

R^2 -gravity quark stars from perturbative QCD*

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We investigate the structure of quark stars in the framework of $f(R) = R + \alpha R^2$ gravity using an **equation of state for cold quark matter obtained from perturbative QCD**, parametrized only by the renormalization scale. We show that a considerably large range of the free parameter α , within and even beyond the constraints previously reported in the literature, yield **non-negligible modifications in the mass and radius** of stars with large central mass densities. Besides, their stability against baryon evaporation is analyzed through the behavior of the associated total binding energies for which we show that these energies are slightly affected by the modified gravity term in the regime of high proper (baryon) masses

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Motivations

$$\mathcal{L}_{\text{EH}} = R \longrightarrow f(R) = R + \alpha R^2$$

- Inflation
- Large-curvature behaviour
- Dense cores: quark stars ?
- Degeneracy with EOS

Modified Einstein Equations

$$I = \frac{1}{16\pi} \int d^4x \sqrt{-g} f(R) + I_m$$

$$f_R R_{\mu\nu} - \frac{1}{2} g_{\mu\nu} f + [g_{\mu\nu} \square - \nabla_\mu \nabla_\nu] f_R = 8\pi T_{\mu\nu}$$

$$3\square f_R(R) + R f_R(R) - 2f(R) = 8\pi T$$

Extra degree of Freedom:

Einstein Frame \rightarrow Scalar Field
 \rightarrow Jordan Frame \rightarrow Physical Interpretation

Metric Static and Spherically symmetric

$$ds^2 = -e^{2\psi(r)} dt^2 + e^{2\lambda(r)} dr^2 + r^2(d\theta^2 + \sin^2 \theta d\phi^2)$$

$$e^{-2\lambda(r)} = 1 - \frac{2m(r)}{r}$$

Astrophysical Mass*: $M = \lim_{r \rightarrow \infty} \frac{r}{2} \left\{ 1 - \exp[-2\lambda(r)] \right\}$

Surface Mass*: $m_{\text{sur}} = \frac{r_{\text{sur}}}{2} \left\{ 1 - \exp[-2\lambda(r_{\text{sur}})] \right\}$

* "Neutron star masses in R² gravity", Fulvio Sbisà *et al.*, Physics of the Dark Universe 27 (2020) 100411

Modified TOV equations

$$\frac{d\psi}{dr} = \frac{1}{4r(1+2\alpha R + \alpha r R')} [r^2 e^{2\lambda} (16\pi p - \alpha R^2) + 2(1+2\alpha R) (e^{2\lambda} - 1) - 8\alpha r R'] ,$$

$$\begin{aligned} \frac{d\lambda}{dr} = & \frac{1}{4r(1+2\alpha R + \alpha r R')} \left\{ 2(1+2\alpha R) (1 - e^{2\lambda}) + \frac{r^2 e^{2\lambda}}{3} [16\pi(2\rho + 3p) + 2R + 3\alpha R^2] \right. \\ & \left. + \frac{2\alpha r R'}{1+2\alpha R} \left[2(1+2\alpha R) (1 - e^{2\lambda}) + \frac{r^2 e^{2\lambda}}{3} (16\pi\rho + R + 3\alpha R^2) + 4\alpha r R' \right] \right\}, \end{aligned}$$

$$\frac{d^2 R}{dr^2} = \frac{e^{2\lambda}}{6\alpha} [R + 8\pi(3p - \rho)] + \left(\lambda' - \psi' - \frac{2}{r} \right) R' ,$$

$$\frac{dp}{dr} = -(\rho + p)\psi' ,$$

$$f' \equiv \frac{df}{dr}$$

Curvature Fluid

$$\left\{ \begin{array}{l} \frac{d}{dr} (r e^{-2\lambda}) = 1 - 8\pi r^2 \rho - 2\alpha \left\{ -R \frac{d}{dr} [r(1 - e^{-2\lambda})] + \frac{r^2}{4} R^2 + \frac{r^2}{e^{2\lambda}} \left[\left(\frac{2}{r} - \lambda' \right) R' + R'' \right] \right\} \\ e^{-2\lambda(r)} = 1 - \frac{2m(r)}{r} \end{array} \right.$$

$$\begin{aligned} m(r') &= 4\pi \int_0^{r'} r^2 \rho dr + \\ &+ \alpha \int_0^{r'} \left\{ \frac{R^2}{4} - \frac{R}{r^2} \frac{d}{dr} [r(1 - e^{-2\lambda})] + \frac{1}{e^{2\lambda}} \left[\left(\frac{2}{r} - \lambda' \right) R' + R'' \right] \right\} r^2 dr \end{aligned}$$

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Quark Matter

E. S. Fraga, A. Kurkela, and A. Vuorinen, *Astrophys. J. Lett.* 781, L25 (2014), arXiv:1311.5154 [nucl-th].

$$T_{\mu\nu} = (\rho + p)u_\mu u_\nu + p g_{\mu\nu}$$

$$p = p_{\text{SB}}(\mu_B) \left(c_1 - \frac{a(X)}{(\mu_B/\text{GeV}) - b(X)} \right)$$

$$p_{\text{SB}}(\mu_B) = (3/4\pi^2)(\mu_B/3)^4$$

Stefan-Boltzmann gas (3 massless quarks)

$$\mu_B$$

Chemical potential

$$X = 3\bar{\Lambda}/\mu_B$$

Dimensionless renormalization scale parameter

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$$a(X) = d_1 X^{-\nu_1}$$

$$\nu_1 = 0.3553$$

$$b(X) = d_2 X^{-\nu_2}$$

$$\nu_2 = 0.9101$$

$$c_1 = 0.9008$$

$$d_1 = 0.5034$$

$$d_2 = 1.452$$

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$$p = p_{\text{SB}}(\mu_B) \left(c_1 - \frac{a(X)}{(\mu_B/\text{GeV}) - b(X)} \right) \quad = \text{FKvX} \\ 1 < X < 4$$

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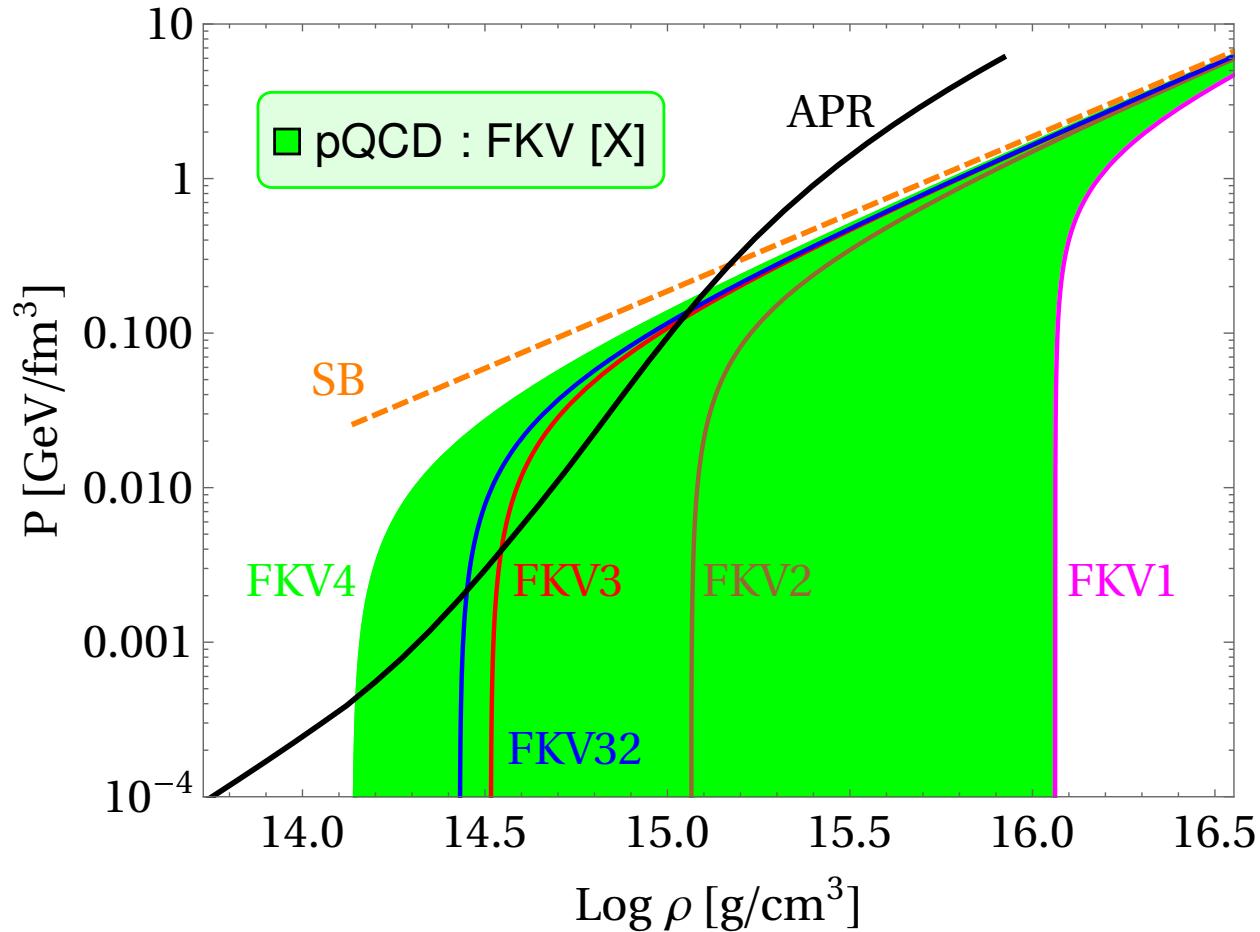
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Equation of State



Ingredients

Modified TOV + Quark Matter =

= Standard TOV + Curvature Fluid + Quark Matter

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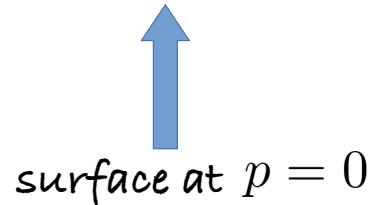
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surface at $p = 0$

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leaks out of the star !

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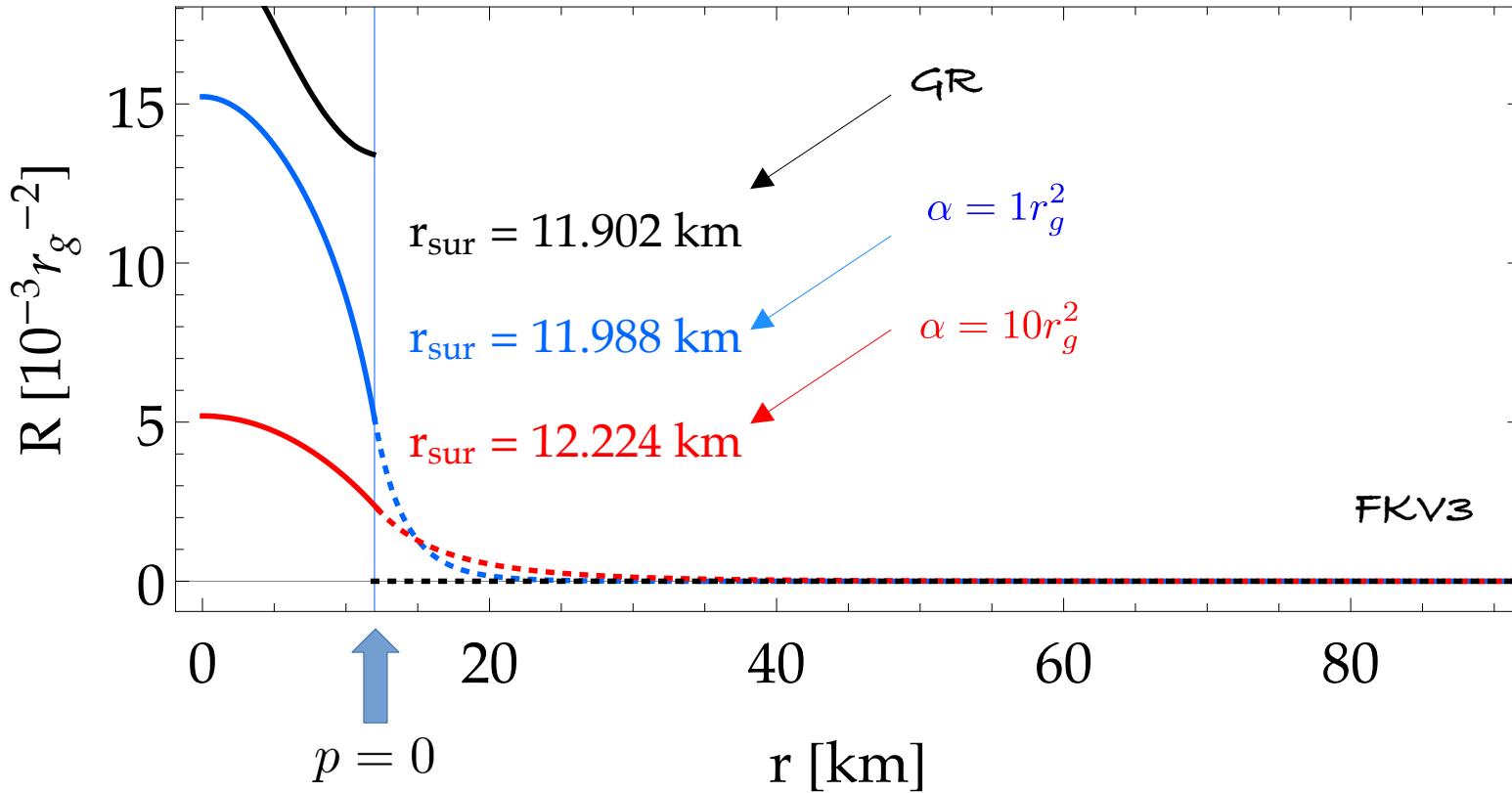


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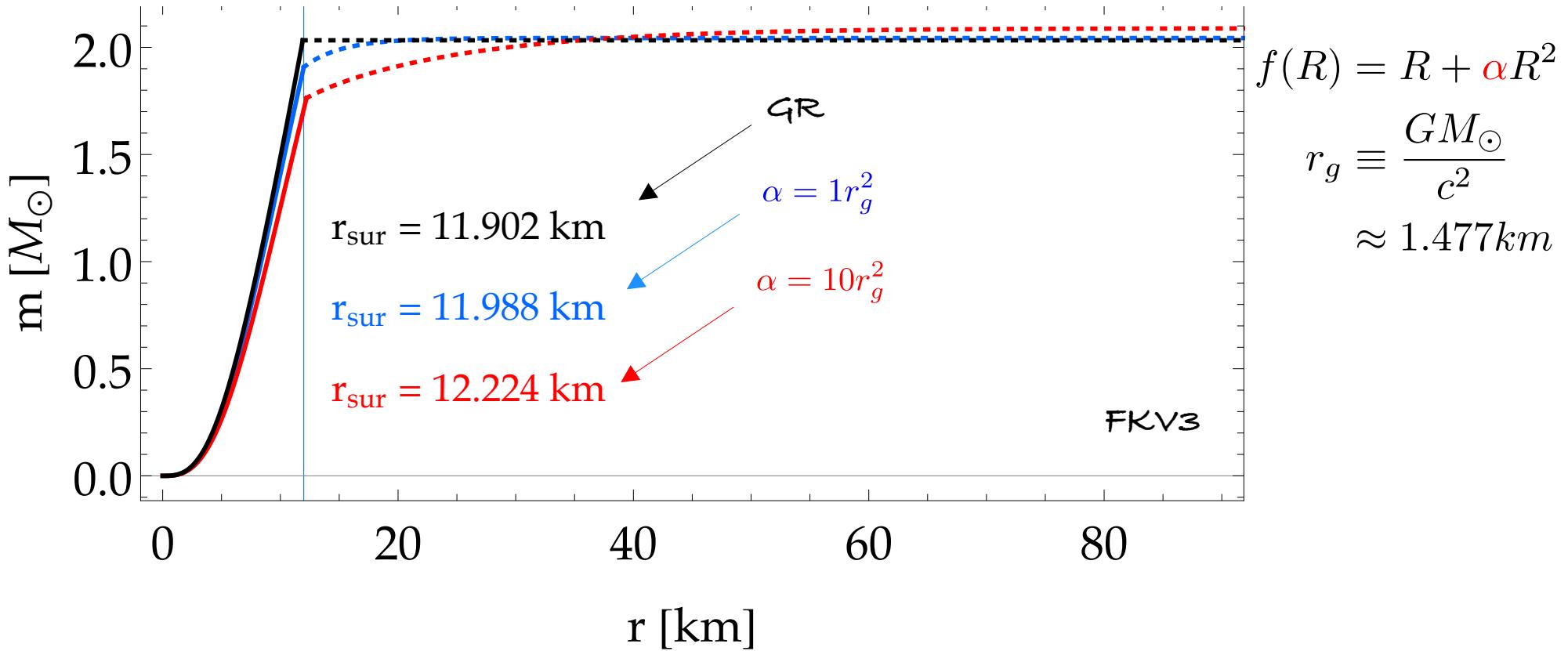
astrophysical mass
(at infinity)

Leaking curvature

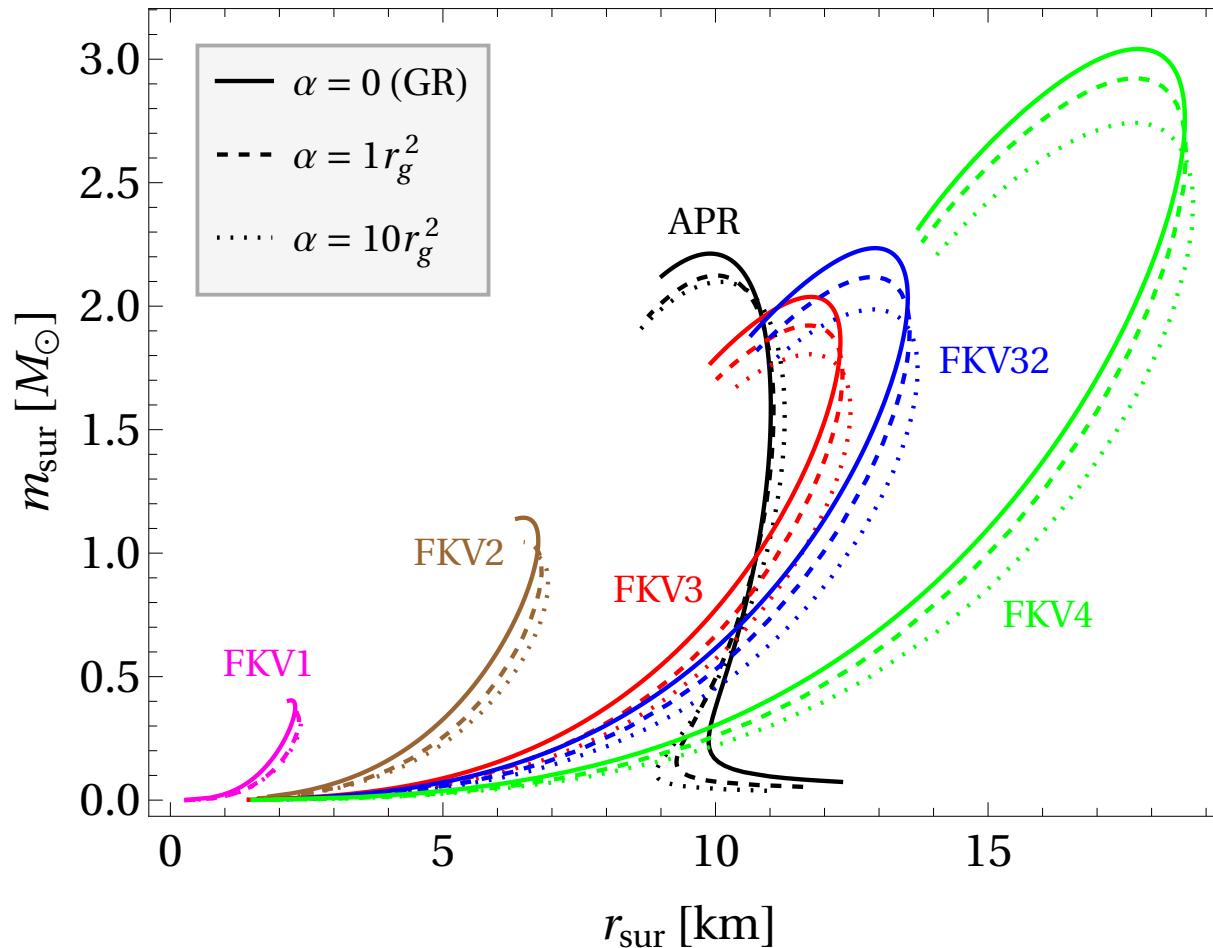


$$f(R) = R + \alpha R^2$$
$$r_g \equiv \frac{GM_\odot}{c^2}$$
$$\approx 1.477 \text{ km}$$

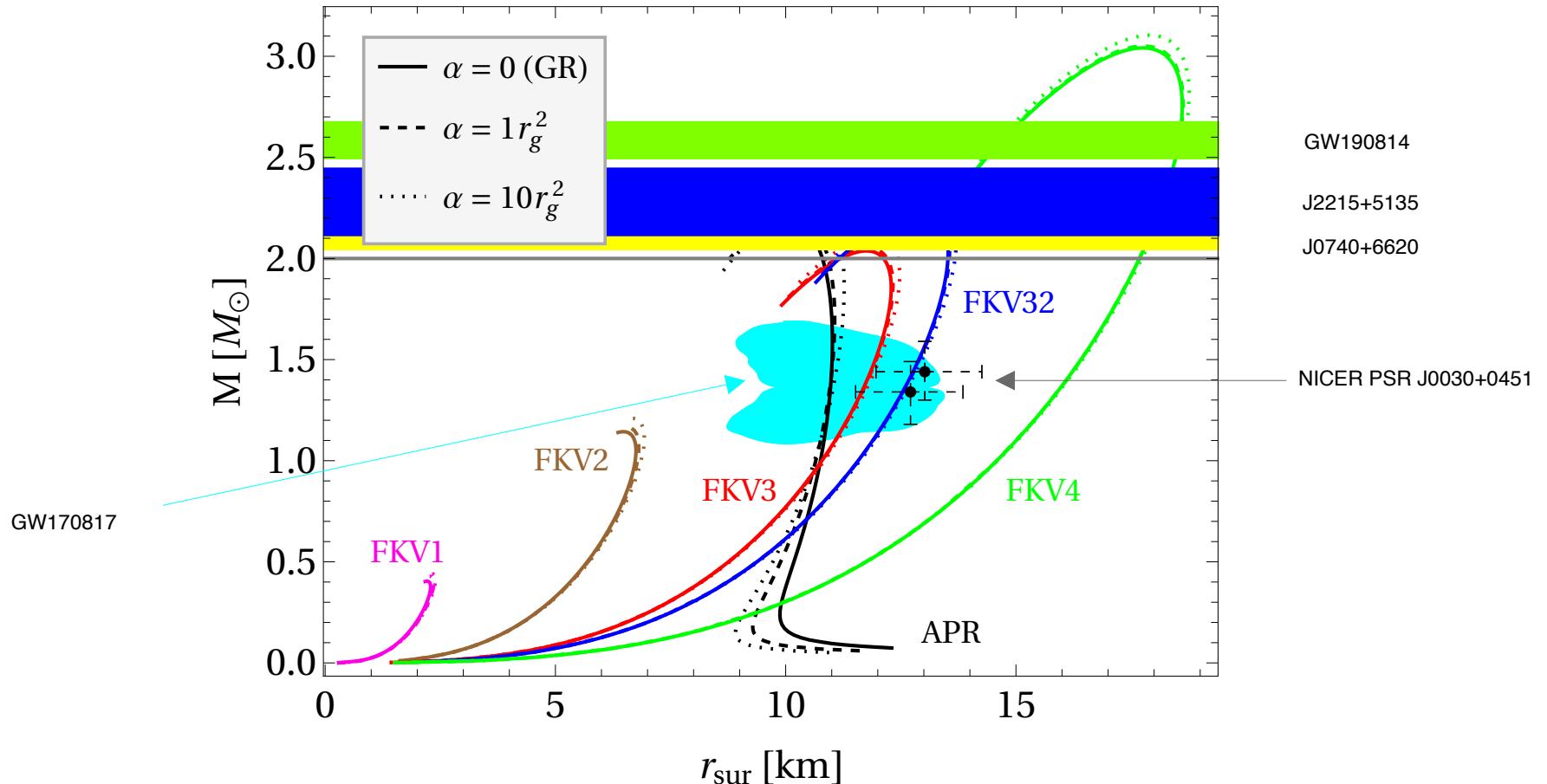
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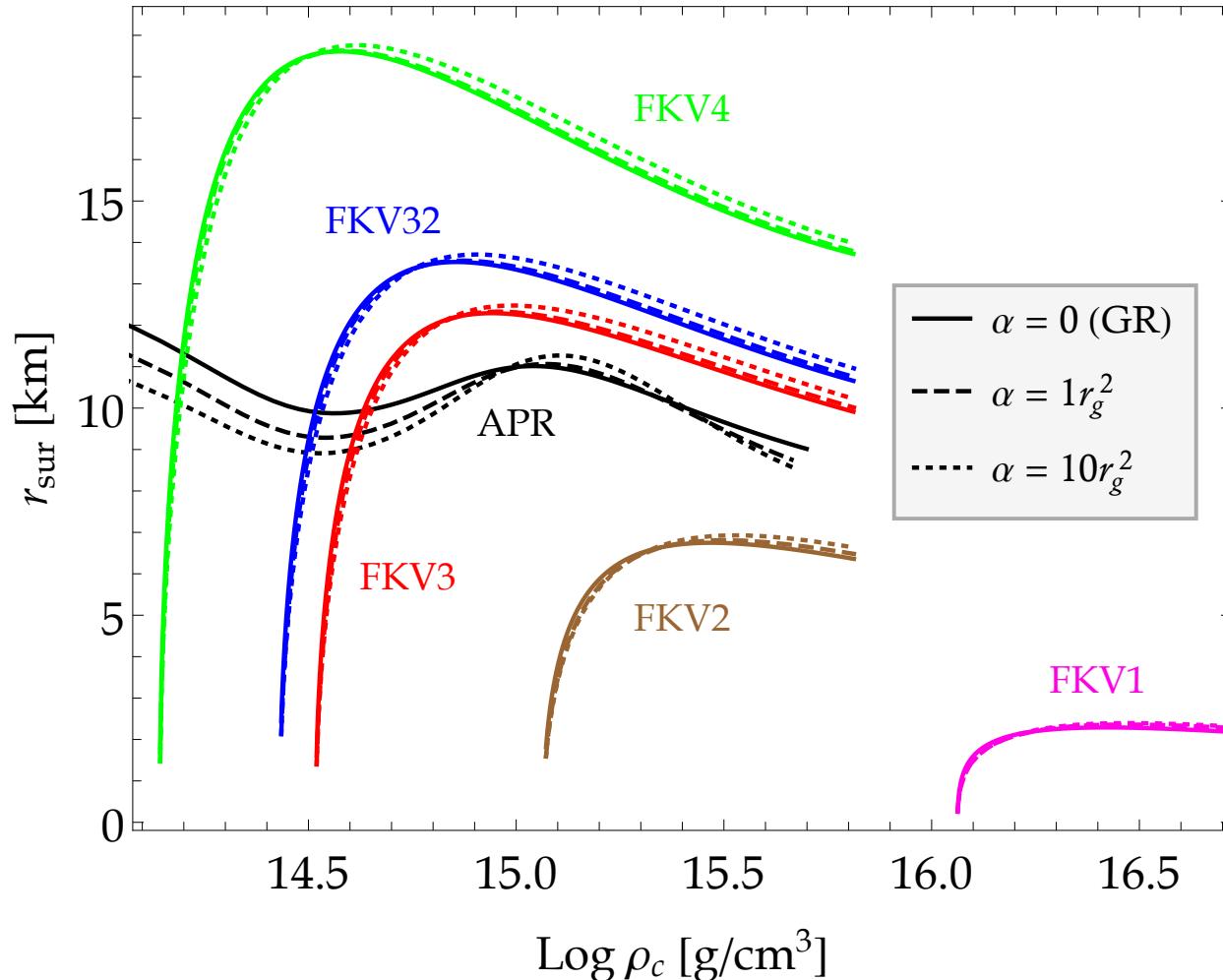


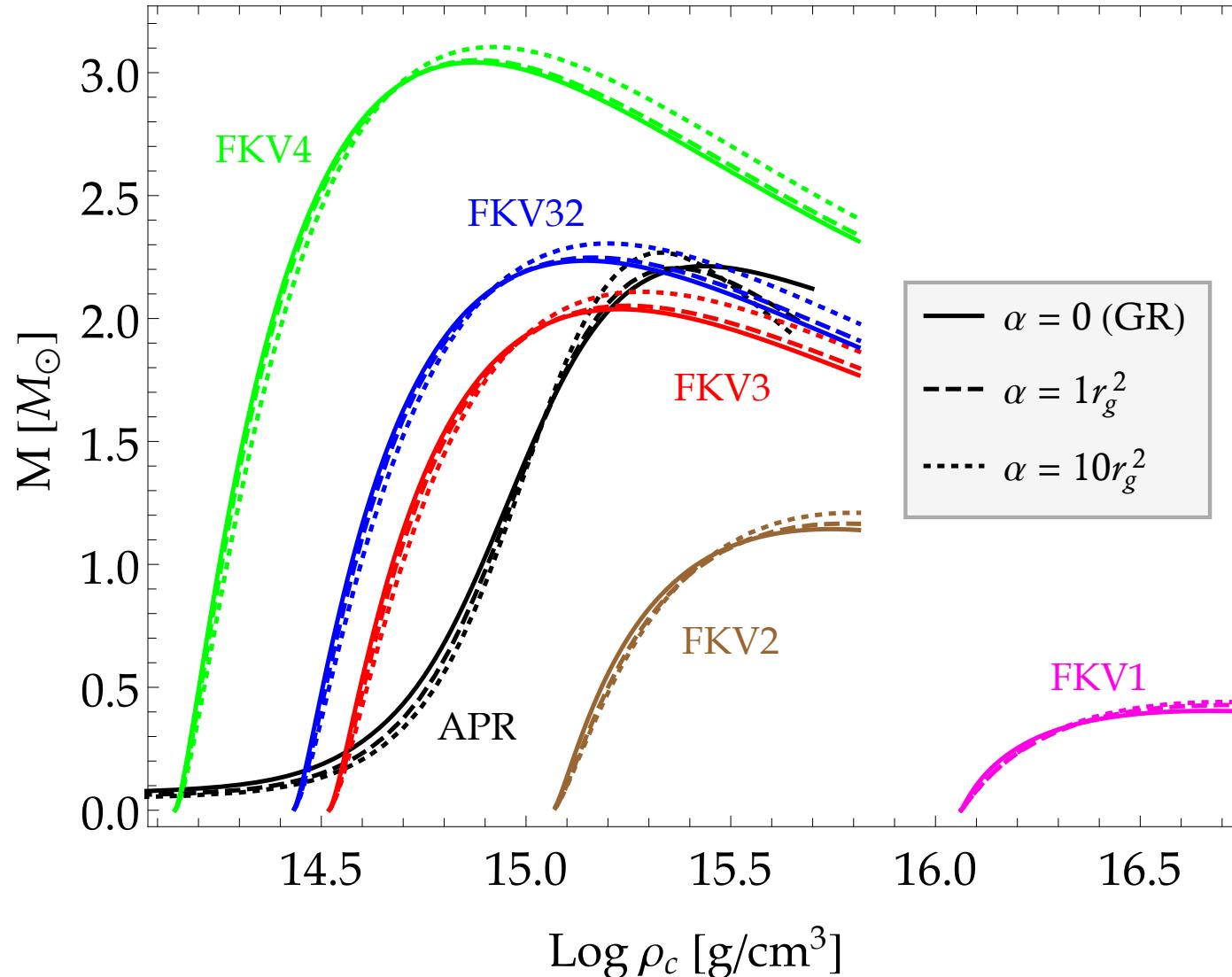
Numerical Results

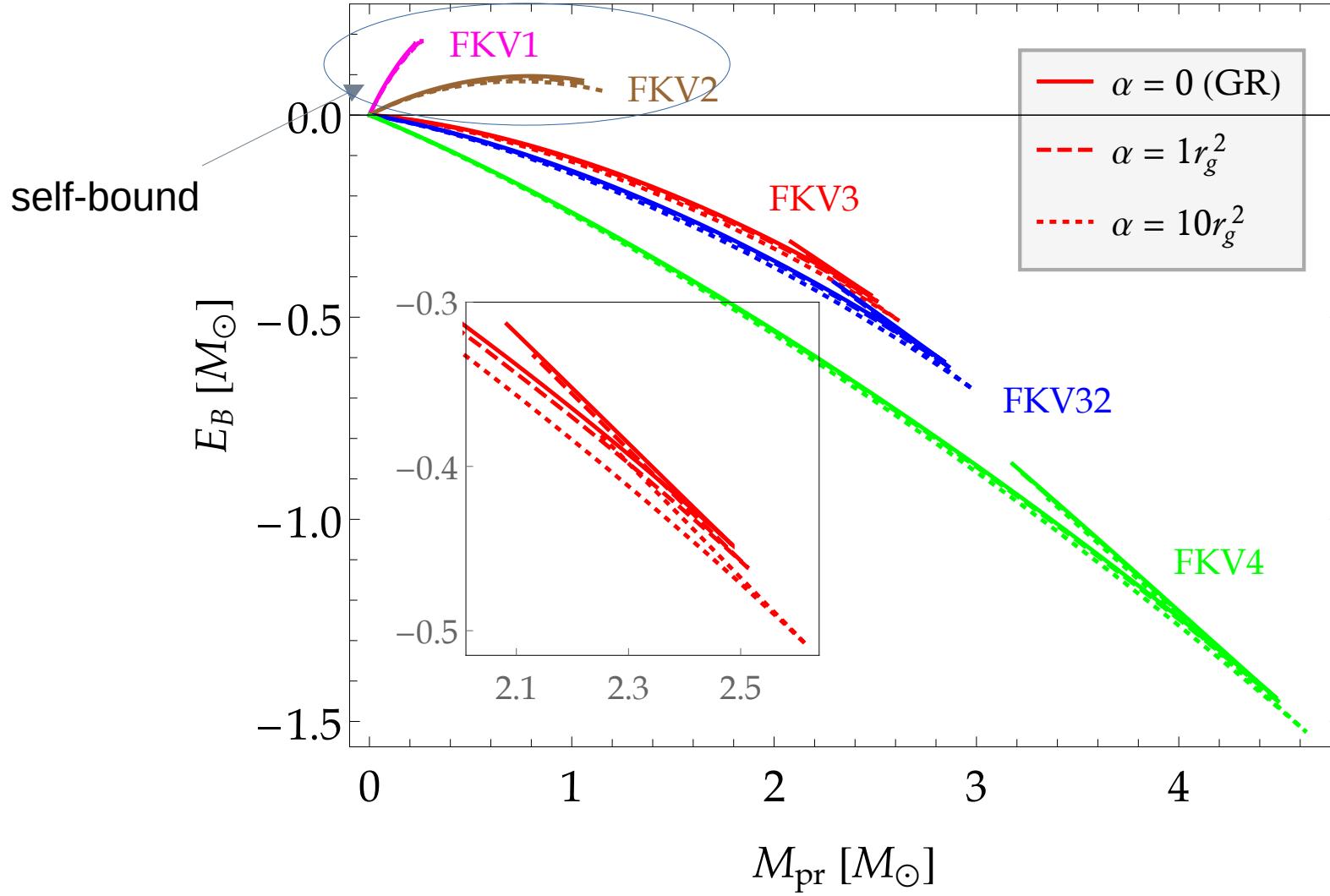


Numerical Results









EoS	α [r_g^2]	R_c [$10^{-3}r_g^{-2}$]	ρ_c [$10^{15}\text{g}/\text{cm}^3$]	r_{sur} [km]	m_{sur} [M_\odot]	M [M_\odot]	E_B [M_\odot]
FKV3	0	21.160	1.693	11.752	2.037	2.037	-0.437
	1	15.735	1.775	11.784	1.921	2.052	-0.462
	10	5.284	1.912	11.939	1.800	2.109	-0.509
FVK32	0	17.687	1.400	12.927	2.235	2.235	-0.612
	1	13.682	1.459	12.956	2.117	2.248	-0.626
	10	4.981	1.587	13.100	1.982	2.305	-0.678
FVK4	0	9.727	0.744	17.755	3.041	3.041	-1.443
	1	8.278	0.763	17.775	2.922	3.051	-1.458
	10	3.925	0.829	17.900	2.738	3.105	-1.526

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Conclusions

- Nonperturbative treatment
- Larger effect in R_c and in m_{sur} ($\sim z_{\text{sur}}$)
- Negligible effects in (astrophysical) mass and radius from modifications in GR for FKVX.
- Stability?
- Hybrid stars?