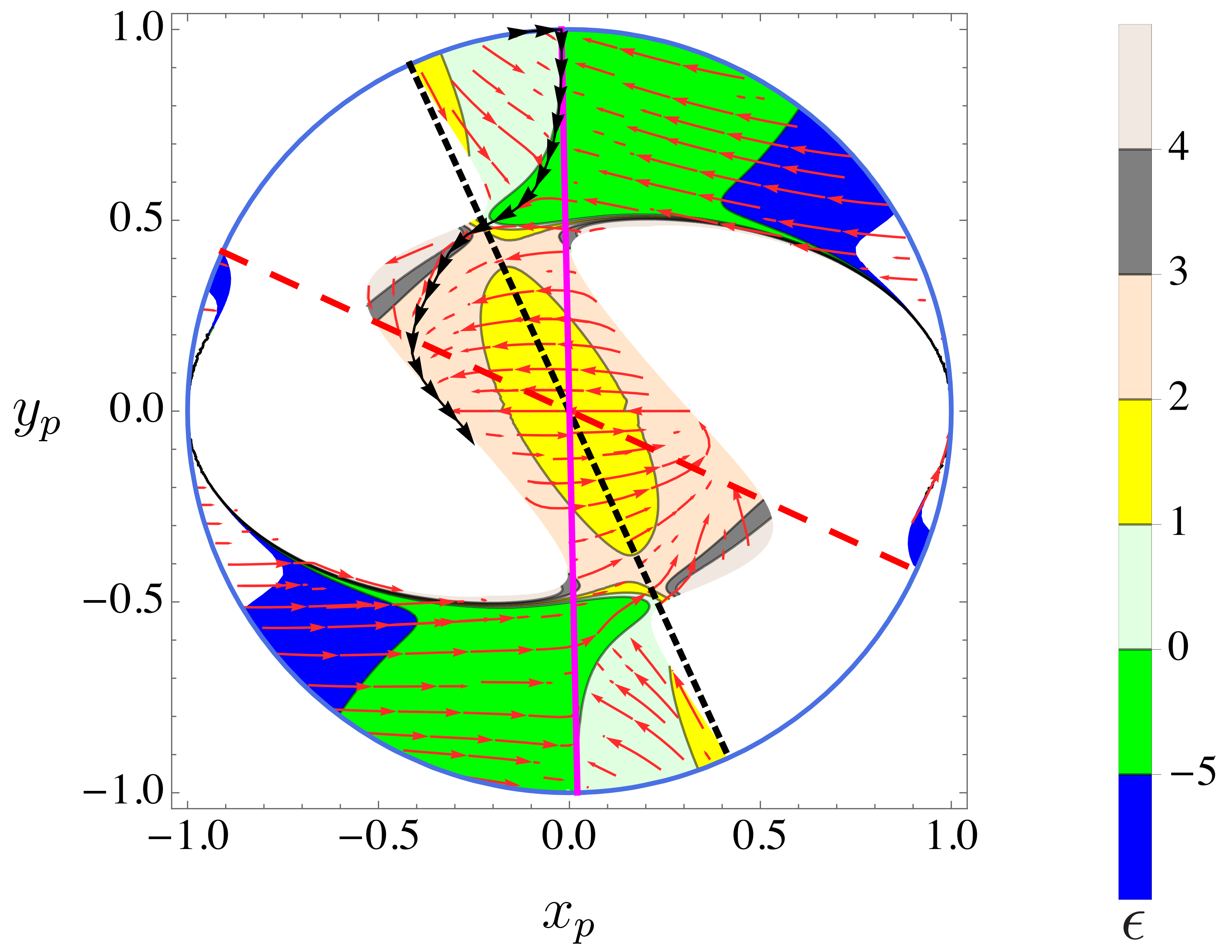
Generalized SU(2) Proca theory and constant-roll inflation



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Sequel

- E. Allys, P. Peter, and Y. Rodríguez, “Generalized Proca action for an Abelian vector field”, JCAP 1602 (2016) 004.
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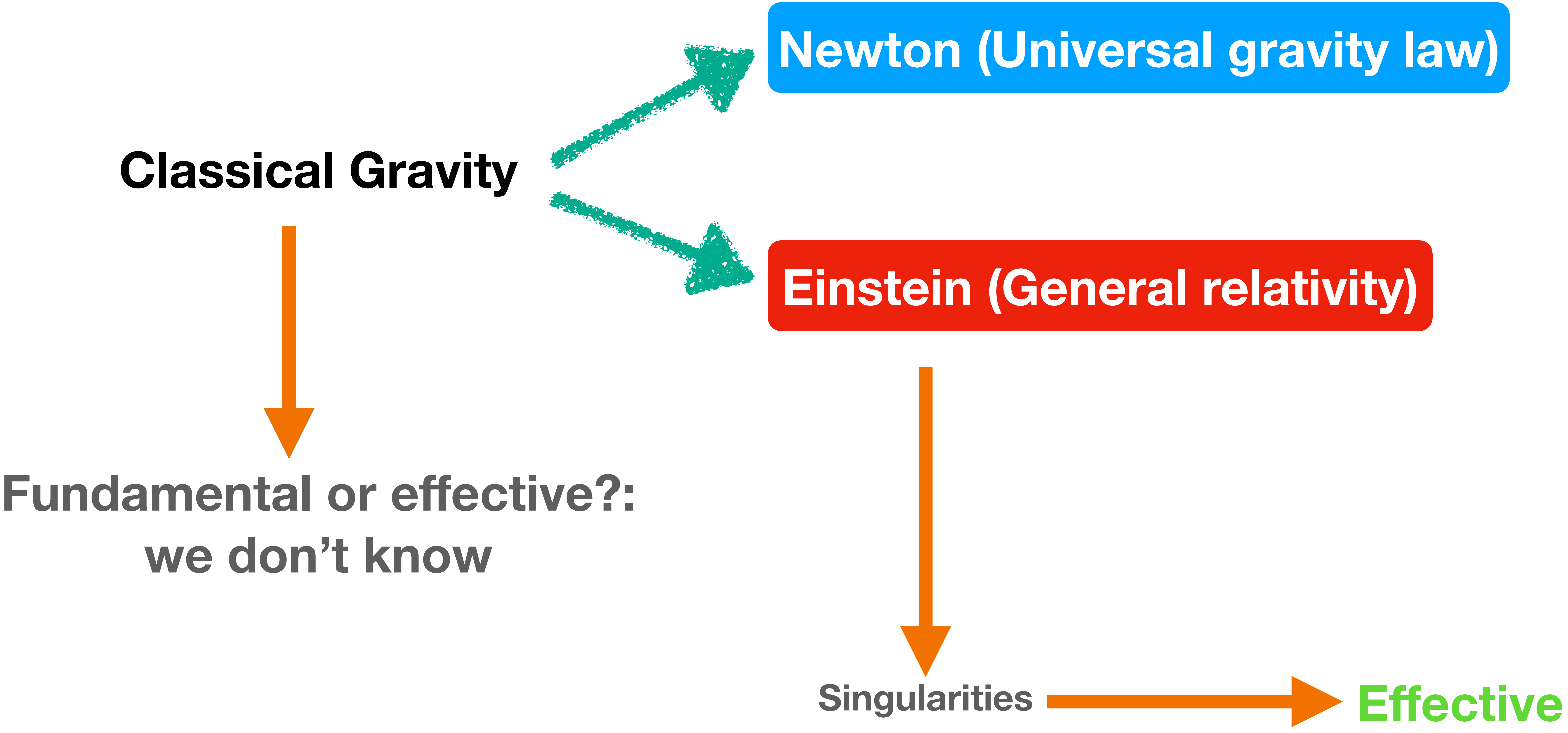
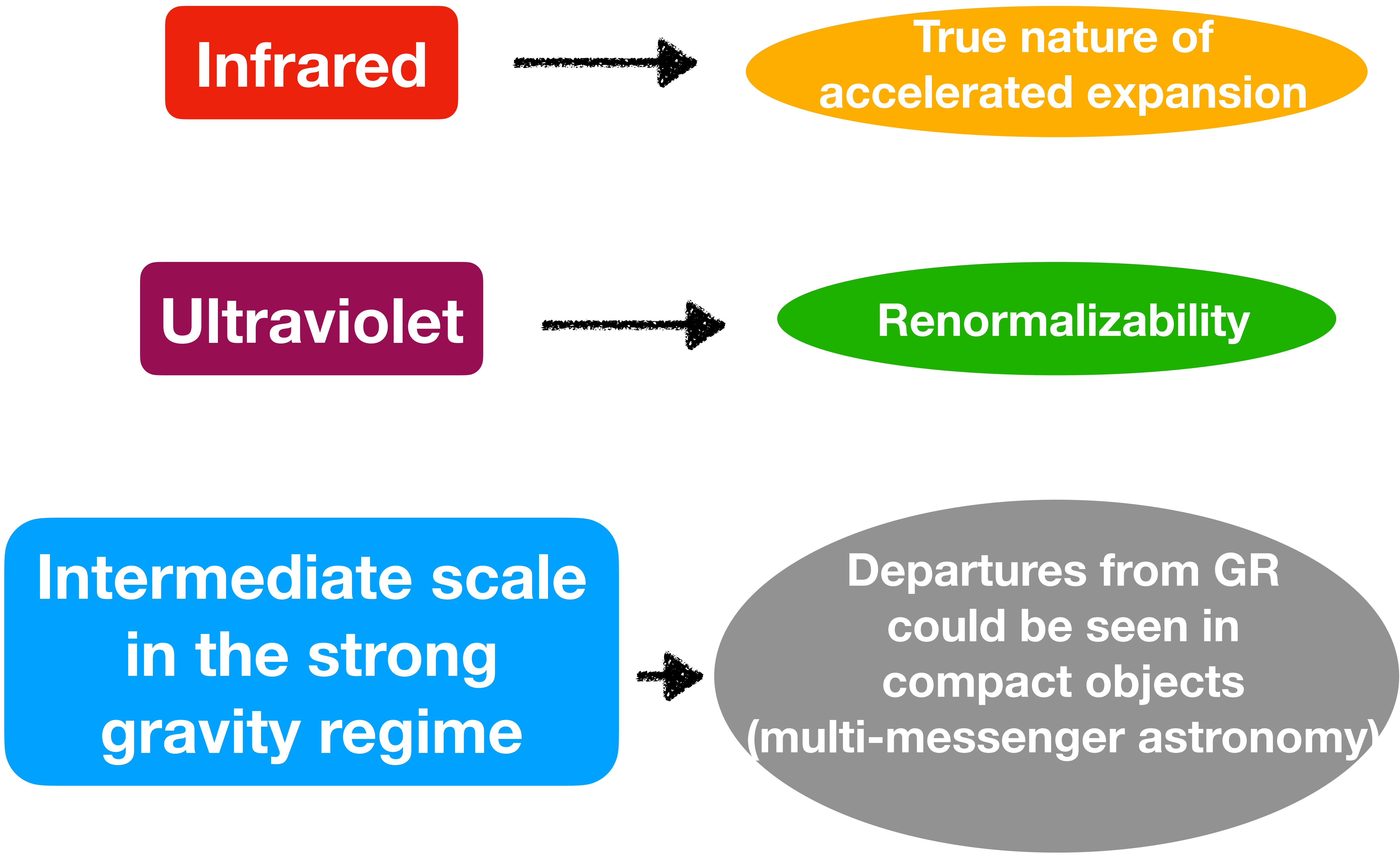
- E. Allys, P. Peter, and Y. Rodríguez, “Generalized SU(2) Proca theory”, Phys. Rev. D 94 (2016) 084041.
- Y. Rodríguez and A. A. Navarro, “Scalar and vector Galileons”, J. Phys. Conf. Ser. 831 (2017) 012004.
- Y. Rodríguez and A. A. Navarro, “Non-Abelian S-term dark energy and inflation”, Phys. Dark Univ. 19 (2018) 129.
- A. Gallego Cadavid and Y. Rodríguez, “A systematic procedure to build the beyond generalized Proca field theory”, Phys. Lett. B 798 (2019) 134958.
- L. G. Gómez and Y. Rodríguez, “Stability conditions in the generalized SU(2) Proca theory”, Phys. Rev. D 100 (2019) 084048.
- A. Gallego Cadavid, Y. Rodríguez, and L. G. Gómez, “Generalized SU(2) Proca theory reconstructed and beyond”, Phys. Rev. D 102 (2020) 104066.
- J. C. Garnica, L. G. Gómez, A. A. Navarro, and Y. Rodríguez, “Constant-roll inflation in the generalized SU(2) Proca theory”, Ann. Phys. (Berlin) 534 (2022) 2100453.
- A. Gallego Cadavid, C. M. Nieto, and Y. Rodríguez, “Decoupling-limit consistency of the generalized SU(2) Proca theory”, Phys. Rev. D 105 (2022) 104051.
- A. Gallego Cadavid, C. M. Nieto, and Y. Rodríguez, “Towards the extended SU(2) Proca theory”, Phys. Rev. D 105 (2022) 124060.

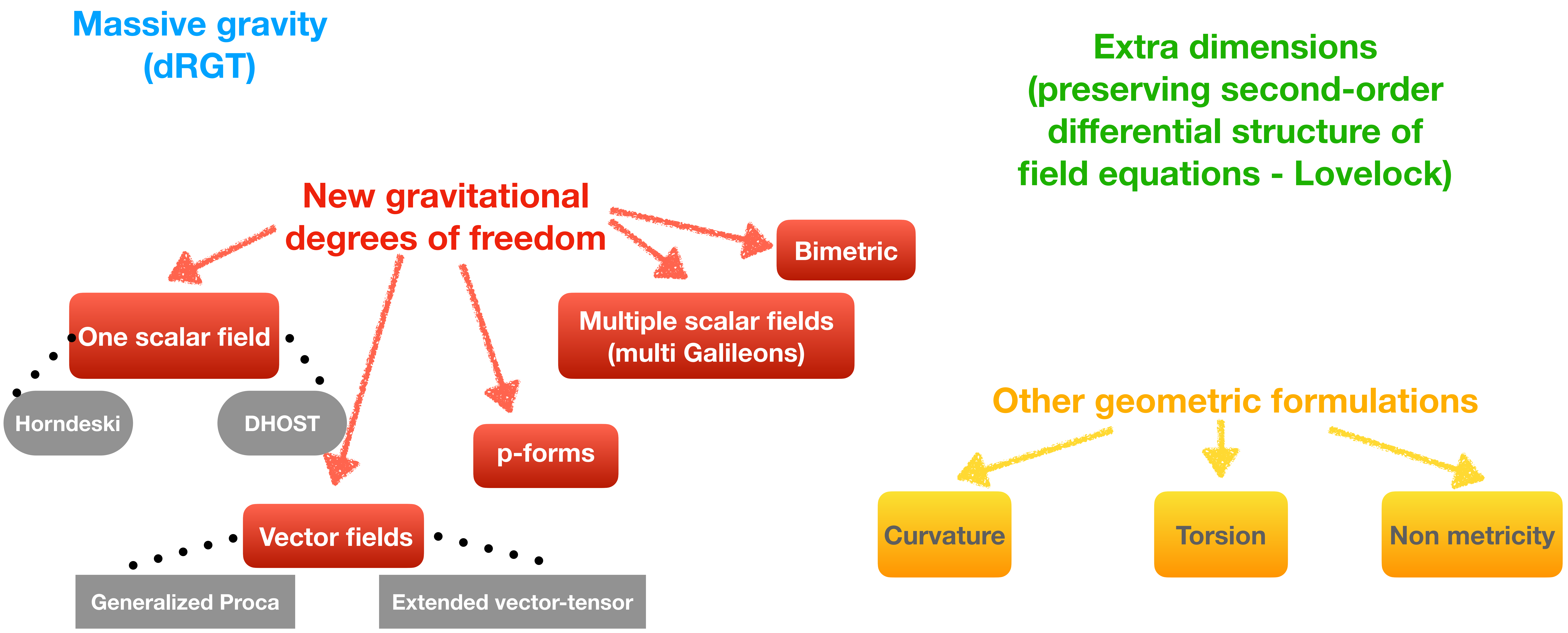

Classical Gravity
Newton (Universal gravity law)
Einstein (General relativity)
Fundamental or effective?:
we don't know
Singularities
Effective

Breakdown of GR:



Are we in the verge of a scientific crisis?

Different possibilities to extend GR:



L. Heisenberg, Phys. Rep. 2019

Vector fields in gravity and/or cosmology

Why to introduce them?

Why not?

Let's be pragmatical:
there are much more vector fields in nature
than fundamental scalar fields

Despite of these problems:

Ghosts, anisotropies
in cosmology, etc.

→

The role of vector fields in gravitation, astrophysics, and cosmology has attracted a lot of interest in recent years

↓

Generalized Proca theory

Generalized Proca theory

G. Tasinato, JHEP 2014
L. Heisenberg, JCAP 2014
E. Allys, P. Peter, and Y. Rodríguez, JCAP 2016
E. Allys, P. Peter, J. P. Beltrán Almeida, and Y. Rodríguez, JCAP 2016
J. Beltrán Jiménez and L. Heisenberg, Phys. Lett. B 2016

$$\mathcal{L} = \frac{m_P^2}{2}R - \frac{1}{4}F_{\mu\nu}F^{\mu\nu} + \frac{1}{2}m^2 B_\mu B^\mu + \boxed{\dots}$$

Proca theory in curved spacetime

Terms that break internal gauge symmetries

Principle of construction:

B_μ has four degrees of freedom but only three can propagate → According to the structure of the irreducible representations of the Poincaré group

Then, it must be degenerate by construction

Its decoupling limit reduces to the Horndeski theory, so it's healthy


Generalized Proca theory in astrophysics and cosmology
L. Heisenberg, Phys. Rep. 2019
Compact objects
Inflation
Dark energy
Too much anisotropy
Restoration of the isotropy
Rapid oscillations
Dilution by a companion scalar field
Suppression of the spatial components against the temporal one
Cosmic triad

Cosmic triad → Particular case of the most general spherically symmetric configuration:

$$B_{0a}(t) = b_0(t)\hat{\bar{r}}_a$$

$$B_{ia}(t) = b_1(t)\hat{r}_i\hat{\bar{r}}_a + b_2(t)[\delta_{ia} - \hat{r}_i\hat{\bar{r}}_a] + b_3(t)\epsilon_{ia}{}^{l}\hat{\bar{r}}_l$$

Except for the suppression of the spatial components against the temporal one, all the other options require global invariance of the action under SU(2)

This is the main motivation for the construction of the generalized SU(2) Proca theory (GSU2P)

E. Witten, Phys. Rev. Lett. 1977
D. Sivers, Phys. Rev. D 1986
P. Forgács and N. S. Manton, Commun. Math. Phys. 1980

GSU2P → Globally invariant under internal SU(2)

Spherically symmetric configuration

It spontaneously breaks:

1. the internal SU(2) symmetry
2. the Lorentz rotational symmetry
3. the Lorentz boosts

This is, anyway, extraordinarily reasonable: it seems to be nature's strategy to produce all patterns we see in condensed matter systems (fluids, superfluids, solids, supersolids)

A. Nicolis, R. Penco, F. Piazza, and R. Rattazzi, JHEP 2015

Building the GSU2P

Implemented by (in strict order):

- The construction of Lorentz-invariant and group-invariant Lagrangian building blocks
- The primary constraint enforcing relation
- The secondary constraint enforcing
- The covariantization
- The remotion of redundant terms via total derivatives
- The decoupling limit and its healthiness

E. Allys, P. Peter, and Y. Rodríguez, Phys. Rev. D 2016
A. Gallego Cadavid, Y. Rodríguez, and L. G. Gómez, Phys. Rev. D 2020
A. Gallego Cadavid, C. M. Nieto, and Y. Rodríguez, Phys. Rev. D 2022

Healthiness: either second-order field equations or degeneracy

The GSU2P and beyond

(up to six space-time indices in the Lagrangian building blocks before contractions)

$$S^a_{\mu\nu} \equiv \nabla_\mu B^a_\nu + \nabla_\nu B^a_\mu$$
$$A^a_{\mu\nu} \equiv \nabla_\mu B^a_\nu - \nabla_\nu B^a_\mu$$

$$\mathcal{L}_2 \equiv \mathcal{L}_2(A^a_{\mu\nu}, B^a_\mu)$$

$$\mathcal{L}_{4,0} \equiv G_{\mu\nu} B^{\mu a} B^\nu_a$$

$$\mathcal{L}_{4,2} \equiv \sum_{i=1}^{6} \frac{\alpha_i}{m_P^2} \mathcal{L}^i_{4,2} + \sum_{i=1}^{4} \frac{\tilde{\alpha}_i}{m_P^2} \tilde{\mathcal{L}}^i_{4,2}$$

$$\tilde{\mathcal{L}}_{5,0} \equiv B^{\nu a} R^\sigma{}_{\nu\rho\mu} B^b_\sigma \tilde{A}^{\mu\rho c} \epsilon_{abc}$$

$$\mathcal{L}^1_{4,2} \equiv (B_b \cdot B^b)[S^{\mu a}_\mu S^\nu_{\nu a} - S^{\mu a}_\nu S^\nu_{\mu a}] + 2(B_a \cdot B_b)[S^{\mu a}_\mu S^{\nu b}_\nu - S^{\mu a}_\nu S^{\nu b}_\mu]$$
$$\mathcal{L}^2_{4,2} \equiv A^a_{\mu\nu} S^{\mu b}_\sigma B^\nu_a B^\sigma_b - A^a_{\mu\nu} S^{\mu b}_\sigma B^\nu_b B^\sigma_a + A^a_{\mu\nu} S^{\rho b}_\rho B^\mu_a B^\nu_b$$
$$\mathcal{L}^3_{4,2} \equiv B^{\mu a} R^\alpha{}_{\sigma\rho\mu} B_{\alpha a} B^{\rho b} B^\sigma_b + \frac{3}{4}(B_b \cdot B^b)(B^a \cdot B_a) R$$
$$\mathcal{L}^4_{4,2} \equiv [(B_b \cdot B^b)(B^a \cdot B_a) + 2(B_a \cdot B_b)(B^a \cdot B^b)] R$$
$$\mathcal{L}^5_{4,2} \equiv G_{\mu\nu} B^{\mu a} B^\nu_a (B^b \cdot B_b)$$
$$\mathcal{L}^6_{4,2} \equiv G_{\mu\nu} B^{\mu a} B^{\nu b} (B_a \cdot B_b)$$
$$\tilde{\mathcal{L}}^1_{4,2} \equiv -2 A^a_{\mu\nu} S^{\mu b}_\sigma B_{\alpha a} B_{\beta b} \epsilon^{\nu\sigma\alpha\beta} + S^a_{\mu\nu} S^{\nu b}_\sigma B_{\alpha a} B_{\beta b} \epsilon^{\mu\sigma\alpha\beta}$$
$$\tilde{\mathcal{L}}^2_{4,2} \equiv A^a_{\mu\nu} S^{\mu b}_\sigma B_{\alpha a} B_{\beta b} \epsilon^{\nu\sigma\alpha\beta} - \tilde{A}^{\alpha\beta}_a S^b_{\rho\alpha} B^{\rho a} B_{\beta b} + \tilde{A}^{\alpha\beta}_a S^\rho_{\rho b} B^a_\alpha B^b_\beta$$
$$\tilde{\mathcal{L}}^3_{4,2} \equiv B^b_\beta R^\alpha{}_{\sigma\rho\mu} B^a_\alpha (B_a \cdot B_b) \epsilon^{\mu\rho\sigma\beta}$$
$$\tilde{\mathcal{L}}^4_{4,2} \equiv B_{\beta a} R^\alpha{}_{\sigma\rho\mu} B^a_\alpha (B^b \cdot B_b) \epsilon^{\mu\rho\sigma\beta}$$

A. Gallego Cadavid, Y. Rodríguez, and L. G. Gómez, Phys. Rev. D 2020

The GSU2P and beyond

$$S^a_{\mu\nu} \equiv \nabla_\mu B^a_\nu + \nabla_\nu B^a_\mu$$
$$A^a_{\mu\nu} \equiv \nabla_\mu B^a_\nu - \nabla_\nu B^a_\mu$$

There are new interactions of purely non-Abelian character!

$$\tilde{\mathcal{L}}_{5,0} \equiv B^{\nu a} R^\sigma{}_{\nu\rho\mu} B^b_\sigma \tilde{A}^{\mu\rho c} \epsilon_{abc}$$

$$\begin{aligned}
\mathcal{L}^1_{4,2} \equiv& (B_b \cdot B^b)[S^{\mu a}_\mu S^\nu_{\nu a} - S^{\mu a}_\nu S^\nu_{\mu a}] \\
&+ 2(B_a \cdot B_b)[S^{\mu a}_\mu S^{\nu b}_\nu - S^{\mu a}_\nu S^{\nu b}_\mu] \\
\mathcal{L}^2_{4,2} \equiv& A^a_{\mu\nu} S^{\mu b}_\sigma B^\nu_a B^\sigma_b - A^a_{\mu\nu} S^{\mu b}_\sigma B^\nu_b B^\sigma_a + A^a_{\mu\nu} S^{\rho b}_\rho B^\mu_a B^\nu_b \\
\mathcal{L}^3_{4,2} \equiv& B^{\mu a} R^\alpha{}_{\sigma\rho\mu} B_{\alpha a} B^{\rho b} B^\sigma_b + \frac{3}{4}(B_b \cdot B^b)(B^a \cdot B_a) R \\
\mathcal{L}^4_{4,2} \equiv& [(B_b \cdot B^b)(B^a \cdot B_a) + 2(B_a \cdot B_b)(B^a \cdot B^b)] R \\
\mathcal{L}^5_{4,2} \equiv& G_{\mu\nu} B^{\mu a} B^\nu_a (B^b \cdot B_b) \\
\mathcal{L}^6_{4,2} \equiv& G_{\mu\nu} B^{\mu a} B^{\nu b} (B_a \cdot B_b) \\
\tilde{\mathcal{L}}^1_{4,2} \equiv& -2 A^a_{\mu\nu} S^{\mu b}_\sigma B_{\alpha a} B_{\beta b} \epsilon^{\nu\sigma\alpha\beta} + S^a_{\mu\nu} S^{\nu b}_\sigma B_{\alpha a} B_{\beta b} \epsilon^{\mu\sigma\alpha\beta} \\
\tilde{\mathcal{L}}^2_{4,2} \equiv& A^a_{\mu\nu} S^{\mu b}_\sigma B_{\alpha a} B_{\beta b} \epsilon^{\nu\sigma\alpha\beta} - \tilde{A}^{\alpha\beta}_a S^b_{\rho\alpha} B^{\rho a} B_{\beta b} \\
&+ \tilde{A}^{\alpha\beta}_a S^\rho_{\rho b} B^a_\alpha B^b_\beta \\
\tilde{\mathcal{L}}^3_{4,2} \equiv& B^b_\beta R^\alpha{}_{\sigma\rho\mu} B^a_\alpha (B_a \cdot B_b) \epsilon^{\mu\rho\sigma\beta} \\
\tilde{\mathcal{L}}^4_{4,2} \equiv& B_{\beta a} R^\alpha{}_{\sigma\rho\mu} B^a_\alpha (B^b \cdot B_b) \epsilon^{\mu\rho\sigma\beta}
\end{aligned}$$

A. Gallego Cadavid, Y. Rodríguez, and L. G. Gómez, Phys. Rev. D 2020

A novel mechanism to implement inflation

- Let's split artificially our vector field fluid into two components: one corresponding to the Yang-Mills term and the other corresponding to the new Proca terms.

- What would happen if, due to the dynamics of the vector field, $\rho_{YM} \to \infty$?

- That means a singularity in the energy-momentum tensor which must be avoided. How?: $\rho_{nP} \to -\infty$ so that $\rho_{nP}/\rho_{YM} \to -1$. Fine tuning??: yes, but the system could "self tune".

Y. Rodríguez and A. A. Navarro, Phys. Dark Univ. 2018

A novel mechanism to implement inflation

- What about the pressure?: since $P_{YM} = \rho_{YM}/3$, we have again a singularity in the energy-momentum tensor unless $P_{nP} \to -\infty$, i.e., $P_{nP}/P_{YM} \to -1$

- The only possible consistent way to do this is if the new Proca component behaves as radiation: $P_{nP} = \rho_{nP}/3$.

- The new Proca component: a radiation-like component with negative energy density and pressure.

Y. Rodríguez and A. A. Navarro, Phys. Dark Univ. 2018

A novel mechanism to implement inflation

- If the two components behave as radiation, could we say that the complete system also behaves as radiation?

- No. 0 divided by 0 could be anything:

$$\omega \equiv \frac{P_{YM} + P_{nP}}{\rho_{YM} + \rho_{nP}}$$

- The actual value of the equation of state parameter depends on the characteristics of the model.

- A realization of this scenario, with $\omega = -1$, i.e., $\epsilon = 0$ and self tuning will be presented in the following.

Y. Rodríguez and A. A. Navarro, Phys. Dark Univ. 2018

Constant-roll inflation ($\ddot{\psi} = \frac{1}{\beta} H \dot{\psi}$) as a non-eternal asymptotic behaviour ($y_p \to \beta x_p$)

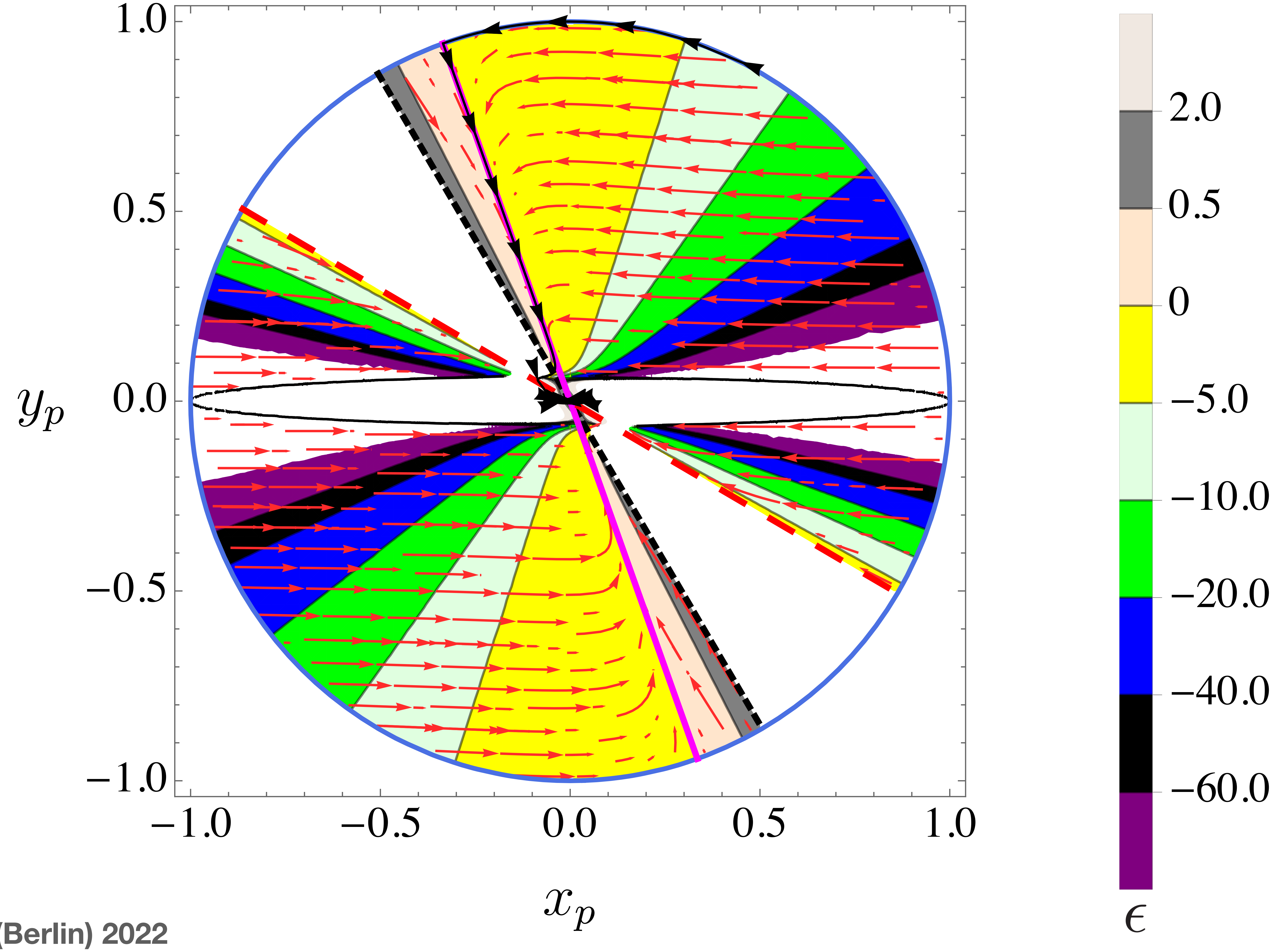
$$x \equiv \frac{\dot{\psi}}{\sqrt{2} m_P H}$$

$$y \equiv \frac{\psi}{\sqrt{2} m_P}$$

- **Cosmic triad:** $B_{0a}(t) = 0$
  $B_{ia}(t) = a(t)\psi(t)\delta_{ia}$

- ***Almost all the available parameter space*** **exhibits a non-eternal asymptotic behaviour that corresponds to an inflationary solution with $\epsilon = 0$.**

- **These results can be made consistent with the gravity waves speed constraint!**



J. C. Garnica, L. G. Gómez, A. A. Navarro, and Y. Rodríguez, Ann. Phys. (Berlin) 2022

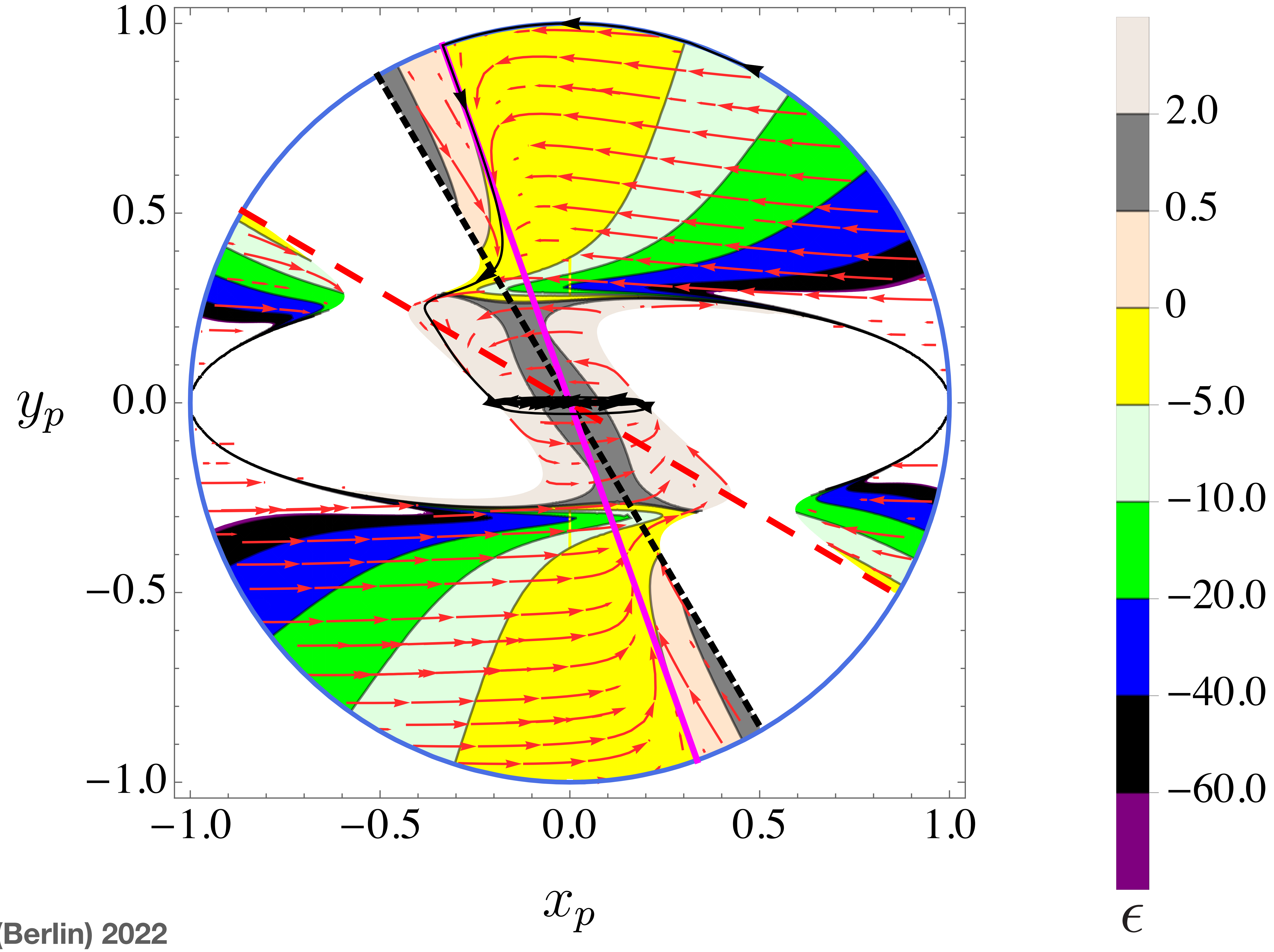
Inflation exit

$$x \equiv \frac{\dot{\psi}}{\sqrt{2} m_P H}$$

$$y \equiv \frac{\psi}{\sqrt{2} m_P}$$

- When x_p and y_p approach 0, solutions leave the asymptotic behaviour and inflation ends through oscillations of ψ around 0 and oscillations of ϵ around 2 (like a scalar inflaton decaying into radiation).

- This scenario is free of ghosts and Laplacian instabilities in the tensor sector since it behaves as general relativity at the perturbative (tensorial) level at least up to second order.



J. C. Garnica, L. G. Gómez, A. A. Navarro, and Y. Rodríguez, Ann. Phys. (Berlin) 2022

Inflation exit

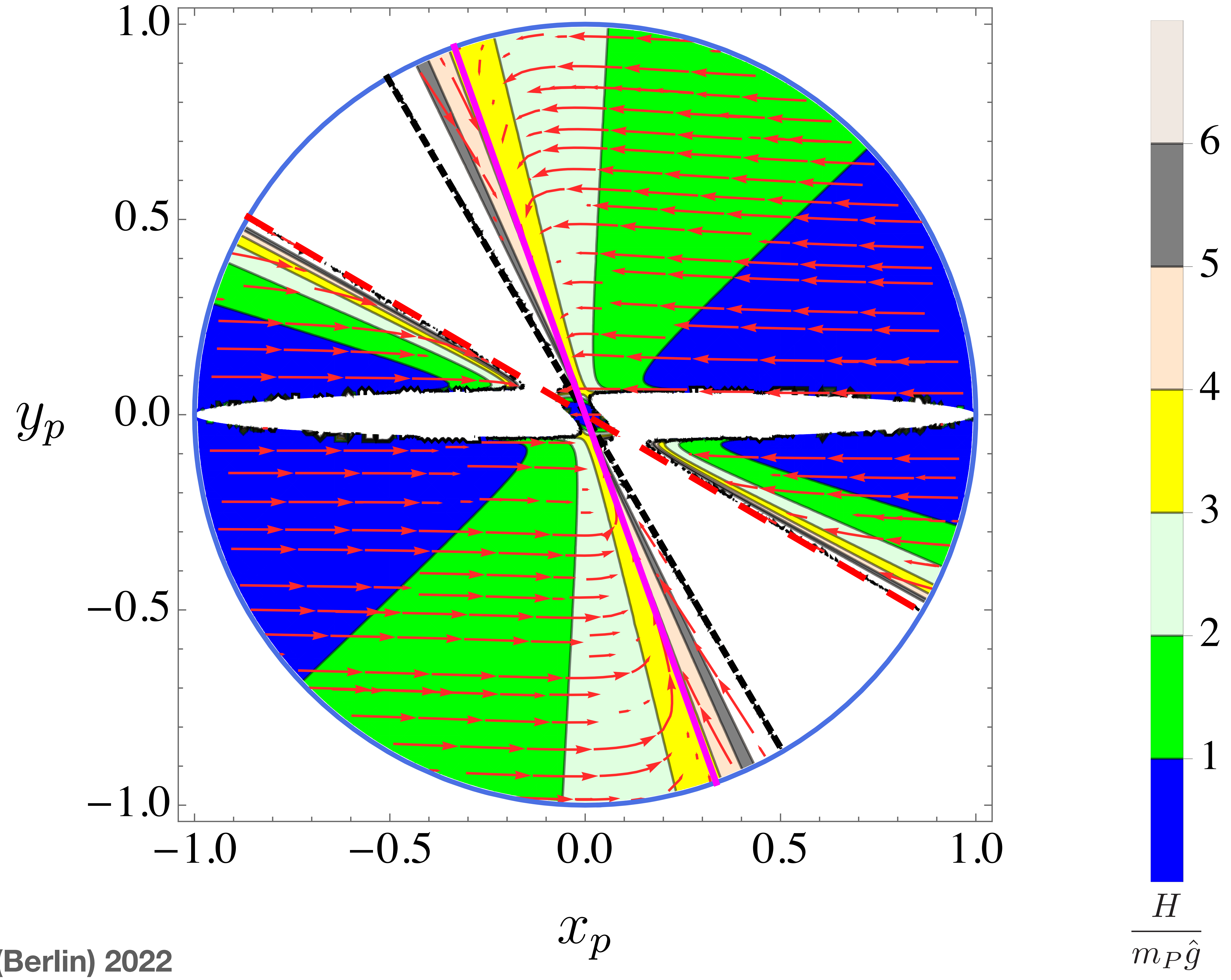
$$x \equiv \frac{\dot{\psi}}{\sqrt{2} m_P H}$$

$$y \equiv \frac{\psi}{\sqrt{2} m_P}$$

- **The Hubble parameter contourplot reveals there are no Big-Bang singularities!**

- **The amount of constant-roll inflation is enough to solve the classical problems of the standard cosmology.**

$$N \approx -\beta \ln |y_{ini}|$$



J. C. Garnica, L. G. Gómez, A. A. Navarro, and Y. Rodríguez, Ann. Phys. (Berlin) 2022

Equations of state parameters

- Analytical results demonstrate that, in the asymptotic limit, when $y \to \beta x$ and $x \to \pm\infty$, the system reveals the following behaviour for energy densities, pressures, and equations of state parameters:

$$\rho_{YM} \to \infty \qquad \rho_{nP} \to -\infty$$

$$P_{YM} \to \infty \qquad P_{nP} \to -\infty$$

$$\omega_{YM} \equiv P_{YM}/\rho_{YM} \to \frac{1}{3} \qquad \omega_{nP} \equiv P_{nP}/\rho_{nP} \to \frac{1}{3}$$

$$\rho_{nP}/\rho_{YM} \to -1 \qquad P_{nP}/P_{YM} \to -1$$

$$\omega \equiv \frac{P_{YM} + P_{nP}}{\rho_{YM} + \rho_{nP}} \to -1$$

- This is consistent with our previous expectations!

Y. Rodríguez and A. A. Navarro, Phys. Dark Univ. 2018

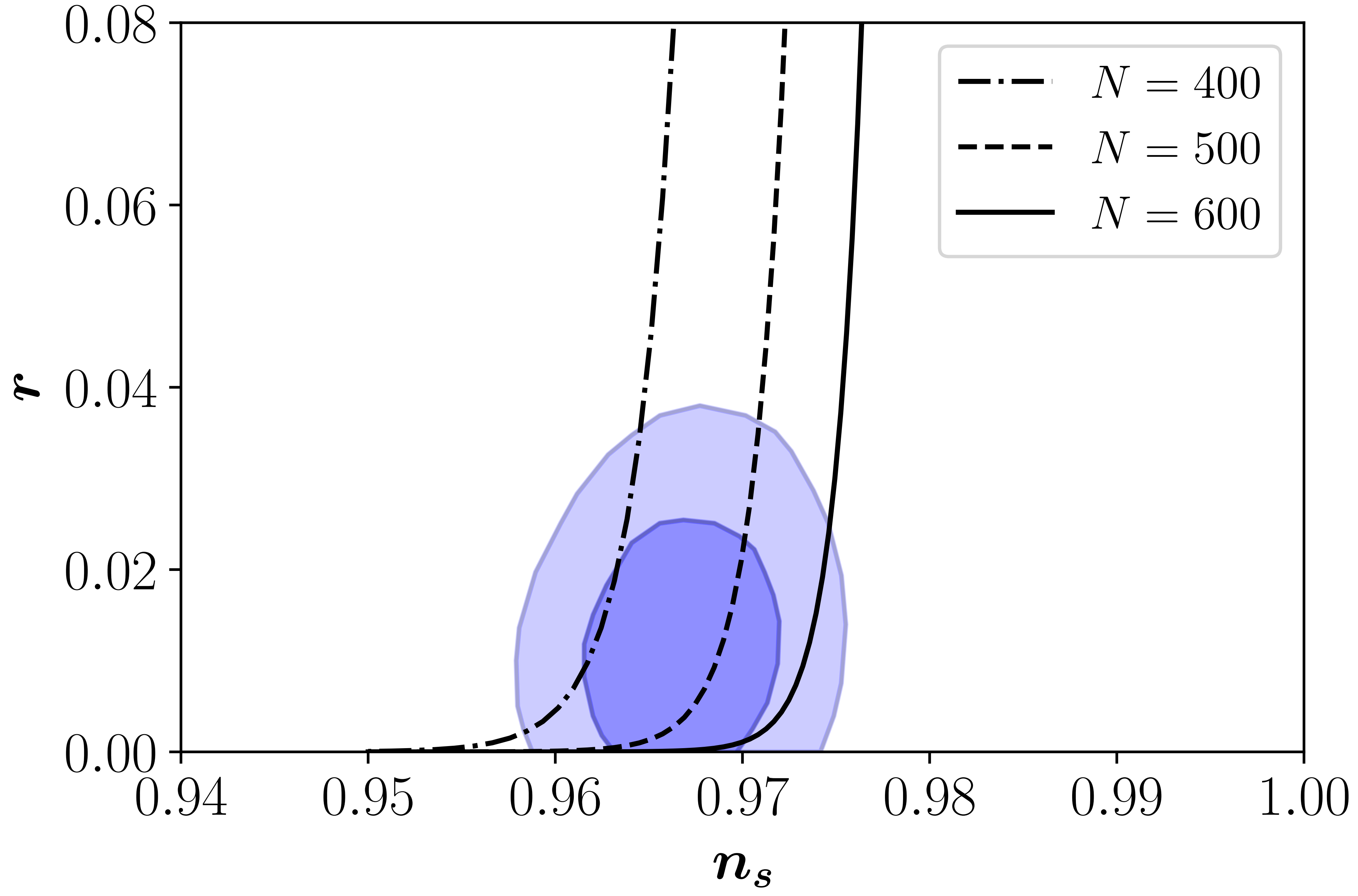
Spectrum of ζ

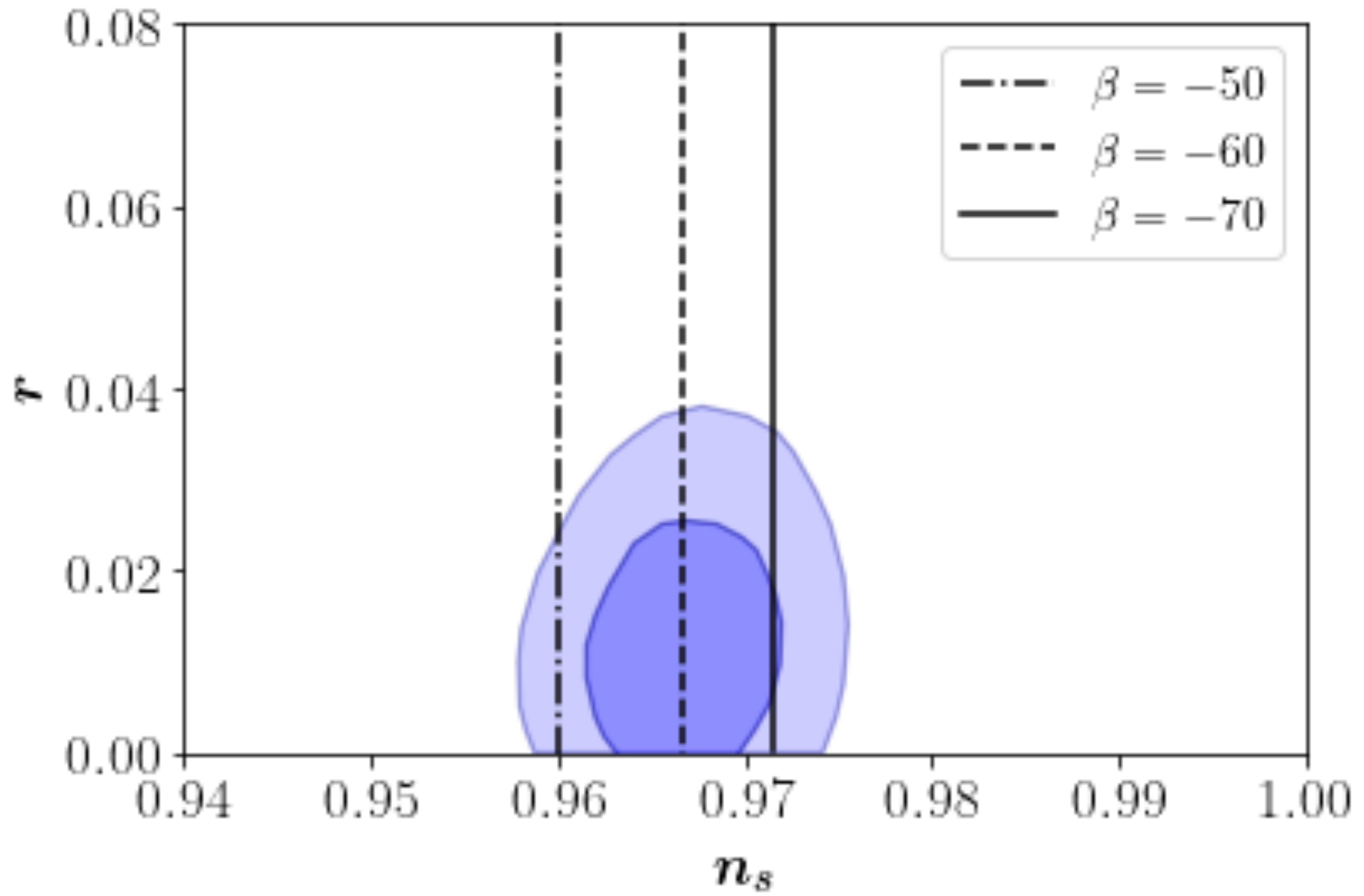
- There is only one degree of freedom in the background: the norm ψ of the fields in the cosmic triad.

- ***Assuming*** the problem can be treated, perturbatively, ***as a single-field scenario***, the spectrum is given by:

$$\mathcal{P}_\zeta = \frac{3\psi_{ini}^2}{2m_P^2\beta^2}\left(\frac{H_*}{2\pi}\right)^2\left(\frac{k}{a_* H_*}\right)^{2\left(\frac{1}{\beta}-\epsilon\right)}$$

- Consistency with observations!





Some open questions

Technical questions:

- Does the theory propagate the right number of degrees of freedom in curved spacetime?
- Stability issues (partially studied in [1] and [2])
- The cutoff scale of the theory

Consistency with observations:

- Does the theory respect the equivalence principle?
- The causal structure of the theory
- Does there exist a screening mechanism?
- Applications to dark energy (partially explored in [3])

[1] L. G. Gómez and Y. Rodríguez, Phys. Rev. D 2019

[2] J. C. Garnica, L. G. Gómez, A. A. Navarro, and Y. Rodríguez, Ann. Phys. (Berlin) 2022

[3] Y. Rodríguez and A. A. Navarro, Phys. Dark Univ. 2018

Thank you!