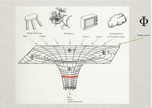

EFT Approach to Black Hole Scalarization and its Compatibility with Cosmic Evolution

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Introduction

- Gravity in the "strong-field" regime is an experimentally unexplored scenario which motivates new physical grounds where modifications of General Relativity (GR) may take place.
- It is well known that within the strong-field regime of gravity, the effects of higher order curvature operators become more relevant.
- "Spontaneous scalarization" is a distinctive manifestation of gravitational interactions in the strong-field regime. Compact objects such as neutron stars [DEF model '70] and black holes can be scalarized [Silva et al '18][Doneva, Yazadjiev '18].
- A theory exhibiting scalarization requires a GR solution with trivial scalar field and a hairy black hole.
- In Einstein scalar-Gauss-Bonnet (ESGB) gravity, spontaneous scalarization takes place whenever the mass of a Schwarzschild black hole is below a critical value. In short, within this regime, the Schwarzchild solution becomes unstable and a new branch of solutions with a nontrivial scalar field bifurcates from the former.

- The aim of this talks is:

  - To address a scalar-tensor effective field theory (EFT) that exhibits curvature-induced scalarization, triggered by a set of suitable invariants made up of Riemann tensor, up to cubic order.

  - To investigate within this framework, how the new operators modify a previously claimed catastrophic instability triggered by quantum fluctuations during the inflationary stage in ESGB theory.

  - To explore the Big Bang Cosmology (BBC) of the model, and check that GR is indeed a late-time cosmological attractor as experiments seem to demand.

- For definiteness, we will consider the so-called Einsteinian Cubic Gravity (ECG) theory [Bueno, Cano '16], which possesses some basic "healthiness" and attractive properties such as:

    - Having a spectrum identical to that of Einstein gravity, i.e., the metric perturbation (on a maximally symmetric background) propagates only a transverse massless graviton.

    - It is neither topological nor trivial in four dimensions.

    - It is defined such that it is independent of the number of dimensions.

- Moreover, ECG admits a spherically symmetric black hole solution [Bueno, Cano '16] and a FLRW solution with a "purely geometric" inflationary period [Arcienaga et al '20].

The Model and Black Hole Scalarization

- We start by recalling the cubic operator $\mathcal{P}$ in ECG theory, which reads

$$\mathcal{P} = 12 R_{\mu\ \nu}^{\ \rho\ \sigma} R_{\rho\ \sigma}^{\ \gamma\ \delta} R_{\gamma\ \delta}^{\ \mu\ \nu} + R_{\mu\nu}^{\ \ \rho\sigma} R_{\rho\sigma}^{\ \ \gamma\delta} R_{\gamma\delta}^{\ \ \mu\nu} - 12 R_{\mu\nu\rho\sigma} R^{\mu\rho} R^{\nu\sigma} + 8 R^{\nu}_{\ \mu} R^{\mu}_{\ \rho} R^{\rho}_{\ \nu},$$

  and the operator $\mathcal{C}$ [Hennigar et al '17] which is given by the combination

$$\mathcal{C} = R_{\mu\nu\rho\sigma} R^{\mu\nu\rho}{}_{\delta} R^{\sigma\delta} - \frac{1}{4} R_{\mu\nu\rho\sigma} R^{\mu\nu\rho\sigma} R - 2 R_{\mu\nu\rho\sigma} R^{\mu\rho} R^{\nu\sigma} + \frac{1}{2} R_{\mu\nu} R^{\mu\nu} R.$$

- In order to explore the phenomenon of scalarization we must include a scalar field φ, while keeping the healthy features of ECG. For simplicity, we impose a $\varphi \to -\varphi$ (discrete) symmetry and a $\varphi \to \varphi +$ constant (shift) symmetry, where the latter is only spoiled by gravitational interactions. The action of the theory is then given by

$$S[g_{\mu\nu}, \varphi] = \int d^4x \sqrt{-g} \left[\frac{M_{\rm Pl}^2}{2} R + \frac{\alpha}{M_{\rm Pl}^2} (\mathcal{P} - 8\,\mathcal{C}) - \frac{1}{2} g^{\mu\nu} \nabla_\mu \varphi \nabla_\nu \varphi + f\left(\varphi/M_{\rm Pl}\right) \mathcal{I} + \cdots \right].$$

  ["Scalar-Einsteinian Cubic Gravity" (SECG)]

- Here, $f(\varphi/M_{\rm Pl})$ is a dimensionless "coupling function" between the canonically normalized scalar field φ and a set of curvature invariants given by

$$\mathcal{I} = -\beta\, M_{\rm Pl}^2 R + \gamma\, \mathcal{G} - \frac{\lambda}{M_{\rm Pl}^2} \left(\mathcal{P} - 8\mathcal{C}\right),$$

  where $\mathcal{G}$ is the well-known Gauss-Bonnet operator $\mathcal{G} \equiv R_{\mu\nu\alpha\beta} R^{\mu\nu\alpha\beta} - 4 R_{\mu\nu} R^{\mu\nu} + R^2$.

- EFT reasoning leads us to expect that the *dimensionless* coupling constants α, β, γ, and λ are $\mathcal{O}(1)$ numbers.
- We will *not* set the (reduced) Planck scale $M_{\rm Pl} = 2.4 \times 10^{18}$ GeV to unity as it is usually done in the literature since we want to keep track of it to easily emphasize its role of being the ultimate EFT cut-off of any gravitational system.
- The equations of motion (EOM) that stem from extremizing the action $S[g_{\mu\nu}, \varphi] = \int d^4x \sqrt{-g}\,\mathcal{L}$ read

$$R^{\alpha\beta\rho}{}_{\mu} P_{\nu\rho\alpha\beta} + 2\nabla^{\alpha}\nabla^{\beta} P_{\alpha\mu\nu\beta} + \frac{1}{2}\nabla_{\mu}\varphi\nabla_{\nu}\varphi + \frac{1}{2}g_{\mu\nu}\mathcal{L} = 0,$$
$$\Box\varphi + f_{,\varphi}\left(\varphi/M_{\rm Pl}\right)\mathcal{I} = 0,$$

  where $P_{\alpha\beta\mu\nu} \equiv \frac{\partial \mathcal{L}}{\partial R^{\alpha\beta\mu\nu}}$.
- The EOM for the scalar field fluctuation $\delta\varphi \equiv \varphi - \varphi_0$ is given by

$$\Big[\Box + f_{,\varphi\varphi}(\varphi_0/M_{\rm Pl})\,\mathcal{I}\Big]\delta\varphi = 0,$$

  where φ_0 is the scalar field background, while the d'Alembertian operator and the curvature invariant $\mathcal{I}$ are computed in a fixed background.

- We need to demand that $f_{,\varphi}(0) = 0$ so that GR vacuum solutions together with $\varphi_0 = 0$ are admissible solutions of the field equations.

- Moreover, $f_{,\varphi\varphi}(0) > 0$ is necessary for the emergence of a tachyonic instability, which triggers the phase transition.

- Finally, one can show that linearized Einstein field equations in SECG are the same as in ECG through these scalarization conditions.

- Without loss of generality we then take $f(x) = \frac{1}{2}x^2 + \ldots$, impliyng a scalar field fluctuation effective mass squared given by

$$m_{\rm eff}^2 = -f_{,\varphi\varphi}\left(\varphi_0/M_{\rm Pl}\right)\mathcal{I} = \beta R - \frac{\gamma}{M_{\rm Pl}^2}\mathcal{G} + \frac{\lambda}{M_{\rm Pl}^4}\left(\mathcal{P} - 8\mathcal{C}\right).$$

- We must recall that we are interested in models that exhibit spontaneous scalarization around compact objects such as Schwarzschild black holes for which $R = 0$ and $\mathcal{G} > 0$. As the cubic operator is further suppressed by the cut-off for natural values of λ, this implies the condition $\gamma > 0$. Hereafter, we will take $\boldsymbol{\gamma > 0}$.

- Perturbations on a fixed Schwarzschild background may be decomposed as

$$\delta\varphi = \frac{u(r)}{r} e^{-i\omega t}\, Y_{lm}\,(\theta, \phi).$$

  In tortoise coordinates r_*, the Klein-Gordon equation becomes "Schrödinger-like", meaning

$$\frac{d^2 u}{dr_*^2} + \omega^2 u = V_{\rm eff}(r) u,$$

  where $V_{\rm eff}$ is some effective potential. It can be shown that there exists a sufficient condition for the existence of an unstable mode given by

$$\int_{r_g}^{\infty} dr\, \frac{V_{\rm eff}(r)}{\left(1 - \frac{r_g}{r}\right)} < 0,$$

  where $r_g \equiv \mathcal{M}/4\pi M_{\rm Pl}^2$, and $\mathcal{M}$ stands for the black hole mass.
- Such a condition implies that

$$\left(\frac{\mathcal{M}}{M_{\rm Pl}}\right)^2 \in [q_-, q_+] \qquad \text{if} \quad 0 < \lambda \leq \frac{48}{175}\,\gamma^2,$$

$$\left(\frac{\mathcal{M}}{M_{\rm Pl}}\right)^2 \in [0, q_+] \qquad \text{if} \quad \lambda \leq 0,$$

  where we have defined

$$q_\pm \equiv \frac{16\,\pi^2}{5}\left(12\,\gamma \pm \sqrt{144\,\gamma^2 - 525\,\lambda}\right).$$

- We see that a Schwarzschild background is unstable for a specific range of masses given by the above bounds. Indeed scalarization may only occur whenever

$$\lambda \leq \frac{48}{175}\gamma^2,$$

which is a non-trivial constraint between otherwise completely independent Wilson coefficients of the EFT. Note that this constraint is still compatible with both γ and λ being $\mathcal{O}(1)$ numbers. However, the sign of λ is not fixed by this condition.

- As it stands, the theory predicts that the maximum mass of Schwarzschild black holes that may become scalarized is

$$\mathcal{M}_{\mathsf{MAX}} \sim 10^{-37} M_{\odot},$$

precluding any possibility of such a version of SECG theory to be compared with observations. We will have more to say about this soon enough.

- Let us now consider how scalarization may occur within a cosmological setting.

Perturbations on a FLRW Background

- In a FLRW background with metric

$$ds^2 = g_{\mu\nu}\, dx^\mu dx^\nu = -dt^2 + a(t)^2 \delta_{ij}\, dx^i dx^j, \quad \text{and} \quad H \equiv \frac{\dot{a}}{a},$$

  it so happens that the EOM for the fluctuation reads

$$\delta\ddot{\varphi} + 3H\delta\dot{\varphi} - \frac{\nabla^2 \delta\varphi}{a^2} + m_{\text{eff}}^2\, \delta\varphi = 0,$$

  which, by Fourier expanding $\delta\varphi \sim \int d\omega\, d^3\boldsymbol{k}\, \delta\varphi(\omega, \boldsymbol{k})\, e^{-i(\omega t - \boldsymbol{k}\cdot\boldsymbol{x})}$, implies that

$$\omega^2 = \frac{k^2}{a^2} + m_{\text{eff}}^2,$$

  when neglecting the slow change of ω on the time scales shorter than H^{-1}.
- Moreover, for a FLRW spacetime

$$R = 6\left(2H^2 + \dot{H}\right), \quad \mathcal{G} = 24H^2\left(H^2 + \dot{H}\right), \quad \mathcal{P} - 8\mathcal{C} = -48H^4\left(2H^2 + 3\dot{H}\right).$$

- In the ESGB theory ($\beta = \lambda = 0$) we get

$$m_{\text{eff}}^2 = -\frac{24\,\gamma}{M_{\text{Pl}}^2} H^2\left(H^2 + \dot{H}\right) = -\frac{24\,\gamma}{M_{\text{Pl}}^2} H^2\, \frac{\ddot{a}}{a},$$

  recalling that we are always taking $\gamma > 0$.

- Then $m_{\text{eff}}^2 < 0 \Longleftrightarrow \ddot{a} > 0$ implying the existence of a tachyonic instability during any (quasi)-de Sitter (dS) phase of our universe.
- In the general case using the standard definition $\epsilon \equiv -\dot{H}/H^2$ we may write

$$m_{\text{eff}}^2 = 12 \left[\beta \left(1 - \frac{\epsilon}{2}\right) - 2\,\gamma\,(1-\epsilon)\,\zeta - 8\,\lambda \left(1 - \frac{3}{2}\,\epsilon\right) \zeta^2 \right] H^2,$$

  as an expansion in the (very) small ratio $\zeta \equiv \left(\frac{H}{M_{\text{Pl}}}\right)^2$.

- Therefore during inflation, a tachyonic instability for which $m_{\text{eff}}^2 < 0$, $\zeta \sim 10^{-11}$, and $\epsilon \ll 1$, will take place whenever
  - $\beta \neq 0, \gamma = 0, \lambda = 0 \implies \beta < 0$
  - $\beta \neq 0, \gamma \neq 0, \lambda = 0 \implies \dfrac{\beta}{\gamma} \lesssim \mathcal{O}(10^{-11})$
  - $\beta \neq 0, \gamma = 0, \lambda \neq 0 \implies \dfrac{\beta}{\lambda} \lesssim \mathcal{O}(10^{-22})$
- In short, our EFT approach to the theory which assigns natural values for all the coupling constants, univocally demands $\boldsymbol{\beta < 0}$ for the tachyonic instability to take place within a cosmological setting.

- Within the "ESGB limit", one can show that the possibility of scalarizing astrophysical compact objects introduces a hierarchy problem since $\boldsymbol{\gamma \sim 10^{74}}$. Sweeping this fact under the rug for a second, we may estimate the ratio of the instability time $t_{\rm inst}$ to the age of the universe $t_0 \sim 1/H_0$ ($H_0 \simeq 10^{-43}$ GeV) obtaining

$$\frac{t_{\rm inst}}{t_0} \sim \frac{H_0}{m_{\rm eff}} \sim \frac{1}{2\sqrt{6\,\gamma}}\frac{M_{\rm Pl}}{H_0} \sim 10^{23}.$$

  Therefore, we may conclude that the instability is not noticeable during current dark energy domination.

- On the other hand, during inflation the estimation delivers

$$\frac{t_{\rm inst}}{t_{\rm inf}} \sim \frac{1}{N}\frac{H_{\rm inf}}{m_{\rm eff}} \sim \frac{1}{2\sqrt{6\,\gamma}N}\frac{M_{\rm Pl}}{H_{\rm inf}} \sim 10^{-34},$$

  where $N \sim 10^2$ is the required number of e-folds to overcome the classical shortcomings of BBC. One may then be tempted to conclude that inflation is *not* compatible with the phenomenon of black hole scalarization.

- However, we argue that there is just *no* reason for setting $\beta = 0$ from an EFT point of view, invalidating such a hazy claim.

- In order to cure the the highly unnatural value for γ we need to introduce a new scale M within the coupling sector operators.
- One can then show that scalarization of astrophysical black holes with characteristic length scale $L \equiv M^{-1} \sim 10$ km, implies

$$M = 1.98 \times 10^{-20}\,\text{GeV} \Rightarrow \gamma \sim 10^{-2},$$

which is a much more sensible number than the one we had before.
- We see that the introduction of the new energy scale M "naturalizes" an otherwise finely tuned EFT. Moreover, such a scale is actually related to the physical extent of the compact object to be scalarized.
- Furthermore, it is possible to show that the maximum mass of Schwarzschikd black holes that may become scalarized grows into

$$\mathcal{M}_{\text{MAX}} \sim 180 M_{\odot},$$

which are clearly way better news for observational prospects.

- However, the ratio between the instability time $t_{\rm inst}$ to the age of the universe t_0 is now given by

$$\frac{t_{\rm inst}}{t_0} \sim \frac{H_0}{m_{\rm eff}} \sim \frac{1}{\sqrt{12\,|\beta|}} \sim \frac{0.29}{\sqrt{|\beta|}}.$$

- Moreover, in the case of inflation, with $N \sim 10^2$, the relevant ratio now goes like

$$\frac{t_{\rm inst}}{t_{\rm inf}} \sim \frac{1}{N}\frac{H_{\rm inf}}{m_{\rm eff}} \sim \frac{1}{\sqrt{12\,|\beta|}N} \sim \frac{2.9\times 10^{-3}}{\sqrt{|\beta|}}.$$

- In short, the above means that the tachyonic instability will arise during both the inflationary period and during dark energy domination.

- The only way out to this problem which preserves the expected hierarchy in the set of curvature operators, admits black hole scalarization of astrophysical black holes, and is compatible with standard big bang cosmology (BBC) is to impose $\boldsymbol{\beta > 0}$, $\boldsymbol{\gamma > 0}$ with $\boldsymbol{\beta \sim \gamma \sim \lambda \sim \mathcal{O}(1)}$.

- Let us finally address how well this model fits within BBC.

General Relativity as a Cosmic Attractor

- The scalar EOM in a FLRW background is given by

$$\ddot{\varphi} + 3H\dot{\varphi} + m_{\text{eff}}^2\,\varphi = 0 \quad \text{where} \quad m_{\text{eff}}^2 = \beta R - \frac{\gamma}{M^2}\mathcal{G} + \frac{\lambda}{M^4}\left(\mathcal{P} - 8\mathcal{C}\right)$$

  while the "t-t" Einstein equation reads

$$M_{\text{Pl}}^2 G_{tt} = \rho_{\text{eff}} + \rho_a,$$

  with

$$\rho_{\text{eff}} \equiv \rho_{\mathcal{PC}} + \rho_\varphi, \quad \rho_{\mathcal{PC}} = -\frac{48\,\alpha}{M_{\text{Pl}}^2}H^6,$$

$$\rho_\varphi = \frac{1}{2}\dot{\varphi}^2 + 6\left(\beta - 4\,\gamma\,\chi - 24\,\lambda\,\chi^2\right)H\varphi\dot{\varphi} + 3\left(\beta + 8\,\lambda\,\chi^2\right)H^2\varphi^2, \text{ and } \chi \equiv (H/M)^2.$$

- We shall demand usual cosmic evolution, i.e.

$$\rho_a \approx 3M_{\text{Pl}}^2 H^2,$$

  with $\rho_a = \{\rho_{\text{r}}, \rho_{\text{m}}, \rho_{\text{de}}\}$, implying that $|\rho_{\mathcal{PC}}| \ll \rho_a$, or equivalently $\alpha \ll (M_{\text{Pl}}/H)^4$, which is always trivially fulfilled.
- Since we do not want φ to play any role in late-time cosmology, we shall assume that

$$\rho_\varphi \ll \rho_a.$$

  We acknowledge that it is mandatory to check if such an assumption is dynamically consistent.

- In order to solve the scalar field EOM and to analize the dynamical evolution of its energy density during BBC, it is useful to trade cosmic time t by the redshift z.
- The numerical analysis is set to start at $z_i = 10^{10}$, right before big bang nucleosynthesis (BBN) epoch. A natural initial value for the dimensionless field $\tilde{\varphi} \equiv \varphi/\sqrt{2}M_{\rm Pl}$ is given by $\tilde{\varphi}_i = \frac{\varphi_i}{\sqrt{2}M_{\rm Pl}} \simeq \frac{H}{\sqrt{2}M_{\rm Pl}} \ll 1$.
- In Figure 1 below we show the evolution of both the dimensionless scalar field φ/φ_i and the dimensionless ratio ρ_φ/ρ_a for $z < z_i$ for different values of β and fixed values of γ and λ.

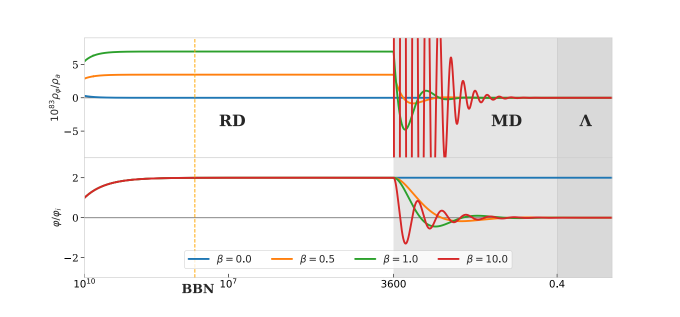

Figure 1: *Top panel*: Effective energy density ρ_φ relative to the energy density of the cosmic fluid ρ_a. *Bottom panel*: Scalar field value relative to its initial value fixed at $z_i = 10^{10}$. The values of the coupling constants are taken to be $\gamma = 1$ and $\lambda = 48/175$.

- As previously noted, the contributions from higher-order curvature terms do become relevant during the very early stage of the universe. We may explicitly observe this, for high redshift $z > z_i$, in Figure 2 below.

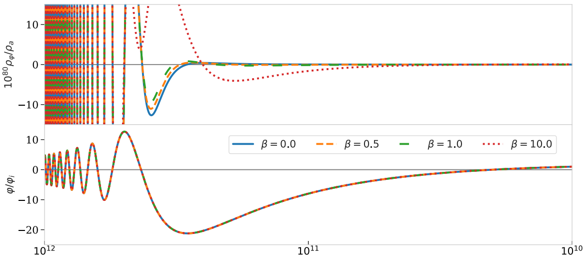

Figure 2: Relative effective density and scalar field value for high redshift. The values of the coupling constants are the same as those of Figure 1.

Comments

- We confirm that the solution is *strongly* consistent with our initial assumption, since $\rho_\varphi(z) \lll \rho_a(z)$ for the whole range of numerical integration which goes from $z = 0$ to $z = 10^{12}$.
- The scalar field φ is basically a constant throughout most of such a range, with the exception of cosmological "phase transitions" redshifts and very early and late times.
- During early times (high redshift) m^2_{eff} dominates over Hubble friction within the scalar field equation. However, as we "move" forward in time, m^2_{eff} decays much faster than the Hubble friction which rapidly takes over, so it is expected that the scalar field freezes to a constant way before entering matter domination (MD) era.
- For even higher redshift values the relative scalar field and the relative energy density oscillate with ever increasing frequency as can be seen in Figure 2.

- During radiation domination (RD) the scalar field is completely insensitive to the value of β as the Ricci scalar identically vanishes, while the relative density does marginally depend on such a constant even though all the curves, for high enough z, eventually converge.

- On the other hand, by the time the MD era begins, the Ricci scalar stops being trivial, and in fact it entirely determines the relative scalar and energy density evolution because the higher-order operators become irrelevant considering that $\chi \sim 10^{-36} \lll 1$ for $z = 3600$.

- Importantly, we do observe that the strict $\beta = 0$ case is actually problematic.

- Within this context, the effective Newton constant becomes $G_{\text{eff}} \equiv G / \left(1 - 2\,\beta\,\tilde{\varphi}^2\right)$ with $\tilde{\varphi} \equiv \varphi/\sqrt{2}M_{\text{Pl}}$, which is a negligible correction to the GR value.

- As it was expected, the scalar field profile in SECG exhibits a manifest deviation from its quadratic counterpart for very high cosmological redshifts, as can be appreciated in the two following figures.

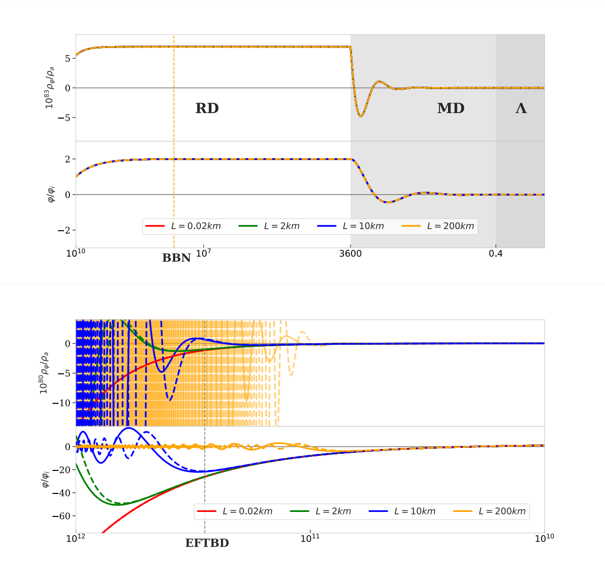

Figure 3: The continuous and dashed curves represent the profile stemming from ESGB and SECG, respectively.

Some More Comments

- ESGB and SECG theories depart when applied to a compact object of size $L = 10$ km., around $z \approx 3.7 \times 10^{11}$ (RD), when the size of the universe was ~ 4 km.
- Clearly, such a redshift value marks the “breakdown” of the EFT expansion (EFTBD), in the sense that perturbativity is lost and we should *not* trust the naive model anymore.
- Any behavior of the system beyond the EFTBD point should not be taken seriously as it does not represent sensible perturbative physics because the system becomes strongly-coupled.
- New non-trivial physics in the form of an ultraviolet (UV) completion is required.
- This implies that the bigger the scale M, the further back in time the EFT is appropriate to describe the phenomenon of scalarization within a cosmological setting.

Ending remarks

- We have followed the principles of EFT in order to address the issue of black hole scalarization and its compatibility with standard cosmic evolution in higher-order theories of gravity, using what we have dubbed "scalar-Einstenian Cubic Gravity" as a well-motivated, healthy toy model.

- By using "naturalness" arguments, a "foreign" new scale in the problem was introduced, which is actually associated with the physical extension of the would-be scalarized compact object.

- After the introduction of such a scale, the scalarization bound was increased from $10^{-37} M_{\odot}$ to $180 M_{\odot}$ in the ESGB limit.

- Unlike ESGB, scalarization in SECG scenario is restricted by an upper bound of the curvature at the event horizon.

- We have confirmed the fact that the relative signs of the dimensionless parameters of the model play a crucial role within the process of spacetime scalarization as they actually determine whether or not scalarized solutions may emerge from dynamics.

- From an EFT perspective picking the right sign for $\varphi^2 R$ operator in the theory is enough to render inflation safe from the tachyonic instability while still scalarizing compact objects through the irrelevant operator $\varphi^2\mathcal{G}$, hence scalarization remains compatible with the inflationary paradigm.

- We integrated the scalar field equation to find that, under very sensible assumptions for the initial conditions of the system, the theory admits GR as a cosmological attractor.

- Departures from GR do become significant for high enough redshift $\left(z \sim 10^{11}\right)$, way before BBN.

Muito Obrigado!