$B \to D\bar{D}X$ for flavour anomalies

A K-matrix approach to $ee \to c\bar{c}$

PWA 12 / ATHOS 7

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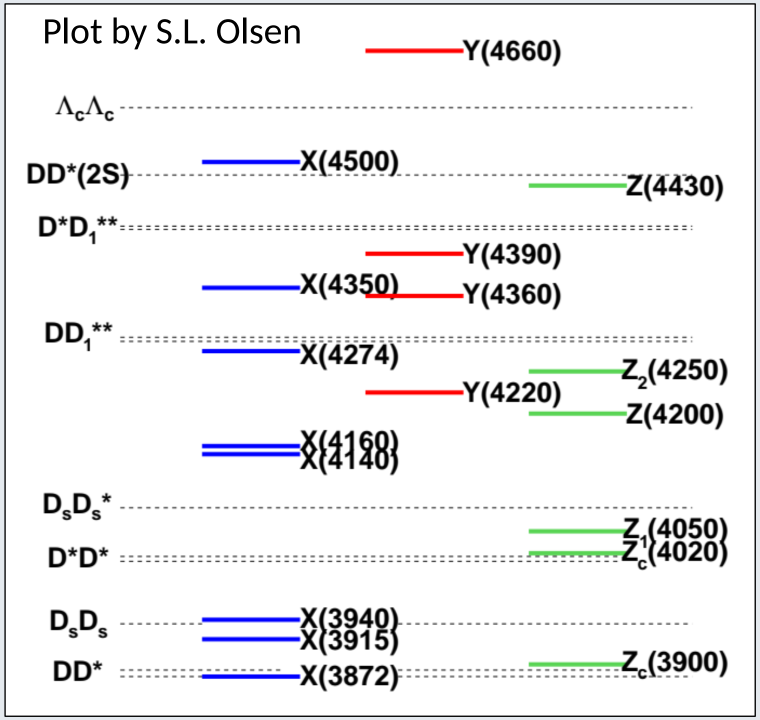
Outline

I. **Why do we study ee → $c\bar{c}$?**
II. **What data do we have?**
III. **What is the link with b → $s\ell\ell$**
IV. **Definition of the model and global fit**
V. **Preliminary results**

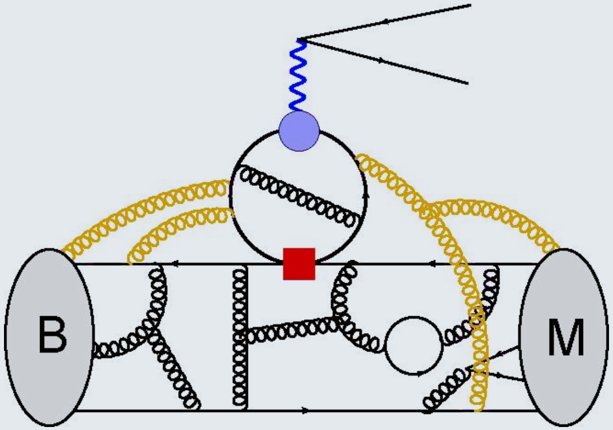
Why do we study ee → $c\bar{c}$?

- **Zoo of resonances** (X, Y, Z…) candidates for exotic hadrons (not only X(3872)…)



- Charm resonances are a **playground for NRQCD**

- Study in relation to **non-local contributions** for $b \to s\ell\ell$



What data do we have?

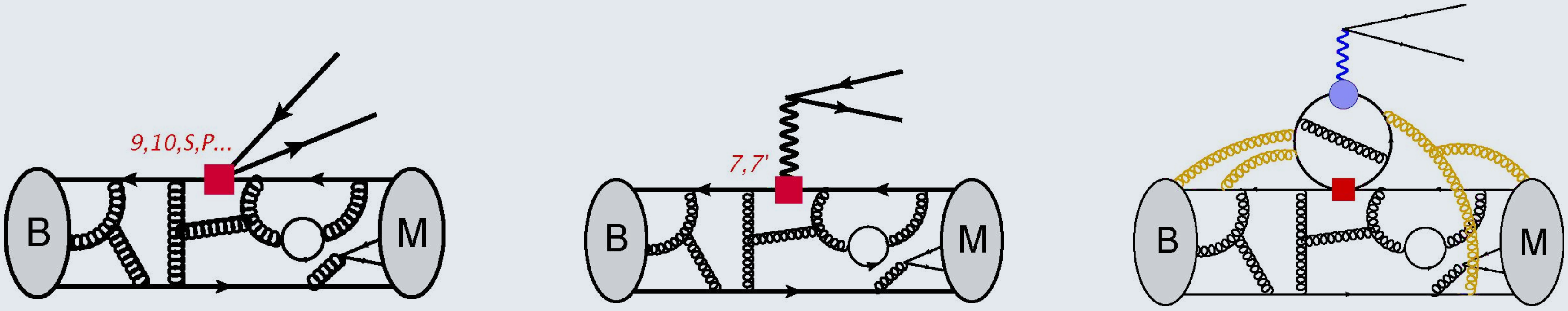
Two main experimental approaches:

- **Fixed energy scans** (CLEO, BES…)
  - Exclusive cross-sections
    e.g. $e^+e^- \to D^{(*)}\overline{D}^{(*)}$, $D_s^{(*)}\overline{D}_s^{(*)}$, $\Lambda_c\Lambda_c$
  - Inclusive measurement

$$R = \frac{\sigma(e^+e^- \to \text{hadrons})}{\sigma(e^+e^- \to \mu\mu)}$$

- **ISR analysis** (BaBar, Belle)
  - mostly exclusive cross-sections
    e.g. $e^+e^- \to D^{(*)}\overline{D}^{(*)}$, $D_s^{(*)}\overline{D}_s^{(*)}$, $\Lambda_c\Lambda_c$
  - one helicity analysis
    $e^+e^- \to D_L^*\overline{D}_L^*$, $D_L^*\overline{D}_T^*$, $D_T^*\overline{D}_T^*$

Digression on b→sℓℓ transitions



$$\mathcal{A}_\lambda^{L,R}(B \to M_\lambda \ell\ell) = \mathcal{N}_\lambda \left\{ (C_9 \mp C_{10}) \mathcal{F}_\lambda(q^2) + \frac{2 m_b M_B}{q^2} \left[C_7 \mathcal{F}_\lambda^T(q^2) - 16\pi^2 \frac{M_B}{m_b} \mathcal{H}_\lambda(q^2) \right] \right\}$$

Non-local form-factors

$$\mathcal{H}_\lambda(q^2) = i \mathcal{P}_\mu^\lambda \int d^4x \, e^{iq\cdot x} \langle \bar{M}_\lambda(k) | T\{ \mathcal{J}_{\text{em}}^\mu(x), \mathcal{C}_i \mathcal{O}_i(0) \} | \bar{B}(q+k) \rangle$$

Factorization approximation [Kruger & Sehgal `96; Lyon & Zwicky '14; Brass, Hiller *et al* '16]

$$\mathcal{H}_\lambda^{\text{KS}}(q^2) = (C_F \mathcal{C}_1 + \mathcal{C}_2)\, \Pi(q^2)\, \mathcal{F}_\lambda(q^2)$$

$$R = \frac{\sigma(e^+e^- \to \text{hadrons})}{\sigma(e^+e^- \to \mu\mu)} \propto \text{Im}\, \Pi(q^2)$$

Relation to b→sℓℓ

Beyond naive factorization, we use a more general approach

$$\mathrm{disc}\,\mathcal{H}_\lambda^{\mathrm{res}}(q^2) \sim \sum_\psi \frac{\mathcal{A}(\psi \to \ell\ell)\,\mathcal{A}(BK^{(*)} \to \psi)}{m_\psi^2 - q^2 + i\Gamma_\psi m_\psi}$$

Fix as many parameters **from data** as possible using:

$$\mathrm{disc}\,\mathcal{A}(e^+e^- \to D\bar{D}) \propto \sum_\psi \frac{\mathcal{A}(\psi \to D\bar{D})\,\mathcal{A}(\psi \to e^+e^-)}{m_\psi^2 - q^2 + i\Gamma_\psi m_\psi}$$

(BES, BaBar, Belle)

$$\mathrm{disc}\,\mathcal{A}(B \to K^{(*)}D\bar{D}) \propto \sum_\psi \frac{\mathcal{A}(\psi \to D\bar{D})\,\mathcal{A}(BK^{(*)} \to \psi)}{m_\psi^2 - q^2 + i\Gamma_\psi m_\psi}$$

(LHCb, BaBar, Belle)

[→ Will Belle II look into $e^+e^- \to D\,\bar{D}$?]

K-matrix approach

- We have a **coupled multichannel problem**

  $\psi \to e^+e^-$, $\psi \to D^{(*)}\bar{D}^{(*)}$, $(\psi \to BK^{(*)})$

- **K-matrix** is the tool to use [Chung, Brose *et al.* '95]

$$S = 1 + 2i\,T = 1 + 2i\,\rho^{1/2}\,\hat{T}\,\rho^{1/2}$$

$$\hat{T} = \hat{K}\,(1 - i\rho\,\hat{K})^{-1}$$

$$\hat{K}_{ij} = \sum_{\psi_r} \frac{g^0_{ri}\,g^0_{rj}}{m^2_{\psi_r} - q^2} + \hat{c}_{ij}$$

Real valued couplings

ee and DD channels

$c\bar{c}$ resonances

Non-resonant contributions

K-matrix approach

- We have a **coupled multichannel problem**

  $\psi \to e^+e^-$, $\psi \to D^{(*)}\bar{D}^{(*)}$, $(\psi \to BK^{(*)})$

- **K-matrix** is the tool to use [Chung, Brose *et al.* '95]

- **Problem**: other decays contribute, including 3-body decays e.g. $\psi(3770) \to J/\psi\,\pi\,\pi$

  → approximate the width due to these decays through **uncoupled effective** 2-body channels (one per resonance)

We end up with (at least) **24 channels** and **5 resonances**!

→ the list is in the backup slides

Step 1: Global $ee \to c\bar{c}$ fit

- The fit we perform is:
  - **global** = we used all experimental data
  - **extendable** = significance of new resonances can be studied
  - Implemented in **EOS**
- Main challenges: **Fit performances**, **estimation of uncertainties**

EOS is a software for a variety of applications in flavour physics. It is written in C++, but provides an interface to Python.

https://eos.github.io/

Features of the implementation

- K-matrix is implemented in **EOS**
  - **Fast numerical evaluation**
    - Written in C++
    - Efficiency due to caching of intermediate results
  - **Versatile**
    - Not limited to $ee \to c\bar{c}$
    - Adjustable number of channels/resonances
    - Polymorphic object for the channels (adjustable phase space factors, centrifugal barrier factors...)
- https://github.com/eos/eos
- New features/observables can be implemented!
  A tutorial based on Jupyter notebook is being written...

Preliminary result – General conclusions

- Need for **non-resonant contributions**
  - Allows to account for $R_{udsc} - R_{uds}$; impacts the exclusive channels
  - $\hat{c}_{0j}$ (i.e. involving e^+e^- channel) are enough to describe the data

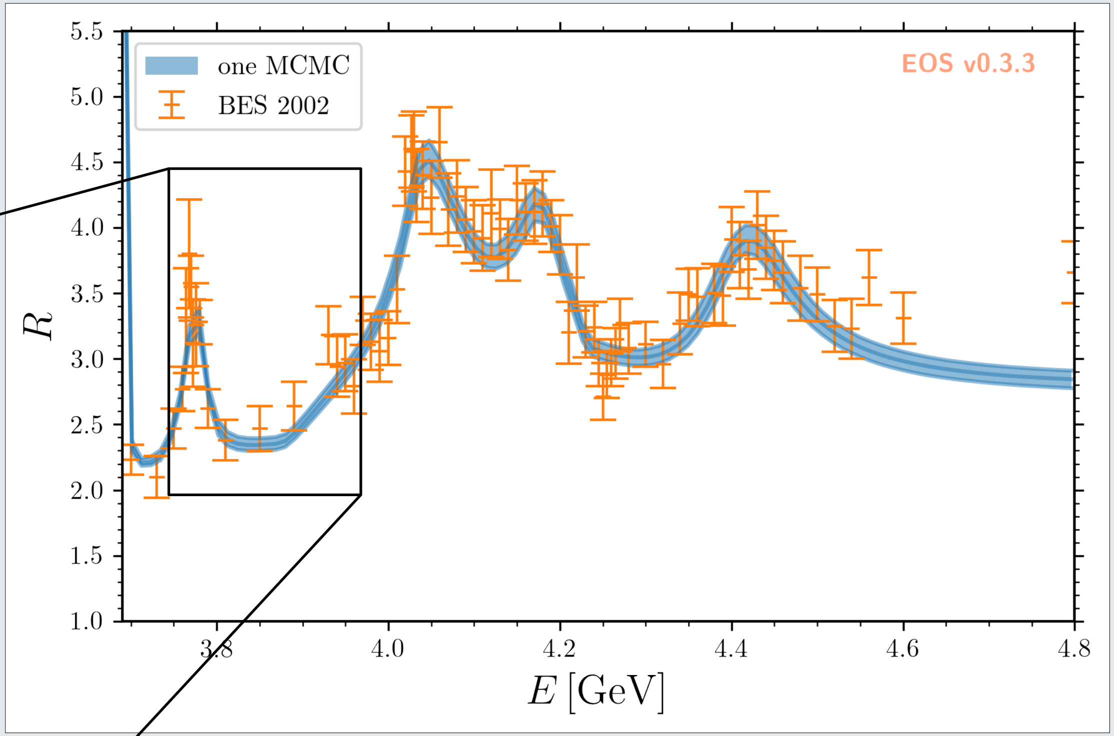
$$\hat{K}_{ij} = \sum_{\psi_r} \frac{g^0_{ri}\, g^0_{rj}}{m^2_{\psi_r} - q^2} + \hat{c}_{ij}$$

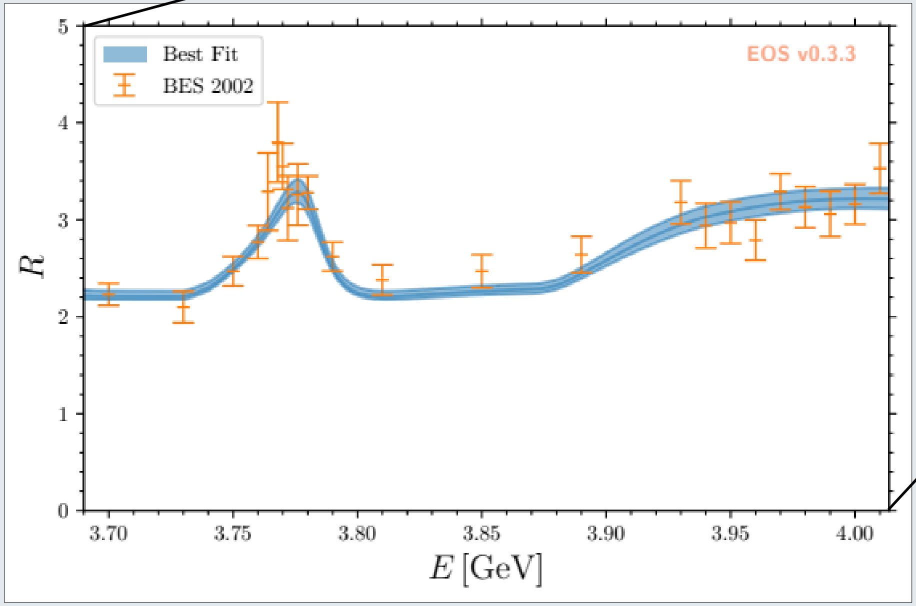
- **Sub-threshold couplings** play a crucial role [Uglov, Kalashnikova *et al.* '19]

- Data seems to be **insufficient to determine all parameters** → needs of assumptions:
  - Isospin relates $D^0\overline{D}^0$ to D^+D^-
  - SU(3) would relate $D^0\overline{D}^0$ to $D_s^+D_s^-$
  - PDG couplings to e^+e^- (from lattice in the future?)

Preliminary result – Numerics

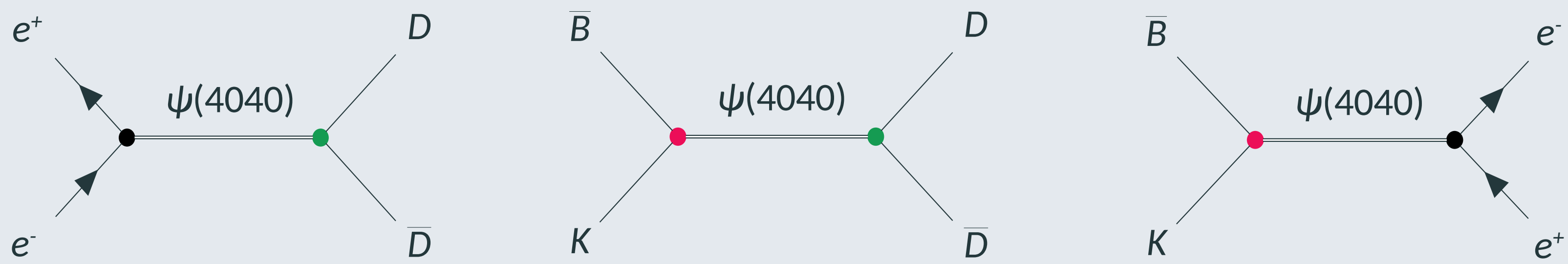
- The fit **converges**!
- **MCMC** are run and **posterior uncertainties** are estimated with a **PMC** method
- This is still in progress…





Conclusion

- **Non-local contributions in b → s$\ell\ell$** are directly related to **ee → c$\bar{c}$**



- We are fitting all these data in a **K-matrix approach**
- The implementation is part of **EOS** and **can be used for other projects**

Thanks!

Back-up slides

List of channels

Dilepton channel

	channel	type	related to channel
0	e^+e^-	PP (P wave)	-
1	eff(2S)	Effective	-
2	eff(3770)	Effective	-
3	eff(4040)	Effective	-
4	eff(4160)	Effective	-
5	eff(4415)	Effective	-
6	$D^0 \bar{D}^0$	PP (P wave)	-
7	$D^+ D^-$	PP (P wave)	6 (isospin)
8	$D^0 \bar{D}^{*0}$	VP (P wave)	-
9	$D^{*0} \bar{D}^0$	VP (P wave)	8 (c.c.)
10	$D^+ D^{*-}$	VP (P wave)	8 (isospin)
11	$D^{*+} D^-$	VP (P wave)	8 (c.c.)
12	$D_s^+ D_s^-$	PP (P wave)	- (*)
13	$D^{*0} \bar{D}^{*0}$	VV (P wave, S=0)	-
14	$D^{*0} \bar{D}^{*0}$	VV (P wave, S=2)	-
15	$D^{*0} \bar{D}^{*0}$	VV (F wave, S=2)	14 (waves)
16	$D^{*+} D^{*-}$	VV (P wave, S=0)	13 (isospin)
17	$D^{*+} D^{*-}$	VV (P wave, S=2)	14 (isospin)
18	$D^{*+} D^{*-}$	VV (F wave, S=2)	14 (waves)
19	$D_s^+ D_s^{*-}$	VP (P wave)	- (*)
20	$D_s^{*+} D_s^-$	VP (P wave)	19 (c.c.)
21	$D_s^{*+} D_s^{*-}$	VV (P wave, S=0)	- (*)
22	$D_s^{*+} D_s^{*-}$	VV (P wave, S=2)	- (*)
23	$D_s^{*+} D_s^{*-}$	VV (F wave, S=2)	22 (waves)

Effective channels (fix the resonances widths)

$D_{(s)}\bar{D}_{(s)}$ channels

$D_{(s)}\bar{D}^*_{(s)}$ channels

$D^*_{(s)}\bar{D}^*_{(s)}$ channels

(*) SU(3) symmetry could be imposed

Centrifugal barrier factors (finite size effects)

[Blatt & Weisskopf '52]

$$\hat{K}_{ij} = \sum_{\psi_r} \frac{g^0_{ri}\, g^0_{rj}\, B^L_{ri}(q, q_\alpha) B^L_{rj}(q, q_\alpha)}{m^2_{\psi_r} - q^2} + \hat{c}_{ij}$$

$$B^{l}_{ai}(q, q_a) = \frac{F_l(q)}{F_l(q_a)}$$

$$F_0(q) = 1$$

$$F_1(q) = \sqrt{\frac{2z}{z+1}}$$

$$F_2(q) = \sqrt{\frac{13z^2}{(z-3)^2 + 9z}}$$

$z = (q/q_R)^2$ and q_R corresponds to the range of interaction.

Data wishlist

- **Experimental** wishlist
  - **Tagged** analysis $D^0 \overline{D^{*0}}$ vs. $D^{*0} \overline{D^0}$
  - **More data** :-)
    Especially larger variety of center-of-mass energies.

- **Theory** wishlist
  - **Lepton decay constant** of as many states as possible would allow to less rely on (correlated) data