E and B modes of the CMB y-type distortions: Polarised kinetic Sunyaev-Zeldovich effect

COSMO 22

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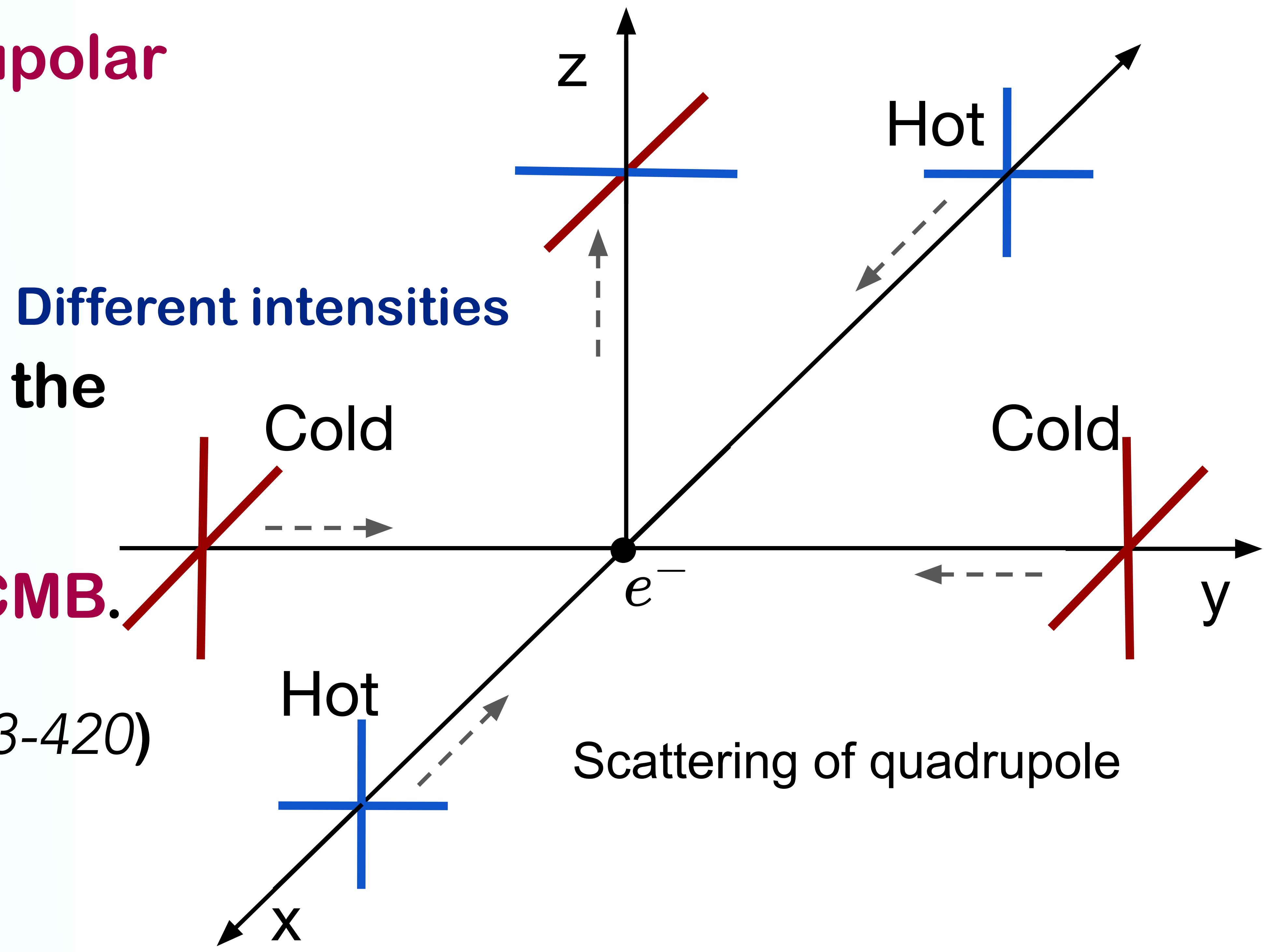
Electron peculiar velocities at second order generate E and B mode polarisation: The pkSZ effect

* Free electrons produced during reionisation, have peculiar velocities $(\vec{\mathbf{v}})$.
* In the electron rest frame, the CMB is not isotropic. Has a quadrupolar anisotropy.
    * Non-linear nature of Relativistic Doppler shift.
    * A non-linear relation between temperature and intensity in the Planck spectrum
* Thomson Scattering generates linear polarisation $\propto \mathbf{v_t}^2$ in the CMB.
* First predicted by Sunyaev and Zeldovich in 1980. (*MNRAS,190:413-420*)

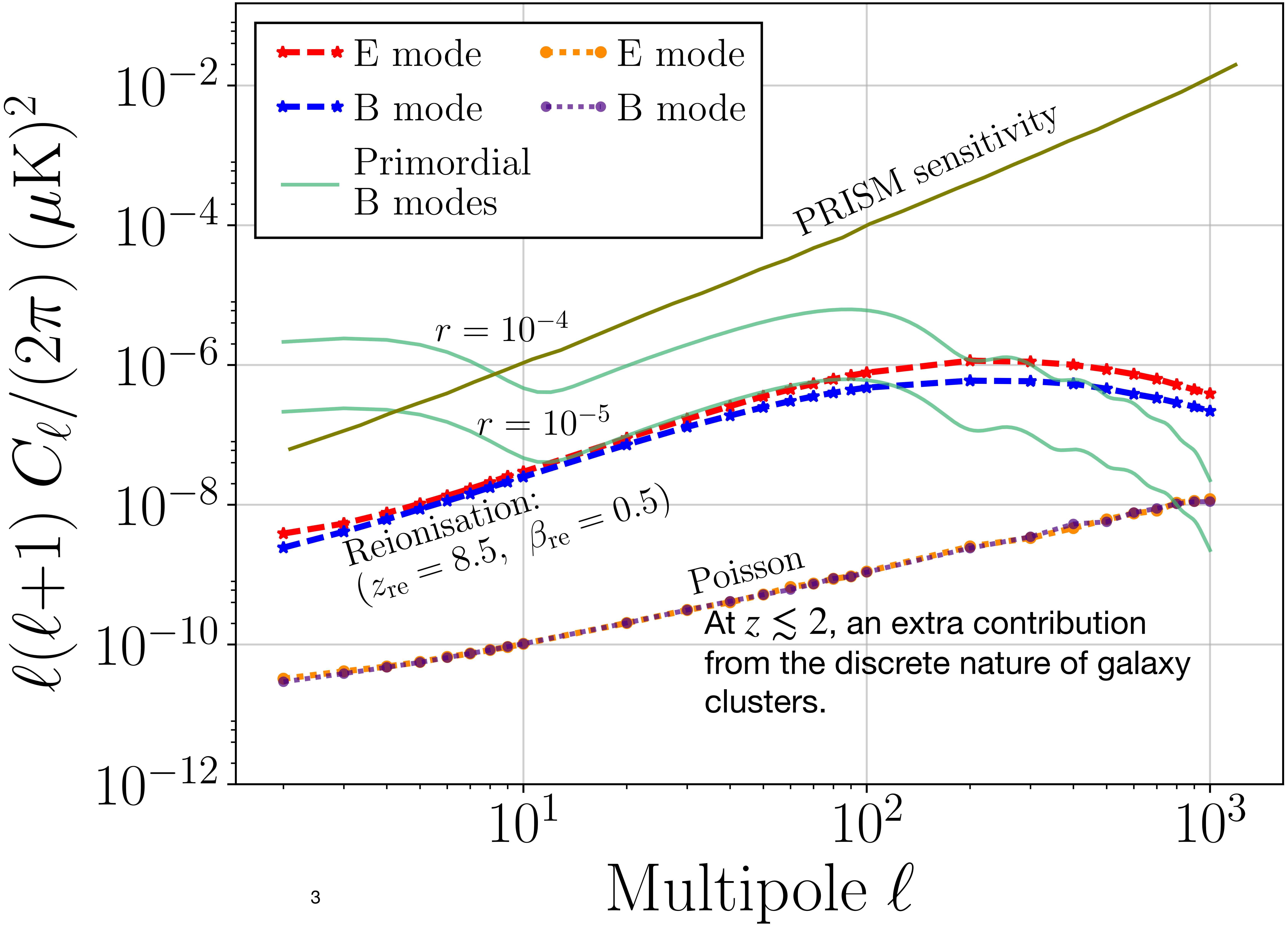


* Previous studies (*Renaux-Petel et al. arXiv:1312.4448*) (*Kamionkowski et al. arXiv: 2203.12503*)

Beating the cosmic variance with pkSZ effect

- **Sensitive to reionisation central redshift, width, and the matter velocity power spectrum.**
- **The spectrum consists of the y-type distortions part and a blackbody part.**
- **Differentiates it from primary CMB signals with blackbody spectrum and other SZ-type signals, which are unpolarised.**
- **Free from the cosmic variance of the primary CMB polarisation signal and lensing B modes.**

Full sky angular power spectra of the E and B modes



The scattered spectrum has a y-type distortion

* Photons from different blackbody spectra with different temperatures mix.
* Scattered spectrum not only has a differential blackbody but also a y-type distortion also.

$$\left(\frac{\delta I}{I} \equiv \mathcal{I}_{\rm sc}\right)\Big|_{\rm (quadrupolar)} = 2\left(\mathbf{v}\cdot\hat{\mathbf{n}}'\right)^2 g(x) + \frac{1}{2} y(x) \left(\mathbf{v}\cdot\hat{\mathbf{n}}'\right)^2$$

Planck Spectrum: $n_\nu(x) = \frac{1}{(e^x - 1)}$ $\quad x = \frac{h\nu}{k_B T_0}$

$$g(x) = \frac{xe^x}{(e^x - 1)}$$

$$y(x) = \frac{xe^x}{(e^x - 1)}\left(x\frac{e^x + 1}{e^x - 1} - 4\right)$$

* Distinguishable from the primary polarisation signals which only have a blackbody spectrum.
* Differentiable from other y-type signals, such as the thermal SZ effect which are unpolarised.
* The blackbody part will act as a foreground for primordial B modes for $r \lesssim 3 \times 10^{-5}$ for $\ell \gtrsim 100$.

Polarisation field and angular power spectra

* **The polarisation field is a spin-2 field.**

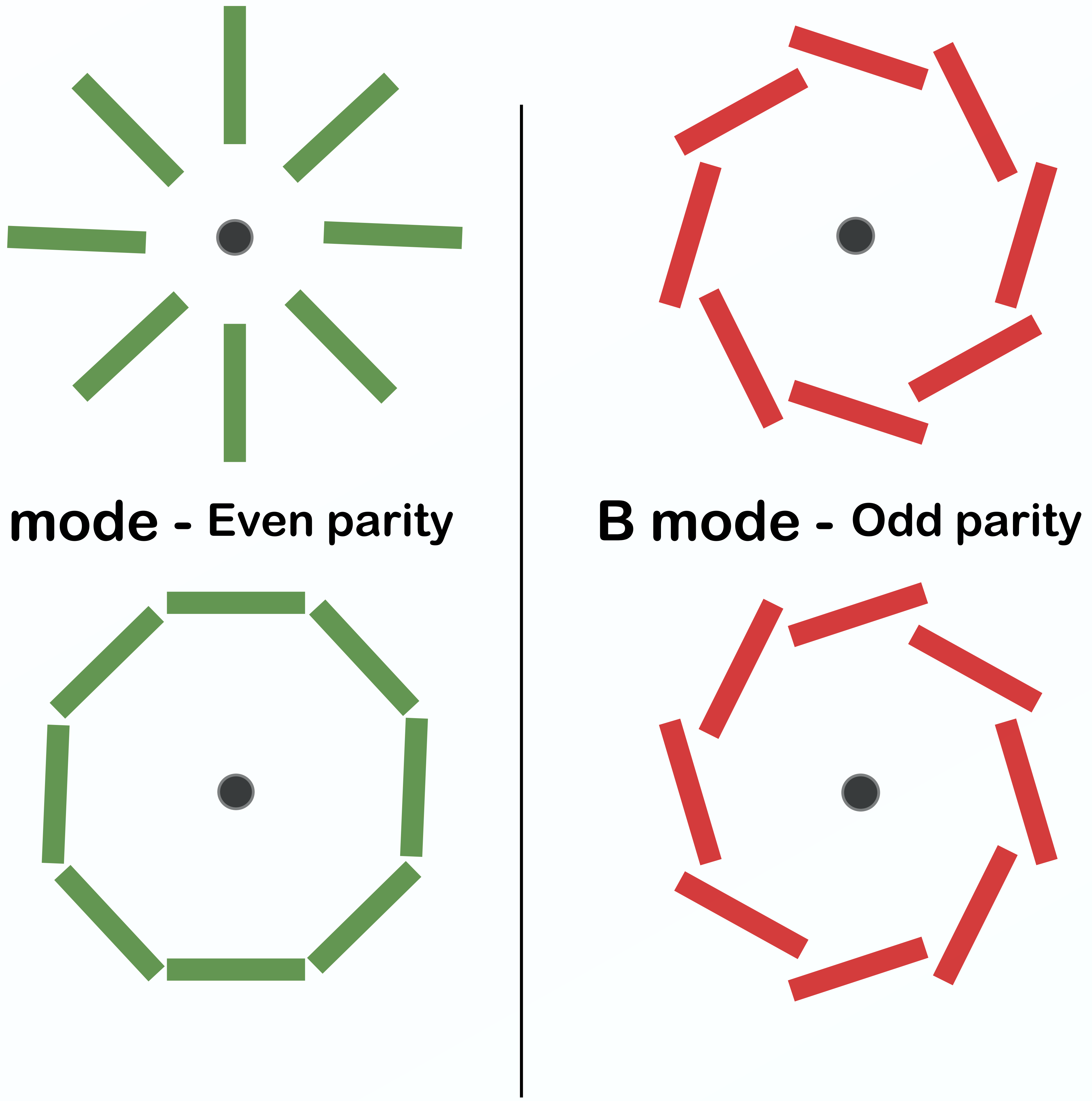
$$\left(\mathcal{Q} \pm i\mathcal{U}\right)\left(\hat{\mathbf{n}}\right) \equiv P_{\pm}\left(\hat{\mathbf{n}}\right)$$

$$a_{\ell m} = \int P_{+}\left(\hat{\mathbf{n}}\right)\ {}_{2}Y^{*}_{\ell m}\left(\hat{\mathbf{n}}\right)\, d^{2}\hat{\mathbf{n}}$$

1. Electron number density - only a function of time.
2. Square of transverse velocity.

* **Construct spin-0 fields related to the polarisation field.**

$$e_{\ell m} = \frac{1}{2}\left(a_{\ell m} + (-1)^{m} a^{*}_{\ell -m}\right) \qquad b_{\ell m} = \frac{-i}{2}\left(a_{\ell m} - (-1)^{m} a^{*}_{\ell -m}\right)$$



* **The E and B mode power spectra :**

$$\langle e_{\ell m} e^{*}_{\ell' m'} \rangle = C^{EE}_{\ell}\ \delta_{\ell,\ell'}\ \delta_{m,m'}$$

$$\langle b_{\ell m} b^{*}_{\ell' m'} \rangle = C^{BB}_{\ell}\ \delta_{\ell,\ell'}\ \delta_{m,m'}$$

The power spectra at second order is a complicated function.

$$C_\ell^{BB} = \frac{T_{CMB}^2}{2}\left[(4\pi)\left(\frac{4\pi}{3}\right)^2\sqrt{\frac{3}{2\pi}}\frac{\sqrt{6}\sigma_T}{10}\right]^2 \sum_{\lambda,\lambda'=-2}^{2}(-1)^{(\lambda+\lambda')}\int_0^{\chi} d\chi\, e^{-\tau(\chi)}\, a(\chi)\int_0^{\chi} d\chi'\, e^{-\tau(\chi')} \times$$

$$a(\chi')\mathrm{n_e}(\chi)\mathrm{n_e}(\chi')\sum_{\substack{L,M\\L',M'}}\sum_{\substack{p_1,p_2\\p_1',p_2'}} i^{(L-L')}\begin{pmatrix}1 & 1 & 2\\ p_1 & p_2 & -\lambda\end{pmatrix}\begin{pmatrix}1 & 1 & 2\\ p_1' & p_2' & -\lambda'\end{pmatrix}\iint\frac{k_1^2 dk_1\, k_2^2 dk_2}{(2\pi)^6}\times$$

$$P_{uu}(k_1)P_{uu}(k_2)\, j_L(k\chi)\, j_{L'}(k'\chi')\int d\Omega_{\mathbf{k_1}}\int d\Omega_{\mathbf{k_2}}\, Y_{LM}^*(\hat{\mathbf{k}})Y_{L'M'}(\hat{\mathbf{k}})Y_{1p_1}^*(\hat{\mathbf{k}}_1)Y_{1p_2}^*(\hat{\mathbf{k}}_2)\times$$

$$Y_{1p_1'}(\hat{\mathbf{k}}_1')Y_{1p_2'}(\hat{\mathbf{k}}_2')\, A_{\ell m}^{\lambda LM}\, A_{\ell m}^{\lambda' L' M'}\left(1-(-1)^{(L+\ell)}\right)\left(1-(-1)^{(L'+\ell)}\right)$$

Matter velocity power spectrum

Electron number density—(Reionisation history)

$$A_{\ell m}^{\lambda LM} = \sqrt{\frac{5(2L+1)(2\ell+1)}{4\pi}}(-1)^{(m)}\begin{pmatrix}L & 2 & \ell\\ 0 & -2 & 2\end{pmatrix}\begin{pmatrix}L & 2 & \ell\\ M & \lambda & -m\end{pmatrix}$$

pkSZ effect is sensitive to the redshift of central reionization

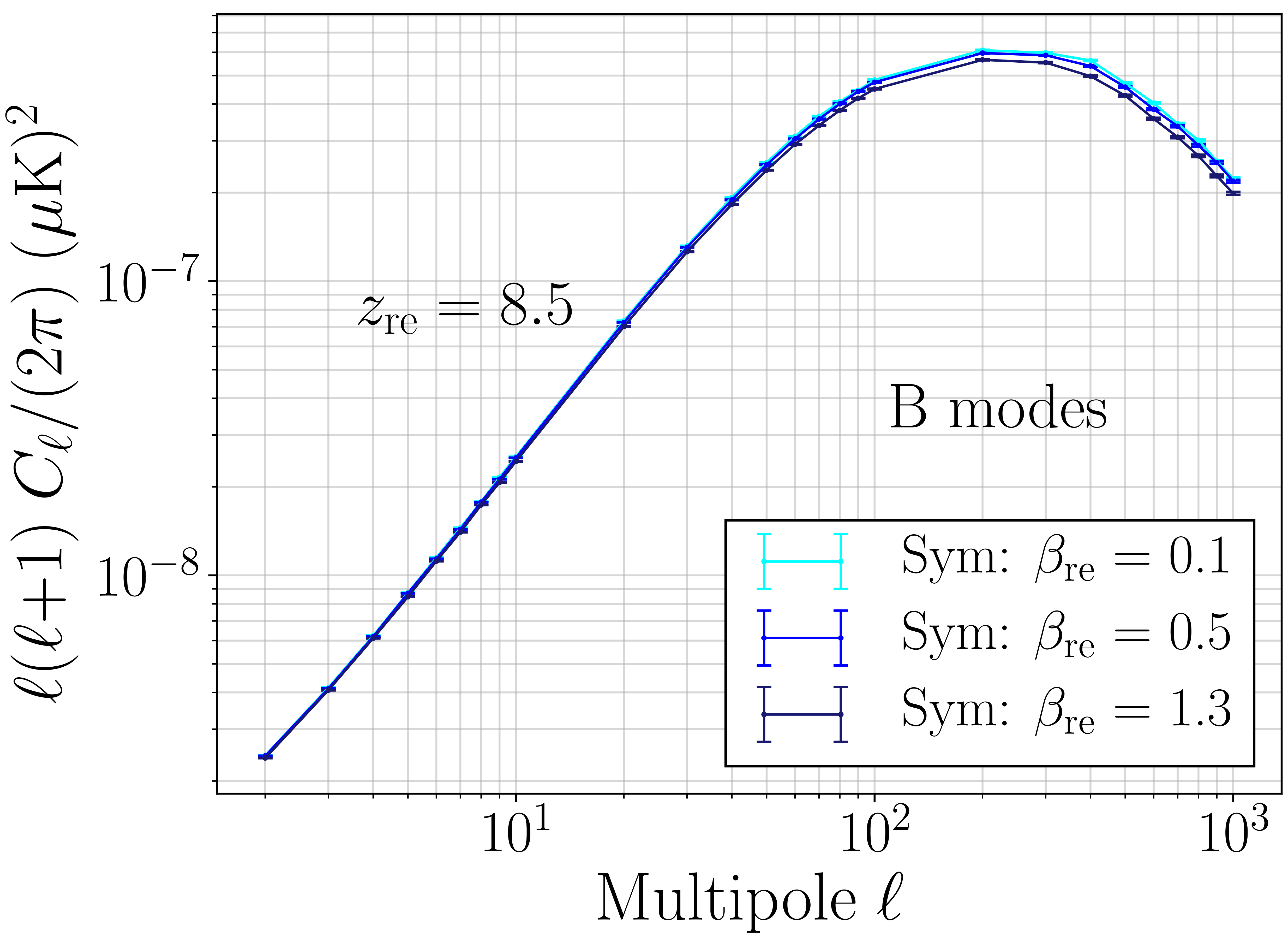
- The power spectra increase with the increase in the central redshift of reionisation.
- Increasing the central redshift increases the total Thomson optical depth.

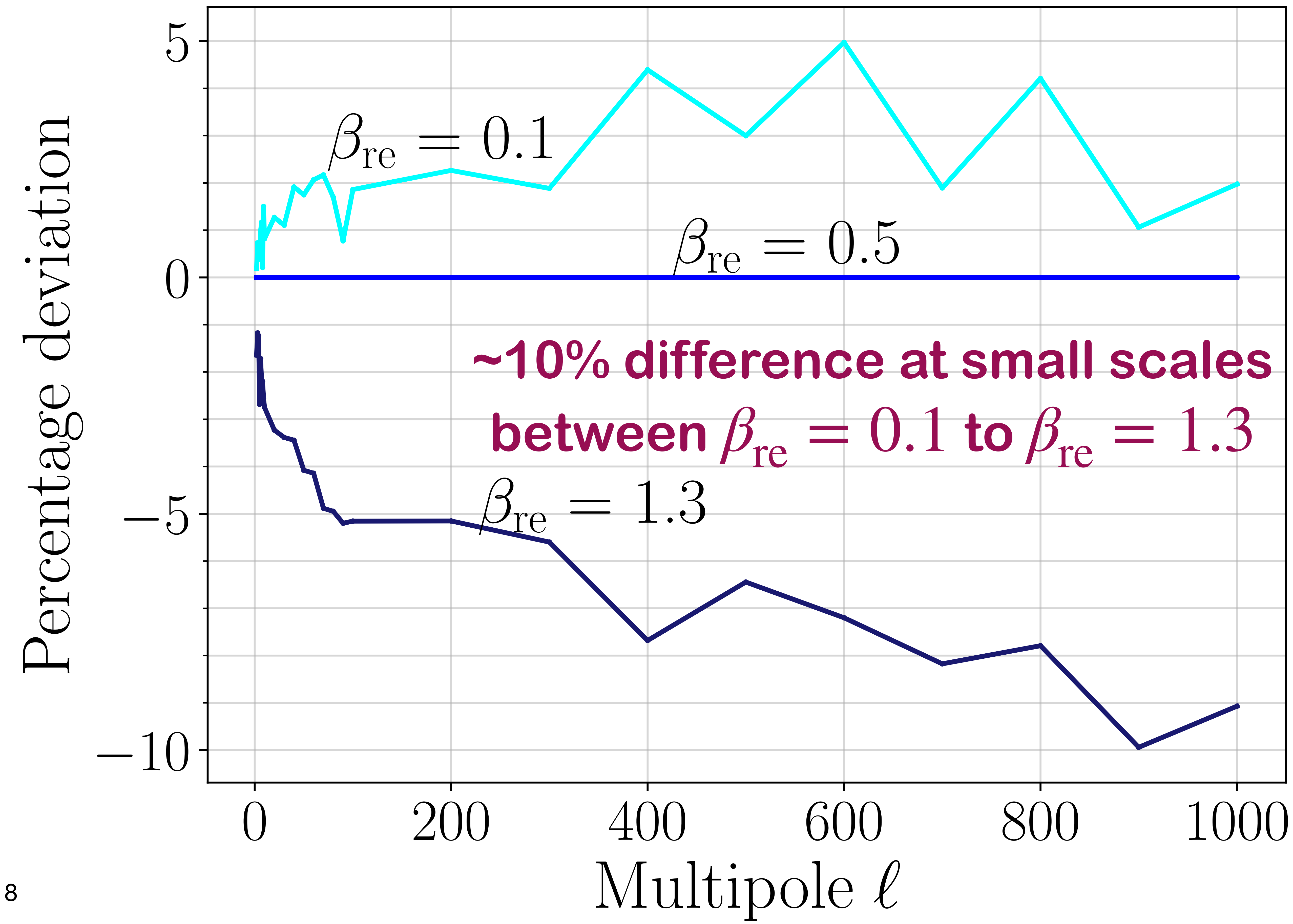
37% increase at peak position between $z_{re} = 7.5$ to $z_{re} = 9.5$

pkSZ effect is sensitive to the reionisation width

- **Changing the width at a fixed central redshift has a negligible effect on the optical depth**
- **The power spectra still decrease with the increase in the duration of reionisation.**

$$\text{Width} = z_{99\%} - z_{10\%}$$





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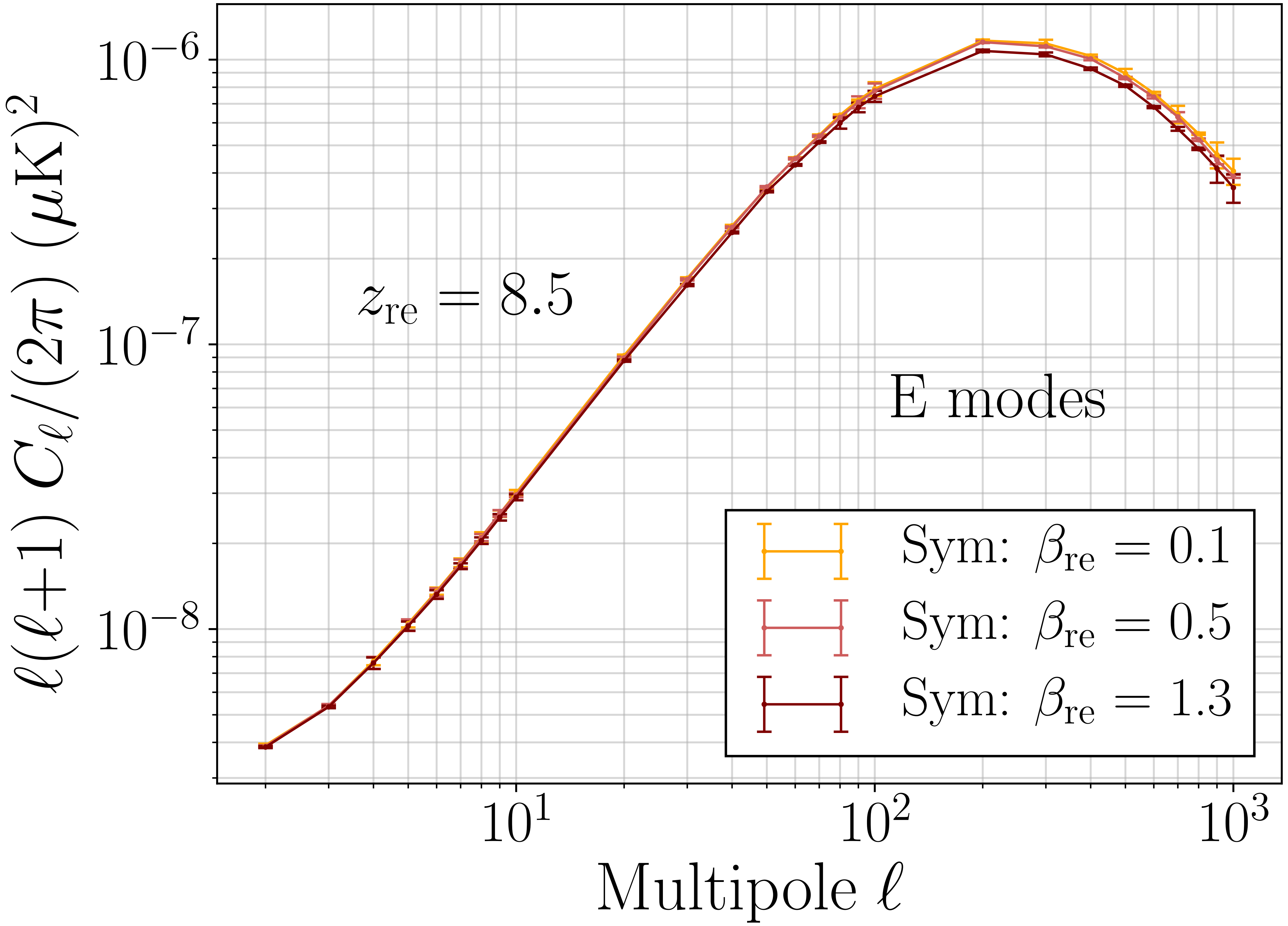

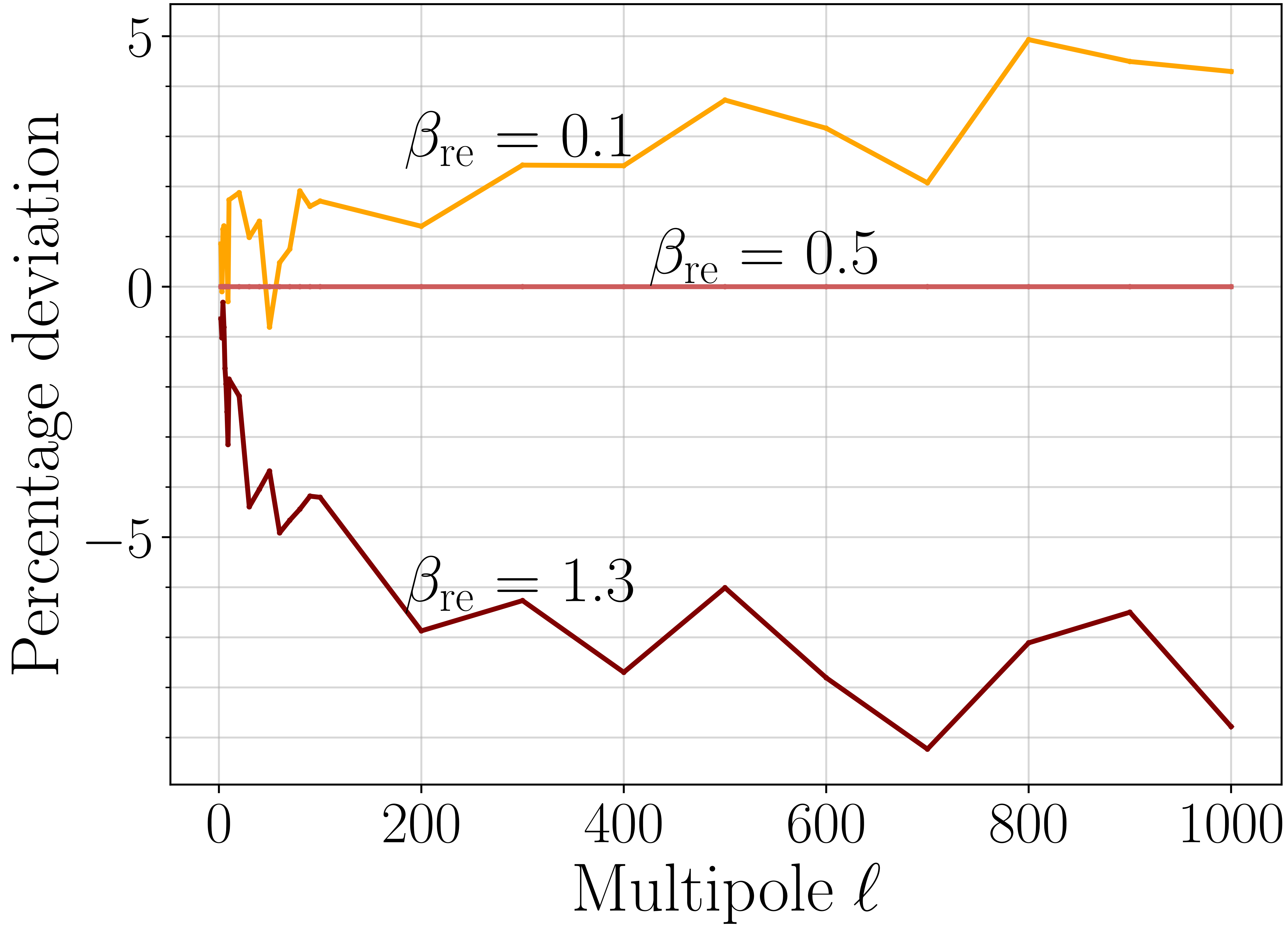

pkSZ effect is sensitive to the rapidity of reionisation

* In the case of asymmetric reionisation, the power spectra are sensitive to how quickly reionisation occurs.

E modes are greater than the B modes

Scalar ($\lambda = 0$), Vector ($\lambda = 1$) and Tensor ($\lambda = 2$) Decomposition

Auto-correlations

Concluding Remarks

- Sensitive to reionisation central redshift, width, rapidity and the matter velocity power spectrum.

- The spectrum consists of the y-type distortions part - A unique signature .

- Differentiates it from primary CMB signals with blackbody spectrum and other SZ-type signals, which are unpolarised.

- Free from the cosmic variance of the primary CMB polarisation signal and lensing B modes.

Thank You !!

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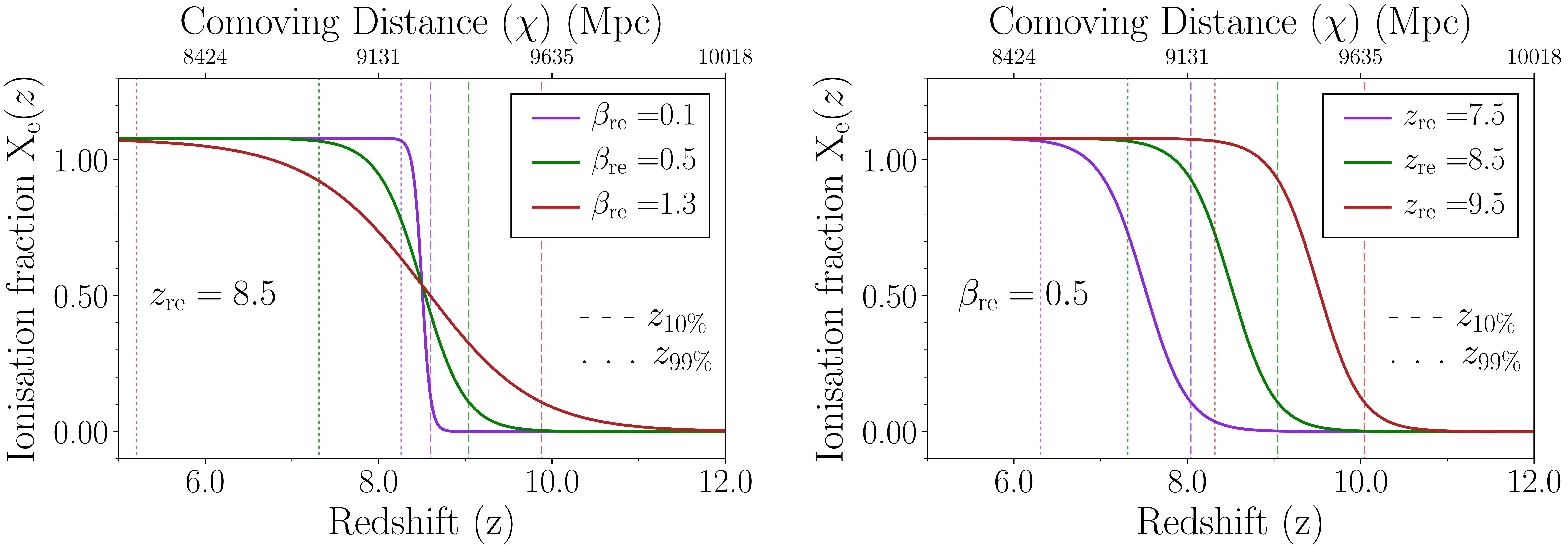
Symmetric Reionisation

$$q(z) = (1+z)^{1.5}$$

$$\Delta q_{\rm re} = 1.5(\sqrt{1+z_{re}})\Delta z_{\rm re}$$

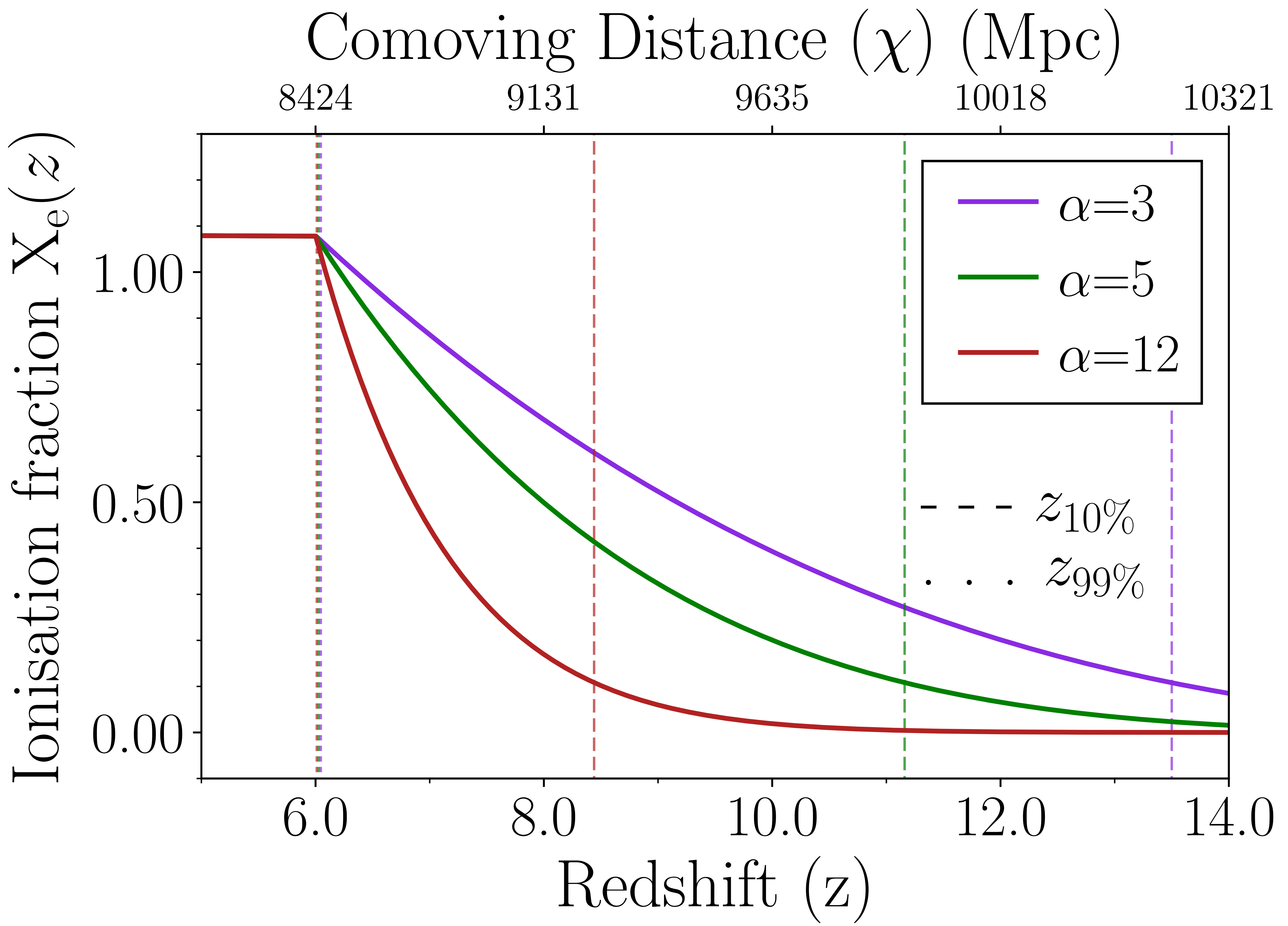
$$f = \left(\frac{\rm m_H}{\rm m_{He}}\frac{\rm X_{He}}{1-\rm X_{He}}\right) \simeq 0.079$$

$$\mathrm{X_e}^{\rm Sym}(z) = \left[\frac{(1+f)}{2}\left\{1+\tanh\left(\frac{q_{\rm re}-q(z)}{\Delta q_{\rm re}}\right)\right\} + \frac{f}{2}\left\{1+\tanh\left(\frac{q_{\rm re}^{\rm HeII}-q(z)}{\Delta q_{\rm re}^{\rm HeII}}\right)\right\}\right]$$

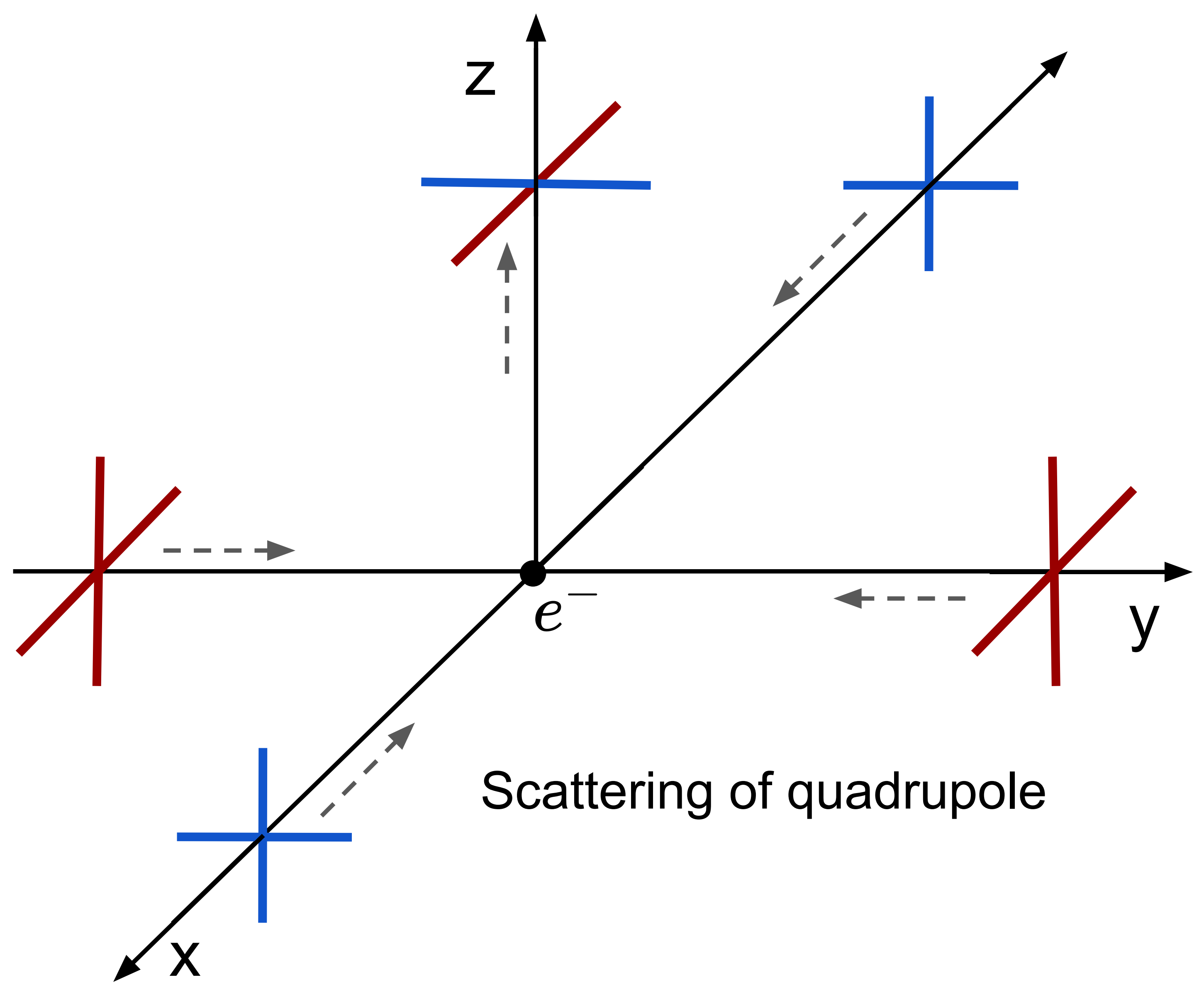


Asymmetric Reionisation

$$X_e^{\mathrm{Asym}}(z) = \left[\begin{cases} (1+f) & z < z_{\mathrm{end}} \\ (1+f)\left(\dfrac{z_{\mathrm{early}} - z}{z_{\mathrm{early}} - z_{\mathrm{end}}}\right)^{\alpha} & z > z_{\mathrm{end}} \end{cases} + \frac{f}{2}\left\{1 + \tanh\left(\frac{q_{\mathrm{re}}^{\mathrm{HeII}} - q}{\Delta q_{\mathrm{re}}^{\mathrm{HeII}}}\right)\right\} \right]$$



Secondary polarisation of CMB: The pkSZ effect



- Free electrons produced during reionisation, have a bulk peculiar velocity $(\mathbf{v})$.
- In the electron rest frame, the CMB is not isotropic, in particular, has a quadrupolar anisotropy $\propto \mathbf{v}^2$.
    - Non-linear nature of Relativistic Doppler shift.
    - A non-linear relation between temperature and intensity in the Planck spectrum
- Thomson Scattering in presence of a quadrupolar anisotropy generates polarisation in the CMB.
- First predicted by Sunyaev and Zeldovich in 1980. (MNRAS,190:413-420)

$$T\left(\mathbf{r}, \hat{\mathbf{n}}', \eta\right) = \frac{T_0(\eta)}{\gamma\left(1 + \mathbf{v}(\mathbf{r}, \eta) \cdot \hat{\mathbf{n}}'\right)} = T_0(\eta)\left[1 + \underbrace{\frac{1}{2}v^2 - \mathbf{v} \cdot \hat{\mathbf{n}}' + \left(\mathbf{v} \cdot \hat{\mathbf{n}}'\right)^2 + \mathcal{O}\left(\left(\mathbf{v} \cdot \hat{\mathbf{n}}'\right)^3\right) + \cdots}_{\theta(\mathbf{r}, \hat{\mathbf{n}}', \eta)}\right]$$

$$\delta n_\nu = \frac{1}{2h\nu^3}\delta I_\nu = \left(\theta + \theta^2\right)\left(T\frac{\partial n_{pl}}{\partial T}\right)\Bigg|_{T_0} + \frac{\theta^2}{2}\left[T^4\frac{\partial}{\partial T}\left(\frac{1}{T^2}\frac{\partial n_{pl}}{\partial T}\right)\right]\Bigg|_{T_0} + \mathcal{O}(\theta^3)\ \cdots$$

Polarisation field and Power spectra

- The polarisation field :

$$(\mathcal{Q} \pm i\mathcal{U})(\hat{\mathbf{n}}) \equiv P_{\pm}(\hat{\mathbf{n}}) = -\frac{\sqrt{6}\sigma_T}{10}\int_0^{\chi} d\chi\, a(\chi)\, e^{-\tau(\chi)}\, \mathrm{n_e}(\chi) \sum_{\lambda=-2}^{2} {}_{\pm 2}Y_{2\lambda}(\hat{\mathbf{n}}) \int d^2\hat{\mathbf{n}}'\, Y^*_{2\lambda}(\hat{\mathbf{n}}')\left(\mathbf{v}(\mathbf{r},\chi)\cdot\hat{\mathbf{n}}'\right)^2.$$

- (yellow) → Electron number density - only a function of time.
- (red) → Shows that polarisation is a spin-2 field.
- (green) → Source term - integral over all incoming photon direction. Extracts the quadrupole.

$$a_{\ell m} = \int P_+(\hat{\mathbf{n}})\; {}_2Y^*_{\ell m}(\hat{\mathbf{n}})\, d^2\hat{\mathbf{n}}$$

$$\mathcal{E}(\hat{\mathbf{n}}) = \frac{1}{2}\left[\left(\eth^*\right)^2 P_+(\hat{\mathbf{n}}) + (\eth)^2 P_-(\hat{\mathbf{n}})\right] = \sum_{\ell,m} e_{\ell m}\sqrt{\frac{(\ell+2)!}{(\ell-2)!}}\, Y_{\ell m}(\hat{\mathbf{n}})$$

$$\mathcal{B}(\hat{\mathbf{n}}) = -\frac{i}{2}\left[\left(\eth^*\right)^2 P_+(\hat{\mathbf{n}}) - (\eth)^2 P_-(\hat{\mathbf{n}})\right] = \sum_{\ell,m} b_{\ell m}\sqrt{\frac{(\ell+2)!}{(\ell-2)!}}\, Y_{\ell m}(\hat{\mathbf{n}})$$

- We define to define spin 0 fields related to the polarisation field through spin raising and lowering operator.

$$b_{\ell m} = \frac{-i}{2}\left(a_{\ell m} - (-1)^m a^*_{\ell -m}\right) \qquad e_{\ell m} = \frac{1}{2}\left(a_{\ell m} + (-1)^m a^*_{\ell -m}\right)$$

- The E and B mode power spectra :

$$\langle e_{\ell m} e^*_{\ell' m'}\rangle = C_\ell^{EE}\,\delta_{\ell,\ell'}\,\delta_{mm'} \qquad \langle b_{\ell m} b^*_{\ell' m'}\rangle = C_\ell^{BB}\,\delta_{\ell,\ell'}\,\delta_{m,m'}$$

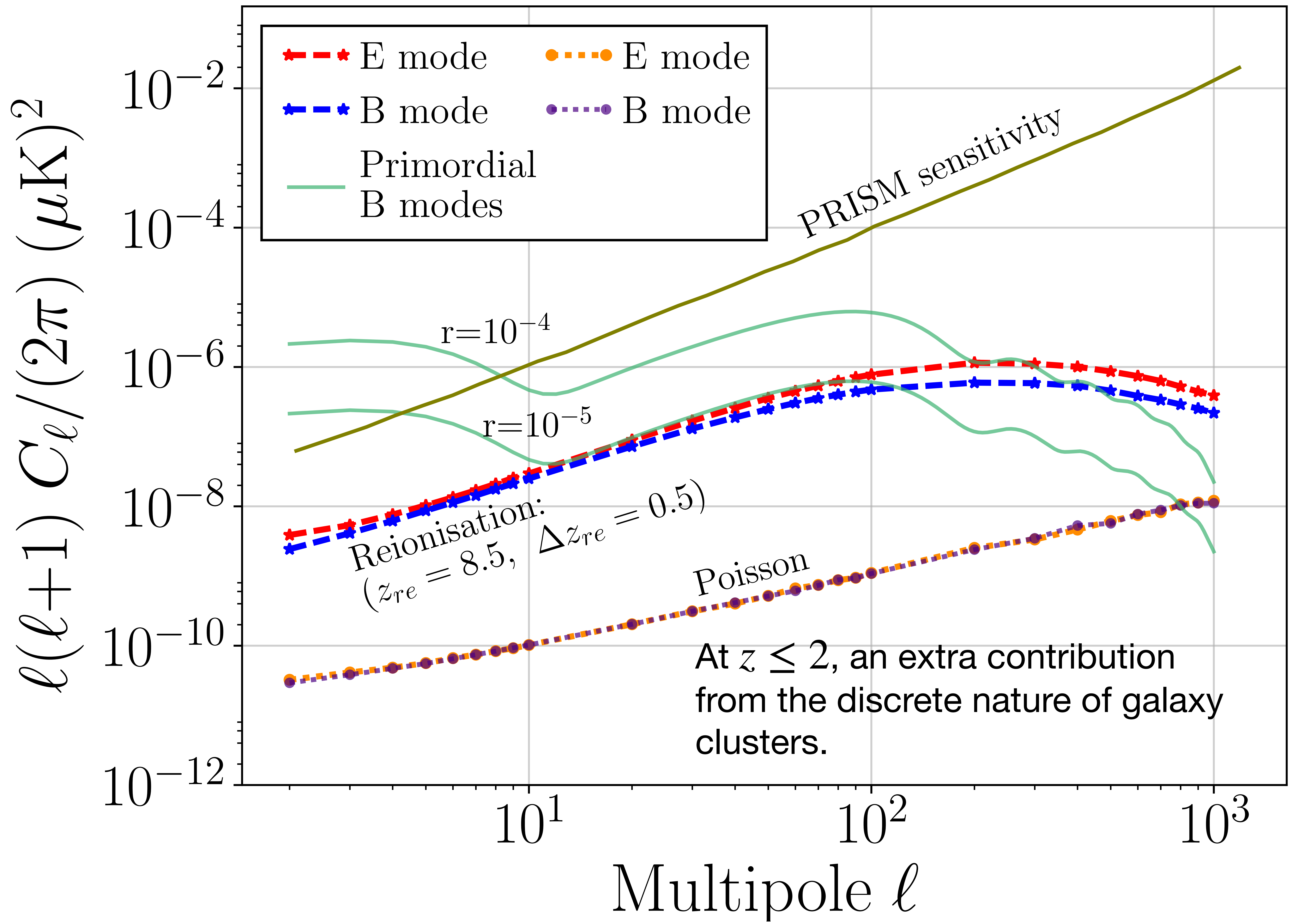
y-type E and B mode power spectra are sensitive to the matter power spectrum

- We selected an asymmetric reionisation history

$$X_e^{Sym}(z) = \left[\frac{(1+f)}{2}\left\{1+\tanh\left(\frac{q_{re}-q}{\Delta q_{re}}\right)\right\} + \frac{f}{2}\left\{1+\tanh\left(\frac{q_{re}^{HeII}-q}{\Delta q_{re}^{HeII}}\right)\right\}\right]$$

$$q(z) = (1+z)^{1.5} \qquad \Delta q_{re} = 1.5(\sqrt{1+z_{re}})\Delta z_{re}$$

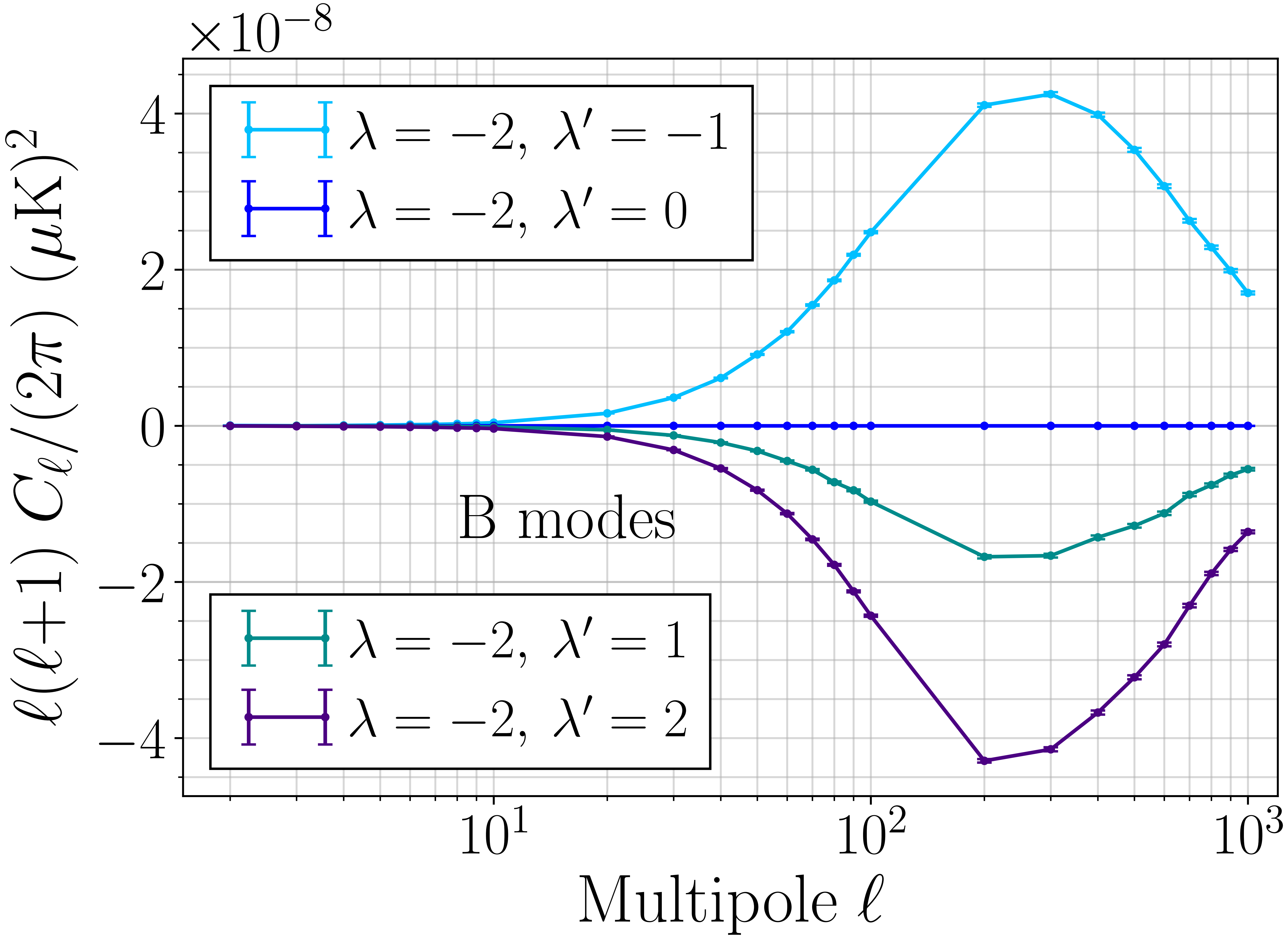
$$f = \left(\frac{m_H}{m_{He}}\frac{X_{He}}{1-X_{He}}\right) \simeq 0.079$$

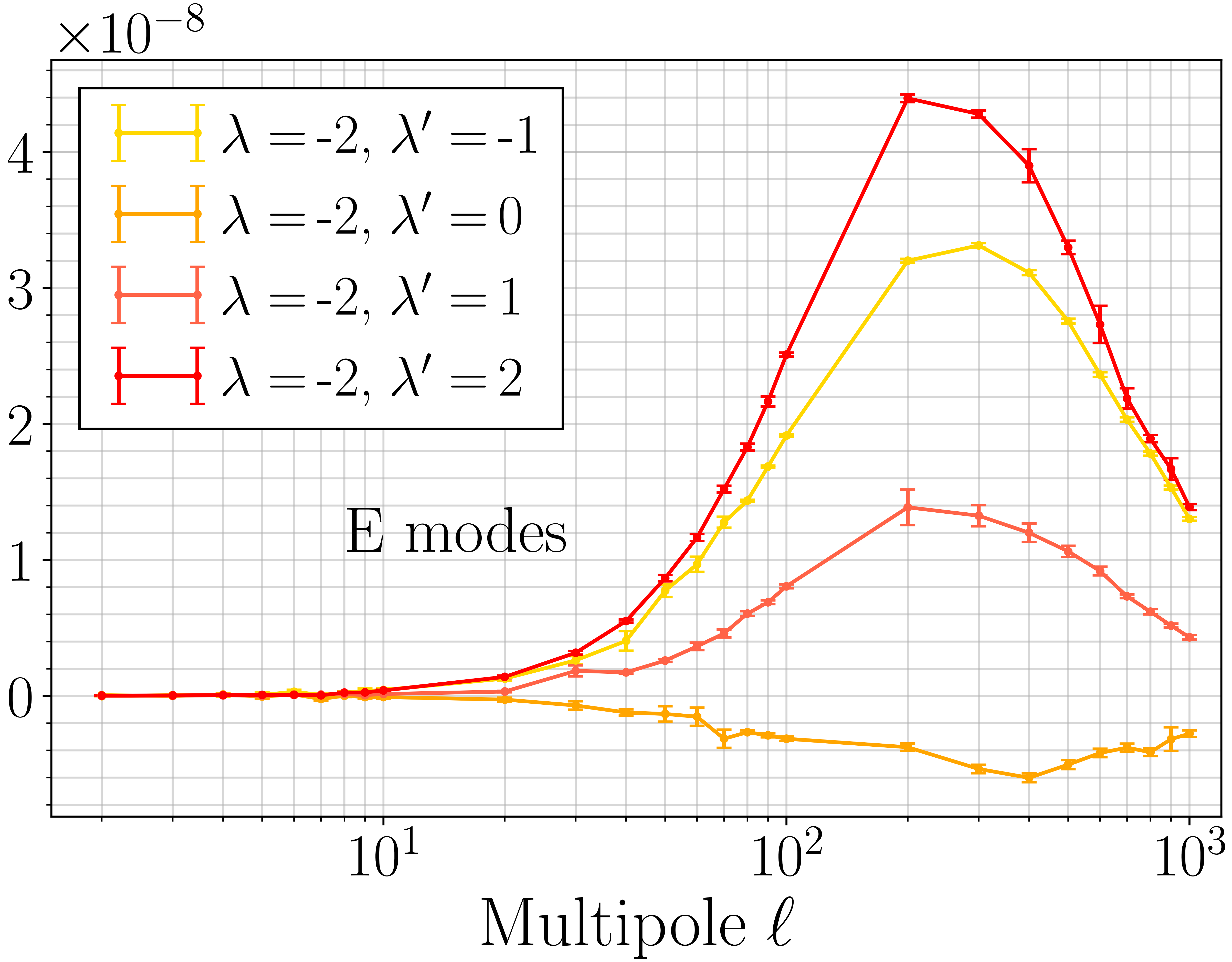


E modes greater than the B modes

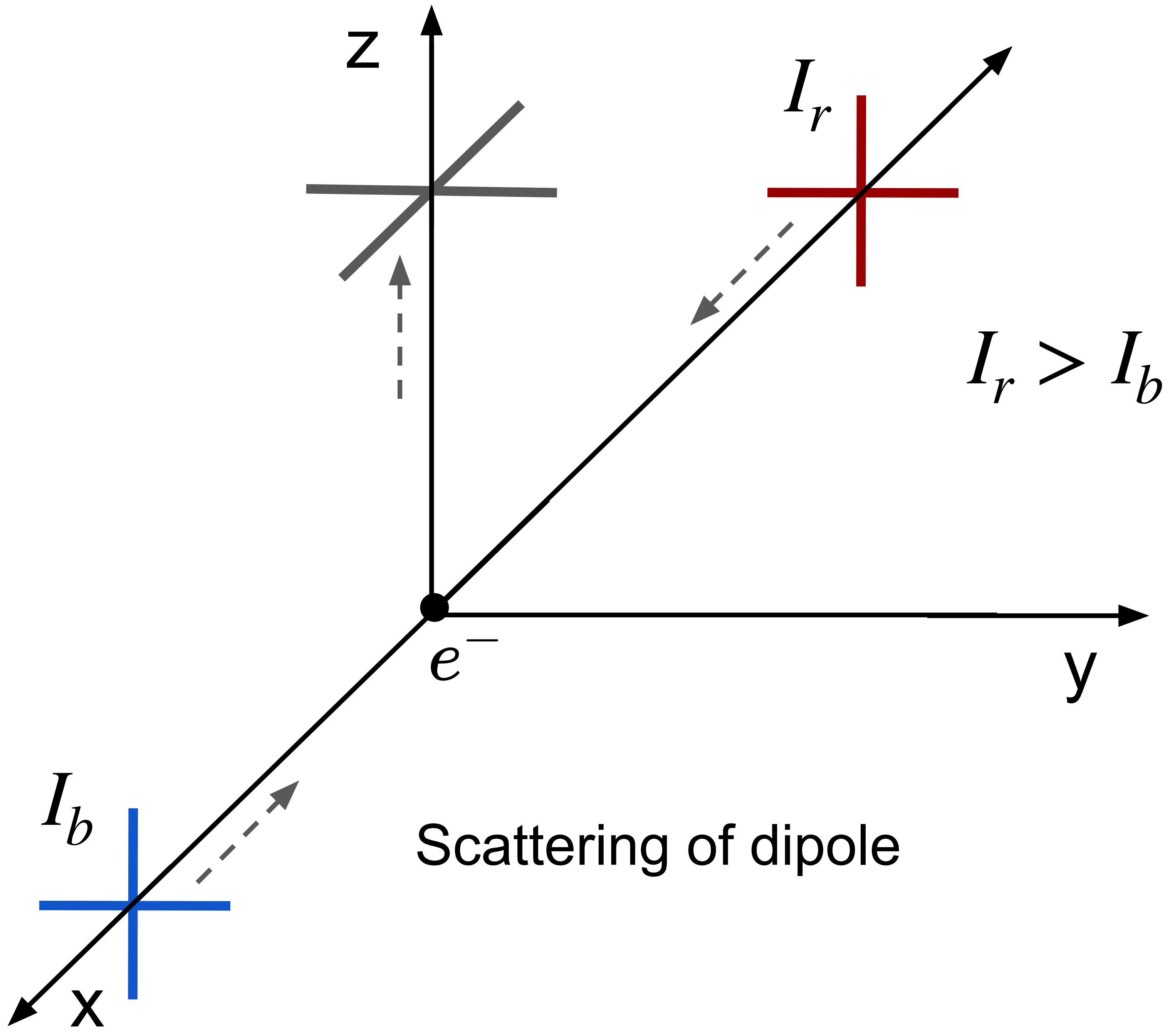
Scalar, Vector and Tensor Decomposition

Cross-correlations



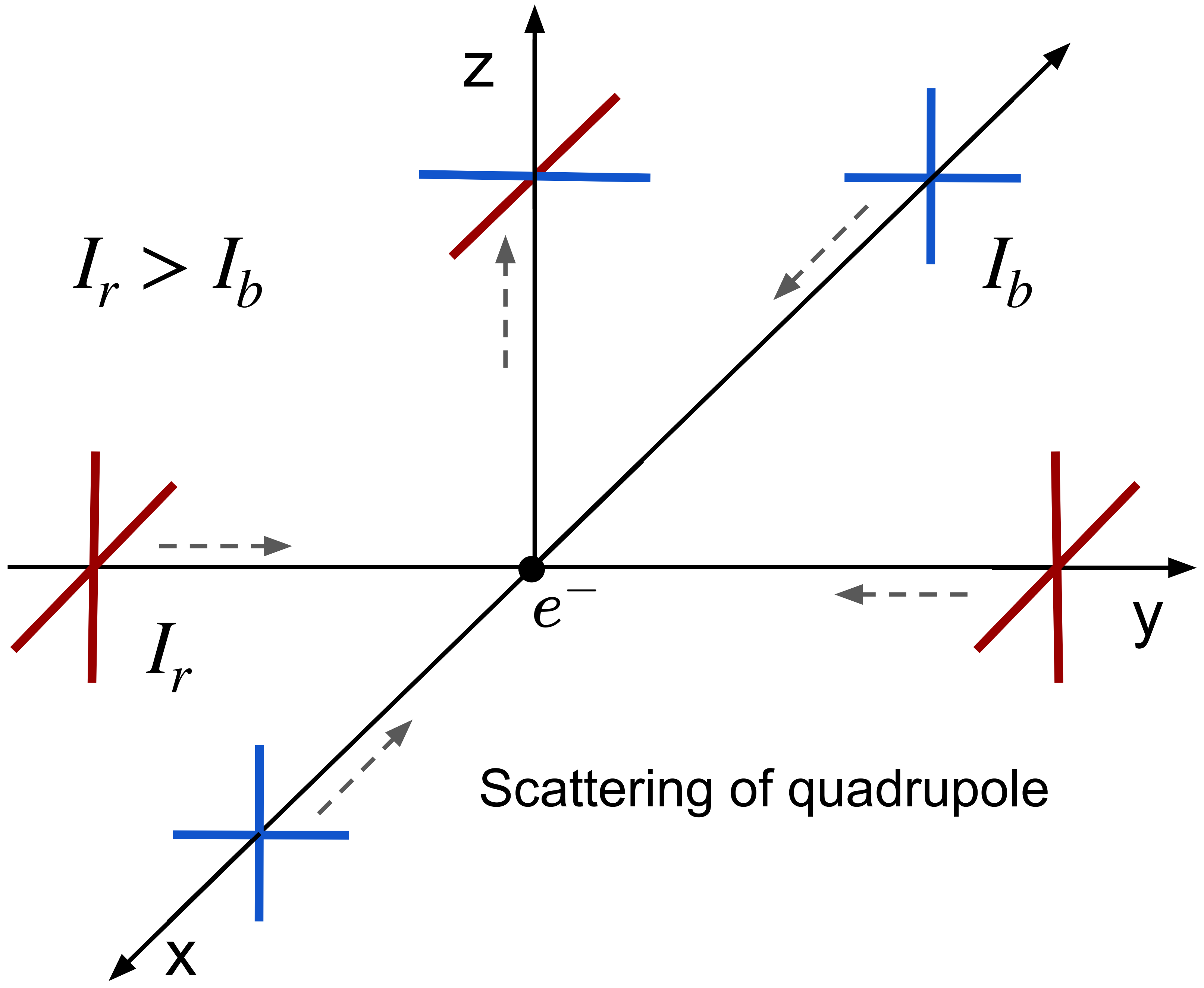


Thomson Scattering generates polarisation if the incoming radiation has a quadrupolar anisotropy



Scattering of dipole

$$\sigma_T \propto \boldsymbol{\epsilon} \cdot \boldsymbol{\epsilon}'$$



Scattering of quadrupole

$$\dot{\tau} = -\, n_e\, a(\chi)\, \sigma_T$$

Considering incoming radiation to be unpolarised

$$P_{\pm}\left(\hat{\mathbf{n}}\right) = -\frac{\sqrt{6}\sigma_{\mathrm{T}}}{10} \int_0^{\chi} d\chi\, a(\chi)\, e^{-\tau(\chi)}\, \mathrm{n_e}(\chi) \sum_{\lambda=-2}^{2} {}_{\pm 2}Y_{2\lambda}\left(\hat{\mathbf{n}}\right) \int d^2\hat{\mathbf{n}}'\, Y^*_{2\lambda}\left(\hat{\mathbf{n}}'\right) \left(\mathbf{v}(\mathbf{r},\chi) \cdot \hat{\mathbf{n}}'\right)^2 .$$

$Y^*_{2\lambda}\left(\hat{\boldsymbol{n}}'\right)$ will pick out the quadrupolar part in $\theta(\boldsymbol{r}, \hat{\boldsymbol{n}}')$