Freed-isobar technique on Dalitz plots

—

News since last time

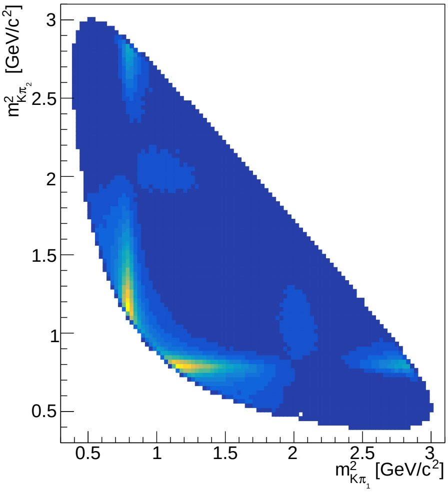
Fabian Krinner

Max Planck Institut für Physik

PWA12/ATHOS7

- Understand the processes leading to the final state

$D^+ \dashrightarrow K^- + \pi^+ + \pi^+$



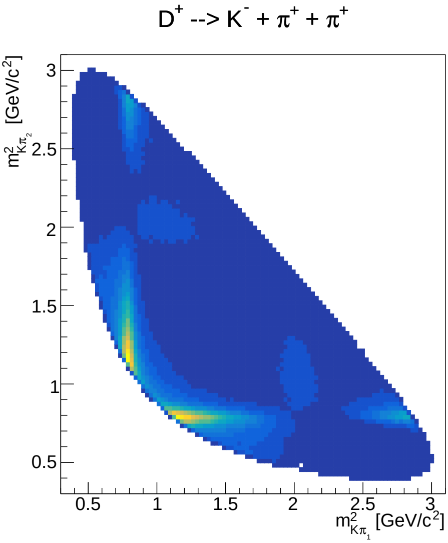
Inspired by CLEO:
Phys. Rev.**D78** 052001 (2008).

Partial-Wave Analysis (PWA)

Some basics

- Understand the processes leading to the final state
- Amplitude analysis:
  Describe the complex-valued amplitude of the process:

$$\sum_{i}^{\text{waves}} \mathcal{T}_i \mathcal{A}_i \left(\vec{\theta}\right)$$



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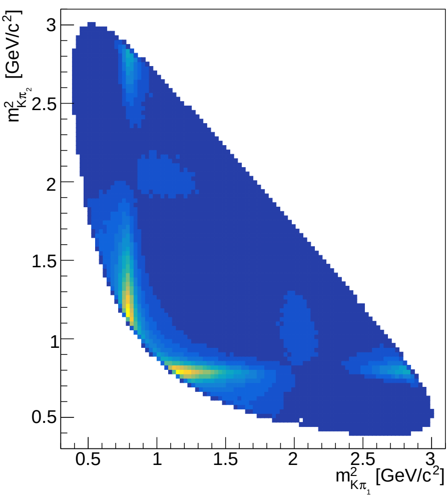

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- Measure only intensity distribution
- Goal: Learn about the amplitude
- Fit intensity distribution to the data (extended unbinned log-likelihood)

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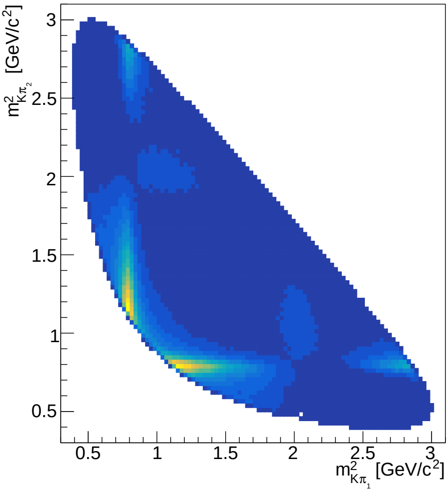

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Partial-Wave Analysis (PWA)
Modelling the amplitude

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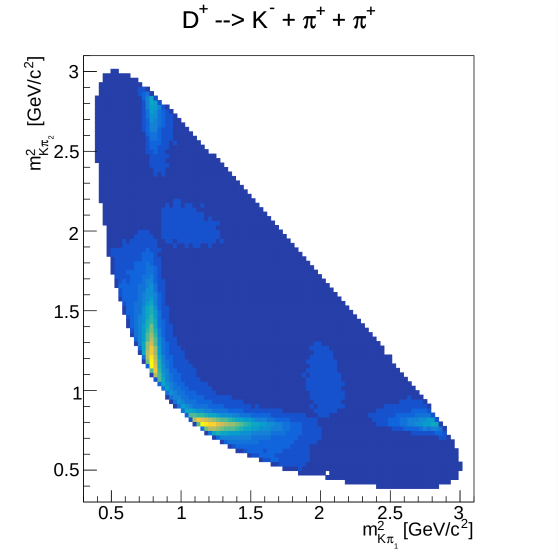

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  - Have to be known beforehand

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  - Re-scattering with 3rd particle
- Effects neglected or falsely attributed
  - “Leakage”

Freed-isobar method
Step-like isobar amplitudes

- Total intensity as function of phase-space variables $\vec{\theta}$:

$$\mathcal{I}(\vec{\theta}) = \left| \sum_{i}^{\text{waves}} \mathcal{T}_i \left[\psi_i\left(\vec{\theta}\right) \Delta_i\left(m_{\text{isob}}\right) + \text{Bose sym.} \right] \right|^2$$

Fit parameters: Production amplitudes $\mathcal{T}_i$

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$$\Delta_i\left(m_{\text{isob}}\right) \to \sum_{\text{bins}} \mathscr{T}_i^{\text{bin}} \Delta_i^{\text{bin}}\left(m_{\text{isob}}\right) \equiv [K\pi]_{J^{PC}}$$

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- Each m_{isob} bin: independent partial wave with $\mathcal{T}_i^{\text{bin}} = \mathcal{T}_i \mathscr{T}_i^{\text{bin}}$:

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Freed-isobar method
Step-like isobar amplitudes

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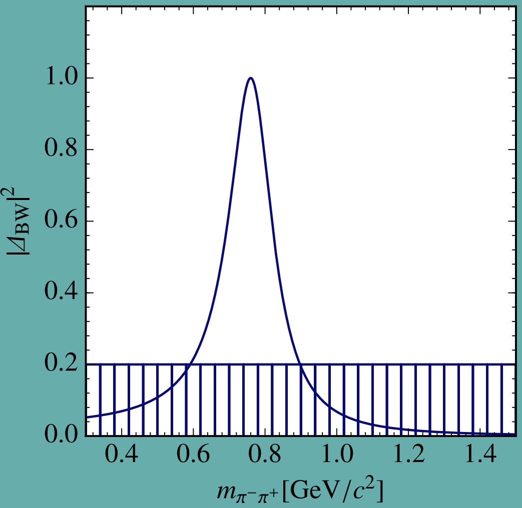
Illustration



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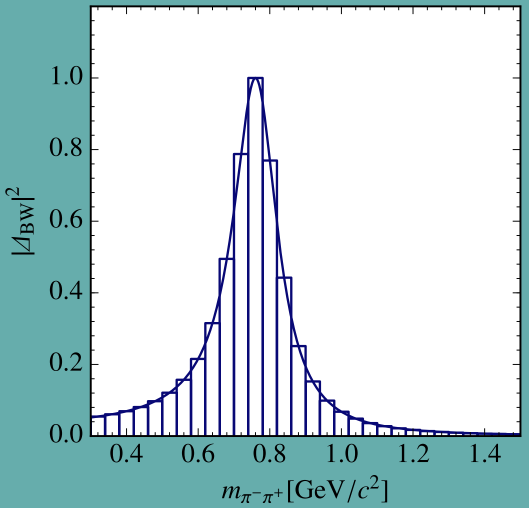
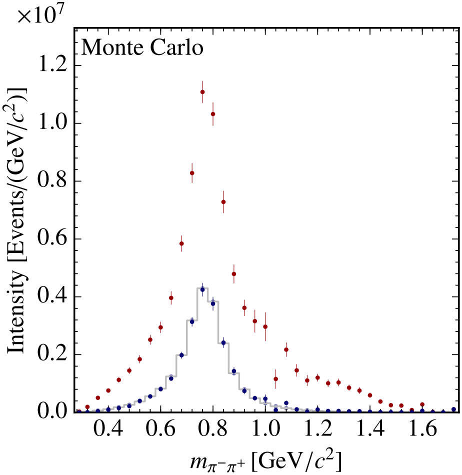
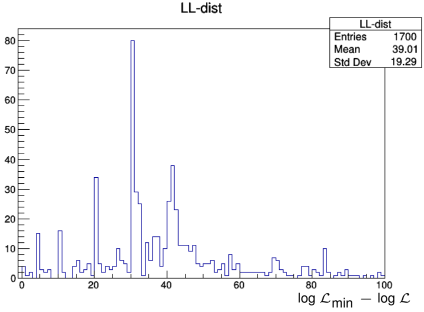


Status last time

- Often: Used for singlemost interesting of complicated waves
- Several waves simultaneously: Fit gets crazy
- $\rightarrow$ Zero modes present:
  - Freed waves can take specific shapes
  - Complete cancellation everywhere in phase-space
  - Complex-valued ambiguity
- Resolve with minimal assumptions
- $n_{\text{bins}} - 1$ degrees of freedom remain



- Fully freed fit of $[K\pi]_S$, $[K\pi]_P$, $[K\pi]_D$
- Random start values
- Many fit attempts
- Likelihood function is multimodal
- How to find best minimum?



Fully free fit
Multimodality

- Start values for all bins independent
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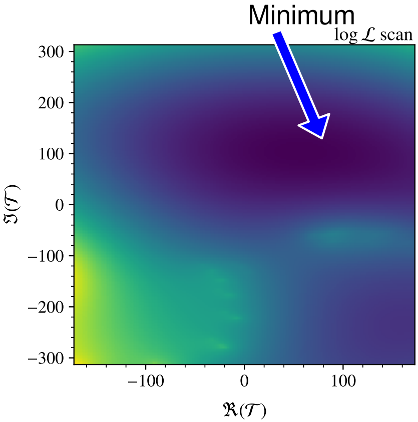

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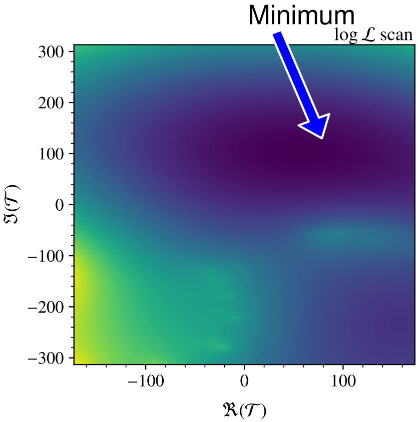

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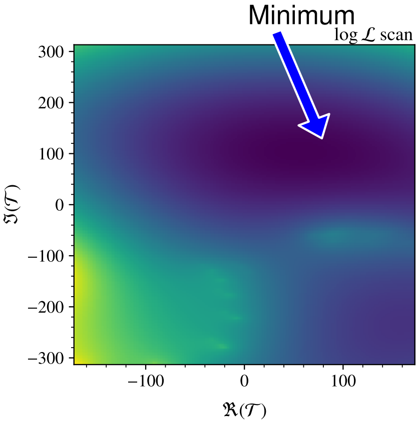

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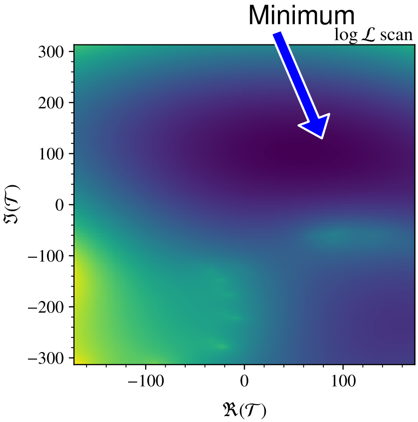

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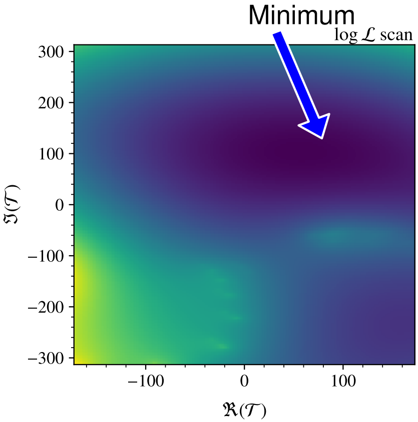
- Similar for other bins
- Approximately 2^{n_bins} local minima



News

News 1: Function basis
Current status

Discussed up to now: Step-like functions

Advantages:

- Fit parameters directly interpretable
- Easy to implement

Disadvantages:

- Discontinuous description of the amplitudes

In principle any function basis could be used to approximate amplitudes

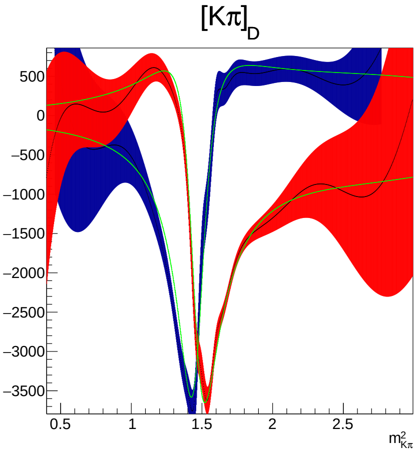
News 1: Function basis
Global functions

Idea 1: Global functions

- Theory: Amplitudes are analytical
- $\rightarrow$ Binned functions are not
- Use global function basis instead
  - Fourier-basis, polynomial basis

Problems:

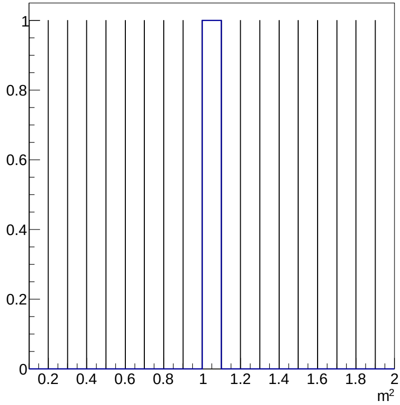
- Large overlap of basis functions:
  - Parameters not directly interpretable
  - Very multimodal
  - Suffer from artifacts
- Hard to adjust to narrow regions



News 1: Function basis
Splines

Compromise: Higher order splines

- Degree $n \to \mathbb{C}^{n-1}$
- Optimize computing time:
  Sum only non-zero functions
- Linear splines "spikes" suffice
  - Degree = 1
  - Fit parameters directly interpretable
- Artefacts for higher degrees (> 2)



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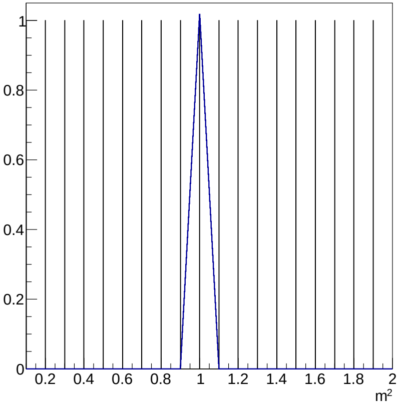

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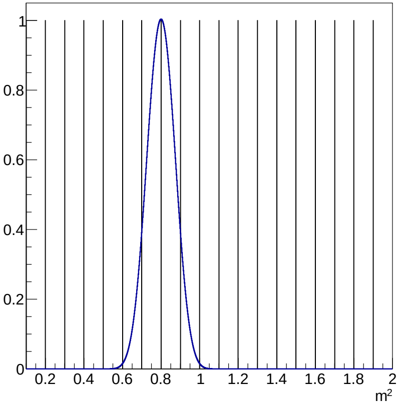

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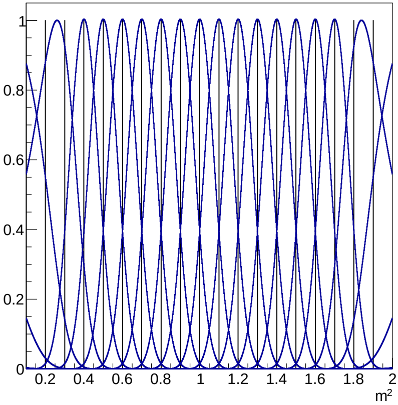

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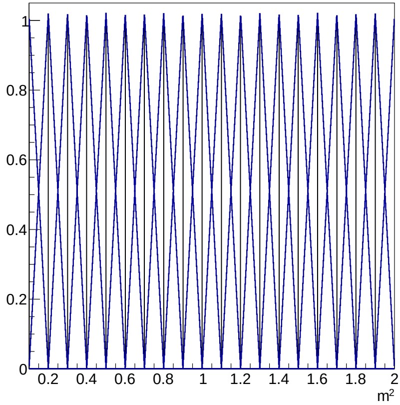

News 2: Adaptive binning

- Usual approach:
  - Equidistant binning
  - Maybe finer binned regions with known narrow resonances
- However: Large regions with no/broad resonances
  - Unnecessary degrees of freedom
- Often: rough resonance content known
  - Deviations and small signals of interest
  - Adapt binning to estimated resonance content

News 2: Adaptive binning
Optimization of the binning

- Goal: best resolution with least number of bins
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  - Adjust bin borders to minimize:

$$\int_{m^2_{\text{min}}}^{m^2_{\text{max}}} \mathrm{d}m^2 \left|\Delta_{\mathrm{BW}}(m^2) - \Delta_{\mathrm{approx}}\left(m^2\right)\right|^2$$

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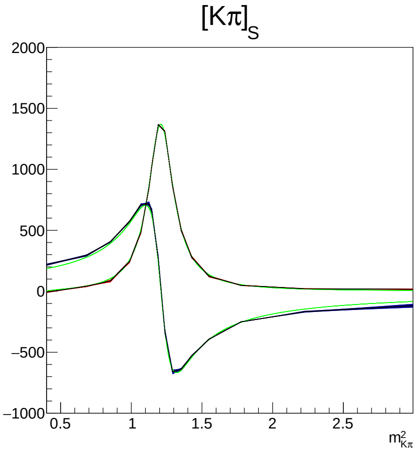
    coefficients in Δ_{approx} optimized analytically
- Avoid over-fitting: number of bins from MC studies
  - Compare fit quality with input model
  - $\sim$ 20 bins suffice

News 2: Adaptive binning

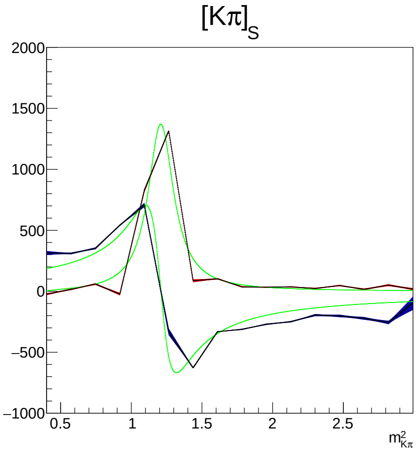
Test case

- Optimize binning for the $[K\pi]_S$ wave
- Fit generated MC data
- Compare to fit with equidistant binning
  - Mismatch over the whole range
  - Event actually finer binned regions



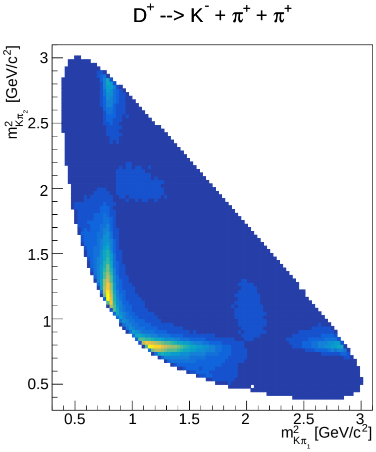
News 2: Adaptive binning
Test case

- Optimize binning for the $[K\pi]_S$ wave
- Fit generated MC data
- Compare to fit with equidistant binning
  - Mismatch over the whole range
  - Event actually finer binned regions



Test case study

- Two data-sets:
  - Only ground-states: $K_0^*(700)$, $K^*(892)$, $K_2^*(1430)$
  - Ground-states plus $K_0^*(1430)$ and $K^*(1410)$
- Optimize binning to ground states only
- Perform freed fits of S and P waves



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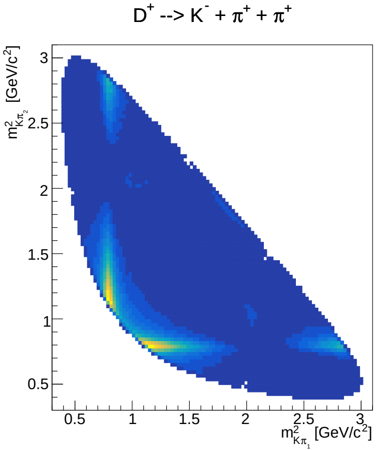
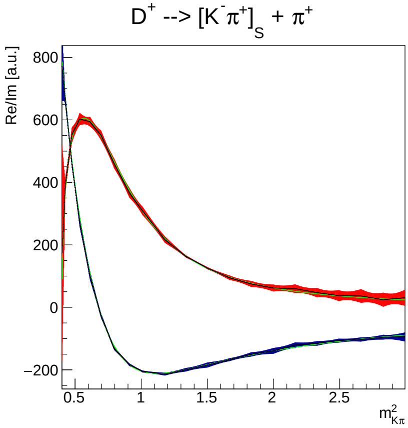


- Both waves reproduced nicely
- Works as expected
- However: Non-freed waves described perfectly



Test case study
Ground states only

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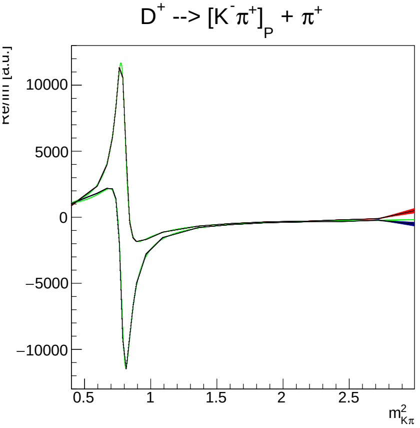
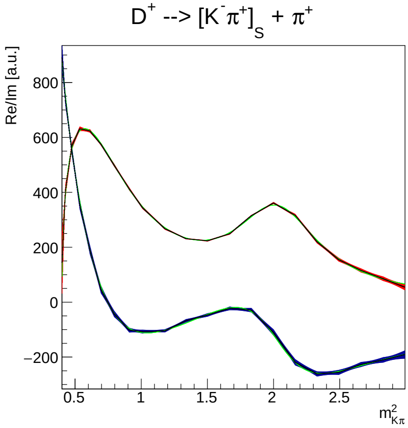


- Use data-set with excited states
- Allow also excited states in fixed waves
- Optimized binning can accomodate for new resonances



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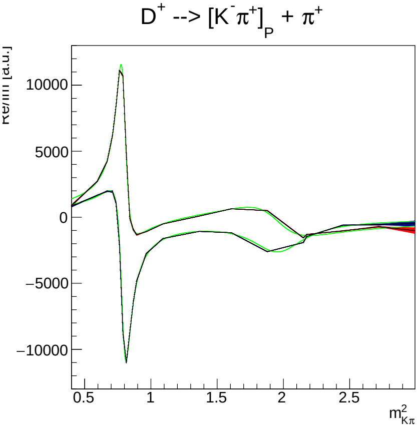

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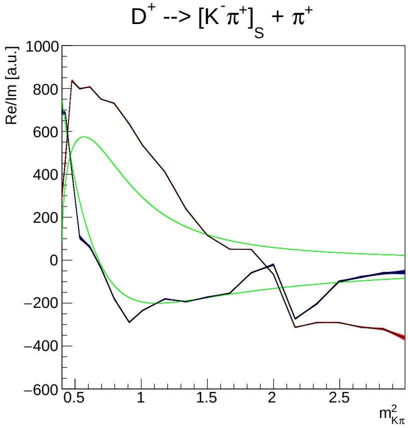
- Model fixed waves with ground-states only
- Freed fits are way off
- Leakage from other waves



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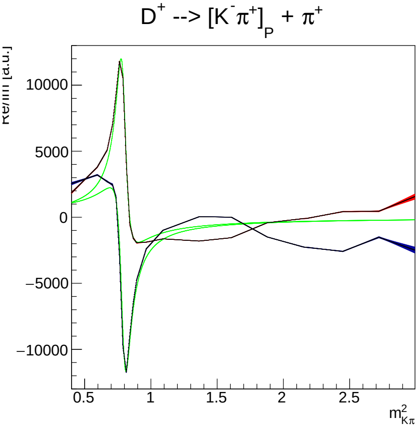
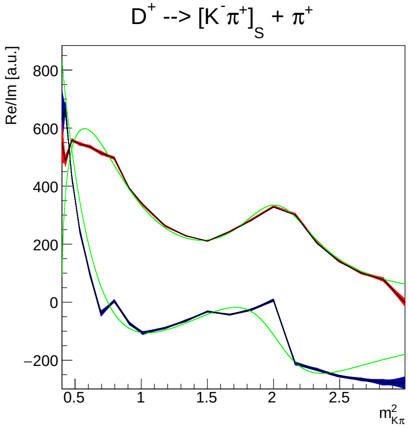


Test case study

Both waves freed

- Simultaneous freed fit recovers excited resonances
- No zero mode effects visible
  - Broader binning in some regions suppress zero mode



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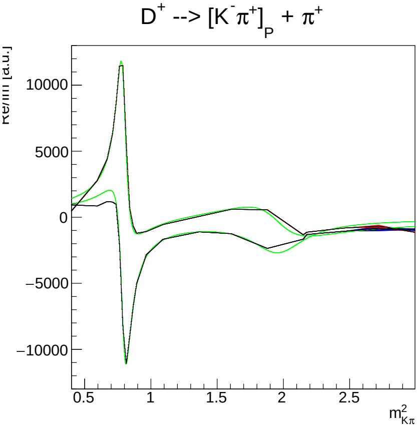

Summary & Conclusion

Freed-isobar PWA

- Versatile tool-set for detailed PWA
  - Adjust depending on analysis goals
- Function basis
  - Spike-functions most suitable
  - Better approximation than step-like functions
  - Higher orders: more overlap $\rightarrow$ artifacts
- Adaptive binning
  - Lower fit-complexity
  - Suppress zero-modes
  - Still resolve resonances
- Test case looks good (however not perfect)
- Freed-isobar method applicable to many cases
  - Diffractive hadron production
  - Dalitz plot decays
  - Hadronic τ decays

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Bad news

- Fully freed fit still not possible
- No *out-of-the-box* solution