An Effective Fluid Description of Scalar-Vector-Tensor Theories Under the Sub-Horizon and Quasi-Static Approximations

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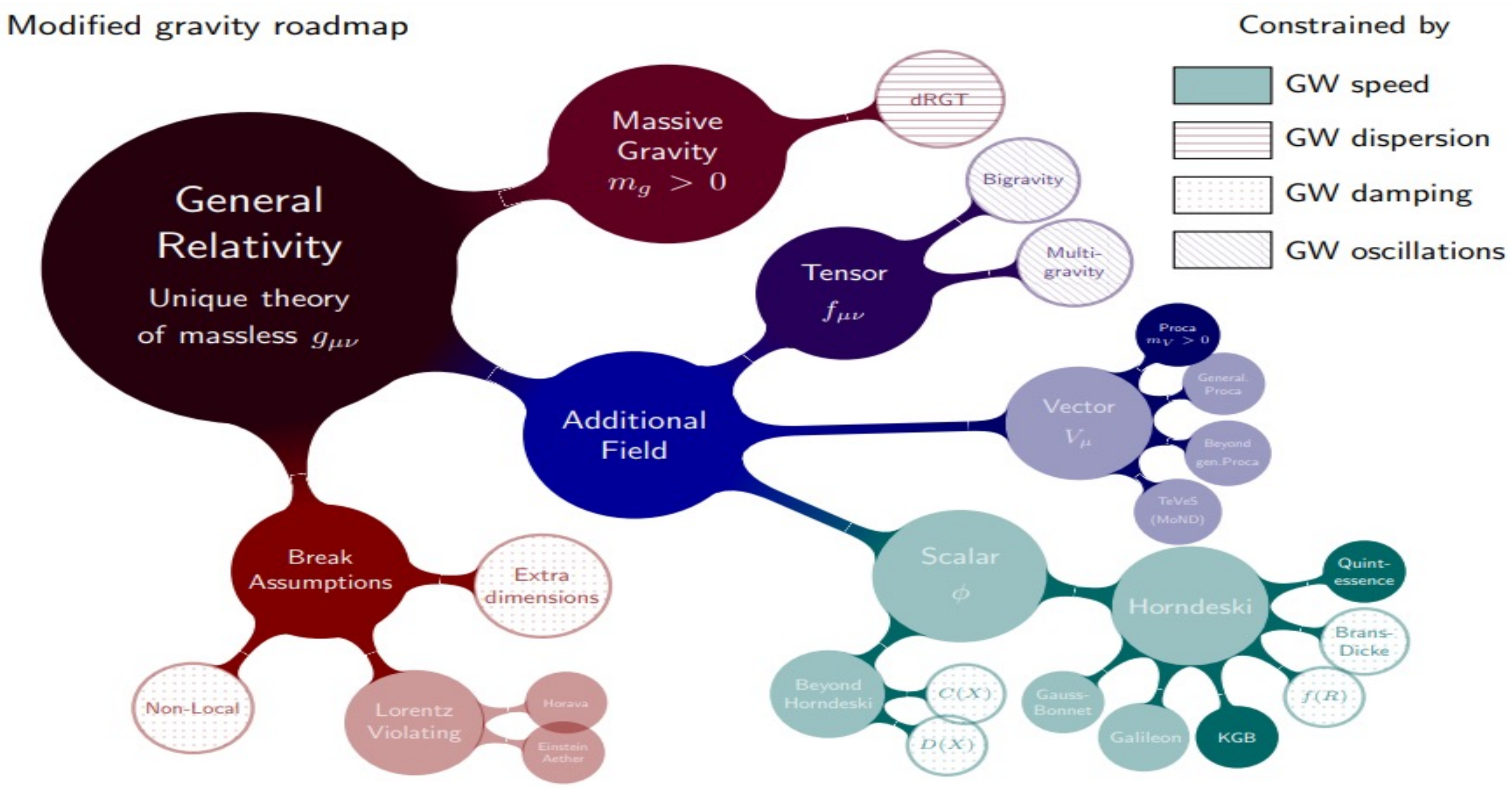
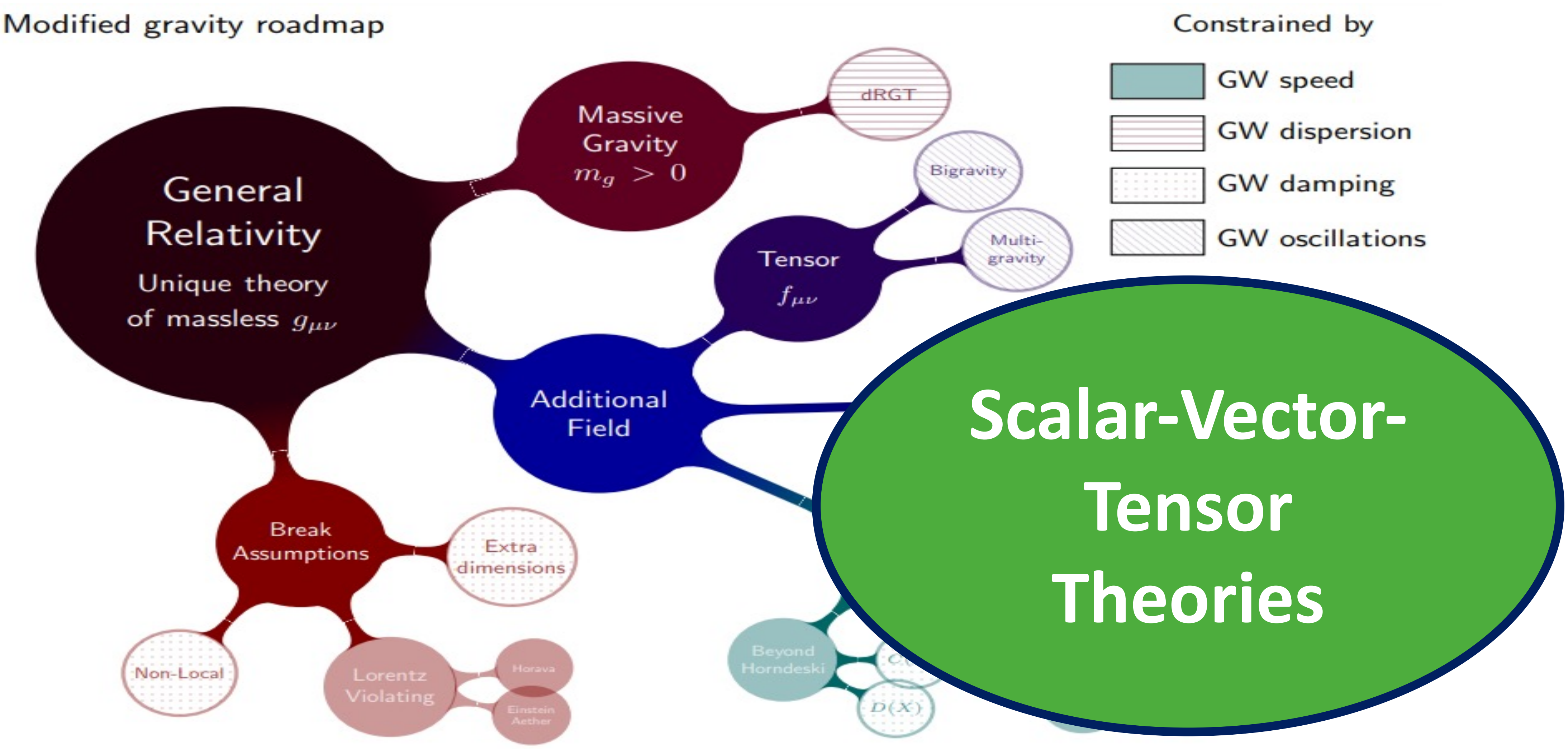


- Ezquiaga, Zumalacarregui: 1807.09241 [astro-ph.CO]



- Ezquiaga, Zumalacarregui: 1807.09241 [astro-ph.CO]
- Heisenberg: 1801.01523 [gr-qc]

SVT Theories

- Heisenberg, Kase, Tsujikawa: 1805.01066 [gr-qc]
- Kase, Tsujikawa: 1805.11919 [gr-qc]

$$\mathcal{L}_2^{\rm ST} = G_2(\varphi, X_1),$$
$$\mathcal{L}_3^{\rm ST} = -G_3(\varphi, X_1)\Box\varphi,$$
$$\mathcal{L}_4^{\rm ST} = G_4(\varphi, X_1)R + G_{4X_1}\left\{(\Box\varphi)^2 - \nabla_\mu\nabla_\nu\varphi\nabla^\nu\nabla^\mu\varphi\right\},$$
$$\mathcal{L}_5^{\rm ST} = G_5(\varphi, X_1)G^{\mu\nu}\nabla_\mu\nabla_\nu\varphi - \frac{1}{6}G_{5X_1}(\varphi, X_1)\left\{(\Box\varphi)^3 - 3(\Box\varphi)\nabla_\mu\nabla_\nu\varphi\nabla^\mu\nabla^\nu\varphi + 2\nabla^\mu\nabla_\sigma\varphi\nabla^\sigma\nabla_\rho\varphi\nabla^\rho\nabla_\mu\varphi\right\}$$

$$\mathcal{L}_2^{\rm SVT} = f_2(\varphi, X_1, X_2, X_3, F, Y_1, Y_2, Y_3),$$
$$\mathcal{L}_3^{\rm SVT} = f_3(\varphi, X_3)g^{\mu\nu}S_{\mu\nu} + \tilde{f}_3(\phi, X_3)A^\mu A^\nu S_{\mu\nu},$$
$$\mathcal{L}_4^{\rm SVT} = f_4(\varphi, X_3)R + f_{4X_3}(\varphi, X_3)\left\{(\nabla_\mu A^\mu)^2 - \nabla_\mu A_\nu\nabla^\mu A^\nu\right\},$$
$$\mathcal{L}_5^{\rm SVT} = f_5(\varphi, X_3)G^{\mu\nu}\nabla_\mu A_\nu + \mathcal{M}_5^{\mu\nu}\nabla_\mu\nabla_\nu\varphi + \mathcal{N}_5^{\mu\nu}S_{\mu\nu} - \frac{1}{6}f_{5X_3}(\varphi, X_3)\left\{(\nabla_\mu A^\mu)^3 - 3(\nabla_\mu A^\mu)\nabla_\rho A_\sigma\nabla^\sigma A^\rho + 2\nabla_\rho A_\sigma\nabla^\tau A^\rho\nabla^\sigma A_\tau\right\},$$
$$\mathcal{L}_6^{\rm SVT} = f_6(\varphi, X_1)L^{\mu\nu\alpha\beta}F_{\mu\nu}F_{\alpha\beta} + \tilde{f}_6(\varphi, X_3)L^{\mu\nu\alpha\beta}F_{\mu\nu}F_{\alpha\beta} + \mathcal{M}_6^{\mu\nu\alpha\beta}\nabla_\mu\nabla_\alpha\varphi\nabla_\nu\nabla_\beta\varphi + \mathcal{N}_6^{\mu\nu\alpha\beta}S_{\mu\nu}S_{\alpha\beta}.$$

GW Constraint

Speed of GW:

$$c_T^2 \equiv \frac{f_4 - \frac{A_0^2 A_0' f_{5X_3}}{2a^4} + G_4 + \frac{A_0^3 f_{5X_3}\mathcal{H}}{2a^4} - \frac{A_0 f_{5\varphi}\varphi'}{2a^2} - \frac{G_{5\varphi}\varphi'^2}{2a^2} + \frac{G_{5X_1}\mathcal{H}\varphi'^3}{2a^4} - \frac{G_{5X_1}\varphi'^2\varphi''}{2a^4}}{f_4 - \frac{A_0^2 f_{4X_3}}{a^2} + G_4 - \frac{A_0^3 f_{5X_3}\mathcal{H}}{2a^4} + \frac{A_0 f_{5\varphi}\varphi'}{2a^2} - \frac{G_{4X_1}\varphi'^2}{a^2} + \frac{G_{5\varphi}\varphi'^2}{2a^2} - \frac{G_{5X_1}\mathcal{H}\varphi'^3}{2a^4}}$$

GW Constraint

$$c_T^2 = 1 \quad \longrightarrow \quad f_4 = f_4(\varphi), \quad f_5 = \text{constant}, \quad G_4 = G_4(\varphi), \quad G_5 = \text{constant}.$$

GW Constraint

Speed of GW:

$$c_T^2 \equiv \frac{f_4 - \frac{A_0^2 A_0' f_{5X_3}}{2a^4} + G_4 + \frac{A_0^3 f_{5X_3}\mathcal{H}}{2a^4} - \frac{A_0 f_{5\varphi}\varphi'}{2a^2} - \frac{G_{5\varphi}\varphi'^2}{2a^2} + \frac{G_{5X_1}\mathcal{H}\varphi'^3}{2a^4} - \frac{G_{5X_1}\varphi'^2\varphi''}{2a^4}}{f_4 - \frac{A_0^2 f_{4X_3}}{a^2} + G_4 - \frac{A_0^3 f_{5X_3}\mathcal{H}}{2a^4} + \frac{A_0 f_{5\varphi}\varphi'}{2a^2} - \frac{G_{4X_1}\varphi'^2}{a^2} + \frac{G_{5\varphi}\varphi'^2}{2a^2} - \frac{G_{5X_1}\mathcal{H}\varphi'^3}{2a^4}}$$

GW Constraint

$$c_T^2 = 1 \quad \longrightarrow \quad f_4 = f_4(\varphi), \quad f_5 = \text{constant}, \quad G_4 = G_4(\varphi), \quad G_5 = \text{constant}.$$

Remaining Theory:

$$\mathcal{L}_2^{\rm SVT} = f_2(\varphi, X_1, X_2, X_3),$$
$$\mathcal{L}_3^{\rm SVT} = f_3(\varphi, X_3) g^{\mu\nu} S_{\mu\nu} + \tilde{f}_3(\varphi, X_3) A^\mu A^\nu S_{\mu\nu},$$

$$\mathcal{L}_3^{\rm ST} = -G_3(\varphi, X_1)\Box\varphi,$$
$$\mathcal{L}_4^{\rm ST} = G_4(\varphi) R,$$

$$X_1 \equiv -\frac{1}{2}\nabla_\mu\varphi\nabla^\mu\varphi, \quad X_2 \equiv -\frac{1}{2}A_\mu\nabla^\mu\varphi, \quad X_3 \equiv -\frac{1}{2}A_\mu A^\mu.$$

Remaining SVT

General Equations:

$$\sum_{i=2}^{3} \mathcal{G}^{(i)}_{\mu\nu} + \sum_{i=3}^{4} \mathscr{H}^{(i)}_{\mu\nu} = \frac{1}{2} T^{(m)}_{\mu\nu},$$

$$\sum_{i=2}^{3} \mathcal{J}_i + \sum_{i=3}^{4} \mathcal{K}_i = 0, \quad \sum_{i=2}^{3} \mathcal{A}^{\mu}{}_{(i)} = 0,$$

Perturbations:

FLRW:

$$ds^2 = a(\eta)^2 \left[-\{1 + 2\Psi(\boldsymbol{x}, \eta)\} d\eta^2 + \{1 + 2\Phi(\boldsymbol{x}, \eta)\} \delta_{ij} dx^i dx^j\right]$$

Fields in SVT:

$$\varphi = \varphi(\eta) + \delta\varphi(\boldsymbol{x}, \eta), \qquad A_\mu = (A_0(\eta) + \delta A_0(\boldsymbol{x}, \eta), \delta A_i(\boldsymbol{x}, \eta)),$$

Remaining SVT

"Time-Time"

$$0 = A_1\frac{\Phi'}{a} + A_2\frac{\delta\varphi'}{a} + A_3\frac{k^2}{a^2}\Phi + A_4\Psi + \left(A_5\frac{k^2}{a^2} - \mu_\varphi\right)\delta\varphi + A_6\frac{\delta A_0}{a} + A_7\frac{k^2}{a^2}\psi - \delta\rho_m,$$

"Time-Space"

$$0 = C_1\frac{\Phi'}{a} + C_2\frac{\delta\varphi'}{a} + C_3\Psi + C_4\delta\varphi + C_5\frac{\delta A_0}{a} + C_6\psi - \frac{a\bar{\rho}_m V_m}{k^2},$$

"Trace Space-Space"

$$0 = B_1\frac{\Phi''}{a^2} + B_2\frac{\delta\varphi''}{a^2} + B_3\frac{\Phi'}{a} + B_4\frac{\delta\varphi'}{a} + B_5\frac{\Psi'}{a} + B_6\frac{k^2}{a^2}\Phi + \left(B_7\frac{k^2}{a^2} + 3\nu_\varphi\right)\delta\varphi + \left(B_8\frac{k^2}{a^2} + B_9\right)\Psi + B_{10}\frac{\delta A_0'}{a^2} + B_{11}\frac{\delta A_0}{a},$$

"Trace-less Space-Space"

$$0 = G_4\left(\Psi + \Phi\right) + G_{4\varphi}\delta\varphi,$$

Remaining SVT

Scalar Field:

$$0 = D_1\frac{\Phi''}{a^2} + D_2\frac{\delta\varphi''}{a^2} + D_3\frac{\Phi'}{a} + D_4\frac{\delta\varphi'}{a} + D_5\frac{\Psi'}{a} + \left(D_7\frac{k^2}{a^2} + D_8\right)\Phi + \left(D_9\frac{k^2}{a^2} - m_\varphi^2\right)\delta\varphi + \left(D_{10}\frac{k^2}{a^2} + D_{11}\right)\Psi + D_{12}\frac{\delta A_0'}{a^2} + D_{13}\frac{\delta A_0}{a} + D_{14}\frac{k^2}{a^2}\psi,$$

Temporal Vector Field

$$0 = F_1\frac{\Phi'}{a^2} + F_2\frac{\delta\varphi'}{a^2} + F_3\frac{\Psi}{a} + F_4\frac{\delta\varphi}{a} + F_5\frac{\delta A_0}{a^2} + F_6\frac{k^2}{a^2}\frac{\psi}{a},$$

Spatial Vector Field

$$0 = H_1\frac{\Psi}{a^2} + H_2\frac{\delta\varphi}{a^2} + H_3\frac{\delta A_0}{a^3} + H_4\frac{\psi}{a^2}.$$

Effective Fluid Approach

$$G_{\mu\nu} = \kappa\left(T^{(m)}_{\mu\nu} + T^{(\mathrm{DE})}_{\mu\nu}\right)$$

Dark Energy Tensor:

$$T^{(\mathrm{DE})}_{\mu\nu} \equiv \frac{1}{\kappa}G_{\mu\nu} - 2\left(\sum_{i=2}^{3}\mathcal{G}^{(i)}_{\mu\nu} + \sum_{i=3}^{4}\mathscr{H}^{(i)}_{\mu\nu}\right)$$

Dark Energy Density:

$$\bar{\rho}_{\mathrm{DE}} = -f_2 + \frac{\varphi'^2 f_{2X_1}}{a^2} + \frac{A_0\varphi' f_{2X_2}}{a^2} + \frac{A_0^2 f_{2X_3}}{a^2} + \frac{2A_0\varphi' f_{3\varphi}}{a^2} - \frac{2A_0^3\varphi'\tilde{f}_{3\varphi}}{a^4} - \frac{\varphi'^2 G_{3\varphi}}{a^2} - \frac{6A_0^3 f_{3X_3}\mathcal{H}}{a^4} - \frac{6A_0^3\tilde{f}_3\mathcal{H}}{a^4} + \frac{3\varphi'^3 G_{3X_1}\mathcal{H}}{a^4} - \frac{6\varphi' G_{4\varphi}\mathcal{H}}{a^2} - \frac{6G_4\mathcal{H}^2}{a^2} + \frac{3\mathcal{H}^2}{\kappa a^2},$$

Dark Energy Pressure:

$$\bar{P}_{\mathrm{DE}} = f_2 + \frac{2A_0^2A_0' f_{3X_3}}{a^4} + \frac{2A_0\varphi' f_{3\varphi}}{a^2} + \frac{2A_0^2A_0'\tilde{f}_3}{a^4} - \frac{\varphi''\varphi'^2 G_{3X_1}}{a^4} - \frac{\varphi'^2 G_{3\varphi}}{a^2} + \frac{2\varphi'' G_{4\varphi}}{a^2} + \frac{2\varphi'^2 G_{4\varphi\varphi}}{a^2} - \frac{2A_0^3 f_{3X_3}\mathcal{H}}{a^4} - \frac{2A_0^3\tilde{f}_3\mathcal{H}}{a^4} + \frac{\varphi'^3 G_{3X_1}\mathcal{H}}{a^4} + \frac{2\varphi' G_{4\varphi}\mathcal{H}}{a^2} + \frac{2G_4\mathcal{H}^2}{a^2} - \frac{\mathcal{H}^2}{\kappa a^2} + \frac{4G_4\mathcal{H}'}{a^2} - \frac{2\mathcal{H}'}{\kappa a^2}.$$

Effective Fluid Approach

Dark Energy Perturbations:

$$\begin{aligned}
\delta\rho_{\rm DE} &= (\cdots)\delta\varphi + (\cdots)\delta\varphi' + (\cdots)\Psi + (\cdots)\Phi + (\cdots)\Phi' + (\cdots)\delta A_0 + (\cdots)\psi, \\
\delta p_{\rm DE} &= (\cdots)\delta\varphi + (\cdots)\delta\varphi' + (\cdots)\delta\varphi'' + (\cdots)\Psi + (\cdots)\Psi' + (\cdots)\Phi \\
&\quad + (\cdots)\Phi' + (\cdots)\Phi'' + (\cdots)\delta A_0 + (\cdots)\delta A_0', \\
V_{\rm DE} &= (\cdots)\delta\varphi + (\cdots)\delta\varphi' + (\cdots)\Psi + (\cdots)\Phi' + (\cdots)\delta A_0 + (\cdots)\psi.
\end{aligned}$$

Effective Fluid Approach

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δρDE = Collect[%, ϕh[LI[1]]]
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$$
\begin{aligned}
&\delta A_0{}^{1}\Bigg(\frac{A_0{}^3 f_{2\,X_3X_3}}{a^4}+\frac{A_0 f_{2\,X_3}}{a^2}-\frac{6A_0{}^4 f_{3\,X_3X_3}\mathcal{H}}{a^6}-\frac{6A_0{}^4 \tilde{f}_{3\,X_3}\mathcal{H}}{a^6}-\frac{18A_0{}^2 f_{3\,X_3}\mathcal{H}}{a^4}-\frac{18A_0{}^2\tilde{f}_3\mathcal{H}}{a^4}-\\
&\quad\frac{2A_0{}^4\tilde{f}_{3\,\varphi X_3}\varphi'}{a^6}+\frac{3A_0{}^2 f_{2\,X_2X_3}\varphi'}{2a^4}+\frac{2A_0{}^2 f_{3\,\varphi X_3}\varphi'}{a^4}-\frac{6A_0{}^2\tilde{f}_{3\,\varphi}\varphi'}{a^4}+\frac{f_{2\,X_2}\varphi'}{2a^2}+\frac{2f_{3\,\varphi}\varphi'}{a^2}+\frac{A_0 f_{2\,X_1X_3}\varphi'^2}{a^4}+\frac{A_0 f_{2\,X_2X_2}\varphi'^2}{2a^4}+\frac{f_{2\,X_1X_2}\varphi'^3}{2a^4}\Bigg)+\\
&\Bigg(-f_{2\,\varphi}+\frac{A_0{}^2 f_{2\,\varphi X_3}}{a^2}-\frac{2G_{4\,\varphi}k^2}{a^2}-\frac{6A_0{}^3 f_{3\,\varphi X_3}\mathcal{H}}{a^4}-\frac{6A_0{}^3\tilde{f}_{3\,\varphi}\mathcal{H}}{a^4}-\frac{6G_{4\,\varphi}\mathcal{H}^2}{a^2}-\frac{2A_0{}^3\tilde{f}_{3\,\varphi\varphi}\varphi'}{a^4}+\frac{A_0 f_{2\,\varphi X_2}\varphi'}{a^2}+\frac{2A_0 f_{3\,\varphi\varphi}\varphi'}{a^2}-\frac{6G_{4\,\varphi\varphi}\mathcal{H}\varphi'}{a^2}+\frac{G_{3\,X_1}k^2\varphi'^2}{a^4}+\frac{f_{2\,\varphi X_1}\varphi'^2}{a^2}-\frac{G_{3\,\varphi\varphi}\varphi'^2}{a^2}+\frac{3G_{3\,\varphi X_1}\mathcal{H}\varphi'^3}{a^4}\Bigg)\\
&\left({}^{(1)}\varphi\right)+\\
&\Bigg(\frac{A_0{}^3 f_{2\,X_2X_3}}{2a^4}-\frac{2A_0{}^3\tilde{f}_{3\,\varphi}}{a^4}+\frac{A_0 f_{2\,X_2}}{2a^2}+\frac{2A_0 f_{3\,\varphi}}{a^2}-\frac{6G_{4\,\varphi}\mathcal{H}}{a^2}+\frac{A_0{}^2 f_{2\,X_1X_3}\varphi'}{a^4}+\frac{A_0{}^2 f_{2\,X_2X_2}\varphi'}{2a^4}+\frac{f_{2\,X_1}\varphi'}{a^2}-\frac{2G_{3\,\varphi}\varphi'}{a^2}+\frac{3A_0 f_{2\,X_1X_2}\varphi'^2}{2a^4}+\frac{9G_{3\,X_1}\mathcal{H}\varphi'^2}{a^4}+\frac{f_{2\,X_1X_1}\varphi'^3}{a^4}-\frac{G_{3\,\varphi X_1}\varphi'^3}{a^4}+\frac{3G_{3\,X_1X_1}\mathcal{H}\varphi'^4}{a^6}\Bigg)\\
&\left({}^{(1)}\varphi'\right)+\Bigg(-\frac{A_0{}^4 f_{2\,X_3X_3}}{a^4}-\frac{A_0{}^2 f_{2\,X_3}}{a^2}+\frac{6A_0{}^5 f_{3\,X_3X_3}\mathcal{H}}{a^6}+\frac{6A_0{}^5\tilde{f}_{3\,X_3}\mathcal{H}}{a^6}+\frac{24A_0{}^3 f_{3\,X_3}\mathcal{H}}{a^4}+\frac{24A_0{}^3\tilde{f}_3\mathcal{H}}{a^4}+\frac{12G_4\mathcal{H}^2}{a^2}-\frac{6\mathcal{H}^2}{\kappa a^2}+\frac{2A_0{}^5\tilde{f}_{3\,\varphi X_3}\varphi'}{a^6}-\frac{2A_0{}^3 f_{2\,X_2X_3}\varphi'}{a^4}-\frac{2A_0{}^3 f_{3\,\varphi X_3}\varphi'}{a^4}+\frac{8A_0{}^3\tilde{f}_{3\,\varphi}\varphi'}{a^4}-\\
&\quad\frac{A_0 f_{2\,X_2}\varphi'}{a^2}-\frac{4A_0 f_{3\,\varphi}\varphi'}{a^2}+\frac{12G_{4\,\varphi}\mathcal{H}\varphi'}{a^2}-\frac{2A_0{}^2 f_{2\,X_1X_3}\varphi'^2}{a^4}-\frac{A_0{}^2 f_{2\,X_2X_2}\varphi'^2}{a^4}-\frac{f_{2\,X_1}\varphi'^2}{a^2}+\frac{2G_{3\,\varphi}\varphi'^2}{a^2}-\frac{2A_0 f_{2\,X_1X_2}\varphi'^3}{a^4}-\frac{12G_{3\,X_1}\mathcal{H}\varphi'^3}{a^4}-\frac{f_{2\,X_1X_1}\varphi'^4}{a^4}+\frac{G_{3\,\varphi X_1}\varphi'^4}{a^4}-\frac{3G_{3\,X_1X_1}\mathcal{H}\varphi'^5}{a^6}\Bigg)\left({}^{(1)}\Phi\right)+\\
&\left(-\frac{2A_0{}^2 f_{3\,X_3}k^2}{a^4}-\frac{2A_0{}^2\tilde{f}_3k^2}{a^4}\right)\psi^{1}+\left(\frac{4G_4k^2}{a^2}-\frac{2k^2}{\kappa a^2}\right)\left({}^{(1)}\Psi\right)+\left(\frac{6A_0{}^3 f_{3\,X_3}}{a^4}+\frac{6A_0{}^3\tilde{f}_3}{a^4}+\frac{12G_4\mathcal{H}}{a^2}-\frac{6\mathcal{H}}{\kappa a^2}+\frac{6G_{4\,\varphi}\varphi'}{a^2}-\frac{3G_{3\,X_1}\varphi'^3}{a^4}\right)\left({}^{(1)}\Psi'\right)
\end{aligned}
$$

Effective Fluid Approach

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δρDE = Collect[%, ϕh[LI[1]]]
```

$$\delta A_0{}^1 \left(\frac{A_0{}^3 f_2\,{}_{x_3 x_3}}{a^4} + \frac{A_0 f_2\,{}_{x_3}}{a^2} - \frac{6 A_0{}^4 f_3\,{}_{x_3 x_3}\mathcal{H}}{a^6} - \frac{6 A_0{}^4 \tilde{f}_3\,{}_{x}\ldots}{a^6} \ldots \right.$$

$$\frac{2 A_0{}^4 \tilde{f}_3\,{}_{\varphi x_3}\varphi'}{a^6} + \frac{3 A_0{}^2 f_2\,{}_{x_2 x_3}\varphi'}{2 a^4} + \frac{2 A_0{}^2 f_3\,{}_{\varphi x_3}\varphi'}{a^4} - \frac{6 A_0{}^2 \ldots}{\ldots} \ldots$$

$$\left(-f_2\,{}_{\varphi} + \frac{A_0{}^2 f_2\,{}_{\varphi x_3}}{a^2} - \frac{2 G_4\,{}_{\varphi} k^2}{a^2} - \frac{6 A_0{}^3 f_3\,{}_{\varphi x_3}\mathcal{H}}{a^4} - \frac{6 A_0{}^3 \tilde{f}_3 \ldots}{a^4} \ldots \frac{\ldots k^2 \varphi'^2}{\ldots{}^4} + \frac{f_2\,{}_{\varphi x_1}\varphi'^2}{a^2} - \frac{G_3\,{}_{\varphi\varphi}\varphi'^2}{a^2} + \frac{3 G_3\,{}_{\varphi x_1}\mathcal{H}\varphi'^3}{a^4} \right)$$

$$\left(\overset{(1)}{\varphi}\right) +$$

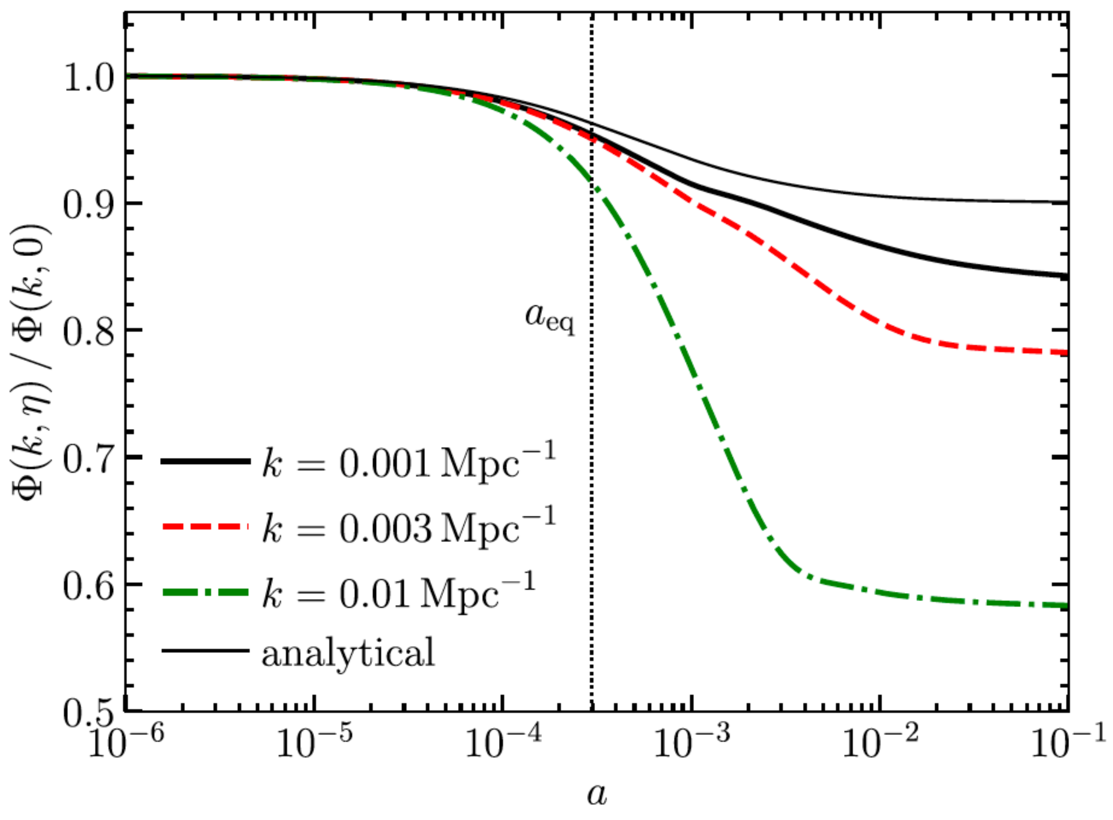
$$\left(\frac{A_0{}^3 f_2\,{}_{x_2 x_3}}{2 a^4} - \frac{2 A_0{}^3 \tilde{f}_3\,{}_{\varphi}}{a^4} + \frac{A_0 f_2\,{}_{x_2}}{2 a^2} + \frac{2 A_0 f_3\,{}_{\varphi}}{a^2} - \frac{6 G_4\,{}_{\varphi}\ldots}{a^2} \ldots \mathcal{H}\varphi'^2 + \frac{f_2\,{}_{x_1 x_1}\varphi'^3}{a^4} - \frac{G_3\,{}_{\varphi x_1}\varphi'^3}{a^4} + \frac{3 G_3\,{}_{x_1 x_1}\mathcal{H}\varphi'^4}{a^6} \right)$$

$$\left(\overset{(1)}{\varphi'}\right) + \left(-\frac{A_0{}^4 f_2\,{}_{x_3 x_3}}{a^4} - \frac{A_0{}^2 f_2\,{}_{x_3}}{a^2} + \frac{6 A_0{}^5 f_3\,{}_{x_3 x_3}\mathcal{H}}{a^6} + 6 \ldots \frac{\ldots{}^3 f_2\,{}_{x_2 x_3}\varphi'}{a^4} - \frac{2 A_0{}^3 f_3\,{}_{\varphi x_3}\varphi'}{a^4} + \frac{8 A_0{}^3 \tilde{f}_3\,{}_{\varphi}\varphi'}{a^4} - \right.$$

$$\left. \frac{A_0 f_2\,{}_{x_2}\varphi'}{a^2} - \frac{4 A_0 f_3\,{}_{\varphi}\varphi'}{a^2} + \frac{12 G_4\,{}_{\varphi}\mathcal{H}\varphi'}{a^2} - \frac{2 A_0{}^2 f_2\,{}_{x_1 x_3}\ldots}{a^4} \ldots \frac{\ldots{}_1 x_1 \varphi'^4}{a^4} + \frac{G_3\,{}_{\varphi x_1}\varphi'^4}{a^4} - \frac{3 G_3\,{}_{x_1 x_1}\mathcal{H}\varphi'^5}{a^6} \right) \left(\overset{(1)}{\Phi}\right) +$$

$$\left(-\frac{2 A_0{}^2 f_3\,{}_{x_3} k^2}{a^4} - \frac{2 A_0{}^2 \tilde{f}_3 k^2}{a^4} \right) \psi^1 + \left(\frac{4 G_4 k^2}{a^2} - \frac{2 k^2}{\kappa a^2} \right) \left(\overset{(1)}{\Psi}\right) \ldots$$

Quasi-Static and Sub-Horizon Approximations



QSA: $\Phi\ ,\ \Psi\ = \text{constant}.$

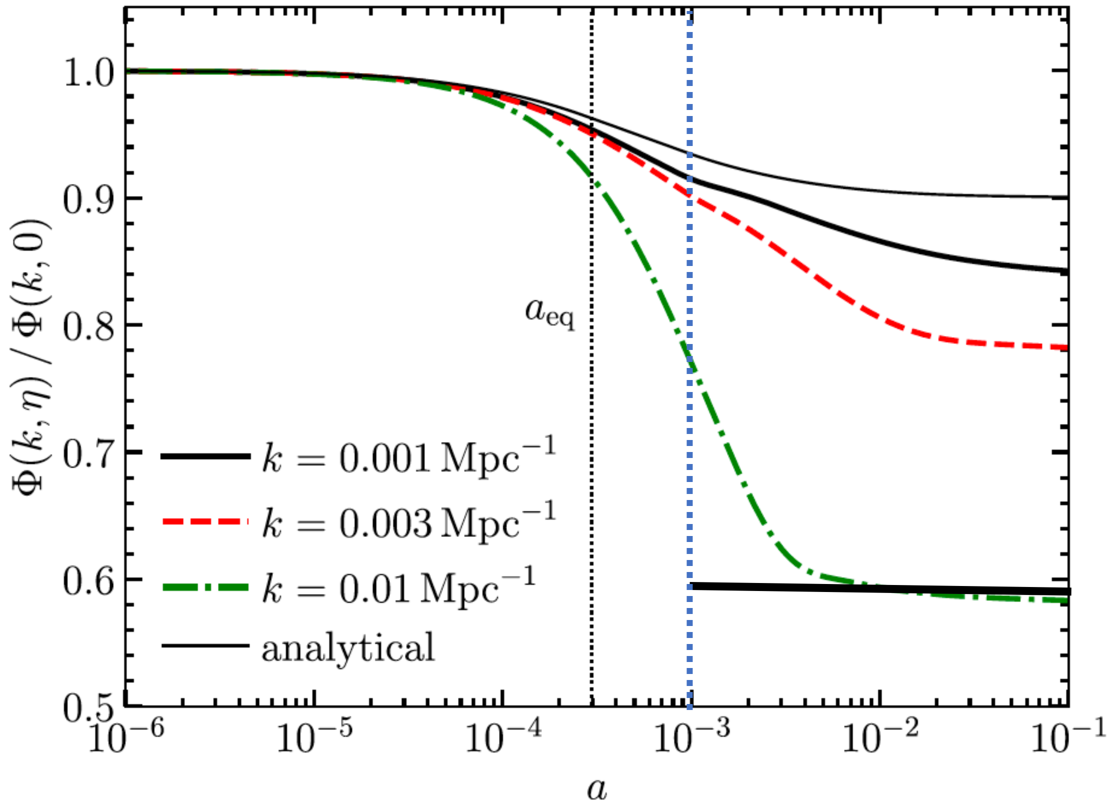
SHA: $k^2 \gg \mathcal{H}^2.$

$\mathcal{H} \times \text{perturbation} = 0,$

Neglect time derivatives of perturbation variables

- Dodelson 2nd Edition

Quasi-Static and Sub-Horizon Approximations



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SHA: $k^2 \gg \mathcal{H}^2.$

- $\mathcal{H} \times \text{perturbation} = 0,$
- Neglect time derivatives of perturbation variables

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Quasi-Static and Sub-Horizon Approximations

Perturbation Eqs are highly simplified under the QSA and the SHA

$$0 = A_3 \frac{k^2}{a^2}\Phi + A_5 \frac{k^2}{a^2}\delta\varphi + A_7 \frac{k^2}{a^2}\psi - \delta\rho_m,$$

$$0 = B_6 \frac{k^2}{a^2}\Phi + B_7 \frac{k^2}{a^2}\delta\varphi + B_8 \frac{k^2}{a^2}\Psi,$$

$$0 = D_7 \frac{k^2}{a^2}\Phi + \left(D_9 \frac{k^2}{a^2} - m_\varphi^2\right)\delta\varphi + D_{10} \frac{k^2}{a^2}\Psi + D_{14} \frac{k^2}{a^2}\psi,$$

$$0 = F_3 \frac{\Psi}{a} + F_4 \frac{\delta\varphi}{a} + F_5 \frac{\delta A_0}{a^2} + F_6 \frac{k^2}{a^2}\frac{\psi}{a},$$

$$0 = H_1 \frac{\Psi}{a^2} + H_2 \frac{\delta\varphi}{a^2} + H_3 \frac{\delta A_0}{a^3} + H_4 \frac{\psi}{a^2}.$$

Perturbation Variables in the QSA and the SHA

Scalar Field:

$$\delta\varphi = \frac{\frac{k^2}{a^2}W_1 + W_2}{\frac{k^4}{a^4}W_3 + \frac{k^2}{a^2}W_4 + W_5}\delta\rho_m,$$

Vector Field:

$$\frac{\delta A_0}{a} = \frac{\frac{k^4}{a^4}W_6 + \frac{k^2}{a^2}W_7 + W_8}{\frac{k^6}{a^6}W_3 + \frac{k^4}{a^4}W_4 + \frac{k^2}{a^2}W_5}\delta\rho_m, \quad \psi = \frac{\frac{k^2}{a^2}W_9 + W_{10}}{\frac{k^6}{a^6}W_3 + \frac{k^4}{a^4}W_4 + \frac{k^2}{a^2}W_5}\delta\rho_m$$

Gravitational Potentials:

$$\frac{k^2}{a^2}\Phi = \frac{\frac{k^4}{a^4}W_{11} + \frac{k^2}{a^2}W_{12} + W_{13}}{\frac{k^4}{a^4}W_3 + \frac{k^2}{a^2}W_4 + W_5}\delta\rho_m, \quad \frac{k^2}{a^2}\Psi = -\frac{\frac{k^4}{a^4}W_{14} + \frac{k^2}{a^2}W_{15} + W_{13}}{\frac{k^4}{a^4}W_3 + \frac{k^2}{a^2}W_4 + W_5}\delta\rho_m$$

DE Perturbations in the QSA and the SHA

One of our main results!

Perturbations under the SHA and the QSA

When there is Anisotropic Stress:

$$\delta\rho_{\rm DE} = \frac{\frac{k^6}{a^6}Z_1 + \frac{k^4}{a^4}Z_2 + \frac{k^2}{a^2}Z_3 + Z_4}{\frac{k^6}{a^6}Z_5 + \frac{k^4}{a^4}Z_6 + \frac{k^2}{a^2}Z_7}\delta\rho_m, \qquad \delta P_{\rm DE} = \frac{1}{3Z_{12}}\frac{\frac{k^6}{a^6}Z_8 + \frac{k^4}{a^4}Z_9 + \frac{k^2}{a^2}Z_{10} + Z_{11}}{\frac{k^6}{a^6}Z_5 + \frac{k^4}{a^4}Z_6 + \frac{k^2}{a^2}Z_7}\delta\rho_m,$$

$$\frac{a\bar{\rho}_{\rm DE}}{k^2}V_{\rm DE} = \frac{\frac{k^4}{a^4}Z_{13} + \frac{k^2}{a^2}Z_{14} + Z_{15}}{\frac{k^6}{a^6}Z_5 + \frac{k^4}{a^4}Z_6 + \frac{k^2}{a^2}Z_7}\delta\rho_m, \qquad \bar{\rho}_{\rm DE}\pi_{\rm DE} = \frac{k^2}{a^2}\frac{\frac{k^2}{a^2}(W_{11} + W_{14}) + W_{12} + W_{15}}{\frac{k^4}{a^4}W_3 + \frac{k^2}{a^2}W_4 + W_5}\delta\rho_m,$$

$$c^2_{s,\rm DE} = \frac{1}{3Z_{12}}\frac{\frac{k^6}{a^6}Z_8 + \frac{k^4}{a^4}Z_9 + \frac{k^2}{a^2}Z_{10} + Z_{11}}{\frac{k^6}{a^6}Z_1 + \frac{k^4}{a^4}Z_2 + \frac{k^2}{a^2}Z_3 + Z_4}$$

$$c^2_{s,\rm eff} \equiv c^2_{s,\rm DE} - \frac{2}{3}\frac{\bar{\rho}_{\rm DE}\pi_{\rm DE}}{\delta\rho_{\rm DE}}$$

DE Perturbations in the QSA and the SHA

One of our main results!

Perturbations under the SHA and the QSA

When there is No Anisotropic Stress:

$$\delta\rho_{\rm DE} = \frac{\frac{k^6}{a^6}Y_1 + \frac{k^4}{a^4}Y_2 + \frac{k^2}{a^2}Y_3 + Y_4}{\frac{k^6}{a^6}Y_5 + \frac{k^4}{a^4}Y_6 + \frac{k^2}{a^2}Y_7}\delta\rho_m, \quad \delta P_{\rm DE} = \frac{1}{3}\frac{\frac{k^4}{a^4}Y_8 + \frac{k^2}{a^2}Y_9 + Y_{10}}{\frac{k^6}{a^6}Y_5 + \frac{k^4}{a^4}Y_6 + \frac{k^2}{a^2}Y_7}\delta\rho_m,$$

$$\frac{a\bar{\rho}_{\rm DE}}{k^2}V_{\rm DE} = \frac{\frac{k^4}{a^4}Y_{11} + \frac{k^2}{a^2}Y_{12} + Y_{13}}{\frac{k^6}{a^6}Y_5 + \frac{k^4}{a^4}Y_6 + \frac{k^2}{a^2}Y_7}\delta\rho_m, \quad c^2_{s,\rm DE} = \frac{1}{3}\frac{\frac{k^4}{a^4}Y_8 + \frac{k^2}{a^2}Y_9 + Y_{10}}{\frac{k^6}{a^6}Y_1 + \frac{k^4}{a^4}Y_2 + \frac{k^2}{a^2}Y_3 + Y_4}.$$

Examples

- $\boldsymbol{f(R)}$ **Theories**

$$f_2 = -\frac{RF - f}{2}, \quad f_{2\varphi} = -\frac{R}{2}, \quad f_{2\varphi\varphi} = -\frac{1}{2F_R}, \quad G_4 = \frac{F}{2}, \quad G_{4\varphi} = \frac{1}{2},$$

$$\delta\rho_{\rm DE} = \frac{(1-F)F + (2-3F)\frac{k^2}{a^2}F_R}{F(F + 3\frac{k^2}{a^2}F_R)}\delta\rho_m, \quad \delta P_{\rm DE} = \frac{1}{3F}\frac{2\frac{k^4}{a^4}F_R + 15\frac{k^2}{a^4}F_R F'' + \frac{3FF''}{a^2}}{3\frac{k^4}{a^4}F_R + \frac{k^2}{a^2}F}\delta\rho_m,$$

$$\bar{\rho}_{\rm DE}V_{\rm DE} = \frac{1}{2F}\frac{(F + 6\frac{k^2}{a^2}F_R)F'}{F + 3\frac{k^2}{a^2}F_R}\delta\rho_m, \quad \bar{\rho}_{\rm DE}\pi_{\rm DE} = \frac{\frac{k^2}{a^2}F_R}{F^2 + 3\frac{k^2}{a^2}FF_R}\delta\rho_m.$$

Examples

- **Quintessence**

$$f_2 = X_1 - V(\varphi), \quad f_{2\varphi} = -V_\varphi, \quad f_{2\varphi\varphi} = -V_{\varphi\varphi}, \quad G_4 = \frac{1}{2},$$

$$\delta\rho_{\rm DE} = \frac{\varphi'\delta\varphi'}{a^2} + V_\varphi\delta\varphi - \frac{\varphi'^2}{a^2}\Psi, \quad \delta P_{\rm DE} = \frac{\varphi'\delta\varphi'}{a^2} - V_\varphi\delta\varphi - \frac{\varphi'^2}{a^2}\Psi, \quad \bar{\rho}_{\rm DE}V_{\rm DE} = k^2 a^{-1}\frac{\varphi'\delta\varphi}{a}$$

$$\delta\rho_{\rm DE} = \delta P_{\rm DE} = \frac{\varphi'^2}{2k^2}\delta\rho_m, \quad \bar{\rho}_{\rm DE}V_{\rm DE} = 0, \quad c^2_{s,\rm DE} = 1.$$

Examples

- **Generalised Proca**

$$f_2 = f_2(X_3), \quad f_3 \to \frac{1}{2} f_3(X_3), \quad G_4 = \frac{1}{2},$$

$$\delta\rho_{\rm DE} = -\frac{A_0^3 f_{2X_3}}{A_0^3 f_{2X_3} + 2k^2 A_0}\delta\rho_m, \quad \bar{\rho}_{\rm DE} V_{\rm DE} = \frac{A_0 f_{2X_3} A_0'}{A_0^2 f_{2X_3} + 2k^2}\mathcal{H}\delta\rho_m \approx 0,$$

$$\delta P_{\rm DE} = \frac{2}{3}\frac{A_0^2 f_{2X_3} - 3a^2 f_2}{A_0^2 f_{2X_3} + 2k^2}\delta\rho_m, \quad c_{s,\rm DE}^2 = -\frac{2}{3} + \frac{2a^2 f_2}{A_0^2 f_{2X_3}},$$

Examples

- Easy to Implement New Models

- 2202.08291: The Microphysics of Early Dark Energy

Models with non-trivial anisotropic stress and sound speed could alleviate some of the tensions in cosmology, like the H_0 and the σ_8 tensions.

SVTDES: Designer Model in SVT

$$f_2 = -3H_0^2\Omega_{\Lambda 0} + \frac{\sqrt{2}H_0\tilde{J}X_1^{1/2}}{\Omega_{m0}}\left[\left(\frac{X_1}{H_0^2}\right)^{-2}\left(\frac{X_3}{H_0^2}\right)^4 - \Omega_{\Lambda 0}\right]$$

$$f_3 = 0, \quad \tilde{f}_3 = \frac{2X_1^{1/2}\tilde{J}\left(\frac{X_1}{H_0^2}\right)^{-1}\left(\frac{X_3}{H_0^2}\right)^2}{3X_3^{3/2}\Omega_{m0}}$$

$$G_{3X_1} = \frac{2\tilde{J}\left(\frac{X_1}{H_0^2}\right)^{-2}\left(\frac{X_3}{H_0^2}\right)^2}{3H_0^2\Omega_{m0}}, \quad G_4 = \frac{1}{2}.$$

Background exactly equal to that of the Standard Model

$$\mathcal{H}^2 = a^2H_0^2(\Omega_{m0}a^{-3} + \Omega_{\Lambda 0})$$

$$\rho_{\mathrm{DE}} = 3H_0^2\Omega_{\Lambda 0}, \quad P_{\mathrm{DE}} = -\rho_{\mathrm{DE}}, \quad w_{\mathrm{DE}} = -1$$

Growth Factor

$$\frac{k^2}{a^2}\Psi = -\frac{1}{2}\frac{G_{\rm eff}}{G_N}\delta\rho_m \quad \longrightarrow \quad \frac{G_{\rm eff}}{G_N} = -2\frac{\frac{k^4}{a^4}W_{11} + \frac{k^2}{a^2}W_{12} + W_{13}}{\frac{k^4}{a^4}W_3 + \frac{k^2}{a^2}W_4 + W_5}$$

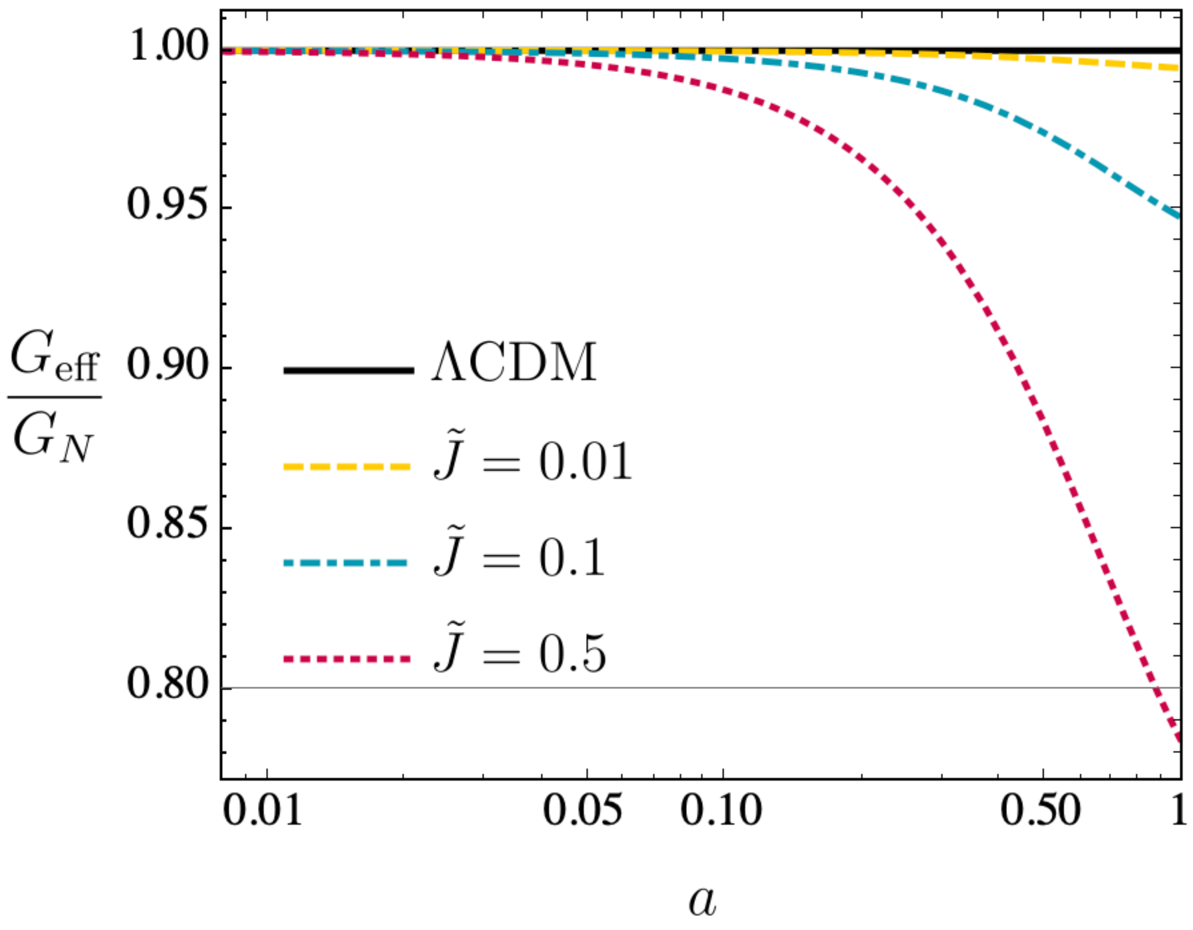
Equation for Matter Contrast:

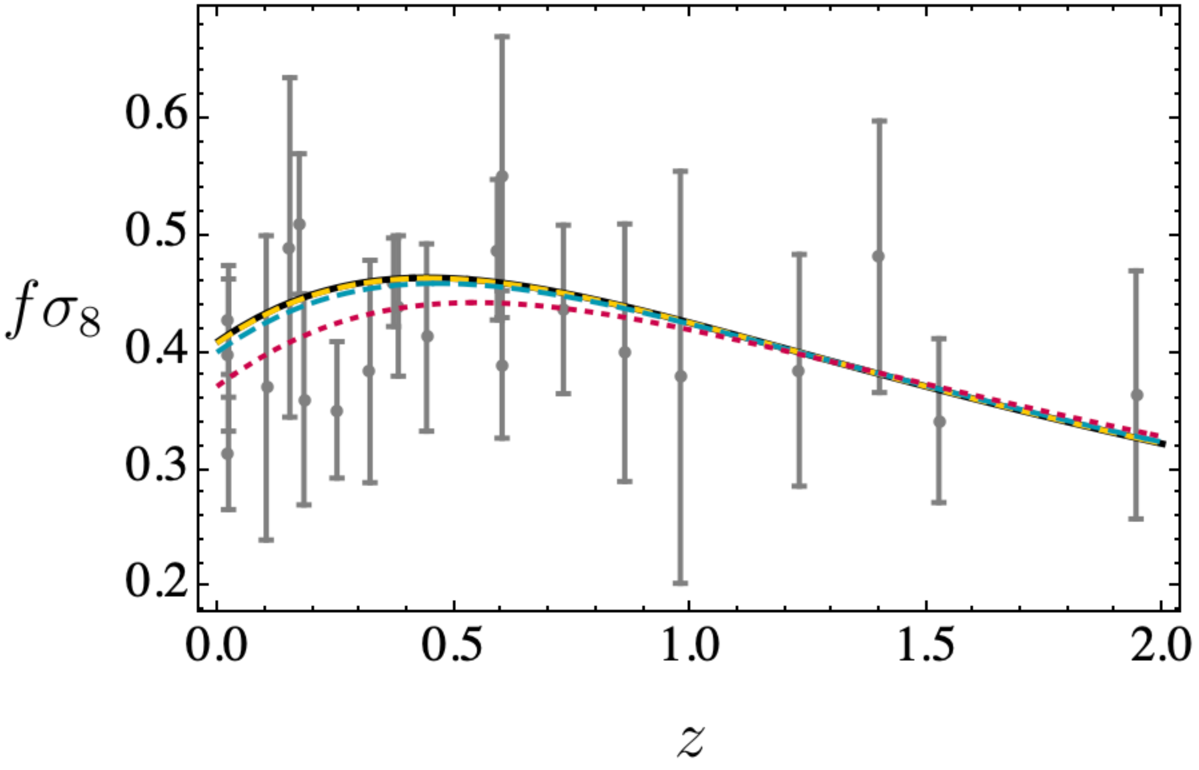
$$\delta''_m(a) + \left(\frac{3}{a} + \frac{H'(a)}{H(a)}\right)\delta'_m(a) - \frac{3}{2}\frac{\Omega_{m0}G_{\rm eff}/G_N}{a^5 H(a)^2/H_0^2}\delta_m(a) = 0$$

The Sigma8 Function:

$$f\sigma_8(a) \equiv \sigma_8 \frac{a\,\delta'_m(a)}{\delta_m(a=1)}$$

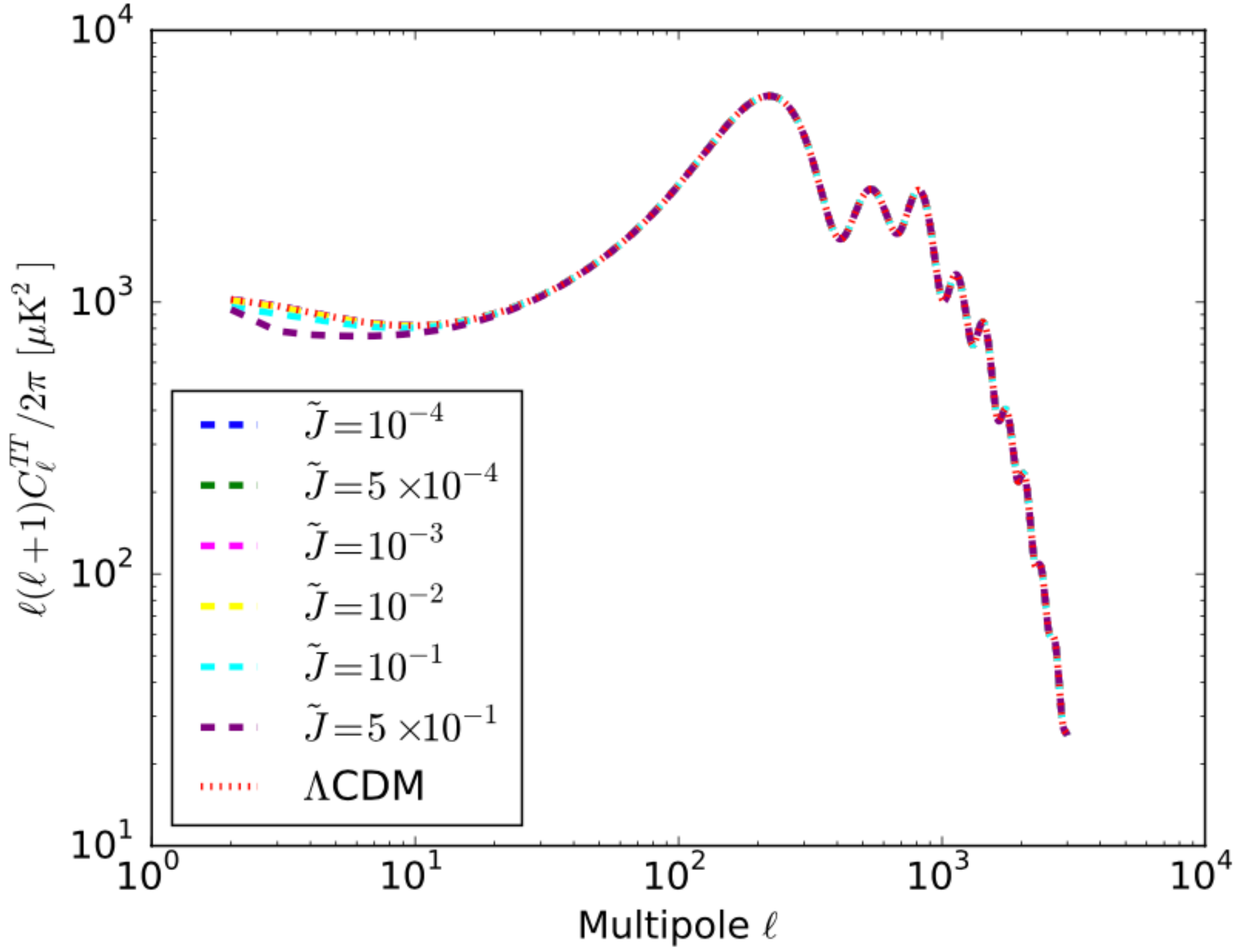
Growth Factor





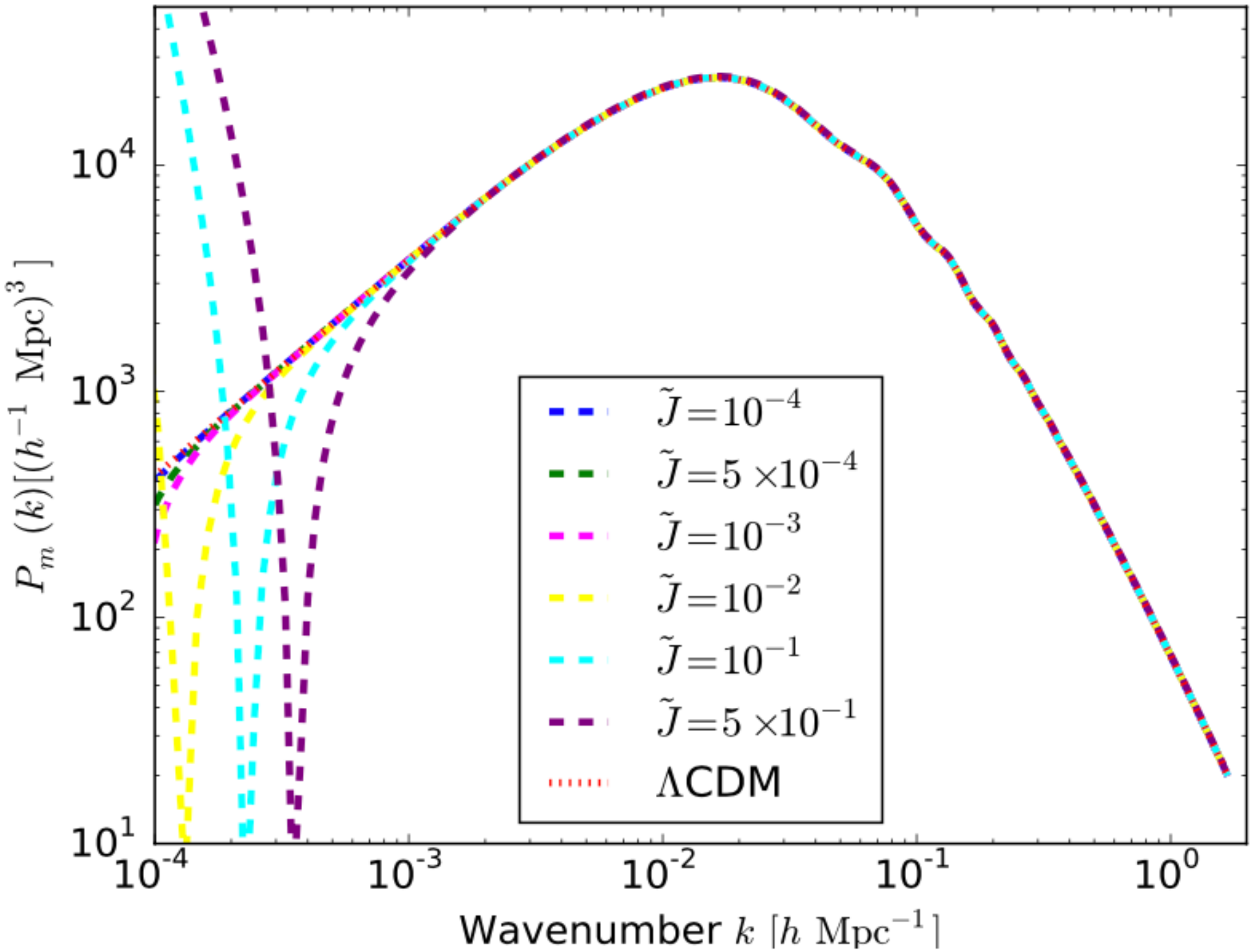
CMB Angular and Matter Power Spectra

CMB Power Spectra



CMB Angular and Matter Power Spectra

Matter Power Spectrum



Summary and Conclusions

- We need systematic ways to study observational signatures of models --- Generalized theories --- SVT .
- We can define a general dark energy fluid for the remaining SVT theories.
- We get simpler analytical expressions for the perturbations under the SHA and the QSA.
- We design SVTDES and study some of its cosmological implications.
- More details in 2206.02895 [astro-ph.CO]… Accepted in JCAP!

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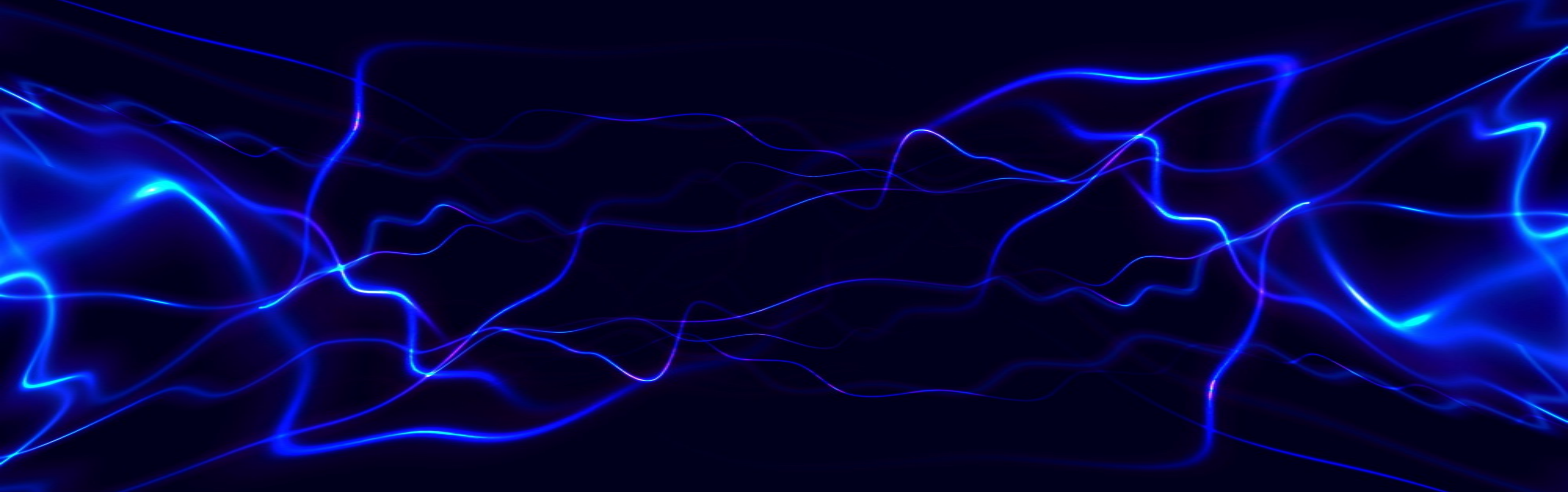
- Next steps? Validation of QSA and SHA. Generalization of CLASS. Higher order perturbations.

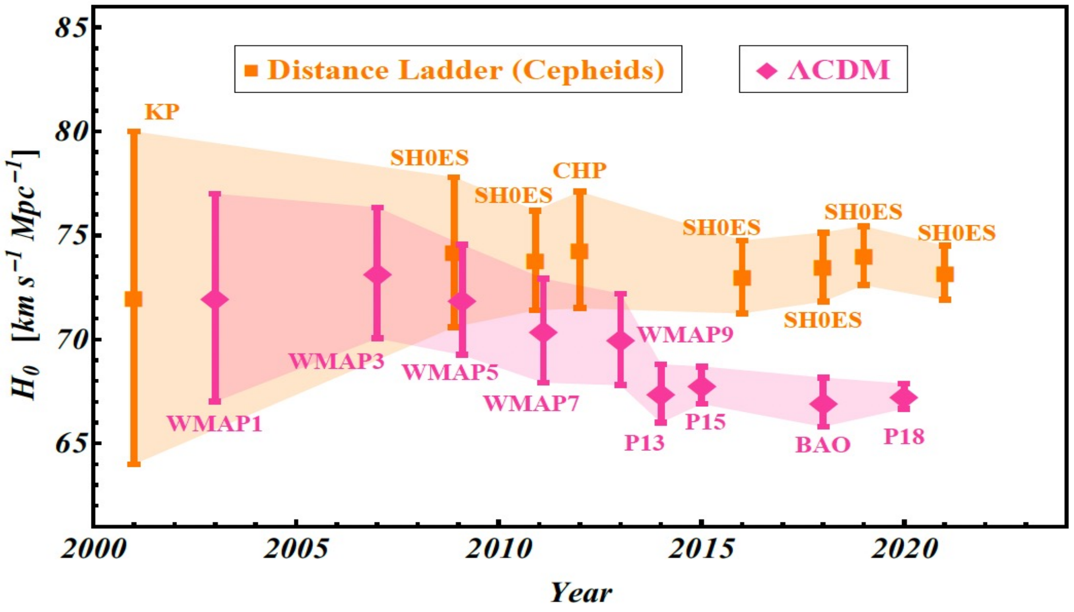


Thank you.

Problems ~ Tensions

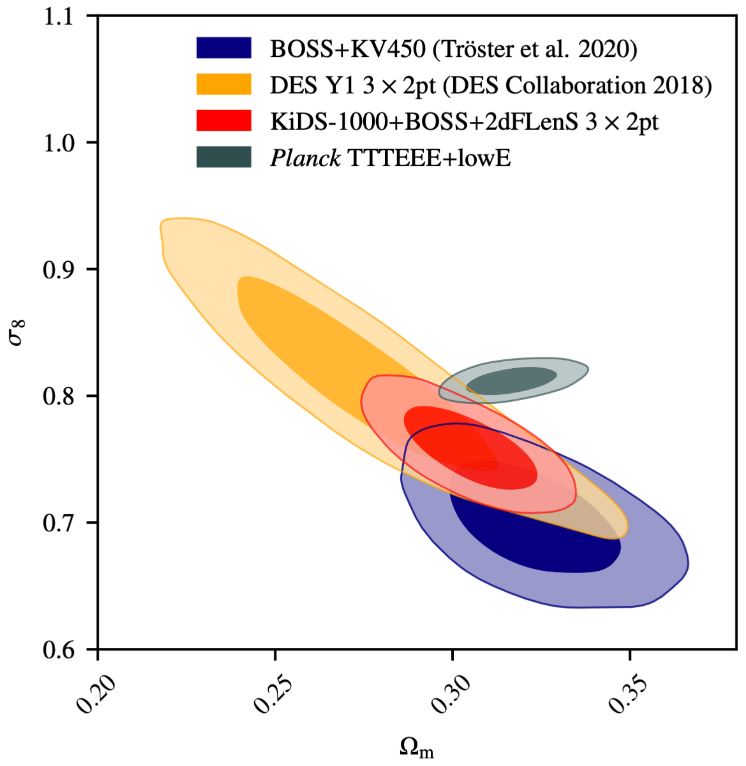
ΛCDM?

Hubble Tension



- Perivolaropoulos, Skara: 2105.05208 [astro-ph.CO]

σ_8 Tension



- KIDS Collaboration: 2007. 15632 [astro-ph.CO]

$$c_{s,\mathrm{DE}}^2 \gtrsim 3 \times 10^{-6} a^{-1},$$

$$\delta'' + (1-6w)\mathcal{H}\delta' + 3\mathcal{H}\left(\frac{\delta P}{\delta\rho}\right)' + 3\left[(1-3w)\mathcal{H}^2 + \mathcal{H}'\right]\left(\frac{\delta P}{\bar{\rho}} - w\delta\right) - 3\mathcal{H}w'\delta$$
$$= -3(1+w)\left[\Phi'' + \left(1 - 3w + \frac{w'/\mathcal{H}}{1+w}\right)\mathcal{H}\Phi'\right] - k^2\left[(1+w)\Psi + \frac{\delta P}{\bar{\rho}} - \frac{2}{3}\pi\right]$$

$$\delta\rho_{\mathrm{DE}} = \frac{\varphi'^2 + 2\frac{A_0^3\varphi'\tilde{f}_{3\varphi}}{a^2}}{2k^2 - 4\frac{A_0^3\varphi'\tilde{f}_{3\varphi}}{a^2}}\delta\rho_m, \quad \delta P_{\mathrm{DE}} = \frac{\varphi'^2 - \frac{4A_0^2A_0'\varphi'\tilde{f}_{3\varphi}}{3a^2\mathcal{H}}}{2k^2 - 4\frac{A_0^3\varphi'\tilde{f}_{3\varphi}}{a^2}}\delta\rho_m,$$

$$\bar{\rho}_{\mathrm{DE}}V_{\mathrm{DE}} = -\frac{1}{ak^2}\frac{2A_0^2\tilde{f}_{3\varphi}\varphi'A_0'}{k^2 - 2\frac{A_0^3\varphi'\tilde{f}_{3\varphi}}{a^2}}\delta\rho_m, \quad c_{s,\mathrm{DE}}^2 = \frac{\varphi'^2 - \frac{4A_0^2A_0'\varphi'\tilde{f}_{3\varphi}}{3a^2\mathcal{H}}}{\varphi'^2 + 2\frac{A_0^3\varphi'\tilde{f}_{3\varphi}}{a^2}}$$

Slip Parameters:

$$\eta \equiv \frac{\Psi + \Phi}{\Phi} = \frac{\frac{k^4}{a^4}(W_{11} - W_{14}) + \frac{k^2}{a^2}(W_{12} - W_{15})}{\frac{k^4}{a^4}W_{11} + \frac{k^2}{a^2}W_{12} + W_{13}},$$

$$\gamma \equiv -\frac{\Phi}{\Psi} = \frac{\frac{k^4}{a^4}W_{11} + \frac{k^2}{a^2}W_{12} + W_{13}}{\frac{k^4}{a^4}W_{14} + \frac{k^2}{a^2}W_{15} + W_{13}};$$

Gravitational and Lensing Potentials:

$$\frac{k^2}{a^2}\Psi = -\frac{1}{2}\frac{G_{\rm eff}}{G_N}\delta\rho_m,$$

$$\frac{k^2}{a^2}\Phi = \frac{1}{2}Q_{\rm eff}\delta\rho_m.$$

Evolution of Perturbations