COSMO22 (22-26 AUGUST 2022) RIO DE JANEIRO, BRAZIL

Parallel Session Lecture Room: Modified Gravity & Dark Energy (22 Aug 2022, 16:30-16:50)

Gravitational-wave polarizations in generic higher-curvature gravity

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(arXiv:2102.05540v2 [gr-qc])

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INTRODUCTION

Modify gravity

General relativity [Einstein (1915)] is very much consistent with observations.

Examples: Bending of light by the sun [Eddington (1919)],

The perihelion shift of Mercury’s orbit [Le Verrier’s (1859)],

Gravitational wave detected directly in 2015 [Adv. LIGO (2016)].

However, it is not a perfect theory of gravity.

Problems: Accelerated expansion of universe (dark energy)

[Supernova Search Team (1998), Supernova Cosmology Project (1998)],

Quantization [’t Hooft and Veltman (1974)], …

To overcome these problems, there have been attempts to modify or extend general relativity, such as massive gravity (MG), higher-curvature gravity (HCG) and so on.

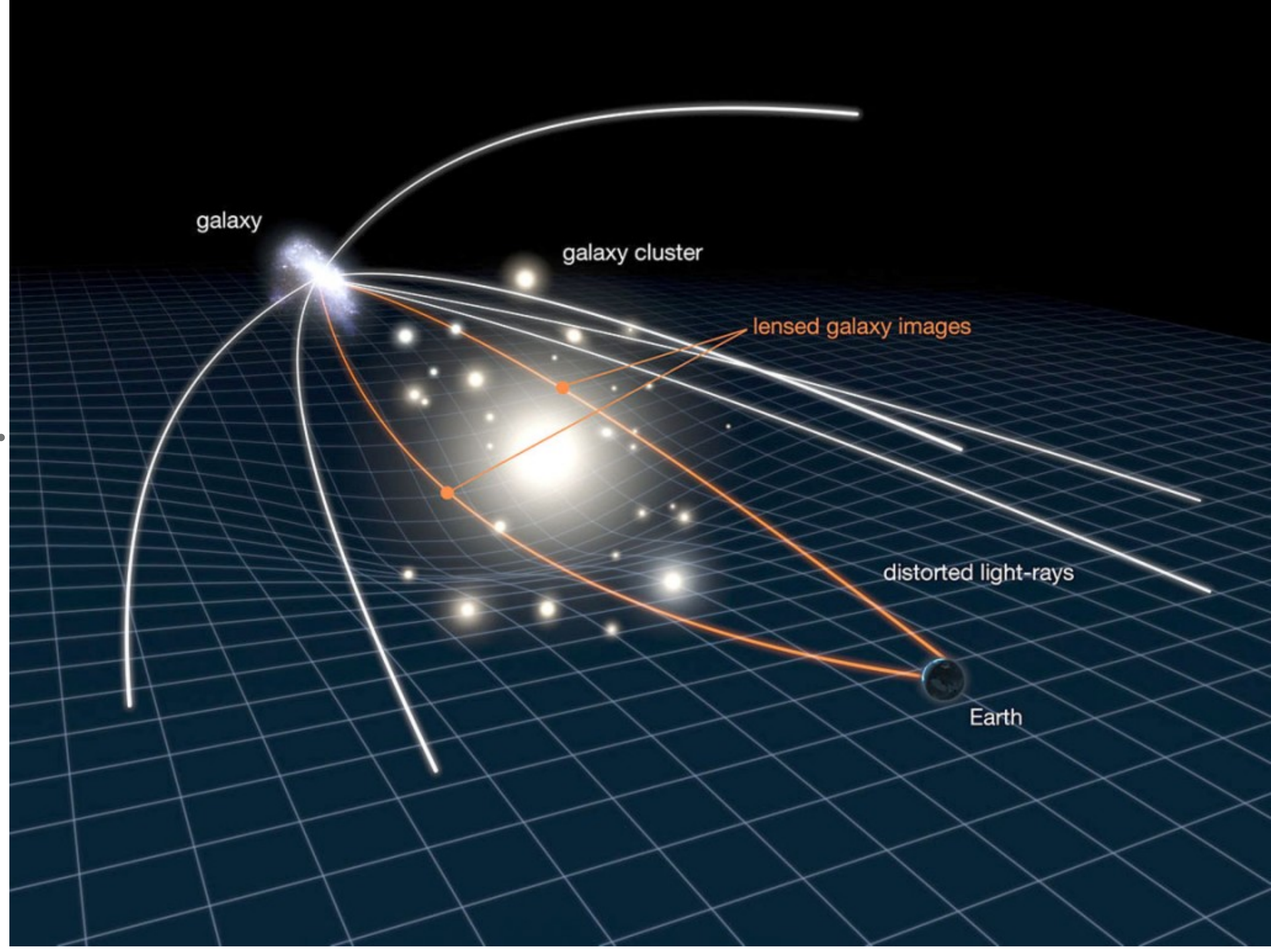
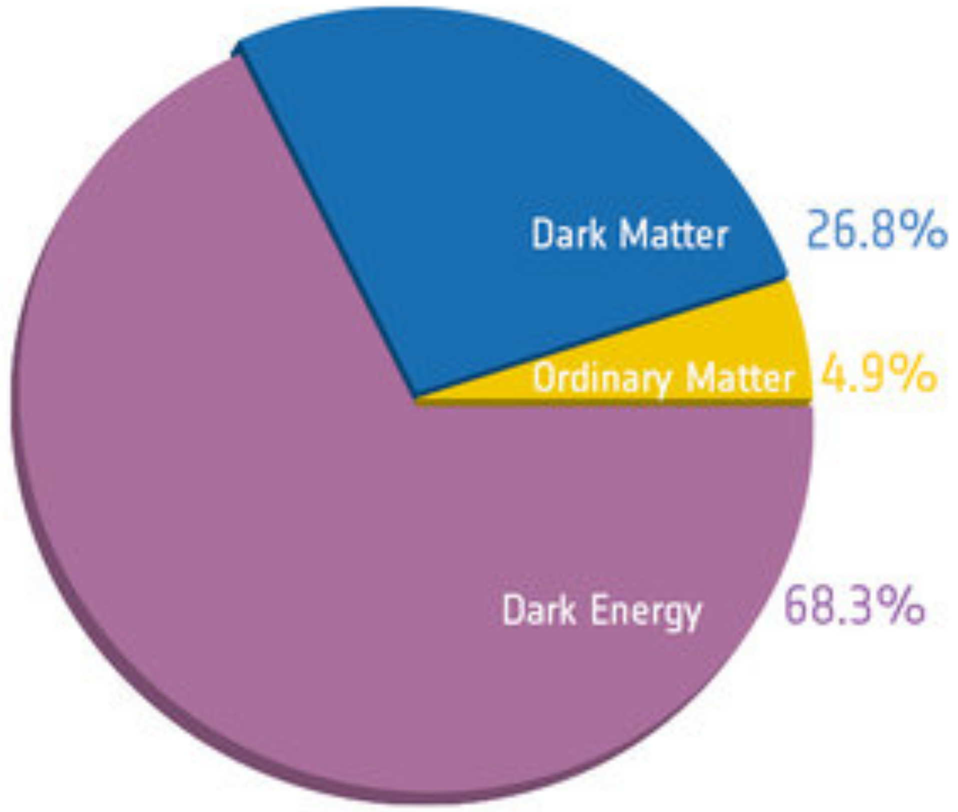


Image: NASA



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INTRODUCTION

Higher-curvature gravity (HCG)

Since we consider GWs, do not require $\mathcal{O}(R_{\mu\nu\rho\sigma})^3$.
Gauss-Bonnet term is topological in $D = 4$.
α and β are positive constants with dimension of $[L]^2$.

- Motivations
  - Renormalizable [Stelle (1977)]
  - Inflation by R^2 [Starobinsky (1980)]
  - Suggestions from string theory [Gross and Witten (1986) etc.]
- Lagrangian

$$\mathcal{L} = R - \alpha\, C_{\mu\nu\rho\sigma}\, C^{\mu\nu\rho\sigma} + \beta\, R^2 + \gamma \left(R_{\mu\nu\rho\sigma}\, R^{\mu\nu\rho\sigma} - 4R_{\mu\nu}\, R^{\mu\nu} + R^2\right) + \mathcal{O}\left(R_{\mu\nu\rho\sigma}\right)^3$$

- Includes massless spin-2, massive spin-2 (Ghost) and massive spin-0 [Stelle (1978)].
- Absence of ghost instability

  Sometimes concluded that HCG is unstable based on Ostrogradsky theorem (1850).

  However, ghost and other degrees of freedom decouple at the linear level [Stelle (1978)].

INTRODUCTION

Gravitational waves (GWs)

- ➤ Ripples in spacetime predicted by Einstein in 1916.
- ➤ Indirect evidence of GWs as energy extraction from the Hulse-Taylor binary pulsar [Hulse and Taylor (1975), Taylor and Weisberg (1982)].
- ➤ Direct observations
  - The first observation is GW150914 [Adv. LIGO (2016)].
  - Limit on deviation $|c_g - c|/c \leq \mathcal{O}(10^{-15})$, interpreted as graviton mass from GW170817 [Adv. LIGO (2017)].
- ➤ Outlook
  - Observations by other facilities than laser interferometers like LIGO.

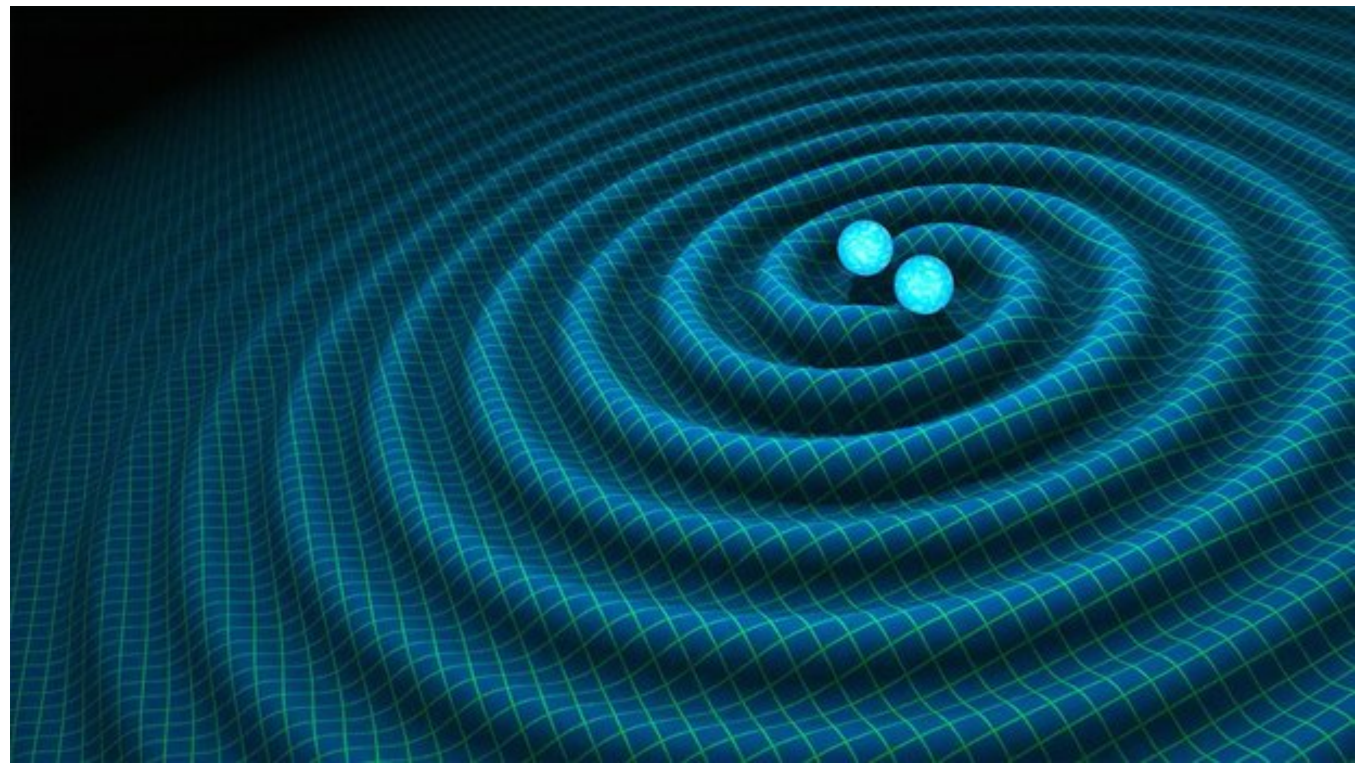
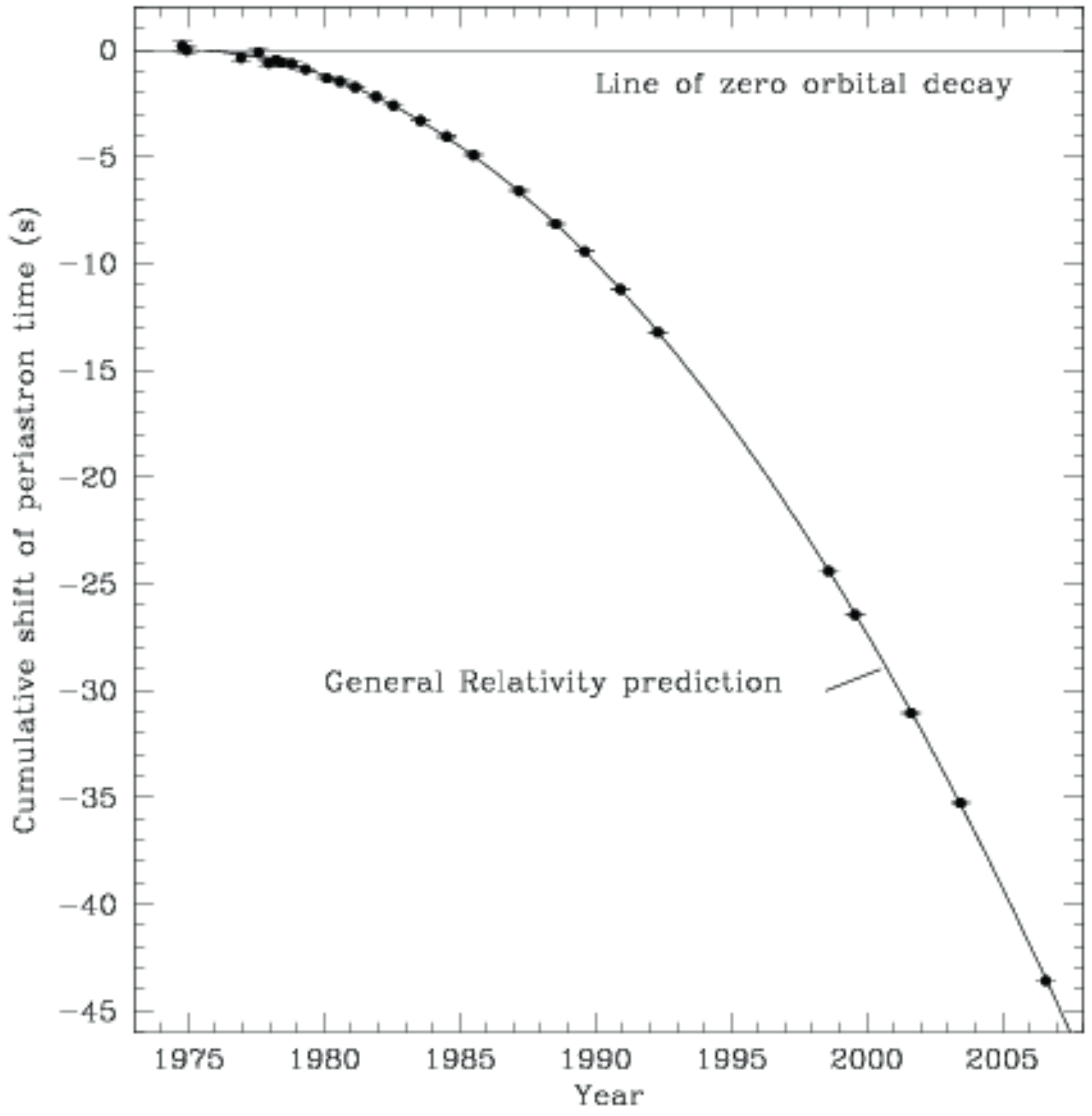


Image: LIGO



Cumulative shift of the periastron time from 1975-2005 [Will (2014)].

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DEGREES OF FREEDOM IN HCG

The action expanded around a flat background to quadratic order

$$S_{\rm HCG}[h_{\mu\nu}] = \frac{1}{2\kappa}\int d^4x \left(-\frac{1}{2}{}^{(1)}G_{\mu\nu}h^{\mu\nu} - \alpha\,{}^{(1)}C_{\mu\nu\rho\sigma}{}^{(1)}C^{\mu\nu\rho\sigma} + \beta\,{}^{(1)}R^2\right)$$ where $\kappa = 8\pi G$ is gravitational constant.

Equations of motion

Tensor

General relativity

$$\begin{cases} \Box\phi_{ij} = 0 \\ (1-2\alpha\Box)\tilde{\phi}_{ij} = 0 \end{cases}$$

Vector

$$(1-2\alpha\Box)\Sigma_i = 0$$

Scalar

$$\begin{cases} (1-2\alpha\Box)\Theta = 0 \\ (1-6\beta\Box)\Xi = 0 \end{cases}$$

where $H_{ij} = \phi_{ij} + \tilde{\phi}_{ij}\,, \Theta = \frac{2}{3}\triangle(\Phi-\Psi)\,, \Xi = -6\Box\Phi + 2\triangle(\Phi-\Psi)$.

We identified all dynamical degrees of freedom satisfying Klein-Gordon type eq.[TT et al. (2021)].

Line element expanded around a flat b.g. to the linear order and construct gauge-invariant variables:

$$g_{\mu\nu}dx^\mu dx^\nu = -(1+2A)dt^2 - (\partial_i B + B_i)dtdx^i + (\delta_{ij} + 2\delta_{ij}C + 2\partial_i\partial_j E + 2\partial_{(i}E_{j)} + 2H_{ij})dx^i dx^j\,,$$

where $\partial_i B^i = \partial_i E^i = 0,\ H_i{}^i = 0,\ \partial_j H^{ij} = 0$.

Gauge-invariant variables are $\Psi \equiv A - (\dot{B} + \ddot{E}),\ \Phi \equiv C,\ \Sigma_i \equiv B_i + \dot{E}_i$ [Kodama and Sasaki (1984)].

DEGREES OF FREEDOM IN HCG

In summary, HCG has eight degrees of freedom (dofs).

Helicity type	Variable (Number of dofs)
2 (Tensor)	$\phi_{ij}\ (2) + \tilde{\phi}_{ij}\ (2)$
1 (Vector)	$\Sigma_i\ (2)$
0 (Scalar)	$\Theta\ (1) + \Xi\ (1)$

$$\begin{cases} \Box \phi_{ij} = 0 \\ (1 - 2\alpha\Box)\tilde{\phi}_{ij} = 0 \end{cases}$$

$$(1 - 2\alpha\Box)\Sigma_i = 0$$

$$\begin{cases} (1 - 2\alpha\Box)\Theta = 0 \\ (1 - 6\beta\Box)\Xi = 0 \end{cases}$$

How to confirm the existence of these extra dofs? ⇨ Gravitational-wave polarizations.

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GRAVITATIONAL-WAVE POLARIZATIONS

- An orthonormal basis of propagating dynamical degrees of freedom (dofs) of gravity.
- Not yet confirmed, but possible in the near future.
- In metric theories GW can have maximally six polarizations [Eardley et al. (1973)].
- Since the contents of polarization modes differ depending on the theory of gravity, it is possible to test the correctness of the theory by observing the polarizations.

Polarization basis

- GWs propagating in z-direction.

  (Latin indices of tensor such as i, j are spatial.)
- GR has only tensor type (plus and cross).

$$e^{+}_{ij} \equiv \begin{pmatrix} 1 & 0 & 0 \\ 0 & -1 & 0 \\ 0 & 0 & 0 \end{pmatrix} \qquad e^{x}_{ij} \equiv \begin{pmatrix} 0 & 0 & 1 \\ 0 & 0 & 0 \\ 1 & 0 & 0 \end{pmatrix} \qquad e^{\mathrm{B}}_{ij} \equiv \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 0 \end{pmatrix}$$

$$e^{\times}_{ij} \equiv \begin{pmatrix} 0 & 1 & 0 \\ 1 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix} \qquad e^{y}_{ij} \equiv \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & 1 \\ 0 & 1 & 0 \end{pmatrix} \qquad e^{\mathrm{L}}_{ij} \equiv \sqrt{2} \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 1 \end{pmatrix}$$

General relativity

Tensor type | Vector type | Scalar type

GRAVITATIONAL-WAVE POLARIZATIONS

Geodesic deviation equation

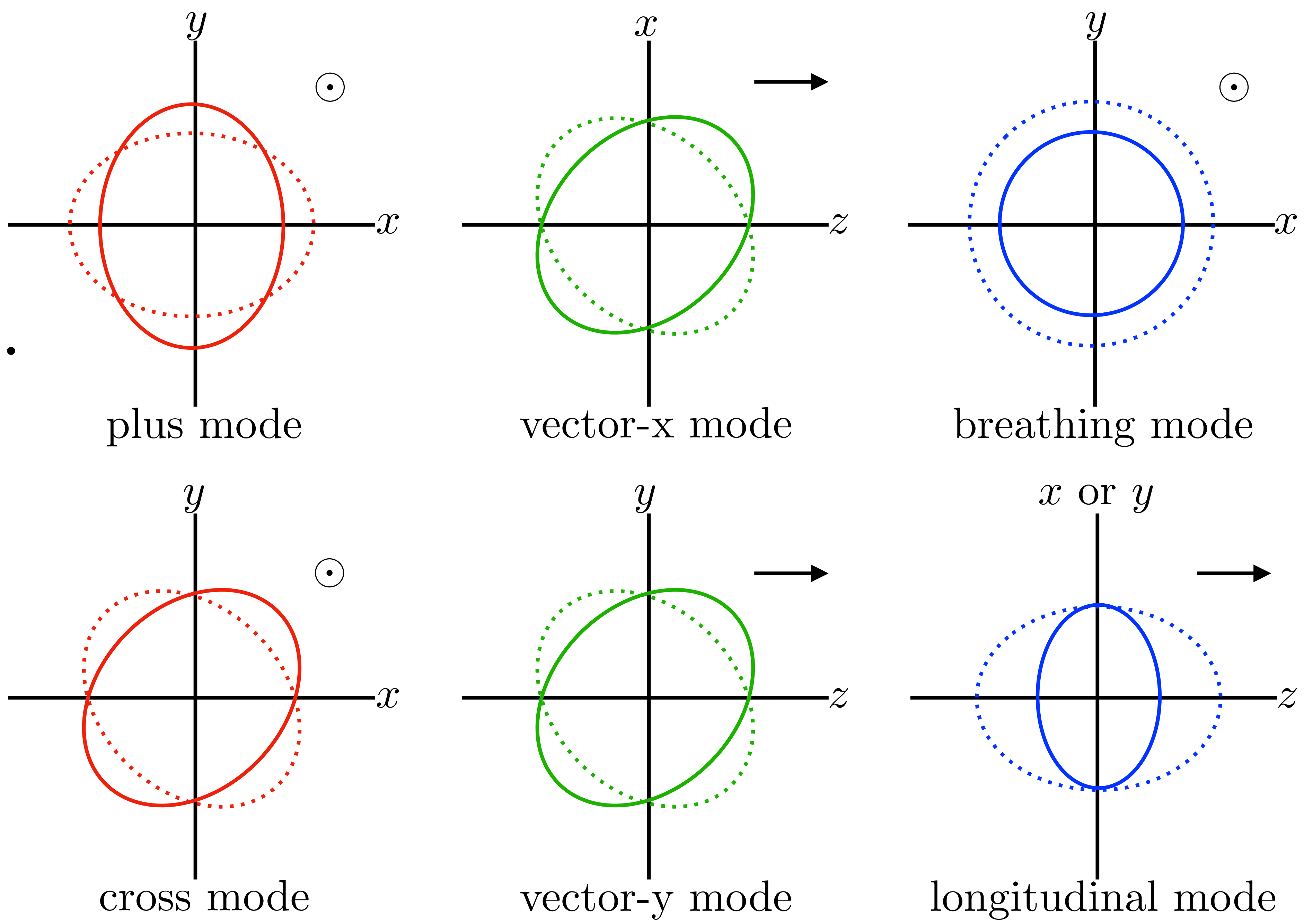
$$\ddot{\zeta}^i = -{}^{(1)}R^i{}_{0j0}\,\zeta^j$$

- Govern the separation of a nearby pair of test bodies at rest ζ^i .
- The principle of detecting GWs.

The linear Riemann tensor

$${}^{(1)}R_{i0j0} = -\ddot{H}_{ij} - \partial_{(i}\dot{\Sigma}_{j)} + \partial_i\partial_j\Psi - \delta_{ij}\ddot{\Phi}$$

- Riemann tensor expanded around a flat b.g. to the linear order written in terms of gauge-invariant variables.
- It can be expressed as a linear combination of the polarization basis.
- A mixture of different spins occurs in the scalar part as we have seen.



Our task

Give a complete characterization of GWs in HCG in terms of the six types of polarizations.

GRAVITATIONAL-WAVE POLARIZATIONS IN HCG

Lagrangian
$$\mathcal{L} = R - \alpha\, C_{\mu\nu\rho\sigma}\, C^{\mu\nu\rho\sigma} + \beta\, R^2$$

Consider **plane wave propagating in z-direction** with different frequencies each spins.

Plug $H_{ij} = \phi_{ij} + \tilde{\phi}_{ij}\,, \Phi = \alpha\Theta - \beta\Xi\,, \Psi = \alpha\Theta - \frac{3}{2}\triangle^{-1}\Theta - \beta\Xi$ into linear Riemann tensor and using eoms

$$^{(1)}R_{i0j0} = -\ddot{H}_{ij} - \partial_{(i}\dot{\Sigma}_{j)} + \partial_i\partial_j\Psi - \delta_{ij}\ddot{\Phi}$$
$$= \omega_1^2\,(\phi_+ e^+_{ij} + \phi_\times e^\times_{ij}) + \omega_2^2\,(\tilde{\phi}_+ e^+_{ij} + \tilde{\phi}_\times e^\times_{ij})$$
$$- \frac{1}{2}\sqrt{\omega_2^2 - \frac{1}{2\alpha}}\omega_2\,(\Sigma_x e^x_{ij} + \Sigma_y e^y_{ij})$$
$$+ \alpha\Theta\left(\omega_2^2 e^{\mathrm{B}}_{ij} - \frac{1}{2\alpha}\sqrt{2}\, e^{\mathrm{L}}_{ij}\right) - \beta\Xi\left(\omega_0^2 e^{\mathrm{B}}_{ij} + \frac{1}{6\beta}\frac{1}{\sqrt{2}} e^{\mathrm{L}}_{ij}\right).$$

General relativity

Weyl squared ($\alpha \neq 0$)

Ricci scalar squared ($\beta \neq 0$)

Polarization tensors

$$e^+_{ij} \equiv \begin{pmatrix} 1 & 0 & 0 \\ 0 & -1 & 0 \\ 0 & 0 & 0 \end{pmatrix} \qquad e^\times_{ij} \equiv \begin{pmatrix} 0 & 1 & 0 \\ 1 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix}$$

$$e^x_{ij} \equiv \begin{pmatrix} 0 & 0 & 1 \\ 0 & 0 & 0 \\ 1 & 0 & 0 \end{pmatrix} \qquad e^y_{ij} \equiv \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & 1 \\ 0 & 1 & 0 \end{pmatrix}$$

$$e^{\mathrm{B}}_{ij} \equiv \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 0 \end{pmatrix} \qquad e^{\mathrm{L}}_{ij} \equiv \sqrt{2}\begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 1 \end{pmatrix}$$

In summary,

- HCG provides six massive polarizations on top of the GR modes.
- Polarizations of scalar dofs are non-trivial linear combinations of the basis e^{B}_{ij} and e^{L}_{ij}.

[TT et al. (2021)]

GRAVITATIONAL-WAVE POLARIZATIONS IN MASSIVE GRAVITY

At linearized level, HCG is equivalent to "GR minus MG" [Stelle (1978)].

Therefore, the analyses of MG can be applied to HCG.

Linear massive gravity action in a flat background

$$S_{\rm MG}[h_{\mu\nu}] = \frac{1}{2\kappa}\int \mathrm{d}^4x \left[-\frac{1}{2}{}^{(1)}G_{\mu\nu}h^{\mu\nu} - \frac{m^2}{4}\left\{h_{\mu\nu}h^{\mu\nu} - (1-\epsilon)h^2\right\}\right].$$

Includes massive spin-2 and massive spin-0 (Ghost).
If we choose $\epsilon = 0$, becomes Fierz—Pauli theory (1939).

Equations of motion

Tensor $\quad \Box H_{ij} - m^2 H_{ij} = 0$

Vector $\quad \Box\Sigma_i - m^2\Sigma_i = 0$

Scalar $\quad \begin{cases} \Box W - m^2 W = 0\,, \\ \epsilon\Box h - \frac{3-4\epsilon}{2}m^2 h = 0 \end{cases}$ where $W = A - \dot{B} - \ddot{E} - C\,, h = 2A + 6C + 2\triangle E\,.$

Plug $\Psi = W - \frac{1}{3m^2}\triangle W + \frac{\epsilon}{6}h\,, \Phi = -\frac{1}{3m^2}\triangle W + \frac{\epsilon}{6}h$ into linear Riemann tensor and using eoms.

GRAVITATIONAL-WAVE POLARIZATIONS IN MASSIVE GRAVITY

Consider **plane wave propagating in z-direction** with different frequencies each spins.

Riemann tensor

$$(1)\quad R_{i0j0} = -\ddot{H}_{ij} - \partial_{(i}\dot{\Sigma}_{j)} + \partial_i\partial_j\Psi - \delta_{ij}\ddot{\Phi}$$
$$= \omega_2^2\left(H_+ e_{ij}^+ + H_\times e_{ij}^\times\right)$$
$$- \frac{1}{2}\sqrt{\omega_2^2 - m^2}\,\omega_2\left(\Sigma_x e_{ij}^x + \Sigma_y e_{ij}^y\right)$$
$$- \frac{1}{3m^2}\triangle W\left(\omega_2^2 e_{ij}^{\mathrm{B}} - \sqrt{2}m^2 e_{ij}^{\mathrm{L}}\right) + \frac{\epsilon}{6}h\left(\omega_0^2 e_{ij}^{\mathrm{B}} + \frac{m_0^2}{\sqrt{2}} e_{ij}^{\mathrm{L}}\right).$$

Massive spin-2 — Massive spin-0 ($\epsilon \neq 0$)

Polarization tensors

$$e_{ij}^+ \equiv \begin{pmatrix}1 & 0 & 0\\ 0 & -1 & 0\\ 0 & 0 & 0\end{pmatrix} \quad e_{ij}^\times \equiv \begin{pmatrix}0 & 1 & 0\\ 1 & 0 & 0\\ 0 & 0 & 0\end{pmatrix}$$

$$e_{ij}^x \equiv \begin{pmatrix}0 & 0 & 1\\ 0 & 0 & 0\\ 1 & 0 & 0\end{pmatrix} \quad e_{ij}^y \equiv \begin{pmatrix}0 & 0 & 0\\ 0 & 0 & 1\\ 0 & 1 & 0\end{pmatrix}$$

$$e_{ij}^{\mathrm{B}} \equiv \begin{pmatrix}1 & 0 & 0\\ 0 & 1 & 0\\ 0 & 0 & 0\end{pmatrix} \quad e_{ij}^{\mathrm{L}} \equiv \sqrt{2}\begin{pmatrix}0 & 0 & 0\\ 0 & 0 & 0\\ 0 & 0 & 1\end{pmatrix}$$

In summary,

- Massive gravity provides six massive polarization modes if $\epsilon \neq 0$.
- Polarizations of scalar dofs are non-trivial linear combinations of the basis e_{ij}^{B} and e_{ij}^{L}.
- Our method works with any epsilon.

[TT et al. (2021)]

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DETERMINATION OF THEORY PARAMETER

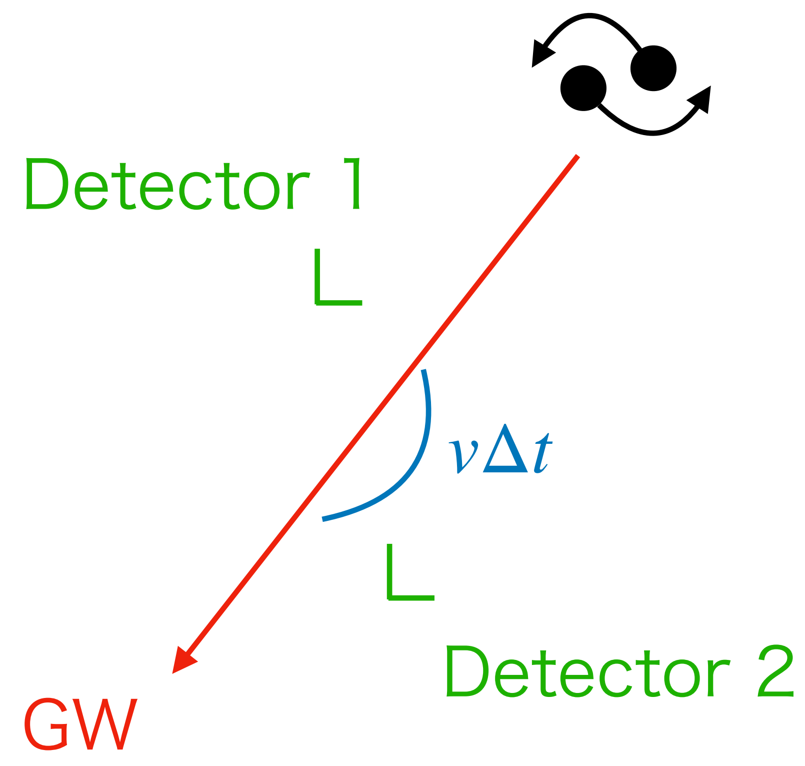
Lagrangian

$$\mathcal{L} = R - \alpha\, C_{\mu\nu\rho\sigma}\, C^{\mu\nu\rho\sigma} + \beta\, R^2$$

Determination of parameter with spin-0 ($\beta \neq 0$) GW

The linear Riemann tensor is $^{(1)}R_{i0j0} \supset -\beta \Xi \left(\omega_0^2\, e^{\mathrm{B}}_{ij} + \frac{1}{6\beta}\frac{1}{\sqrt{2}}\, e^{\mathrm{L}}_{ij} \right)$.

Massive spin-0 (Ricci scalar squared)



If the amplitude of each polarization A_{B} and A_{L} are detected,

the parameter β can be determined from the ratio $\beta = \frac{1}{6\omega_0^2}\frac{A_B(t)}{A_L(t)}$.

Our method does not require measuring the velocity [TT et al. (2021)].

It is also possible to determine α with spin-2 GWs. However, we need more detectors.

We consider two independent signals.

$$\begin{pmatrix} S^1(t) \\ S^2(t+\Delta t) \end{pmatrix} = \mathcal{F} \begin{pmatrix} A_B(t) \\ A_L(t) \end{pmatrix}, \mathcal{F} \equiv \begin{pmatrix} F_B^1 & F_L^1 \\ F_B^2 & F_L^2 \end{pmatrix}$$

where S is signal and F is antenna pattern function.
To solve the amplitude each polarization,
the inverse of F must exist $(\det|\mathcal{F}| \neq 0)$.

⭕ Pulsar timing array

$$F_B = \frac{1}{2}\frac{\sin^2\theta}{1+v\cos\theta}, \quad F_L = \frac{1}{\sqrt{2}}\frac{\cos^2\theta}{1+v\cos\theta}$$

❌ Laser interferometer

$$F_B = -\frac{1}{2}\sin^2\theta\cos 2\phi, \quad F_L = \frac{1}{\sqrt{2}}\sin^2\theta\cos 2\phi$$

[Yunes and Siemens (2013),
Qin, Boddy and Kamionkowski (2021)]

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CONCLUSION

I showed

- All helicity states in HCG and MG satisfy Klein-Gordon type equations.
- HCG and MG provide massive polarizations.
  - HCG provides six massive polarizations on top of the GR modes.
  - MG provides six massive polarizations ($\epsilon \neq 0$).
  - Polarizations of scalar dofs are non-trivial linear combinations of the basis e_{ij}^{B} and e_{ij}^{L}.
- Method to determine theory parameters (masses) by observations.
  - We need to detect the amplitudes of both scalar modes.
  - However, our method does not require measuring GW velocity.

Outlook

- Develop a method to detect GW polarizations.
- Consider interaction with matter fields (energy extraction from binary systems).