Light-Cone Sum Rules for $B \to K\pi$ (S-wave) Form Factors

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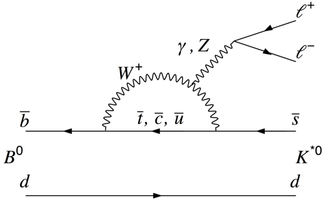
in collaboration with S. Descotes-Genon, A. Khodjamirian and J. Virto

Work in progress
arXiv:2021.xxxx

Motivation: Why $B \to \pi K$ form factors?

Important inputs

- Searches for new physics
  - Rare decays: $B \to K^* \mu\mu$



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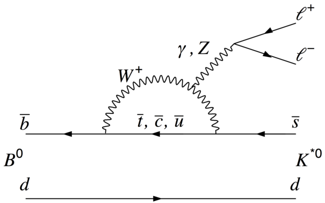

Vector mesons (ρ, K^*) are not stable particles

- Form factor calculations are done in the narrow-width limit
- Naively finite-width effect scale as: $\mathcal{W} \sim 1 + \mathrm{coeff.}\ \Gamma/M$ where $\Gamma/M \sim 20\%(\rho), 6\%(K^*)$
- Also higher-resonances and S/D wave contributions important

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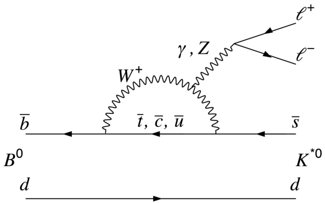
- Searches for CP violation
  - Pure hadronic decays: $B \to K\pi + \pi$ and $B \to \pi\pi + \pi$ see e.g. [KKV, Mannel, Virto, Klein, Olschewsky [1708.02047, 2003.12053]]



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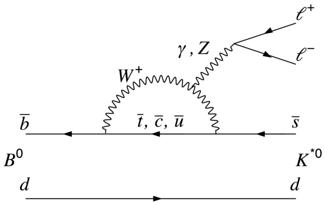
- Amplitude analyses
  - Information on lineshape required



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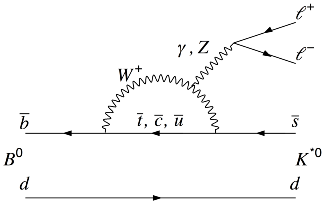

Beyond narrow-width limit

- Recent discussion of $B \to \pi K \ell\ell$ [Algueró, Alvarez Cartelle, Mclean Marshall, Masjuan, Matias, McCann, Patel, Petridis, Smith [2107.05301]]
- P-wave $B \to \pi\pi$ form factors studied using LCSRs [Cheng, Khodjamirian, Virto JHEP 05 (2017) 157 [1701.01633]] [Cheng, Khodjamirian, Virto Phys.Rev.D 96 (2017) 5, 051901 [1709.00173]]
- This talk: similar approach for $B \to \pi K$ (both S and P wave)

The plan: P and S wave πK states

- Goal: Constrain $B \to K\pi$ form factors by imposing what we know from QCD
- Light-cone sum rule analysis

P-wave $B \to \pi K$ form factors

[J. Virto, A. Khodjamirian, S. Descotes-Genon JHEP 1912, 083 (2019)] [arXiv:1908.02267]

- Improvement over assuming K^* is a stable state
- Finite width effects in P wave at 20% level for BR
- Higher resonances large impact $\to$ can be constrained by moment analysis

S-wave $B \to \pi K$ form factors [S. Descotes-Genon, A. Khodjamirian, J. Virto, KKV] [in progress..]

- S wave even more challenging; generally broad resonances
- Requires coupled-channel analysis?

$B \to K\pi$ form factors

- Generated by the (axial-)vector and (pseudo)tensor $b \to s$ transition currents

$$j_A^\mu = \bar{s}\gamma^\mu(\gamma_5)b\,, \quad j_T^\mu = \bar{s}\sigma^{\mu\nu}q_\nu(\gamma_5)b\,.$$

- Form factors $F_i(k^2, q^2, q\cdot\bar{k})$ defined as

$$\begin{aligned}
i\langle K^-(k_1)\pi^+(k_2)|\bar{s}\gamma^\mu b|\bar{B}^0(p)\rangle &= F_\perp\, k_\perp^\mu\,,\\
-i\langle K^-(k_1)\pi^+(k_2)|\bar{s}\gamma^\mu\gamma_5 b|\bar{B}^0(p)\rangle &= F_t\, k_t^\mu + F_0\, k_0^\mu + \mathrm{F}_\parallel\, k_\parallel^\mu\,,\\
\langle K^-(k_1)\pi^+(k_2)|\bar{s}\sigma^{\mu\nu}q_\nu b|\bar{B}^0(p)\rangle &= F_\perp^T k_\perp^\mu \|\,,\\
\langle K^-(k_1)\pi^+(k_2)|\bar{s}\sigma^{\mu\nu}q_\nu\gamma_5 b|\bar{B}^0(p)\rangle &= F_0^T k_0^\mu + F_\parallel^T\, k_\parallel^\mu\,,
\end{aligned}$$

- Isolate P or S-wave part via partial wave expansion:

$$\begin{aligned}
F_{0,t}(k^2, q^2, q\cdot\bar{k}) &= \sum_{\ell=0}^{\infty}\sqrt{2\ell+1}\; F_{0,t}^{(\ell)}(k^2, q^2)\; P_\ell^{(0)}(\cos\theta_K)\,,\\
F_{\perp,\parallel}(k^2, q^2, q\cdot\bar{k}) &= \sum_{\ell=1}^{\infty}\sqrt{2\ell+1}\; F_{\perp,\parallel}^{(\ell)}(k^2, q^2)\; \frac{P_\ell^{(0)}(\cos\theta_K)}{\sin\theta_k}\,,
\end{aligned}$$

A brief dive into Light-Cone Sum Rules (LCSR)

- Example: Strange scalar current to interpolate the πK state: $j_S = (m_s - m_d)\bar{s}d$
- Start with correlation function:

$$\mathcal{S}_b(k,q) = i\int d^4x\, e^{ik\cdot x}\langle 0|\mathrm{T}\left\{j_S^\dagger(x), j_b(0)\right\}|\bar{B}^0(q+k)\rangle\,,$$

- $\mathcal{S}$ calculated using light-cone OPE in terms of B-meson LCDAs for $k^2 < 0$ and $q^2 \ll m_b^2$
- Use dispersion relation in the variable k^2:

$$\mathcal{S}^{(\mathrm{OPE})}(k^2,q^2) = \frac{1}{\pi}\int_{s_{\mathrm{th}}=(m_K+m_\pi)^2}^{\infty} ds\, \frac{\mathrm{Im}\mathcal{S}(s,q^2)}{s-k^2}\,.$$

- Obtain spectral density by inserting a full set of states

$$2\,\mathrm{Im}\mathcal{S}_b^{(K\pi)}(k,q) = \sum_{K\pi}\int d\tau_{K\pi}\langle 0|j_S^\dagger|K(k_1)\pi(k_2)\rangle^*\langle K(k_1)\pi(k_2)|j_b|\bar{B}^0(q+k)\rangle\,,$$

$$\mathrm{Im}\mathcal{S}(s,q^2) = \mathrm{Im}\mathcal{S}^{(K\pi)}(s,q^2) + \mathrm{Im}\mathcal{S}^{(h)}(s,q^2)\theta(s-s_h)\,.$$

- $\mathcal{S}^{(h)}$ all contributions above s_{th}

- Assume quark-hadron duality for the states above threshold

$$\int_{s_h}^{\infty} ds\, \frac{\mathrm{Im}\mathcal{S}^{(h)}(s,q^2)}{s-k^2} = \int_{s_0}^{\infty} ds\, \frac{\mathrm{Im}\mathcal{S}^{(OPE)}(s,q^2)}{s-k^2}\,,$$

- Perform Borel transformation in the variable k^2

$$\frac{1}{\pi}\int\limits_{(m_K+m_\pi)^2}^{s_0} ds\, e^{-s/M^2}\,\mathrm{Im}\mathcal{S}^{(K\pi)}(s,q^2) = \frac{1}{\pi}\int\limits_{m_s^2}^{s_0} ds\, e^{-s/M^2}\,\mathrm{Im}\mathcal{S}^{(OPE)}(s,q^2) \equiv \mathcal{S}^{(\mathrm{OPE})}(q^2,s_0,M^2)$$

- Borel trafo suppressed the effect of higher-order resonances
- $\mathcal{S}^{\mathrm{OPE}}(q^2,s_0,M^2)$ OPE expression after subtracting the above-threshold contribution from the dispersive integral
- s_0 and M^2 can be determined from two-point sum rule

LCSR for $B \to (K\pi)_S$

$$\int_{(m_K+m_\pi)^2}^{s_0} ds\, e^{-s/M^2} \omega_{0,t}(s,q^2) F_S(s) F_{0,t}^{(\ell=0)}(s,q^2) = \mathcal{S}_{0,t}^{(\mathrm{OPE})}(q^2, s_0, M^2)$$

- s_0 effective threshold
- $\omega_{0,t}(s,q^2)$ kinematic factors
- $F_S(s)$ scalar form factor:$(m_s - m_d)\langle K^-(k_1)\pi^+(k_2)|\bar{s}d|0\rangle \equiv F_S((k_1+k_2)^2)$
- $\mathcal{S}_{0,t}^{(\mathrm{OPE})}$ pert. calculable in terms of B-LCDA parameters
- Analogous expressions for P wave [J. Virto, A. Khodjamirian, S. Descotes-Genon JHEP 1912, 083 (2019)] [arXiv:1908.02267]

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Key points:

- No closed expression for the $F_{0,t}^{(\ell=0)}(s,q^2)$!
- Only information on a weighted integral over the $K\pi$ invariant mass
- Use sum rule to constrain parameters of your favourite $K\pi$ P/S-wave model

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Inputs:

- $F_S(s)$ from data
- s_0 from two-point sum rule using $K\pi$ form factor from data

What do LCSR tell us about $B \to \pi K$ form factors?

LCSR for $B \to \pi K$ form factors

Descotes-Genon, Khodjamirian, Virto, JHEP 1912 (2019) 083

Descotes-Genon, Khodjamirian, Virto, KKV, in progress

- Simple ansatz: $K\pi$ states decays via set of Breit-Wigner-type resonances:

$$\langle K(k_1)\pi(k_2)|\bar{s}\Gamma d|X\rangle = \sum_R BW_R(k^2)\langle K(k_1)\pi(k_2)|R(k)\rangle\langle R(k)|\bar{s}\Gamma d|X\rangle$$

$$BW_R(s) = \frac{1}{m_R^2 - s - i\sqrt{s}\,\Gamma_R(s)} \qquad \Gamma_R(s) = \Gamma_R^{\rm tot}\left[\frac{\lambda_{K\pi}(s)}{\lambda_{K\pi}(m_R^2)}\right]^{1/2}\frac{m_R^3}{s^{3/2}}\,\theta(s - s_{\rm th})\,.$$

- πK form factor becomes ϕ_R chosen to reproduce the $K\pi$ form factor phase shift

$$F_S(s) = \frac{1}{m_K^2 - m_\pi^2}\sum_R \frac{m_R^2\, f_R\, g_{RK\pi}\ \ e^{i\phi_R(s)}}{m_R^2 - s - i\sqrt{s}\,\Gamma_R(s)}$$

- $B \to \pi K$ form factor parametrized as $f_R, g_{RK\pi}$ decay constants, $X_{R,i}$ kinematical factors

$$F_i^{(\ell=0)}(s,q^2) = \sum_R \frac{X_{R,i}(s,q^2)\, g_{RK\pi}\, \mathcal{G}_{R,i}(q^2)\, e^{i\phi_R(s)}}{m_R^2 - s - i\sqrt{s}\,\Gamma_R(s)}$$

- $\mathcal{G}_{R,i}(q^2)$ independent of s!

P-wave example

- Sum rule allows to determine model parameters $\mathcal{G}_{R,i}(q^2)$ $c_{R,i}$ kinematical factors

$$\sum_R \mathcal{G}_{R,i}(q^2)\, c_{R,i}(q^2)\, H_R(s_0, M^2) = \mathcal{S}_i^{\mathrm{OPE}}(q^2, s_0, M^2)$$

$$H_R(s_0, M^2) = \frac{1}{16\,\pi^2} \int_{s_{\mathrm{th}}}^{s_0} ds\; e^{-s/M^2}\; \frac{g_{RK\pi}\; \lambda_{K\pi}^{1/2}(s)\; |F_S(s)|}{s\sqrt{(m_R^2 - s)^2 + s\,\Gamma_R^2(s)}}$$

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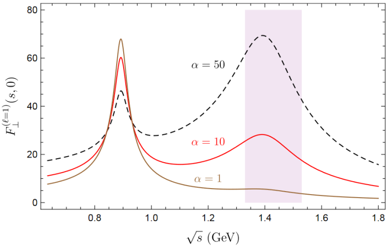
Application: [Virto, Khodjamirian, Descotes-Genon JHEP 1912, 083 (2019)]

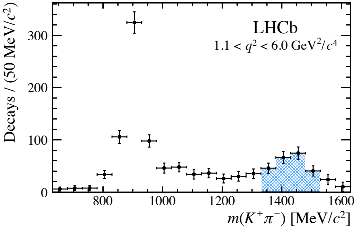
- Study finite-width effects for single K^* resonance:
  Width ratio $\mathcal{W} \equiv \frac{\mathcal{G}}{\mathcal{G}|_{\Gamma\to 0}} = 1 + 1.9\frac{\Gamma}{M} \sim 1.09 \to 20\%$ effect on BRs!
- Study effect of higher resonances beyond the $K^*(892)$: $\mathcal{G}_{K^*(1410)} = \alpha \mathcal{G}_{K^*(892)}$



What can data do for us?

LHCb [JHEP12(2016)065] [arXiV:1609.04736]

- LHCb measured 41 moments depending on S, P, D waves around $m_{K\pi} \in [1.3, 1.5]$ GeV with $q^2 \in [1.1, 6]$ GeV^2
- 4 combinations only depend on P-wave [Virto, Khodjamirian, Descotes-Genon JHEP 1912, 083 (2019)]



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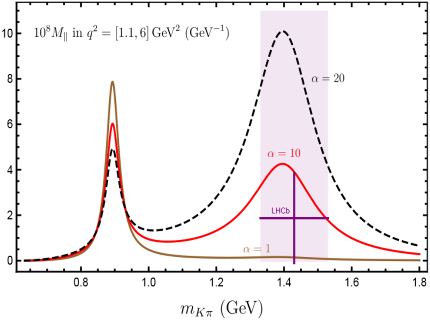


- Example of use of the data to constrain higher-partial waves
- Simultaneous analysis of S and P wave gives more information (in progress!)

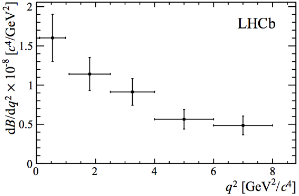
What can data do for us?

[Virto, Khodjamirian, Descotes-Genon JHEP 1912, 083 (2019)]

- Differential branching ratio also limits P- (and S-)wave

$$\frac{d\Gamma}{dq^2 dk^2} = \tilde{\Gamma}_1 = |\widehat{S}^L|^2 + |\widehat{S}^R|^2 + |\widehat{A}^L_{\|}|^2 + |\widehat{A}^R_{\|}|^2 + |\widehat{A}^L_{\perp}|^2 + |\widehat{A}^R_{\perp}|^2 + |\widehat{A}^L_0|^2 + |\widehat{A}^R_0|^2 + \ldots$$

- Considering only P wave gives:



$$10^8 \cdot \langle \mathcal{B} \rangle_{[0.10,0.98]} = 1.41 \pm 0.27 \rightarrow \alpha \lesssim 5$$
$$10^8 \cdot \langle \mathcal{B} \rangle_{[1.10,2.50]} = 1.60 \pm 0.29 \rightarrow \alpha \lesssim 6$$
$$10^8 \cdot \langle \mathcal{B} \rangle_{[2.50,4.00]} = 1.37 \pm 0.26 \rightarrow \alpha \lesssim 5$$
$$10^8 \cdot \langle \mathcal{B} \rangle_{[4.00,6.00]} = 1.12 \pm 0.26 \rightarrow \alpha \lesssim 4$$
$$10^8 \cdot \langle \mathcal{B} \rangle_{[6.00,8.00]} = 0.98 \pm 0.23 \rightarrow \alpha \lesssim 3$$

- Simultaneous analysis with S-wave in progress

QCD to constrain S wave models

- Use light-cone sum rules to constrain $B \to (K\pi)_S$ parametrizations/models
- Simple sum of Breit-Wigners (used for P-wave case) does not suffice

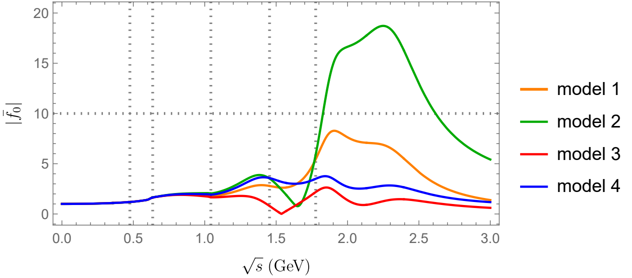
Model requirements:

- appropriate analytical properties
- poles corresponding to known resonances
- cuts for the relevant open channels
- simple (linear) dependence on the parameters to be constrained by the sum rules

S wave πK form factor

von Detten, Noël, Hanhart, Hoferichter, Kubis, Eur. Phys. J. C 81 (2021) 420 [ArXiV:2103.01966]

- Based on rescattering πK phase shifts using Omnes parametrization at low energies
- Includes inelastic effects through higher resonances
- Applied to $\tau^- \to K_S \pi^- \nu_\tau$ data to fit resonance parameters (both for P and S wave)
- Four different fit assumptions for source term give four scalar form factors



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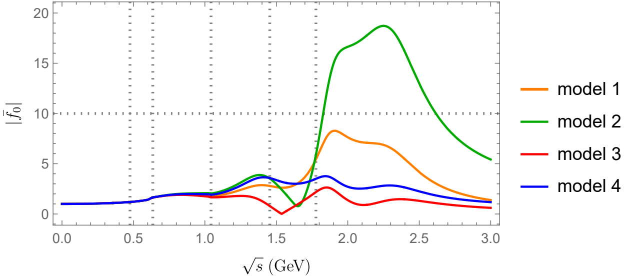


In progress:

- Use these parametrization of F_S ($K\pi$ S wave form factor) to obtain s_0 and Borel parameters (required LCSR parameters)

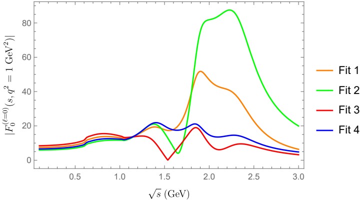
[Preliminary] $B \to \pi K$ S wave form factors

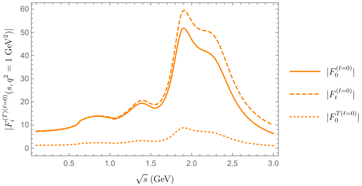
- Ansatz:

$$F_i^{(\ell=0)}(s, q^2) = \sqrt{\lambda}\rho_i(q^2) F_S(s)$$

- ρ parameters only depend on q^2 and are fixed by sum rule

$$\int_{s_{\rm th}}^{s_0} ds\ e^{-s/M^2}\omega_i(s, q^2)|F_S(s)|^2\rho_i(q^2) = \mathcal{S}_i^{\rm OPE}(q^2, s_0, M^2)$$





- Next different q^2 points and combined S and P wave

Discussion:

$B \to \pi K$ P wave:

- Studied using LCSRs [Virto, Khodjamirian, Descotes-Genon JHEP 1912, 083 (2019)]
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- Higher resonances have a large influence on $B \to K^*$ peak

Measurements of angular moments in bins across q^2 and k^2 spectra very useful!

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- Applications to $B \to K\pi\mu\mu$
- Combined analysis of the differential rate and angular observables in $K^*(1410)$ region

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Thank you for your attention!