UNIVERSITÉ DE GENÈVE

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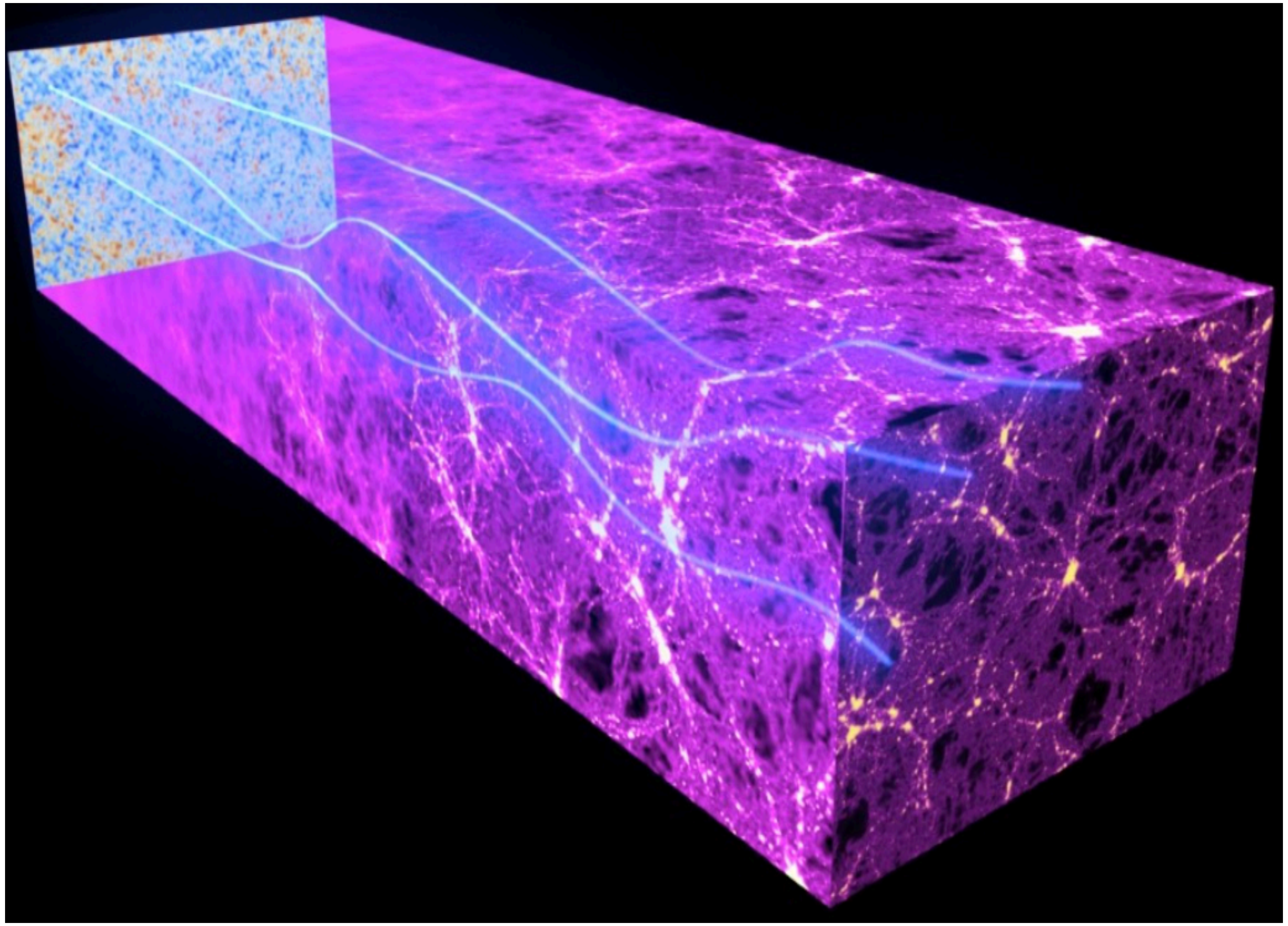
Legrand and Carron 2022
PRD 2112.05764

CMB LENSING SPECTRUM WITH DEEP POLARISATION SURVEYS

CMB E modes lensed - unlensed

CMB GRAVITATIONAL LENSING

- CMB is an extended light source at z=1100
- CMB photons are lensed by the large scale structures created by gravitational evolution of matter



CMB LENSING

- Lensing acts as a remapping of the primordial CMB fields

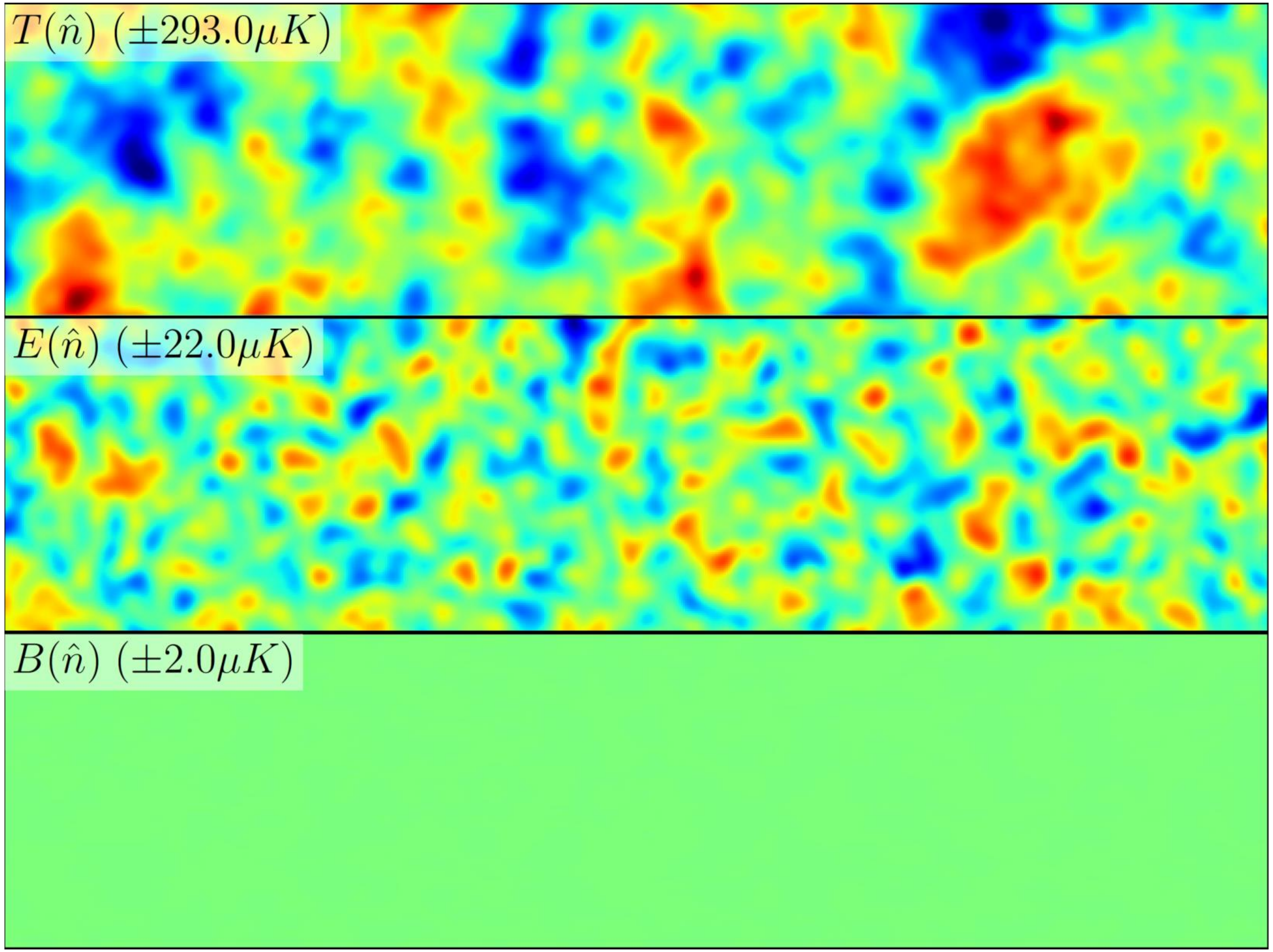
$$X^{\mathrm{len}}(\boldsymbol{n}) = X^{\mathrm{unl}}(\boldsymbol{n} + \boldsymbol{\alpha}(\boldsymbol{n}))$$

$$\boldsymbol{\alpha} = \vec{\nabla}\phi$$

$$\phi(\boldsymbol{n}) = -2\int_0^{\chi_*} d\chi \frac{\chi_* - \chi}{\chi_* \chi} \Psi\left(\chi\boldsymbol{n}; \eta_0 - \chi\right)$$

- It creates statistical anisotropies and correlation between different scales

- It creates B modes



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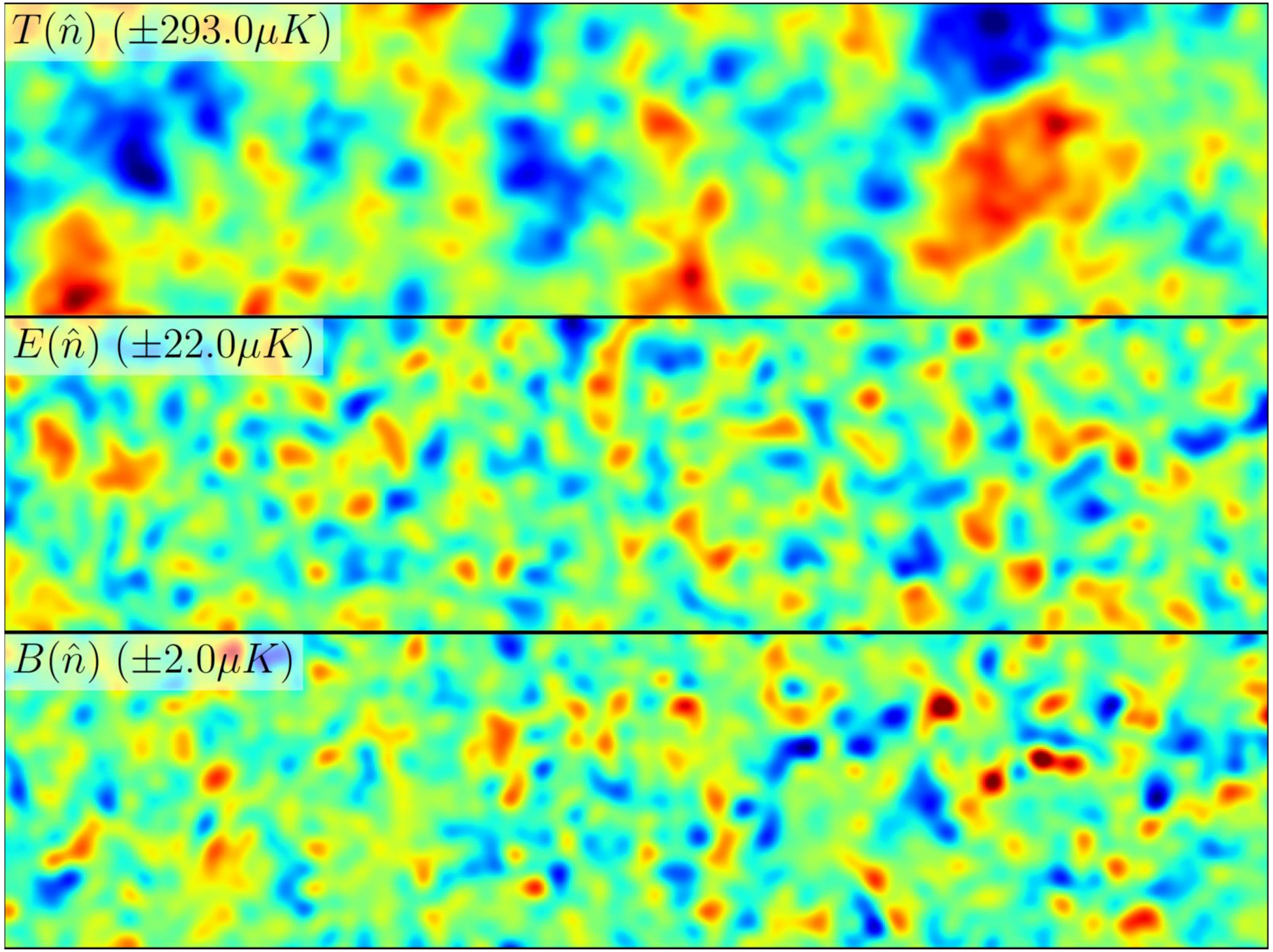
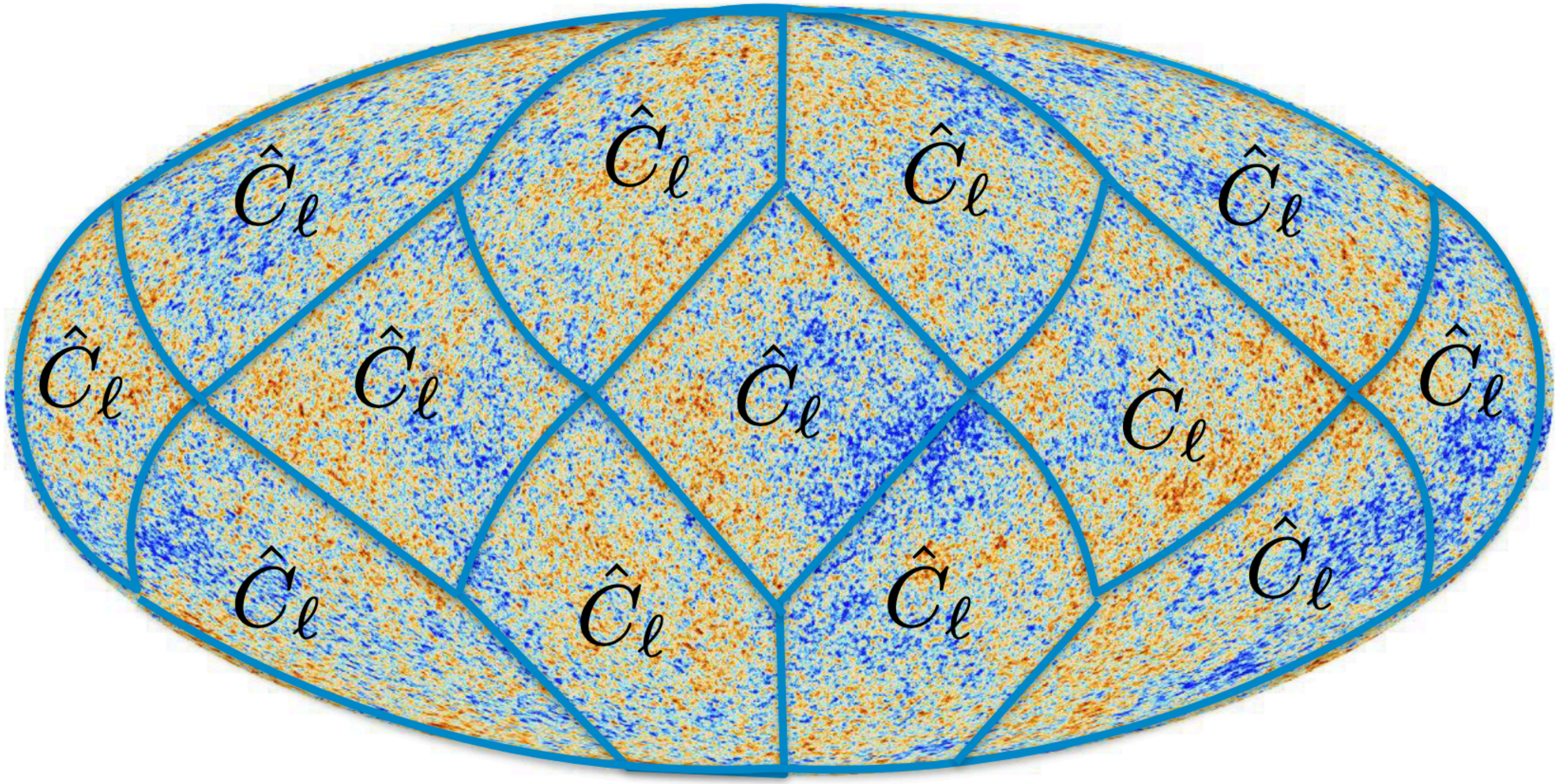
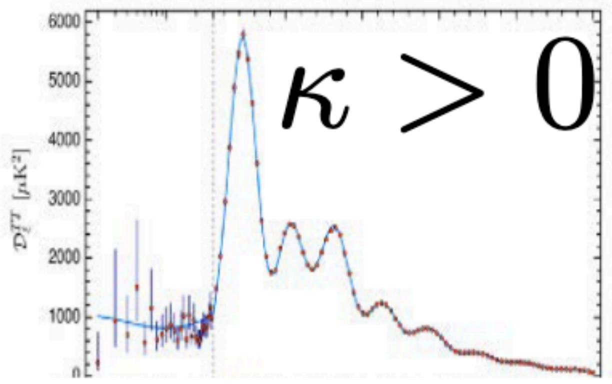


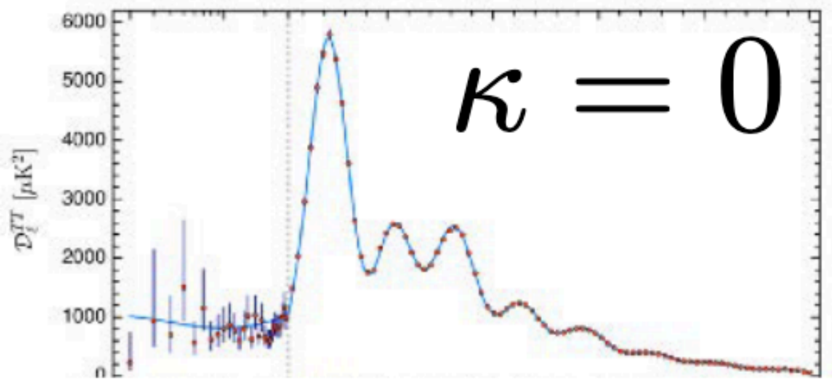
HOW CAN WE MEASURE THE CMB LENSING POTENTIAL?

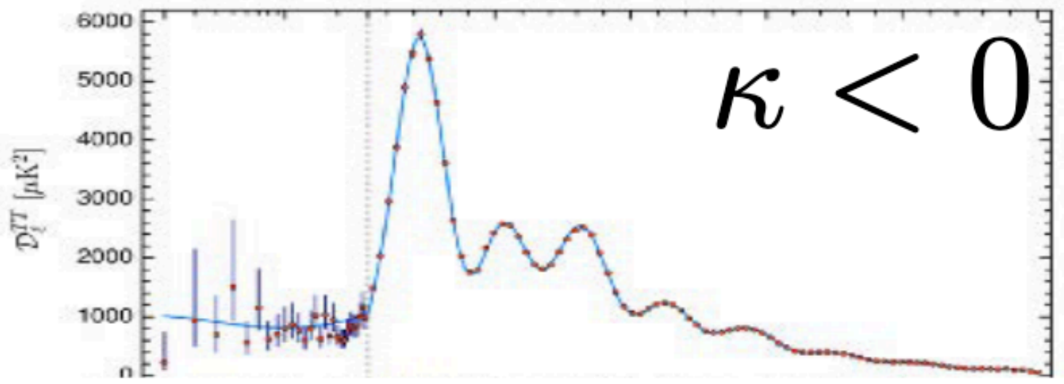
BASIC IDEA OF QUADRATIC ESTIMATION

- Lensing creates statistical anisotropies: patches of the sky will have different spectra
- We can measure these deviations, and estimate the lensing potential field









QUADRATIC ESTIMATOR (QE)

- Lensing creates correlations between different multipole moments

$$\left\langle X^{\text{len}}(\boldsymbol{l}) Y^{\text{len}\,*}(\boldsymbol{l}') \right\rangle_{\substack{\text{fixed lensed} \\ \boldsymbol{l} \neq \boldsymbol{l}',\, \boldsymbol{L} = \boldsymbol{l} + \boldsymbol{l}'}} = f_{XY}(\boldsymbol{l}, \boldsymbol{l}')\, \phi(\boldsymbol{L})$$

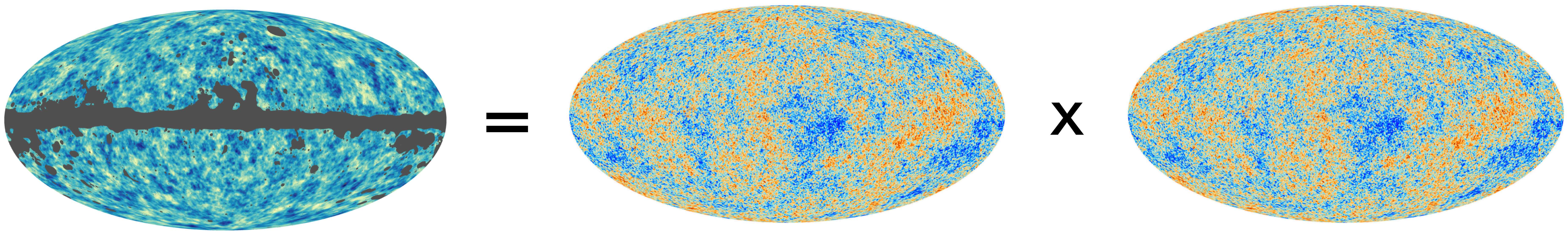
Lensing induced correlations

- The QE combines scales of two CMB fields (Hu & Okamoto 2002)

$$\hat{\phi}(\boldsymbol{L}) = \frac{1}{R_L^{XY}} \int \frac{d^2\boldsymbol{l}}{2\pi}\, f^{XY}(\boldsymbol{l}, \boldsymbol{L})\, \bar{X}(\boldsymbol{l})\, \bar{Y}^*(\boldsymbol{l} - \boldsymbol{L})$$

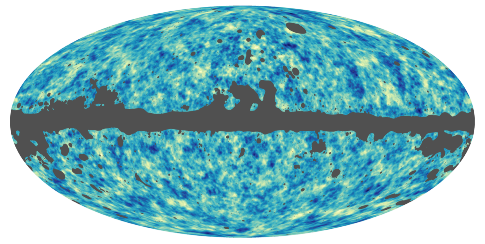
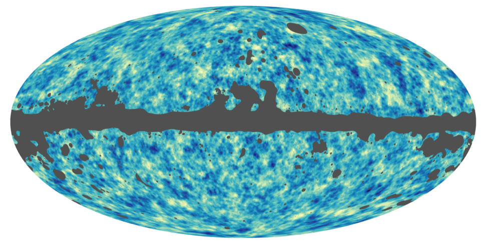
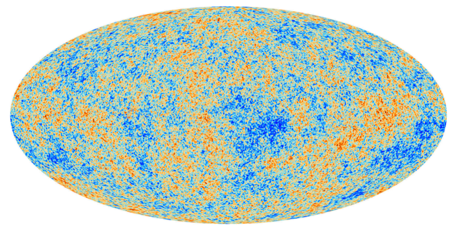
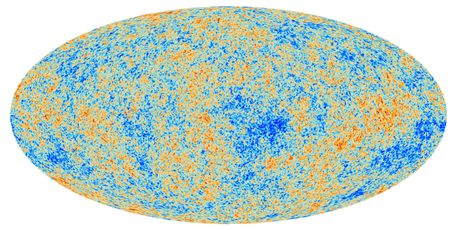
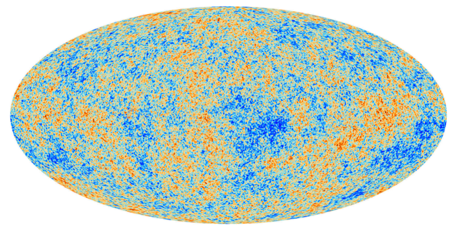
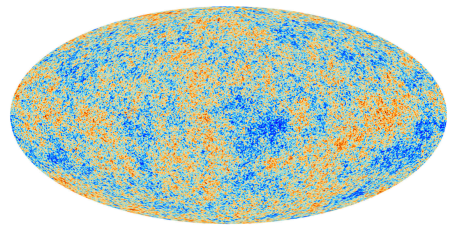
Normalisation (response of the estimator)

Inverse variance filtered CMB fields



NOISY RECONSTRUCTION

- The power spectrum of the estimated lensing potential is a 4 point functions of the maps

$C_L^{\hat{\phi}\hat{\phi}} =$  x  =  x  x  x 

- Chance correlations between different scales can mimic the lensing effect

The signal we want

$$C_L^{\hat{\phi}\hat{\phi}} = C_L^{\phi\phi} + N_L^0 + N_L^1 + \dots$$

Disconnected (gaussian) contractions of the lensed CMB fields

Non gaussian secondary contractions created by lensing (proportional to $C^{\phi\phi}$)

POWER SPECTRUM BIASES

$$\hat{\phi}_L \simeq \int_l X_l^{\rm len} Y_{L-l}^{\rm len}$$

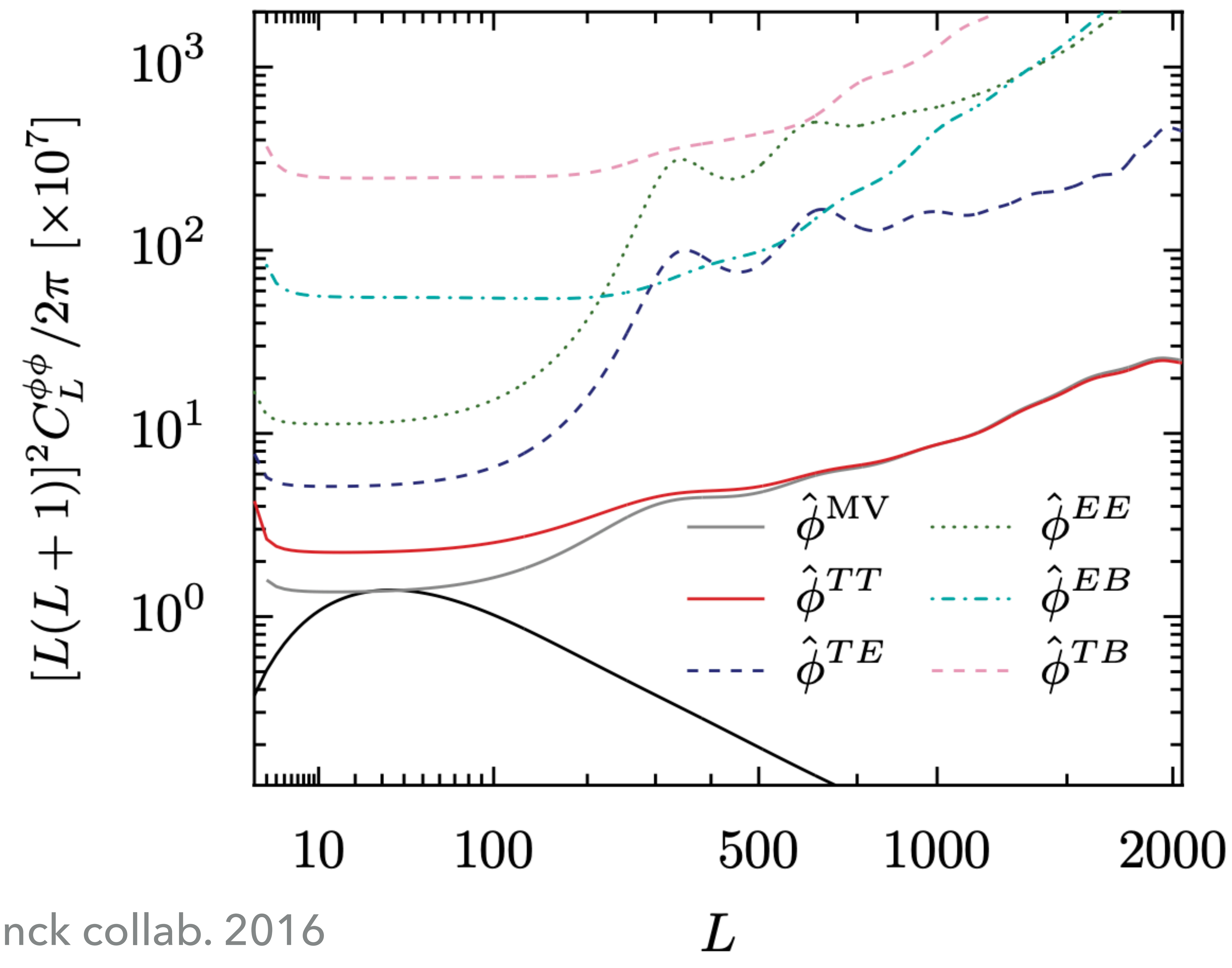
$$X^{\rm len}(n) = X^{\rm unl}(n + \nabla\phi) \sim X^{\rm unl}(n) + \nabla\phi\nabla X(n)$$

+ all combinations with order 0 in phi: N0 bias

$$< \hat{\phi}\hat{\phi}' > \simeq < (X + \nabla\phi\nabla X)(Y + \nabla\phi\nabla Y)(X' + \nabla\phi'\nabla X')(Y' + \nabla\phi'\nabla Y') >$$

+ all combinations with order 2 in phi: Cpp and N1 bias

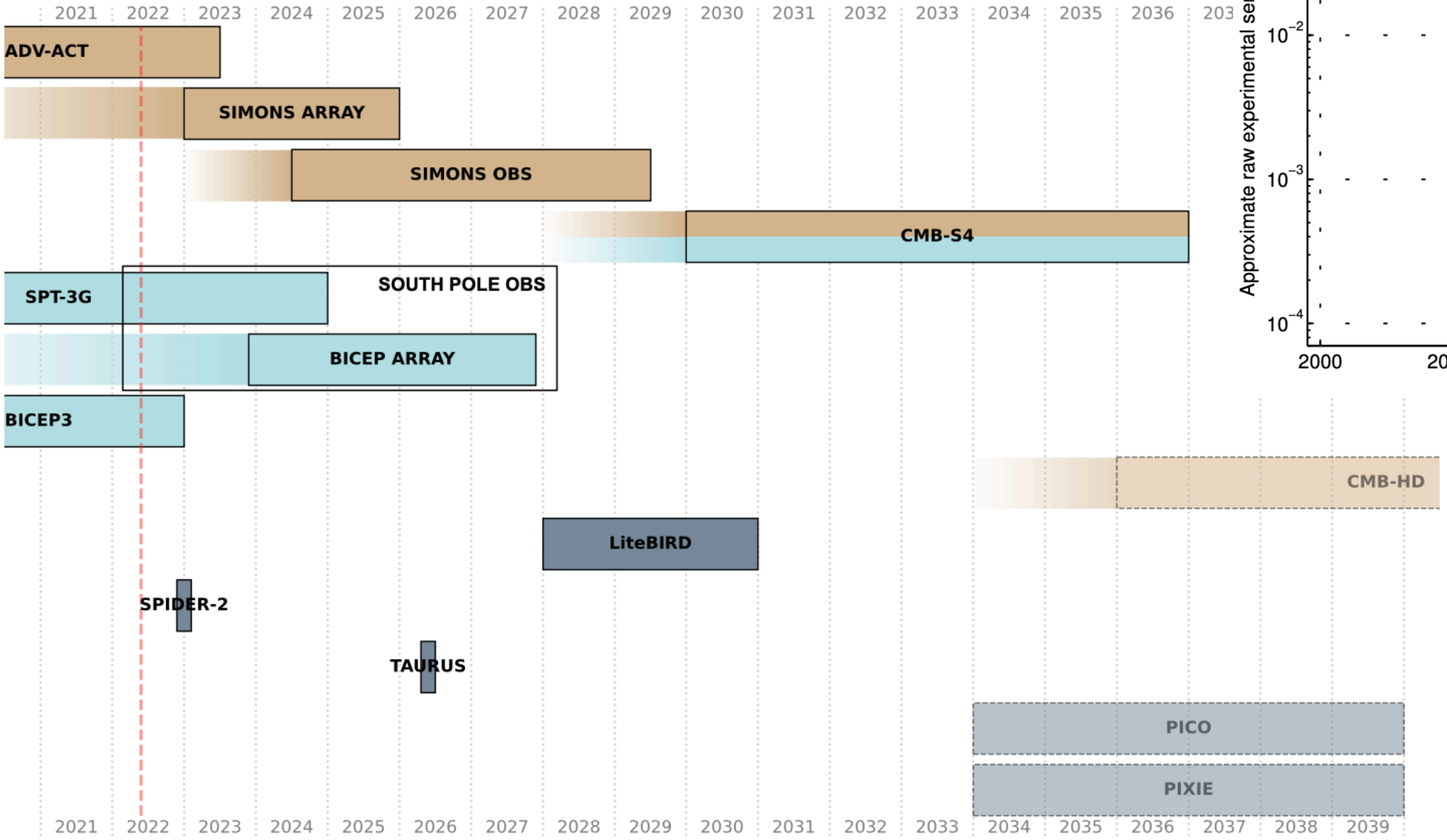
NOISY RECONSTRUCTION

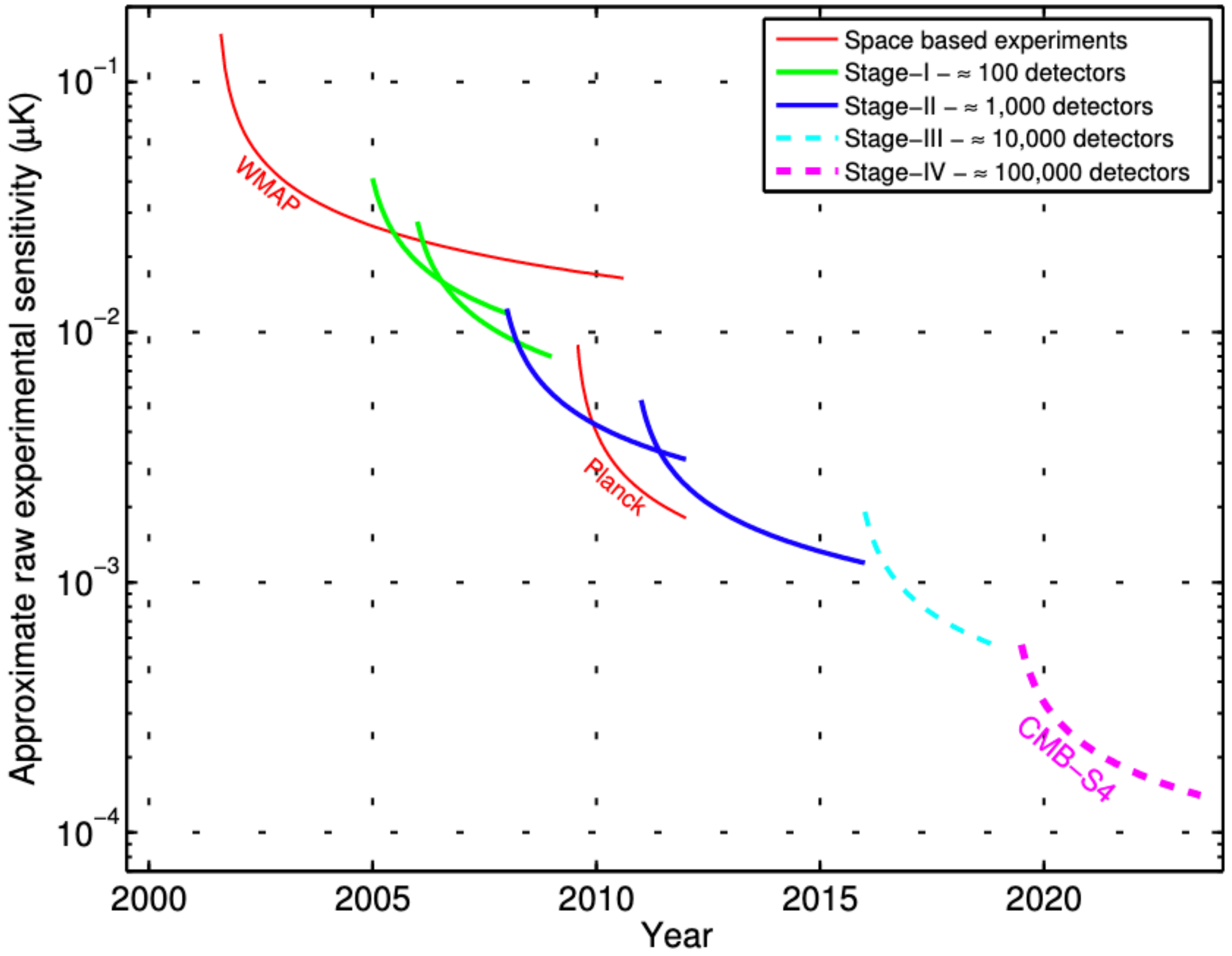


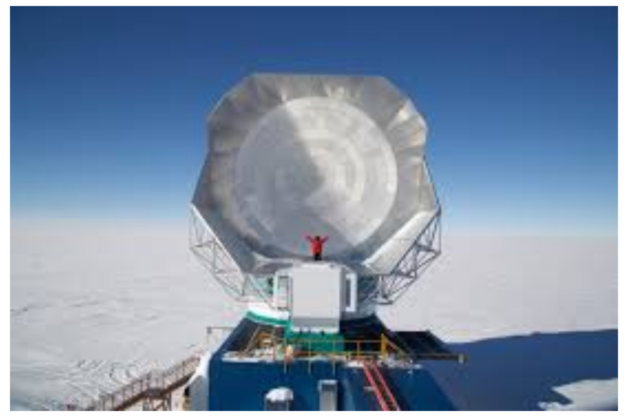
Planck collab. 2016

- Planck lensing power spectrum is dominated by the N0 bias at all scales
- Combining all pairs of maps into a minimum variance estimator
- TT estimator is dominating in Planck

NEXT GENERATION SURVEYS



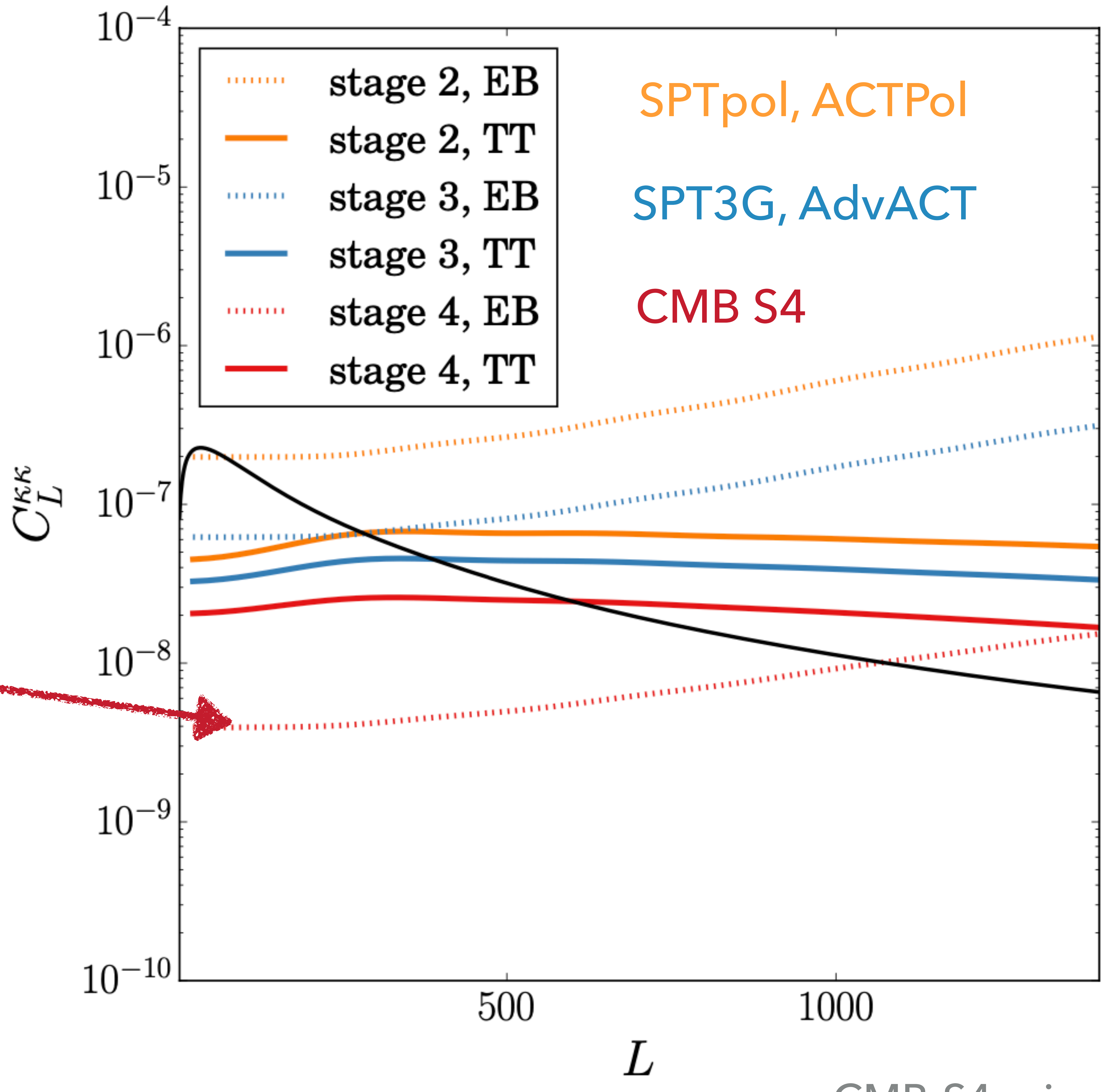




Chang et al 2022

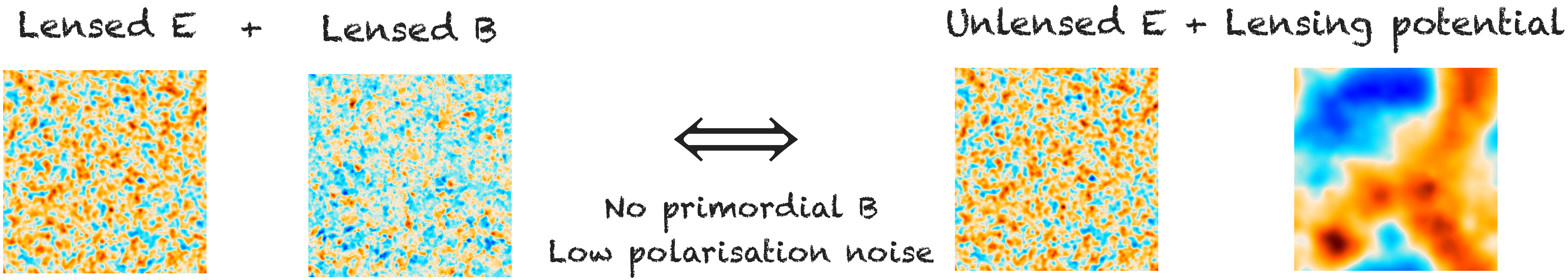
NEXT GENERATION CMB SURVEYS

EB (polarisation) estimator will be dominant for CMB S4



CMB-S4 science book 2016

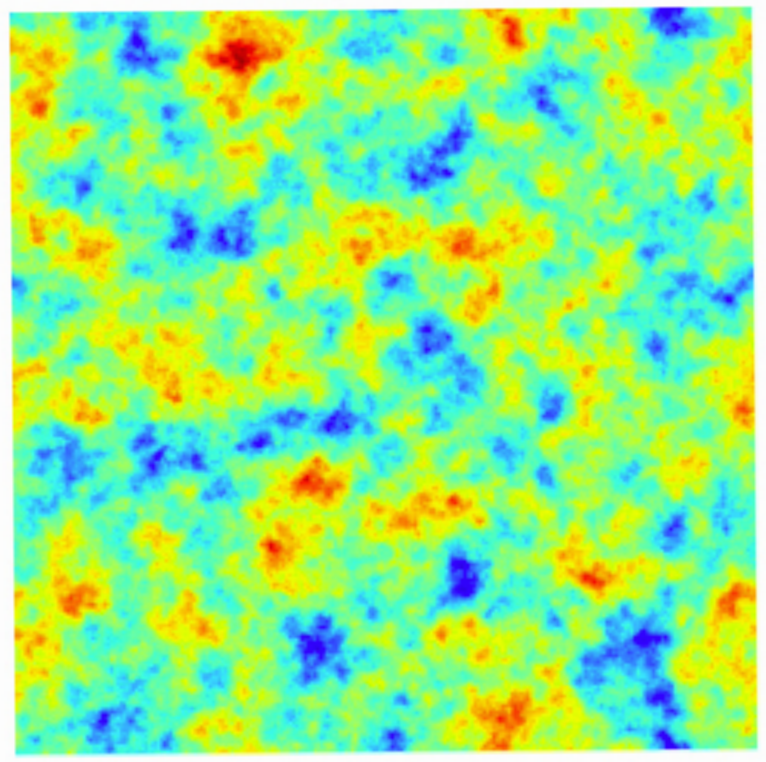
OPTIMAL ESTIMATORS



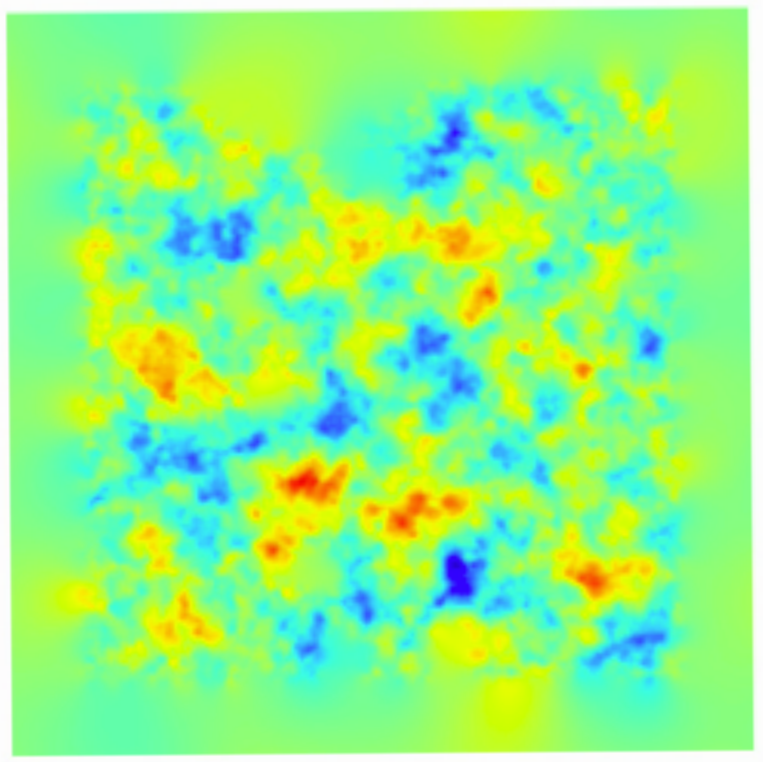
- Likelihood based approach, first introduced in Hirata & Seljak 2003: $P(\phi\,|\,\mathrm{data})$
- Implementations:
  - Millea et al. 2020, Millea and Seljak 2021
  - Carron & Lewis 2017

ITERATIVE APPROACH

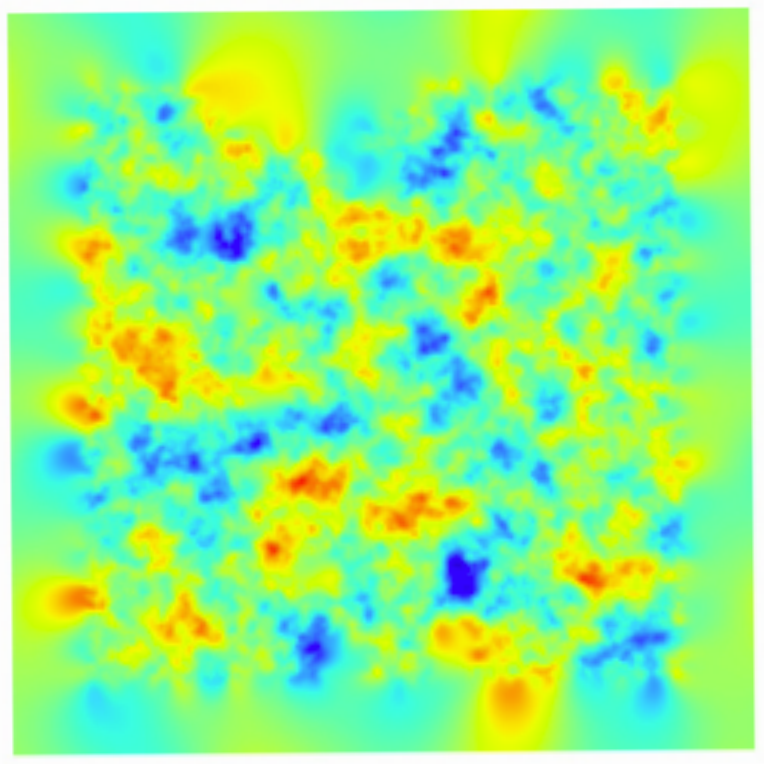
- Newton-Raphson iterations on the likelihood
- In practice at each step:
  - delens the data using the deflection estimate
  - apply a quadratic estimator on the resulting maps to obtain the maximum a posteriori (MAP) lensing field
  - start again until convergence
- Advantage: fast and based on a well known tool: the quadratic estimator
- From this MAP lensing map we want to estimate the lensing power spectrum



Input phi



QE



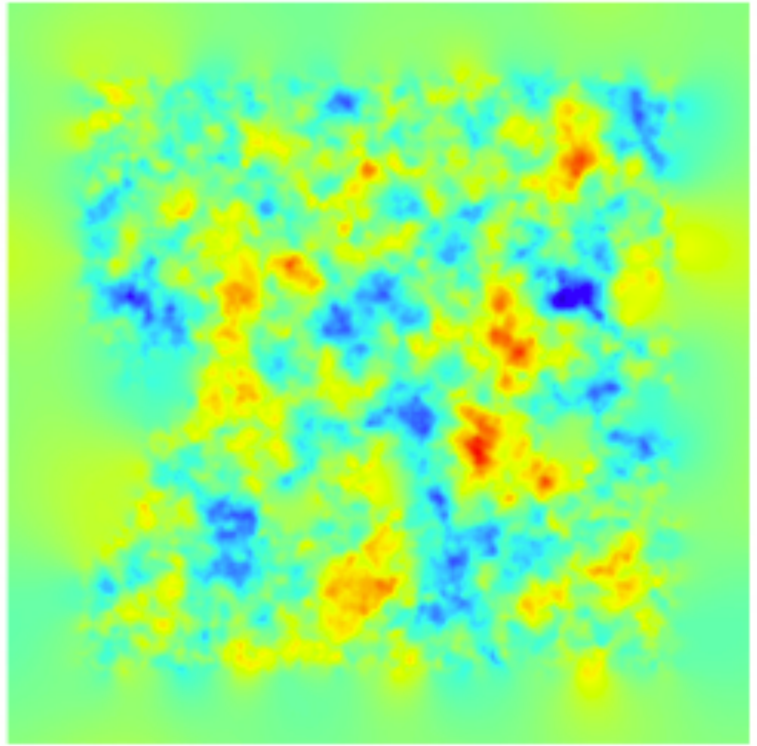
MAP

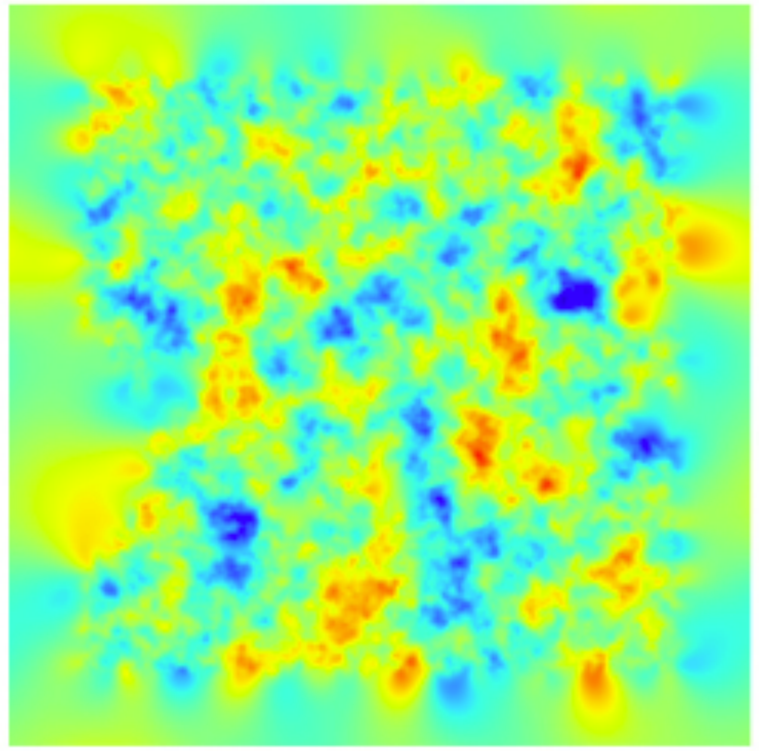
Carron&Lewis 2017

OPTIMAL LENSING POWER SPECTRUM ESTIMATION

- Cannot track analytically the 4 point function of the delensed CMB maps over all iterations
- What are the normalisation and the biases of the two point function of the MAP lensing potential ?

QE





MAP

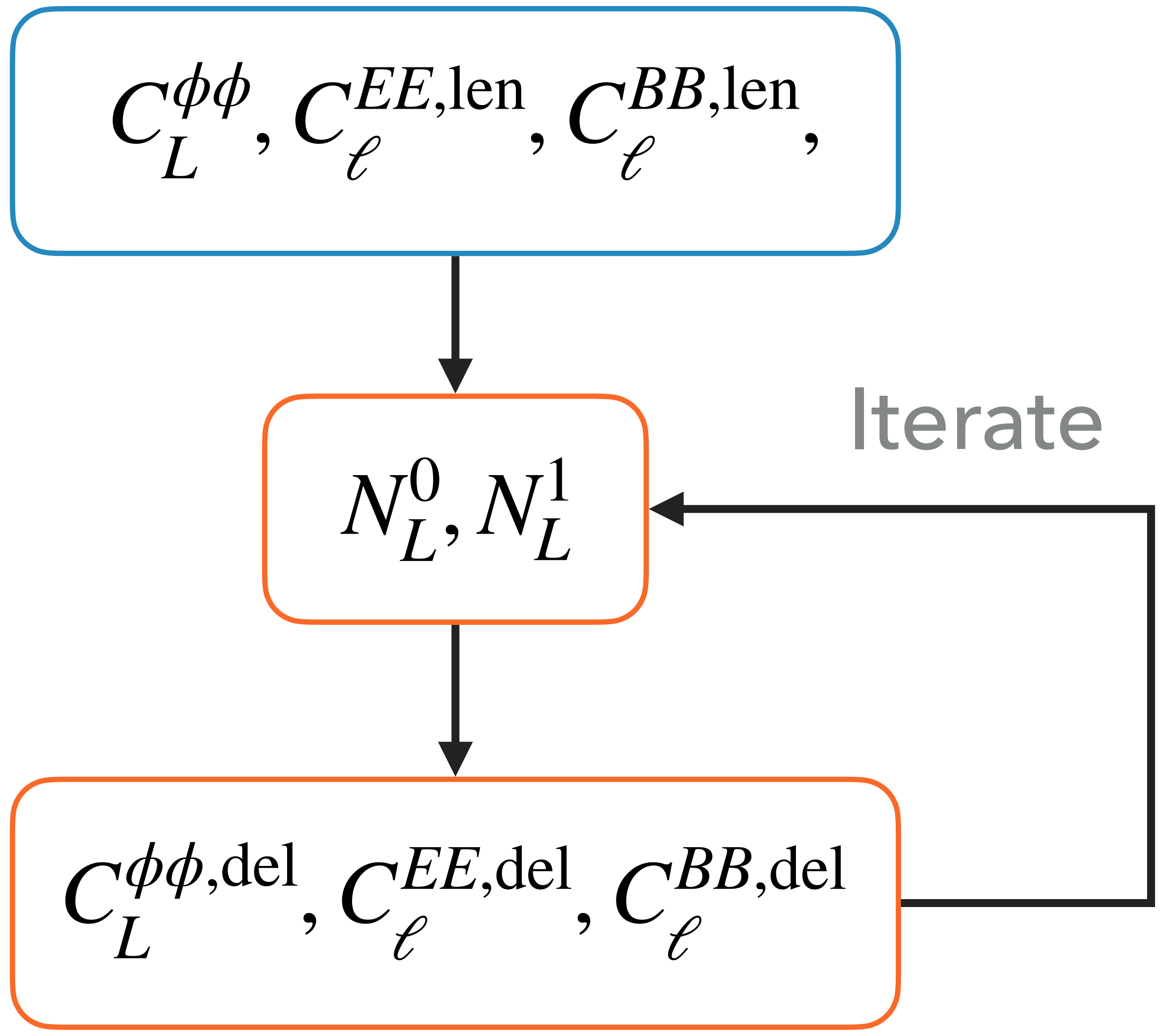
$$C_L^{\hat\phi\hat\phi} = C_L^{\phi\phi} + N_L^0 + N_L^1 + \dots$$

$$C_L^{\hat\phi\hat\phi} = C_L^{\phi\phi} + N_L^{0,\mathrm{MAP}} + N_L^{1,\mathrm{MAP}} + \dots$$

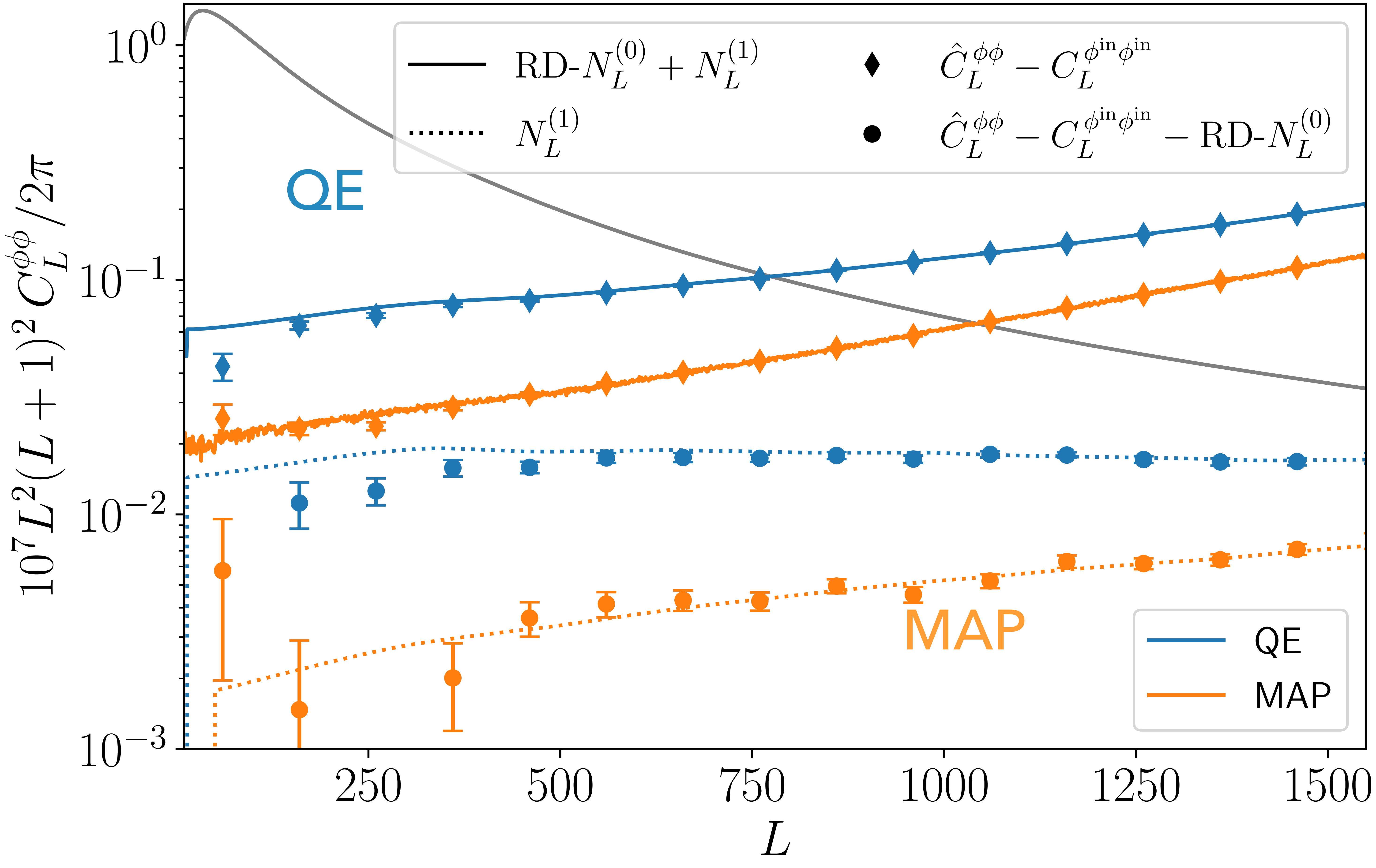
ITERATIVE BIASES

- Same expression of the QE but with partially delensed CMB spectra in the weights, obtained iteratively

$$C_L^{\hat\phi\hat\phi} = C_L^{\phi\phi} + N_L^{0,\mathrm{MAP}} + N_L^{1,\mathrm{MAP}}$$



CMB-S4 noise levels, polarisation only



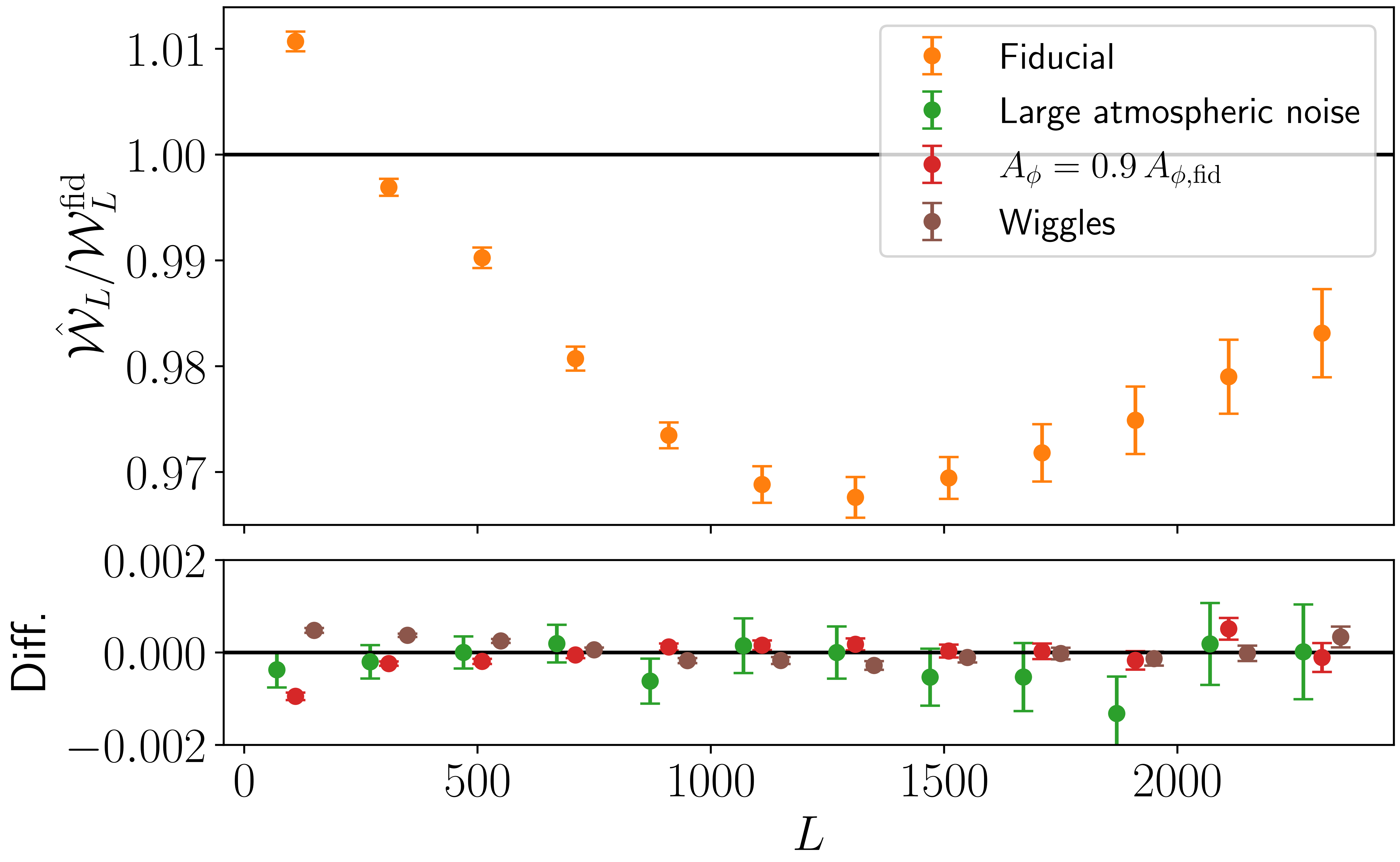
NORMALISATION

$$\hat{\phi}(\boldsymbol{L}) = \frac{1}{R_L^{XY}} \int \frac{d^2\boldsymbol{l}}{2\pi} f^{XY}(\boldsymbol{l}, \boldsymbol{L})\, \bar{X}(\boldsymbol{l})\, \bar{Y}^*(\boldsymbol{l} - \boldsymbol{L})$$

- The MAP lensing potential is a Wiener filtered lensing estimate (due to our prior)

$$W_L = \frac{C_L^{\phi\phi}}{C_L^{\phi\phi} + N_L^{0,\mathrm{MAP}}}$$

- Correct percent level bias with simulations



REALISATION DEPENDENT DEBIASING

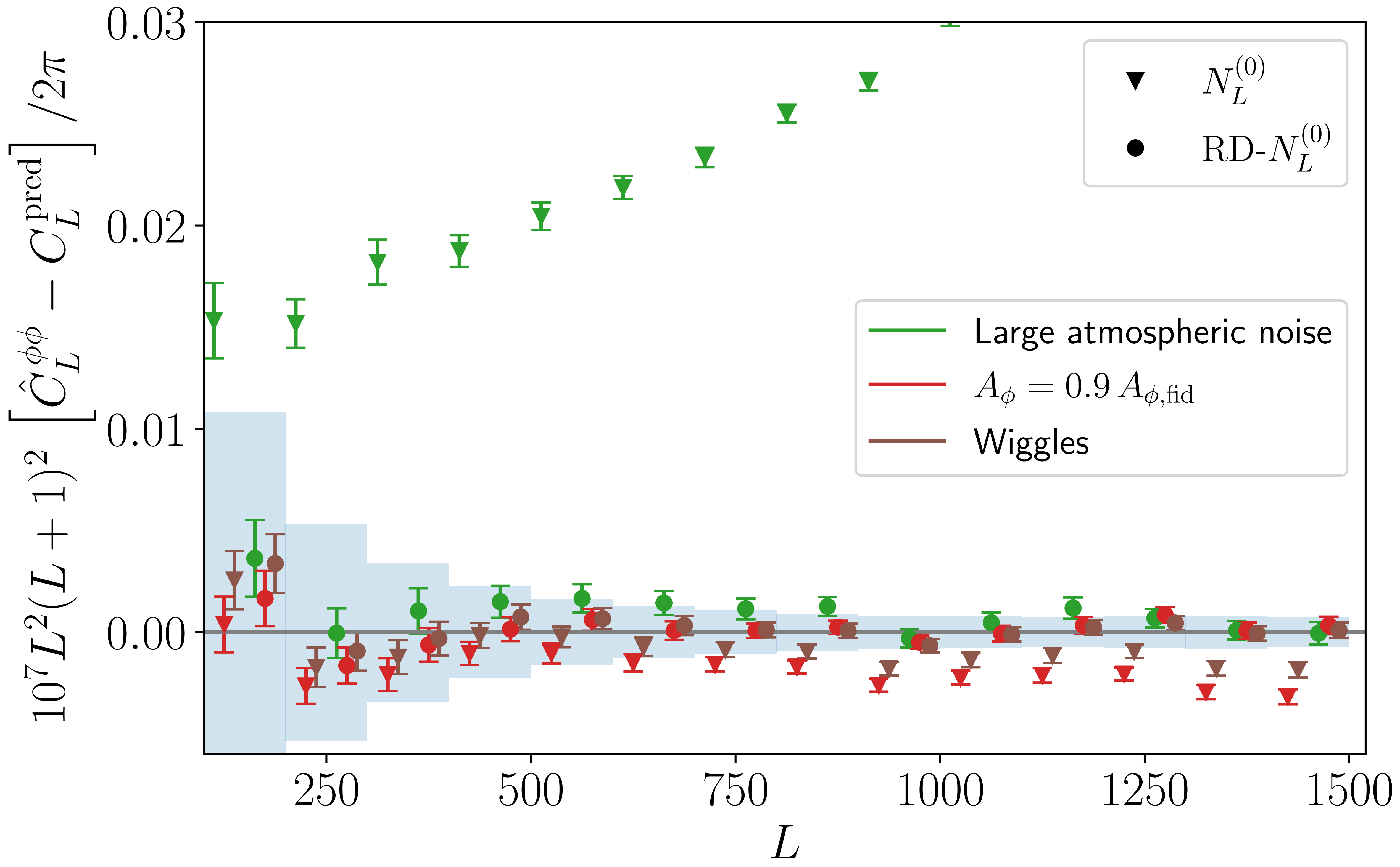
- Because N0 bias dominates the signal at small scales, it needs to be evaluated very accurately, but it depends strongly on the assumptions made on the CMB spectra and noise
- The RD-N0 is a combination of QEs with one leg on the data map (resp. a simulated map), and the other leg on a simulated map (resp. a different simulation)

$$\text{RD-}N_L^{(0)} \equiv \left\langle 4\hat{C}_L^{di} - 2\hat{C}_L^{ij} \right\rangle_{N_{\text{bias}}} ,$$

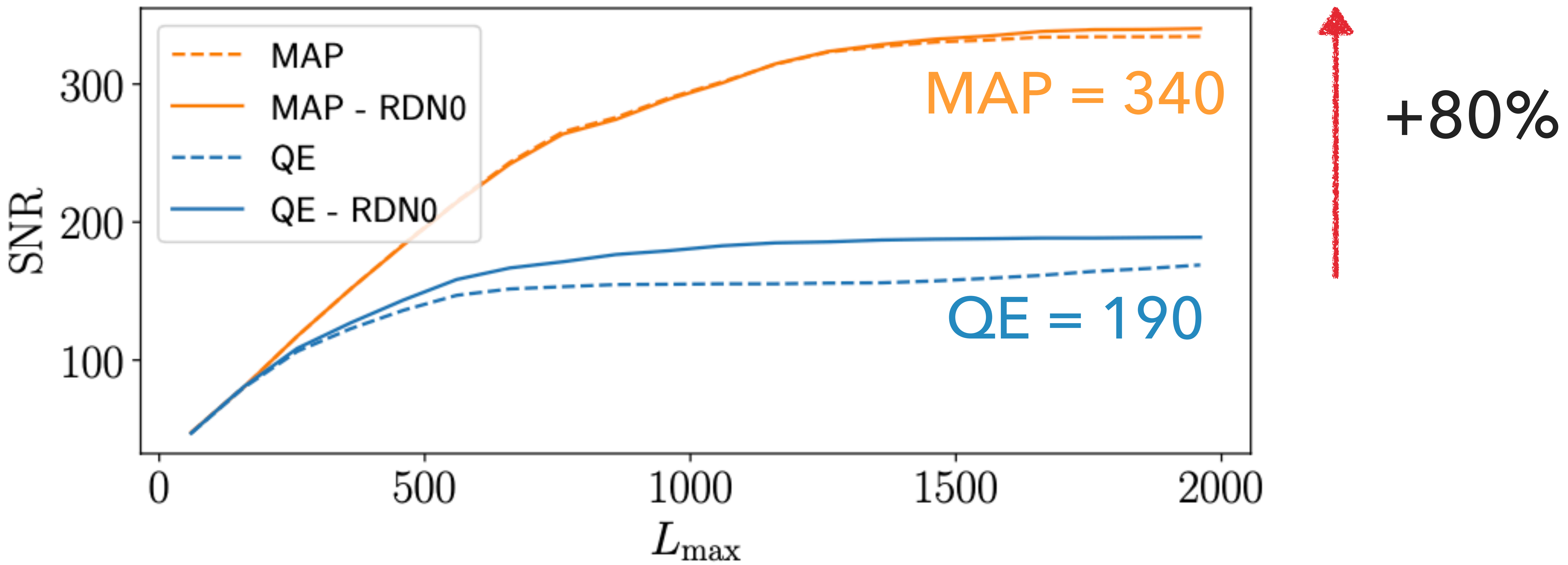
- Averaging over simulations, this RDN0 is **insensitive at first order to mismatch** between the fiducial and the real CMB spectra and noise

RDN0 FOR THE MAP

- Similar combination as for the QE, but all simulations are lensed with the MAP lensing potential (instead of being random)



SIGNAL TO NOISE RATIO

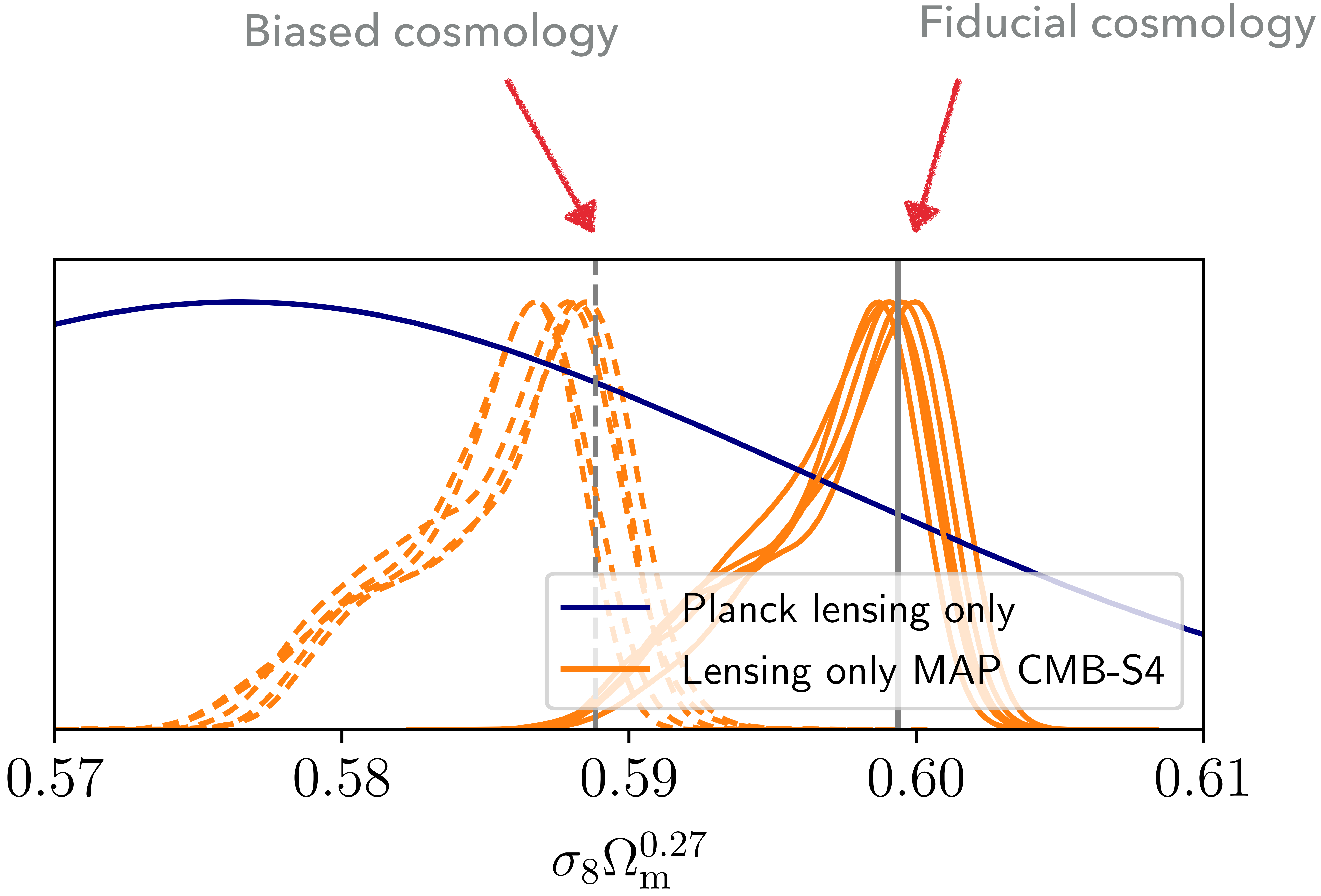


$$\mathrm{SNR}(L_{\max}) = \sqrt{\sum_{L_{\min}}^{L_{\max}} C_L^{\phi\phi,\mathrm{fid}} \mathrm{Cov}_{LL'}^{-1} C_L^{\phi\phi,\mathrm{fid}}}$$

- Information gain saturates above 1000 for QE and 1500 for MAP

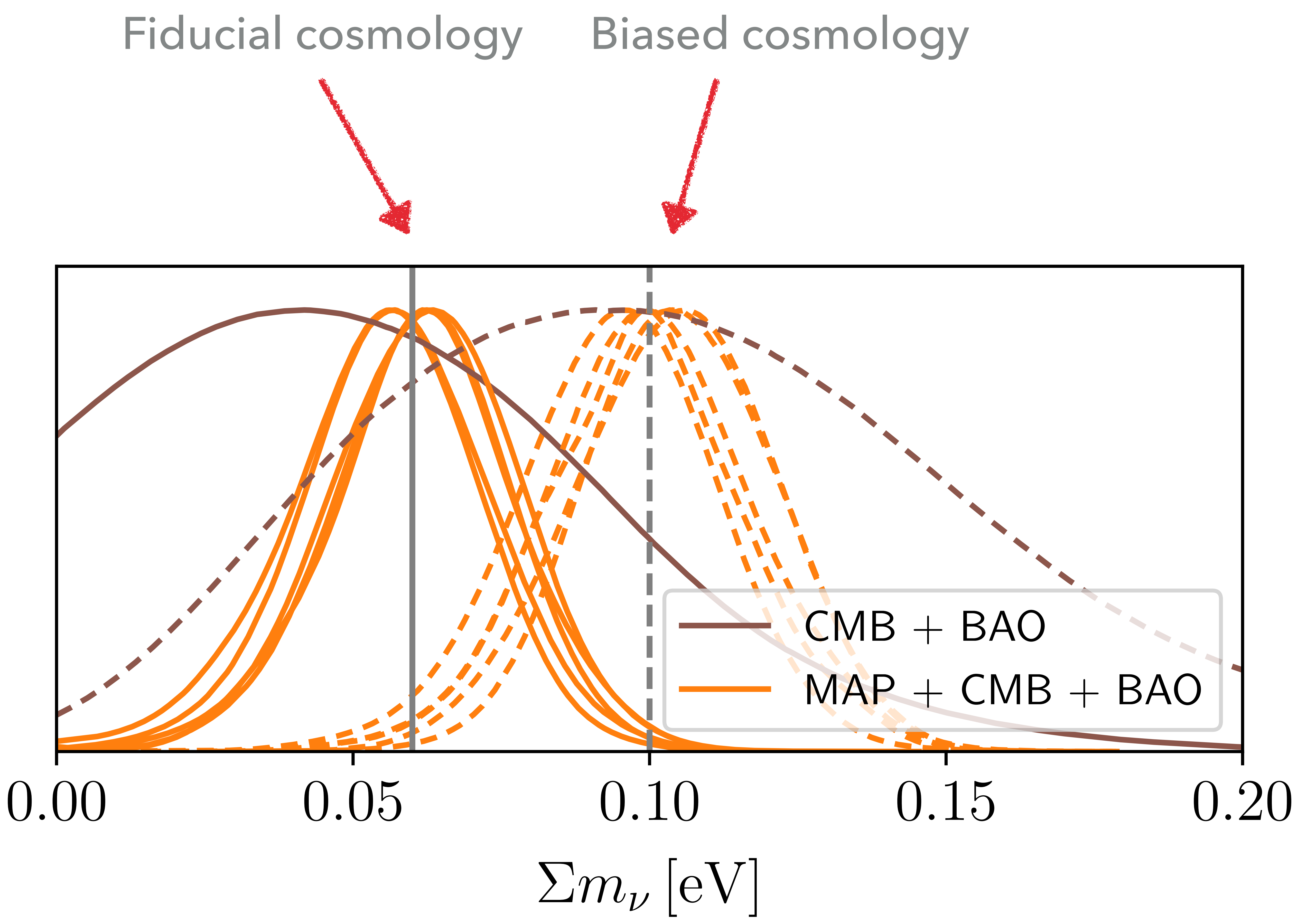
LENSING LIKELIHOOD

- Lensing likelihood similar to the Planck pipeline, sampled by MCMC
- Two datasets:
  - One in the fiducial cosmology used for the reconstruction
  - One with less matter and more massive neutrinos
- Sampling 6 LCDM parameters



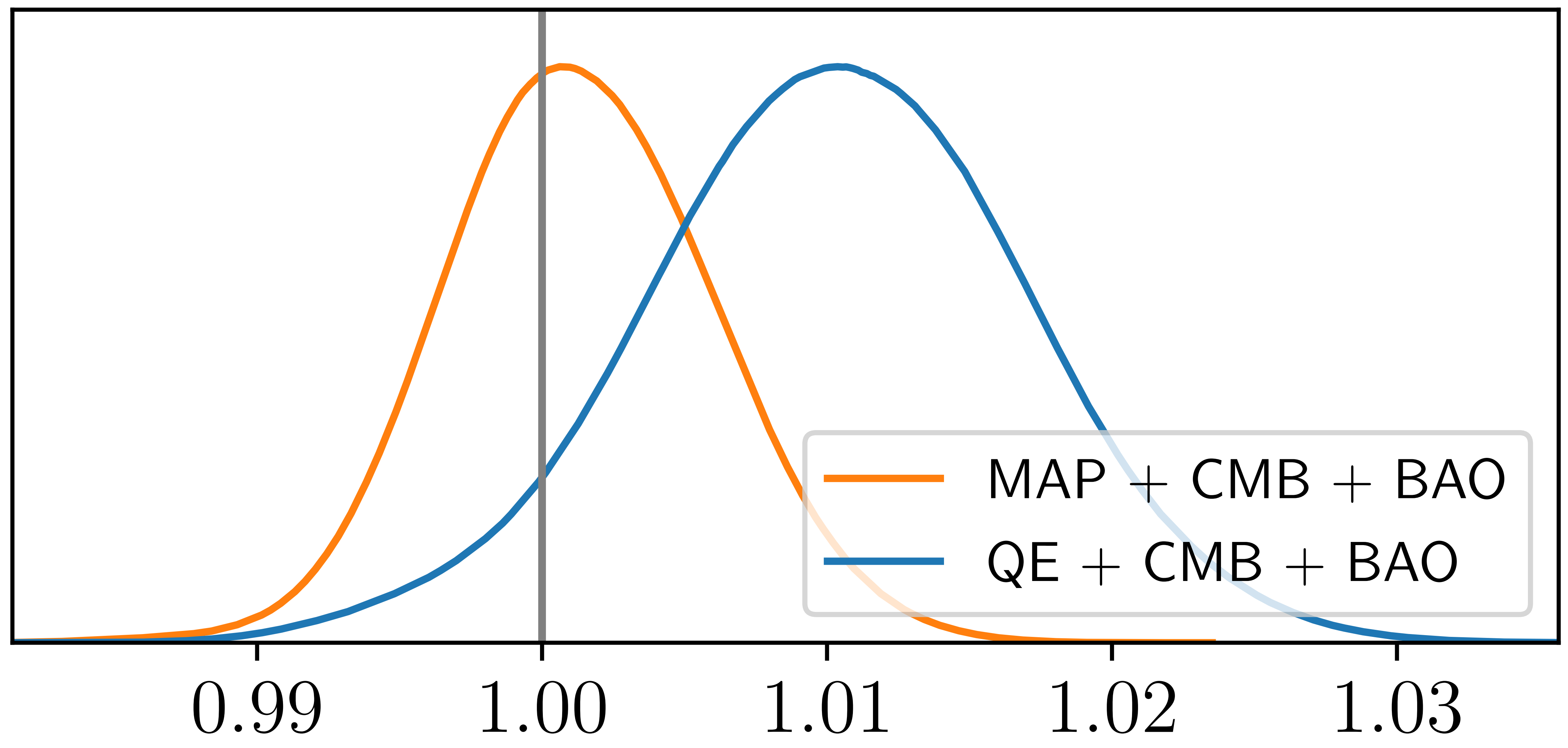
NEUTRINO MASS ESTIMATES

- Combines MAP lensing likelihood with:
  - CMB-S4 unlensed
  - DESI BAO
- Varying 6 LCDM parameters + Sum of the neutrino mass
- Fiducial $\sum m_\nu = 0.06$ eV
- Strong prior on tau: $\sigma(\tau) = 0.002$ (LiteBIRD constraints)
- CMB-S4 will allow for a 4σ detection of massive neutrinos



COMPARING MAP TO QE CONSTRAINTS

- Both QE and MAP spectra get $\sigma_{M_\nu} = 0.016$
- Coming from remaining degeneracies between parameters
- We perform a PCA on $\sum m_\nu, \Omega_\mathrm{m}, \tau$ combination
- Improvement of 40%



$$I = \left(1 + \sum m_\nu - \left(\sum m_\nu\right)^{\mathrm{fid}}\right)\left(\Omega_m/\Omega_m^{\mathrm{fid}}\right)^{-1.7}\left(\tau/\tau^{\mathrm{fid}}\right)^{-0.18}$$

CONCLUSION

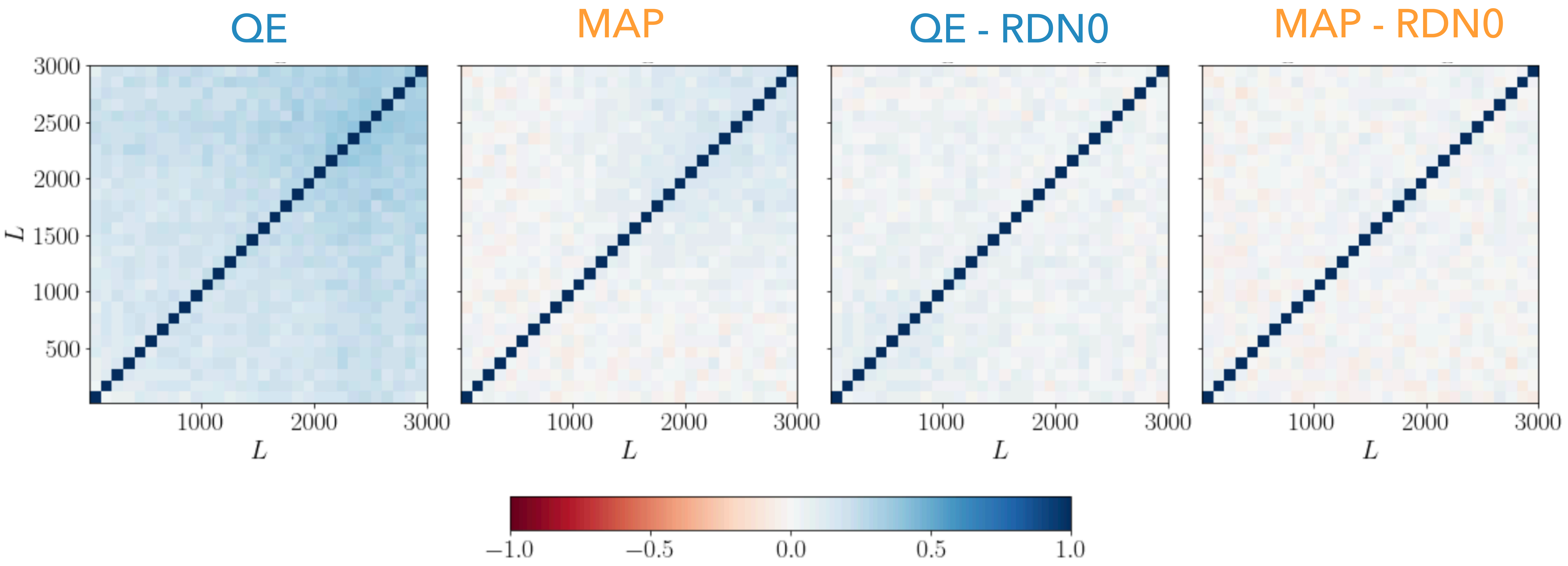
- Iterative methods will reconstruct optimally the lensing potential for next generation CMB surveys
- We introduced a **simple and robust** end-to-end pipeline to get an **optimal estimation of the lensing spectrum**
- Increases the signal to noise ratio of the lensing amplitude by 80%
- Robust to uncertainties in fiducial cosmology and observational noise
- Can get improved constraints on cosmological parameters of interest, such as the sum of neutrino mass, but limited by cosmological degeneracies

LIKELIHOOD ANALYSIS

$$\ln L(\theta \,|\, \hat{\phi}) = -\frac{1}{2}\left(C_L^{\hat{\phi}\hat{\phi}} - RD\text{-}N_L^0 - C^{\phi\phi}(\theta) - N_L^1(\theta)\right)^T \mathbf{Cov}^{-1}\left(C_L^{\hat{\phi}\hat{\phi}} - RD\text{-}N_L^0 - C^{\phi\phi}(\theta) - N_L^1(\theta)\right)$$

- Estimate most likely cosmology by sampling the parameter space with a MCMC
- Do not re-estimate the lensing potential for each step of the sampling
- Introduce a possible bias (mismatch between the fiducial and the true cosmology)
- We correct this bias at first order
  - on the normalisation of the lensing potential
  - on the N1 bias

CORRELATION MATRICES



- 1024 flat sky CMB-S4 like simulations

POWER SPECTRUM BIASES

$$X^{\rm len} \sim X^{\rm unl} + \nabla\phi\nabla X$$

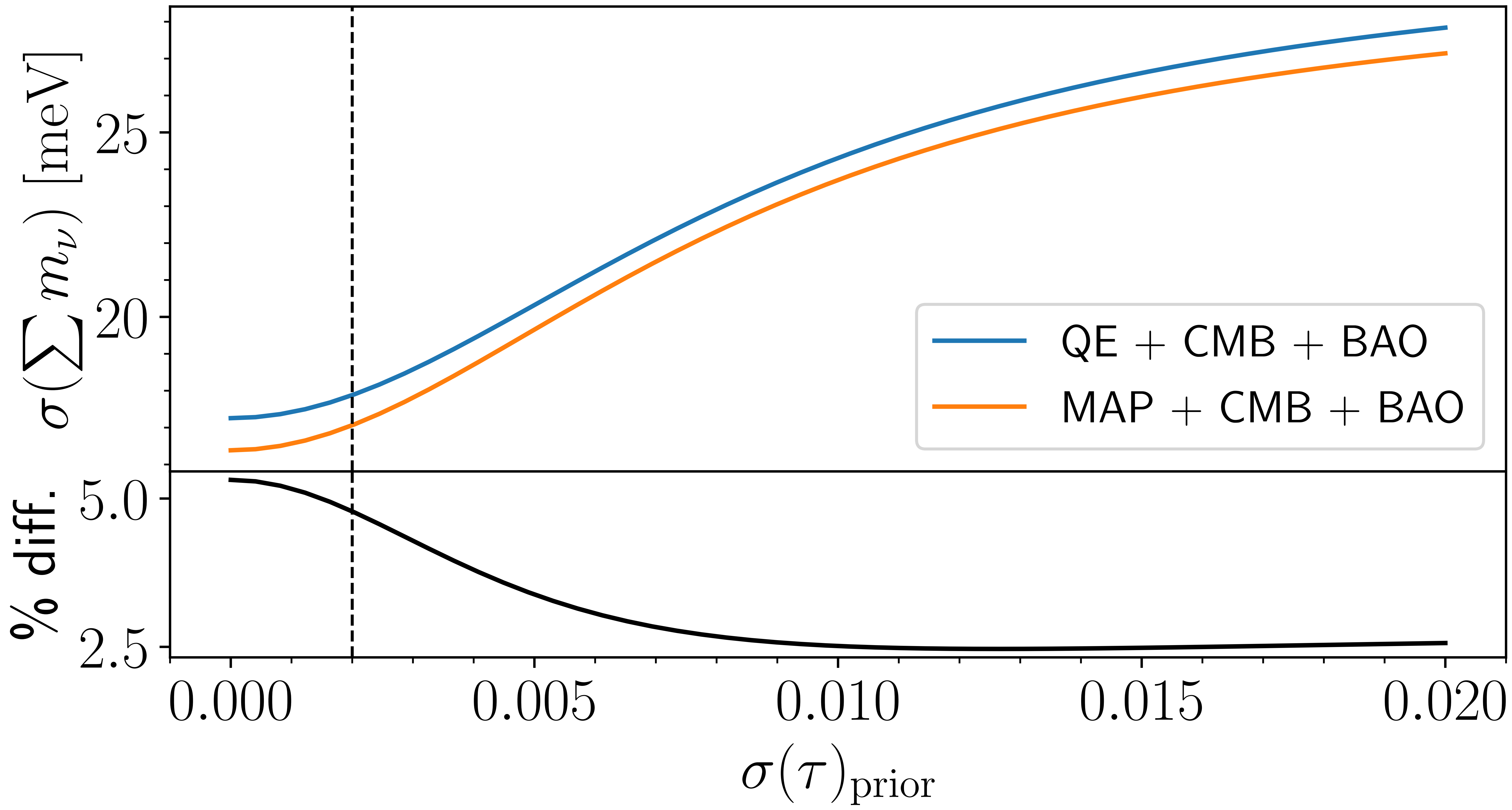
$$\hat{\phi}_L \simeq \int_l X_l Y_{L-l}$$

+ all combinations with order 0 in phi: N0 bias

$$< \hat{\phi}\hat{\phi}' > \simeq < (X + \nabla\phi\nabla X)(Y + \nabla\phi\nabla Y)(X' + \nabla\phi'\nabla X')(Y' + \nabla\phi'\nabla Y') >$$

+ all combinations with order 2 in phi: Cpp and N1 bias

IMPACT OF TAU PRIOR



BIASES EXPRESSIONS FOR QE (FLAT SKY)

$$\hat{\phi}^{XY}_{\rm QE}(\boldsymbol{L}) = \frac{1}{\mathcal{R}^{XY}_L}\int \frac{\mathrm{d}^2\boldsymbol{l}_1}{(2\pi)^2} X(\boldsymbol{l}_1)Y(\boldsymbol{l}_2)F^X_{\ell_1}F^Y_{\ell_2}W^{XY}(\boldsymbol{l}_1,\boldsymbol{l}_2)$$

$$\mathcal{R}^{XY}_L = \int \frac{\mathrm{d}^2\boldsymbol{l}_1}{(2\pi)^2}\frac{f^{XY}(\boldsymbol{l}_1,\boldsymbol{l}_2)}{1+\delta_{XY}}F^X_{\ell_1}F^Y_{\ell_2}W^{XY}(\boldsymbol{l}_1,\boldsymbol{l}_2)\,.$$

$$\hat{C}^{\hat{\phi}^{XY},\hat{\phi}^{IJ}}_L = \hat{C}^{\phi\phi}_L + N^{(0)XYIJ}_L + N^{(1)XYIJ}_L\,,$$

$$F^X_\ell = 1/C^{XX}_\ell.$$

$$N^{(0)XYIJ}_L = \frac{1}{\mathcal{R}^{XY}_L}\frac{1}{\mathcal{R}^{IJ}_L}\int \frac{\mathrm{d}^2\boldsymbol{l}_1}{(2\pi)^2}F^X_{\ell_1}F^Y_{\ell_2}W^{XY}(\boldsymbol{l}_1,\boldsymbol{l}_2) \times\Big(F^I_{\ell_1}F^J_{\ell_2}W^{IJ}(\boldsymbol{l}_1,\boldsymbol{l}_2)C^{XI}_{\ell_1}C^{YJ}_{\ell_2} + F^I_{\ell_2}F^J_{\ell_1}W^{IJ}(\boldsymbol{l}_2,\boldsymbol{l}_1)C^{XJ}_{\ell_1}C^{YI}_{\ell_2}\Big)\,,$$

$$N^{(1)XYIJ}_L = \frac{1}{\mathcal{R}^{XY}_L}\frac{1}{\mathcal{R}^{IJ}_L}\int \frac{\mathrm{d}^2\boldsymbol{l}_1\mathrm{d}^2\boldsymbol{l}'_1}{(2\pi)^4}F^X_{\ell_1}F^Y_{\ell_2}F^I_{\ell'_1}F^J_{\ell'_2} \times W^{XY}(\boldsymbol{l}_1,\boldsymbol{l}_2)W^{IJ}(\boldsymbol{l}'_1,\boldsymbol{l}'_2) \times\Big[C^{\phi\phi}_{|\boldsymbol{l}_1+\boldsymbol{l}'_1|}f^{XI}(\boldsymbol{l}_1,\boldsymbol{l}'_1)f^{YJ}(\boldsymbol{l}_2,\boldsymbol{l}'_2) + C^{\phi\phi}_{|\boldsymbol{l}_1+\boldsymbol{l}'_2|}f^{XJ}(\boldsymbol{l}_1,\boldsymbol{l}'_2)f^{YI}(\boldsymbol{l}_2,\boldsymbol{l}'_1)\Big]\,;$$