Primordial Black Holes from Multifield Inflation with Non-minimal Couplings

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My collaborators!

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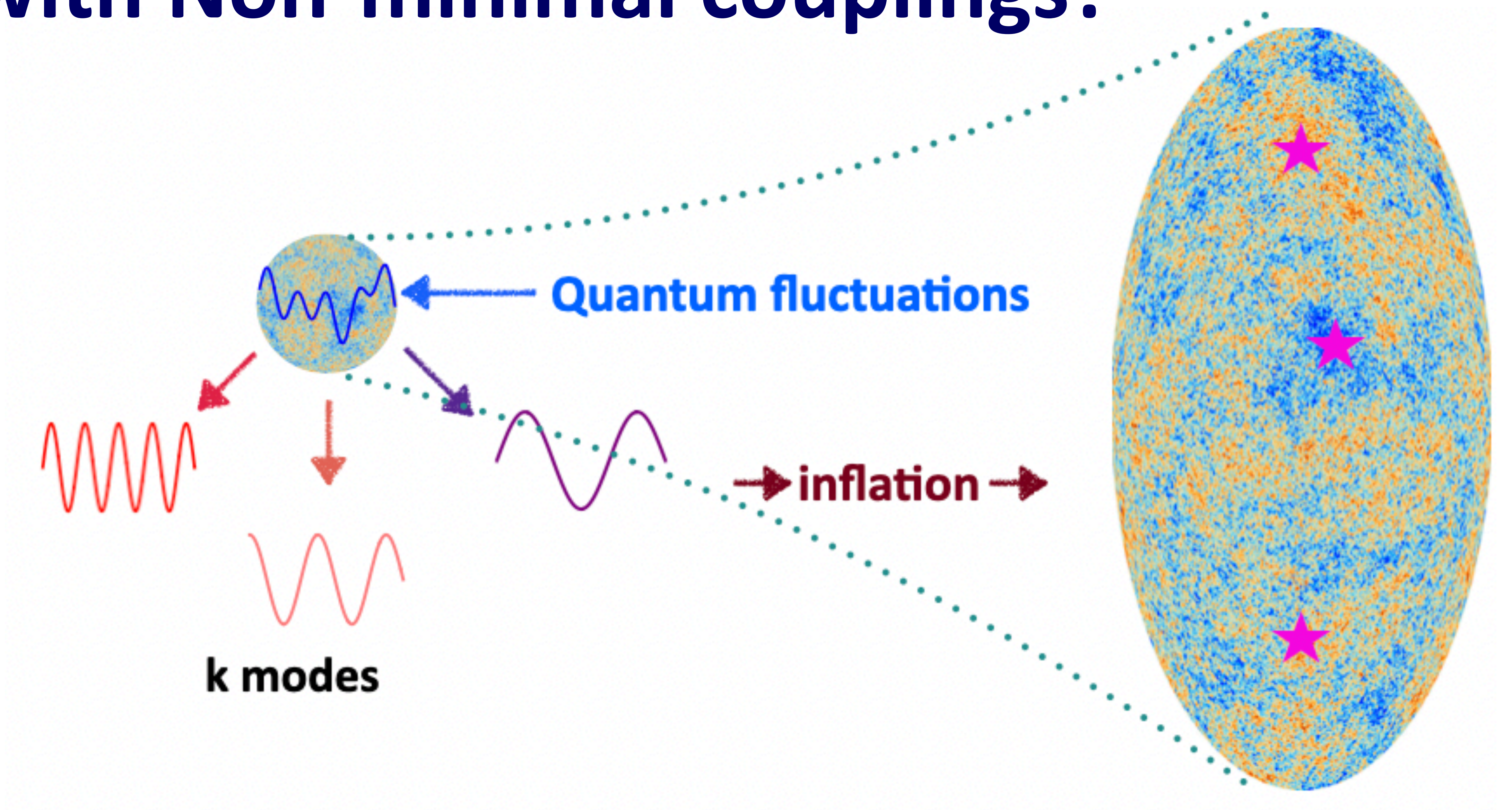
(arXiv: 2205.04471)

Overview

Main Question:

How generically and under what constraints will Primordial Black Holes (PBHs) near the Dark Matter mass range ($10^{17} \sim 10^{22}$g) form from primordial density perturbations following Multifield Inflation (MFI) with Non-minimal couplings?



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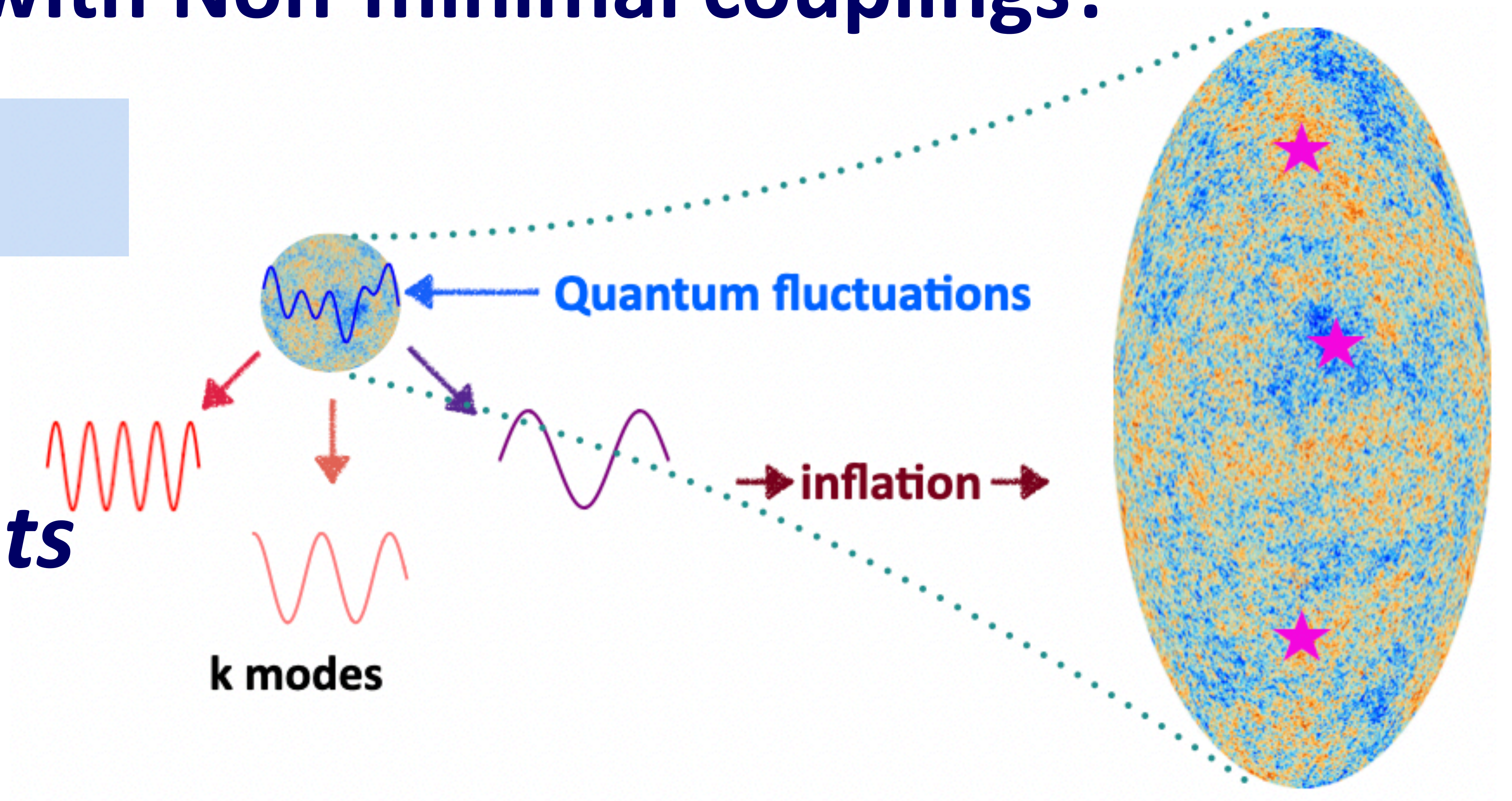
PBHs as Dark Matter

PBHs from *realistic high energy theory ingredients*



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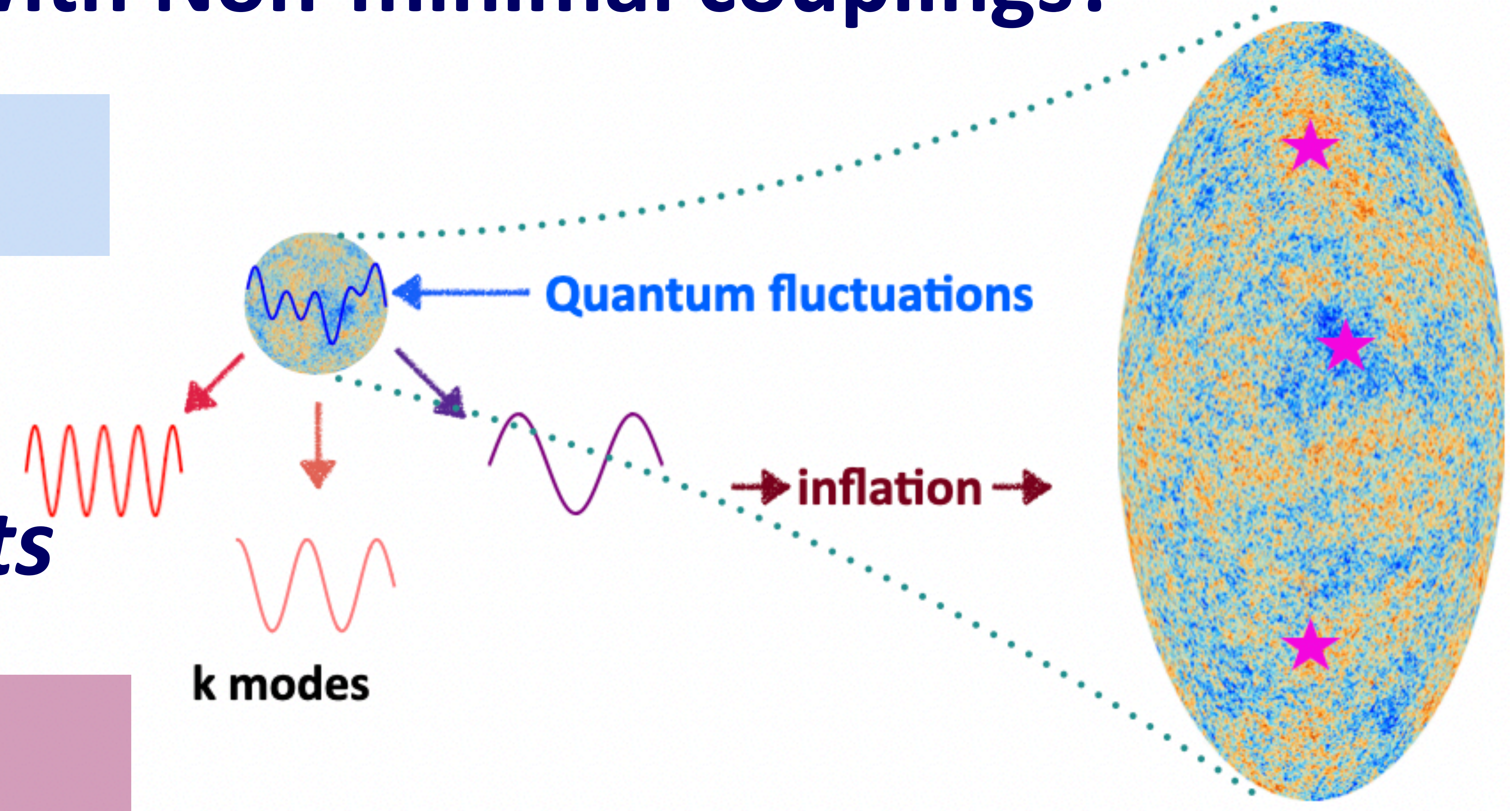

Result:

We can produce PBHs near relevant mass range! We match observables/constraints from data

Curvature Perturbations in Multifield Inflation

During inflation, quantum fluctuations get ***stretched and amplified***

Linear perturbations around flat FLRW metric:

$$g_{\mu\nu} \rightarrow \tilde{g}_{\mu\nu} + h_{\mu\nu}$$

Scalar perturbations (longitudinal gauge):

$$ds^2 = -(1+2A)dt^2 + a^2(1-2\psi)\delta_{ij}dx^i dx^j$$

Scalar field =background+ perturbation:

$$\phi_I(x^\mu) = \varphi_I(t) + \delta\phi_I(x^\mu)$$

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Mukhanov-Sasaki is a gauge invariant quantifying perturbations:

$$Q^I = \delta\phi^I + \frac{\dot{\varphi}^I}{H}\psi$$

$$Q^I = \underbrace{\hat{\sigma}^I Q_\sigma}_{\text{Adiabatic}} + \underbrace{\sqrt{|\mathcal{G}_{IJ}|}\,\epsilon^{IJ}\hat{\sigma}_J Q_s}_{\text{Isocurvature}}$$

Power Spectrum:

$$\mathcal{P}_R(k) \equiv \frac{k^3}{2\pi^2}\left|\frac{Q_\sigma}{M_{\text{pl}}\sqrt{2\epsilon}}\right|^2 = A\left(\frac{k}{k_*}\right)^{n_s - 1}$$

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Primordial Black Holes from Critical Collapse

Adiabatic modes with freq. k

↓

Cross outside Hubble horizon k<aH ("Super-Hubble")

↓

"freeze out"

↓

Cross back into Hubble patch k>aH "Sub-Hubble"

Mode k_{PBH} crosses back at time t_c

$$k_{\mathrm{PBH}} = a(t_c)H(t_c)$$

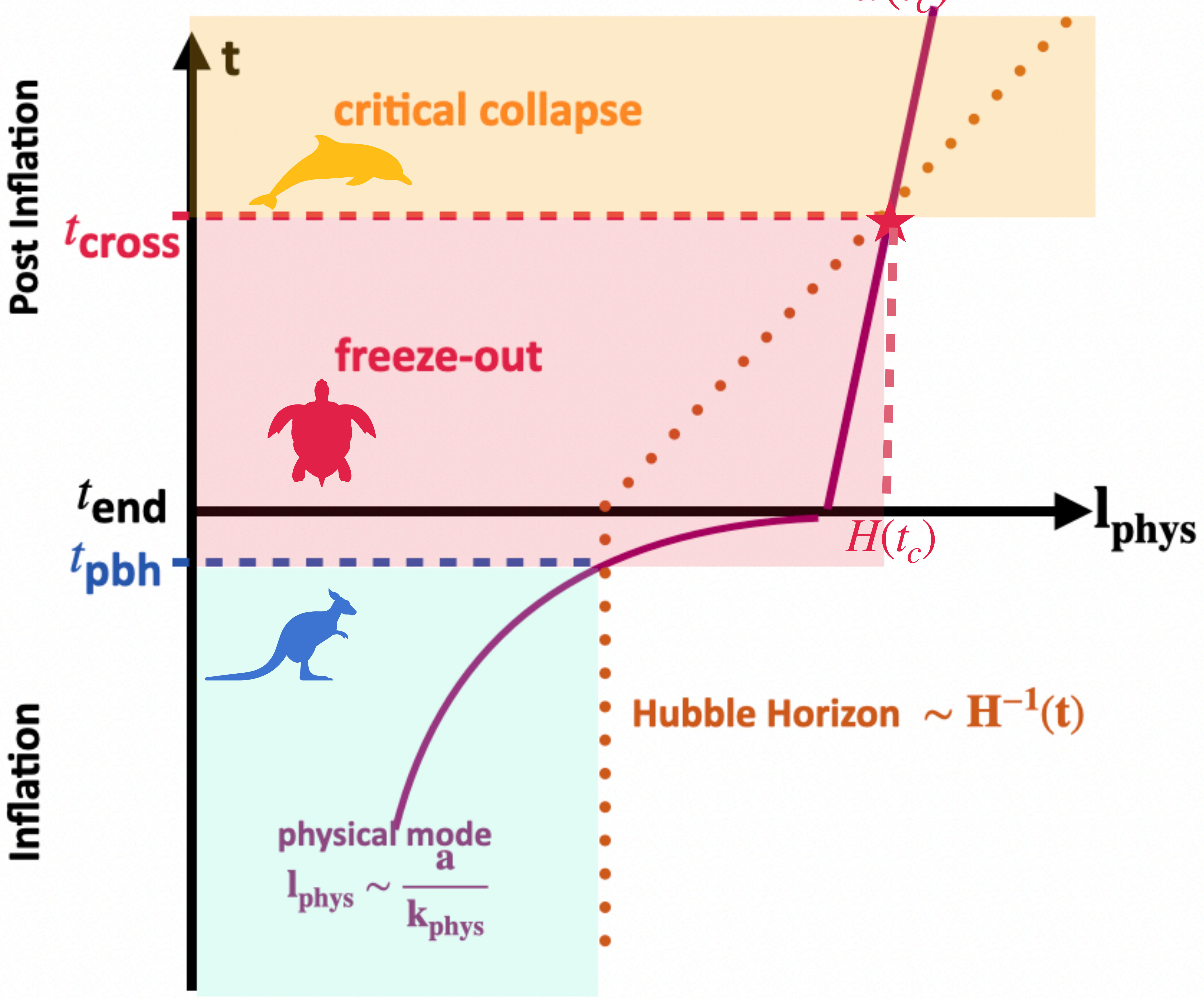
Corresponds to threshold for

$$\mathscr{P}_R(k_{\mathrm{PBH}}) \geq 10^{-3}$$

Mass distribution centered around

$$\bar{M} = \gamma M_{\mathrm{H}}(t_c),\ \gamma \sim .2$$

$$\star\ \frac{k_{\mathrm{pbh}}}{a(t_c)} = H(t_c)$$



Primordial Black Holes as Dark Matter

Massive Compact Halo Objects (MACHOs)

$\approx$ Non-interacting

Wide range of possible PBH masses

Avoid need to posit one or more BSM fields

source: Green and Kavanagh 2007.10722v3

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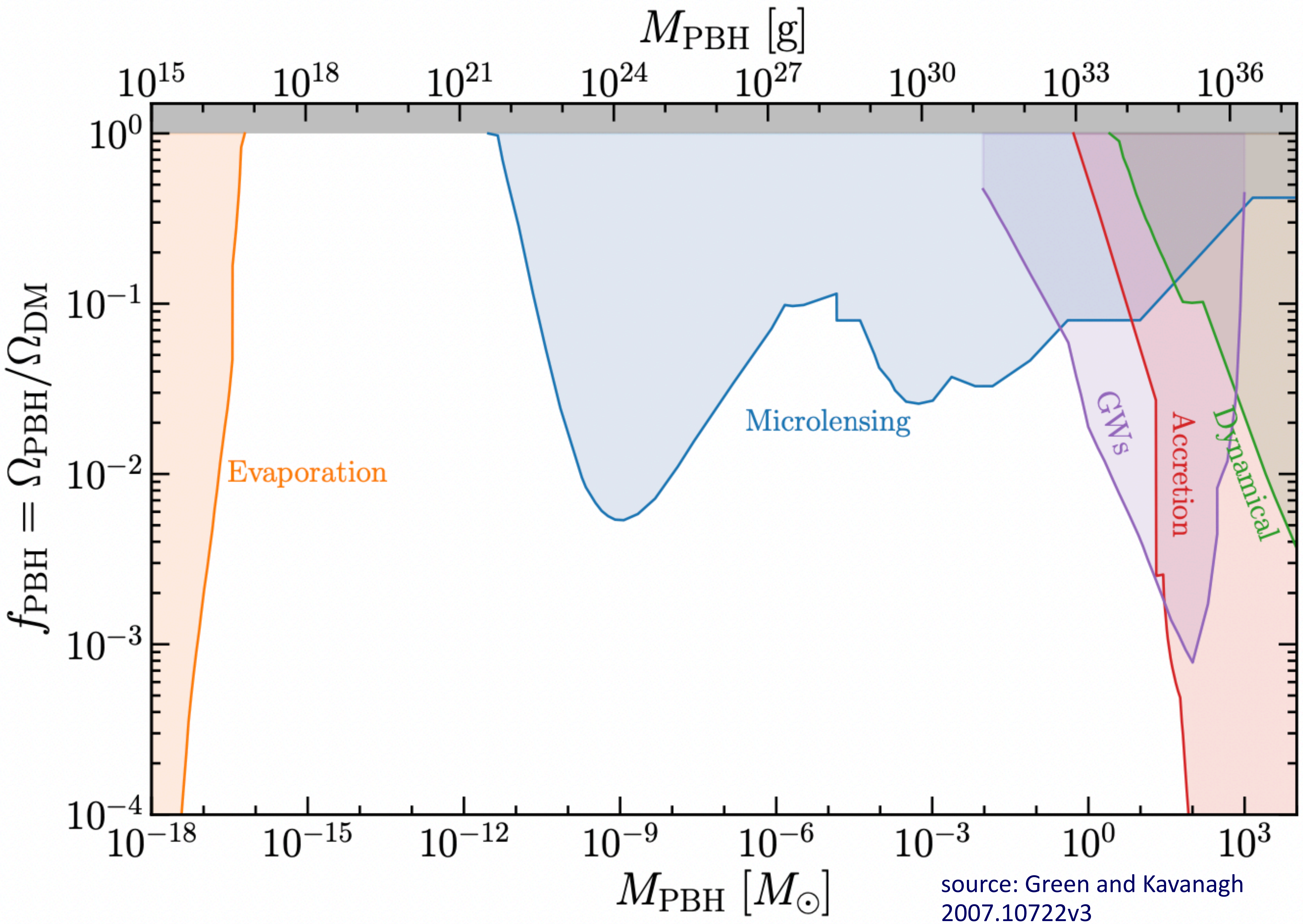

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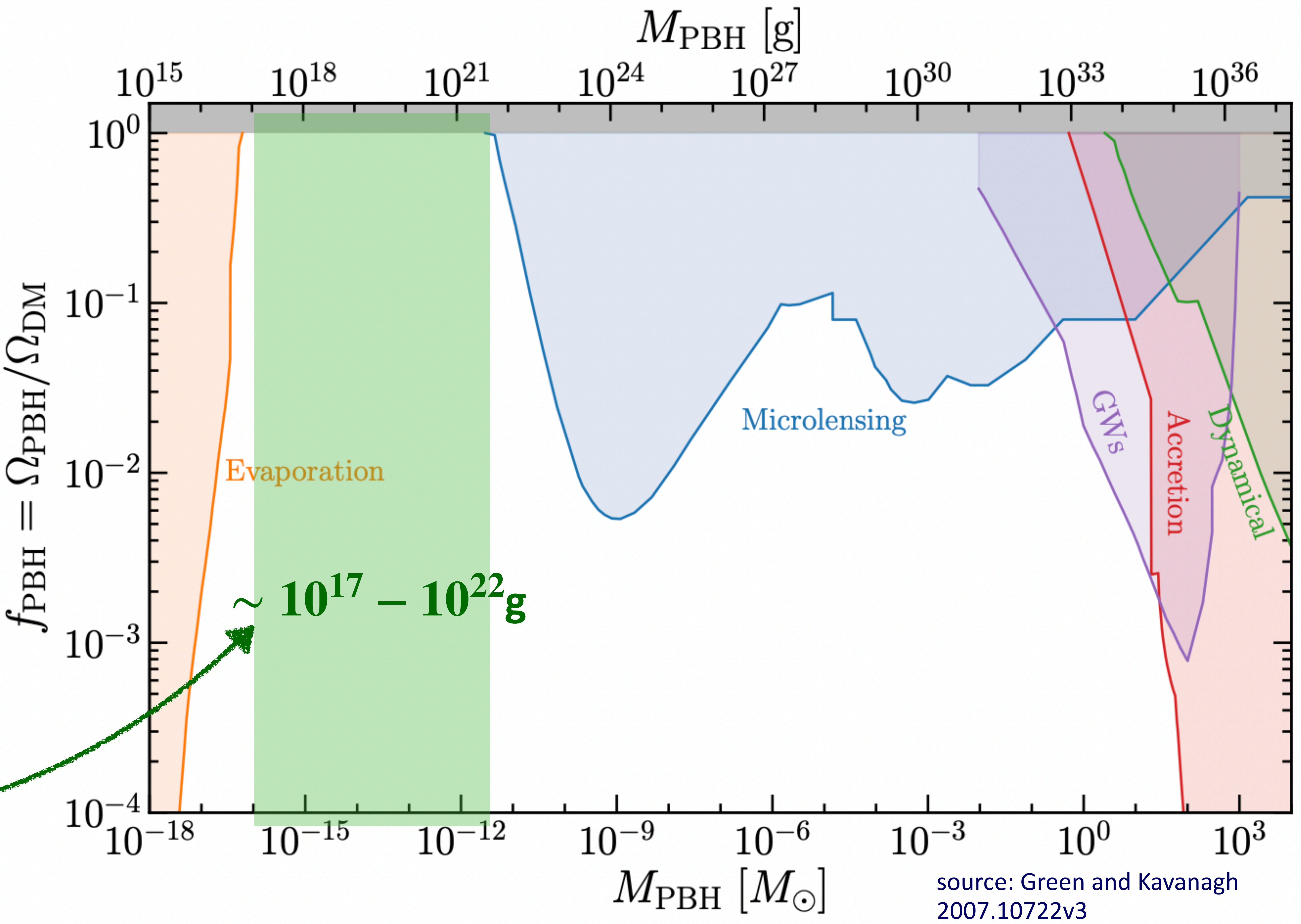

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PBHs in this mass range could constitute $\mathcal{O}(1)$ fraction of Dark Matter

Ingredients from High Energy Theory
Multiple fields and Non-minimal Couplings

Multifield Models $\sim \phi^I(x^\mu)$

Field theories (FTs) at high energies generically have > 1 scalar d.o.f.

BSM theories have more, for example, MSSM $\ni$ 7 Chiral Superfields

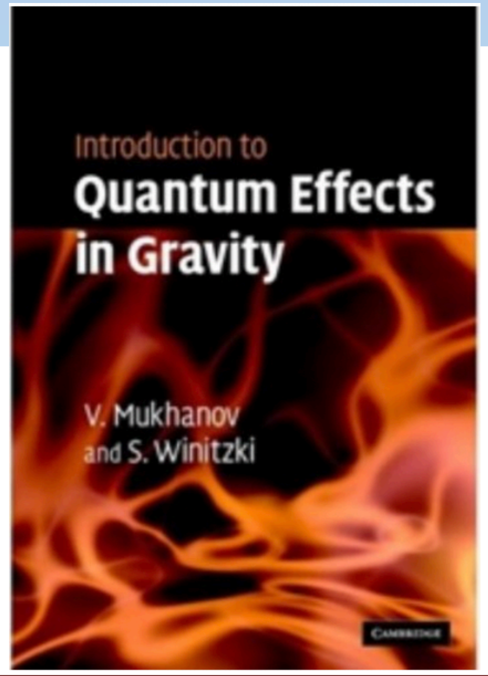
Non-minimal Couplings

Self interacting scalar fields in curved spacetime generically induce non-minimal couplings

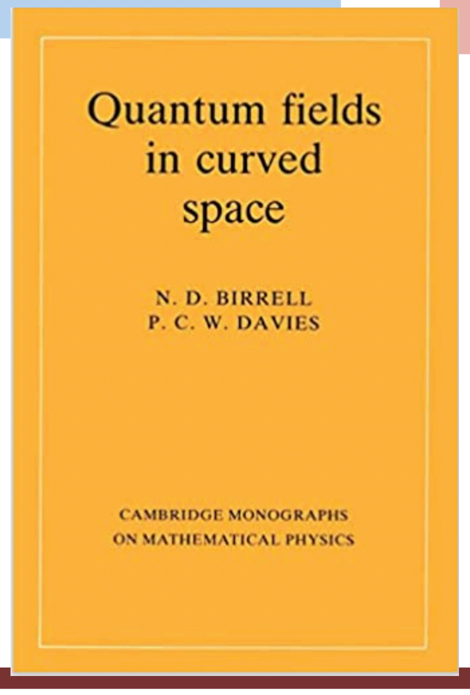
EFT point of view: well-behaved dim 4 operators that should be included in S

RG: The couplings increase with energy scale with no UV fixed point

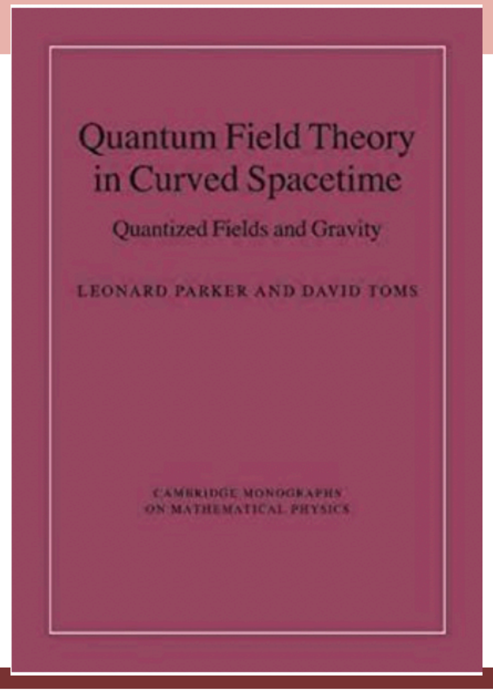
See:



Mukhanov & Winitzki



Birrell & Davies



Parker & Toms

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$$\tilde{\mathbf{S}} = \int \mathrm{d}^4\mathbf{x}\sqrt{-\tilde{\mathbf{g}}}\left[\mathbf{f}\left(\phi^{\mathbf{I}}\right)\tilde{\mathbf{R}} - \frac{1}{2}\delta_{\mathbf{IJ}}\tilde{\mathbf{g}}^{\mu\nu}\partial_\mu\phi^{\mathbf{I}}\partial_\nu\phi^{\mathbf{J}} - \tilde{\mathbf{V}}\left(\phi^{\mathbf{I}}\right)\right]$$

$$\mathbf{f}(\phi^{\mathbf{I}}) = \frac{1}{2}\left[\mathbf{M}_{\mathrm{pl}}^2 + \sum_{\mathbf{I}=1}^{\mathbf{N}}\xi_{\mathbf{I}}(\phi^{\mathbf{I}}(\mathbf{x}^\mu))^2\right]$$

The Field Space in Multifield Inflation

Jordan Frame:

$$\tilde{S} = \int d^4x \sqrt{-\tilde{g}} \left[f\left(\phi^I\right) \tilde{R} - \frac{1}{2} \delta_{IJ} \tilde{g}^{\mu\nu} \partial_\mu \phi^I \partial_\nu \phi^J - \tilde{V}\left(\phi^I\right) \right]$$

Kaiser 1003.1159v2

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$$\tilde{g}^{\mu\nu} \rightarrow g^{\mu\nu} = \Omega^{-2}(x)\tilde{g}^{\mu\nu}$$

Einstein Frame:

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Induces non-canonical kinetic terms $\Longrightarrow$ curved field space

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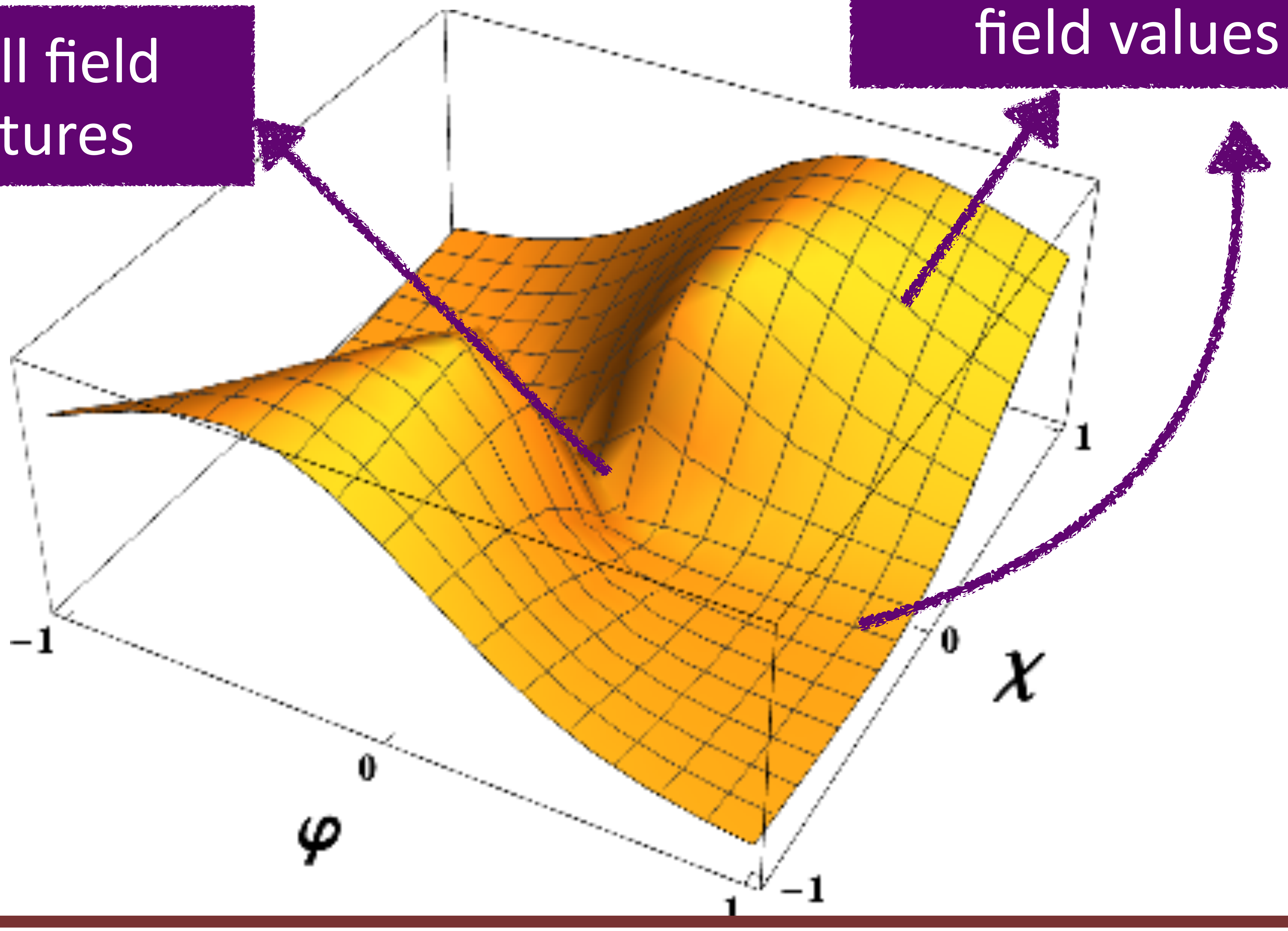

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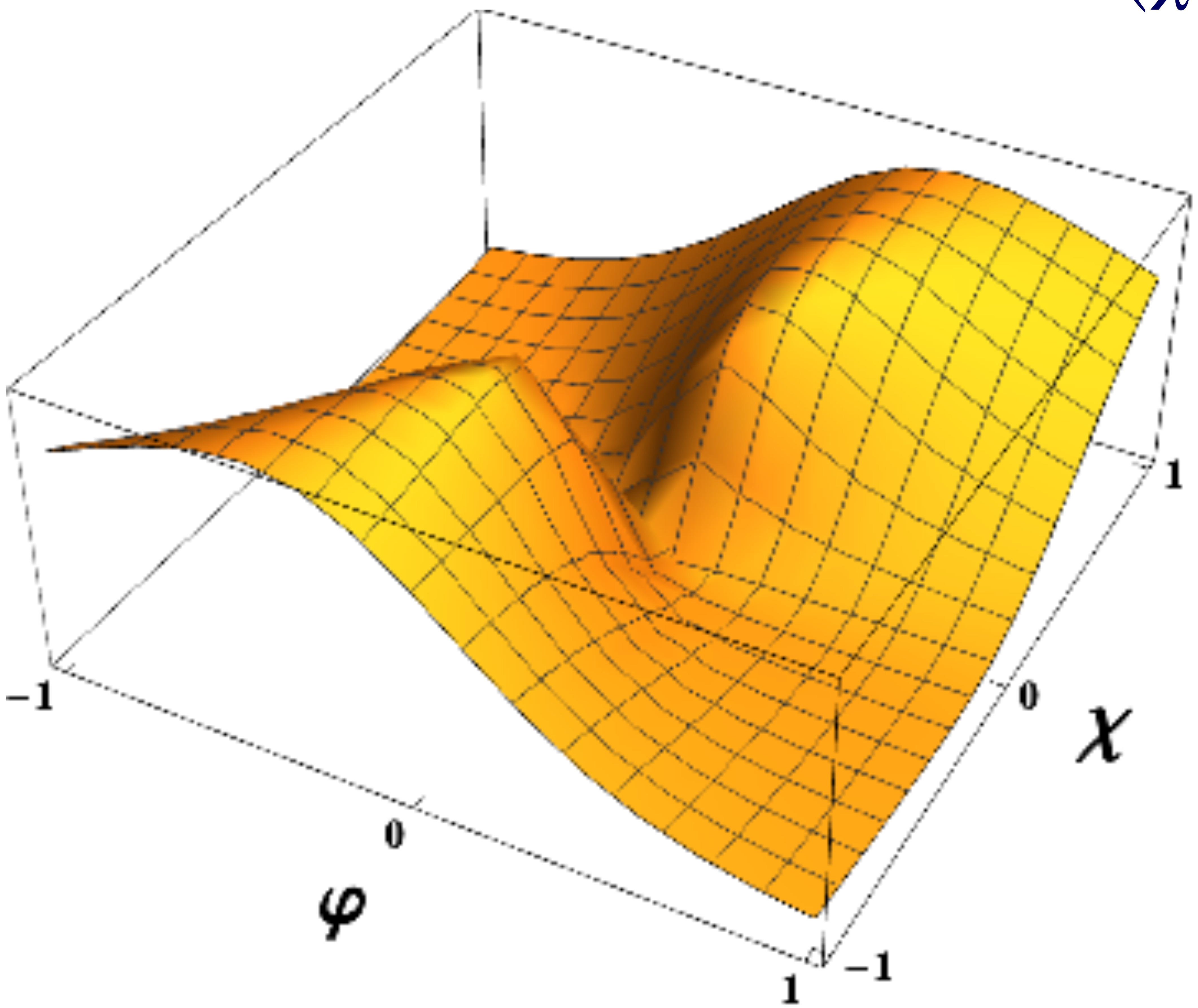
The 2-Field Inflaton Potential

2-Dimensional Field Space: $\phi_I = \begin{pmatrix} \varphi \\ \chi \end{pmatrix}$



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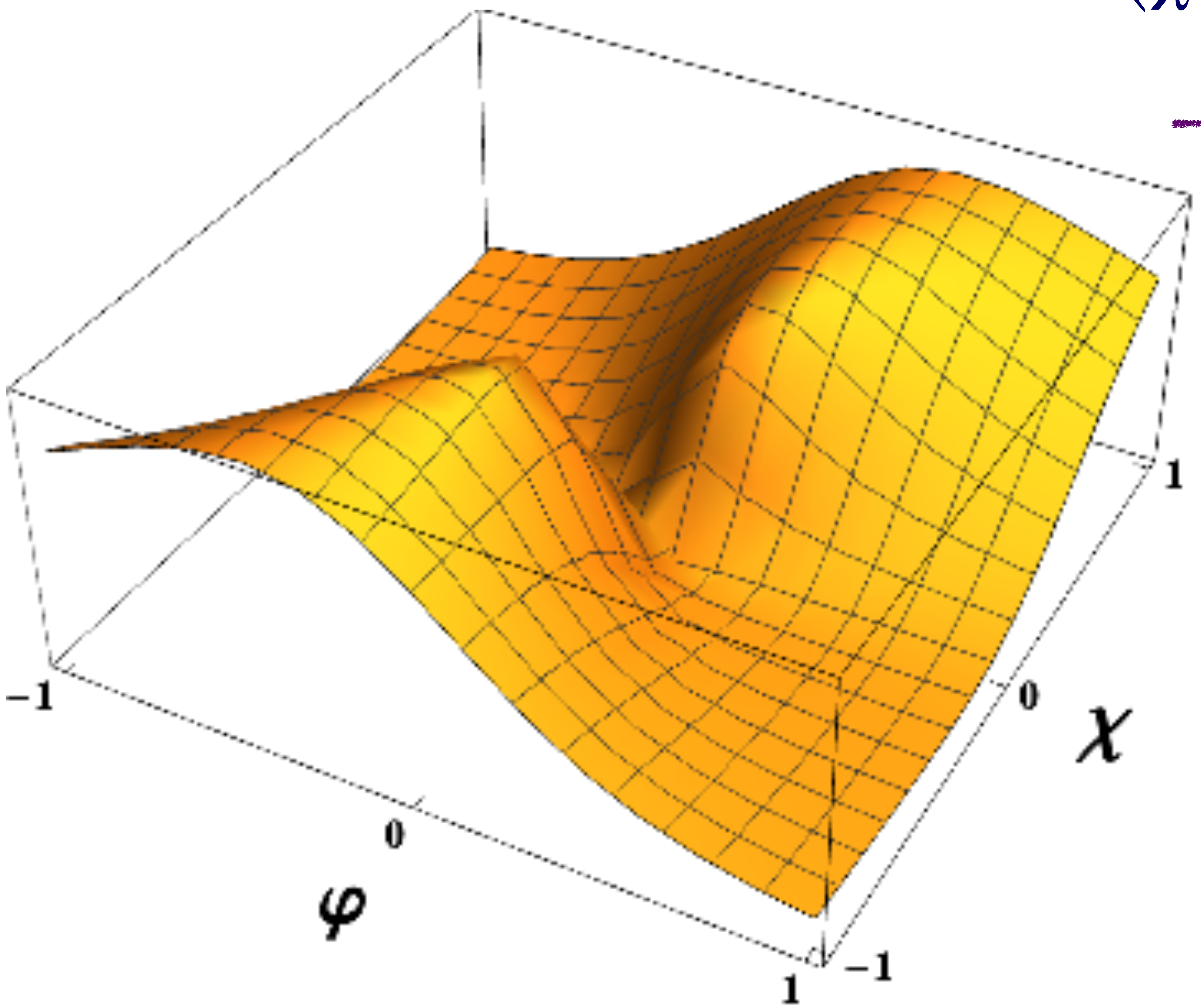
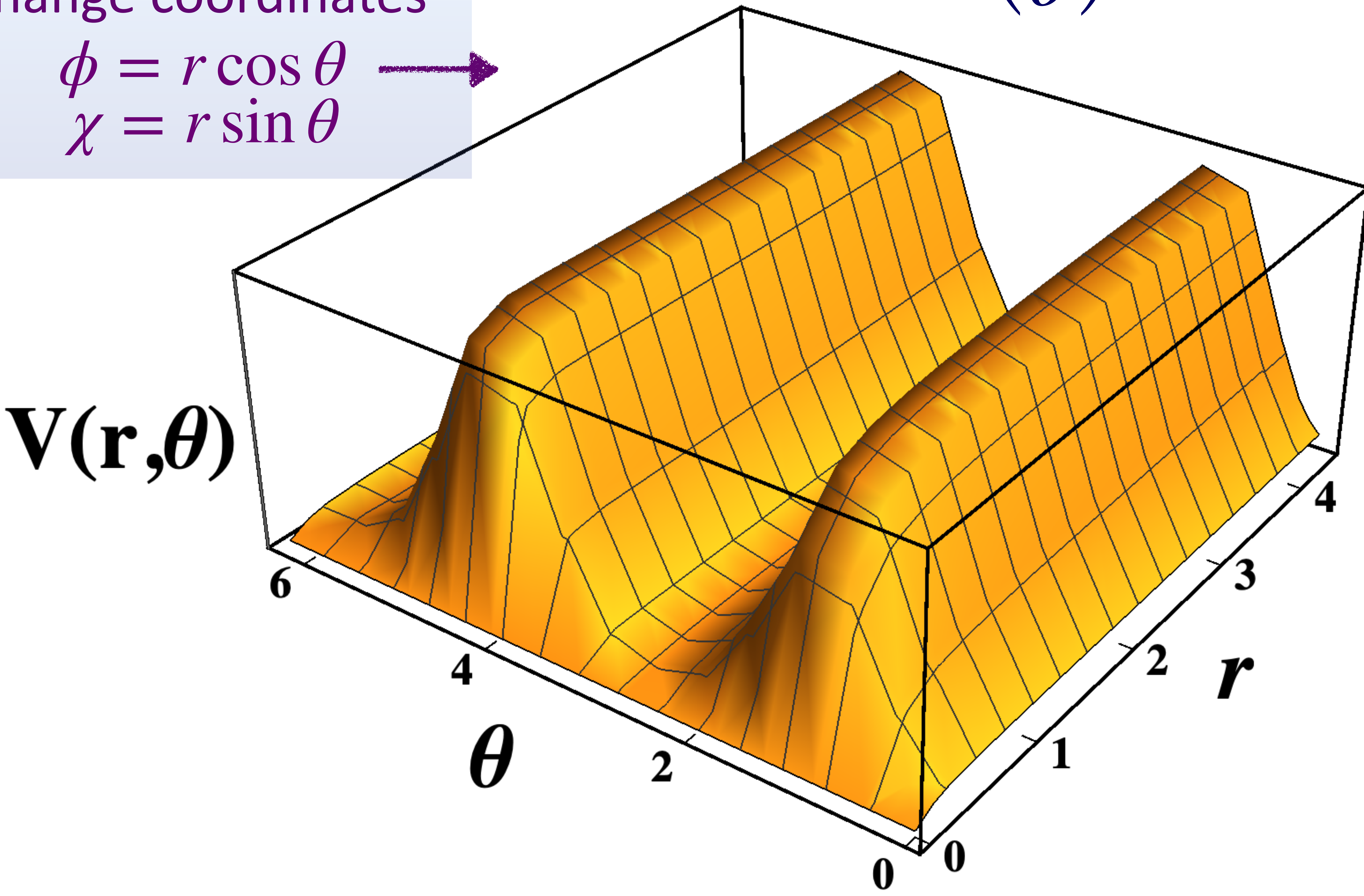


change coordinates

$$\phi = r\cos\theta$$
$$\chi = r\sin\theta$$

$\phi_{I'} = \begin{pmatrix} r \\ \theta \end{pmatrix}$



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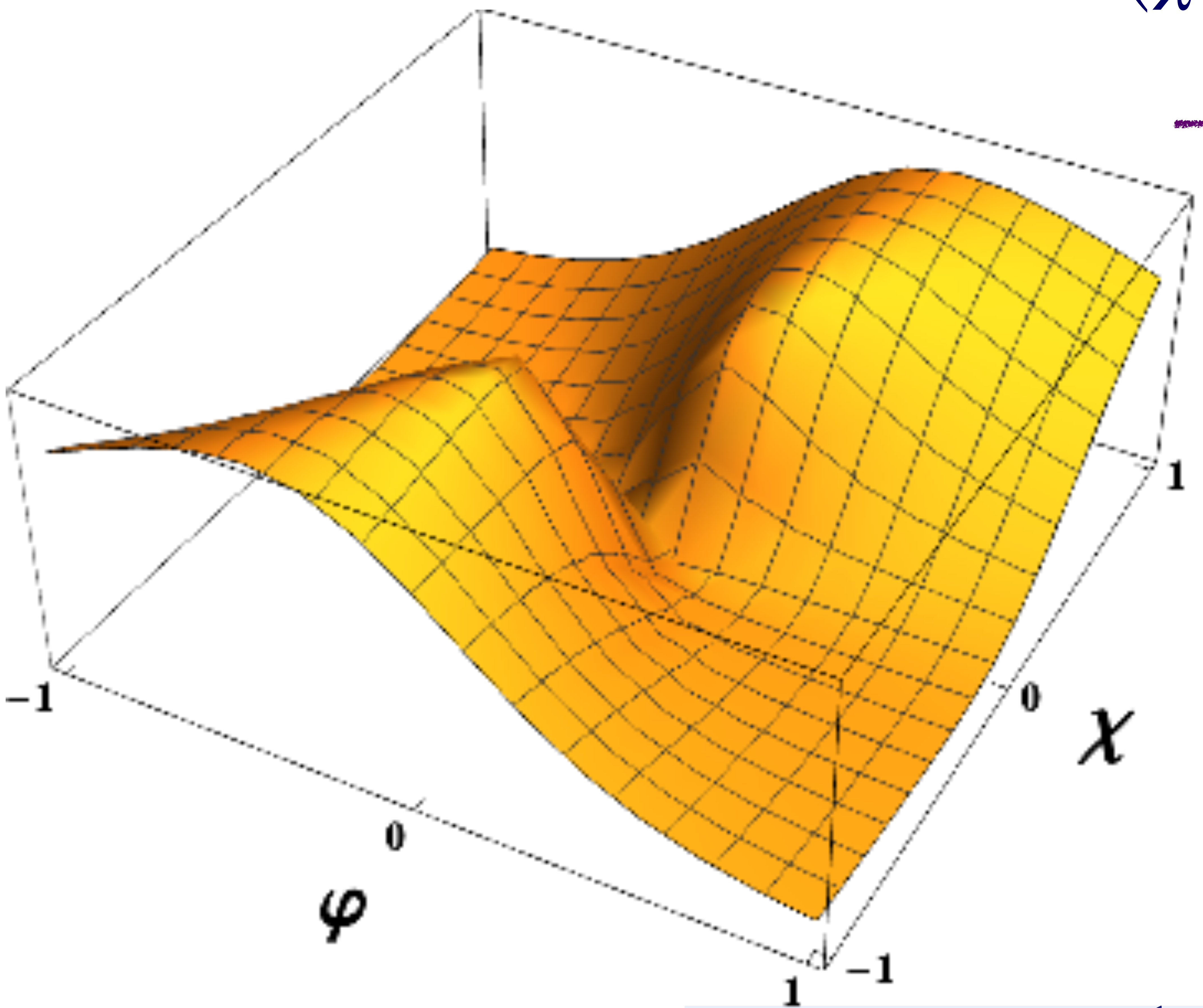

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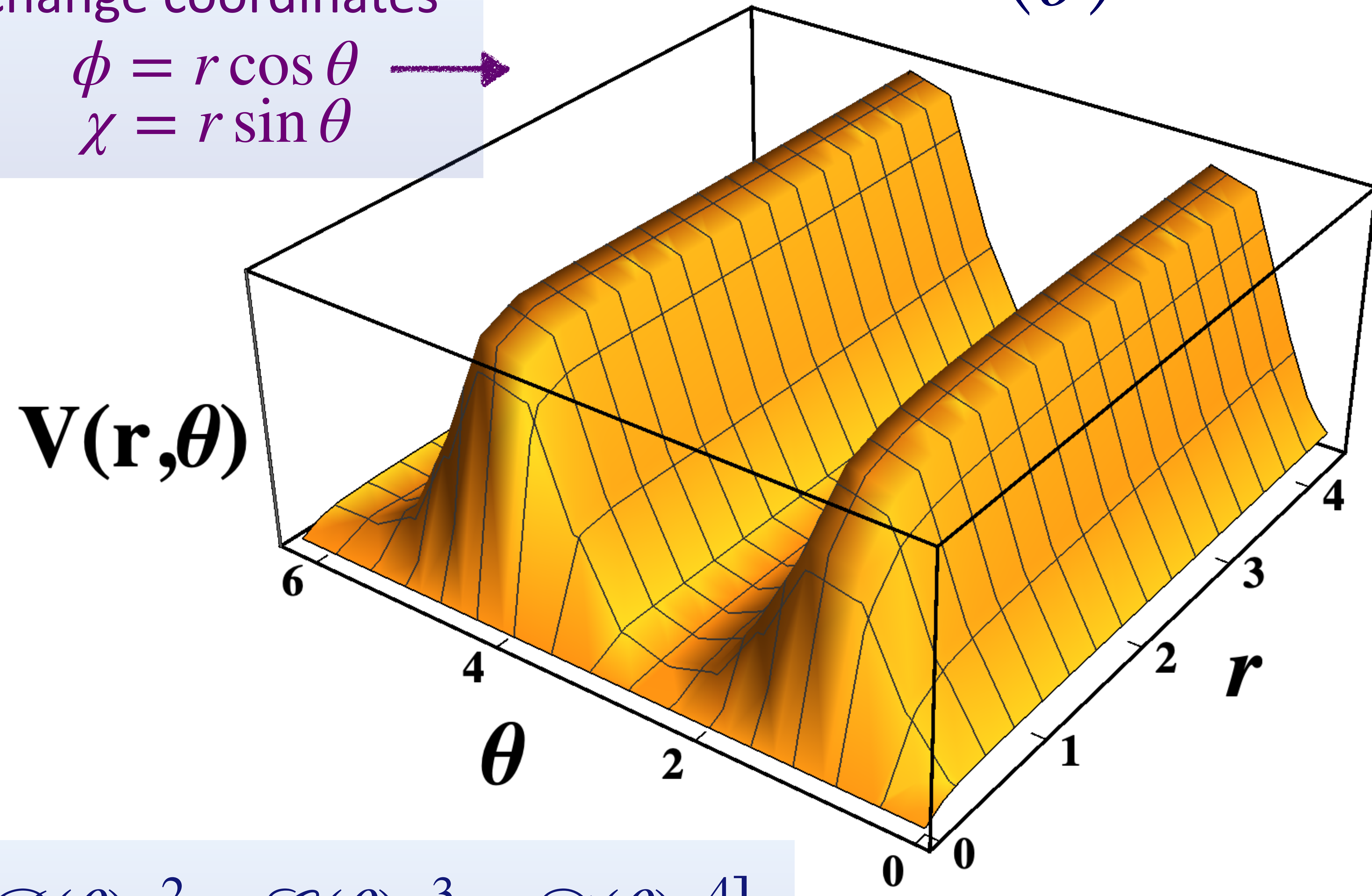


$$V(r,\theta) = \frac{1}{4f(r)^2}\left[\mathscr{B}(\theta)r^2 + \mathscr{C}(\theta)r^3 + \mathscr{D}(\theta)r^4\right]$$

(Exact) Inflationary Trajectories

$V(r, \theta)$ depends on parameters:

$b_1, b_2 \rightarrow$ mass coefficients

$c_1, c_2, c_3, c_4 \rightarrow$ "Yukawa" couplings

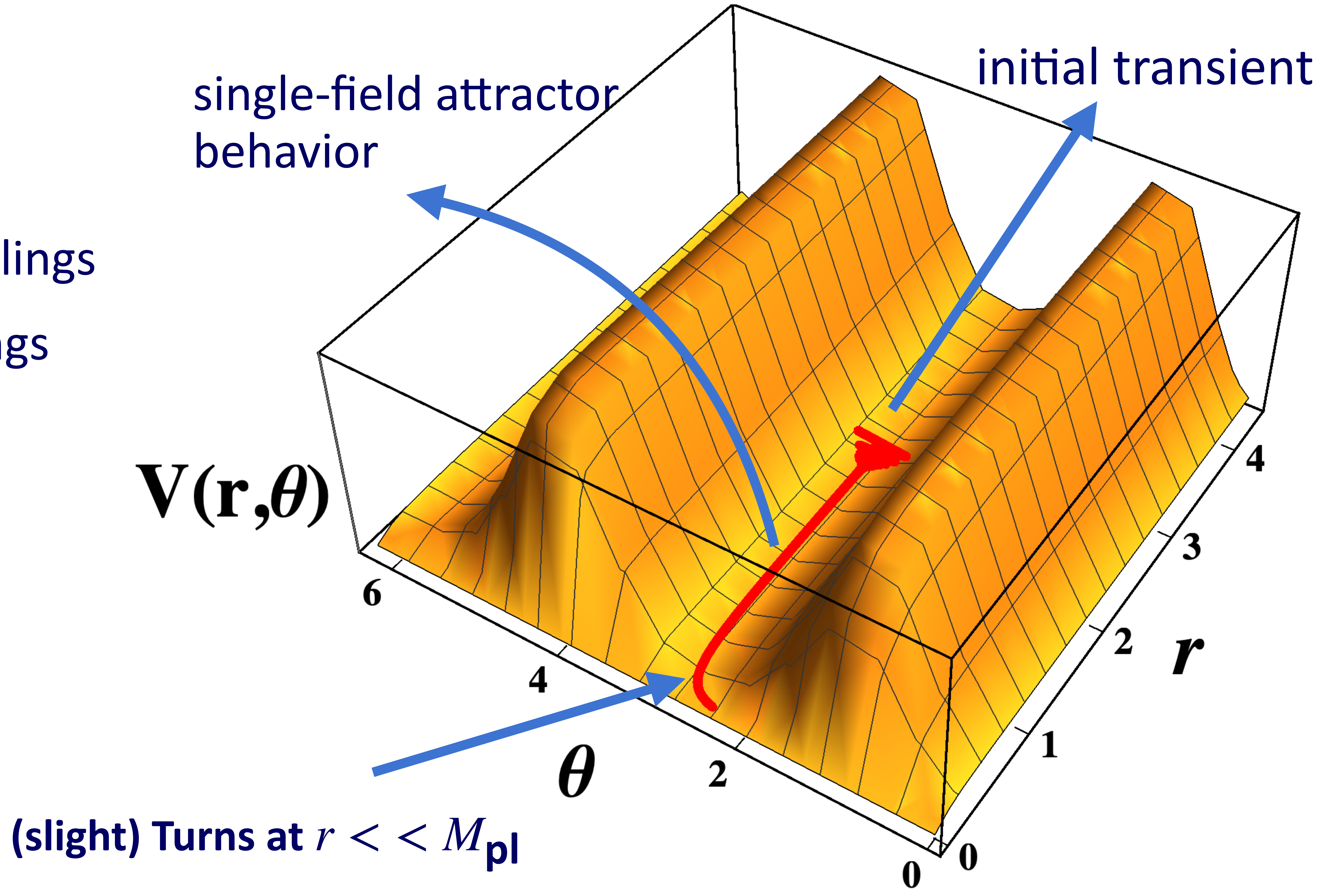
$\xi_\varphi, \xi_\chi \rightarrow$ non-minimal couplings

Consider the symmetries:

$$\xi_\phi = \xi_\chi = \xi$$

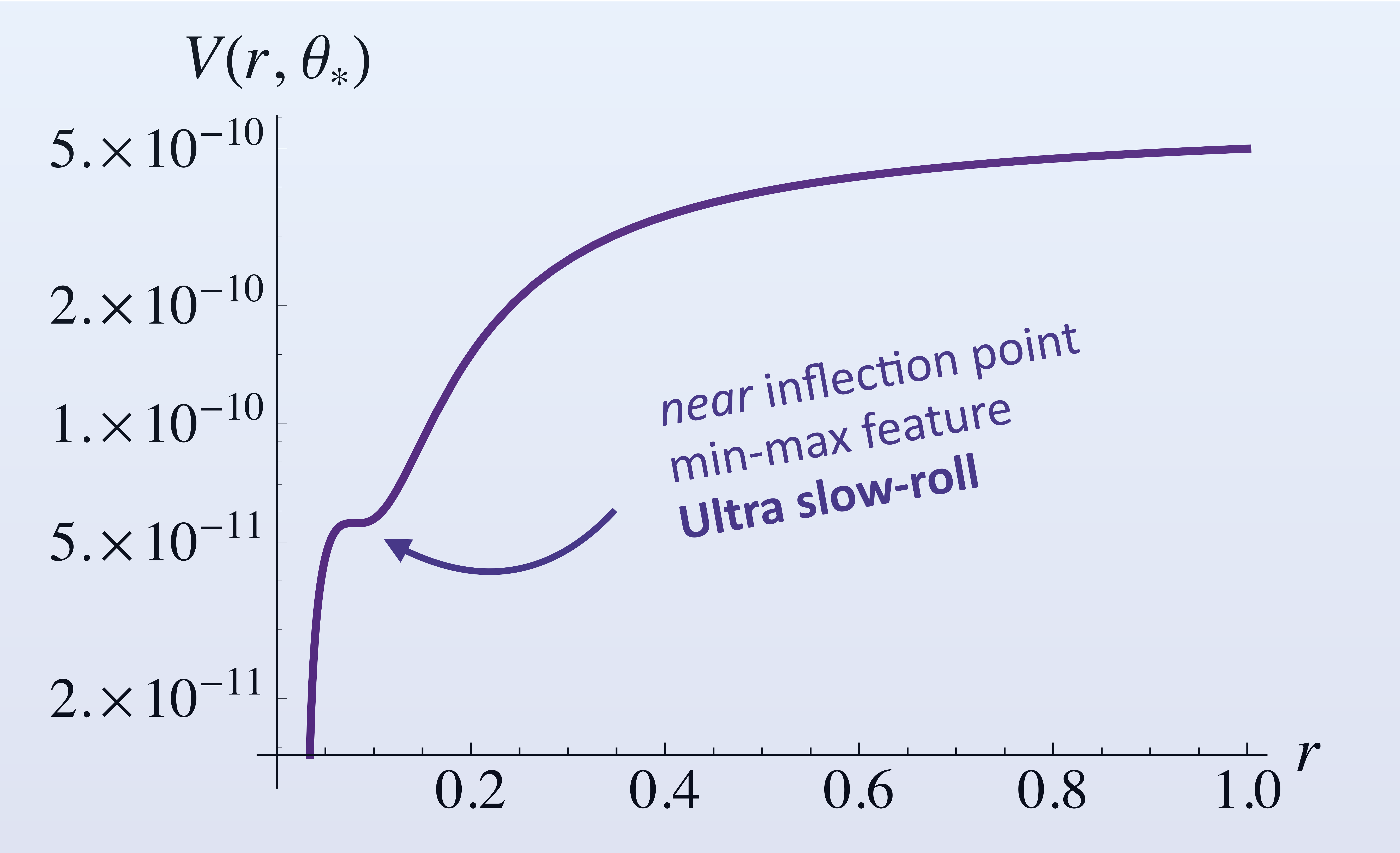
$$c_2 = c_3$$

$$b_1 = b_2 = b$$

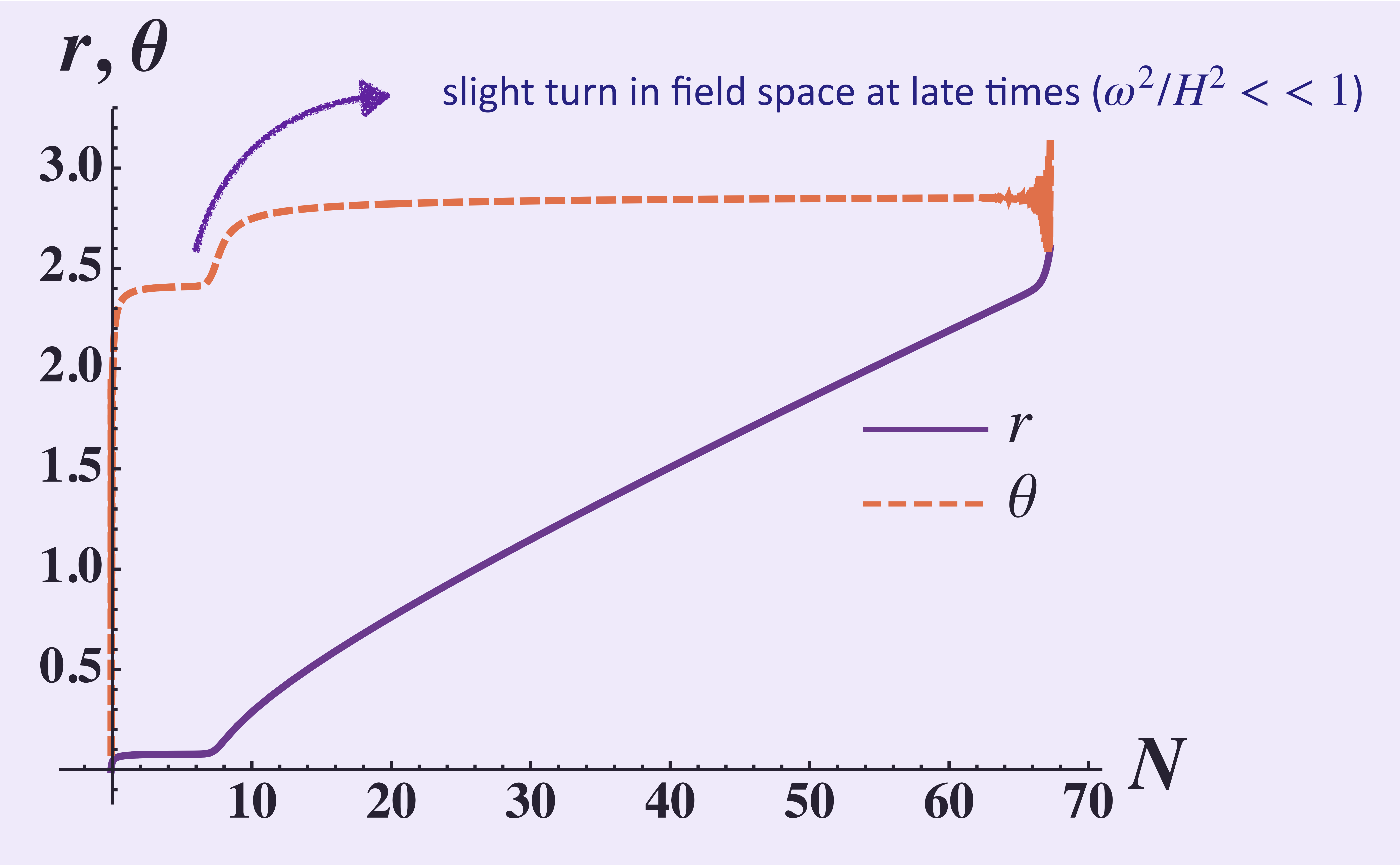


(Exact) Inflationary Trajectories

The potential evaluated along an exact inflationary trajectory $\theta_*^{\pm}(r)$ for $b_1 = b_2, c_2 = c_3, \xi_\varphi = \xi_\chi$

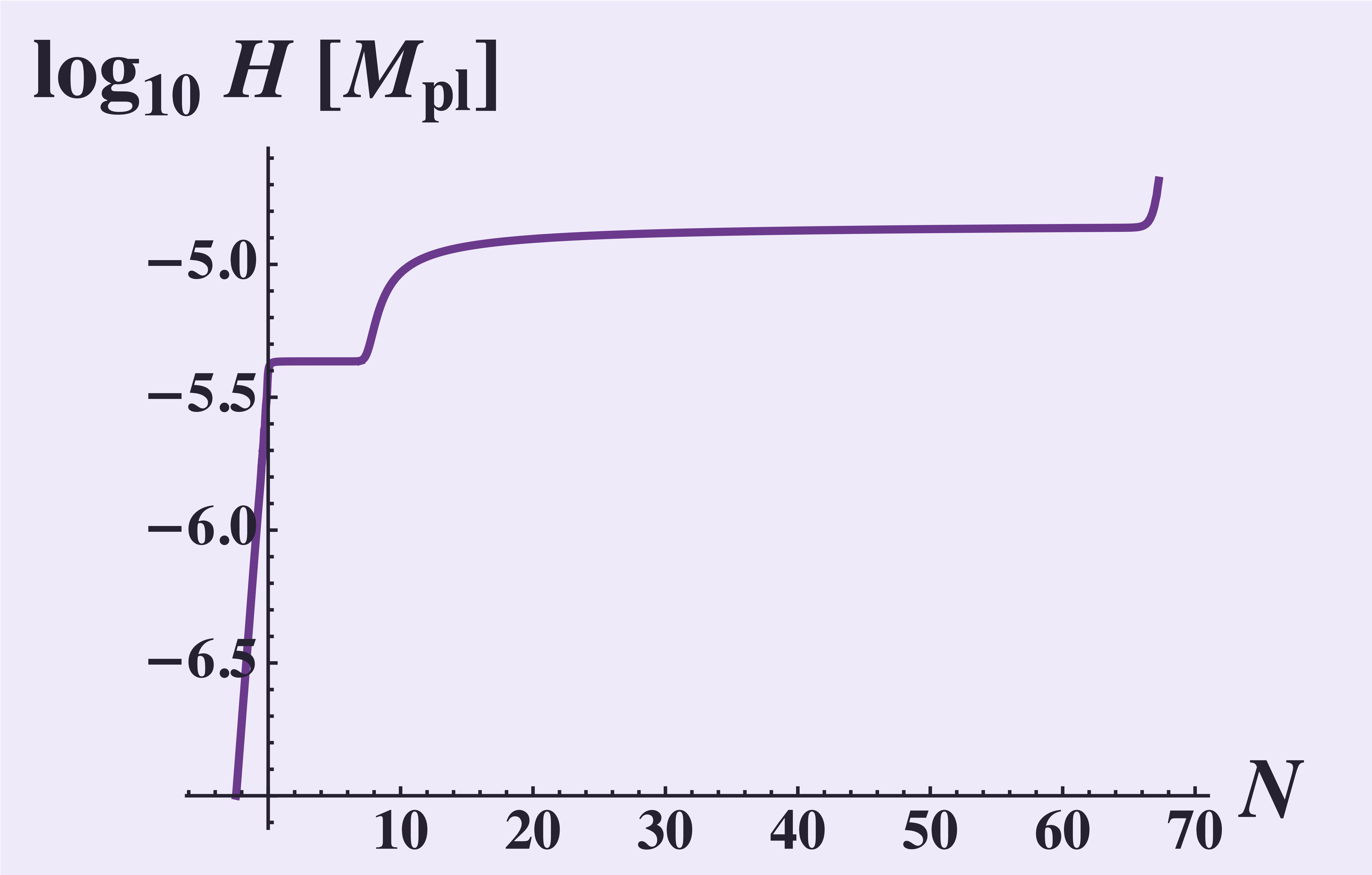


Time evolution of fields



$\mu = M_{\rm pl}, b_1 = b_2 = -1.8 \times 10^{-4}, c_1 = 2.5 \times 10^{-4}, c_2 = c_3 = 3.57 \times 10^{-3}, c_4 = 3.9 \times 10^{-3}, \xi_\phi = \xi_\chi = 100$

Time evolution of Hubble parameter



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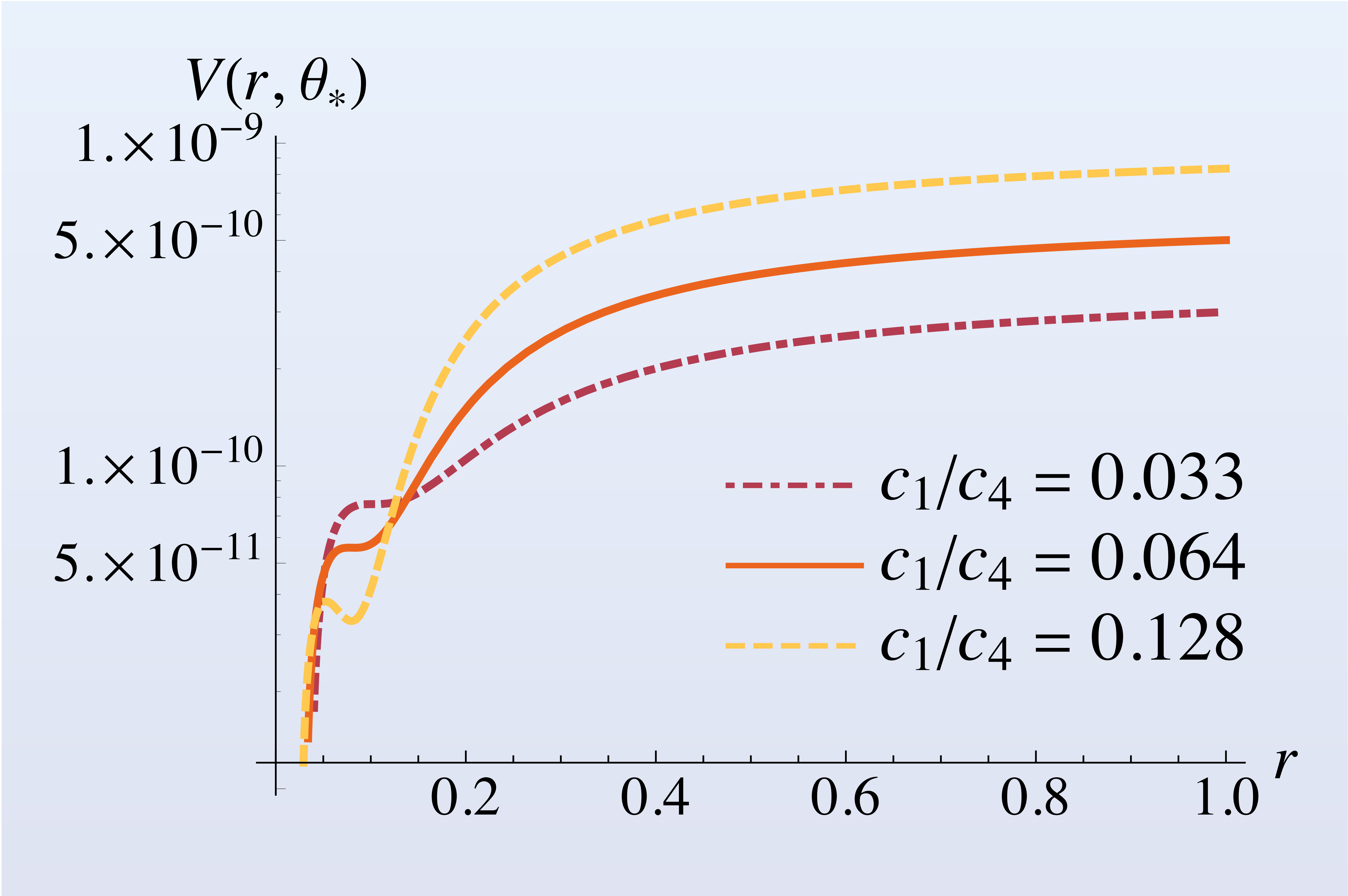
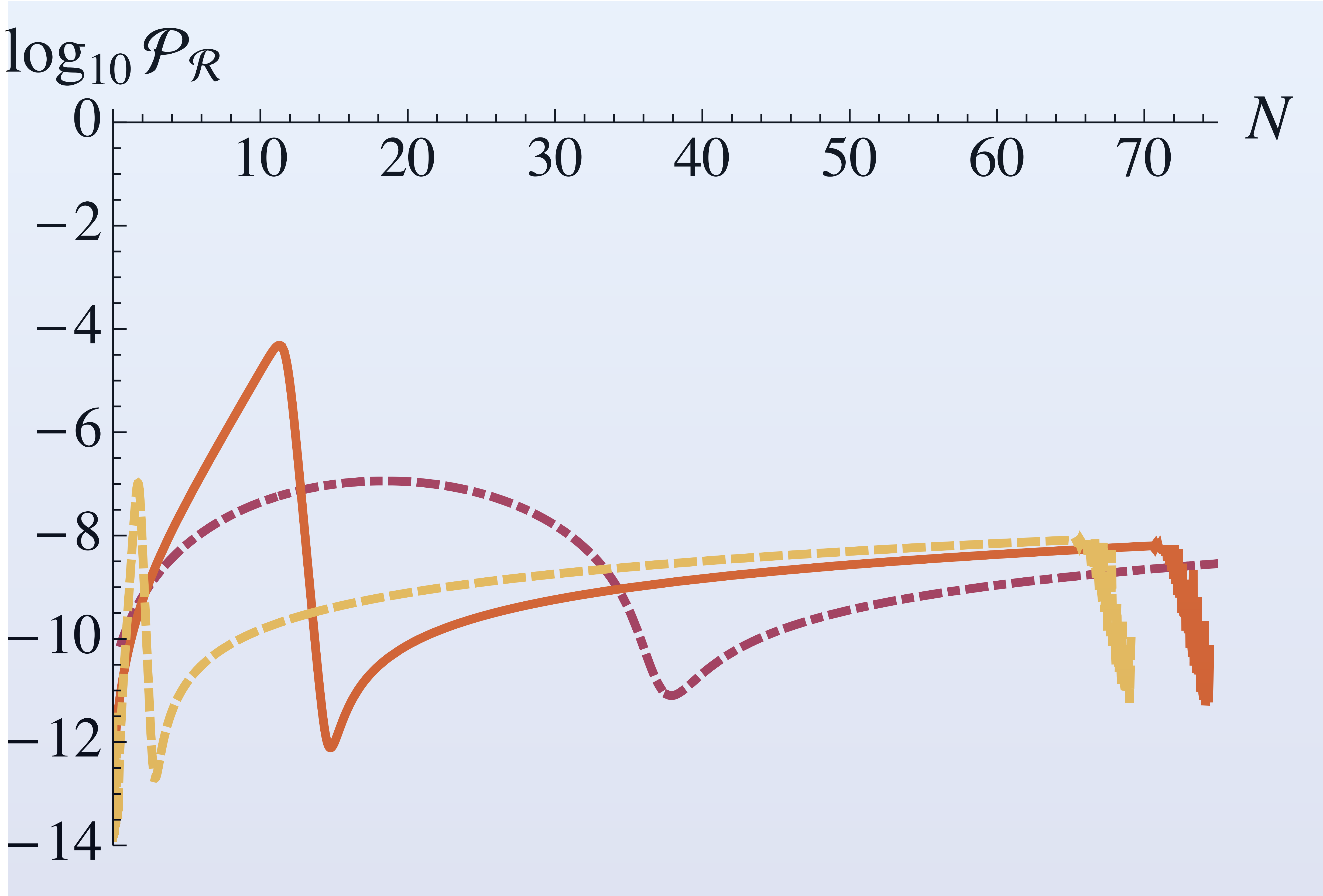
Parameter dependence: the min/max feature and power spectrum



Min/Max feature can be adjusted by varying ratio of couplings.
In each case, $c_2 \sim c_4$



$\mu = M_{\rm pl}$ $\quad \xi = 100 \quad$ $b = -1.8\times10^{-4}$

In all cases must have sufficient kinetic energy to classically escape local minimum.

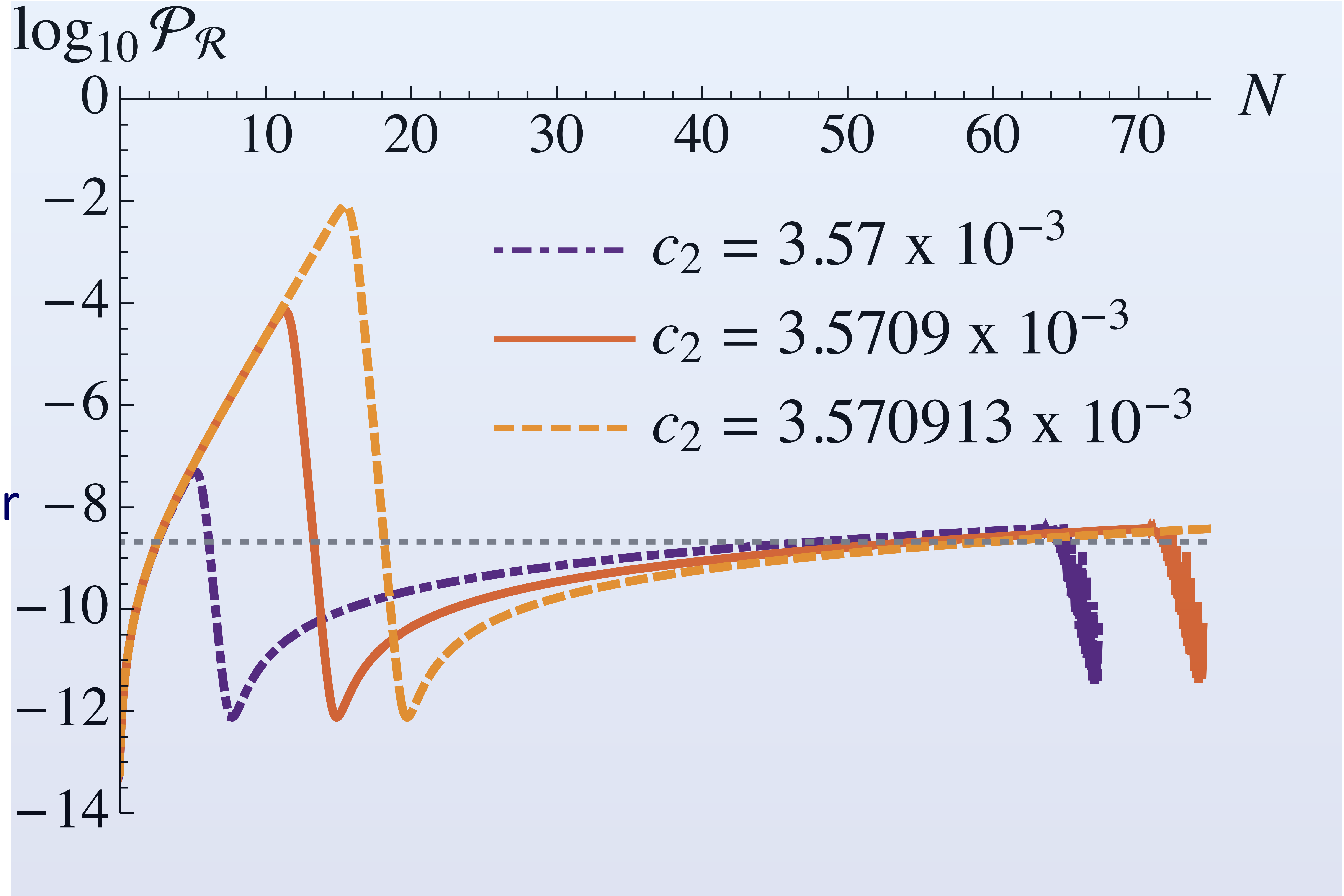
Fine tuning of (≥ 1) coupling(s)

Parameter set F1= $\{\mu = M_{\rm pl}, b_1 = b_2 = -1.8\times10^{-4}, c_1 = 2.5\times10^{-4}, c_2 = c_3, c_4 = 3.9\times10^{-3}, \xi_\phi = \xi_\chi = 100\}$

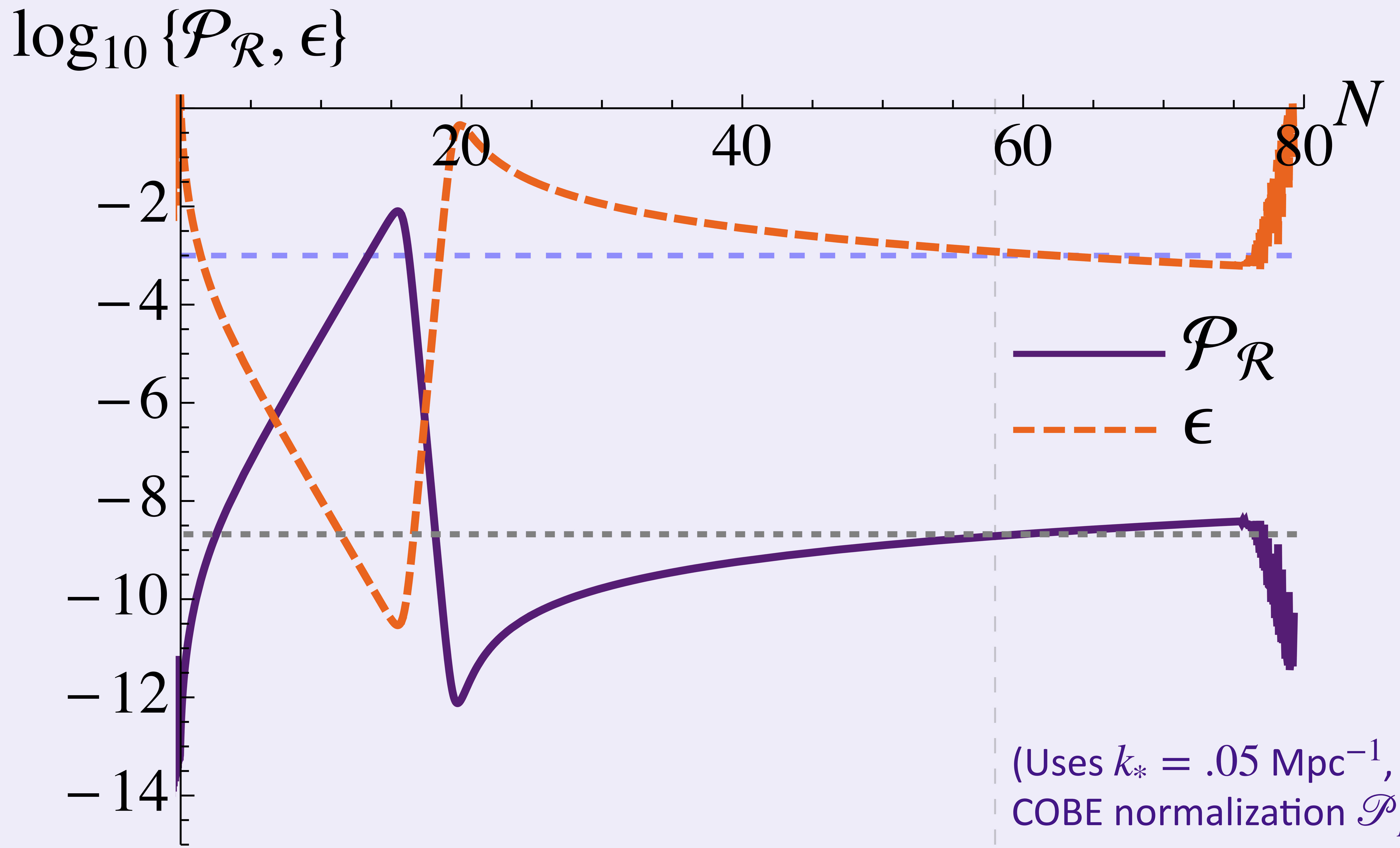
Fine tuning one parameter increases length of ultra slow-roll

As ultra slow-roll gets longer, ϵ gets smaller and peak in $\mathscr{P}_R(k)$ gets larger

(Uses $k_* = .05$ Mpc^{-1} and COBE normalization $\mathscr{P}_R(k_*) = 2.1\times10^{-9}$



The Power Spectrum from primordial perturbations that seed near-DM mass PBHs



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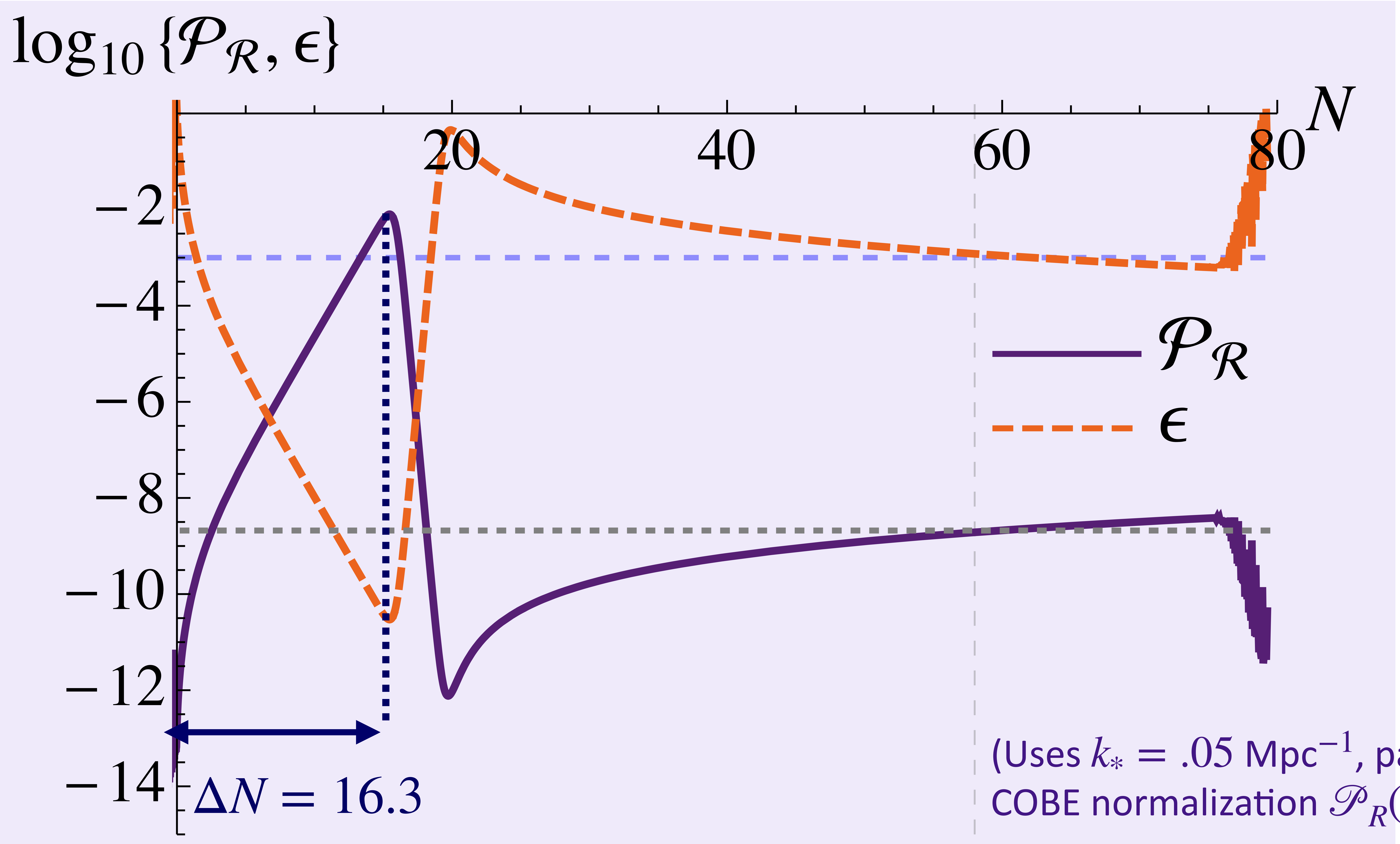

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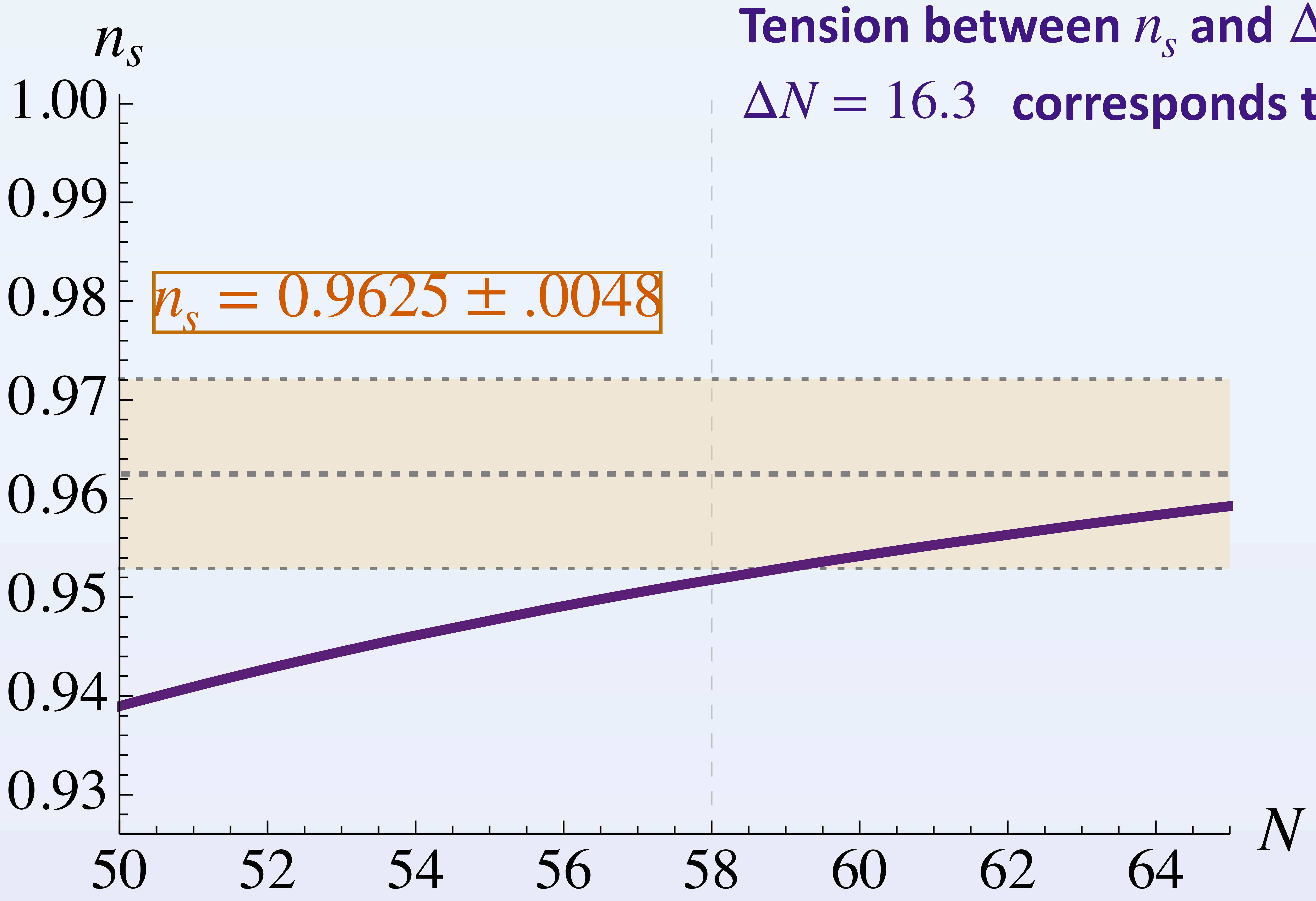
Spectral Index at CMB Pivot Scale



Tension between n_s and ΔN

$\Delta N = 16.3$ **corresponds to n_s at low end of 2σ range**

*our bounds include running

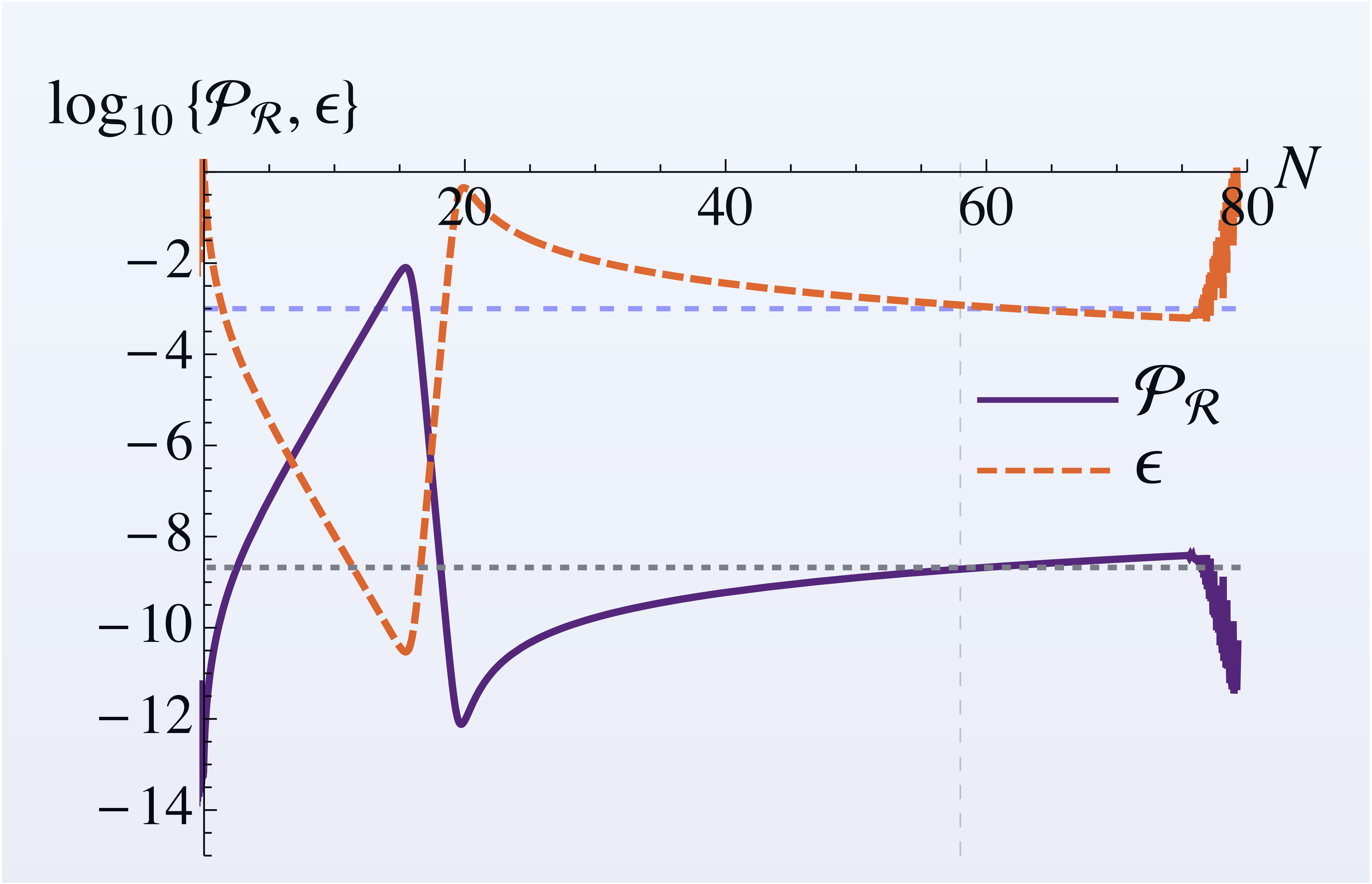
$$\alpha_s = \frac{dn_s}{d\log(k)}$$

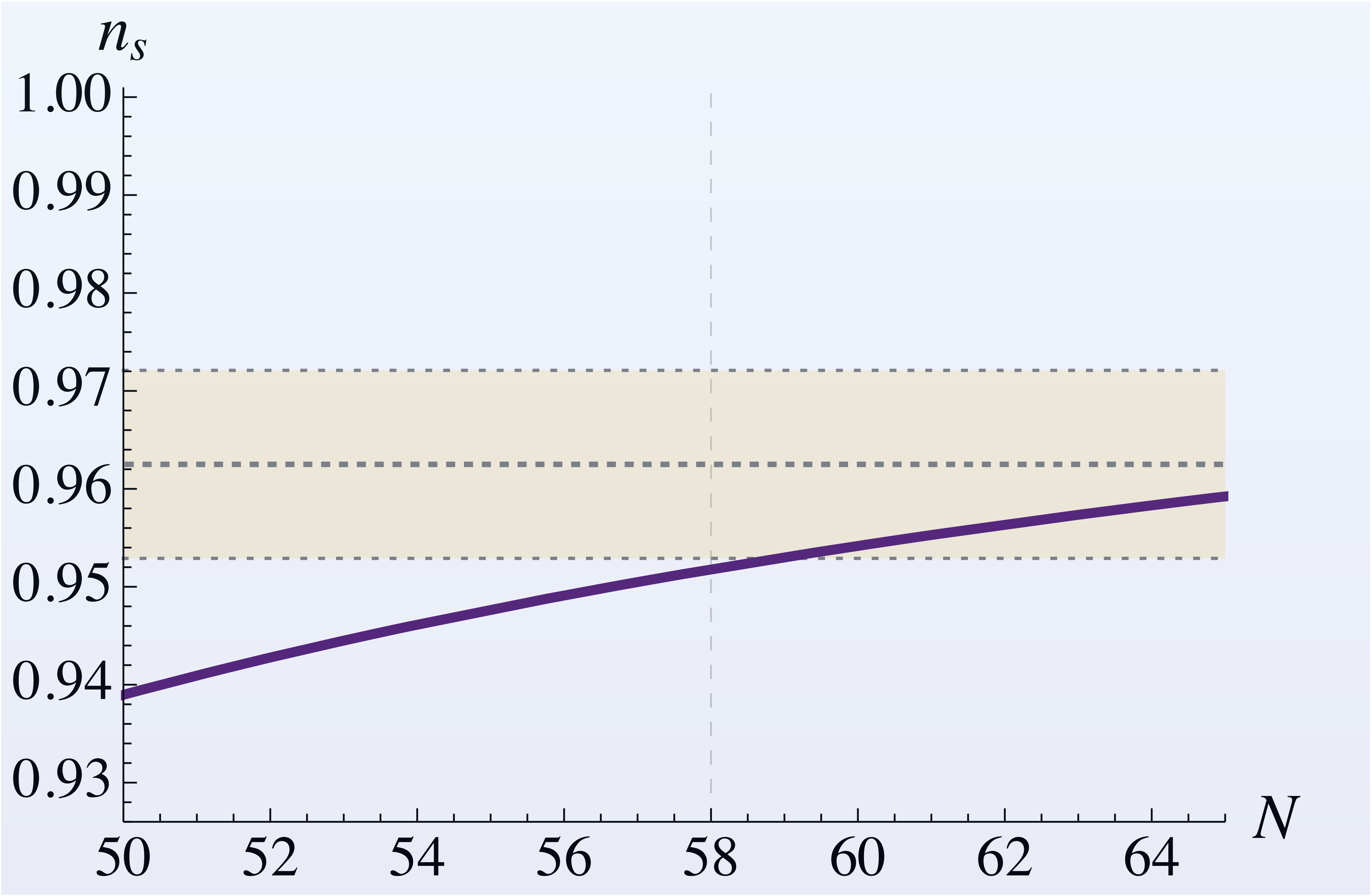
(Uses $k_* = .05\ \text{Mpc}^{-1}$, parameters F1, and COBE normalization $\mathscr{P}_R(k_*) = 2.1 \times 10^{-9}$

The Power Spectrum and Spectral Index for perturbations leading to near- DM mass PBHs

$\mu = M_{\rm pl}, b_1 = b_2 = -1.8 \times 10^{-4}, c_1 = 2.5 \times 10^{-4}, c_4 = 3.9 \times 10^{-3}, \xi_\phi = \xi_\chi = 100,$ $c_2 = c_3 = 3.570193 \times 10^{-3}$

CMB pivot scale $k_* \simeq 58$ e-folds before end of inflation





$\Delta N = 16.3$ when $\mathcal{P}_R$ first crosses 10^{-3}

2σ bounds on $n_s = 0.9625 \pm .0048$

(Uses $k_* = .05\ \text{Mpc}^{-1}$ and
COBE normalization $\mathcal{P}_R(k_*) = 2.1 \times 10^{-9}$

bounds include running $\alpha(k_*) = .002 \pm 0.010$, adapted from Planck 2018

Scaling relations for non-minimal couplings

$\mu = \mathbf{M_{pl}}, \mathbf{b_1} = \mathbf{b_2} = -1.8 \times 10^{-4}, \hat{\mathbf{c}}_1 = 2.5 \times 10^{-4}, \hat{\mathbf{c}}_2 = \hat{\mathbf{c}}_3 = 3.5709 \times 10^{-3}, \hat{\mathbf{c}}_4 = 3.9 \times 10^{-3}$

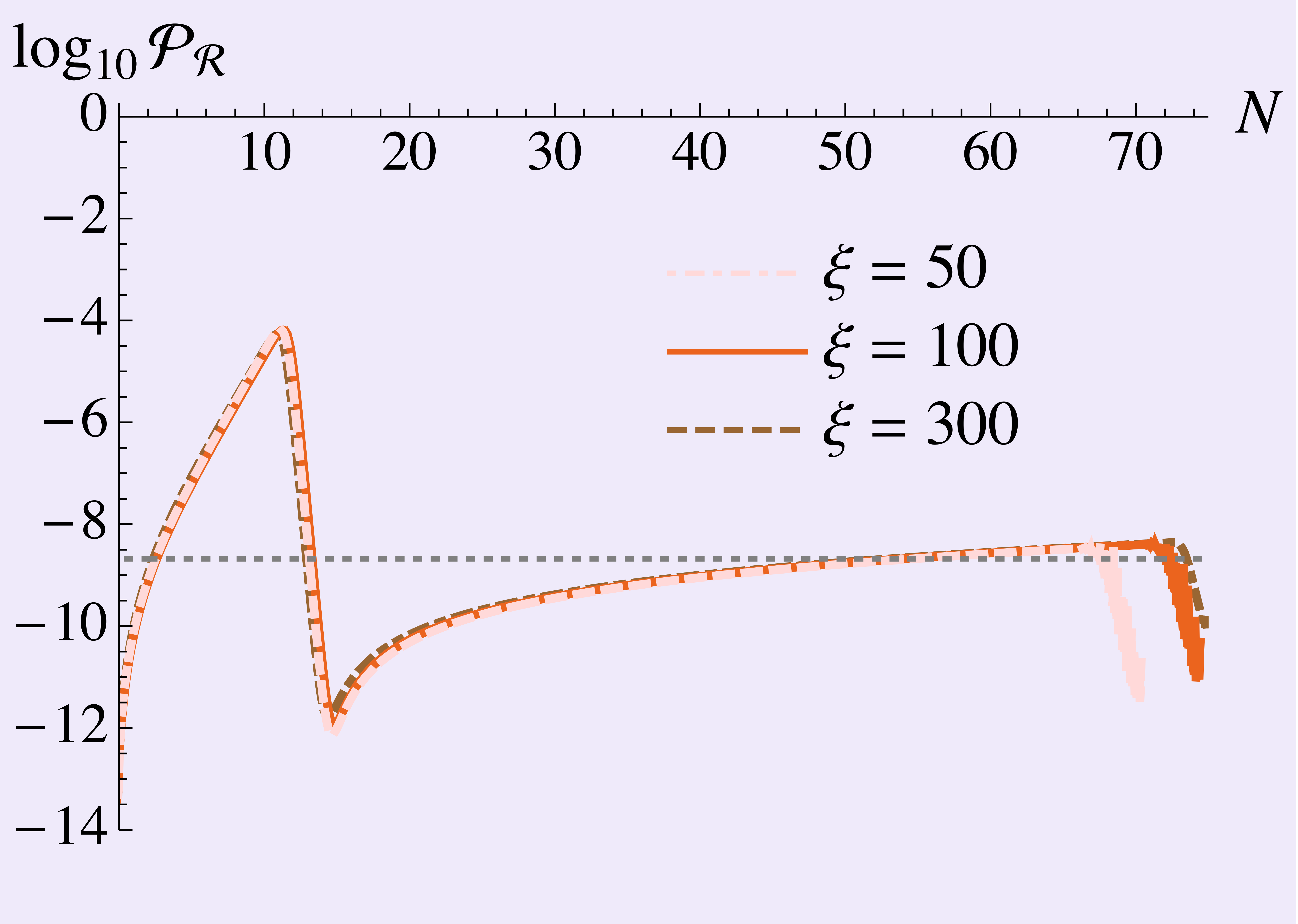
$b = y\hat{b}, c_i = y\hat{c}_i, y > 0$

Scaling relations:

Fixing $\hat{b}\sqrt{\xi}$ = constant, $\frac{\xi}{y}$ = constant

$V(r, \theta_*)$ and $\mathscr{P}_R$ show self-similarity at various values of ξ

(Uses $k_* = .05\ \mathrm{Mpc}^{-1}$ and COBE normalization $\mathscr{P}_R(k_*) = 2.1 \times 10^{-9}$



Observables and Parameters

"With four parameters I can fit an elephant and with five I can make him wiggle his trunk"
Enrico Fermi to John Von Neumman (https://www.nature.com/articles/427297a)

Counting Observables and Parameters

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8 Observables to match: $\Omega_k,\ n_s(k_*),\ \alpha(k_*),\ r(k_*),\ \beta_{\rm iso}(k_*),\ f_{\rm NL},\ \mathcal{P}_R(k_{\rm pbh})$

At first glance…13 degrees of freedom:

$\{\xi_\varphi, \xi_\chi\} \rightarrow$ Non-minimal couplings **2**

$\{b_1, b_2, b_3\}, \{c_1, c_2, c_3, c_4\} \rightarrow$ Self couplings **3+4**

$r(t_i), \theta(t_i), \dot{r}(t_i), \dot{\theta}(t_i) \rightarrow$ Initial conditions **4**

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$\{\xi_\varphi, \xi_\chi\} \rightarrow$ Non-minimal couplings ~~2~~

$\{b_1, b_2, b_3\}, \{c_1, c_2, c_3, c_4\} \rightarrow$ Self couplings ~~3~~+~~4~~

$r(t_i), \theta(t_i), \dot{r}(t_i), \dot{\theta}(t_i) \rightarrow$ Initial conditions ~~4~~

Applying constraints…

$\xi_\varphi = \xi_\chi$ ⟶ **1**

$b_1 = b_2, b_3 = 0$ ⟶ **2+3**

only $r(t_i)$ ⟶ **1**

Conclusion and ongoing research

Multifield inflation with non-minimal couplings *generically* gives inflation with single-field attractor behavior that fits CMB data.

A few e-folds of Ultra Slow-Roll towards end of inflation can seed density perturbations that will collapse to PBHs around the mass range $10^{15} \sim 10^{22}$g.

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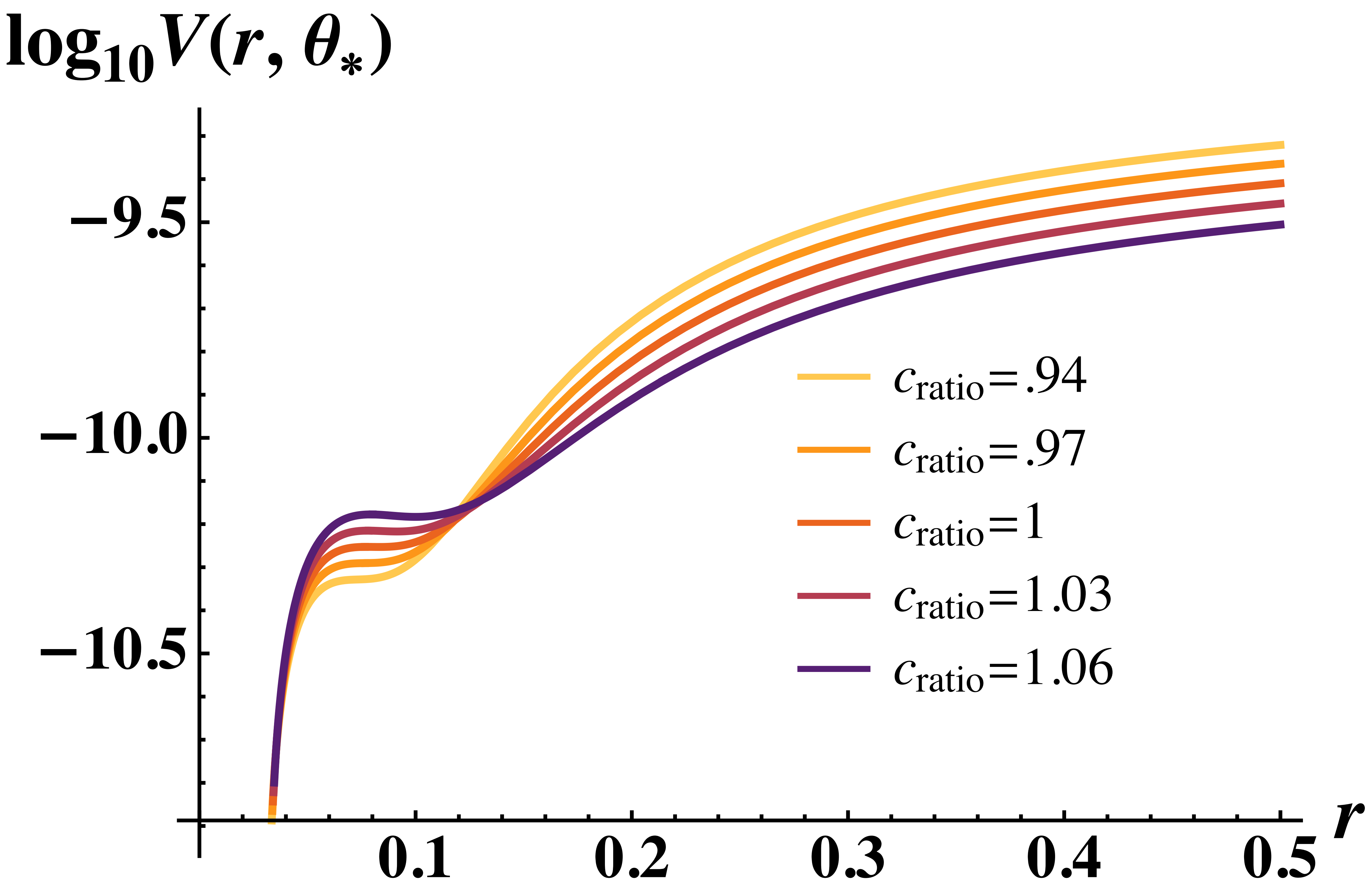
Effects of broken symmetries



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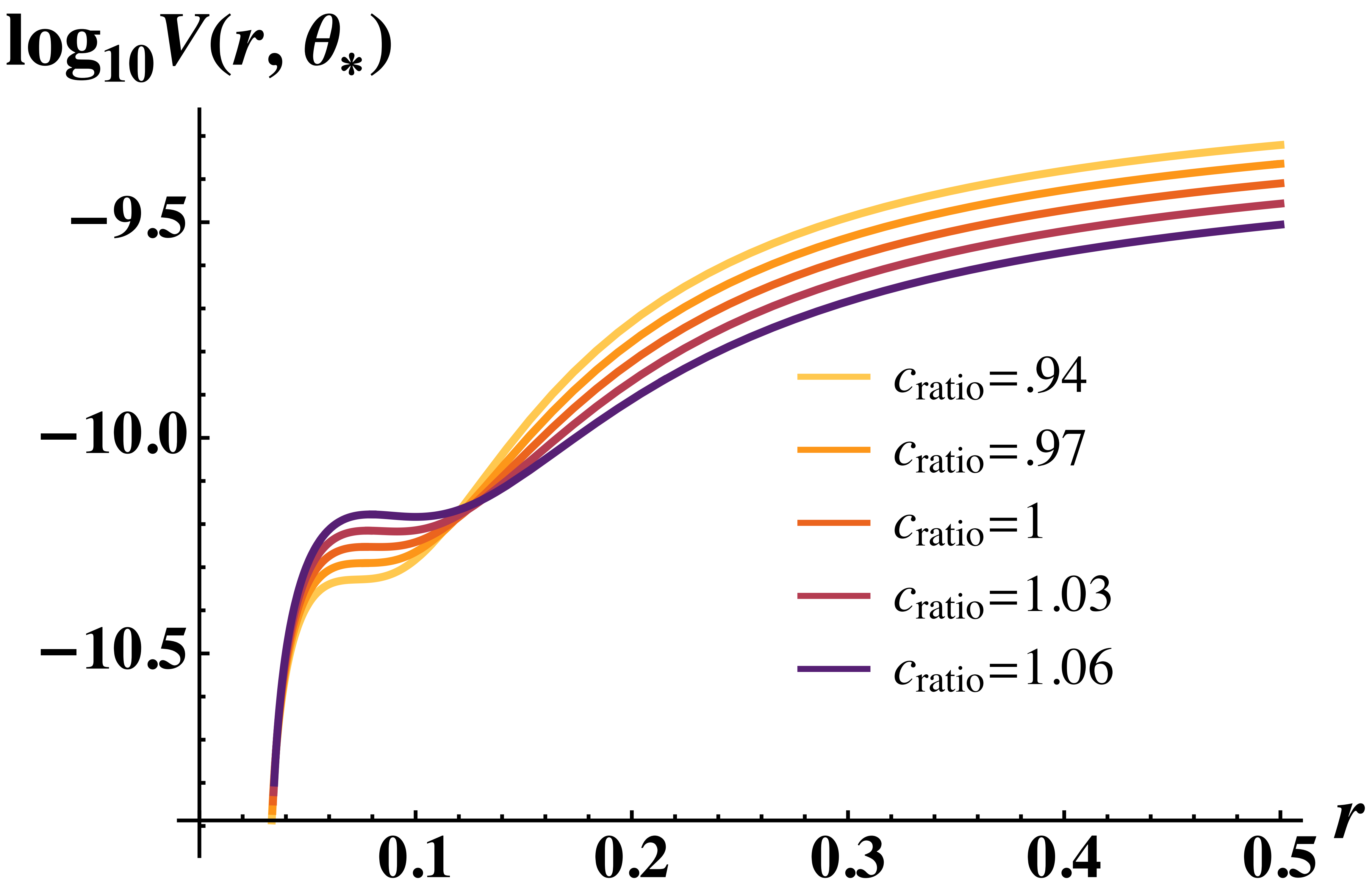

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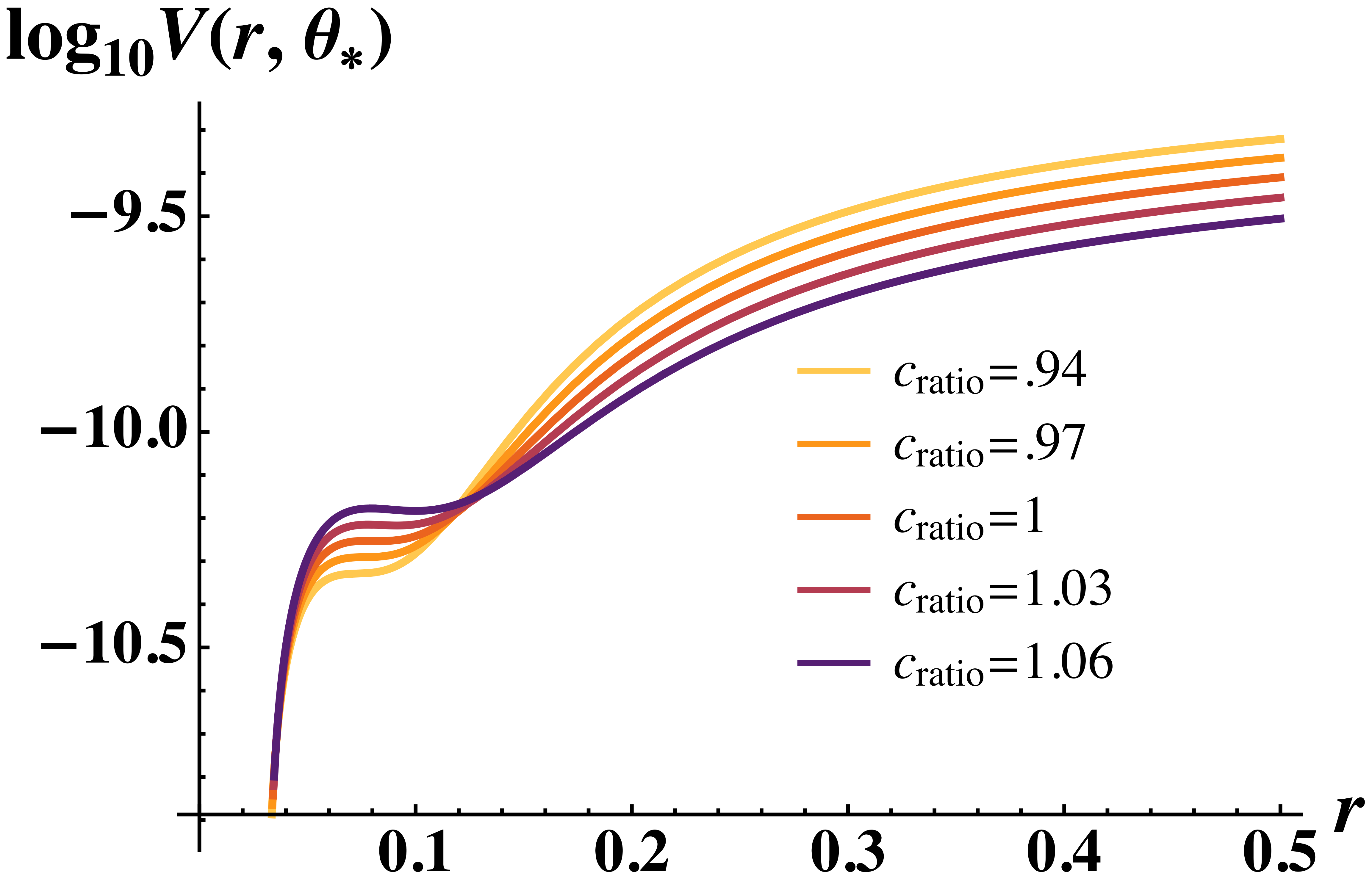

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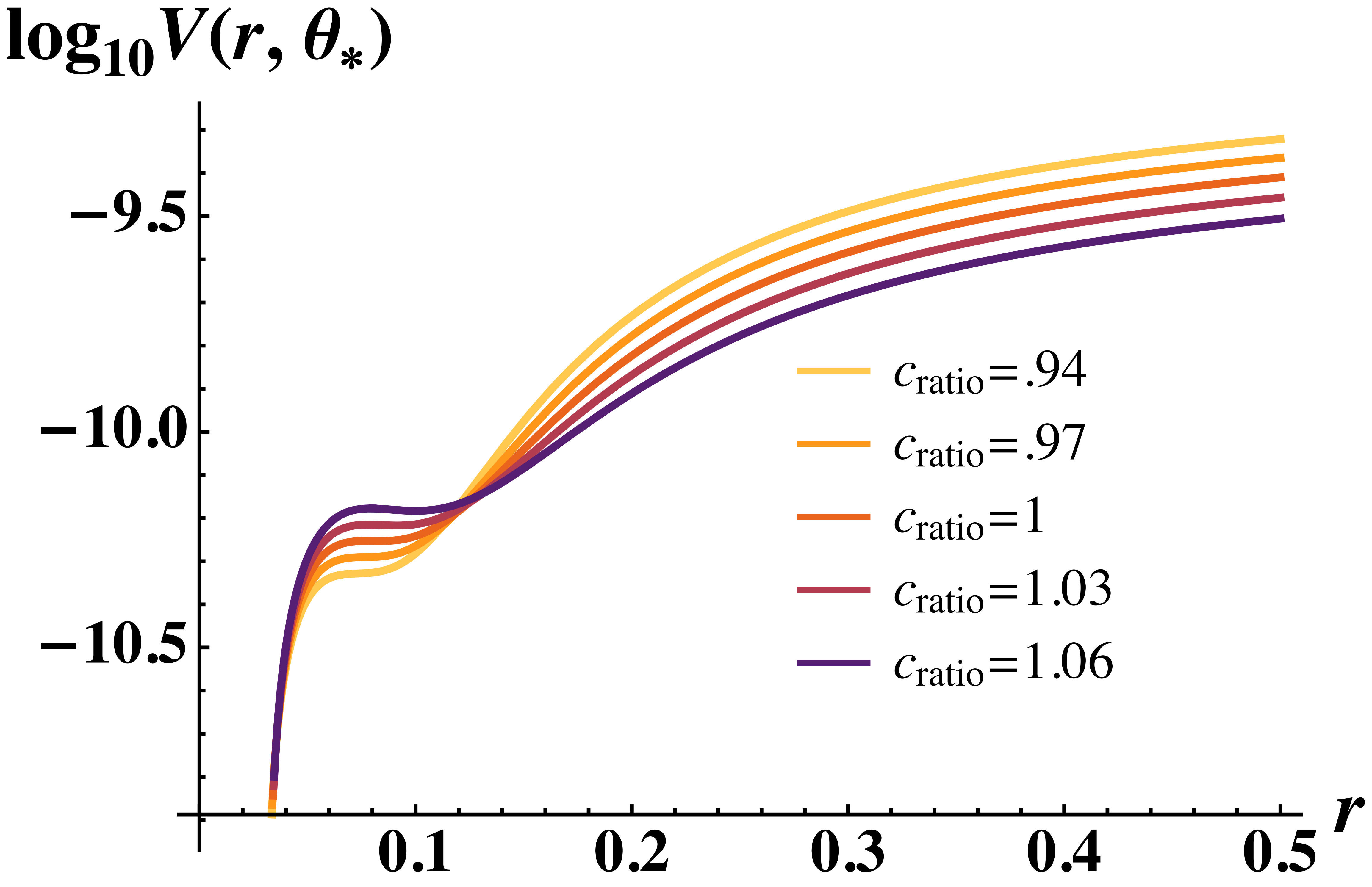
Tunneling rates and mass spectra



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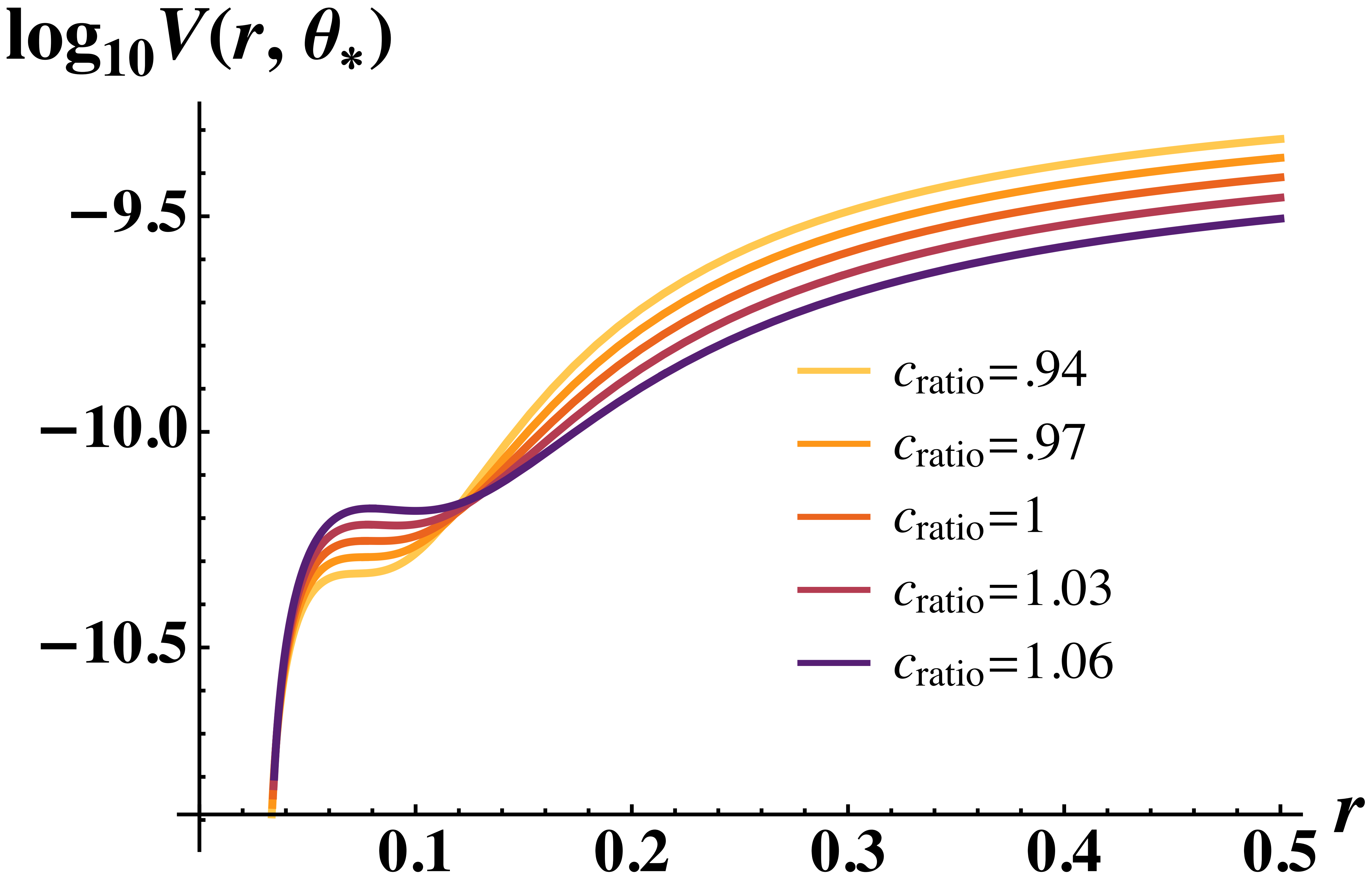

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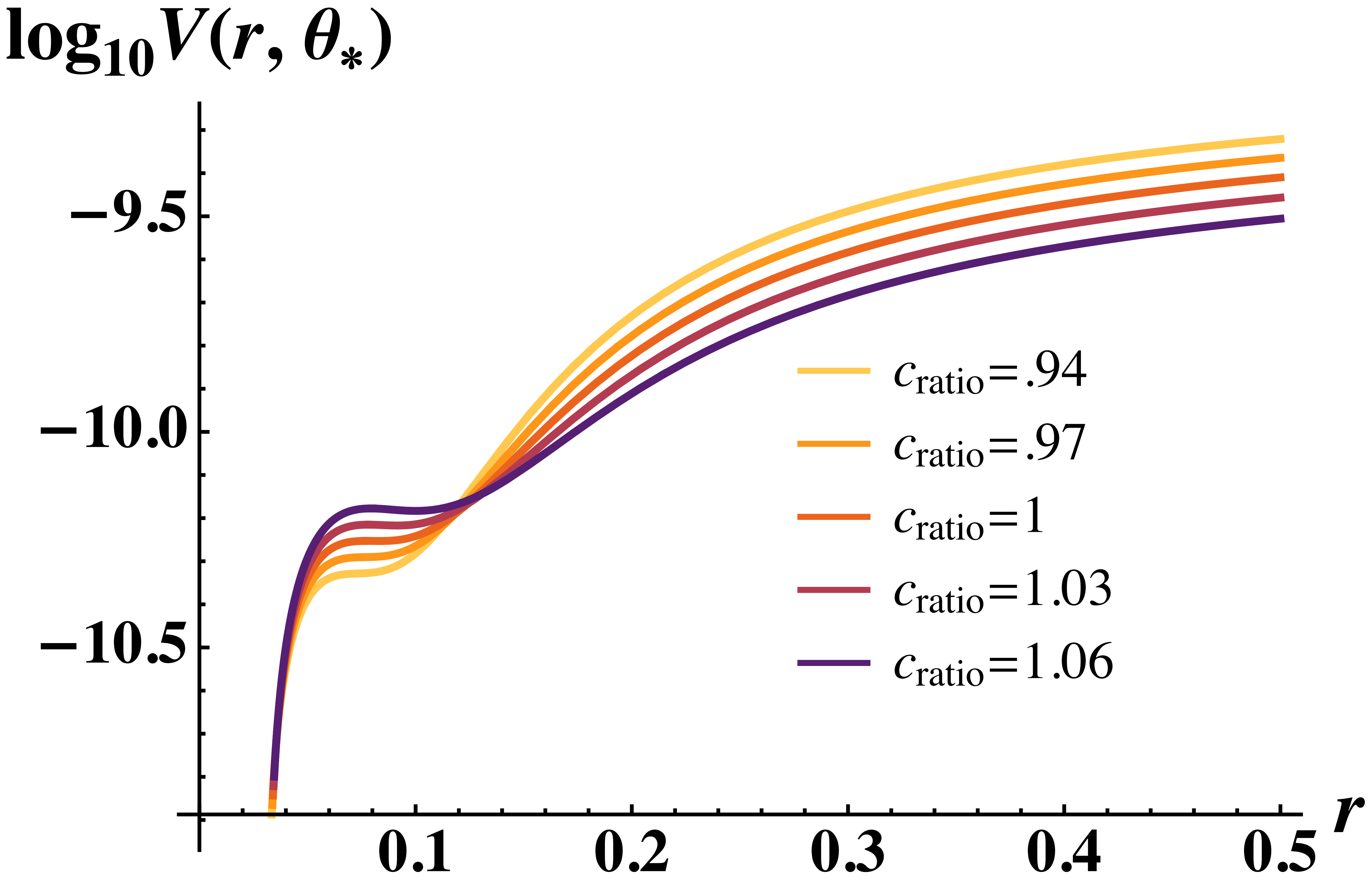
N+2 fields with GUT scale SSB



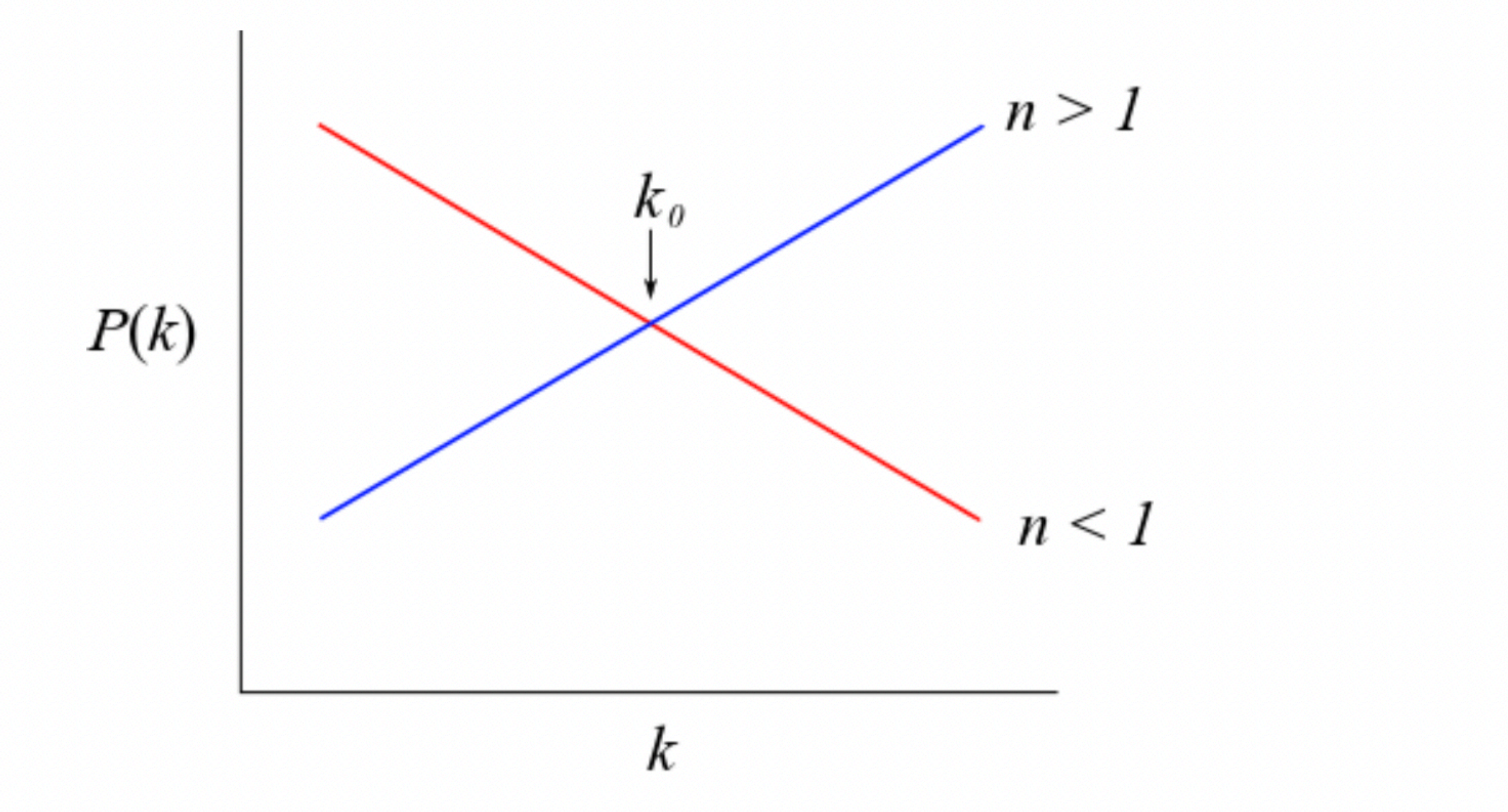
EXTRA/Q&A SLIDES

The CMB Pivot Scale

Why is it called the pivot scale?

$\mathscr{P}_R(k) = A\left(\frac{k}{k_0}\right)^{n-1}$ (Power spectrum is a power law in k)

$\ln \mathscr{P}_R(k) = \ln A + (nk_0 - 1)\ln\left(\frac{k}{k_0}\right) + \frac{1}{2}\alpha \ln\left(\frac{k}{k_0}\right)^2$ (k_0 is the "pivot scale" i.e. the reference scale at which A is measured)



$n = n_s$ is the spectral index and $\alpha = \frac{d \ln n_s}{d \ln k}$ is the running of the spectral index

When n changes the spectra will **pivot** about the point $k = k_0$

source: bapowell

PBHs as Dark Matter: The Available Parameter Space

Constraints from Femto-lensing?

A Gould (1992) proposed gamma-ray bursts could be used to constrain PBHs in the range $10^{17} \sim 10^{20}$ g via interference fringes. Later work (Katz et al.) showed constraints should be discounted because 1. gamma ray bursts too large for point sources and 2. need to consider wave optics (Source: Green and Kavanagh 2020)

Subaru HSC Constraints?

"High cadence optical observation of M31 constraints…are weaker than initially found due to finite sources and wave optics effects."
(Source: Green and Kavanagh 2020)

The 2-Field Inflaton Potential

Take a generic superpotential with two Chiral superfields $I = \{1,2\}$

$$\Phi = \Phi(x) + \ldots$$

Complex Scalar: $\Phi(x) = \dfrac{\phi(x)}{\sqrt{2}} e^{i\psi(x)}$

$$\tilde{W} = \mu b_{IJ} \mathbf{\Phi^I \Phi^J} + c_{IJK} \mathbf{\Phi^I \Phi^J \Phi^K} + \mathcal{O}\left(\frac{\Phi_I^4}{M_{\text{pl}}}\right)$$

$$= b_1 \mu (\mathbf{\Phi_1})^2 + b_2 \mu (\mathbf{\Phi_2})^2 + c_1 (\mathbf{\Phi_1})^3 + c_2 (\mathbf{\Phi_1})^2 \mathbf{\Phi_2} + c_3 \mathbf{\Phi_1} (\mathbf{\Phi_2})^2 + c_4 (\mathbf{\Phi_2})^3 + \mathcal{O}\left(\frac{\mathbf{\Phi}_I^4}{M_{\text{pl}}}\right)$$

In the low energy limit ($|\Phi^I|^2 / M_{\text{pl}}^2 \to 0$), this gives a potential for the real part of the complex scalar field $\Phi(x)$

$$\tilde{V}(\phi_i) = \sum_i \left| \frac{\partial \tilde{W}}{\partial \Phi_i} \right|^2_{\Phi_i \to \phi_i} = \frac{1}{4 f(r)^2} \left[\mathscr{B}(\theta) r^2 + \mathscr{C}(\theta) r^3 + \mathscr{D}(\theta) r^4 \right]$$

Two Field Inflaton Potential and SUSY/SUGRA Motivations

(Generic) Superpotential with 2 Chiral superfields $\mathbf{\Phi_1}, \mathbf{\Phi_2}$:

$$\tilde{W} = \mu b_{IJ}\mathbf{\Phi^I\Phi^J} + c_{IJK}\mathbf{\Phi^I\Phi^J\Phi^K} + \mathcal{O}\left(\frac{\Phi_I^4}{M_{\text{pl}}}\right)$$

$$= b_1\mu(\mathbf{\Phi_1})^2 + b_2\mu(\mathbf{\Phi_2})^2 + c_1(\mathbf{\Phi_1})^3 + c_2(\mathbf{\Phi_1})^2\mathbf{\Phi_2} + c_3\mathbf{\Phi_1}(\mathbf{\Phi_2})^2 + c_4(\mathbf{\Phi_2})^3 + \mathcal{O}\left(\frac{\mathbf{\Phi}_I^4}{M_{\text{pl}}}\right)$$

(note: *chose* $b_{12} = 0$)

Superfield field content:

$$\Phi^I(x) = \Phi(x)^I + \sqrt{2}\theta\chi^I(x) + \theta\theta F^I(x)$$

Complex Scalar: $\Phi(x) = \dfrac{\phi(x)}{\sqrt{2}}e^{i\psi(x)}$

Kähler Potential: $K(\mathbf{\Phi}, \bar{\mathbf{\Phi}}) = -\dfrac{1}{2}\sum_I \left(\mathbf{\Phi^I} - \bar{\mathbf{\Phi}}^{\mathbf{I}}\right)^2$ (imaginary part remains heavy)

Low energy limit: $\lim_{\zeta\to 0}$ SUGRA → SUSY where $\zeta = \dfrac{|\Phi^I|^2}{M_{\text{pl}}^2}$, and integrate over auxiliary fields.

Remaining real scalar fields can drive inflation, with potential

$$\tilde{V}(\phi_i) = \sum_i \left|\frac{\partial \tilde{W}}{\partial \Phi_i}\right|^2_{\Phi_i\to\phi_i}$$

Each scalar field is non-minimally coupled in action $\sim \xi_I(\phi^I)^2\tilde{R}$

SUGRA and SUSY Background of Inflaton Potential (1)

Start with $\mathcal{N} = 1$ 4-dimensional supergravity with 2 chiral superfields

$$\boldsymbol{\Phi}(y)^I = \Phi(y) + \sqrt{2\theta}\psi(y) + \theta\theta F(y)$$

One next integrates out the auxiliary fields, get the Lagrangian we

complex scalar field — fermion — auxiliary field

started with: $\mathscr{L} = \mathscr{G}_{IJ} g^{\mu\nu} \partial_\mu \Phi^I \partial_\nu \bar{\Phi}^{\bar{J}} - V(\Phi, \bar{\Phi})$ With a generic choice of superpotential (linear terms dropped - unless Φ^I is gauge singlet.)

$$\tilde{W} = \mu b_{IJ} \Phi^I \Phi^J + c_{IJK} \Phi_I \Phi_J \Phi_K + \mathcal{O}\left(\frac{\Phi_I^4}{M_{\rm pl}}\right)$$

$$= b_1(\Phi_1)^2 + b_2(\Phi_2)^2 + c_1(\Phi_1)^3 + c_2(\Phi_1)^2\Phi_2 + c_3\Phi_1(\Phi_2)^2 + c_4(\Phi_2)^3 + \mathcal{O}\left(\frac{\Phi_I^4}{M_{\rm pl}}\right)$$

In (local) SUGRA we also choose a Kähler potential (such that imaginary part of Φ^I remains heavy/decoupled)

$$K(\Phi, \bar{\Phi}) = \sum_{I,J} (\Phi^I - \bar{\Phi}^I)^2$$

The potential for the scalar field part of $W(\boldsymbol{\Phi}, \bar{\boldsymbol{\Phi}})$ is:

$$V(\Phi, \bar{\Phi}) = \exp\left(\frac{K(\Phi, \bar{\Phi})}{M_{\rm pl}^2}\right)\left(\mathscr{G}^{I\bar{J}} \nabla_I W(\Phi) \nabla_{\bar{J}} \bar{W}(\bar{\Phi}) - \frac{3}{M_{\rm pl}^2} W(\Phi) \bar{W}(\bar{\Phi})\right)$$

where $\nabla_I = \partial_I + \frac{1}{M_{\rm pl}^2} K_{,I}$

(McDonough,Long,Kolb), (Linde),(Bertolami, Ross)

SUGRA and SUSY Background of Inflaton Potential (2)

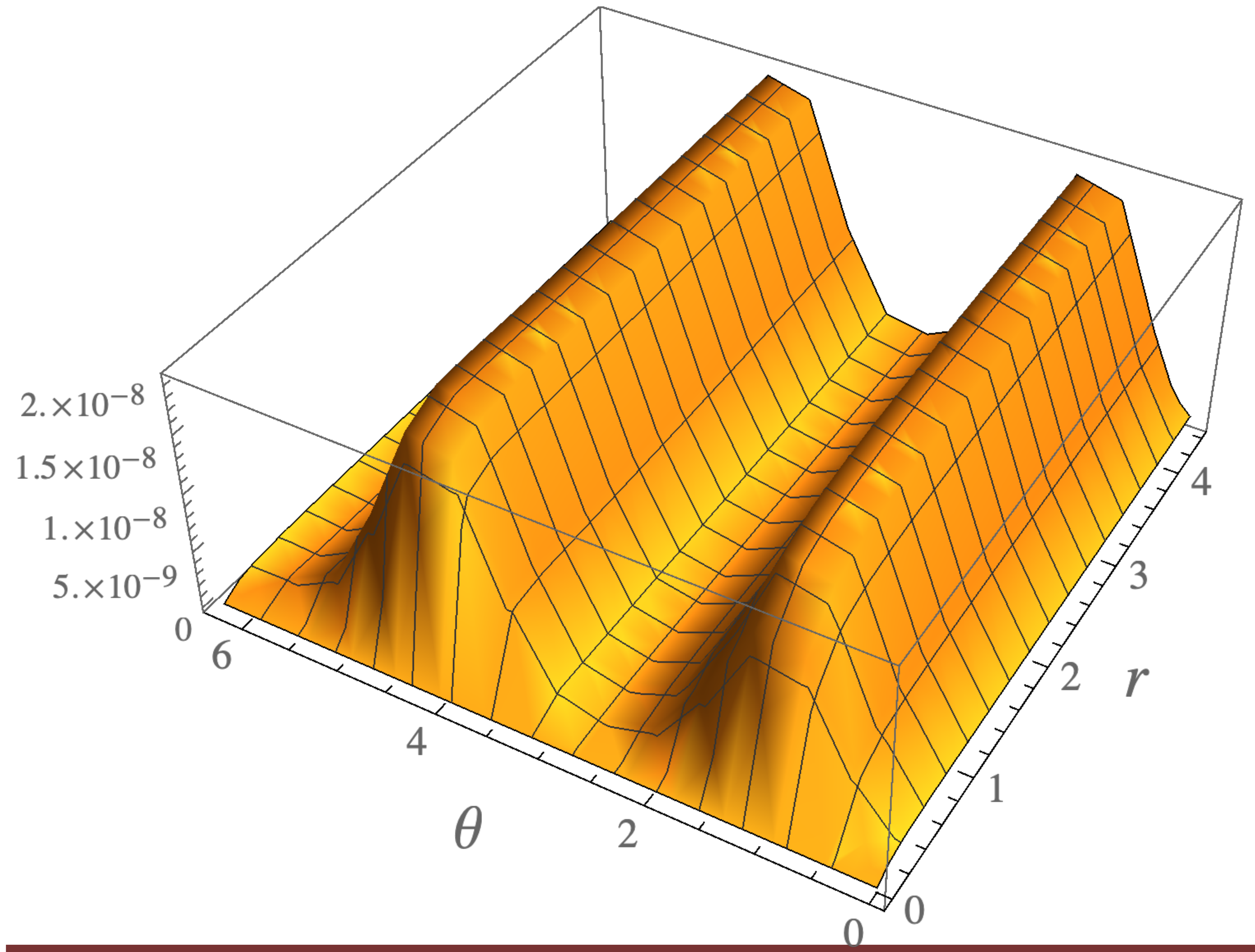
$$V(\Phi,\bar{\Phi}) = \exp\left(\frac{K(\Phi,\bar{\Phi})}{M_{\rm pl}^2}\right)\left(\mathcal{G}^{I\bar{J}}\nabla_I W(\Phi)\nabla_{\bar{J}}\bar{W}(\bar{\Phi}) - \frac{3}{M_{\rm pl}^2}W(\Phi)\bar{W}(\bar{\Phi})\right)$$ where $$\nabla_I = \partial_I + \frac{1}{M_{\rm pl}^2}K_{,I}$$

Take the limit of $V(\Phi,\bar{\Phi})$ as $\frac{|\Phi^I|^2}{M_{\rm pl}^2} \to 0$ to get the expression for $V(\phi)$. The ψ dependence drops out because of the choice of Kähler potential which makes the imaginary part of the complex scalar field heavy- it decouples for all of inflation.

(McDonough,Long,Kolb), (Linde),(Bertolami, Ross)

(Exact) Inflationary Trajectories

Exact trajectories are extrema of $V_{,\theta}(r, \theta_*) = 0$, 'i.e. system evolves along path $\theta_*(r)$ in field space



$$V(r,\theta) = \frac{1}{4f^2(r,\theta)}\left(\mathscr{B}(\theta)r^2 + \mathscr{C}(\theta)r^3 + \mathscr{D}(\theta)r^4\right)$$

=

$$\frac{1}{4f(r,\theta)^2}\Big[\left[4b_1^2\cos^2\theta + 4b_2^2\sin^2\theta\right]r^2$$

$$+\left[12b_1c_1\cos^3\theta + 4(2b_1+b_2)c_2\cos^2\theta\sin\theta + 4(b_1+2b_2)c_3\cos\theta\sin^2\theta + 12b_2c_4\sin^3\theta\right]r^3$$

$$+\Big[(9c_1^2+c_2^2)\cos^4\theta + 4c_2(3c_1+c_3)\cos^3\theta\sin\theta + (4c_2^2+6c_1c_3+6c_2c_4+4c_3^2)\cos^2\theta\sin^2\theta + 4c_3(c_2+3c_4)\cos\theta\sin^3\theta + (9c_4^2+c_3^2)\sin^4\theta\Big]r^4$$

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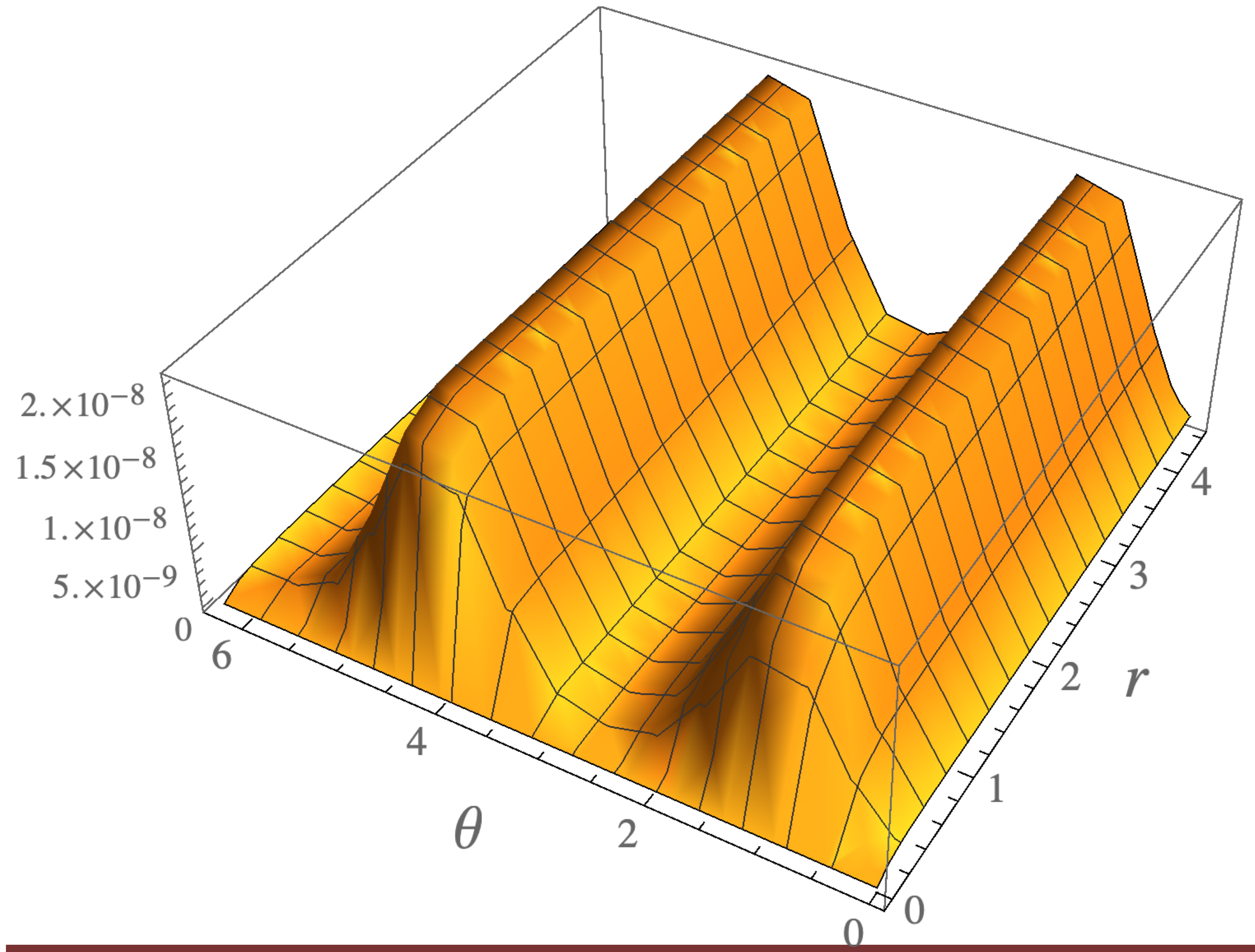

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$$+ \left[12b_1c_1\cos^3\theta + 4(2b_1 + b_2)c_2\cos^2\theta\sin\theta + 4(b_1 + 2b_2)c_3\cos\theta\sin^2\theta + 12b_2c_4\sin^3\theta\right] r^3$$

$$+ \left[(9c_1^2 + c_2^2)\cos^4\theta + 4c_2(3c_1 + c_3)\cos^3\theta\sin\theta + (4c_2^2 + 6c_1c_3 + 6c_2c_4 + 4c_3^2)\cos^2\theta\sin^2\theta + 4c_3(c_2 + 3c_4)\cos\theta\sin^3\theta + (9c_4^2 + c_3^2)\sin^4\theta\right] \Big] r^4$$

Power Spectrum Peaks in Our 2-field Model

Adiabatic and Isocurvature modes **decouple** for $\omega = 0$

Large turns $\Longrightarrow$ **transfer of power** from isocurvature modes to adiabatic modes

$$\mathscr{R}_k = \frac{H}{\dot\sigma} Q_\sigma = \frac{Q_\sigma}{M_{\rm pl}\sqrt{2\epsilon}}$$

$$\mathscr{P}_{\mathbf{R}}(\mathbf{k}) \equiv \frac{\mathbf{k}^3}{2\pi^2} |\mathscr{R}_{\mathbf{k}}|^2$$

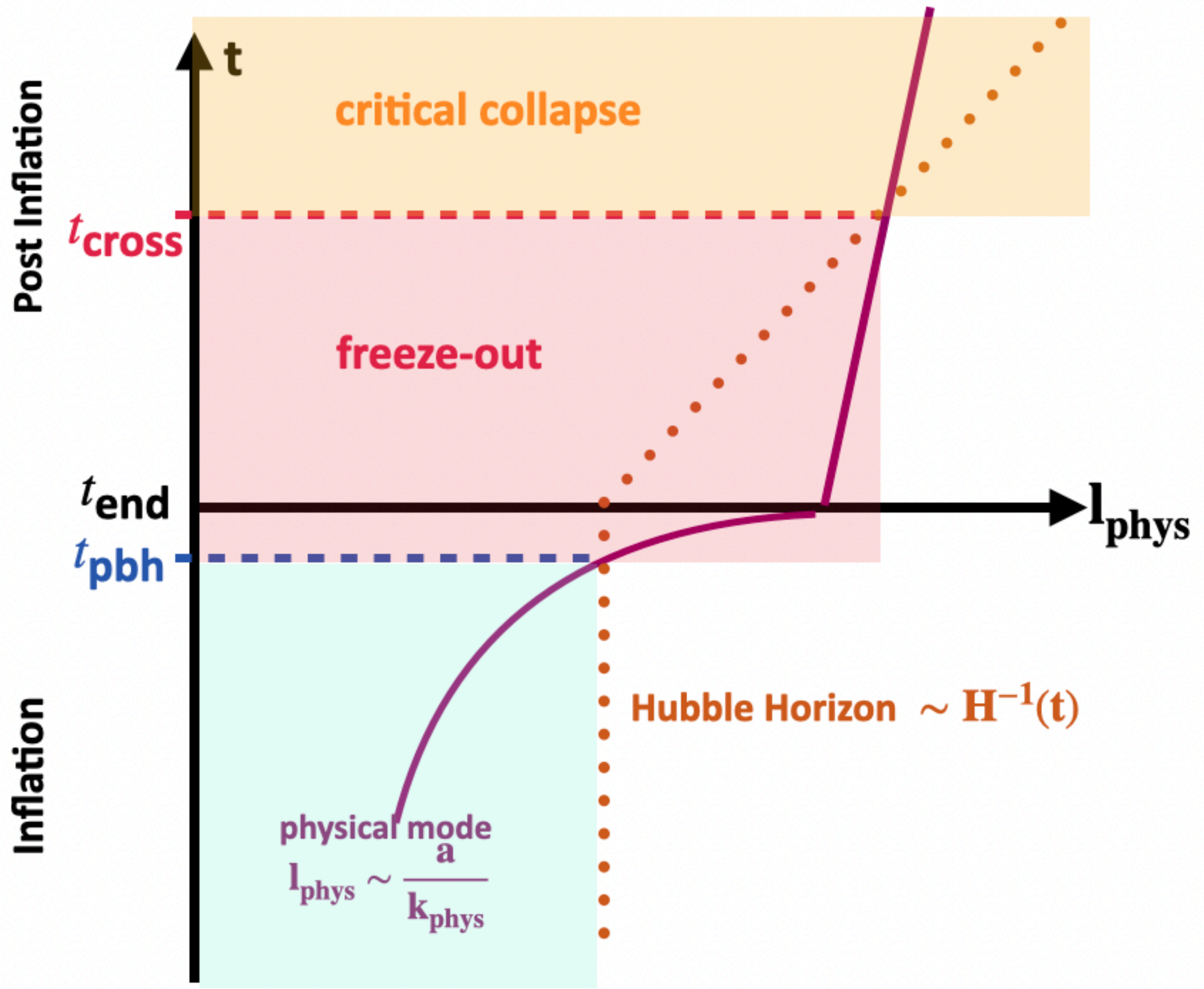
Multifield effects **heavily constrained** by experiment!

Main idea: multi-field model with **slight** turns while keeping isocurvature modes small -$\mathscr{P}_{\mathbf{R}}$ **amplified for modes** $k_{\mathbf{pbh}}(t_{USR})$

How to spike the power spectrum (revisited)?

Numerator gets larger:
(1) tachyonic modes (hybrid inflation)
(2) turns in field space (multifield seeds)

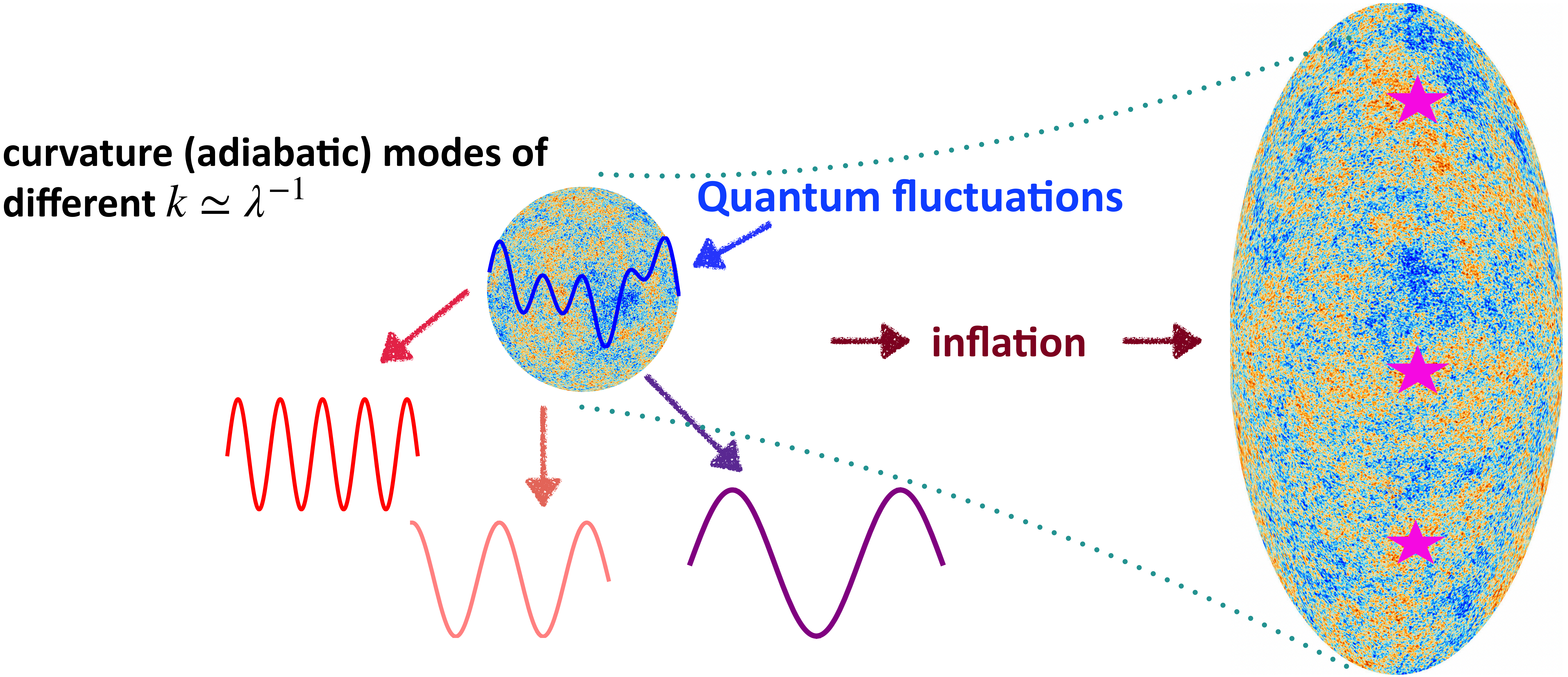
Denominator gets smaller:
Brief phase of Ultra slow-roll



Overview - A cartoon picture of PBH formation from Primordial Density Perturbations

During inflation fluctuations are stretched and amplified to cosmic scales.

Modes seed density perturbations which then cause collapse if $\delta_{\text{density}} \geq \delta_{\text{critical}}$

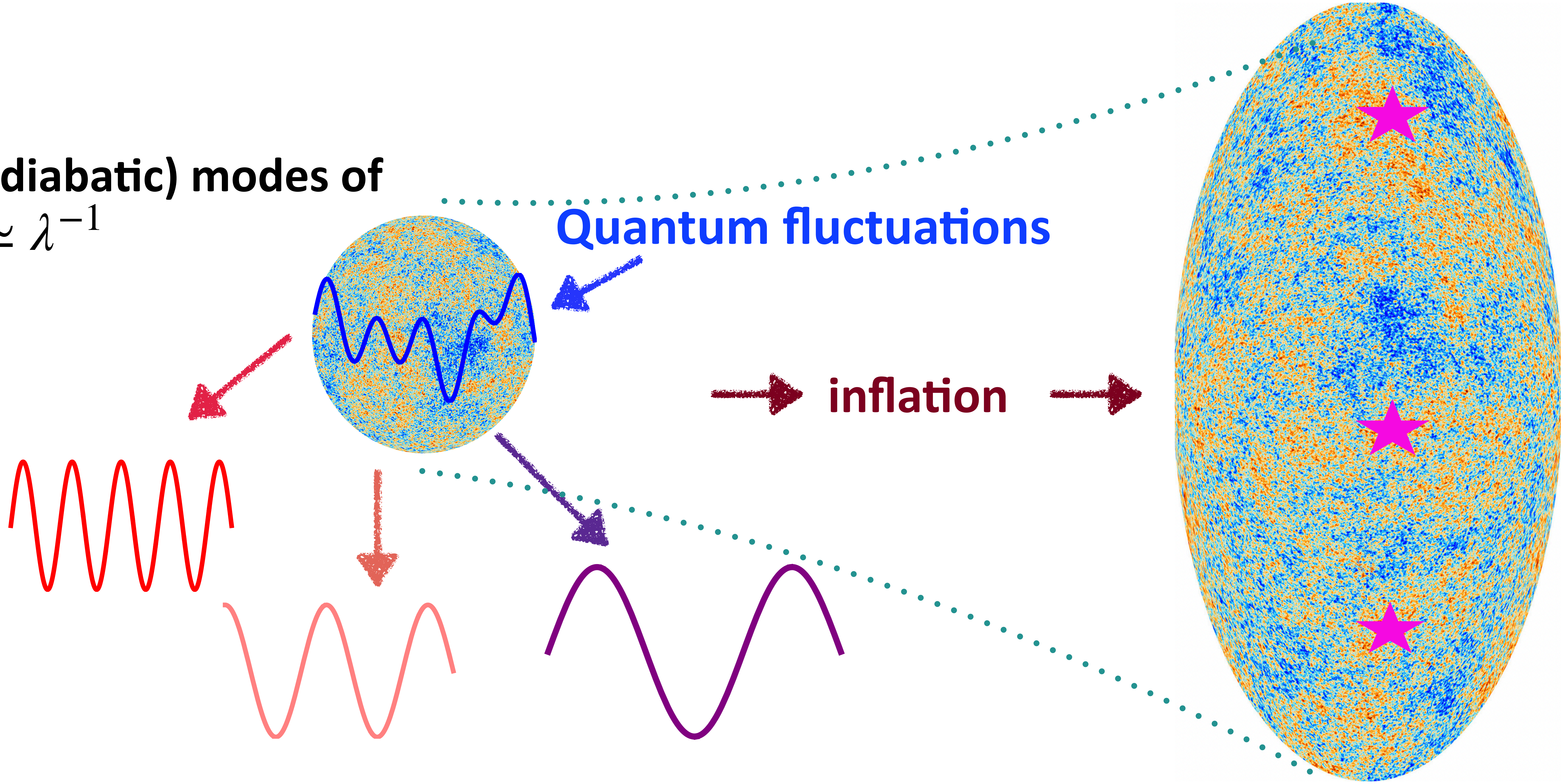


Overview - A cartoon picture of PBH formation from Primordial Density Perturbations

Modes seed density perturbations which then cause collapse if $\delta_{\text{density}} \geq \delta_{\text{critical}}$

Diagnose these perturbations by seeing spikes in the curvature power spectrum $\mathscr{P}_{\mathbf{R}}(\mathbf{k})$

curvature (adiabatic) modes of different $k \simeq \lambda^{-1}$



More details on the curvature perturbations

perturb about the FLRW metric to linear order : $g_{\mu\nu} = \tilde{g}_{\mu\nu} + h_{\mu\nu}$ where $\tilde{g}_{\mu\nu}$ is flat FLRW.

$$ds^2 = -(1+2A)dt^2 + 2a(t)(\partial_i B - S_i)dt dx^i + a^2(t)\left[(1-2\psi)\delta_{ij} + 2\partial_i\partial_j E + 2(\partial_i F_j + \partial_j F_i) + \mathscr{H}_{ij}\right] dx^i dx^j$$

tensor perturbations $\mathscr{H}_{ij}$=gravitational waves → decouple at linear order

vector perturbations F_i, S_i, fall off as $\dfrac{1}{a^2(t)}$ during radiation domination

Scalar perturbations E, A, ψ, B: due to gauge redundancy, only have 2 independent scalar d.o.f

Metric with just scalar perturbations to linear order:

$$ds^2 = -(1+2A)dt^2 + 2a(t)(\partial_i B)dt dx^i + a^2\left[(1-2\psi)\delta_{ij} + 2\partial_i\partial_j E\right] dx^i dx^j$$

Power Spectrum Peaks in Our 2-field Model

(Multifield) Gauge Invariant Mukhanov-Sasaki variables

$$Q^I = \delta\phi^I + \frac{\dot{\varphi}^I}{H}\psi$$

Split into two modes: Adiabatic and Isocurvature

$$Q^I = \underbrace{\hat{\sigma}^I Q_\sigma}_{\text{Adiabatic}} + \underbrace{\sqrt{|\mathcal{G}_{IJ}|}\,\epsilon^{IJ}\hat{\sigma}_J Q_s}_{\text{Isocurvature}}$$

In multifield inflation: trajectory can turn and perturbations can couple

Covariant turn rate vector:

$$\omega^I \equiv \mathcal{D}_t \hat{\sigma}^I = \dot{\phi}^J \mathcal{D}_J \hat{\sigma}^I \quad \textbf{where} \quad \hat{\sigma}^I \equiv \frac{\dot{\varphi}}{\sqrt{\mathcal{G}_{IJ}\dot{\varphi}^I\dot{\varphi}^J}}$$

Adiabatic:
fields have equal fraction over/under-densities

Isocurvature:
overall density uniform not in chemical equilibrium

under-density

over-density

Vegemite over-densities

Margarine over-density

inspiration: Katelin Schutz

$\omega^2 << H^2 \rightarrow$ (only slight turning)

$\frac{\mu_s^2}{H^2} >> 1 \rightarrow$ Isocurvature modes heavy

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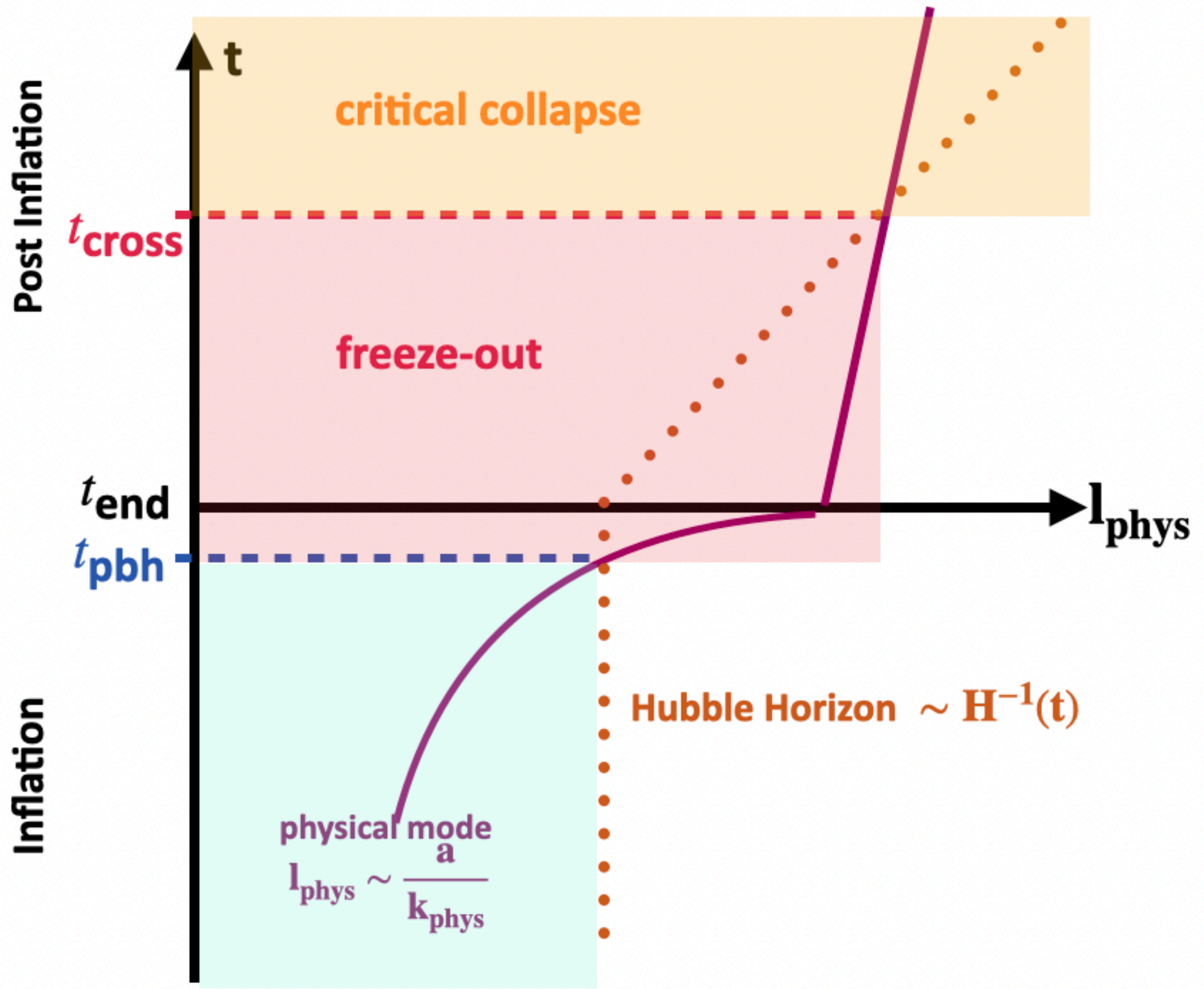
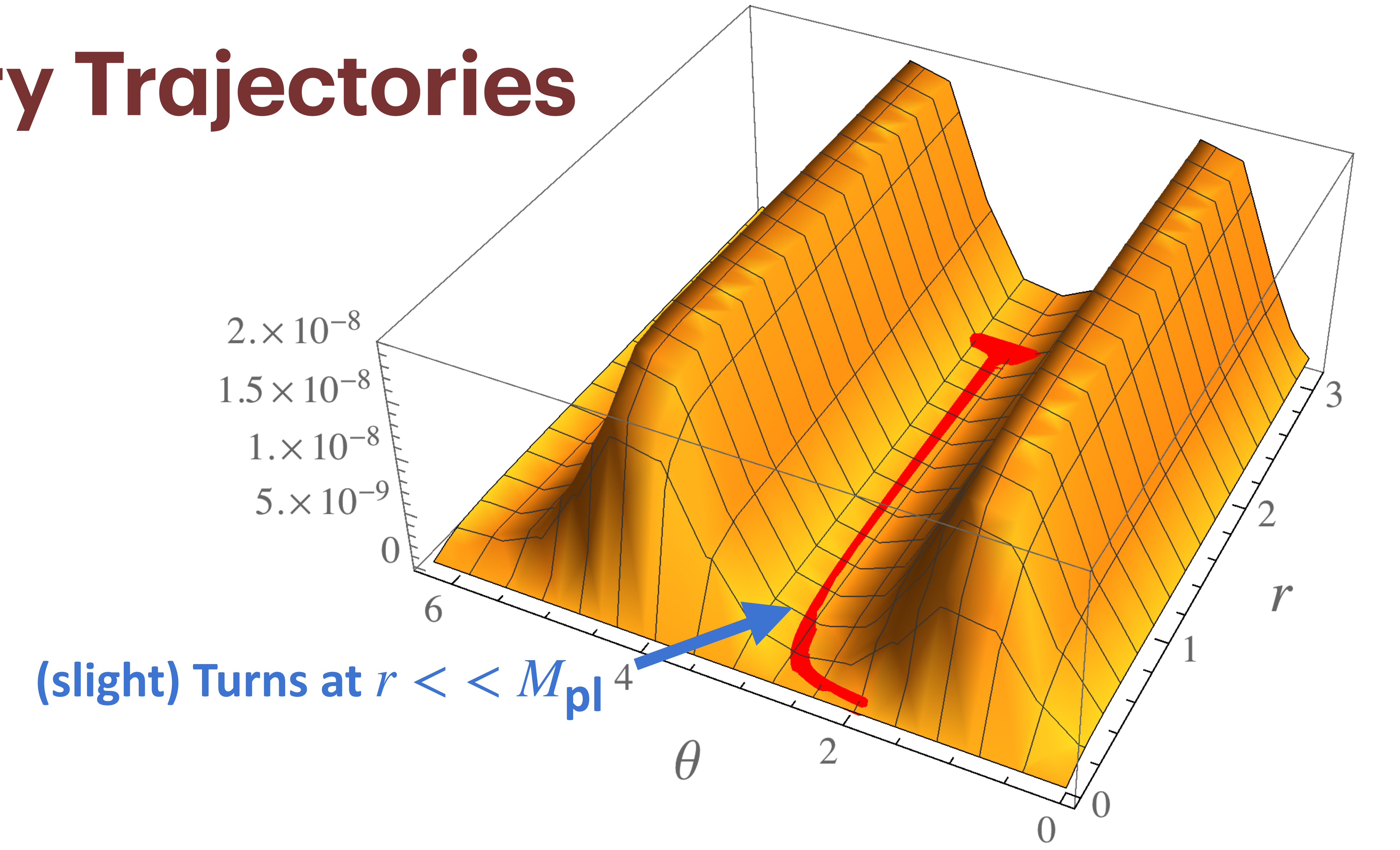


Exact Solutions for Inflationary Trajectories

Potential in "polar" field space coordinates:

$$\mathbf{V}(\mathbf{r},\theta) = \frac{1}{\left(1+\mathbf{r}^2\left(\xi_\phi\cos^2\theta+\xi_\chi\sin^2\theta\right)\right)^2}\left[\mathscr{B}(\theta)\mathbf{r}^2+\mathscr{C}(\theta)\mathbf{r}^3+\mathscr{D}(\theta)\mathbf{r}^4\right]$$



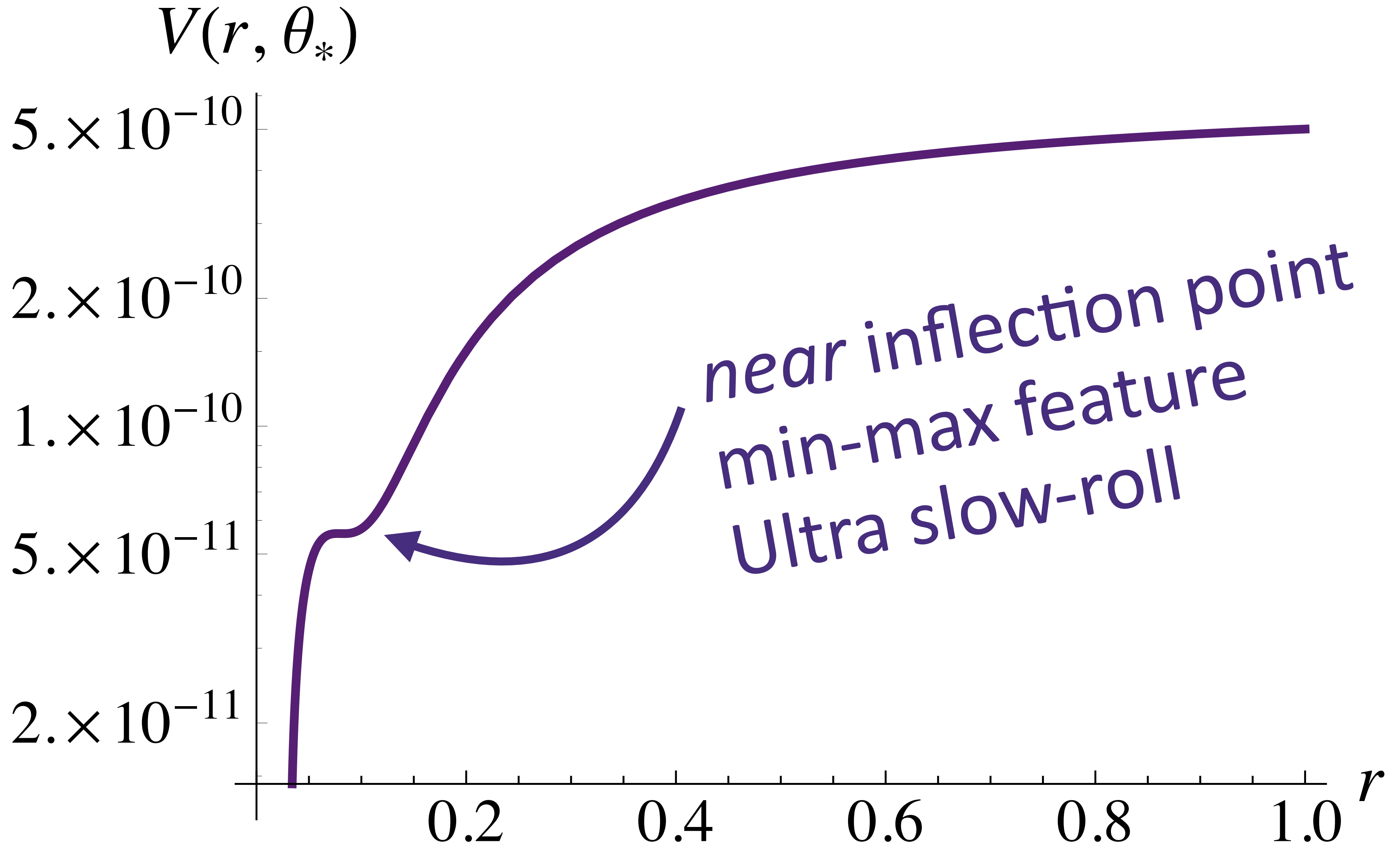
Impose the constraints: $\xi_\phi = \xi_\chi = \xi \quad c_2 = c_3 \quad b_1 = b_2 = b$

Exact trajectories are extrema of $V_{,\theta}(r,\theta_*) = 0$,
'i.e. system evolves along path $\theta_*^\pm(r)$ in field space

$$x^\pm(r) = \frac{-d_1 \pm |d_4|\sqrt{-1+R^2}}{R\sqrt{d_1^2+d_4^2}} \qquad \theta_*^\pm(r) = \arccos(x^\pm(r))$$

where we define: $d_1 \equiv c_1 + \frac{c_2}{3}, \quad d_4 \equiv c_4 + \frac{c_2}{3}, \quad \theta \equiv \arccos(x)$

$$r_{\text{imag}} \equiv \frac{b\mu}{\sqrt{d_1^2+d_4^2}}, \qquad R \equiv \frac{r}{r_{\text{imag}}}$$



Examples with Broken "Extra" Constraints $\xi_\phi \neq \xi_\chi, b_1 \neq b_2$ and/or $c_2 \neq c_3$

The full form of the potential in "polar" field space coordinates $V(r,\theta) = \frac{1}{4f^2(r,\theta)}\left[\mathscr{B}(\theta)r^2 + \mathscr{C}(\theta)r^3 + \mathscr{D}(\theta)r^4\right]$

where the non-minimal coupling function is $f = \frac{1}{2}\left(M_{\text{pl}}^2 + r^2\left(\xi_\phi \cos^2\theta + \xi_\chi \sin^2\theta\right)\right)$

$$\mathscr{B}(\theta) = 4b_1^2\cos^2\theta + 4b_2^2\sin^2\theta$$

$$\mathscr{C}(\theta) = 12b_1c_1\cos^3\theta + 4(2b_1 + b_2)c_2\cos^2\theta\sin\theta \quad +4(b_1 + 2b_2)c_3\cos\theta\sin^2\theta + 12b_2c_4\sin^3\theta$$

$$\mathscr{D}(\theta) = 9(c_1^2 + c_2^2)\cos^4\theta + 4c_2(3c_1 + c_3)\cos^3\theta\sin\theta + (4c_2^2 + 6c_1c_3 + 6c_2c_4 + 4c_3^2)\cos^2\theta\sin^2\theta$$
$$+4c_3(c_2 + 3c_4)\cos\theta\sin^3\theta + (9c_4^2 + c_3^2)\sin^4\theta$$

Reparametrize the potential by $\frac{\xi_\phi}{\xi_\chi} = 1 + \xi_{\text{ratio}}, \frac{b_1}{b_2} = 1 + b_{\text{ratio}}, \frac{c_2}{c_3} = 1 + c_{\text{ratio}}$ and see how the small-field feature varies with small perturbations around $\xi_{\text{ratio}}, b_{\text{ratio}}, c_{\text{ratio}} = 0$.

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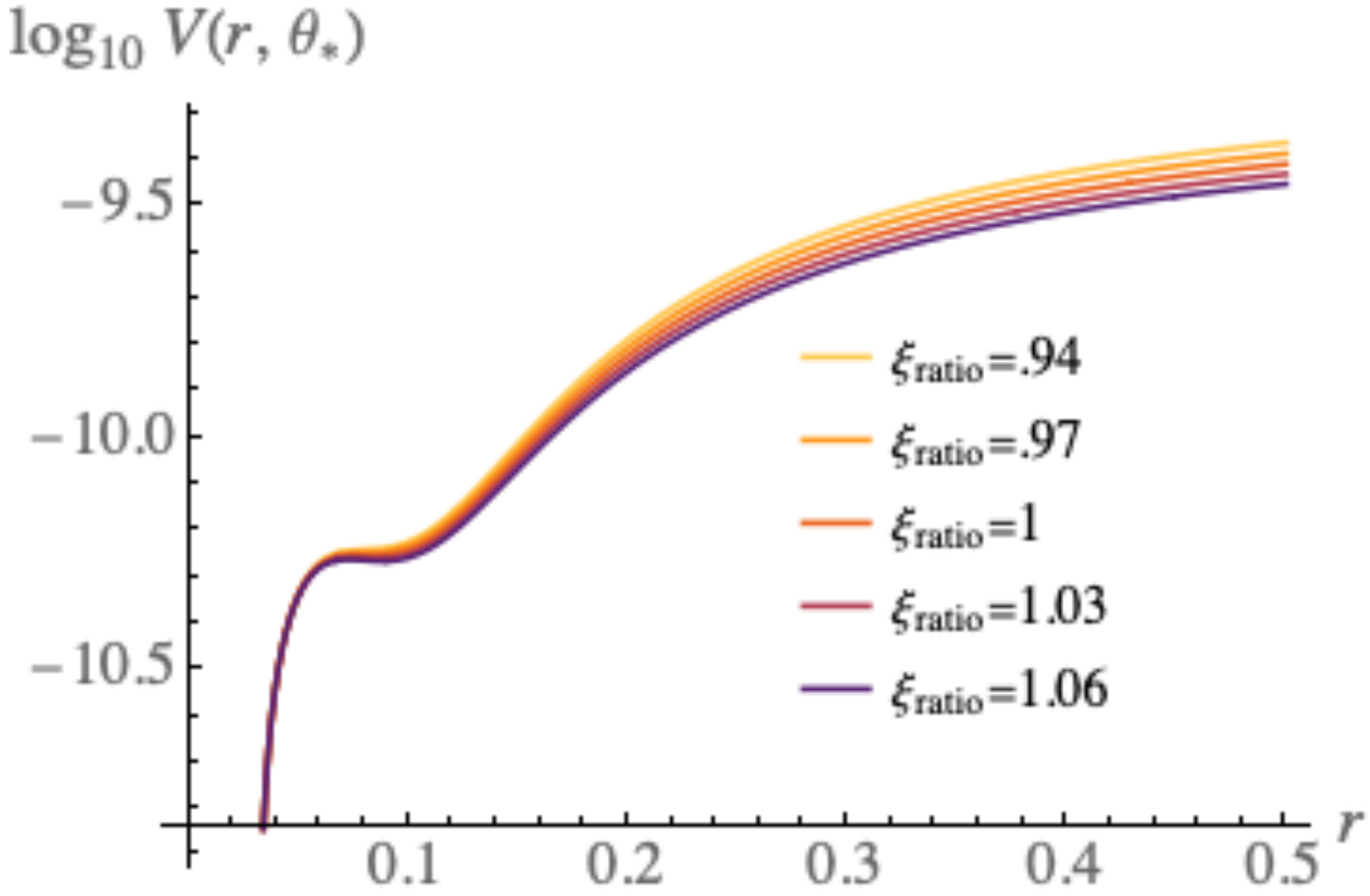
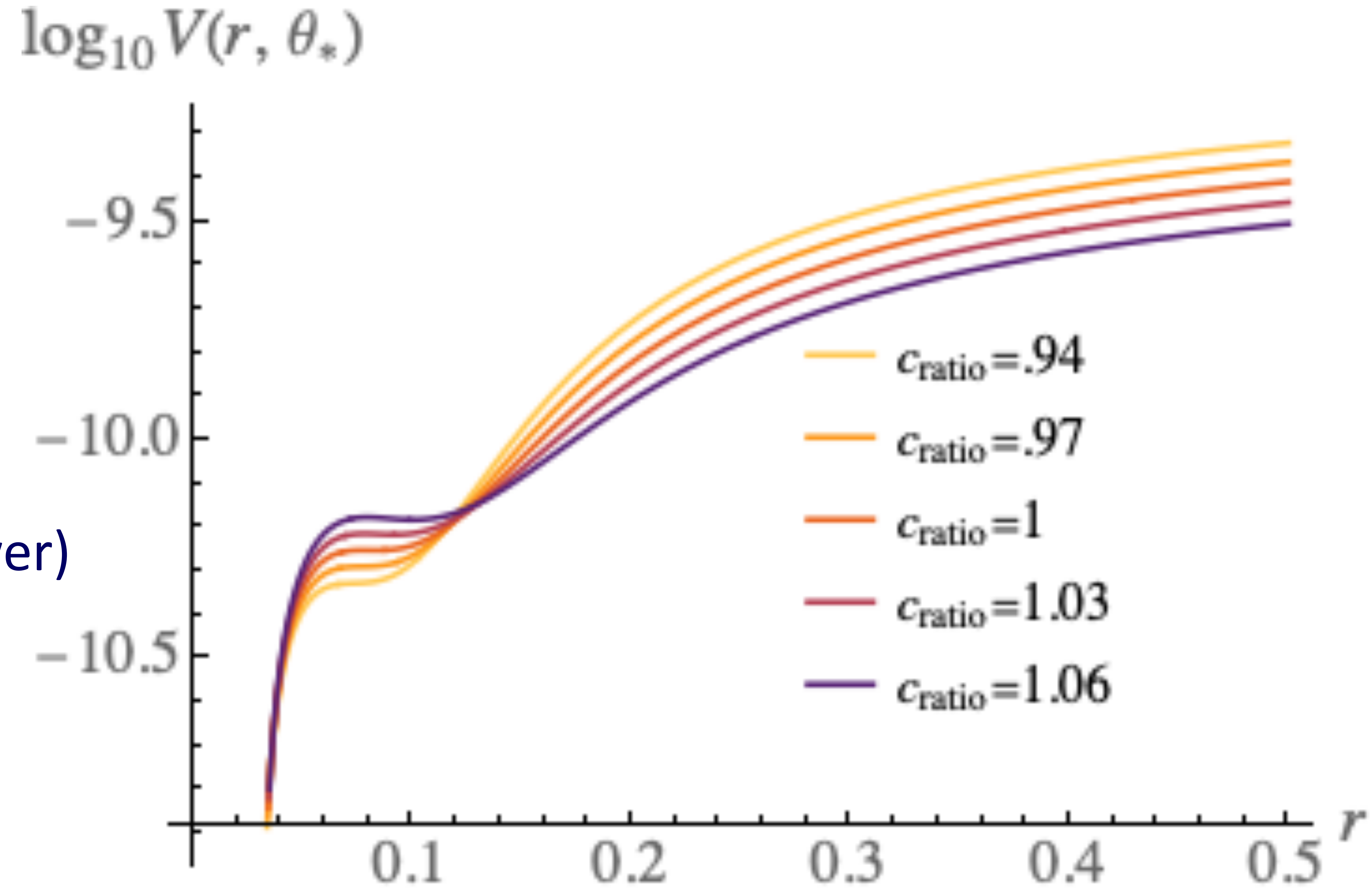


Reparametrize $V(r,\theta)$ by

$$\frac{\xi_\phi}{\xi_\chi} = 1 + \xi_{\rm ratio}, \frac{b_1}{b_2} = 1 + b_{\rm ratio}, \frac{c_2}{c_3} = 1 + c_{\rm ratio}$$

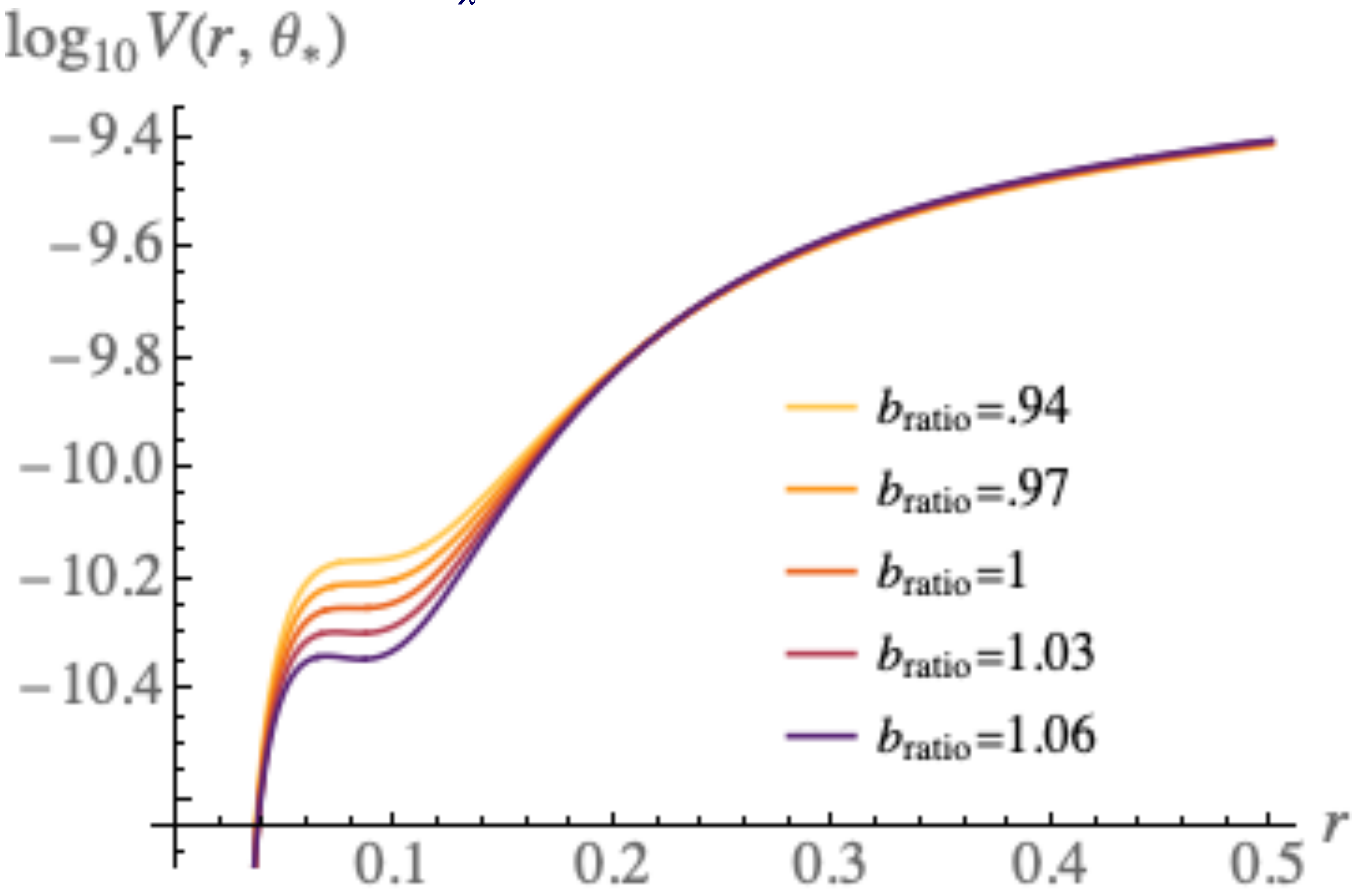


as $\xi_{\rm ratio}, c_{\rm ratio}$ change, local min feature deeper (shallower) relative to plateau $\Longrightarrow$ more (less) KE going from large to small r

$c_{\rm ratio}$ also shift feature along r

Examples with Broken "Extra" Constraints $\xi_\phi \neq \xi_\chi, b_1 \neq b_2$ and/or $c_2 \neq c_3$

Reparametrize $V(r, \theta)$ by $\frac{\xi_\phi}{\xi_\chi} = 1 + \xi_{\text{ratio}}, \frac{b_1}{b_2} = 1 + b_{\text{ratio}}, \frac{c_2}{c_3} = 1 + c_{\text{ratio}}$



Examples with Broken "Extra" Constraints $\xi_\phi \neq \xi_\chi, b_1 \neq b_2$ and/or $c_2 \neq c_3$

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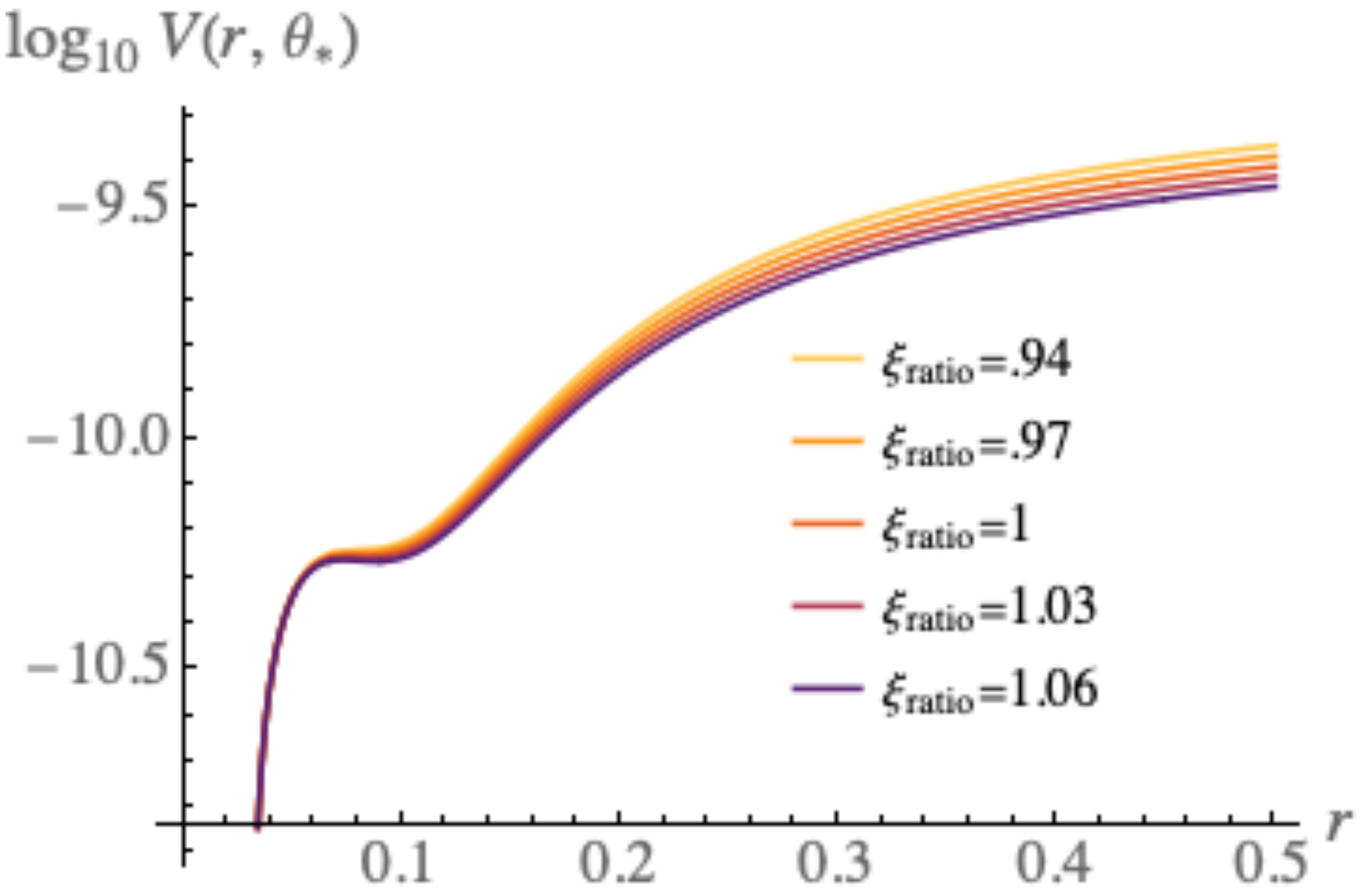
- Caveat: Varying away from constraints e.g. $\xi_\phi = \xi_\chi$ in one direction infinitesimally will reduce KE enough to get field stuck in local min.

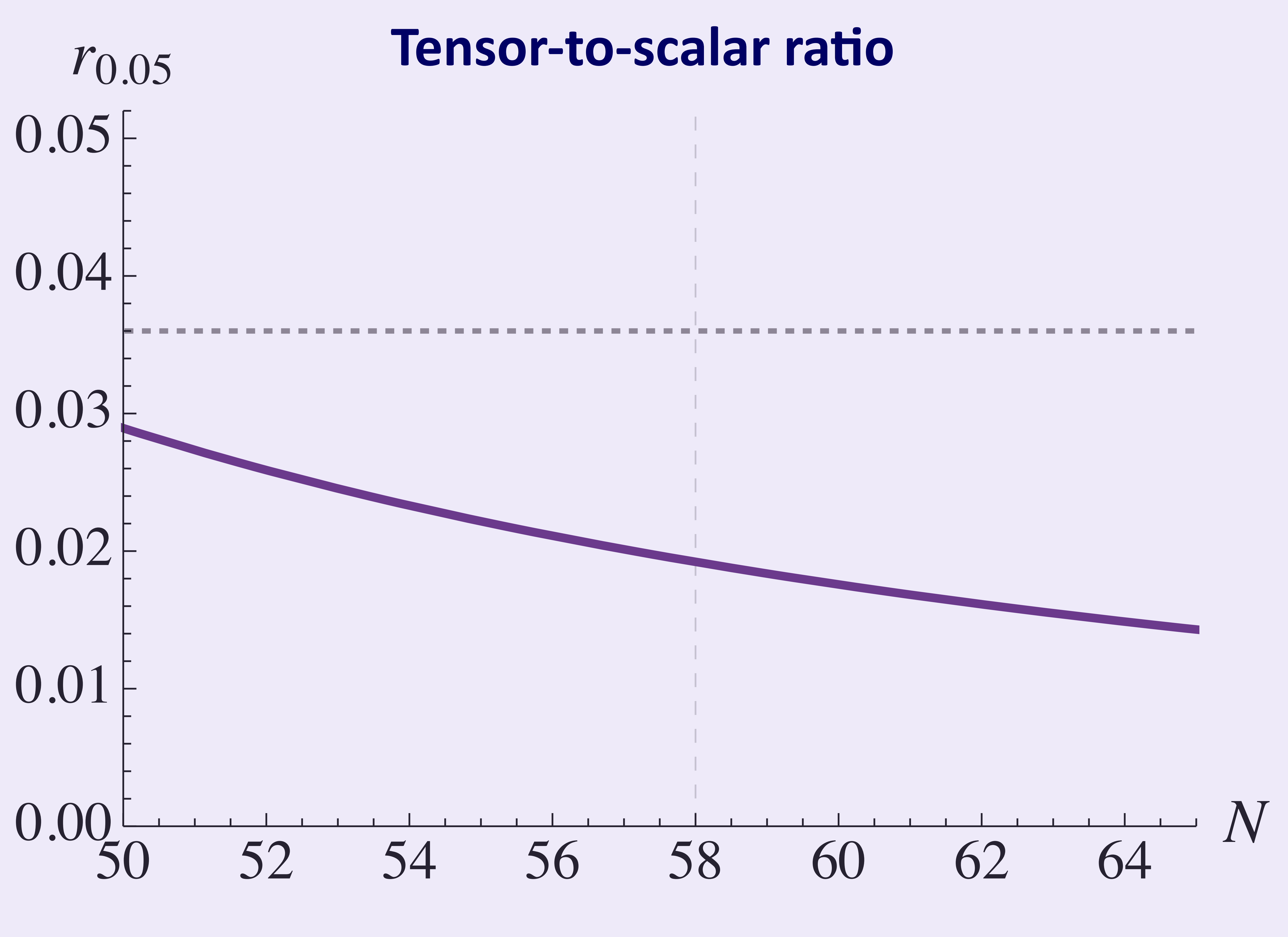
- Leads to 1st order PT when quantum
- effects take over during USR.

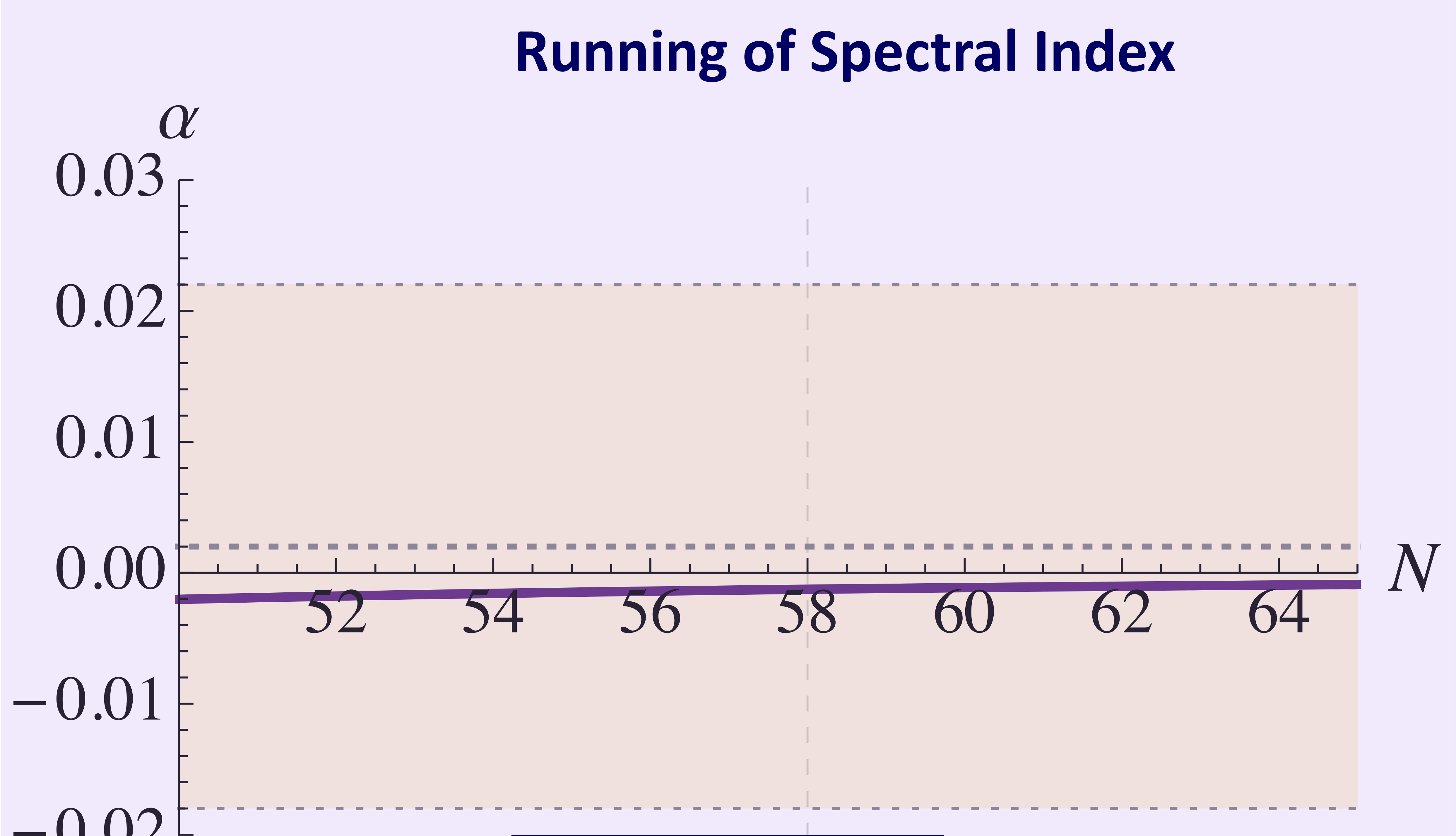
- Plotting parametrically, t
- These trajectories don't proceed past USR.



Running of the Spectral Index and Tensor-to-Scalar Ratio

$\mu = M_{\rm pl}, b_1 = b_2 = -1.8 \times 10^{-4}, c_1 = 2.5 \times 10^{-4}, c_4 = 3.9 \times 10^{-3}, \xi_\phi = \xi_\chi = 100,$ $c_2 = c_3 = 3.570193 \times 10^{-3}$





$$\alpha(k_*) = \left.\frac{dn_s(k)}{d\ln k}\right|_{k_*}$$

Non-Gaussianities: constraints and our model

Equation of motion for the Adiabatic Modes:

$$\ddot{Q}_\sigma + 3H\dot{Q}_\sigma + \left[\frac{k^2}{a^2} + \mathcal{M}_{\sigma\sigma} - \omega^2 - \frac{1}{M_{\rm pl}^2 a^3}\frac{d}{dt}\left(\frac{a^3\dot{\sigma}^2}{H}\right)\right]Q_\sigma = 2\frac{d}{dt}(\omega Q_s) - 2\left(\frac{V_{,\sigma}}{\dot{\sigma}} + \frac{\dot{H}}{H}\right)(\omega Q_s)$$

Modes couple only when $\omega \neq 0$! Scalar turn rate acts as a source term

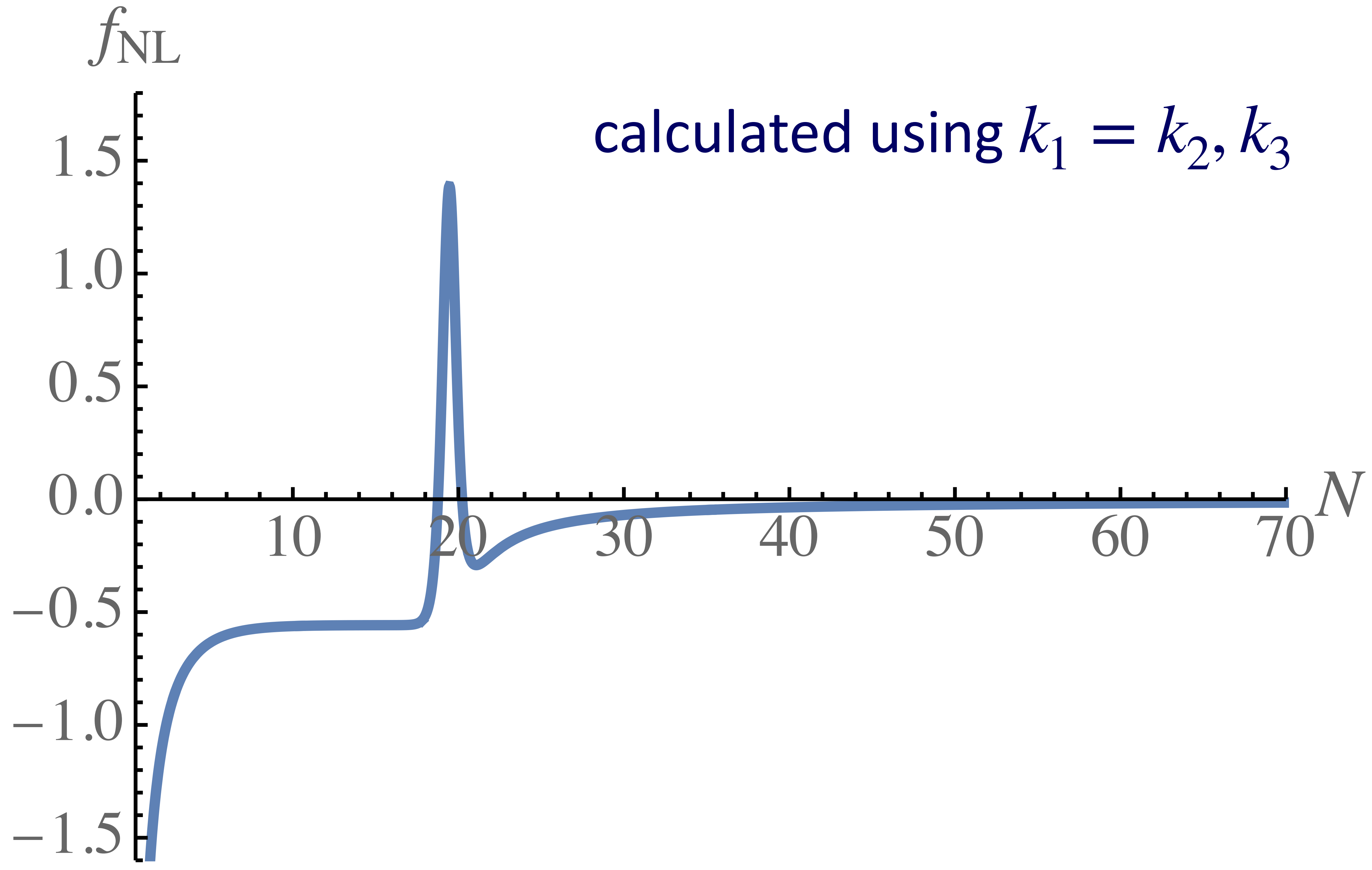
Equation of motion for the Isocurvature Modes:

$$\ddot{Q}_s + 3H\dot{Q}_s + \left[\frac{k^2}{a^2} + \mu_s^2\right]Q_s = 4M_{\rm pl}^2\frac{\omega}{\dot{\sigma}}\frac{k^2}{a^2}(\psi + a^2H(\dot{E} - Ba^{-1}))$$

$f_{\rm NL}$ is defined in terms of power spectrum and bispectrum:

$$f_{\rm NL}(k_1,k_2,k_3) = \frac{5}{6}\frac{\mathcal{B}_\zeta(k_1,k_2,k_3)}{\mathcal{P}_\zeta(k_1)\mathcal{P}_\zeta(k_2) + \mathcal{P}_\zeta(k_2)\mathcal{P}_\zeta(k_3) + \mathcal{P}_\zeta(k_2)\mathcal{P}_\zeta(k_3)}$$

where $\zeta = -\psi - \frac{H}{\dot{\rho}}\delta\rho$

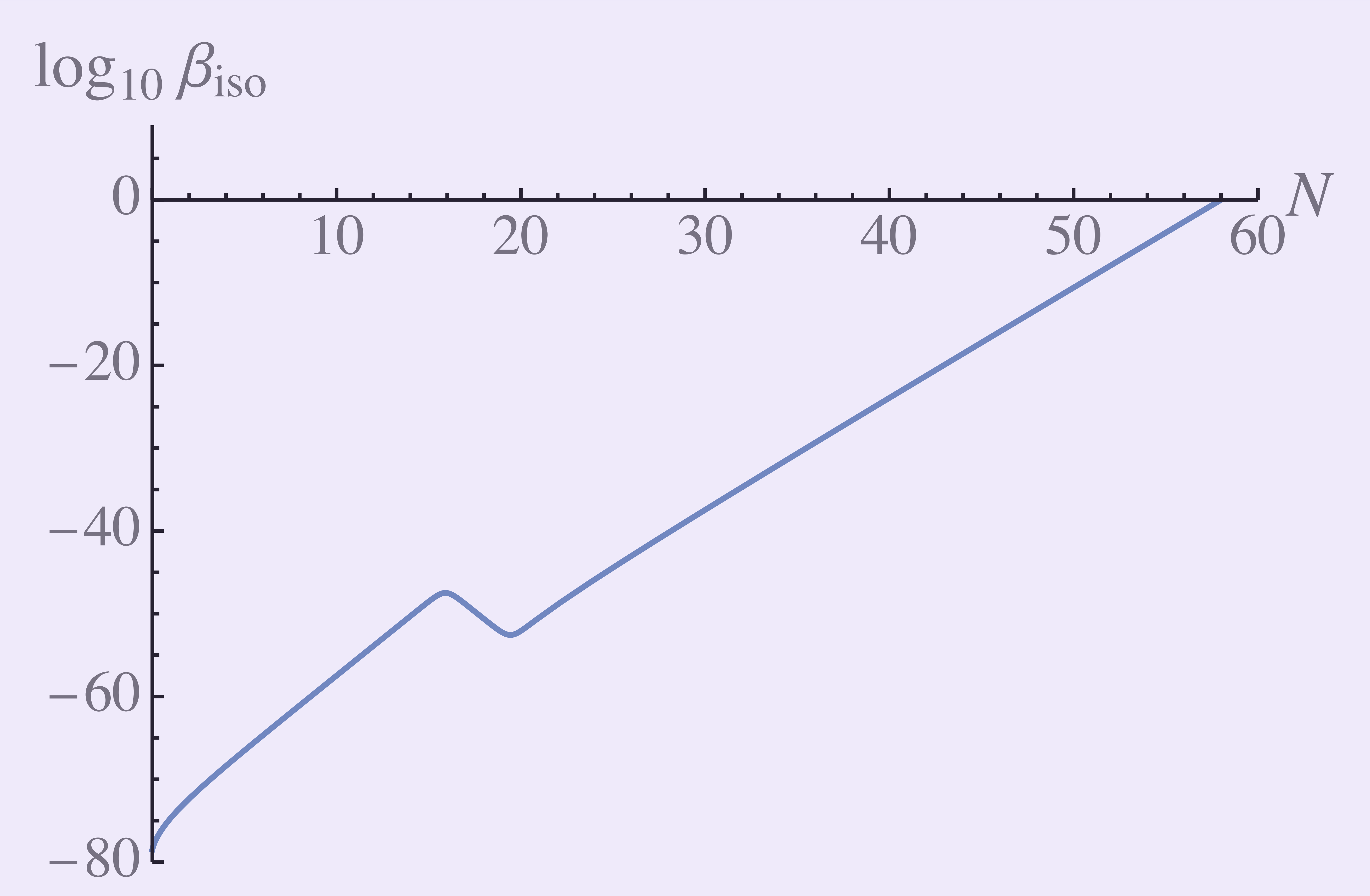


$\mu = M_{\rm pl}, b_1 = b_2 = -1.8\times10^{-4}, c_1 = 2.5\times10^{-4}, c_4 = 3.9\times10^{-3}, \xi_\phi = \xi_\chi = 100,$ $c_2 = c_3 = 3.570193\times10^{-3}$

Non-Gaussianities: β_{iso}

In our model, β_{iso} consistently remains small:

$$\beta_{\text{iso}} \lesssim 10^{-8}$$



$\mu = M_{\text{pl}}, b_1 = b_2 = -1.8 \times 10^{-4}, c_1 = 2.5 \times 10^{-4}, c_4 = 3.9 \times 10^{-3}, \xi_\phi = \xi_\chi = 100,$ $c_2 = c_3 = 3.570193 \times 10^{-3}$

Reheating in Multifield Models with Non-minimal couplings

Reheating has been studied in such models using lattice simulations

Our model $N_{\text{reh}} \sim \mathcal{O}(1)$ e-folds.
Between t_{end} and t_{rd}, energy red-shifts as

$$\rho(t_{\text{rd}}) = \rho(t_{\text{end}}) e^{-3N_{\text{reh}}}$$

$$\Delta N = \frac{1}{2} \log \left[\frac{2H^2(t_{\text{pbh}})}{H(t_{\text{end}})} e^{-N_{\text{reh}}/4} t_c \right]$$

Radiation domination ($w \simeq 1/3$) within 1-3 e-folds $\implies$ $18 \lesssim \Delta N \lesssim 25$

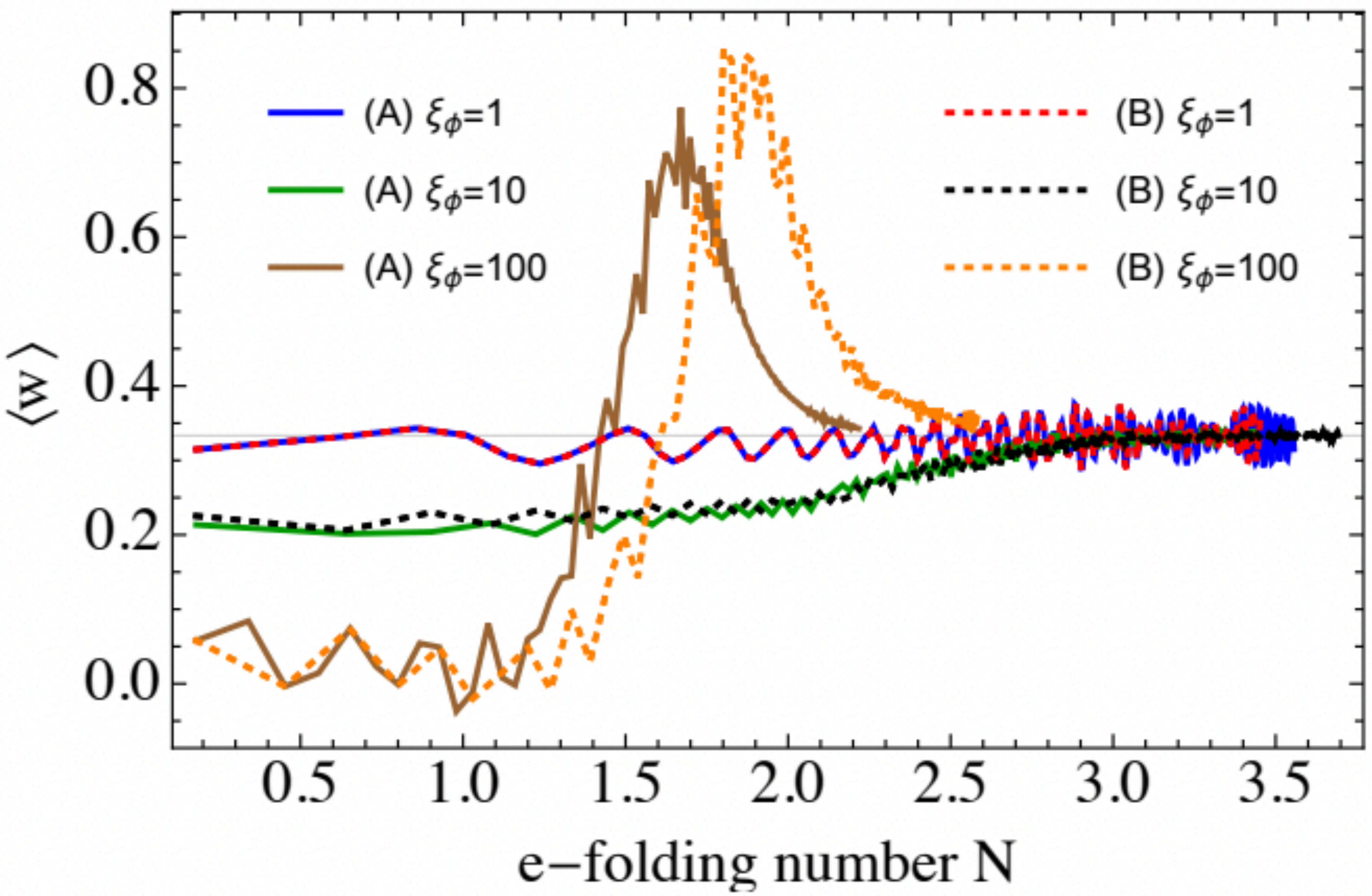


FIG. 5. The averaged effective equation of state $\langle w \rangle$ for $\xi_\phi = 1, 10, 100$ and the two representative cases, "generic" (A) and symmetric (B).

1905.12562v2 Nguyen, van de Vis, Sfakianakis, Giblin, Kaiser 2019

Quantum Diffusion During Ultra-Slow Roll Phase

Main idea:
1. During Ultra Slow-roll, quantum fluctuations must not make field zoom past the min/max feature ($V_{,\sigma} \simeq 0$) too quickly or $\mathscr{P}_R$ will not get large enough for PBH formation.
2. Also can't have insufficient kinetic energy for the field to classically pass through the local minimum or quantum diffusion effects become dominant

The condition that must be satisfied for us to ignore quantum diffusion effects during slow roll is:

$$\mathscr{P}_R(k) < 1/6$$

Approach: Back-reaction from quantum fluctuations → variance in kinetic energy density:

$$\langle (\Delta K)^2 \rangle \simeq \frac{3H^4}{4\pi^2}\rho_{\text{kin}} \qquad (\rho_{\text{kin}} = \dot{\sigma}^2/2)$$

Classical evolution >> Quantum diffusion during ultra slow-roll **IF** $\rho_{\text{kin}} > \sqrt{\langle (\Delta K)^2 \rangle}$. Equivalent to

Idea: Use $\Delta E \Delta t \leq \hbar/2$ as bound to determine when system will tunnel. Tunnel to right →restart inflation, tunnel left → first order phase transition ends inflation.

More on the non-minimal couplings…

1. Why isn't $\xi = -1/6$?

$-1/6$ is a fixed point of the β-function, but any nonzero value will work for renormalization. If we start with $\xi \neq -1/6$ then the RG $\Longrightarrow \xi$ will run to higher values in the UV. If at tree level, $\xi = -1/6$, it will stay there for any energy scale.

2. How does renormalization work in this context?

Renormalization of a QFT is possible in a **fixed** curved background, not in dynamical curved background.

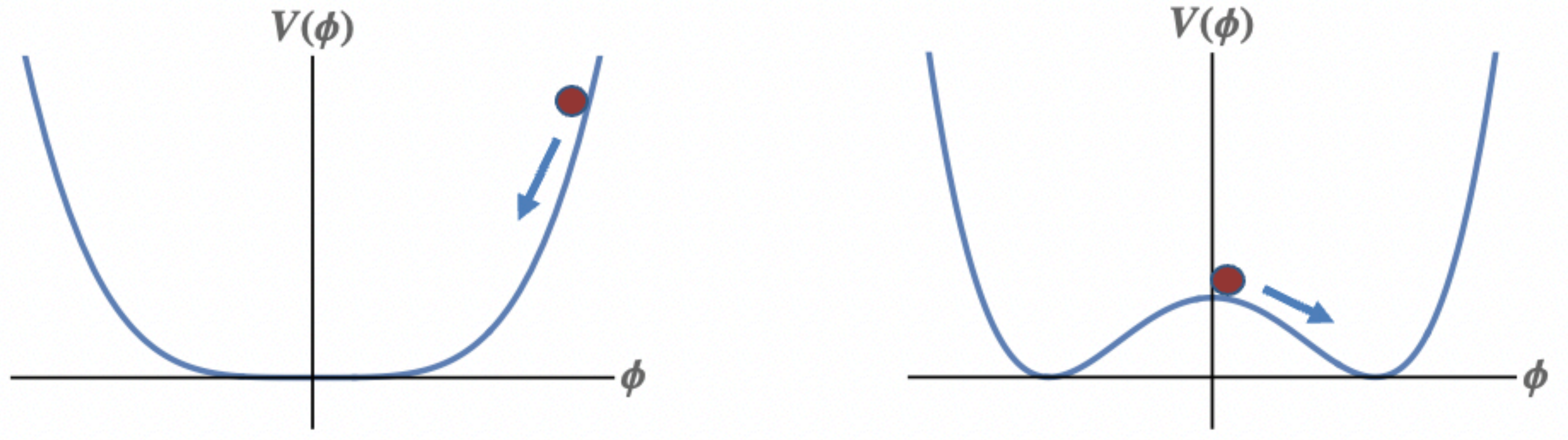
IF we set aside renormalization of the gravitational sector, and consider an EFT for self interacting scalar fields in 3+1 dimensions, then we must include the $f(\phi)\tilde{R} \in \mathscr{L}$ and ξ can be any dimensionless free parameter

$$\mathscr{L} \ni f(\phi)\tilde{R} \sim \left(M^2 + \sum_I \xi_I (\phi^I)^2\right)\tilde{R}$$

Does Inflation Itself Require Fine-Tuning of the Initial Conditions?

eg. a smooth patch of size $r > r_{\mathrm{H}} \sim \frac{1}{H}$? Numerical simulations have been done but are limited by difficulty of putting these simulations onto computers.
Most are 1+1 dimensional.

Some 3+1 dimensional Numerical Relativity Sims have been done recently e.g. Clough, Lim, Flauger 1712.07352



Large-field inflation is robust even amid large initial inhomogeneities; small-field inflation requires more special initial conditions, but is still more robust than analytic estimates had suggested.

Source: David Kaiser Jan. 2021

For recent review of Inflation see:
Inflation after Planck: Judgement Day Chowdhury, Martin, Ringeval, Vennin

Work by Kaiser, Fitzpatrick, Bloomfield, Hilbert
(arXiv:1906.08651) simulated