

Cosmology With Bright Standard Sirens

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*Cosmo 22, Rio de Janeiro, Brazil
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Bright Standard Sirens



Spacetime interval

$$dS^2 = -dt^2 + 2h_{\times} dxdt + (1+h_{+})dx^2 + (1-h_{+})dy^2 + dz^2$$

Polarizations

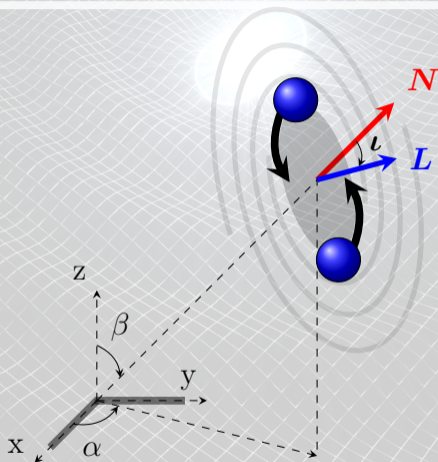
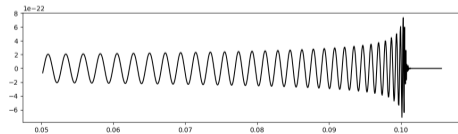
$$h_{+} \propto \frac{1}{d_L} \left(\frac{1 + \cos^2 \iota}{2} \right)$$

$$h_{\times} \propto \frac{1}{d_L} \cos \iota$$

$$h(t) = F_{+} h_{+}(t) + F_{\times} h_{\times}(t) : \text{Signal}$$

$$h \propto \frac{1}{d_L}$$

$$\Delta L \approx h L_0$$



First Measurement of H_0 with GW170817

Source Properties:

Total Mass $2.74^{+0.04}_{-0.01} M_{\odot}$;

Luminosity Distance: $d_L = 40^{+8}_{-14} \text{ Mpc}$;

Inclination: $\iota \leq 55^\circ$;

From EM-Counterpart

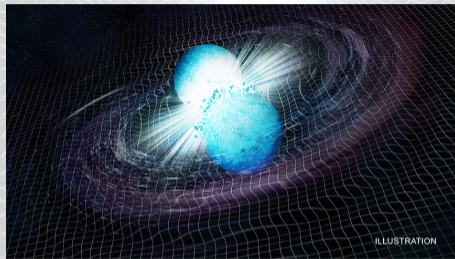
Host Galaxy: NGC 4993;

Redshift: $z = 0.00973$ (host galaxy);

H_0 Measurement: $H_0 = 70^{+12}_{-8} \text{ km/s/Mpc}$.

Abbott, B. P., et al., PhysRevLett.119.161101

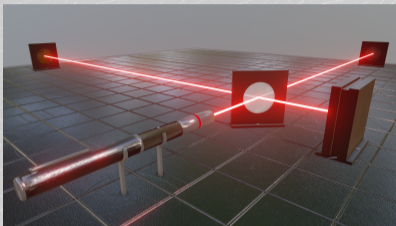
Uncertainty of 12% with respect to the measurement.



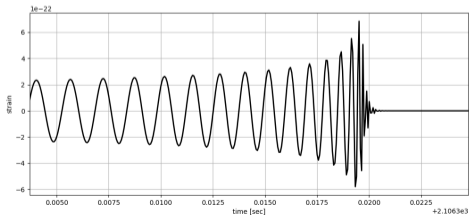
Credit: [<https://chandra.harvard.edu/photo/2018/gw170817/>]

Gravitational Waves Detectors (Second Generation)

Interferometers



Credit: Author.



$$L_{LIGO} = 4\text{km}, \quad L_{Virgo} = 3\text{km}, \quad L_{KAGRA} = 3\text{km}$$



Credit: [<https://www.virgo-gw.eu/>], [<https://www.ligo.caltech.edu/>], [<https://gwcenter.icrr.u-tokyo.ac.jp/en/>].

Gravitational Waves Detectors (Second Generation)

Detector Output:

$$\mathcal{O}(t) = \underset{\substack{\uparrow \\ \text{Signal}}}{h(t)} + \underset{\substack{\uparrow \\ \text{Noise}}}{n(t)}$$

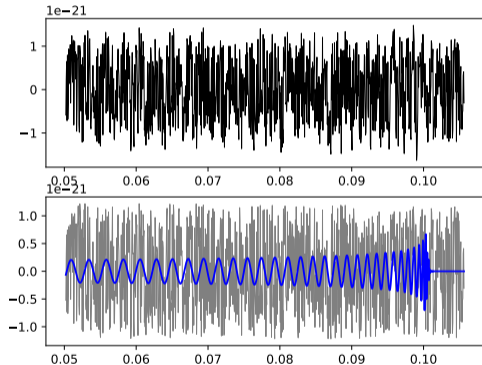
Signal to Noise Ratio [Matched Filtering]:

$$SNR = 2 \left[\text{Re} \int_0^\infty \frac{\tilde{\mathcal{O}}^*(f) \tilde{h}(f)}{S_n(f)} df \right]^{1/2}$$

$$\langle \tilde{n}(f) \tilde{n}^*(f') \rangle = \frac{1}{2} \overset{\substack{\text{Noise Power Spectrum} \\ \text{Density}}}{S_n(f)} \delta(f - f')$$

Adimensional Quantities

$$h_c = 2f |\tilde{h}(f)|, \quad S_c = \sqrt{f \cdot S_n(f)}$$



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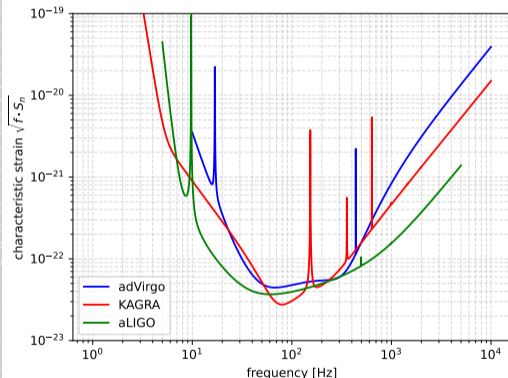
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Noise Power Spectrum Density

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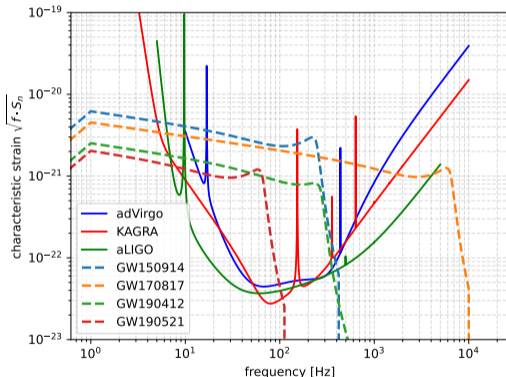
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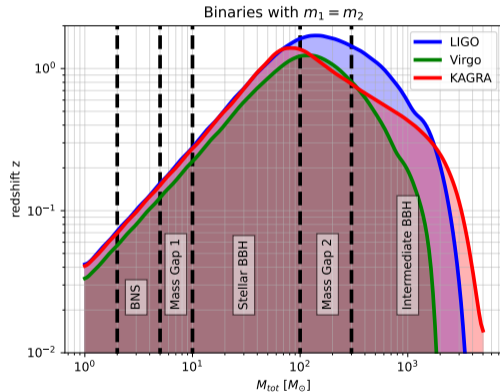
Gravitational Waves Detectors (Second Generation)

For which redshift/distance range can we detect binaries of neutron stars and black holes?

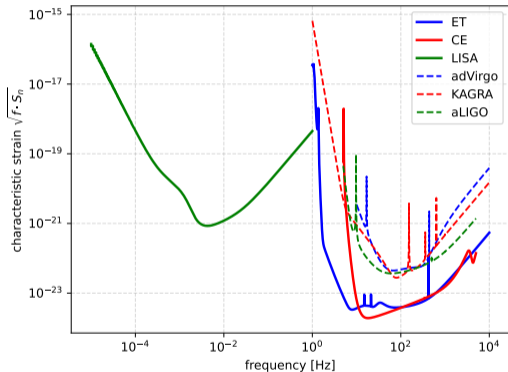
Aside we show the sensibility region of detectors in mass/redshift space of detections with **SNR bigger than 8**.

$$SNR(M_{tot}, z) = 2 \left[\int_0^\infty \frac{|\tilde{h}(f, M_{tot}, z)|^2}{S_n(f)} df \right]^{1/2}$$

$$\langle \tilde{n}(f) \tilde{n}^*(f') \rangle = \frac{1}{2} S_n(f) \delta(f - f')$$

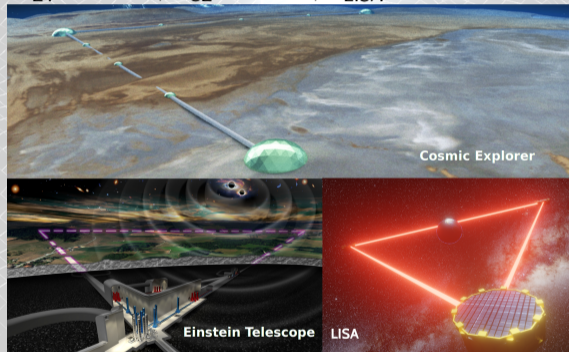


Gravitational Waves Detectors (Ground 3G and Space 1G)



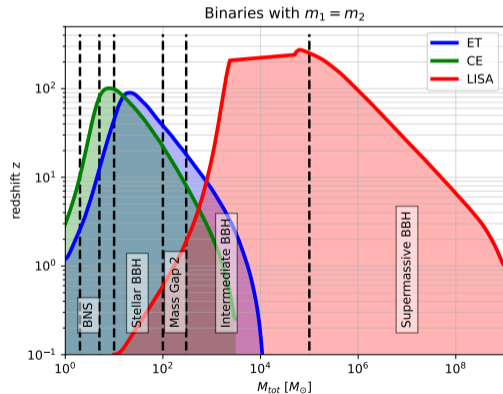
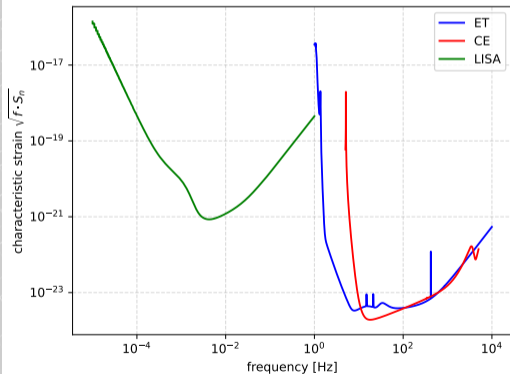
Curves of ET and CE retrived from [<https://dcc.ligo.org/LIGO-T1500293/public>]

$L_{ET} = 10\text{km}$, $L_{CE} = 40\text{km}$, $L_{LISA} = 2.500.000\text{km}$

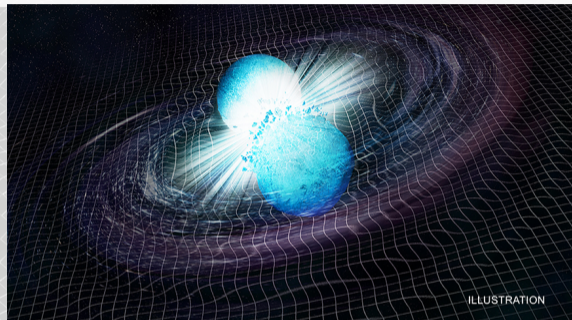
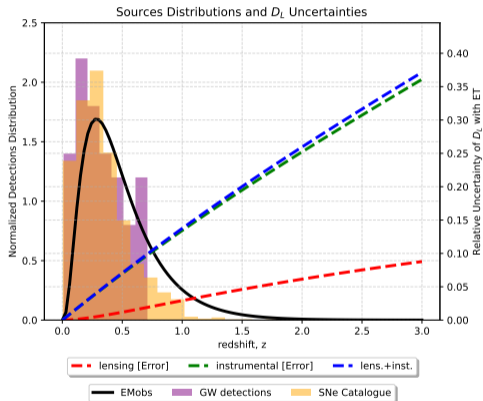


Credits: [<https://gwic.ligo.org/3Gsubcomm/>], [<http://www.et-gw.eu/>] e Author (figura do LISA).

Gravitational Waves Detectors (Third Generation)

Range Power of Detectors (SNR > 8): $SNR(z, M_{tot})$ 

GW+shGRBs with Einstein Telescope



Credit: [<https://chandra.harvard.edu/photo/2018/gw170817/>]

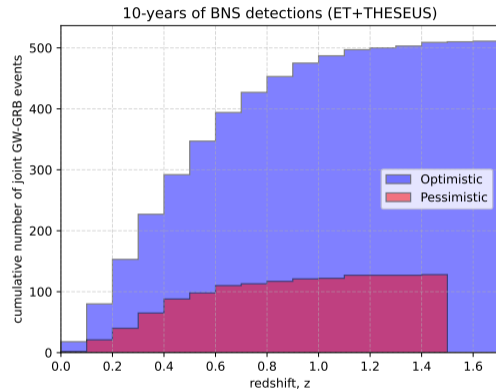
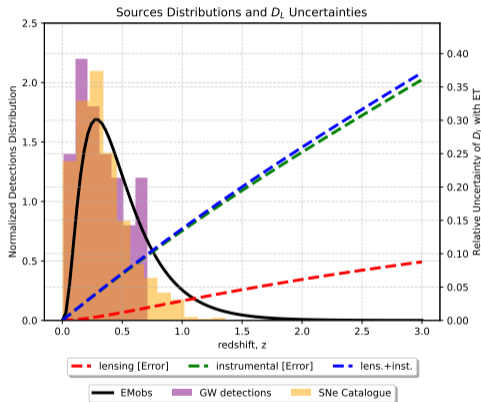
Distributions and Uncertainties in d_L :

Belgacem, E., et al. *JCAP*, 2019(08), 015.

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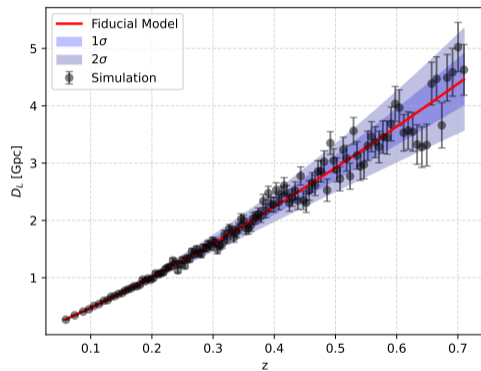
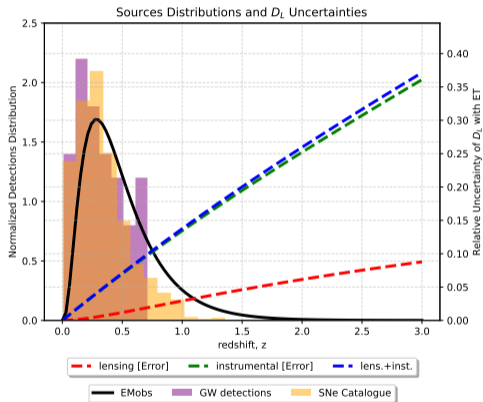
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Bayesian Model Selection [ET]

Bayes Theorem:

$$\underbrace{P(\{\theta_i\} | data, M)}_{\text{Posterior}} = \frac{\underbrace{P(data | \{\theta_i\}, M)}_{\text{Likelihood}} \underbrace{P(\{\theta_i\}, M)}_{\text{Prior}}}{\underbrace{P(data)}_{\text{Evidence}}}$$

Bayesian Evidence:

$$\mathcal{Z} = P(data) \equiv \int P(data | \{\theta_i\}, M) P(\{\theta_i\}, M) d\theta_1 \dots d\theta_N$$

Bayes Factor:

$$B_{0,1} \equiv \frac{\mathcal{Z}_0 \leftarrow \text{Evidence for the model } M_0}{\mathcal{Z}_1 \leftarrow \text{Evidence for the model } M_1}$$

Jeffrey's Scale

$\log B_{0,1}$	Conclusion
< 1	Inconclusive
$> 1, < 2.5$	Weak Evidence
$> 2.5, < 5.0$	Moderate Evidence
> 5.0	Strong Evidence

Trotta, R. *Contemporary Physics*, 49(2), 71-104.

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Dark Energy Equation of State Parametrizations

$$P_{DE} = \omega_{DE} \rho_{DE}$$

$$\Lambda \text{CDM Model} \quad \omega_{DE} = -1;$$

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Physics of the Dark Universe 32 (2021) 100830



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journal homepage: www.elsevier.com/locate/dark



Cosmological model selection from standard siren detections by third-generation gravitational wave observatories

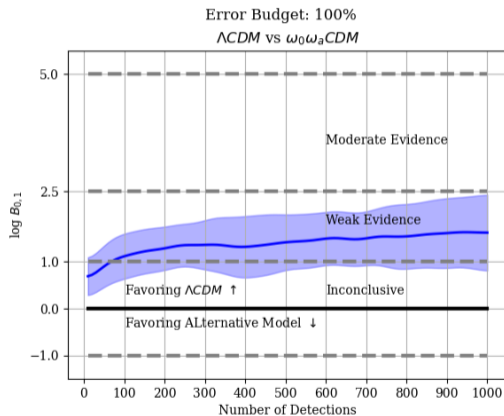
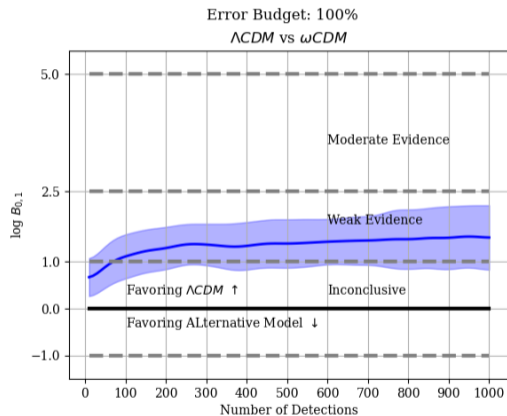
Josiel Mendonça Soares de Souza, Riccardo Sturani*

Departamento de Física Teórica e Experimental, Universidade Federal do Rio Grande do Norte, Natal-RN 59078-970, Brazil
International Institute of Physics, Universidade Federal do Rio Grande do Norte, Campus Universitário, Lagoa Nova, Natal-RN 59078-970, Brazil



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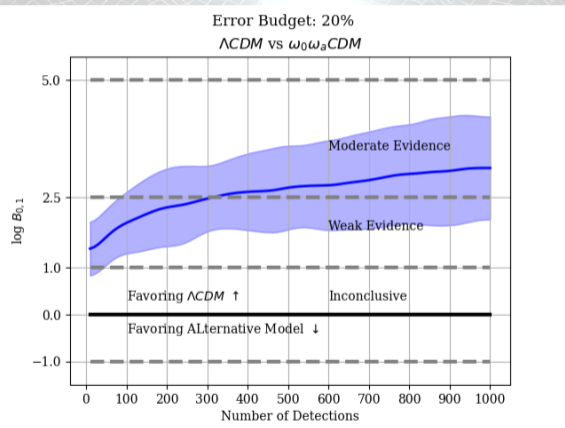
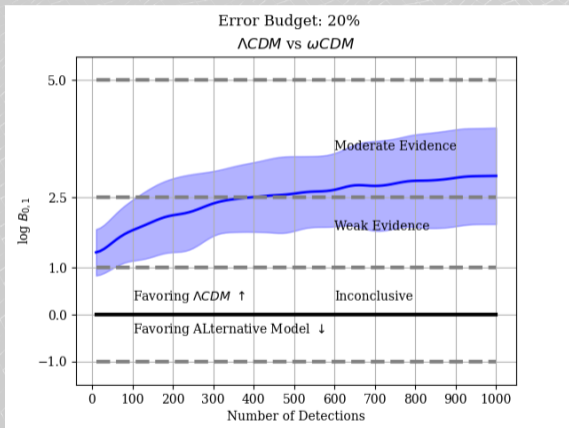
The simulations were performed 50 times. The plots show the averages over 50 realizations of the Bayes factor computations.



[Souza, J. M. S., & Sturani, R. *Physics of the Dark Universe*, 32, 100830.]

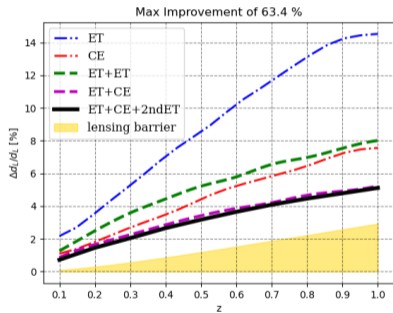
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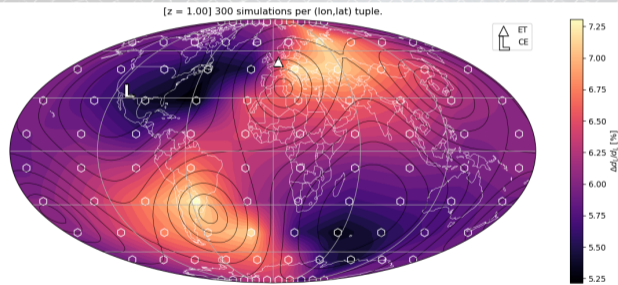


[Souza, J. M. S., & Sturani, R. *Physics of the Dark Universe*, 32, 100830.]

Improving $D_L(z)$ Measurements With Multiple 3G Ground Detectors



Estimations from Fisher Matrix Approach.



Cosmography with Standard Sirens [ET]

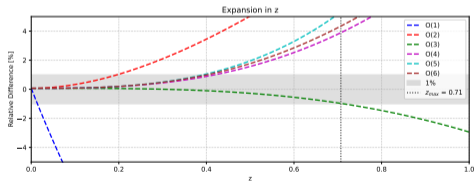
Cosmographic Expansion:

$$d_L(z) = \frac{1}{H_0} \left[z + \frac{1}{2}(1 - q_0)z^2 + \frac{1}{6}(q_0 + 3q_0^2 - j_0 - 1)z^3 + O(z^4) \right]$$

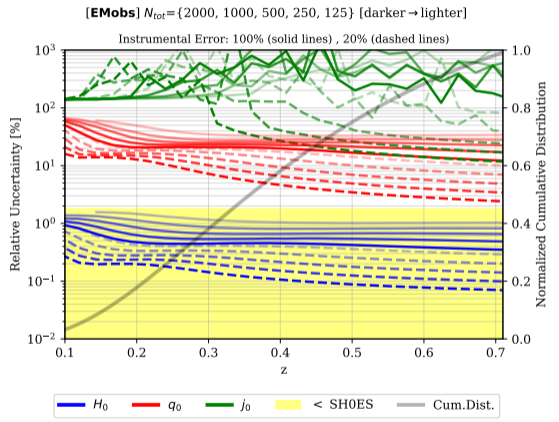
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Convergence of the Series:



The simulations were performed up to $z = 0.71$ with 125, 250, 500, 1000 e 2000 bright standard sirens detections.



Journal of Cosmology and Astroparticle Physics 2022.03 (2022): 025

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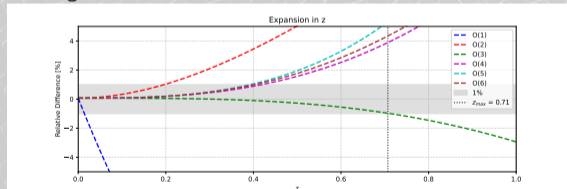
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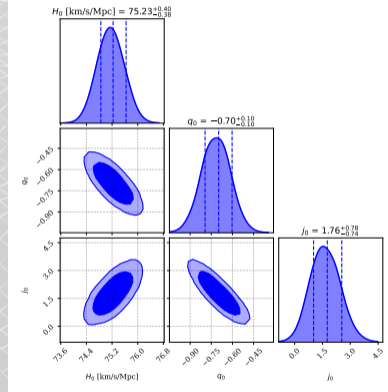
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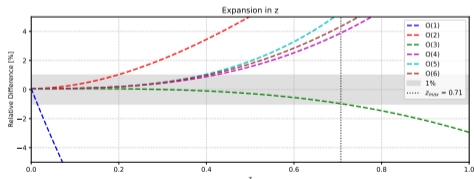
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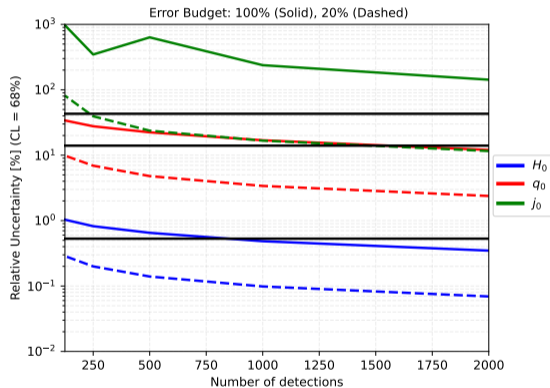
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Thank You

