Hybrid Lagrangian Bias Expansion for Covariance Matrices

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Fitting a Model to Data

Model: $x_i(\theta)$

Data: $\hat{x}_i$

Quality of the Fit →

$$\chi^2 = \sum_{i,j} (\hat{x}_i - x_i(\theta)) C_{ij}^{-1} (\hat{x}_k - x_j(\theta))$$

Tightness of the Constraints →

$$\mathcal{L}(\theta) = \frac{1}{(2\pi)^{n/2}\sqrt{\det C}} \exp -\frac{1}{2} \sum_{i,j} (\hat{x}_i - x_i(\theta)) C_{ij}^{-1} (\hat{x}_k - x_j(\theta))$$

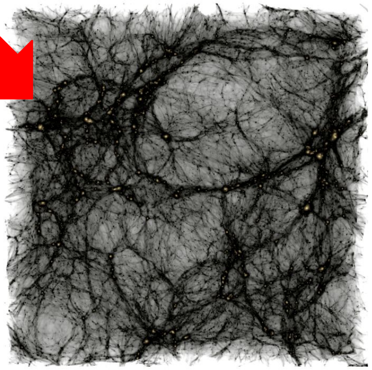
Maximum Likelihood Point ↗

Assuming the wrong covariance can lead to:

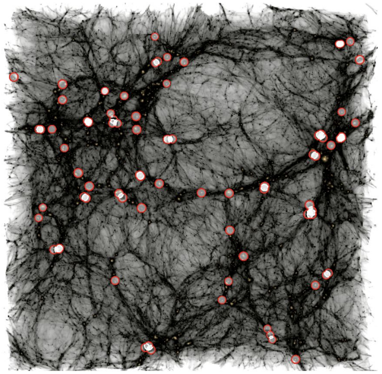
- **Poor evaluation on quality of fit**
- **Distorted confidence regions**
- **Biased constraints on parameters**

Covariances in Cosmology

Assume cosmology



Preferred galaxy formation model (HOD, SHAM, semi-analytics)



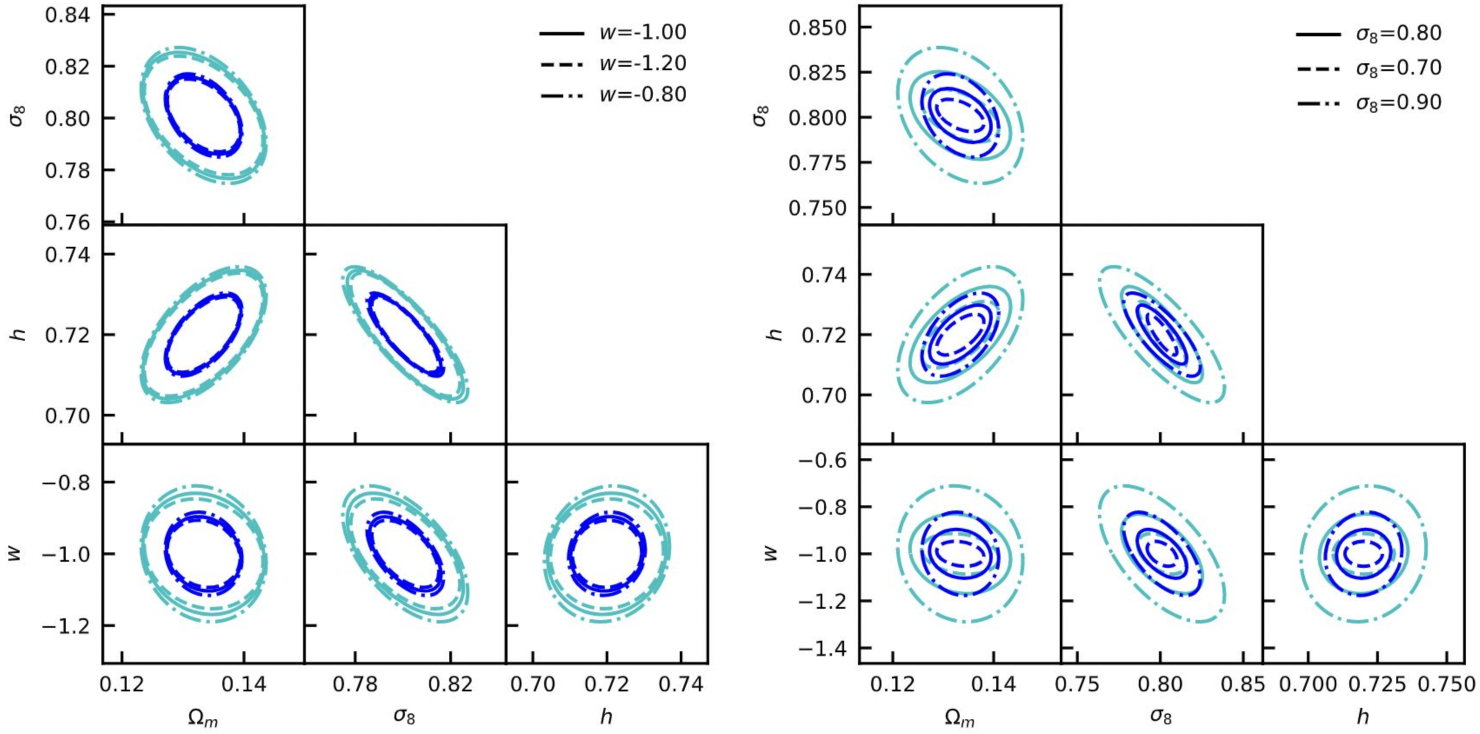
Repeat $\mathcal{O}(10^3)$ times

Estimate covariance from them

$$C(k, k') = \frac{1}{N-1} \sum_{i=0}^{N} (P_i(k) - \langle P(k) \rangle)(P_i(k') - \langle P(k') \rangle)$$

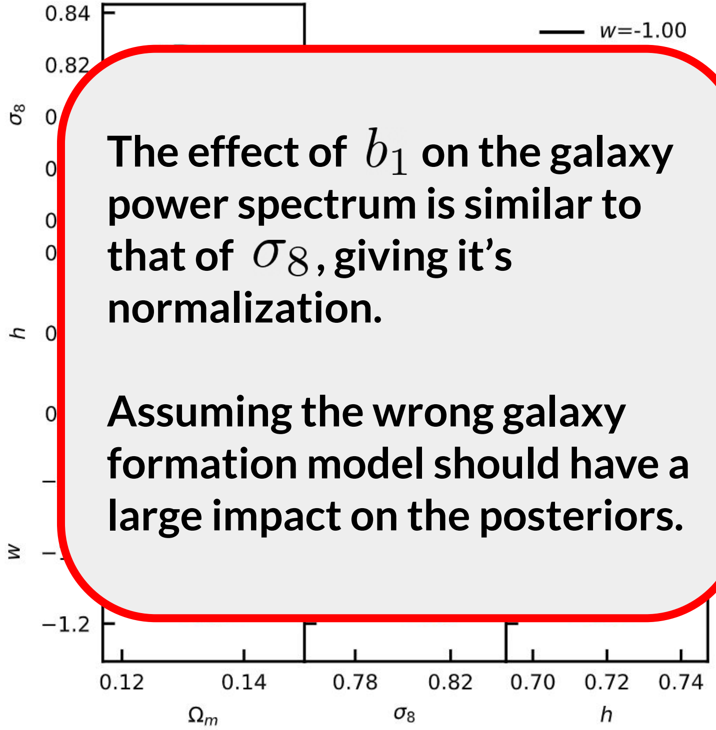
figure credit: Wechsler & Tinker

Choosing Wrong Fiducial Cosmology



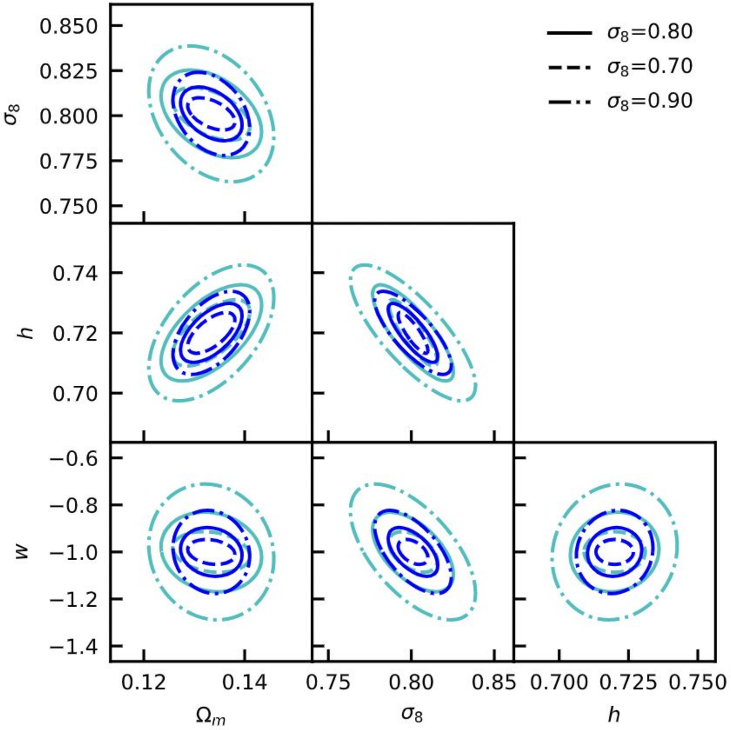
Extracted from Blot et. al (2021)

Choosing Wrong Fiducial Cosmology



The effect of b_1 on the galaxy power spectrum is similar to that of σ_8, giving it's normalization.

Assuming the wrong galaxy formation model should have a large impact on the posteriors.



Extracted from Blot et. al (2021)

Bias Expansion for Covariance Matrices

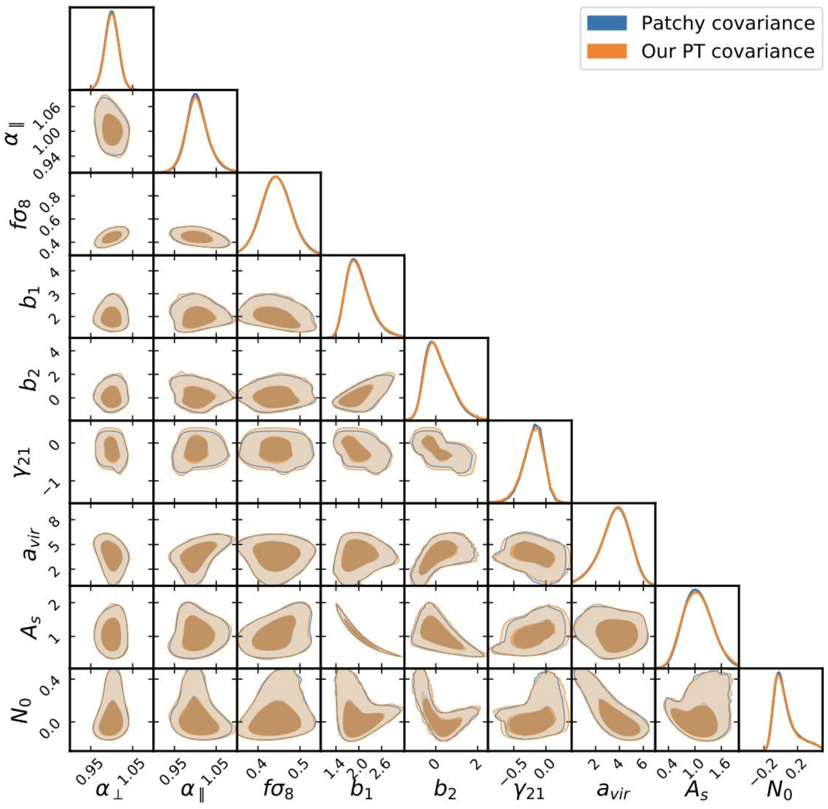
Bias Expansion for Covariances

$$C_{gg} = \sum_{ij,lm} b_i b_j b_l b_m C_{ij,lm}$$

+

Perturbative calculation of the DM statistics

Works very well until scales of $k \approx 0.3[h/Mpc]$

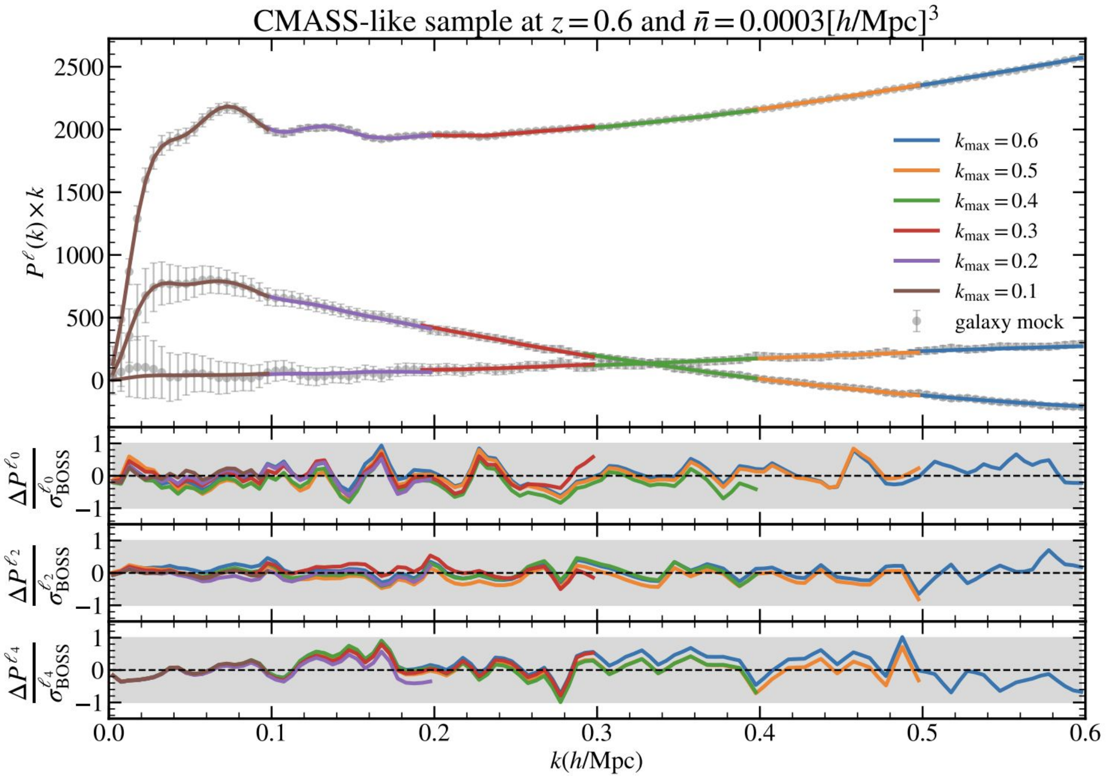


Taken from Wadekar & Scoccimarro (2019)

Hybrid Lagrangian Bias Model

We have developed a model which is accurate until very small scales.

Thus, we need a covariance matrix that is also accurate in this regime.



Pellejero-Ibanez, Angulo, Zennaro, Stuecker, Contreras, Arico, **FM** **(2022)**

Hybrid Lagrangian Bias Model

Second-order Lagrangian bias expansion plus non-local term

$$1 + \delta_g(\mathbf{q}) = 1 + b_1 \delta_L + b_2 \delta_L^2 + b_{s^2} s^2 + b_\nabla \nabla^2 \delta_L + \mathcal{O}(\delta_L^3)$$

use displacements of particles computed from N-Body simulations to move tracers from Lagrangian to Eulerian space.

$$\rho_E(\mathbf{x}) = \int d^3\mathbf{q} \rho_L(\mathbf{q}) \delta^D(\mathbf{x} - \mathbf{q} - \psi(\mathbf{q}))$$

The expression for the galaxy density in Eulerian space then becomes:

$$\delta_g(\mathbf{x}) \approx \delta + b_1 \delta_\delta + b_2 \delta_{\delta^2} + b_{s^2} s^2 + b_\nabla \delta_{\nabla^2}$$

and computing the galaxy power spectrum, one can write it as a linear combination of 15 auto and cross-spectra between the basis fields

$$P_{gg}(k) = \sum_{i,j \in [1, \delta, \delta^2, s^2, \nabla^2 \delta]} b_i b_j P_{ij}(k) + \frac{1}{\bar{n}}$$

Choosing Wrong Fiducial Cosmology

$$P_{gg}(k) = \sum_{i,j\in[1,\delta,\delta^2,s^2,\nabla^2\delta]} b_i b_j P_{ij}(k) + \frac{1}{\bar{n}}$$

➡

$$C_{gg}(k,k') = \langle (P_{gg}(k) - \langle P_{gg}(k)\rangle)\,(P_{gg}(k') - \langle P_{gg}(k')\rangle)\rangle$$

$$C_{gg} = \sum_{ij,lm} b_i b_j b_l b_m C_{ij,lm} + \text{shot-noise}$$

$$C_{ij,lm}(k,k') = \frac{1}{N-1}\sum_{n=1}^{N} (P_{ij}^{(n)}(k) - \langle P_{ij}(k)\rangle)(P_{lm}^{(n)}(k') - \langle P_{lm}(k')\rangle)$$

Choosing Wrong Fiducial Cosmology

Fiducial Cosmology

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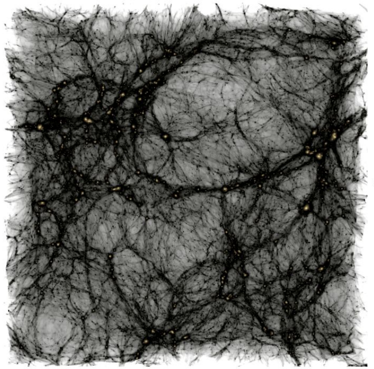


credit: Wechsler & Tinker

$$P_{ij}$$

$$i, j \in [1, \delta, \delta^2, s^2, \nabla^2 \delta]$$

Repeat $\mathcal{O}(10^3)$ times

$$C_{gg} = \sum_{ij,lm} b_i b_j b_l b_m C_{ij,lm} + \text{shot-noise}$$

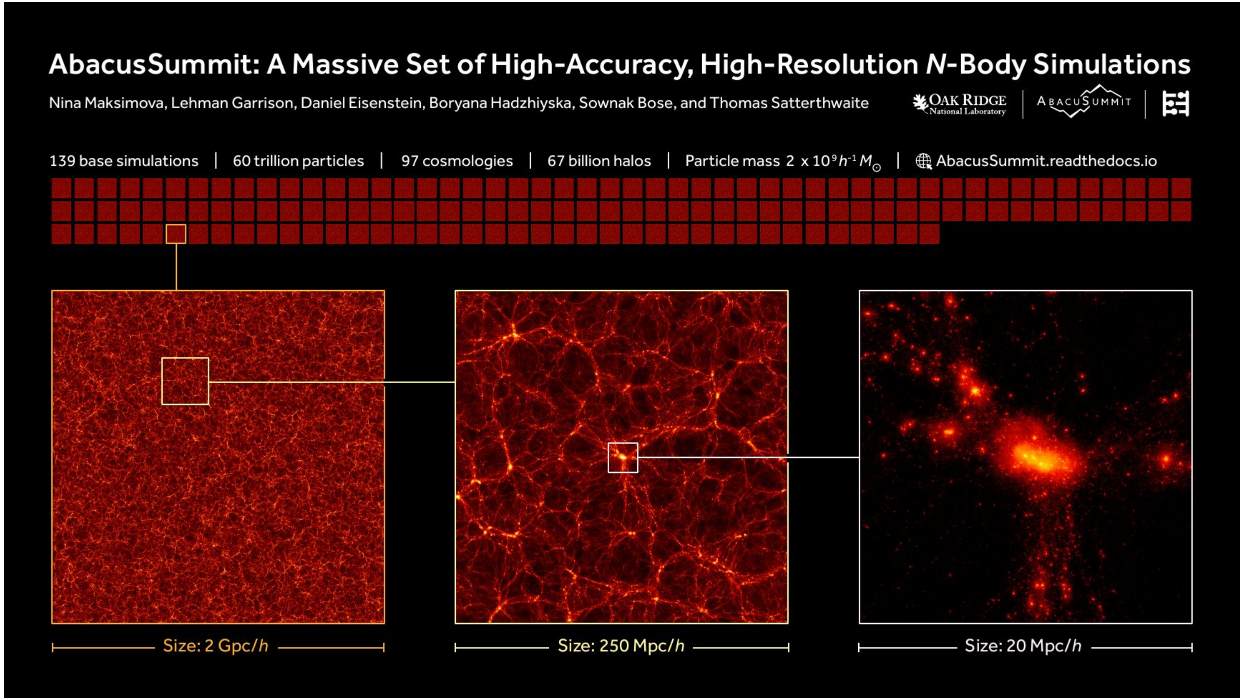
$$C_{ij,lm}(k, k') = \frac{1}{N-1} \sum_{n=1}^{N} (P_{ij}^{(n)}(k) - \langle P_{ij}(k) \rangle)(P_{lm}^{(n)}(k') - \langle P_{lm}(k') \rangle)$$

AbacusSummit

Set of 1800 simulations run with Planck 2018 cosmology

$$L = 500[h/\text{Mpc}]$$

$$N_p = 536^3$$

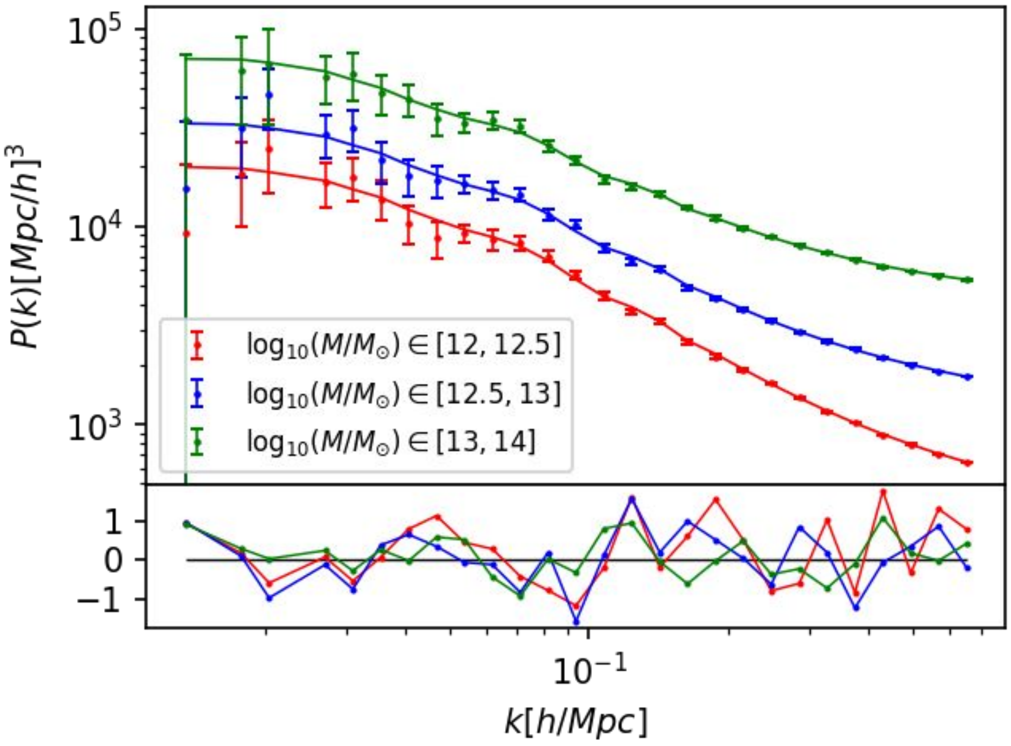


Bias Expansion for Covariances: Halos

Halos from the **AbacusSummit** suite of simulations divided into three mass bins.

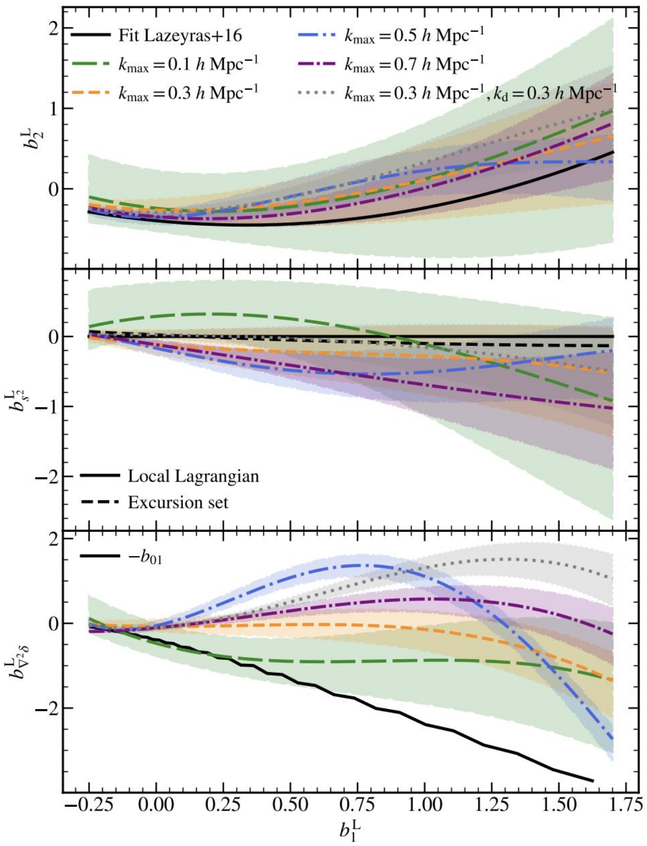
We fit their power spectra using the coevolution relations from Zennaro et. al. (2021)

$$P_{hh}(k) = \sum_{i,j} b_i b_j P_{ij} + \frac{A_{SN}}{\bar{n}}$$



Bias Expansion for Covariances: Halos

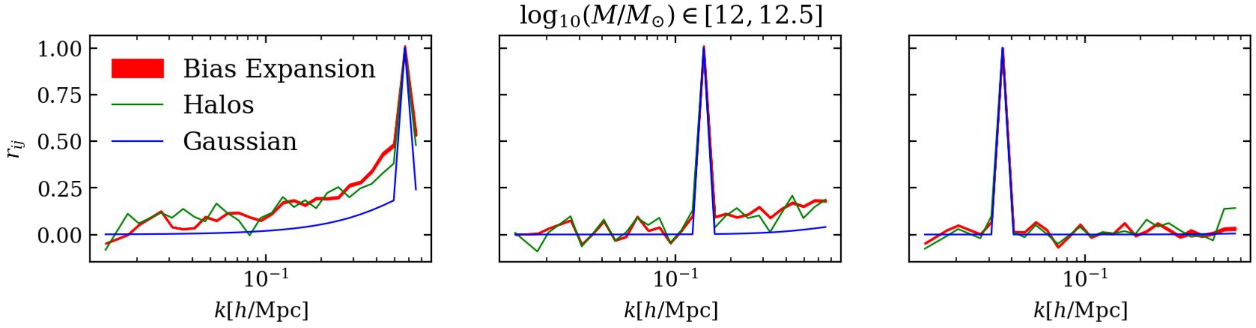
Haloes, $k_{\rm d} = 0.75h\,{\rm Mpc}^{-1}$, $k_{\rm max} = 0.7h\,{\rm Mpc}^{-1}$	
$b_2^{\rm L}(b_1^{\rm L}) =$	$-0.09143(b_1^{\rm L})^3 + 0.7093(b_1^{\rm L})^2 - 0.2607b_1^{\rm L} - 0.3469$
$b_{s^2}^{\rm L}(b_1^{\rm L}) =$	$0.02278(b_1^{\rm L})^3 - 0.005503(b_1^{\rm L})^2 - 0.5904b_1^{\rm L} - 0.1174$
$b_{\nabla^2\delta}^{\rm L}(b_1^{\rm L}) =$	$-0.6971(b_1^{\rm L})^3 + 0.7892(b_1^{\rm L})^2 + 0.5882b_1^{\rm L} - 0.1072$
Galaxies, $k_{\rm d} = 0.75h\,{\rm Mpc}^{-1}$, $k_{\rm max} = 0.7h\,{\rm Mpc}^{-1}$	
$b_2^{\rm L}(b_1^{\rm L}) =$	$0.01677(b_1^{\rm L})^3 - 0.005116(b_1^{\rm L})^2 + 0.4279b_1^{\rm L} - 0.1635$
$b_{s^2}^{\rm L}(b_1^{\rm L}) =$	$-0.3605(b_1^{\rm L})^3 + 0.5649(b_1^{\rm L})^2 - 0.1412b_1^{\rm L} - 0.01318$
$b_{\nabla^2\delta}^{\rm L}(b_1^{\rm L}) =$	$0.2298(b_1^{\rm L})^3 - 2.096(b_1^{\rm L})^2 + 0.7816b_1^{\rm L} - 0.1545$



Zennaro, Angulo, Contreras, Pellejero-Ibanez, **FM (2022)**

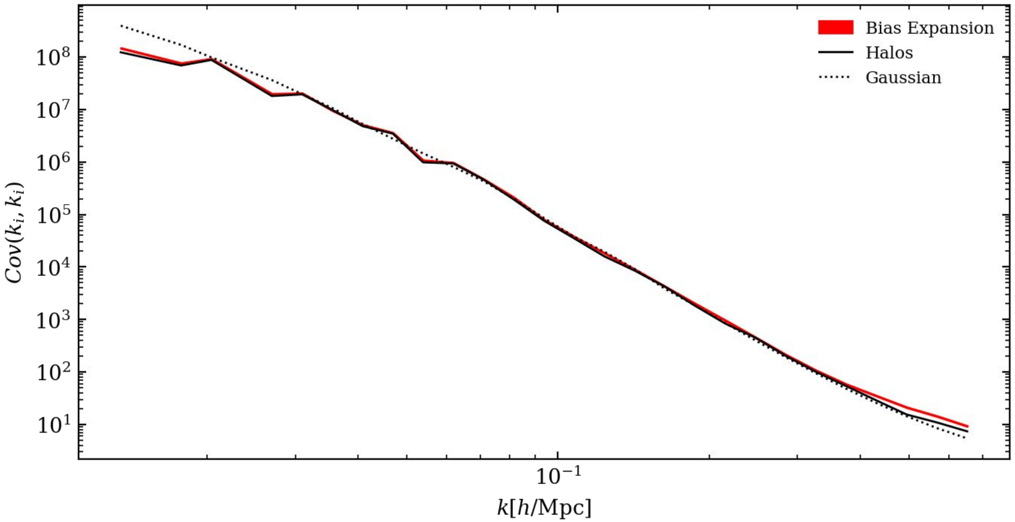
Results

$$r_{ij} = \frac{C(k_i,k_j)}{\sqrt{C(k_i,k_i)C(k_j,k_j)}}$$



Good description of the covariance matrix for scales such that

$$P(k) > \frac{1.5}{\bar{n}}$$



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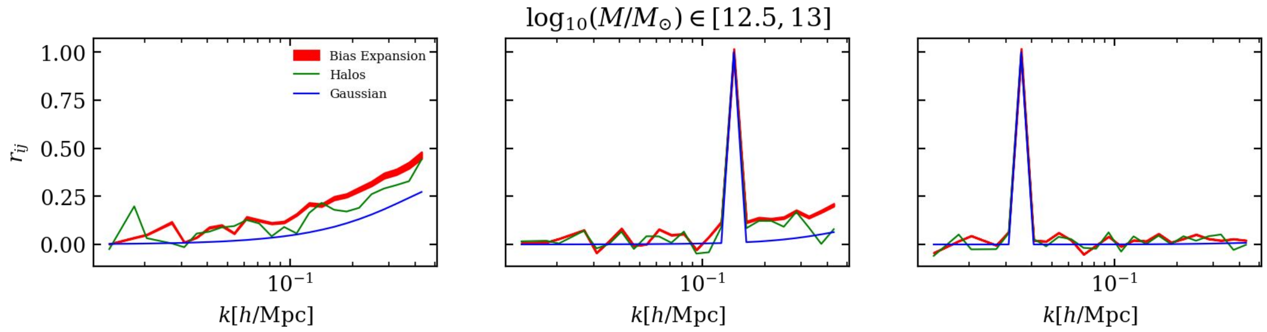

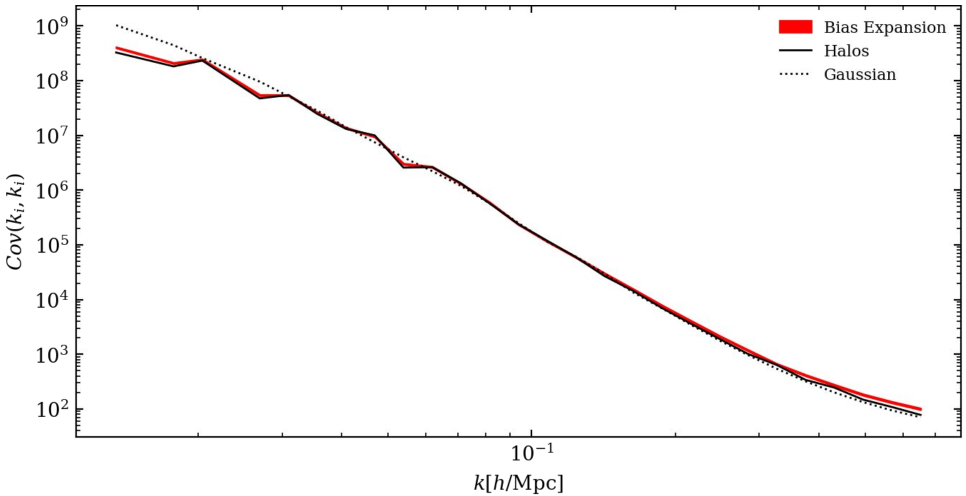

Results

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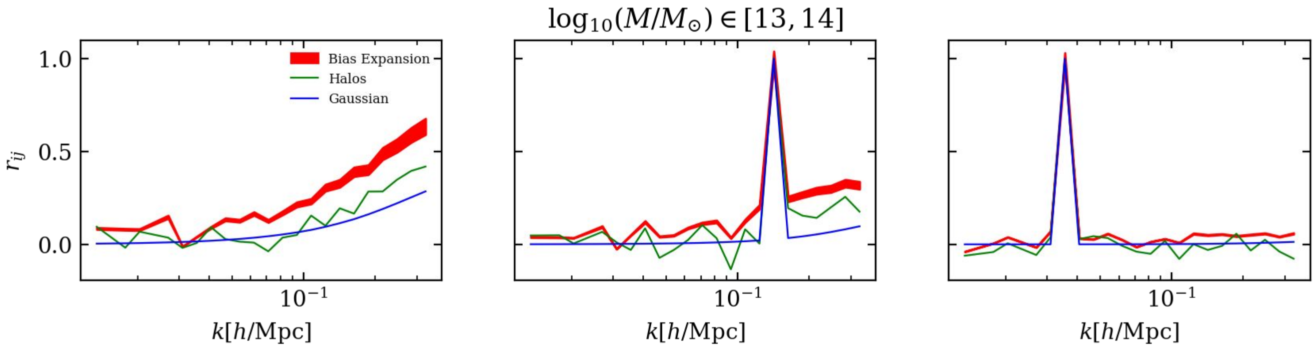
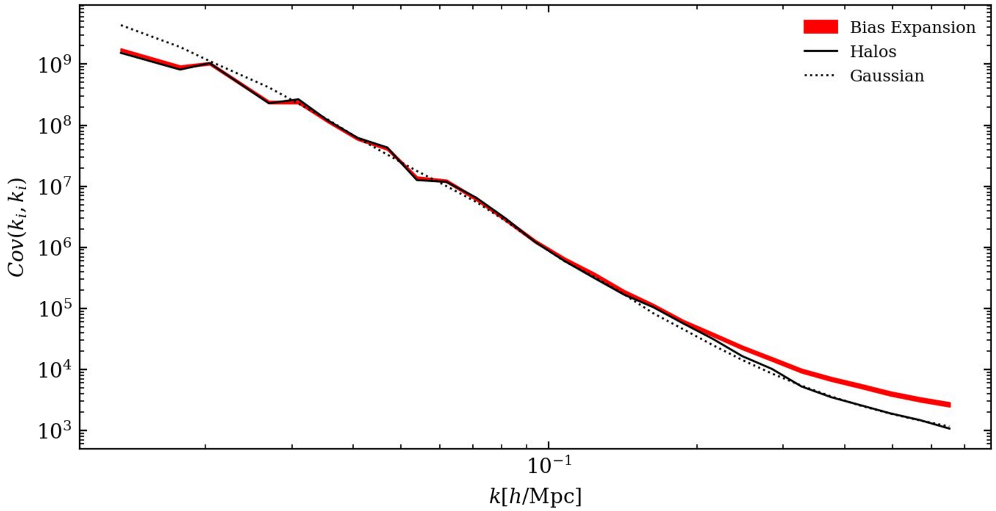
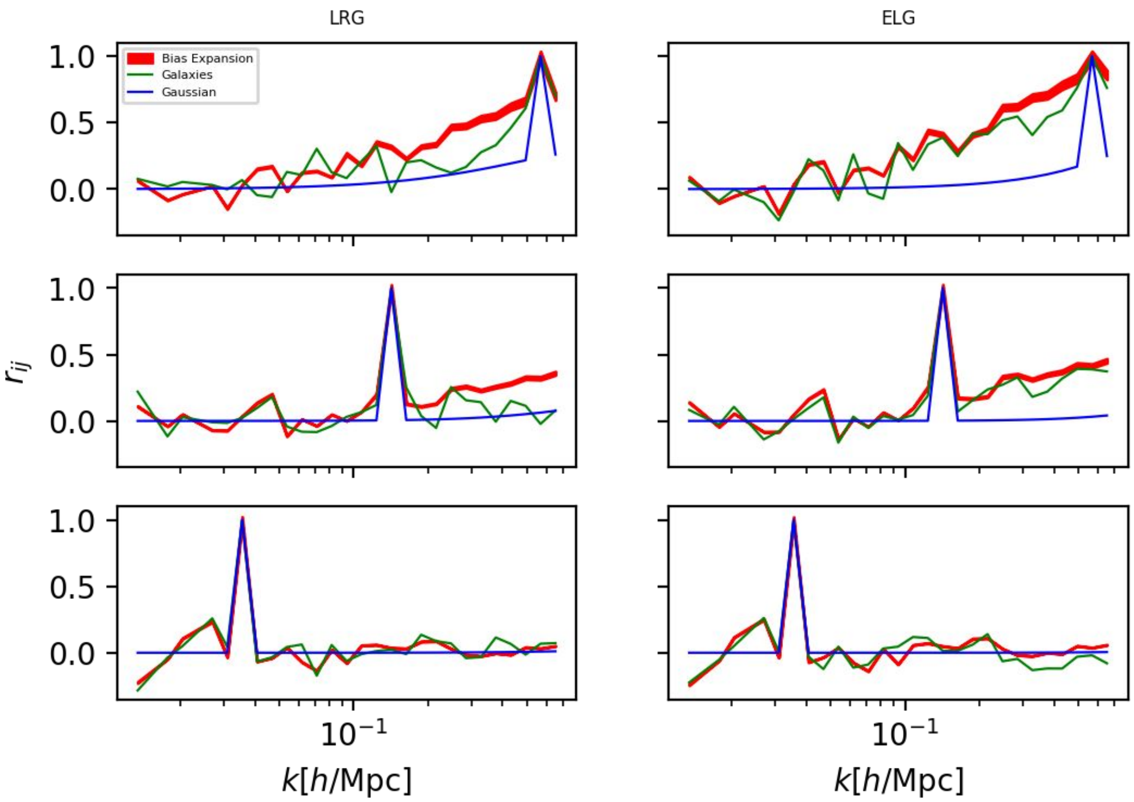




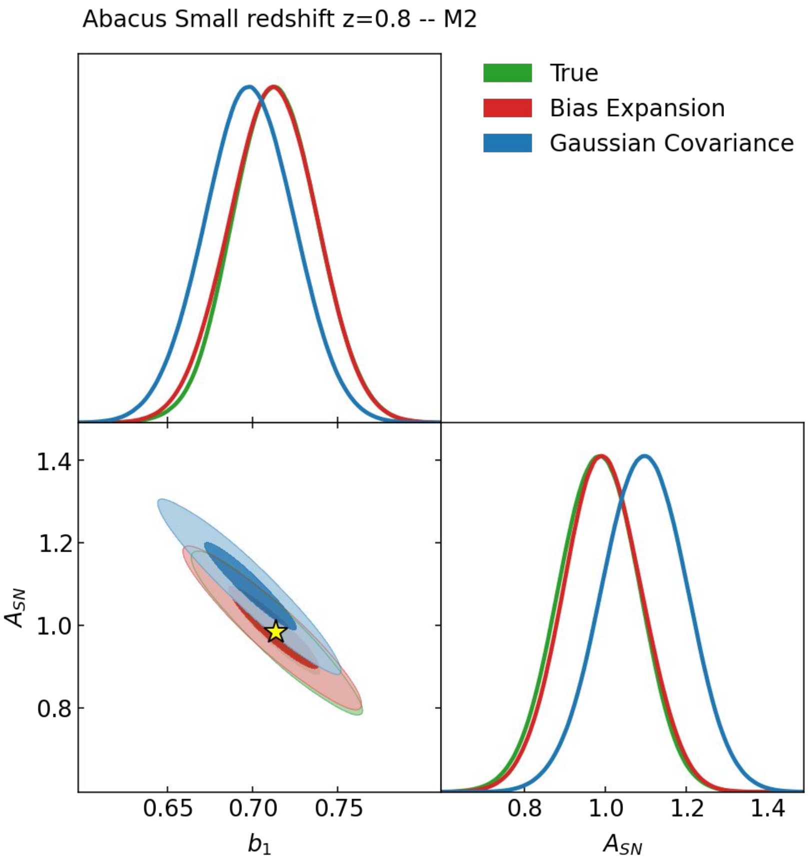
Results

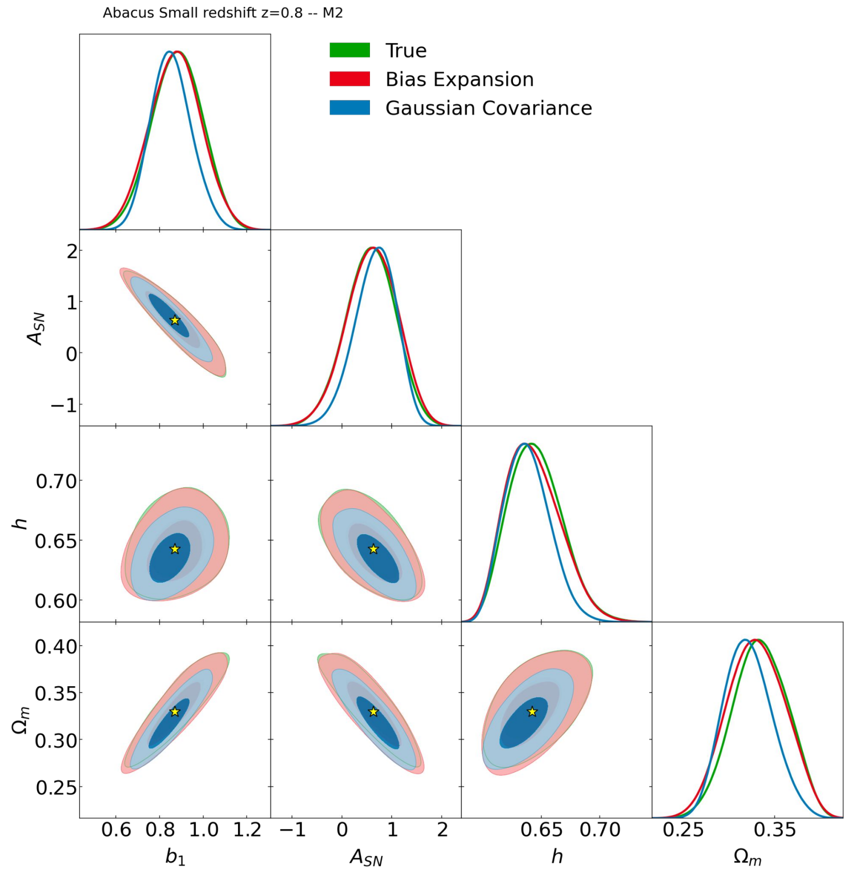
Using the Multi-Trace HOD implemented for AbacusUtils, we can populate the simulations with galaxies. This HOD is based in Alam et al. (2021).

$$r_{ij} = \frac{C(k_i,k_j)}{\sqrt{C(k_i,k_i)C(k_j,k_j)}}$$

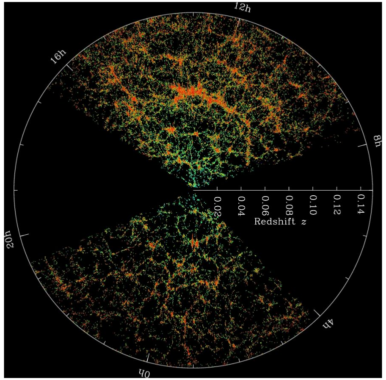


Bias Expansion for Covariances: Halos





Bias Expansion for Covariances: Halos

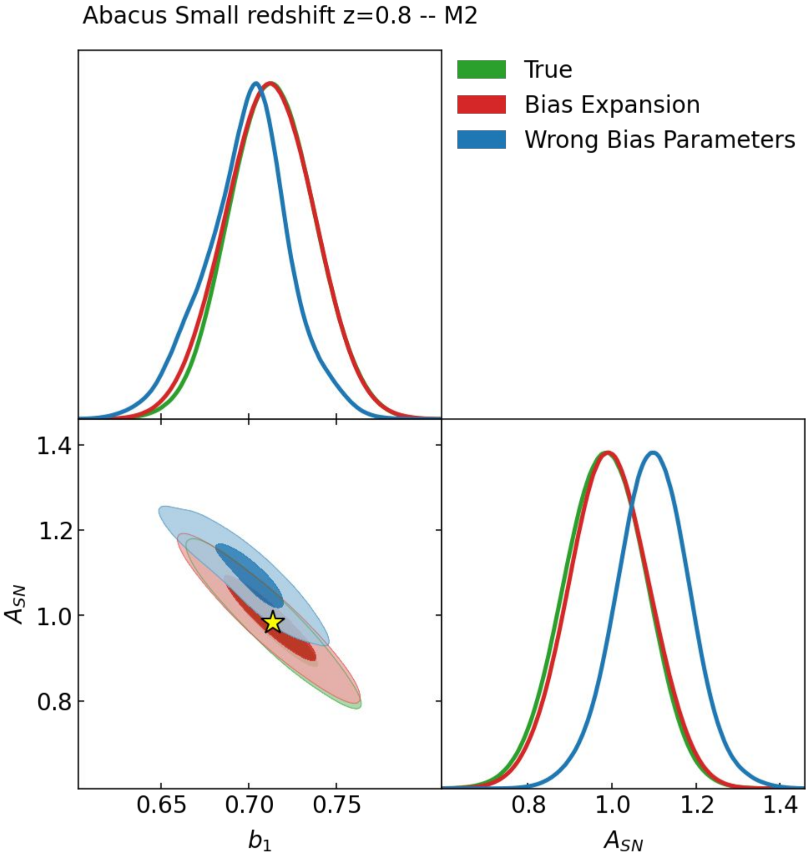


$$\widehat{P}_{gg}(k)$$

$$\mathcal{L}(\theta)$$

$$b_i$$

$$C_{gg} = \sum_{ij,lm} b_i b_j b_l b_m C_{ij,lm}$$



Conclusions

Upshots

- Hybrid Lagrangian Bias Expansion describes well the covariance matrix of simulated halos
- Correct posterior regions are recovered with good precision
- Allows for great flexibility in the galaxy-formation model
- Works well even in mildly non-linear scales.

Future Developments

- Include third-order operators in the bias expansion
- Test the covariances for realistic analyses
- Further quantify the deviations caused by assuming wrong galaxy formation model

Bias Expansion for Covariances: Halos

Counting noise for a distribution of halos with Poisson statistics:

$$
\begin{aligned}
C_{SN} =& \frac{1}{\bar{n}} \left[B(\mathbf{k}, -\mathbf{k}) + B(\mathbf{k}, \mathbf{k}') + B(\mathbf{k}, -\mathbf{k}') \right. \\
& \left. + B(-\mathbf{k}, \mathbf{k}') + B(-\mathbf{k}, -\mathbf{k}') + B(\mathbf{k}', -\mathbf{k}') \right] \\
& + \frac{1}{\bar{n}^2} \left[P(\mathbf{k} + \mathbf{k}') + P(0) + P(\mathbf{k}' - \mathbf{k}) + P(\mathbf{k}) + P(-\mathbf{k}) + P(\mathbf{k}') + P(-\mathbf{k}') \right] \\
& + \frac{1}{\bar{n}^3}
\end{aligned}
$$