Gauge Invariance in the Light-Cone

Based on JCAP02(2021)014

In collaboration with G. Fanizza, G. Marozzi, G. Schiaffino

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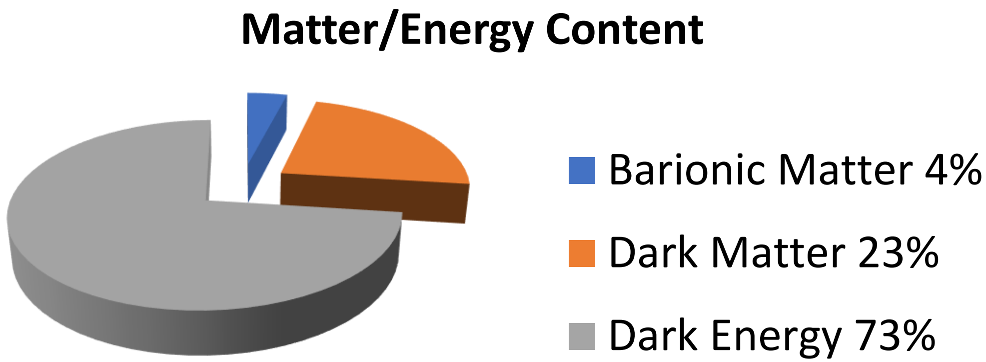
Outline and Motivations

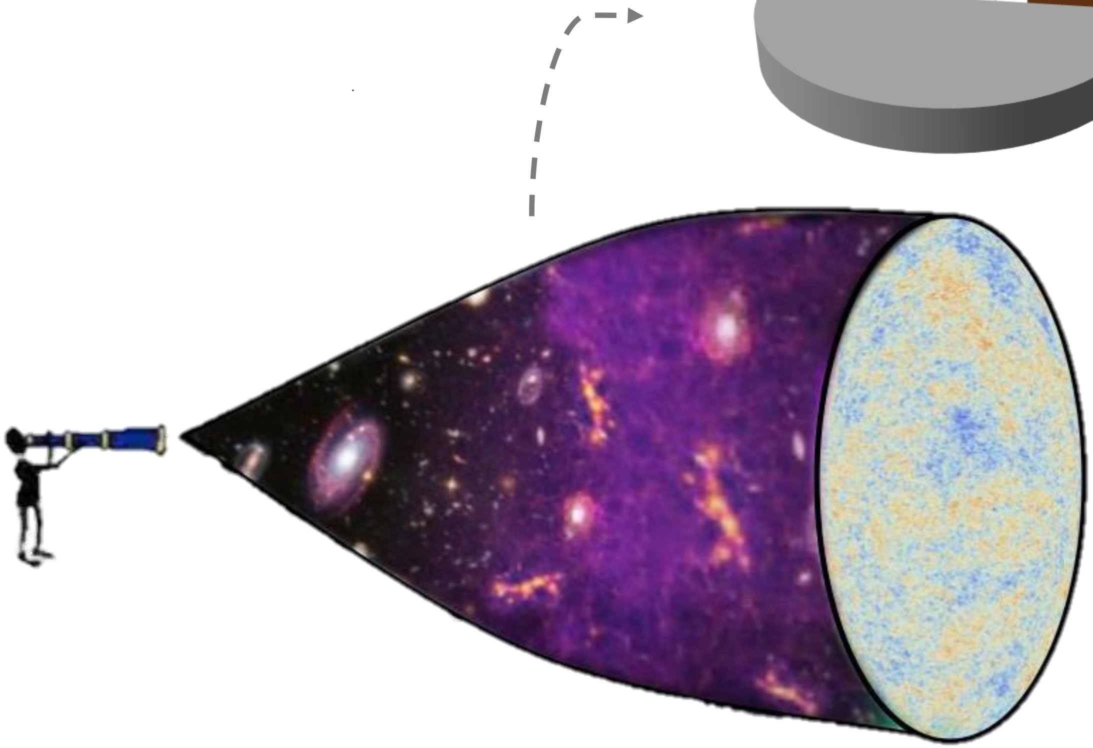
- Introduction
  - The non-linear Geodesic Light Cone (GLC) gauge
  - Standard Cosmological Perturbation Theory
- GLC Perturbation Theory
- Standard FLRW/GLC relation
- Scalar-Vector-Tensor/Scalar-Pseudo-Scalar relation
- Gauge Invariant Tensor
- Helicity decomposition
- Conclusion

Introduction

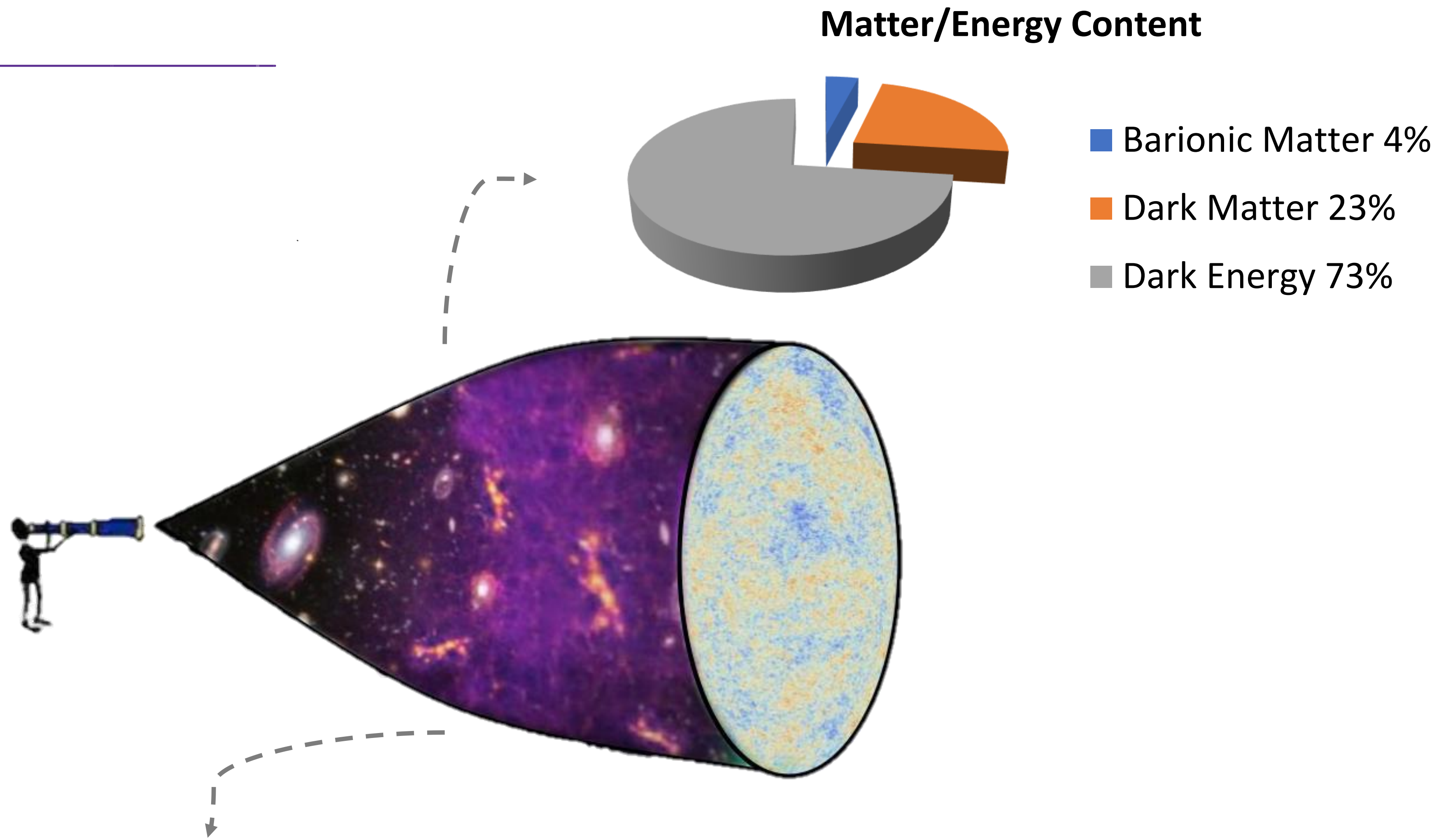
Introduction

Introduction





Introduction



A statistically homogeneous and isotropic space-time with dynamics given by Einstein’s equations

Introduction

All cosmological observables are measured on our past light-cone!!!

However, the standard framework to compute cosmological observables at perturbative level, relies on decomposing the perturbations on the spatial hypersurface due to the SO(3) background symmetries.

Does a past-light-cone adapted framework provides a simplification on the description of cosmological observables?

Testing the cosmological principle with coordinates adapted to observations instead of symmetries

- Temple's optical coordinates: *New Systems of Normal Co-ordinates for Relativistic Optics*, Royal Society of London Proceedings Series A **168** (1938).
- Observational coordinates: *Ideal observational cosmology*, G. F. R. Ellis, S. D. Nel, R. Maarstens, W. R. Stoeger, A. P. Withman, Phys. Rep. **124** (1985)
- Geodesic Light-Cone (GLC) coordinates: M. Gasperini, G. Marozzi, F. Nugier, and G. Veneziano, *Light-cone averaging in cosmology: Formalism and applications*, JCAP **1107** (2011)

Introduction

Does a framework adapted to the past-light-cone simplifies the description of cosmological observables?

It was showed by Fanizza, Gasperini, Marozzi and Veneziano, that in the GLC gauge the Jacobi Map can be solved explicitly in terms of the GLC metric entries for a general geometry of space-time.

- G. Fanizza, M. Gasperini, G. Marozzi, G. Veneziano, *An exact Jacobi map in the geodesic light-cone gauge*, JCAP **11** (2013)

Even though the GLC program offers simple and non-approximative equations for cosmological observables. The dynamics are very involved. Therefore, a relation between the standard SVT and the GLC perturbations are welcome, which additionally may help to disentangle the physical effects in observables expressions!!

- G. Fanizza, M. Gasperini, G. Marozzi, G. Veneziano, *An exact Jacobi map in the geodesic light-cone gauge*, JCAP **11** (2013)

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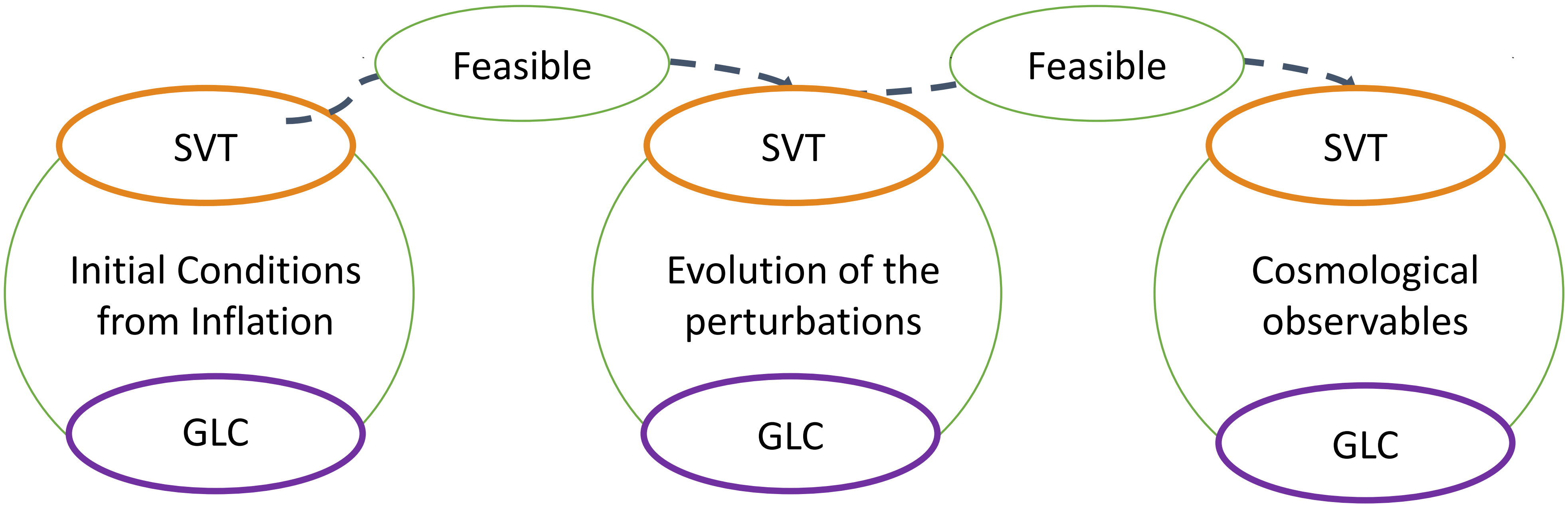
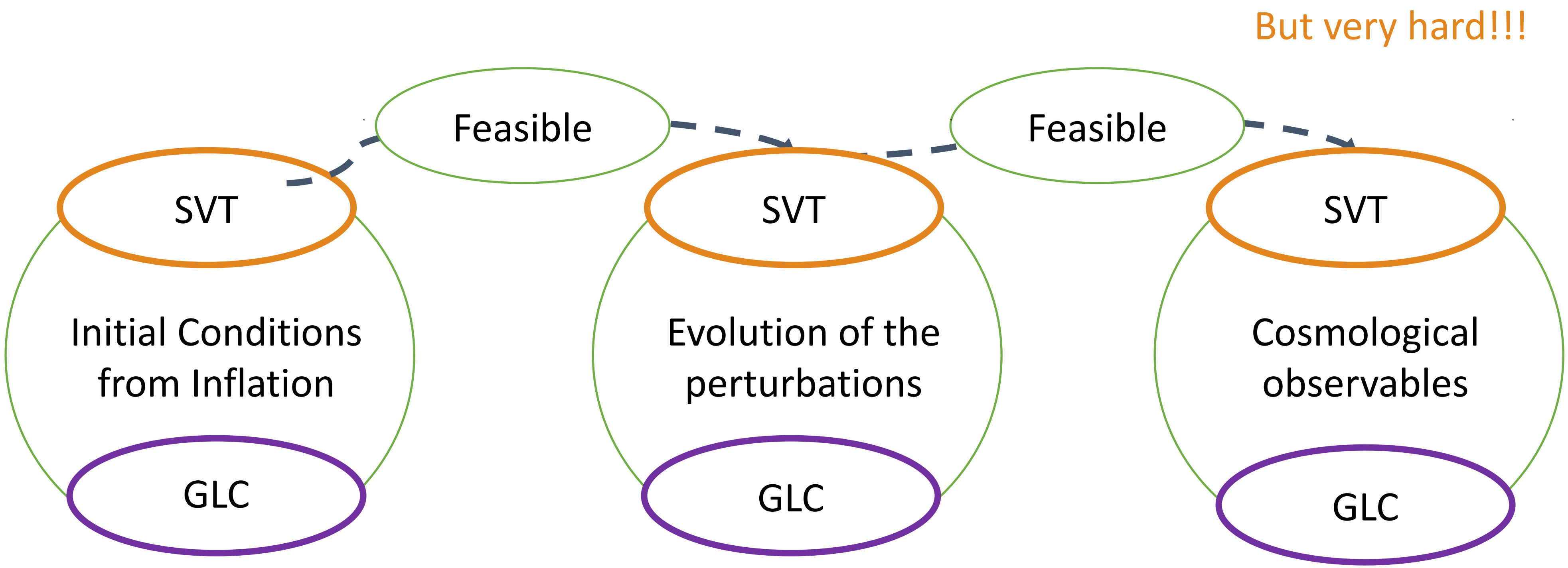
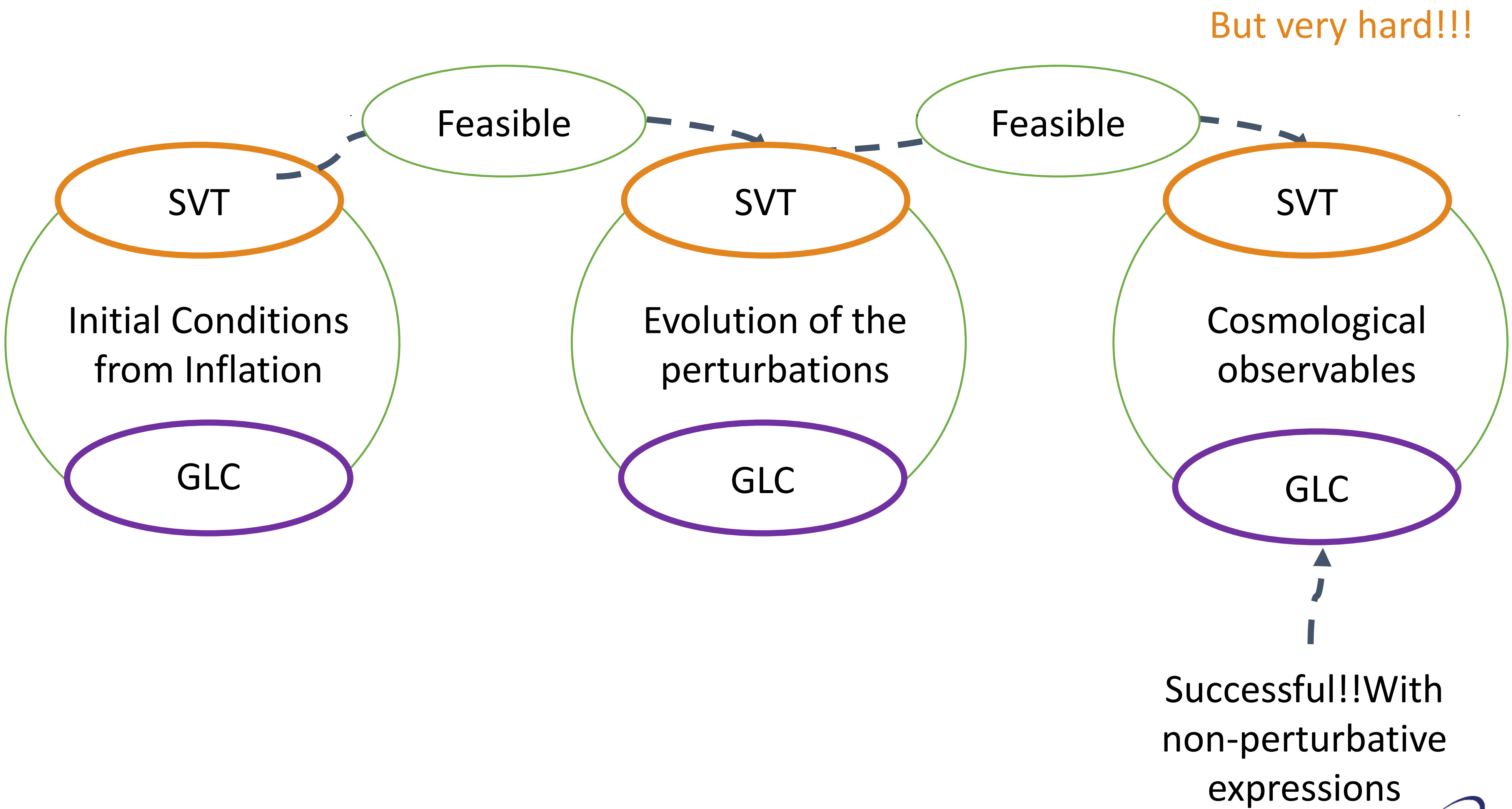



But very hard!!!
Feasible
Feasible
SVT
SVT
SVT
Initial Conditions from Inflation
Evolution of the perturbations
Cosmological observables
GLC
GLC
GLC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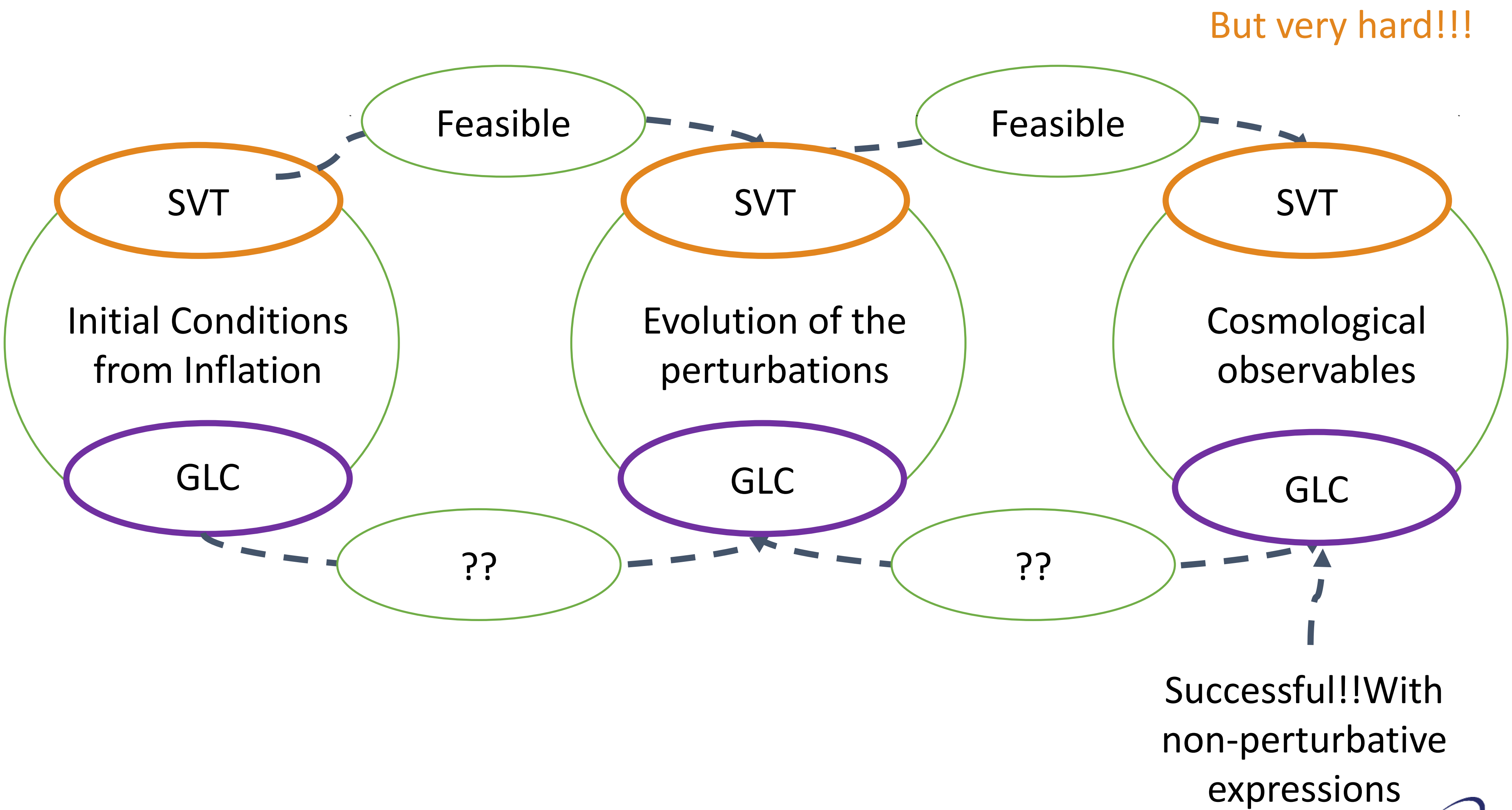
Successful!!With non-perturbative expressions

Introduction



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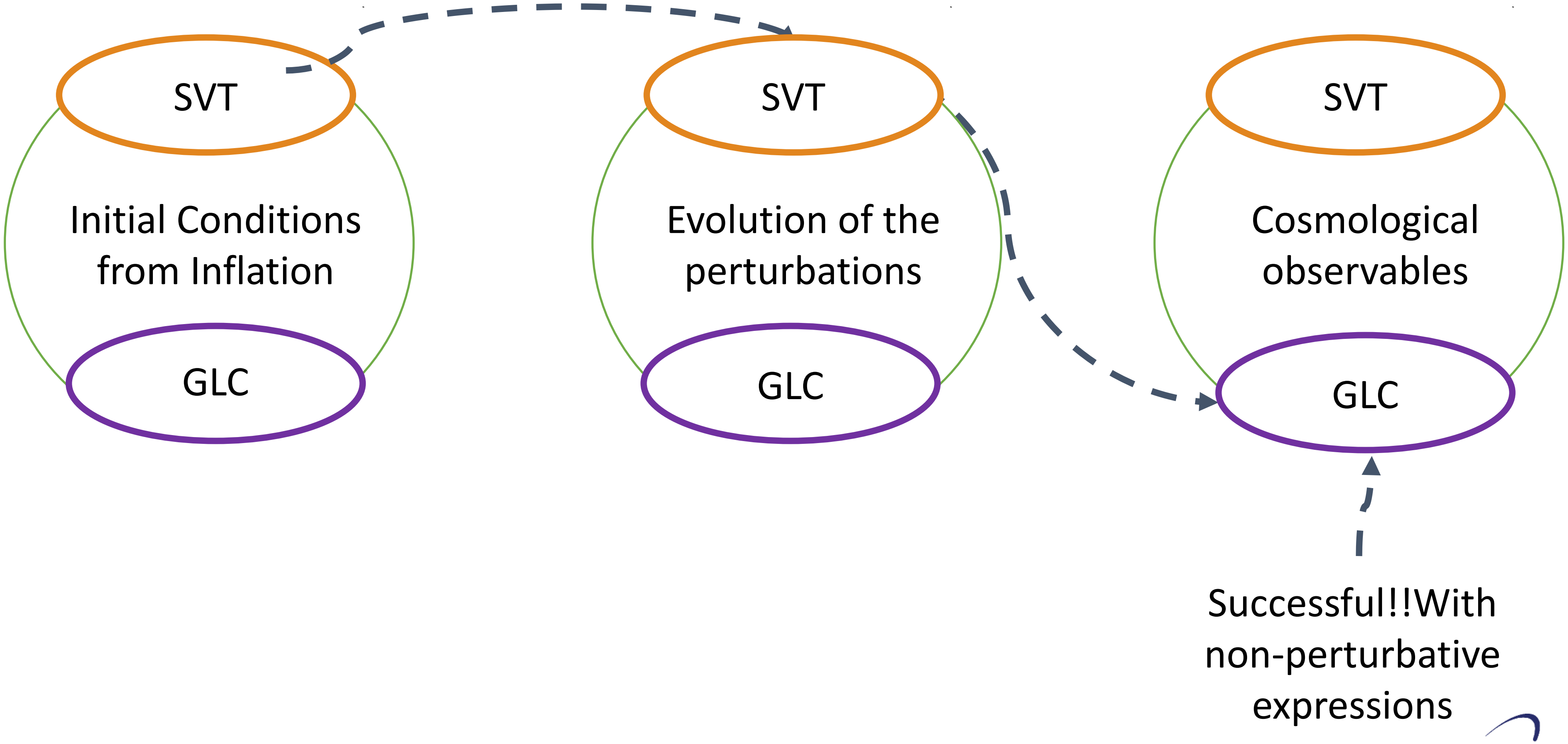
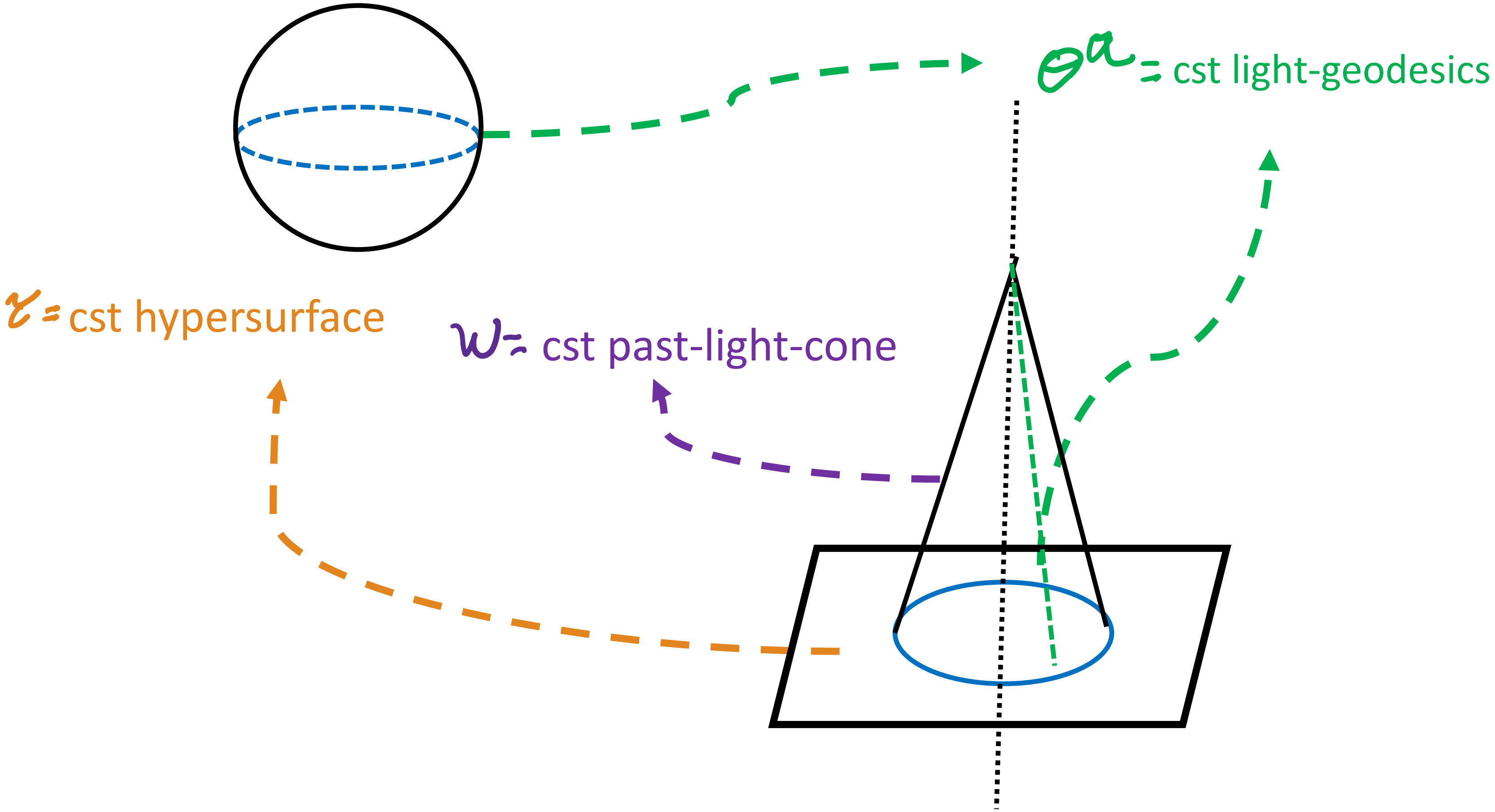


GLC Gauge

The non-linear GLC gauge



The non-linear GLC gauge

$$ds^2 = \Upsilon^2 dw^2 - 2\Upsilon dw d\tau + \gamma_{ab}\left(d\theta^a - U^a dw\right)\left(d\theta^b - U^b dw\right)$$

- Simplify the light-like geodesics

$$u_\mu = -\partial_\mu \tau = -\delta^\tau_\mu \qquad k^\mu = \omega\left(u^\mu + d^\mu\right)$$

$$k^\mu = \omega \delta^\mu_\tau \qquad \omega = k^\mu u_\mu$$

- M. Gasperini, G. Marozzi, F. Nugier, G. Veneziano, JCAP07(2011)008

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$$(1+z) = \frac{\Upsilon_o}{\Upsilon_s}$$

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Standard Cosmological Perturbation Theory

Metric Perturbations

- In the standard perturbation theory in FLRW geometry

$$g_{\mu\nu}\left(\eta, x^i\right) = \bar{g}_{\mu\nu}\left(\eta\right) + \delta g_{\mu\nu}\left(\eta, x^i\right)$$

$$\delta g_{\mu\nu}\left(\eta, x^i\right) = a^2\left(\bar{\eta}\right)\begin{pmatrix} -2\phi & -\mathcal{B}_i \\ -\mathcal{B}_j & C_{ij} \end{pmatrix}$$

$$\bar{g}_{\mu\nu}\left(\bar{\eta}\right) = a^2\left(-1, \bar{\gamma}_{ij}\right)$$

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- Due to the SO(3) symmetries of $\bar{\gamma}_{ij}$, we may decompose the perturbations as Scalars Vectors and Tensors (SVT)

$$\mathcal{B}_i = B_i + \partial_i B \qquad \nabla_i B^i = 0$$

$$C_{ij} = -2\psi\bar{\gamma}_{ij} + 2\bar{D}_{ij}E + 2\nabla_{(i}F_{j)} + 2h_{ij}$$

$$\nabla_i F^i = 0 \qquad \nabla^i h_{ij} = 0 \qquad \bar{\gamma}^{ij} h_{ij} = 0$$

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$$D_{ij} = \nabla_{(i}\partial_{j)} - \frac{1}{3}\bar{\gamma}_{ij}\Delta_3$$

Metric Perturbations

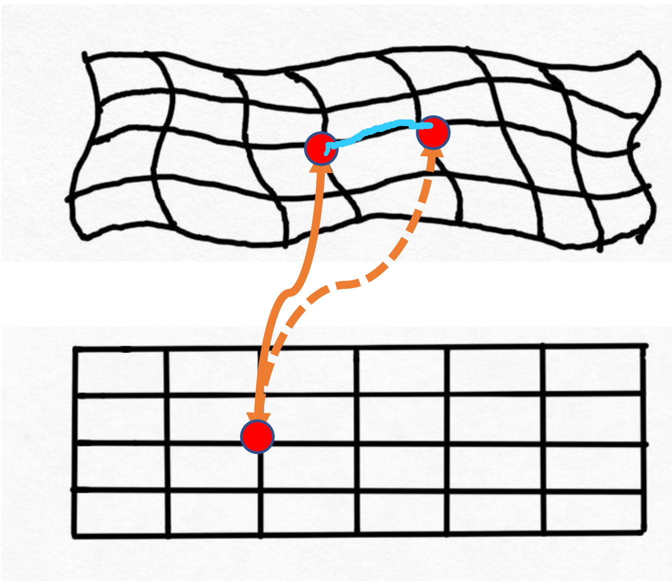
- Due to diffeomorphism invariance of GR, we have freedom in choosing ϵ^{η} and ϵ^{i}. Also, the physics should not depend on this parameters.

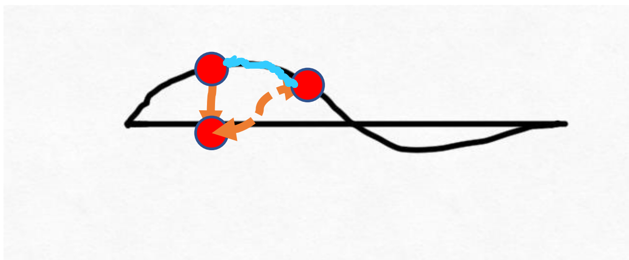
$$\eta \to \tilde{\eta} = \eta + \epsilon^{\eta}$$

$$x^i \to \tilde{x}^i = x^i + \epsilon^i \qquad \epsilon^i = e^i + \partial^i \epsilon \qquad \nabla_i e^i = 0$$

- Under a coordinate transformation.

$$\tilde{\delta g}_{\mu\nu} = \delta g_{\mu\nu} + \mathcal{L}_{\epsilon} g_{\mu\nu}$$





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- Under a coordinate transformation.

$$\tilde{\delta g}_{\mu\nu} = \delta g_{\mu\nu} + \mathcal{L}_\epsilon g_{\mu\nu}$$

$$\tilde{\phi} = \phi - \mathcal{H}\epsilon^\eta - \partial_\eta \epsilon^\eta$$

$$\tilde{B} = B - \epsilon^\eta + \partial_\eta \epsilon$$

$$\tilde{B}_i = B_i + \partial_\eta \left(\frac{e_i}{a^2} \right)$$

$$\tilde{\psi} = \psi + \mathcal{H}\epsilon^\eta + \frac{1}{3}\Delta_3 \epsilon$$

$$\tilde{E} = E - \epsilon$$

$$\tilde{F}_i = F_i - e_i$$

$$\tilde{h}_{ij} = h_{ij}$$

GLC Perturbations

GLC perturbation theory

How the GLC metric entries relates with the SVT decomposition?

In order to answer this question, we need to refer to a background with SO(3) symmetries.

$$\bar{f}^{GLC}_{\mu\nu}\left(\bar{\tau}, \bar{w}, \bar{\theta}^a\right) = \begin{pmatrix} 0 & -a & 0 \\ -a & a^2 & 0 \\ 0 & 0 & \bar{\gamma}_{ab} \end{pmatrix}$$

$$\bar{\tau} = \int \frac{d\eta}{a}$$

$$\bar{w} = r + \eta$$

$$\theta^a_{GLC} = \theta^a_{FLRW}$$

$$\bar{\gamma}^{GLC}_{ab} = \bar{\gamma}^{FLRW}_{ab}$$

GLC perturbation theory

- Adding general perturbations to the FLRW geometry

$$\delta f_{\mu\nu}(\tau, w, \theta^a) = a^2 \begin{pmatrix} L & M & V_a \\ M & N & \mathcal{U}_a \\ V_a & \mathcal{U}_a & \gamma_{ab} \end{pmatrix}$$

- Scalar/Pseudo-Scalar (SPS) decomposition

$$V_a = r^2 \left(D_a v + \tilde{D}_a \hat{v} \right)$$

$$U_a = r^2 \left(D_a u + \tilde{D}_a \hat{u} \right)$$

$$\delta\gamma_{ab} = 2 \left[\bar{\gamma}_{ab} \nu + r^2 \left(D_{ab} \mu + \tilde{D}_{ab} \hat{\mu} \right) \right]$$

- An infinitesimal change in the coordinates

$$\tilde{x}^{\mu} = x^{\mu} + \xi^{\mu} \qquad \delta \tilde{f}_{\mu\nu} = \delta f_{\mu\nu} - 2\nabla_{(\mu}\xi_{\nu)}$$

- Leads to gauge transformations

$$\tilde{L} = L + \frac{2}{a}\partial_{\tau}\xi^{w}$$

$$\tilde{M} = M + \partial_{\tau}\left(\frac{\xi^{\tau}}{a} - \xi^{w}\right) + \frac{1}{a}\partial_{w}\xi^{w}$$

$$\tilde{N} = N - 2H\xi^{\tau} + 2\partial_{w}\left(\frac{\xi^{\tau}}{a} - \xi^{w}\right)$$

- The gauge transformations are given by

$$\tilde{V}_a = V_a + \frac{1}{a}\partial_a \xi^w - \bar{\gamma}_{ab}\partial_\tau \xi^b$$

$$\tilde{U}_a = U_a + \partial_a\left(\frac{\xi^\tau}{a} - \xi^w\right) - \bar{\gamma}_{ab}\partial_w \xi^b$$

$$\tilde{\delta\gamma}_{ab} = \delta\gamma_{ab} - 2\bar{\gamma}_{ab}H\xi^\tau + \frac{2\bar{\gamma}_{ab}}{r}\left(\frac{\xi^\tau}{a} - \xi^w\right) - \left(\bar{\gamma}_{ac}D_b + \bar{\gamma}_{bc}D_a\right)\xi^c$$

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- Adding general perturbations to the FLRW geometry

$$\delta f_{\mu\nu}\left(\tau, w, \theta^a\right) = a^2 \begin{pmatrix} L & M & V_a \\ M & N & \mathcal{U}_a \\ V_a & \mathcal{U}_a & \gamma_{ab} \end{pmatrix}$$

- Fixing the GLC gauge

$$f^{GLC}_{\mu\nu} \overset{!}{=} \bar{f}_{\mu\nu} + \delta f_{\mu\nu} \longrightarrow \begin{cases} L = 0 \\ V_a = 0 \\ N + 2aM = 0 \end{cases}$$

- With the GLC gauge fixed we can obtain the redshift and the angular distance redshift-relation.

$$1+z=\frac{a_o}{a_s}\left(1+\frac{1}{2}N|_s^o\right) \qquad d_A = ar\frac{\left[1+\nu-\frac{1}{2}\left(1-\frac{1}{arH}\right)N|_o^z\right]}{\left(1+\nu-ar\partial_\tau\nu\right)_o}$$

- We note absence of integral terms along the geodesics.
- Therefore, we can interpret the GLC gauge as the gauge where integral effects on the angular distance-redshift relation vanishes.

More details in, G. Fanizza, G. Marozzi, MM, G. Schiaffino, *The Cosmological Perturbation Theory on the Geodesic Light-Cone background*, **JCAP** 02 (2021)

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SVT/GLC Relation

FLRW/GLC relation

- Through a coordinate transformation

$$\delta g_{\eta\eta} \dashrightarrow \quad \phi = -\frac{1}{2}(N + 2aM + L)$$

$$\delta g_{\eta r} \dashrightarrow \mathcal{B}_r(B,\ B_r) = -(N + aM)$$

$$\delta g_{\eta a} \dashrightarrow \mathcal{B}_a(B,\ B_a) = -(U_a + aV_a)$$

$$\delta g_{rr} \dashrightarrow C_{rr}(\psi,\ E,\ F_i,\ h_{rr}) = N$$

$$\delta g_{ra} \dashrightarrow C_{ra}(E,\ F_i\ , h_{ra}) = U_a$$

$$\delta g_{ab} \dashrightarrow C_{ab}(\psi,\ E,\ F_i,\ h_{ab}) = \delta\gamma_{ab}$$

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SVT/GLC relation

Acting with the operators $\left(\bar{\gamma}^{ij},\, \nabla^i,\, \bar{D}^{ij}\right)$

we may extract the SVT d.o.f. from the standard perturbations.

$$\left(\bar{\gamma}^{ij},\, \nabla^i,\, \bar{D}^{ij}\right) C_{ij} \propto \left(\psi,\, F_j + \partial_j \Delta_3 E + \partial_j \psi,\, \Delta_3^2 E + \Delta_3 \psi\right)$$

Re-writing it in terms of GLC, we obtain a relation between SVT/GLC perturbations, decomposing the GLC perturbations in SPS we obtain a SVT/SPS relation.

$$\bar{\gamma}^{ij} C_{ij} = -6\psi \rightarrow C_{rr} + 4\bar{\gamma}^{ab} C_{ab} = N + 4\nu$$

$$\psi = -\frac{1}{6}\left(N + 4\nu\right)$$

$$C_{ij} \rightarrow \psi \rightarrow E \rightarrow F_i \rightarrow h_{ij}$$

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SVT/GLC relation

- From the divergence of $\mathcal{B}_i$

$$\nabla^i \mathcal{B}_i \to \Delta_3 B = -\left(\partial_w + \frac{2}{r}\right)(N + aM) - D^a\,(U_a + aV_a)$$

$$\Delta_3 B \to B_r = -(N + aM) - \partial_r B$$

$$B_a = -(U_a + aV_a) - \partial_a B$$

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SVT/GLC relation

- Analogously for C_{ij}

$$\bar{\gamma}^{ij} C_{ij} \qquad\qquad \psi = -\frac{1}{6}\left(N + 4\nu\right)$$

$$\bar{D}^{ij} C_{ij}$$

$$\Delta_3\left(\psi + \frac{1}{3}\Delta_3 E\right) = \frac{1}{2}\left(r^{-1}\partial_w + r^{-2} - \frac{r^{-2}}{2}D^2\right)N + \frac{1}{2r^2}\left(D^2\right)^2\mu + \frac{1}{4r^2}D^2\mu$$
$$\frac{r^{-2}}{2}\left(\partial_w + \frac{3}{r}\right)r^2 D^2 u - \left(\partial_w^2 + \frac{3}{r}\partial_w + r^{-2} + \frac{r^{-2}}{2}D^2\right)\nu$$

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$$\frac{r^{-2}}{2}\left(\partial_w + \frac{3}{r}\right)r^2 D^2 u - \left(\partial_w^2 + \frac{3}{r}\partial_w + r^{-2} + \frac{r^{-2}}{2}D^2\right)\nu$$

SVT/GLC relation

- Analogously for C_{ij}

$$\nabla^i C_{ij} \dashrightarrow \Delta_3 F_r = \left(\partial_w + \frac{2}{r}\right) N + D^2 u - \frac{4}{r}\nu + 6\partial_w \psi - 4\partial_w \left(\psi + \frac{1}{3}\Delta_3 E\right)$$

$$\nabla^i C_{ij} \dashrightarrow \Delta_3 F_a = \left(\partial_w + \frac{2}{r}\right) U_a + r^{-2} D^b \delta\gamma_{ba} + 6\partial_a \psi - 4\partial_a \left(\psi + \frac{1}{3}\Delta_3 E\right)$$

SVT/GLC relation for tensor modes

- Using the previous results

$$h_{rr} = \frac{1}{2}N + \psi - D_{ww}E - \nabla_w F_r$$

$$h_{ar} = \frac{1}{2}U_a - D_{wa}E - \nabla_{(a}F_{r)}$$

$$h_{ab} = \frac{1}{2}\delta\gamma_{ab} + \bar{\gamma}_{ab}\psi - D_{ab}E - \nabla_{(a}F_{b)}$$

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$$h_{ab} = \frac{1}{2}\delta\gamma_{ab} + \bar{\gamma}_{ab}\psi - \underbrace{D_{ab}E - \nabla_{(a}F_{b)}}$$

$$\nabla_{(a}\chi_{b)}$$

Helicity Decomposition

Tensor Modes (Spin-2 Graviton)

We see that the expression for the gauge invariant tensor in GLC are very complicated and depends on previous expressions for Scalars and Vectors

Also, at first look becomes tantamount to identify the true spin-2 degrees of freedom.

Projecting the SVT/GLC-SPS relation we may simplify this relation and obtain the formulae easily decomposed in E/B modes.

Tensor Modes (Spin-2 Graviton)

We will be using the Sachs basis in GLC which are also used to project the Jacobi Map in the Screen Space.

$$s_1^a = \delta_1^a \qquad s_2^a = \delta_2^a sin^{-1}\theta$$

From which we build the helicity operators

$$s_\pm^a = \frac{1}{\sqrt{2}}\left(s_1^a \pm i s_2^a\right) \qquad \bar{\gamma}_{ab} s_\pm^a s_\pm^b = 0$$

$$s_\pm^a s_\pm^b D_{ab} f = \frac{\not{\partial}_\pm^2}{2} f$$

$$s_\pm^a \partial_a f = -\frac{1}{\sqrt{2}} \not{\partial}_\pm f$$

$$s_\pm^a s_\pm^b \tilde{D}_{ab} \hat{f} = \mp i \frac{\not{\partial}_\pm^2}{2} \hat{f}$$

Tensor Modes (Spin-2 Graviton)

- With the helicity decomposition, we may project the tensor modes

$$h_{\pm} \equiv s_{\pm}^{a} s_{\pm}^{b} h_{ab} = s_{\pm}^{a} s_{\pm}^{b} \left(\frac{\delta\gamma_{ab}}{2} - \nabla_{(a}\chi_{b)} \right)$$

$$h_{\pm} = \frac{r^2}{2} \eth_{\pm}^{2} \left(\mu \mp i\hat{\mu}\right) + \frac{1}{\sqrt{2}} \eth_{\pm}\chi_{\pm}$$

Tensor Modes (Spin-2 Graviton)

- With the helicity decomposition, we may project the tensor modes

$$\tilde{\chi}_a = \chi_a - \epsilon_a = \chi_a - r^2 \left(D_a \chi + \tilde{D}_a \hat{\chi} \right)$$

$$\tilde{\mu} = \mu - \chi$$

$$\tilde{\hat{\mu}} = \hat{\mu} - \hat{\chi}$$

$$\tilde{h}_\pm = \frac{r^2}{2} \eth_\pm^2 \left[\mu - \chi \mp i \left(\hat{\mu} - \hat{\chi} \right) \right] + \frac{1}{\sqrt{2}} \eth_\pm \chi_\pm + \frac{r^2}{2} \eth_\pm^2 \left(\chi \mp i \hat{\chi} \right) = h_\pm$$

Tensor Modes (Spin-2 Graviton)

- The advantage is that the **B**-modes are simple

$$h^B_\pm \equiv \frac{h_\pm - h^*_\pm}{2} = \frac{1}{2}\eth^2_\pm \left\{ r^2 \hat{\mu} - \frac{1}{\Delta_3}\left[\left(\partial_w + \frac{2}{r}\right)(r^2 \hat{u}) + D^2 \hat{\mu} + 2\hat{\mu} \right] \right\}$$

Tensor Modes (Spin-2 Graviton)

- The advantage is that the **B**-modes are simple

$$h^B_\pm \equiv i\frac{h_\pm - h^*_\pm}{2} = \frac{1}{2}\eth^2_\pm \left\{ r^2\hat{\mu} - \underbrace{\frac{1}{\Delta_3}\left[\left(\partial_w + \frac{2}{r}\right)(r^2\hat{u}) + D^2\hat{\mu} + 2\hat{\mu}\right]\right\}}_{\eth_\pm F^{\mathbf{B}}_\pm}$$

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$$\underbrace{\frac{1}{\Delta_3}\left[\left(\partial_w + \frac{2}{r}\right)(r^2\hat{u}) + D^2\hat{\mu} + 2\hat{\mu}\right]}_{\eth_\pm F^{\mathbf{B}}_\pm}$$

- In the early-universe fixing the GLC gauge and neglecting vector modes

$$\mu = \mathcal{M}$$

$$\hat{\mu} = \hat{\mathcal{M}}$$

$$h^B_\pm = \frac{r^2}{2}\eth^2_\pm\hat{\mathcal{M}}$$

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Tensor Modes (Spin-2 Graviton)

- In the UCG

$$\tilde{\chi}_a^{UCG} = \chi_a - \epsilon_a = \chi_a - r^2 \left(D_a \chi + \tilde{D}_a \hat{\chi} \right) = 0$$

$$\frac{1}{\sqrt{2}} \eth_\pm \tilde{\chi}_\pm |_{UCG} = \frac{1}{\sqrt{2}} \eth_\pm \chi_\pm + \frac{r^2}{2} \eth_\pm^2 \left(\chi \mp i \hat{\chi} \right) = 0$$

$$\tilde{h}_\pm = \frac{r^2}{2} \eth_\pm^2 \left[\mu \mp i \hat{\mu} \right]_{UCG}$$

Tensor Modes (Spin-2 Graviton)

- Unfixing the UCG

$$\tilde{h}_{\pm} = \frac{r^2}{2} \eth_{\pm}^2 \left[\mathcal{M} + \Delta_{\chi} \mp i \left(\hat{\mathcal{M}} - \hat{\Delta}_{\hat{\chi}} \right) \right]$$

$$\theta^a_{GLC} = \theta^a_{UCG} + D^a \Delta_{\chi} + \tilde{D}^a \hat{\Delta}_{\hat{\chi}}$$

Tensor Modes (Spin-2 Graviton)

- What Δ_χ and $\hat{\Delta}_{\hat{\chi}}$ measures?

$$\theta^a_{GLC} = \theta^a_{UCG} + D^a \Delta_\chi + \tilde{D}^a \hat{\Delta}_{\hat{\chi}}$$

- If now we consider the angular position of a source, and remember that the angles are constant along the geodesics in GLC, we have

$$\theta^a_{GLC}\Big|_O^S = \theta^a_{UCG}\Big|_O^S + \left(D^a \Delta_\chi + \tilde{D}^a \hat{\Delta}_{\hat{\chi}}\right)_O^S = 0$$

Deflection Angle $= \theta^a_{UCG}\Big|_O^S = -\left(D^a \Delta_\chi + \tilde{D}^a \hat{\Delta}_{\hat{\chi}}\right)_O^S$

Tensor Modes (Spin-2 Graviton)

- Different ways to express the tensor modes in the light-cone

SVT-GLC relation

$$h_{ab} = \frac{1}{2}\delta\gamma_{ab} + \bar{\gamma}_{ab}\psi - \underbrace{D_{ab}E - \nabla_{(a}F_{b)}}_{\text{To complicated in GLC!!!}}$$

Helicity decomposition

$$h_{\pm} = \frac{r^2}{2}\eth_{\pm}^2\left(\mu \mp i\hat{\mu}\right) + \underbrace{\frac{1}{\sqrt{2}}\eth_{\pm}\chi_{\pm}}_{\text{Still complicated!!!}}$$

- Alternatives: Exploit different gauge fixings

$$\tilde{h}_{\pm} = \frac{r^2}{2}\eth_{\pm}^2\left[\mu \mp i\hat{\mu}\right]_{UCG}$$

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Tensor Modes (Spin-2 Graviton)

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Helicity decomposition

$$h_{\pm} = \frac{r^2}{2}\eth_{\pm}^2\left(\mu \mp i\hat{\mu}\right) + \underbrace{\frac{1}{\sqrt{2}}\eth_{\pm}\chi_{\pm}}_{\text{Still complicated!!!}}$$

- Alternatives: In the early-universe neglect long-wavelength contributions

$$\tilde{h}_{\pm} = \frac{r^2}{2}\eth_{\pm}^2\left[\mu \mp i\hat{\mu}\right]$$

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Tensor Modes (Spin-2 Graviton)

- Different ways to express the tensor modes in the light-cone

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Helicity decomposition

$$h_{\pm} = \frac{r^2}{2}\eth_{\pm}^2\left(\mu \mp i\hat{\mu}\right) + \underbrace{\frac{1}{\sqrt{2}}\eth_{\pm}\chi_{\pm}}_{\text{Still complicated!!!}}$$

- Alternatives: The E- and B- modes disentangle in scalars and pseudo-scalars. B-modes are simple

$$h_{\pm}^{B} \equiv i\frac{h_{\pm} - h_{\pm}^{*}}{2} = \frac{1}{2}\eth_{\pm}^2\left\{r^2\hat{\mu} - \frac{1}{\Delta_3}\left[\left(\partial_w + \frac{2}{r}\right)\left(r^2\hat{u}\right) + D^2\hat{\mu} + 2\hat{\mu}\right]\right\}$$

Tensor Modes (Spin-2 Graviton)

- Different ways to express the tensor modes in the light-cone

SVT-GLC relation

$$h_{ab} = \frac{1}{2}\delta\gamma_{ab} + \bar{\gamma}_{ab}\psi - \underbrace{D_{ab}E - \nabla_{(a}F_{b)}}_{\text{To complicated in GLC!!!}}$$

Helicity decomposition

$$h_{\pm} = \frac{r^2}{2}\eth^2_{\pm}\left(\mu \mp i\hat{\mu}\right) + \underbrace{\frac{1}{\sqrt{2}}\eth_{\pm}\chi_{\pm}}_{\text{Still complicated!!!}}$$

- Alternatives: Express it as a gauge transformation

$$\tilde{h}_{\pm} = \frac{r^2}{2}\eth^2_{\pm}\left[\mathcal{M} + \Delta_{\chi} \mp i\left(\hat{\mathcal{M}} - \hat{\Delta}_{\hat{\chi}}\right)\right]$$

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General perspectives

- The GLC gauge greatly simplifies the description of cosmological observables.
- The angular distance-redshift relation have a simple form in the GLC gauge, with the absence of integral effects along the geodesics.
- The GLC perturbations are compatible with the SVT decomposition.
- We provided the expression for the gauge invariant tensor modes in terms of GLC perturbations.
- We also saw that the SPS decomposition offers an easy interpretation of E and B modes.
- We provided a gauge invariant expression for the E and B tensor modes, as a spin-2 operator acting respectively in scalars and pseudo-scalars from GLC perturbations.

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