+\text{Loop}} \\
&+32\pi P_0(\cos\theta_s)\frac{m^{00}_{\eta'\to\eta\pi\pi}(s)^{(2)+\text{Res}+\text{Loop}}}{1+g_{\pi\pi}(s)t^{00}_{\pi\pi}(s)^{(2)+\text{Res}+\text{Loop}}} \\
&+160\pi P_2(\cos\theta_s)\frac{m^{02}_{\eta'\to\eta\pi\pi}(s)^{(2)+\text{Res}+\text{Loop}}}{1+g_{\pi\pi}(s)t^{02}_{\pi\pi}(s)^{(2)+\text{Res}+\text{Loop}}} \\
&-32\pi P_0(\cos\theta_s)m^{00}_{\eta'\to\eta\pi\pi}(t)^{(2)+\text{Res}+\text{Loop}} - 160\pi P_2(\cos\theta_s)m^{02}_{\eta'\to\eta\pi\pi}(u)^{(2)+\text{Res}+\text{Loo}} \\
&+32\pi P_0(\cos\theta_t)\frac{m^{10}_{\eta'\to\eta\pi\pi}(t)^{(2)+\text{Res}+\text{Loop}}}{1+g_{\pi\eta}(t)t^{10}_{\pi\eta}(t)^{(2)+\text{Res}+\text{Loop}}} \\
&+32\pi P_0(\cos\theta_u)\frac{m^{10}_{\eta'\to\eta\pi\pi}(u)^{(2)+\text{Res}+\text{Loop}}}{1+g_{\pi\eta}(u)t^{10}_{\pi\eta}(u)^{(2)+\text{Res}+\text{Loop}}} \\
&-32\pi P_0(\cos\theta_t)m^{10}_{\eta'\to\eta\pi\pi}(t)^{(2)+\text{Res}+\text{Loop}} - 32\pi P_0(\cos\theta_u)m^{10}_{\eta'\to\eta\pi\pi}(u)^{(2)+\text{Res}+\text{Loop}}
\end{aligned}
$$

Fits to A2 data: $\pi\pi$ and $\pi\eta$ FSI

- Fit restrictions: $M_S = M_{S_8} = M_{S_1} = M_{a_0}$ and $a_{\pi\eta} = 2.0^{+3.1}_{-3.4}$ (Guo'11)

Parameter	Analysis I		Analysis II	
	Fit 1	Fit 2	Fit 1	Fit 2
M_S	1017(68)(24)	999(33)(23)	1040(79)(28)	1020(48)(28)
c_d	30.4(4.8)(9)	29.1(2.4)(1.6)	32.0(5.3)(9)	30.9(3.4)(2.2)
c_m	$= c_d$	$= 41.1(1)$	$= c_d$	$= 41.1(1)$
$\tilde{c}_d$	17.6(2.8)(5)	16.8(1.4)(9)	18.5(2.8)(5)	17.8(2.0)(1.3)
$\tilde{c}_m$	$= \tilde{c}_d$	$= 18.9(9)$	$= \tilde{c}_d$	$= 18.9(9)$
$a_{\pi\pi}$	0.76(61)(6)	0.34(22)(19)	0.98(58)(9)	0.57(38)(20)
χ^2_{dof}	1.12	1.12	1.23	1.23
$a[Y]$	$-$ **0.074(7)(8)**	$-$**0.073(6)(9)**	$-$**0.071(6)(8)**	$-$**0.070(6)(9)**
$b[Y^2]$	$-$**0.049(1)(2)**	$-$**0.054(1)(2)**	$-$**0.050(2)(1)**	$-$**0.054(1)(1)**
$d[X^2]$	$-$**0.047(8)(4)**	$-$**0.047(2)(4)**	$-$**0.055(6)(4)**	$-$**0.055(6)(4)**
$\kappa_{03}[Y^3]$	0.001	0.003	0.001	0.002
$\kappa_{21}[YX^2]$	-0.004	-0.005	-0.005	-0.005
$\kappa_{22}[Y^2X^2]$	0.001	0.002	0.002	0.002

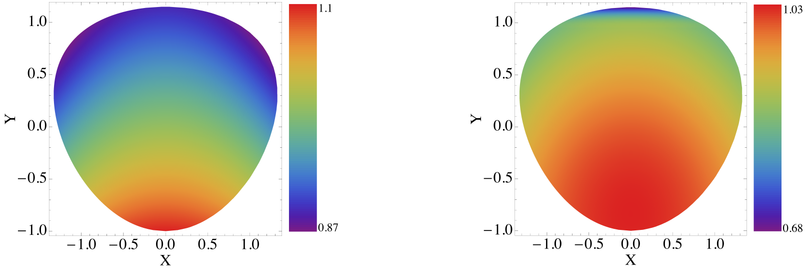
Fits to A2 data: $\pi\pi$ and $\pi\eta$ FSI



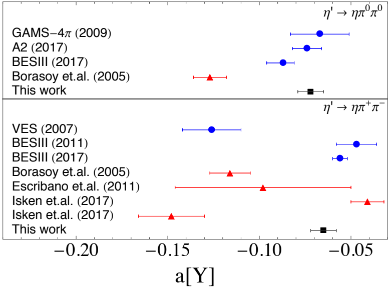
Fit results

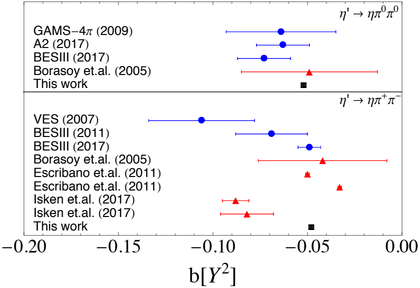
- Dalitz plot parameters:

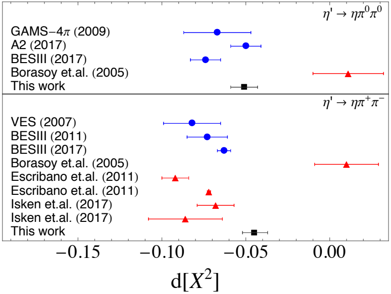
Dalitz	$\pi\pi + \pi\eta$	$\pi\eta$	$\pi\pi$	ChPT	A2 coll.
$a[Y]$	$-0.072(7)(8)$	$-0.094(6)(9)$	$-0.075(9)$	$-0.095(6)$	$-0.074(8)(6)$
$b[Y^2]$	$-0.052(1)(2)$	$0.005(1)(1)$	$-0.051(1)$	$0.005(1)$	$-0.063(14)(5)$
$d[X^2]$	$-0.051(8)(6)$	$-0.031(5)(4)$	$-0.049(14)$	$-0.037(5)$	$-0.050(9)(5)$



Dalitz Plot Parameters



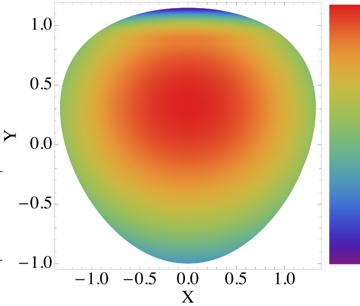




THE ROLE OF THE D-WAVE $\pi\pi$ FSI

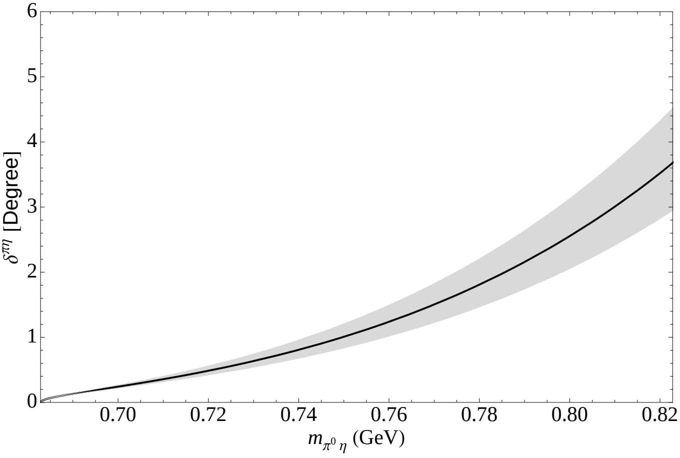
Parameter	Analysis I Fit 1 (with D-wave)	Fit 1 (w/o D-wave)
M_S	1017(68)(24)	996(66)(25)
c_d	30.4(4.8)(9)	23.3(3.5)(1.5)
c_m	$= c_d$	$= c_d$
$\tilde{c}_d$	17.6(2.8)(5)	13.5(2.0)(9)
$\tilde{c}_m$	$= \tilde{c}_d$	$= \tilde{c}_d$
$a_{\pi\pi}$	0.76(61)(6)	2.01(1.61)(71)
χ^2_{dof}	1.12	1.24
$a[Y]$	**−0.074(7)(8)**	**−0.091(9)(4)**
$b[Y^2]$	**−0.049(1)(2)**	**−0.013(1)(5)**
$d[X^2]$	**−0.047(8)(4)**	**−0.031(6)(3)**

$|M(X,Y)_{\mathrm{Full}}|^2/|M(X,Y)_{\mathrm{D-wave=0}}|^2$



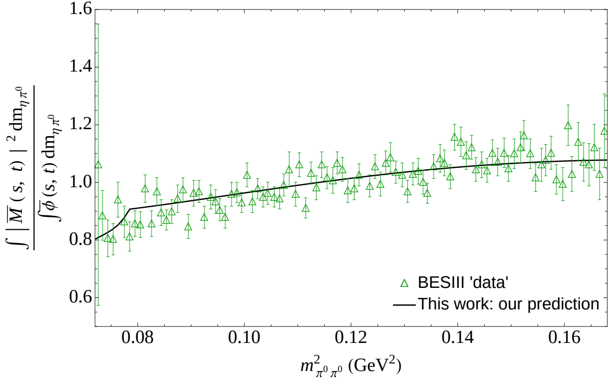
$\pi\eta$ PHASE

- Extraction of the $\pi\eta$ phase shift within the phase space



Comparison with BESIII 2017 experimental data

- Data is not publicly available [BESIII Coll. 1709.04627]
- Less events than A2 (351016 vs $1.2 \cdot 10^5$)
- No cusp structure seen
- Contrary to A2 both $\eta' \to \eta\pi^0\pi^0/\eta\pi^+\pi^-$ are measured



$\chi^2_{\text{dof}} \sim 1.1$

- $\eta' \to \eta\pi\pi$ ideal to test ChPT extensions
- $\eta' \to \eta\pi\pi$ analyzed within $U(3)$ ChPT at one-loop with resonances
- Two-particle FSI through N/D
- Dalitz plot parameters:
  - Y-variable is linear in s: Importance of $\pi\pi$ FSI
  - X-variable appear in the form $\cos\theta_s = Xf(Y)$: Importance of the D-wave
  - $\pi\eta$ FSI effects are small
- **Important experimental activities**: A2, BESIII, GlueX
- A lot of **interesting physics** to be done in the η-η' sector

Unitarity Violations