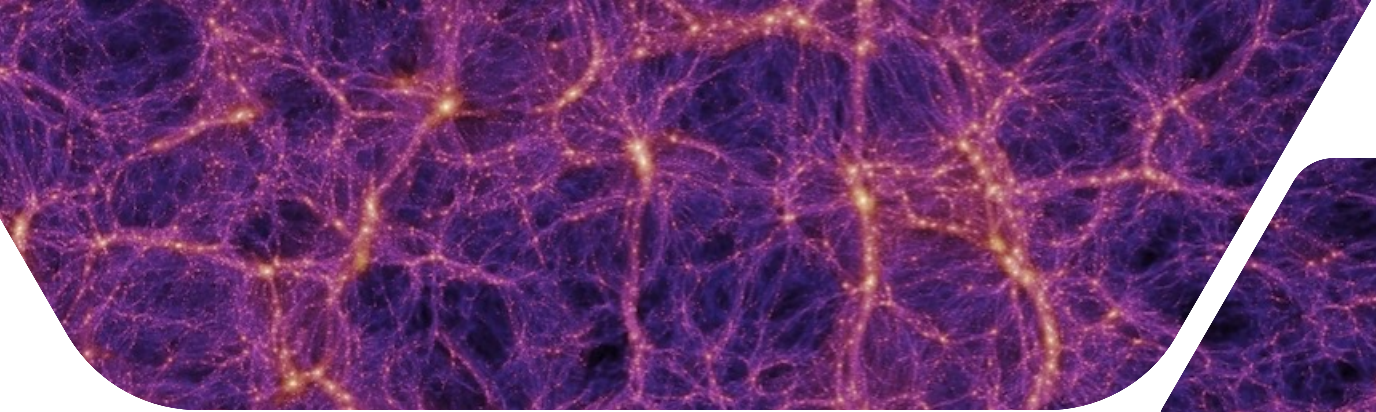
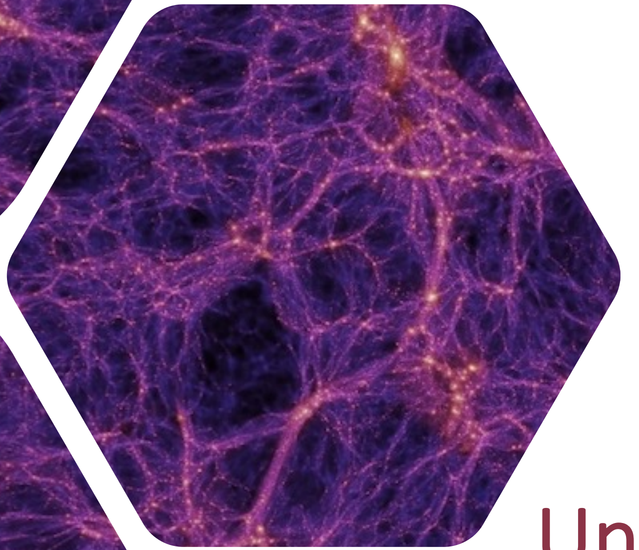
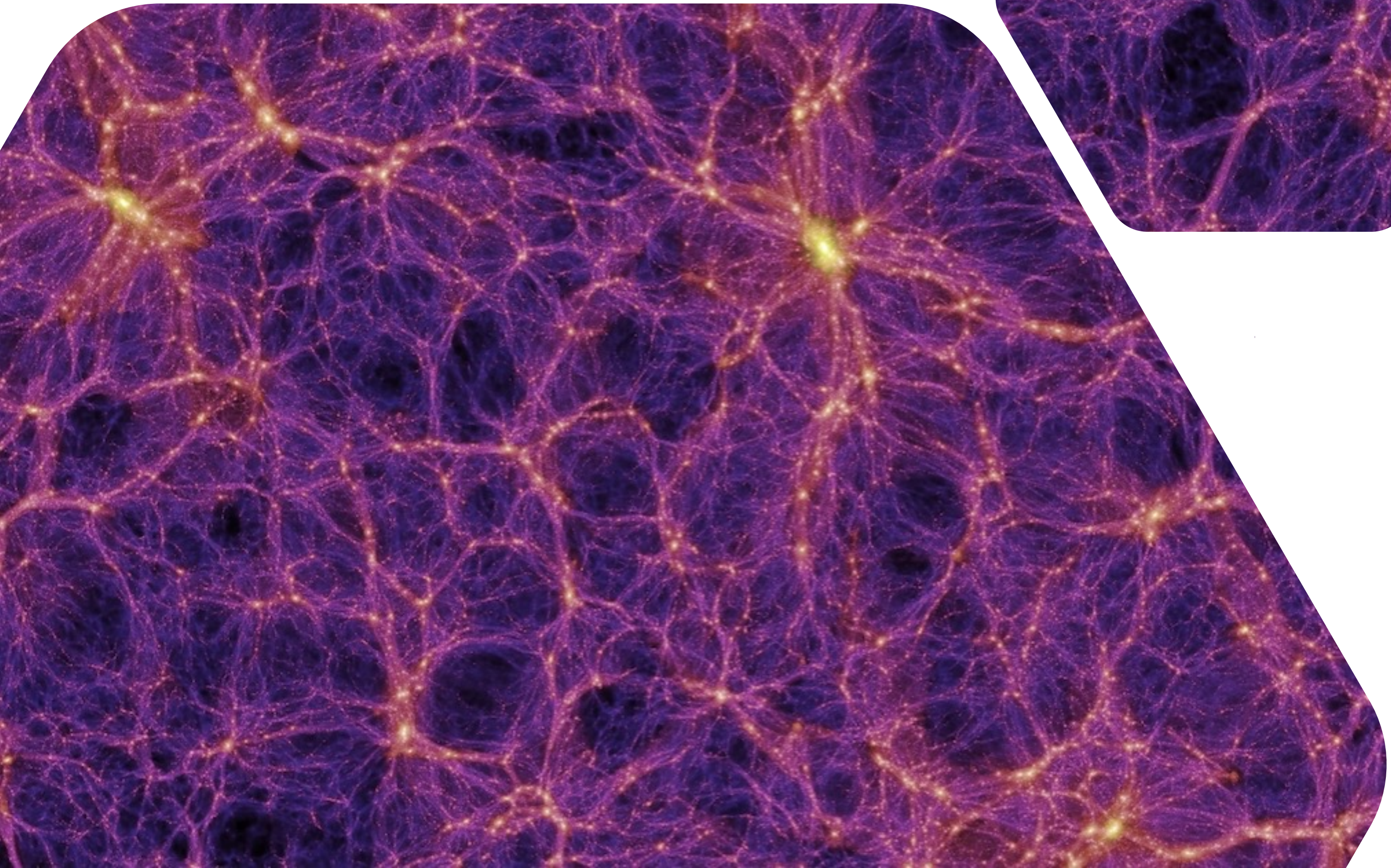






Unified description of corpuscular and Fuzzy Scalar Dark Matter

Alex Soto

Newcastle University

COSMO 22

Work with N. Proukakis and G. Rigopoulos

Outline

- Motivations for the work
- The model: Equations and limits
- Summary and conclusions

Motivations: Dark Matter

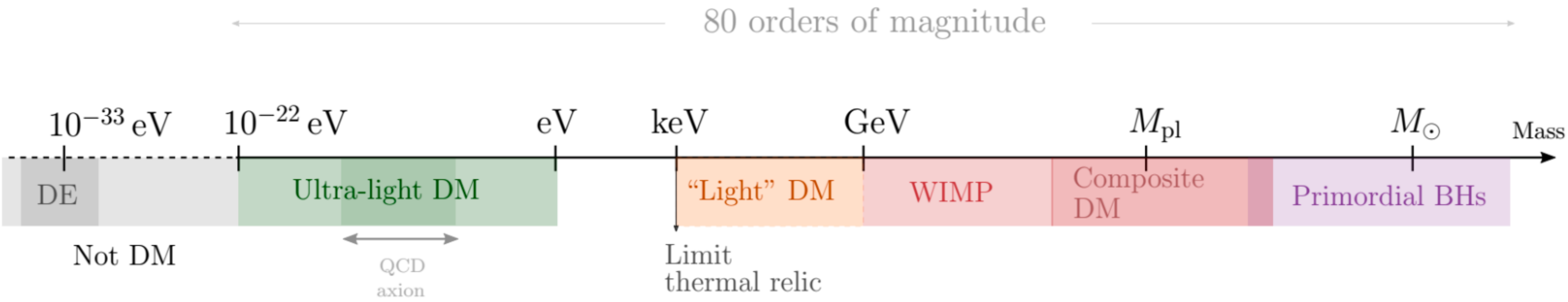


Image from: Ferreira, 2021

Motivations: CDM and FDM

Small scale crisis in ΛCDM

- Missing satellites problem
- Cusp-Core problem
- Etc.

Fuzzy Dark matter (FDM)
(Ferreira, 2021; Hui, 2021)

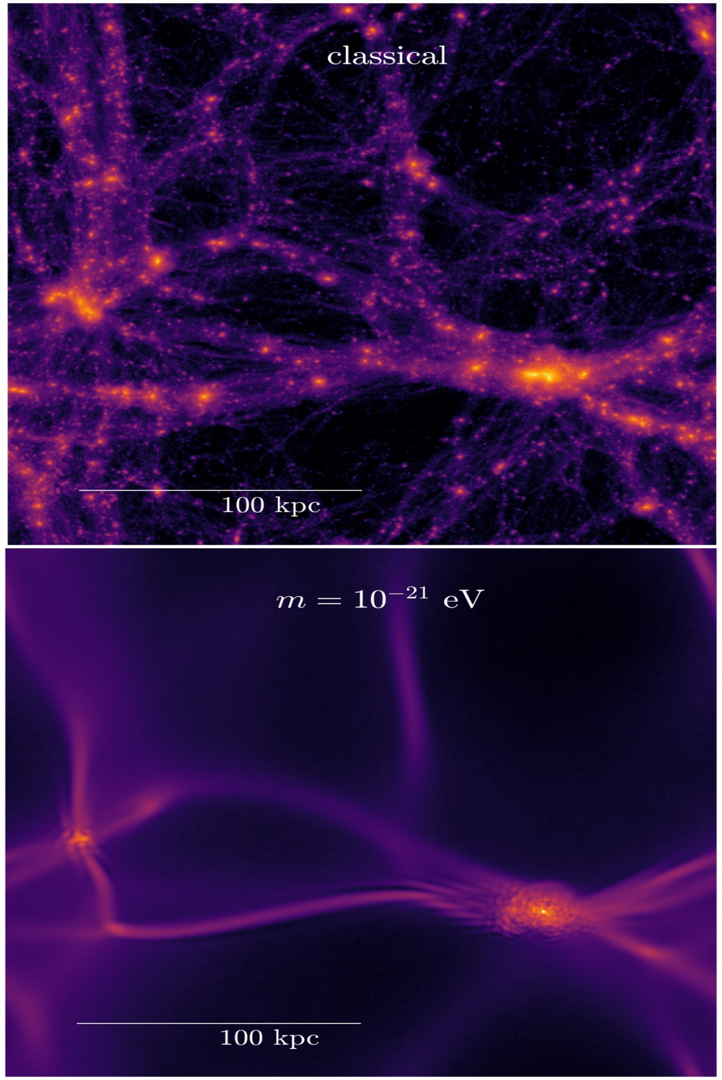
Ultra light bosons → QCD axion, ALPs

It suppresses substructures and limits central densities

$$i\partial_t\Phi = -\frac{1}{2m\,a^2}\nabla^2\Phi + mV\Phi$$

$$\nabla^2 V = \frac{4\pi G m}{a}|\Phi|^2$$

Extensions with self-interaction (SIFDM)
(Chavanis, 2016; Salehian *et al.*, 2021)



Images from: Mocz *et al.*, 2018

Motivations: Bosons and finite temperature

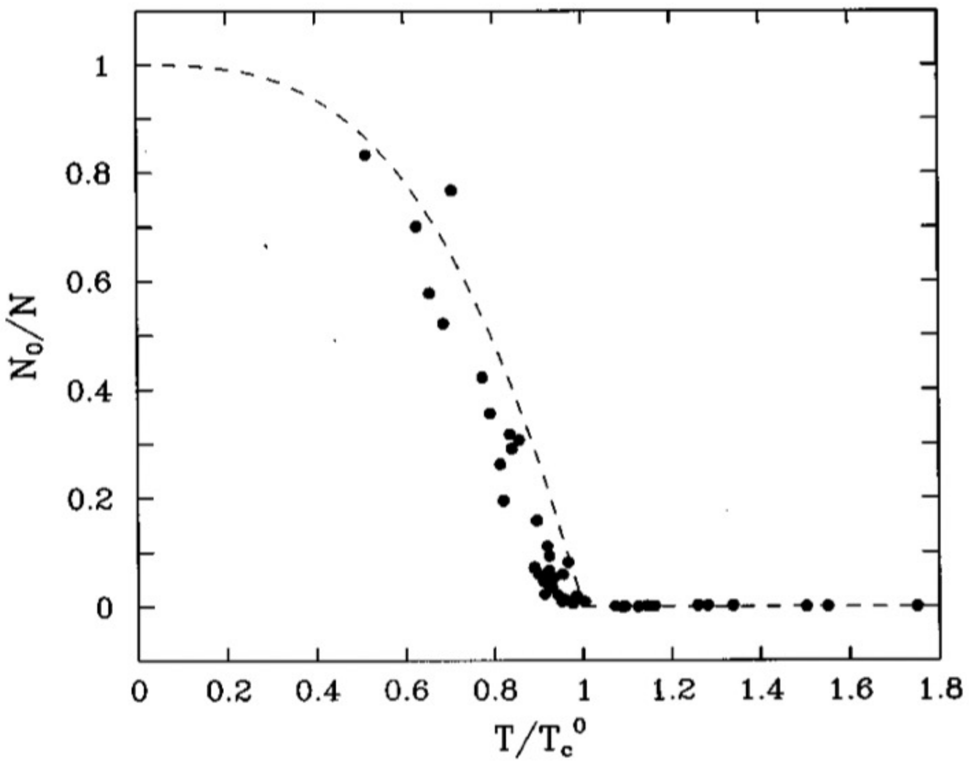


Image from: Dalfovo *et al.*, 1998

For bosons

$$\frac{N_0}{N} = 1 - \left(\frac{T}{T_c}\right)^{1+\tau}$$

$\tau = \frac{1}{2}$ Free bosons

$\tau = 2$ Harmonical trap

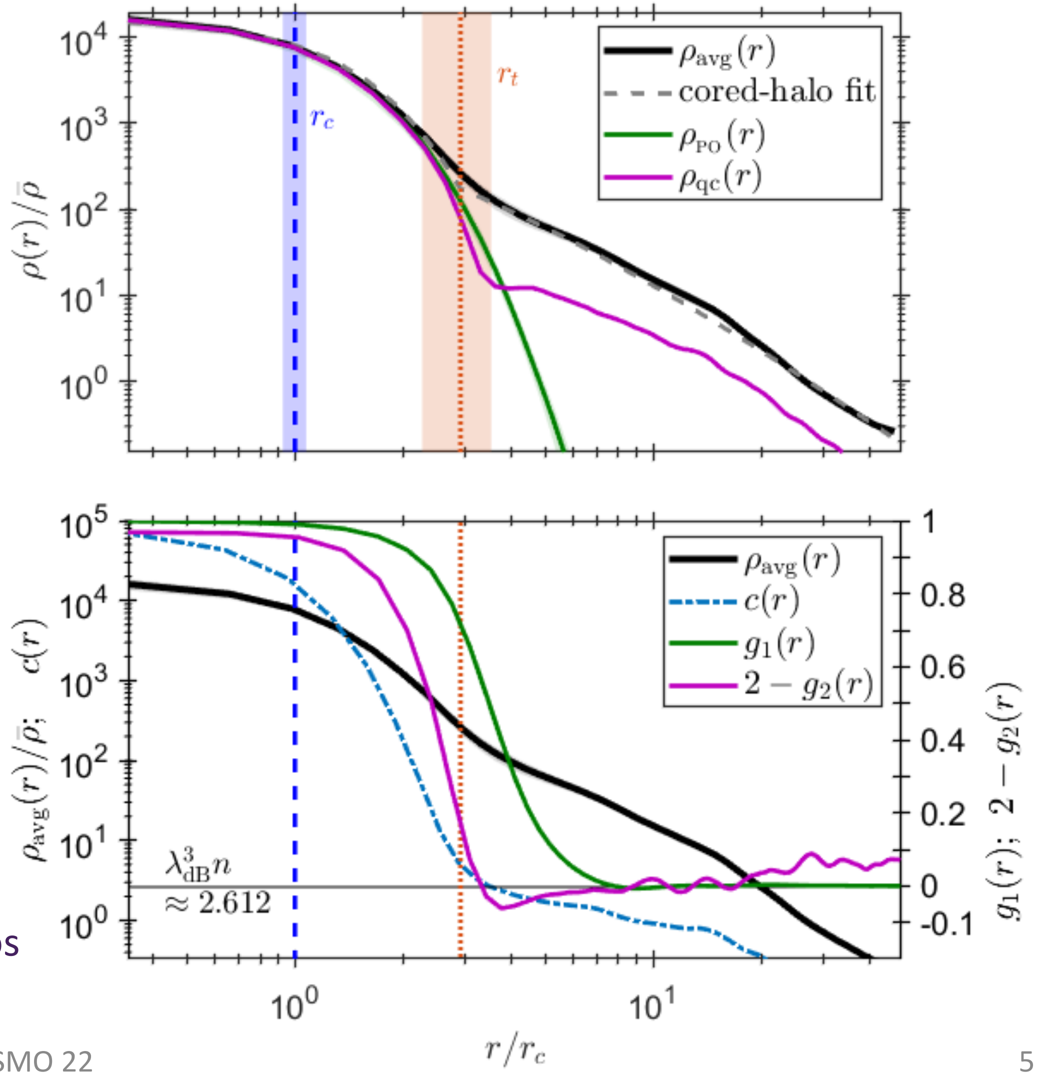
There are non condensed bosons in non-zero temperature

Motivations: Coherence and distance

New work shows the coexistence between coherent and incoherent parts
(Liu *et al.*, 2022 to appear)

There is a inner region of condensate,
And an external incoherent region

Images from: G. Liu, N. Proukakis and G. Rigopoulos



The Model

✓ Pick a non-relativistic boson with self-interaction g and with gravity

Splitting the boson
in slow (condensate)
and fast (particles) parts

Some approximations

Schwinger-Keldysh
Formalism

Perturbation theory
up to order g^2

The Model: Main equations

We have three equations. One for the condensate, particles and gravitational potential

$$i\partial_t\Phi_0 = \left[-\frac{1}{2m\,a^2}\nabla^2 + \left(mV + \frac{g}{a^3}(n_c + 2\tilde{n})\right) - i\,R\right]\Phi_0$$

$$\left(\frac{\partial}{\partial t} + \frac{k}{m\,a^2}\frac{\partial}{\partial r} - \frac{\partial}{\partial r}\left(mV + 2\frac{g}{a^3}(n_c + \tilde{n})\right)\frac{\partial}{\partial k}\right)f = \frac{1}{2}(I_a + I_b)$$

$$\nabla^2 V = \frac{4\pi G\,m}{a}(n_c + \tilde{n})$$

The Model: Main equations

We have three equations. One for the condensate, particles and gravitational potential

$$i\partial_t\Phi_0 = \left[-\frac{1}{2ma^2}\nabla^2 + \left(mV + \frac{g}{a^3}(n_c + 2\tilde{n})\right) - iR\right]\Phi_0$$

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$$\nabla^2 V = \frac{4\pi G m}{a}(n_c + \tilde{n})$$

Condensate wavefunction

Scale factor

Boson mass

Gravitational potential

Condensate Density number

Particle Density number

Coupling Self-interaction

Particle distribution function

The Model: Main equations

We have three equations. One for the condensate, particles and gravitational potential

$$i\partial_t \Phi_0 = \left[-\frac{1}{2m\,a^2}\nabla^2 + \left(mV + \frac{g}{a^3}(n_c + 2\tilde{n})\right) - i\,R\right]\Phi_0$$

$$\left(\frac{\partial}{\partial t} + \frac{k}{m\,a^2}\frac{\partial}{\partial r} - \frac{\partial}{\partial r}\left(mV + 2\frac{g}{a^3}(n_c + \tilde{n})\right)\frac{\partial}{\partial k}\right) f = \frac{1}{2}(I_a + I_b)$$

$$\nabla^2 V = \frac{4\pi G\,m}{a}(n_c + \tilde{n})$$

Collisional terms

$$I_b = 4\,\frac{g^2}{a^6}\int \frac{d^3p_2 d^3p_3 d^3p_4}{(2\pi)^5}\delta(\varepsilon_{\boldsymbol{p}_3} + \varepsilon_{\boldsymbol{p}_4} - \varepsilon_{\boldsymbol{p}_2} - \varepsilon_{\boldsymbol{p}})\delta(\boldsymbol{p} + \boldsymbol{p}_2 - \boldsymbol{p}_3 - \boldsymbol{p}_4)$$
$$\times[f_3 f_4 (f+1)(f_2+1) - f f_2 (f_3+1)(f_4+1)]$$

Particle-Particle collisions

The Model: Main equations

We have three equations. One for the condensate, particles and gravitational potential

$$i\partial_t\Phi_0 = \left[-\frac{1}{2m\,a^2}\nabla^2 + \left(mV + \frac{g}{a^3}(n_c + 2\tilde{n})\right) - i\,R\right]\Phi_0$$

$$\left(\frac{\partial}{\partial t} + \frac{k}{m\,a^2}\frac{\partial}{\partial r} - \frac{\partial}{\partial r}\left(mV + 2\frac{g}{a^3}(n_c + \tilde{n})\right)\frac{\partial}{\partial k}\right)f = \frac{1}{2}(I_a + I_b)$$

$$\nabla^2 V = \frac{4\pi G\,m}{a}(n_c + \tilde{n})$$

Collisional terms

$$I_a = 4\frac{g^2}{a^6}n_c\int\frac{d^3p_1\,d^3p_2\,d^3p_3}{(2\pi)^2}\delta(\varepsilon_{\boldsymbol{q}} + \varepsilon_{\boldsymbol{p}_1} - \varepsilon_{\boldsymbol{p}_2} - \varepsilon_{\boldsymbol{p}_3})\delta(\boldsymbol{p}_2 - \boldsymbol{p}_1 - \boldsymbol{q} + \boldsymbol{p}_3)$$
$$\times(\delta(\boldsymbol{p}_1 - \boldsymbol{p}) - \delta(\boldsymbol{p}_2 - \boldsymbol{p}) - \delta(\boldsymbol{p}_3 - \boldsymbol{p}))((1 + f_1)f_2f_3 - f_1(1 + f_2)(1 + f_3))$$

Particle-Condensate collisions

The Model: Main equations

We have three equations. One for the condensate, particles and gravitational potential

$$i\partial_t \Phi_0 = \left[-\frac{1}{2ma^2}\nabla^2 + \left(mV + \frac{g}{a^3}(n_c + 2\tilde{n}) \right) - iR \right] \Phi_0$$

Collisional term

$$\left(\frac{\partial}{\partial t} + \frac{k}{ma^2}\frac{\partial}{\partial r} - \frac{\partial}{\partial r}\left(mV + 2\frac{g}{a^3}(n_c + \tilde{n}) \right) \frac{\partial}{\partial k} \right) f = \frac{1}{2}(I_a + I_b)$$

$$\nabla^2 V = \frac{4\pi G m}{a}(n_c + \tilde{n})$$

$$R = \frac{1}{4n_c}\int \frac{d^3p}{(2\pi)^3} I_a$$ → Particle-Condensate collisions

The condensate can grow or decrease

The Model: Limits

For certain limits we recover known cases

$$i\partial_t\Phi_0 = \left[-\frac{1}{2m\,a^2}\nabla^2 + \left(mV + \frac{g}{a^3}(n_c + 2\tilde{n})\right) - i\,R\right]\Phi_0$$

$$\left(\frac{\partial}{\partial t} + \frac{k}{m\,a^2}\frac{\partial}{\partial r} - \frac{\partial}{\partial r}\left(mV + 2\frac{g}{a^3}(n_c + \tilde{n})\right)\frac{\partial}{\partial k}\right)f = \frac{1}{2}(I_a + I_b)$$

$$\nabla^2 V = \frac{4\pi G\,m}{a}(n_c + \tilde{n})$$

For $g = 0$ and all the bosons are condensed → We recover Fuzzy Dark Matter (Schrodinger-Poisson)

For $g = 0$ and there are no condensate → We recover the Vlasov-Poisson equations for CDM

No gravity and $a = 1$ → We recover ZNG model

Summary and Conclusions

- We have presented a general model for bosonic Dark Matter combining a condensate and a non-coherent part. Self-Interaction makes richer the phenomenology.

- Under some limits we recover the Fuzzy Dark matter, Vlasov-Poisson CDM, and ZNG models.

- The model can be extended to the study of fluctuations on the condensate and the gravitational potential

- Next: Applications

References

- Ferreira, E.G.M., *Astron Astrophys Rev* **29,** 7 (2021), arXiv:2005.03254 **[astro-ph.CO]**.
- Hui, L., Annual Review of Astronomy and Astrophysics 2021 59:1, 247-289, arXiv:2101.11735 **[astro-ph.CO]**
- Mocz P., Lancaster L., Fialkov A., Becerra F., and Chavanis P., Phys. Rev. D **97**, 083519 (2018), arXiv:1801.03507 **[astro-ph.CO]**
- Chavanis P., Phys. Rev. D **94**, 083007, arXiv:1604.05904 **[astro-ph.CO]**
- Salehian, B., Zhang, HY., Amin, M.A. *et al.*, *J. High Energ. Phys.* **2021,** 50 (2021), arXiv:2104.10128 **[astro-ph.CO]**
- F. Dalfovo, S. Giorgini, L. P. Pitaevskii, and S. Stringari, Rev. Mod. Phys. **71**, 463 (1998), arXiv:cond-mat/9806038