Hybrid to standard bottomonium transitions

Jaume Tarrús Castellà

Institut de Física d'Altes Energies $\rightarrow$ Indiana University

with: Emilie Passemar (Indiana University)

based on: **Phys.Rev.D 104 (2021) 3, 034019**

contact: jtarrus@ifae.es

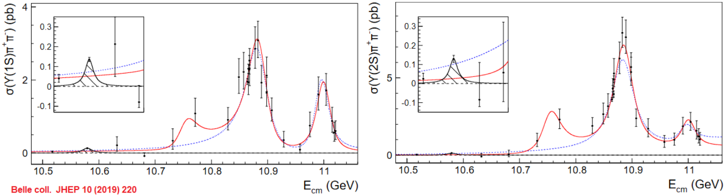
PWA12/ATHOS7, September 6th 2021.

What is the nature of exotic quarkonium?

- Not many J^{PC} are easily accessible experimentally:
  - It is difficult to use spectrum predictions to validate different approaches.
  - Cannot test heavy quark spin symmetry multiplets in different approaches.

$\Rightarrow$ The nature of many exotic quarkonium states is still not settled.

- On the other hand information on decay channels is always available.
- Many exotic states discovered in channels with standard quarkonium and light-quark mesons.



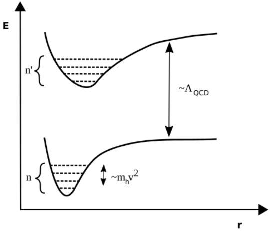
Belle coll. JHEP 10 (2019) 220

- We have studied these transitions in nonrelativistic EFT incorporating the multipole expansion.

Exotic Quarkonium in Effective Field theory

Energy gaps between scales

- Nonrelativistic heavy quarks: $m_Q \gg \Lambda_{\rm QCD}$ (QCD $\to$ NRQCD).
- Adiabatic expansion: $\Lambda_{\rm QCD} \gg m_Q v^2$ (NRQCD $\to$ BOEFT).



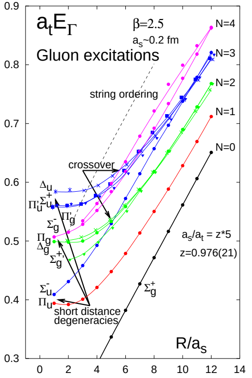
Born-Oppenheimer picture of exotic quarkonium:

- Static energies: levels of the light d.o.f for static heavy quarks.
- Heavy quarks bound states in the static energies.

Short Distance

- Multipole expansion: $m_Q v \gg \Lambda_{\rm QCD}$ (weakly coupled pNRQCD).

- Quenched lattice NRQCD.



Juge, Kuti, Morningstar Phys.Rev.Lett.90 (2003) Recent precision computation: Capitani et al Phys.Rev.D 99 (2019)

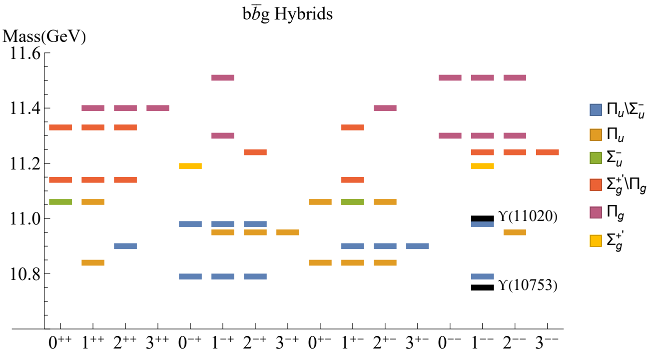
- The static energies of a $Q\bar{Q}$ system are defined as the energies of the eigenstates of NRQCD in the static limit.

$$E_{\kappa^p\Lambda_\eta}(\boldsymbol{r}) = \lim_{t\to\infty}\frac{i}{t}\ln\left\langle \mathcal{Q}^\dagger_{\kappa^p} \quad \mathcal{Q}_{\kappa^p}\right\rangle$$

- Nonperturbative quantity.
- Cylindrical symmetry group $D_{\infty\,h}$.

Heavy Hybrids: Bottomonium Spectrum at leading order

Berwein, Brambilla, JT, Vairo Phys.Rev.D92 (2015); Pineda, JT Phys.Rev.D 100 (2019)



There are only 3 neutral exotic bottomonium states, all 1^{--}:

- $\Upsilon(10753)$ mass within 40 MeV of ground state hybrid.
- $\Upsilon(10860)$ lays very close to B mesons pair threshold, likely a molecular state.
- $\Upsilon(11020)$ mass within 20 MeV of first excited hybrid.

Quarkonium states in pNRQCD

- Potential NRQCD (pNRQCD) is and EFT which incorporates the heavy quark mass and multipole expansions. Pineda, Soto Nucl.Phys.B Proc.Suppl. 64 (1998); Brambilla, Pineda, Soto, Vairo Nucl.Phys.B 566 (2000)
- S heavy quark pair singlet field, O heavy quark pair octet field.
- The standard quarkonium static states are just

$$|\mathbf{R},\mathbf{r};\Sigma_g^+\rangle = S^\dagger(\boldsymbol{R},\boldsymbol{r})\,|0\rangle\,.$$

- The static potential corresponds to the Σ_g^+ static energy

$$V^{(0)}_{\Sigma_g^+}(r) = \lim_{t\to\infty}\frac{i}{t}\ln\langle\mathbf{R},\mathbf{r};\Sigma_g^+;t/2|\mathbf{R},\mathbf{r};\Sigma_g^+;-t/2\rangle = E^{(0)}_{\Sigma_g^+}(r)$$

- A general standard quarkonium state

$$|S_m\rangle = \int d^3\boldsymbol{r}\,d^3\boldsymbol{R}\,\phi^{(m)}(\boldsymbol{R},\boldsymbol{r})|\boldsymbol{R},\boldsymbol{r};\Sigma_g^+\rangle\,,$$

- The wave function $\phi^{(m)}$ is obtained solving the Shrödinger eq. with $V^{(0)}_{\Sigma_g^+}(r)$.

Hybrid quarkonium states in pNRQCD

- The Hybrid static states contain a 1^{+-}, color octet, gluonic operator $\boldsymbol{G}^a_B \sim Z_B^{-1/2}\boldsymbol{B}^a + \ldots$:

$$|\mathbf{R},\mathbf{r};\lambda\rangle = \hat{\boldsymbol{r}}_\lambda \cdot \boldsymbol{G}^a_B(\boldsymbol{R})O^{a\,\dagger}(\boldsymbol{R},\boldsymbol{r})\,|0\rangle$$

- The static potentials

$$V^{(0)}_\lambda(r) = \lim_{t\to\infty}\frac{i}{t}\ln\langle\mathbf{R},\mathbf{r};\lambda;t/2|\mathbf{R},\mathbf{r};\lambda;-t/2\rangle = E^{(0)}_{|\lambda|}(r)$$

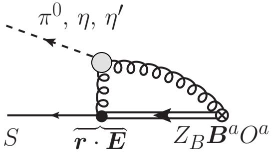
$E^{(0)}_0(r) = E^{(0)}_{\Sigma_u^-}(r)$ and $E^{(0)}_{|\pm1|}(r) = E^{(0)}_{\Pi_u}(r)$ from the lattice.

- A general hybrid state

$$|H_n\rangle = \int d^3\boldsymbol{r}d^3\boldsymbol{R}\sum_\lambda \psi^{(n)}_\lambda(\boldsymbol{R},\boldsymbol{r})|\boldsymbol{R},\boldsymbol{r};\lambda\rangle\,.$$

- The wave functions $\psi^{(n)}_\lambda$ are obtained from the coupled Shrödinger eqs. with $V^{(0)}_{\Sigma_u^-}(r)$ and $V^{(0)}_{\Pi_u}(r)$ Berwein, Brambilla, JTC, Vairo Phys.Rev.D92 (2015).
- Due to the mixing the hybrid wave functions are eigenstates of $(\boldsymbol{L}_{\bar{Q}Q}+\boldsymbol{S}_1)^2$ with eigenvalue $\ell(\ell+1)$.

LO Transitions



- Transitions from the LO singlet-octet operator:

$$\langle S_m \mathcal{O}_\pi | g \mathrm{Tr}\left[\mathrm{S}^\dagger \boldsymbol{r}\cdot\boldsymbol{E}\,\mathrm{O}\right]|H_n\rangle \sim Z_B^{-1/2} \underbrace{\langle \mathcal{O}_\pi | g^2 \boldsymbol{E}\cdot\boldsymbol{B}|0\rangle}_{\text{l.q. meson production}} \underbrace{\langle \phi^{(m)} | \boldsymbol{r}\cdot\hat{\boldsymbol{r}}_\lambda | \psi^{(n)}_\lambda\rangle}_{\text{Heavy quark matrix element}}$$

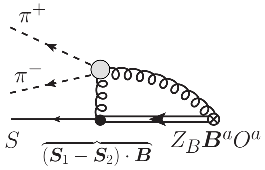
- Z_B can be related to the gluon condensate.
- Heavy quark matrix element:
  - Selection rules for final quarkonium states: $\Delta s = 0$, $\ell = l$.
  - $\Upsilon(10753)$ and $\Upsilon(11020)$ decay into $h_b(m^1P_1)$ and light quark mesons.
- Light-quark meson production:
  - Allowed final light-quark states: 0^{-+}, $I = 0$ such as π^0, η, η', η-like resonances or odd numbers of pseudoscalar mesons.
  - Matrix elements for production of π^0, η, η' can be determined from $U(1)_A$ anomaly and a mixing scheme. Feldmann, Kroll, Stech, Phys.Rev.D58 (1998); Kroll, Mod. Phys. Lett. A20 (2005)

LO Transitions

- LO Transition widths:

$$\Gamma_{\Upsilon(10753)\to h_b(1P)\pi^0} = 2.57(\pm 1.03)_{\rm m.e.}(\pm 0.14)_{Z_B}(\pm 0.16)_{\omega_{\pi^0}} \text{ keV}$$
$$\Gamma_{\Upsilon(10753)\to h_b(1P)\eta} = 2.29(\pm 0.92)_{\rm m.e.}(\pm 0.13)_{Z_B}(\pm 0.08)_{\omega_\eta} \text{ MeV}$$
$$\Gamma_{\Upsilon(10753)\to h_b(2P)\pi^0} = 0.168(\pm 0.067)_{\rm m.e.}(\pm 0.009)_{Z_B}(\pm 0.010)_{\omega_{\pi^0}} \text{ keV}$$
$$\Gamma_{\Upsilon(11020)\to h_b(1P)\pi^0} = 2.04(\pm 0.82)_{\rm m.e.}(\pm 0.11)_{Z_B}(\pm 0.13)_{\omega_{\pi^0}} \text{ keV}$$
$$\Gamma_{\Upsilon(11020)\to h_b(1P)\eta} = 2.04(\pm 0.81)_{\rm m.e.}(\pm 0.11)_{Z_B}(\pm 0.07)_{\omega_\eta} \text{ MeV}$$
$$\Gamma_{\Upsilon(11020)\to h_b(1P)\eta'} = 9.23(\pm 3.69)_{\rm m.e.}(\pm 0.51)_{Z_B}(\pm 0.39)_{\omega_{\eta'}} \text{ MeV}$$
$$\Gamma_{\Upsilon(11020)\to h_b(2P)\pi^0} = 0.104(\pm 0.042)_{\rm m.e.}(\pm 0.006)_{Z_B}(\pm 0.006)_{\omega_{\pi^0}} \text{ keV}$$
$$\Gamma_{\Upsilon(11020)\to h_b(2P)\eta} = 81.8(\pm 32.7)_{\rm m.e.}(\pm 4.6)_{Z_B}(\pm 2.7)_{\omega_\eta} \text{ keV}$$

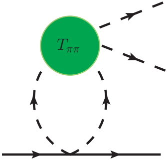
- Uncertainties labeled by the origin m.e.=multipole expansion, ω=Production matrix element.



- Transitions from the NLO singlet-octet operator

$$\langle S_m \mathcal{O}_{\pi\pi}|\frac{g c_F}{m_Q}\mathrm{Tr}\left[\mathrm{S}^\dagger(\boldsymbol{S}_1-\boldsymbol{S}_2)\cdot\boldsymbol{B}\,\mathrm{O}\right]|H_n\rangle \sim \frac{c_F}{m_Q}Z_B^{-1/2}\underbrace{\langle\mathcal{O}_{\pi\pi}|\boldsymbol{B}^2|0\rangle}_{\text{l.q. meson production}}\underbrace{\langle\phi^{(m)}|(\boldsymbol{S}_1-\boldsymbol{S}_2)\cdot\hat{\boldsymbol{r}}_\lambda|\psi_\lambda^{(n)}\rangle}_{\text{Heavy quark matrix element}}$$

- Heavy quark matrix element:
  - Selection rules: $\Delta s = 1$, $l = \ell \pm 1$.
  - $\Upsilon(10753)$ and $\Upsilon(11020)$ decay into $\Upsilon(m^3S_1)$ or $\Upsilon(m^3D_1)$ and light quark mesons.
- Light-quark meson production:
  - Allowed final light-quark states: 0^{++} and $I=0$ such as $\pi^+\pi^-$, K^+K^-, pairs of π^0 or η as well as f_0 resonances.
  - We use a dispersive representation for the matrix elements for the production $\pi^+\pi^-$, K^+K^-. Donoghue, Gasser, Leutwyler, Nucl.Phys.B343 (1990); Moussallam, Eur.Phys.J.C 14 (2000); Celis, Cirigliano, Passemar, Phys.Rev.D89 (2014)



- Decompose $\langle \mathcal{O}_{\pi\pi} | \boldsymbol{B}^2 | 0 \rangle$ into S ($F^{(0)}$) and D ($F^{(2)}$) wave pieces.
- From **Watson's Theorem** we obtain the imaginary part of $F^{(I)}$ corresponding to **two-pion and two-kaon rescattering**.

Muskhelishvili-Omnès problem:

Muskhelishvili, *Singular integral equations*; Omnes, Nuovo Cim.8 (1958)

- Imaginary part is known.
- Analytic in the complex s-plane, except on the cuts.
- Real in the real s axis below the cuts.

$\Rightarrow$ A general form of the form factors is

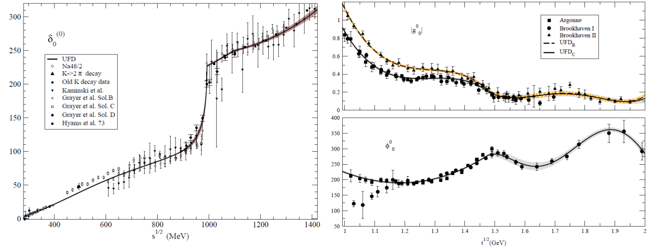
$$n_P F_P^{(I)}(s) = \Omega_{PP'}^{(I)}(s) Q_{P'}^{(I)}(s)\,, \qquad P, P' = \pi,\, K$$

Dispersive representation: Omnès Functions

- Ω-matrix satisfies the singular integral equations

$$\boldsymbol{\Omega}(s) = \frac{1}{\pi}\int_{4m_\pi^2}^{\infty}\frac{ds'}{s'-s}\left(\boldsymbol{T}_l^0(s')\right)^*\boldsymbol{\Sigma}(s')\boldsymbol{\Omega}(s')$$

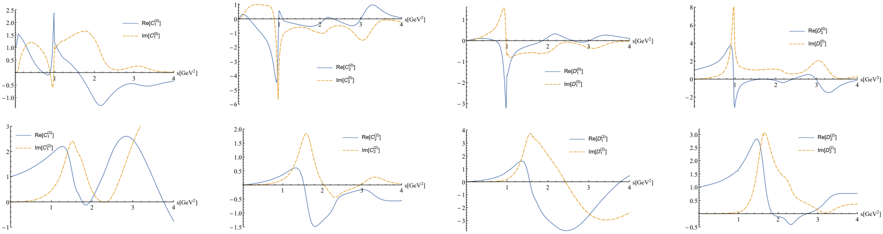
- Input needed: $\pi\pi \to \pi\pi$ and $\pi\pi \to K\bar{K}$ T-matrix for each partial wave.



- Parametrizations (and figure) from Garcia-Martin et al Phys.Rev.D83 (2011); Pelaez, Rodas Eur.Phys.J. C78 (2018)

Dispersive representation

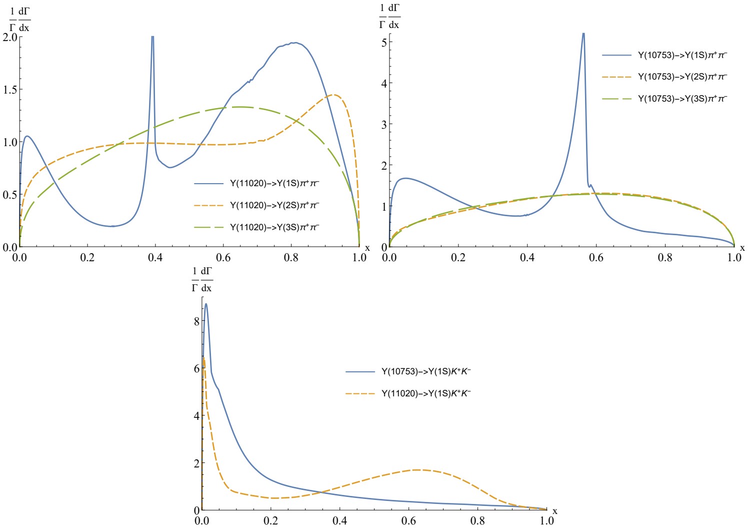
- (Numerical) $\Omega^{(I)}$ solutions Moussallam, Eur.Phys.J.C 14 (2000); Descotes-Genon Ph.D. thesis (2000)



- $\boldsymbol{Q}^{(I)}(s) = (Q_1^{(I)}, Q_2^{(I)})$ are subtraction polynomials obtained by matching to a chiral representation.
- Chiral low-energy constants determined using the Scale anomaly, Feynman-Hellmann theorem, and one free parameter from quarkonium transitions.

  Voloshin, Zakharov, Phys.Rev.Lett.45 (1980); Novikov, Shifman, Z.Phys.C8 (1981); Chivukula et al, Annals Phys. 192 (1989); Pineda, JTC, Phys.Rev.D100 (2019)

NLO Transitions differential widths



$x = (s - 4m^2_{\rm GB})/(m_{\bar{Q}Qg} - m_{\bar{Q}Q} - 4m^2_{\rm GB})$

NLO Transitions widths

- NLO transition widths:

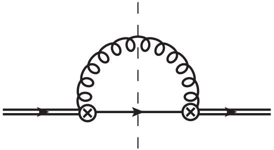
$$\Gamma_{\Upsilon(10753)\to\Upsilon(1S)\pi^+\pi^-} = 43.4(\pm 17.3)_{\rm m.e.}(\pm 2.4)_{Z_B}(\pm 8.6)_{\alpha_s}({}^{+0.5}_{-0.0})_{\kappa}\ \mathrm{keV}$$
$$\Gamma_{\Upsilon(10753)\to\Upsilon(2S)\pi^+\pi^-} = 2.75(\pm 1.10)_{\rm m.e.}(\pm 0.15)_{Z_B}(\pm 0.55)_{\alpha_s}({}^{+0.13}_{-0.12})_{\kappa}\ \mathrm{keV}$$
$$\Gamma_{\Upsilon(10753)\to\Upsilon(3S)\pi^+\pi^-} = 0.98(\pm 0.39)_{\rm m.e.}(\pm 0.05)_{Z_B}(\pm 0.19)_{\alpha_s}(\pm 0.03)_{\kappa}\ \mathrm{eV}$$
$$\Gamma_{\Upsilon(11020)\to\Upsilon(1S)\pi^+\pi^-} = 99.1(\pm 39.6)_{\rm m.e.}(\pm 5.5)_{Z_B}(\pm 19.7)_{\alpha_s}({}^{+26.3}_{-21.8})_{\kappa}\ \mathrm{keV}$$
$$\Gamma_{\Upsilon(11020)\to\Upsilon(2S)\pi^+\pi^-} = 3.96(\pm 1.58)_{\rm m.e.}(\pm 0.22)_{Z_B}(\pm 0.70)_{\alpha_s}({}^{-0.16}_{+0.17})_{\kappa}\ \mathrm{keV}$$
$$\Gamma_{\Upsilon(11020)\to\Upsilon(3S)\pi^+\pi^-} = 1.33(\pm 0.53)_{\rm m.e.}(\pm 0.07)_{Z_B}(\pm 0.27)_{\alpha_s}(\pm 0.02)_{\kappa}\ \mathrm{keV}$$
$$\Gamma_{\Upsilon(10753)\to\Upsilon(1S)K^+K^-} = 3.98(\pm 1.59)_{\rm m.e.}(\pm 0.22)_{Z_B}(\pm 0.79)_{\alpha_s}({}^{-0.50}_{+0.67})_{\kappa}\ \mathrm{keV}$$
$$\Gamma_{\Upsilon(11020)\to\Upsilon(1S)K^+K^-} = 5.93(\pm 2.37)_{\rm m.e.}(\pm 0.33)_{Z_B}(\pm 1.18)_{\alpha_s}({}^{+1.75}_{-1.18})_{\kappa}\ \mathrm{keV}$$

- Experimental **ranges** for the widths Belle col. JHEP 10, 220 (2019)

$$\Gamma^{\rm exp}_{\Upsilon(11020)\to\Upsilon(1S)\pi^+\pi^-} = 85^{+33}_{-36}\ \mathrm{keV} \Leftarrow \textbf{Promising agreement!}$$
$$\Gamma^{\rm exp}_{\Upsilon(11020)\to\Upsilon(2S)\pi^+\pi^-} = 120^{+105}_{-107}\ \mathrm{keV}$$
$$\Gamma^{\rm exp}_{\Upsilon(11020)\to\Upsilon(3S)\pi^+\pi^-} = 61^{+37}_{-38}\ \mathrm{keV}$$

Semi-inclusive widths

- If the energy gap between a hybrid and a standard quarkonium state is large semi-inclusive transition widths can be computed. Oncala, Soto Phys.Rev.D96 (2017)

$$\Gamma_{\text{s.i.}} = \langle H_n|\text{Im}[$$  $$]|H_n\rangle$$

$$\Gamma^{\text{LO}}_{\Upsilon(11020)\to h_b(1P)} = 20(\pm 9)_{\alpha_s}\ \text{MeV}$$
$$\Gamma^{\text{NLO}}_{\Upsilon(10753)\to \Upsilon(1S)} = 9.7(\pm 3.8)_{\alpha_s}\ \text{MeV}$$
$$\Gamma^{\text{NLO}}_{\Upsilon(11020)\to \Upsilon(1S)} = 7.3(\pm 2.5)_{\alpha_s}\ \text{MeV}$$
$$\Gamma^{\text{NLO}}_{\Upsilon(11020)\to \Upsilon(2S)} = 1.1(\pm 0.5)_{\alpha_s}\ \text{MeV}$$

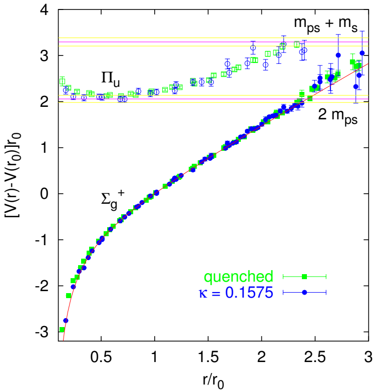
- The sum of semi-inclusive widths for $\Gamma^{\text{LO+NLO}}_{\Upsilon(11020)} = 28.4 \pm 9.4$ MeV is compatible with the experimental value of the total width $\Gamma^{\text{exp}}_{\Upsilon(11020)} = 24^{+8}_{-6}$ MeV.

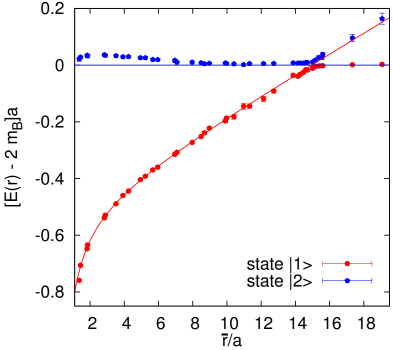
Summary

- We have computed the transition widths of $\Upsilon(10753)$ and $\Upsilon(11020)$ into standard botomonium states and light quark mesons.
- We have assumed that these two states are the first two lowest laying 1^{--} hybrid bottomonium states.
- EFT incorporating the heavy quark mass and multipole expansions (pNRQCD).
  - LO : $\Upsilon(10753)/\Upsilon(11020)$ to $h_b(mP)$ and π^0, η, η'.
  - NLO: $\Upsilon(10753)/\Upsilon(11020)$ to $\Upsilon_b(mS)$ and $\pi^+\pi^-$, K^+K^-.
- Total width, mass and transition width to $\Upsilon(1S)\pi^+\pi^-$ are consistent with the interpretation of $\Upsilon(11020)$ as a hybrid in the Born-Oppenheimer picture.

Thank you for your attention

SESAM/TCL Col. Phys.Rev.D62 (2000), Phys.Rev.D71 (2005)





- No significant difference between quenched and unquenched results.
- However with dynamic light quarks new states appear such as thresholds.