# Cosmological bootstrap in slow motion and the low speed collider


Sébastien Renaux-Petel

*CNRS - Institut d'Astrophysique de Paris*

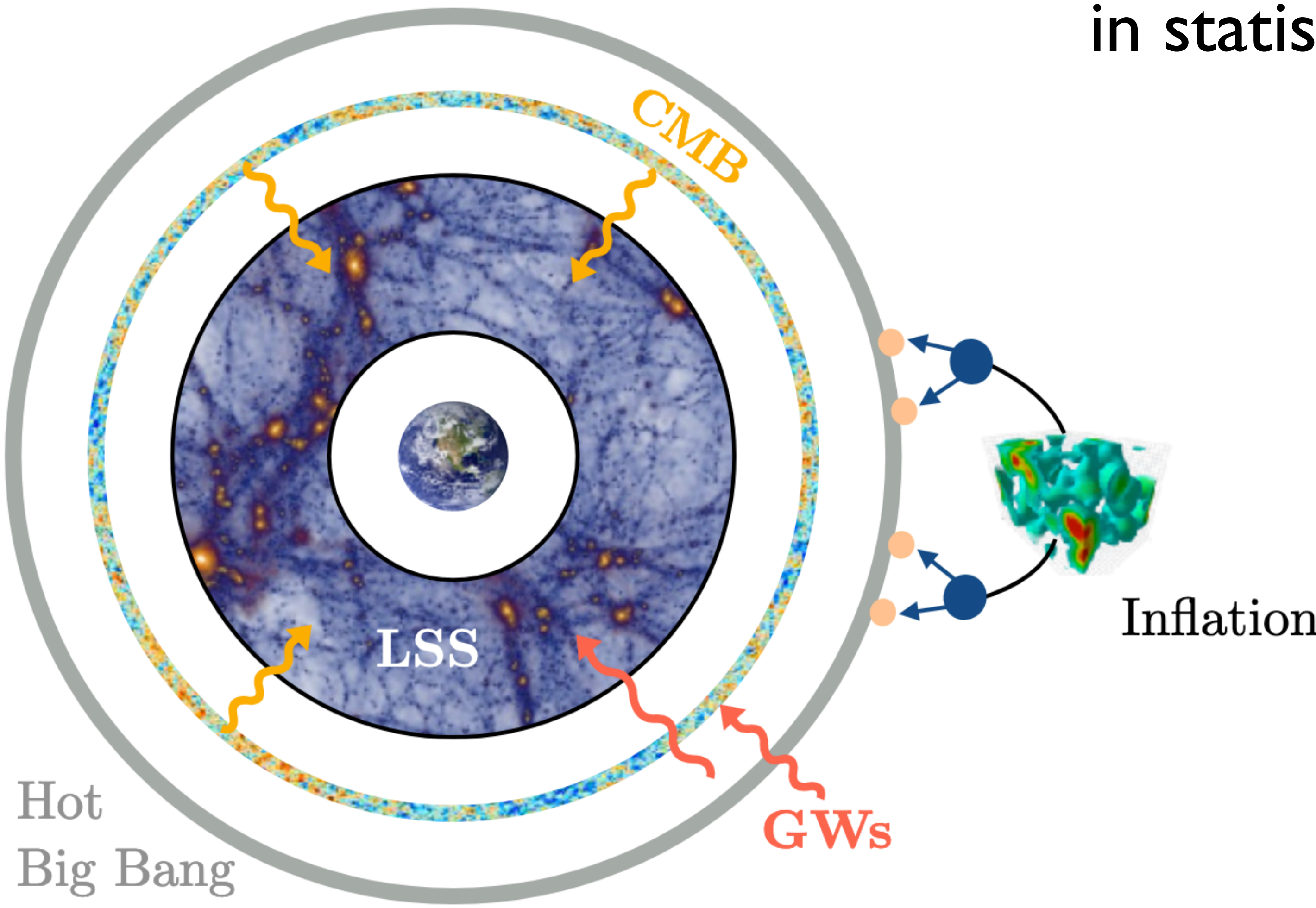

Cosmo 2022
Rio de Janeiro, August 22nd 2022

based on arXiv: 2205.10340 [hep-th] with Sadra Jazayeri

European Research Council
Established by the European Commission

GEODESI

# *Probing inflation*




Physics at extremely high-energies encoded in statistical properties of primordial fluctuations

density fluctuations & gravitational waves

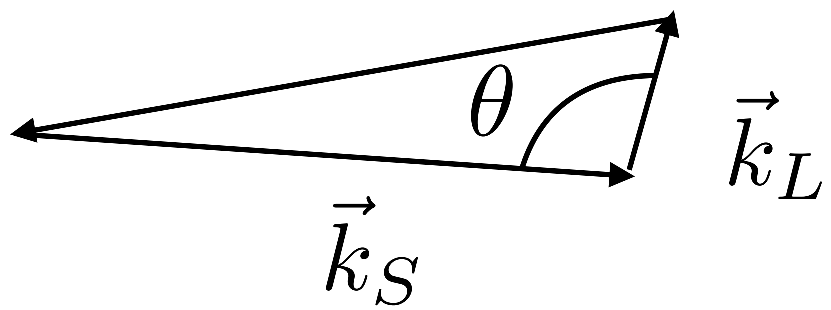
2pt, 3pt, n-pt
Even full pdf

see G. Palma's talk

Treasure of information to extract

# *Inflation as a cosmological collider*

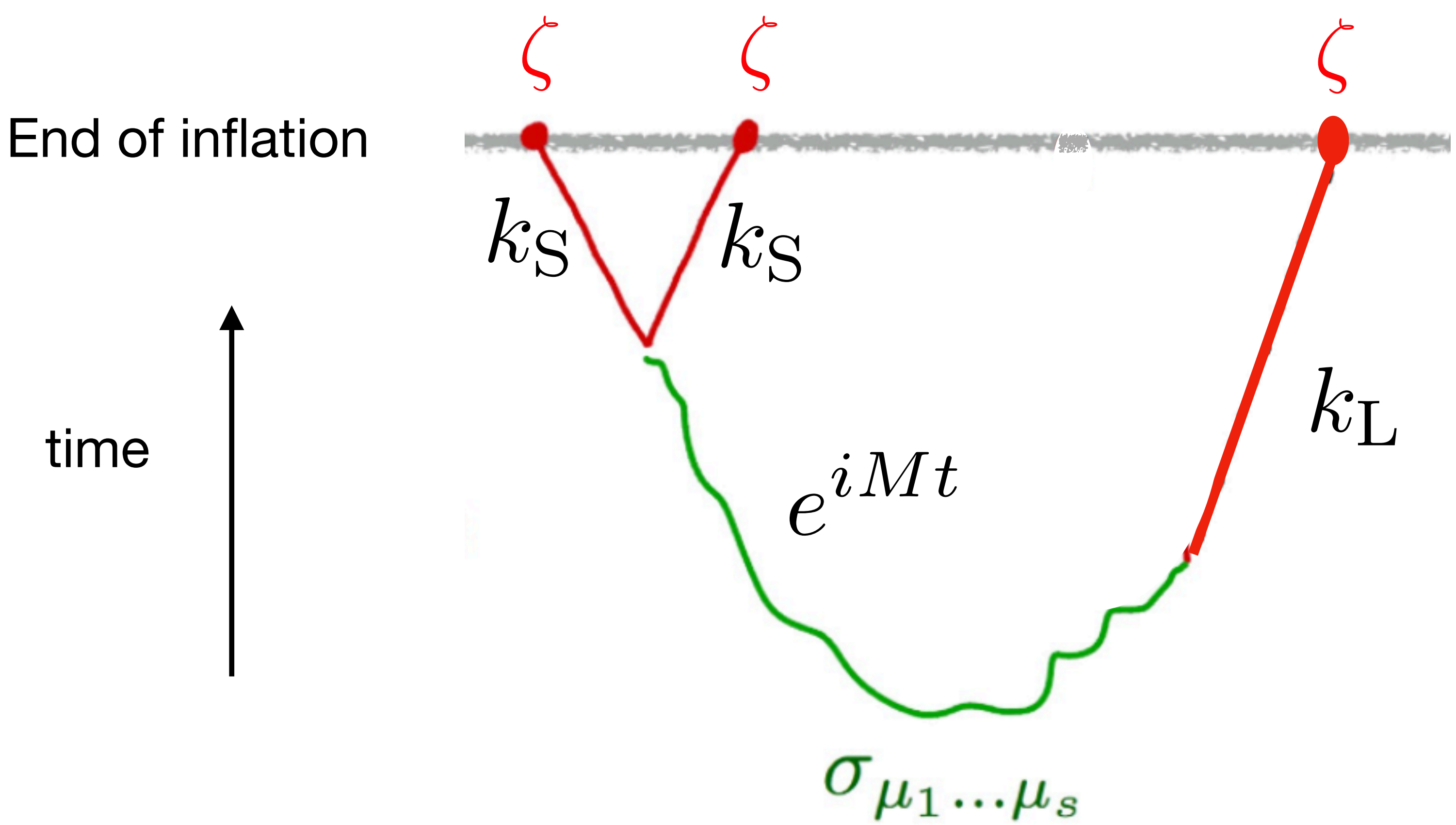
**3pt**




$$\lim_{k_L\to 0}\langle\zeta_{\vec{k}_L}\zeta_{\vec{k}_1}\zeta_{\vec{k}_2}\rangle \propto \left(\frac{k_L}{k_S}\right)^{3/2}\cos\left[\frac{M}{H}\ln\left(\frac{k_L}{k_S}\right)+\delta\right]P_S(\cos\theta)$$

Mass & Spin

of heavy exchanged particle



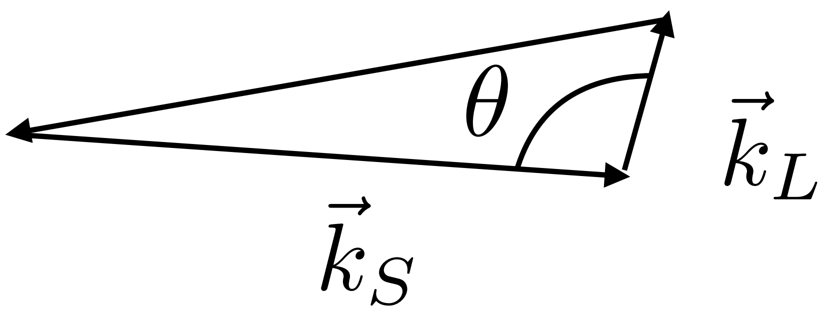

Chen, Wang 2009
Baumann Green 2011
Noumi, Yamaguchi, Yokohama 2012
Arkani-Hamed, Maldacena 2015
Lee, Bauman, Pimentel 2016
Arkani-Hamed, Baumann, Lee, Pimentel 2018
+ many works

Which mass actually? Inflationary flavor oscillations and cosmic spectroscopy

Pinol, Aoki, Renaux-Petel, Yamaguchi, 2112.05710

# *Inflation as a cosmological collider*

## 3pt

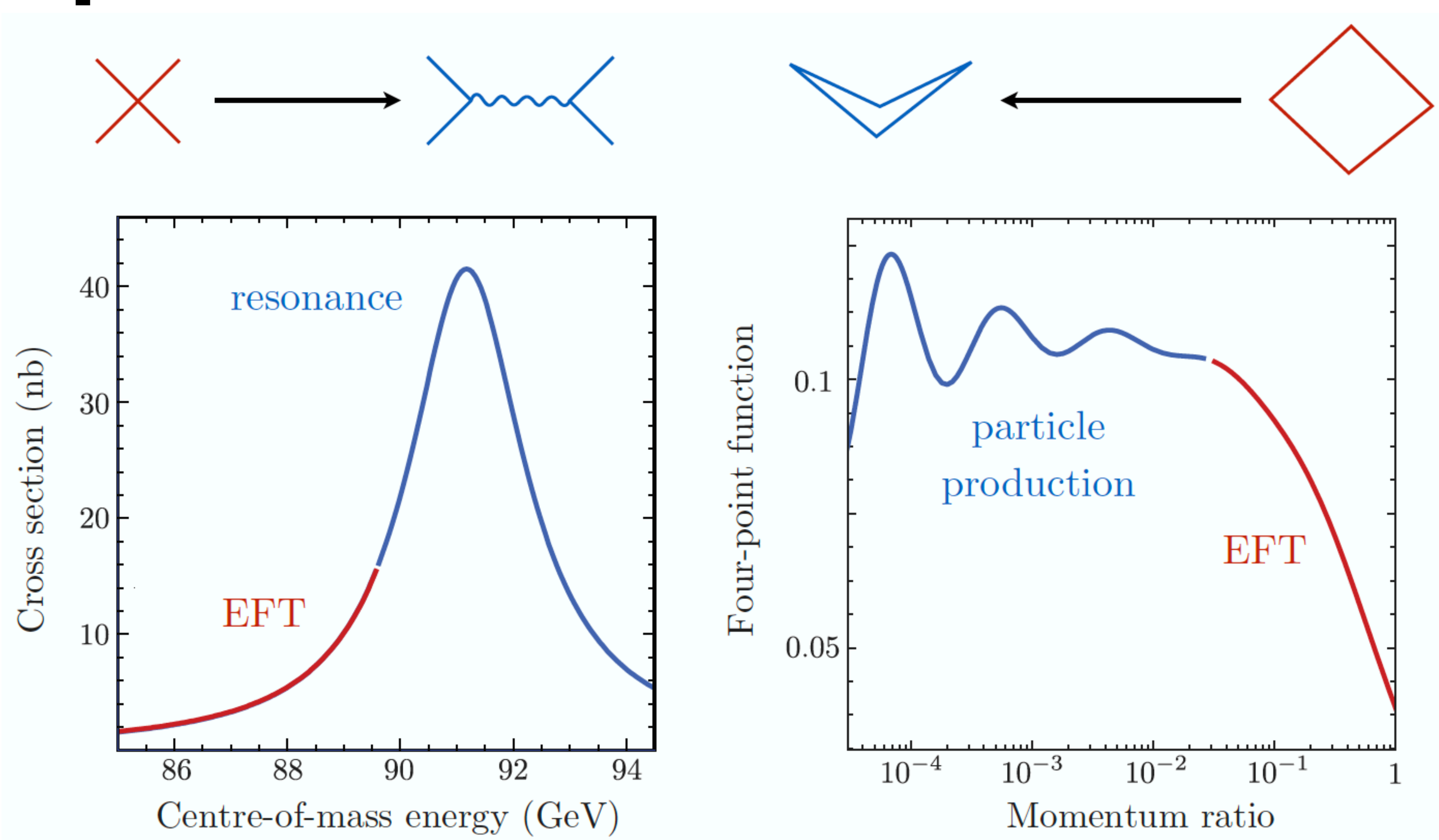



$$\lim_{k_L\to 0}\langle \zeta_{\vec k_L}\zeta_{\vec k_1}\zeta_{\vec k_2}\rangle \propto \left(\frac{k_L}{k_S}\right)^{3/2}\cos\left[\frac{M}{H}\ln\left(\frac{k_L}{k_S}\right)+\delta\right]P_S(\cos\theta)$$

Mass & Spin

of heavy exchanged particle

## 4pt

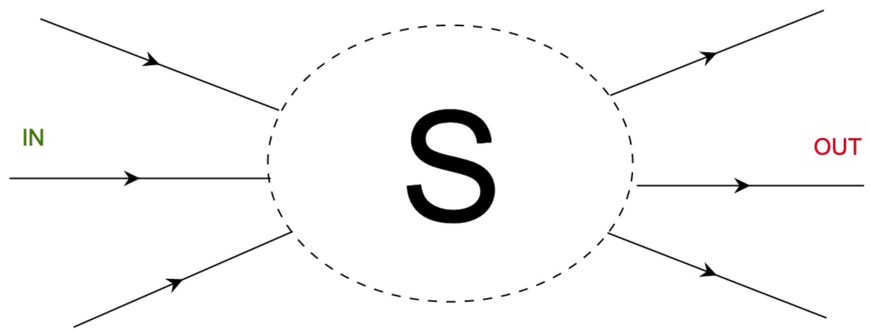



From 1811.00024

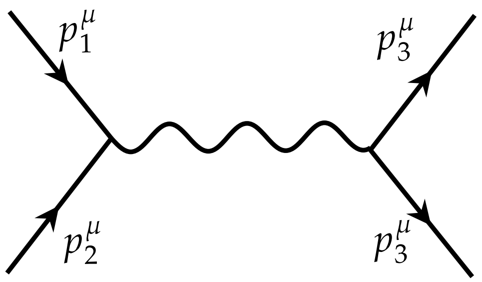
Chen, Wang 2009
Baumann Green 2011
Noumi, Yamaguchi, Yokohama 2012
Arkani-Hamed, Maldacena 2015
Lee, Bauman, Pimentel 2016
Arkani-Hamed, Baumann, Lee, Pimentel 2018
+ many works

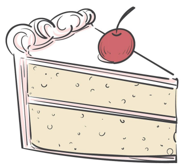
# ***Time-dependent perturbation theory is hard!***

## Flat space



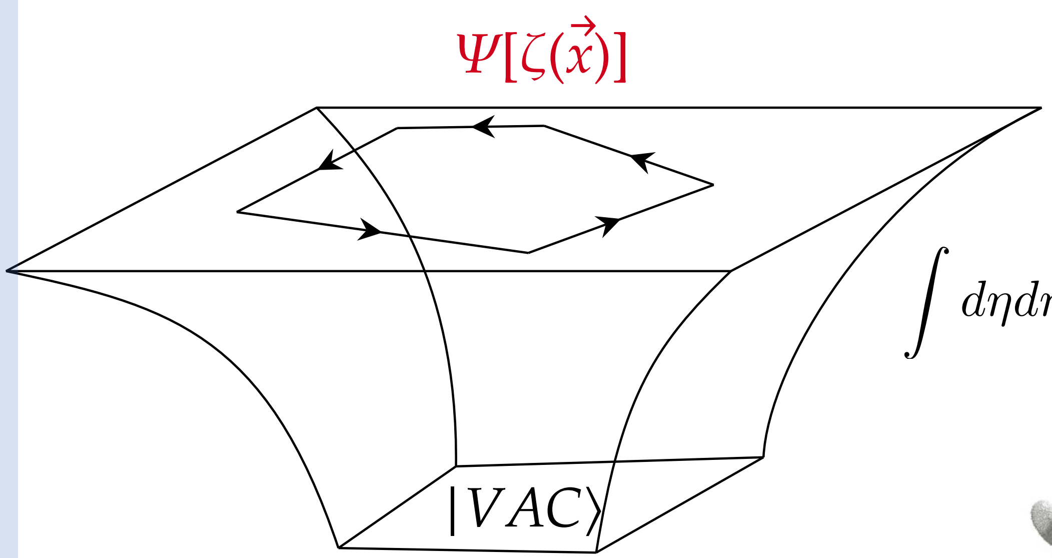

$$\int dtdt' \, e^{iE_1 t} e^{iE_2 t'} \subset$$



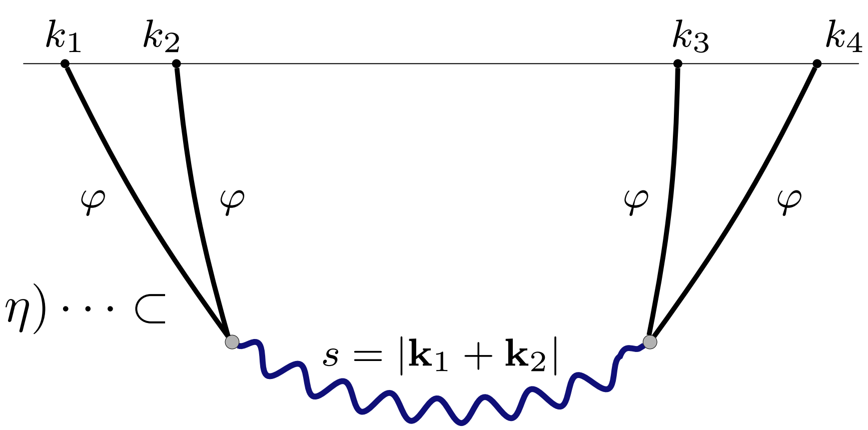
$$\mathcal{A}_{1,2\to 3,4} = \frac{1}{(p_1+p_2)^2 - m^2} + t - u \text{ channels}$$

## Cosmology




$$\Psi[\zeta(\mathbf{x}), t = -\infty] = \text{Gaussian}$$

$$\int d\eta d\eta' a^p(\eta) \times e^{i|\mathbf{k}|\eta} \times H_{i\mu}(-|\mathbf{k}|\eta) \cdots \subset$$




Not until **2018** by Arkani-Hamed et al

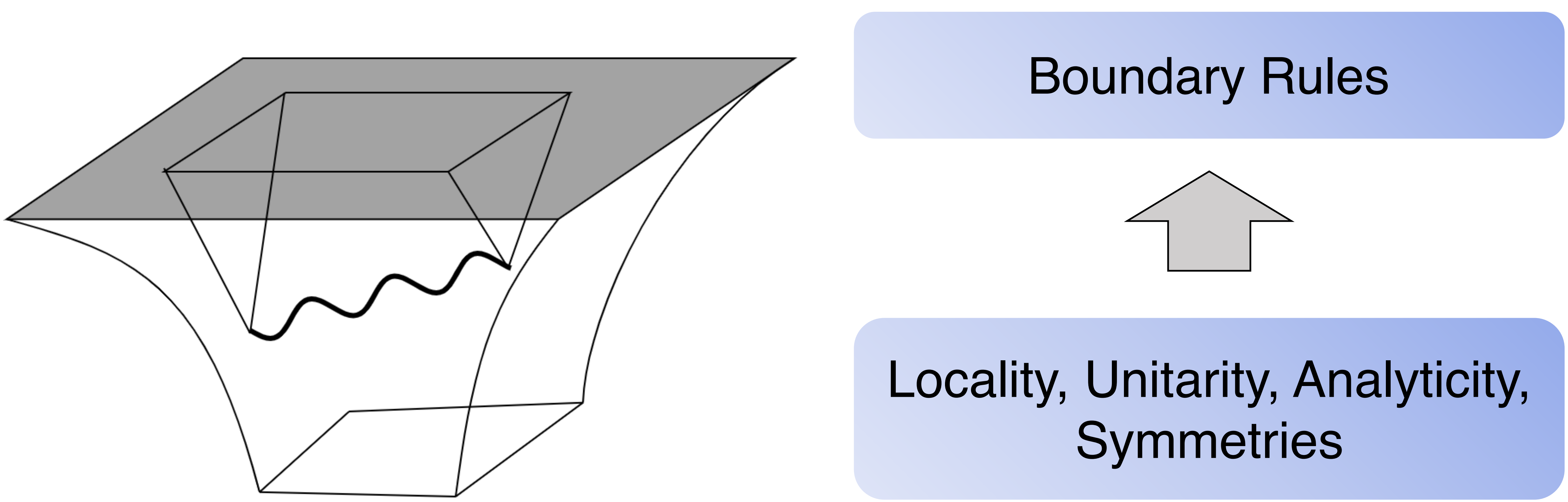
# *Cosmological bootstrap*

Shifting the perspective on cosmological correlators: finding them without directly following the bulk time evolution. Active field.

Recent review, Baumann et al, 2203.08121

2017-2022: Arkani-Hamed, Baumann, Benincasa, Duaso Pueyo, Goodhew, Gorbenko, Jazayeri, Joyce, Lee, Meltzer, Melville, Pajer, Penedones, Pimentel, Renaux-Petel, Sleight, Salehi-Vaziri, Stefanyszyn, Tarona ….

Earlier works: Bzowski et al (2011,2012, 2013), Raju (2012), Kundo et al (2013, 2015), Maldacena and Pimentel(2011)



# ***Our work***

Cosmological collider + cosmological bootstrap + breaking dS boosts

Production of heavy particles

Exact solution from first principles

Different propagation speeds

Observational consequences and physical understanding

Theoretical methods

**I Low speed collider**

**II Cosmological bootstrap in slow motion**

# *The setup*

- Curvature perturbation $\zeta = -H\pi$ with a reduced sound speed $c_s$

$$S_\pi = \int d\eta\, d^3\boldsymbol{x}\, a^2 \epsilon H^2 M_{\rm Pl}^2 \left[ \frac{1}{c_s^2}\left(\pi'^2 - c_s^2(\partial_i\pi)^2\right) - \frac{1}{a}\left(\frac{1}{c_s^2}-1\right)\left(\pi'(\partial_i\pi)^2 + \frac{A}{c_s^2}\pi'^3\right) + \dots \right]$$

- Additional relativistic heavy field

$$S_\sigma^{(2)} = \int d\eta d^3\boldsymbol{x}\, a^2 \left(\frac{1}{2}\sigma'^2 - \frac{1}{2}(\partial_i\sigma)^2 - \frac{1}{2}m^2a^2\sigma^2\right) \qquad \mu = \sqrt{\frac{m^2}{H^2}-\frac{9}{4}} > 0$$

- Quadratic and cubic couplings

$$S_{\pi\sigma} = \int d\eta d^3\mathbf{x}\, a^2 \left(\rho a \pi_c'\sigma + \frac{1}{\Lambda_1}\pi_c'^2\sigma + \frac{c_s^2}{\Lambda_2}(\partial_i\pi_c)^2\sigma\right) \qquad \pi_c = \sqrt{2\epsilon} H M_{\rm Pl} c_s^{-1}\pi$$

# I Low speed collider

# Qualitative picture

Two characteristic times in dynamics

Event 1: sound horizon crossing for pi $k/a = H/c_s$

Event 2: mass crossing for the heavy field $k/a = m$

For $m > H/c_s$ event 2 is before 1, not qualitatively different from cs=1

For $m < H/c_s$ event 1 is before 2, unusual:

Between 1 and 2, curvature perturbation outside sound horizon quantum-mechanically interacts with sigma still in the Bunch-Davies vacuum

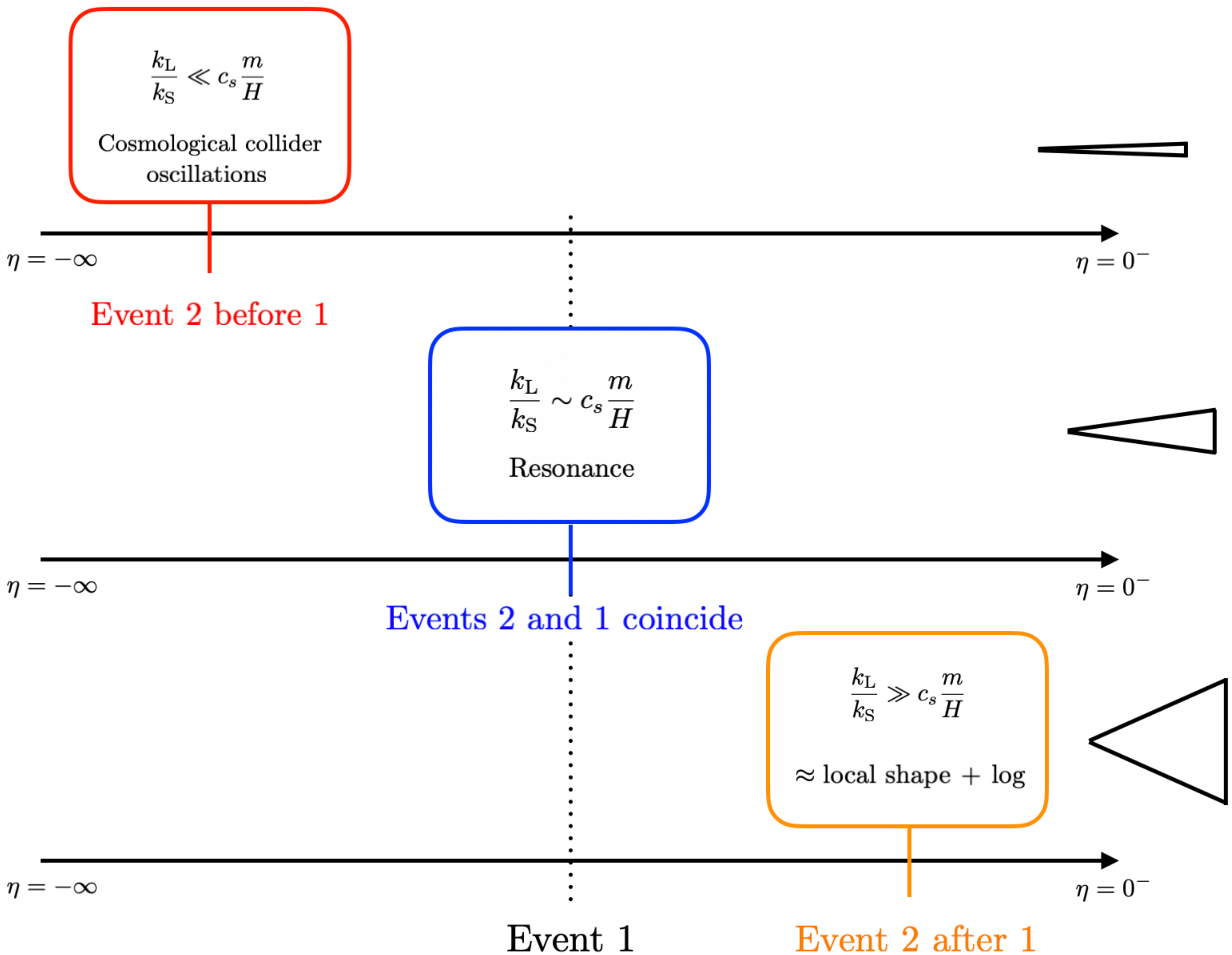
$\longrightarrow$

Growth of curvature power spectrum during $-\log(c_s m/H)$ e-folds

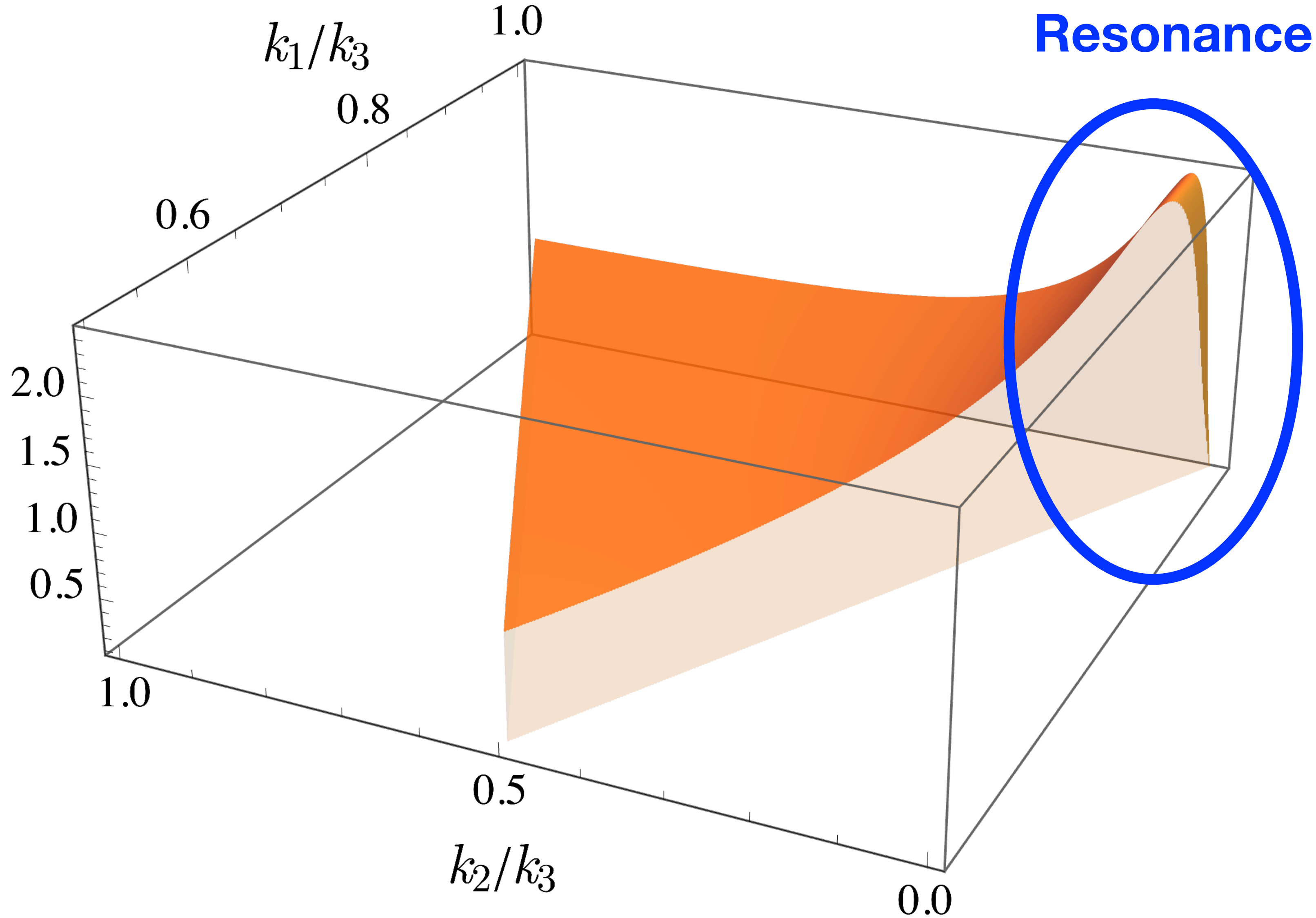
# *Qualitative picture (bispectrum)*

Event 1: sound horizon crossing for pi's short mode $k_{\rm S}/a(t_1) = H/c_s$

Event 2: mass crossing of long mode $k_{\rm L}/a(t_2) = m$



# New signature of heavy fields: the low speed collider




Bispectrum shape (normalised)

$$f_{\rm NL}^{\rm eq} \sim \left(\frac{\rho}{H}\right)^2 \lesssim \left(\frac{m}{H}\right)^2$$

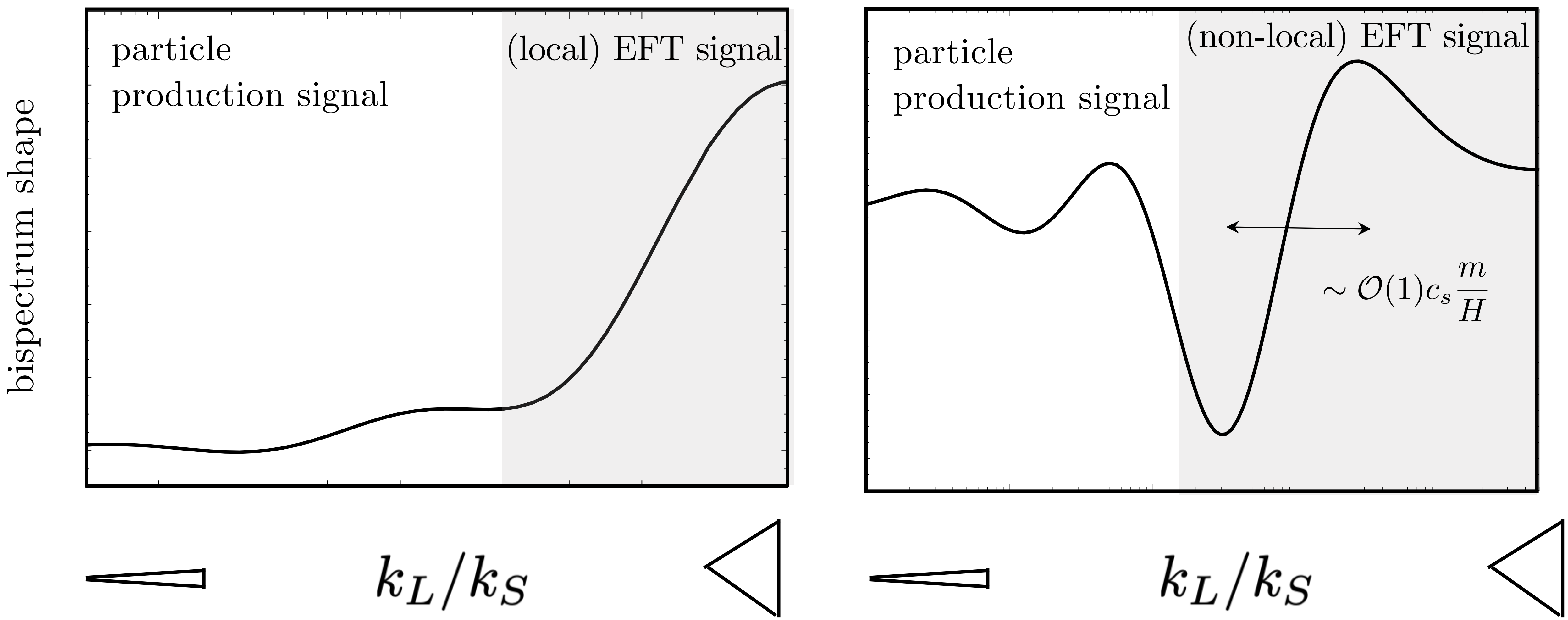
weak mixing

$$c_s = 0.03, \mu = 2$$

# *New signature of heavy fields: the low speed collider*

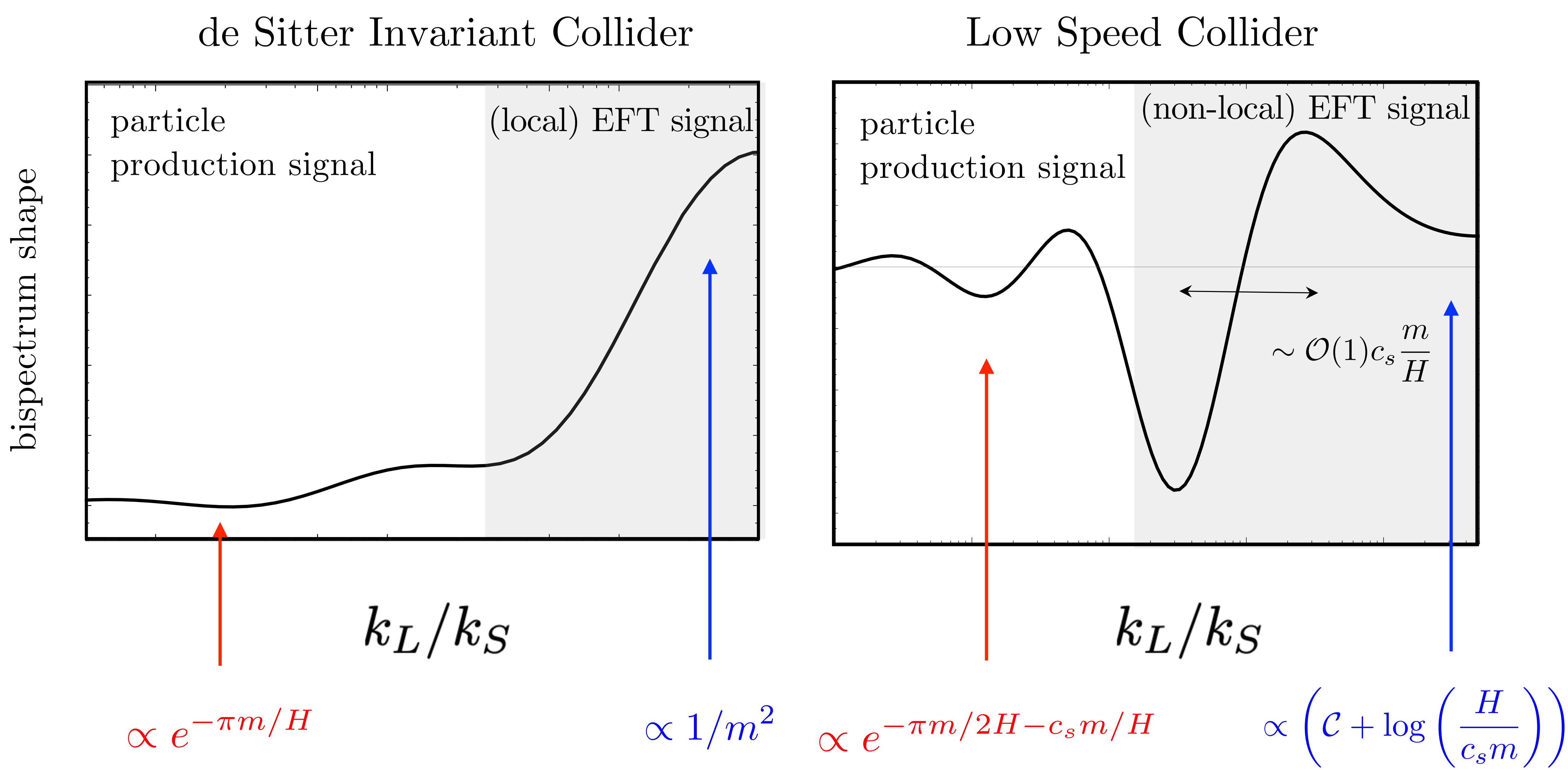
$c_s = 1,\, m \gg H$ | $c_s \ll 1,\, H \ll m \ll H/c_s$

de Sitter Invariant Collider | Low Speed Collider



# New signature of heavy fields: the low speed collider

$$c_s = 1,\ m \gg H \qquad\qquad c_s \ll 1,\ H \ll m \ll H/c_s$$



# Non-local single-field EFT

For a low sound speed, the heavy supersonic field instantaneously responds to the dynamics of the curvature perturbation

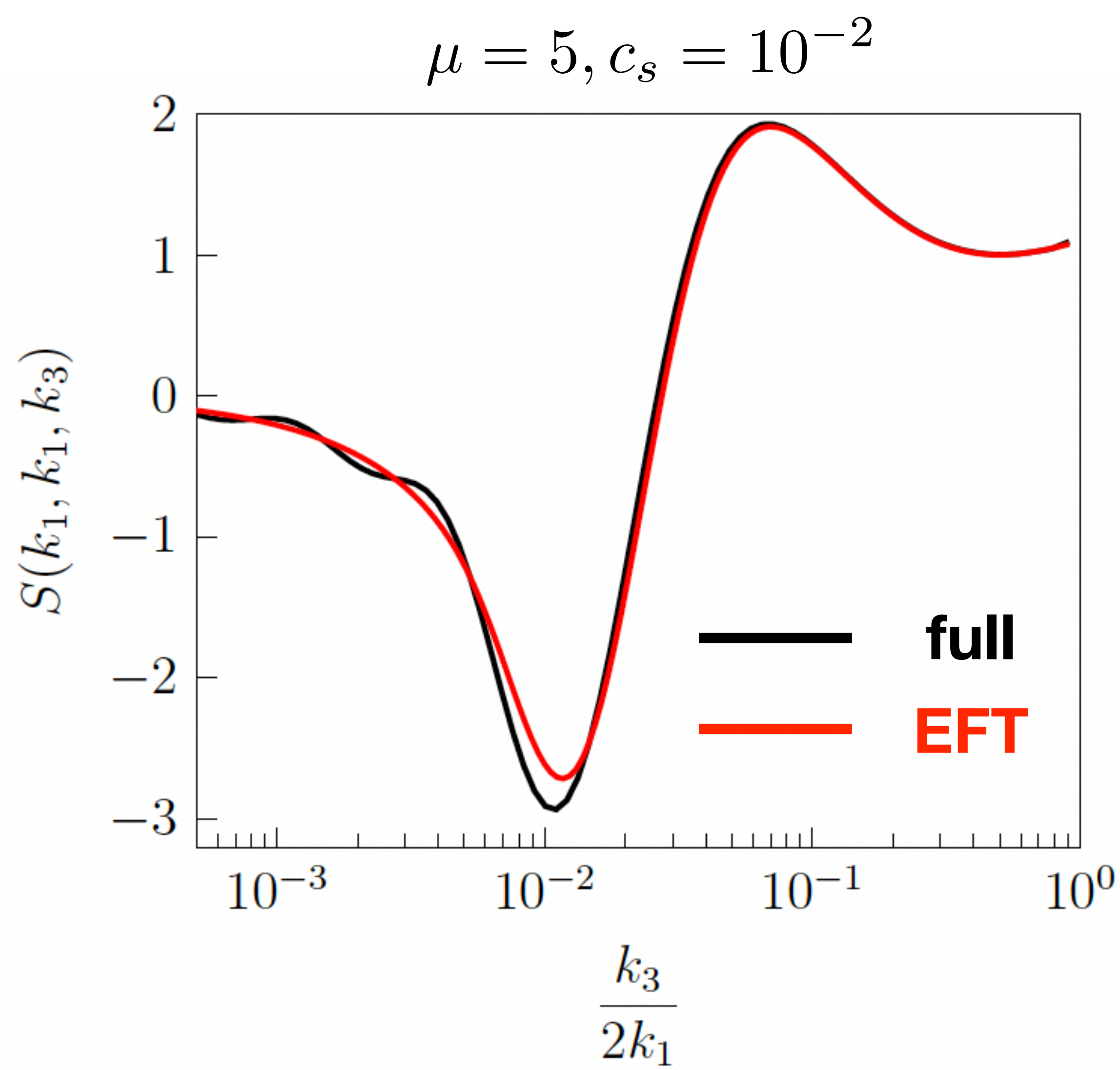
The heavy field can be integrated out, but in a non-standard manner (the field is relativistic at sound horizon crossing), yielding a (spatially) non-local single-field EFT

see also 1210.3020
Gwyn, Palma, Sakellariadou, Sypsas

$$S_{\pi,\text{induced}} = \int d\eta\, d^3\mathbf{x}\, a^2(\eta) \left( \frac{\rho^2}{2} \pi_c' \frac{1}{m^2 - 2H^2 - H^2\eta^2\nabla^2} \pi_c' + \frac{\rho}{a(\eta)\Lambda_1} \pi_c'^2 \frac{1}{m^2 - 2H^2 - H^2\eta^2\nabla^2} \pi_c' + \frac{\rho c_s^2}{a(\eta)\Lambda_2} (\partial_i \pi_c)^2 \frac{1}{m^2 - 2H^2 - H^2\eta^2\nabla^2} \pi_c' \right)$$

# ***Non-local single-field EFT***

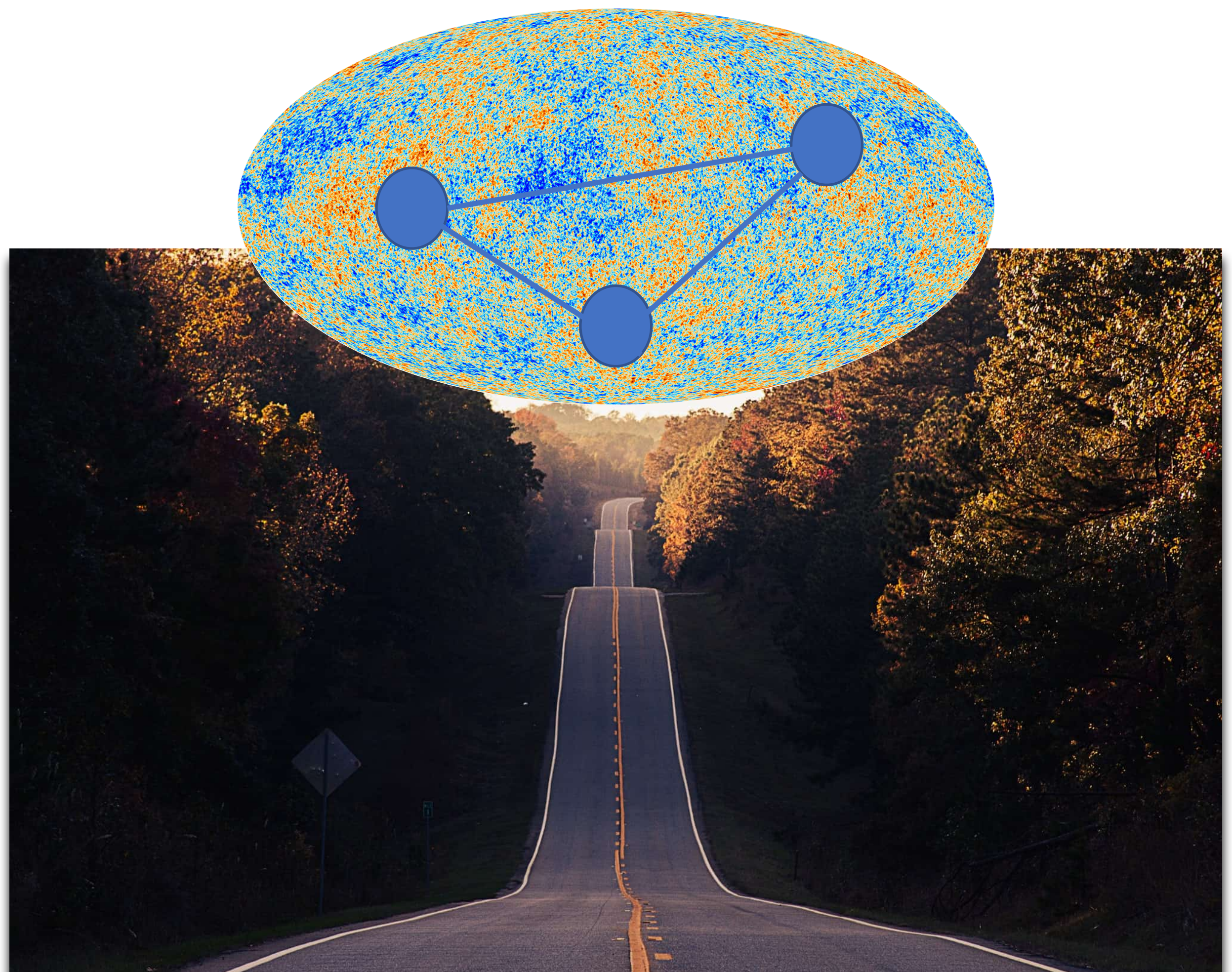
Simple analytical formulae: one-parameter family of shapes, depending on order parameter $\alpha = c_s(\mu^2 + 1/4)^{1/2} \approx c_s m/H$

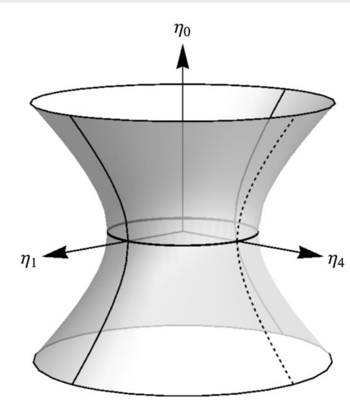



- ✔ Boil down to standard equilateral shapes for large alpha
- ✔ Reproduce resonances of low speed collider for small alpha
- ✘ Misses particle production effects
- ✘ Breaks down for masses too close to 3H/2

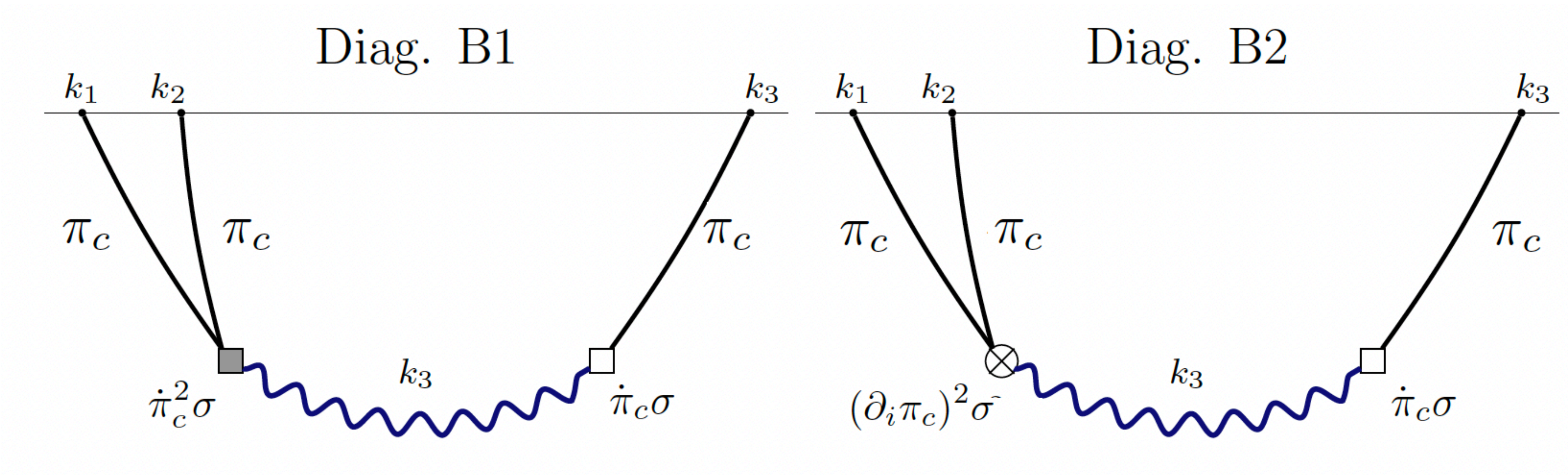
# II Cosmological bootstrap in slow motion

# From a de Sitter seed four-point to inflationary correlators





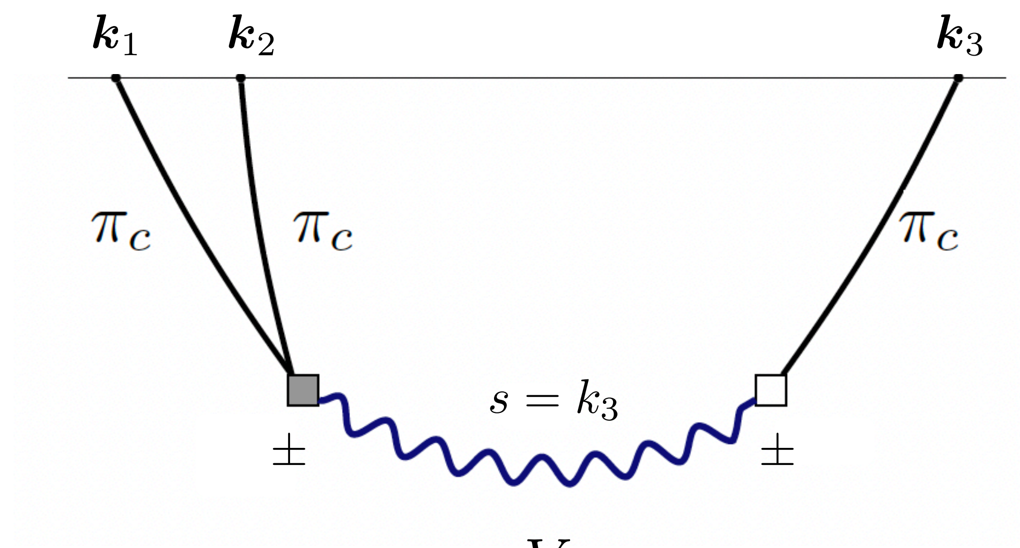
# *Correlation functions of interest*




with same method: 2 pt, 4 pt, higher order derivatives (see paper)

See 2205.00013 for another method, for bispectrum only

# The In-in Computation is difficult



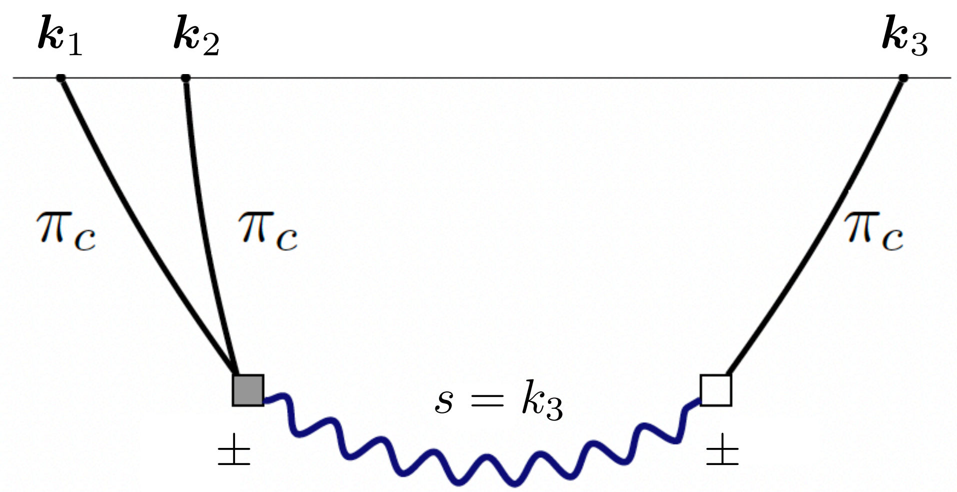

$$= \sum_{\pm \text{ at each vertex}} \int \prod_{i=1}^{V} d\eta_i \;\; \text{vertex}_i(\boldsymbol{k}_i, \eta_i)$$

$$\prod_{j=1}^{n} \pi_c^{\mp}(k_j, \eta_i)\pi_c^{\pm}(k_j, \eta_0) \times \prod_{l=1}^{L} G_{\pm\pm}(s_l, \eta_{1l}, \eta_{2l})$$

bulk-to-boundary

bulk-to-bulk

# The In-in Computation is difficult




$$= \sum_{\pm \text{ at each vertex}} \int \prod_{i=1}^{V} d\eta_i \ \ \text{vertex}_i(\boldsymbol{k}_i, \eta_i)$$

$$\prod_{j=1}^{n} \pi_c^{\mp}(k_j, \eta_i)\pi_c^{\pm}(k_j, \eta_0)$$

bulk-to-boundary

$$\times \prod_{l=1}^{L} G_{\pm\pm}(s_l, \eta_{1l}, \eta_{2l})$$

bulk-to-bulk

mode functions

$$\pi_c^{\pm}(k,\eta) = \frac{iH}{\sqrt{2c_s^3k^3}}(1 \pm ic_s k\eta)\exp(\mp ic_s k\eta)$$

$$\sigma_+(k,\eta) = \frac{\sqrt{\pi}H}{2}\exp(-\pi\mu/2)\ \exp(i\pi/4)(-\eta)^{3/2}H_{i\mu}^{(1)}(-k\eta)$$

bulk-to-bulk propagators

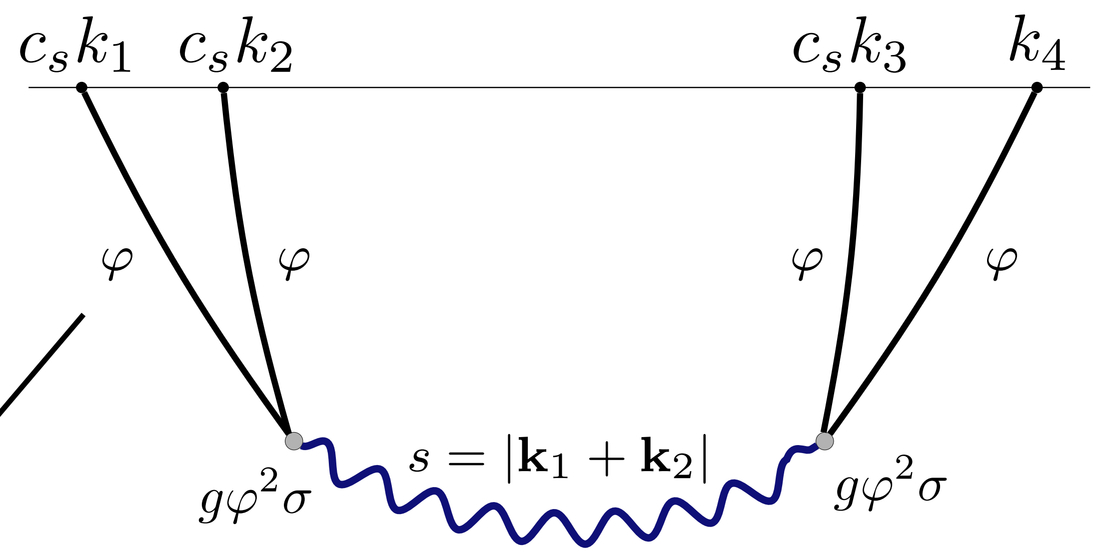
$$G_{++}(s,\eta,\eta') = \sigma_-(s,\eta')\sigma_+(s,\eta)\theta(\eta-\eta') + \sigma_-(s,\eta)\sigma_+(s,\eta')\theta(\eta'-\eta)$$

$$G_{+-}(s,\eta,\eta') = \sigma_+(s,\eta')\sigma_-(s,\eta)$$

# *Bispectra from a dS four-point*

All our diagrams (2-,3-,4-pt) can be related to a **de Sitter-invariant seed four-point function** of a conformally coupled field, e.g.:

$$B(k_1, k_2, k_3) = W(k_i, \partial_{k_i}) \lim_{k_4 \to 0}$$




$$\boxed{\varphi_\pm(k, \eta) = -\frac{H}{\sqrt{2k}}\eta \, \exp(\mp ik\eta)}$$

(relativistic dispersion relation)

*Boost breaking* manifests itself both

- in the weight-shifting operators (*boost breaking vertices*)

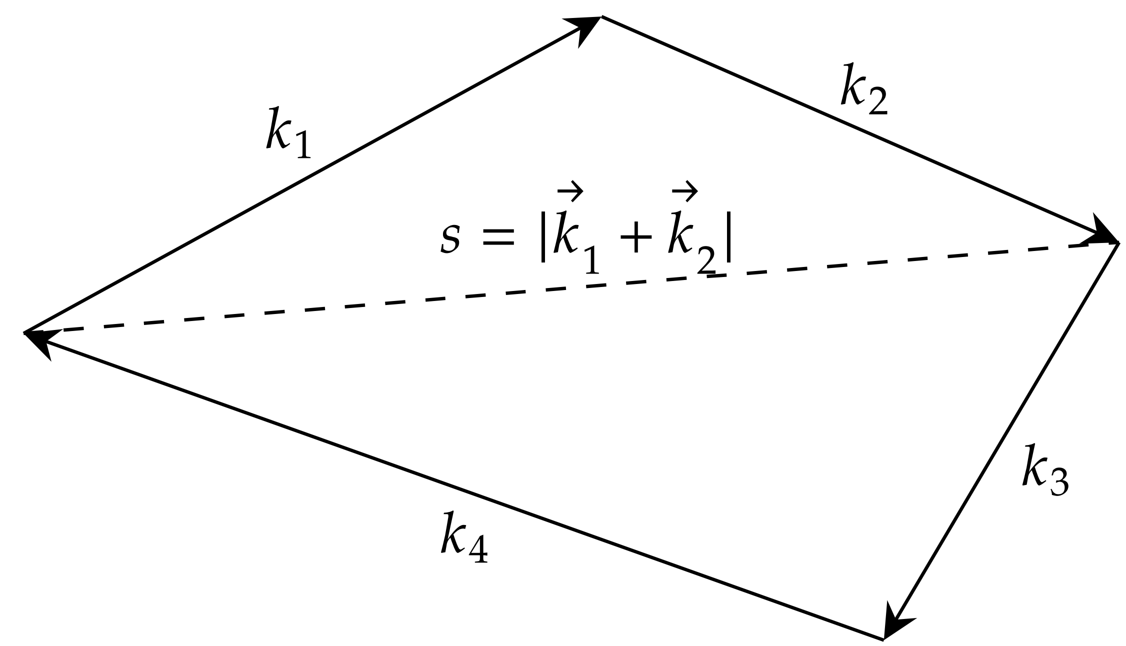
- and also in the argument of the four-point function (*different speeds of propagation*) $k_i \; (i = 1, 2, 3) \to c_s k_i$

# Bispectra from a dS four-point

The seed correlator $\hat{F}(u, v)$ has been found in 1811.00024

$$u = \frac{s}{k_1 + k_2} \leq 1$$

$$v = \frac{s}{k_3 + k_4} \leq 1$$

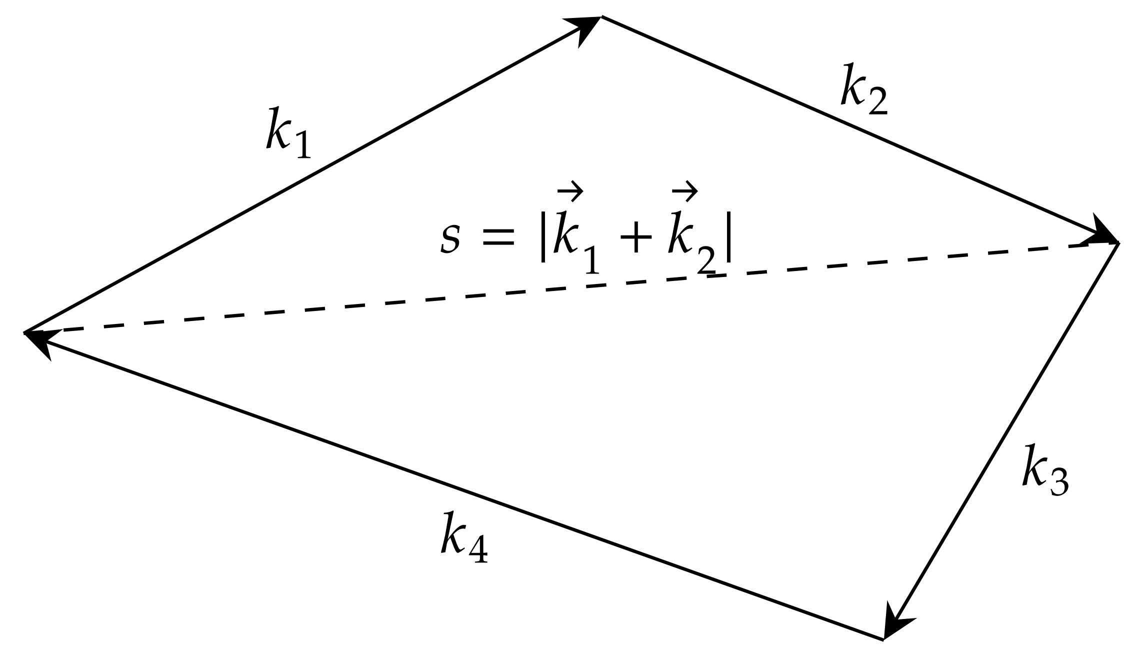


# *Bispectra from a dS four-point*

The seed correlator $\hat{F}(u,v)$ has been found in 1811.00024

$$u = \frac{s}{k_1 + k_2} \leq 1$$

$$v = \frac{s}{k_3 + k_4} \leq 1$$



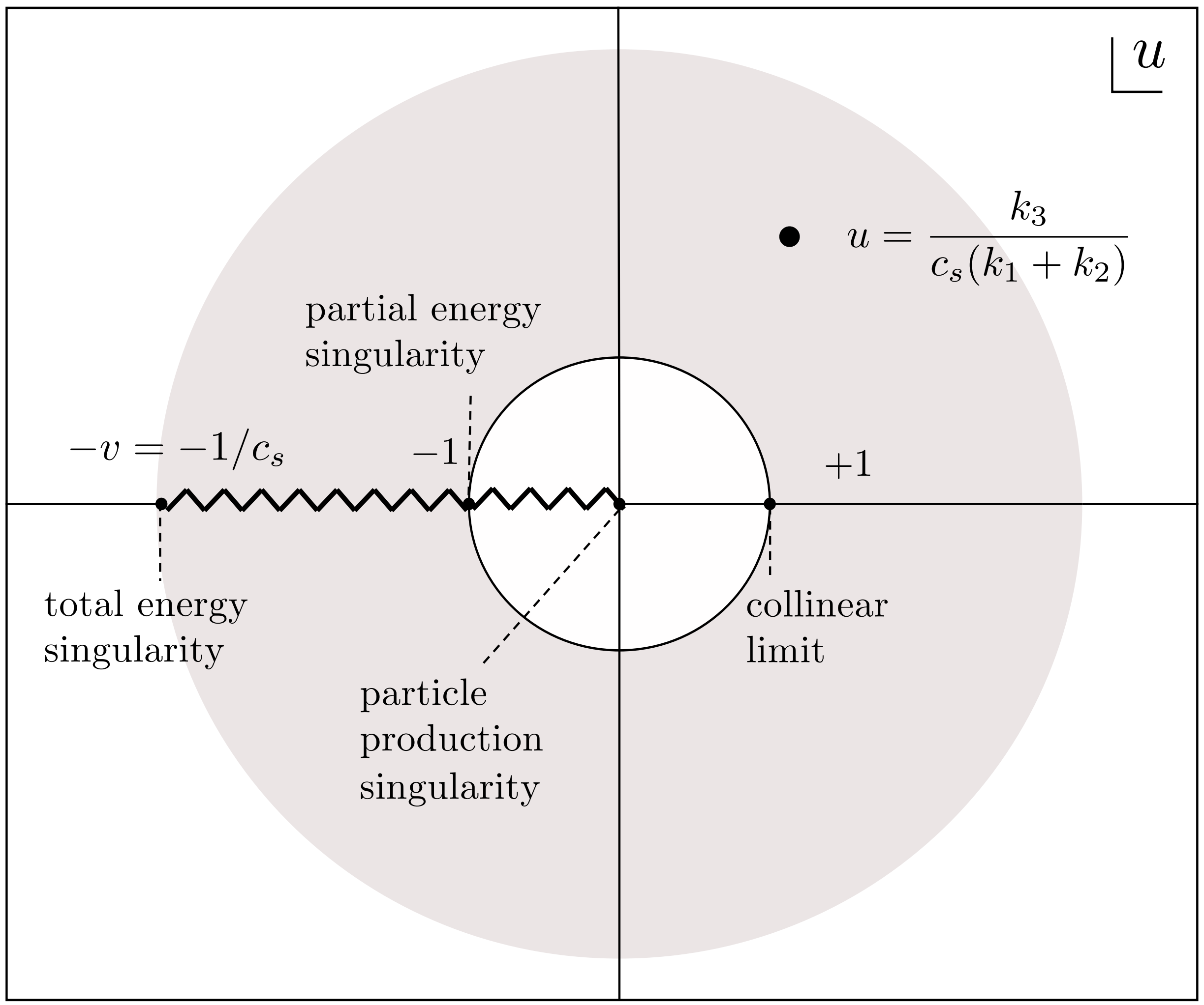

… but we need its analytical continuation beyond the kinematically allowed region

$$k_i (i = 1,2,3) \to c_s k_i,\ k_4 \to 0,\ s = |\mathbf{k}_3 + \mathbf{k}_4| \to k_3$$

$$u \to \frac{k_3}{c_s(k_1 + k_2)}, v \to \frac{1}{c_s}$$

# *Seed correlator beyond physical domain*

Bootstrap techniques only work if we further analytically continue to complex (external) "energies" $k_i \in \mathbb{C}$




(u,v) inside the unit circle in 1811.00024

Outside the unit circle is another world!

e.g. series expansion inside the unit circle in 1811.00024 do not converge outside

We need to bootstrap it from scratch

# Building blocks of the seed correlator

$$\hat{F} = \underbrace{\hat{F}_{++} + \hat{F}_{--}}_{} + \underbrace{\hat{F}_{+-} + \hat{F}_{-+}}_{}$$

Difficult

Truly nested integrals

Easiest part

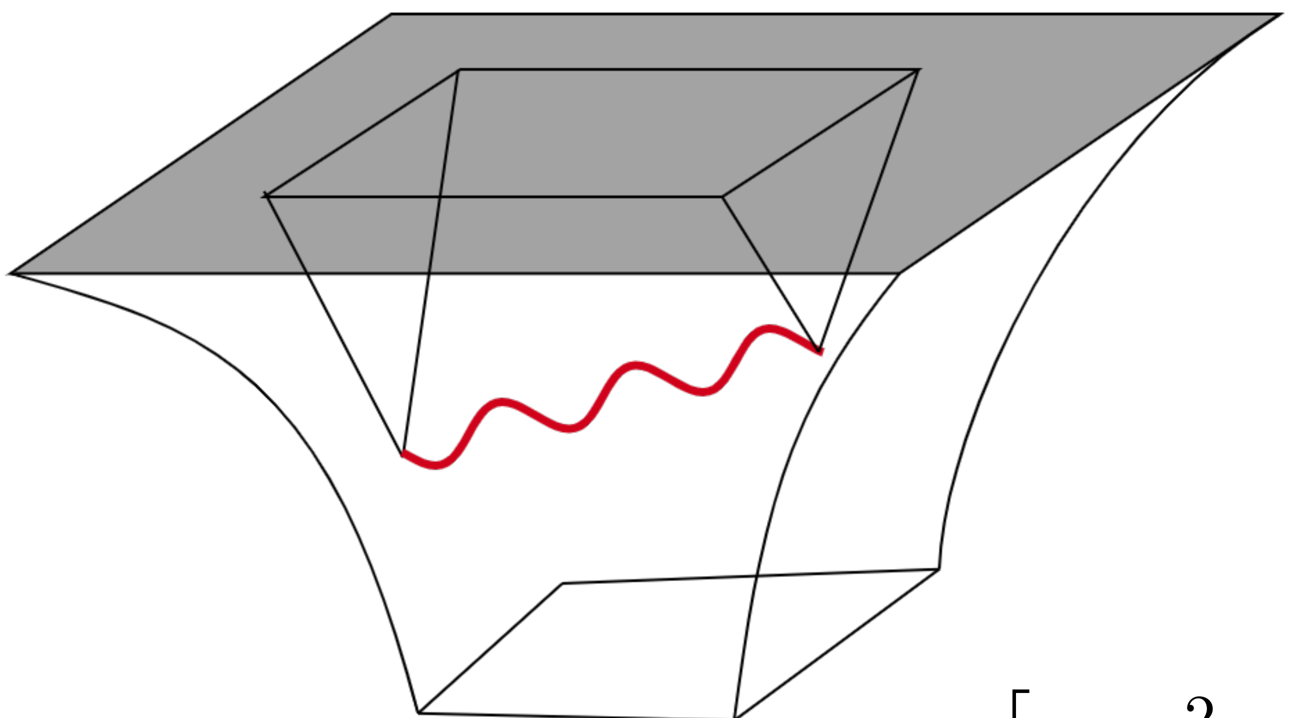
Factorized time integrals

$$\hat{F}_{\pm\pm}(u,v) = -\frac{g^2 s}{2H^2}\int_{-\infty(1\mp i\epsilon)}^{\eta_0}\frac{d\eta}{\eta^2}\int_{-\infty(1\mp i\epsilon)}^{\eta_0}\frac{d\eta'}{\eta'^2}e^{\pm i(k_1+k_2)\eta}\,e^{\pm i(k_3+k_4)\eta'}\,G_{\pm\pm}(s,\eta,\eta')$$

The bulk time integral for the ++ components defines an analytic function of external energies in the lower complex half-plane (reverse for - -)

$$\mathrm{Im}(k_i) < 0 \Rightarrow \mathrm{Im}(u,v) > 0$$

# *Bootstrap. I Locality*

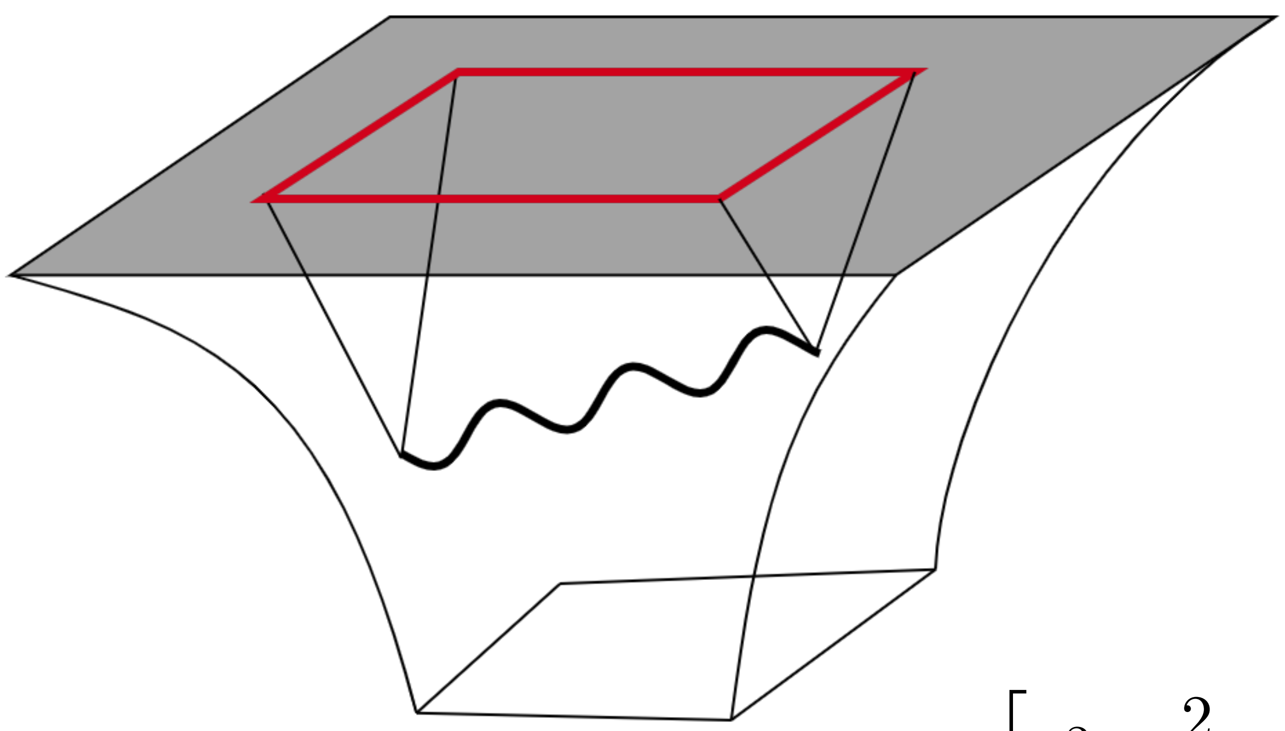


$$\left[\partial_\eta^2 - \frac{2}{\eta}\partial_\eta + k^2 + \frac{m^2}{\eta^2 H^2}\right] G_{\pm\pm}(s,\eta,\eta') = (\eta' H)^2 \delta(\eta - \eta')$$

Bulk local differential equation

# Bootstrap. I Locality

$$\left[u^2(1-u^2)\partial_u^2 - 2u^3\partial_u + \left(\mu^2+\frac{1}{4}\right)\right]\hat{F}_{\pm\pm}(u,v) = g^2\frac{u\,v}{2(u+v)}$$



Boundary differential equation

$$\left[\partial_\eta^2 - \frac{2}{\eta}\partial_\eta + k^2 + \frac{m^2}{\eta^2H^2}\right]G_{\pm\pm}(s,\eta,\eta') = (\eta' H)^2\delta(\eta-\eta')$$

Bulk local differential equation

# Bootstrap. I Locality

$$\left[u^2(1-u^2)\partial_u^2 - 2u^3\partial_u + \left(\mu^2+\frac{1}{4}\right)\right]\hat{F}_{\pm\pm}(u,v) = g^2\frac{u\,v}{2(u+v)}$$

$$\hat{F}_{++}(u,v) = \underbrace{\sum_{m,n}\left(a_{m,n}+b_{m,n}\log(u)\right)\frac{1}{u^m}\left(\frac{u}{v}\right)^n}_{} + \underbrace{\sum_{\pm\pm}\beta_{\pm\pm}f_\pm(u)f_\pm(v)}_{}\,, \qquad 1<|u|<|v|$$

Suitable particular solution
“from ‘infinity”

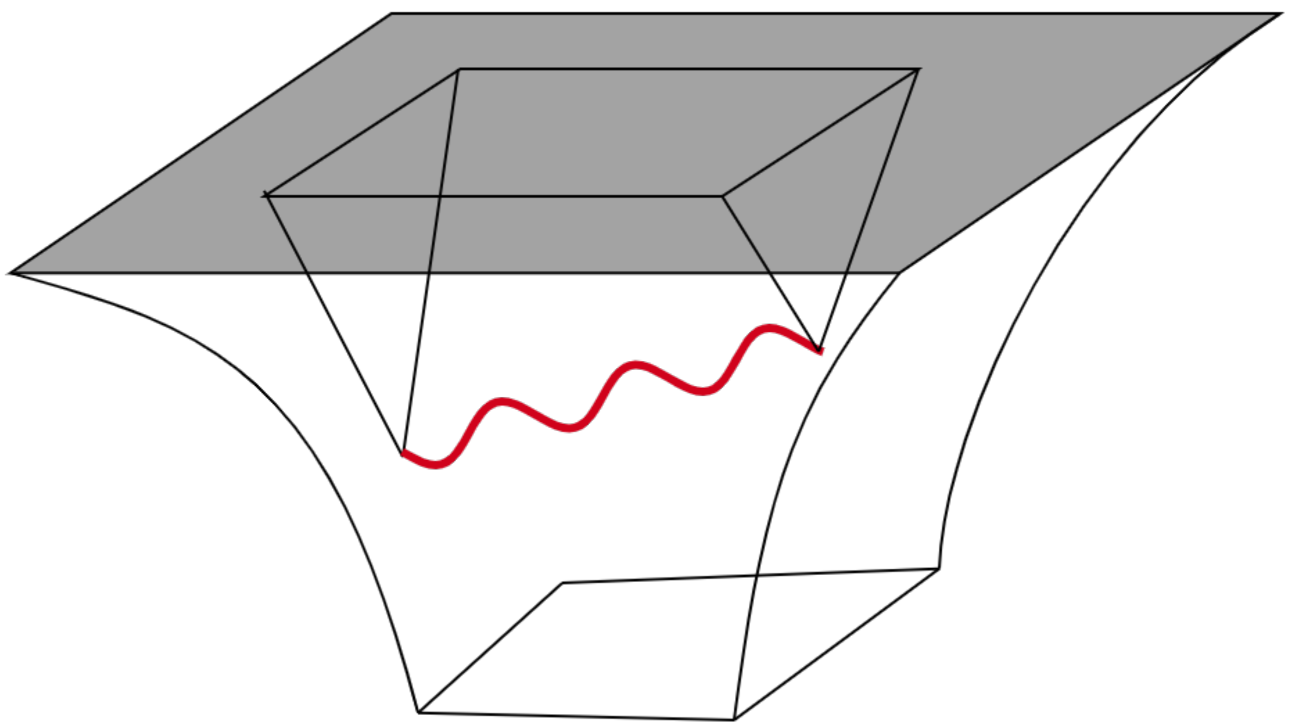
Homogeneous solution with
four free parameters to determine

Series coefficients
and partial resummation

$$f_+(u) = {}_2F_1\left(\frac{1}{4}-\frac{i\mu}{2},\frac{1}{4}+\frac{i\mu}{2};\frac{1}{2};\frac{1}{u^2}\right)$$

$$f_-(u) = \frac{2}{u}\times{}_2F_1\left(\frac{3}{4}-\frac{i\mu}{2},\frac{3}{4}+\frac{i\mu}{2};\frac{3}{2};\frac{1}{u^2}\right)$$

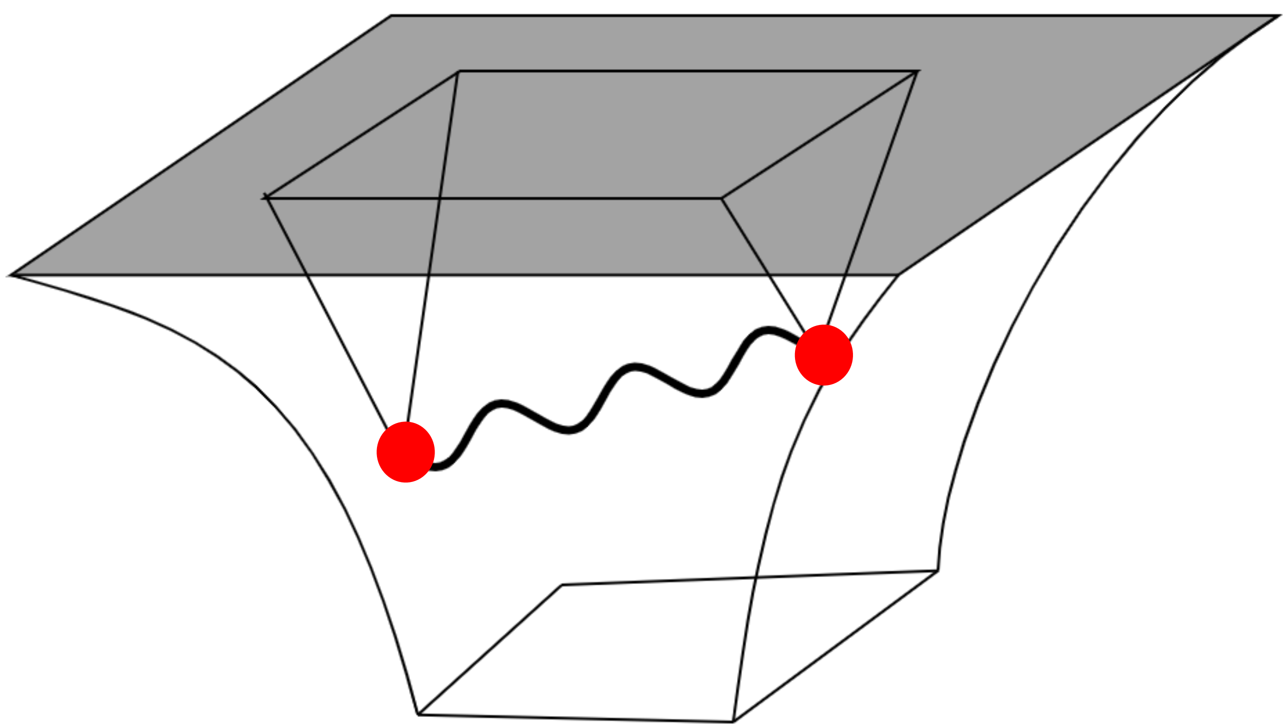
# Bootstrap. 2 Unitarity



Cutting rule for the bulk to bulk propagator (energy positivity)

$$G^*_{++}(s,\eta,\eta') + G_{++}(s,\eta,\eta') = \sigma_-(s,\eta)\sigma_+(s,\eta') + \eta \leftrightarrow \eta'$$

# *Bootstrap. 2 Unitarity*

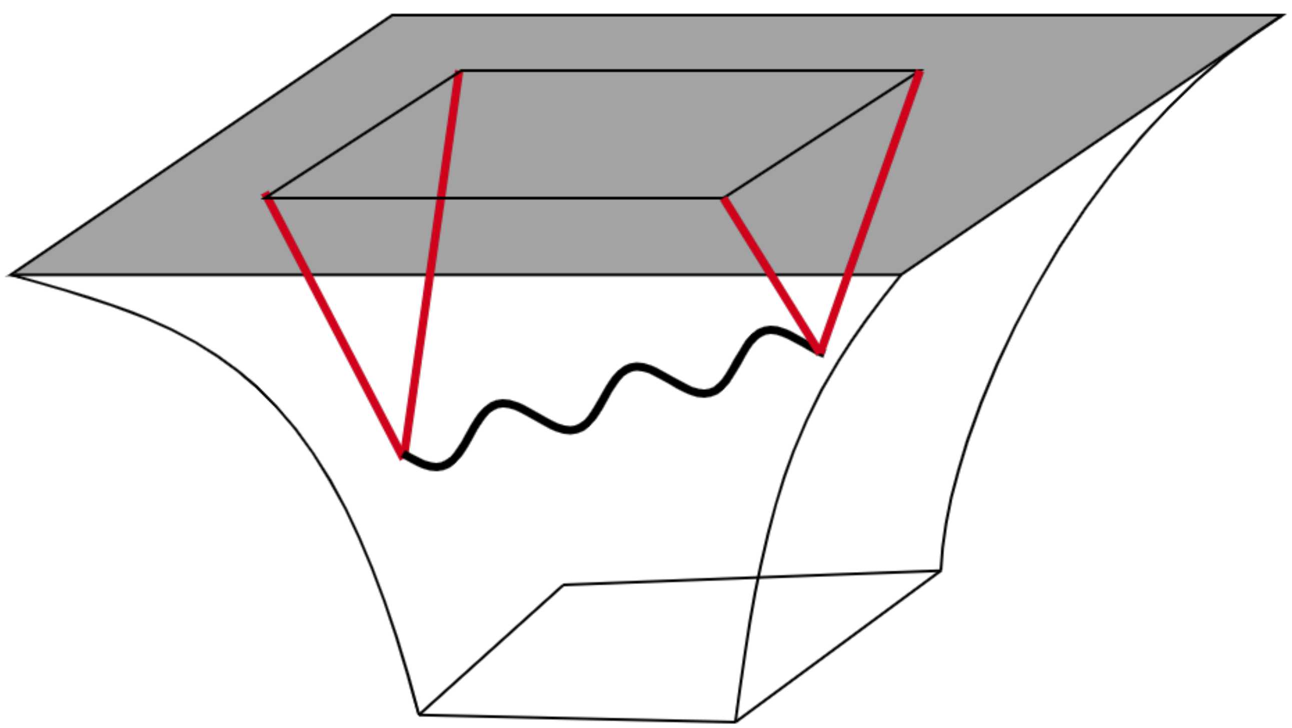


Reality of the couplings $g* = g$

Cutting rule for the bulk to bulk propagator (energy positivity)

$$G^*_{++}(s,\eta,\eta') + G_{++}(s,\eta,\eta') = \sigma_-(s,\eta)\sigma_+(s,\eta') + \eta \leftrightarrow \eta'$$

# Bootstrap. 2 Unitarity



Hermitian analyticity of the bulk to boundary propagator

$$\varphi_+^*(k,\eta) = \varphi_+(-k,\eta)$$

Reality of the couplings $g* = g$

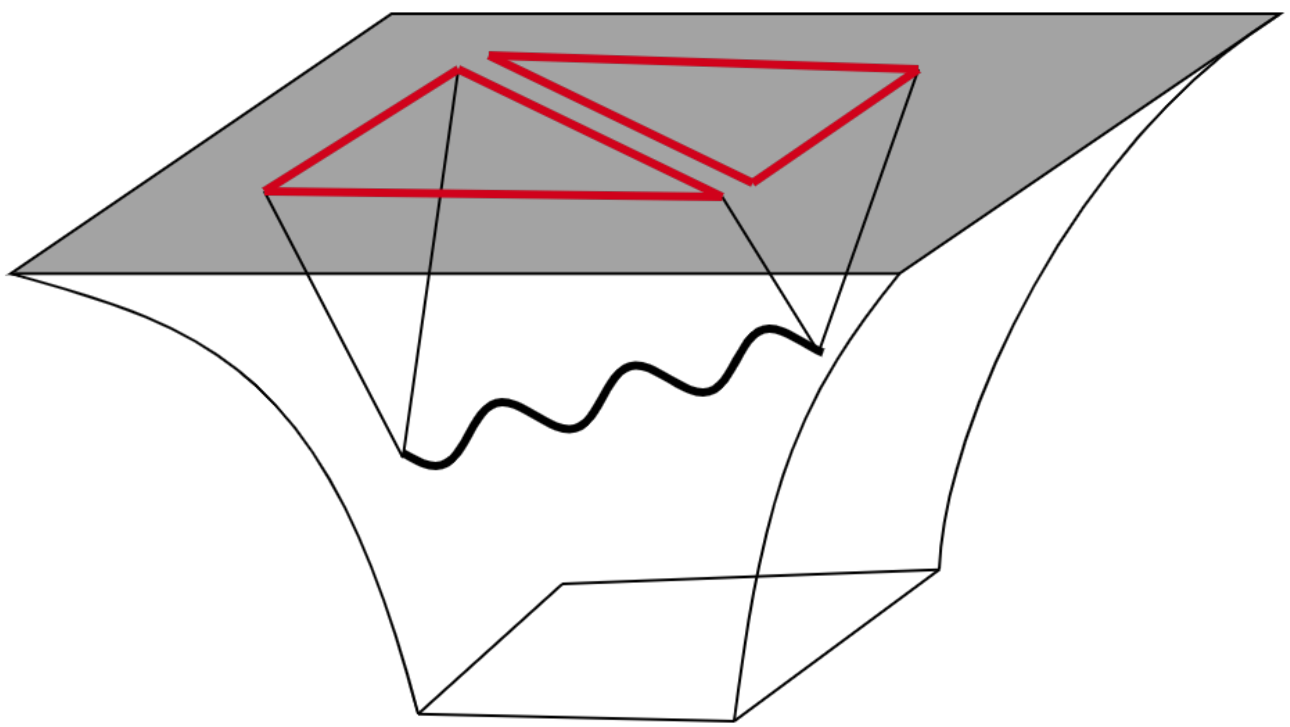
Cutting rule for the bulk to bulk propagator (energy positivity)

$$G_{++}^*(s,\eta,\eta') + G_{++}(s,\eta,\eta') = \sigma_-(s,\eta)\sigma_+(s,\eta') + \eta \leftrightarrow \eta'$$

# Bootstrap. 2 Unitarity

## Cosmological Cutting Rule

$$\hat{F}_{++}(u,v) + \hat{F}^*_{++}(-u^*,-v^*) = -\frac{1}{2}\,\hat{f}_3(u)\hat{f}^*_3(-v^*) - \frac{1}{2}\hat{f}_3(v)\hat{f}^*_3(-u^*)$$



Hermitian analyticity of the bulk to boundary propagator

$$\varphi^*_+(k,\eta) = \varphi_+(-k,\eta)$$

Reality of the couplings $g* = g$

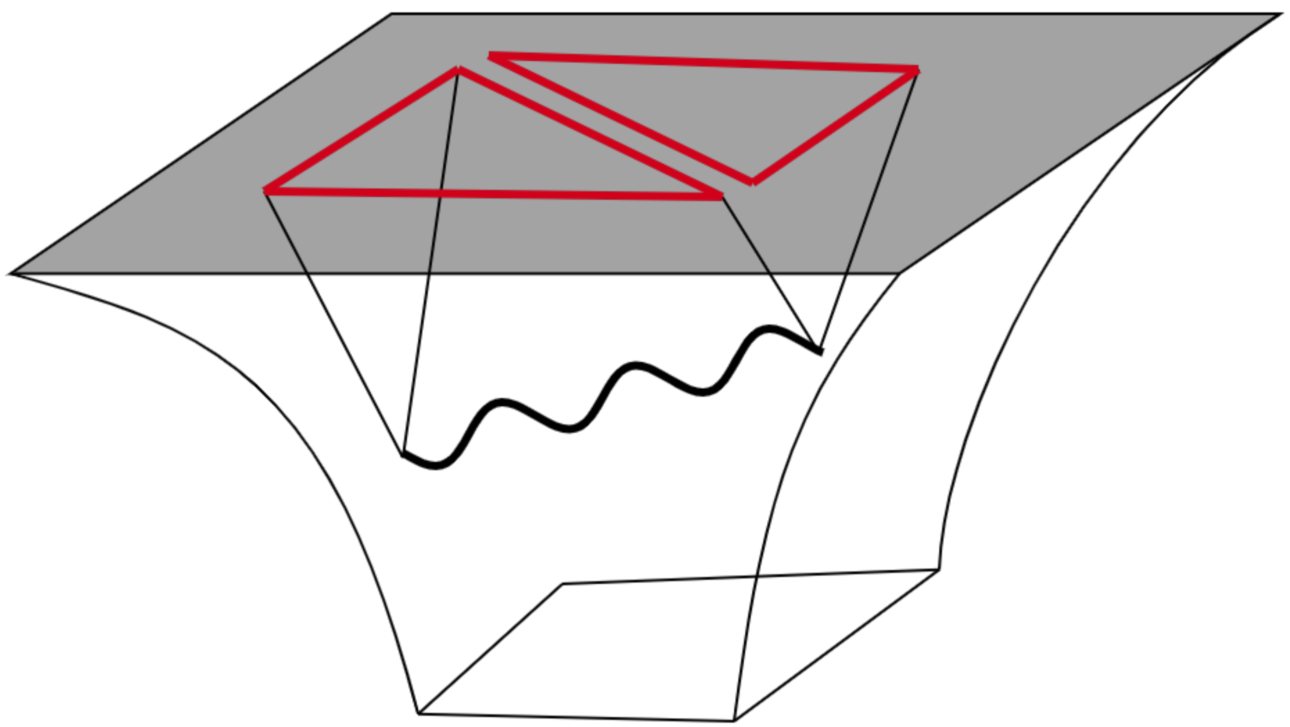
Cutting rule for the bulk to bulk propagator (energy positivity)

$$G^*_{++}(s,\eta,\eta') + G_{++}(s,\eta,\eta') = \sigma_-(s,\eta)\sigma_+(s,\eta') + \eta \leftrightarrow \eta'$$

# Bootstrap. 2 Unitarity

## Cosmological Cutting Rule

$$\hat{F}_{++}(u,v) + \hat{F}^*_{++}(-u^*,-v^*) = -\frac{1}{2}\,\hat{f}_3(u)\hat{f}^*_3(-v^*) - \frac{1}{2}\hat{f}_3(v)\hat{f}^*_3(-u^*)$$



Hermitian analyticity of the bulk to boundary propagator

$$\varphi^*_+(k,\eta) = \varphi_+(-k,\eta)$$

Reality of the couplings $\quad g* = g$

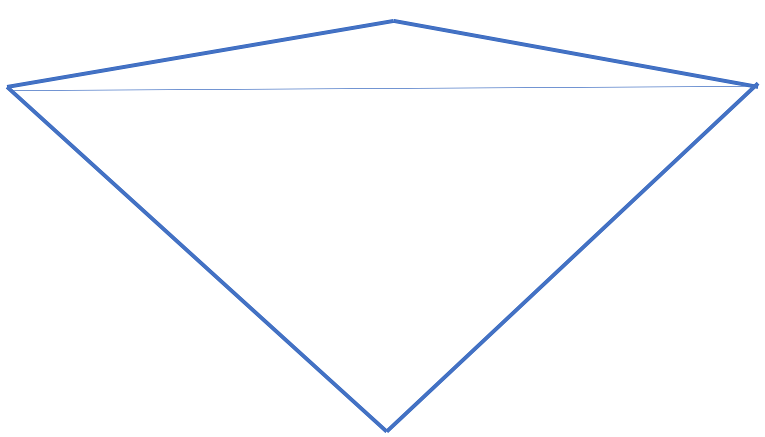
with $\quad \hat{f}_3(u) = \frac{ig}{2\sqrt{2\pi}}\left(|\Gamma\left(1/4+i\mu/2\right)|^2 f_+(u) - |\Gamma\left(3/4+i\mu/2\right)|^2 f_-(u)\right)$

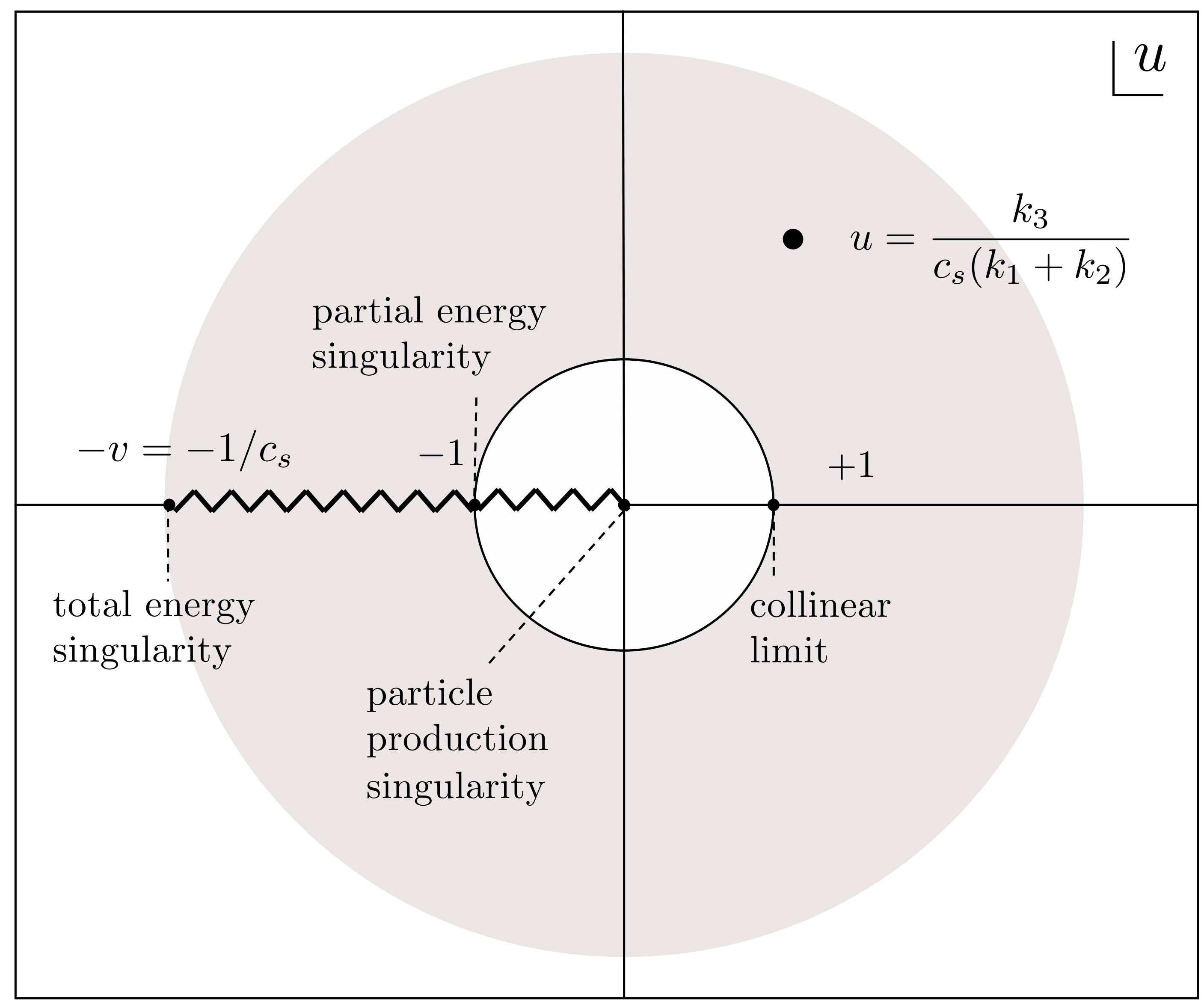
# ***Bootstrap. 3 Analyticity***

For Bunch-Davies initial conditions, no singularity can arise in the physical domain of any correlator

Requiring absence of singularity in the collinear (folded) limit

$$u \to 1$$





# *Concluding remarks*

Cosmological bootstrap: new methods and new insight into cosmological correlators

We extended the reach of cosmological collider + bootstrap beyond de Sitter invariant setups

Identification of new signatures of heavy fields: low speed collider with resonances in squeezed limit

Non-local EFT gives useful insight but breaks down for masses too close to Hubble, by contrast to exact bootstrap results