Radial oscillations of quark stars admixed with dark matter

Eduardo S. Fraga

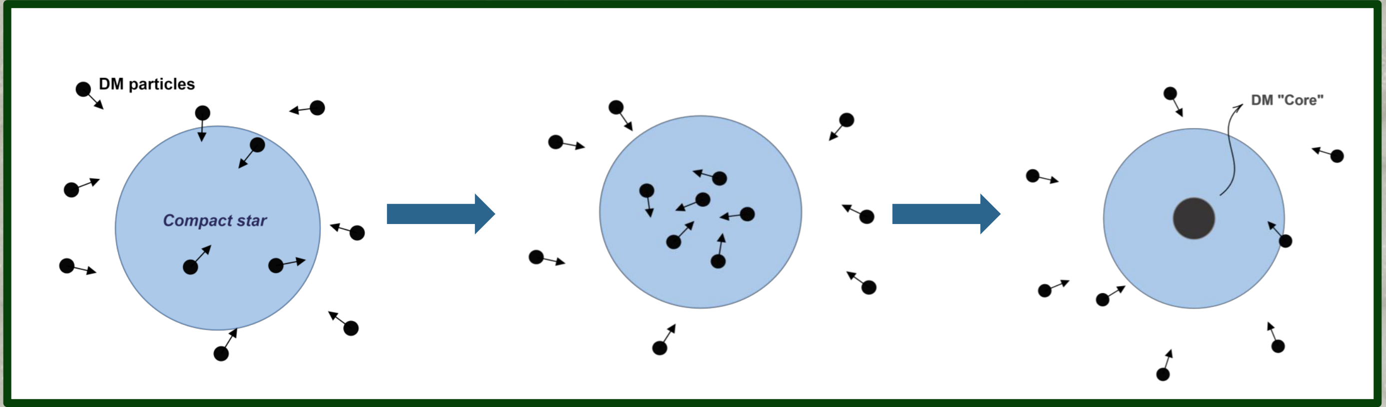
Instituto de Física
Universidade Federal do Rio de Janeiro

[NASA]

Cold dark matter & compact stars

★ DM is hard to probe, so one needs extreme gravitational interactions.

★ In principle, DM particles could collide with neutrons and other components of NS, loose energy, be gravitationally trapped, and accumulate in their cores.



[Kouvaris et al (2008]

★ NB: nucleon interactions (not ideal Fermi gas) + momentum dependence of the hadronic form factors -> significant suppression of DM capture rate in NS [Bell et al (2021)].

★ For high enough central densities, one expects to find either hybrid stars, i.e., neutron stars with a quark matter core, or even more exotic objects, such as quark stars.

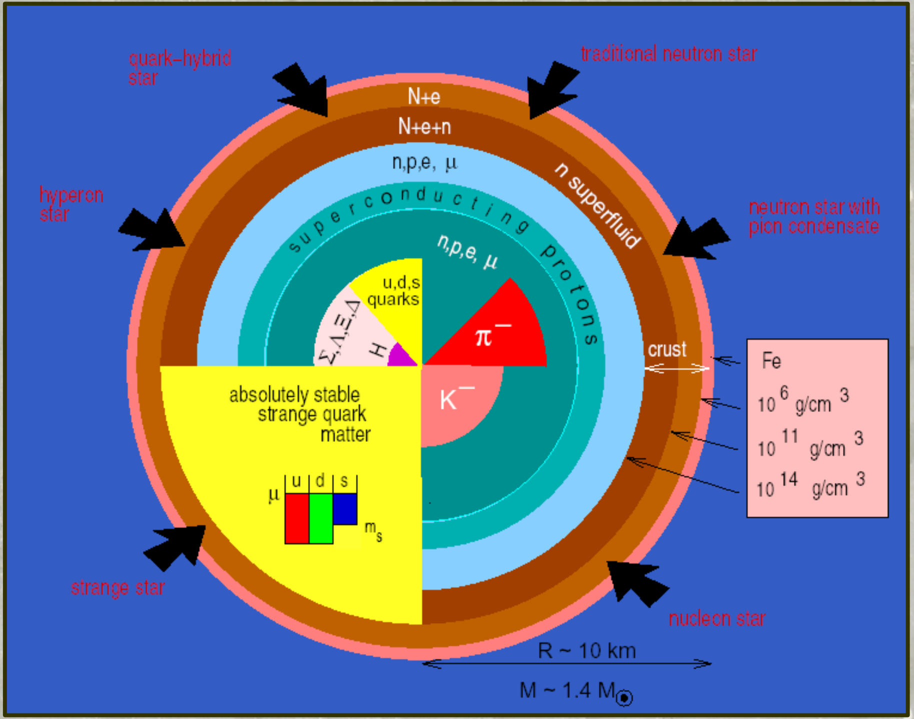
★ If quark stars are to be found in the universe, they have most likely accumulated some amount of dark matter over the course of their lives.

★ What is the effect of the presence of cold fermionic DM on:

➡ the structure of quark stars (mass, radius, etc) ?

➡ their stability w.r.t. radial oscillations ?

➡ quark magnetars with very high magnetic fields ?



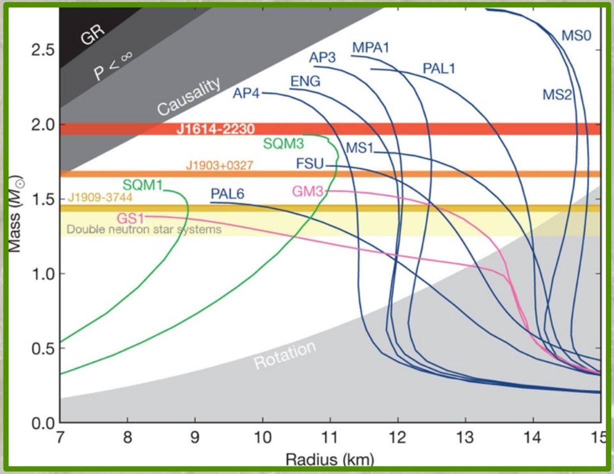
[F. Weber, 2000]

★ Which ingredients do we need?

➡ Equations of state for cold DM and cold QM.

➡ Stellar structure from TOV equations for two-fluid stars.

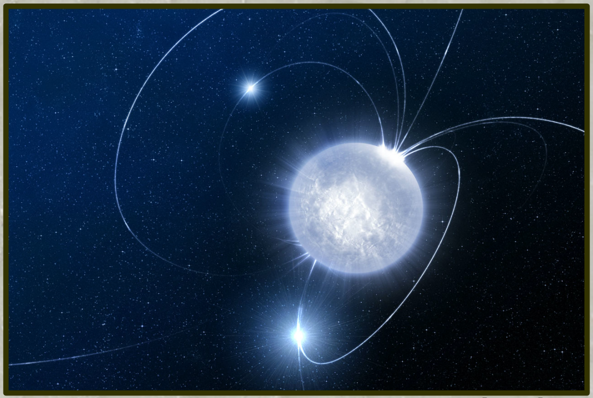
pressure(T,μ,B,etc) + TOV ➡



[Demorest et al (2010)]

➡ Stability equations & behavior of fundamental frequency.

➡ Incorporation of large magnetic fields in the EoS for QM.



[ESO]

Equations of state

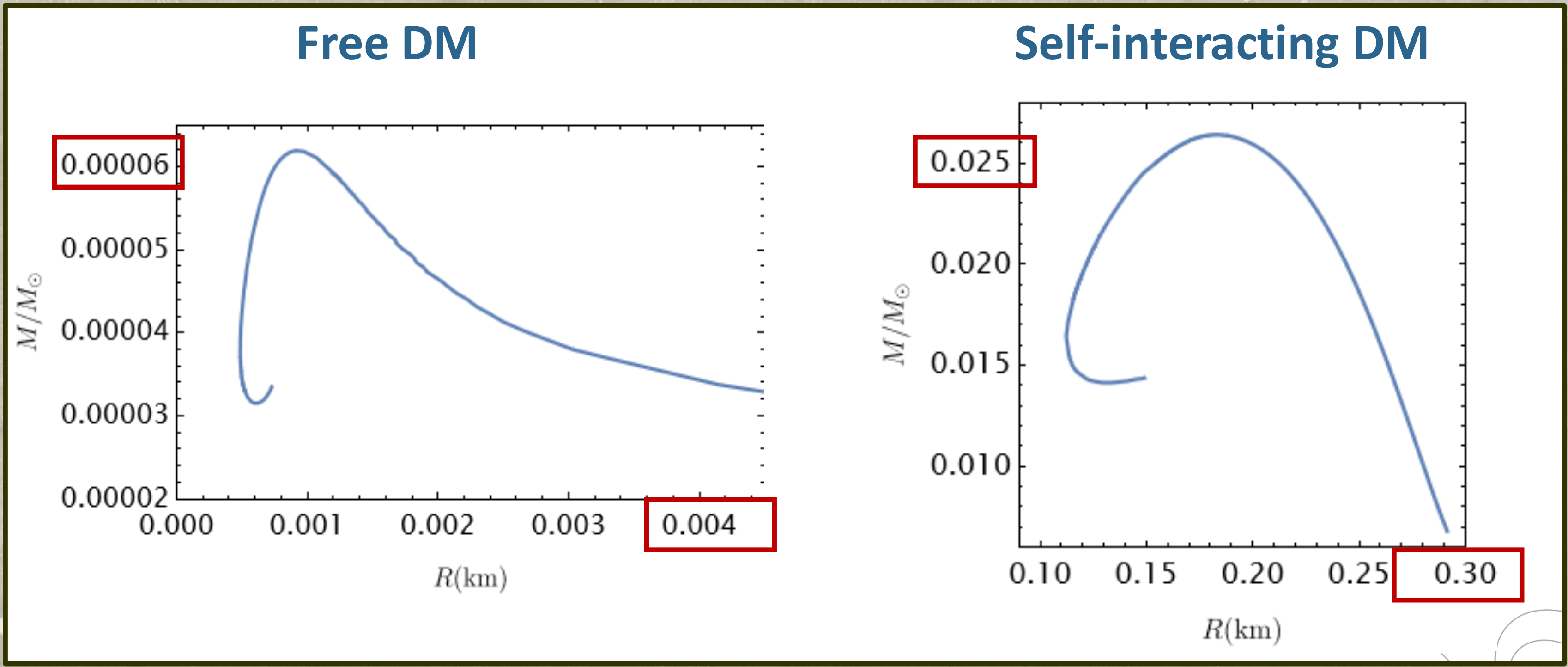
Self-interacting CDM

[Narain, Schaffner-Bielich & Mishustin (2006); Mukhopadhyay & Schaffner-Bielich (2016)]

- ★ Fermi gas + two-body self-repulsion between fermions
- ★ Useful dimensionless quantities: $z = k_F / m_D$; $y = m_D / m_I$
- ★ m_I : interaction mass scale
- ★ m_D = 1, 10, 50, 100, 200, 500 GeV (dark fermion mass)
- ★ $y = 0.1$ (weak DM); $y = 10^3$ (strong DM)
- ★ Pressure:

$$\frac{p_{\rm DM}}{m_{\rm D}^4} = \frac{1}{24\pi^2}\left[(2z^3 - 3z)\sqrt{1+z^2} + 3\sinh^{-1}(z)\right] + \left(\frac{1}{3\pi^2}\right)^2 y^2 z^6$$

Effects from DM self-interaction

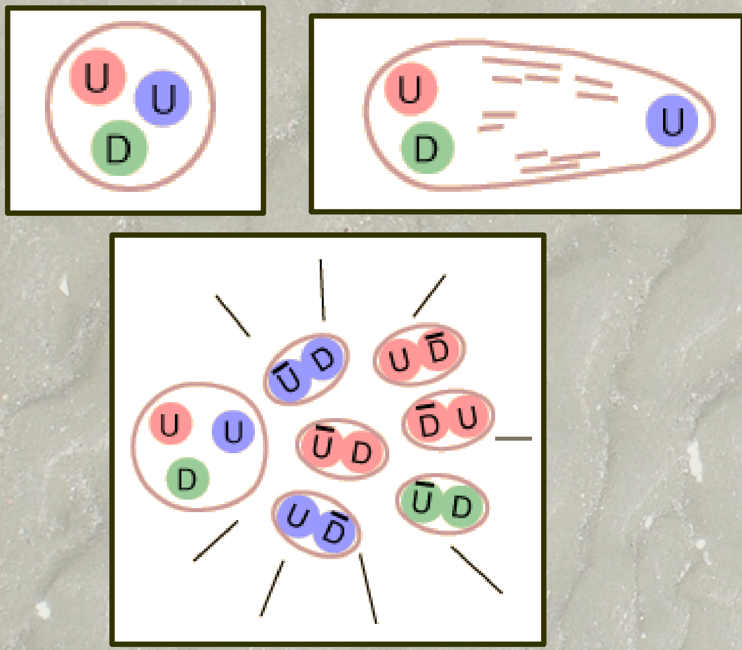


[Ferreira & ESF (in prep.)]

★ Self-interacting case corresponds to much larger masses and radii.

Cold QM

★ MIT bag model, perhaps the most popular approach to QM in NS.



Asymptotic freedom + confinement in the simplest and crudest fashion: bubbles (bags) of perturbative vacuum in a confining medium.
+ eventual corrections $\sim \alpha_s$

- Asymptotic freedom: free quarks and gluons inside color singlet bags
- Confinement: vector current vanishes on the boundary

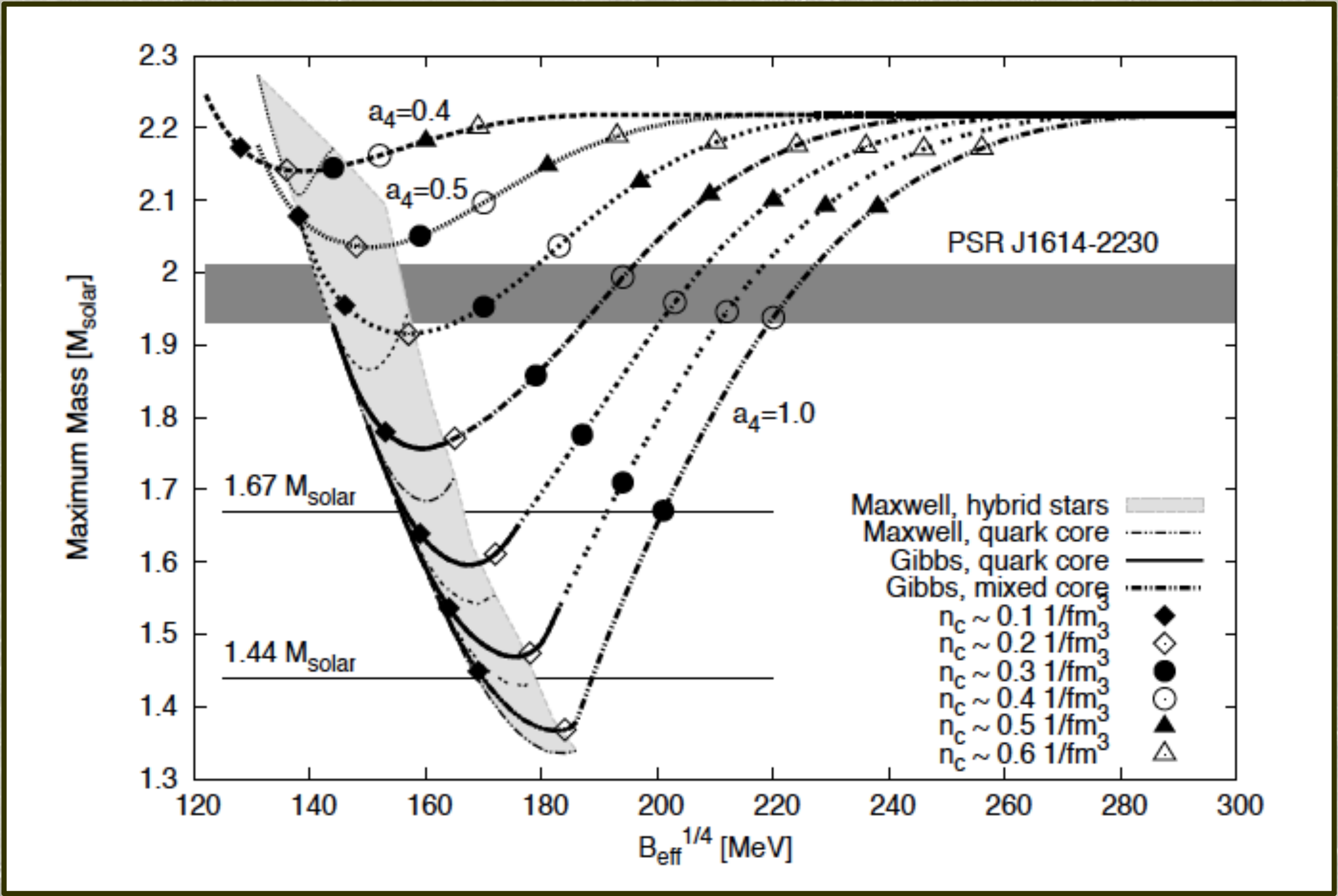
★ $B = (145\text{MeV})^4 \approx 57\text{MeV/fm}^3$ is the bag constant chosen to surpass the two-solar mass limit.

★ Pressure:

$$p_{\rm QM} = \frac{3\mu_q^4}{4\pi^2} - B$$

(μ_q : quark chemical potential)

Dependence on the choice of the bag constant



[Weissenborn, Sagert, Pagliara, Hempel, Schaffner-Bielich (2011)]

Stellar structure of one-fluid stars

★ From the TOV equations

[Einstein's GR field equations + spherical symmetry + hydrostatic equilibrium]

$$\frac{dp}{dr} = -\frac{G\mathcal{M}(r)\epsilon(r)}{r^2\left[1-\frac{2G\mathcal{M}(r)}{r}\right]}\left[1+\frac{p(r)}{\epsilon(r)}\right]\left[1+\frac{4\pi r^3 p(r)}{\mathcal{M}(r)}\right]$$

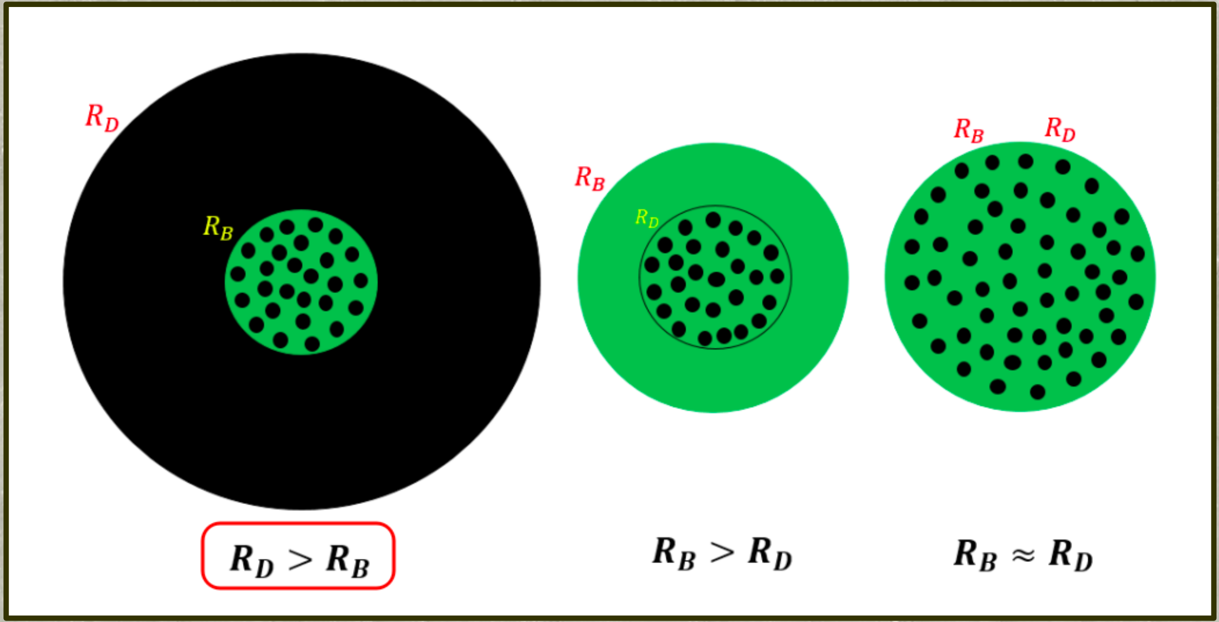
$$\frac{d\mathcal{M}}{dr} = 4\pi r^2 \epsilon(r) \;\; ; \quad \mathcal{M}(R) = M$$

m_D	$M_{max}(M_\odot)$	R_{min}	**Compact Star**
100 GeV	10^{-4}	1 m	neutralino star (cold DM)
1 GeV	1	10 km	neutron star
1 GeV/0.5 MeV	1	10^3 km	white dwarf
10 keV	10^{10}	10^{11} km	sterile neutrino star
1 keV	10^{12}	10^{13} km	axino star (warm DM)
1 eV	10^{18}	10^{19} km	neutrino star
10^{-2} eV	10^{22}	10^{23} km	gravitino star

[Mukhopadhyay & Schaffner-Bielich (2016)]

Quark stars admixed with DM

★ Three possible configurations for dark compact stars



[Karkevandi et al. (2021)]

Stellar structure of two-fluid stars

★ Two-fluid TOV equations

[Sandin & Ciarcelluti (2009)]

$$\frac{dp_{\rm QM}}{dr} = -\frac{(p_{\rm QM}+\epsilon_{\rm QM})}{2}\frac{d\nu}{dr}, \qquad \frac{dm_{\rm QM}}{dr} = 4\pi r^2 \epsilon_{\rm QM},$$

$$\frac{dp_{\rm DM}}{dr} = -\frac{(p_{\rm DM}+\epsilon_{\rm DM})}{2}\frac{d\nu}{dr}, \qquad \frac{dm_{\rm DM}}{dr} = 4\pi r^2 \epsilon_{\rm DM},$$

$$\frac{d\nu}{dr} = 2\frac{(m_{\rm QM}+m_{\rm DM}) + 4\pi r^3 (p_{\rm QM}+p_{\rm DM})}{r(r-2(m_{\rm QM}+m_{\rm DM}))},$$

★ Boundary conditions:

- $m_{QM}(r \to 0) = m_{DM}(r \to 0) \to 0$
- $R_{QM} > R_{DM}$: first $p_{DM}(R_{DM}) \to 0$; later $p_{QM}(R_{QM}) \to 0$
- $R_{DM} > R_{QM}$: first $p_{QM}(R_{QM}) \to 0$; later $p_{DM}(R_{DM}) \to 0$

Radial oscillations

[Jiménez & ESF (2022)]

★ $\Delta r/r \equiv \xi$ & Δp are the independent variables ; Γ: adiabatic index

[Gondek et al. (1997)]

★ For two-fluid stars one can write the total Lagrangian variables as $\xi \equiv \xi_{QM} + \xi_{DM}$ and $\Delta p \equiv \Delta p_{QM} + \Delta p_{DM}$

★ Two-fluid radial pulsating equations

$$\frac{d\xi_{\rm QM/DM}}{dr} \equiv -\frac{1}{r}\left(3\xi_{\rm QM/DM} + \frac{\Delta p_{\rm QM}}{\Gamma p}\right) - \frac{dp}{dr}\frac{\xi_{\rm QM/DM}}{(p+\epsilon)} \,,$$

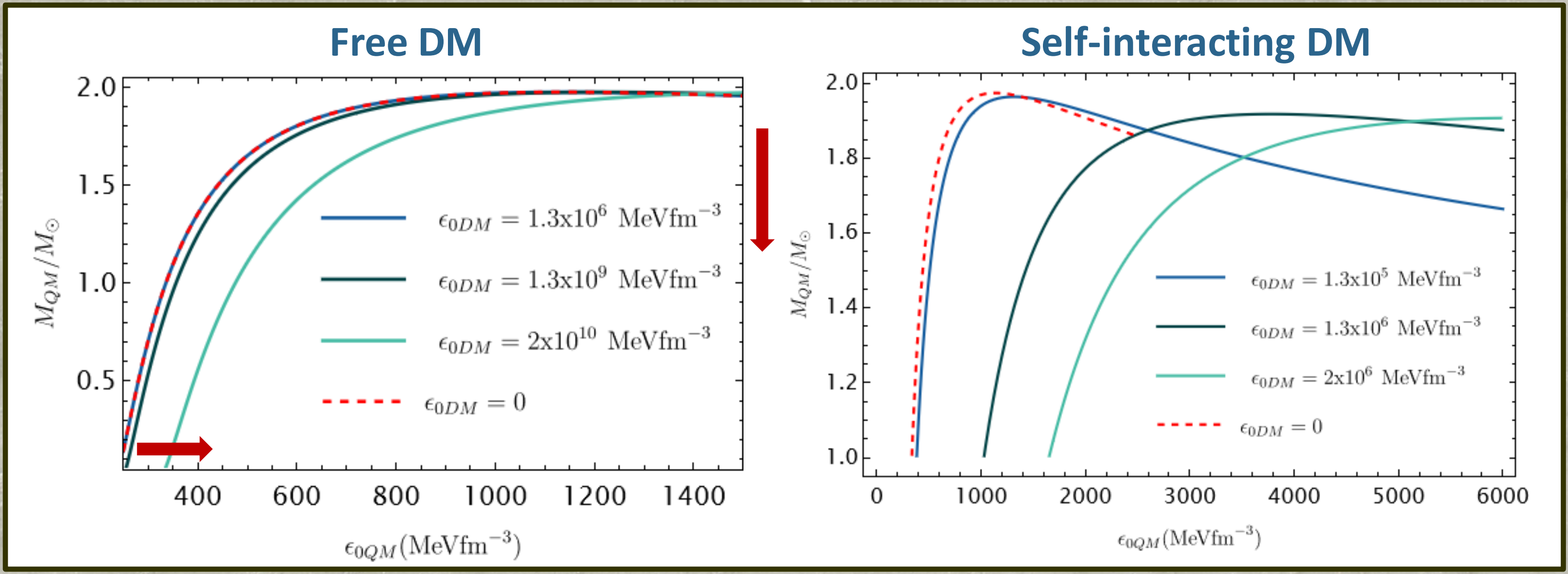
$$\frac{d\Delta p_{\rm QM/DM}}{dr} \equiv \xi_{\rm QM/DM}\left\{\omega^2 e^{\lambda-\nu}(p+\epsilon)r - 4\frac{dp}{dr}\right\} + \xi_{\rm QM/DM}\left\{\left(\frac{dp}{dr}\right)^2\frac{r}{(p+\epsilon)} - 8\pi e^{\lambda}(p+\epsilon)pr\right\} + \Delta p_{\rm QM/DM}\left\{\frac{dp}{dr}\frac{1}{p+\epsilon} - 4\pi(p+\epsilon)re^{\lambda}\right\}$$

$$\lambda(r) = -\ln(1 - 2(m_{\rm QM}(r) + m_{\rm DM}(r))/r)$$

★ ω: oscillation frequency ; $\lambda(R_{QM}) = -\nu(R_{QM})$ and $\lambda(R_{DM}) = -\nu(R_{DM})$

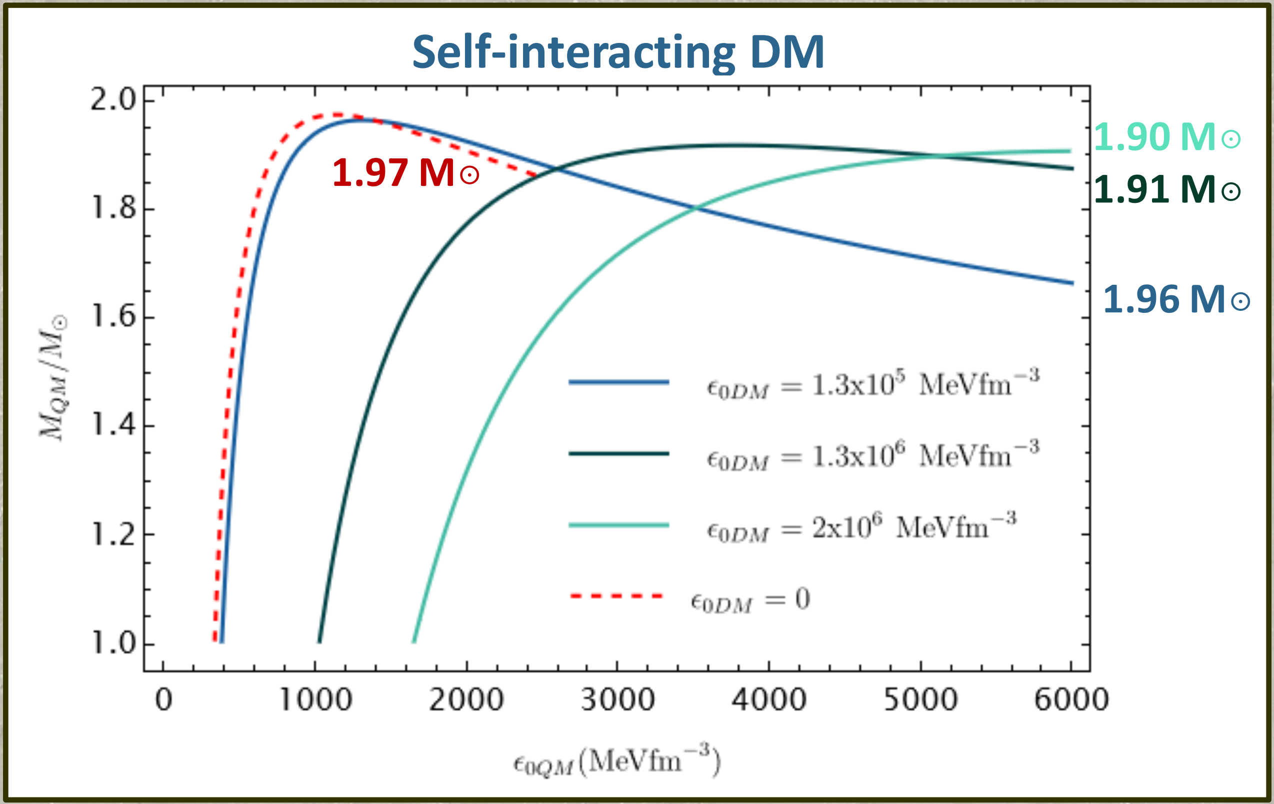
Results for structure and stability

Results for m_D = 100 GeV for illustration: [Ferreira & ESF (in prep.)]



★ The increase in DM central energy density does not change the maximum mass and radius very much, but shifts the curves towards higher central energy densities.

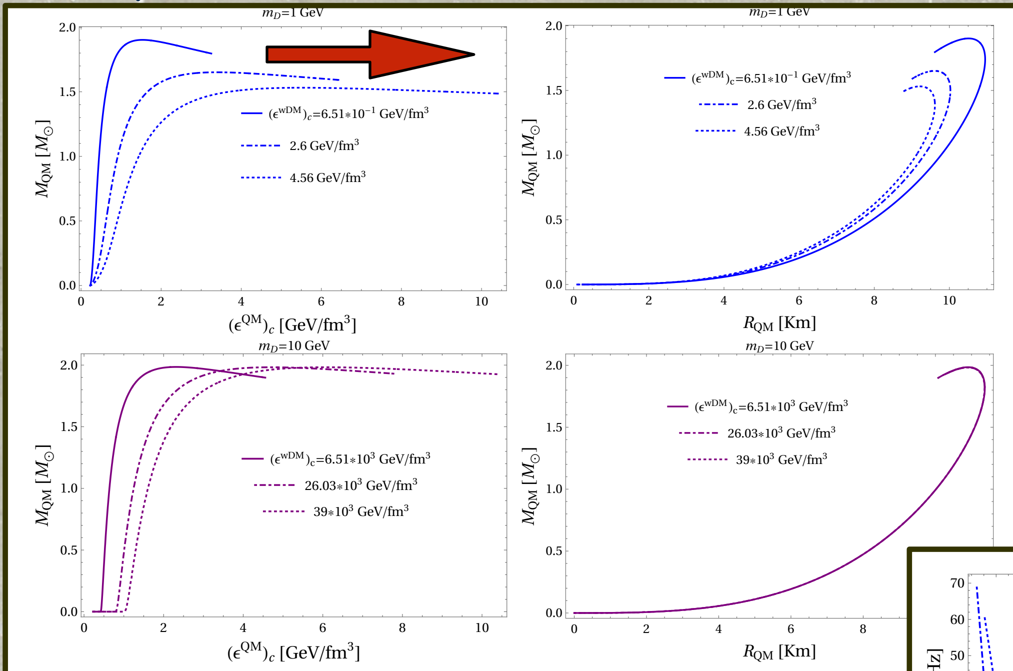
★ The range of stable configurations occurs at higher central energy densities.



★ Slight decrease of maximum mass with the increase of DM central energy density.

Results for different values of m_D – structure and stability of quark stars admixed with DM

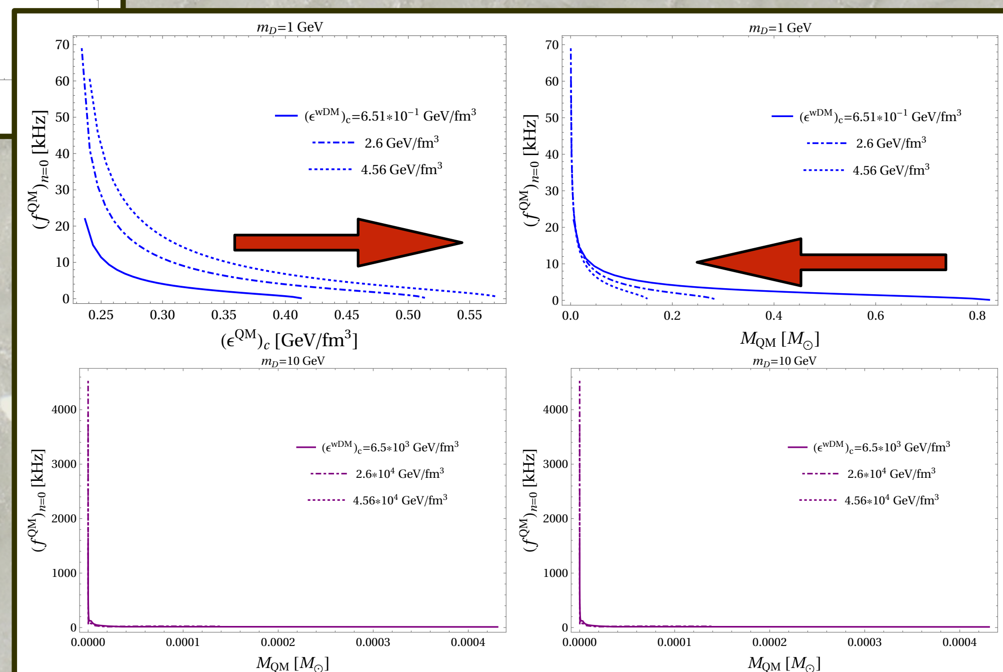
[Jiménez & ESF (2022)]



wDM: y = 0.1 ; m_D = 1, 10 GeV

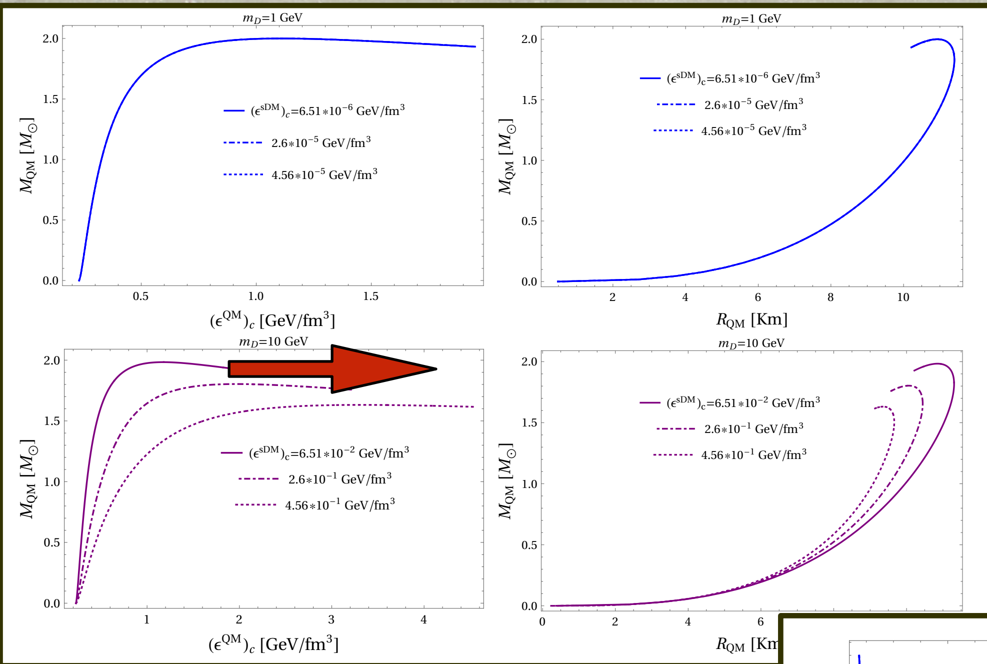
★ Mass-radius visible modifications only for small m_D.

★ Higher QM energy densities to compensate for the extra gravitational pull from DM.

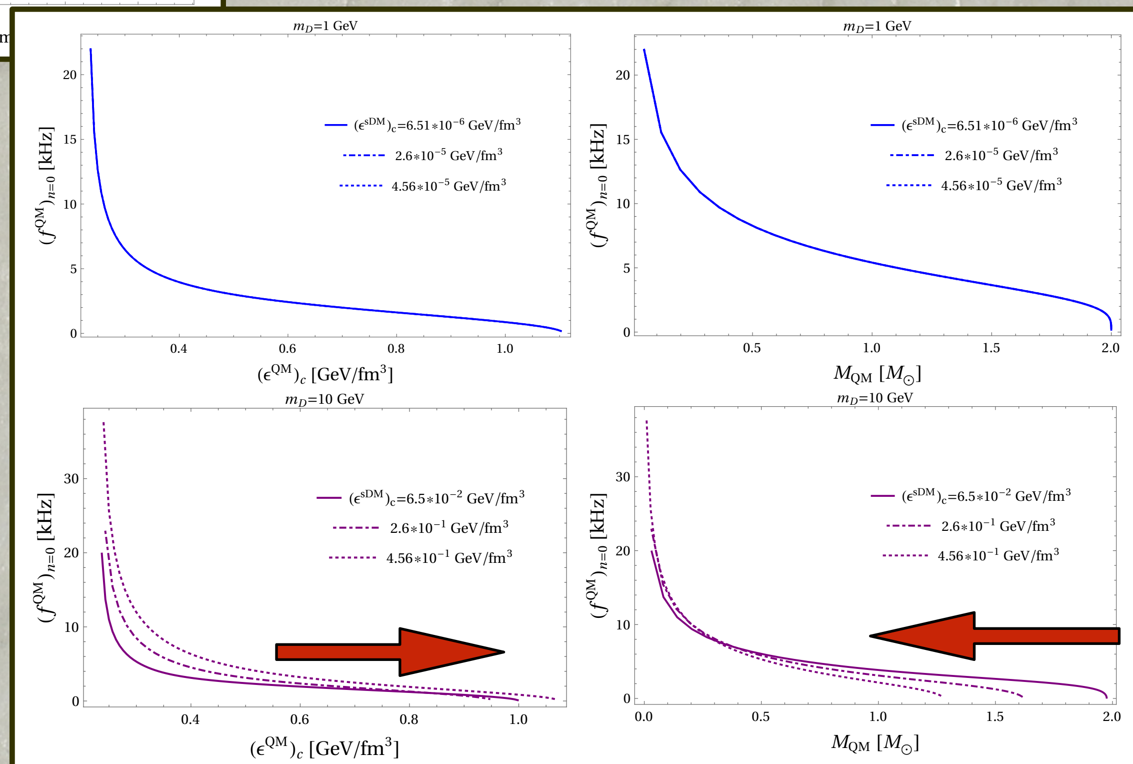


★ Stability window of ultra-light quark stars (surrounded by DM): 10^{-18}–10^{-4} $M_\odot$, depending on m_D -> dark strange "planets" and strangelets.

sDM: y = 10^3 ; m_D = 1, 10 GeV



- ★ As for wDM, in most of the cases M, R and central energy densities of the QM core are not appreciably affected.
- ★ As we increase m_D, the fundamental frequency is strongly affected.



- ★ Increasing DM central densities, the maximum QM central densities are increased by a factor of ~20 in some cases.
- ★ Results very sensitive to m_D.

Effects from high magnetic fields

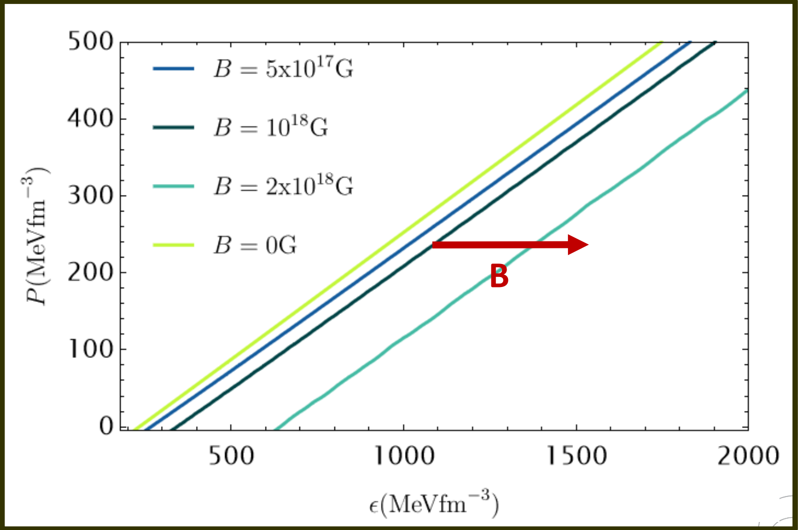
[Ferreira & ESF (in prep.)]

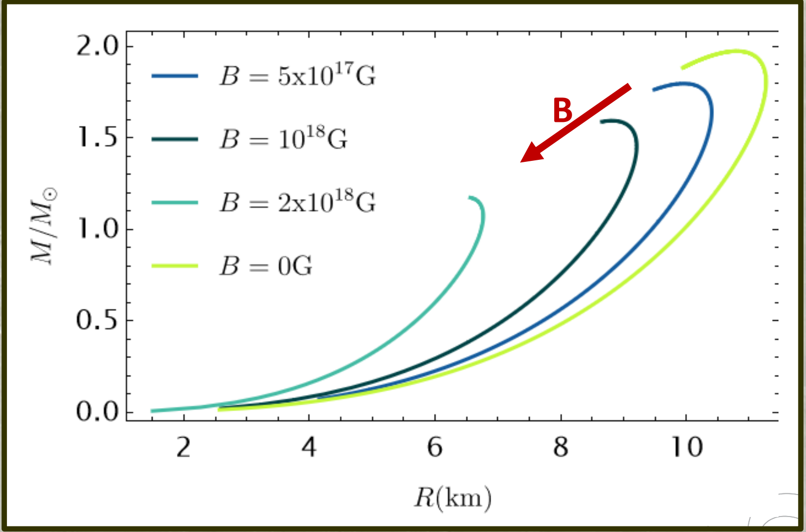
Soft gamma repeater (SGR) in 1979
(Mazets et al., 1979 [7])
(Cline et al, 1980 [8])

Anomalous X-ray pulsar (AXP)
(Mereghetti & Stella, 1995 [9])

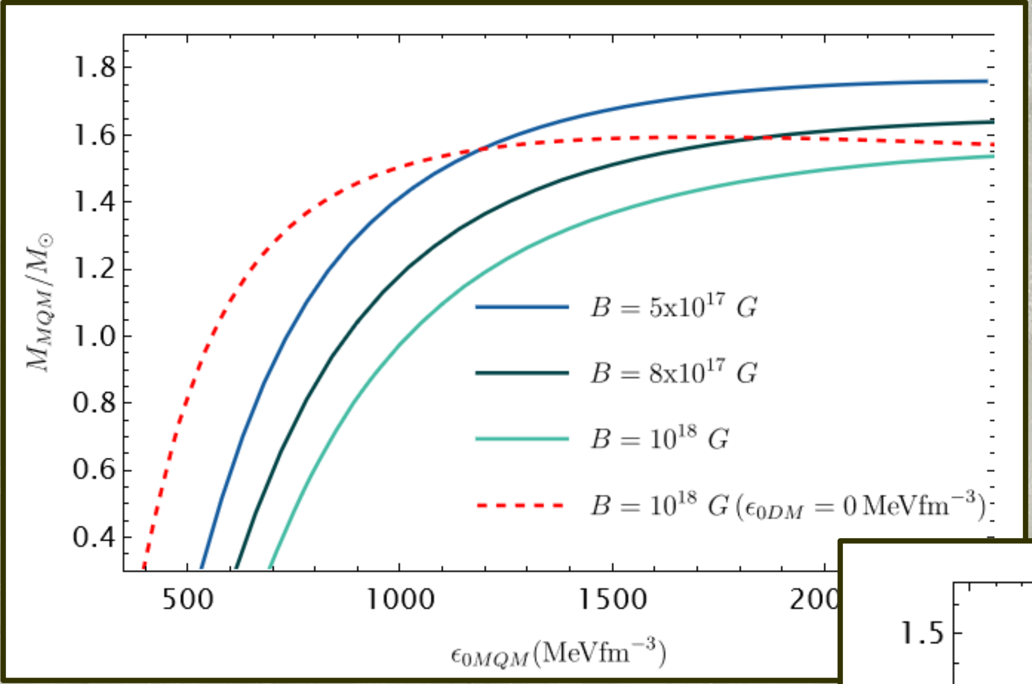
→ **Magnetars** surface magnetic fields of the order of 10^{14} G - 10^{15} G.

- ★ Magnetic fields inside magnetars may reach values B ~ 10^{18} G.
  [Cardall, Prakash & Lattimer (2001)]
- ★ For quark magnetars:





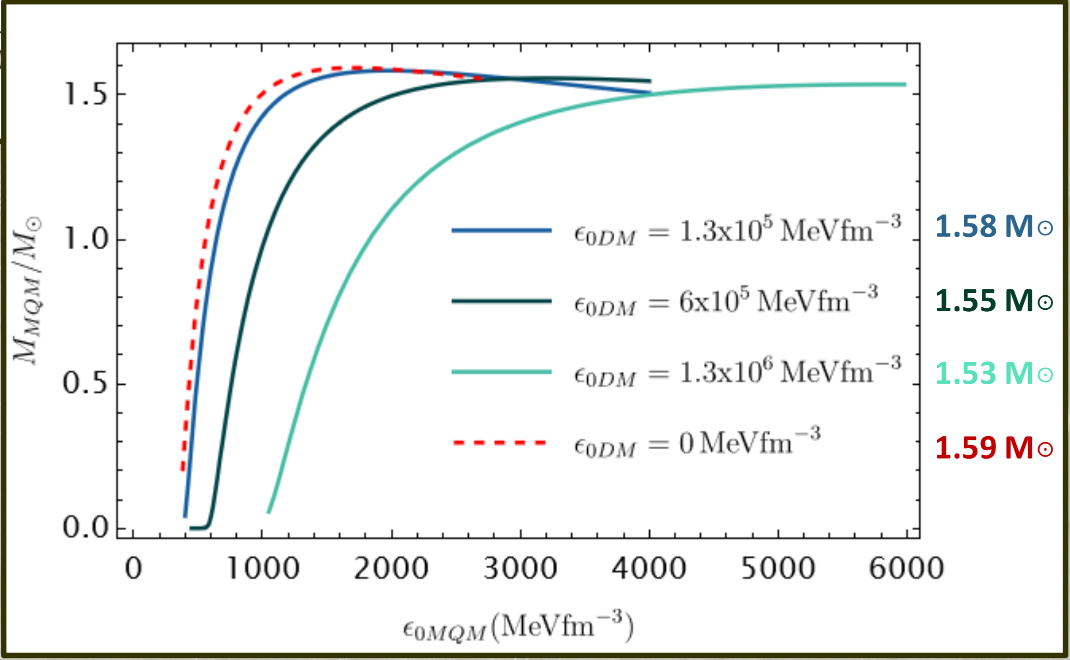
$\epsilon_{0DM} = 6\text{x}10^5\ \text{MeVfm}^{-3}$



★ Magnetic fields tend to "pull" in the same direction as DM: larger central energy densities, smaller masses.

★ NB: red curves have no DM.

★ There was no concern in producing $2M_\odot$ stars here (which is possible).



1.58 $M_\odot$
1.55 $M_\odot$
1.53 $M_\odot$
1.59 $M_\odot$

Summary and outlook

★ We investigated effects of weakly ($y = 0.1$) and strongly ($y = 10^3$) self-interacting DM on the structure of quark stars for dark fermion masses $m_D = 1, 10, 50, 100, 200, 500$ GeV.

★ We developed a framework which deals with the oscillation equations of two-fluid stars where one assumes a disturbance of just one fluid. The other being indirectly affected by the thermodynamic coupling.

★ Results are very sensitive to (m_D,y). In most situations, central QM densities are increased by the presence of DM (extra gravitational pull).

★ Stability window of ultra-light quark stars (surrounded by DM): 10^{-18}-10^{-4} $M_\odot$, depending on m_D -> dark strange "planets" and strangelets

★ Strong magnetic fields make quark matter softer and magnetars made of quark matter would be more compact.

★ Next steps: quark matter EoS from cold and dense pQCD, hybrid stars, include magnetic field effects on TOV.

Back up slides

Boundary conditions

★ Demanding:

➡ smoothness at the QM or DM stellar center

➡ Vanishing $p_{QM/DM}$ at $R_{QM/DM}$

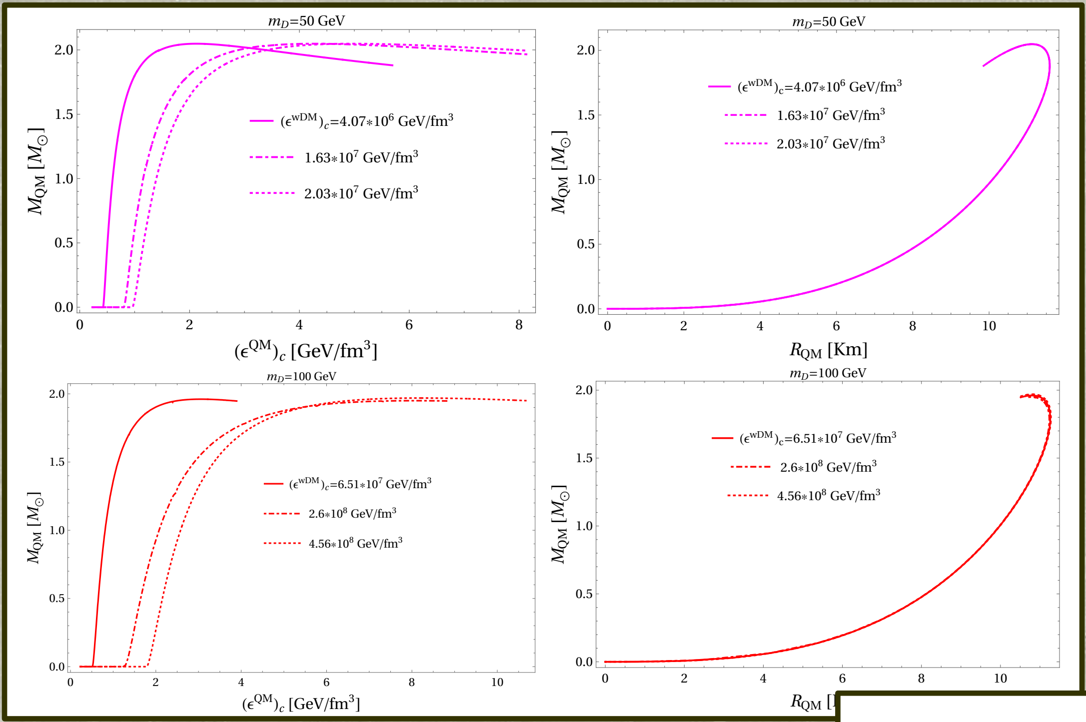
$$\nu(R_{\rm QM}) = \ln\left(1 - \frac{2(M_{\rm QM} + m_{\rm DM}(R_{\rm QM}))}{R_{\rm QM}}\right)$$

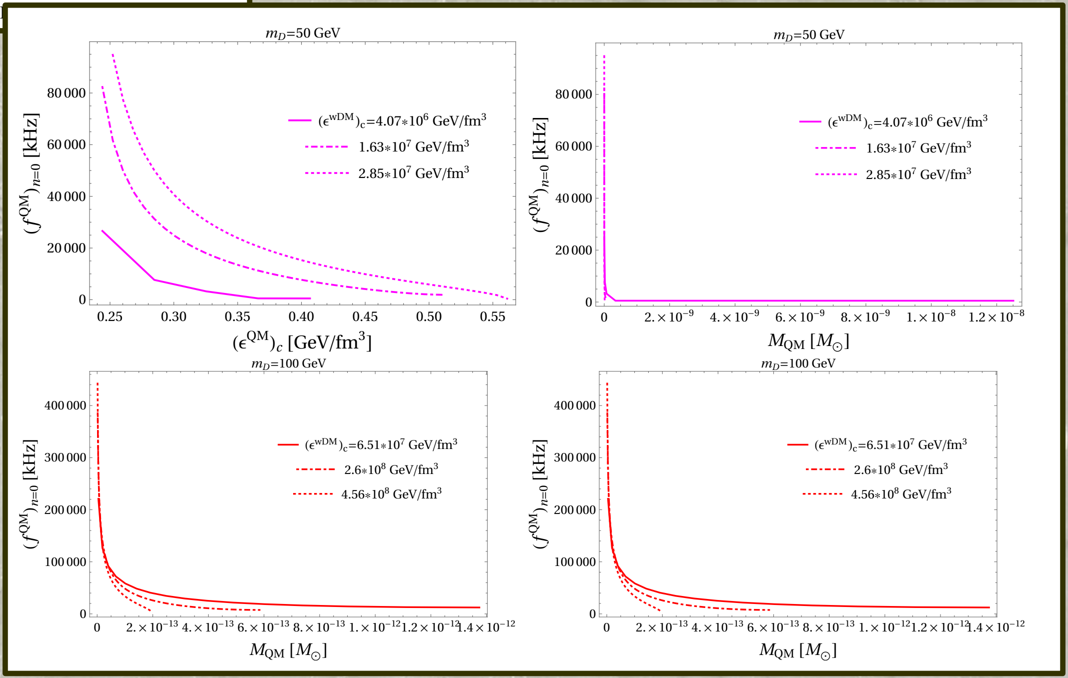
$$\nu(R_{\rm DM}) = \ln\left(1 - \frac{2(m_{\rm QM}(R_{\rm DM}) + M_{\rm DM})}{R_{\rm DM}}\right)$$

$$(\Delta p_{\rm QM/DM})_{\rm center} \equiv -3(\xi_{\rm QM/DM}\Gamma p_{\rm QM/DM})_{\rm center}$$

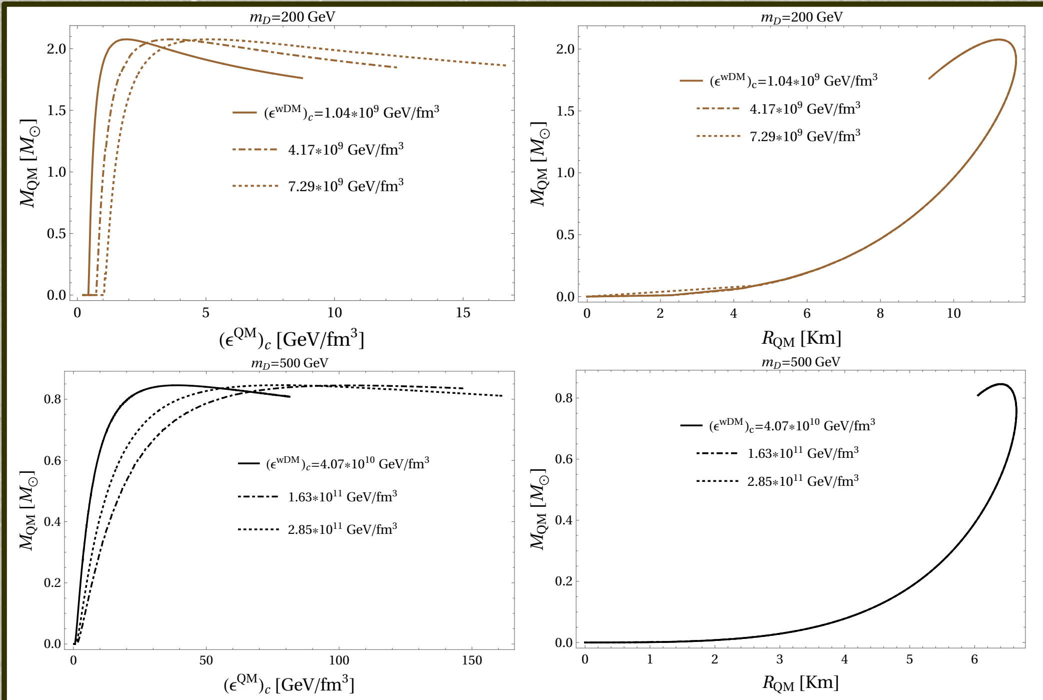
$$(\Delta p_{\rm QM/DM})_{\rm surface} \equiv 0$$

★ We define $\omega^2 \to \omega^2_{QM/DM}$ if we are dealing with a QM/DM oscillating core in the admixed star.

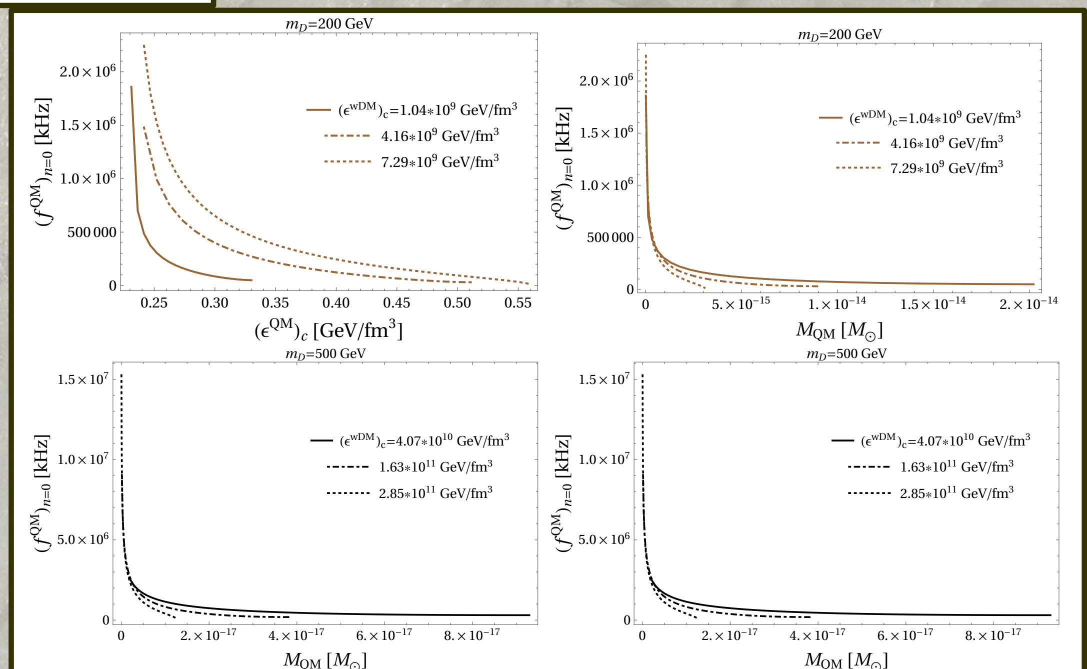


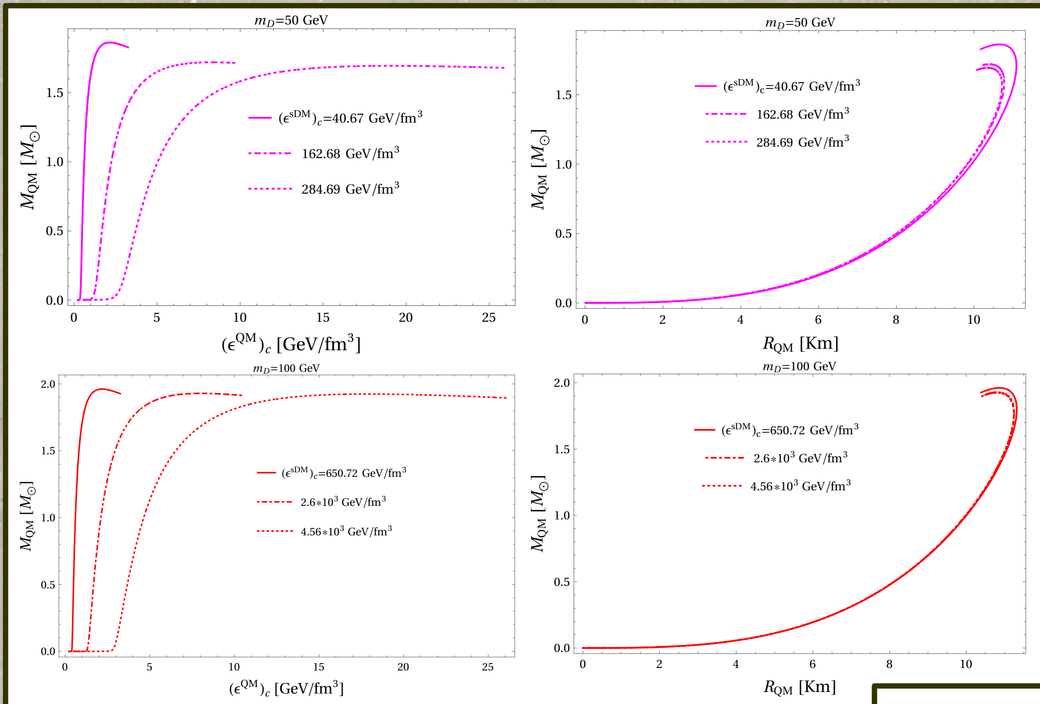


- ★ y = 0.1
- ★ m_D = 50, 100 GeV

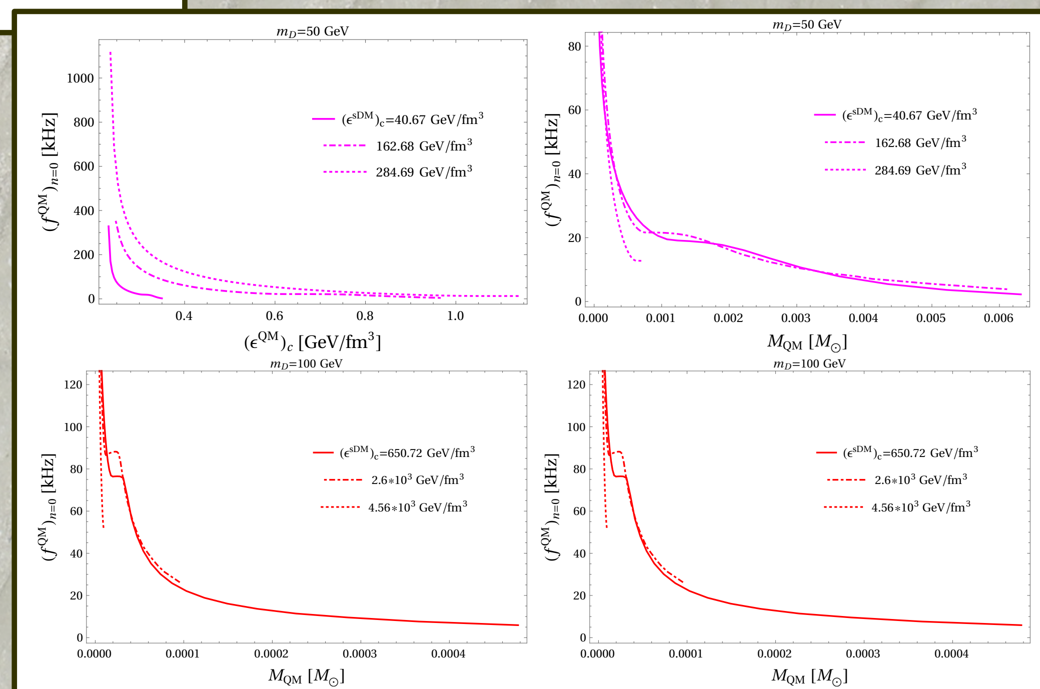


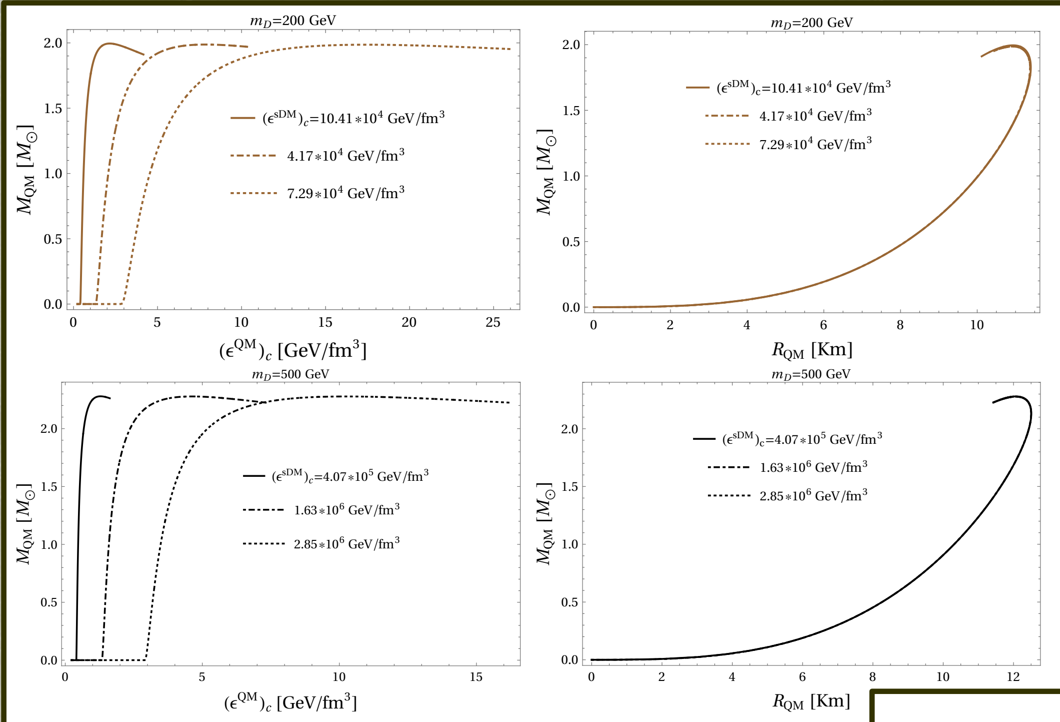
- ★ y = 0.1
- ★ m_D = 200, 500 GeV





- ★ $y = 10^3$
- ★ m_D = 50, 100 GeV





- ★ $y = 10^3$
- ★ m_D = 200, 500 GeV